UE19MA251 LINEAR ALGEBRA AND ITS APPLICATIONS

Unit-1-Matrices and Gaussian Elimination:
Introduction, The Geometry of Linear Equations, Gaussian Elimination, Singular Cases, Elimination Matrices, Triangular Factors and Row Exchanges, Inverses and Transposes, Inverse by Gauss -Jordan method. **Self Learning Component:** Algebra of Matrices.

Class No.	Portions to be covered
1	Introduction to Linear Algebra
2-3	The Geometry of Linear Equations – Row and Column Pictures
4	Singular cases in two and three dimensions
5-6	Gaussian Elimination,
7	The breakdown of elimination
8	**Scilab Class Number1 – Gaussian Elimination**
9	Elementary Matrices
10	Triangular Factors
11	Row Exchanges and Permutation Matrices
12	Inverse by Gauss -Jordan Method, Transposes
13	Supplementary problems
14-15	**Scilab Class Number 2&3- LU Decomposition and Inverses**

Classwork problems:

1.	Do the three planes $x + 2y + z = 4$, $y - z = 1$ and $x + 3y = 0$ have at least one common point of intersection ? Explain. Is the system consistent if the last equation is changed to $x + 3y = 5$? If so, solve the system completely. **Answer:** The planes do not have a common point. If the last equation is changed as $x + 3y = 5$ then the system is consistent with infinity of solutions. The solution set is $x = 2 - 3k$, $y = 1 + k$ and $z = k$ where k is real.
2.	Solve the following system of equations using Gaussian Elimination: (i) $x+y+2z+3t=13$, $x-2y+z+t=8$, $3x+y+z-t=1$ **Answer:** $x=-2+k$, $y=-1$, $z=8-2k$, $t=k$ (ii) $x+y+z=1$, $x+y-2z=3$, $2x+y+z=2$ **Answer:** $x=-1$, $y=2/3$, $z=-2/3$ (iii) $x-2y+3z=4$, $2x-y-3z=5$, $3x+z=2$, $3x-3y=7$ **Answer:** No solution

3.	Determine the values of a and b for which the system of equations x+y+az=2b, x+3y+(2+2a)z=7b, 3x+y +(3+3a)z=11b will have (i)trivial solution (ii)unique non-trivial solution (iii) no solution)(iv)infinity of solutions. **Answer:**(i)a$\neq$-5 and any b (ii)a$\neq$-5 and b=0 (iii)a=-5 and b$\neq$0 (iv)a=-5 and b=0.
4.	Determine the equation of the quadratic polynomial y=f(x) whose graph passes through the points (1,2), (3,3), (5,8). **Answer:** y= $(1/2)x^2$- (3/2) x + 3
5.	Write down the elementary matrices E,F,G associated with the system of equations 2u+3v-4w=3, u+2v+2w=10. u+v+w=7. Also find A=LU factorization for A if possible. **Answer:** L=$\begin{pmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ -1/2 & 1 & 1 \end{pmatrix}$U=$\begin{pmatrix} 2 & 3 & -4 \\ 0 & 1/2 & 4 \\ 0 & 0 & 7 \end{pmatrix}$
6.	Compute LU and LDU factorization for asymmetric matrix $\begin{pmatrix} a & r & r & r \\ a & b & s & s \\ a & b & c & t \\ a & b & c & d \end{pmatrix}$ Find the four conditions on the entries of A to get A=LU with four pivots. **Answer:** $L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix} U = \begin{pmatrix} a & r & r & r \\ 0 & b-r & s-r & s-r \\ 0 & 0 & c-s & t-s \\ 0 & 0 & 0 & d-t \end{pmatrix}$ $L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix} D = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b-r & 0 & 0 \\ 0 & 0 & c-s & 0 \\ 0 & 0 & 0 & d-t \end{pmatrix} U = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ The conditions to get four pivots are $a \neq 0, b \neq r, c \neq s, d \neq t$. Here $L=U^T$ and $U=L^T$.

7.	Apply elimination to produce the factors L and U for A= $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & -2 & 4 \end{pmatrix}$. Is A=LU possible? Explain. Write down the permutation matrices if any. **Answer:** A=LU is not possible since we need to use permutation matrix P_{23} which gives PA=LU. i.e., $\begin{pmatrix} 1 & 2 & 3 \\ 1 & -2 & 4 \\ 2 & 4 & 6 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 0 & -4 & 1 \\ 0 & 0 & 0 \end{pmatrix}$
8.	Compute inverse of the following matrices by Gauss Jordan method. (a) $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & 2 & 4 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 4 \end{pmatrix}$ **Answer:** (a) $\begin{pmatrix} 10 & -2 & -7 \\ -7 & 1 & 5 \\ 1 & 0 & -1 \end{pmatrix}$ (b) $\begin{pmatrix} 3/2 & -1 & 1/2 \\ 1/2 & 0 & -1/2 \\ -3/2 & 1 & 1/2 \end{pmatrix}$
9.	A furniture manufacturer makes chairs, coffee tables and dining-room tables. Each chair requires 10 minutes of sanding, 6 minutes of staining and 12 minutes of varnishing. Each coffee table requires 12 minutes of sanding, 8 minutes of staining and 12 minutes of varnishing.Each dining-room table requires 15 minutes of sanding,12 minutes of staining and 18 minutes of varnishing. The sanding bench is available 16 hours per week, the staining bench 11 hours per week and the varnishing bench 18 hours per week. How many (per week) of each type of furniture should be made so that the benches are fully utilized?(Use Gaussian elimination) **Answer:** 30 chairs, 30 coffee tables and 20 dining-room tables
10.	Boron sufhide reacts violently with water to form boric acid and hydrogen sulfide gas. The unbalanced equation is $B_2S_3 + H_20 \rightarrow H_3BO_3 + H_2S$. Balance the given chemical equation using Gaussian elimination. **Answer:** $B_2S_3 + 6H_20 \rightarrow 2H_3BO_3 + 3H_2S$