PRINCIPLES OF POINT ESTIMATION

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STATISTICS FOR DATA SCIENCE

Maximum Likelihood Estimation

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Topics to be covered...

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Maximum Likelihood Estimate → method

(MLE)

Best Point Estimator

$X \sim Bin(n, p)$

n: known, p is unknown

p: proportion of success in population

$X : 0 \quad 1 \quad 2 \; \ldots \; n$

Maximum Likelihood for Discrete Distributions

$X = 4$

X: No. of successes

n: no. of trials

$n = 10$

$x_1 \quad x_2 \quad x_3 \; \ldots \; x_{10}$

$1 \quad 0 \quad 0$

x_1, x_3, x_{10}, x_4

$$\boxed{p(x) = \binom{n}{x} \cdot p^x (1-p)^{n-x}}$$

p: unknown

$P(X = x)$:

Using Sample Estimate p

$p = ?$

$\hat{p} = \frac{X}{n} = \frac{4}{10}$

$\hat{p} = \frac{4}{10} = 0.4$ point estimate

$$\boxed{\hat{p} = \frac{X}{n}}$$

point estimator

Maximum Likelihood Estimate (MLE)

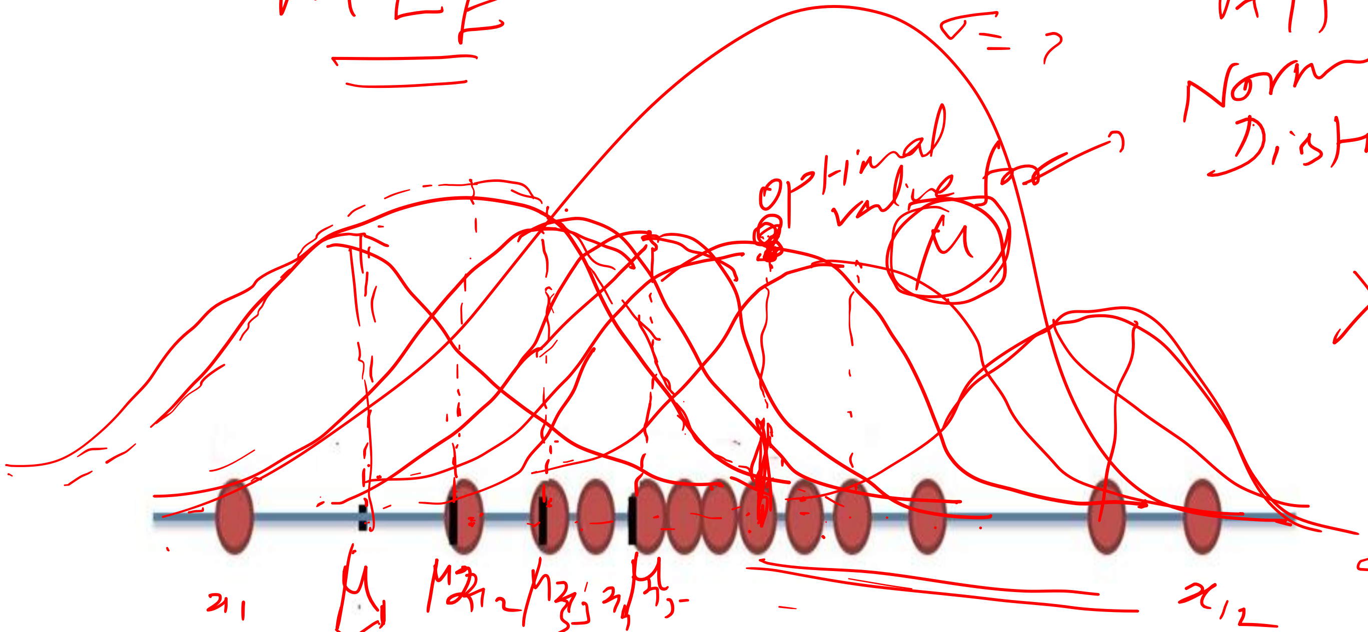
MLE

$\mu = ?$

$\sigma = ?$

Approximately Normal Distributed.

optimal value for μ



Mean of the Distribution

μ

Sample mean of the sample $\bar{x}$

1. Identify the Distribution
   - unknown • Normal
   - unknown • Exponential
   - • Gamma

$X \sim N(\mu, \sigma^2)$

99.7%

68%

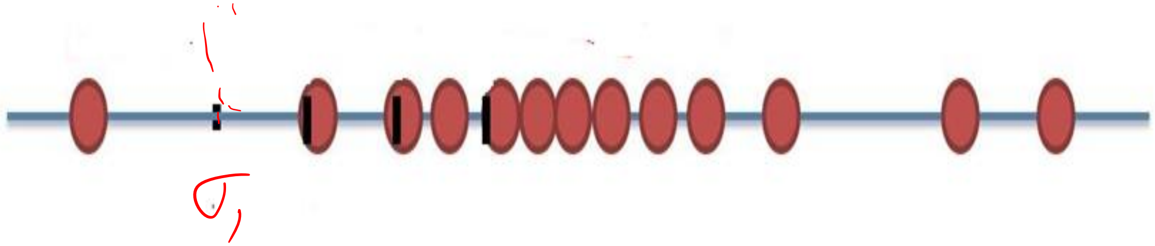
95.7%

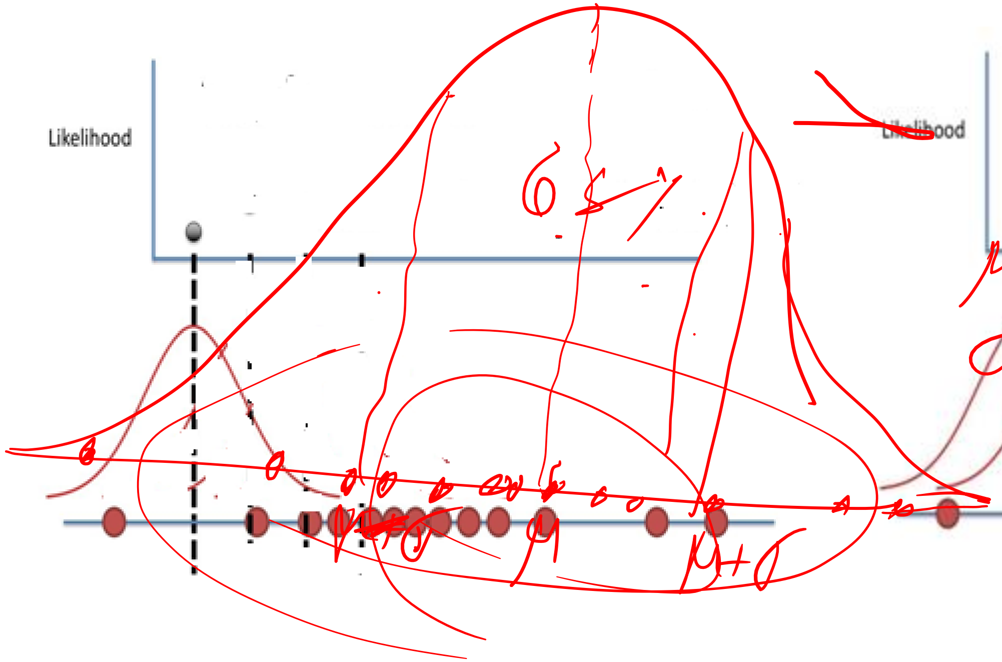
-3σ -2σ $-\sigma$ μ σ 2σ 3σ

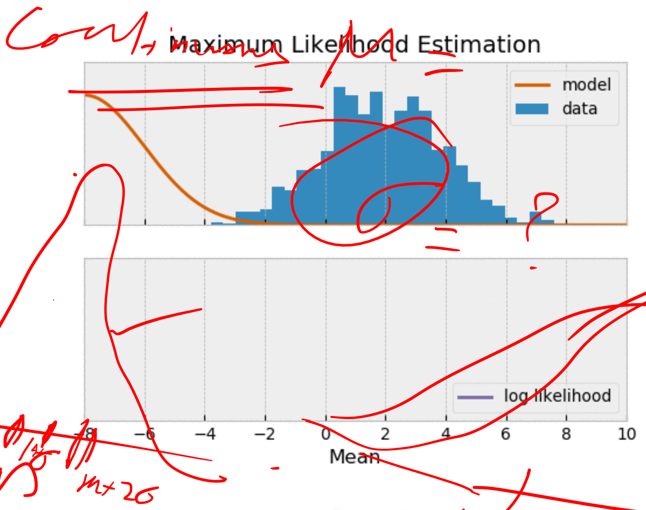
2. (shape) → where the μ is located σ

Source Images: www.StatQuest.com

Maximum Likelihood Estimate (MLE)







σ = large

$\mu-\sigma$ μ $\mu+\sigma$ $\mu+2\sigma$

Sample

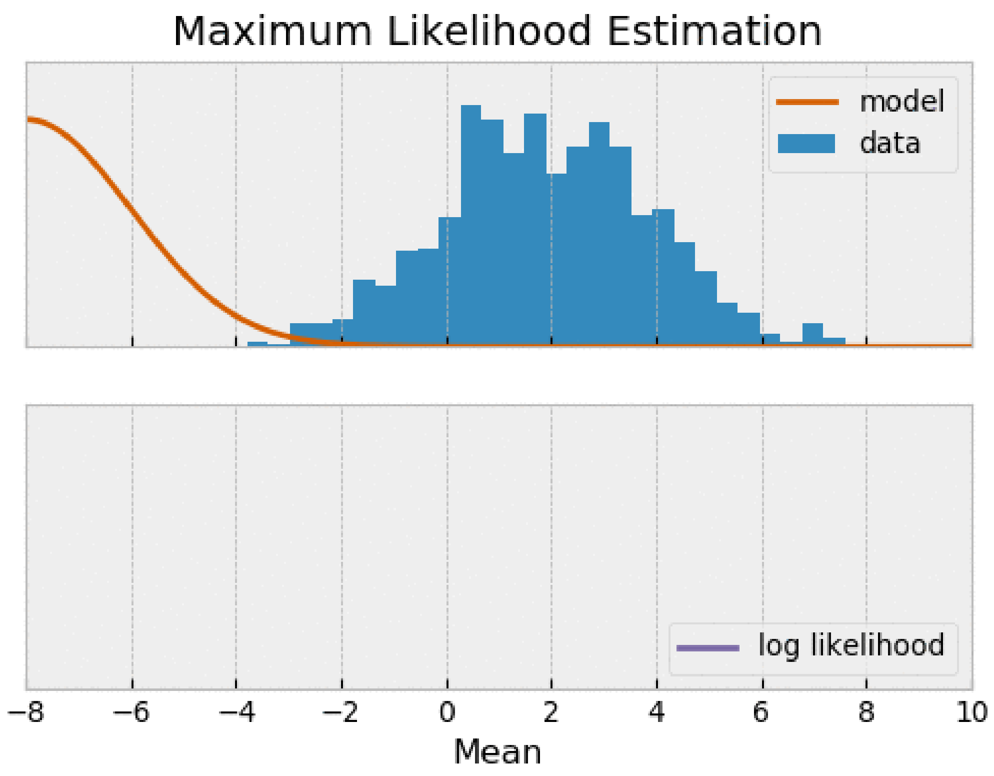
Bernoulli (p)

$p \rightarrow$ unknown

$\hat{p} = \frac{x}{n}$

Method to Construct Good Estimator

Maximum Likelihood Estimate(MLE)



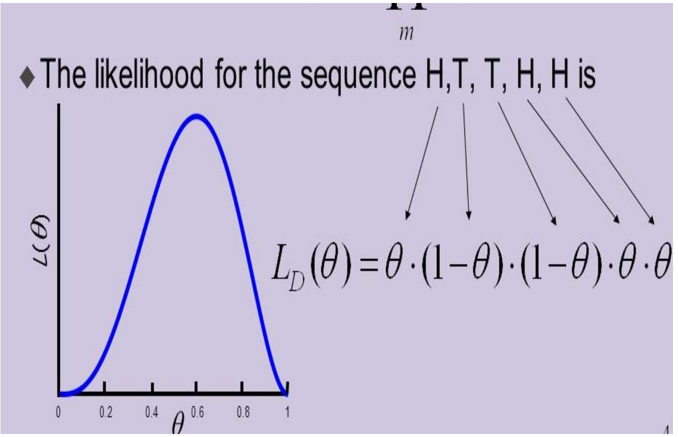
To estimate the parameter with the value that makes the observed data most likely.

Source: towardsdatascience.com

Method to Construct Good Estimator

MLE (maximum likelihood estimator) is regarded as the **best method** to point estimation.

Likelihood function is the probability of obtaining observed value.



$$L_D(\theta) = P(D\,|\,\theta) = \prod_m P(x[m]\,|\,\theta)$$

Note: If single observation is made, Likelihood function is just the probability of obtaining that value. (There will be no product involved)

Source: apollo.lsc.vsc.edu, webpages.uidaho.edu

Maximum Likelihood Estimator

MLE is that value of the estimator which when substituted in place of the parameter, maximizes the Likelihood function.

$$L_D(\theta) = P(D \mid \theta) = \prod_m P(x[m] \mid \theta)$$

Method to Construct Good Estimator

Method of Maximum Likelihood(MLE) is regarded as the best method to point estimation.

Likelihood function is the probability of obtaining observed value.

If single observation is made, Likelihood function is just the probability of obtaining that value.

MLE is that value of the estimator which when substituted in place of the parameter, maximizes the Likelihood function.

$L(p \mid \underbrace{x_1, x_2 \ldots x_n}_{\text{sample}}) =$

$p = ?$

Joint Prob. Mass

" Prob Density Fn

$x = 4$

$L = P(x = 4)$

General Steps to proceed with MLE.

Step 1: Write down the likelihood function.

Step 2: Take natural log of likelihood function.
(Reason: the quantity that maximizes log of a function is always the same quantity that maximizes the function itself)

Step 3: Differentiate log-likelihood function with respect to the parameter being estimated.

Step 4: Set the derivative equal to 0 to get MLE.

General Steps to proceed with MLE.

$L(\theta \mid x_1, x_2 \cdots x_n)$

Construct a Point Estimator

$L(\mu \mid x_1, x_2 \ldots x_n)$

$L(p \mid x_1, x_2 \ldots x_n)$

$L(\sigma \mid x_1, \ldots x_n)$

Step 1: Write down the likelihood function.

$\ln L(\theta \mid x_1, \ldots x_n)$

Step 2: Take natural log of likelihood function.

(Reason: the quantity that maximizes log of a function is always the same quantity that maximizes the function itself)

$$\frac{d}{d\theta}(\ln L) = 0 \Rightarrow \boxed{\hat{\theta} = ?}$$

$\bar{X}$ s_n

MLE of θ

Step 3: Differentiate log-likelihood function with respect to the parameter being estimated.

$\hat{\theta} =$

MLE of $p \Rightarrow \hat{p} = ?$

MLE of $\mu \Rightarrow \hat{\mu} = ?$

" " $\sigma \Rightarrow \hat{\sigma}$

Step 4: Set the derivative equal to 0 to get MLE.

Points to Remember

- The **maximum likelihood estimate** is the value of the estimators that when substituted in for the parameters maximizes the likelihood function.

- The likelihood function can be a **probability density function** or a **probability mass function**.

- It can also be a **joint probability** density or mass function, and is often the joint density or mass function of independent random variables.

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Maximum Likelihood Estimator for Bernoulli Distribution

Let $X_1, X_2, X_3 \ldots X_n$ be a random sample from a population with Bernoulli(p) distribution. Find MLE of p.

$X_i \sim$ Bernoulli(p) for $i = 1$ to n

Distribution	X :	0	1
$p(x)$ or $P(X=x)$	:	$1-p$	p

$$P(X = x_i) = p^{x_i}(1-p)^{1-x_i}$$

1. Write down the likelihood function $L(\theta)$
2. Take natural log of likelihood function.
3. Differentiate log-~~likely~~hood function w.r.to the parameter being estimated.
4. Set the derivative $= 0$ to get MLE.

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Maximum Likelihood Estimator for Bernoulli Distribution

$$P(X=x_i) = p^{x_i}(1-p)^{1-x_i}$$

$P(X$

H T T H H

$$L = f(p \mid x_1, x_2 \ldots x_n)$$

$$L(\phi) = p \cdot (1-p) \cdot (1-p) \cdot p \cdot p$$

$$L(\Phi) =$$

$$L = P(X=x_1) * P(X=x_2) * \ldots * P(X=x_n)$$

$$= p^{x_1}(1-p)^{1-x_1} * p^{x_2}(1-p)^{1-x_2} * \ldots * p^{x_n}(1-p)^{1-x_n}$$

$$① \; L = \prod_{i=1}^{n} p^{x_i}(1-p)^{1-x_i} \simeq p^{\sum_{i=1}^{n} x_i}(1-p)^{n-\sum_{i=1}^{n} x_i}$$

$$② \; \ln L = \sum x_i \ln p + (n - \sum x_i) \ln(1-p)$$

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Maximum Likelihood Estimator for Bernoulli Distribution

3. $\frac{d \ln L}{dp} = 0$ $\qquad \ln L = \sum x_i \ln p + (n - \sum x_i) \ln(1-p)$

$\Rightarrow \sum x_i \frac{d \ln p}{dp} + (n - \sum x_i) \frac{d \ln(1-p)}{dp} = 0$

$\Rightarrow \frac{\sum x_i}{p} + (n - \sum x_i) \frac{(-1)}{1-p} = 0$ $\qquad \Rightarrow \sum x_i - p\sum x_i - np + p\sum x_i = 0$

$\Rightarrow \sum x_i \qquad)$ $\qquad)$

$\Rightarrow \frac{\qquad)}{p(1-p)}$

MLE $\Rightarrow$ $\hat{p} = \bar{x}$

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Maximum Likelihood Estimator for Bernoulli Distribution

$X \sim Bern$

$X \sim Bernoulli(p)$

$n = 5$

$L(p|x)$ $L(p) =$ $p(x) = p^x (1-p)^{1-x}$

$p = ?$

H, H, T, H, T

x_1, x_2, x_3, x_4, x_5

Let X_1 X_2 X_3 . . X_n which

$X_i \sim Bernoulli(p)$ unknown

$L(p | H, H, T, H, T) = p * p * (1-p) * p(1-p)$

Find MLE of p.

$p = P(X = 1)$

$P(X_1 = H, X_2 = H, X_3 = T, X_4 = H, X_5 = T) = ?$

$$L(p | x_1, x_2 \ldots x_n) = \prod_{i=1}^{n} p^{x_i} (1-p)^{1-x_i}$$

Maximum Likelihood Estimator for Bernoulli Distribution

Find MLE of p.

Let $X_1, X_2, \ldots X_n$ be a simple random sample which follows Bernoulli(p) distn.

$X_i \sim \text{Bernoulli}(p)$

X_i :	0	1
$P(X_i = x_i)$:	$1-p$	p

$p(x) = p^x (1-p)^{1-x}$

1. $L(p \mid x_1, \ldots x_n) = \prod_{i=1}^{n} p^{x_i}(1-p)^{1-x_i}$

$= p^{x_1}(1-p)^{1-x_1} * p^{x_2}(1-p)^{1-x_2} * \ldots * p^{x_n}(1-p)^{1-x_n}$

$L(p \mid x_1 \ldots x_n) = p^{\sum_{i=1}^{n} x_i} * (1-p)^{n-\sum_{i=1}^{n} x_i}$

2. $\ln L(p \mid \ldots) = \sum_{i=1}^{n} x_i \ln p + \left(n - \sum_{i=1}^{n} x_i\right) \ln(1-p)$

3. $\frac{d}{dp}\left(\ln L(p)\right) = 0 \Rightarrow \sum x_i \times \frac{1}{p} + \left(n - \sum x_i\right)\frac{1 \times (-1)}{1-p} = 0$

$\Rightarrow \frac{\sum x_i}{p} - \frac{(n - \sum x_i)}{1-p} = 0$

$\rightarrow \frac{\sum x_i}{p} - \frac{(n-\sum x_i)}{1-p} = 0$

multiply by $p(1-p)$

$(1-p)\sum x_i - p(n - \sum x_i) = 0$

$\sum x_i - p\sum x_i - np + p\sum x_i = 0$

$\sum x_i - np = 0$

$\Rightarrow p = \frac{\sum x_i}{n} = \bar{x} \ni$ MLE of p is $\boxed{\hat{p} = \bar{x}}$

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Maximum Likelihood Estimator for Binomial Distribution

Let X be a Binomial r.v with $X \sim Bin(n, p)$ follows Binomial (n, p) distribution where p is unknown. Find MLE of p.

$p(x) = \binom{n}{x} p^x (1-p)^{n-x}$

1. $L(p \mid x) = \frac{n!}{x!\,(n-x)!} * p^x \cdot (1-p)^{n-x}$

2. $\ln L(p \mid x) = \ln\left(\frac{n!}{x!\,(n-x)!}\right) + x \ln(p) + (n-x)\ln(1-p)$

3. $\frac{d}{dp}(\ln L) = 0 \Rightarrow \frac{d}{dp}\left(\ln n! - \ln x! - \ln (n-x)! + x \ln(p) + (n-x)\ln(1-p)\right) = 0$

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Maximum Likelihood Estimator for Binomial Distribution

$$\frac{d}{dp}(\ln L) = 0 \Rightarrow \frac{d}{dp}\left(x \ln(p) + (n-x)\ln(1-p)\right) = 0$$

$$\Rightarrow \frac{x}{p} + \frac{n}{1-p}(-1) - \frac{x}{(1-p)}(-1) = 0$$

$$\Rightarrow \frac{x}{p} - \frac{n}{1-p} + \frac{x}{1-p} = 0$$

Multiplying by $p(1-p)$

$$x(1-p) - np + xp = 0$$

$$x - xp - np + xp = 0$$

$$x = np \Rightarrow p = \frac{x}{n}$$

$\therefore$ MLE of p is

$$\boxed{\hat{p} = \frac{X}{n}}$$

Maximum Likelihood Estimator for Poisson Distribution

Let $X_1, X_2, \ldots X_n$ be a simple random sample from a population which follows $X_i \sim \text{Poisson}(\lambda)$ distribution where λ is unknown. Find MLE of λ.

$P(X_1 = x_1 ; X_2 = x_2 \ldots X_n = x_n)$

$X_i \sim \text{Poisson}(\lambda)$

$$p(x_i) = \prod_{t=1}^{n} \frac{e^{-\lambda} \cdot \lambda^{x_t}}{x_t!}$$

$$L(\lambda \mid x_1, x_2, \ldots x_n) = \prod_{t=1}^{n} \frac{e^{-\lambda} \cdot \lambda^{x_t}}{x_t!} = \frac{e^{-\lambda} \cdot \lambda^{x_1}}{x_1!} * \frac{e^{-\lambda} \cdot \lambda^{x_2}}{x_2!} * \cdots * \frac{e^{-\lambda} \cdot \lambda^{x_n}}{x_n!}$$

$$\ln L(\lambda \mid x) = -n\lambda + (\Sigma x_t) \cdot \ln \lambda - \sum_{t=1}^{n} \ln x_t!$$

$$\frac{d}{d\lambda}(\ln L(\lambda)) = 0 \Rightarrow -n \cdot 1 + (\Sigma x_t)\frac{1}{\lambda} = 0 \Rightarrow -n + \frac{\Sigma x_t}{\lambda} = 0$$

$$\Rightarrow n = \frac{\Sigma x_t}{\lambda} \Rightarrow \lambda = \frac{\Sigma x_t}{n} = \bar{x}$$

$\therefore$ MLE of λ is $\boxed{\hat{\lambda} = \bar{X}}$

Maximum Likelihood Estimator for Normal Distribution

$x_1, x_2, \ldots, x_n$ — {1, 5, 7, ..., 13} Sample set

$n=10$ $X \sim N(\mu, \sigma^2)$

Normal

$P(a \le X \le b) = \int_a^b f(x)\,dx$ prob. density fn.

$L(\mu, \sigma \mid x_1, x_2 \ldots x_n) = \prod f(x_i; \mu, \sigma)$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

pdf

$P(X_1 = 1;\ X_2 = 5;\ X_3 = 7, \ldots X_{10} = 13)$

Joint and Prob

$P(X = x_1;\ Y = y_1)$

$= P(X = x_1) * P(Y = y_1)$

$$L(\mu, \sigma) = \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

$$= \left(\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_1-\mu)^2}{2\sigma^2}}\right) * \left(\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_2-\mu)^2}{2\sigma^2}}\right) * \cdots * \left(\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_n-\mu)^2}{2\sigma^2}}\right)$$

$$L = \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n \cdot e^{-\frac{1}{2\sigma^2}\sum (x_i-\mu)^2} \Rightarrow n \ln\left(\frac{1}{\sigma\sqrt{2\pi}}\right) = -n \ln \sigma - \frac{1}{2\sigma^2} \sum \ln (x_i - \mu)^2$$

Maximum Likelihood Estimator for Normal Distribution

Let $X_1, X_2, \ldots X_n$ be a random sample from a $N(\mu, \sigma^2)$ distribution. Find MLE of μ and σ.

$x_1, \ldots, x_n$

If $X \sim N(\mu, \sigma^2)$ $\quad f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$X_i \sim N(\mu, \sigma^2)$

$$L(\mu, \sigma \mid x_1, \ldots x_n) = \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

$$L(\mu, \sigma) = \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n \cdot e^{-\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2\sigma^2}}$$

$$\ln L(\mu, \sigma) = -n \ln \sigma - \frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2\sigma^2} \quad \text{——} \ (1)$$

Maximum Likelihood Estimator for Normal Distribution

$$\ln L(\mu,\sigma) = -n\ln\sigma - \sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\sigma^2}$$

Case i) To find MLE of μ

$$\frac{\partial}{\partial\mu}\left(\ln L(\mu,\sigma)\right)=0 \Rightarrow 0-\frac{1}{2\sigma^2}*2\sum_{i=1}^{n}(x_i-\mu)\cdot(-1)=0$$

$$\Rightarrow \frac{\sum_{i=1}^{n}(x_i-\mu)}{\sigma^2}=0$$

$$\Rightarrow \sum_{i=1}^{n}(x_i-\mu)=0$$

$$\Rightarrow \sum_{i=1}^{n}x_i-n\mu=0$$

$$\Rightarrow n\mu=\sum_{i=1}^{n}x_i \Rightarrow \mu=\frac{\sum_{i=1}^{n}x_i}{n}=\bar{x}$$

$\therefore$ MLE of μ is $\boxed{\hat{\mu}=\bar{X}}$

Maximum Likelihood Estimator for σ

case ii)

$$\ln L(\mu, \sigma \mid x_1, \dots x_n) = -n \ln \sigma - \frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2\sigma^2}$$

$$\frac{\partial}{\partial \sigma}\left(\ln L(\mu, \sigma \mid x)\right) = 0 \Rightarrow -n * \frac{1}{\sigma} - \frac{\sum (x_i-\mu)^2}{2} * \frac{(-2)}{\sigma^3} = 0$$

$$\Rightarrow -\frac{n}{\sigma} + \frac{\sum (x_i-\mu)^2}{\sigma^3} = 0$$

$$\Rightarrow -n\sigma^2 + \sum (x_i-\mu)^2 = 0$$

$$\Rightarrow n\sigma^2 = \sum (x_i-\mu)^2$$

$$\Rightarrow \boxed{\sigma^2 = \frac{\sum (x_i-\mu)^2}{n}}$$

MLE of σ

$$\boxed{\hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n}}}$$

Maximum Likelihood Estimator: Problems

Let $X \sim Bin(20, p)$. Suppose we observe the value of $x = 7$. Find MLE of p.

X : No. of successes

$$p(x) = {}^nC_x\, p^x (1-p)^{n-x}$$

$n = 20$

$X = x = 7$

$$\hat{p} = \frac{X}{n} = \frac{7}{20}$$

$$L(p \mid x=7) = {}^{20}C_7\, p^7 (1-p)^{20-7} \simeq p^7 (1-p)^{13}$$

Estimator for p is

$$\hat{p} = \frac{X}{n} = \frac{7}{20}$$

① $L(p \mid x=7) = p^7(1-p)^{13}$ $\qquad \ln\left(p^7(1-p)^{13}\right)$

② $\ln L(p \mid x=7) = 7\ln p + 13\ln(1-p)$

③ $\dfrac{d \ln L}{dp} = 0 \Rightarrow 7 \cdot \dfrac{1}{p} + 13\dfrac{1}{1-p}(-1) = 0$

$$\Rightarrow \frac{7}{p} - \frac{13}{1-p} = 0 \Rightarrow 7(1-p) - 13p = 0$$

$$\Rightarrow 7 - 7p - 13p = 0$$

$$7 - 20p = 0$$

$$20p = 7$$

$$p = \frac{7}{20}$$

$$\therefore \hat{p} = \frac{7}{20}$$

Maximum Likelihood Estimator: Problems

Suppose number of claims in a month from a certain portfolio of policies has a Poisson distribution with mean λ.

a) If no. of claims observed in one particular month is 8. Find MLE of λ. $X \sim Poisson(\lambda)$

X: No. of claims x $X \sim Poisson(8)$

$$p(x) = \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$

$$L(\lambda \mid x=8) = \frac{e^{-\lambda} \cdot \lambda^{8}}{8!} \simeq e^{-\lambda} \cdot \lambda^{8}$$

$$\ln L(\lambda \mid x=8) = -\lambda + 8 \ln \lambda$$

$$\frac{d}{d\lambda}(\ln L(\lambda)) = 0 \Rightarrow -1 + \frac{8}{\lambda} = 0 \Rightarrow \frac{8}{\lambda} = 1 \Rightarrow \lambda = 8$$

$$\hat{\lambda} = \frac{\sum x_i}{n} = \bar{x}$$

$$\boxed{\hat{\lambda} = \bar{X}}$$

$8 \rightarrow \hat{\lambda} = 8$

$\hat{\lambda} = \bar{x}$ $\hat{\lambda} =$

8 2 7 5 3 7 → Sample

8 6 $\hat{\lambda} = 7$

$$\therefore \boxed{\hat{\lambda} = 8}$$

Suppose number of claims in a month from a certain portfolio of policies has a Poisson distribution with mean λ.

b) In one month 8 claims were incurred and in next month 10 claims were incurred.

Find MLE of λ.

$$L(\lambda \mid x_1 = 8;\ x_2 = 10) = \prod_{i=1}^{2} \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$

$$= P(X = 8) \cdot P(X = 10)$$

$$L(\lambda \mid x_1, x_2) = \frac{e^{-\lambda} \cdot \lambda^{8}}{8!} * \frac{e^{-\lambda} \cdot \lambda^{10}}{10!}$$

$$L(\lambda \mid x_1, x_2) = e^{-2\lambda} \cdot \lambda^{18}$$

$$\boxed{\hat{\lambda} = \bar{X}}$$

$$\ln L(\lambda \mid x_1, x_2) = -2\lambda + 18 \ln \lambda$$

$$\frac{d \ln L}{d\lambda} = 0 \Rightarrow -2 + \frac{18}{\lambda} = 0$$

$$\Rightarrow -2\lambda + 18 = 0$$

$$\Rightarrow 2\lambda = 18$$

$$\Rightarrow \lambda = 9$$

$\therefore$ MLE of λ is $\hat{\lambda} = 9$

Maximum Likelihood Estimator: Problems

Let $X_1, X_2, \ldots X_n$ be a random sample from $N(\mu, 1)$ population. Find MLE of μ.

$X_i \sim N(\mu, \sigma^2)$ pdf $\Rightarrow f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

Given $\sigma^2 = 1$

$x_i \sim N(\mu, 1)$ $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2}}$

$$L(\mu, 1 \mid x_1, \ldots x_n) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{(x_i-\mu)^2}{2}}$$

$$L(\mu, 1) x_1, \ldots x_n) = e^{-\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{2}}$$

$$\ln L(\mu, 1 / x_1 \ldots x_n) = -\frac{1}{2}\sum_{i=1}^{n} (x_i-\mu)^2$$

$\frac{d \ln L}{d\mu} = 0 \Rightarrow$

$-\frac{1}{2} \cdot 2 \sum_{i=1}^{n}(x_i-\mu) \cdot (-1) = 0$

$\Rightarrow \sum_{i=1}^{n}(x_i-\mu) = 0$

$\Rightarrow \sum_{i=1}^{n} x_i - n\mu = 0$

$\Rightarrow n\mu = \sum x_i \Rightarrow \mu = \frac{\sum x_i}{n}$

$\mu = \bar{x}$

$\therefore$ MLE of μ is $\hat{\mu} = \bar{x}$

Maximum Likelihood Estimator: Problems

Let $X_1, X_2, \ldots X_n$ be a random sample from a $N(0, \sigma^2)$ population. Find MLE of σ.

Example - Poisson Distribution

Problem:

The following data are the observed frequencies of occurrence of domestic accidents: we have n = 647 data as follows

Number of Accidents	Frequency
0	447
1	132
2	42
3	21
4	3
5	2

Problem Source - http://wwwf.imperial.ac.uk/

Example - Poisson Distribution

Solution:

$$\lambda = \frac{1}{\lambda}\sum_{i=1}^{n} x_i = \bar{X}$$

The MLE of λ is $\hat{\lambda} = \bar{X}$

$$\hat{\lambda} = \frac{1}{n}\sum_{i=1}^{n} x_i = \bar{X}$$

$$= \frac{(447*0)+(132*1)+(42*2)+(21*3)+(3*4)+(2*5)}{674}$$

$$= 0.465$$

Example – Maximum Likelihood Estimate for Mean (μ)

Problem:

A random sample of 10 weights (in pounds) of Annie's class mates are given as 115 122 130 127 149 160 152 138 149 180.

Solution:

We know that, the MLE of μ is $\hat{\mu} = \bar{X}$

$$\hat{\mu} = \sum_{i=1}^{n} \frac{x_i}{n} = \bar{X}$$

$$\hat{\mu} = \frac{1}{10}(115 + 122 + 130 + 127 + 149 + 160 + 152 + 138 + 149 + 180)$$

$$= 142.2$$

Numerical Data Source: https://online.stat.psu.edu/stat414/

THANK YOU

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