1) Check if singular or non-singular

$$u+v+w=1$$
$$2u+2v+5w=2$$
$$4u+6v+8w=3$$

$$A:b = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 2 & 2 & 5 & 2 \\ 4 & 6 & 8 & 3 \end{array}\right] \xrightarrow[R_3 \to R_3 - 4R_1]{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & 0 & 3 & 0 \\ 0 & 2 & 4 & -1 \end{array}\right]$$

$$\Big\downarrow R_2 \leftrightarrow R_3$$

$$\rho(A:b) = \rho(A) = 3 = n \qquad \left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & 2 & 4 & -1 \\ 0 & 0 & 3 & 0 \end{array}\right]$$

$\therefore$ system is consistent, non-singular

2) $u+v+w=1$ If singular, discuss solution

$$2u+2v+5w=2$$
$$4u+4v+8w=3$$

$$A:b = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 2 & 2 & 5 & 2 \\ 4 & 4 & 8 & 3 \end{array}\right] \xrightarrow[R_4 \to R_4 - 4R_1]{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 4 & -1 \end{array}\right]$$

$$\Big\downarrow R_3 \to R_3 - 2R_2$$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & -1 \end{array}\right]$$

$\rho(A) = 2$

$\rho(A:b) = 3 = n$

$\therefore$ system is singular, inconsistent

3) $x + y - 2z = -3$

$2x + 5y + 3z = 11$

$-x + 3y + z = 5$

Singularity?

$$[A:b] = \left[\begin{array}{ccc:c} 1 & 1 & -2 & -3 \\ 2 & 5 & 3 & 11 \\ -1 & 3 & 1 & 5 \end{array}\right] \xrightarrow[R_3 \to R_3 + R_1]{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc:c} 1 & 1 & -2 & -3 \\ 0 & 3 & 7 & 17 \\ 0 & 4 & -1 & 2 \end{array}\right]$$

$$R_3 \to R_3 - 4/3\, R_2 \Big\downarrow$$

$$\rho(A:b) = \rho(A) = n = 3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -2 & -3 \\ 0 & 3 & 7 & 17 \\ 0 & 0 & -31/3 & -62/3 \end{array}\right]$$

$\therefore$ solution is consistent, non-singular and unique

$$\frac{-31}{3} z = \frac{-62}{3}$$

$$3y + 14 = 17$$

$$x + 1 - 4 = -3$$
$$x - 3 = -3$$

$\boxed{z = 2}$ $\boxed{y = 1}$ $\boxed{x = 0}$

4) $y - 2z = 4$
$x + 3y + 2z = 1$
$-2x + 3y + z = 2$

Singularity?

$$A:b = \left[\begin{array}{ccc|c} 0 & 1 & -2 & 4 \\ 1 & 3 & 2 & 1 \\ -2 & 3 & 1 & 2 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 1 \\ 0 & 1 & -2 & 4 \\ -2 & 3 & 1 & 2 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 + 2R_1}$$

$$\begin{bmatrix} 1 & 3 & 2 & : & 1 \\ 0 & 1 & -2 & : & 4 \\ 0 & 0 & 23 & : & -32 \end{bmatrix} \xleftarrow{R_3 \to R_3 - 9R_2} \begin{bmatrix} 1 & 3 & 2 & : & 1 \\ 0 & 1 & -2 & : & 4 \\ 0 & 9 & 5 & : & 4 \end{bmatrix}$$

$$\rho(A:b) = \rho(A) = n = 3$$

$\therefore$ non-singular, consistent, unique

$$23z = -32$$

$$\boxed{z = \frac{-32}{23}}$$

$$y + \frac{64}{23} = 4$$

$$y = \frac{92-64}{23}$$

$$\boxed{y = \frac{28}{23}}$$

$$x + \frac{84}{23} - \frac{64}{23} = 1$$

$$x + \frac{20}{23} = \frac{23}{23}$$

$$\boxed{x = \frac{3}{23}}$$

5) Singularity

$$u+v+w=0$$
$$u+2v+3w=0$$
$$3u+5v+7w=1$$

$$A:b=\left[\begin{array}{ccc:c} 1 & 1 & 1 & 0 \\ 1 & 2 & 3 & 0 \\ 3 & 5 & 7 & 1 \end{array}\right] \xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to R_2 - R_1} \left[\begin{array}{ccc:c} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 2 & 4 & 1 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 2R_2} \left[\begin{array}{ccc:c} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$$\rho(A) = 2$$
$$\rho(A:b) = 3 = n$$

$\therefore$ system inconsistent, singular

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