

PES University, Bangalore

Department of Computer Science and Engineering

Automata Formal Languages & Logic

Q&A for First Order Logic- Quantifiers, Terms, Axioms

1. Given Vocabulary statement, write it in Clausal Forms in FOL

All people who are not poor and are smart are happy. Those people who read are not stupid.
John can read and is wealthy. Happy people have exciting lives.

Solution:-

$\forall X (\neg \text{poor}(X) \wedge \text{smart}(X) \rightarrow \text{happy}(X))$

$\forall Y (\text{read}(Y) \rightarrow \text{smart}(Y))$

$\text{read}(\text{john}) \wedge \neg \text{poor}(\text{john})$

$\forall Z (\text{happy}(Z) \rightarrow \text{exciting}(Z))$

2. Consider the following axioms:

- Every child loves anyone who gives the child any present.
 $\forall x \forall y \forall z (\text{CHILD}(x) \wedge \text{PRESENT}(y) \wedge \text{GIVE}(z,y,x) \rightarrow \text{LOVES}(x,z))$
- Every child will be given some present by Santa if Santa can travel on Christmas eve.
 $\text{TRAVEL}(\text{Santa}, \text{Christmas}) \rightarrow \forall x (\text{CHILD}(x) \rightarrow \exists y (\text{PRESENT}(y) \wedge \text{GIVE}(\text{Santa}, y, x)))$
- It is foggy on Christmas eve.
 $\text{FOGGY}(\text{Christmas})$
- Anytime it is foggy, anyone can travel if he has some source of light.
 $\forall x \forall t (\text{FOGGY}(t) \rightarrow (\exists y (\text{LIGHT}(y) \wedge \text{HAS}(x,y)) \rightarrow \text{TRAVEL}(x,t)))$
- Any reindeer with a red nose is a source of light.
 $\forall x (\text{RNR}(x) \rightarrow \text{LIGHT}(x))$
- (Conclusion) If Santa has some reindeer with a red nose, then every child loves Santa.
 $(\exists x (\text{RNR}(x) \wedge \text{HAS}(\text{Santa}, x))) \rightarrow \forall y (\text{CHILD}(y) \rightarrow \text{LOVES}(y, \text{Santa}))$

3. Given Vocabulary statement, write it in Clausal Forms in FOL and convert in CNF form.

- Harry, Ron and Draco are students of Hogwarts school of wizards
- Every student is either wicked or is a good Quidditch player, or both
- No Quidditch player likes rain and all wicked students like potions
- Draco dislikes whatever Harry likes and likes whatever Harry dislikes
- Draco likes rain and potions



From Fact 1:

C1 - Student(Harry)

C2 - Student(Ron)

C3 - Student(Draco)

From Fact 2:

$\forall x: \text{Student}(x) \Rightarrow \text{Wicked}(x) \vee \text{Quidditch}(x)$

C4 - $\sim \text{Student}(x) \vee \text{Wicked}(x) \vee \text{Quidditch}(x)$

From Fact 3:

$\forall x: \text{Quidditch}(x) \Rightarrow \sim \text{Likes}(x, \text{Rain}) \wedge$

$\forall x: \text{Wicked}(x) \Rightarrow \text{Likes}(x, \text{Potions})$

C5 - $\sim \text{Quidditch}(x) \vee \sim \text{Likes}(x, \text{Rain})$

C6 - $\sim \text{Wicked}(x) \vee \text{Likes}(x, \text{Potions})$

From Fact 4:

$\forall x: \text{Likes}(\text{Harry}, x) \Leftrightarrow \sim \text{Likes}(\text{Draco}, x)$

C7 - $\sim \text{Likes}(\text{Harry}, x) \vee \sim \text{Likes}(\text{Draco}, x)$

C8 - $\text{Likes}(\text{Harry}, x) \vee \text{Likes}(\text{Draco}, x)$

From Fact 5:

C9 - $\text{Likes}(\text{Draco}, \text{Rain})$

C10 - $\text{Likes}(\text{Draco}, \text{Potions})$