STATISTICS FOR DATA SCIENCE

HYPOTHESIS and INFERENCE

Dr. Deepa Nair
Department of Science and Humanities

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UNIT-4 HYPOTHESIS and INFERENCE

Session-11

Fixed Level Testing

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Fixed Level Testing

- A hypothesis test measures the plausibility of the null hypothesis by producing a P-value.

- The smaller the P −value, the less plausible the null. We have pointed out that there is no scientifically valid dividing line between plausibility and implausibility, Sometimes, however, a decision has to be made.

Fixed Level Testing

- **Fixed-level testing is just like the hypothesis testing we have been discussing so far, except that a firm rule is set ahead of time for rejecting the null hypothesis.**

Fixed Level Testing

To conduct a fixed-level test:

- Choose a number α, where $0 < \alpha < 1$. This is called the significance level, or the level, of the test.
- Compute the P-value in the usual way.
- If $P \leq \alpha$, reject H_0. If $P > \alpha$, do not reject H_0.

Example:

- The mean wear in a sample of 45 steel balls was $X = 673.2\mu m$, and the standard deviation was $s = 14.9\mu m$.
- Let μ denote the population mean wear. A test of $H_1 : \mu \geq 675$ versus $H_0 : \mu < 675$ yielded a *P*-value of 0.209.
- Can we reject H_0 at the 25% level? Can we reject H_0 at the 5% level?

Solution:

- The P –value of 0.209 is less than 0.25
- So if we had chosen a significance level of $\alpha = 0.25$,we would reject H_0.
- Thus we reject H_0 at the 25% level.
- Since $0.209 > 0.05$, we do not reject H_0 at the 5% level.

Example:

A process for a certain type of ore is designed to reduce the concentration of impurities to less than 2%.

- It is known that the standard deviation of impurities for processed ore is 0.6%.

- Let μ represent the mean impurity level, in percent, for ore specimens treated by this process.

Example:

- The impurity of 80 ore specimens is measured, and a test of the hypothesis $H_0: \mu \geq 2$ versus $H_1: \mu < 2$ will be performed.

a. If the test is made at the 5% level, what is the rejection region?

b. If the sample mean impurity level is 1.85, will H_0 be rejected at the 10% level?

Solution 2 (a)

- H_0: $\mu \geq 2$ versus H_1: $\mu < 2$
- $\sigma = 0.6, \quad n = 80$
- $\frac{\sigma}{\sqrt{n}} = 0.06708$
- Null distribution of $\bar{X}$:

$$\bar{X} \sim N(2, 0.6^2/80)$$

- $\alpha = 0.05$

Solution 2 (a)

- $Z = -1.645$ (Critical value for 5%)

The rejection region is

$$=> \bar{X} = \frac{z * s}{\sqrt{n}} + 2$$

$$=> \bar{X} = \frac{-1.645 * 0.6}{\sqrt{80}} + 2$$

$$=> \bar{X} = 1.89$$

Solution 2 (a)

- Hence, H_0 will be rejected if $\bar{X} \leq 1.890$.

- The rejection region

consists of all values of $\bar{X}$ less than or equal to 1.890.

Solution Using Rejection Region Approach:

$H_0: \mu \geq 2$ versus $H_1: \mu < 2$

- Null distribution of $\bar{X}$:

$$\bar{X} \sim N(2, 0.6^2/80)$$

- $\alpha = 0.10$

- $Z = -1.28$ (Critical value)

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Solution Using Rejection Region Approach:

$$=> \bar{X} = \frac{z * s}{\sqrt{n}} + 2$$

$$=> \bar{X} = \frac{-1.28 * 0.6}{\sqrt{80}} + 2$$

$$=> \bar{X} = 1.9141$$

Since $1.85 < 1.9141$

$=> H_0$ will be rejected at the 10% level

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Solution Using P Value Approach:

$H_0: \mu \geq 2$ versus $H_1: \mu < 2$

- Null distribution of $\bar{X}$:

$$\bar{X} \sim N(2, 0.6^2/80)$$

- $\alpha = 0.10$

$$\bar{X} = 1.85,$$

- Finding z-score for 1.85
- $z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$

Solution Using P Value Approach:

$$Z = (1.85 - 2)/(0.6/\sqrt{80}) = -2.24$$

$$=> P = P(Z < -2.24) = 0.0125$$

$=> P < \alpha => H_0$ will be rejected at the 10% level

Example:

Small Sample test for population mean:

- Before a substance can be deemed safe for landfilling, its chemical properties must be characterized. The article "Landfilling Ash/Sludge Mixtures" (J. Benoit, T. Eighmy, and B. Crannell, *Journal of Geotechnical and Geoenvironmental Engineering*, 1999: 877–888) reports that in a sample of six replicates of sludge from a New Hampshire wastewater treatment plant, the mean pH was 6.68 with a standard deviation of0.20.
- Can we conclude that the mean pH is less than 7.0?

Solution:

Small Sample test for population mean:

$$n = 6$$

$$H_0: \mu \geq 7.0 \ versus \ H_1: \mu < 7.0$$

Under H_0, the test statistic

$$t = \frac{\bar{X} - 7.0}{s/\sqrt{n}}$$

has a Student's *t* distribution with n-1 degrees of freedom.

Solution:

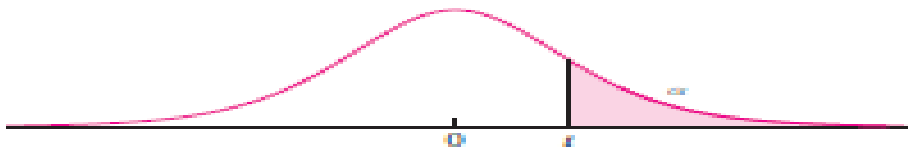
Small Sample test for population mean:

- Has a Student's t distribution with five degrees of freedom. Substituting $\overline{X} = 6.68, s = 0.20, and\ n = 6$, the value of the test statistic is
- $t = \frac{6.68-7.0}{0.2/\sqrt{6}} = -3.910$

STATISTICS FOR DATA SCIENCE

Fixed Level Testing

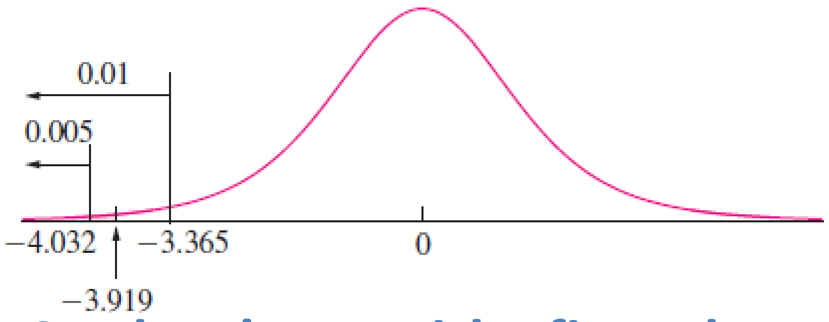
TABLE A.3 Upper percentage points for the Student's t distribution



	α								
ν	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	318.309	636.619
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.385	3.646
35	0.255	0.682	1.306	1.690	2.030	2.438	2.724	3.340	3.591
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	0.254	0.677	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Solution:

Small Sample test for population mean:



- The null distribution is Student's t with five degrees of freedom. The observed value of t is −3.919.
- If H_0 is true, the probability that t takes on a value as extreme as or more extreme than that observed is between 0.005 and 0.01.

Solution:

Small Sample test for population mean:

- Consulting the t table, we find that the value $t = -3.365$ cuts off an area of 0.01 in the left-hand tail, and the value $t = -4.033$ cuts off an area of 0.005.

- We conclude that the P-value is between 0.005 and 0.01. There is strong evidence that the mean pH is less than 7.0.

Dr. Deepa Nair
Department of Science and Humanities

deepanair@pes.edu