LINEAR ALGEBRA AND ITS APPLICATIONS UE19MA251

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Transformations Represented by Matrices

If we know Ax for each vector in a basis then we know Ax for each vector in the entire vector space.

For example, if x = (1, 0) goes to (1 , 3, 5) and (0, 1) is taken to (3, 7, 0) under some transformation then the matrix associated with this transformation is

$$A = \begin{bmatrix} 1 & 3 \\ 3 & 7 \\ 5 & 0 \end{bmatrix}$$

Starting with a different basis (1, 1) and (2 , 1) this same A is also the only linear transformation with A (1, 1) = (4,10,5) and A (2, 1) = (5, 13, 10).

Transformations Represented by Matrices

Matrix Representation of Differntiation:

- Consider differentiation that goes from P_3 to P_2.
- A basis for P_3 is $u = 1$, $v = t$, $w = t^2$, $z = t^3$
- The derivatives of these basis are 0, 1, 2t, $3t^2$
- Hence, $Au = 0$, $Av = 1$, $Aw = 2t$, $Az = 3t^2$

  i.e $Au = 0$, $Av = u$, $Aw = 2v$, $Az = 3w$.

Transformations Represented by Matrices

We thus get the matrix of differentiation as

$$A_{diff} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}_{3x4}$$

Transformations Represented by Matrices

Matrix Representation of Integration:

Similarly , it can be proved that the matrix that represents Integration that brings P_2 back to P_3 is given by

$$A_{\text{int}} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/3 \end{bmatrix}_{4x3}$$

Note: A_{diff} is a left inverse of A_{int}

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THANK YOU