STATISTICS FOR DATA SCIENCE

Confidence Intervals for Small Samples

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Topics to be covered...

- Confidence Intervals for population mean of small samples
- Student's t Distribution
- Confidence Intervals using t Distribution
- Student's t Distribution Is Appropriate?
- One-Sided CI for Small Samples

- If the sample size is small, standard deviation (s) of the sample may not be close to σ (population standard deviation). Hence $\overline{X}$ (sample_mean) may not be aproximately normal.
- However, if the population from which the sample is drawn is known to be approximately normal (can be confirmed using normal probability plot).

- It turns out that we can still use the quantity.
- $(\overline{X} - \mu) / (s/\sqrt{n})$,

  but since s is not necessarily close to σ , the quantity will not have a normal distribution.
- Instead it has Student's t distribution with $n - 1$ degrees of freedom, denoted as t_{n-1} .

t - Distribution

- The t distribution is a theoretical probability distribution.
- It is symmetrical, bell-shaped, and similar to the standard normal curve.
- It differs from the standard normal curve, however, in that it has an additional parameter, called **degrees of freedom**, which changes its shape.

  df = sample size – 1

- Setting the value of df defines a particular member of the family of t distributions. (df > 0 => Sample Size > 1)

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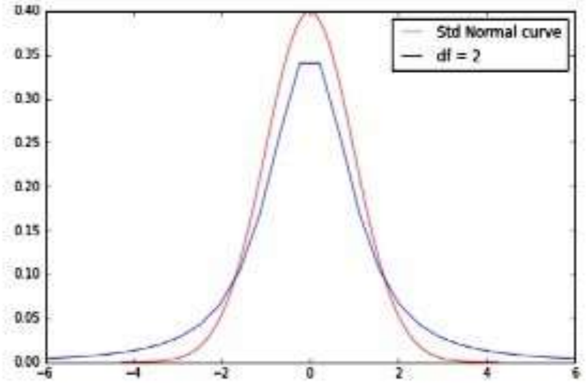
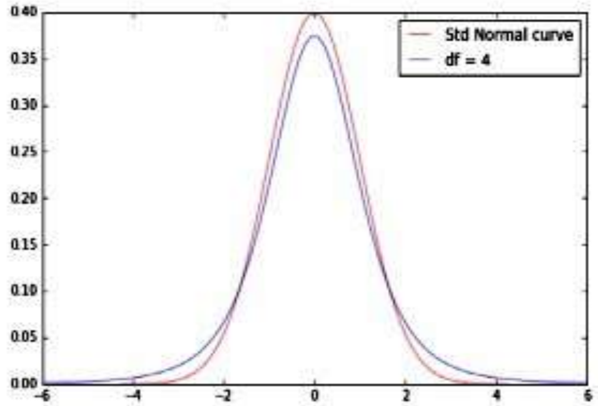
Students t Distribution

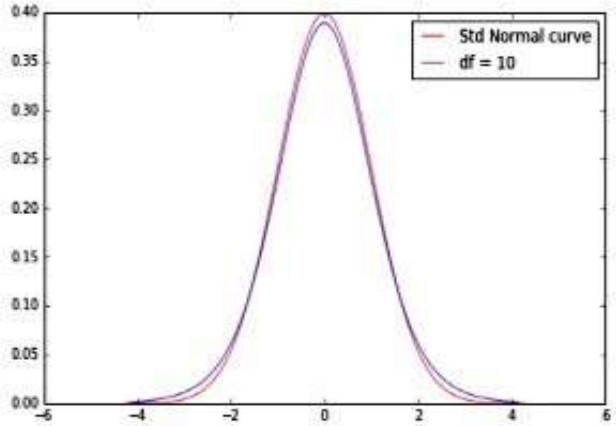
1) df = 2



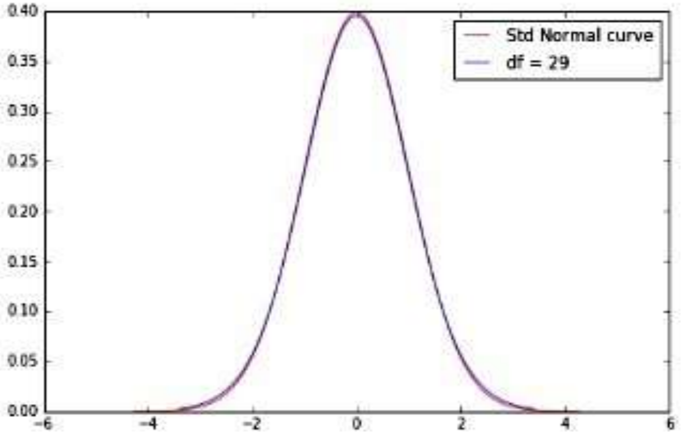
2) df =4



3) df = 10



4) df = 30



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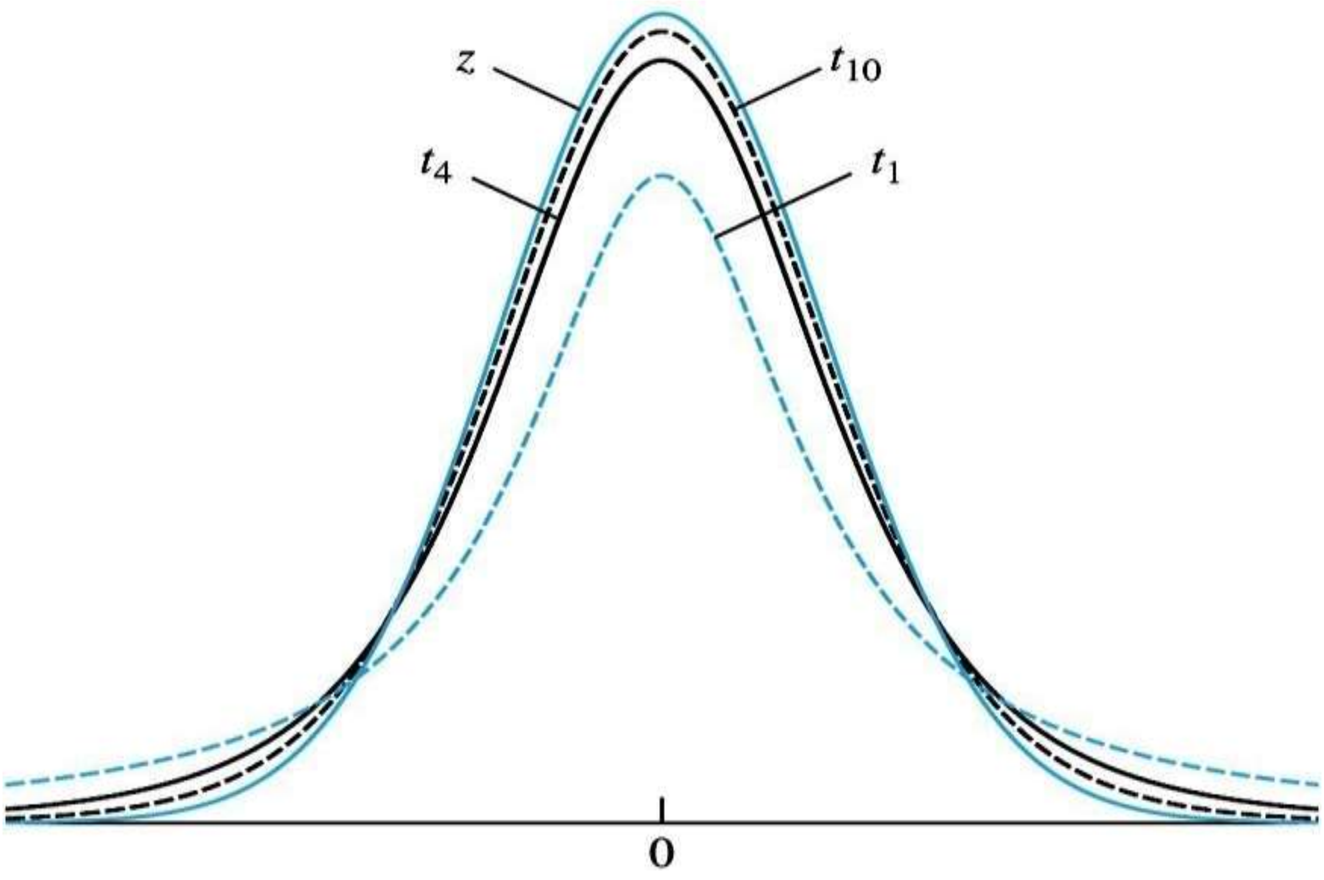
PDF for Students t curve

Note that the smaller the distribution function, the flatter the shape of the distribution, resulting in greater area in the tails of the distribution.



Relationship to the normal curve

- As the df increase, the t distribution approaches the standard normal distribution (μ=0.0, σ=1.0).
- The standard normal curve is a special case of the t distribution when df= infinity.
- For practical purposes, the t distribution approaches the standard normal distribution relatively quickly, such that when df=30 the two are almost identical.

Using t table

- **We use t table to find probabilites associated with t distribution.**

- **Row headings** – denotes degree of freedom
- **Column headings** – denotes the area to the right(probabilities)

- The value in particular row and column specifies the t-score where,

$$P(t > t\text{-}tscore) = col\_heading$$

Examples

1) A random sample of size 10 is drawn from a normal distribution with mean 4.

a) Find $P(t > 1.833)$

b) Find $P(t > 1.5)$

Solution

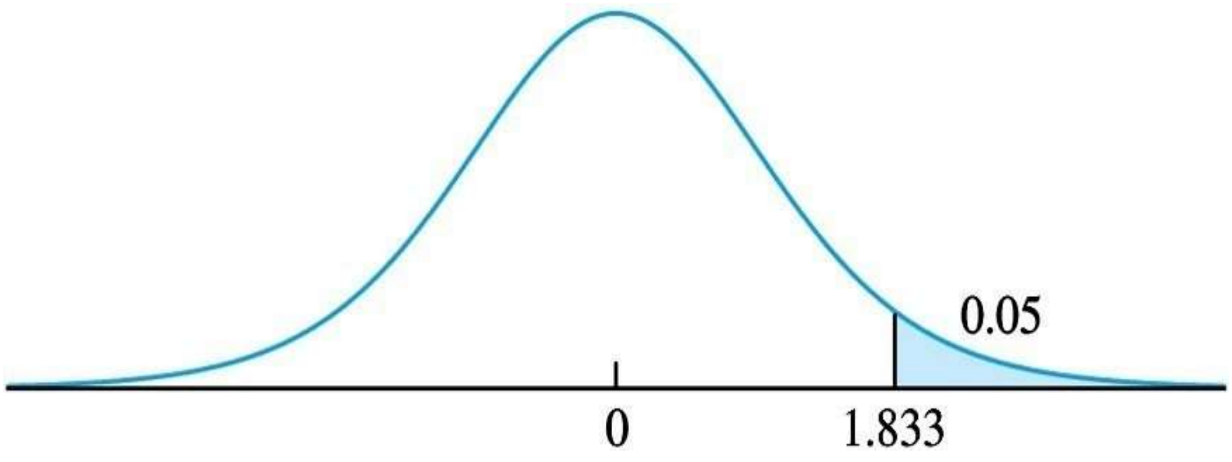
a) Find P(t >1.833)

df = 9 (row_heading)

t-score = 1.833

corresponding col_heading = 0.05

P(t >1.833) = 0.05



Solution

b) Find P(t > 1.5)

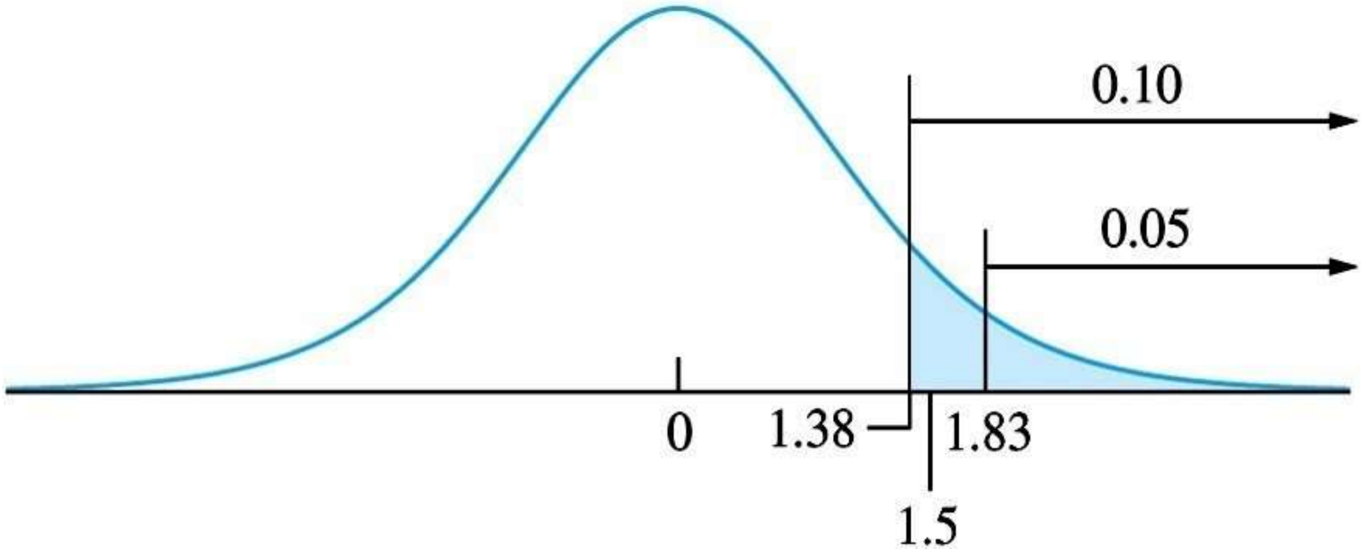
df = 9 (row_heading)

t-score = 1.5 [does not correspond to any of the values in that row]

but we do have t-scores 1.383, 1.833 corresponding to upper tail probabilties 0.10 and 0.05 respectively. That is,

P(t >1.383) = 0.10 and P(t >1.833) = 0.05

Since 1.383 < 1 .5 < 1.833 => 0.05 < P(t > 1.5) < 0.10



Confidence Interval for Small Samples using t distribution:

The quantity,

$$\frac{\overline{X} - \mu}{S/\sqrt{n}}$$

has a t distribution with n – 1 degrees of freedom.

We can generate a (1 – α) 100% Confidence Interval for μ as

$$\overline{X} \pm t_{n-1,\alpha/2}\frac{s}{\sqrt{n}}$$

Student's t Distribution is Appropriate when

- Sample size is small (n < 30)
- Sample comes from a population that is approximately normal.
- In many cases, we must examine the sample for normality,by constructing a box plot or normal probability plot.
- Unfortunately, when the sample size is small, departures from normality may be hard to detect.
- If these plots do not reveal a strong asymmetry or any outliers, then in most cases the Student’s *t* distribution will be reliable.

One-Sided Confidence Intervals for small samples

We can generate a (1 – a) 100% Upper Confidence bound for μ as:

$$X\_bar + \mathbf{t}_{n-1,\alpha} * s/sqrt(n)$$

We can generate a (1 – a) 100% Lower Confidence bound for μ as:

$$X\_bar - \mathbf{t}_{n-1,\alpha} * s/sqrt(n)$$

Example1

Find the value of $t_{n-1,\,\alpha/2}$ needed to construct a two-sided confidence interval of the given level with the given sample size:

a) 90% with sample size 12

b) 95% with sample size 7

Solution

a) 90% with sample size 12

df = 11

alpha = 0.10 => alpha/2 = 0.05

=> in t table : row_heading = 11, col_heading = 0.05 => $\mathbf{t}_{11,\,0.05}$ = 1.796

b) 95% with sample size 7

df = 6

alpha = 0.05 => alpha/2 = 0.025

=> in t table : row_heading = 6, col_heading = 0.025 => $\mathbf{t}_{6,\,0.025}$ = 2.447

Use z,Not t, if σ is known

If it is known that the sample indeed was drawn from a **normal population**, also the **standard deviation of the population is known**, use z not t distribution to find out the confidence interval irrespective of the sample size.

Summary

Let $X_1, \ldots, X_n$ be a random sample (of any size) from a *normal* population with mean μ. If the standard deviation σ is known, then a level $100(1-\alpha)\%$ confidence interval for μ is

$$\overline{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \tag{5.12}$$

THANK YOU

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