LINEAR ALGEBRA AND ITS APPLICATIONS

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MATRICES AND
GAUSSIAN ELIMINATION

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THE GEOMETRY OF LINEAR EQUATIONS:

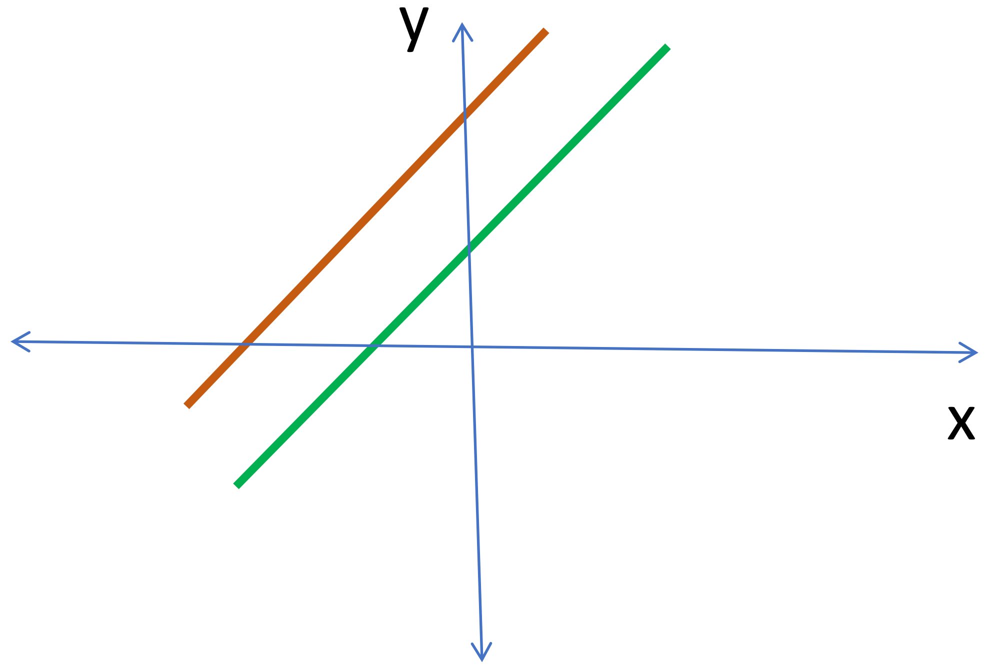
Course Content: Singular Cases

- **SINGULAR CASES in Two dimensions:** A system of linear equations is said to be singular $(|A| = 0)$ if it has no solution or has infinite number of solutions.

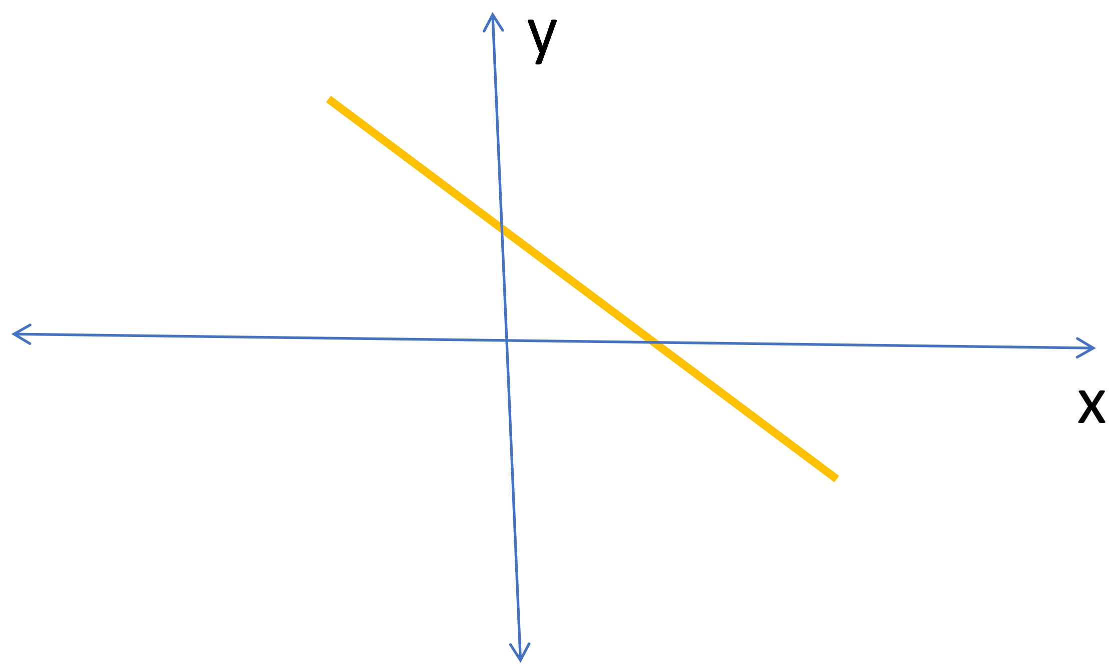
- **(i) ROW PICTURE (Two dimensions):** In two dimensions the lines are parallel if they have no solution and coincident if they ha e infinite number of solutions. In such a case the matrix A will have dependent row /column and det (A)=0. Such a matrix is called Singular Matrix.

(2 lines):

Lines parallel (no solution)



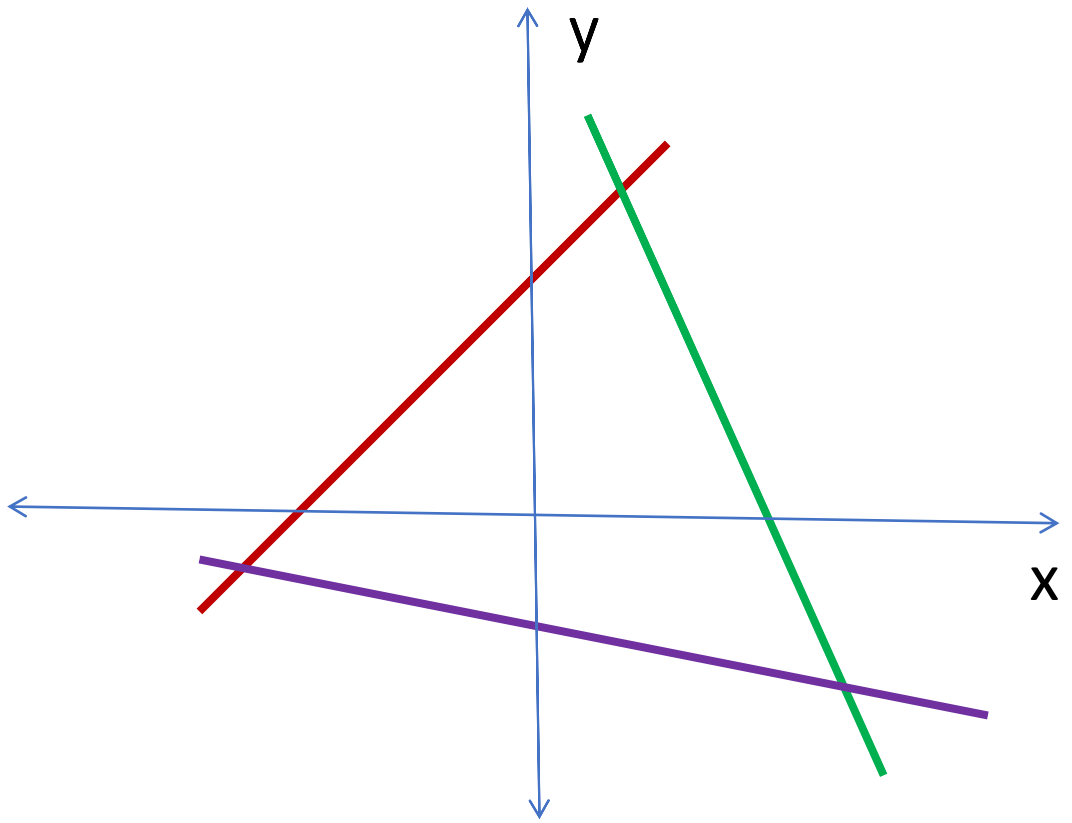
Lines coincident (infinite no. of solutions)



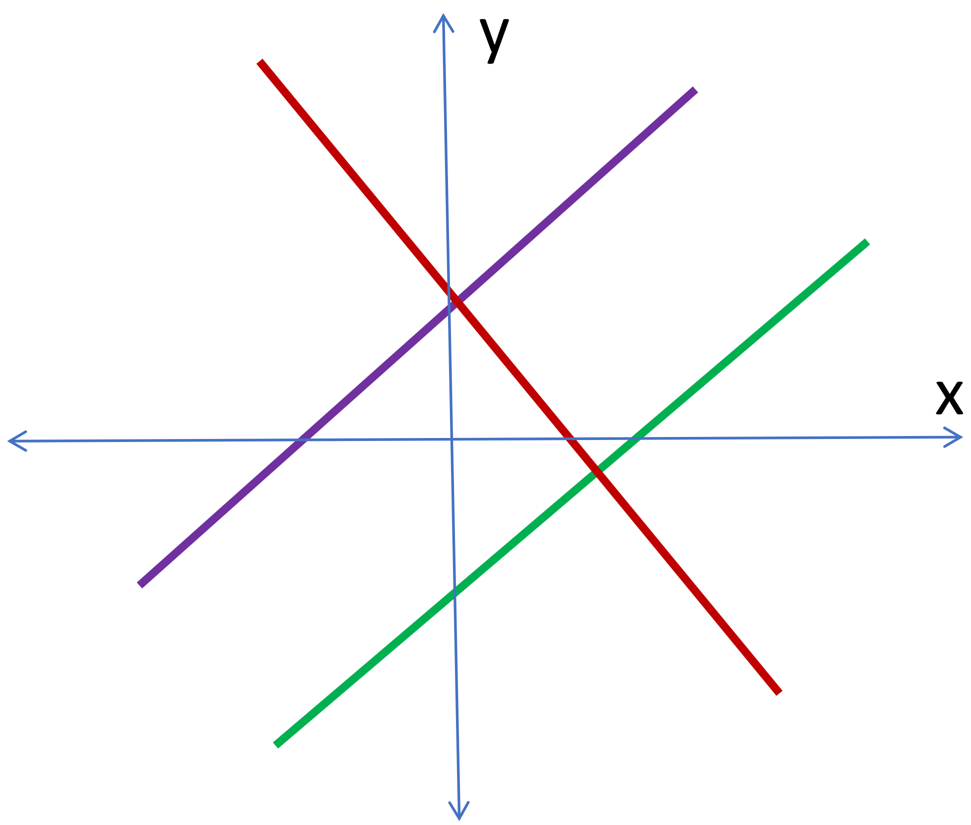
THE GEOMETRY OF LINEAR EQUATIONS:

(3 lines):

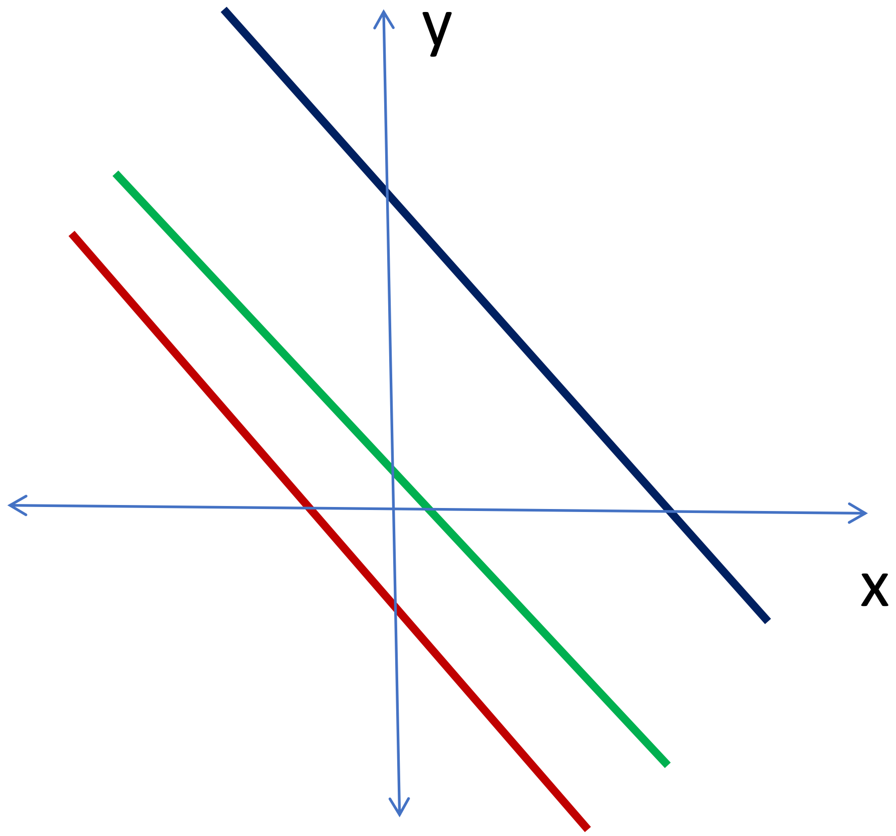
Lines intersecting in pairs (no solution)



2 Lines parallel & one intersecting(no solution)



All 3 parallel lines (no solution)

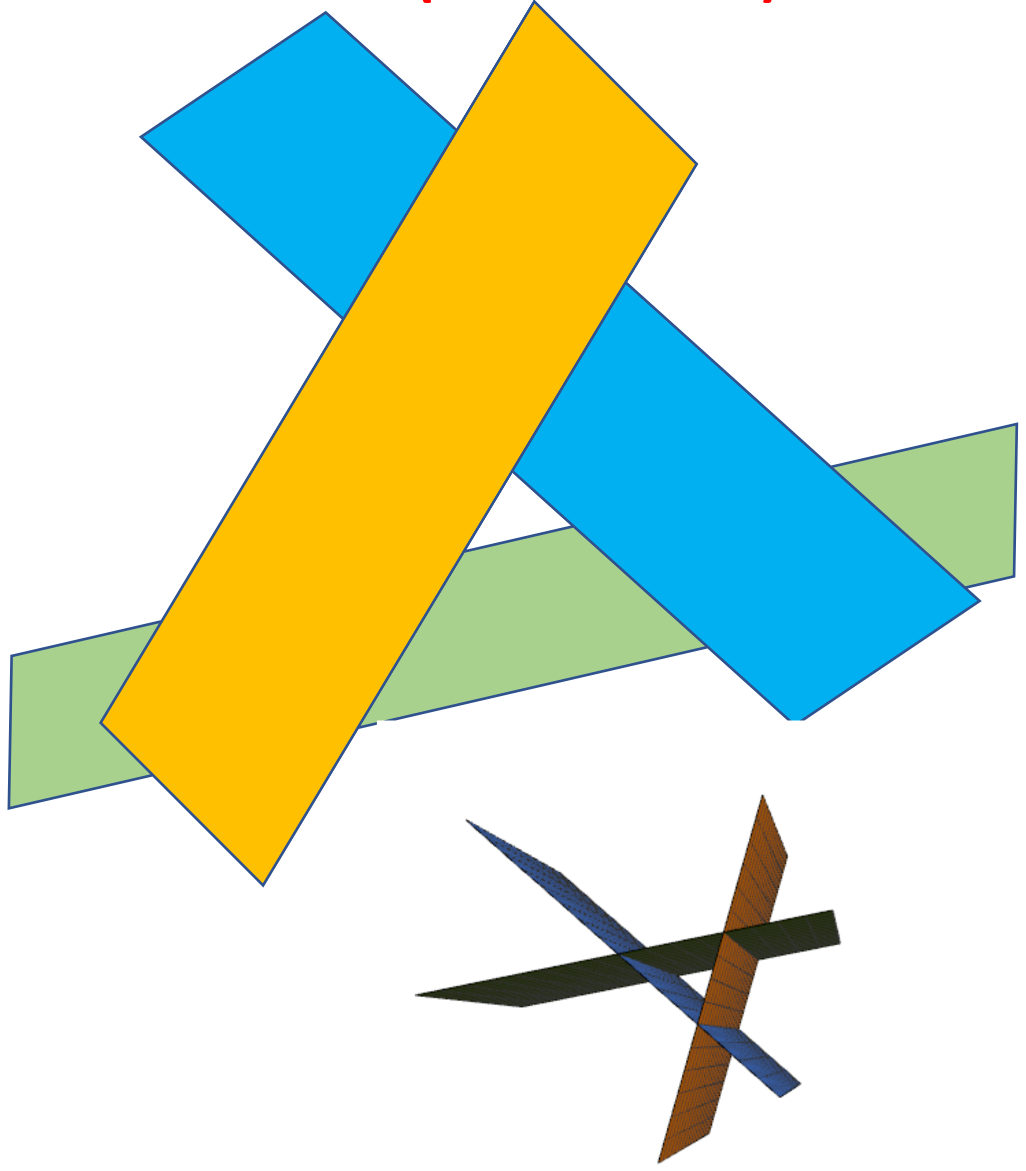


THE GEOMETRY OF LINEAR EQUATIONS:

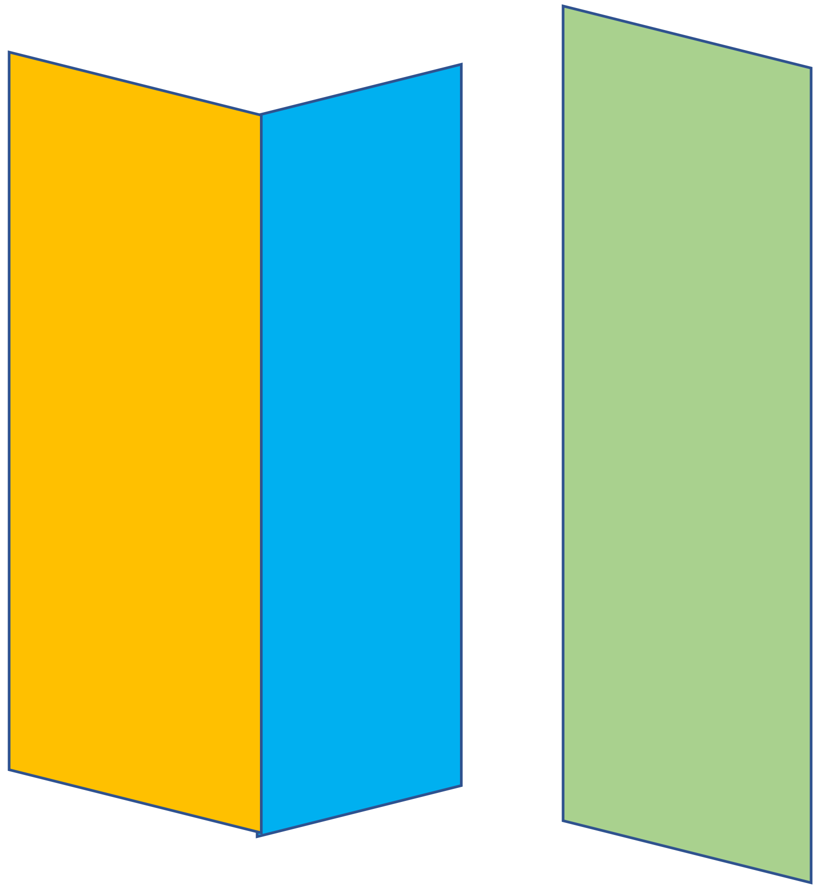
- **ROW PICTURE(Three dimensions):** In three dimensions if the 3 Planes do not intersect then we have the following cases :

(3 planes):

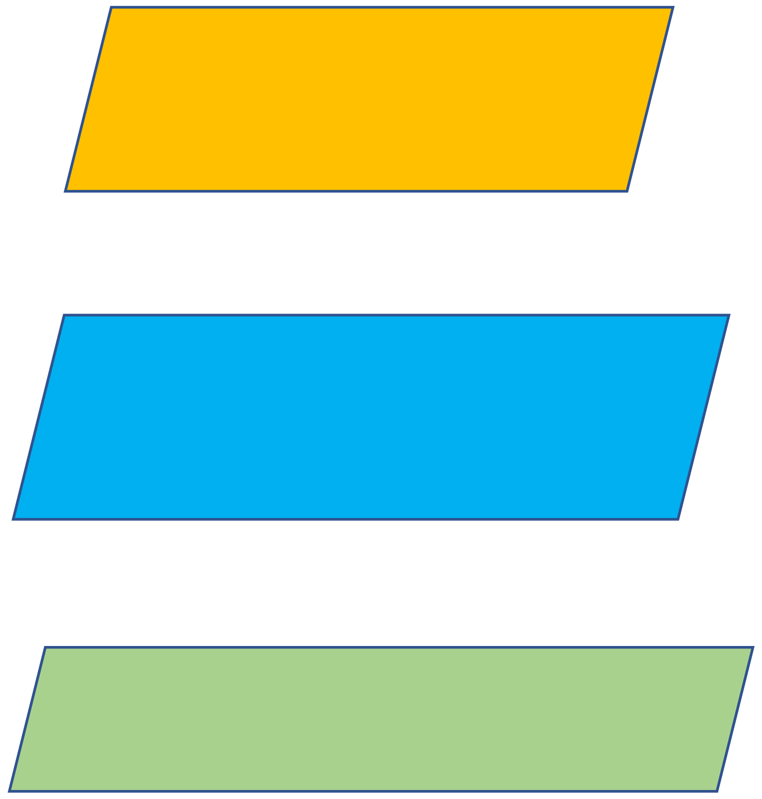
Every pair of planes intersecting in a line (no solution)



2 planes intersecting in a line and 3rd is parallel to this line (no solution)



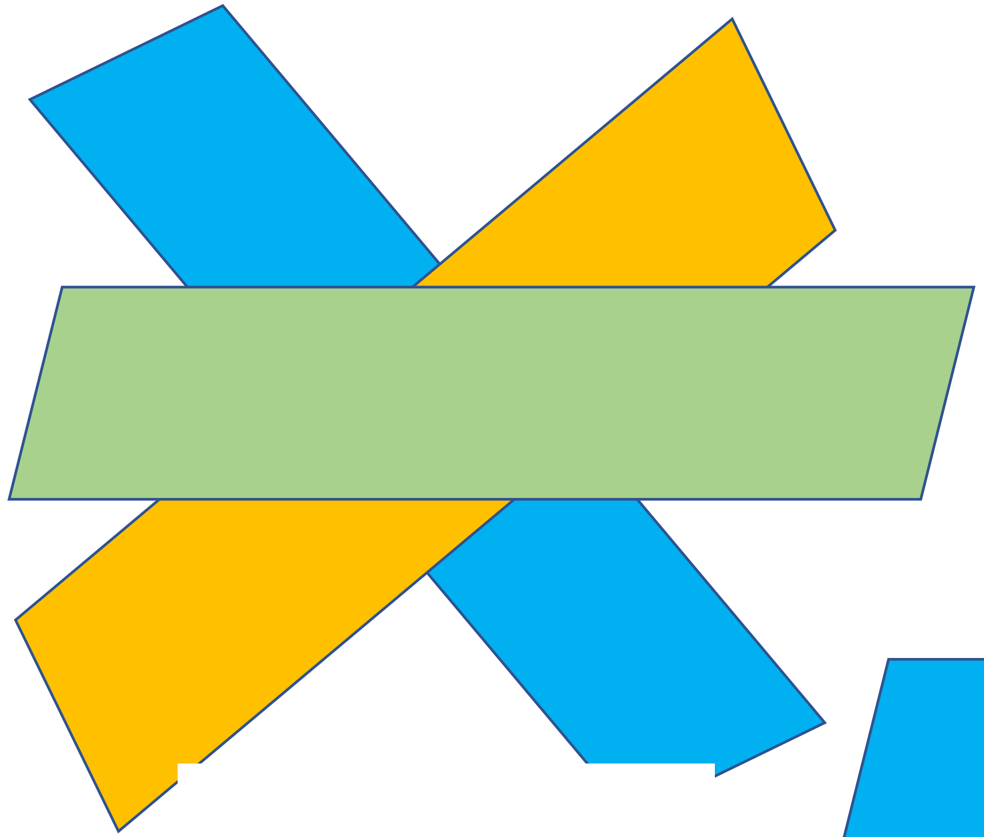
All 3 parallel planes (no solution)

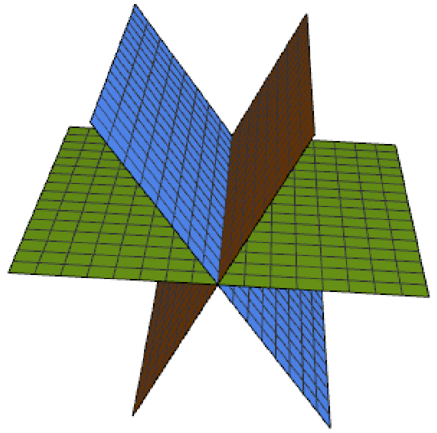


THE GEOMETRY OF LINEAR EQUATIONS:

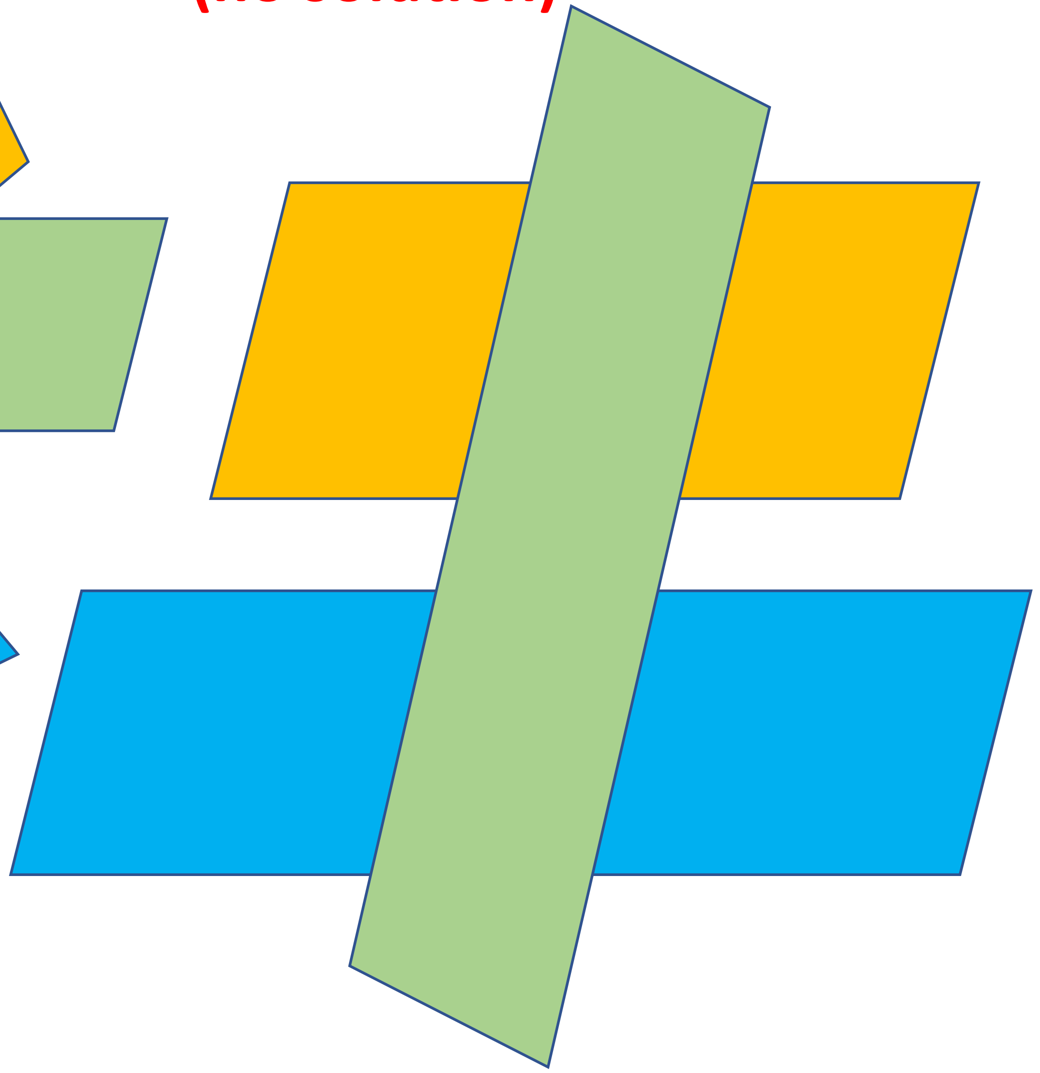
All 3 overlapping planes
(infinite no. of solutions)

All 3 planes intersecting in a line (infinite no solutions)





2 parallel planes and third plane intersecting them
(no solution)

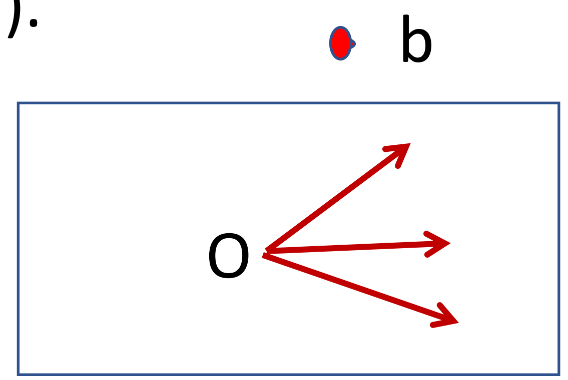


THE GEOMETRY OF LINEAR EQUATIONS:

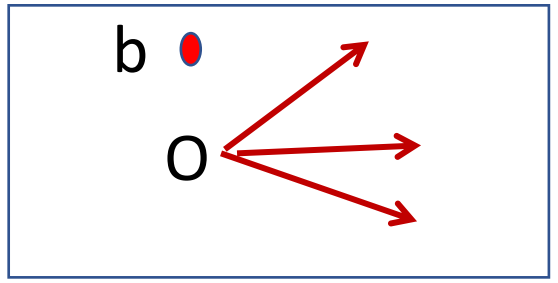
- **(ii) COLUMN PICTURE (Three dimensions):** Consider the column picture for a system of 3equations in 3 variables

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3 \end{aligned} \Rightarrow x\begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \end{pmatrix} + y\begin{pmatrix} a_{12} \\ a_{22} \\ a_{32} \end{pmatrix} + z\begin{pmatrix} a_{13} \\ a_{23} \\ a_{33} \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Each of these column vectors is a position vector with origin as a point, These column vectors lie in a plane (as they pass through the origin). Then every combination of these vectors on LHS lie in the same plane (3 vectors are coplanar).



If the vector b is not in that plane, then solution is impossible. This system is singular and has no solution.



If the vector b lies in the plane, (i.e b is also coplanar), then there are too many solutions. The 3 columns combine in infinitely many ways to produce b. This system is singular and has infinite no. of solutions.

References/Links:

https://upload.wikimedia.org/wikipedia/commons/c/c0/Intersecting_Lines.svg

Google search: Graphs of row and column pictiure for a system of linear equations

THANK YOU

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