## Unit 2 – Vector Spaces

Vector Spaces and Subspaces ( definitions only ) , Linear Independence, Basis and Dimensions, The Four Fundamental Subspaces.

Self Learning Component : Examples of vector spaces and subspaces

15-17	Vector Spaces and Subspaces ( Definition only ), Column Space and Null Space,
	Examples
18-20	Echelon Form, Row Reduced Form, Pivot Variables , Free variables
21- 24	Linear Independence, Basis and Dimensions
25	Scilab Class Number 4 – Span of Column Space of A
26-27	The Four Fundamental Subspaces
28	Existence of Inverses
29	Scilab Class Number 5 –Four Fundamental Subspaces of A

## **Class work Problems:**

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1	Describe ( geometrically ) the column space and null space of the following matrices: ( do not do a detailed procedure )			
	(i) $\begin{bmatrix} 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 0 & -3 \end{bmatrix}$ (iii) $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$ (iv) $\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$ (v) $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$			
	(vi) $\begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix}$ Sample Answer: (i) $C(A) = Z$ , $N(A) = R^1$			
2	Find the column space and null space of $A = \begin{bmatrix} 1 & 0 \\ 2 & 7 \\ 5 & 3 \end{bmatrix}$ . Give an example of a matrix			
	whose column space is the same as that of A but null space is different.  Answer: C(A) is a 2d plane in R <sup>3</sup> and N(A) is the origin in R <sup>2</sup> . The matrix			
	$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 7 & 9 \\ 5 & 3 & 8 \end{bmatrix}$ has same C(A) but its N(A) is a line in R <sup>3</sup> passing through (1, 1, -1)			
3	( this is same as finding $C(A)$ )			
	(i) $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ (ii) $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ Answer: (i) all b (ii) $b_3 = 0$			
	(iii) $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ -1 & -2 & -3 \end{bmatrix}$ (iv) $A = \begin{bmatrix} 1 & 2 \\ 2 & 5 \\ -1 & -2 \end{bmatrix}$ Answer: (iii) $b_2 = 2b_1$ , $b_3 = -b_1$ , (iv) $b_3 = -b_1$			

Reduce the matrix $A = \begin{bmatrix} 1 & 2 & -3 & 1 & 2 \\ 2 & 4 & -4 & 6 & 10 \\ 3 & 6 & -6 & 9 & 13 \end{bmatrix}$ to its echelon form U and hence find its rank. Identify the pivot and free variables. Find the special solutions to $Ax = 0$ by reducing A to its row reduced form R.  Answer: $U = \begin{bmatrix} 1 & 2 & -3 & 1 & 2 \\ 0 & 0 & 2 & 4 & 6 \\ 0 & 0 & 0 & 0 & -2 \end{bmatrix}$ , $R = \begin{bmatrix} 1 & 2 & 0 & 7 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ , rank $r = 3$ Special solutions are $(-2, 1, 0, 0, 0)$ and $(-7, 0, -2, 1, 0)$ .  Reduce these matrices to their echelon form to find their rank. Also find a special solution to each of the free variables. $\begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ (i) & A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix}$ Answer: $(i) (-2, 1, 0, 0, 0)$ , $(0, 0, -2, 1, 0)$ , $(0, 0, -3, 0, 1)$ (ii) $(1, -1, 1)$ For every c, find R and the special solutions to $Ax = 0$ : $\begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & 2 & 4 & 4 \\ 1 & c & 2 & 2 \end{bmatrix}$ Answer: Perfer Gilbert Strang book.  7 Choose the number q so that (if possible) the ranks are (i) 1 (ii) 2 (iii) 3 $A = \begin{bmatrix} 6 & 4 & 2 \\ -3 & -2 & -1 \\ 9 & 6 & q \end{bmatrix}$ Answer: For $A$ , $a = 3$ gives rank 1 and every other q gives rank 2. For B, $a = 6$ gives rank 1 and every other q gives rank 2.  8 Which vectors $(b_1, b_2, b_3)$ are in the column space of A? Which combination of the rows of A give 0? $\begin{bmatrix} 1 & 2 & 1 \\ (i) & A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 6 & 3 \\ 0 & 2 & 5 \end{bmatrix}$ (ii) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 2 & 4 & 8 \end{bmatrix}$ Answer: (i) all b and no combination of the rows (ii) $b_3 = 2b_2$ and $a_3 - 2R_2 = 0$ 9 Examine if the following sets of vectors are linearly independent. When the set is dependent find a relation between the vectors $1 = (1, 2, 0, 1, 0, 3, 1), (-1, 0, 1)$ . Answer: independent $1 = (1, 2, 0, 1, 0, 3, 1), (-1, 0, 1)$ . Answer: independent. The set $(1, 1, 2, 0, 3, 1, 3, 2, -2, 4, 1, 1, 3, 3, 3, 3, 2, 2, 3, 3, 1, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3,$	_			
rank. Identify the pivot and free variables. Find the special solutions to $Ax = 0$ by reducing A to its row reduced form R. $ Answer: U = \begin{bmatrix} 1 & 2 & -3 & 1 & 2 \\ 0 & 0 & 2 & 4 & 6 \\ 0 & 0 & 0 & 0 & -2 \end{bmatrix}, R = \begin{bmatrix} 1 & 2 & 0 & 7 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, rank r = 3 $ Special solutions are $(-2, 1, 0, 0, 0)$ and $(-7, 0, -2, 1, 0)$ .  Reduce these matrices to their echelon form to find their rank. Also find a special solution to each of the free variables. $ \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix} (ii) A = \begin{bmatrix} 2 & 4 & 2 \\ 0 & 4 & 4 \\ 0 & 8 & 8 \end{bmatrix} $ Answer: $(i) (-2, 1, 0, 0, 0), (0, 0, -2, 1, 0), (0, 0, -3, 0, 1) (ii) (1, -1, 1).  For every c, find R and the special solutions to Ax = 0:   \begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & 2 & 4 & 4 \\ 1 & c & 2 & 2 \end{bmatrix}  Answer: PI refer Gilbert Strang book.  7 Choose the number q so that (if possible) the ranks are (i) = 1 (ii) = 2 (iii) = 3 Answer: For A, A = 3 gives rank 1 and every other q gives rank 2. For B, A = 6 gives rank 1 and every other q gives rank 2. For B, A = 6 gives rank 1 and every other q gives rank 2.  8 Which vectors (b_1, b_2, b_3) are in the column space of A? Which combination of the rows of A give A?  (i) A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 6 & 3 \\ 0 & 2 & 5 \end{bmatrix} (ii) A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 2 & 4 & 8 \end{bmatrix} Answer: (1) = 1 and (1) = 1 a$	4	1 2 -3 1 2		
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rank. Identify the pivot and free variables. Find the special solutions to $Ax = 0$ by reducing A to its row reduced form R. $ Answer: U = \begin{bmatrix} 1 & 2 & -3 & 1 & 2 \\ 0 & 0 & 2 & 4 & 6 \\ 0 & 0 & 0 & 0 & -2 \end{bmatrix}, R = \begin{bmatrix} 1 & 2 & 0 & 7 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, rank r = 3 $ Special solutions are $(-2, 1, 0, 0, 0)$ and $(-7, 0, -2, 1, 0)$ .  Reduce these matrices to their echelon form to find their rank. Also find a special solution to each of the free variables. $ \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix} (ii) A = \begin{bmatrix} 2 & 4 & 2 \\ 0 & 4 & 4 \\ 0 & 8 & 8 \end{bmatrix} $ Answer: $(i) (-2, 1, 0, 0, 0), (0, 0, -2, 1, 0), (0, 0, -3, 0, 1) (ii) (1, -1, 1).  For every c, find R and the special solutions to Ax = 0:   \begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & 2 & 4 & 4 \\ 1 & c & 2 & 2 \end{bmatrix}  Answer: PI refer Gilbert Strang book.  7 Choose the number q so that (if possible) the ranks are (i) = 1 (ii) = 2 (iii) = 3 Answer: For A, A = 3 gives rank 1 and every other q gives rank 2. For B, A = 6 gives rank 1 and every other q gives rank 2. For B, A = 6 gives rank 1 and every other q gives rank 2.  8 Which vectors (b_1, b_2, b_3) are in the column space of A? Which combination of the rows of A give A?  (i) A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 6 & 3 \\ 0 & 2 & 5 \end{bmatrix} (ii) A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 2 & 4 & 8 \end{bmatrix} Answer: (1) = 1 and (1) = 1 a$		2 6 6 0 12		
A to its row reduced form R.   Answer: $U = \begin{bmatrix} 1 & 2 & -3 & 1 & 2 \\ 0 & 0 & 2 & 4 & 6 \\ 0 & 0 & 0 & 0 & -2 \end{bmatrix}$ , $R = \begin{bmatrix} 1 & 2 & 0 & 7 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ , rank r = 3  Special solutions are $(-2, 1, 0, 0, 0)$ and $(-7, 0, -2, 1, 0)$ .  Reduce these matrices to their echelon form to find their rank. Also find a special solution to each of the free variables. $\begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix}$ (ii) $A = \begin{bmatrix} 2 & 4 & 2 \\ 0 & 4 & 4 \\ 0 & 8 & 8 \end{bmatrix}$ Answer: (i) $(-2, 1, 0, 0, 0)$ , $(0, 0, -2, 1, 0)$ , $(0, 0, -3, 0, 1)$ (ii) $(1, -1, 1)$ For every c, find R and the special solutions to $Ax = 0$ : $\begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & 2 & 4 & 4 \\ 1 & c & 2 & 2 \end{bmatrix}$ (ii) $A = \begin{bmatrix} 1 - c & 2 \\ 0 & 2 - c \end{bmatrix}$ Answer: PI refer Gilbert Strang book.  7 Choose the number q so that ( if possible ) the ranks are (i) 1 (ii) 2 (iii) 3 $A = \begin{bmatrix} 6 & 4 & 2 \\ -3 & -2 & -1 \\ 9 & 6 & q \end{bmatrix}$ Answer: For A, q = 3 gives rank 1 and every other q gives rank 2. For B, q = 6 gives rank 1 and every other q gives rank 2.  8 Which vectors ( $b_1$ , $b_2$ , $b_3$ ) are in the column space of A? Which combination of the rows of A give 0?  (i) $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 6 & 3 \\ 0 & 2 & 5 \end{bmatrix}$ (ii) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 2 & 4 & 8 \end{bmatrix}$ Answer: (i) all b and no combination of the rows (ii) $b_3 = 2b_2$ and $R_3 - 2R_2 = 0$ 9 Examine if the following sets of vectors are linearly independent. When the set is dependent find a relation between the vectors 1. { (1, 2, 0), (0, 3, 1), (-1, 0, 1)}. Answer: independent, v3 = 2v1 - v2  10 If $v_1$ , $v_2$ , $v_3$ are vectors and a and b are scalars show that the set $\{v_1, v_2, v_3\}$ is linearly dependent. Find a maximal linearly independent set of vectors from the set $v_1 = (2, -2, -4)$ , $v_2 = (1, 9, 3)$ , $v_3 = (-2, -4, 1)$ , $v_4 = (3, 7, -1)$				
Answer: $U = \begin{bmatrix} 1 & 2 & -3 & 1 & 2 \\ 0 & 0 & 2 & 4 & 6 \\ 0 & 0 & 0 & 0 & -2 \end{bmatrix}$ , $R = \begin{bmatrix} 1 & 2 & 0 & 7 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ , rank r = 3  Special solutions are (-2, 1, 0, 0, 0) and (-7, 0, -2, 1, 0)  Reduce these matrices to their echelon form to find their rank. Also find a special solution to each of the free variables. $ \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix} $ (ii) $A = \begin{bmatrix} 2 & 4 & 2 \\ 0 & 4 & 4 \\ 0 & 8 & 8 \end{bmatrix}$ Answer: (i) (-2, 1, 0, 0, 0, 0), (0, 0, -2, 1, 0), (0, 0, -3, 0, 1) (ii) (1, -1, 1)  For every c, find R and the special solutions to $Ax = 0$ : $ \begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & 2 & 4 & 4 \\ 1 & c & 2 & 2 \end{bmatrix} $ (ii) $A = \begin{bmatrix} 1 - c & 2 \\ 0 & 2 - c \end{bmatrix}$ Answer: PI refer Gilbert Strang book.  7 Choose the number q so that (if possible) the ranks are (i) 1 (ii) 2 (iii) 3 $ A = \begin{bmatrix} 6 & 4 & 2 \\ -3 & -2 & -1 \\ 9 & 6 & q \end{bmatrix},  B = \begin{bmatrix} 3 & 1 & 3 \\ q & 2 & q \end{bmatrix}$ Answer: For A, q = 3 gives rank 1 and every other q gives rank 2. For B, q = 6 gives rank 1 and every other q gives rank 2. Which vectors (b <sub>1</sub> , b <sub>2</sub> , b <sub>3</sub> ) are in the column space of A? Which combination of the rows of A give 0? $ \begin{bmatrix} 1 & 2 & 1 \\ 2 & 6 & 3 \\ 0 & 2 & 5 \end{bmatrix} $ (ii) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 2 & 4 & 8 \end{bmatrix}$ Answer: (i) all b and no combination of the rows (ii) b <sub>3</sub> = 2b <sub>2</sub> and R <sub>3</sub> - 2R <sub>2</sub> = 0  9 Examine if the following sets of vectors are linearly independent. When the set is dependent find a relation between the vectors  1. { (1, 2, 0), (0, 3, 1), (-1, 0, 1)}. Answer: independent. Set V <sub>1</sub> , V <sub>2</sub> , V <sub>3</sub> is linearly dependent whenever the set { V <sub>1</sub> + 4V <sub>2</sub> + 4V <sub>3</sub> + 4V <sub>3</sub> , V <sub>3</sub> , V <sub>3</sub> , V <sub>3</sub> , V <sub>3</sub> } is linearly dependent. Find a maximal linearly independent set of vectors from the set V <sub>1</sub> = (2, -2, -4), V <sub>2</sub> = (1, 9, 3), V <sub>3</sub> = (-2, -4, 1), V <sub>4</sub> = (3, 7, -1)				
Special solutions are $(-2, 1, 0, 0, 0)$ and $(-7, 0, -2, 1, 0)$ Reduce these matrices to their echelon form to find their rank. Also find a special solution to each of the free variables.  (i) $A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix}$ (ii) $A = \begin{bmatrix} 2 & 4 & 2 \\ 0 & 4 & 4 \\ 0 & 8 & 8 \end{bmatrix}$ Answer: (i) $(-2, 1, 0, 0, 0)$ , $(0, 0, -2, 1, 0)$ , $(0, 0, -3, 0, 1)$ (ii) $(1, -1, 1)$ 6 For every c, find R and the special solutions to $Ax = 0$ :  (i) $A = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & 2 & 4 & 4 \\ 1 & c & 2 & 2 \end{bmatrix}$ (ii) $A = \begin{bmatrix} 1-c & 2 \\ 0 & 2-c \end{bmatrix}$ Answer: PI refer Gilbert Strang book.  7 Choose the number q so that ( if possible ) the ranks are (i) 1 (ii) 2 (iii) 3 $A = \begin{bmatrix} 6 & 4 & 2 \\ -3 & -2 & -1 \\ 9 & 6 & q \end{bmatrix},  B = \begin{bmatrix} 3 & 1 & 3 \\ q & 2 & q \end{bmatrix}$ Answer: For A, $q = 3$ gives rank 1 and every other q gives rank 2. For B, $q = 6$ gives rank 1 and every other q gives rank 2.  8 Which vectors ( $b_1$ , $b_2$ , $b_3$ ) are in the column space of A? Which combination of the rows of A give 0?  (i) $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 6 & 3 \\ 0 & 2 & 5 \end{bmatrix}$ (ii) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 2 & 4 & 8 \end{bmatrix}$ Answer: (i) all b and no combination of the rows (ii) $b_3 = 2b_2$ and $R_3 - 2R_2 = 0$ 9 Examine if the following sets of vectors are linearly independent. When the set is dependent find a relation between the vectors  1. { (1, 2, 0), (0, 3, 1), (-1, 0, 1)}. Answer: independent 2. { (1, 2, 1), (3, 0, -1), (-5, 4, 3)} Answer: dependent, $\sqrt{3} = 2\sqrt{1} - \sqrt{2}$ 10 If $\sqrt{1}$ , $\sqrt{2}$ , $\sqrt{3}$ are vectors and a and b are scalars show that the set $\sqrt{1}$ , $\sqrt{2}$ , $\sqrt{3}$ is linearly dependent. Find a maximal linearly independent set of vectors from the set $\sqrt{1}$ , $\sqrt{2}$ , $\sqrt{3}$ , is linearly dependent. Find a maximal linearly independent set of vectors from the set $\sqrt{1}$ ( $2, -2, -4$ ), $\sqrt{2}$ ( $2, -1, -1$ ), $\sqrt{4}$ ( $3, -7, -1$ )				
Special solutions are ( -2, 1, 0, 0, 0) and ( -7, 0, -2, 1, 0)  Reduce these matrices to their echelon form to find their rank. Also find a special solution to each of the free variables.  (i) $A = \begin{bmatrix} 1 & 2 & 2 & 4 & 6 \\ 1 & 2 & 3 & 6 & 9 \\ 0 & 0 & 1 & 2 & 3 \end{bmatrix}$ (ii) $A = \begin{bmatrix} 2 & 4 & 2 \\ 0 & 4 & 4 \\ 0 & 8 & 8 \end{bmatrix}$ Answer: (i) ( -2, 1, 0, 0, 0), (0, 0, -2, 1, 0), (0, 0, -3, 0, 1) (ii) (1, -1, 1)  6 For every c, find R and the special solutions to $Ax = 0$ :  (i) $A = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & 2 & 4 & 4 \\ 1 & c & 2 & 2 \end{bmatrix}$ (ii) $A = \begin{bmatrix} 1-c & 2 \\ 0 & 2-c \end{bmatrix}$ Answer: PI refer Gilbert Strang book.  7 Choose the number q so that ( if possible ) the ranks are (i) 1 (ii) 2 (iii) 3 $A = \begin{bmatrix} 6 & 4 & 2 \\ -3 & -2 & -1 \\ 9 & 6 & q \end{bmatrix}, B = \begin{bmatrix} 3 & 1 & 3 \\ q & 2 & q \end{bmatrix}$ Answer: For A, q = 3 gives rank 1 and every other q gives rank 2. For B, q = 6 gives rank 1 and every other q gives rank 2.  8 Which vectors ( b <sub>1</sub> , b <sub>2</sub> , b <sub>3</sub> ) are in the column space of A ? Which combination of the rows of A give 0 ?  (i) $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 6 & 3 \\ 0 & 2 & 5 \end{bmatrix}$ (ii) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 2 & 4 & 8 \end{bmatrix}$ Answer: (i) all b and no combination of the rows (ii) b <sub>3</sub> = 2b <sub>2</sub> and R <sub>3</sub> – 2R <sub>2</sub> = 0  9 Examine if the following sets of vectors are linearly independent. When the set is dependent find a relation between the vectors  1. { (1, 2, 0), (0, 3, 1), (-1, 0, 1)}. Answer: independent 2. { (1, 1, 2, 1), (3, 0, -1), (-5, 4, 3)} Answer: dependent, v3 = 2v1 – v2  10 If v <sub>1</sub> , v <sub>2</sub> , v <sub>3</sub> are vectors and a and b are scalars show that the set {v <sub>1</sub> , v <sub>2</sub> , v <sub>3</sub> } is linearly dependent. Find a maximal linearly independent set of vectors from the set v <sub>1</sub> = (2, -2, -4), v <sub>2</sub> = (1, 9, 3), v <sub>3</sub> = (-2, -4, 1), v <sub>4</sub> = (3, 7, -1)		$\begin{bmatrix} 1 & 2 & -3 & 1 & 2 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 0 & 7 & 0 \end{bmatrix}$		
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(i) $A = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & 2 & 4 & 4 \\ 1 & c & 2 & 2 \end{bmatrix}$ (ii) $A = \begin{bmatrix} 1-c & 2 \\ 0 & 2-c \end{bmatrix}$ Answer: PI refer Gilbert Strang book.  7 Choose the number q so that ( if possible ) the ranks are (i) 1 (ii) 2 (iii) 3 $A = \begin{bmatrix} 6 & 4 & 2 \\ -3 & -2 & -1 \\ 9 & 6 & q \end{bmatrix},  B = \begin{bmatrix} 3 & 1 & 3 \\ q & 2 & q \end{bmatrix}$ Answer: For A, q = 3 gives rank 1 and every other q gives rank 2. For B, q = 6 gives rank 1 and every other q gives rank 2.  8 Which vectors ( b <sub>1</sub> , b <sub>2</sub> , b <sub>3</sub> ) are in the column space of A ? Which combination of the rows of A give 0 ?  (i) $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 6 & 3 \\ 0 & 2 & 5 \end{bmatrix}$ (ii) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 2 & 4 & 8 \end{bmatrix}$ Answer: (i) all b and no combination of the rows (ii) $b_3 = 2b_2$ and $B_3 - 2B_2 = 0$ 9 Examine if the following sets of vectors are linearly independent. When the set is dependent find a relation between the vectors  1. { (1, 2, 0), (0, 3, 1), (-1, 0, 1)}. Answer: independent 2. { (-1, 2, 1), (3, 0, -1), (-5, 4, 3)} Answer: dependent, v3 = 2v1 - v2  10 If $v_1$ , $v_2$ , $v_3$ are vectors and a and b are scalars show that the set { $v_1$ , $v_2$ , $v_3$ } is linearly dependent whenever the set { $v_1$ + $av_2$ + $bv_3$ , $v_2$ , $v_3$ } is linearly dependent. Find a maximal linearly independent set of vectors from the set $v_1 = (2, -2, -4)$ , $v_2 = (1, 9, 3)$ , $v_3 = (-2, -4, 1)$ , $v_4 = (3, 7, -1)$		Answer: (i) (-2, 1, 0, 0, 0), (0, 0, -2, 1, 0), (0, 0, -3, 0, 1) (ii) (1, -1, 1)		
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$ \begin{array}{c} 8 & \text{Which vectors (b_1,b_2,b_3) are in the column space of A ? Which combination of the rows of A give 0?} \\ & \begin{bmatrix} 1 & 2 & 1 \\ 2 & 6 & 3 \\ 0 & 2 & 5 \end{bmatrix} & \text{(ii)}  A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 2 & 4 & 8 \end{bmatrix} \\ & \text{Answer: (i) all b and no combination of the rows (ii) }  b_3 = 2b_2 \text{ and } R_3 - 2R_2 = 0 \\ 9 & \text{Examine if the following sets of vectors are linearly independent. When the set is dependent find a relation between the vectors} \\ & 1. \; \{ \; (1,2,0) \;, (0,3,1) \;, (-1,0,1) \}.  \text{Answer: independent} \\ & 2. \; \{ \; (-1,2,1) \;, (3,0,-1) \;, (-5,4,3) \}  \text{Answer: dependent, v3 = 2v1 - v2} \\ 10 & \text{If } v_1, v_2, v_3 \text{ are vectors and a and b are scalars show that the set } \{ \; v_1, v_2 \;, v_3 \} \text{ is linearly dependent} \\ & \text{dependent whenever the set } \{ \; v_1 + av_2 + bv_3 \;, \; v_2 \;, v_3 \} \text{ is linearly dependent.} \\ 11 & \text{Find a maximal linearly independent set of vectors from the set } v_1 = (\; 2\;, \; -2\;, \; -4\;) \;, v_2 = (\; 1,\; 9,\; 3\;) \;, v_3 = (\; -2\;, \; -4\;, \; 1\;) \;, v_4 = (\; 3,\; 7\;, \; -1\;) \\ \end{array}$		Answer: For A, $q = 3$ gives rank 1 and every other q gives rank 2. For B, $q = 6$ gives rank		
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dependent find a relation between the vectors  1. { (1,2,0), (0,3,1), (-1,0,1)}. Answer: independent 2. { (-1,2,1), (3,0,-1), (-5,4,3)} Answer: dependent, v3 = 2v1 - v2  10  If v <sub>1</sub> , v <sub>2</sub> , v <sub>3</sub> are vectors and a and b are scalars show that the set { v <sub>1</sub> , v <sub>2</sub> , v <sub>3</sub> } is linearly dependent whenever the set { v <sub>1</sub> + av <sub>2</sub> + bv <sub>3</sub> , v <sub>2</sub> , v <sub>3</sub> } is linearly dependent.  11  Find a maximal linearly independent set of vectors from the set v <sub>1</sub> = (2, -2, -4), v <sub>2</sub> = (1,9,3), v <sub>3</sub> = (-2, -4, 1), v <sub>4</sub> = (3,7, -1)				
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<ul> <li>2. { (-1, 2, 1), (3, 0, -1), (-5, 4, 3) } Answer: dependent, v3 = 2v1 – v2</li> <li>10 If v<sub>1</sub>, v<sub>2</sub>, v<sub>3</sub> are vectors and a and b are scalars show that the set { v<sub>1</sub>, v<sub>2</sub>, v<sub>3</sub> } is linearly dependent whenever the set { v<sub>1</sub> + av<sub>2</sub> + bv<sub>3</sub>, v<sub>2</sub>, v<sub>3</sub> } is linearly dependent.</li> <li>11 Find a maximal linearly independent set of vectors from the set v<sub>1</sub> = (2, -2, -4), v<sub>2</sub> = (1, 9, 3), v<sub>3</sub> = (-2, -4, 1), v<sub>4</sub> = (3, 7, -1)</li> </ul>				
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	' '			
1 / Wild Wol I / Willy LWO OF THE GIVETT VECTORS				
		Transmort. Trans two or the given vectors		

12	Find the condition on a, b, c so that the vector (a, b, c) belongs to the space spanned by				
	$u = (2, 1, 0), v = (1, -1, 2)$ and $w = (0, 3, -4)$ . Do the vectors u, v, w span $\mathbb{R}^3$ ?				
13	Answer: $2a - 4b - 3c = 0$ . The vectors u, v, w do not span $R^3$				
13	For what value of $\lambda$ will the vectors (1, 3, -5), (0, 5, $\lambda$ ) and (-2, -1, 0) span a two dimensional subspace? For this value of $\lambda$ , (i) express (-2, -1, 0) as a linear				
	combination of the other two vectors and (ii) find a vector in R <sup>3</sup> that is not in the span of				
	these vectors.				
	Answer: For $\lambda = -10$ , the vectors span a 2-d subspace.				
	(i) $(-2, -1, 0) = -2(1, 3, -5) + 1(0, 5, -10)$				
	(ii) Any vector (a, b, c) that satisfies -a+2b+c ≠ 0 will not be in the span of the given				
	vectors.				
14	Let $v_1 = (1, 2, 1)$ , $v_2 = (3, 1, 5)$ and $v_3 = (3, -4, 7)$ . Show that the subspaces				
45	spanned by $\{v_1, v_2\}$ and $\{v_1, v_2, v_3\}$ are the same.				
15	If $V_1 = \{(a, b, 0) / a, b \text{ are real }\}$ and $V_2$ is spanned by $(1, 2, 3)$ and $(1, -1, 1)$ find a				
16	nonzero vector in $\mathbb{R}^3$ that is in both $V_1$ and $V_2$ . Answer: any multiple of $(-2, 5, 0)$ If the set vectors $\{(1, x, 1), (x, 1, 0), (0, 1, x)\}$ is linearly dependent find x.				
10					
17	Answer: $x = 0$ , $\pm \sqrt{2}$ .				
17	Show that the vectors $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ , $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ , $\begin{bmatrix} 0 & 2 \\ 0 & -1 \end{bmatrix}$ generate the vector $\begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix}$				
	$\begin{bmatrix} 1 & 0 \end{bmatrix}$ , $\begin{bmatrix} 1 & 1 \end{bmatrix}$ , $\begin{bmatrix} 0 & -1 \end{bmatrix}$				
	in the vector space of all 2 x 2 matrices.				
18	Expand the set { (1, 2, 0), (1, -2, 4)} to a basis for R <sup>3</sup> by choosing an appropriate				
	vector from the set { ( -2 , 4 , -8 ) , ( 2, -1, -5 ) , ( 3 , -6 , 0 ) }. Justify your choice.				
4.0	Answer: $(-2, 4, -8)$ . The right combination is $-2a + b + c = 0$ .				
19	Find a basis and the dimension of the subspaces $V = \{(a, b, 0) / a \text{ and } b \text{ are real} \}$				
	numbers $\}$ , W = { (0, b, c) / b and c are real $\}$ and V $\cap$ W. Answer: Basis for V is { (1, 0, 0), (0, 1, 0) $\}$ , dim V = 2				
	Basis for W is $\{(0, 1, 0), (0, 1, 0)\}$ , dim W = 2				
	Basis for V ∩ W is { ( 0, 1, 0 ) } , dim = 1				
20	Let $V = \{ (a, b, c, d) / b + c + d = 0 \}$ and $W = \{ (a, b, c, d) / a + b = 0 \text{ and } c = 2d \}$ be				
	subspaces of $R^4$ . Find the dimension of $V \cap W$ .				
	Answer: (3, -3, 2, 1) is a vector in the intersection. Dim = 1.				
21	If the column space of A is spanned by the vectors (1, 4, 2), (2, 5, 1) and (3, 6, 0)				
	find all those vectors that span the left null space of A. Determine whether or not the				
	vector $b = (4, -2, 2)$ is in that subspace. What are the dimensions of $C(A^T)$ and $N(A^T)$ ?				
	Answer: Solutions of $A^{T} x = 0$ are (2, -1, 1). The vector b is in $N(A^{T})$ .				
	Dim $C(\Lambda^T) = 2$ and dim $N(\Lambda^T) = 1$				
22	Find the four fundamental subspaces, their dimensions and a basis given				
	$A = \begin{bmatrix} 1 & -1 & 2 & -2 & 3 \\ -2 & 2 & 0 & 4 & -2 \\ 0 & 3 & 1 & -1 & 6 \\ -1 & -2 & -3 & 3 & -9 \end{bmatrix}$				
	$\begin{vmatrix} -2 & 2 & 0 & 4 & -2 \end{vmatrix}$				
	A =   0 2 1 1 6				
	_				
	Answer: Basis for C(A) is columns 1, 2, 3; Basis for $C(A_T^T)$ is rows 1, 2, 3				
	Basis for N(A) is $\{ (7, 1, 0, 3, 0), (-8, -5, -3, 0, 3) \}; N(A^T) = \{ c (1, 0, 1, 1) \}$				

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Obtain the four fundamental subspaces of A =

Answer: Basis for C(A) is columns 1, 2, 3, 4; Basis for  $C(A^T)$  is all four rows; Basis for

N(A) is (0,-1,-1,0,1) and  $N(A^T) = Z$ . Find a left inverse / right inverse for

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(i) 
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

(ii) 
$$A = \begin{vmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{vmatrix}$$

Answer: (i)  $\begin{bmatrix} 2/3 & -1/3 \\ 1/3 & 1/3 \\ -1/3 & 2/3 \end{bmatrix}$  (ii)  $\begin{bmatrix} 2/3 & 1/3 & -1/3 \\ -1/3 & 1/3 & 2/3 \end{bmatrix}$ 

(ii) 
$$\begin{bmatrix} 2/3 & 1/3 & -1/3 \\ -1/3 & 1/3 & 2/3 \end{bmatrix}$$