Digital Design and Computer Organization

Boolean Algebra, Identities - 1

Reetinder Sidhu

Department of Computer Science and Engineering

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Course Outline

- Digital Design
  - Combinational logic design
    - **Boolean Algebras, Identities**
  - Sequential logic design
- Computer Organization
  - Architecture (microprocessor instruction set)
  - Microarchitecure (microprocessor operation)

Concepts covered

- Truth table from Logic Circuit
- Boolean Formulas
- Combinational Logic Circuits

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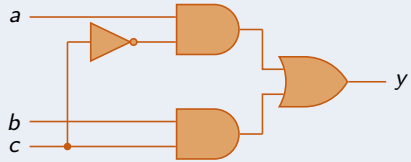
What is a Logic Circuit?

Logic Circuit

Multiple logic gates combined together, with the output of one gate being connected to the input of another, form a *logic circuit*

Example Logic Circuit



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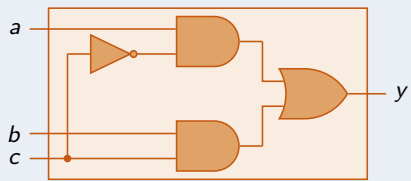

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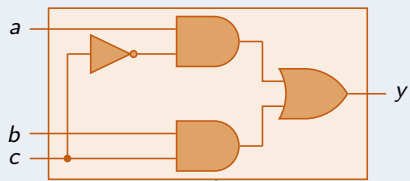

Boolean function

a	b	c	y
0	0	0	
0	0	1	
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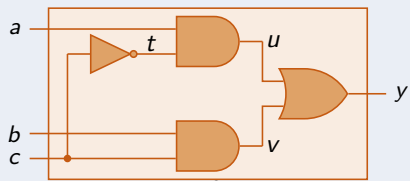

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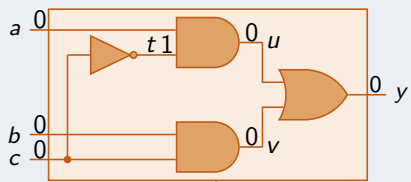

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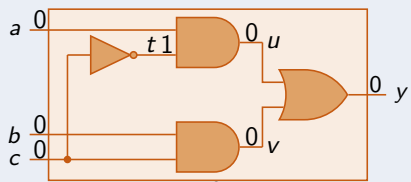

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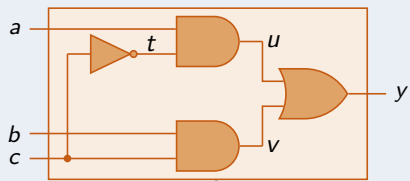

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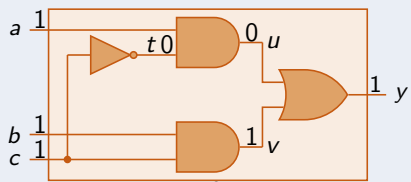

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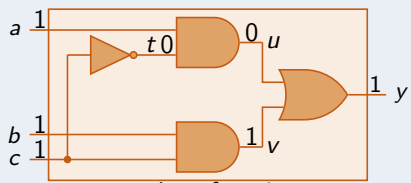

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Boolean Formula Syntax

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Boolean Formula Syntax

Syntax Rules for Boolean Formulas

1. A Boolean constant (0 or 1) is a Boolean Formula
2. A Boolean variable (say x) is a Boolean formula
3. If P and Q are Boolean formulas then so are:
   a. $(P \cdot Q)$
   b. $(P + Q)$
   c. $\overline{P}$

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(i) From rule 2, Boolean variable a is a Boolean formula

(ii) From rule 2, Boolean variable b is a Boolean formula

(iii) From rule 2, Boolean variable c is a Boolean formula

(iv) From rule 3c and step (iii) above, $\overline{c}$ is a Boolean formula

(v) From rule 3a, and steps (i) and (iv) above, $(a \cdot \overline{c})$ is a Boolean formula

(vi) From rule 3a, and steps (ii) and (iii) above, $(b \cdot c)$ is a Boolean formula

(vii) From rule 3b, and steps (v) and (vi) above, $((a \cdot \overline{c}) + (b \cdot c))$ is a Boolean formula

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Meaning of Boolean Formulas

Each Boolean formula “means” a Boolean function as well as a logic circuit

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Meaning of Boolean Formulas

1. The Boolean constants and variables form the inputs of the Boolean functions (or logic circuit)
2. $\cdot$ means AND Boolean function (or logic gate)
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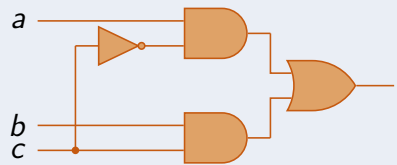
Example

Consider the Boolean formula $((a \cdot \overline{c}) + (b \cdot c))$. Using the rules on the left, it is converted to:



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Simplifying Notation in Boolean Formulas

- Boolean formulas can be difficult to read because of many brackets
- Number of brackets can be reduced as follows:
  - $\cdot$ has higher precedence than $+$ (like in arithmetic, $\times$ has higher precedence than $+$)
    - So $((a \cdot \overline{c}) + (b \cdot c))$ can be written as $(a \cdot \overline{c} + b \cdot c)$
  - Outermost brackets can be dropped
    - So $(a \cdot \overline{c} + b \cdot c)$ can be written as $a \cdot \overline{c} + b \cdot c$
  - $\cdot$ can be dropped (like in arithmetic, $a \times b$ can be written as ab)
    - So $a \cdot \overline{c} + b \cdot c$ can be written as $a\overline{c} + bc$

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But only those logic circuits that can be represented by Boolean formulas are called **combinational logic circuits** (or just combinational logic)

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Combinational Logic Circuits

But only those logic circuits that can be represented by Boolean formulas are called **combinational logic circuits** (or just combinational logic)

- In other words, combinational logic circuits are those that can be specified as Boolean functions
- Logic circuits that cannot be represented by a Boolean formula or function:
  - Logic circuits where the output of a gate is fed back as an input
  - Logic circuits where the outputs of two gates are connected together

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Think About It

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- Consider the following two Boolean formulas:
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  - $((a + b) \cdot (\bar{a} + \bar{b}))$

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- Are they really Boolean formulas? How will you check
- What do their simplified notations look like
- Write the truth table for each formula
  - Is there any relation between the two truth tables
  - Also compare with truth tables for basic gates

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- Draw the combinational logic circuit for each formula