AUTOMATA, FORMAL LANGUAGES AND LOGIC

Dr Pooja Agarwal

Professor,

Department of Computer Science & Engineering

AUTOMATA, FORMAL LANGUAGES AND LOGIC

MODULE 5

Propositional Logic

Dr Pooja Agarwal
Department of Computer Science & Engineering

Outline

- Inference and proofs
  - Theorem proving
    - Logical equivalence
    - Validity
    - Satisfiability
  - Inference and Proofs
    - Modus Ponens
    - And-Elimination

Theorem proving

- Applying rules of inference directly to the sentences in our knowledge base, to construct a proof of the desired sentence without consulting models

Logical equivalence

Validity

Satisfiability

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Logical equivalence

- Two sentences α and β are logically equivalent if they are true in the same set of models.
- We write this as **α ≡ β.**
- For example, P ∧ Q and Q ∧ P are logically equivalent

Inference and Proofs

- Two sentences are logically equivalent iff true in same models: α ≡ ß iff α ╞ β and β ╞ α

$$
\begin{array}{rcll}
(\alpha \wedge \beta) & \equiv & (\beta \wedge \alpha) & \text{commutativity of } \wedge \\
(\alpha \vee \beta) & \equiv & (\beta \vee \alpha) & \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) & \equiv & (\alpha \wedge (\beta \wedge \gamma)) & \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) & \equiv & (\alpha \vee (\beta \vee \gamma)) & \text{associativity of } \vee \\
\neg(\neg\alpha) & \equiv & \alpha & \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) & \equiv & (\neg\beta \Rightarrow \neg\alpha) & \text{contraposition} \\
(\alpha \Rightarrow \beta) & \equiv & (\neg\alpha \vee \beta) & \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) & \equiv & ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) & \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) & \equiv & (\neg\alpha \vee \neg\beta) & \text{de Morgan} \\
\neg(\alpha \vee \beta) & \equiv & (\neg\alpha \wedge \neg\beta) & \text{de Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) & \equiv & ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) & \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) & \equiv & ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) & \text{distributivity of } \vee \text{ over } \wedge
\end{array}
$$

Validity

- A sentence is valid if it is true in all models.
- For example, the sentence **$P \lor \neg P$** is valid
- Validity is connected to inference via **deduction theorem**
- For any sentences α and β, $\alpha \models \beta$ if and only if the sentence $(\alpha \Rightarrow \beta)$ is valid.

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Satisfiability

- A sentence is satisfiable if it is true in, or satisfied by, some model
- Satisfiability can be checked by enumerating the possible models *until one is found* that satisfies the sentence.

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Inference and Proofs

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Examples

- Decide whether each of the following sentences is valid, unsatisfiable, or satisfiable

a. Smoke ⇒ Smoke *Valid*

b. Smoke ⇒ Fire *Satisfiable*

c. ((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∧ (Heat ⇒ Fire)) *Valid*

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Inference Rules

- **Modus Ponens**
- **And-Elimination**

Inference Rules

Modus Ponens

$$\frac{\alpha \Rightarrow \beta,\ \alpha}{\beta}$$

- Whenever any sentences of the form $\alpha \Rightarrow \beta$ and α are given, then the sentence β can be inferred

- For example, if *(WumpusAhead ∧WumpusAlive) ⇒ Shoot* and *(WumpusAhead ∧ WumpusAlive)* are given, then *Shoot* can be inferred
- If today is Sunday (p) then it is a holiday(q).
- Today is Sunday(p). It infers, It is a holiday

Inference Rules

And-Elimination

- from a conjunction, **any of the** conjuncts can be inferred

$$\frac{\alpha \land \beta}{\alpha}$$

- For example, from *(WumpusAhead ∧ WumpusAlive)*, *WumpusAlive* can be inferred.

Inference and Proofs

R_1: $\neg P_{1,1}$

R_2: $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$

R_3: $B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$

R4: $\neg B_{1,1}$

R5: $B_{2,1}$

- Example
  - We have KB = R1 ∧ R2 ∧ R3 ∧ R4 ∧ R5.
    We want to prove ¬P1,2

$R_6 : (B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$ by bicond. elim R_2

$R_7 : ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$ by And-Elimination to R_6

$R_8 : (\neg B_{1,1} \Rightarrow \neg(P_{1,2} \vee P_{2,1}))$ by contrapositives

$R_9 : \neg(P_{1,2} \vee P_{2,1})$ by Modus Ponens with R_8 and R_4

$R_{10} : \neg P_{1,2} \wedge \neg P_{2,1}$ by De Morgan's rule

That is: Neither [1,2] nor [2,1] contains a pit.

THANK YOU

Dr Pooja Agarwal
Department of Computer Science and Engineering
poojaagarwal@pes.edu