AUTOMATA, FORMAL LANGUAGES AND LOGIC

MODULE 5

Propositional Logic

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Outline

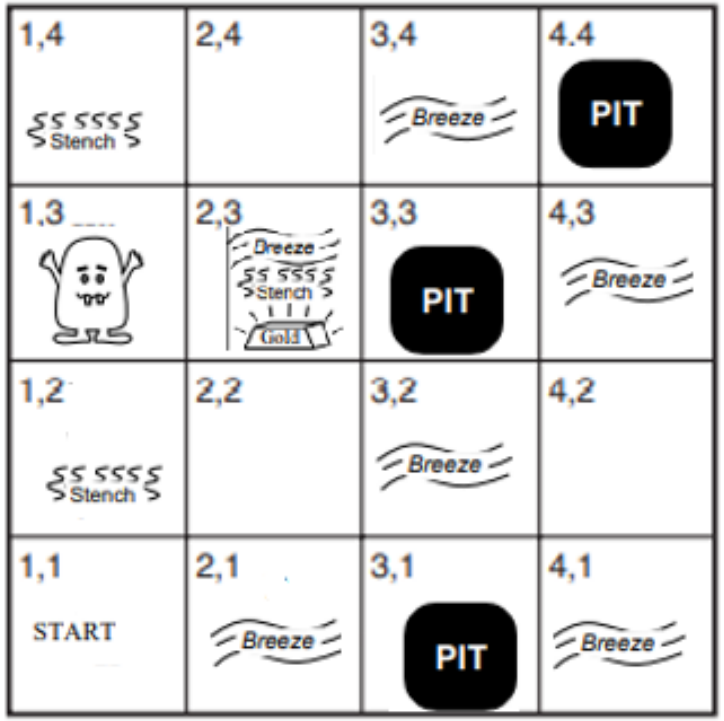
- Propositional logic - A very Simple Logic
  - Syntax and Semantics
  - **A Simple Knowledge Base**
  - **A Simple Inference Procedure**

Knowledge Base

To construct the **knowledge base**, let us focus on immutable aspects of wumpus world

We need the following symbols for each [i, j] location:

- $P_{i,j}$ be true if there is a pit in [i, j].
- $B_{i,j}$ be true if there is a breeze in [i, j].
- $W_{i,j}$ be true if there is a wumpus in [i, j], dead or alive.
- $S_{i,j}$ be true if agent perceives a stench in [i, j].



Propositional Logic (A simple Knowledge Base)

1. There is no pit in [1,1],

R_1: $\neg P_{1,1}$

2. A square is **breezy iff** there is **pit in its neighboring squares**

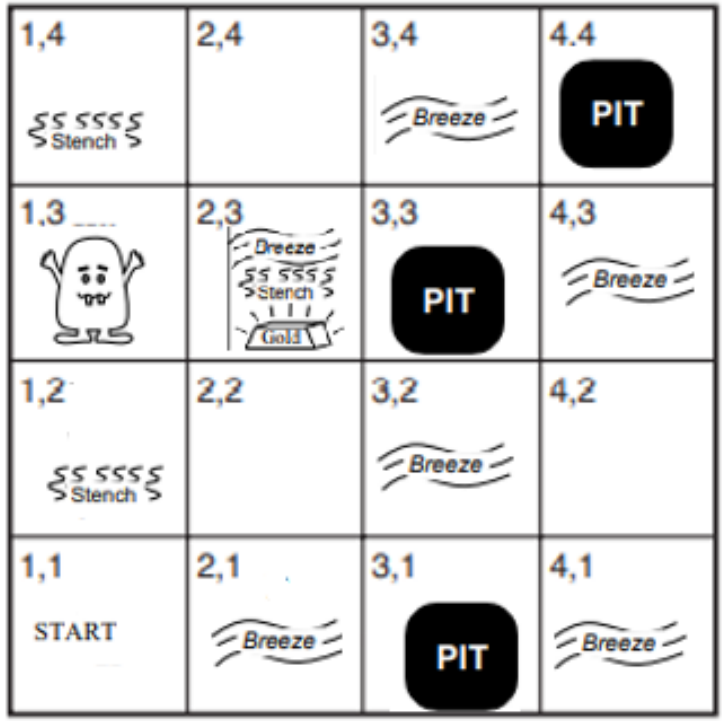
R_2: $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

R_3: $B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$

3. Let us include the **breeze** percept for the 1st two squares, leading to the situation

R4: $\neg B_{1,1}$

R5: $B_{2,1}$



The knowledge base might be considered as **conjunction of one or more statements**

Here the *PROPOSITION SYMBOLS* are :

$B_{1,1}$, $B_{2,1}$, $P_{1,1}$, $P_{1,2}$, $P_{2,1}$, $P_{2,2}$, $P_{3,1}$

Propositional Logic (A simple Inference Procedure)

Goal: If $\alpha 1 = \neg P_{1,2}$ and $\alpha 2 = P_{2,2}$

Whether KB $\models \alpha 1$ or KB $\models \alpha 2$

for some sentence $\alpha 1$ and $\alpha 2$.

Is $\neg P_{1,2}$ entailed by our Knowledge Base?

Is $P_{2,2}$ entailed by our Knowledge Base?

Use Model Checking Approach

1. Enumerate the models
2. Check for α being true, in every model in which KB is true

For Wumpus world *PROPOSITION SYMBOLS* are :

$B_{1,1}$, $B_{2,1}$, $P_{1,1}$, $P_{1,2}$, $P_{2,1}$, $P_{2,2}$, $P_{3,1}$

So with 7 symbols there are $2^7 = 128$ possible models

For 3 symbols there are $2^3 = 8$ models

A	B	C
True	True	True
True	False	False
False	True	False
False	False	True
True	True	False
True	False	True
False	True	True
False	False	False

Truth Table for inference

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*

Is $\neg P_{1,2}$ entailed by our KB?

Here $\neg P_{1,2}$ is true

Hence, There is no pit in [1,2].

Propositional Logic (A simple Inference Procedure)

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*

P	Q	¬P	P ∧ Q	P ∨ Q	P ⇒ Q	P ⇔ Q
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

1. R_1: $\neg P_{1,1}$
2. R_2: $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$
3. R_3: $B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$
4. R4: $\neg B_{1,1}$
5. R5: $B_{2,1}$

Propositional Logic (A simple Inference Procedure)

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*

KB = R1 ∧ R2 ∧ R3 ∧ R4 ∧ R5

1. R_1: $\neg P_{1,1}$
2. R_2: $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$
3. R_3: $B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$
4. R4: $\neg B_{1,1}$
5. R5: $B_{2,1}$

Truth Table for inference

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*

Is $P_{2,2}$ entailed by our Knowledge Base?

Here $P_{2,2}$ is true in 2 of the three models and false in one.

Hence, We can't yet tell whether there is a pit in [2,2].

THANK YOU

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