UE19MA251 LINEAR ALGEBRA AND ITS APPLICATIONS

.Unit 5: **Singular Value Decomposition**

Quadratic Forms, Tests for Positive Definiteness, Positive Definite Matrices and Least Squares, Semi Definite Matrices, Singular Value Decomposition, Applications, Curve Fitting, Covariance of matrices (2x2).

Class No.	Portions to be covered
55	Quadratic Forms
56	Tests for Positive Definiteness
57	Positive Definite Matrices and Least Squares
58	Semi definite Matrices
59-60	The Singular Value Decomposition of a Matrix
61-62	Problems on SVD
63	Applications of SVD, Curve Fitting' Covariance of matrices(2x2)
64	Supplementary Problems
65-67	**Scilab – In Semester Assessment**
68-70	**Revision**

Classwork problems:

1.	Write the symmetric matrix which corresponds to the following quadratic forms: $(i)Q(x)=5x_1^2+3x_2^2+2x_3^2-x_1x_2+8x_2x_3$ $(ii)Q(x)=8x_1^2+7x_2^2-3x_3^2-6x_1x_2+4x_1x_3-2x_2x_3$ $(iii)Q(x)=10x_1^2-6x_1x_2-3x_2^2$
2.	Compute the quadratic form x^TAx for A= $\begin{pmatrix} 4 & 3 & 0 \\ 3 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ and (a) $x=\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ (b) $x=\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$ (c) $x=\begin{pmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{pmatrix}$
3.	Decide for or against the positive definiteness of these matrices and write the corresponding quadratic form f= x^TAx. $\begin{pmatrix} 1 & 3 \\ 3 & 5 \end{pmatrix}$ $\begin{pmatrix} -1 & 2 \\ 2 & -8 \end{pmatrix}$ $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ $\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$ $\begin{pmatrix} 3 & 2 & 0 \\ 2 & 2 & 2 \\ 0 & 2 & 1 \end{pmatrix}$. $\begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}^2$

4.	For which s and t do A and B have all $\lambda>0$ and are therefore positive definite. $A=\begin{pmatrix} s & -4 & -4 \\ -4 & s & -4 \\ -4 & -4 & s \end{pmatrix}$ $B=\begin{pmatrix} 1 & 2 & 4 \\ 2 & t & 8 \\ 4 & 8 & 7 \end{pmatrix}$ Answer : For s > 4, A is positive definite, no value of t makes B positive definite .
5.	Find the 3x3 matrix A and its pivots, rank, eigen values and determinant? $(x_1 \quad x_2 \quad x_3)\begin{pmatrix} & & \\ & \mathrm{A} & \\ & & \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=(-x_1+x_2+x_3)^2$ Answer : Pivot is 1, rank=1, eigen values are 0,1,2 and |A|=0.
6.	If A=$Q\Delta Q^T$ is symmetric positive definite, then R=$Q\sqrt{\Delta}Q^T$ is its symmetric positive definite square root. Why does R have positive eigen values? Compute R and verify R^2=A for $A=\begin{pmatrix} 10 & 6 \\ 6 & 10 \end{pmatrix}$ $A=\begin{pmatrix} 10 & -6 \\ -6 & 10 \end{pmatrix}$
7.	Compute A^TA and AA^T, and their eigen values and unit eigenvectors, for A=$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$. Verify if A=$U\Sigma V^T$.
8.	Find SVD of the following matrices: $\begin{pmatrix} 4 & 4 \\ -3 & 3 \end{pmatrix}$ $\begin{pmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{pmatrix}$ $\begin{pmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{pmatrix}$
9.	Find the Covariance Matrix for $a_1=\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, $a_2=\begin{pmatrix} 4 \\ 2 \\ 13 \end{pmatrix}$, $a_3=\begin{pmatrix} 7 \\ 8 \\ 1 \end{pmatrix}$, $a_4=\begin{pmatrix} 8 \\ 4 \\ 5 \end{pmatrix}$
10.	The following table lists the weights and heights of 5 boys: <table><tr><td>Boy</td><td># 1</td><td># 2</td><td># 3</td><td># 4</td><td># 5</td></tr><tr><td>Weight(lbs)</td><td>120</td><td>125</td><td>125</td><td>135</td><td>145</td></tr><tr><td>Height(in)</td><td>61</td><td>60</td><td>64</td><td>68</td><td>72</td></tr></table> Find the covariance matrix for the above data.