



LINEAR ALGEBRA AND ITS APPLICATIONS

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MATRICES AND GAUSSIAN ELIMINATION

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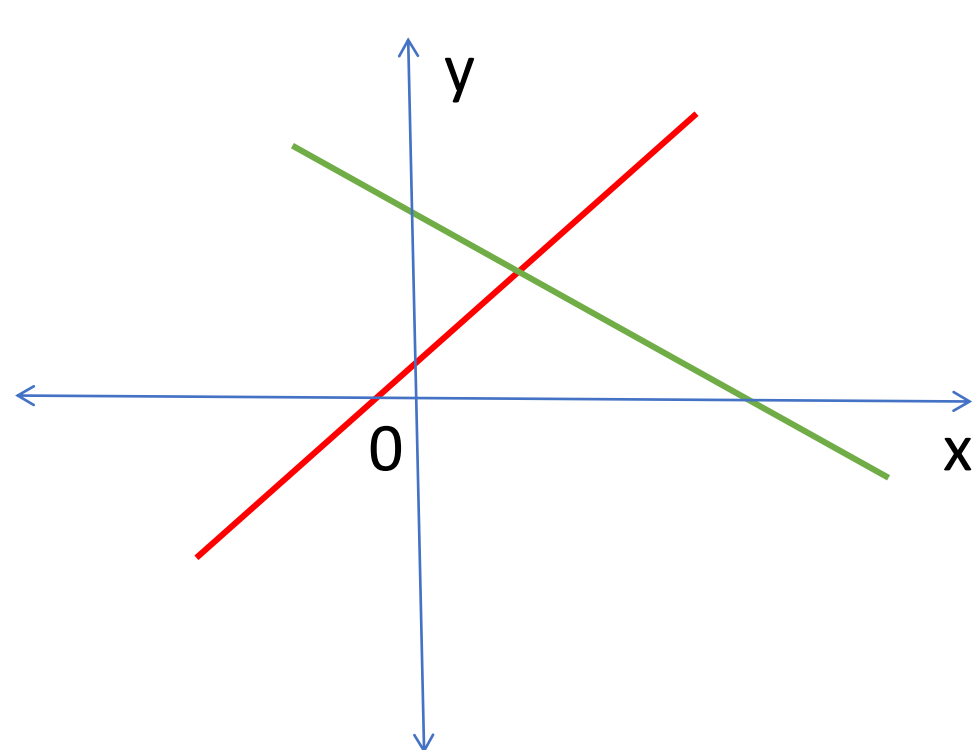
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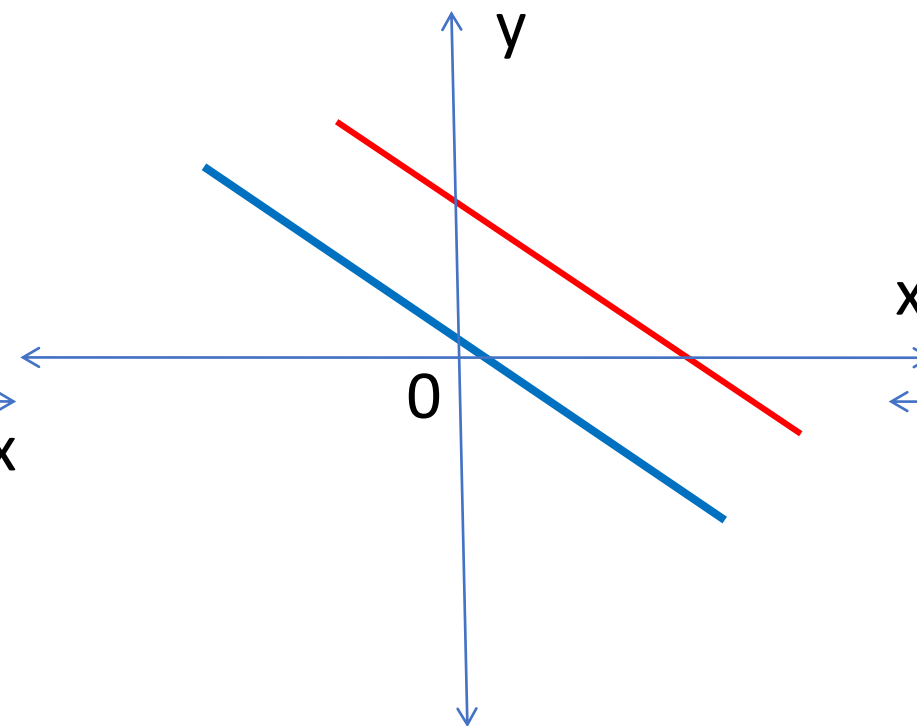
THE GEOMETRY OF LINEAR EQUATIONS:

Course Content: The Geometry of Linear Equations

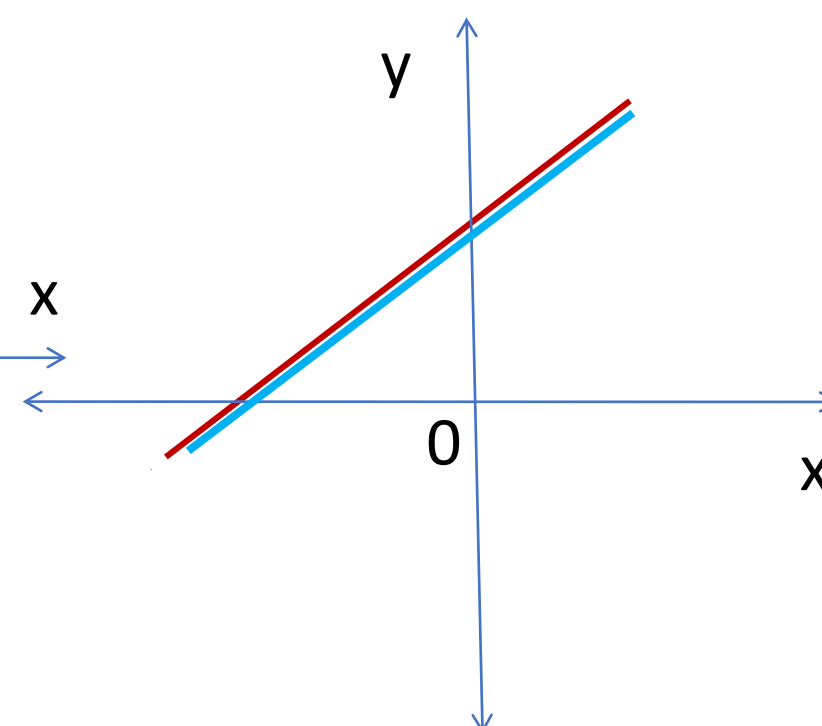
❖ SYSTEM OF 2 EQUATIONS WITH 2 VARIABLES: $a_1x + b_1y = c_1$
 $a_2x + b_2y = c_2$



Intersecting lines
Unique solution



Parallel lines
No Solution



Overlapping lines
Infinite no. of solutions

System of m linear equations with n unknowns have either a unique solution or infinite solutions or no solution.

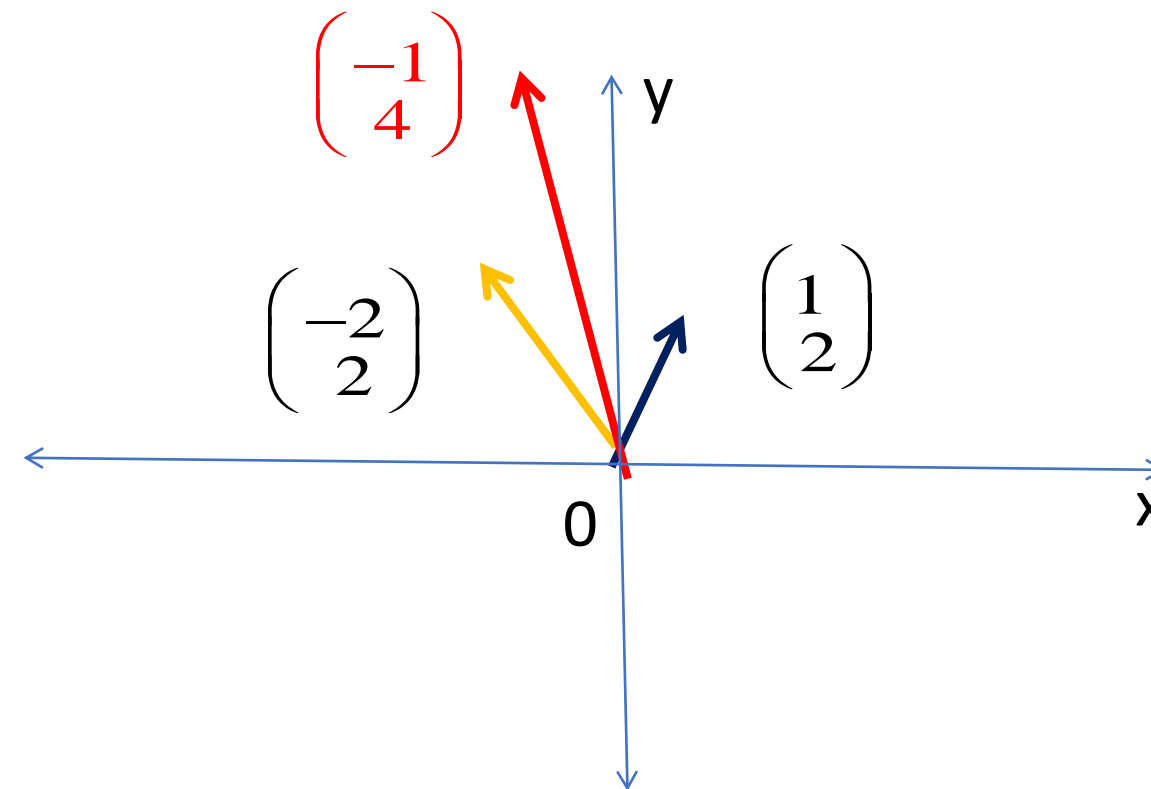
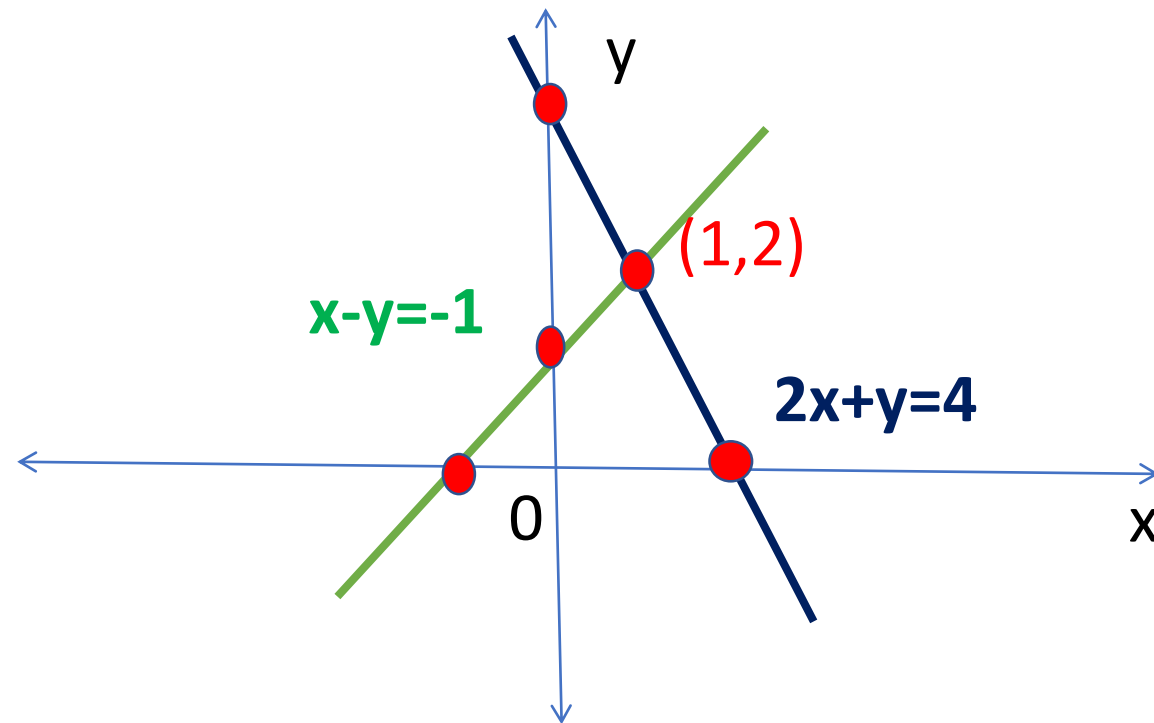
LINEAR ALGEBRA AND ITS APPLICATIONS

THE GEOMETRY OF LINEAR EQUATIONS:

- (i) Consider a system of 2 equations with 2 variables
- $$\begin{aligned}x - y &= -1 \\ 2x + y &= 4\end{aligned}$$
- **Row Picture:** Solving these 2 equations we get (1,2) as the point of intersection of the 2 lines. Hence this system has **a unique solution**.

❖ **Column Picture:** $x \begin{pmatrix} 1 \\ 2 \end{pmatrix} + y \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \end{pmatrix} \Rightarrow 1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} + 2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$
x=1 and y=2 will satisfy this equation.

Hence the linear combination of the column vectors on LHS produces the vector on RHS.



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THE GEOMETRY OF LINEAR EQUATIONS:

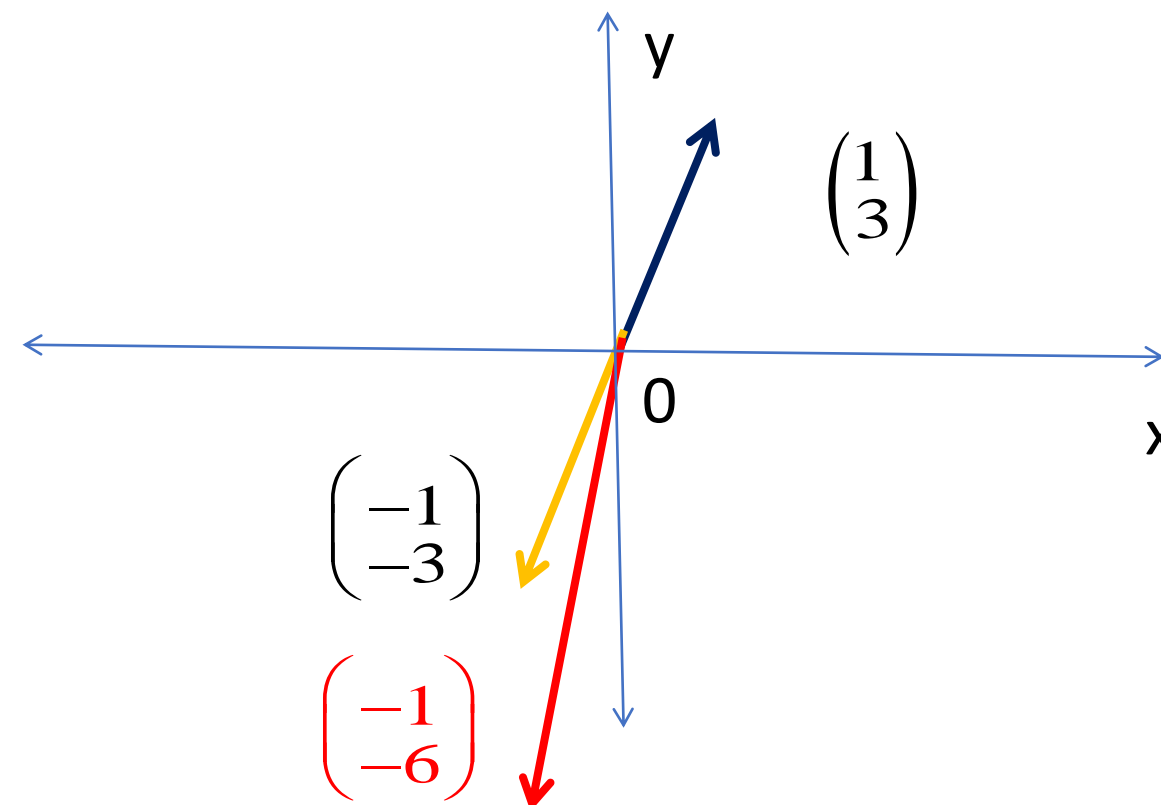
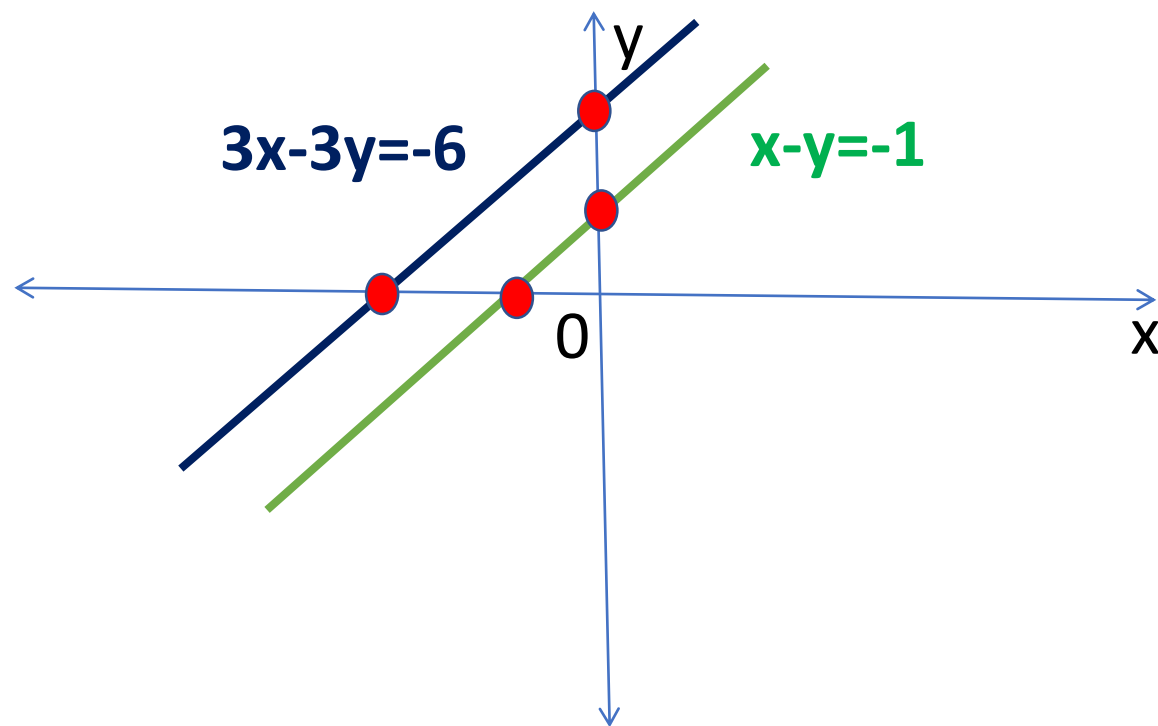
- (ii) Consider the system
- $$\begin{aligned}x - y &= -1 \\ 3x - 3y &= -6\end{aligned}$$

➤ **Row Picture:** These are parallel lines and so has no point of intersection. Hence this system has **no solution**.

❖ **Column Picture:**

$$x \begin{pmatrix} 1 \\ 3 \end{pmatrix} + y \begin{pmatrix} -1 \\ -3 \end{pmatrix} = \begin{pmatrix} -1 \\ -6 \end{pmatrix}$$

There is no linear combination of the column vectors on LHS which produces the vector on RHS.



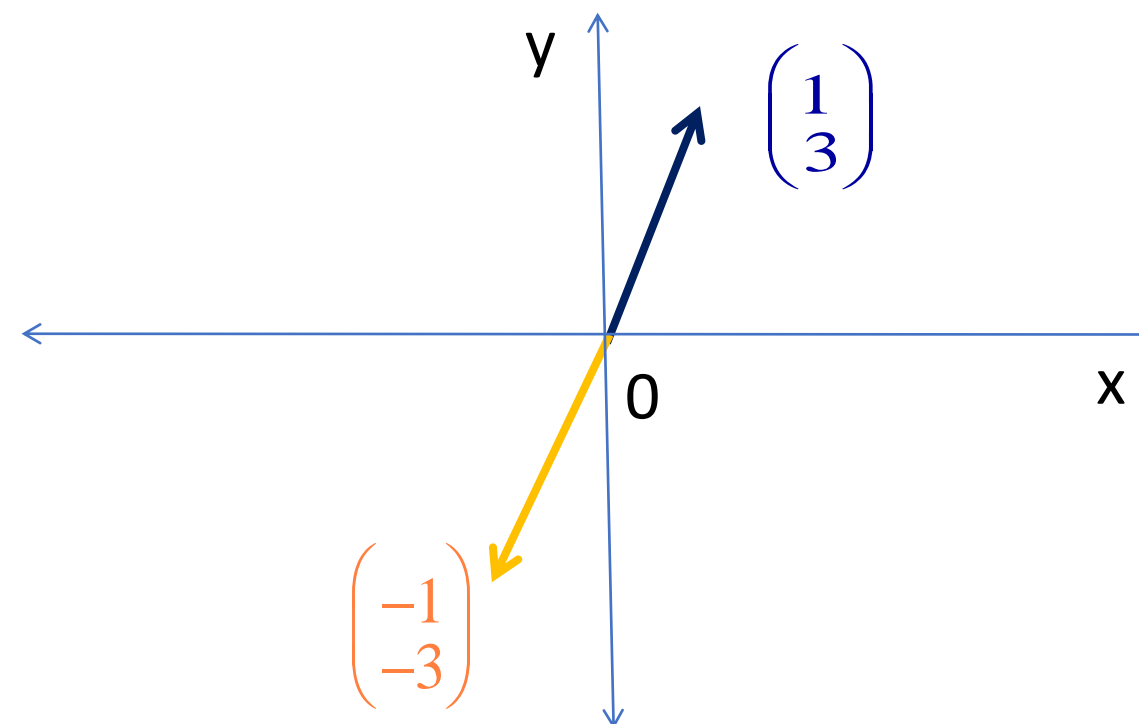
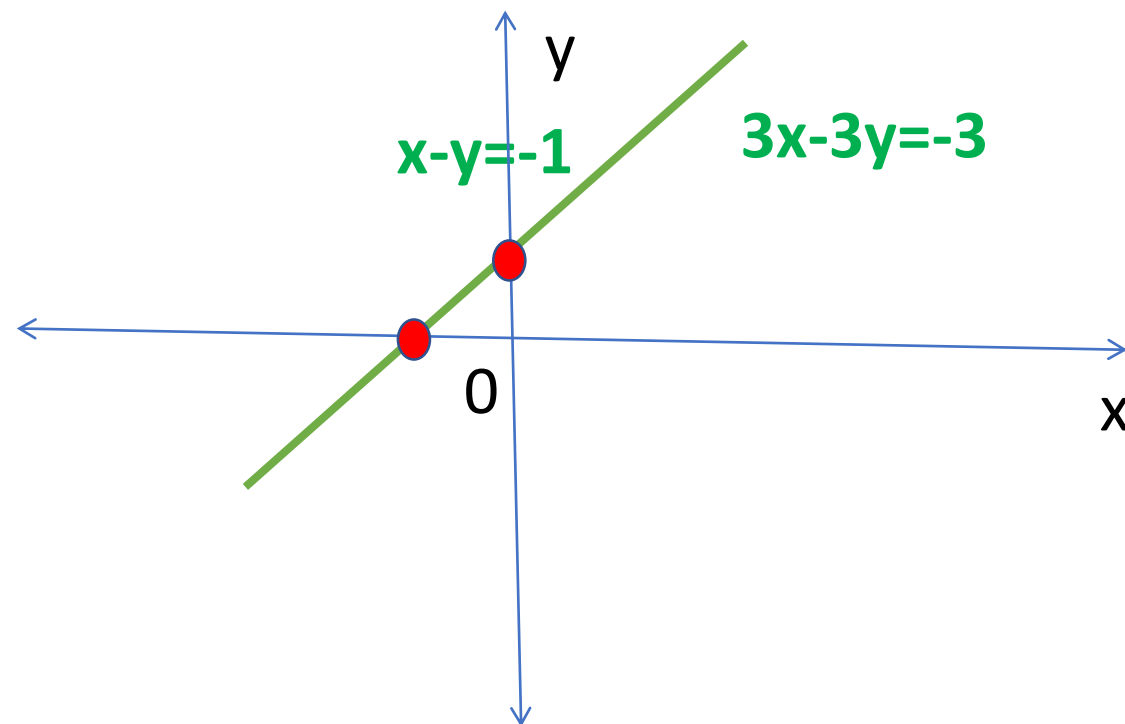
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THE GEOMETRY OF LINEAR EQUATIONS:

- (iii) Consider the system
- $$\begin{aligned}x - y &= -1 \\ 3x - 3y &= -3\end{aligned}$$
- **Row Picture:** These are coincident (overlapping) lines and so has infinite number of solutions. Hence this system has **infinitely many solutions**.

❖ **Column Picture:** $x \begin{pmatrix} 1 \\ 3 \end{pmatrix} + y \begin{pmatrix} -1 \\ -3 \end{pmatrix} = \begin{pmatrix} -1 \\ -3 \end{pmatrix}$

There are infinite number of linear combinations of the column vectors on LHS which produces the vector on RHS.



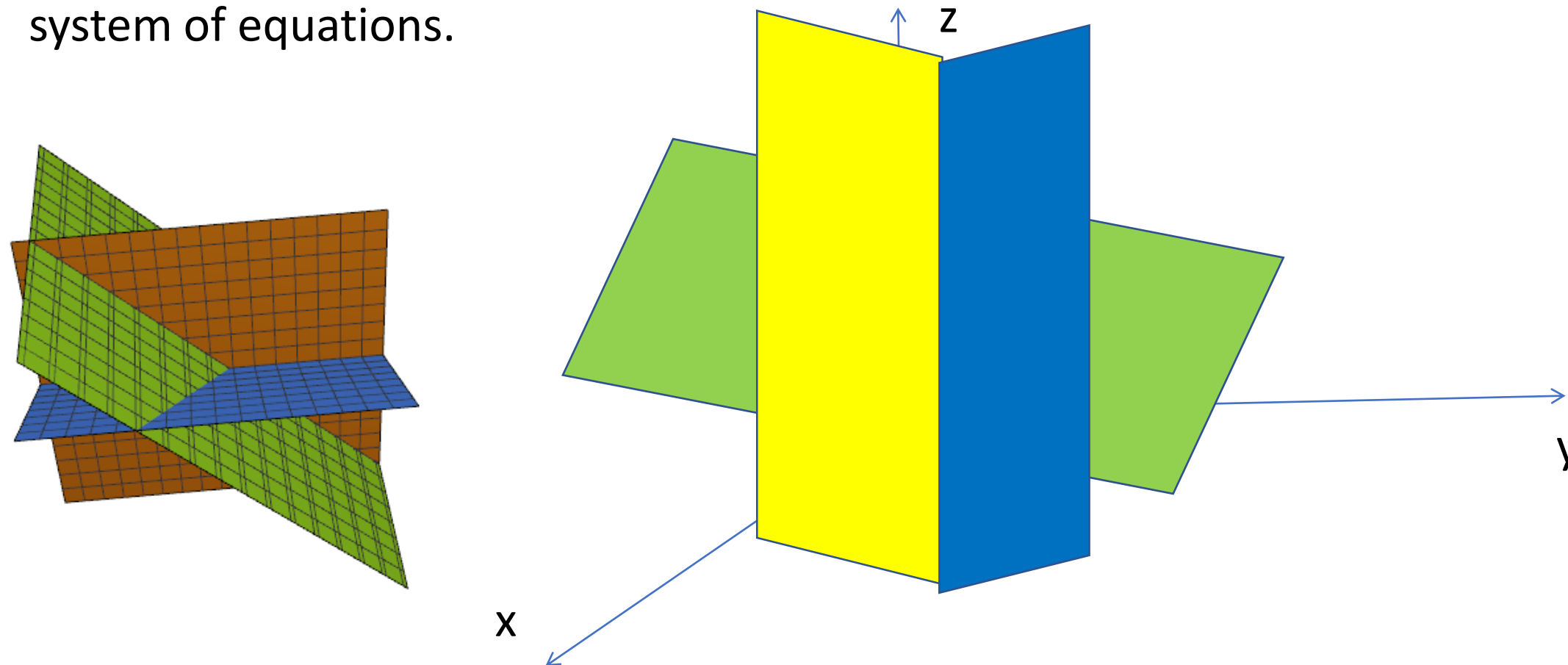
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THE GEOMETRY OF LINEAR EQUATIONS:

❖ **SYSTEM OF 3 EQUATIONS WITH 3 VARIABLES:** Consider the system

$$\begin{aligned}x + y + 2z &= 1 \\x + 2y - z &= -2 \\x + 3y + z &= 5\end{aligned}$$

❖ **Row Picture:** Each equation describes a 2-dimensional plane in \mathbb{R}^3 . The first 2 planes intersect along a line and this line intersects with the third plane to produce a point $(-6, 3, 2)$ which is the unique solution (point of intersection of the three planes) to the system of equations.



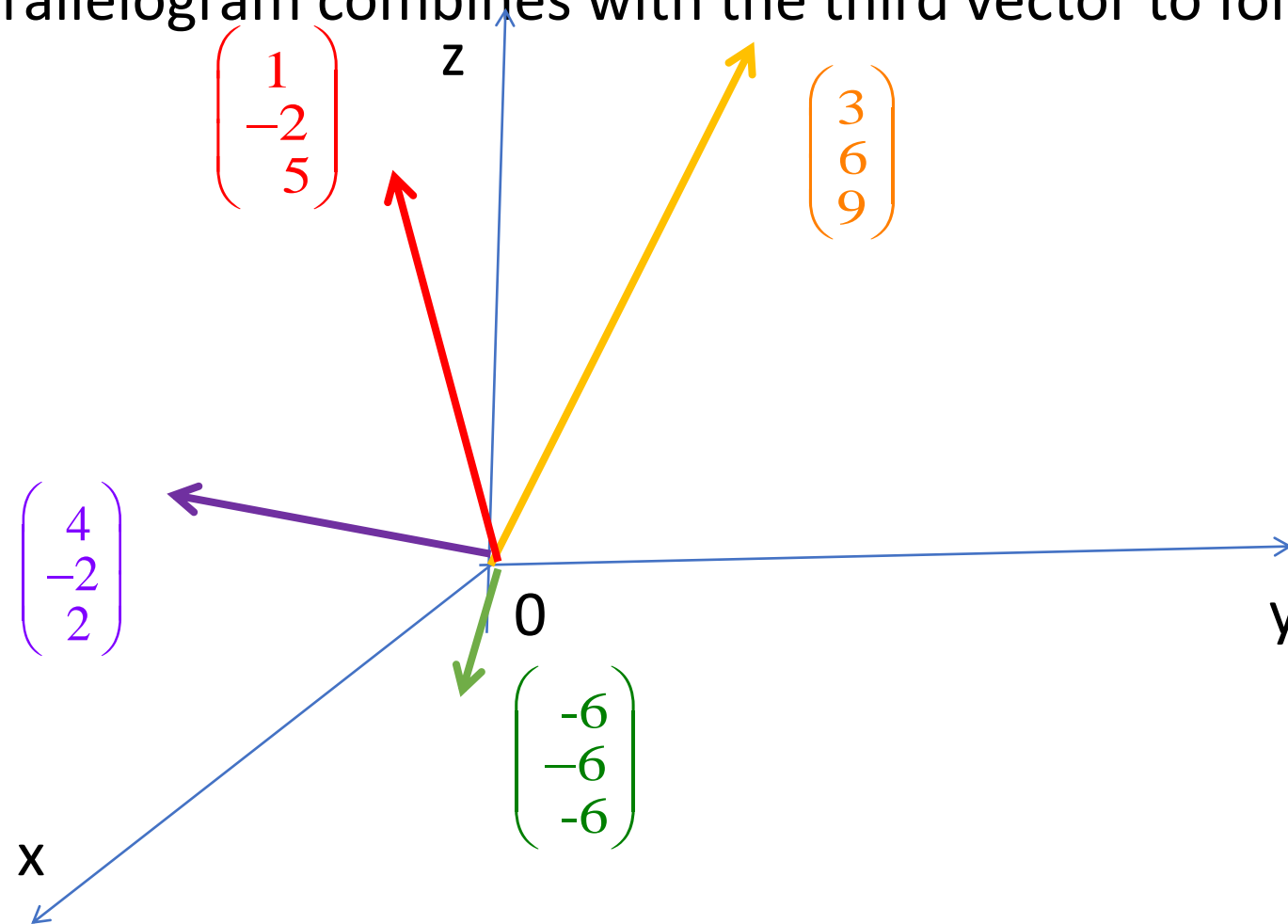
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THE GEOMETRY OF LINEAR EQUATIONS:

❖ **Column Picture:** The vector equation for the given system can be written as

$$x \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + y \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + z \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix} \Rightarrow -6 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + 3 \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \\ 3 \end{pmatrix} + 2 \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}$$

The first two vectors on the LHS combine to form a Parallelogram and the diagonal of this parallelogram combines with the third vector to form a Parallelepiped.



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THE GEOMETRY OF LINEAR EQUATIONS:

- ❖ **Row Picture:** Intersection of Lines/Planes.
- ❖ **Column Picture:** Combination of Columns.

- ❖ **Row Picture:** A Line requires 2 equations in 3-dimensional space. Similarly a Line requires 3 equations in 4-dimensional space. Hence a Line requires $(n-1)$ equations in n -dimensional space.
The first equation represents a $(n-1)$ dimensional plane in n dimensions. The second plane(equation) intersects it in a smaller set of dimension $(n-2)$. Thus every new plane reduces the dimension by one. At the end when all the ' n ' planes are accounted for the intersection has dimension zero. It is a point, which lies on all the planes and its co-ordinates satisfy all ' n ' equations. This point is the solution.



ENGINEERING MATHEMATICS-III

References/Links:

https://upload.wikimedia.org/wikipedia/commons/c/c0/Intersecting_Lines.svg

Google search: Graphs of row and column picture for a system of linear equations





THANK YOU

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