

Q1. Which matrices E_{ij} put A into upper triangular form U ?

$$x + y + z = 6$$

$$x + 2y - z = 2$$

$$2x + y - z = 1$$

Find M such that $MA = U$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -1 \\ 2 & 1 & -1 \end{bmatrix} \xrightarrow[\substack{R_2 \rightarrow R_2 - R_1 \\ R_3 \rightarrow R_3 - 2R_1}]{} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -2 \\ 0 & -1 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & -5 \end{bmatrix} \xleftarrow{R_3 \rightarrow R_3 + R_2}$$

$$E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$$E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$M = E_{32} \cdot E_{31} \cdot E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 1 & 1 \end{bmatrix}$$

perform same transformations on I

Q2. Which E converts A to U

$$A = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \quad R_2 \rightarrow R_2 + \frac{1}{2} R_1$$

$$R_3 \rightarrow R_3 + \frac{2}{3} R_2 \quad \begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 3/2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 3/2 & -1 & 0 \\ 0 & 0 & 4/3 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \quad R_4 \rightarrow R_4 + \frac{3}{4} R_3$$

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 3/2 & -1 & 0 \\ 0 & 0 & 4/3 & -1 \\ 0 & 0 & 0 & 5/4 \end{bmatrix}$$

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1/2 & 1 & 0 & 0 \\ 0 & 2/3 & 1 & 0 \\ 0 & 0 & 3/4 & 1 \end{bmatrix}$$

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