DIGITAL DESIGN AND COMPUTER ORGANIZATION

Boolean Functions

Reetinder Sidhu

Department of Computer Science and Engineering

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Digital Design and Computer Organization

Course Outline

- Digital Design
  - Combinational logic design
    - **Boolean Functions**
  - Sequential logic design
- Computer Organization
  - Architecture (microprocessor instruction set)
  - Microarchitecure (microprocessor operation)

Boolean Functions

What is a Boolean Function?

- You have learnt logic gates and truth tables in your first year
  - What are they? Where do they come form?

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  - Domain and range are set of real numbers

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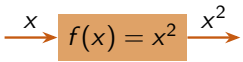
  - Specified as a box:



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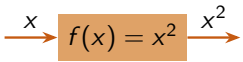

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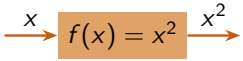

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  - Example: AND function/gate

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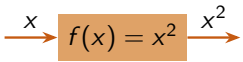

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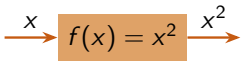

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a	b	y
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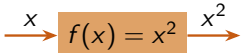

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  - Specified as a logic gate:

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Boolean Constants and Variables

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- In general, inputs and outputs of mathematical functions are represented by variables (like $y = x^2$)
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What does the Truth Table Mean?

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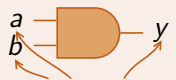

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0	0	0
0	1	0
1	0	0
1	1	1

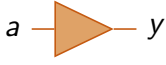
The truth table for and n input function contains 2^n rows

Boolean Functions
Basic Functions / Gates

- Boolean function / gate name: **BUFFER**
- Truth table:

a	y
0	0
1	1

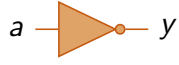
- Symbol:



- Boolean function / gate name: **NOT**
- Truth table:

a	y
0	1
1	0

- Symbol:

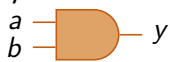


Boolean Functions
Basic Functions / Gates

- Boolean function / gate name: **AND**
- Truth table:

a	*b*	*y*
0	0	0
0	1	0
1	0	0
1	1	1

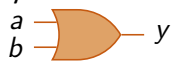
- Symbol:



- Boolean function / gate name: **OR**
- Truth table:

a	*b*	*y*
0	0	0
0	1	1
1	0	1
1	1	1

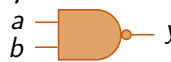
- Symbol:



- Boolean function / gate name: **NAND**
- Truth table:

a	*b*	*y*
0	0	1
0	1	1
1	0	1
1	1	0

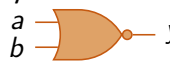
- Symbol:



- Boolean function / gate name: **NOR**
- Truth table:

a	*b*	*y*
0	0	1
0	1	0
1	0	0
1	1	0

- Symbol:



Boolean Functions
Basic Functions / Gates

- Boolean function / gate name: **XOR**
- Truth table:

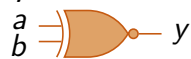
a	b	y
0	0	0
0	1	1
1	0	1
1	1	0

- Symbol:

- Boolean function / gate name: **XNOR**
- Truth table:

a	b	y
0	0	1
0	1	0
1	0	0
1	1	1

- Symbol:



Boolean Functions
What is a Logic Gate?

Boolean Functions
What is a Logic Gate?

- Abstract mathematical level

a	b	y
0	0	1
0	1	1
1	0	1
1	1	0

  - A two input NAND gate can denote a Boolean function which can be specified as a truth table
  - Logic minimization is a task during which gates are typically thought of as Boolean functions

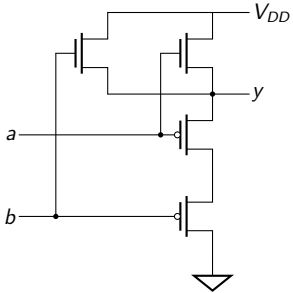
What is a Logic Gate?

- Logic design level

  - A two input NAND gate can denote a component in a logic circuit (may be optionally associated with a value that specifies the gate delay)
  - Such a view of a logic gate is useful during design and analysis of digital circuits

Boolean Functions
What is a Logic Gate?

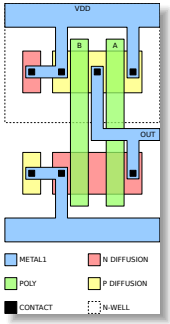
- VLSI design level (standard CMOS implementation)



  - A two input NAND gate can be represented by a transistor level circuit diagram
  - The circuit functions in a digital manner with the transistors acting as switches which are typically in on or off state

Boolean Functions
What is a Logic Gate?

- VLSI layout level

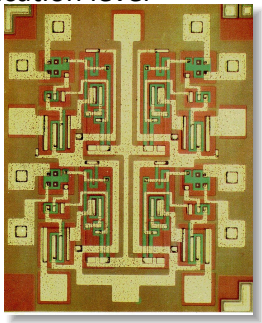


Source: Wikimedia

  - A VLSI designer (“silicon programmer”) specifies layout of various layers (silicon, metal etc.) that compose the required logic

Boolean Functions
What is a Logic Gate?

- VLSI fabrication level



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  - A “chip” (integrated circuit) such as an Intel microprocessor is fabricated in a VLSI fab and may have more than a billion gates on it

What is a Logic Gate?

To summarize:

- Depending on the context (level of abstraction) the term “logic gate” could refer to any of the above interpretations
- The Boolean function and gate levels, and aspects of VLSI design and layout levels, are in the domain of computer science and engineering
- This course will primarily focus on the Boolean function and gate levels

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What is a Logic Gate?

Is a logic gate. . .

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- a digital electronic circuit?

Boolean Functions
What is a Logic Gate?

Is a logic gate. . .

- a Boolean function?
- a digital electronic circuit?
- It is both
  - This wonderful fact enables us to create the machines that perform mathematics, which we call computers