Q1. Solve $\begin{aligned} x+2y-z&=6 \\ 2x+y+z&=3 \\ x-y+z&=-2 \end{aligned}$ by GE

$$[A:b] = \left[\begin{array}{ccc:c} 1 & 2 & -1 & 6 \\ 2 & 1 & 1 & 3 \\ 1 & -1 & 1 & -2 \end{array}\right] \xrightarrow[R_2 \to R_2 - 2R_1]{R_3 \to R_3 - R_1} \left[\begin{array}{ccc:c} 1 & 2 & -1 & 6 \\ 0 & -3 & 3 & -9 \\ 0 & -3 & 2 & -8 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - R_2} \left[\begin{array}{ccc:c} 1 & 2 & -1 & 6 \\ 0 & -3 & 3 & -9 \\ 0 & 0 & -1 & 1 \end{array}\right]$$

$-z = 1$

$\boxed{z=-1}$

$-3y-3=-9$

$3y+3=9$

$\boxed{y=2}$

$x+4+1=6$

$x+5=6$

$\boxed{x=1}$

$(1, 2, -1)$ is solution ✓

Q2. $u+v+w=2$

$2u+2v+5w=7$

$4u+6v+8w=16$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 2 \\ 2 & 2 & 5 & 7 \\ 4 & 6 & 8 & 16 \end{array}\right] \xrightarrow[R_3 \to R_3 - 4R_1]{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc:c} 1 & 1 & 1 & 2 \\ 0 & 0 & 3 & 3 \\ 0 & 2 & 4 & 8 \end{array}\right]$$

$$\xrightarrow{R_3 \leftrightarrow R_2} \left[\begin{array}{ccc:c} 1 & 1 & 1 & 2 \\ 0 & 2 & 4 & 8 \\ 0 & 0 & 3 & 3 \end{array}\right]$$

$3z = 3$

$\boxed{z=1}$

$2y+4 = 8$

$2y = 4$

$\boxed{y=2}$

$x+2+1=2$

$\boxed{x=-1}$

$(-1, 2, 1)$ is sol

Q3.

$$x + 2y + z = 3$$
$$2x + 5y - z = -4$$
$$3x - 2y - z = 5$$

$$\left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 2 & 5 & -1 & -4 \\ 3 & -2 & -1 & 5 \end{array}\right] \xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 0 & 1 & -3 & -10 \\ 0 & -8 & -4 & -4 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 + 8R_2} \left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 0 & 1 & -3 & -10 \\ 0 & 0 & -28 & -84 \end{array}\right]$$

$$z = \frac{84}{28} = 3$$

$$y - 9 = -10$$
$$y = -1$$

$$x - 2 + 3 = 3$$
$$x = 2$$

$$(2, -1, 3)$$

Q4. $x+y-2z+3t=4$

$2x+3y+3z-t=3$

$5x+7y+4z+t=5$

$$\left[\begin{array}{cccc:c} 1 & 1 & -2 & 3 & 4 \\ 2 & 3 & 3 & -1 & 3 \\ 5 & 7 & 4 & 1 & 5 \end{array}\right] \xrightarrow[R_3 \to R_3 - 5R_1]{R_2 \to R_2 - 2R_1} \left[\begin{array}{cccc:c} 1 & 1 & -2 & 3 & 4 \\ 0 & 1 & 7 & -7 & -5 \\ 0 & 2 & 14 & -14 & -15 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 2R_2} \left[\begin{array}{cccc:c} 1 & 1 & -2 & 3 & 4 \\ 0 & 1 & 7 & -7 & -5 \\ 0 & 0 & 0 & 0 & -5 \end{array}\right]$$

inconsistent

Q5. $$\left[\begin{array}{ccc:c} 1 & 1 & -1 & 2 \\ 1 & 2 & 1 & 3 \\ 1 & 1 & (a^2-5) & a \end{array}\right] \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - R_1} \left[\begin{array}{ccc:c} 1 & 1 & -1 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & a^2-4 & a-2 \end{array}\right]$$

(1) Case 1: $a = 2$

$$\begin{bmatrix} 1 & 1 & -1 & : & 2 \\ 0 & 1 & 2 & : & 1 \\ 0 & 0 & 0 & : & 0 \end{bmatrix}$$

consistent

$y + 2z = 1$

Let $z = k$

$y = 1 - 2k$

$x + (1 - 2k) - k = 2$

$x = 2 + k + 2k - 1$

$x = 1 + 3k$

$(1+3k, 1-2k, k)$

(2) Case 2: $a = -2$

inconsistent

(3) case 3 : $a \neq \pm 2$

$$\left[\begin{array}{ccc:c} 1 & 1 & -1 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & a^2-4 & a-2 \end{array}\right]$$

$$(a^2-4)z = a-2$$

$$z = \frac{a-2}{(a+2)(a-2)} = \frac{1}{a+2}$$

$$y + 2\left(\frac{1}{a+2}\right) = 1$$

$$y = \frac{a}{a+2} \qquad x + \frac{a}{a+2} - \frac{1}{a+2} = 2$$

$$\left(\frac{a+5}{a+2}, \frac{a}{a+2}, \frac{1}{a+2}\right) \qquad x = \frac{2a+4-a+1}{a+2}$$

$$x = \frac{a+5}{a+2}$$

Q6. Investigate the values of λ and μ such that

$$x + 3y + 5z = 9$$
$$x - y + 2z = 1$$
$$2x + 2y + \lambda z = \mu$$

(i) has unique sol
(ii) infinitely many solutions
(iii) no solutions

$$\left[\begin{array}{ccc:c} 1 & 3 & 5 & 9 \\ 1 & -1 & 2 & 1 \\ 2 & 2 & \lambda & \mu \end{array}\right] \xrightarrow[R_3 \to R_3 - 2R_1]{R_2 \to R_2 - R_1} \left[\begin{array}{ccc:c} 1 & 3 & 5 & 9 \\ 0 & -4 & -3 & -8 \\ 0 & -4 & \lambda-10 & \mu-18 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - R_2} \left[\begin{array}{ccc:c} 1 & 3 & 5 & 9 \\ 0 & -4 & -3 & -8 \\ 0 & 0 & \lambda-7 & \mu-10 \end{array}\right]$$

(i) unique solution: $\lambda \neq 7$ <u>and</u> $\mu \neq 10$

(ii) infinite solutions: $\lambda = 7$ and $\mu = 10$

$$-4y - 3z = -8$$

let $z = k$

$$-4y = -8 + 3k$$

$$y = 2 - \frac{3}{4}k$$

$$x + 6 - \frac{9}{4}k + 5k = 9$$

$$x = 3 - 5k + \frac{9}{4}k$$

$$x = 3 - \frac{11}{4}k$$

$$\left(3 - \frac{11}{4}k,\ 2 - \frac{3}{4}k,\ k\right)$$

(iii) no solution: $\lambda = 7$ and $\mu \neq 10$

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