PES University, Bangalore

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UE19CS203 – STATISTICS FOR DATA SCIENCE

Unit-4 - Hypothesis and Inference

QUESTION BANK - SOLVED

Exercises for section 6.1: [Text Book Exercise 6.1– Pg. No. [403 – 405]]

1. A simple random sample consists of 65 lengths of piano wire that were tested for the amount of extension under a load of 30 N. The average extension for the 65 lines was 1.102mmand the standard deviation was 0.020 mm. Let μ represent the mean extension for all specimens of this type of piano wire.

a. Find the P-value for testing $H_0: \mu \leq 1.1$ versus $H_1: \mu > 1.1$.

b. Either the mean extension for this type of wire is greater than 1.1 mm, or the sample is in the most extreme------------ % of its distribution.

[Text Book Exercise – Section 6.1 – Q. No.2 – Pg. No. 403]

Solution:

Given $n = 65, \quad \bar{x} = 1.102, \; \sigma = 0.020$

$$H_0: \mu \leq 1.1$$

$$H_1: \mu > 1.1$$

The sampling distribution of the sample mean has mean $\boldsymbol{\mu}$ and standard deviation

$$\frac{\sigma}{\sqrt{n}}$$

Determine the z statistic:

$$z = \frac{\bar{x} - \bar{\mu}}{\sigma/\sqrt{n}} = \frac{1.102 - 1.1}{0.020/\sqrt{65}} \approx 0.81$$

The P-value is the probability of obtaining a value more extreme or equal to the standardized test statistic z, assuming that the null hypothesis is true.

Determine the probability using the normal probability table

$$P(Z > 0.81) = 1 - P(Z < 0.81) = 1 - 0.7910 = 0.2090 = 20.90\%$$

(b) By part (a), we know that if the null hypothesis H_0 is true, then the given sample is within the most extreme 20.90% of its distribution.

2. A certain type of stainless steel powder is supposed to have a mean particle diameter of $\mu = 15\,\mu$m. A random sample of 87 particles had a mean diameter of 15.2 μm, with a standard deviation of $1.8\ \mu m$. A test is made of $H_0 : \mu = 15$ versus $H_1 : \mu \neq 15$.

a. Find the P –value.
b. Do you believe it is plausible that the mean diameter is $15\ \mu m$, or are you convinced that it differs from $15\ \mu m$? Explain your reasoning.

[Text Book Exercise – Section 6.1 – Q. No.6 – Pg. No. 404]

Solution:

Given $n = 87, \quad \bar{x} = 15.2, \ \sigma = 1.8$

$$H_0: \mu = 15$$

$$H_1: \mu \neq 15$$

The sampling distribution of the sample mean has mean $\boldsymbol{\mu}$ and standard deviation $\frac{\sigma}{\sqrt{n}}$

Determine the z statistic:

$$z = \frac{\bar{x} - \bar{\mu}}{\sigma/\sqrt{n}} = \frac{15.2 - 15}{1.8/\sqrt{87}} \approx 1.04$$

The P-value is the probability of obtaining a value more extreme or equal to the standardized test statistic z, assuming that the null hypothesis is true.

Determine the probability using the normal probability table

$$P(Z < -1.04 \; or \; Z > 1.04) = 2P(Z < -1.04) = 2(0.1492) = 0.2984$$

(b) The probability in part(a) is not small (larger than 0.05), which indicates that the null hypothesis is not false and thus it is plausible that the mean diameter is 15 $\boldsymbol{\mu}$m.

3. Fill in the blank: In a test of $H_0: \mu \geq 10$ versus $H_1: \mu < 10$, the sample mean was $\bar{X} = 8$ and the P-value was 0.04. This means that if $\mu = 10$, and the experiment were repeated 100 times, we would expect to obtain a value of $\bar{X}$ of 8 or less approximately ------------times.

i. 8
ii. 0.8
iii. 4
iv. 0.04
v. 80

[Text Book Exercise – Section 6.1 – Q. No.12 – Pg. No. 405]

Solution:

Given $P = 0.04 = 4\%$, $\bar{x} = 8$.

$$H_0: \mu \geq 10$$

$$H_1: \mu < 10$$

The P-value is the probability to obtain the value of the sample mean or more extreme, if the null hypothesis is true.

This mean that if $\boldsymbol{\mu}$ =10, then we expect to obtain a sample mean of 8 or less in about 4% of the samples.

When we have 100 samples(repetitions), we then $4\% \times 100 = 0.0 \times 100 = 4$ of the sample means to have a sample mean of 8 or less.

So, the answer is (iii) 4.