STATISTICS FOR DATA SCIENCE

POWER OF TEST AND SIMPLE LINEAR REGRESSION

SIVASANKARI V
Department of Science & Humanities

STATISTICS FOR DATA SCIENCE

Unit 5 : Power of test and Simple linear regression

Session : 1

Sub Topic : Power of test

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- Power computation

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Power of a Hypothesis Test :

The power of a test is the probability of rejecting H_0 when it is false.

Recall- type I error and type II error

Hypothesis testing : H_0 vs H_1

Statistical Conclusion	**Actual State of Reality**	
Researcher Decision	H_0 is true	H_0 is false
Reject H_0	Type I error (α)	Correct Decision (1 – β)
Fail to reject H_0	Correct Decision ($1 - \alpha$)	Type II error (β)

Power of a Hypothesis Test :

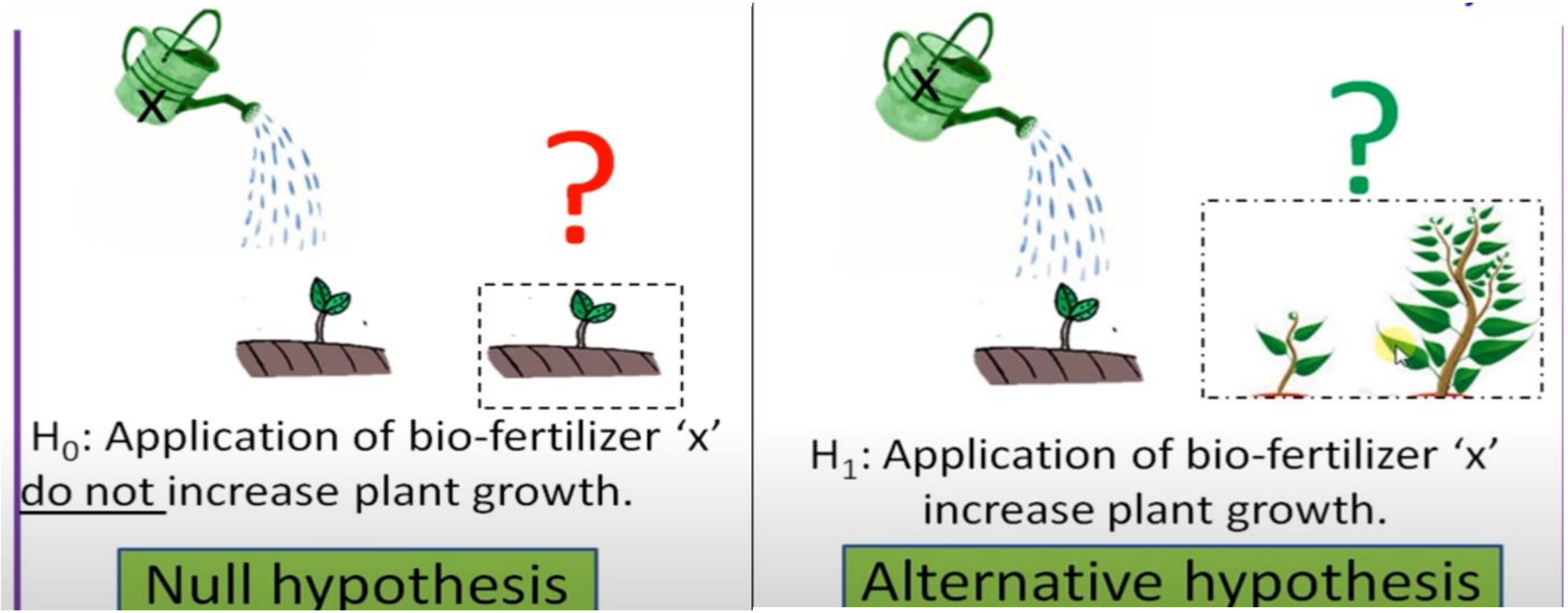
Power = 1 - P(type II error)

= 1 – β.

80% power means you have 80% chance of getting a significant results when the effect is real.

Power of test

Effect of bio-fertilizer 'x' on plant growth



Power is 0.80(or 80%) → there is an 80% chance of rejecting the null hypothesis(false) when conducting the study.

Source: www.majordifferences.com

Why is Power Important?

Power calculations are important to ensure that the experiments have the potential to provide useful calculations.

As researchers, we put a lot of effort into designing and conducting our research. This effort may be wasted if we do not have sufficient power in our studies to find the effect of interest.

How large the power must be for a test ?

In general, tests with power greater than 0.80 or perhaps 0.90 are considered acceptable, but there are no well-established rules of thumb.

Power of test

Analysis of power is performed:

1) Before gathering data

To determine the **minimal sample size** needed to have desired power in statistical testing (to detect a particular effect size).

2) After gathering data

To determine the **magnitude of power** that your statistical test will have given the sample parameters (**n** and **s**) and the magnitude of the effect that you want to detect.

Note: *Statistical power has relevance only when the null is false.*

Power of test

Power calculations are generally done before data are collected.

Computing the power involves two steps:

1. Compute the rejection region.

2. Compute the probability that the test statistic falls in the rejection region if the alternate hypothesis is true.

This is the power.

Example of a power calculation

Assume that a new chemical process has been developed that may increase the yield over that of the current process. The current process is known to have a mean yield of 80 and a standard deviation of 5, where the units are the percentage of a theoretical maximum. If the mean yield of the new process is shown to be greater than 80, the new process will be put into production.

Let μ denote the mean yield of the new process. It is proposed to run the new process 50 times and then to test the hypothesis

$H_0: \mu \leq 80$ versus $H_1 : \mu > 80$ at a significance level of 5%.

Problem 1 :

Find the power of the 5% level test of

$\boldsymbol{H_0}$ **: μ ≤ 80 versus** $\boldsymbol{H_1}$ **: μ > 80**

for the mean yield of the new process under the

alternative μ = 81, assuming n = 50 and σ = 5.

Solution:

Null distribution of $\bar{X}$:

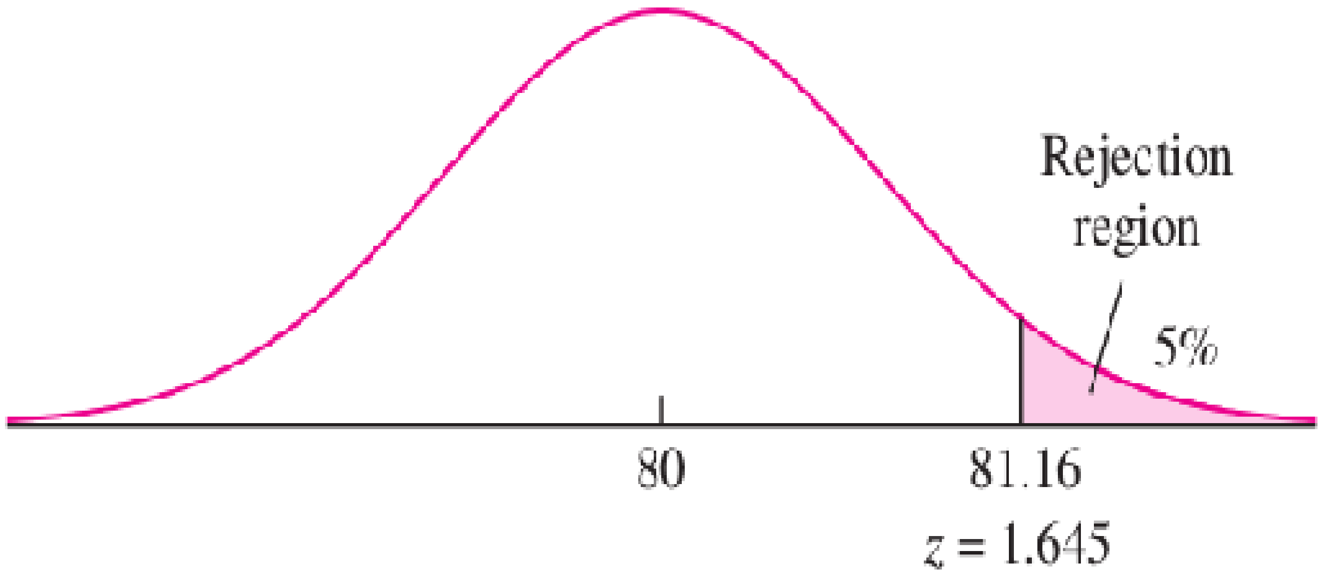
$$\bar{X} \sim N\left(\mu, \sigma_{\bar{X}}^2\right) where\ \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

Power of test

Null distribution of $\bar{X}$:

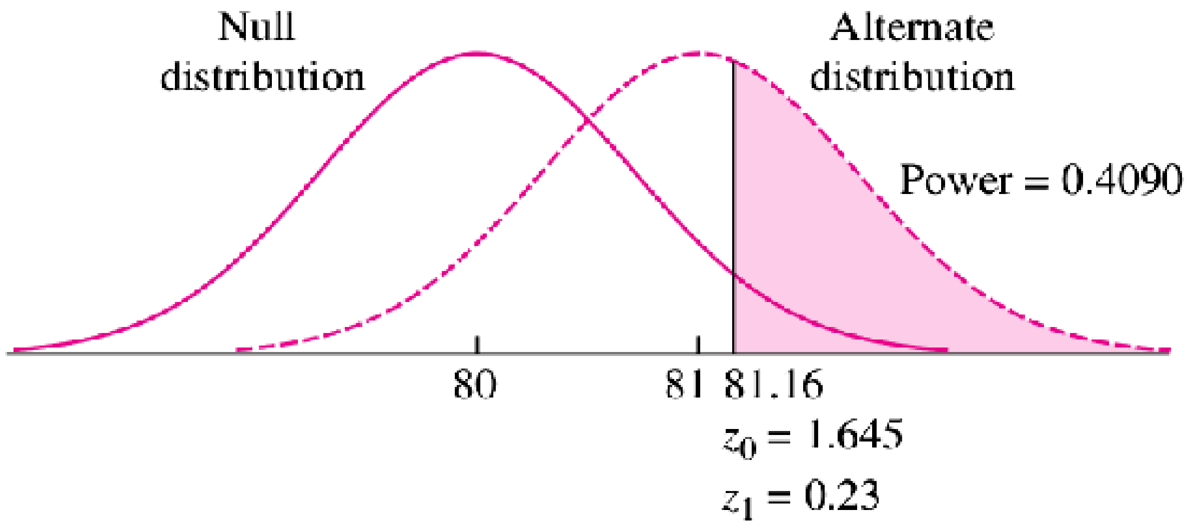
$$\bar{X} \sim N(80, 0.707^2)$$

The **critical point** has a z-score of 1.645, so its value is $\bar{X}$ = 80 + (1.645)(0.707) = 81.16.
The rejection region consists of all values of $\bar{X} \geqslant 81.16$



Alternate distribution of $\overline{X}$:

$$\bar{X} \sim N(81, 0.707^2)$$



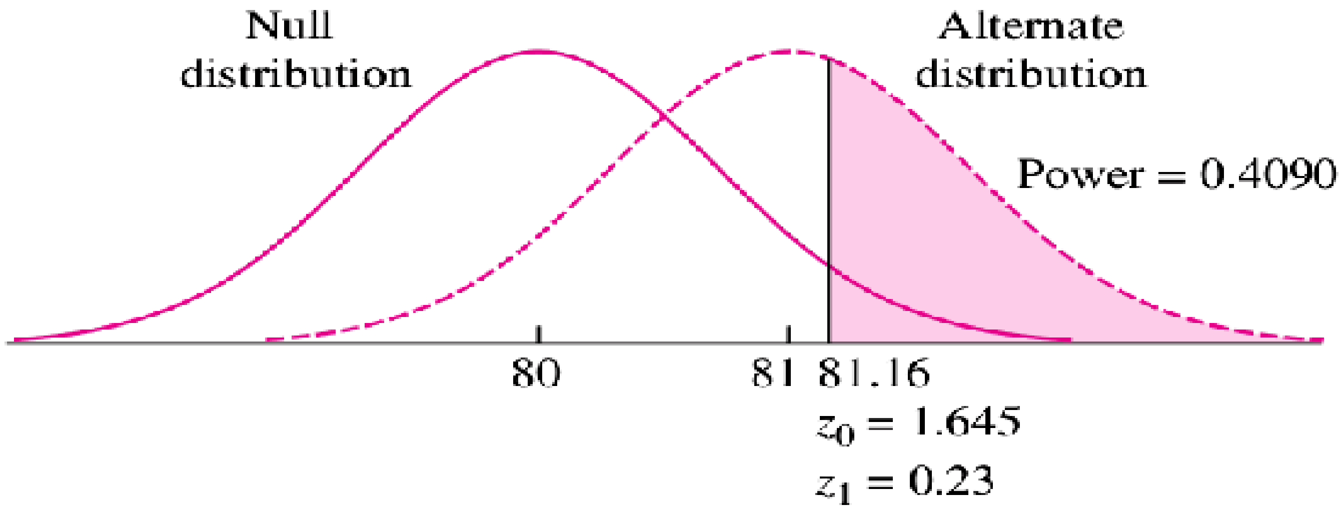
Computing the power

z -Score under $\boldsymbol{H_1}$ for the critical point 81.16 is

$$z = \frac{\bar{X} - \mu}{\sigma} = \frac{81.16 - 81}{0.707} = 0.23$$

The area to the right of $z = 0.23$ is **0.4090.**

This is the **power** of the test.



Calculation of Power

Problem 2 :

Find the power of the 5% level test of

H_0 **: μ ≤ 80 versus** H_1 **: μ > 80**

for the mean yield of the new process under the alternative μ = 82, assuming n = 50 and σ = 5.

Solution:

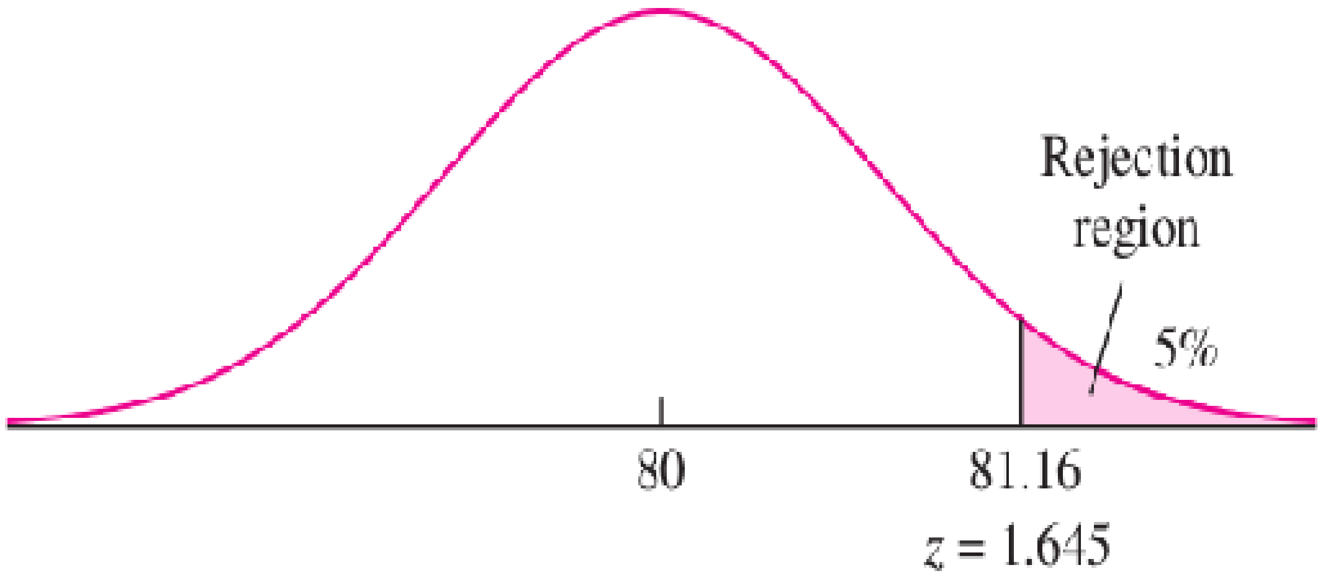
Null distribution of $\bar{X}$:

$$\bar{X} \sim N\left(\mu, \sigma_{\bar{X}}^{2}\right) \; where \; \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

Null distribution of $\bar{X}$:

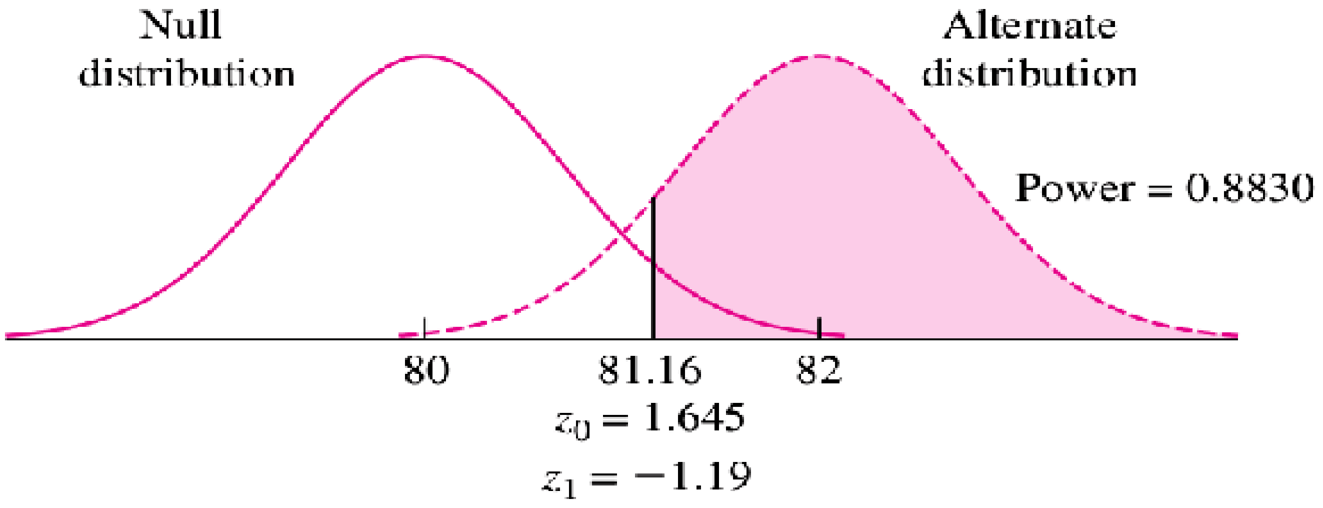
$$\bar{X} \sim N(80, 0.707^2)$$

The **critical point** has a z-score of 1.645, so its value is
$\bar{X}$ = 80 + (1.645)(0.707) = 81.16.
The rejection region consists of all values of $\bar{X} \geqslant 81.16$



Alternate distribution of $\overline{X}$:

$$\bar{X} \sim N(82, 0.707^2)$$



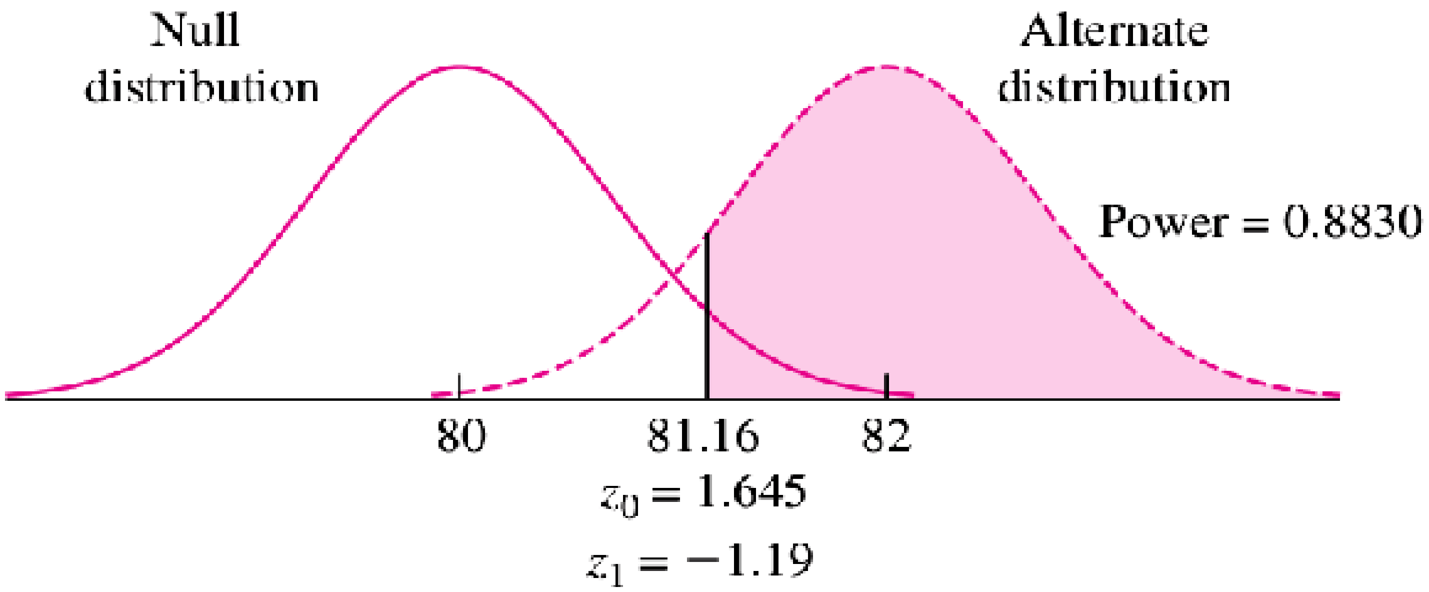
Computing the power

z -Score under H_1 for the critical point 81.16 is

$$z = \frac{\bar{X} - \mu}{\sigma} = \frac{81.16 - 82}{0.707} = -1.19$$

The area to the right of $z = -1.19$ is **0.8830.**

This is the **power** of the test.



Observations:

Power is different for different values of μ

- **if μ is close to H_0** : the power will be small

- **if μ is far from H_0** : the power will be large

Power of test

When power is not large enough, it can be increased by increasing the sample size.

When planning an experiment, one can determine the sample size necessary to achieve a desired power.

Power of test

Problem 3 :

In testing the hypothesis H_0 **: μ ≤ 80 versus** H_1 **: μ > 80** regarding the mean yield of the new process, how many times must the new process be run so that a test conducted at a significance level of 5% will have power 0.90 against the alternative μ = 81, if it is assumed that σ = 5?

Solution:

Let n represent the necessary sample size.

Power of test

Null distribution of $\bar{X}$:

$$\bar{X} \sim N\left(\mu, \sigma_{\bar{X}}^2\right) where\ \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

Critical point : $80 + 1.645\left(\frac{5}{\sqrt{n}}\right)$

Consider the **alternate distribution of $\bar{X}$.**

Given Power is 0.90. The power of the test is the area of the rejection region under the alternate curve. This area must be 0.90.

Therefore, Z-score is -1.28.

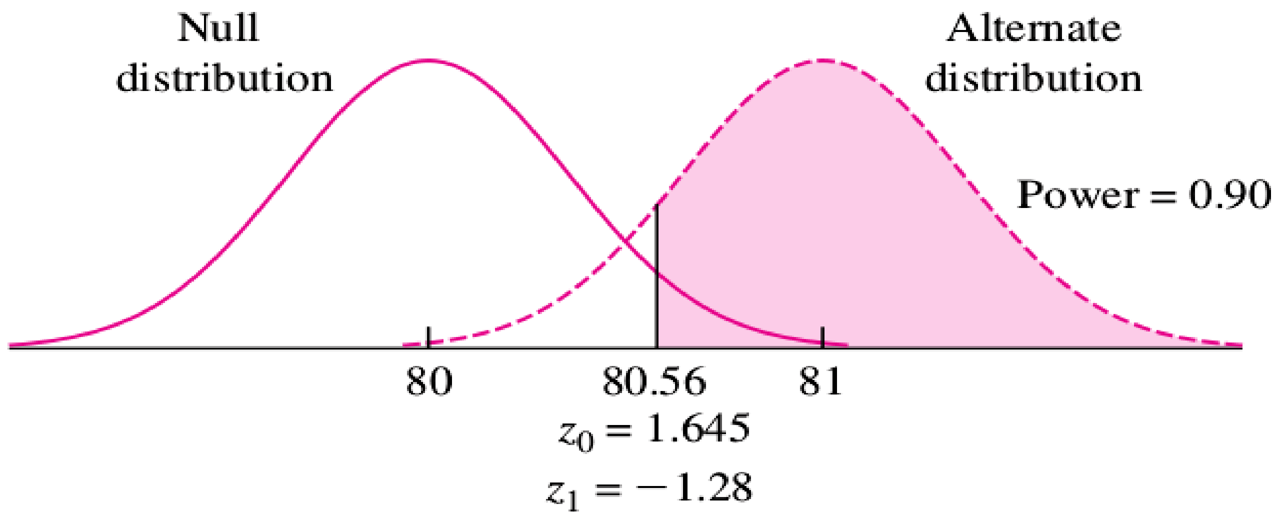
Critical point : $81 - 1.28\left(\frac{5}{\sqrt{n}}\right)$

We now have two different expression for the critical point. Since there is only one critical point, these two expressions are equal.

Power of test

Set them equal and solve for n

$$80 + 1.645\left(\frac{5}{\sqrt{n}}\right) = 81 - 1.28\left(\frac{5}{\sqrt{n}}\right)$$
$$\rightarrow n \approx 214.$$



The critical point is 80.56 (The critical point can by computed by substituting this value for n into either side of the equation).

THANK YOU

SIVASANKARI V
Department of Science & Humanities

sivasankariv@pes.edu