PES University, Bangalore

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UE19CS203 – STATISTICS DATA SCIENCE

Unit-3 – Probability Distribution

Question Bank - SOLVED

Confidence Intervals for the Difference between Two Means

Exercises for Section 5.4

1. The article "Hatching Distribution of Eggs Varying in Weight and Breeder Age" (S. Viera, J. Almeida,et al., *Brazilian Journal of Poultry Science* 2005:(73–78) presents the results of a study in which the weights of 296 eggs from 27 week-old breeding hens averaged 54.1 g with a standard deviation of 4.4 g, and weights of 296 eggs from 59 week-old hens averaged 72.7 g with a standard deviation of 4.7 g. Find a 95% confidence interval for the difference between the mean weights.

   [Text Book Exercise – Section 5.4 – Q. No. 4 – Pg. No. 357]
   Solution:

   $\bar{x}$ = 54.1, S_x = 4.4, n_x = 296, $\bar{y}$ = 72.7, S_y = 4.7, n_y = 296, z.025 = 1.96.

   The confidence interval is $72.7-54.1 \pm 1.96\sqrt{\frac{4.4^2}{296}+\frac{4.7^2}{296}}$, or (17.8665, 19.3334).

2. In a study of the effect of cooling rate on the hardness of welded joints, 50 welds cooled at a rate of 10◦C/s had an average Rockwell (B) hardness of 91.1 and a standard deviation of 6.23, and 40 welds cooled at a rate of 30◦C/s had an average hardness of 90.7 and a standard deviation of 4.34.

   a) Find a 95% confidence interval for the difference in hardness between welds cooled at the different rates.
   b) Someone says that the cooling rate has no effect on the hardness. Do these data contradict this claim? Explain.

   [Text Book Exercise – Section 5.4 – Q. No. 11 – Pg. No. 358]
   Solution:

   (a) X = 91.1, sX = 6.23, nX = 50, Y = 90.7, sY = 4.34, nY = 40, z.025 = 1.96.

   The confidence interval is $91.1-90.7 \pm 1.96\sqrt{\frac{6.232^2}{50}+\frac{4.34^2}{40}}$, or (−1.789,2.589).

(b) No. Since 0 is in the confidence interval, it may be regarded as being a plausible value for the mean difference in hardness.

3. Refer to Exercise 2. Ten more welds will be made in order to increase the precision of the confidence interval. Which would increase the precision the most, cooling all 10 welds at the rate of 10◦C/s, cooling all 10 welds at the rate of 30◦C/s, or cooling 5 welds at 10◦C/s and 5 at 30◦C/s? Explain.

[Text Book Exercise – Section 5.4 – Q. No. 12 – Pg. No. 358]

Solution:

The standard deviation of the difference between the means is

$$\sqrt{\sigma_X^2/n_X + \sigma_Y^2/n_Y}.$$

Estimate $\sigma X \approx sX = 6.23$ and $\sigma Y \approx sY = 4.34$.

If all 10 new welds are cooled at 10◦C, then $nX = 60$ and $nY = 40$.

The standard deviation of the difference between the means is then $\sqrt{\frac{6.232^2}{60} + \frac{4.34^2}{40}} = 1.057$.

If all 10 new welds are cooled at 30◦C, then $nX = 50$ and $nY = 50$.

The standard deviation of the difference between the means is then $\sqrt{\frac{6.232^2}{50} + \frac{4.34^2}{50}} = 1.074$.

If 5 of the new welds are cooled at 10◦C and 5 at 30◦C, then $nX = 55$ and $nY = 45$.

The standard deviation of the difference between the means is $\sqrt{\frac{6.232^2}{55} + \frac{4.34^2}{45}} = 1.060$.

Therefore the confidence interval would be most precise if all 10 new welds were cooled at 10◦C.