# LINEAR ALGEBRA AND ITS APPLICATIONS
# UE19MA251

**VRINDA KAMATH**
Department of Science and Humanities

# Unit 3. Linear Transformations and Orthogonality

Topics

1. **Linear Transformations**
2. **Orthogonal vectors and Subspaces**
3. **Cosines and Projections onto lines**
4. **Projections and Least Squares**

## *Linear Transformations*

*Definition*:

Let A be a matrix of order n. When A multiplies a n- dimensional vector x, it transforms x to a n-dimensional vector Ax. This happens at every x in $R^n$. The whole space $R^n$ is ***transformed or mapped*** into itself by the matrix A. The matrix A induces a transformation of $R^n$.

## *Linear Transformations*

Few Examples.....

1. $$A = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}$$

If x = ( x, y ) then Ax = ( cx, cy ).

A multiple of the identity matrix A = *cI* ***stretches*** every vector by the scale factor c. The whole space expands or contracts.

2. $$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

If x = ( x, y ) then Ax = ( -y, x ).

The matrix A ***rotates*** every vector about the origin through a right angle in the counter clockwise direction.

## *Linear Transformations*

3. $$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

If x = ( x, y ) then Ax = ( y, x ).

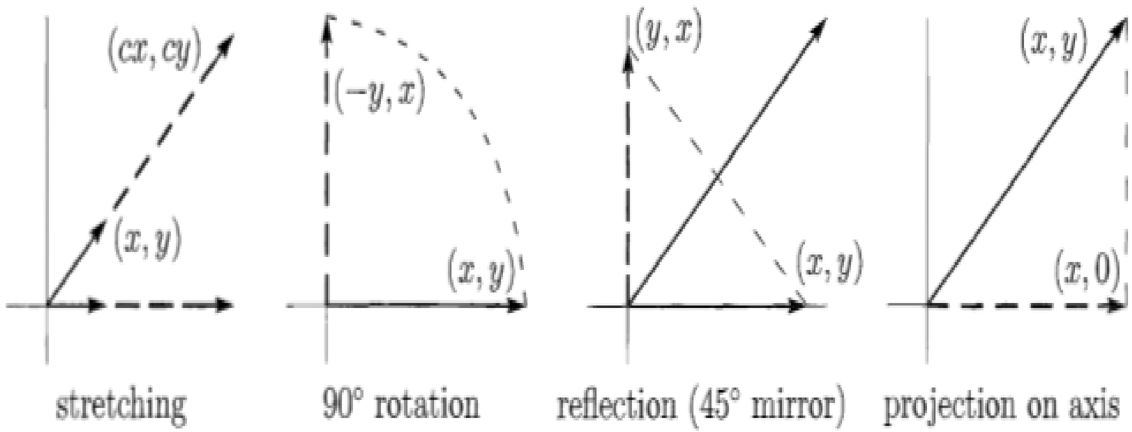
The matrix A ***reflects*** every vector on the line y = x. It is also a permutation matrix.

4. $$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

If x = ( x , y ) then Ax = ( x , 0 ).

The matrix A ***projects*** every vector onto the x axis.

## *Linear Transformations*



### *Linear Transformations*

*Note*

- A transformation can now be understood as a function ( or a mapping ) $f : A \to B$ defined by $f(x) = y$. In terms of matrices we have the rule

$A : R^n \to R^m$ defined by $Ax = b$.

## Unit 3. Linear Transformations and Orthogonality

### *Linear Transformations*

*Definition* :

A transformation T on $R^n$ is said to be ***linear*** if

it satisfies the ***rule of linearity***

$$T(cx + dy) = c(Tx) + d(Ty)$$

for all scalars c, d and vectors x, y.

***Note*** :

1. If T is linear then $T(0) = 0$ i.e T preserves origin. The converse may or may not be true.

2. If A is a matrix of order m x n then A induces a transformation from $R^n$ to $R^m$.

## *Linear Transformations*

Few examples.....

Let $v = (v_1, v_2)$ . Then,

1. $T(v) = (v_2, v_1)$ is linear
2. $T(v) = (v_1, v_1)$ is not linear
3. $T(v) = (0, v_1)$ is not linear
4. $T(v) = (0, 1)$ is not linear
5. $T(v) = (v_1, v_2)$ is linear

***Note*** :

If a transformation preserves origin it may or may not be linear!!

## THANK YOU