UNIT - 2 (VECTOR SPACES)

Q1) Given vectors $(1,1,2), (1,2,4), (2,4,8)$ and $b = (2,3,5)$

$$[A\ b] = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 1 & 2 & 4 & 3 \\ 2 & 4 & 8 & 5 \end{bmatrix}$$

performing row operations,

$R_2 \to R_2 - (1) R_1$, $R_3 \to R_3 - (2) R_1$

$$[A\ b] \sim \begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 2 & 4 & 1 \end{bmatrix}$$

$R_3 \to R_3 - (2) R_2$

$$[A\ b] \sim \begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

The third Component "5" should be replaced by "6" so, that $Ax = b$ has infinitely many solutions.

The new vector $b = (2,3,6)$.

Let, $c_1 \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} + c_3 \begin{bmatrix} 2 \\ 4 \\ 8 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 6 \end{bmatrix}$

$$c_1 + c_2 + 2c_3 = 2$$

$$c_1 + 2c_2 + 4c_3 = 3$$

$$2c_1 + 4c_2 + 8c_3 = 6 \Rightarrow c_1 + 2c_2 + 4c_3 = 3$$

Let, $c_3 = k$ an arb $(k \in R)$

by solving the above equations we get,

$c_1 = 1$, $c_2 = 1 - 2k$, $c_3 = k$.

$$\therefore \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + (1-2k) \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} + k \begin{bmatrix} 2 \\ 4 \\ 8 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}, \ k \in R$$

Q2) Given, $u = (2, 1, 0)$, $v = (1, -1, 2)$ and $w = (0, 3, -4)$.

and $b = (a, b, c)$

$$[A \; b] = \begin{bmatrix} 2 & 1 & 0 & a \\ 1 & -1 & 3 & b \\ 0 & 2 & -4 & c \end{bmatrix}$$

performing row operations,

$$R_2 \rightarrow R_2 - (1/2) R_1$$

$$[A \; b] \sim \begin{bmatrix} 2 & 1 & 0 & a \\ 0 & -3/2 & 3 & b - a/2 \\ 0 & 2 & -4 & c \end{bmatrix}$$

$$R_3 \rightarrow R_3 - (-4/3) R_2$$

$$[A \; b] \sim \begin{bmatrix} 2 & 1 & 0 & a \\ 0 & -3/2 & 3 & b - a/2 \\ 0 & 0 & 0 & c + 4b/3 - 2a/3 \end{bmatrix}$$

since, columns ① & ② contain the pivots so, the columns ① & ② are the independent vectors. From the last row, the system is consistent if and only if $2a - 4b - 3c = 0$. Only u & v are independent vectors so, the vectors u, v, w does not span R^3.

Q3) Null space:-

Let, A be a matrix of order $m \times n$, the set of all solutions x of $Ax = 0$ is a set, is known as Null space of A denoted by $N(A)$.

i.e; $N(A) = \{x / Ax = 0\} \subseteq R^n$.

Given, to construct a matrix with $(1,0,1)$ and $(1,2,0)$.

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 1 \\ 2 & 4 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Let, $b = (a, b, c)$

$$[A \;\; b] = \begin{bmatrix} 2 & 2 & 1 & a \\ 2 & 4 & 0 & b \\ 1 & 0 & 1 & c \end{bmatrix}$$

performing row operations, $R_2 \to R_2 - (1)R_1$, $R_3 \to R_3 - (1/2)R_1$

$$[A \;\; b] \sim \begin{bmatrix} 2 & 2 & 1 & a \\ 0 & 2 & -1 & b-a \\ 0 & -1 & -1/2 & c - a/2 \end{bmatrix}$$

$R_3 \to R_3 - (-1/2) R_2$

$$[A \;\; b] \sim \begin{bmatrix} 2 & 2 & 1 & a \\ 0 & 2 & -1 & b-a \\ 0 & 0 & 0 & c - b/2 \end{bmatrix}$$

The basis of $N(A) = \{(0,-1,2)\}$.

$\therefore \dim C(A^T) = 2 = C(A)$ and $n = 3$, Therefore $\dim N(A) = 1$. The given vectors cannot be a basis for row space and Null space since, in that case $\dim C(A^T) + \dim N(A)$

$= 2 + 2 = 4 \neq 3$

Scanned by CamScanner

Q4) Given,

$$A = \begin{bmatrix} 2 & 4 & 6 & 4 \\ 2 & 5 & 7 & 6 \\ 2 & 3 & 5 & 2 \end{bmatrix}$$

Performing row operations,

$R_2 \to R_2 - (1)R_1$, $R_3 \to R_3 - (1)R_1$

$$A \sim \begin{bmatrix} 2 & 4 & 6 & 4 \\ 0 & 1 & 1 & 2 \\ 0 & -1 & -1 & -4 \end{bmatrix}$$

$R_3 \to R_3 - (-1)R_1$

$$A \sim \begin{bmatrix} 2 & 4 & 6 & 4 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

Since, the given matrix A has two pivots, basis of the $C(A) = \{(2,2,2), (4,5,3)\}$

$$U = \begin{bmatrix} 2 & 4 & 6 & 4 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

$R_1 \to \frac{R_1}{2}$

$$U \sim \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & -2 \end{bmatrix} = R$$

$Rx = 0$

$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x + 2y + 3z + 2t = 0$

$y + z + 2t = 0$

$-2t = 0$

so, $\begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = z\begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 2 \\ -2 \\ 0 \\ 1 \end{bmatrix}$

A basis of $N(A) = \{(-1,-1,1,0), (2,-2,0,1)\}$

Q5) Given vectors $(1,4,2)$, $(2,5,1)$ and $(3,6,0)$.

Let, $b = (a,b,c)$

$$[A \ b] = \begin{bmatrix} 1 & 2 & 3 & a \\ 4 & 5 & 6 & b \\ 2 & 1 & 0 & c \end{bmatrix}$$

$R_2 \to R_2 - (4)R_1$, $R_3 \to R_3 - (2)R_1$

$$[A \ b] \sim \begin{bmatrix} 1 & 2 & 3 & a \\ 0 & -3 & -6 & b-4a \\ 0 & -3 & -6 & c-2a \end{bmatrix}$$

$R_3 \to R_3 - (-1)R_2$

$$[A \ b] \sim \begin{bmatrix} 1 & 2 & 3 & a \\ 0 & -3 & -6 & b-4a \\ 0 & 0 & 0 & 2a-b+c \end{bmatrix}$$

Solutions of $A^T x_1 = 0$ are $(2,-1,1)$. The vector b is in $N(A^T)$. since, there are 2 ~~&~~ independent vectors dim $C(A^T) = 2$ and $N(A^T) = 1$.

Q6) Given vectors $(1,3,-5)$, $(0,5,\lambda)$ and $(-2,-1,0)$

let, $b = (a,b,c)$

$$[A\ b] = \begin{bmatrix} 1 & 0 & -2 & a \\ 3 & 5 & -1 & b \\ -5 & \lambda & 0 & c \end{bmatrix}$$

performing row operations,

$R_2 \to R_2 - (3) R_1$, $R_3 \to R_3 - (-5) R_1$

$$[A\ b] \sim \begin{bmatrix} 1 & 0 & -2 & a \\ 0 & 5 & 5 & b-3a \\ 0 & \lambda & -10 & c+5a \end{bmatrix}$$

$R_3 \to R_3 - (-2) R_2$

$$[A\ b] \sim \begin{bmatrix} 1 & 0 & -5 & a \\ 0 & 5 & 5 & b-3a \\ 0 & \lambda+10 & 0 & c+2b-a \end{bmatrix}$$

$\lambda + 10 = 0 \Rightarrow \boxed{\lambda = -10}$

If $\lambda = -10$ then the third column becomes dependent giving a 2D plane in $\mathbb{R}^3$.

For $\lambda = -10$,

$(-2,-1,0) = c_1(1,3,-5) + c_2(0,5,-10)$

By solving, $c_1 = -2$ and $c_2 = 1$

$\therefore (-2,1,0) = -2(1,3,-5) + (0,5,-10)$

$$[A\ b] \sim \begin{bmatrix} 1 & 0 & -5 & a \\ 0 & 5 & 5 & b-3a \\ 0 & 0 & 0 & c+2b-a \end{bmatrix}$$

A vector in $\mathbb{R}^3$ i.e; not in the span of the 2D-subspace is (a,b,c) such that $a-2b-c \neq 0$.

Q7) $A = \begin{bmatrix} 2 & 4 & 6 & 4 \\ 2 & 5 & 7 & 6 \\ 2 & 3 & 5 & c \end{bmatrix}$

Performing gaussian elimination.

i) $R_2 \to R_2 - R_1$ $\qquad$ $R_3 \to R_3 - R_1$

$$A \sim \begin{bmatrix} 2 & 4 & 6 & 4 \\ 0 & +1 & 1 & 2 \\ 0 & -1 & -1 & c-4 \end{bmatrix}$$

ii) $R_3 \to R_3 + R_2$

$$U = \begin{bmatrix} 2 & 4 & 6 & 4 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & c-2 \end{bmatrix}$$

i) It will be a plane in R^3, if ther is a dependent, non-pivot containg column.

$\therefore$ $c-2=0$

$c=2$.

ii) It will be the whole of R^3, if its a full rank matrix.

$\therefore$ $c-2 \neq 0$

$c \neq 2$.

Transforming U into R.

i) $R_2 \to R_2 - \left(\frac{2}{c-2}\right) R_3$ $\qquad$ $R_1 \to R_1 - \left(\frac{4}{c-2}\right) R_3$

$$U \sim \begin{bmatrix} 2 & 4 & 6 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & c-2 \end{bmatrix}$$

ii) $R_1 \to R_1 - 4R_2$ $\qquad$ iii) $R_1 \to R_1 \times 1/2$, $R_3 \to R_3 \times 1/(c-2)$

$$R \sim \begin{bmatrix} 2 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & c-2 \end{bmatrix} \qquad \therefore R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ x_1, x_2, x_4 are pivot variables, x_3 is a free variable

It is evident, that $N(A) = \{(-1,-1,1,0)\}$

8) Given matrix $A \begin{bmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 7 \\ -1 & -3 & 3 & 4 \end{bmatrix}$

Augmenting with $b = (b_1 \ b_2 \ b_3)$

$$[A \ b] = \begin{bmatrix} 1 & 3 & 3 & 2 & b_1 \\ 2 & 6 & 9 & 7 & b_2 \\ -1 & -3 & 3 & 4 & b_3 \end{bmatrix}$$

Row transformations:

i) $R_2 \to R_2 - 2R_1$ ii) $R_3 \to R_3 + R_1$

$$[A \ b] \sim \begin{bmatrix} 1 & 3 & 3 & 2 & b_1 \\ 0 & 0 & 3 & 3 & b_2 - 2b_1 \\ 0 & 0 & 6 & 6 & b_3 + b_1 \end{bmatrix}$$

iii) $R_3 \to R_3 - \left(\frac{6}{3}\right) R_2$

$$[U \ c] \sim \begin{bmatrix} 1 & 3 & 3 & 2 & b_1 \\ 0 & 0 & 3 & 3 & b_2 - 2b_1 \\ 0 & 0 & 0 & 0 & b_3 + b_1 - 2(b_2 - 2b_1) \end{bmatrix}$$

for consistency, $b_3 + b_1 - 2(b_2 - 2b_1) = 0$

$$5b_1 - 2b_2 + b_3 = 0$$

$\therefore$ Basis for $N(A^T)$ is $\{(5, -2, 1)\}$

$\dim(N(A^T)) = 1$

$N(A^T)$ is a line in R^3.

$\rightarrow$ The null space of A

To find $N(A)$, we need to solve $Rx = 0$.

$$U = \begin{bmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 7 \\ -1 & -3 & 3 & 4 \end{bmatrix}$$

$R_1 \longrightarrow R_1 - R_2$

$$U \sim \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 \rightarrow 1/1 \times R_1$ $\qquad$ $R_2 \rightarrow 1/3 \times R_2$

pivot variables

$$R = \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

free variables

x and z are pivot variables, because the 1^{st} and 3^{rd} column have pivots.

Hence $Rx = 0$, yields

$$x + 3y - w = 0 \longrightarrow x = w - 3y$$
$$z + w = 0 \longrightarrow z = -w$$

$\therefore$ the set of all the solutions, is:

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} w-3y \\ y \\ -w \\ w \end{bmatrix} = y \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + w \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

The independent vectors, $(-3,1,0,0)$ and $(1,0,-1,1)$ are called special solutions of $Ax = 0$. $\therefore$ A basis of $N(A)$ is $\{(-3,1,0,0), (1,0,-1,1)\}$

$$\dim(N(A)) = 2$$

$N(A)$ is a 2-D plane in R^4.

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Performing Gaussian Elimination.

$R_2 \rightarrow R_2 - R_1$

$$A \sim \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

ii) $R_3 \rightarrow R_3 + R_2$

$$A \sim \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

iii) $R_4 \rightarrow R_4 - R_3$

$$U = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Only c_1 c_2 c_3 (columns) contain pivots. Therefore c_4 is linearly dependendent.

$C(A) = \{(1,1,0,0), (1,0,1,0), (0,0,1,1)\}$

$\therefore$ It spans a 3-D plane in R^4.
It DOES NOT span R^4.

$\dim(A) = 3$

Q10) $A = \begin{bmatrix} 2 & -6 & -8 \\ -4 & 12 & a \\ 1 & b & 2 \end{bmatrix}$

Performing Gaussian Elimination:

i) $R_2 \rightarrow R_2 - \left(\frac{-4}{2}\right) R_1$ $\quad$ $R_3 \rightarrow R_3 - \left(\frac{1}{2}\right) R_1$

$$A \sim \begin{bmatrix} 2 & -6 & -8 \\ 0 & 0 & a-16 \\ 0 & b+3 & 6 \end{bmatrix}$$

ii) $R_2 \longleftrightarrow R_3$

$$U = \begin{bmatrix} 2 & -6 & -8 \\ 0 & b+3 & 6 \\ 0 & 0 & a-16 \end{bmatrix}$$

i) C(A) will span entire R^3, only if all columns contain pivots.

$\therefore$ $b+3 \neq 0$ $\quad\quad$ $a-16 \neq 0$

$b \neq -3$ $\quad\quad$ $a \neq 16$

$\therefore$ C(A) is whole of R^3, if $b \neq -3$ and $a \neq 16$

ii) C(A) will span a 2-D subspace in R^3, if one column does not contain pivot

$\therefore$ ~~$a-16=0$~~ $\quad\quad$ $\therefore$ $b+3=0$

~~$a=16$~~ $\quad\quad$ $b=-3$

C(A) is a 2 dimensional plane if ~~$a=16$~~. $b=-3$

iii) Such a and b do not exist.

iv) Substituting $b=-3$ and $a=22$ in U.

$$U = \begin{bmatrix} 2 & -6 & -8 \\ 0 & 0 & 6 \\ 0 & 0 & 6 \end{bmatrix}$$

i) $R_1 \rightarrow R_1 - \left(\frac{-8}{6}\right) R_2$

$$R \sim \begin{bmatrix} 2 & -6 & 0 \\ 0 & 0 & 6 \\ 0 & 0 & 6 \end{bmatrix}$$

ii) $R_1 \rightarrow R_1 \times 1/2$ $\quad\quad$ $R_2 \rightarrow R_2 \times 1/6$

$$\therefore R = \begin{bmatrix} 1 & -3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 6 \end{bmatrix}$$

$\therefore$ $y \begin{bmatrix} -(-3) \\ 1 \\ 0 \end{bmatrix}$ is the null space.

$N(A) = \{(3,1,0)\}$

Q11) $Ax=0$, indicates that we have to find the null space of A.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Obtaining Row reduced matrix (R)

i) $R_1 \leftrightarrow R_2$

$$A \sim \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = U$$

$\therefore r(A) = 3$

ii) $R_1 \rightarrow R_1 \times (1/-1)$

$$U \sim \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = R$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

pivot variables

free variable

x, y, w are pivot variables, because 1^{st}, 2^{nd} and 4^{th} colums contain pivots.

Hence $Rx=0$, yields:

$x - z = 0 \rightarrow x = z$

$y = 0 \rightarrow y = 0$

$w = 0 \rightarrow w = 0$

$\therefore$ The set of all solutions is

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} z \\ 0 \\ z \\ 0 \end{bmatrix} = z \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$\therefore$ The basis for $N(A)$ is $(1,0,1,0)$

$\dim(N(A)) = 1$. $(= n-r)$

where $n = 4$

$r = 3$

) $A = \begin{bmatrix} 0 & 3 & -1 & -2 & 6 \\ -2 & 1 & 2 & 1 & -3 \\ 1 & -1 & 2 & -2 & 3 \end{bmatrix}$

ugment it with a b vector $(b_1\ b_2\ b_3)$

$[A\ b] = \begin{bmatrix} 0 & 3 & -1 & -2 & 6 & b_1 \\ -2 & 1 & 2 & +1 & -3 & b_2 \\ 1 & -1 & 2 & -2 & 3 & b_3 \end{bmatrix}$

rforming Gaussian elimination:

) $R_1 \leftrightarrow R_2$

$[A\ b] \sim \begin{bmatrix} -2 & 1 & 2 & 1 & -3 & b_2 \\ 0 & 3 & -1 & -2 & 6 & b_1 \\ 1 & -1 & 2 & -2 & 3 & b_3 \end{bmatrix}$

i) $R_3 \to R_3 - \left(\frac{1}{-2}\right) R_1$

$[A\ b] \sim \begin{bmatrix} -2 & 1 & 2 & 1 & -3 & b_2 \\ 0 & 3 & -1 & -2 & 6 & b_1 \\ 0 & -0.5 & 3 & -1.5 & 4.5 & b_3 + 0.5 b_2 \end{bmatrix}$

iii) $R_3 \to R_3 - \left(\frac{-0.5}{3}\right) R_2$

$[U\ c] = \begin{bmatrix} -2 & 1 & 2 & 1 & -3 & b_2 \\ 0 & 3 & -1 & -2 & 6 & b_1 \\ 0 & 0 & 3.167 & -2 & 3 & (b_3 + 0.5 b_2) + \frac{1}{6} b_1 \end{bmatrix}$

We, observe that its a full rank matrix.

$\therefore N(A^T) = \{\}$

$\therefore C(A) = \{(0,-2,1), (3,1,-1), (-1,2,2)\}$

$\therefore C(A^T) = \{(0,3,-1,-2,6), (-2,1,2,1,-3), (1,-1,2,-2,3)\}$

Performing Row transformations, to obtain R

$R = \begin{bmatrix} 1 & 0 & 0 & -0.47 & 0.63 \\ 0 & 1 & 0 & 0.78 & -2.05 \\ 0 & 0 & 1 & -0.36 & 0.15 \end{bmatrix}$

Obtaining solutions for $Rx = 0$.

$$\text{Rre } 0 \Rightarrow \begin{bmatrix} 1 & 0 & 0 & -0.47 & 0.63 \\ 0 & 1 & 0 & 0.78 & -2.05 \\ 0 & 0 & 1 & 0.36 & 0.15 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

x_1 x_2 x_5 are pivot variables.
x_4 x_5 are free variables.

Solving for that yields:

$x_1 - 0.47x_4 + 0.63x_5 = 0 \longrightarrow x_1 = 0.47x_4 - 0.63x_5$

$x_2 + 0.78x_4 - 2.05x_5 = 0 \longrightarrow x_2 = -0.78x_4 + 2.05x_5$

$x_3 + (-0.36)x_4 + 0.15x_5 = 0 \longrightarrow x_3 = 0.36x_4 - 0.15x_5$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0.47x_4 - 0.63x_5 \\ -0.78x_4 + 2.05x_5 \\ 0.36x_4 - 0.15x_5 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0.47 \\ -0.78 \\ 0.36 \\ 1 \\ 0 \end{bmatrix} x_4 + \begin{bmatrix} -0.63 \\ 2.05 \\ -0.15 \\ 0 \\ 1 \end{bmatrix} x_5$$

$\therefore N(A) = \{(0.47, -0.78, 0.36, 1, 0), (-0.63, 2.05, -0.15, 0, 1)\}$

$r(A) = 3$, It's a 3×5 matrix.

Summary:

Subspace	Basis	Dimension	Geometry
$C(A)$	$(0,-2,1)$ $(3,1,-1)$ $(-1,2,+2)$	3 (r)	R^3
$N(A^T)$	$\{\}$	0 (m-r)	Origin
$C(A^T)$	$(0,3,-1,-2,+6)$ $(-2,1,2,1,-3)$ $(1,-1,2,-2,3)$	3 (r)	3-D plane in R^5
$N(A)$	$(0.47,-0.78,0.36,1,0)$ $(-0.63,2.05,-0.15,0,1)$	2 (n-r)	2-D plane in R^5