CONFIDENCE INTERVALS

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STATISTICS FOR DATA SCIENCE

CONFIDENCE INTERVALS FOR LARGE SAMPLES

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Confidence Intervals

- Its an interval estimate for a population parameter and is how much uncertainty there is with any particular statistic.
- Based on sample data and provides a range of plausible values for a parameter.
- Confidence Interval differs from sample to sample (taken from same population).
- Confidence intervals are intrinsically connected to confidence levels.

Confidence Intervals

- Confidence intervals are often used with a margin of error.

- For example: we may be 95% confident that μ (population parameter) lies in the interval (-0.2, 3.1). Here, being 95% is the confidence level associated with the Confidence Interval (-0.2, 3.1).

Confidence Intervals vs. Confidence Levels

- Confidence levels are expressed as a percentage(for example, a 95% confidence level).

- Confidence intervals are your results, usually numbers.

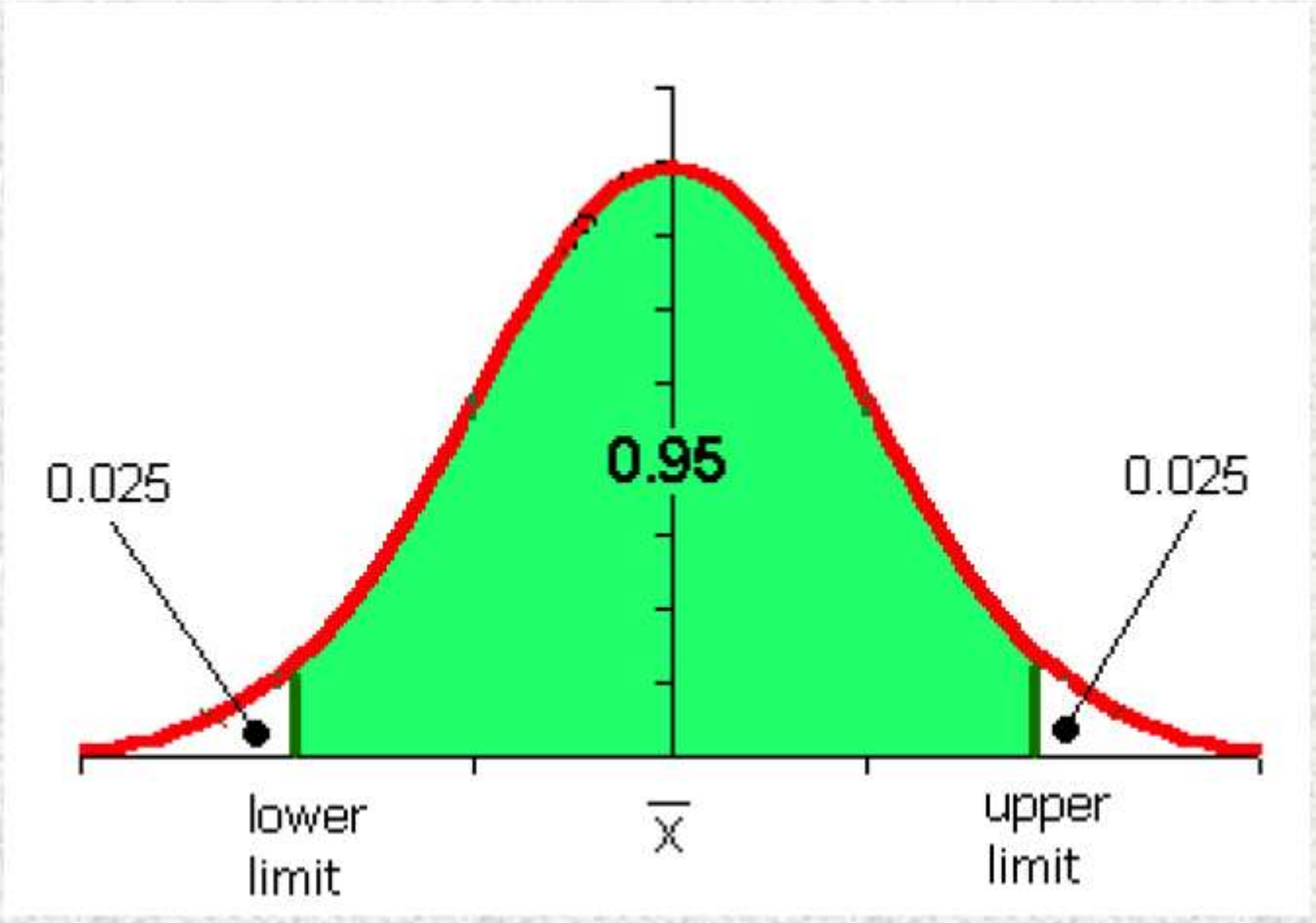
Confidence Intervals vs. Confidence Levels

You survey a group of pet owners to see how many cans of dog food they purchase a year.

You test your statistics at the 99 percent confidence level and get a confidence interval of (200,300).

That means you think they buy between 200 and 300 cans a year. You're super confident (99% is a very high level!) that your results are sound, statistically.

95% Confidence Interval



Confidence Co-efficient

- The confidence coefficient is the confidence level stated as a proportion, rather than as a percentage.

- For example, if you had a confidence level of 99%, the confidence coefficient would be .99.

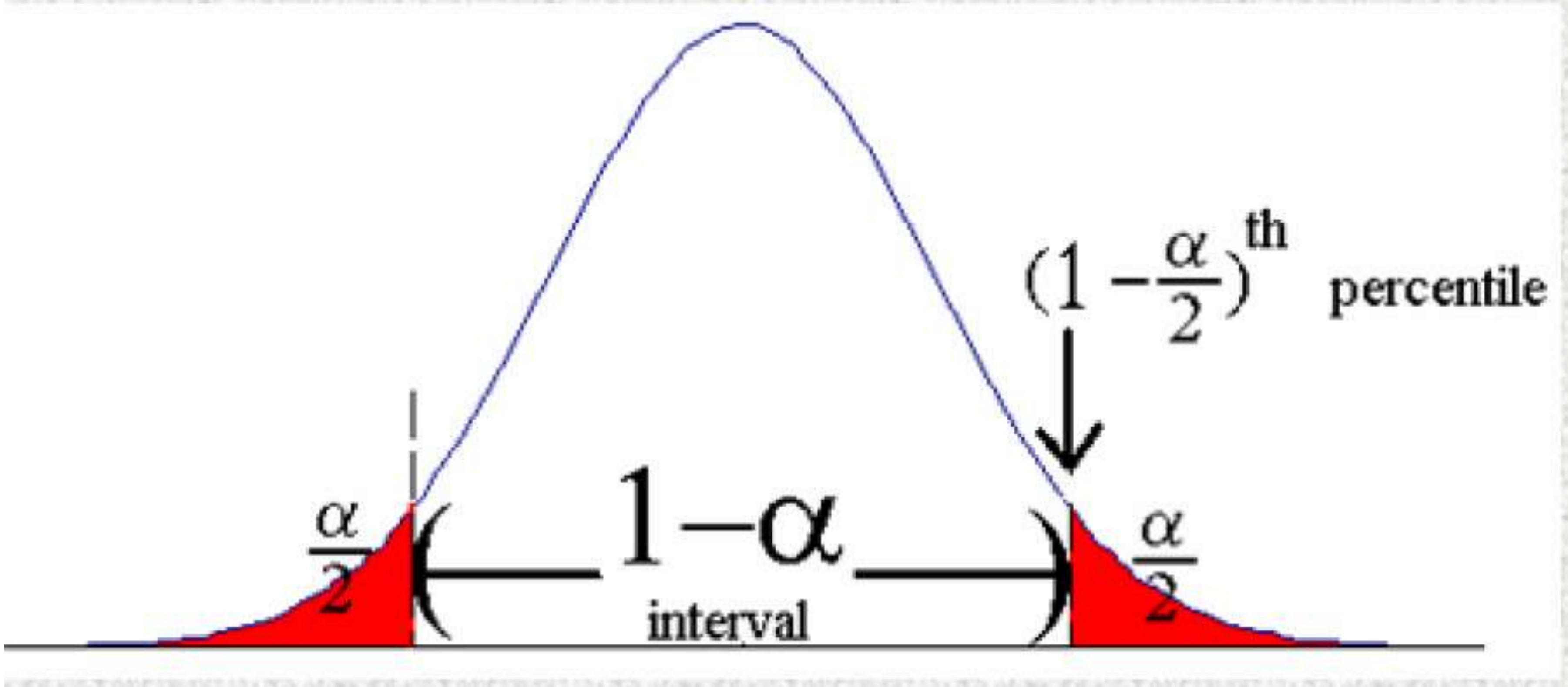
- In general, the higher the coefficient, the more certain you are that your results are accurate.

Confidence Co-efficient

The following table lists confidence coefficients and the equivalent confidence levels.

Confidence coefficient $(1 – \alpha)$	Confidence level $(1 – \alpha * 100\%)$
0.90	90%
0.95	95%
0.99	99%

Confidence Co-efficient



Confidence Intervals for population mean of large samples

Let $X_1, \ldots, X_n$ be a *large* ($n > 30$) random sample from a population with mean μ and standard deviation σ, so that $\overline{X}$ is approximately normal. Then a level $100(1-\alpha)\%$ confidence interval for μ is

$$\overline{X} \pm z_{\alpha/2}\sigma_{\overline{X}} \tag{5.1}$$

where $\sigma_{\overline{X}} = \sigma/\sqrt{n}$. When the value of σ is unknown, it can be replaced with the sample standard deviation s.

In particular,

- $\overline{X} \pm \frac{s}{\sqrt{n}}$ is a 68% confidence interval for μ.
- $\overline{X} \pm 1.645\frac{s}{\sqrt{n}}$ is a 90% confidence interval for μ.
- $\overline{X} \pm 1.96\frac{s}{\sqrt{n}}$ is a 95% confidence interval for μ.
- $\overline{X} \pm 2.58\frac{s}{\sqrt{n}}$ is a 99% confidence interval for μ.
- $\overline{X} \pm 3\frac{s}{\sqrt{n}}$ is a 99.7% confidence interval for μ.

Confidence Intervals for population mean of large samples

point estimate ± Margin of error

Confidence Interval for μ will be of the form:

X_bar ± Margin of error

Margin of error:

$$\pm 1.96 \frac{s}{\sqrt{n}}$$

Confidence Intervals for population mean of large samples

A $(1 - \alpha)$ 100% Confidence Interval for μ is given by:

$$\bar{X} \pm z_{\alpha/2} (\sigma / \sqrt{n})$$

where, the quantity $z_{\alpha/2} (\sigma / \sqrt{n})$ is the Margin of error.

Note:

$(\sigma / \sqrt{n})$ is the standard deviation of Sampling distribution of sample mean (X_bar).

Example1

Find the value of $z_{\alpha/2}$ to use to construct a confidence interval with level:

a) 95%

b) 98%

c) 99%

d) 80%

Solution

a)95% : X_bar ± 1.96 (σ/√n)

b)98% : X_bar ± 2.33 (σ/√n)

c)99% : X_bar ± 2.57 (σ/√n) or

X_bar ± 2.58 (σ/√n)

d)80% : X_bar ± 1.28 (σ/√n)

Example2

Find the levels of confidence intervals that have the following values of $z_{\alpha/2}$:

a) $z_{\alpha/2} = 2.17$

b) $z_{\alpha/2} = 3.28$

Examples

a) $z_{\alpha/2} = 2.17$

$$P(-z_{0.015} < Z < z_{0.015}) = 1 - 0.03$$

Hence the confidence level is 97%

b) $z_{\alpha/2} = 3.28$

$$P(-z_{0.0005} < Z < z_{0.0005}) = 1 - 0.001$$

Hence the confidence level is 99.9%

Interpreting confidence intervals

- To interpret the confidence interval of the mean, you must assume that:
- All the values were **independently and randomly sampled** from a population whose values are distributed according to a **Gaussian(Normal) distribution.**

- A confidence interval is calculated from one given sample. It either covers or misses the true parameter. Since the true parameter is unknown, you'll never know which one is true.

Interpreting confidence intervals

- If independent samples are taken repeatedly from the same population, and a confidence interval calculated for each sample, then a certain percentage (**confidence level**) of the intervals will include the unknown population parameter.

- The **confidence level** associated with a confidence interval is the success rate of the confidence interval.

Example

A random sample of n = 50 males showed a mean average daily intake of dairy products equal to 756 grams with a standard deviation of 35 grams. Find a 95% confidence interval for the population average μ?

Solution

$$\bar{x} \pm 1.96\frac{s}{\sqrt{n}} \Rightarrow 756 \pm 1.96\frac{35}{\sqrt{50}} \Rightarrow 756 \pm 9.70$$

$$\text{or } 746.30 < \mu < 765.70 \text{grams.}$$

Probability Vs Confidence

- It is correct to say that there is a 95% chance that the confidence interval you calculated contains the true population mean.
- It is not quite correct to say that there is a 95% chance that the population mean lies within the interval.
- The population mean has one value.
- In contrast, the confidence interval you compute depends on the data you happened to collect.

Example

A 90% confidence interval for the mean diameter (in cm) of steel rods manufactured on a certain extrusion machine is computed to be (14.73, 14.91). True or false: The probability that the mean diameter of rods manufactured by this process is between 14.73 and 14.91 is 90%.

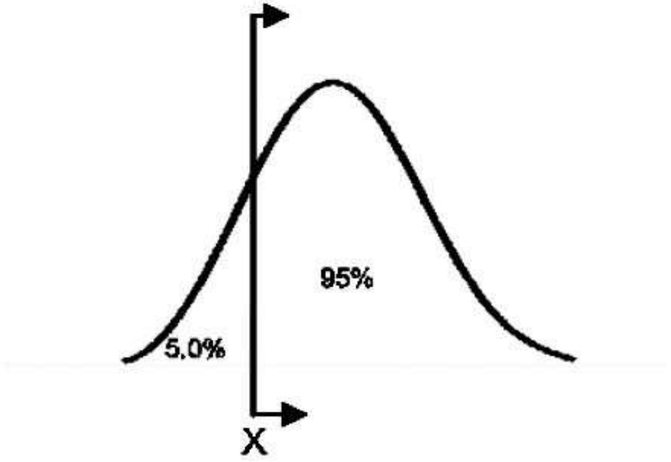
Solution

False. A specific confidence interval is given. The mean is either in the interval or it isn't. We are 90% confident that the population mean is between 14.73 and 14.91. The term *probability* is inappropriate.

One-Sided Confidence Intervals

1)An upper one-sided bound defines a point that a certain percentage of the population is less than.

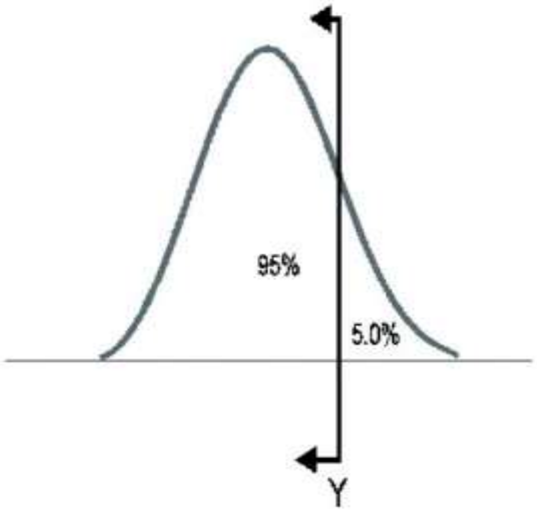
For example, if X is a 95% upper one-sided bound, this would indicate that 95% of the population is less than X.



One-Sided Confidence Intervals

2)A lower one-sided bound defines a point that a specified percentage of the population is greater than.

For example, If X is a 95% lower one-sided bound, this would indicate that 95% of the population is greater than X.



One-Sided Confidence Intervals

Let $X_1, \ldots, X_n$ be a *large* ($n > 30$) random sample from a population with mean μ and standard deviation σ, so that $\overline{X}$ is approximately normal. Then level $100(1-\alpha)\%$ lower confidence bound for μ is

$$\overline{X} - z_\alpha \sigma_{\overline{X}} \qquad (5.2)$$

and level $100(1-\alpha)\%$ upper confidence bound for μ is

$$\overline{X} + z_\alpha \sigma_{\overline{X}} \qquad (5.3)$$

where $\sigma_{\overline{X}} = \sigma/\sqrt{n}$. When the value of σ is unknown, it can be replaced with the sample standard deviation s.

One-Sided Confidence Intervals

In particular,

- $\overline{X} + 1.28\dfrac{s}{\sqrt{n}}$ is a 90% upper confidence bound for μ.
- $\overline{X} + 1.645\dfrac{s}{\sqrt{n}}$ is a 95% upper confidence bound for μ.
- $\overline{X} + 2.33\dfrac{s}{\sqrt{n}}$ is a 99% upper confidence bound for μ.

The corresponding lower bounds are found by replacing the "+" with "−."

Example1

In a sample of 80 ten-penny nails, the average weight was 1.56g and the standard deviation was 0.1g.

a) Find a 90% upper confidence bound for the mean weight.

b) Find a 80% lower confidence bound for the mean weight.

c) Someone says that the mean weight is less than 1.585g. With what level of confidence can this statement be made?

Solution

a) 90% upper confidence bound for the mean weight.

= 1.5743

b) Find a 80% lower confidence bound for the mean weight.

= 1.551

c) Someone says that the mean weight is less than 1.585g. With what level of confidence can this statement be made?

Hence we can make the statement with 98.75% confidence.

Example2

One step in the manufacture of a certain metal clamp involves the drilling of four holes. In a sample of 150 clams, the average time needed to complete this step was 72 seconds and the standard deviation was 10 seconds.

An efficiency expert says that the mean time is greater than 70 seconds. With what level of confidence can this statement be made?

Solution

Given:

mean = 72, sigma = 10, n = 150

Mean > 70

That means the lower confidence bound = 70

mean – lower_bound = 72 - 70 = 2

=> - z * (10/sqrt(150)) = 2

=> z = - 2.449 = - 2.45

=> Area to right of -2.45 = Area to the left of 2.45 = 0.9929

Hence we can make the statement with 99.29% confidence.

THANK YOU

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