UE19MA251 LINEAR ALGEBRA AND ITS APPLICATIONS

<u>Unit-2-Vector Spaces:</u>

Vector Spaces and Subspaces (definitions only) , Linear Independence, Basis and Dimensions, The Four Fundamental Subspaces.
Self Learning Component: Examples of Vector Spaces and Subspaces.

Class No.	Portions to be covered
16	Vector Spaces and Subspaces (Definition & Examples)
17	Echelon Form, Row Reduced Form, Pivot Variables , Free variables
18	Problems
19-20	Linear Dependence, Independence, Basis and Dimensions
21-22	The Four Fundamental Subspaces-Column Space and Row Space
23	**Scilab Class Number 4 – Span of Column Space of A**
24	Null Space
25	Left Null Space
26	Problems on Four Fundamental Subspaces
27	Supplementary problems
28	Uniqueness and Existence, Right and Left Inverses, Matrix of Rank 1
29	**Scilab Class Number 5 –Four Fundamental Subspaces of A**

Classwork problems:

1. Find the Column space and Null space for the following matrices:

$$\begin{pmatrix} -2 & 4 & -2 & -4 \\ 2 & -6 & 3 & 1 \\ -3 & 8 & 2 & -3 \end{pmatrix} \qquad \begin{pmatrix} 1 & 2 & 0 & 2 \\ 3 & 5 & -1 & 6 \\ 2 & 4 & 1 & 2 \\ 2 & 0 & -7 & 11 \end{pmatrix}$$

Answer: C(A) is a 3-d plane in $\sim^{3}$ and N(A) is a line in $\sim^{4}$

C(A) is a 4-d plane in $\sim^{4}$ and N(A) is origin in $\sim^{4}$

2. Let $A = \begin{pmatrix} -6 & 12 \\ -3 & 6 \end{pmatrix}$ and $w = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, Determine if w is in Column space of A . Is W in Null space of A.

Answer: W lies in both Column and Null space of A.

3.	Examine if the following sets of vectors are linearly independent. When the set is dependent find a relation between the vectors: (a) $\{(1,0,1,2),(0,1,1,2),(1,1,1,3)\}$ (b) $\{(1,2,-1),(1,-2,1),(3,-2,1)\}$ (c) $\{t^2+t+2,\ 2t^2+t,\ 3t^2+2t+2\}$ (d) $\left\{\begin{pmatrix}1 & 0\\ -2 & 1\end{pmatrix},\begin{pmatrix}0 & -1\\ 1 & 1\end{pmatrix},\begin{pmatrix}-1 & 2\\ 1 & 0\end{pmatrix},\begin{pmatrix}2 & 1\\ -4 & 4\end{pmatrix}\right\}$ in $M_{2x2}(R)$ **Answer:** (a)independent (b) dependent $v_1+2v_2=v_3$ (c)dependent, $p_1(t)+p_2(t)=p_3(t)$. (d) dependent, $3M_1+M_2+M_3=M_4$.
4.	Reduce the following matrices to Row Reduced Echelon form and determine their ranks $\begin{pmatrix}2 & 4 & -3 & -2\\ -2 & -3 & 2 & -5\\ 1 & 3 & -2 & 2\end{pmatrix}$. $\begin{pmatrix}0 & 2 & 3 & 7\\ 1 & 1 & 1 & 1\\ 1 & 3 & 4 & 8\\ 0 & 0 & 0 & 1\end{pmatrix}$ Identify the pivot variables and free variables. Find the special solutions to Ax=0. **Answer:** (-1/2,1,1,0); (5/2,-7/2,0,1)
5.	For which vectors b=(a,b,c) do the following systems Ax=b have a solution? x+2y=a; -x+y+2z=b; 3x-4z=c **Answer:** C+2b-a=0
6.	Which vectors (b_1,b_2,b_3) are in the column space of A? Which combination of the rows of A give 0? $A=\begin{pmatrix}1 & 2 & 0 & 3\\ 0 & 0 & 0 & 0\\ 2 & 4 & 0 & 1\end{pmatrix}$ **Answer:** For $b_2=0$ and any b_1 ,b_3.No combination of rows give zero.
7.	If the set of vectors {u,v,w} are linearly independent vectors, then show that the set $\{u+v-2w,\ u-v-w,\ u+w\}$ is linearly independent.
8.	Find the conditions on a,b,c so that the vector (a,b,c) belong to the space spanned by the vectors $v_1=(1,0,-2)$, $v_2=(3,2,-4)$, $v_3=(-3,-5,1)$. Do the vectors v_1, v_2 v_3 span $\sim^3$. **Answer:** c+2a-b=0. The vectors v_1, v_2, v_3 do not span $\sim^3$
9.	Find a basis for the set of vectors in $\sim^3$ in the plane x+2y+z=0. **Answer:** {(-1,0,1), (-2,1,0)}

10.	Show that the vectors $A=\begin{pmatrix}1 & 1\\ -1 & 1\end{pmatrix}, B=\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}, C=\begin{pmatrix}1 & 1\\ 4 & 5\end{pmatrix}$ generate the vector $M=\begin{pmatrix}4 & 7\\ 7 & 9\end{pmatrix}$ in the vector space of all 2x2 matrices. **Answer:** M=2A+3B-C
11.	Find a basis and the dimension of the subspaces $V=\{(a,b,c,d)/b-2c+d=0\}$ and $W=\{(a,b,c,d)/a=d, b=2c\}$ and $V\cap W$ in $\sim^4$
12.	Let V be the set of all vectors of the form $\begin{pmatrix}a+2b+4c\\ b+2c\\ -a+3b+6c\end{pmatrix}$ where a,b,c are arbitrary.(i) Find vectors u_1, u_2, u_3 such that V=span(u_1, u_2, u_3) . (ii) Is V a subspace of $\sim^3$? (iii) Find a basis and the dimension of V. **Answer:** (i)V=Span of {(1,0,-1),(2,1,3),(4,2,6)} (ii)Yes (iii) Basis={(u_1,u_2)}
13.	If the column space of A is spanned by the vectors (1,2,0), (-2,3,-7), (5,2,8) find all those vectors that span the null space of A, Determine whether or not the vector b=(-4,2,2) is in that subspace What are the bases and dimensions of C(A^T) and N(A^T). **Answer:** (2,1,1), $b\in N(A^T)$, Basis for C(AT)={(1,2,0),(-2,3,-7)}, Basis for N(AT)={(-2,1,1)}
14.	Obtain the four fundamental subspaces , their basis and dimension given $\begin{pmatrix}-2 & 2 & 3 & 7 & 1\\ -2 & 2 & 4 & 8 & 0\\ -3 & 3 & 2 & 8 & 4\\ 4 & -2 & 1 & -5 & -7\end{pmatrix}$. Also describe the four fundamental subspaces.
15.	Find left / right inverse (whichever possible) for the following matrices (i) $\begin{pmatrix}2 & 1 & 0\\ 1 & 0 & -2\end{pmatrix}$ (ii) $\begin{pmatrix}2 & 1\\ -1 & 0\\ 1 & -1\end{pmatrix}$