A periodic $f(x) = |x|$ of period 4 is defined as $-2 < x < 2$. Find its FS expansion.

Sol⁻

$$f(x) = \begin{cases} -x & -2 < x < 0 \text{ or } (-2,0) \\ x & 0 < x < 2 \text{ or } (0,2) \end{cases}$$

$2-(-2) = 4 \quad \therefore 2l = 4$
$$\Rightarrow l = 2$$

$\therefore \phi(x) = -x \quad \& \quad \psi(x) = x$

$\therefore \phi(2l-x) = \phi(4-x) = -(4-x)$

$\phi(-x) = -(-x) = x = \psi(x) \Rightarrow f(x)$ is even fn

$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{2}$$

$$a_0 = \frac{2}{l} \int_0^2 f(x)\,dx = \int_0^2 x\,dx = \frac{x^2}{2}\Big]_0^2 = 2$$

$$a_n = \frac{2}{2} \int_0^2 x \cos\left(\frac{n\pi x}{2}\right)dx$$

$$= x \left(\frac{\sin \frac{n\pi x}{2}}{n\pi/2}\right) - (1)\left(\frac{-\cos \frac{n\pi x}{2}}{n^2\pi^2/4}\right)\Big]_0^2$$

$$= \frac{4}{n^2\pi^2}\left[(-1)^n - 1\right]$$

$$\boxed{\therefore f(x) = 1 + \frac{4}{\pi^2}\sum_{n=1}^{\infty} \frac{\left[(-1)^n-1\right]}{n^2} \cdot \cos\left(\frac{n\pi x}{2}\right)}$$