STATISTICS FOR DATA SCIENCE

POWER OF TEST AND SIMPLE LINEAR REGRESSION

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STATISTICS FOR DATA SCIENCE

Unit 5 : Power of test and Simple linear regression

Session : 3

Sub Topic : Factors affecting Power of a test

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Power of a test - Recall

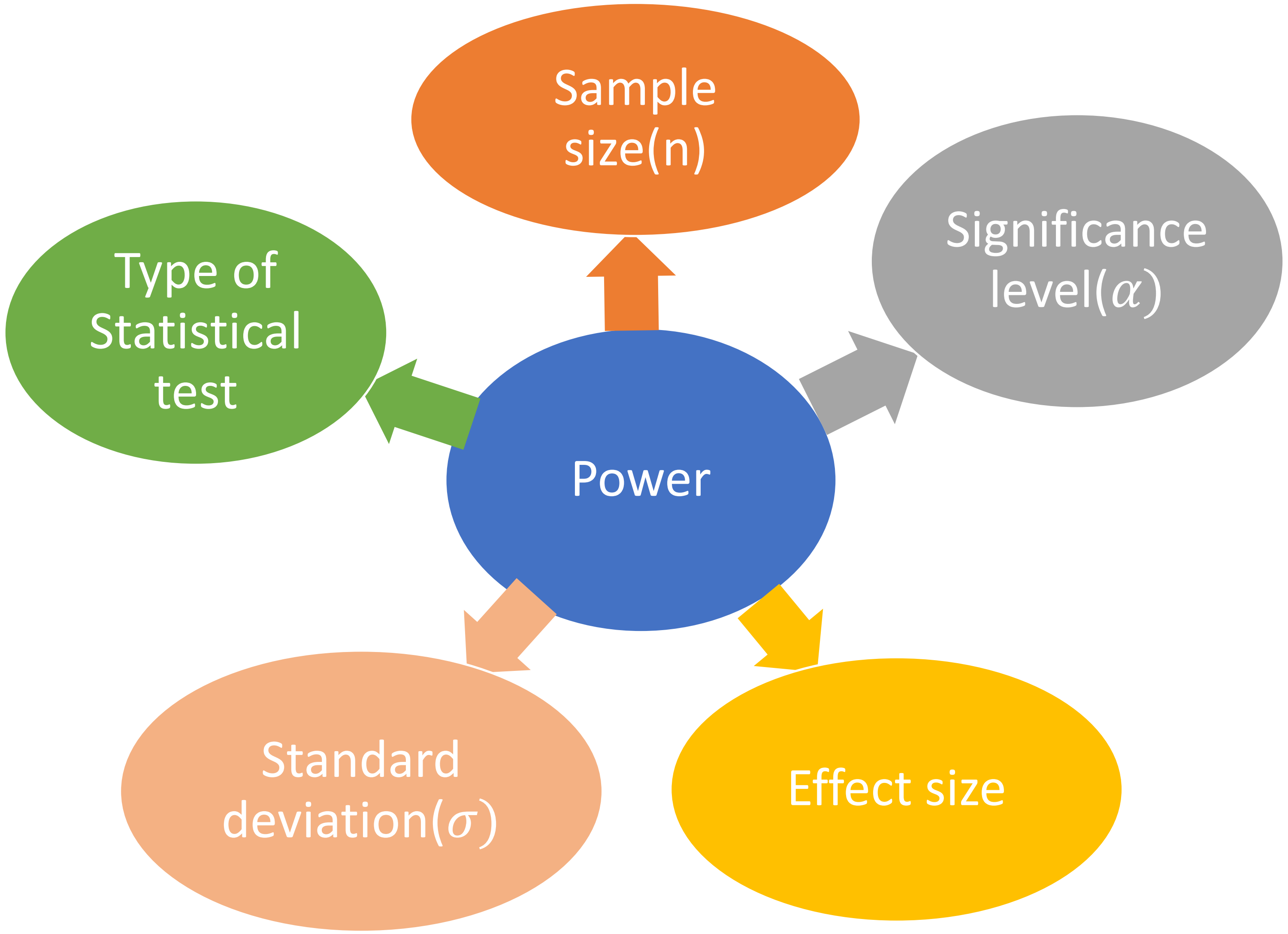
Power of a Hypothesis Test :

The power of a test is the probability of rejecting H_0 when it is false.

Power = 1 - P(type II error)
= 1 – β.

Note: *Statistical power has relevance only when the null is false.*

Factors affecting Statistical Power of test



Factors affecting Statistical Power of test – Sample size

Example:

A random sample of n people’s wight whose mean and standard deviation are 168 lbs and 7.2 lbs. Can we conclude that the mean of the population is 165lb?

$$H_0: \mu = 165$$
$$H_1: \mu \neq 165$$

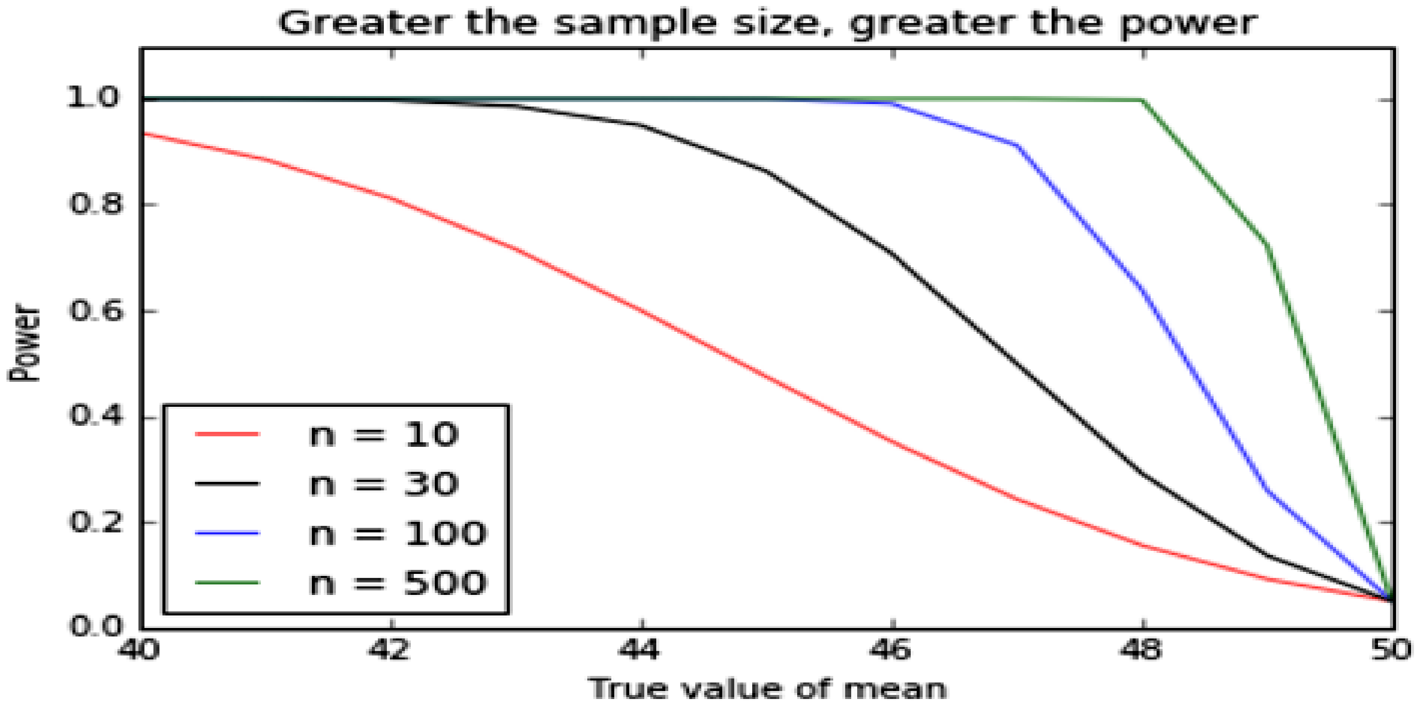
$$z = \frac{168 - 165}{7.2/\sqrt{n}} = \frac{(168 - 165)\sqrt{n}}{7.2}$$

Factors affecting Statistical Power of test- Sample size

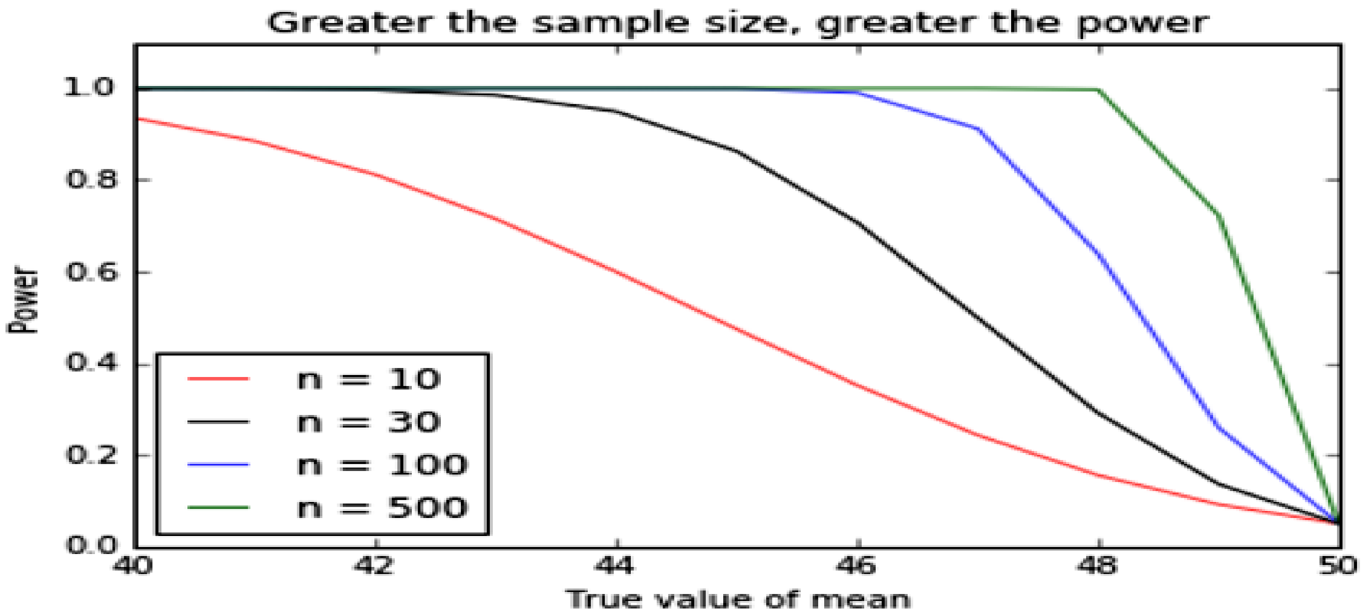
Power

Sample size(n)

The larger the sample size, the higher the power.



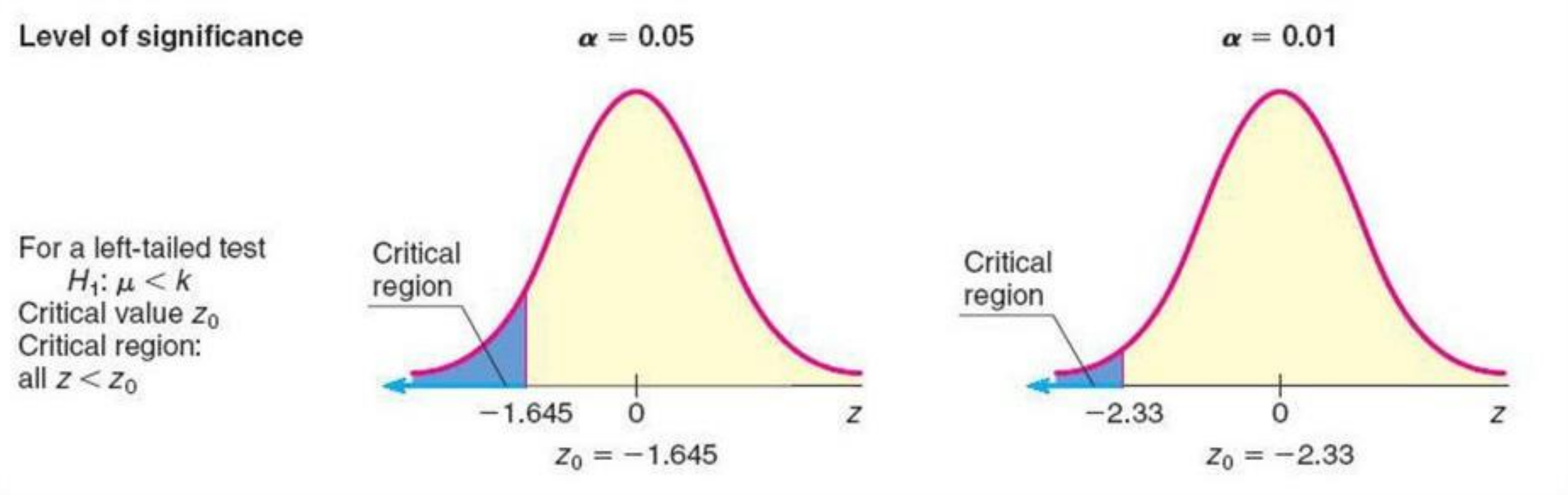
Factors affecting Statistical Power of test – Sample size



The above figure shows that the larger the sample size, the higher the power. Since sample size is typically under an experimenter's control, increasing sample size is one way to increase power. However, it is sometimes difficult and/or expensive to use a large sample size.

Factors affecting Statistical Power of test – Significance level

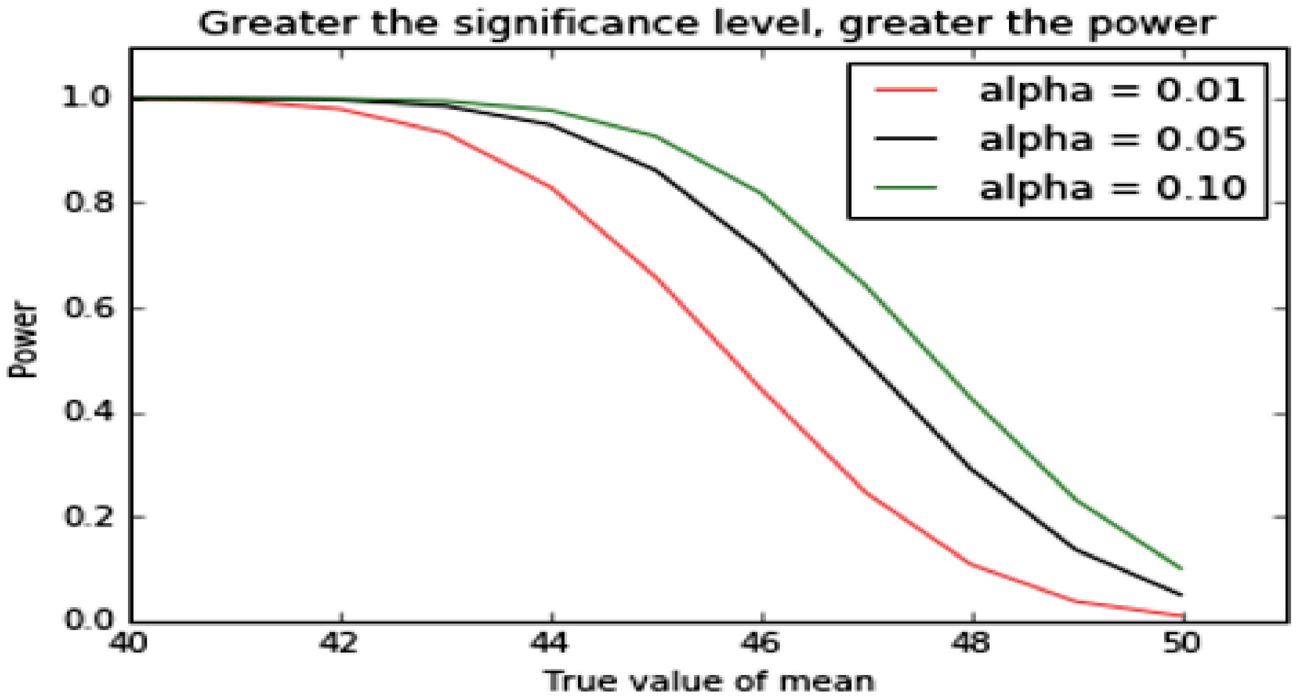
Critical Values z_0 for $\alpha = 0.05$ and $\alpha = 0.01$: Left-tailed Test



Source:Houghton Miffin Company

Factors affecting Statistical Power of test- Significance level

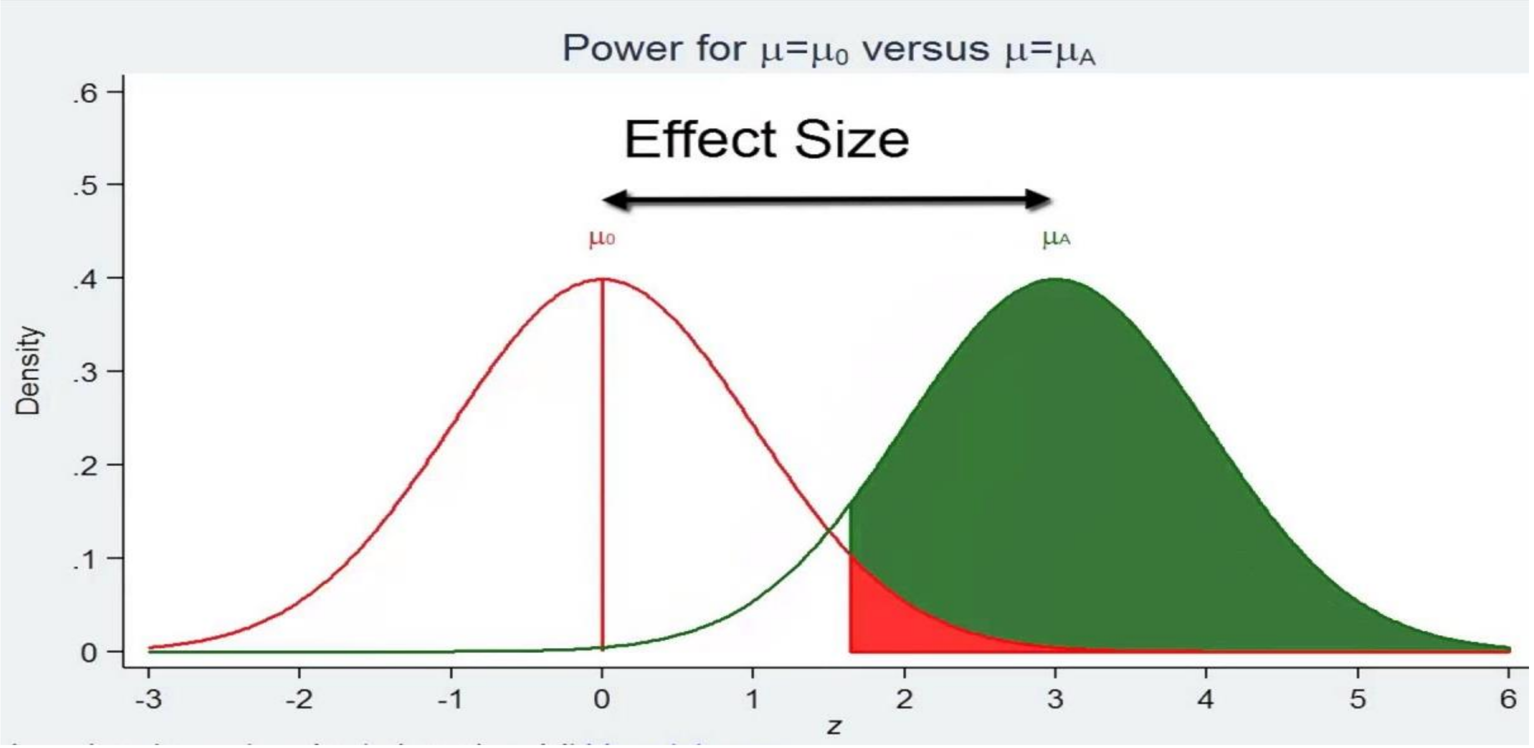
Power → Significance level(α) → **The larger the significance level, the higher the power.**



Factors affecting Statistical Power of test- Effect Size

Effect size = True Mean - Hypothesized Mean

$$= \mu_A - \mu_0$$



Example of a power calculation

Assume that a new chemical process has been developed that may increase the yield over that of the current process. The current process is known to have a mean yield of 80 and a standard deviation of 5, where the units are the percentage of a theoretical maximum. If the mean yield of the new process is shown to be greater than 80, the new process will be put into production.

Let μ denote the mean yield of the new process. It is proposed to run the new process 50 times and then to test the hypothesis

$H_0: \mu \leq 80$ versus $H_1 : \mu > 80$ at a significance level of 5%.

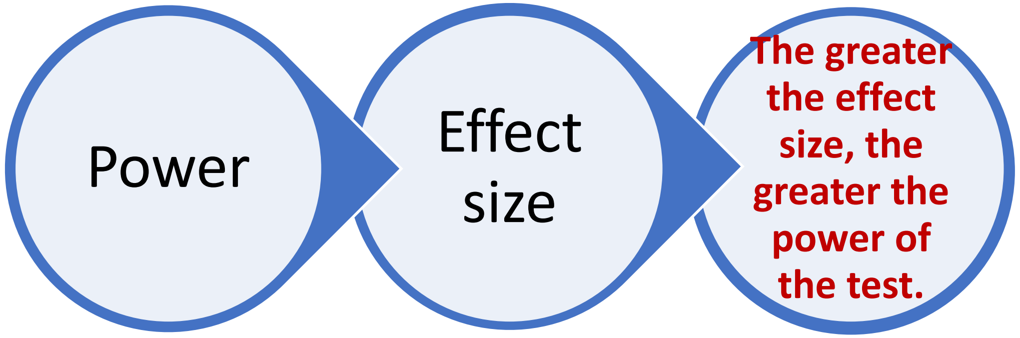
if μ is close to μ_0 : the power will be small

(when μ =81, Power=0.4090)

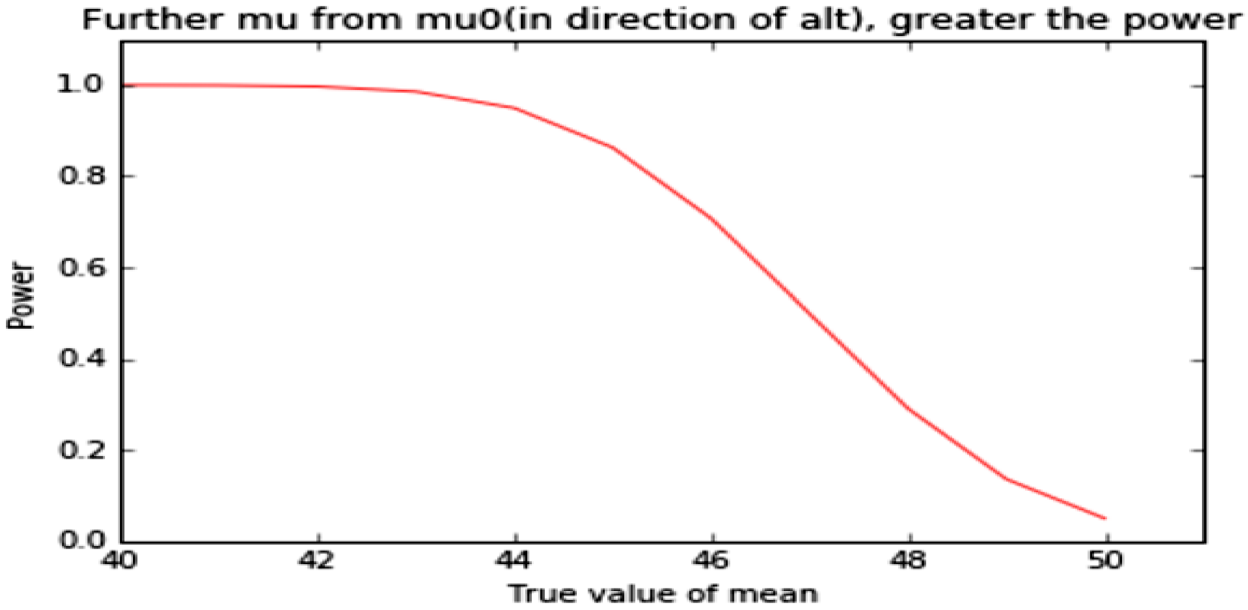
if μ is far from μ_0 : the power will be large

(when μ =82, Power=0.8830)

Factors affecting Statistical Power of test- Effect Size



Effect size = True value - Hypothesized value



Factors affecting Statistical Power of test –Standard deviation

Example:

A random sample of 200 people's wight whose mean is 168 lbs. Can we conclude that the mean of the population is 165lb?

$$H_0 : \mu = 165$$
$$H_1 : \mu \neq 165$$

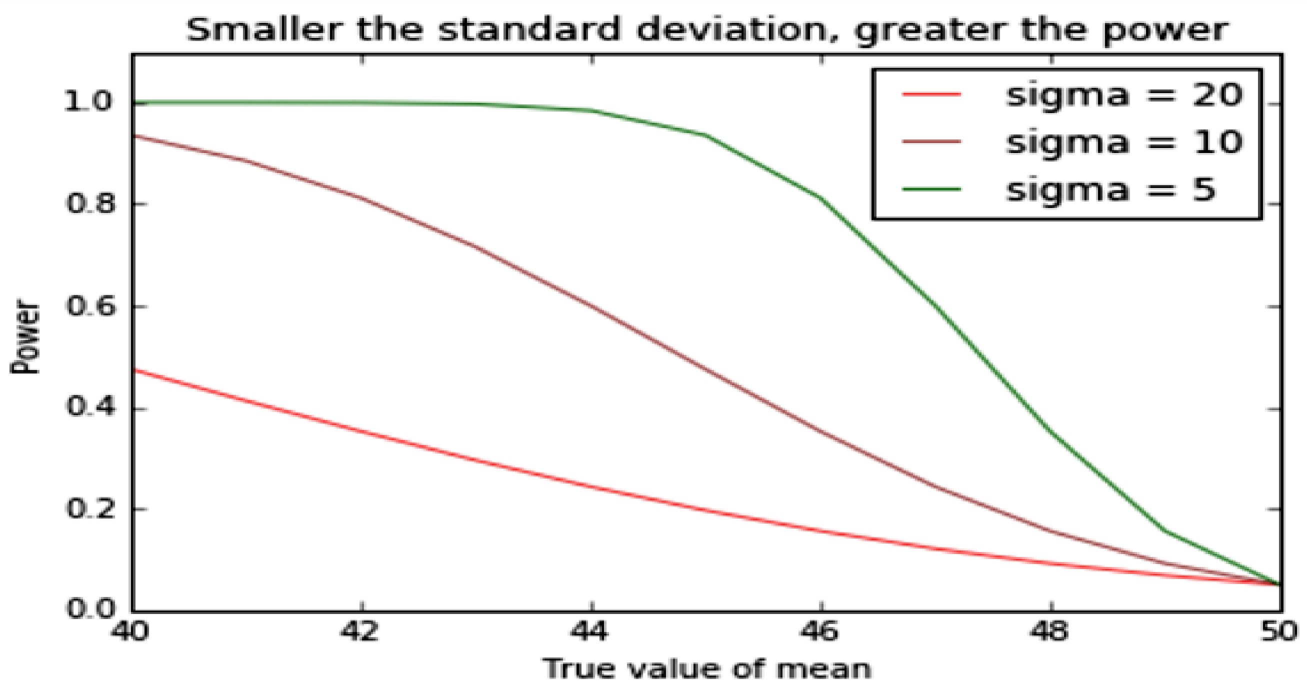
$$z = \frac{168 - 165}{\sigma/\sqrt{200}} = \frac{(168 - 165)\sqrt{200}}{\sigma}$$

Factors affecting Statistical Power of test- Standard deviation

Power

Standard deviation

Smaller the standard deviation, greater the power of the test.



STANDARD DEVIATION

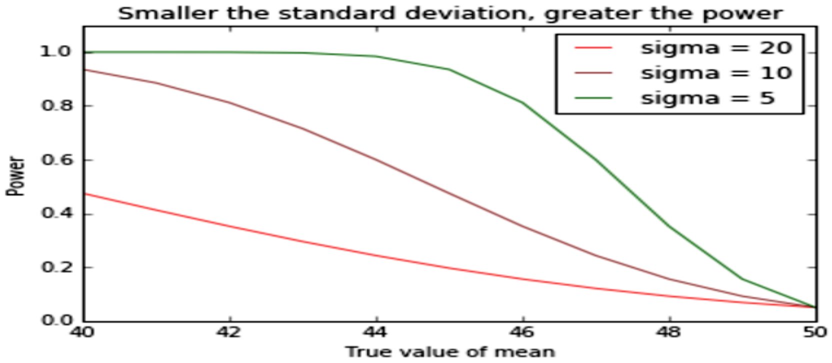
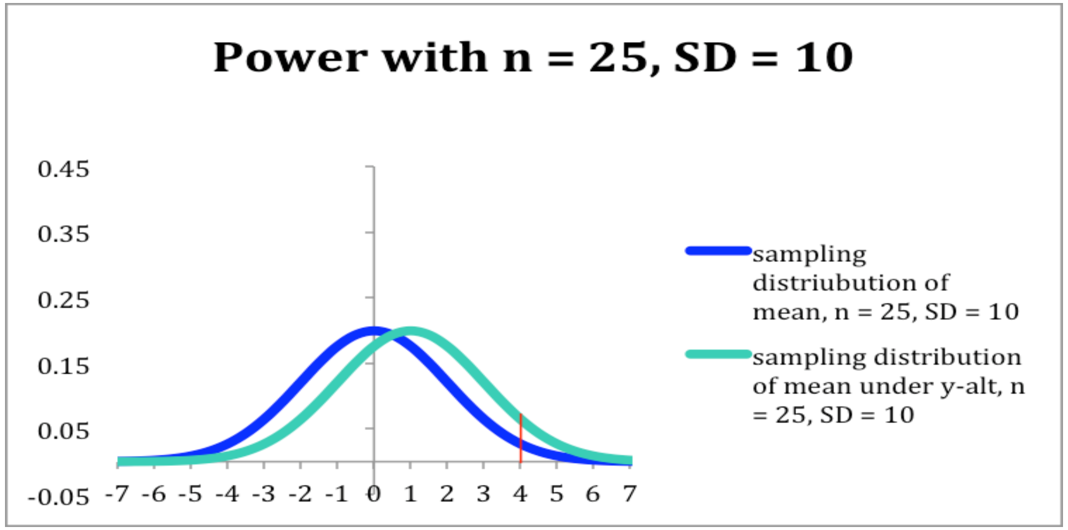
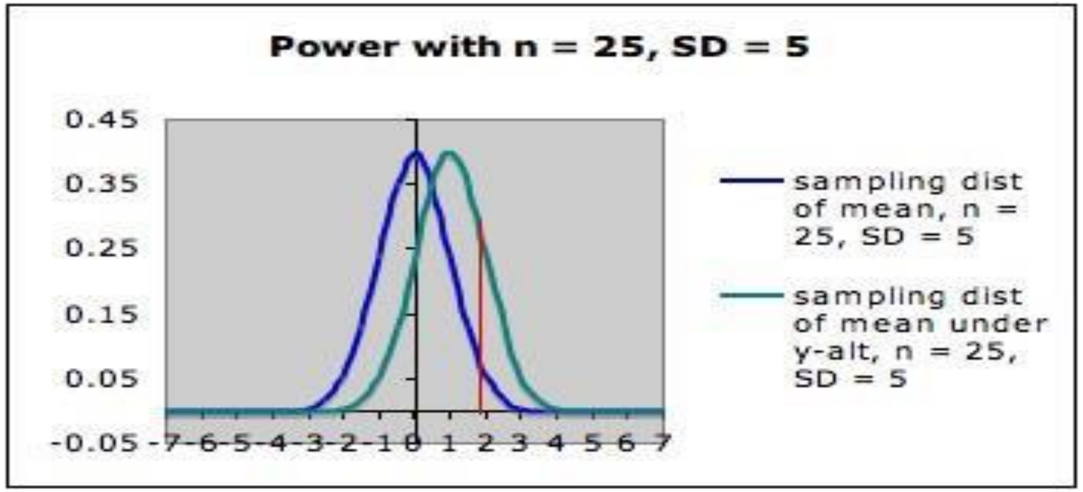


Figure also shows that power is higher when the standard deviation is small than when it is large. For all values of N, power is higher for the standard deviation of 10 than for the standard deviation of 15 (except, of course, when N = 0). Experimenters can sometimes control the standard deviation by sampling from a homogeneous population of subjects, by reducing random measurement error, and/or by making sure the experimental procedures are applied very consistently.

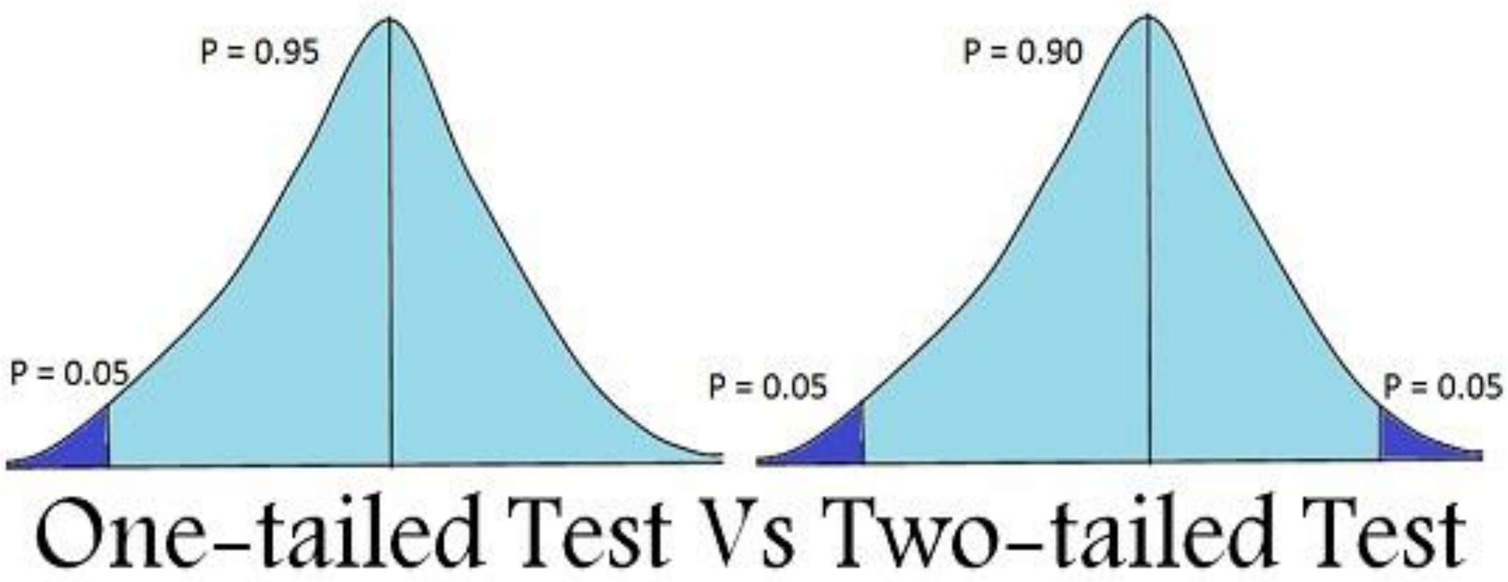
Factors affecting Statistical Power of test – Standard deviation



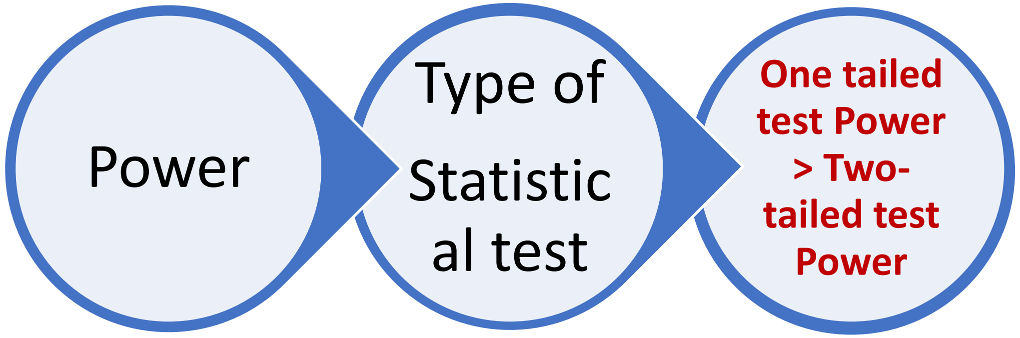


In each picture, the area under the *green* curve to the right of the red line is the power of the test against the alternate depicted. Note that this area is *larger* in the second picture (the one with smaller standard deviation) than in the first picture.

Factors affecting Statistical Power of test – Type of Statistical test



Factors affecting Statistical Power of test- Type of Statistical test



ONE- VERSUS TWO-TAILED TESTS

Power is higher with a *one-tailed* test than with a *two-tailed* test as long as the hypothesized direction is correct. A one-tailed test at the 0.05 level has the same power as a two-tailed test at the 0.10 level. A one-tailed test, in effect, raises the significance level.

Factors affecting Statistical Power of test- Summary

- Greater the sample size, the higher the power.
- The larger the significance level, the higher the power.
- The greater the effect size, the greater the power of the test..
- Smaller the standard deviation, greater the power of the test.
- One tailed test Power > Two-tailed test Power

THANK YOU

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