LINEAR ALGEBRA AND ITS APPLICATIONS

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Unit-4

Orthogonalization , Eigen Values and Eigen Vectors

Topics in the Module:

- Orthogonal Bases, orthogonal matrices and properties
- Rectangular matrices with orthonormal columns
- The Gram- Schmidt Orthogonalization
- A=QR factorization
- Introduction to Eigenvalues and Eigenvectors
- Properties of Eigenvalues and Eigenvectors and C-H theorem
- Problems on Eigenvalues and Eigenvectors and C-H theorem
- Symmetric Matrices and Diagonalization of a Matrix
- Problems on diagonalization of a matrix
- Powers and products of matrices

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CLASS-1

ORTHOGONAL BASES, ORTHOGONAL MATRICES AND PROPERTIES

In an **orthogonal basis**, every vector is perpendicular to every other vector. The coordinate axes are mutually orthogonal.

Mutually perpendicular unit vectors are called **Orthonormal** vectors.

For the vector space R^2,

1. The set (2, 0) , (0, 2) is an orthogonal basis.
2. The set (1, -2) , (2 , 1) is an orthogonal basis.
3. The set (1, 0), (0, 1) is an orthonormal basis.

- A matrix with Orthonormal columns will be called Q.
- A square matrix with Orthonormal columns is called an ***Orthogonal matrix*** denoted by Q.

Ex: Rotation matrix , any permutation matrix .

Properties of Q

- If Q (square or rectangular) has orthonormal columns, then $Q^T Q = I$.
- An orthogonal matrix is a square matrix with orthonormal columns. Then Q^T is Q^{-1}.
- If Q is rectangular then Q^T is left inverse of Q.
- Multiplication by any Q preserves length. The norms of x and Qx are equal.

Properties of Q (Continued…..)

- Also, Q preserves inner products and angles, since $(Q\,x)^T (Qy) = x^T Q^T Q y = x^T y$.

If $q_1, q_2 \ldots . q_n$ are orthonormal basis of R^n then any vector b from R^n can be expressed as

$$b = x_1 q_1 + x_2 q_2 + \ldots\ldots + x_n q_n \qquad \ldots\ldots \text{Eqn (1)}$$

Multiply both sides by q_1^T. Then $q_1^T b = x_1$.

Similarly, $x_2 = q_2^T b, \ldots\ldots\ldots , x_n = q_n^T b$.

Hence, $b = (q_1^T b) q_1 + (q_2^T b) q_2 + \ldots\ldots + (q_n^T b) q_n$

= sum of one dimensional projections on to q_i's.

The matrix form of equation (1) is $Qx = b$ and the solution of this system of equations is

$$x = Q^{-1} b = Q^T b$$

Properties of Q (Continued…..)

- The rows of a square matrix are orthonormal whenever the columns are

Orthonormal columns
Orthonormal rows

$$Q = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & 0 & -2/\sqrt{6} \\ 1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6} \end{bmatrix}.$$

THANK YOU

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