PES UNIVERSITY, Bangalore

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Department of Computer Science & Engineering

Automata Formal Languages & Logic

Question and answers-Functions and Relations

1. Diagram the following functions and mention whether they are one-to-one, onto or bijective:

    a. f : {a, b, c, d} → {1, 2, 3, 4}

    f(a) = 1

    f(b) = 2

    f(c) = 3

    f(d) = 4

    b. g : {a, b, c, d} → {1, 2, 3, 4}

    g(a) = 1

    g(b) = 1

    g(c) = 4

    g(d) = 4

Solution:

a .Bijective.

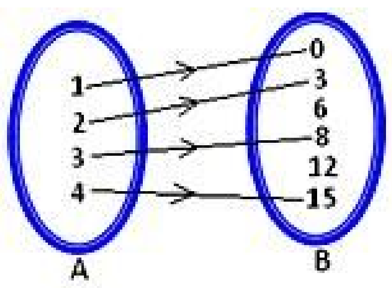
b.one to one

2. Let A = {1, 2, 3, 4} and B = {0, 3, 6, 8, 12, 15}.Consider a rule f (x) = x^2 - 1, x∈A, then
   a. show that f is a mapping from A to B.
   b. draw the arrow diagram to represent the mapping.
   c. represent the mapping in the roster form.
   d. write the domain and range of the mapping.

Solution:

a. Using f (x) = x^2 - 1, x ∈ A we have
   f(1) = 0,
   f(2) = 3,
   f(3) = 8,
   f(4) = 15
   We observe that every element in set A has unique image in set B.
   Therefore, f is a mapping from A to B.

b. Arrow diagram which represents the mapping is given below.



c. Mapping can be represented in the roster form as
   f = {(1, 0); (2, 3); (3, 8); (4, 15)}

d. Domain (f) = {1, 2, 3, 4} Range (f) = {0, 3, 8, 15}

3. Find all real values of x such that f(x) = g(x) where f and g are functions given by f(x) = 3x + $\sqrt{(x)}$ and g(x) = 2x + 6

Solution:

f(x) = g(x) leads to an equation.

$3x + \sqrt{(x)} = 2x + 6$

Rewrite the equation as follows

$\sqrt{(x)} = - x + 6$

Square both sides and simplify

$x = (- x + 6)\ 2$

$x = x^2 + 36 - 12x$

Rewrite in standard form.

$x^2 - 13x + 36 = 0$

Solve the above quadratic equation.

x = 4 and x = 9

Since we squared both sides of the equation, extraneous solutions may be introduced but can be eliminated by checking. After checking, x = 4 is the only value of x that makes f(x) = g(x).

4. Identify if each of the following is one to one function ,onto function or both.

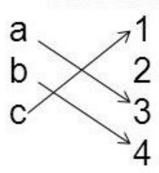


Fig 1

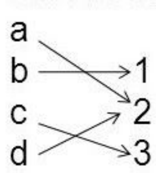


Fig 2

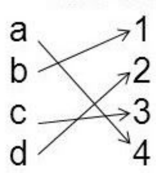


Fig 3

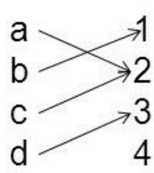


Fig 4

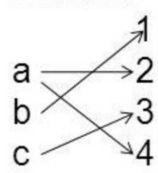


Fig 5

Solution:

Fig 1 one-to-one not onto

Fig 2 onto not one-to-one

Fig 3 one-to-one and onto

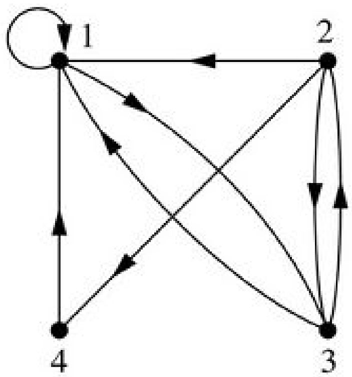
Fig 4 neither

Fig 5 not a function

5. For the set A = {1, 2, 3, 4},show the matrix and digraph representation of the relation R = {(1, 1), (1, 3), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (4, 1)}.

Solution:

	1	2	3	4
1	1	0	1	0
2	1	0	1	1
3	1	1	0	0
4	1	0	0	0



6. Let A = {1, 2, 3, 4, 5} and B = {a, b, c, d}. Which of the following arrow diagram(s) defines onto functions? Explain.

Diagram 1

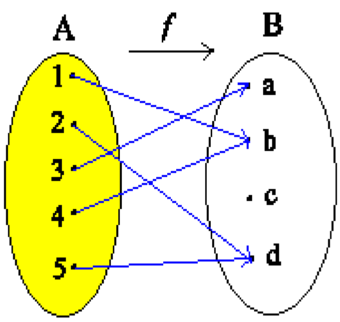


Diagram 2

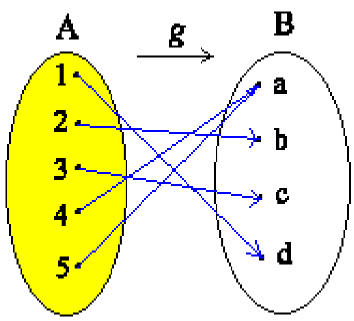
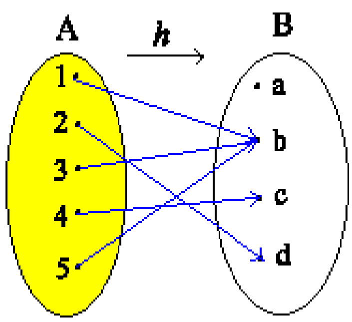


Diagram 3



7. Define functions f from $\mathbf{Z}$ to $\mathbf{Z}$ and g from $\mathbf{R}$ to $\mathbf{R}$ by the formulas: for all $y \in \mathbf{Z}$ and $x \in \mathbf{R}$,

   $f(y) = y^2$ and $g(x) = 2x + 1$

   a. Is f onto? Prove or disprove by giving a counter example.

   b. Is g onto? Prove or disprove by giving a counter example.

Solution:

a.Counter example

Let f(y)=z,let z=3, them f(y)=3

f(y)=y^2 by the definition of f. So,y^2=3.

But the square root of 3 is not an integer, so y$\notin$ Z,Hence there is no integer y for which $f(y)$ = 3, and so f is not onto.

b. g is onto.

Let g(x) = p, p$\in$R.

Let x = (p - 1)/2. Then x$\in$R since subtractions and quotients (other than by 0) of real numbers are real numbers.

It follows that g(x) = 2((p - 1)/2) + 1 by substitution and definition of g

= (p - 1) + 1 by basic algebra

= p

Hence, g is onto.

8. Given relations on the set A={1,2,3} identify if each of the relations is reflexive,symmetric, and transitive.
   a. R1={(1,1),(2,2),(3,3)}
   b. R2((2,2),(2,3),(3,2))
   c. R3{(2,3),(3,2)}
   d. R4={(1,2),(1,3),(2,3)}

Solution:

a.Reflexive,Symmetric,Transitive

b.symmetric

c.symmetric

d. transitive

9. Let R be a relation on the set of real numbers such that aRb iff a-b is an integer. Prove whether R is an equivalence relation.

   Solution:
   a-a=0 and $0 \in Z$
   That is, $\forall$a (aRa). $\therefore$R is reflexive.
   Let a-b = k be an integer.
   Then, b-a = -k, which is also an integer.
   That is, if aRb, then bRa. $\therefore$R is symmetric.
   Let a-b=k and b-c=m where k and m are integers.
   Then, a-c = (a-b)-(c-b) = k-(-m), which is an integer.
   That is, if aRb and bRc, then aRc. $\therefore$R is transitive.

   Because R is reflexive, symmetric and transitive,
   R is an equivalence relation.

10. Let A = {1, 2, 3, 4} and R = {(1,1), (1,2), (2,1), (2,2), (3,4), (4,3), (3,3), (4,4)} be a relation on A. Verify that R is an equivalence relation.

Solution:
R is reflexive since it contains (1,1), (2,2), (3,3) and (4,4).
That is, for every x ,$(x,x) \in R$
R is symmetric since it contains (1,2), (2,1), (3,4), (4,3) and no (a,b) where (b,a) is not in R.
That is, for every x and y if $(x,y) \in R$ then $(y,x) \in R$
R is transitive since for every pair of (x,y) and (y,z), there is (x,z) in R.
That is, for every $(x,y) \in R$ and $(y,z) \in R$ then $(x,z) \in R$
Therefore, R is an equivalence relation.