PES University, Bangalore

(Established under Karnataka Act No. 16 of 2013)

UE19CS203 – STATISTICS FOR DATA SCIENCE

Unit-2 - Random Variables

QUESTION BANK

Bernoulli Dustribution

Exercises for Section 4.1
[Text Book Exercise – Section 2.5 – Q. No. [1 – 7] – Pg. No. [202 - 203]]

1. After scoring a touchdown, a football team may elect to attempt a two-point conversion, by running or passing the ball into the end zone. If successful, the team scores two points. For a certain football team, the probability that this play is successful is 0.40.

   a) Let X = 1 if successful, X = 0 if not. Find the mean and variance of X.
   b) If the conversion is successful, the team scores 2 points; if not the team scores 0 points. Let Y be the number of points scored. Does Y have a Bernoulli distribution? If so, find the success probability. If not, explain why not.
   c) Find the mean and variance of Y .

2. A certain brand of dinnerware set comes in three colors: red, white, and blue. Twenty percent of customers order the red set, 45% order the white, and 35% order the blue. Let X = 1 if a randomly chosen order is for a red set, let X = 0 otherwise; let Y = 1 if the order is for a white set, let Y = 0 otherwise; let Z = 1 if it is for either a red or white set, and let Z = 0 otherwise.

   a) Let p_X denote the success probability for X. Find p_X.
   b) Let p_Y denote the success probability for Y . Find p_Y .
   c) Let p_Z denote the success probability for Z. Find p_Z.
   d) Is it possible for both X and Y to equal 1?
   e) Does $p_Z = p_X + p_Y$?
   f) Does Z = X + Y? Explain.

3. When a certain glaze is applied to a ceramic surface, the probability is 5% that there will be discoloration, 20% that there will be a crack, and 23% that there will be either discoloration or a crack, or both. Let X = 1 if there is discoloration, and let X = 0

otherwise. Let $Y = 1$ if there is a crack, and let $Y = 0$ otherwise. Let Z = 1 if there is either discoloration or a crack, or both, and let Z = 0 otherwise.

a) Let p_X denote the success probability for X. Find p_X.
b) Let p_Y denote the success probability for Y . Find p_Y .
c) Let p_Z denote the success probability for Z. Find p_Z.
d) Is it possible for both X and Y to equal 1?
e) Does $p_Z = p_X + p_Y$?
f) Does Z = X + Y? Explain.

4. Let X and Y be Bernoulli random variables. Let Z = X + Y.

    a) Show that if X and Y cannot both be equal to 1, then Z is a Bernoulli random variable.
    b) Show that if X and Y cannot both be equal to 1, then $p_Z = p_X + p_Y$.
    c) Show that if X and Y can both be equal to 1, then Z is not a Bernoulli random variable.

5. A penny and a nickel are tossed. Both are fair coins. Let X = 1 if the penny comes up heads, and let X = 0 otherwise. Let Y = 1 if the nickel comes up heads, and let Y = 0 otherwise. Let Z = 1 if both the penny and nickel come up heads, and let Z = 0 otherwise.

    a) Let p_X denote the success probability for X. Find p_X .
    b) Let p_Y denote the success probability for Y. Find p_Y .
    c) Let p_Z denote the success probability for Z. Find p_Z .
    d) Are X and Y independent?
    e) Does $p_Z = p_X \ p_Y$?
    f) f. Does Z = XY? Explain.

6. Two dice are rolled. Let $X = 1$ if the dice come up doubles and let $X = 0$ otherwise. Let $Y = 1$ if the sum is 6, and let $Y = 0$ otherwise. Let $Z = 1$ if the dice come up both doubles and with a sum of 6 (that is, double 3), and let $Z = 0$ otherwise.

    a) Let p_X denote the success probability for X. Find p_X .
    b) Let p_Y denote the success probability for Y. Find p_Y .
    c) Let p_Z denote the success probability for Z. Find p_Z .
    d) Are X and Y independent?
    e) Does $p_Z = p_X \ p_Y$?
    f) Does Z = XY? Explain

7. Let X and Y be Bernoulli random variables. Let $Z = XY$.

   a) Show that Z is a Bernoulli random variable.
   b) Show that if X and Y are independent, then $p_Z = p_X \; p_Y$.