

Introduction

Wednesday, October 25, 2023

5:17 AM

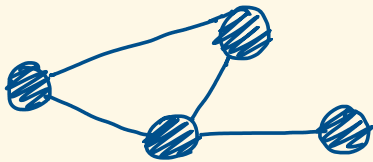
What is graph theory?

Applications of graphs in real world situations.

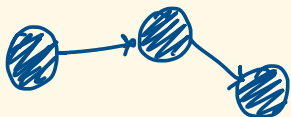
- Social network
- Map, etc.

Types of graphs:

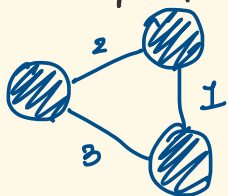
1. Undirected graphs: edges have no orientations.
eg. Nodes as cities & edges as roads.



2. Directed graphs: edges have orientations
eg. people buying gifts for each other.

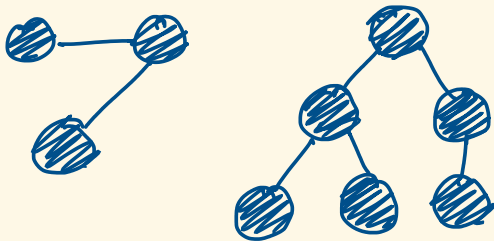


3. Weighted graph: edges have weights



4. Trees: undirected graphs with no cycles

- A tree with N nodes is a connected graph with $N-1$ edges.



A tree has no cycles!

A path between every node in a graph.

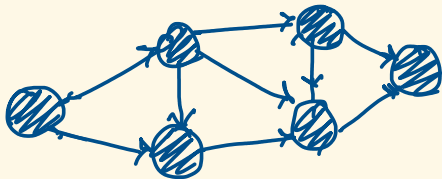
→ A Rooted tree.

- A tree with every node pointing towards the root. ← Arboriculture (out-tree)
- or pointing away from the root. ← Anti-arboriculture (in-tree)

5. Directed Acyclic Graphs (DAGs)

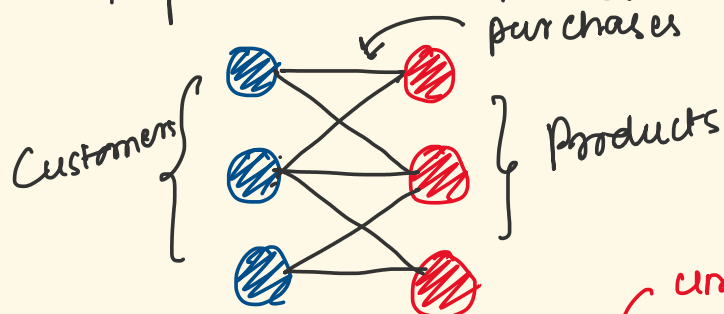
- Common for representing dependencies.
eg. task scheduling, class prerequisites, etc.

- All DAGs are out trees but not the other way round.



6. Bipartite Graph

- Vertices can be split into two groups (U & V)
- Every edge connects between U & V .
- Graph is two colorable.

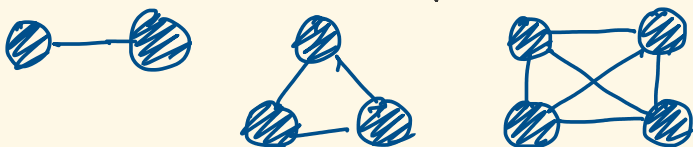


They have nothing in common.

No edges between nodes of the same group

unique edge

7. Complete graph: Edge between every pair of nodes.



How to represent graphs?

1. Adjacency matrix

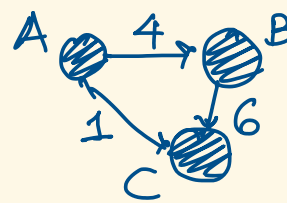
- $m[i][j]$ → cost of i to j
- Space efficient for dense graphs
- lookup time is $O(1)$
- Requires V^2 space & time.

2. Adjacency list.

- Map of lists / list of lists.

$A \rightarrow [(B, 4), (C, 1)]$

$B \rightarrow [(C, 6)]$



- Space efficient for sparse graphs.
- less efficient for dense graphs.
- Lookup time → $O(E)$ (Edges)