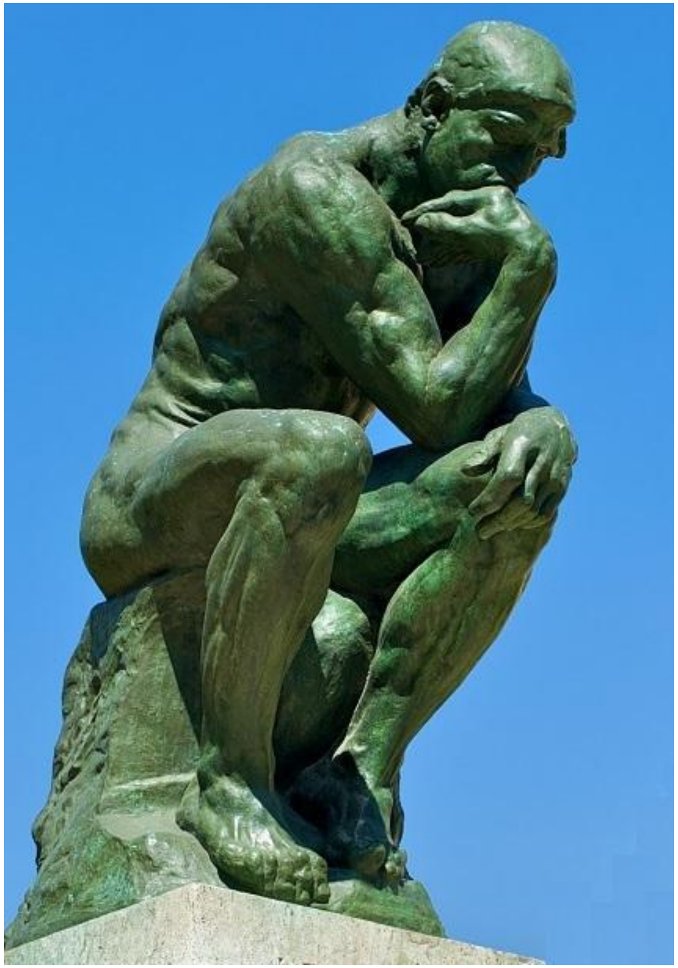
What is Logic?

"Le Penseur"



Sir, "I love you"
and "You love your daughter"
Therefore, it implies that
"I love your daughter"

"All **IIT**ians are talented"
"Arvind is an **IIT**ian"
Therefore, "Arvind is talented"

"If one is crazy, one drills teeth"
"My dentist drills teeth"
Therefore, "My dentist is crazy"

Does this argument makes sense?

- “If Superman exists, then he is neither powerless nor evil-minded”
- “If Superman were able and willing to prevent evil, then he would prevent evil”
- “If Superman were unable to prevent evil, then he would be powerless”
- “If Superman were unwilling to prevent evil, then he would be evil-minded”
- “Superman does not prevent evil”

Therefore, **“Superman does not exists”**.

What is Logic?

logic

/ˈlɒdʒɪk/

noun

1. reasoning conducted or assessed according to strict principles of validity.
   "experience is a better guide to this than deductive logic"
   synonyms: science of reasoning, science of deduction, science of thought, dialectics, argumentation, ratiocination
   "the study of logic"
2. a system or set of principles underlying the arrangements of elements in a computer or electronic device so as to perform a specified task.

Logic | Definition of Logic by Merriam-Webster

www.merriam-webster.com/dictionary/**logic**

Simple **Definition of logic**. : a proper or reasonable way of thinking about or understanding something. : a particular way of thinking about something. : the science that studies the formal processes used in thinking and reasoning.

What is Logic?

Logic: Science dealing with principles of valid arguments and reasoning. Logic is rooted in basic cause and effect.

Common Sense: Ability to see the world in a way which is common to most people.

Fallacies!

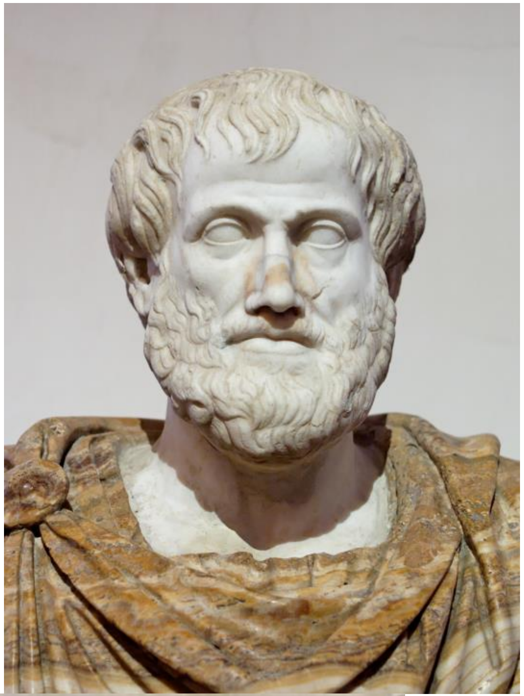
A failure in reasoning which renders an argument invalid.

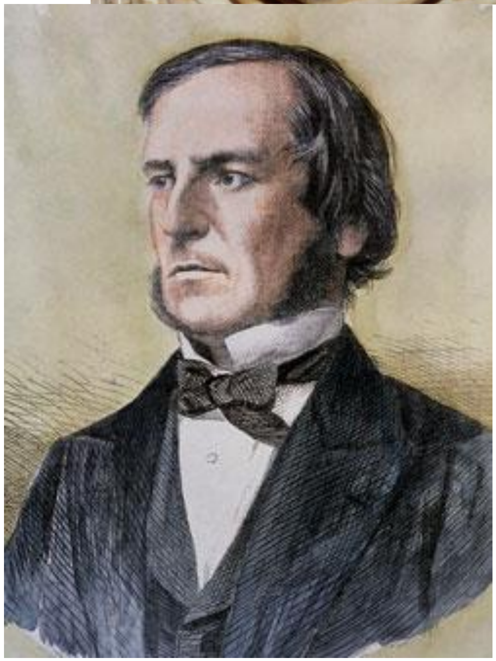
Aristotle 384-322BC: Organon collection

George Boole (1815-64): The Laws of Thought

Proposition: A proposition is a declarative sentence that is either true or false, but not both.

Propositional Logic (**Propositional Calculus**): The area of logic that deals with propositions.





Proposition or not?

- Bangalore is the capital city of India.
- New Delhi is the capital city of India.
- Do you like coffee?
- Get me a cup of coffee.
- $1 + 2 = 3$
- $2 + 3 = 4$
- $x + 3 = 10$
- Mahatma Gandhi was born in the year 1869.
- It’s raining today.
- Earth is flat.

Propositions:

- New Delhi is the capital city of India. // true
- Bangalore is the capital city of India. // false
- 1 + 2 = 3 // true
- 2 + 3 = 4 // false
- Mahatma Gandhi was born in 1869. // true
- It’s raining today. // false
- Earth is flat. // false

These are NOT propositions:

- Get me a cup of coffee.//not a declarative sentence
- Do you like coffee?//not a declarative sentence
  - Though the response to the question could be yes or no; a binary response.
- x + 1 = 2 // has a variable
  - It’s a declarative sentence, but depending on the value of ‘x’, it can be true or false.
  - It can be turned into a proposition by giving a value for ‘x’.
- x + y = z. // has variables

Compound Propositions: are propositions formed from existing propositions using logical operators.

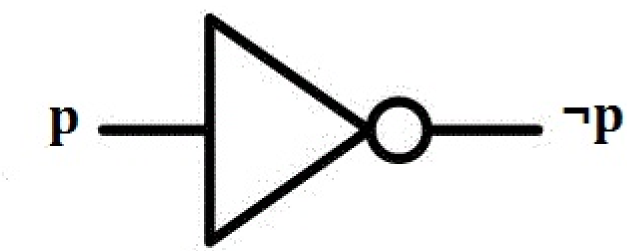
Logical Operators:

- Negation ($\neg$)
- Conjunction ($\wedge$)
- Disjunction ($\vee$)
- Exclusive OR ($\oplus$)
- Conditional Statement ($\rightarrow$)
- Biconditional Statement ($\leftrightarrow$)

Negation: Let p be a proposition. The negation of p, denoted by **¬p**, is the statement "It is not the case that p". The proposition ¬p is read "not p". The truth value of the negation of p is the opposite of the truth value of p.

Eg: Let p be "Today is Friday"
¬p: "It's not the case that today is Friday"

¬p: "Today is not Friday"



Truth table of Negation:

p	**¬p**
F	T
T	F

What’s the **negation** of the following propositions?

- “Bengaluru is the capital city of India”
- “Indian Cricket team won at least three matches in the series”
- “Number 5 turned up when the dice is rolled”
- $\neg p$
- “I don’t love you”

Examples:
“**Not** knowing traffic rules is **not** an excuse for **not** following them”.

“**Not** attending the class is **not** an excuse for **not** completing the assignment”.

A scene from The Big Bang Theory (sitcom)..

Sheldon: “I would **not object** to us **no** longer characterizing you as **not** my girlfriend”

Amy: Interesting! Now try without quadruple negation.

Sheldon: I am fine with you being my girlfriend.

Sheldon of The Big Bang Theory (sitcom) says..

“I would **not object** to us **no** longer characterizing you as **not** my girlfriend”

→ I am fine with us **no** longer characterizing you as **not** my girlfriend.

→ I am fine with you being my girlfriend.

Careful while cancelling out double negations:

“I do not disagree” is not same as
“I agree”.

“A nonnegative number” is not same as
“A positive number”.

“Not a nonnegative number”
is …

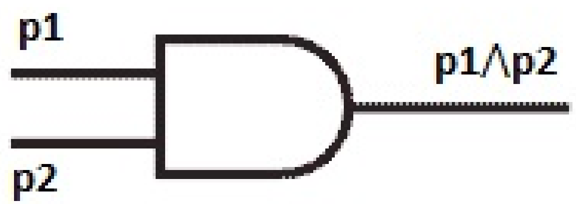
Incorrect usage of double negation:

The IT Crowd (sitcom) …

Roy: ♮ **We don't need no education** ♮

Moss: **"Yes you do. You've just used a double negation."**

Conjunction: Let p and q be propositions. The conjunction of p and q, denoted by **p∧q**, is the proposition "**p and q**". The conjunction p∧q is true when both p and q are true and is false otherwise.



p	q	p∧q
F	F	F
F	T	F
T	F	F
T	T	T

Examples for Conjunction:

$p \wedge q$: “He brought pen and notebook to the class”

p: “He brought pen to the class”

q: “He brought notebook to the class”

If he brings just pen, but not notebook, he is not standing by the statement.

$p \wedge q$: “Tea and Coffee are served in the conference”

p: “Tea is served in the conference”

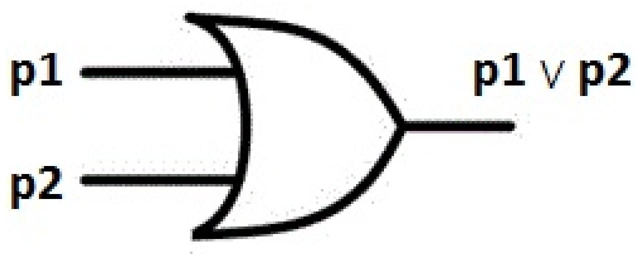
q: “Coffee is served in the conference”

“Don’t drink and drive”

Don't drink and drive.

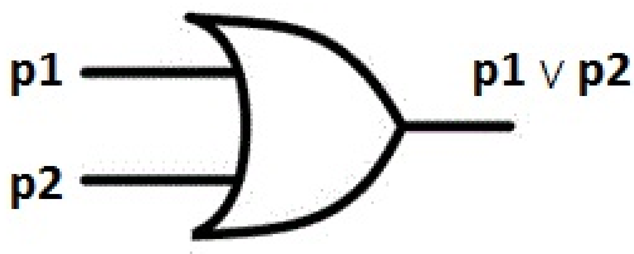
drin**K**	dri**V**e	**¬K ∧ V**	**¬(K ∧ V)**
F	F	F	T
F	T	T	T
T	F	F	T
T	T	F	F

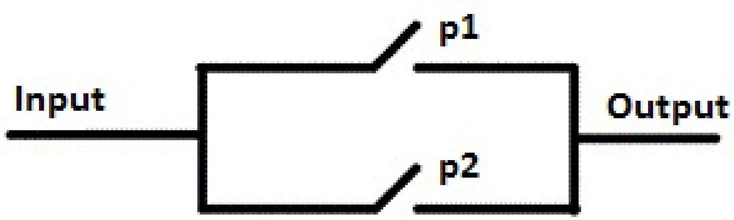
Disjunction: Let p and q be propositions. The disjunction of p and q, denoted by p∨q, is the proposition “p or q”. The disjunction p∨q is false when both p and q are false and is true otherwise.



p	**q**	**p∨q**
F	F	F
F	T	T
T	F	T
T	T	T

Disjunction: Let p and q be propositions. The disjunction of p and q, denoted by p∨q, is the proposition "p or q". The disjunction p∨q is false when both p and q are false and is true otherwise.



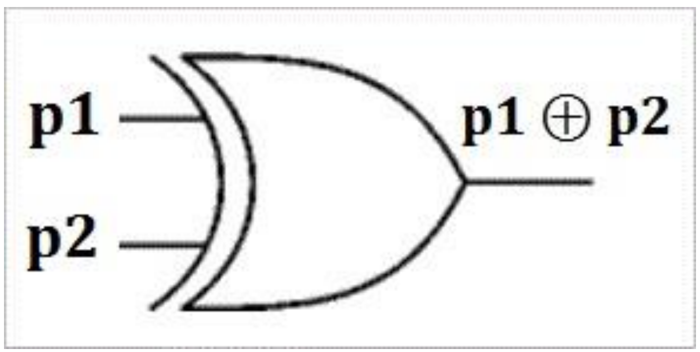


p	**q**	**p∨q**
F	F	F
F	T	T
T	F	T
T	T	T

Examples for Disjunction:
pvq: “He has an apple or an orange”
p: “He has an apple”
q: “He has an orange”
Note: Having both an apple and an orange is okay too.

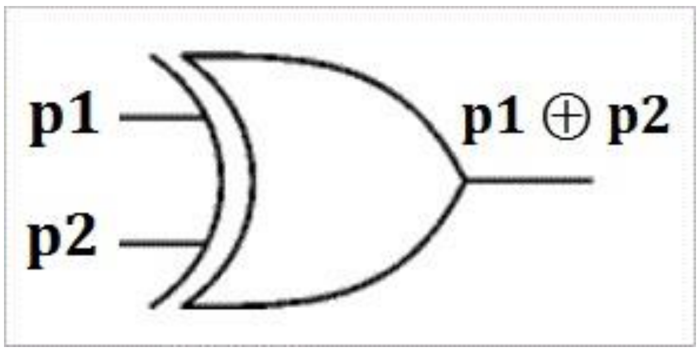
Condition to play the roller-coaster:
pvq: “you are tall or you are an adult”
p: “You are tall”
q: “You are an adult”
That is, only short kids are not allowed.

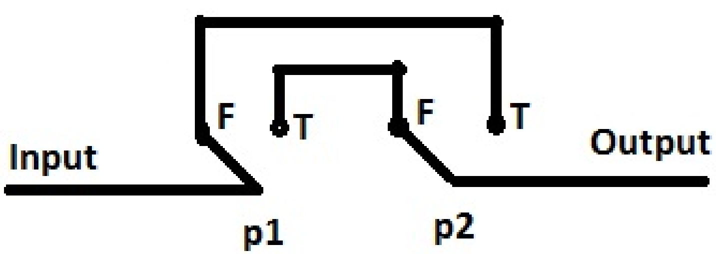
Exclusive OR: Let p and q be propositions. The "exclusive or" of p and q, denoted by $p \oplus q$, is the proposition "either p or q, but not both". The "exclusive or" $p \oplus q$ is true when exactly one of p and q is true and is false otherwise.



p	**q**	**$p \oplus q$**
F	F	F
F	T	T
T	F	T
T	T	F

Exclusive OR: Let p and q be propositions. The "exclusive or" of p and q, denoted by $p \oplus q$, is the proposition "either p or q, but not both". The "exclusive or" $p \oplus q$ is true when exactly one of p and q is true and is false otherwise.





p	q	$p \oplus q$
F	F	F
F	T	T
T	F	T
T	T	F

Examples for Exclusive OR:

$p \oplus q$: “Last night he was in Chennai or Mumbai”
p: “Last night he was in Chennai”
q: “Last night he was in Mumbai”
Note: Obviously you are not expecting the case of he being in both Mumbai and Chennai.

$p \oplus q$: “Dharmendra likes to marry Seeta or Geeta”
Note: If he likes to marry both, the statement would be false. It’s false even when he doesn’t like to marry either.

$p \oplus q$: “Do or die”

Inclusive OR or Exclusive OR ?

- “Experience with C++ or Java is required.”
- “Coffee or tea comes with dinner.”
- “Prerequisite for the course is a course in Discrete Math or Digital Logic.”
- “For identification, you need to bring your passport or voter id card.”
- “In the Identification field, mention your passport number or voter id number”

		Negation	**Conjunction**	**Disjunction**	**Exclusive OR**
p	**q**	**$\neg$p**	**p$\wedge$q**	**p$\vee$q**	**p$\oplus$q**
F	F	**T**	F	F	F
F	T	**T**	F	**T**	**T**
T	F	F	F	**T**	**T**
T	T	F	**T**	**T**	F

Conditional Statement: Let p and q be propositions. The “conditional statement” $p \to q$ is false when p is true and q is false, and true otherwise. Conditional statements are also called implications.

Truth table of Conditional Statement:

p	**q**	**$p \to q$**
F	F	T
F	T	T
T	F	F
T	T	T

In $p \to q$,

p is called hypothesis,
premise
or antecedent

q is called conclusion
or consequence

Note: A useful way to understand the truth value of a conditional statement is to think of it as **an obligation** or **a contract**.

Eg: A pledge made by a politician running for an office, “If I am elected, then I will lower the taxes”.

“If I am elected”	**“I will lower the taxes”**	**“I am elected” → “I will lower the taxes”**
F	F	**T** (Pledge is intact. Can’t expect to lower taxes when the politician is not elected.)
F	T	**T** (Pledge is intact. The politician might have used his influence to lower the taxes even though he is not elected.)
T	F	**F** (**Pledge is broken.**)
T	T	**T** (Pledge is intact. Taxes are lowered.)

“If you top the class, then you will find a job”.
“If you are good at problem solving, then you will find a job”.
“If you top the class, then you will get an award”.

Let **top** be “You top the class”,
job: “You will find a job”,
good: “You are good at problem solving”,
award: “You will get an award”.

$$\begin{array}{lll} \textbf{good} & & \\ & & \searrow \\ \textbf{top} & \rightarrow & \textbf{job} \\ & \searrow & \\ & & \textbf{award} \end{array}$$

The following forms are equivalent to $p \rightarrow q$.

- p implies q
- if p, then q
- if p, q
- q if p
- q when p
- q whenever p
- q follows from p
- **p only if q**
- **q unless $\neg$p**
- **p is sufficient for q**
- a sufficient condition for q is p
- **q is necessary for p**
- a necessary condition for p is q

Converse, Inverse and Contrapositive

of a Conditional Statement

Conditional Statement $\mathbf{p \to q}$	**Converse** $\mathbf{q \to p}$
Inverse $\mathbf{\neg p \to \neg q}$	**Contrapositive** $\mathbf{\neg q \to \neg p}$

p	q	$\mathbf{p \to q}$	$q \to p$	$\neg p \to \neg q$	$\mathbf{\neg q \to \neg p}$
F	F	**T**	T	T	**T**
F	T	**T**	F	F	**T**
T	F	**F**	T	T	**F**
T	T	**T**	T	T	**T**

Eg: Let p: “You top the class” and
q: “You will find a job”.

- Conditional statement $\mathbf{p \rightarrow q}$
  - “If you top the class, then you will find a job”.
- Contrapositive $\mathbf{\neg q \rightarrow \neg p}$
  - “If you did not find a job, then you did not top the class”.

- Inverse $\mathbf{\neg p \rightarrow \neg q}$
  - “If you do not top the class, then you will not find a job”.
- Converse $\mathbf{q \rightarrow p}$
  - “If you found a job, then you must’ve topped the class”.

Biconditional Statement: Let p and q be propositions. The biconditional statement $p \leftrightarrow q$ is the proposition “p if and only if q”. The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise.

Biconditional statements are also called as bi-implications.

p	**q**	**$p \leftrightarrow q$**
F	F	**T**
F	T	**F**
T	F	**F**
T	T	**T**

p if and only if q

(p if q) and (p only if q)

$(q \rightarrow p) \land (p \rightarrow q) \equiv (p \leftrightarrow q)$

Are converse of the following statements are also true?

“If two integers are odd then their product is odd”

“If two integers are even then their product is even”

Eg:

$p \rightarrow q$: “if the switch is on, then the current flows”.

$p \leftrightarrow q$: “current flows if and only if the switch is on”.

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p\rightarrow q)\wedge(q\rightarrow p)$	$p\leftrightarrow q$
F	F	T	T	**T**	**T**
F	T	T	F	**F**	**F**
T	F	F	T	**F**	**F**
T	T	T	T	**T**	**T**

$\therefore$ $p\leftrightarrow q$ is equivalent to $(p\rightarrow q)\wedge(q\rightarrow p)$

Eg: Let p: “I have time” and

q: “I will go to movie”

p→**q**: “**If** I have time, then I will go to movie”

(or “I will go to movie **if** I have time”)

It’s a promise you make to your friend.

q→**p**: “I will go to movie **only if** I have time”

It’s a promise you make to your dad.

p↔**q**: “I will go to movie **if and only if** I have time”

Eg: “You can access the C lab only if you are a computer science major or you are a freshman”.

Let **a** be “you can access the C lab”,
c be “you are a computer science major”, and
f be “you are a freshman”.

$a \rightarrow (c \vee f)$ is required expression.
It doesn’t mean “if you are a computer science major or you are a freshman then you’ll always get access to the C lab”.

Conditional Love!

“He loves her if she loves him”

“He loves her only if she loves him”

“He loves her if and only if she loves him”

“He loves her whether she loves him or not”

Conditional Love!

Let **he**: “He loves her”, **she**: “She loves him”

she → **he:** “He loves her if she loves him”

he → **she:** “He loves her only if she loves him”

she ↔ **he:** “He loves her if and only if she loves him”

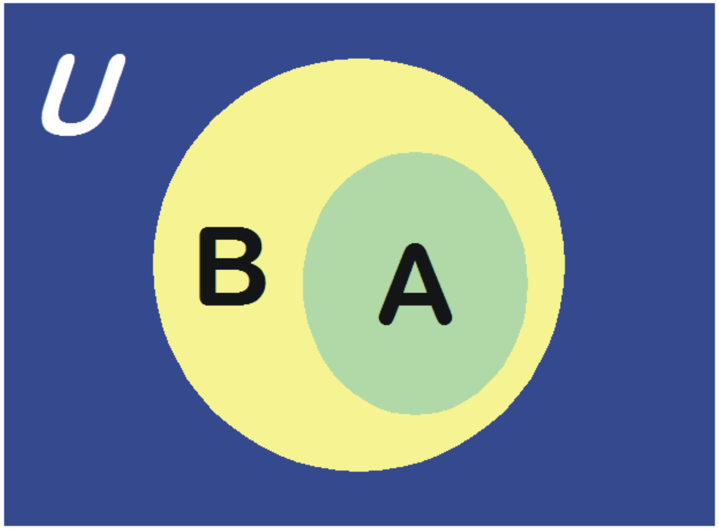
(she∨**¬she)**→**he:** “He loves her whether she loves him or not”

he	**she**	**she→he**	**he→she**	**she↔he**	**(she ∨ ¬she)→he**
F	F	T	T	T	F
F	T	F	T	F	F
T	F	T	F	F	T
T	T	T	T	T	T

“It is sufficient to wash your boss’s car to get promoted”
i.e., Washing boss’s car → Getting promotion

“It is necessary to wash your boss’s car to get promoted”
i.e., Got promoted → Washed boss’s car

“It is necessary and sufficient to wash your boss’s car to get promoted”
Got promoted ↔ Washed boss’s car



“You cannot ride the roller coaster if you are under 4 feet tall unless you are at least 16 years old”.

noride: “you cannot ride the roller coaster”
short: “you are under 4 feet tall” (you are short)
adult: “you are at least 16 years old” (you are an adult)

“noride if short unless adult”.

“You cannot ride the roller coaster if you are short unless you are an adult”.

“noride if short unless adult”.

short→**noride**: “You cannot ride if you are short”.
i.e., “If you are short, then you cannot ride”

¬adult→**(short** → **noride)**: “(short → noride) unless you are an adult”

(short ∧¬adult)→**noride**: “If you are **short** and **not an adult**, then you **cannot** ride the roller coaster”

noride	**short**	**adult**	**¬adult→(short → noride)** **(short ∧ ¬adult) → noride**
ride	**¬short**	**¬adult**	T (Hypothesis is F because you are not short)
ride	**¬short**	**adult**	T (... not short)
ride	**short**	**¬adult**	**F** (They are breaking the rules)
ride	**short**	**adult**	T (... Hypothesis is F because you are adult)
noride	**¬short**	**¬adult**	T (Consequence is T, so stt is T irrespective of the hypothesis)
noride	**¬short**	**adult**	T
noride	**short**	**¬adult**	T
noride	**short**	**adult**	T

“Don’t drink and drive”

“Do or die”

“An apple a day, keeps the doctor away”
Apple a day → Healthy life

"Jinke ghar sheeshe ke hote hain woh doosron ke gharon par patthar nahi phenkna chahiye"
sheeshe ke ghar → patthar nahi phenkna

Let p, q, and r be the propositions
p: You have the flu.
q: You miss the final examination.
r: You pass the course.

Express the propositions as English sentences:

- $p \rightarrow q$
- $\neg q \leftrightarrow r$
- $q \rightarrow \neg r$
- $(p \rightarrow \neg r) \lor (q \rightarrow \neg r)$
- $(p \land q) \lor (\neg q \land r)$

Let p and q be the propositions

p: You drive over 60 kmph. **q:** You get a speeding ticket.

Write the following propositions using p, q and logical connectives.

a. You do not drive over 60 kmph
b. You drive over 60 kmph, but you do not get a speeding ticket.
c. You will get a speeding ticket if you drive over 60 kmph.
d. If you do not drive over 60 kmph, then you will not get a speeding ticket.
e. Driving over 60 kmph per hour is sufficient for getting a speeding ticket.
f. You get a speeding ticket, but you do not drive over 60 kmph.
g. Whenever you get a speeding ticket, you are driving over 60 kmph.

Determine the truth values of the following propositions.

a. $2+2=4$ if and only if $1+1=2$.
b. $1+1=2$ if and only if $2+3=4$.
c. $0>1$ if and only if $2>1$.
d. If $1+1=2$, then $2+2=5$.
e. If $1+1=3$, then $2+2=4$.
f. If $1+1=3$, then $2+2=5$.
g. If $2+2=4$, then $1+2=3$.
h. $1+1=3$ if and only if monkeys can fly.
i. If monkeys can fly, then $1+1=3$
j. If $1+1=3$, then unicorns exist.
k. If $1+1=3$, then dogs can fly.
l. If $1+1=2$, then dogs can fly.

Tautology: is a compound proposition that is **always true** no matter what the truth values of the propositions that occur in it.

Contradiction: is a compound proposition that is **always false** no matter …

Contingency: neither tautology nor contradiction **($\neg$tautology $\wedge$ $\neg$contradiction)**

p	**q**	**$(p \to q) \wedge (q \to p) \leftrightarrow (p \leftrightarrow q)$**	**$\neg(p \to q) \wedge \neg(p \wedge \neg q)$**	**$\neg p \vee q$**
F	F	T	F	T
F	T	T	F	T
T	F	T	F	F
T	T	T	F	T

Logical Equivalence: The compound propositions p and q are called logically equivalent if $p \leftrightarrow q$ is a tautology. The notation $\mathbf{p \equiv q}$ denotes that p and q are logically equivalent.

In other words, compound propositions that have the same truth values in all possible cases are called logically equivalent. $\mathbf{(p \rightarrow q) \land (q \rightarrow p) \equiv (p \leftrightarrow q)}$

Note: The symbol $\equiv$ is **not** a logical connective.
Hence "$p \equiv q$" is **not** a compound proposition.
"$p \equiv q$" means $p \leftrightarrow q$ is a tautology.

Eg: Using truth table, show that

$(p \leftrightarrow q) \equiv (p \rightarrow q) \land (q \rightarrow p)$.

p	q	**$p \leftrightarrow q$**	**$(p \rightarrow q) \land (q \rightarrow p)$**	**$(p \leftrightarrow q) \leftrightarrow (p \rightarrow q) \land (q \rightarrow p)$**
F	F			
F	T			
T	F			
T	T			

Eg: Using truth table, show that

$(p \leftrightarrow q) \equiv (p \rightarrow q) \land (q \rightarrow p)$.

p	q	**$p \leftrightarrow q$**	**$(p \rightarrow q) \land (q \rightarrow p)$**	**$(p \leftrightarrow q) \leftrightarrow (p \rightarrow q) \land (q \rightarrow p)$**
F	F	T	T	T
F	T	F	F	T
T	F	F	F	T
T	T	T	T	T

Truth table demonstrates,

$(p \leftrightarrow q) \leftrightarrow (p \rightarrow q) \land (q \rightarrow p)$ is a tautology.

Therefore, $(p \leftrightarrow q) \equiv (p \rightarrow q) \land (q \rightarrow p)$

Eg: Using truth tables, show that p→q and ¬p∨q are logically equivalent.

p	**q**	**¬p**	**p→q**	**¬p∨q**	**(p→q) ↔ (¬p∨q)**
F	F				
F	T				
T	F				
T	T				

Eg: Using truth tables, show that $p \rightarrow q$ and $\neg p \vee q$ are logically equivalent.

$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$ is a tautology.

Therefore, $(p \rightarrow q) \equiv (\neg p \vee q)$

p	**q**	**$\neg p$**	**$p \rightarrow q$**	**$\neg p \vee q$**	**$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$**
F	F	T	T	T	T
F	T	T	T	T	T
T	F	F	F	F	T
T	T	F	T	T	T

Eg: Using truth tables, prove the De Morgan's laws in Logic.

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

p	**q**	**¬(p∧q)**	**¬p∨¬q**	**¬(p∧q) ↔ ¬p∨¬q**	**¬(p∨q)**	**¬p∧¬q**	**¬(p∨q) ↔ ¬p∧¬q**
F	F						
F	T						
T	F						
T	T						

Eg: Using truth tables, prove the De Morgan's laws in Logic.

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

p	**q**	**¬(p∧q)**	**¬p∨¬q**	**¬(p∧q) ↔ ¬p∨¬q**	**¬(p∨q)**	**¬p∧¬q**	**¬(p∨q) ↔ ¬p∧¬q**
F	F	T	T	T	T	T	T
F	T	T	T	T	F	F	T
T	F	T	T	T	F	F	T
T	T	F	F	T	F	F	T

LOGICAL EQUIVALENCES :

$p \wedge T \equiv p$
$p \vee F \equiv p$
Identity laws

$p \vee T \equiv T$
$p \wedge F \equiv F$
domination laws

$p \vee p \equiv p$
$p \wedge p \equiv p$
Idempotent laws

$\neg(\neg p) \equiv p$ double negation law

$p \vee q \equiv q \vee p$
$p \wedge q \equiv q \wedge p$
commutative laws

$(p \vee q) \vee r \equiv p \vee (q \vee r)$
$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
assosiative laws

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
distributive laws

$\neg(p \wedge q) \equiv \neg p \vee \neg q$
$\neg(p \vee q) \equiv \neg p \wedge \neg q$
De Morgan's laws

$p \vee (p \wedge q) \equiv p$
$p \wedge (p \vee q) \equiv p$
Absorption laws

$p \vee \neg p \equiv T$
$p \wedge \neg p \equiv F$
negation laws.

Involving Conditional Statements:

$p \to q \equiv \neg p \vee q$

$p \to q \equiv \neg q \to \neg p$

$p \vee q \equiv \neg p \to q$

$p \wedge q \equiv \neg(q \to \neg p)$

$\neg(p \to q) \equiv p \wedge \neg q$

$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$

$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$

$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

Involving Biconditionals:

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$

$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q.$

Q: Prove the following logical equivalences using the laws of logic (without using the truth table).

1. $\neg(p \rightarrow q) \equiv p \land \neg q$

Soln: LHS $\equiv \neg(p \rightarrow q)$

$\equiv$

Q: Prove the following logical equivalences using the laws of logic (without using the truth table).

1. $\neg(p \rightarrow q) \equiv p \wedge \neg q$

Soln: LHS $\equiv \neg(p \rightarrow q)$

$\equiv \neg(\neg p \vee q)$ $\quad\because (p \rightarrow q) \equiv \neg p \vee q$

$\equiv \neg(\neg p) \wedge \neg q$ $\quad$ De Morgan's law

$\equiv p \wedge \neg q$ $\quad$ Double negation

$\therefore \neg(p \rightarrow q) \equiv p \wedge \neg q$

2. $p \wedge (p \rightarrow q)) \equiv p \wedge q$

Soln: LHS $\equiv p \wedge (p \rightarrow q))$

$\equiv$

2. $p \land (p \rightarrow q)) \equiv p \land q$

Soln: LHS $\equiv p \land (p \rightarrow q))$

$\equiv p \land (\neg p \lor q))$ $\because (p \rightarrow q)$

$\equiv (\neg p \lor q)$

$\equiv (p \land \neg p) \lor (p \land q)$ Distribution law

$\equiv F \lor (p \land q)$

Negation law

$\equiv p \land q$

Identity law

$\therefore p \land (p \rightarrow q)) \equiv p \land q$

3. $\neg(p \lor (\neg p \land q)) \equiv \neg(p \lor q)$

Soln: LHS $\equiv \neg(p \lor (\neg p \land q))$

$\equiv$

3. $\neg(p \lor (\neg p \land q)) \equiv \neg(p \lor q)$

Soln: LHS $\equiv \neg(p \lor (\neg p \land q))$

$\equiv \neg p \land \neg(\neg p \land q))$ De Morgan's law

$\equiv \neg p \land (\neg(\neg p) \lor (\neg q))$ De Morgan's law

$\equiv \neg p \land (p \lor \neg q)$

Double negation

$\equiv (\neg p \land p) \lor (\neg p \land \neg q)$ Distribution law

$\equiv F \lor (\neg p \land \neg q)$

Negation law

$\equiv \neg p \land \neg q$

Identity law

Q: Prove the following expressions are tautologies using the laws of logic (without using the truth table)

1. $(p \land q) \rightarrow (p \lor q)$

   Soln: $(p \land q) \rightarrow (p \lor q)$

   $\equiv$

Q: Prove the following expressions are tautologies using laws of logic (without using truth table)

1. $(p \land q) \to (p \lor q)$

Soln: $(p \land q) \to (p \lor q)$

$\equiv \neg(p \land q) \lor (p \lor q)$ $\qquad \because (p \to q) \equiv (\neg p \lor q)$

$\equiv (\neg p \lor \neg q) \lor (p \lor q)$ $\qquad$ De Morgan law

$\equiv \neg p \lor \neg q \lor p \lor q$ $\qquad$ Associative law

$\equiv \neg p \lor p \lor \neg q \lor q$ $\qquad$ Commutative law

$\equiv T \lor T$ $\qquad$ Negation law

$\equiv T$

Idempotent law

$\therefore (p \land q) \to (p \lor q)$ is a tautology

2. $(p \wedge q) \rightarrow (p \rightarrow q)$

Soln: $(p \wedge q) \rightarrow (p \rightarrow q)$

$\equiv$

2. $(p \wedge q) \rightarrow (p \rightarrow q)$

Soln: $(p \wedge q) \rightarrow (p \rightarrow q)$

$\equiv \neg(p \wedge q) \vee (\neg p \vee q)$ $\because (p \rightarrow q) \equiv (\neg p \vee q)$

$\equiv (\neg p \vee \neg q) \vee (\neg p \vee q)$ De Morgan's law

$\equiv \neg p \vee \neg q \vee \neg p \vee q$ Associative law

$\equiv \neg p \vee \neg p \vee \neg q \vee q$ Commutative law

$\equiv \neg p \vee \neg q \vee q$ Idempotent law

$\equiv \neg p \vee T$ Negation law

$\equiv T$

Domination law

$\therefore (p \wedge q) \rightarrow (p \rightarrow q)$ is a tautology