Automata Formal Languages & Logic

Preet Kanwal

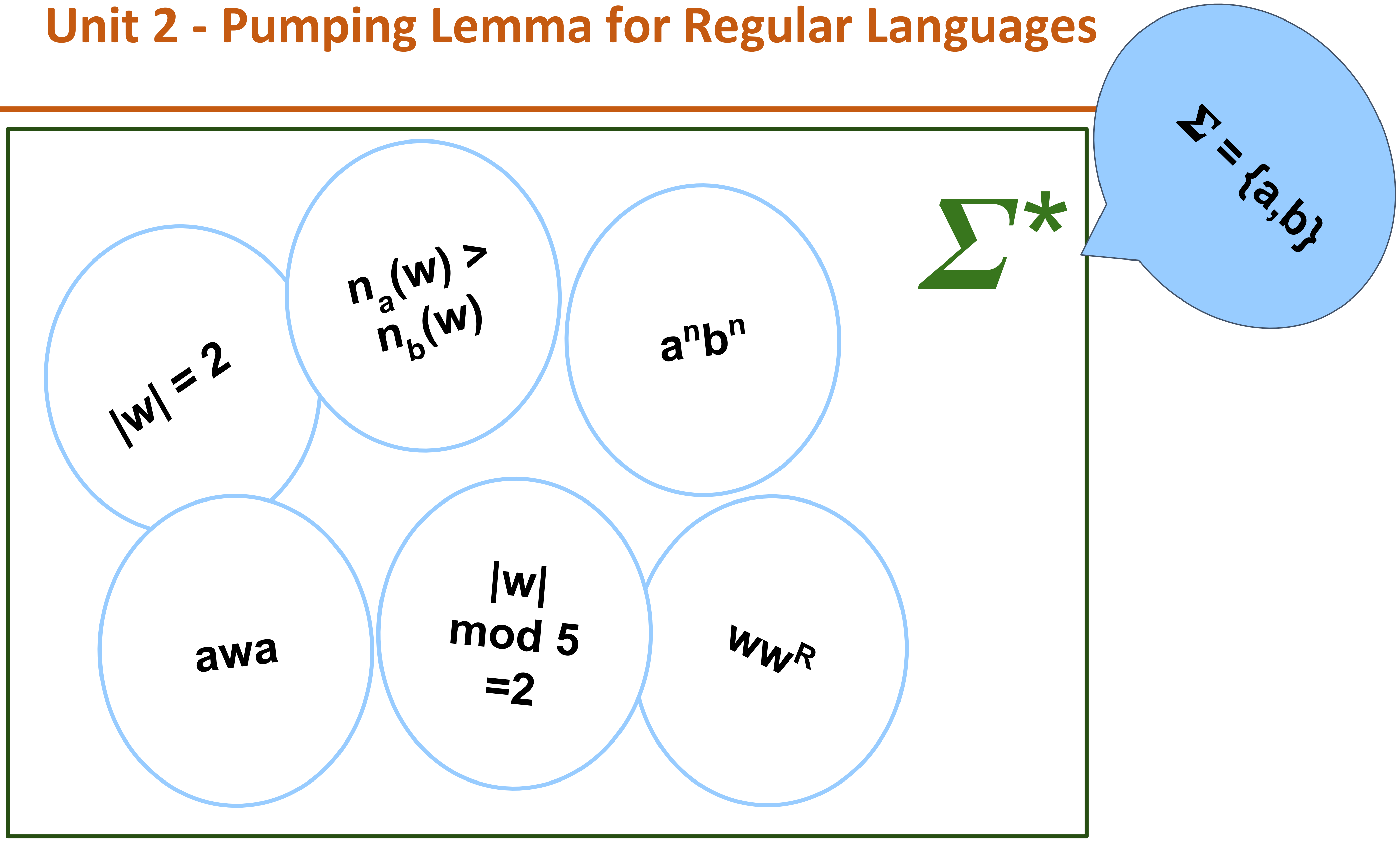
Department of Computer Science & Engineering

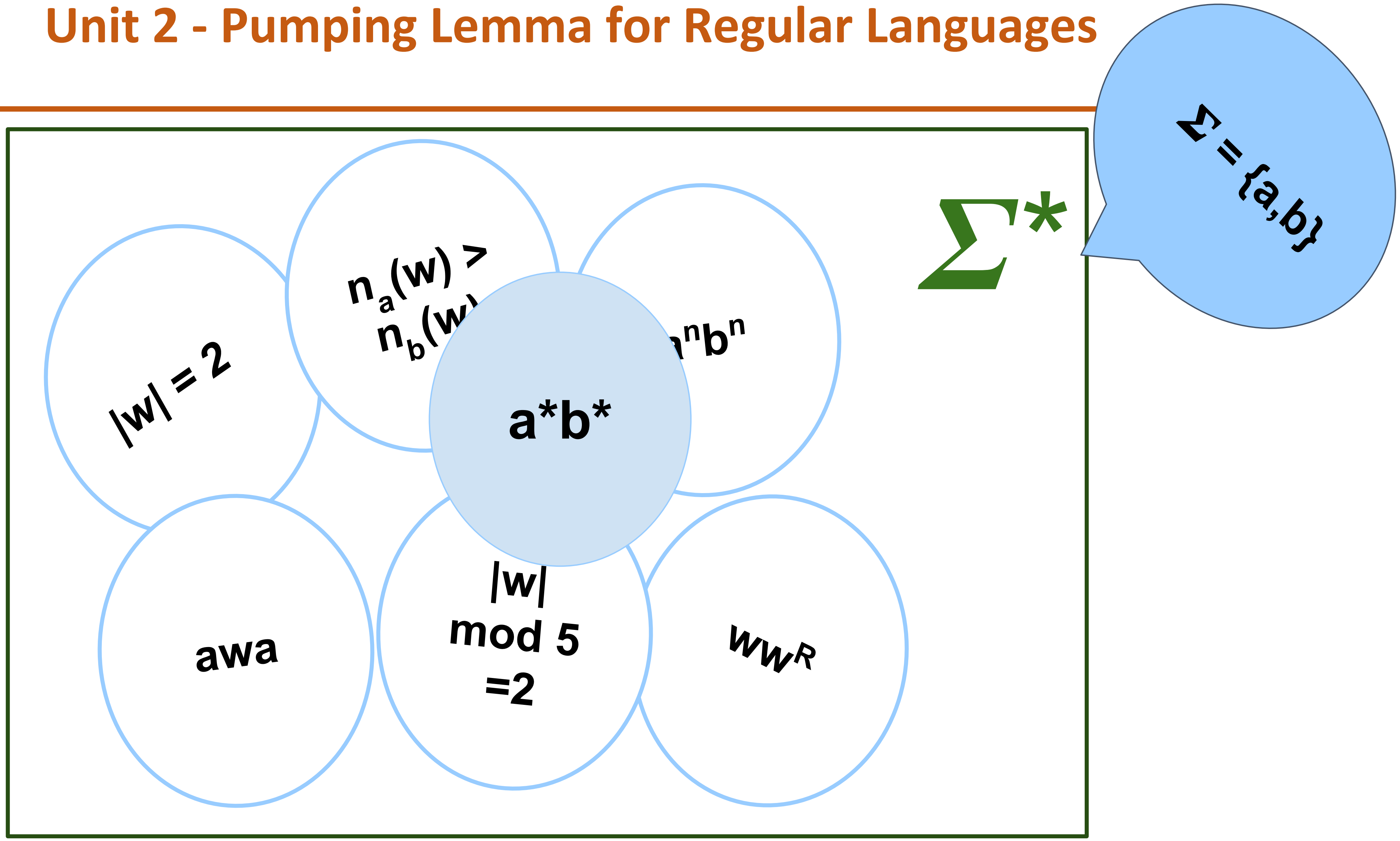
Automata Formal Languages & Logic

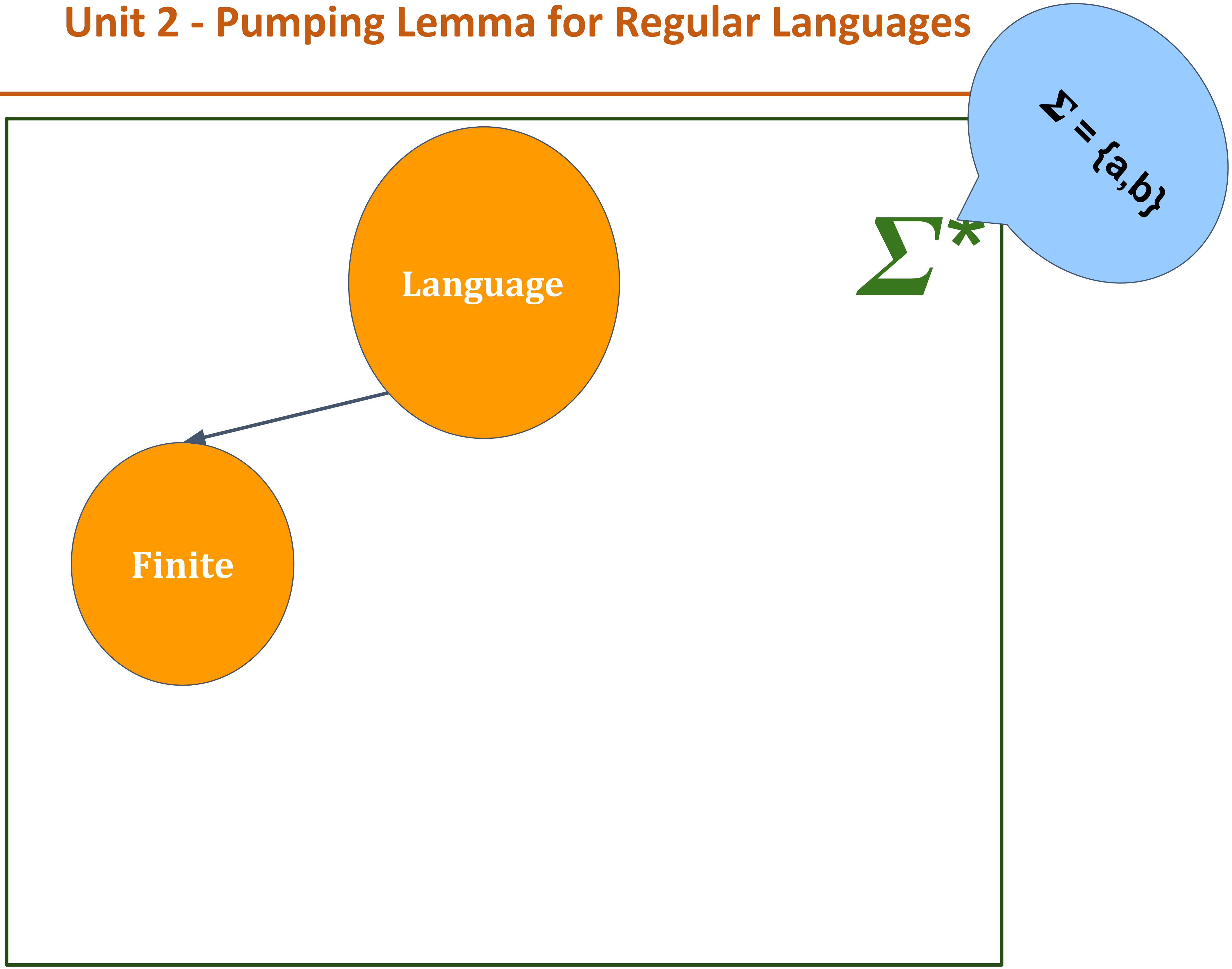
Unit 2

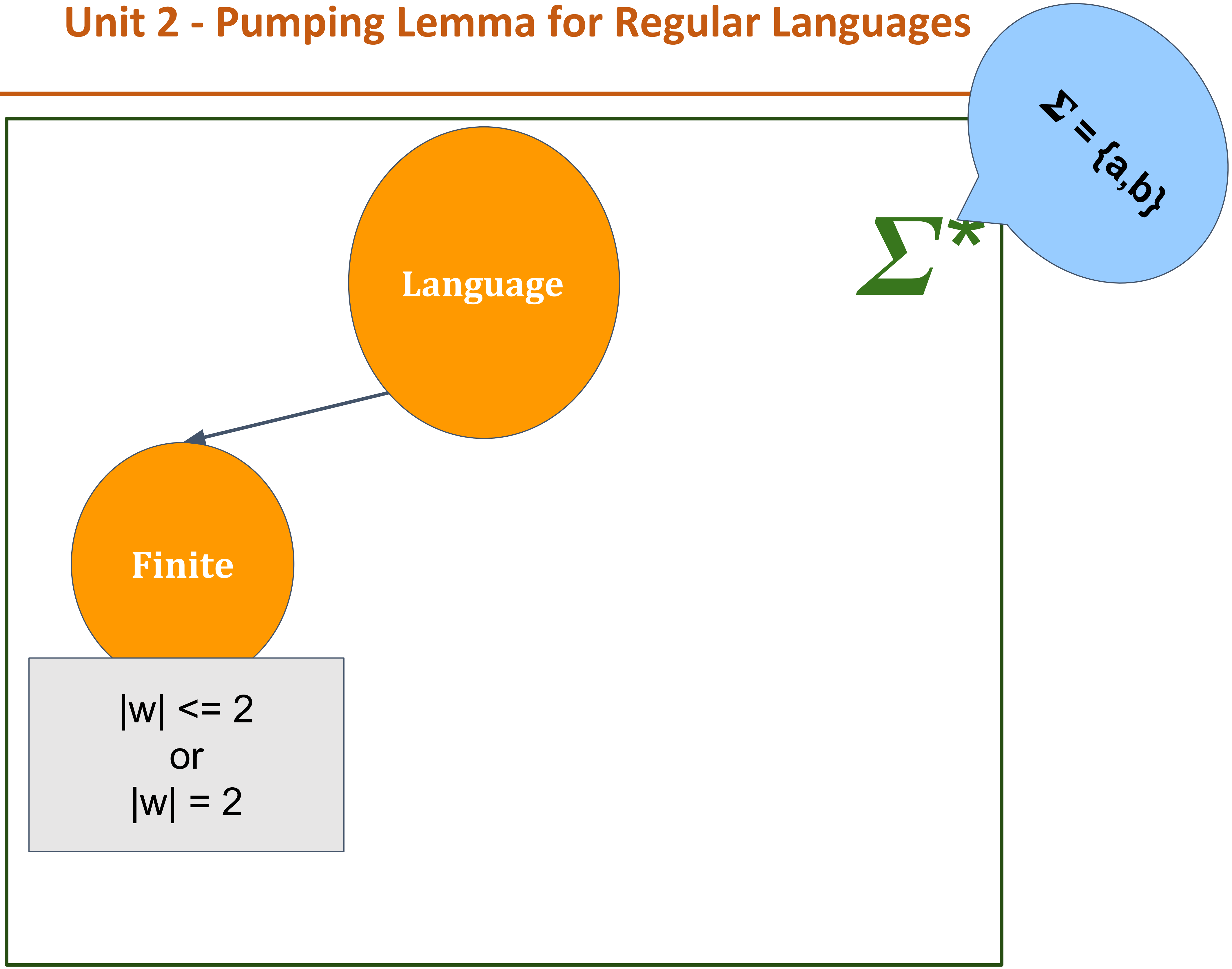
Preet Kanwal

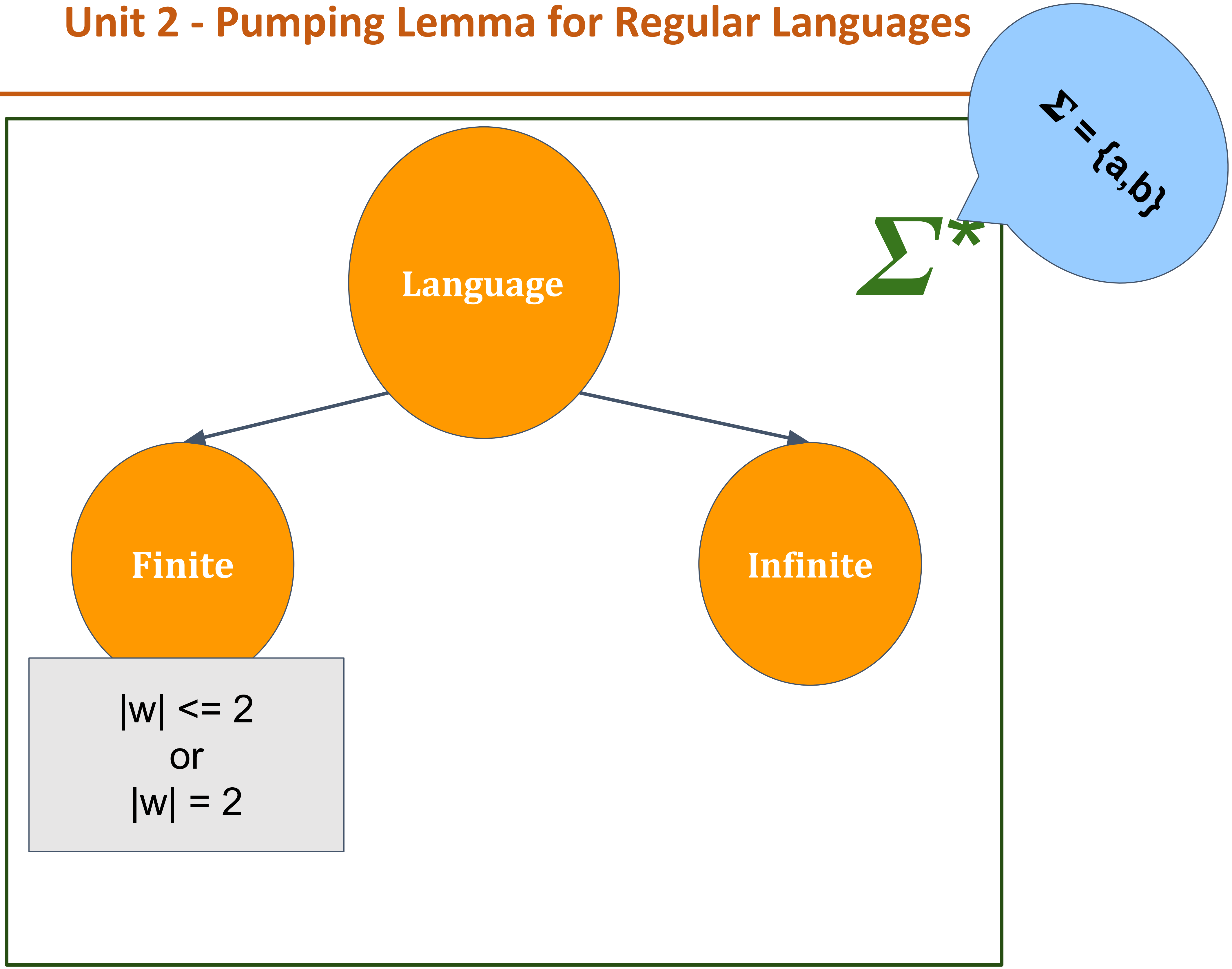
Department of Computer Science & Engineering


Σ^*
$\Sigma = \{a,b\}$
$|w| = 2$
$n_a(w) > n_b(w)$
a^nb^n
awa
$|w| \bmod 5 = 2$
ww^R


Σ^*
$\Sigma = \{a,b\}$
$|w| = 2$
$n_a(w) > n_b(w)$
a^nb^n
a*b*
awa
|w| mod 5 =2
ww^R

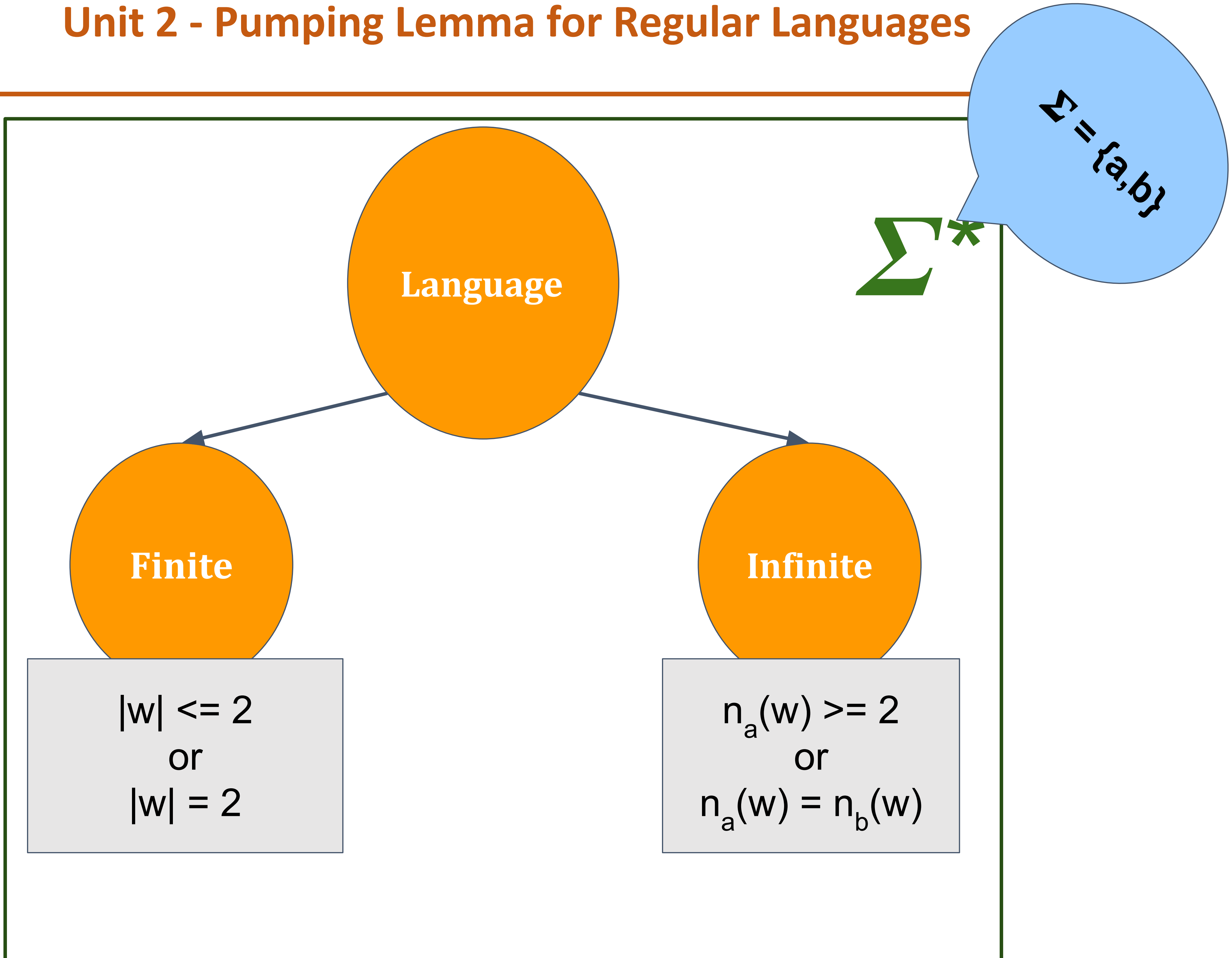

Σ = {a,b}
Σ*
Language
Finite


Σ = {a,b}
Σ*
Language
Finite
|w| <= 2
or
|w| = 2


Σ = {a,b}
Σ*
Language
Finite
Infinite
|w| <= 2
or
|w| = 2

Automata Formal Languages and Logic

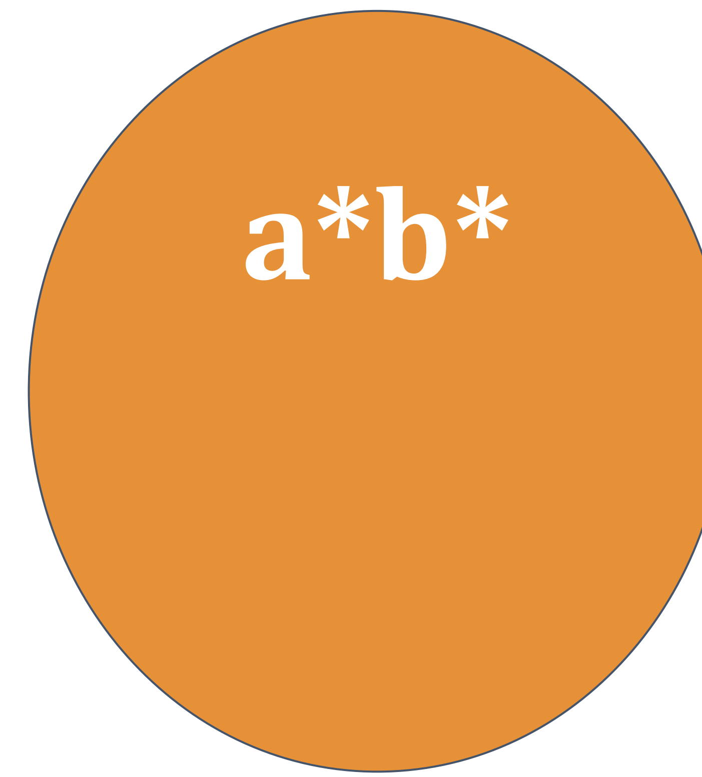
Unit 2 - Pumping Lemma for Regular Languages



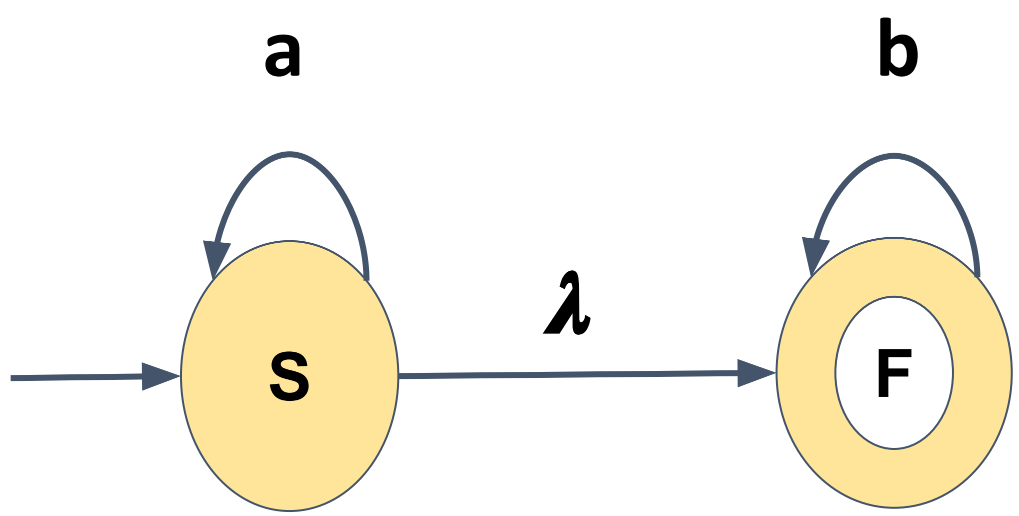
Is there any infinite language for which we cannot construct a Finite Automata?

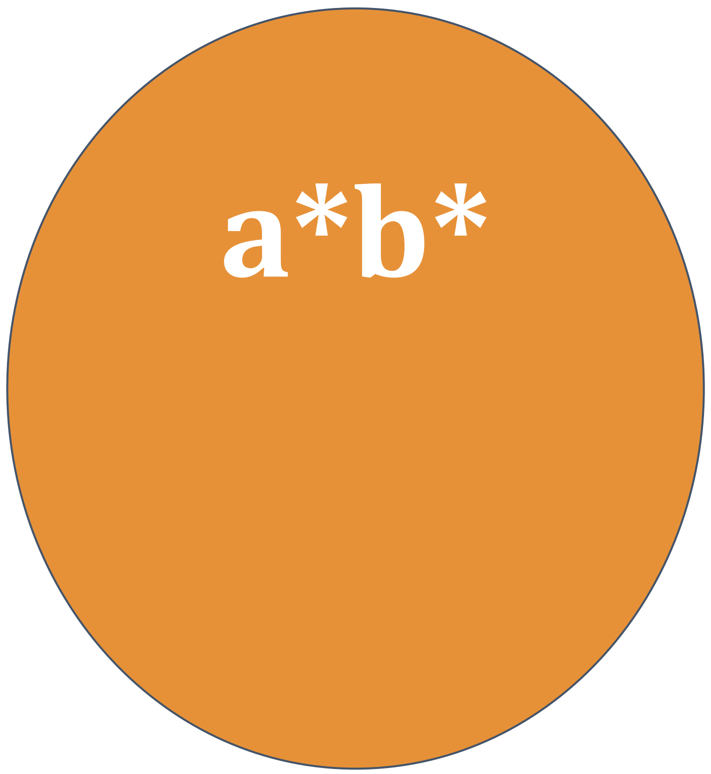
That means, a language which is not regular?

Let's look at an example :

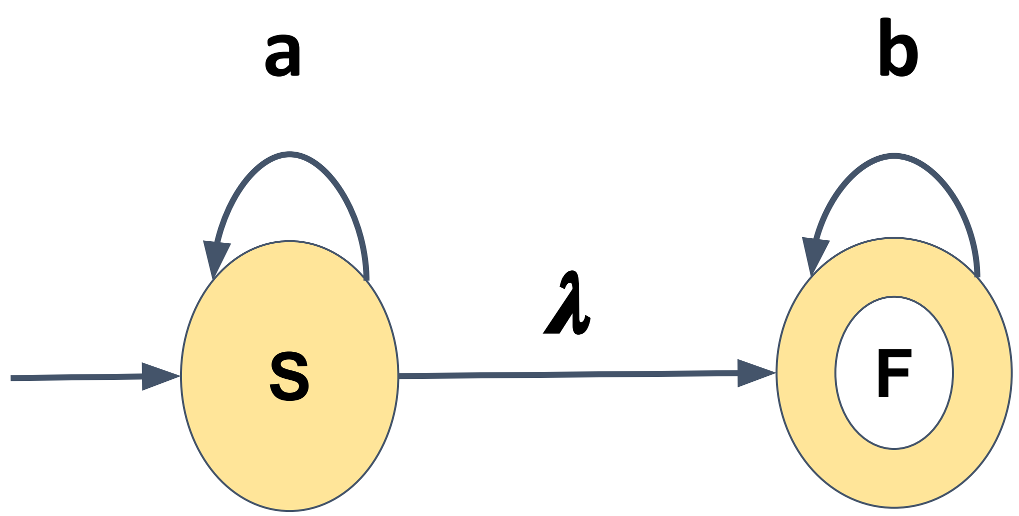


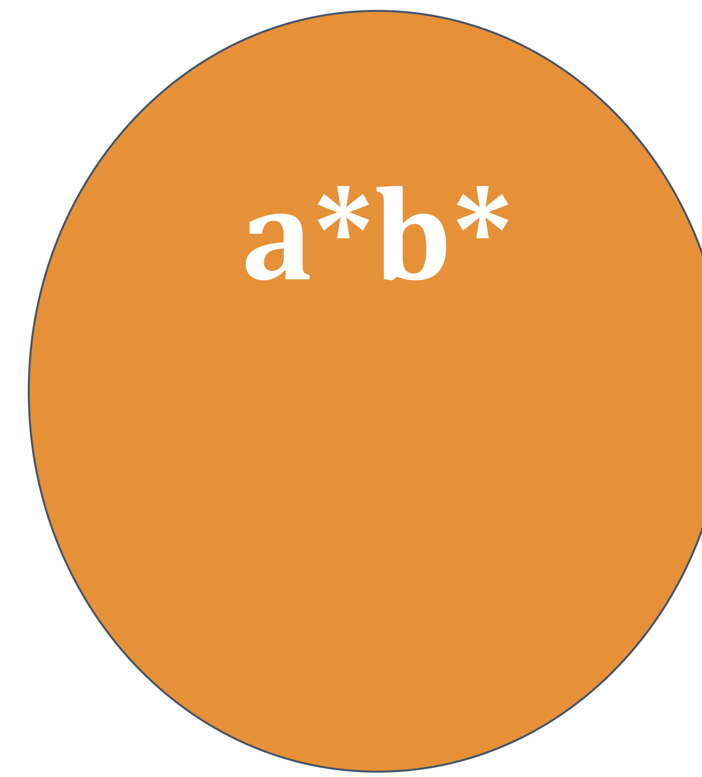
Let's look at an example :





Let's look at an example :

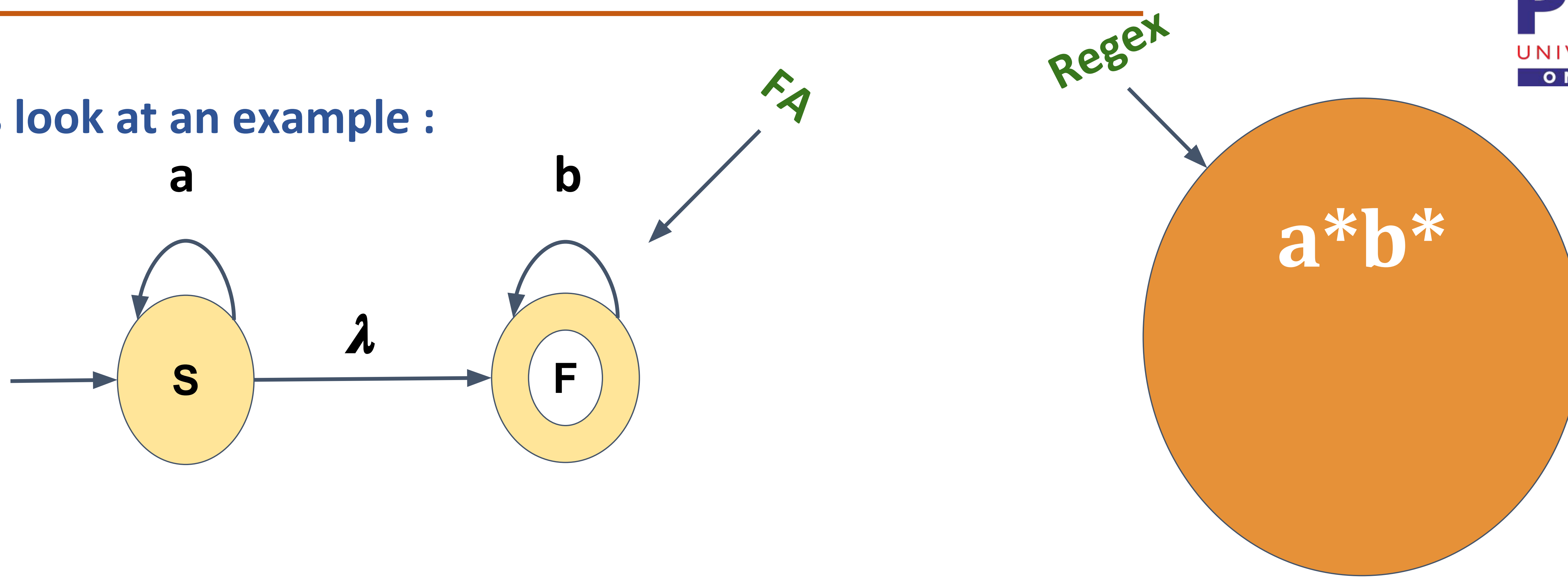




S -> aS | F

F -> bF | λ

Let's look at an example :



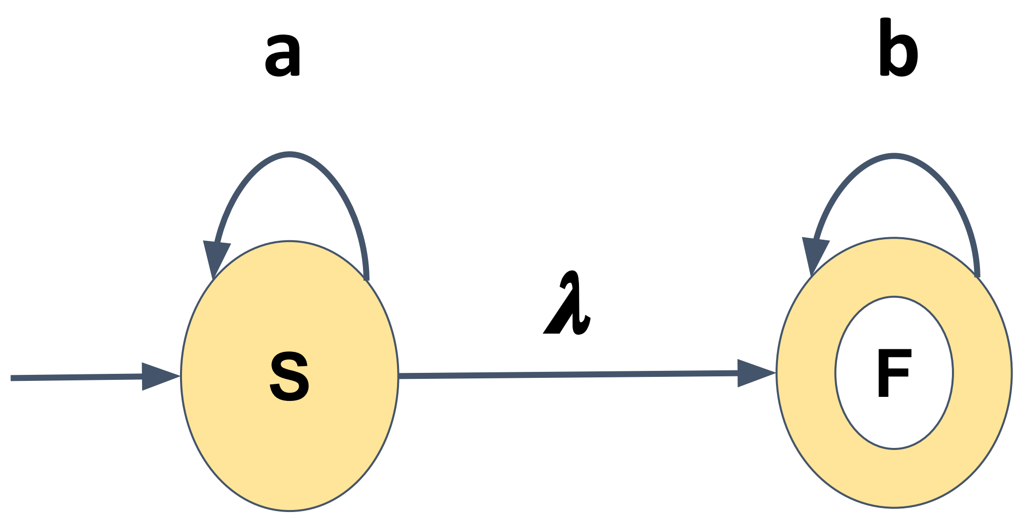
S -> aS | F

F -> bF | λ

Regular Grammar

Let's look at an example :

FA



Regex

a*b*

Definitely Regular !

S -> aS | F

F -> bF | λ

Regular Grammar

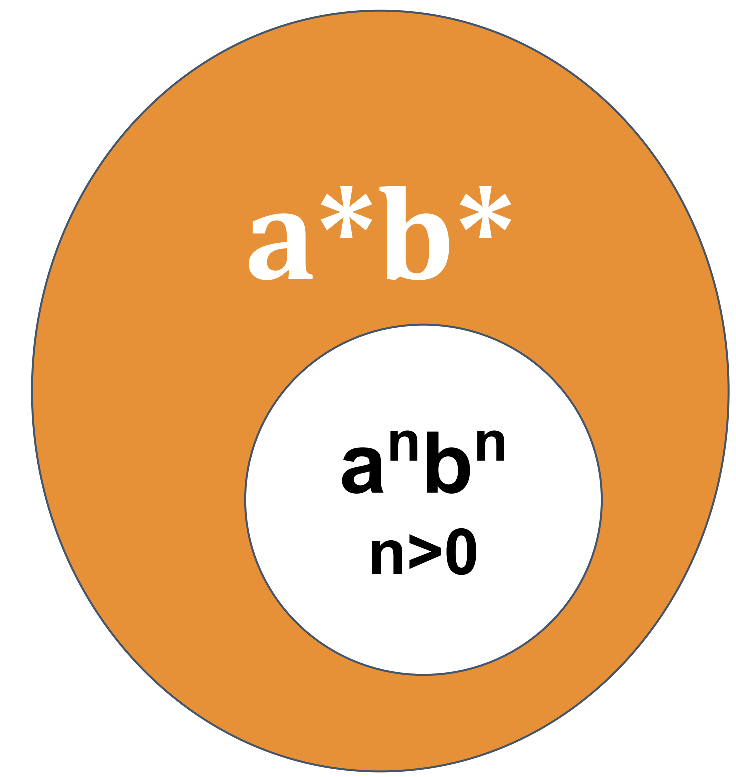
For the language a^nb^n Can we construct either of :

Finite Acceptor or

Regular Grammar or

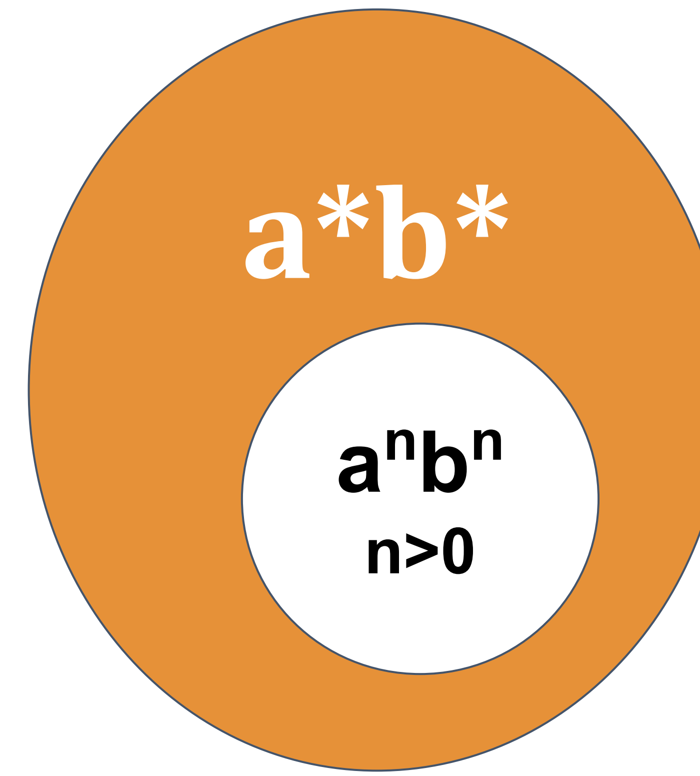
Regular Expression

????????????



aaaaaaaaaaaaaaaaaaaaaaaa bbbbbbbbbbbbbbbbbbbbbb

state q

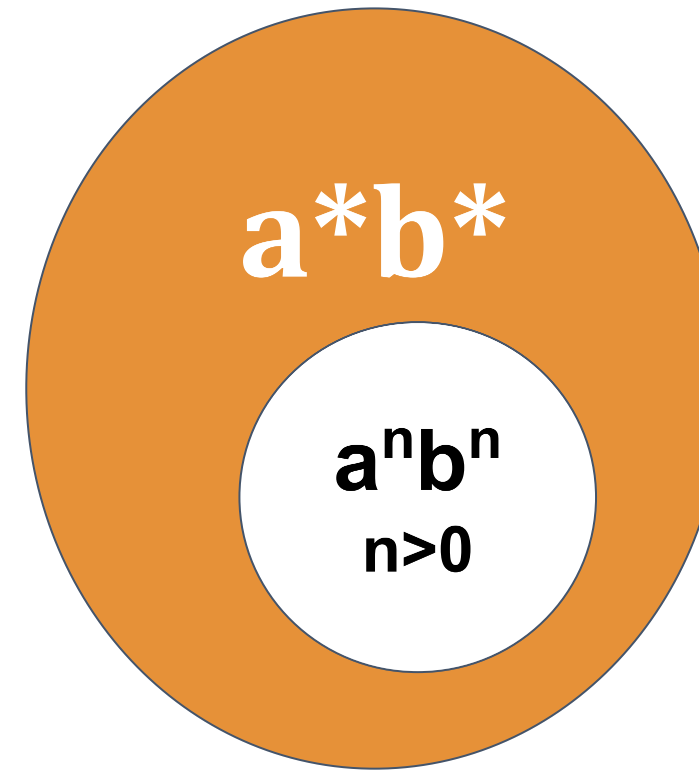


Basically,

There is no way to remember how many a's you have seen to compare with the upcoming b's !

The value of n could be anything!

We cannot come up with a FA that takes care of all n!



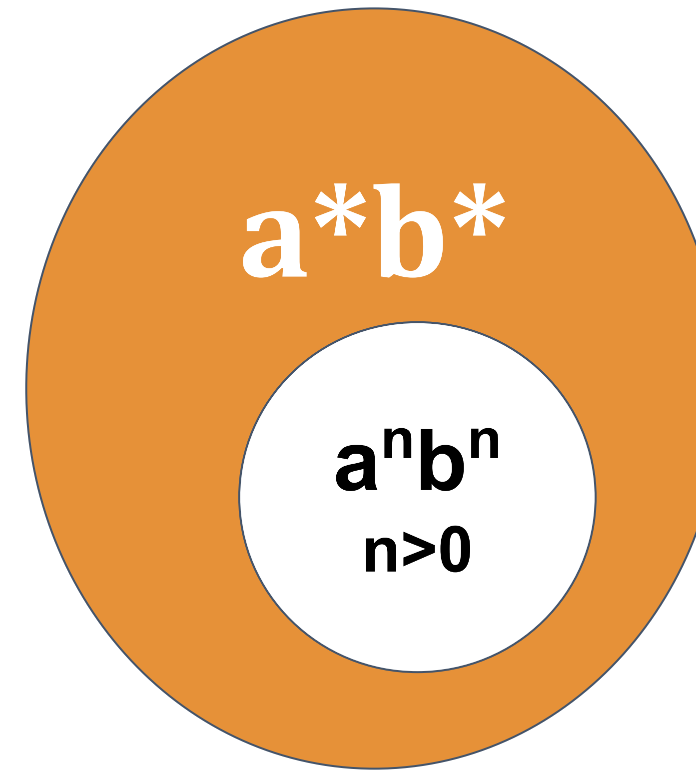
Basically,

There is no way to remember ho[…] you have seen to compare with th[…]

Definitely Non-Regular !

The value o[f] n could be anything!

We cannot come up with a FA that takes care of all n!



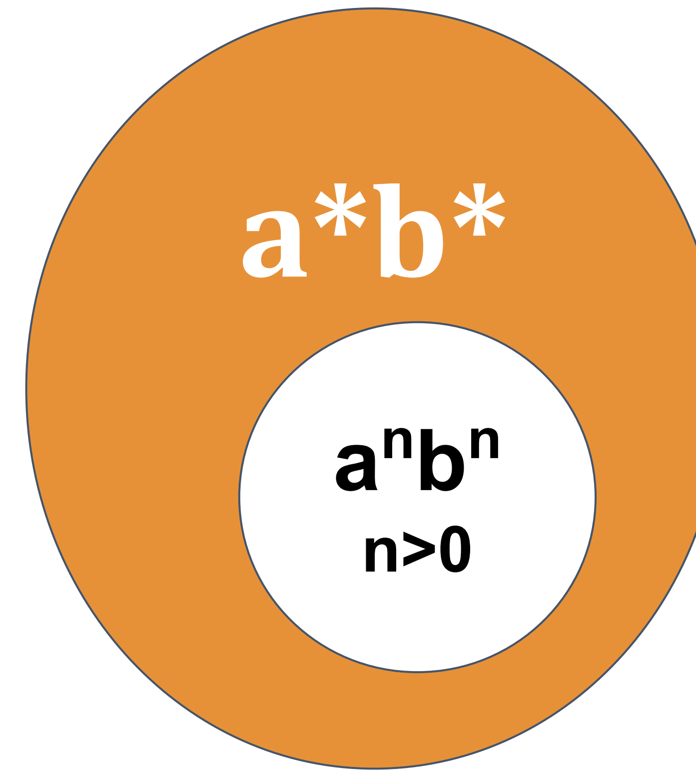
Basically,

There is no way to re… you have seen to compa…

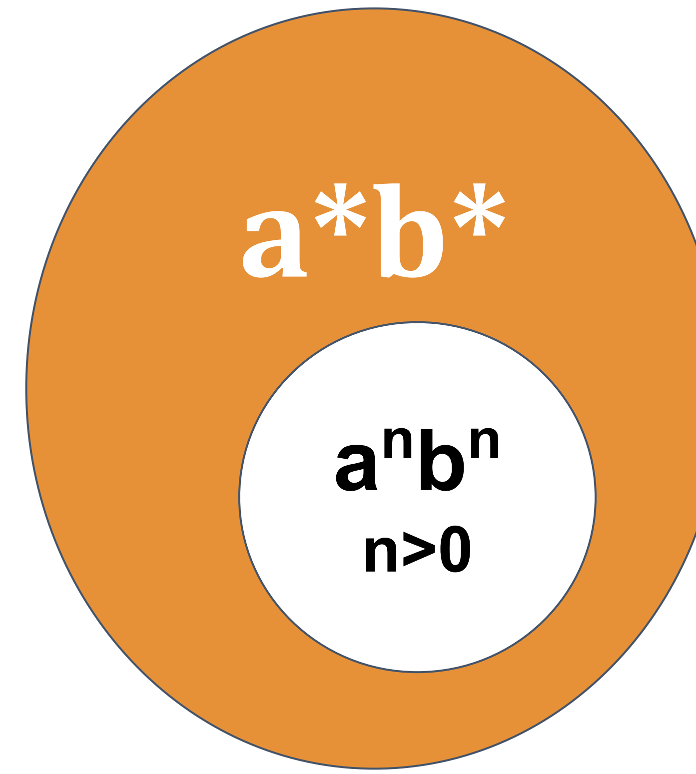
The value … n cou…

We cannot come up w…h a FA that takes care of all n!

Finite Automata has limits !



Limits of Finite Automata :

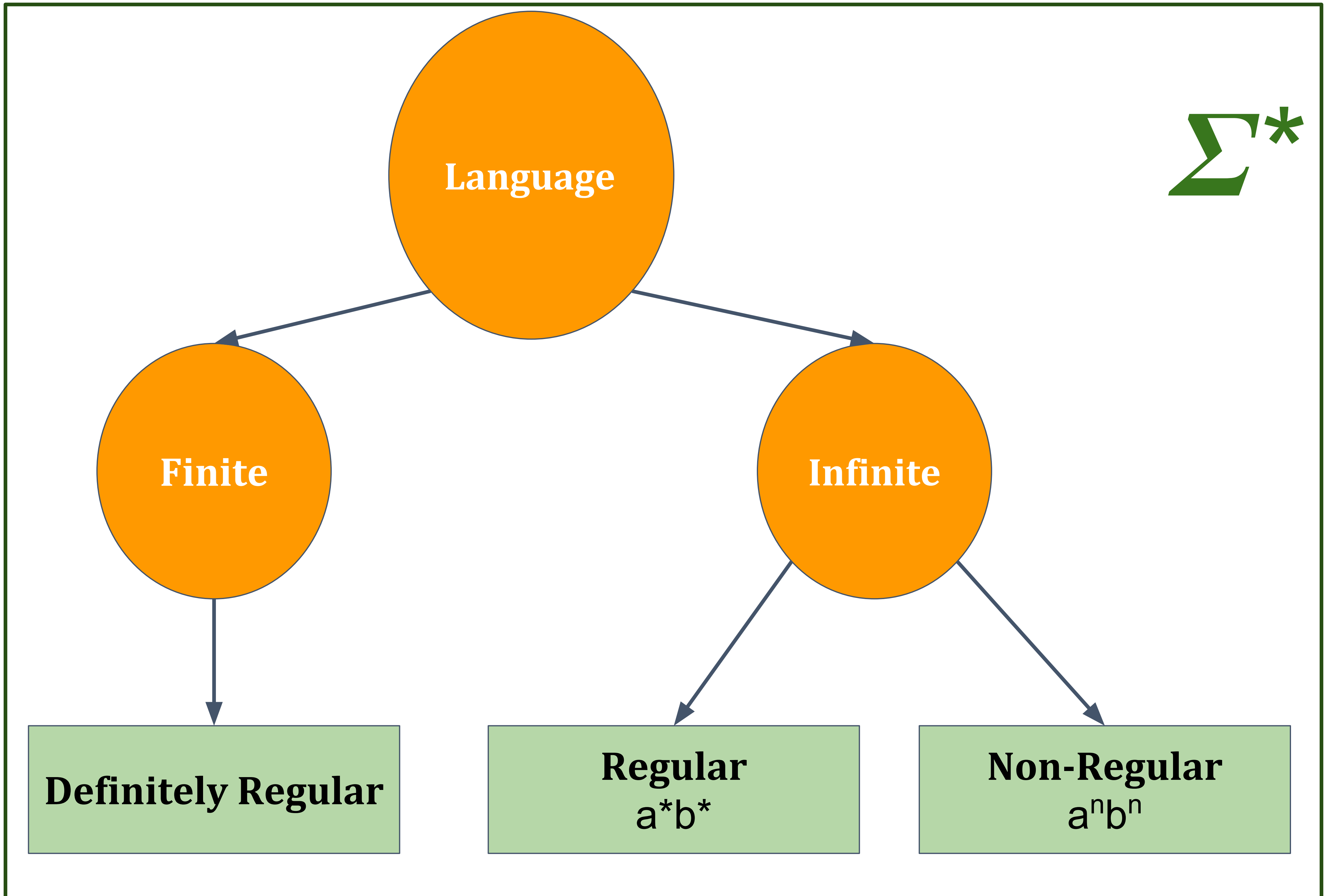


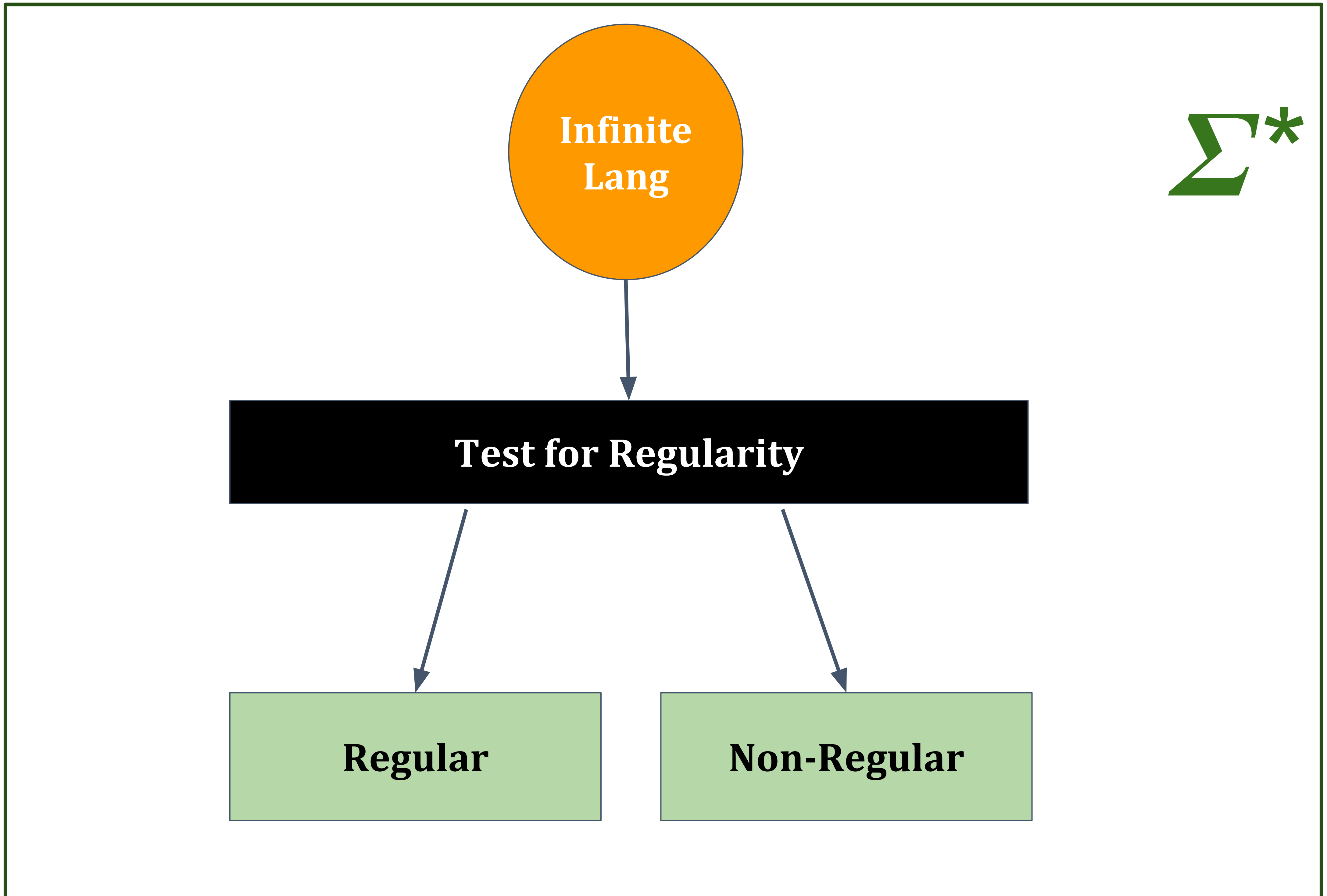
- Finite Memory !

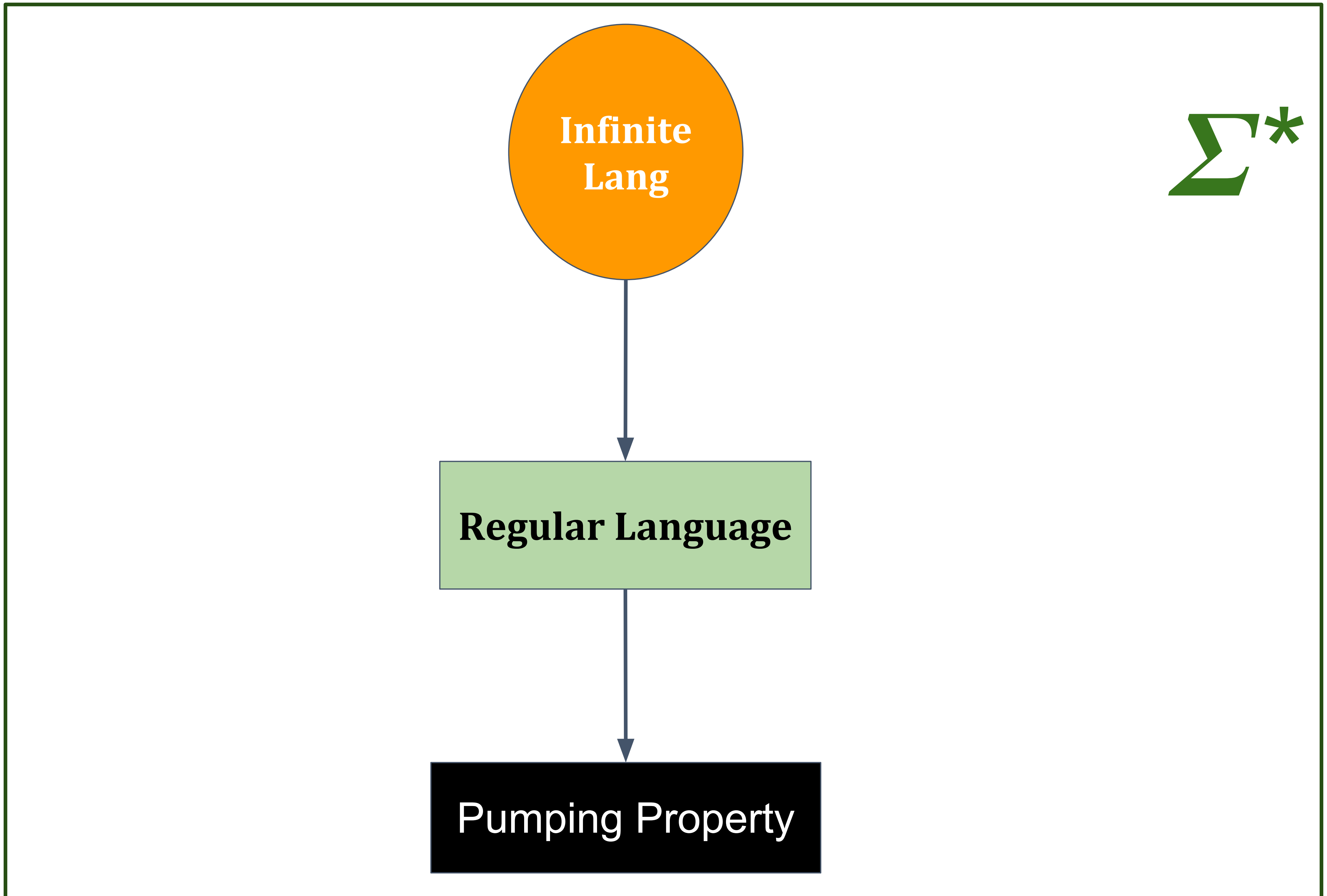
- String comparison not possible - A finite automata can only "count" ; that is, It can maintain a counter, where different states correspond to different values of the counter.

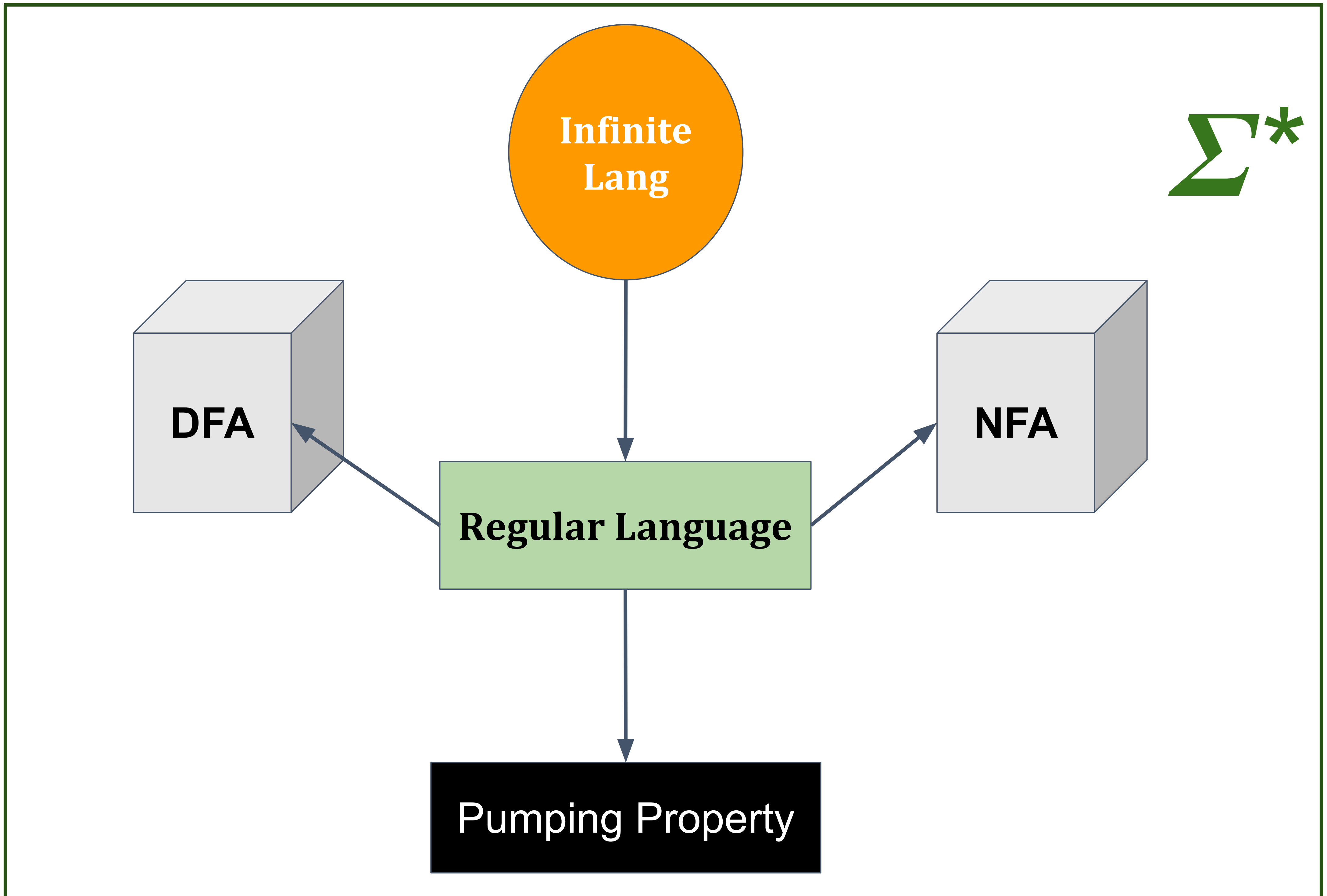
- Linear Power :

  a^n n>0 is possible but a^{n^2}, a^{2^n}, a^i where i is prime is not

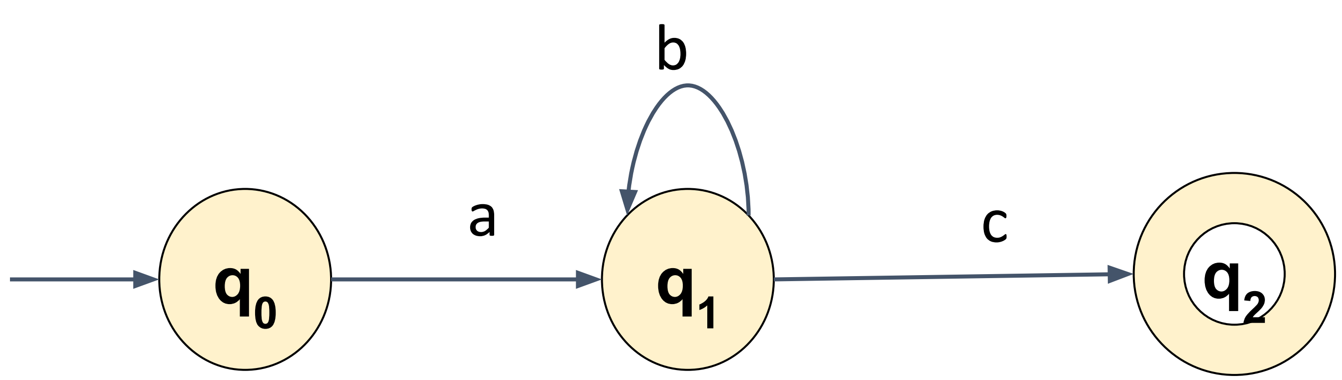

Σ^*
Language
Finite
Infinite
Definitely Regular
Regular
a*b*
Non-Regular
a^nb^n


Infinite
Lang
Test for Regularity
Regular
Non-Regular
Σ*

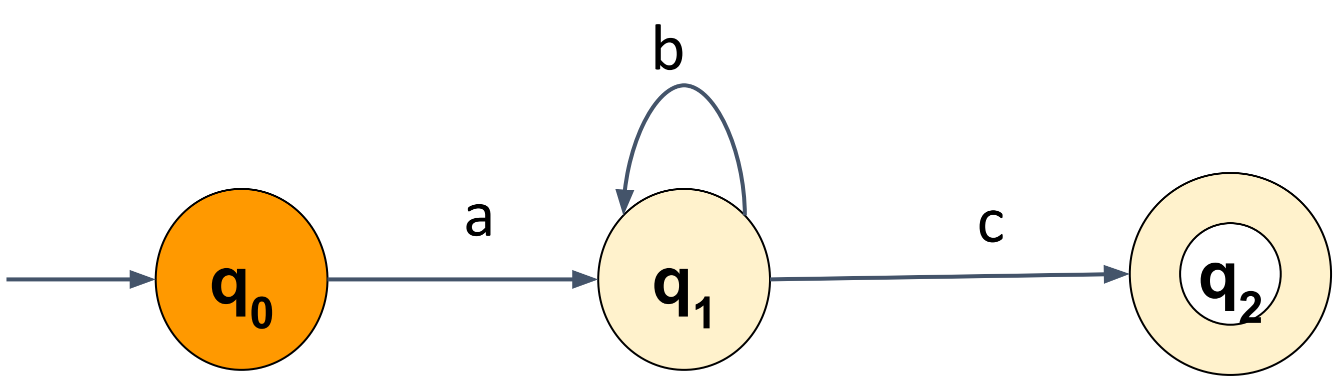

Infinite Lang
Regular Language
Pumping Property
Σ^*


Infinite Lang
DFA
NFA
Regular Language
Pumping Property
Σ^*

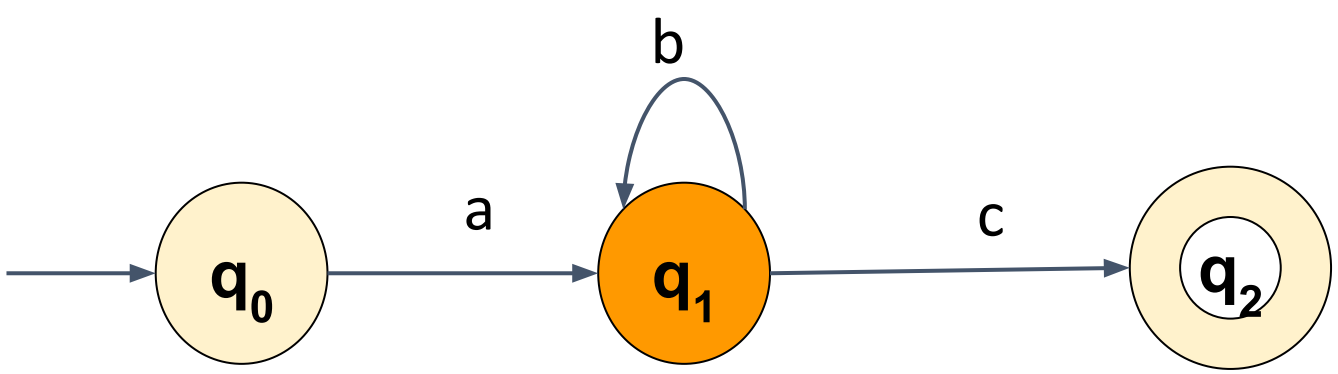
Let’s take an example of infinite regular language ab*c

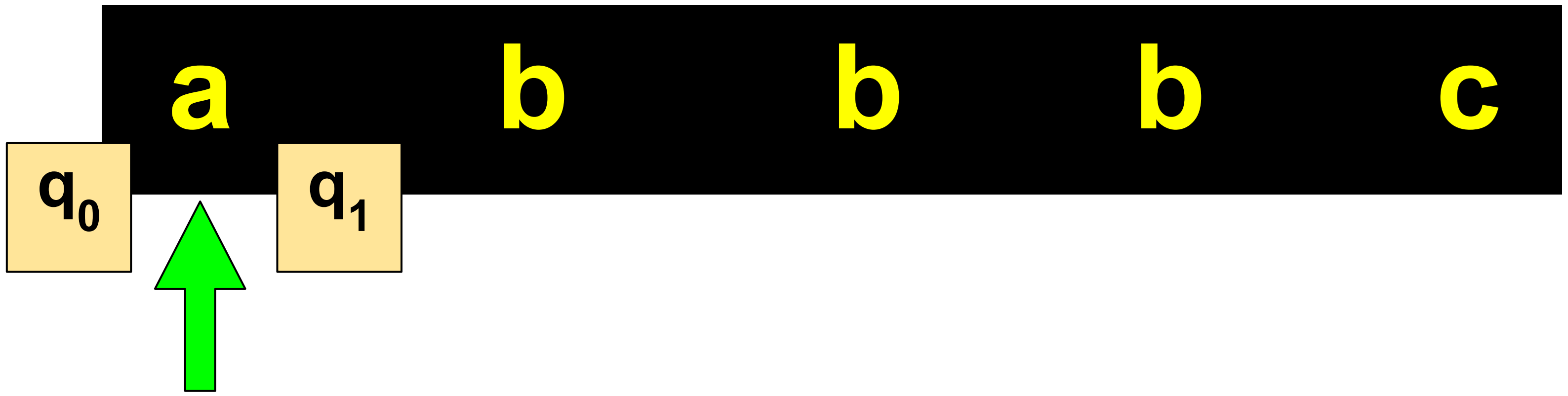


Let’s take an example of infinite regular language ab*c

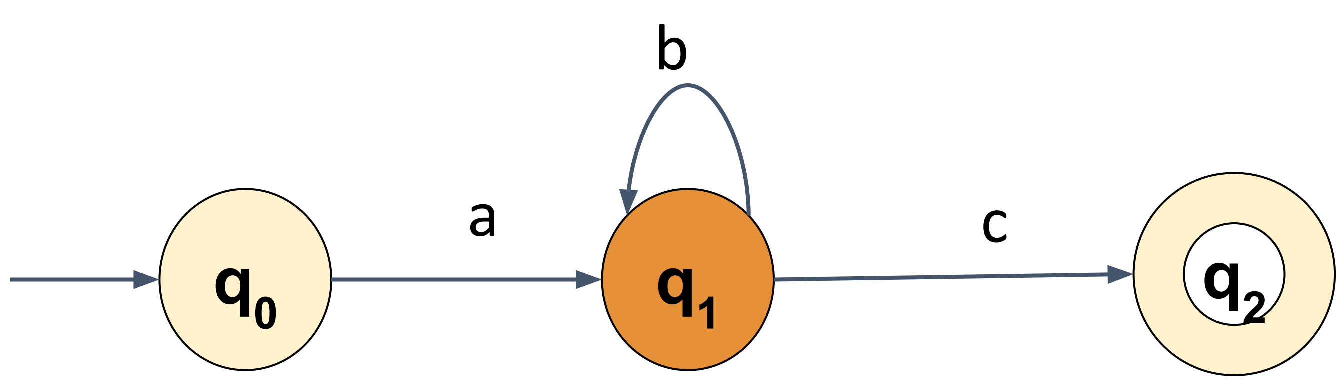


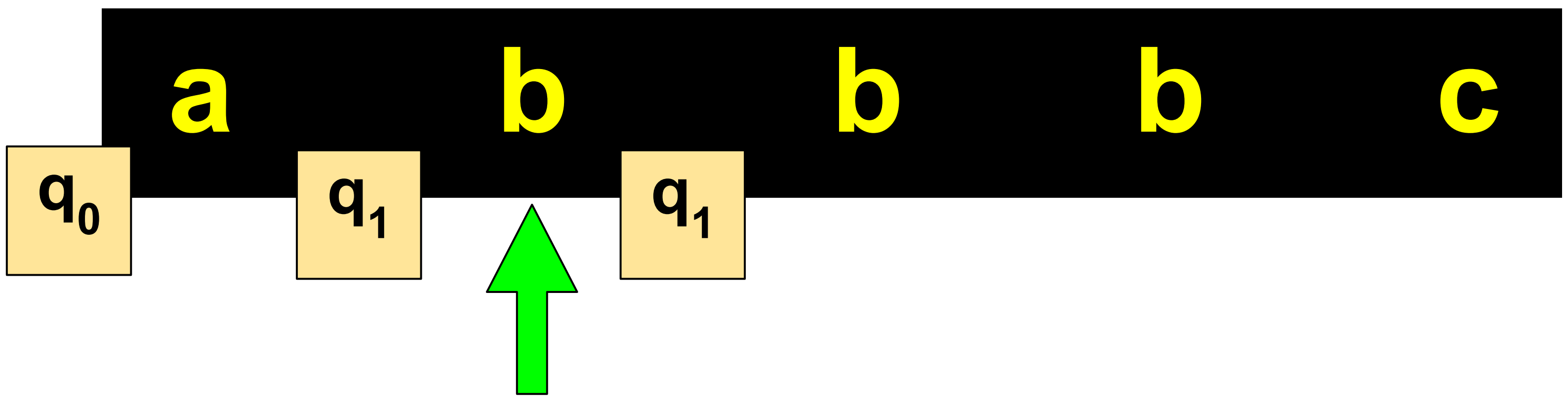
Let’s take an example of infinite regular language ab*c



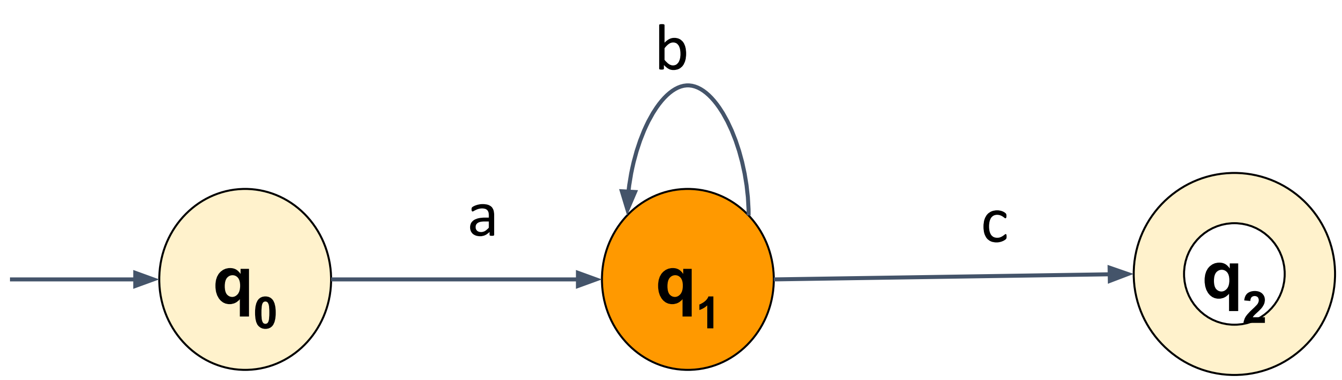


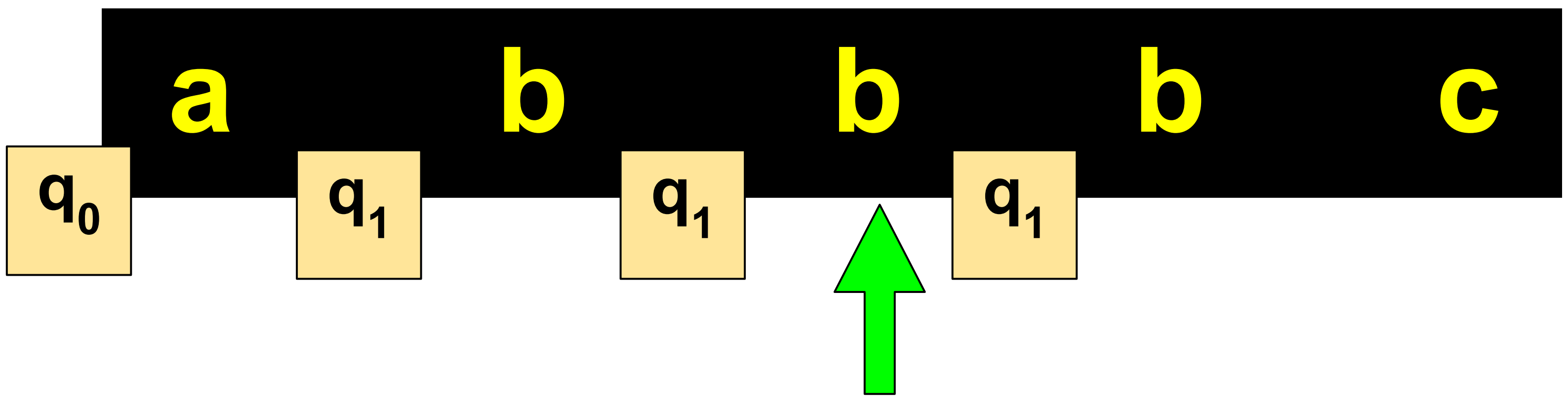
Let’s take an example of infinite regular language ab*c



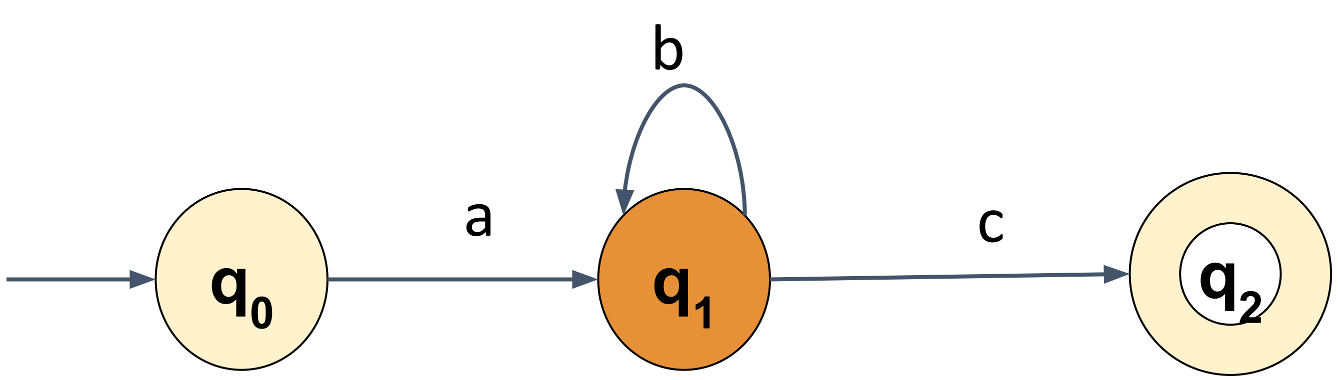


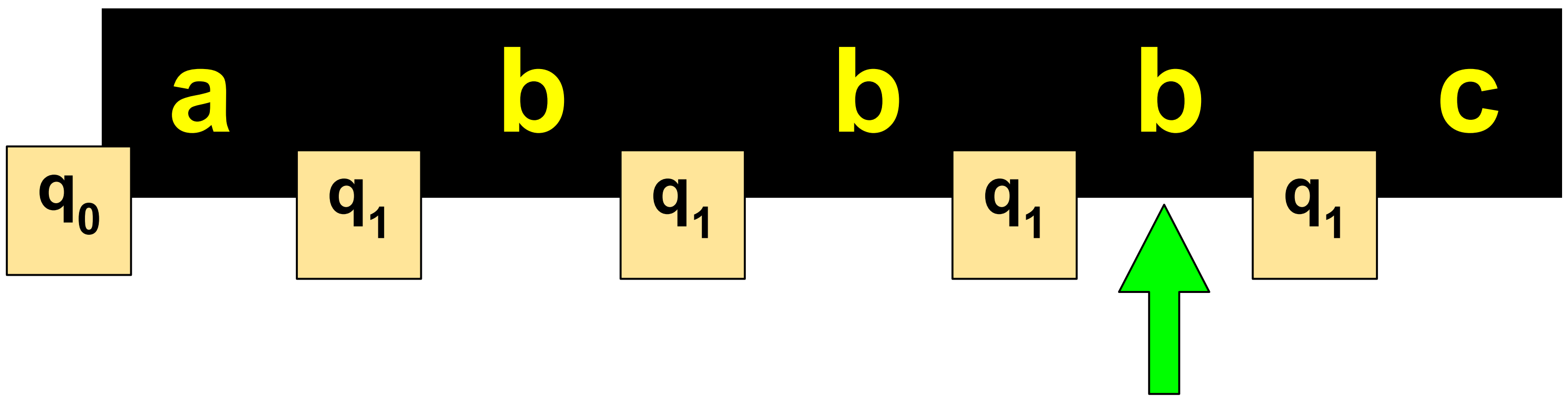
Let's take an example of infinite regular language ab*c



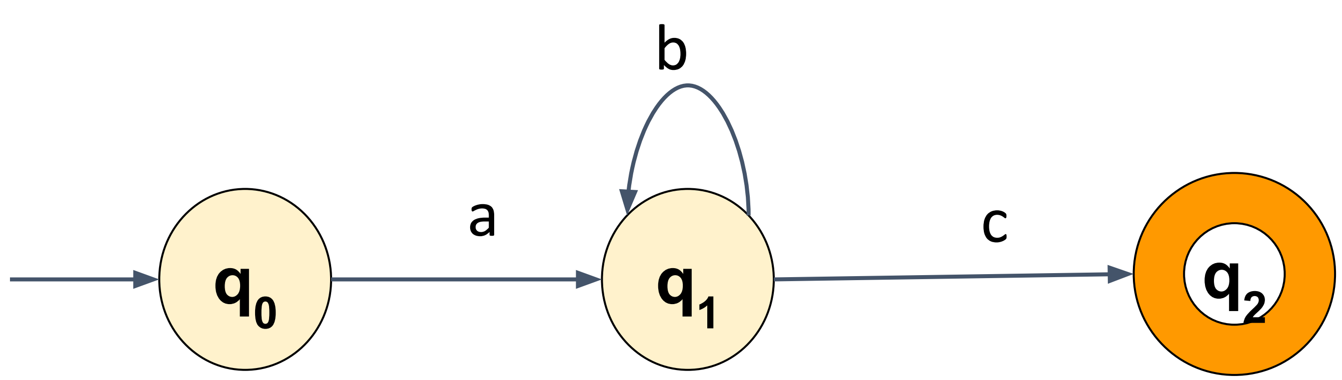


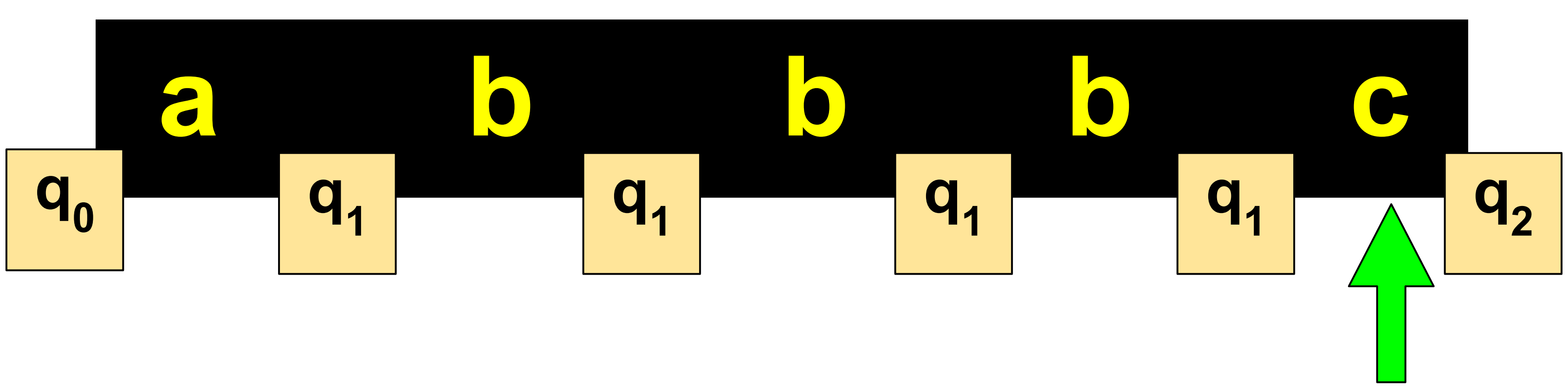
Let's take an example of infinite regular language ab*c





Let’s take an example of infinite regular language ab*c





if |w| >= n

we visit a set of states more than once

if |w| >= n

we visit a set of states more than once

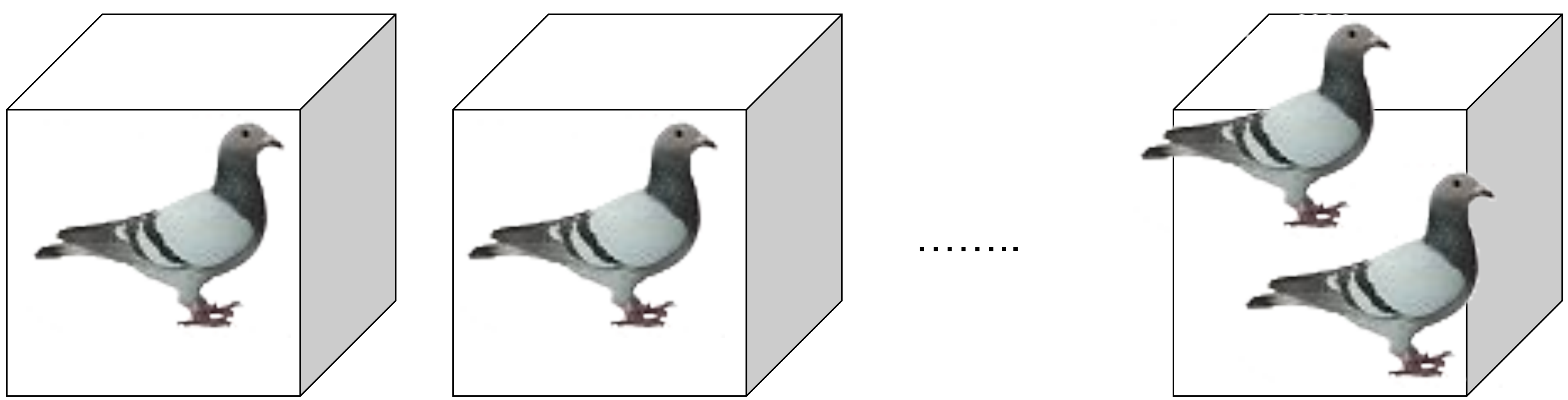
The Pigeonhole Principle

The Pigeonhole Principle

$$n > m$$

pigeons pigeonholes

There is a pigeonhole with more than 1 pigeon



The Pigeonhole Principle

$n > m$

pigeons

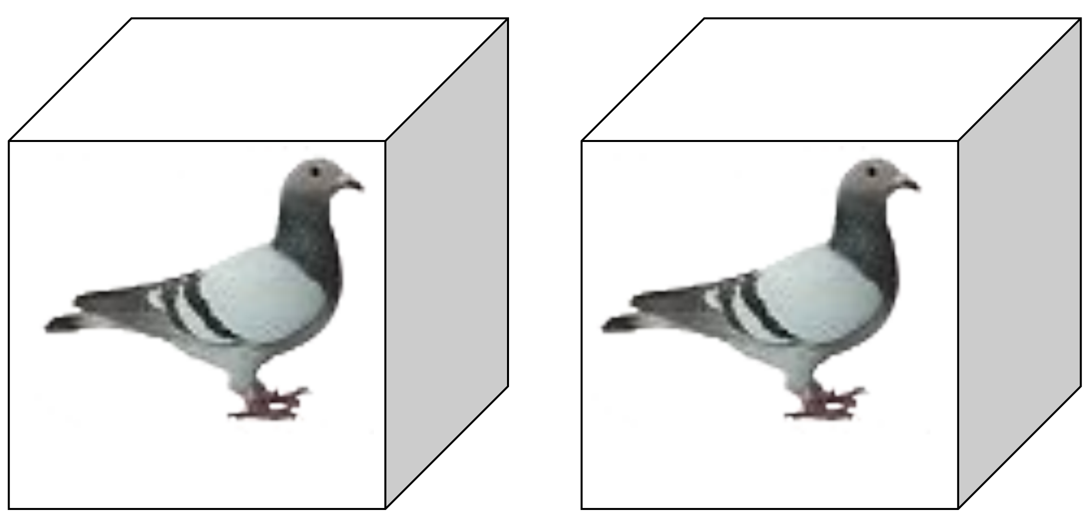
String

pigeonholes

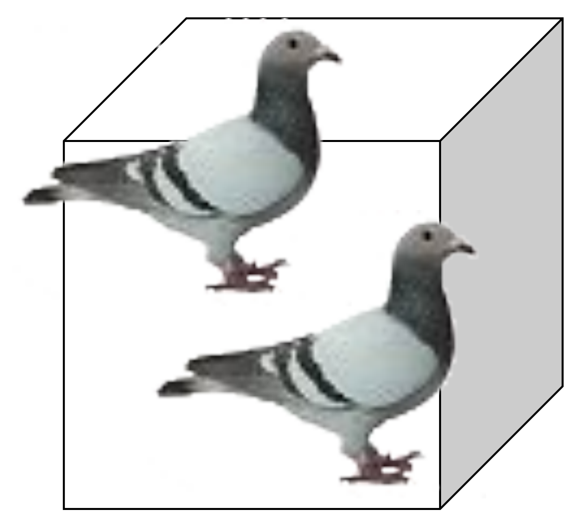
 # States n

if |w| > n
A set of states will be visited more than once

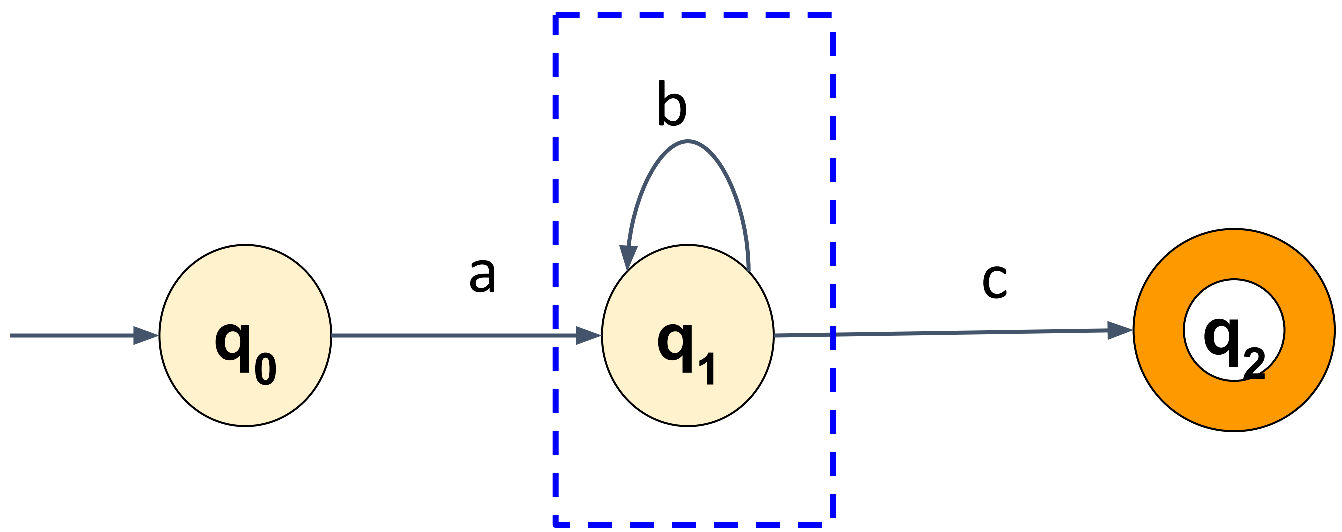
There is a pigeonhole with more than 1 pigeon

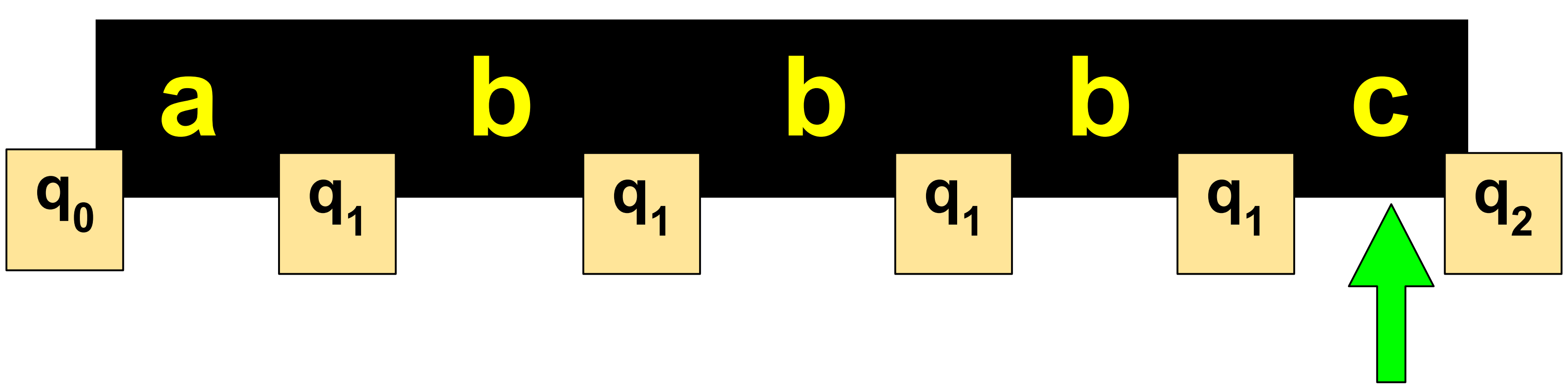


....



Let’s take an example of infinite regular language ab*c





if |w| >= n

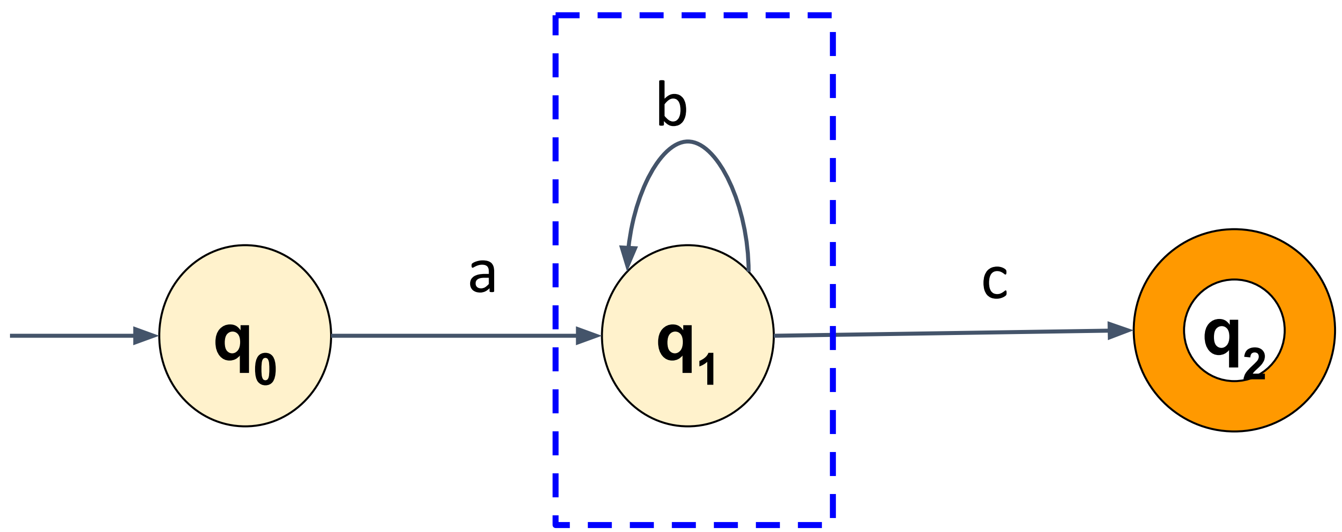
we visit a set of states more than once which means,

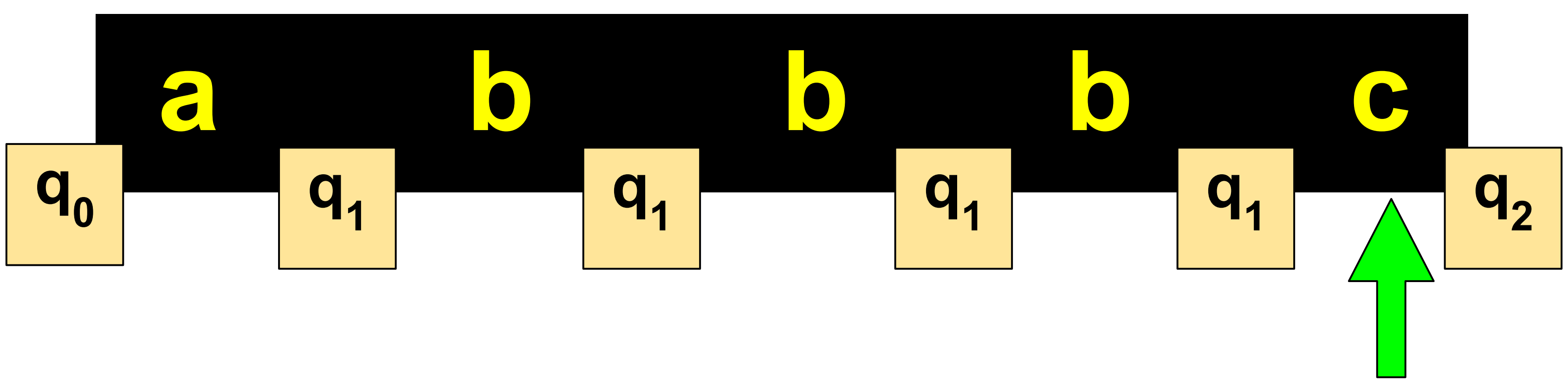
there exists a loop in our Automata (within these n states)

if we pump that loop 0 or more no. of times,

the resultant string will always be in the language

Let’s take an example of infinite regular language ab*c





There exists 3 parts to a string w:

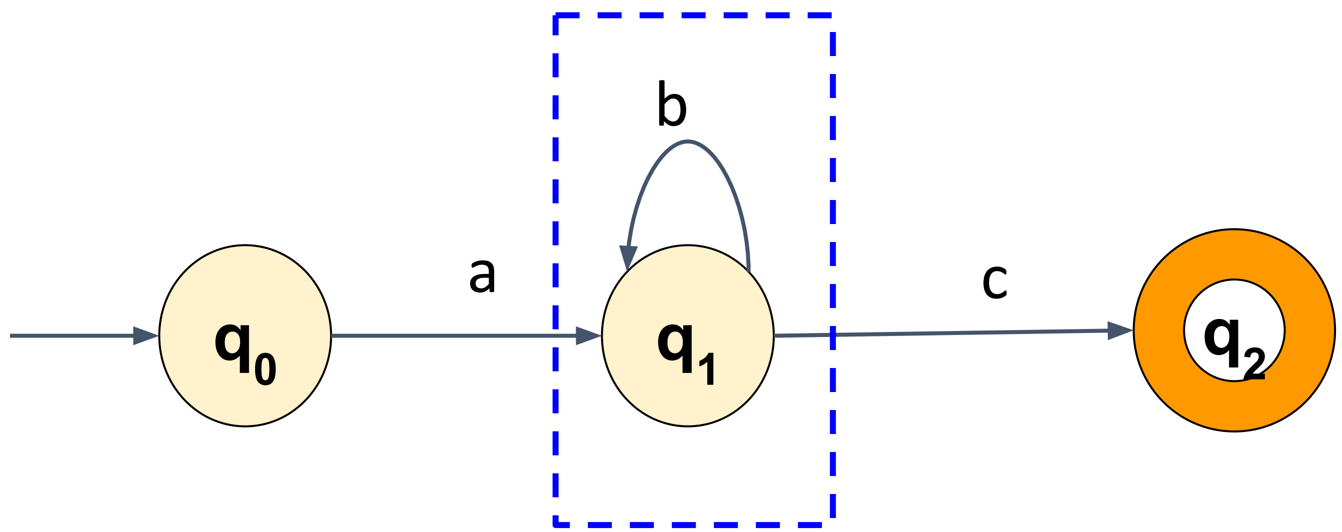
w = x y z

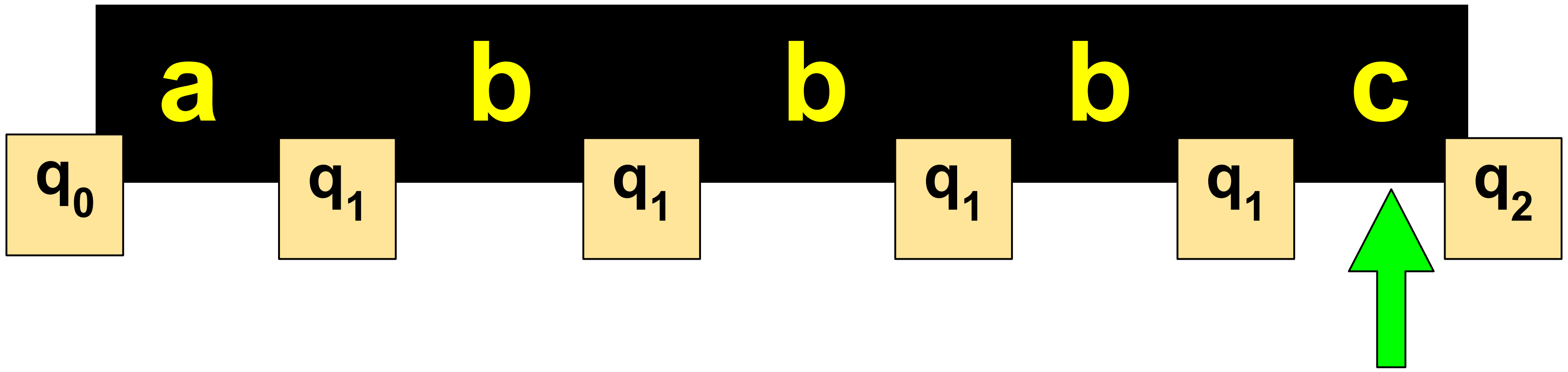
Part before the loop

The loop

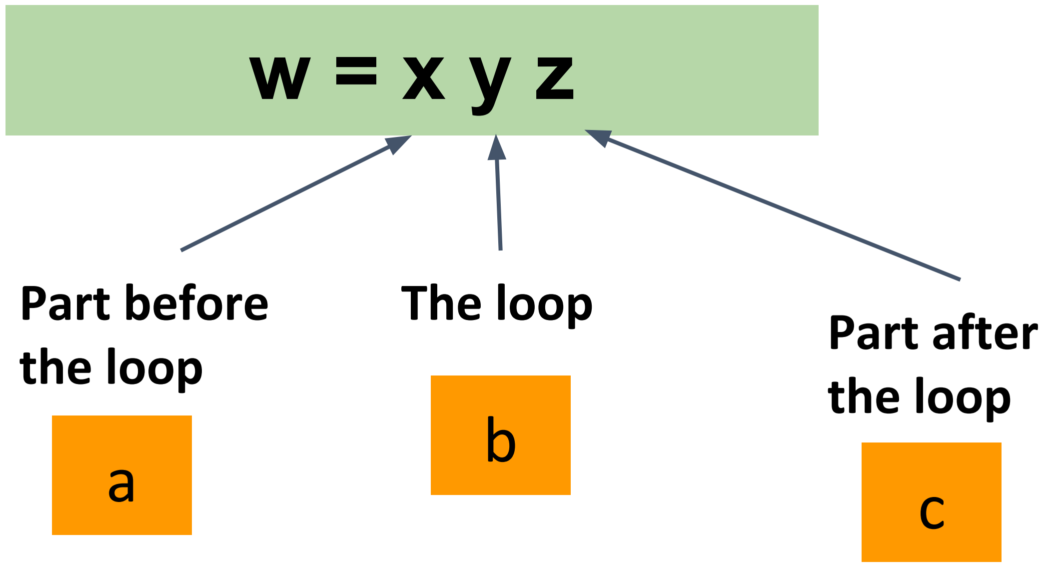
Part after the loop

Let's take an example of infinite regular language ab*c

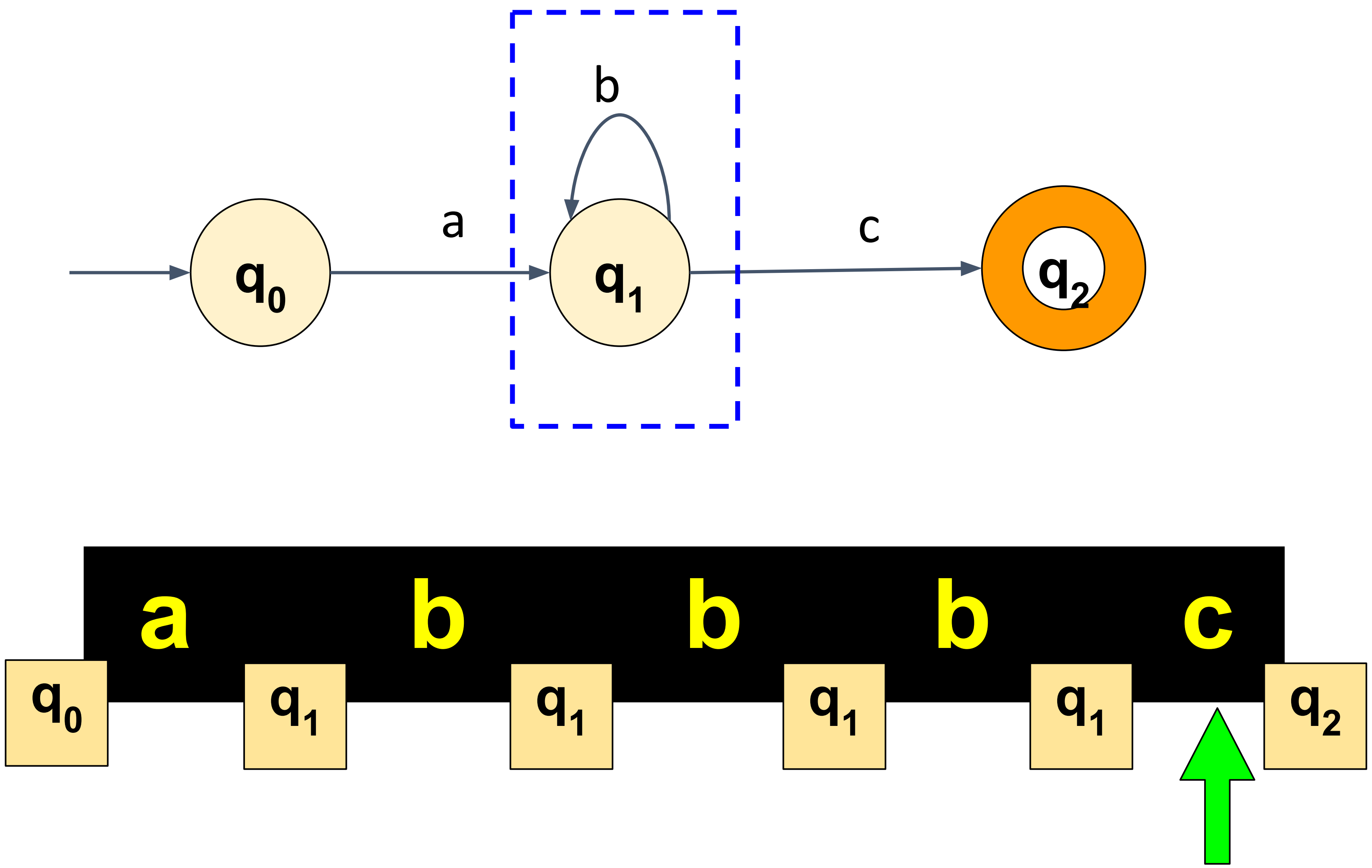




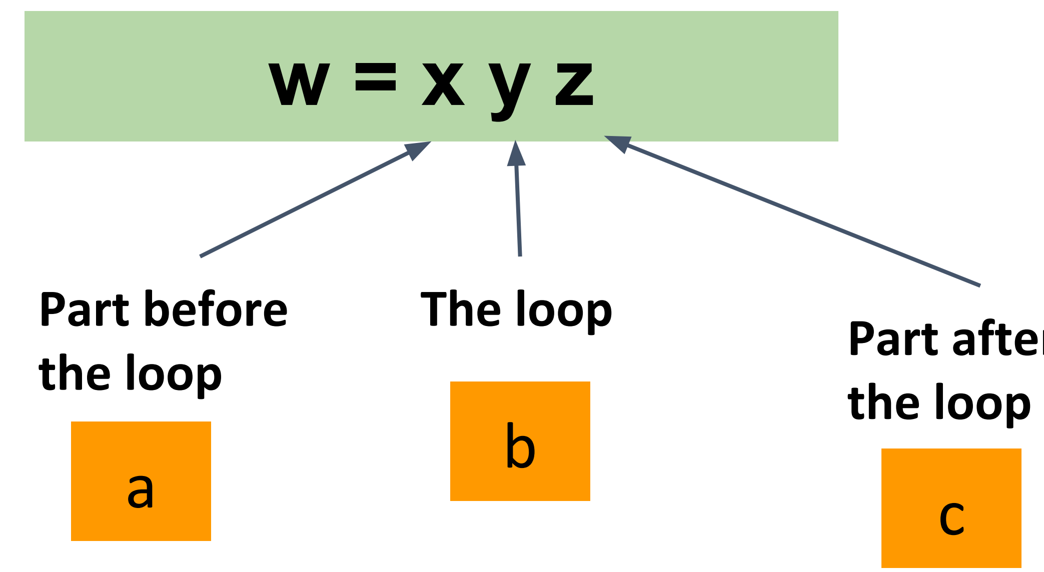
There exists 3 parts to a string w:



Let's take an example of infinite regular language ab*c



There exists 3 parts to a string w:



$y \neq \epsilon$ that is $|y| >= 1$

Unit 2 - Pumping Lemma for Regular Languages

The Pumping property States,

For every Regular language L, (infinite)

there exists n where n is the # states in Finite Automata for L

The Pumping property States,

For every Regular language L, (infinite)

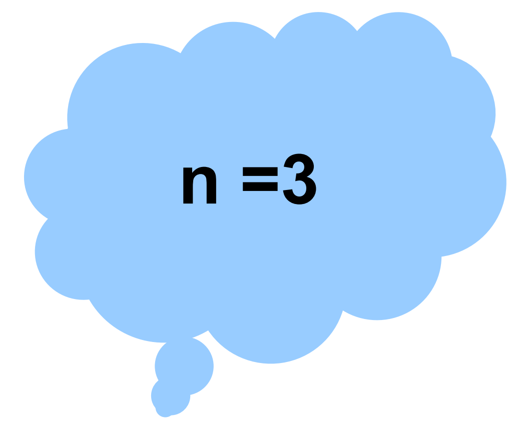
there exists n where n is the # states in Finite Automata for L

In our example of infinite regular language ab*c

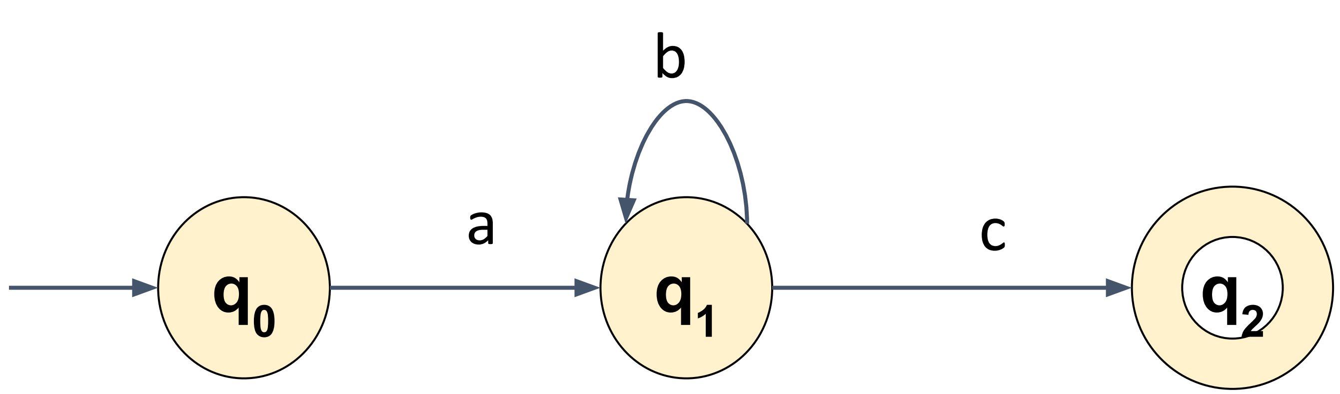
The Pumping property States,

For every Regular language L, (infinite)

there exists n where n is the # states in Finite Automata for L



In our example of infinite regular language ab*c



The Pumping property States,

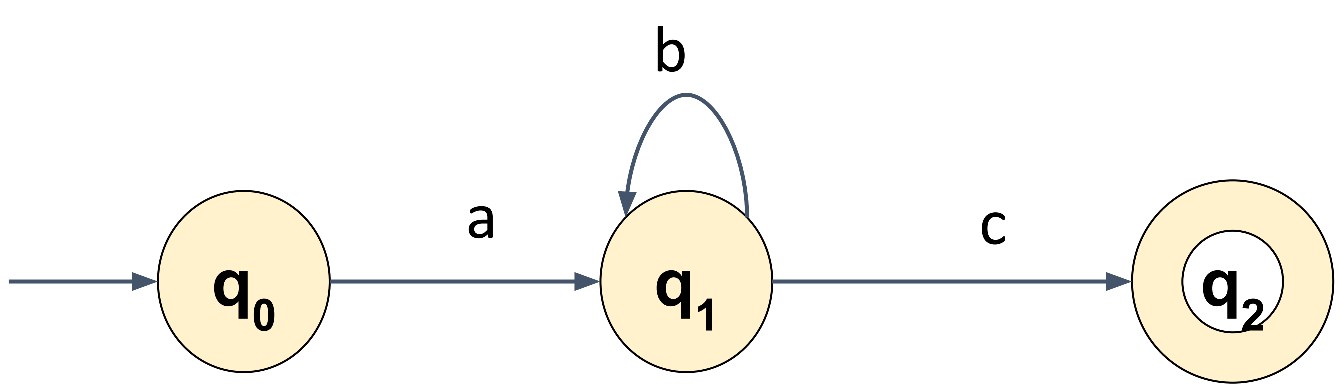
For every Regular language L, (infinite)

there exists n where n is the # states in Finite Automata for L

n =3

n is also called Pumping Constant

In our example of infinite regular language ab*c



The Pumping property States,

For every Regular language L, (infinite)

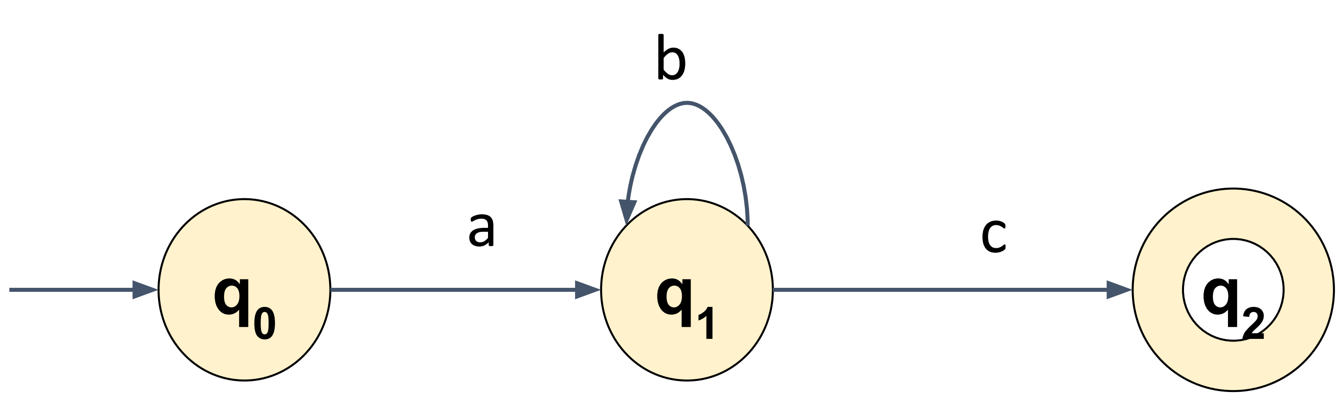
there exists n where n is the # states in Finite Automata for L

For every string w that belongs to L such that,

|w| >= n

n =3

In our example of infinite regular language ab*c



The Pumping property States,

For every Regular language L, (infinite)

there exists n where n is the # states in Finite Automata for L

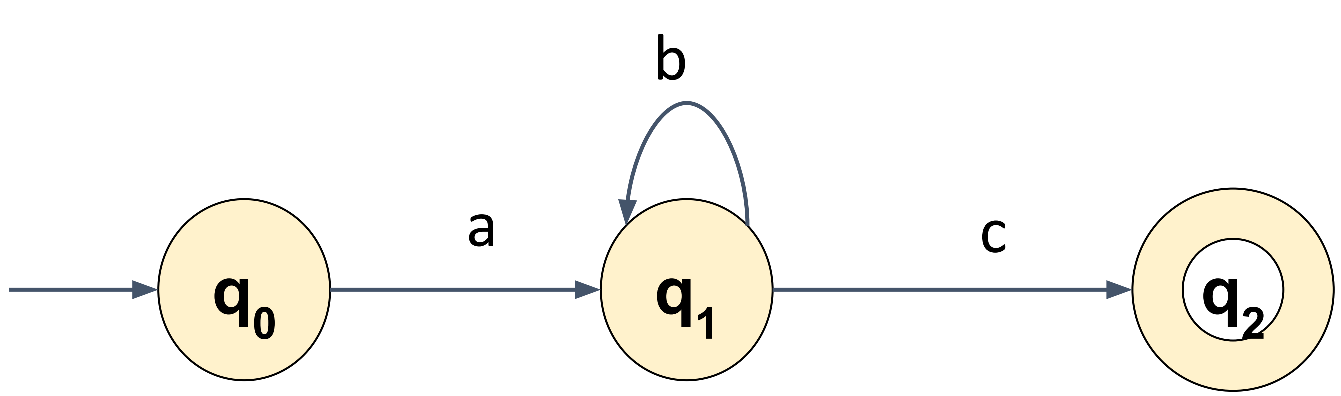
For every string w that belongs to L such that,

|w| >= n

n =3

w = abbbc
|w| = 5 > n

In our example of infinite regular language ab*c



The Pumping property States,

For every Regular language L, (infinite)

there exists n where n is the # states in Finite Automata for L

For every string w that belongs to L such that,

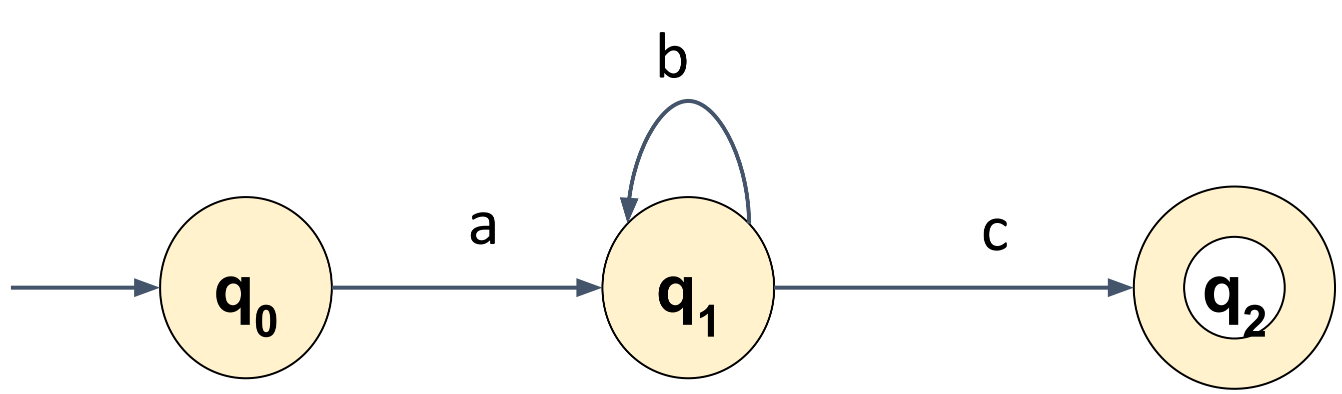
|w| >= n

There exists a break up of the string in three parts w = xyz such that |y| >=1 and |xy| <= n

n =3

w = abbbc
|w| = 5 > n

In our example of infinite regular language ab*c



The Pumping property States,

For every Regular language L, (infinite)

there exists n where n is the # states in Finite Automata for L

For every string w that belongs to L such that,

|w| >= n

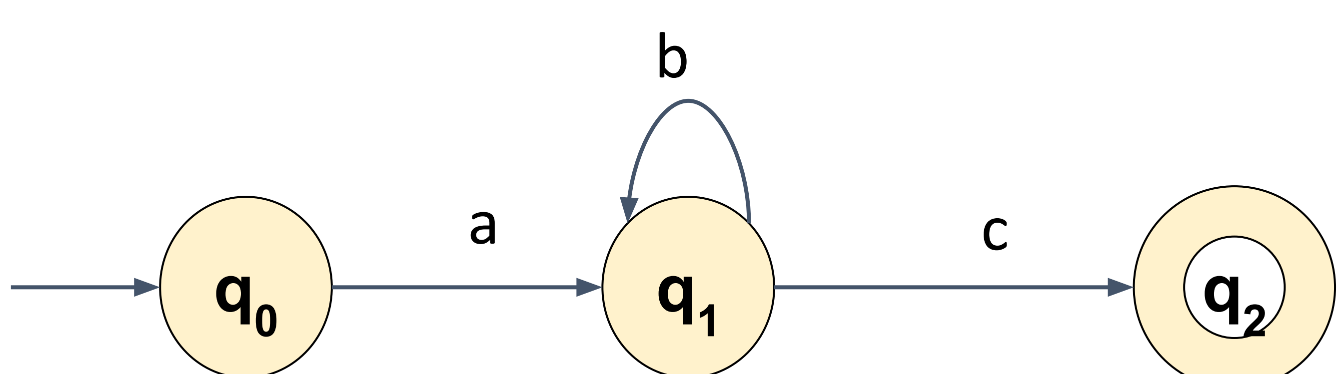
There exists a break up of the string in three parts w = xyz such that |y| >=1 and |xy| <= n

w = abbbc
|w| = 5 > n

n =3

w = abc
x = a
y = b
z = c

In our example of infinite regular language ab*c



The Pumping property States,

For every Regular language L, (infinite)

there exists n where n is the # states in Finite Automata for L

For every string w that belongs to L such that,

$|w| >= n$

There exists a break up of the string in three parts w = xyz
such that $|y| >=1$ and $|xy|<=n$,
for every $i >= 0$,

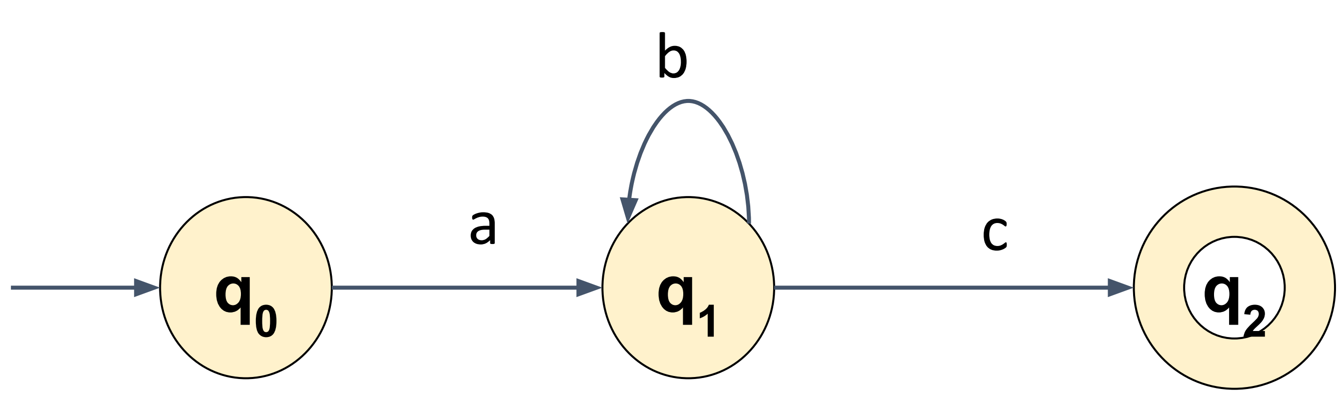
xy^iz belongs to L

w = abbbc
|w| = 5 > n

n =3

w = abc
x = a
y = b
z = c

In our example of infinite regular language ab*c



The Pumping property States,

For every Regular language L, (infinite)

there exists n where n is the # states in Finite Automata for L

For every string w that belongs to L such that,

$|w| >= n$

There exists a break up of the string in three parts w = xyz s
uch that $|y| >=1$ and $|xy|<=n$,
for every $i >= 0$,

xy^iz belongs to L

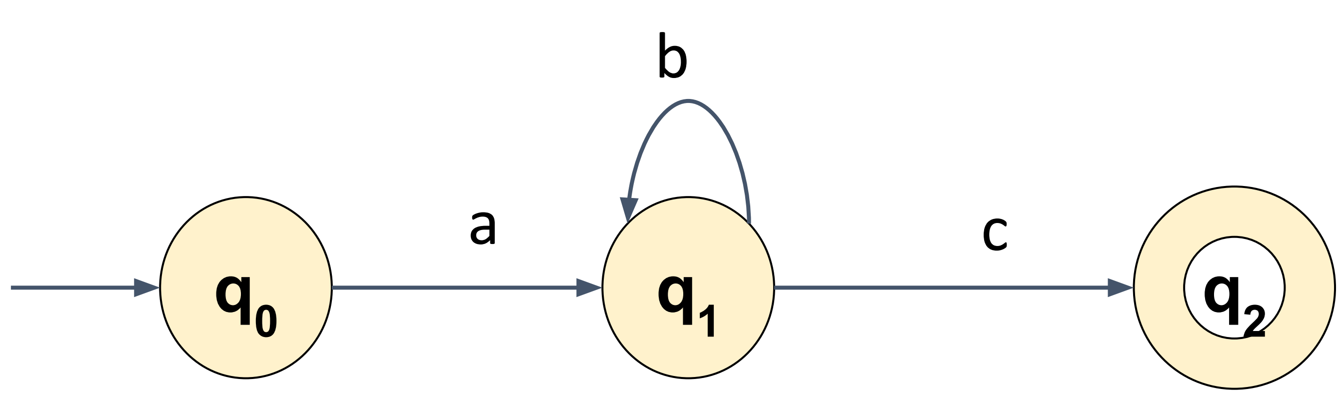
n =3

w = abbbc
$|w| = 5 > n$

w = abc
x = a
y = b
z = c

for i>=0,
ab^ic is in lang ab*c

In our example of infinite regular language ab*c



PES UNIVERSITY ONLINE

For Regular Languages (infinite)

Pumping Property

For every Regular language L,

there exists n where n is the # states in Finite Automata for L

For every string w that belongs to L such that,

$|w| >= n$

There exists a break up of the string in three parts w = xyz such that $|y| >= 1$ and $|xy| <= n$,

for every $i >= 0$,

xy^iz belongs to L

For Regular Languages (infinite)

Pumping Property

For every Regular language L,

there exists n where n is the # states in Finite Automata for L

For every string w that belongs to L such that,

$|w| >= n$

There exists a break up of the string in three parts $w = xyz$ such that $|y| >= 1$ and $|xy|<=n$,

for every $i >= 0$,

xy^iz belongs to L

Replace

For every ----> ∀

There exists ----> ∃

belongs to ----> ∈

For Regular Languages (infinite)

Pumping Property

$\forall$ Regular language L,

$\exists$ n where n is the # states in Finite Automata for L

$\forall$ string w $\in$ L such that,

|w| >= n

$\exists$ w = xyz such that |y| >= 1 and |xy|<=n,

$\forall$ i >= 0,

$xy^iz \in L$

Replace

For every ----> $\forall$

There exists ----> $\exists$

belongs to ----> $\in$

PES UNIVERSITY ONLINE

For Regular Languages (infinite)

Pumping Property

$\forall$ Regular language L,

$\exists$ n where n is the # states in Finite Automata for L

$\forall$ string w $\in$ L such that,

$|w| >= n$

$\exists$ w = xyz such that $|y| >= 1$ and $|xy| <= n$,

$\forall$ $i >= 0$,

$xy^iz \in L$

To Prove a lang is Non-Regular

~Pumping Property

PES UNIVERSITY ONLINE

For Regular Languages (infinite)

Pumping Property

$\forall$ Regular language L,

$\exists$ n where n is the # states in Finite Automata for L

$\forall$ string w $\in$ L such that,

$|w| >= n$

$\exists$ w = xyz such that $|y| >=1$ and $|xy|<=n$,

$\forall$ i >= 0,

$xy^iz \in L$

$\sim\forall = \exists$

$\sim\exists = \forall$

To Prove a lang is Non-Regular

~Pumping Property

For Regular Languages (infinite)

Pumping Property

$\forall$ Regular language L,

$\exists$ n where n is the # states in Finite Automata for L

$\forall$ string w $\in$ L such that,

|w| >= n

$\exists$ w = xyz such that |y| >= 1 and |xy|<=n,

$\forall$ i >= 0,

$xy^iz \in L$

$\sim\forall = \exists$

$\sim\exists = \forall$

To Prove a lang is Non-Regular

~Pumping Property

$\exists$ a language L which is claimed to be regular,

$\forall$ n where n is the # states in Finite Automata for L

$\exists$ string w $\in$ L such that,

|w| >= n

$\forall$ w = xyz such that |y| >= 1 and |xy|<=n,

$\exists$ i >= 0,

$xy^iz \notin L$

This contradicts the claim made, hence proving that the language is not regular

For Regular Languages (infinite)

Pumping Property

$\forall$ Regular language L,

$\exists$ n where n is the # states in Finite Automata for L

$\forall$ string w $\in$ L such that,

$|w| >= n$

$\exists$ w = xyz such that $|y| >= 1$ and $|xy| <= n$,

$\forall$ $i >= 0$,

$xy^iz \in L$

$\sim\forall = \exists$

$\sim\exists = \forall$

To Prove a lang is Non-Regular

~Pumping Property

$\exists$ a language L ... be regular,

$\forall$ n whe... ...ata for L

$\exists$ str...

$\forall$ w = ...

$\exists$ i >= 0...

This contradic... ...e proving that the language is not ...

Proof by Contradiction

PES UNIVERSITY ONLINE

For Regular Languages (infinite)

Pumping Property

∀ Regular language L,

∃ n where n is the # states i

L

∀ string w ∈ L such that

|w| >= n

∃ w = xyz such that |y| >= 1 and |xy| <=n,

∀ i >= 0,

xy^iz ∈ L

Pumping Lemma

~ ∀ =

~ ∃ =

∀

To Prove a lang is Non-Regular

~Pumping Property

∃ a language L ... be regular,

∀ n wher ... ata for L

∃ stri

∀ w =

∃ i >= 0

This contradic ... proving that the language is not

Proof by Contradiction

Procedure to prove a language is Not regular :

1. Assume the opposite: L is regular

2. Use Pumping Lemma to obtain a contradiction

   It suffices to show that only one string gives a contradiction

3. Thereby proving L is not regular

Procedure to prove a language is Not regular :

1. Assume the opposite: L is regular

2. Use Pumping Lemma to obtain a contradiction

! String must be chosen appropriately

It suffices to show that only one *string* gives a contradiction

3. Thereby proving L is not regular

For Regular Languages

↓

Pumping Property

↓

Always Pass

For Regular Languages

↓

Pumping Property

↓

Always Pass

Non-Regular Languages

↓

Pumping Property

↙ ↘

Fails → **Definitely Non-Regular**

Pass → **Can you construct a FA??**

For Regular Languages

N n-Regular Languages

Pump

roperty

Pumping lemma is a necessary but not sufficient condition for a language to be regular

Always Pas

is

Pass

Definitely Non-Regular

Can you construct a FA??

Pumping lemma as a game

Pumping lemma is a game between

You vs. Adversary

The role

You

Adversary

Claims L is regular

The role

You

Okay! Gimme the no. of states in your machine for L

Adversary

Claims L is regular

The role

You

Adversary

Claims L is regular

Okay! Gimme the no. of states in your machine for L

There are n states in my automata for L

The role

You

Okay! Gimme the no. of states in your machine for L

Okay! here is the string w from L such that
|w| >= n
Could you tell me where is the loop in your machine?

Adversary

Claims L is regular

There are n states in my automata for L

The role

You

Okay! Gimme the no. of states in your machine for L

Okay! here is the string w from L such that
|w| >= n
Could you tell me where is the loop in your machine?

Adversary

Claims L is regular

There are n states in my automata for L

The loop is xy^iz

The role

You

Adversary

Claims L is regular

Okay! Gimme the no. of states in your machine for L

The are n states in my automata for L

Okay! here is the string w from L such that
|w| >= n
Could you tell me where is the loop in your machine?

The loop is xy^iz

Find some i, so that the resultant string is not in L

The role

You | **Adversary**

Adversary: Claims L is regular

You: Okay! Gimme the no. of states in your machine for L

Adversary: The are n states in my automata for L

You: Okay! here is the string w from L such that
$|w| >= n$
Could you tell me where is the loop in your machine?

Adversary: The loop is xy^iz

You: Find some i, so that the resultant string is not in L

Adversary: Oh no! I lost!!!!
L is not regular

The role

You

Adversary

laims L is regular

Okay! Gimme the
states in your machi

e are n states in my
automata for L

Okay! here is the st
from L such th
|w| >= n
Could you tell me w
the loop in your machine?

The loop is xy^iz

Okay! but for some i, the
resultant string is not in L

Oh no! I lost!!!!
L is not regular

Using Pumping lemma prove that the language $a^n b^n$ is not regular

The role

You

Adversary

Claims $L = a^n b^n$ is regular

The role

You

Adversary

Claims L = $a^n b^n$ is regular

Okay! Gimme the no. of states in your machine for L

The role

You

Okay! Gimme the no. of states in your machine for L

Adversary

Claims L = a^nb^n is regular

There are 10 states in my automata for L

PES UNIVERSITY ONLINE

The role

You

Okay! Gimme the no. of states in your machine for L

okay! I'll choose the string
a^6b^6
$|w| >= 10$
Now tell me where is the loop in your automata?

Adversary

Claims L = a^nb^n is regular

There are 10 states in my automata for L

Where is the loop ?

aaaaaabbbbbb

Where is the loop ?

aaaaabbbbbb

Loop must
be within first
10 symbols

Where is the loop ?

aaaaaabbbbbb

$a^5(a)^ib^6$

PES UNIVERSITY ONLINE

Where is the loop ?

aaaaaabbbbbb

Pump down, $i=0 \rightarrow a^5b^6 \notin L$

$a^5(a)^0b^6$

Where is the loop ?

aaaaaabbbbbb

$a^5(a)^2b^6$

Pump down, i=0 $\rightarrow a^5b^6 \notin L$

Pump up, i=2 $\rightarrow a^7b^6 \notin L$

Where is the loop ?

aaaaaabbbbbb

$a^5(a)^i b^6$

Pump do… L

…$i=2 \rightarrow a^7b^6 \notin L$

i = 1 doesn't help !!

Where is the loop ?

aaaaaabbbbbb

Pump up, i=2 $\rightarrow$ $a^5ababb^5 \notin L$

$$a^5(ab)^ib^5$$

Where is the loop ?

aaaaaabbbbbb

Pump up, $i=2 \rightarrow a^5ababb^5 \notin L$

i = 0 or i = 1 doesn't help !!

$$a^5(ab)^ib^5$$

Where is the loop ?

aaaaaabbbbbb

Pump down, i=0 $\rightarrow a^6b^5 \notin L$

Pump up, i=2 $\rightarrow a^6b^7 \notin L$

$a^6(b)^ib^5$

Where is the loop ?

aaaaaabbbbbb

Pump down, i=0 → a^6b^5

Pump up, i=2 → $a^6b^7 \notin L$

i = 1 doesn't help !!

$a^6(b)^ib^5$

Where is the loop ?

aaaaaabbbbbb

For every break up possible we got some i that will result in a string ∉ to L

The role

You

Okay! Gimme the no. of states in your machine for L

okay! I'll choose the string
a^6b^6
$|w| >= 10$
Now tell me where is the loop in your automata?

Adversary

Claims L = a^nb^n is regular

There are 10 states in my automata for L

We saw and explored different possibilities where the loop could be

The role

You

Okay! Gimme the no. of states in your machine for L

okay! I'll choose the string
a^6b^6
$|w| >= 10$
Now tell me where is the loop in your automata?

Okay! but for some i, nothing worked out!

Adversary

Claims L = a^nb^n is regular

There are 10 states in my automata for L

We saw and explored different possibilities where the loop could be

The role

You

Okay! Gimme the no. of states in your machine for L

okay! I'll choose the string a^6b^6
$|w| >= 10$
Now tell me where is the loop in your automata?

Okay! but for some i, nothing worked out!

Adversary

Claims $L = a^nb^n$ is regular

There are 10 states in my automata for L

We saw and explored different possibilities where the loop could be

Oh no! I lost!!!!
L is not regular

The role

You

Adversary

ms L = a^nb^n is regular

Okay! Gimme the
states in your machi

e are 10 states in my
automata for L

okay! I'll choose the
a^6b^6
$|w| >= 10$
Now tell me where is the
loop in your automata?

e saw and explored
different possibilities where
the loop could be

Okay! but for some i, nothing
worked out!

Oh no! I lost!!!!
L is not regular

Using Pumping lemma prove that the language of the form ww over $\{a,b\}^*$ is not regular

The role

You

Adversary

Claims L = ww is regular

The role

You

Adversary

Claims L = ww is regular

Okay! Gimme the no. of states in your machine for L

The role

You

Okay! Gimme the no. of states in your machine for L

Adversary

Claims L = ww is regular

There are n states in my automata for L

The role

You

Okay! Gimme the no. of states in your machine for L

okay! I'll choose the string
$a^n a^n$
$|w| >= n$
Now tell me where is the loop in your automata?

Adversary

Claims L = ww is regular

There are n states in my automata for L

Where is the loop ?

a...a..aa..aa...a..aa..a

Where is the loop ?

a...a.aa..aa...a.aa..a

Loop must exist within first n symbols

Where is the loop ?

a...a..aa..aa...a..aa..a

$a^{n-2}(aa)^{i}a^{n}$

Where is the loop ?

a...a..aa..aa...a..aa..a

$$a^{n-2}(aa)^i a^n$$

Let's Pump down, i=0

$$= a^{n-2}a^{n}$$
$$= a^{n-2}a^{2}a^{n-2}$$
$$= a^{n-1}a^{n-1}$$

PES UNIVERSITY ONLINE

Where is the loop ?

a...a..aa..aa...a..aa..a

$$a^{n-2}(aa)^i a^n$$

Let's Pump up, i=3

$$= a^{n-2}(aa)^3 a^n$$
$$= a^{n-2}(a^2)^3 a^n$$
$$= a^{n-2}a^6 a^n$$
$$= a^{n-2}a^4 a^2 a^n$$
$$= a^{n-2+4}a^{n+2}$$
$$= a^{n+2}a^{n+2}$$

Where is the loop ?

a...a..aa..aa...a..aa..a

$a^{n-2}(aa)^i a^n$

Pump up or Pump down,
resultant string will always belong to L

Where is the loop ?

a...a 

a..a

$a^{n-2}(aa)^i a^n$

Pump up or Pump down,
resultant string will always belong to L

Where is the loop ?

a...a ... a..a

$a^{n-2}(aa)^{i}a^{n}$

YOU LOSE!

You chose a wrong string

...r Pump down,
...ring will always belong to L

THANK YOU

Preet Kanwal

Department of Computer Science & Engineering

preetkanwal@pes.edu

+91 80 6666 3333 Extn 724