Digital Design and Computer Organization

Wallace Tree Multiplier

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Department of Computer Science and Engineering

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Wallace Tree Multiplier

Course Outline

- Digital Design
  - Combinational logic design
  - Sequential logic design
    - ★ **Wallace Tree Multiplier**
- Computer Organization
  - Architecture (microprocessor instruction set)
  - Microarchitecure (microprocessor operation)

Concepts covered

- Carry Save Addition

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Addition of Three Numbers

- We have studied how to add two binary numbers

WALLACE TREE MULTIPLIER

Addition of Three Numbers

- We have studied how to add two binary numbers
- How about adding three numbers?

$$\begin{array}{cccccc} & & 0 & 1 & 1 & 1 \\ & & 1 & 0 & 1 & 1 \\ + & & 0 & 1 & 1 & 0 \\ \hline \end{array}$$

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Addition of Three Numbers

- We have studied how to add two binary numbers
- How about adding three numbers?

$$
\begin{array}{cccccc}
 & & & 1 & & \\
 & 0 & 1 & 1 & 1 \\
 & 1 & 0 & 1 & 1 \\
+ & 0 & 1 & 1 & 0 \\
\hline
 & & & & 0
\end{array}
$$

Addition of Three Numbers

- We have studied how to add two binary numbers
- How about adding three numbers?

		10	1	
	0	1	1	1
	1	0	1	1
+	0	1	1	0
			0	0

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Addition of Three Numbers

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- How about adding three numbers?

	10	10	1	
	0	1	1	1
	1	0	1	1
+	0	1	1	0
		0	0	0

WALLACE TREE MULTIPLIER

Addition of Three Numbers

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		0	1	1	1
		1	0	1	1
+		0	1	1	0
	1	1	0	0	0

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Logic to Add Three Numbers

- What logic is required to add three numbers?

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Logic to Add Three Numbers

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  - One *n*-bit adder

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Logic to Add Three Numbers

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Logic to Add Three Numbers

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Logic to Add Three Numbers

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  - So $(2n+1)t_{FA}$ time for ripple carry adders

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Logic to Add Three Numbers

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  - Even if parallel prefix adder used, factor of two remains

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Logic to Add Three Numbers

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  - Even if parallel prefix adder used, factor of two remains
- Can we do better?

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Logic to Add Three Numbers

- What logic is required to add three numbers?
  - One n-bit adder
  - One $(n+1)$-bit adder
- How long does it take?
  - n-bit ripple carry adder takes nt_{FA}
  - So $(2n+1)t_{FA}$ time for ripple carry adders
  - Even if parallel prefix adder used, factor of two remains
- Can we do better?
- Can we add three numbers in only slightly longer than time required to add two numbers?

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Carry Save Addition

Carry Save Example

$$\begin{array}{cccc} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ + \quad 0 & 1 & 1 & 0 \\ \hline \end{array}$$

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Carry Save Addition

Carry Save Example

	0	1	1	1
	1	0	1	1
+	0	1	1	0
Sum				
Carry				

Wallace Tree Multiplier
Carry Save Addition

Carry Save Example

		0	1	1	1
		1	0	1	1
+		0	1	1	0
Sum					0
Carry					

Wallace Tree Multiplier
Carry Save Addition

Carry Save Example

	0	1	1	1
	1	0	1	1
+	0	1	1	0
Sum				0
Carry				1

Wallace Tree Multiplier
Carry Save Addition

Carry Save Example

	0	1	1	1
	1	0	1	1
+	0	1	1	0
Sum			1	0
Carry				1

Wallace Tree Multiplier
Carry Save Addition

Carry Save Example

	0	1	1	1
	1	0	1	1
+	0	1	1	0
Sum			1	0
Carry			1	1

Wallace Tree Multiplier
Carry Save Addition

Carry Save Example

	0	1	1	1
	1	0	1	1
+	0	1	1	0
Sum		0	1	0
Carry			1	1

Wallace Tree Multiplier
Carry Save Addition

Carry Save Example

	0	1	1	1
	1	0	1	1
+	0	1	1	0
Sum		0	1	0
Carry		1	1	1

Wallace Tree Multiplier
Carry Save Addition

Carry Save Example

	0	1	1	1
	1	0	1	1
+	0	1	1	0
Sum	1	0	1	0
Carry		1	1	1

Wallace Tree Multiplier
Carry Save Addition

Carry Save Example

	0	1	1	1
	1	0	1	1
+	0	1	1	0
Sum	1	0	1	0
Carry	0	1	1	1

Wallace Tree Multiplier
Carry Save Addition

Carry Save Example

		0	1	1	1
		1	0	1	1
+		0	1	1	0
Sum		1	0	1	0
Carry	0	1	1	1	

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Carry Save Addition

Carry Save Example

		0	1	1	1
		1	0	1	1
+		0	1	1	0
Sum		1	0	1	0
Carry	0	1	1	1	0

Wallace Tree Multiplier
Carry Save Addition

Carry Save Example

		0	1	1	1
		1	0	1	1
+		0	1	1	0
Sum		1	0	1	0
Carry	0	1	1	1	0

Wallace Tree Multiplier
Carry Save Addition

Carry Save Example

		0	1	1	1
		1	0	1	1
+		0	1	1	0
Sum		1	0	1	0
Carry	0	1	1	1	0
					0

Wallace Tree Multiplier
Carry Save Addition

Carry Save Example

		0	1	1	1
		1	0	1	1
+		0	1	1	0
Sum		1	0	1	0
Carry	0	1	1	1	0
				0	0

Wallace Tree Multiplier
Carry Save Addition

Carry Save Example

		0	1	1	1
		1	0	1	1
+		0	1	1	0
Sum		1	0	1	0
Carry	0	1	1	1	0
			0	0	0

Wallace Tree Multiplier
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		0	1	1	1
		1	0	1	1
+		0	1	1	0
Sum		1	0	1	0
Carry	0	1	1	1	0
	1	1	0	0	0

Wallace Tree Multiplier
Carry Save Addition

Carry Save Example

		0	1	1	1
		1	0	1	1
+		0	1	1	0
Sum		1	0	1	0
Carry	0	1	1	1	0
	1	1	0	0	0

Carry Save Addition

Given three n-bit numbers

1. Compute two n-bit numbers (n sum bits and n carry bits)

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Carry Save Addition

Carry Save Example

		0	1	1	1
		1	0	1	1
+		0	1	1	0
Sum		1	0	1	0
Carry	0	1	1	1	0
	1	1	0	0	0

Carry Save Addition

Given three n-bit numbers

1. Compute two n-bit numbers (n sum bits and n carry bits)
2. Add $n-1$ leftmost sum bits to n carry bits obtaining $n+1$ bit result and attach remaining sum bit as LSB

Wallace Tree Multiplier
Carry Save Addition

Carry Save Example

		0	1	1	1
		1	0	1	1
+		0	1	1	0
Sum		1	0	1	0
Carry	0	1	1	1	0
	1	1	0	0	0

Carry Save Addition

Given three n-bit numbers

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   - Time required: nt_{FA}

Wallace Tree Multiplier
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Carry Save Example

		0	1	1	1
		1	0	1	1
+		0	1	1	0
Sum		1	0	1	0
Carry	0	1	1	1	0
	1	1	0	0	0

Carry Save Addition

Given three n-bit numbers

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Wallace Tree Multiplier
Carry Save Addition

Carry Save Example

		0	1	1	1
		1	0	1	1
+		0	1	1	0
Sum		1	0	1	0
Carry	0	1	1	1	0
	1	1	0	0	0

Carry Save Addition

Given three n-bit numbers

1. Compute two n-bit numbers (n sum bits and n carry bits)
   - **Time required:** t_{FA}
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   - Time required: nt_{FA}

Total time required: $(n+1)t_{FA}$

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Carry Save Addition

Carry Save Example

		0	1	1	1
		1	0	1	1
+		0	1	1	0
Sum		1	0	1	0
Carry	0	1	1	1	0
	1	1	0	0	0

Carry Save Addition

Given three n-bit numbers

1. Compute two n-bit numbers (n sum bits and n carry bits)
   - **Time required:** t_{FA}
2. Add $n-1$ leftmost sum bits to n carry bits obtaining $n+1$ bit result and attach remaining sum bit as LSB
   - Time required: nt_{FA}

Total time required: $(n+1)t_{FA}$

Total time required (ripple carry addition) reduced from $(2n+1)t_{FA}$ to $(n+1)t_{FA}$

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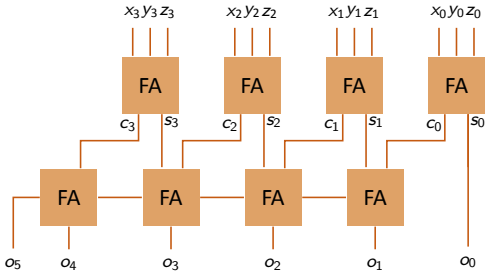
Carry Save Adder

- Logic for carry save addition of three numbers:



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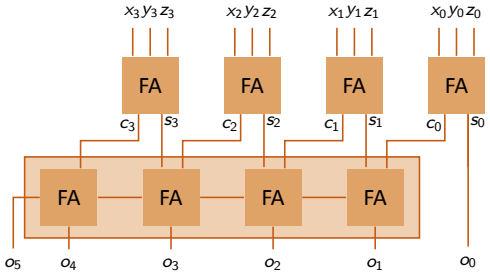
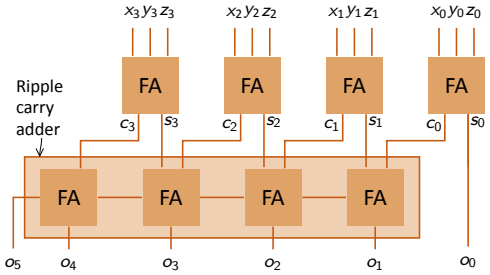
Carry Save Adder

- Logic for carry save addition of three numbers:



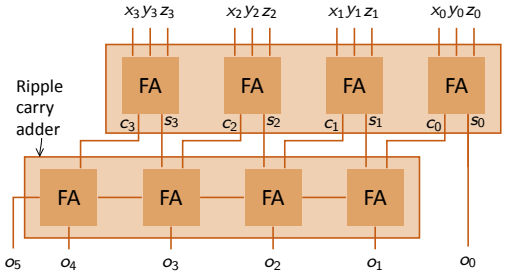
Wallace Tree Multiplier
Carry Save Adder

- Logic for carry save addition of three numbers:



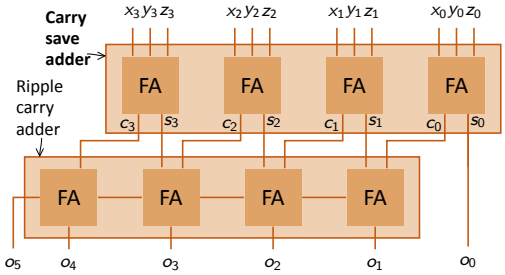
Wallace Tree Multiplier
Carry Save Adder

- Logic for carry save addition of three numbers:



Wallace Tree Multiplier
Carry Save Adder

- Logic for carry save addition of three numbers:



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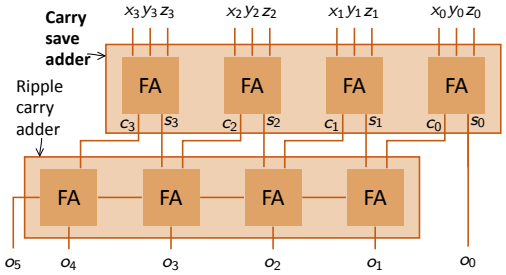
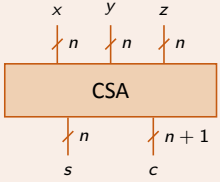
Carry Save Adder

- Logic for carry save addition of three numbers:



Basic Carry Save Adder



- Contains n full adders
- **Inputs** Three n-bit numbers
- **Outputs**
  - One n-bit number
  - One $(n+1)$-bit number (whose LSB is 0)

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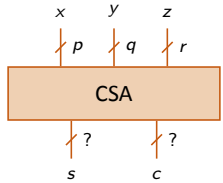
Think About It



- What are the output sizes if input sizes are different?
  - $q = p + 1, r = q + 1$
  - $q = p, r = q + 1$
- Can carry save handle two's complement numbers?