UE19MA251: LINEAR ALGEBRA AND ITS APPLICATIONS

Question Bank: Unit 1

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| 1. | Explain the row approach to solve the system $2x - 3y = 3, x + 3y = 6$ with a neat diagram. What happens to the solution when the second equation isreplacedby $x + 3y = -6$ ? **Answer**: Solution is $x = 3, y = 1$; New solution is $x = -1, y = -5/3$. |
| 2. | Explain the column approach to solve the system $x - 2y = 3, 2x + y = 1$ with a neat diagram. What happens to the solution if the second equation is replaced by $2x - 4y = 5$ ? **Answer:**$x = 1, y = -1$. If the second equation is replaced by $2x - 4y = 5$ then the system is singular. |
| 3 | Solve the following systems of equations using Gaussian elimination:<br>(i) $2x + y + 3z = 1, 2x + 6y + 8z = 3, 6x + 8y + 18z = 5$<br>**Answer**: $x = 3/10, y = 2/5, z = 0$<br>(ii) $x + 2y - z = 6,\ 2x + y + z = 3,\ x - y + z = -2$<br>**Answer**: $x = 1, y = 2, z = -1$<br>(iii) $3x + y - 6z = -10, 2x + y - 5z = -8, 6x - 3y + 3z = 0$<br>**Answer**: $x = k - 2, y = 3k - 4, z = k$ where $k$ is a scalar<br>(iv) $x + z = 1, x + y + z = 2, x - y + z = 1$. What if the right hand side is $(1, 2, 0)$ ?<br>**Answer** : Inconsistent system ; $x = 1 - k, y = 1, z = k, k \epsilon R$ |
| 4 | Investigate the values of $\lambda$ and μ such that $$x + 3y + 5z = 9$$ $$x - y + 2z = 1$$ $$2x + 2y + \lambda z = \mu$$ has (i) unique solution (ii) infinitely many solution (iii) no solution<br>**Answer:** (i) unique solution when$\lambda \neq 7$ (ii) $\lambda = 7,\ \mu = 10$ (iii) $\lambda$ should be equal to 7 $(r(A) = 2)$ and $\mu - 10 \neq 0$ i.e., $\mu \neq 10$ $(r(A{:}b) = 3)$. |
| 5. | Determine the values of $a$ and $b$ for which the system of equations $x + y + az = 2b$, $x + 3y + (2 + 2a)z = 7b, 3x + y + (3 + 3a)z = 11b$willhave<br>(i) unique nontrivialsolution (ii)trivialsolution ( iii )nosolution (iv) infinity ofsolutions.<br>**Answer** : (i) $a \neq -5$ and any $b$ (ii) $a \neq -5$ and $b = 0$ (iii) $a = -5$ and $b \neq 0$<br>(iv) $a = -5$and $b = 0$. |
| 6. | Let $A = \begin{bmatrix} 3 & -6 & 2 & -1 \\ -2 & 4 & 1 & 3 \\ 0 & 0 & 1 & 1 \\ 1 & -2 & 1 & 0 \end{bmatrix}$and $b = (b_1,\ b_2,\ b_3,\ b_4)$. Use the method of Gaussian Elimination to find a condition on thecomponentsof $b$ so that thesystem$Ax = b$ is consistent.<br>When $b = (2, 1, 1, 1)$, if $(x, 0, 0, 1)$ is a solution of the system $Ax = b$ findx.<br>**Answer**:$7b_4 - b_2 - 3b_1 = 0$ and $7b_3 - 3b_2 - 2b_1 = 0$.<br>When $b = (2, 1, 1, 1)$then $3x - 6y + 2z - t = 2, z + t = 1$, Solving we get $x = 1$. |
| 7. | Determine the equation of the polynomial $y = f(x)$ of degree 2 whose graph passes through thepoints $(1, 6)$, $(2, 3)$ and $(3, 2)$. **Answer** :$y = 11 - 6x + x^2$. |
| 8. | Which three matrices $E_{21}, E_{31}, E_{32}$ put $A$ into a triangular form $A = \begin{bmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{bmatrix}$.<br>Multiply those $E$'s to get one matrix $M$ thatdoeselimination $MA = U$. |

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| | **Answer :** $M = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 10 & -2 & 1 \end{bmatrix}$ |
| 9. | Write down the elementary matrices $E$ , $F$ , $G$ associated with the system of equations $2u + v + 3w = -1$ , $4u + v + 7w = 5$ , $-6u - 2v - 12w = -2$ . Also find the $LU$ decomposition of $A$.<br>**Answer :** $L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & -1 & 1 \end{bmatrix}$ , $U = \begin{bmatrix} 2 & 1 & 3 \\ 0 & -1 & 1 \\ 0 & 0 & -2 \end{bmatrix}$ |
| 10. | Find $L$ and $U$ for the matrix $A = \begin{bmatrix} 1 & -1 & 0 & 3 \\ 2 & -2 & 0 & 8 \\ -1 & -1 & 4 & -2 \\ -2 & -2 & 6 & -3 \end{bmatrix}$.<br>Write down the permutation matrices, if any, used in the process of elimination.<br>**Answer**: $L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -2 & 2 & 1 & 0 \\ 2 & 0 & 0 & 1 \end{bmatrix}$ , $U = \begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & -2 & 4 & 1 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$<br>The permutation matrices used are $P_{23}$ and $P_{34}$ |
| 11. | Find $LU$ and $LDU$ factorization for $A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix}$<br>**Answer:**<br>$$A = LDU = \begin{bmatrix} 1 & 0 & 0 \\ \frac{2}{3} & 1 & 0 \\ \frac{1}{3} & \frac{-5}{11} & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & -\frac{11}{3} & 0 \\ 0 & 0 & -\frac{24}{33} \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & 1 & \frac{7}{11} \\ 0 & 0 & 1 \end{bmatrix}$$ |
| 12. | Find $PA = LDU$ factorization for $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 2 & 3 & 4 \end{bmatrix}$ , $B = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 1 \end{bmatrix}$<br>**Answer**:<br>$$PA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$<br>$$PB = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ |
| 13. | Find the symmetric factorization of $A$ in the form $L\,D\,L^T$ and find conditions on $a, b, c$ , $d$ to get $A = LU$ with four pivots. $A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}$ |
| | Suppose $A$ is a 4 x 4 identity matrix except for a vector $v$ in column 2: Factor $A$ into $LU$ assuming |

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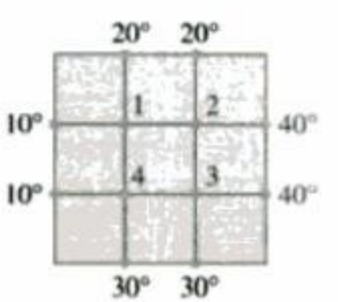
| 14. | $v_2 \neq 0. A = \begin{bmatrix} 1 & v_1 & 0 & 0 \\ 0 & v_2 & 0 & 0 \\ 0 & v_3 & 1 & 0 \\ 0 & v_4 & 0 & 1 \end{bmatrix}$ |
| 15. | Use the Gauss – Jordan method to invert the followingmatrices<br>$.i. A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$ $ii\ \ A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ $iii\ \ A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$<br><br>**Answer**: i. $A^{-1} = \frac{1}{14}\begin{bmatrix} 3 & -1 & 5 \\ 5 & 3 & -1 \\ -1 & 5 & 3 \end{bmatrix}$ ii.$A^{-1} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$ iii.$A =$<br>$\begin{bmatrix} -2/8 & 4/8 & 2/8 \\ 3/8 & -2/8 & 1/8 \\ 7/8 & -2/8 & -3/8 \end{bmatrix}$ |
| 16. | Producing $x$ trucks and $y$ planes requires $x + 50\,y$ tons of steel, $40\,x + 1000\,y$ pounds of rubber and $2x + 50\,y$months of labor. If the unit costs $u, v, w$ are \$ $700$ per ton, \$ $3$ per pound and \$ $3000$ per month, what are the values of one truck and one plane?<br>**Answer**: 6820 , 188000 |
| 17. | Assume that the plate shown in the figure represents a cross section of a metal beam with negligible heat flow in the direction perpendicular to the plate. Let $T_1, T_2, T_3$ and $T_4$ denote the temperatures at the four interior nodes of the mesh. The temperature at a node is approximately equal to the average of the four nearest nodes – to the right, left, above and below. For instance $T_1 = (10 + 20 + T_2 + T_4)\,/\,4$ or $4\,T_1 - T_2 - T_4 = 30$. Write a system of 4 equations whose solution gives estimates for the temperatures$T_1$, $T_2$, $T_3$and $T_4$.Hence find its solution.<br> |
| 18. | Propane is a common gas used for cooking and home heating. Each molecule of propane is comprised of 3 atoms of carbon, and 8 atoms of hydrogen written as $C_3H_8$. When propane burns, it combines with oxygen gas $O_2$ to form carbon dioxide $CO_2$ and water $H_2O$. Balance the chemical equation $C_3H_8 + O_2 \rightarrow CO_2 + H_2O$ that describes this process.<br>**Answer**: $2\,C_3H_8 + 10\,O_2 \rightarrow 6\,CO_2 + 8\,H_2O$ |