(2) Find Fourier series of x^2 in $(-\pi, \pi)$. Use Parsevals identity to prove that $\frac{\pi^4}{90} = 1 + \frac{1}{2^4} + \frac{1}{3^4} + \cdots$ (h)

Sol:- $x^2 = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^n}{n^2} \cos nx \longrightarrow (1)$

Where $a_0 = \frac{2\pi^2}{3}$, $a_n = \frac{4(-1)^n}{n^2}$, $b_n = 0$.

Using Parsevals identity to (1)

$$\int_{-\pi}^{\pi} [x^2]^2 dx = 2 l \left[\frac{a_0^2}{4} + \sum_{n=1}^{\infty} (a_n^2 + b_n^2) \right]$$

$$= 2\pi \left[\frac{\pi^4}{9} + \frac{1}{2} \sum_{n=1}^{\infty} \frac{16}{n^4} \right]$$

$$\left. \frac{x^5}{5} \right]_{-\pi}^{\pi} = \frac{2\pi^5}{9} + \pi \sum_{n=1}^{\infty} \frac{16}{n^4}$$

$$\Rightarrow \frac{2\pi^5}{5} - \frac{2\pi^5}{9} = \pi + \sum_{n=1}^{\infty} \frac{16}{n^4}$$

or $$\frac{\pi^4}{90} = \sum_{n=1}^{\infty} \frac{1}{n^4}$$

$$\frac{\pi^4}{90} = 1 + \frac{1}{2^4} + \frac{1}{3^4} + \cdots$$

h: $f(x) = x^2$

$f(-x) = (-x)^2 = x^2 = f(x)$

even fn $\therefore a_0 = \frac{2}{\pi} \int_0^{\pi} f(x) dx$ & $a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx \, dx$.