



PES UNIVERSITY, Bangalore
(Established under Karnataka Act No. 16 of 2013)

UE19CS205

Scheme & Solution

IN SEMESTER ASSESSMENT (ISA-1)- B.TECH III SEMESTER
October, 2020

Automata Formal Languages & Logic

Time: 2 Hrs

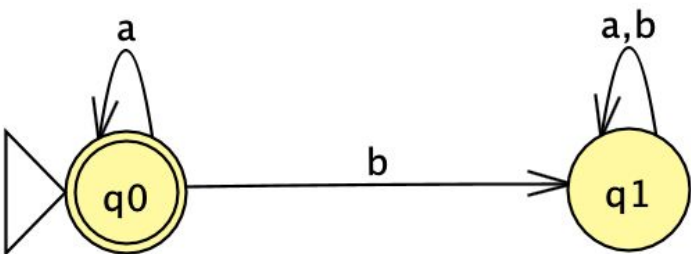
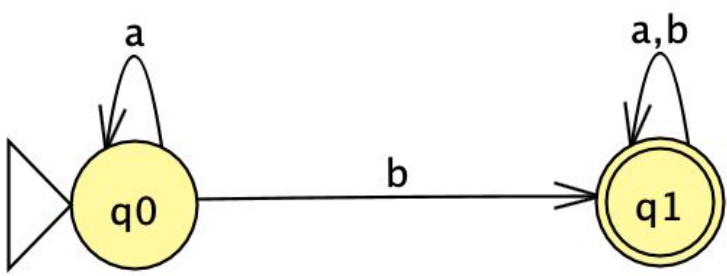
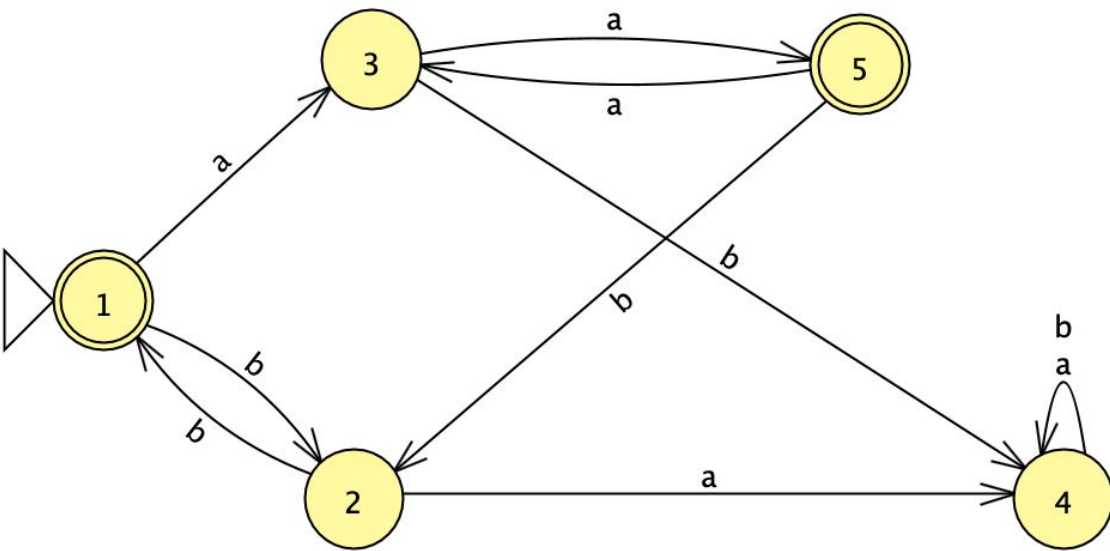
Answer All Questions

Max Marks: 60

Note:

- Read all the Questions carefully before answering.
- The Question paper spans over 3 sheets and contains 6 Questions.
- Each Question carries 10 Marks and contains exactly 2 sub-parts (part a and part b).

1	a	<p>For the alphabet $\Sigma = \{1, 2, 3, 4\}$, construct an NFA for the language :</p> <p>$L = \{ w \in \Sigma^* \mid \text{the last character of } w \text{ appears nowhere else in the string, and } w > 1 \}$</p> <p>Solution:</p>	5
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	b	<p>Let $\Sigma = \{a, b\}$. Consider the language $L = \{a^*\}$.</p> <p>Construct a DFA for the language L^c which accepts the complement of the Language L.</p> <p>Solution:</p> <p>DFA that accepts the language $L = \{a^*\}$ is given as:</p>  <p>DFA that accepts the complement of the Language L given as:</p>  <p>DFA for $L = \{a^*\}$: A DFA with two states, q_0 and q_1. q_0 is the start state and an accepting state (double circle). There is a self-loop on q_0 labeled 'a'. There is a transition from q_0 to q_1 labeled 'b'. There is a self-loop on q_1 labeled 'a, b'.</p> <p>DFA for L^c: A DFA with two states, q_0 and q_1. q_0 is the start state. q_1 is an accepting state (double circle). There is a self-loop on q_0 labeled 'a'. There is a transition from q_0 to q_1 labeled 'b'. There is a self-loop on q_1 labeled 'a, b'.</p>	5
2	a	<p>Minimize the following DFA of 5 States:</p>  <p>DFA with 5 states: 1 (start, accepting), 2, 3, 4, 5 (accepting). Transitions: 1 to 3 on 'a', 1 to 2 on 'b', 2 to 1 on 'b', 2 to 4 on 'a', 3 to 5 on 'a', 5 to 3 on 'a', 3 to 2 on 'b', 4 to 2 on 'b', 4 to 5 on 'b', 4 to 4 on 'a' and 'b'.</p> <p>Solution:</p>	5

Transition Table of the given DFA :

	a	b
$\rightarrow 1^*$	3	2
2	4	1
3	5	4
4	4	4
5*	3	2

We minimize the DFA using the Table Filling algorithm:

2	X			
3	X	X		
4	X	X	X	
5*		X	X	X
	1*	2	3	4

Distinguishable pairs (Pair of Final and Non-Final State) are marked in green color.

We must mark the following states as distinguishable due to the following reasons:

$$\delta((2,4), b) = \{1,4\}$$

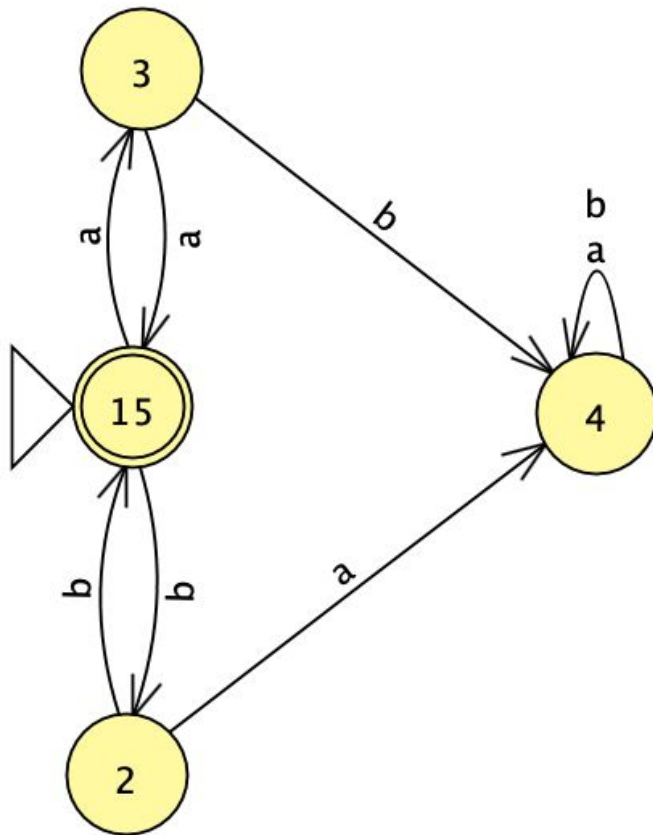
$$\delta((2,3), a) = \{4,5\}$$

$$\delta((3,4), a) = \{4,5\}$$

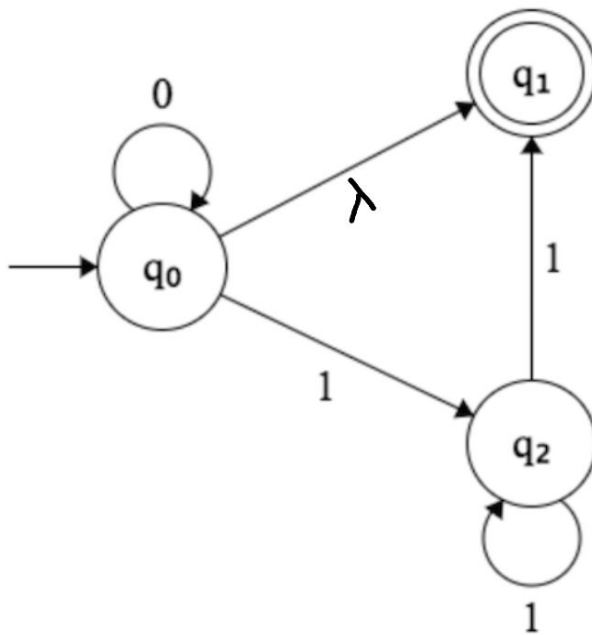
The two final states can be merged as:

$$\delta((1,5), a) = \{3\} \text{ and } \delta((1,5), b) = \{2\}$$

Hence the minimized DFA is given as:



b Convert the following NFA to DFA:



Solution:

5

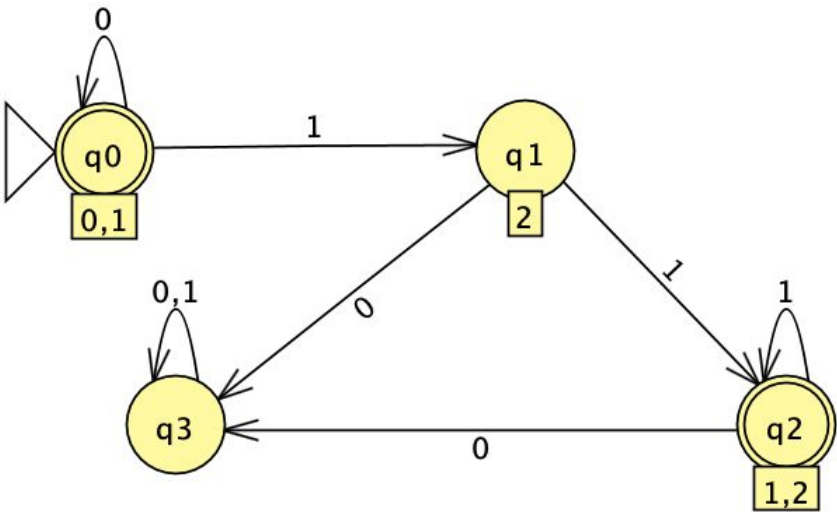
Transition Table of the given λ -NFA :

	0	1	λ -Closur e
→ q0	q0	q2	q0, q1
q1*	Φ	Φ	q1
q2	Φ	q1, q2	q2

Transition Table of the given DFA :

	0	1
→ q0 q1	q0 q1	q2
q2	Φ	q1, q2
q1q2	Φ	q1, q2
Φ	Φ	Φ

DFA Transition Diagram :

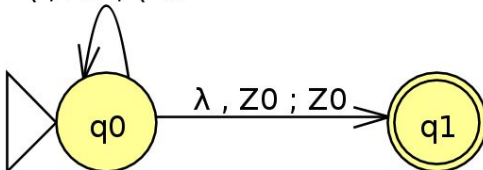


3	a	<p>Draw a NFA that accepts the language corresponding to the regular expression: ((01)* + (12)*)01</p> <p>Solution:</p>	5
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		<pre> graph TD q0((q0)) -- lambda --> q1((q1)) q0 -- lambda --> q2((q2)) q1 -- 0 --> q3((q3)) q2 -- 0 --> q3 q1 -- 0 --> q5((q5)) q5 -- 1 --> q1 q2 -- 2 --> q6((q6)) q6 -- 1 --> q2 q3 -- 1 --> q4(((q4))) </pre>	
	b	<p>Let $\Sigma = \{1, 2, \leq\}$ and let L be the language defined as follows: <math>L = \{ w \in \Sigma^* \mid w \text{ is a valid chain of inequalities relating the numbers 1 and 2} \}</math>. For example: The following strings belong to the language : $1 \leq 2$, $1 \leq 1 \leq 2 \leq 2$, $2 \leq 2 \leq 2$, $1 \leq 1 \leq 1 \leq 1$, $1 \leq 1 \leq 2$ but, the following doesn't: $2 \leq \leq$, ≤ 2 , λ , 1 , $12 \leq 22$ Note in particular that inequalities involving numbers like 12, 222, 121212, etc. whose digits are 1 and 2 aren't allowed i.e. $121 \leq 112$ (the inequality should only relate the numbers that is, single digit 1 and 2) and any individual number itself isn't allowed(i.e., $1 \notin L$ and $2 \notin L$). Construct a regular expression for L. Solution: $(1 \leq 1) (\leq 1)^* + (1 \leq 2) (\leq 2)^* + (2 \leq 2) (\leq 2)^*$</p>	5
4	a	<p>Using Pumping lemma, Determine whether the following language on $\Sigma = \{a,b\}$ is regular or not. $L = \{a^n b^n : n \geq 1\}$ Solution:</p> <ul style="list-style-type: none"> The opponent claims that the language $L = \{a^n b^n : n \geq 1\}$ is regular. Let the number of states in the opponent's hypothetical automata for language L is n (Pumping length). 	5

		<ul style="list-style-type: none"> We choose a string $w = a^n b^n$ such that, $w > n$ (length of the string is greater than the number of states in the machine) and $w \in L$. $\forall w = xyz$ (for any break up of the string in 3 parts) such that, $xy \leq n$ (y- loop is within the n states) and $y \geq 1$ (loop is made up of at least one symbol) In our string $w = a^n b^n$, the first n symbols are made up only of a's. Hence if we assume the loop is made up of single 'a', we can break the string as: $a^{n-1} (a)^i b^n$ We see that, if we pump down the loop that is choose $i=0$, the resultant string does not belong to the language L, as the number of a's and b's become unequal i.e., $a^{n-1} b^n \notin L$ <p>Hence proved that the language L is not regular.</p>	
	b	<p>Convert the following Finite Automata to Regular Grammar:</p> <pre> graph LR S((S)) -- a --> A((A)) A -- a --> A A -- a --> C(((C))) A -- a --> B((B)) B -- b --> C C -- λ --> A style S fill:#ffff00 style A fill:#ffff00 style B fill:#ffff00 style C fill:#ffff00 </pre> <p>Solution: $S \rightarrow aA$ $A \rightarrow aA \mid aC \mid aB \mid \lambda$ $B \rightarrow bC$ $C \rightarrow A \mid \lambda$</p>	5
5	a	<p>Construct a Context free grammar to generate variable declaration statements in a C language. For example, your grammar should be able to generate strings of the following kind:</p> <pre> int a; int a, b, c, d; int a, b=2, c=5, d; int d=8; </pre> <p>Assume you are handling only the basic types int and float. The Terminals in your grammar are : {int, float, id, num, =, , , ;} Here id denotes an identifier (that is a variable name) and num denotes a number. Assume the Start symbol is D.</p> <p>Solution:</p>	5

	<div>$D \rightarrow T L ;$ $T \rightarrow \text{int} \mid \text{float}$ $L \rightarrow L, X \mid X$ $X \rightarrow \text{id} \mid \text{id} = \text{num}$</div>																																					
b	<div>Let G be the grammar below.</div> <div>$\left\{ \begin{array}{l} S \rightarrow AB \mid SS \mid a \\ A \rightarrow BS \mid CD \mid b \\ B \rightarrow DD \mid b \\ C \rightarrow DE \mid a \mid b \\ D \rightarrow a \\ E \rightarrow SS \end{array} \right.$</div> <div>Use CYK algorithm to answer the following membership question: Does the string $w = \text{"abaab"}$ belong to $L(G)$?</div> <table><tr><td>5</td><td>S, E</td><td></td><td></td><td></td><td></td></tr><tr><td>4</td><td>S, E</td><td>S, A</td><td></td><td></td><td></td></tr><tr><td>3</td><td>Φ</td><td>S, A</td><td>S</td><td></td><td></td></tr><tr><td>2</td><td>Φ</td><td>A</td><td>S, E, A, B</td><td>Φ</td><td></td></tr><tr><td>1</td><td>S, C, D</td><td>A, B, C</td><td>S, C, D</td><td>S, C, D</td><td>A, B, C</td></tr><tr><td></td><td>a</td><td>b</td><td>a</td><td>a</td><td>b</td></tr></table>	5	S, E					4	S, E	S, A				3	Φ	S, A	S			2	Φ	A	S, E, A, B	Φ		1	S, C, D	A, B, C	S, C, D	S, C, D	A, B, C		a	b	a	a	b	5
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	a	b	a	a	b																																	
6	<div>a</div> <div>With an example, explain what is an Ambiguous grammar?</div> <div>Solution:</div> <div>A grammar $G = (V, T, P, S)$ is called an Ambiguous grammar if there exists 2 different leftmost derivations or 2 different rightmost derivations (basically 2 different structures) for the same string $w \in L(G)$.</div> <div>For example :</div> <div>$S \rightarrow aS \mid a \mid \lambda$ is an ambiguous grammar as we can obtain string a in two different ways:</div> <table><tr><td>LMD 1 $S \Rightarrow aS$ $S \Rightarrow a$ (using $S \rightarrow \lambda$)</td><td>LMD 2 $S \Rightarrow a$</td></tr></table>	LMD 1 $S \Rightarrow aS$ $S \Rightarrow a$ (using $S \rightarrow \lambda$)	LMD 2 $S \Rightarrow a$	3																																		
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b	<p>Let $\Sigma = \{ (, [,] \}$. Consider the language L as</p> <p>$L = \{ \text{properly nested strings from } \Sigma^* \}$.</p> <p>So $([] ())$ is in L, but not $([])$ and not $([]]$.</p> <p>I. Construct a PDA to accept the language L. [4 Marks]</p> <p>II. Do a short trace of the state sequence and sequence of stack contents as this machine recognizes the string “[() ()]”. [3 Marks]</p> <p>Solution:</p> <p>I.</p> <div><div><div>[, (; ([</div><div>(, [; [(</div><div>[,] ; λ</div><div>(,) ; λ</div><div>[, [; [[</div><div>(, (; ((</div><div>[, Z0 ; [Z0</div><div>(, Z0 ; (Z0</div></div><div><pre>graph LR start(()) --> q0((q0)) q0 --> q0 q0 -- "λ, Z0 ; Z0" --> q1(((q1)))</pre></div><p>II.Trace of String : [() ()]</p><table><tr><td>$\delta(q_0, [() ()], z_0)$</td><td>(Push [)</td></tr><tr><td>$\vdash \delta(q_0, () ()], [z_0)$</td><td>(Push ()</td></tr><tr><td>$\vdash \delta(q_0,) ()], ([z_0)$</td><td>(Match (and))</td></tr><tr><td>$\vdash \delta(q_0, ()], [z_0)$</td><td>(Push ()</td></tr><tr><td>$\vdash \delta(q_0,)], ([z_0)$</td><td>(Match (and))</td></tr><tr><td>$\vdash \delta(q_0,], [z_0)$</td><td>(Match [and])</td></tr><tr><td>$\vdash \delta(q_0, \lambda, z_0)$</td><td></td></tr><tr><td>$\vdash (q_1, \lambda, z_0)$</td><td></td></tr></table><p>Since q1 is a final state and input is completely processed, the string is accepted by the PDA.</p></div>	$\delta(q_0, [() ()], z_0)$	(Push [)	$\vdash \delta(q_0, () ()], [z_0)$	(Push ()	$\vdash \delta(q_0,) ()], ([z_0)$	(Match (and))	$\vdash \delta(q_0, ()], [z_0)$	(Push ()	$\vdash \delta(q_0,)], ([z_0)$	(Match (and))	$\vdash \delta(q_0,], [z_0)$	(Match [and])	$\vdash \delta(q_0, \lambda, z_0)$		$\vdash (q_1, \lambda, z_0)$		7
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