DESIGN AND ANALYSIS OF ALGORITHMS

Surabhi Narayan
Department of Computer Science & Engineering

DESIGN AND ANALYSIS OF ALGORITHMS

DECREASE AND CONQUER

Surabhi Narayan

Department of Computer Science & Engineering

PES UNIVERSITY ONLINE

Fake-Coin Problem

Among n identical-looking coins, one is fake. With a balance scale, we can compare any two sets of coins.

The problem is to design an efficient algorithm for detecting the fake coin.

The most natural idea for solving this problem is to divide n coins into two piles of $n/2$ coins each, leaving one extra coin aside if n is odd, and put the two piles on the scale.

If the piles weigh the same, the coin put aside must be fake; otherwise, we can proceed in the same manner with the lighter pile, which must be the one with the fake coin.

$W(n) = W(n/2) + 1$ for $n > 1$, $W(1) = 0$.

$W(n) = \log_2 n$.

It would be more efficient to divide the coins not into two but into *three* piles of about $n/3$ coins each.

Russian Peasant Multiplication

Let n and m be positive integers whose product we want to compute, and let us measure the instance size by the value of n.
if n is even, an instance of half the size has to deal with $n/2$, and we have an obvious formula relating the solution to the problem's larger instance to the solution to the smaller one:
$n \,.\, m = (n/2\ ) * 2m$

If n is odd, we need only a slight adjustment of this formula:
$n \,.\, m = ((n - 1)/2\ ).\ 2m + m.$
Using these formulas and the trivial case of $1 \,.\, m = m$ to stop, we can compute product $n \,.\, m$ either recursively or iteratively.

n	m	
50	65	
25	130	
12	260	(+130)
6	520	
3	1040	
1	2080	(+1040)
	2080	+(130 + 1040) = 3250

(a)

n	m	
50	65	
25	130	130
12	260	
6	520	
3	1040	1040
1	2080	2080
		3250

(b)

FIGURE 4.11 Computing $50 \cdot 65$ by the Russian peasant method.

Josephus problem

Named for Flavius Josephus, a famous Jewish historian who participated in and chronicled the Jewish revolt of 66–70 c.e. against the Romans. Josephus, as a general, managed to hold the fortress of Jotapata for 47 days, but after the fall of the city he took refuge with 40 diehards in a nearby cave. There, the rebels voted to perish rather than surrender. Josephus proposed that each man in turn should dispatch his neighbor, the order to be determined by casting lots. Josephus contrived to draw the last lot, and, as one of the two surviving men in the cave, he prevailed upon his intended victim to surrender to the Romans.

Josephus Problem

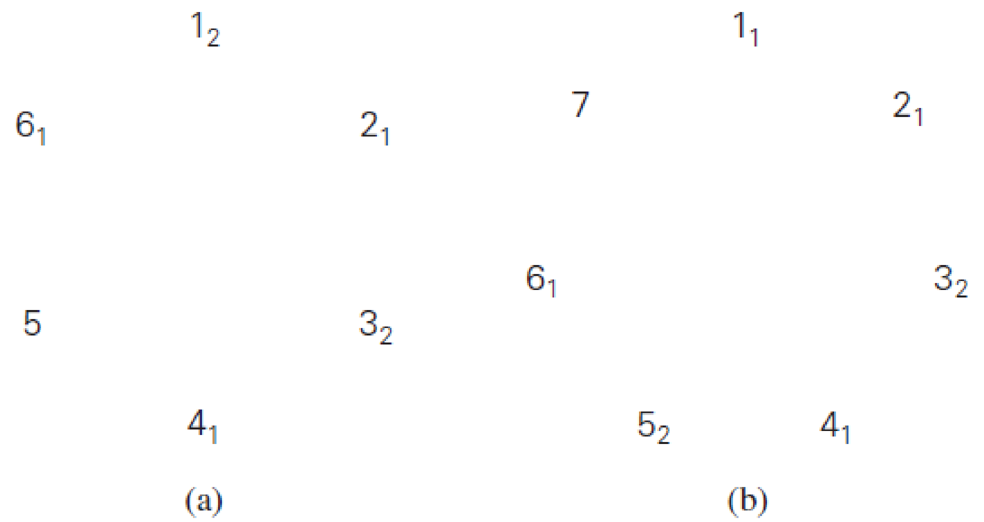

FIGURE 4.12 Instances of the Josephus problem for (a) $n = 6$ and (b) $n = 7$. Subscript numbers indicate the pass on which the person in that position is eliminated. The solutions are $J(6) = 5$ and $J(7) = 7$, respectively.

THANK YOU

Surabhi Narayan
Department of Computer Science &Engineering

surabhinarayan@pes.edu