

DESIGN AND ANALYSIS OF ALGORITHMS

Backtracking

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UNIT 5: Limitations of Algorithmic Power and Coping with the Limitations

ISPES UNIVERSITY ONLINE

- Dynamic Programming
 - Computing a Binomial Coefficient
 - The Knapsack Problem
 - Memory Functions
 - Warshall's and Floyd's Algorithms
 - Optimal Binary Search Trees
- Limitations of Algorithmic Power
 - Lower-Bound Arguments
 - Decision Trees
 - P, NP, and NP-Complete, NP-Hard Problems
- Coping with the Limitations
 - Backtracking
 - Branch-and-Bound

Concepts covered

- Backtracking
 - Introduction
 - N Queens
 - Hamiltonian Circuit
 - Subset Sum
 - Algorithm



BACKTRACKING Tackling Difficult Combinatorial Problems



- There are two principal approaches to tackling difficult combinatorial problems (NP-hard problems):
 - Use a strategy that guarantees solving the problem exactly but doesn't guarantee to find a solution in polynomial time
 - Use an approximation algorithm that can find an approximate (sub-optimal) solution in polynomial time

BACKTRACKING Exact Solution Strategies



- Exhaustive search (brute force)
 - useful only for small instances
- Dynamic programming
 - applicable to some problems (e.g., the knapsack problem)
- Backtracking
 - eliminates some unnecessary cases from consideration
 - yields solutions in reasonable time for many instances but worst case is still exponential
- Branch-and-bound
 - further refines the backtracking idea for optimization problems

Introduction

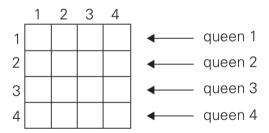


- Construct the state-space tree
 - nodes: partial solutions
 - edges: choices in extending partial solutions
- Explore the state space tree using depth-first search
- "Prune" nonpromising nodes
 - ▶ DFS stops exploring subtrees rooted at nodes that cannot lead to a solution and backtracks to such a node's parent to continue the search

Example: *N*-Queens Problem

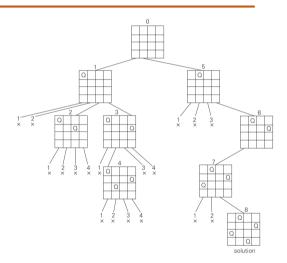


• Place N queens on an $N \times N$ chess board so that no two of them are in the same row, column, or diagonal



State-Space Tree of the 4-Queens Problem





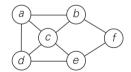
Example: Hamiltonian Circuit Problem



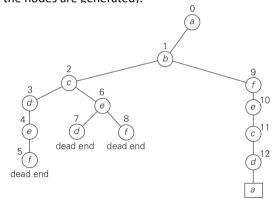
Hamiltonian Circuit

A Hamiltonian circuit is defined as a cycle that passes through all the vertices of the graph exactly once.

• Example graph:



 State-space tree for finding a Hamiltonian circuit (numbers above the nodes of indicate the order in which the nodes are generated):



Example: Subset Sum Problem

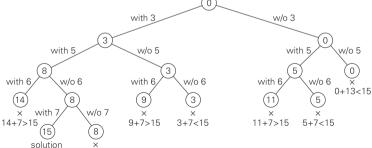
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Subset Sum Problem

Given set $A = \{a_1, \dots, a_n\}$ of n positive integers, find a subset whose sum is equal to a given positive integer d

• State space tree for $A = \{3, 5, 6, 7\}$ and d = 15 (number in each node is the sum so far):



Algorithm



Backtrack Algorithm

```
1: procedure BACKTRACK(X[1...i])
       \triangleright Input: X[1...i] specifies first i promising components of a solution
       Dutput: All the tuples representing the problem's solutions
3:
       if X[1 \dots i] is a solution then
4:
           write X[1 \dots i]
5:
       else
6:
           for each element x \in S_{i+1} consistent with X[1 \dots i] and the constraints do
7:
               X[i+1] \leftarrow x
8:
               Backtrack (X[1 \dots i+1])
9:
```

- Output: n-tuples $(x_1, x_2, ..., x_n)$
- Each $x_i \in S_i$, some finite linearly ordered set

Think About It



- Continue the backtracking search for a solution to the four-queens problem, to find the second solution to the problem
- Explain how the board's symmetry can be used to find the second solution to the four-queens problem