Fourier Series – A Trignometric Infinite series

Jean-Baptiste Joseph Fourier

* JACQUES FOURIER is a French Mathematician & physicist.
* Fourier was taught by Lagrange and Laplace.
* Fourier was scientific advisor for Neopolean army.
* Fourier series came into existence while solving heat equation.

Heat equation describes the distribution of heat or variation in temp in a given region over time.

i.e $\frac{\partial u}{\partial t} - \alpha \nabla^2 u = 0$ or $\frac{\partial u}{\partial t} - \alpha\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right] = 0$

* Fourier series is mainly used to analyse signals.

Ex: In audio system FS is used to modulate & demodulate voice signals.

* Weak signals can't carry the information for long distance, but we can express weak signals interms of Sine & Cosines & extend the signals i.e information.
  * Give example of sound or music in celebration
  * Recording in MP3.

* i) Taylor series of $f(x)$ is a series in ascending powers of $(x-a)$

Maclaurins series is a series in ascending powers of x.

∴ Fourier series is a series containing Cosine and Sine terms which belongs to a class of functions called periodic functions.

(It is a frequency domain representation)

∴ What are the advantages of Fourier series ∴

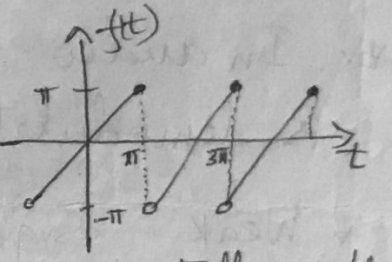
i) Discontinuous function can be represented by Fourier series. (This is not true for Taylor series)

ii) The fourier series is useful in expanding the periodic functions since outside the closed interval, there exists a periodic extension of the function.

iii) Linear combination is easy to handle. (So preferred over LT etc).

signal is in the form of saw tooth.

How Fourier Series works:



* The sawtooth waveform with period 2π as shown in the figure is given by particular combination i.e

$$f(t) = 2\left(\sin t - \frac{1}{2}\sin 2t + \frac{1}{3}\sin 3t - \frac{1}{4}\sin 4t + \frac{1}{5}\sin 5t - \cdots\right) \rightarrow (1)$$

Sawtooth waveform

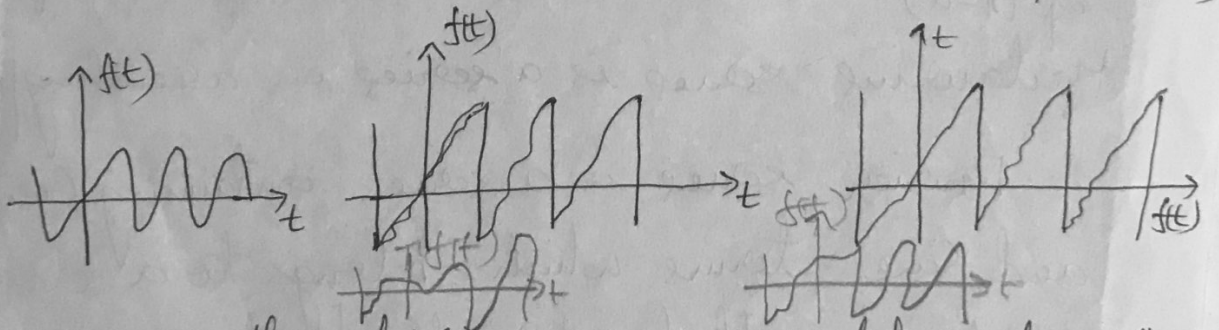
* Following (a), (b) & (c) graphs show the effect of including more & more terms in the series.

As more terms are taken we see that the series approaches the desired sawtooth waveform.

a) $f(t) = 2\sin t$ ($f(t) = 1$ (1 term))

b) $f(t) = 2\left(\sin t - \frac{1}{2}\sin 2t + \frac{1}{3}\sin 3t\right)$

c) $f(t) = 2\left(\sin t - \frac{1}{2}\sin 2t + \frac{1}{3}\sin 3t - \frac{1}{4}\sin 4t + \frac{1}{5}\sin 5t\right)$



* In this example only sine waves were used to construct the function. (but we need both sine & cosine waves).

Where did this series come from i.e series given by (1)

It's from FS, the sum of trigonometric functions.

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website: Real world applications of FS

graph interactive

In this chapter we express a given function interms of Sine and/or cosine functions instead of using a polynomial (like Taylor & Maclaurin series).

* Any kind of wave can be written as a sum of sines & cosines.

For example:

$$\text{Voice} = \sin x + \frac{1}{10}\sin 2x + \frac{1}{100}\sin 3x + \cdots$$

* Almost any kind of wave can be approximated using FS

For Ex: saw tooth; triangular wave, rectangular wave.

*: If we can approximate we can send it to particular destination in a etc.

* MP-3 format uses FS for audio compression.

* In JPEG also they use FS.