STATISTICS FOR DATA SCIENCE

Bernoulli and Binomial Distribution

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Mean and Variance of Bernoulli and Binomial Distribution - Derivation

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Mean and Variance of a Bernoulli Distribution

For Bernoulli Distribution:

Mean = p

Variance = p(1-p)

For Binomial Distribution:

Mean = np

Variance = np(1-p)

Mean and Variance of a Bernoulli Distribution:

$X \sim \text{Bernoulli}(p)$

X:	0	1
$P(X=x)$:	$1-p$	p
	$p^0(1-p)^{1-0}$	$p^1(1-p)^{1-1}$

$p(x) = p^i(1-p)^{1-i}$

when $i=0$, $p(x) = p^0(1-p)^{1-0} = 1-p$

when $i=1$, $p(x) = p^1(1-p)^{1-1} = p$

$$\mu_X \text{ or } E(X) = \sum_x x\,p(x)$$

$$= 0\left[p^0(1-p)^{1-0}\right] + 1\left[p^1(1-p)^{1-1}\right]$$

$$= 0 + p^1(1-p)^0 = p \quad \therefore \boxed{\mu_X \text{ or } E(X) = p}$$

Mean and Variance of a Bernoulli Distribution:

$$Var(X) = \sigma_x^2 = E[(X-\mu)^2]$$

$$Var(X) = \sum_x (x-\mu)^2 p(x) \quad \text{or} \quad E(X^2) - [E(X)]^2$$

$$E(X^2) = \sum_x x^2 p(x)$$

$$= 0^2 [p^0 (1-p)^{1-0}] + 1^2 [p^1 (1-p)^0]$$

$$= p$$

$$Var(X) = E(X^2) - (E(X))^2$$

$$= p - (p)^2$$

$$Var(X) = p(1-p)$$

$$\therefore \boxed{\begin{aligned} \mu_x &= p \\ \sigma_x^2 &= p(1-p) \end{aligned}}$$

Mean and Variance of a Binomial Distribution:

$\mu_X = np \qquad \sigma_X^2 = npq \quad$ or $\quad \sigma_X^2 = np(1-p)$

Let $U_1, U_2, U_3, \ldots U_n$ be the n independent Bernoulli r. v. s.

$E(U_i) = p \qquad \sigma^2(U_i) = p(1-p)$

Let $X = U_1 + U_2 + \cdots + U_n$

$\mu_X = E(X) = E(U_1 + U_2 + \cdots + U_n)$

$= E(U_1) + E(U_2) + \cdots + E(U_n)$

$= p + p + \cdots + p$

$\therefore \boxed{\mu_X = np}$

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Mean and Variance of a Binomial Distribution:

$$\sigma_x^2 = Var(x) = Var(U_1 + U_2 + \cdots + U_n)$$

$$= Var(U_1) + Var(U_2) + \cdots + Var(U_n)$$

$$= p(1-p) + p(1-p) + \cdots + p(1-p)$$

$$\boxed{\sigma_x^2 = np(1-p)}$$

THANK YOU

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