Digital Design and Computer Organization

Carry-lookahead and Prefix adders - 3

Reetinder Sidhu

Department of Computer Science and Engineering

Digital Design and Computer Organization

Carry-lookahead and Prefix adders - 3

Reetinder Sidhu

Department of Computer Science and Engineering

Carry-lookahead and Prefix adders - 3

Course Outline

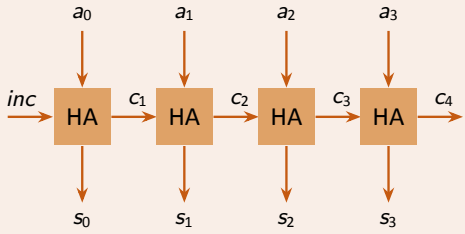
- Digital Design
  - Combinational logic design
  - Sequential logic design
    - ★ **Carry-lookahead and Prefix adders - 3**
- Computer Organization
  - Architecture (microprocessor instruction set)
  - Microarchitecure (microprocessor operation)

Concepts covered

- Parallel Prefix Incrementer

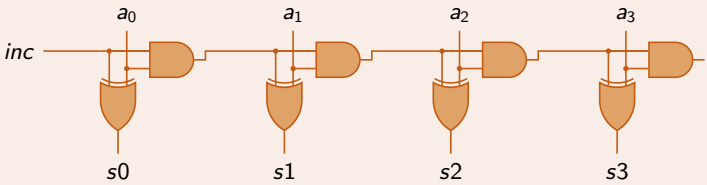
Carry-lookahead and Prefix adders - 3

4-Bit Incrementer



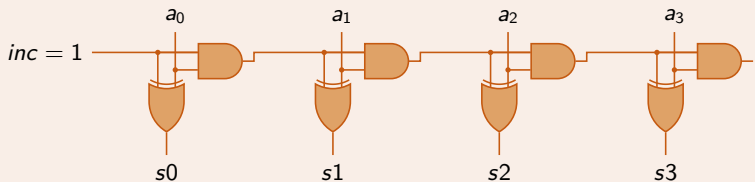
Carry-lookahead and Prefix adders - 3

4-Bit Incrementer



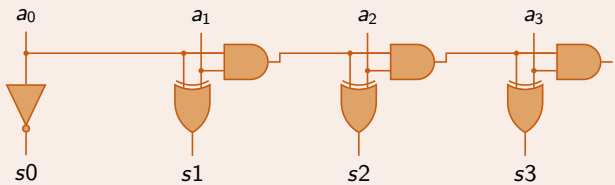
Carry-lookahead and Prefix adders - 3

4-Bit Incrementer (without *inc*)



Carry-lookahead and Prefix adders - 3

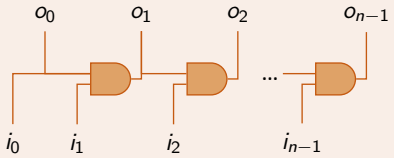
4-Bit Incrementer (without *inc*)



Carry-lookahead and Prefix adders - 3

The Problem

Incrementer Carry Chain (n-bits)

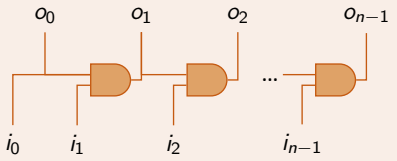


- Assuming t_g and a_g to be respectively the time and area of a two input gate, for the above carry chain:
  - Area: $(n-1)a_g$
  - Time: $(n-1)t_g$

Carry-lookahead and Prefix adders - 3

The Problem

Incrementer Carry Chain (n-bits)



- Assuming t_g and a_g to be respectively the time and area of a two input gate, for the above carry chain:
  - Area: $(n-1)a_g$
  - Time: $(n-1)t_g$

A Prefix Problem

- Given n inputs $i_0 \ldots i_{n-1}$ and n outputs $o_0 \ldots o_{n-1}$, compute:

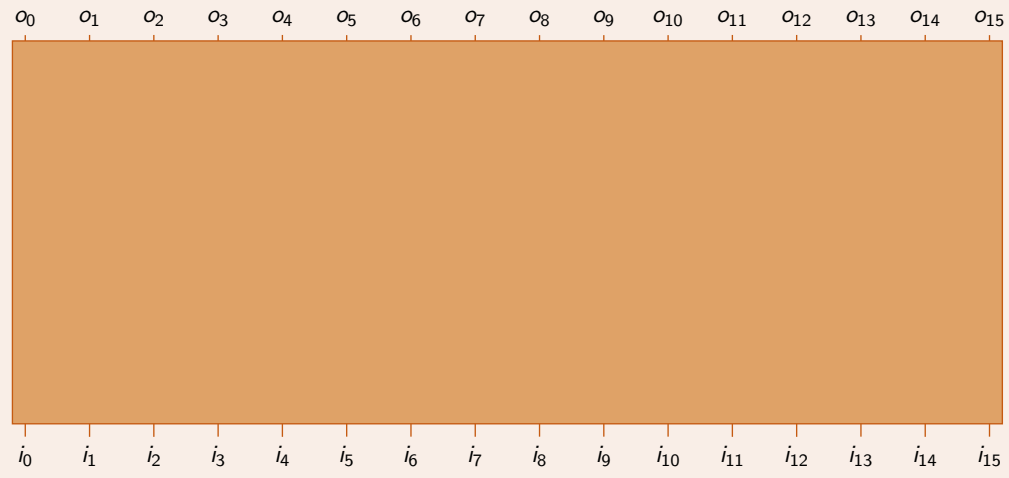
$$
\begin{aligned}
o_0 &= i_0 \\
o_1 &= i_0 \cdot i_1 \\
o_2 &= i_0 \cdot i_1 \cdots i_2 \\
&\vdots \\
o_{n-1} &= i_0 \cdot i_1 \cdot \ldots \cdot i_{n-1}
\end{aligned}
$$

- A problem of the above type where each output in a sequence is computed based on the inputs received so far (which is a prefix of the input sequence) is called a **prefix** problem
  - Ex: matrix multiply prefix problem

Carry-lookahead and Prefix adders - 3

Parallel Prefix Solution

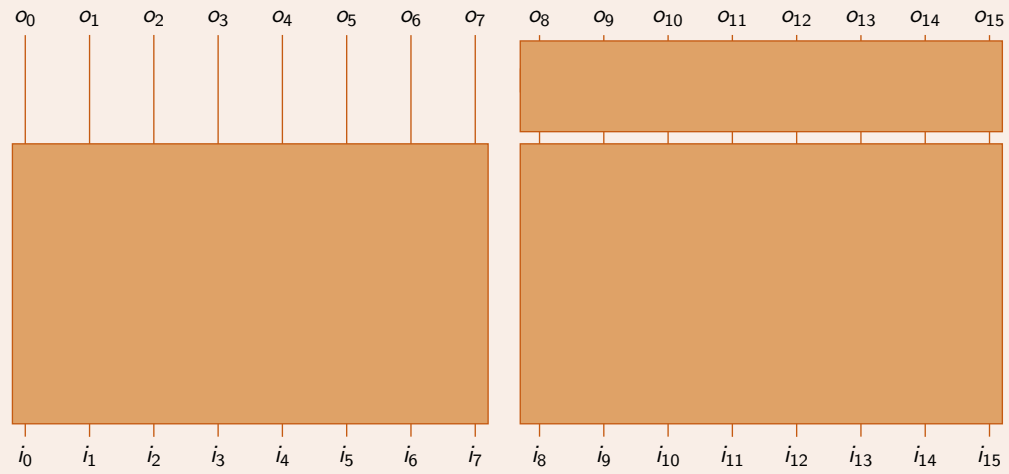
Parallel Prefix Computation of Incrementer Carry Chain



Carry-lookahead and Prefix adders - 3

Parallel Prefix Solution

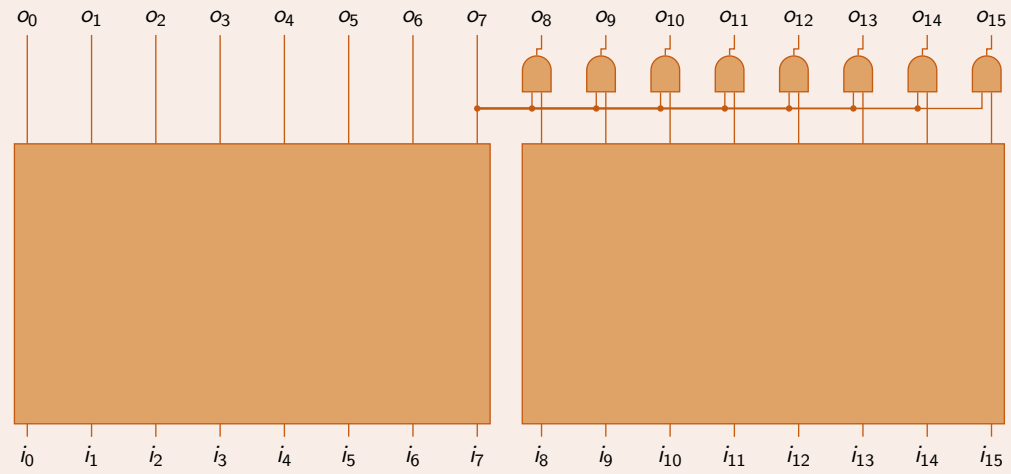
Parallel Prefix Computation of Incrementer Carry Chain



Carry-lookahead and Prefix adders - 3

Parallel Prefix Solution

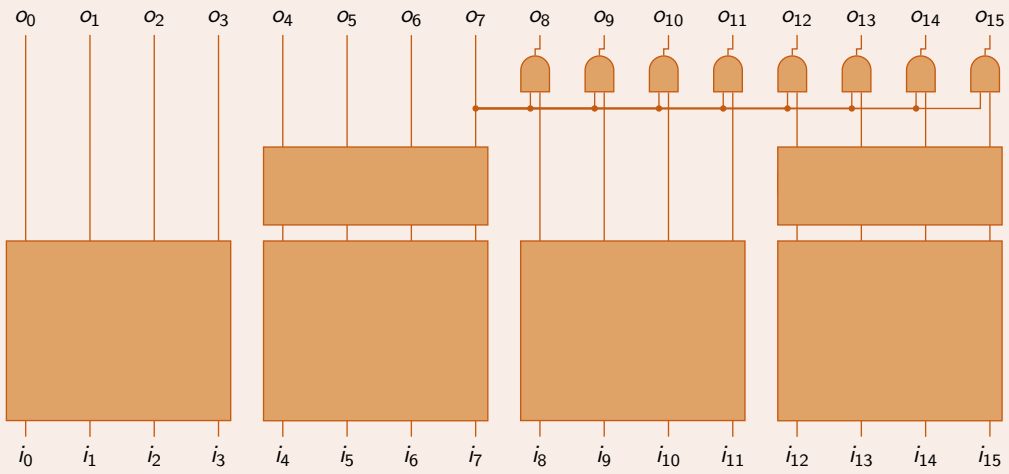
Parallel Prefix Computation of Incrementer Carry Chain



Carry-lookahead and Prefix adders - 3

Parallel Prefix Solution

Parallel Prefix Computation of Incrementer Carry Chain

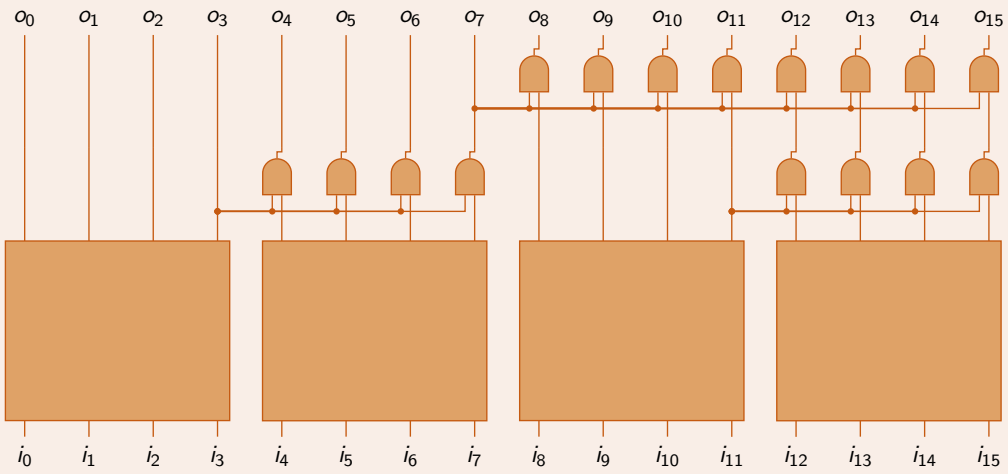


Carry-lookahead and Prefix adders - 3

Parallel Prefix Solution

PES

Parallel Prefix Computation of Incrementer Carry Chain

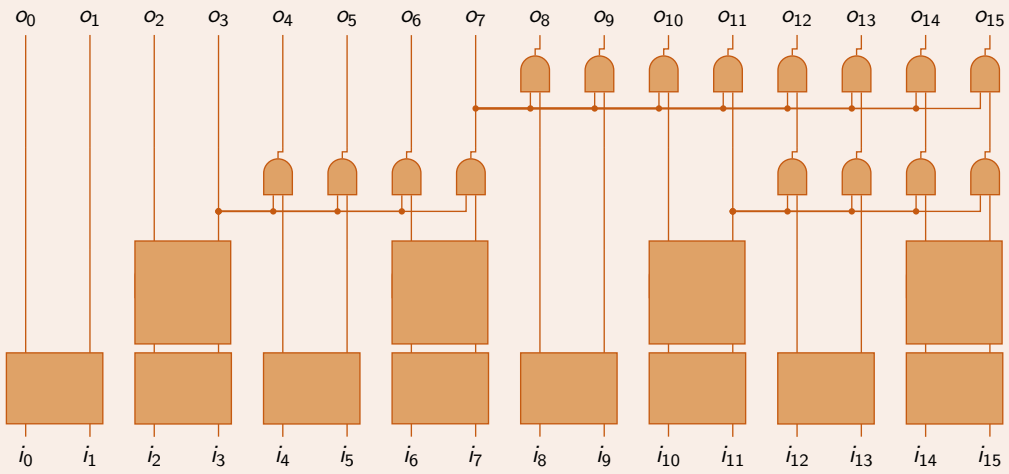


Carry-lookahead and Prefix adders - 3

Parallel Prefix Solution

PES

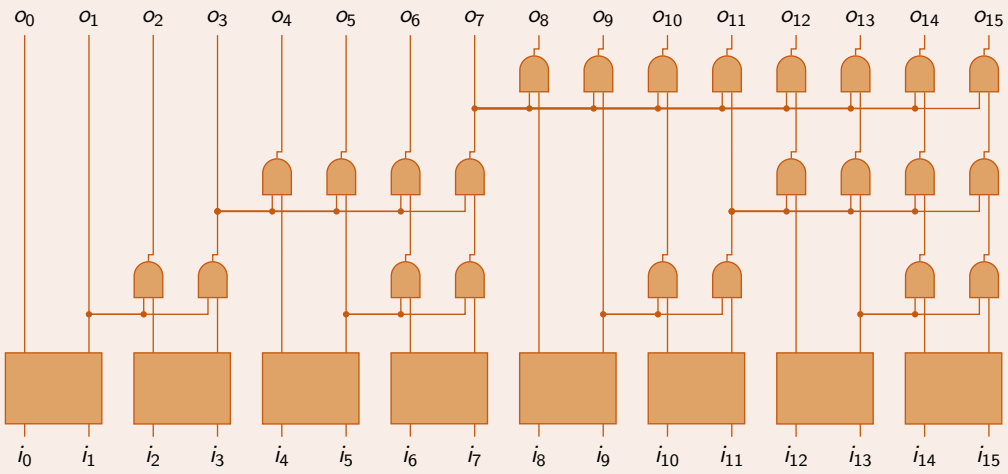
Parallel Prefix Computation of Incrementer Carry Chain



Carry-lookahead and Prefix adders - 3

Parallel Prefix Solution

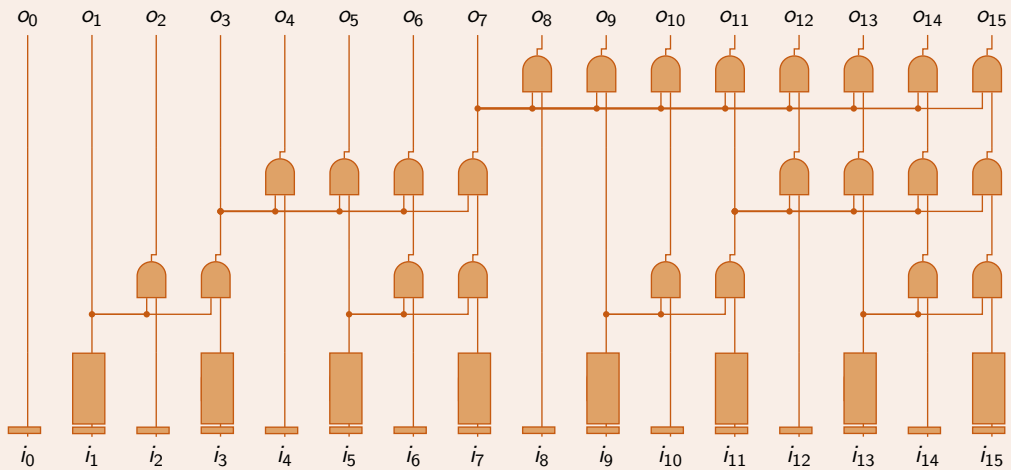
Parallel Prefix Computation of Incrementer Carry Chain



Carry-lookahead and Prefix adders - 3

Parallel Prefix Solution

Parallel Prefix Computation of Incrementer Carry Chain

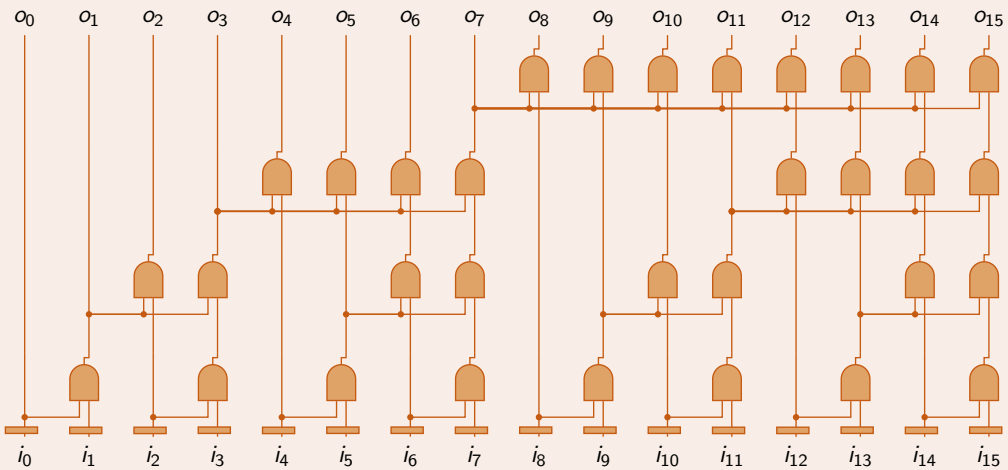


Carry-lookahead and Prefix adders - 3

Parallel Prefix Solution

PES

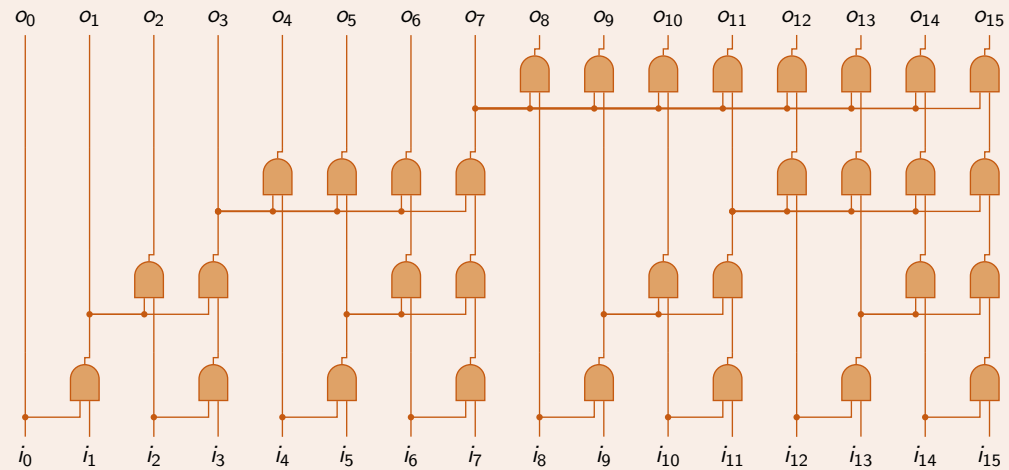
Parallel Prefix Computation of Incrementer Carry Chain



Carry-lookahead and Prefix adders - 3

Parallel Prefix Solution

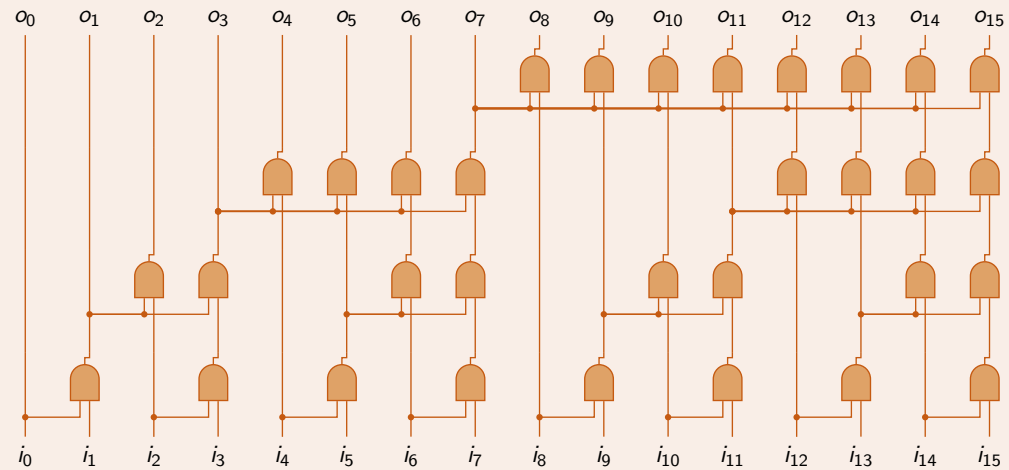
Parallel Prefix Computation of Incrementer Carry Chain



Carry-lookahead and Prefix adders - 3

Parallel Prefix Solution

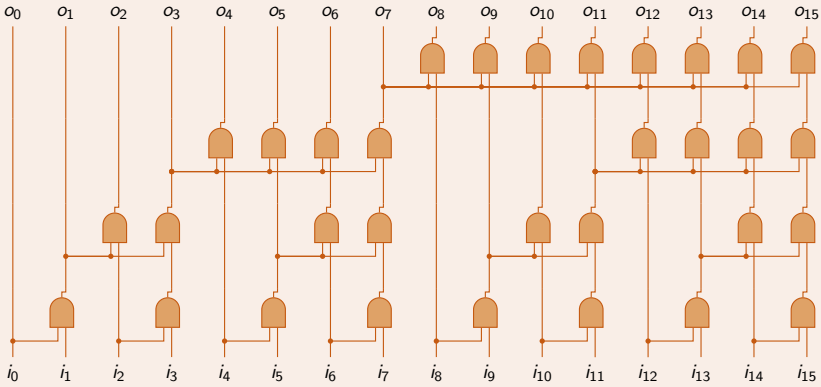
Parallel Prefix Computation of Incrementer Carry Chain



Carry-lookahead and Prefix adders - 3

Parallel Prefix Area and Time

Parallel Prefix Computation of Incrementer Carry Chain

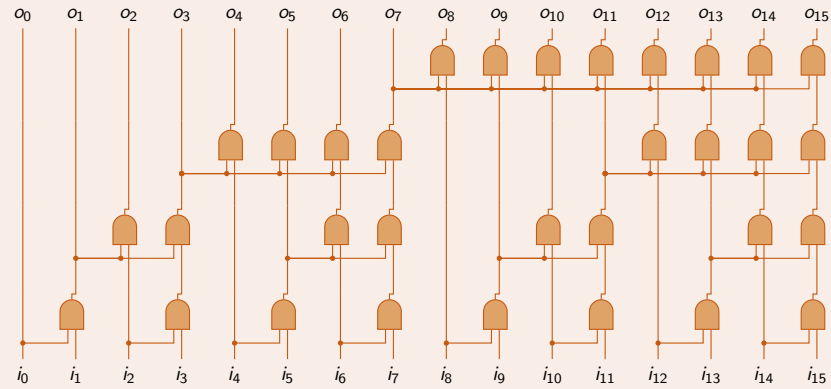


- For 16 input problem:

Carry-lookahead and Prefix adders - 3

Parallel Prefix Area and Time

Parallel Prefix Computation of Incrementer Carry Chain

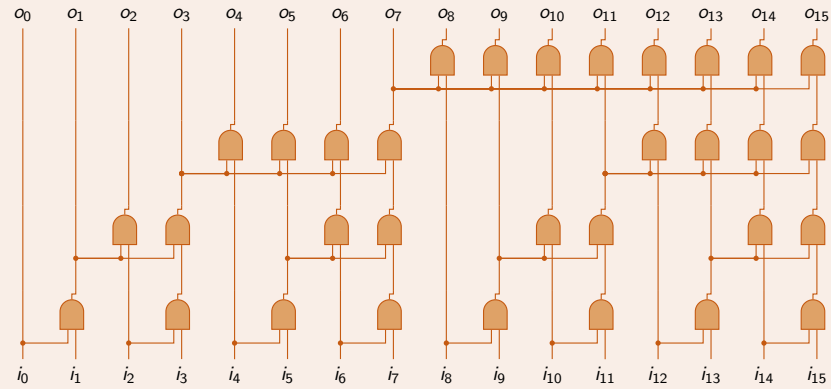


- For 16 input problem:
  - Area: $32a_g$

Carry-lookahead and Prefix adders - 3

Parallel Prefix Area and Time

Parallel Prefix Computation of Incrementer Carry Chain

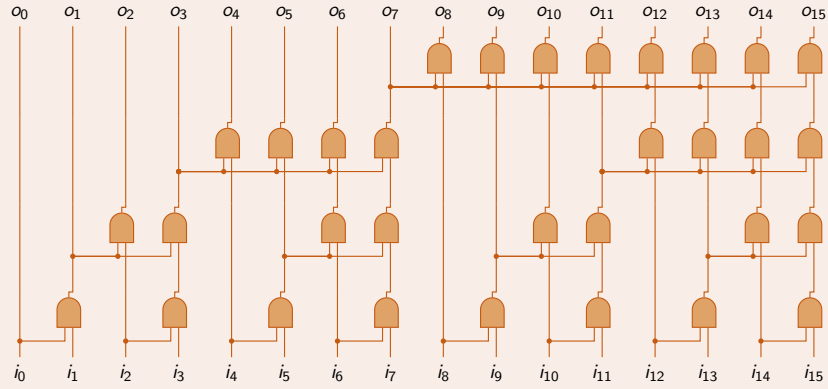


- For 16 input problem:
  - Area: $32a_g$
  - Time: $4t_g$

Carry-lookahead and Prefix adders - 3

Parallel Prefix Area and Time

Parallel Prefix Computation of Incrementer Carry Chain

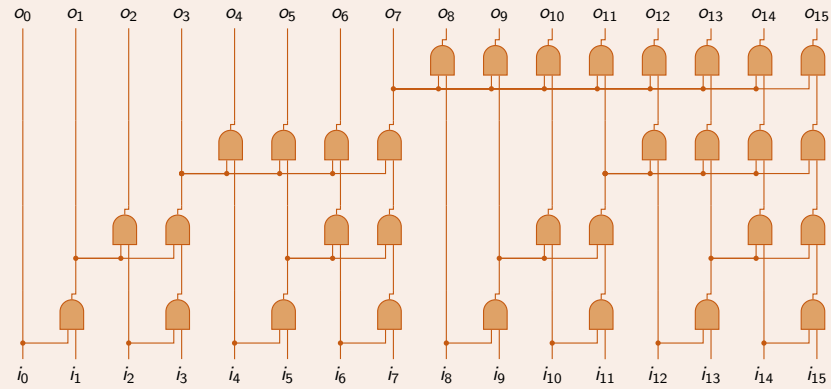


- For 16 input problem:
  - Area: $32a_g$
  - Time: $4t_g$
- For n input problem:

Carry-lookahead and Prefix adders - 3

Parallel Prefix Area and Time

Parallel Prefix Computation of Incrementer Carry Chain

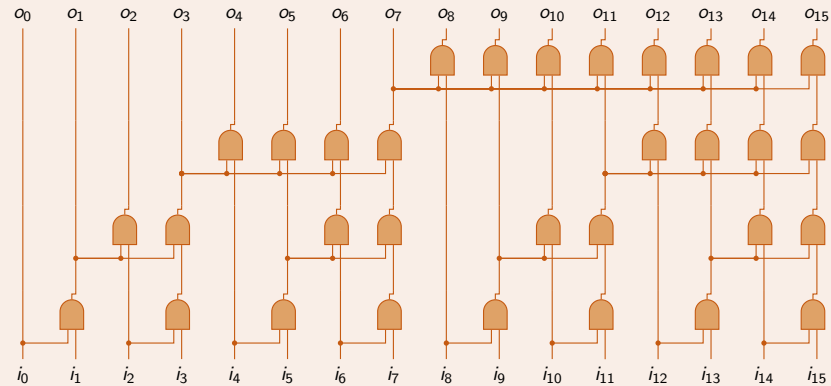


- For 16 input problem:
  - Area: $32a_g$
  - Time: $4t_g$
- For n input problem:
  - Area: $\frac{n}{2}(\log_2 n)a_g$

Carry-lookahead and Prefix adders - 3

Parallel Prefix Area and Time

Parallel Prefix Computation of Incrementer Carry Chain

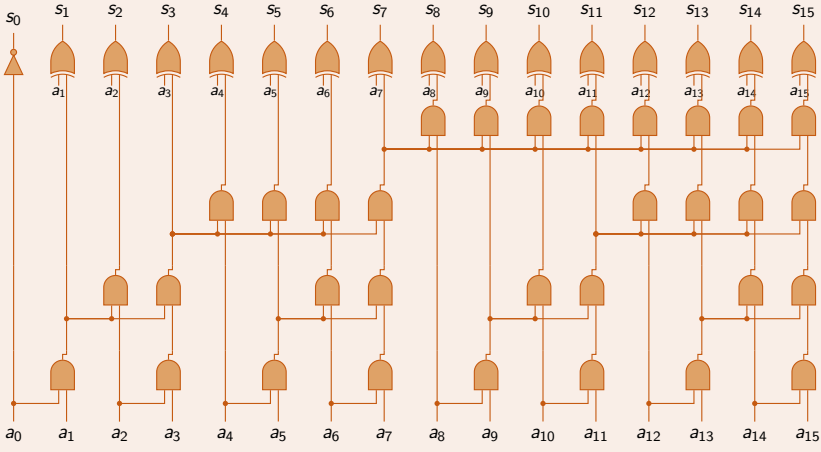


- For 16 input problem:
  - Area: $32a_g$
  - Time: $4t_g$
- For n input problem:
  - Area: $\frac{n}{2}(\log_2 n)a_g$
  - Time: $(\log_2 n)t_g$

Carry-lookahead and Prefix adders - 3

Parallel Prefix Incrementer

Parallel Prefix Incrementer



Carry-lookahead and Prefix adders - 3

Ripple Carry and Parallel Prefix Incrementer Comparison

	Area	Time
Ripple carry	$2(n-1)a_g$	$(n-1)t_g$
Parallel prefix	$\frac{n}{2}(\log_2 n)a_g + (n-1)a_g$	$(\log_2 n + 1)t_g$

Carry-lookahead and Prefix adders - 3

Think About It

- Parallel prefix technique worked for AND gate
- Can it work for any gate (or Boolean function)?