PES University, Bangalore

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UE19CS203 – STATISTICS FOR DATA SCIENCE

Unit-2 - Random Variables

QUESTION BANK

Continuous Random Variables

Exercises for Section 2.4

[Text Book Exercise – Section 2.4 – Q. No. [13 – 26] – Pg. No. [114 - 116]]

1. Resistors labeled 100Ω have true resistances that are between 80Ω and 120Ω. Let *X* be the mass of a randomly chosen resistor. The probability density function of *X* is given by

$$f(x) = \begin{cases} \dfrac{x-80}{800} & 80 < x < 120 \\ 0 & otherwise \end{cases}$$

   a) What proportion of resistors have resistances less than 90Ω?
   b) Find the mean resistance.
   c) Find the standard deviation of the resistances.
   d) Find the cumulative distribution function of the resistances.

2. Elongation (in percent) of steel plates treated with aluminum are random with probability density function

$$f(x) = \begin{cases} \dfrac{x}{250} & 20 < x < 30 \\ 0 & otherwise \end{cases}$$

   a) What proportion of steel plates have elongations greater than 25%?
   b) Find the mean elongation.
   c) Find the variance of the elongations.
   d) Find the standard deviation of the elongations.
   e) Find the cumulative distribution function of the elongations.
   f) A particular plate elongates 28%. What proportion of plates elongate more than this?

3. The lifetime in months of a transistor in a certain application is random with probability density function.

$$f(x) = \begin{cases} e^{-0.1t} & t > 0 \\ 0 & t \leq 0 \end{cases}$$

a) Find the mean lifetime.
b) Find the standard deviation of the lifetimes.
c) Find the cumulative distribution function of the lifetime.
d) Find the probability that the lifetime will be less than 12 months.

4. A process that manufactures piston rings produces rings whose diameters (in centimeters) vary according to the probability density function

$$f(x) = \begin{cases} 3[1 - 16\,(x - 10)^2] & 9.75 < x < 10.25 \\ 0 & otherwise \end{cases}$$

a) Find the mean diameter of rings manufactured by this process.
b) Find the standard deviation of the diameters of rings manufactured by this process. (Hint: Equation 2.36 may be easier to use than Equation 2.37.)
c) Find the cumulative distribution function of piston ring diameters.
d) What proportion of piston rings have diameters less than 9.75 cm?
e) What proportion of piston rings have diameters between 9.75 and 10.25 cm?

5. Refer to Exercise 16. A competing process produces rings whose diameters (in centimeters) vary according to the probability density function.

$$f(x) = \begin{cases} 15[1 - 25\,(x - 10.05)^2]/4 & 9.85 < x < 10.25 \\ 0 & otherwise \end{cases}$$

Specifications call for the diameter to be 10.0 ± 0.1 cm. Which process is better, this one or the one in Exercise 16? Explain.

6. The lifetime, in years, of a certain type of pump is a random variable with probability density function

$$f(x) = \begin{cases} \dfrac{64}{(x+2)^5} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

a) What is the probability that a pump lasts more than two years?
b) What is the probability that a pump lasts between two and four years?

c) Find the mean lifetime.
d) Find the variance of the lifetimes.
e) Find the cumulative distribution function of the lifetime.
f) Find the median lifetime.
g) Find the 60th percentile of the lifetimes.

7. The level of impurity (in percent) in the product of a certain chemical process is a random variable with probability density function

$$f(x) = \begin{cases} \frac{3}{64}x^2(4-x) & 0 < x < 4 \\ 0 & otherwise \end{cases}$$

a) What is the probability that the impurity level is greater than 3%?
b) What is the probability that the impurity level is between 2% and 3%?
c) Find the mean impurity level.
d) Find the variance of the impurity levels.
e) Find the cumulative distribution function of the impurity level.

8. The main bearing clearance (in mm) in a certain type of engine is a random variable with probability density function

$$f(x) = \begin{cases} 625x & 0 < x < 0.04 \\ 50 - 625x & 0.04 < x \le 0.08 \\ 0 & otherwise \end{cases}$$

a) What is the probability that the clearance is less than 0.02 mm?
b) Find the mean clearance.
c) Find the standard deviation of the clearances.
d) Find the cumulative distribution function of the clearance.
e) Find the median clearance.
f) The specification for the clearance is 0.015 to 0.063 mm. What is the probability that the specification is met?

9. The error in the length of a part (absolute value of the difference between the actual length and the target length), in mm, is a random variable with probability density function

$$f(x) = \begin{cases} 12(x^2 - x^3) & 0 < x < 1 \\ 0 & otherwise \end{cases}$$

a) What is the probability that the error is less than 0.2 mm?
b) Find the mean error.
c) Find the variance of the error.
d) Find the cumulative distribution function of the error.
e) The specification for the error is 0 to 0.8 mm. What is the probability that the specification is met?

10. The concentration of a reactant is a random variable with probability density function.

$$f(x) = \begin{cases} \dfrac{2e^{-2x}}{1 - e^{-2}} & 0 < x < 1 \\ 0 & otherwise \end{cases}$$

a) What is the probability that the concentration is greater than 0.5?
b) Find the mean concentration.
c) Find the probability that the concentration is within ± 0.1 of the mean.
d) Find the standard deviation σ of the concentrations.
e) Find the probability that the concentration is within $\pm\sigma$ of the mean.
f) Find the cumulative distribution function of the concentration.

11. The thickness of a washer (in mm) is a random variable with probability density function

$$f(x) = \begin{cases} \dfrac{3}{52}\, x(6 - x) & 2 < x < 4 \\ 0 & otherwise \end{cases}$$

a) What is the probability that the thickness is less than 2.5 m?
b) What is the probability that the thickness is between 2.5 and 3.5 m?
c) Find the mean thickness.
d) Find the standard deviation σ of the thicknesses.
e) Find the probability that the thickness is within $\pm\sigma$ of the mean.
f) Find the cumulative distribution function of the thickness.

12. Particles are a major component of air pollution in many areas. It is of interest to study the sizes of contaminating particles. Let *X* represent the diameter, in micrometers, of a randomly chosen particle. Assume that in a certain area, the probability density function of *X* is inversely proportional to the volume of the particle; that is, assume that,

$$f(x) = \begin{cases} \dfrac{c}{x^3} & x \geq 1 \\ 0 & x < 1 \end{cases}$$

where c is a constant.

a) Find the value of c so that f (x) is a probability density function.
b) Find the mean particle diameter.
c) Find the cumulative distribution function of the particle diameter.
d) Find the median particle diameter.
e) The term PM10 refers to particles 10 μm or less in diameter. What proportion of the contaminating particles are PM10?
f) The term PM2.5 refers to particles 2.5 μm or less in diameter. What proportion of the contaminating particles are PM2.5?
g) What proportion of the PM10 particles are PM2.5?

13. The repair time (in hours) for a certain machine is a random variable with probability density function

$$f(x) = \begin{cases} xe^{-x} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

a) What is the probability that the repair time is less than 2 hours?
b) What is the probability that the repair time is between 1.5 and 3 hours?
c) Find the mean repair time.
d) Find the cumulative distribution function of the repair times.

14. The diameter of a rivet (in mm) is a random variable with probability density function

$$f(x) = \begin{cases} 6\,(x - 12)(13 - x) & 12 < x \leq 13 \\ 0 & otherwise \end{cases}$$

a) What is the probability that the diameter is less than 12.5 mm?
b) Find the mean diameter.
c) Find the standard deviation of the diameters.
d) Find the cumulative distribution function of the diameter.
e) The specification for the diameter is 12.3 to 12.7 mm. What is the probability that the specification is met?