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UE19CS251: Design and Analysis of Algorithms (4-0-0-4-4)

Merge Sort

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Merge Sort

Mergesort is a perfect example of a successful application of the divide-and-conquer technique. It sorts a given array A[0 .. n - 1] by dividing it into two halves A[0 .. $\lfloor n/2 \rfloor$ -1] and A[$\lfloor n/2 \rfloor$ … n-1], sorting each of them recursively, and then merging the two smaller sorted arrays into a single sorted one.

ALGORITHM Mergesort(A[0 .. n - 1])
//Sorts array A[0 .. n - 1] by recursive mergesort
//Input: An array A[0 .. n - 1] of orderable elements
//Output: Array A[0 .. n - 1] sorted in nondecreasing order
if n > 1
copy A[0 .. $\lfloor n/2 \rfloor$ -1] to B[0 .. $\lfloor n/2 \rfloor$ -1]
copy A[$\lfloor n/2 \rfloor$.. n -1] to C[0 .. $\lceil n/2 \rceil$ -1]
Mergesort(B[0 .. $\lfloor n/2 \rfloor$ - 1])
Mergesort(C[0 .. $\lceil n/2 \rceil$ -1])
Merge(B, C, A)

The merging of two sorted arrays can be done as follows. Two pointers (array indices) are initialized to point to the first elements of the arrays being merged. The elements pointed to are compared, and the smaller of them is added to a new array being constructed; after that, the index of the smaller element is incremented to point to its immediate successor in the array it was copied from. This operation is repeated until one of the two given arrays is exhausted, and then the remaining elements of the other array are copied to the end of the new array.

ALGORITHM Merge(B[0 .. p- 1], C[0 .. q -1], A[0 .. p + q -1])
//Merges two sorted arrays into one sorted array
//Input: Arrays B[0 .. p -1] and C[0 .. q -1] both sorted
//Output: Sorted array A[0 .. p + q -1] of the elements of B and C
i←0; j←0; k←0
while i < p and j < q do
    if B[i] ≤ C[j]
        A[k] ← B[i]; i ← i + 1
    else A[k] ←C[j]; j← j + 1
    k←k+1

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if i =  p
        copy C[j .. q-1] to A[k .. p + q - 1]
else
        copy B[i .. p -1] to A[k .. p + q -1]
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The operation of the algorithm on the list 8, 3, 2, 9, 7, 1, 5, 4 is illustrated in Fig. 1.

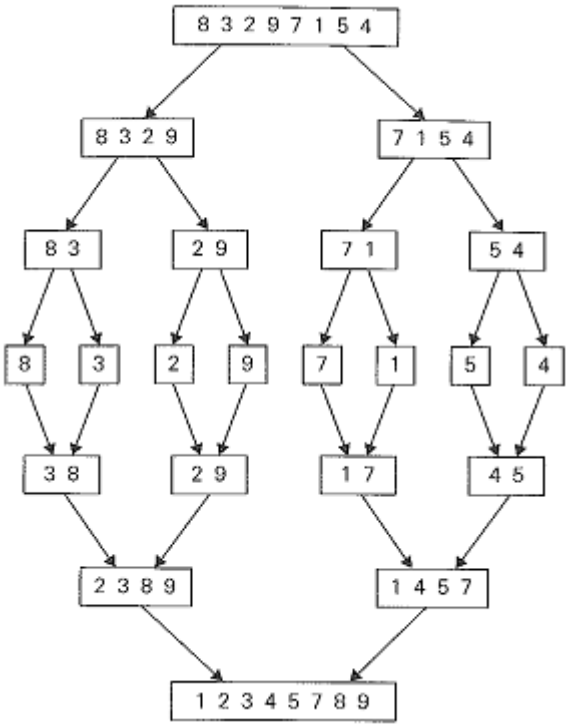

Fig. 1: Example of mergesort operation

Assuming for simplicity that n is a power of 2, the recurrence relation for the number of key comparisons C(n) is:

$C(n) = 2C(n/2) + C_{merge}(n)$ [for $n > 1$], $C(1) = 0$

The number of key comparisons performed during the merging stage in the worst case is:

$C_{merge}(n) = n - 1$

Using the above equation:

$C_{worst}(n) = 2C_{worst}(n/2) + n - 1$ [for $n > 1$], $C_{worst}(1) = 0$

Applying Master Theorem to the above equation:

$C_{worst}(n) \in \Theta (n \log n)$