AUTOMATA, FORMAL LANGUAGES AND LOGIC

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MODULE 5

Propositional Logic

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Outline

- Propositional logic - A very Simple Logic
  - Syntax and Semantics
  - A Simple Knowledge Base
  - A Simple Inference Procedure

Propositional Logic (Syntax and Semantics)

Propositions

Propositions are declarative sentence which is either universally true or universally false but not both. Usually denoted by P or Q or X or Y.

Propositions are **building blocks of logic.**

e.g. 1. The president of India is a woman. (P)

2. Toronto is the capital of Canada. (P)

3. Watch a movie. (~P)

4. What is your name? (~P)

Propositional Logic - Syntax

Syntax defines the sentences in the language.

- Atomic Sentences
- Complex Sentences

Atomic Sentences – consists of a ***single proposition symbol.***

Each such symbol stands for a proposition that can be true or false.

Two proposition symbol with fixed meaning -

TRUE – Always true proposition

FALSE – Always false proposition

Propositional Logic - Syntax

Syntax defines the sentences in the language.

- Atomic Sentences
- Complex Sentences

Complex Sentences – are constructed from simple sentences.
There are ***five logical connectives*** in common use:-

($\vee$ (or) , $\wedge$ (and) , $\neg$ (not) , $\Rightarrow$ (implies) , $\Leftrightarrow$ (if and only if))

Disjunction Conjunction Negation Implies Bidirectional

Propositional Logic - Semantics

The Semantics defines the rules for determining the truth of a sentence *with respect to* a particular model.

Each model specifies true/false for each proposition symbol

E.g. $P_{1,2}$ $P_{2,2}$ $P_{3,1}$

One possible model will be

m1 = {$P_{1,2}$ = false, $P_{2,2}$ = false, $P_{3,1}$ = true}

With these three proposition symbols, 8 possible models, can be enumerated automatically.

Propositional Logic - Semantics

Semantics for propositional logic must specify how to compute the truth value of any sentence, given a model m.

$\neg S$	is true iff	S is false	
$S_1 \wedge S_2$	is true iff	S_1 is true and	S_2 is true
$S_1 \vee S_2$	is true iff	$S_1$is true or	S_2 is true
$S_1 \Rightarrow S_2$	is true **unless** S_1 is true and S_2 is false in m.		
$S_1 \Leftrightarrow S_2$	is true iff	$S_{1,}$ S_2 are both true or both false.	

Propositional Logic - Semantics

- The order from highest to lowest:

$$\text{Ø}, \wedge, \vee, \Rightarrow and \Leftrightarrow$$

Propositional Logic - Semantics

BNF (Backus- Naur form) grammar, Like CFG has four components:-

- A set of terminal symbols
- A set of non-terminal symbols
- A start symbol
- A set of rewrite rules of the form

LHS -> RHS

Zero or more symbols

A nonterminal symbol

Propositional Logic - Semantics

- BNF Grammar for simple arithmetic expressions:-

  Expr $\longrightarrow$ Expr operator Expr | (Expr) | Number

  Number $\longrightarrow$ Digit | Number Digit

  Digit $\longrightarrow$ 0|1|2|3|4|5|6|7|8|9

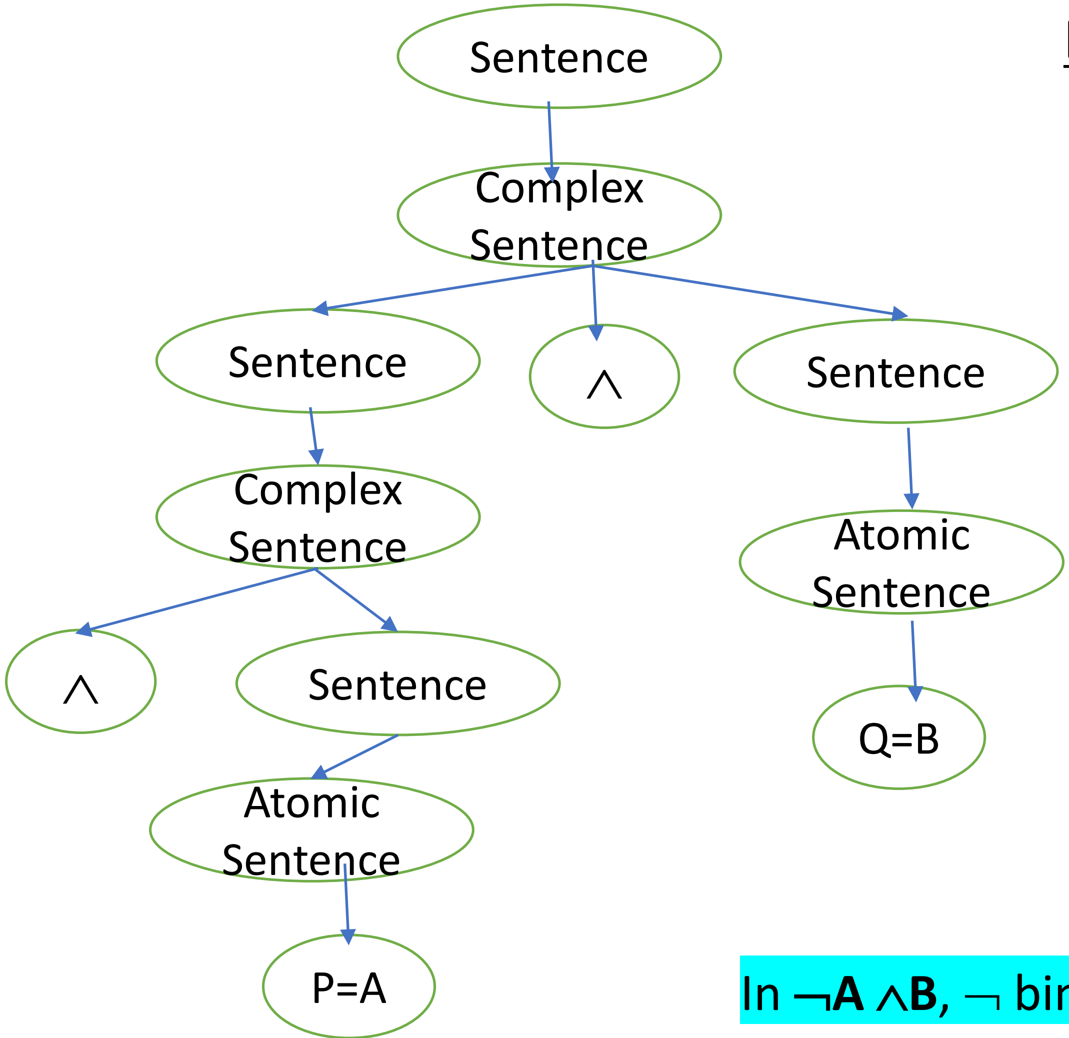
  Operator $\longrightarrow$ +|-| /| *

Propositional Logic - Semantics

BNF grammar of sentences in propositional logic, along with operator precedence, from highest to lowest

$$
\begin{array}{rcl}
\mathit{Sentence} & \rightarrow & \mathit{AtomicSentence} \mid \mathit{ComplexSentence} \\
\mathit{AtomicSentence} & \rightarrow & \mathit{True} \mid \mathit{False} \mid P \mid Q \mid R \mid \ldots \\
\mathit{ComplexSentence} & \rightarrow & (\ \mathit{Sentence}\ ) \mid [\ \mathit{Sentence}\ ] \\
& \mid & \neg\ \mathit{Sentence} \\
& \mid & \mathit{Sentence} \wedge \mathit{Sentence} \\
& \mid & \mathit{Sentence} \vee \mathit{Sentence} \\
& \mid & \mathit{Sentence} \Rightarrow \mathit{Sentence} \\
& \mid & \mathit{Sentence} \Leftrightarrow \mathit{Sentence} \\
\\
\textsc{Operator Precedence} & : & \neg, \wedge, \vee, \Rightarrow, \Leftrightarrow
\end{array}
$$

Example: $\neg A \wedge B$



<u>BNF grammar of sentences in propositional logic</u>

Sentence $\rightarrow$ AtomicSentence | ComplexSentence

AtomicSentence $\rightarrow$ True | False | P | Q | R|…

ComplexSentence $\rightarrow$ (Sentence) | [Sentence]
| $\neg$ Sentence
| Sentence $\wedge$ *Sentence*
| Sentence $\vee$ *Sentence*
| Sentence $\Rightarrow$ *Sentence*
| Sentence $\Leftrightarrow$ *Sentence*

In **$\neg$A $\wedge$B**, $\neg$ binds most tightly as **($\neg$A) $\wedge$B** rather than $\neg$(A) $\wedge$B

Truth table of Five Logical Connectives

Negation ~ Or ¬

P	~P
T	F
F	T

e.g.

1. P: Ram is a good boy
   ¬P: Ram is not a good boy.

2. P: 8+1 = 5
   ¬P: 8+1 ≠ 5

Truth table of Five Logical Connectives

Conjunction ∧

P	Q	P∧Q
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction V

P	Q	PvQ
T	T	T
T	F	T
F	T	T
F	F	F

Truth table of Five Logical Connectives

Conjunction ∧

e.g.
P: Today is Friday.
Q: It is raining today.
P∧Q : Today is Friday and it is raining today.

Disjunction V

e.g.
1. P: Today is Friday.
   Q: It is raining today.
PVQ : Today is Friday or it is raining today.

2. P: Sam is an Architect.
   Q: Sam is a draftsman.
PVQ : Sam is an Architect or a draftsman.

Truth table of Five Logical Connectives

Conditional Statement (Implication/ Implies) $\Rightarrow$

(If..then)

$P \Rightarrow Q$

Antecedent

Consequent

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

is true **unless** P is true and Q is false in model.

e.g.

1. P: Emma learns AI.

   Q: Emma will find a good job.

$P \Rightarrow Q$: If Emma learns AI, then she will find a good job.

2. If you do hard work, then you **would** get 90% on final.

   If you do not do hard work, then you **may or may not** get 90% on final.

Truth table of Five Logical Connectives

Conditional Statement (Implication/ Implies) $\Rightarrow$

(If..then)

$P \Rightarrow Q$

Antecedent

Consequent

is true **unless** P is true and Q is false in model.

If it is raining, then home team wins

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

P implies Q also means

- P only if Q
- Q whenever P
- Q is necessary condition for P
- Q follows from P
- Q unless ¬P

The home team wins whenever it is raining

The home team wins unless it is not raining

Truth table of Five Logical Connectives

Biconditional Operator (If and Only If) $\Leftrightarrow$

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$	$P \Leftrightarrow Q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

P $\Leftrightarrow$ Q means $P \Rightarrow Q$ and $Q \Rightarrow P$

P $\Leftrightarrow$ Q is a conjunction of $P \Rightarrow Q$, $Q \Rightarrow P$

P $\Leftrightarrow$ Q Ξ $P \Rightarrow Q \wedge Q \Rightarrow P$

e.g.

P: You can take the flight

Q: You buy a ticket.

Then

P $\Leftrightarrow$ Q : You can take the flight if and only if you buy a ticket .

Truth table of Five Logical Connectives

P	Q	¬P	P ∧ Q	Q ∨ P	P ⇒ Q	P ⇔ Q
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

Simple recursive process evaluates an arbitrary sentence, e.g.,

$$\neg P_{1,2} \wedge (P_{2,2} \vee P_{3,1}) = \textit{true} \wedge (\textit{true} \vee \textit{false}) = \textit{true} \wedge \textit{true} = \textit{true}$$

Example

- A square is breezy *if* a neighboring square has a pit.

- A square is breezy *only if* a neighboring square has a pit.

  => means the presence of pits if there is a breeze.

Biconditional,

$B_{1,1} \Leftrightarrow P_{1,2}$ or $P_{2,1}$

- $\Leftrightarrow$ means (requires) the absence of pits if there is no breeze.

Tautology

- A tautology is when you use different words to repeat the same idea.

e.g. 1. The phrase

"It was adequate enough" is a tautology. The words adequate and enough convey the same meaning.

2. $P \Rightarrow Q \equiv \neg P \vee Q$

Example

1. Translate the following English sentence into a logical Expression.

"You can access the Internet from campus ONLY IF you are a computer Science major OR you are not a fresher."

2. Prove the logical Equivalence of

$P \Rightarrow Q$ Ξ $\neg P \vee Q$

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3. Prove the logical Equivalence for Biconditional

P ⇔ Q Ξ $(P \wedge Q) \vee (\neg P \wedge \neg Q)$

P	Q				

THANK YOU

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