# LINEAR ALGEBRA AND ITS APPLICATIONS

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## MATRICES AND GAUSSIAN ELIMINATION

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## GAUSSIAN ELIMINATION:

**Supplementary Problems:**

1. For what values of a and b does the following system have (i)a unique solution (ii) Infinitely many solutions (iii) No solution.

$$x+2y+3z=2$$
$$-x-2y+az=-2$$
$$2x+by+6z=5$$

$$\begin{pmatrix} 1 & 2 & 3 : 2 \\ -1 & -2 & a : -2 \\ 2 & b & 6 : 5 \end{pmatrix} \xrightarrow[R_3-2R_1]{R_2+R_1} \begin{pmatrix} 1 & 2 & 3 : 2 \\ 0 & 0 & a+3 : 0 \\ 0 & b-4 & 0 : 1 \end{pmatrix} \xrightarrow{R_3 \leftrightarrow R_2} \begin{pmatrix} 1 & 2 & 3 : 2 \\ 0 & b-4 & 0 : 1 \\ 0 & 0 & a+3 : 0 \end{pmatrix}$$

(i) If $a \neq -3$ and $b \neq 4$ then r(A)=r(A:b)=3=n hence system will be **consistent** and will have **a unique solution**.

(ii) If $a=-3$ then r(A)=r(A:b)=2<n(=3) hence system will be **consistent** and will have infinitely **many solutions**.

(iii) If $a=-3,\ b=4\ (\text{or } a \neq -3,\ b=4)$ , then r(A)=1(or 2) and r(A:b)=2(or 3) hence system will be **inconsistent** and will have **no solution**.

## GAUSSIAN ELIMINATION:

2. For what values of a and b does the following system have (i)a trivial solution

(ii) Infinitely many solutions.

$$x+2y+3z=0$$
$$-x-2y+az=0$$
$$2x+by+6z=0$$

$$\begin{pmatrix} 1 & 2 & 3 \\ -1 & -2 & a \\ 2 & b & 6 \end{pmatrix} \xrightarrow[R_3-2R_1]{R_2+R_1} \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & a+3 \\ 0 & b-4 & 0 \end{pmatrix} \xrightarrow{R_3 \leftrightarrow R_2} \begin{pmatrix} 1 & 2 & 3 \\ 0 & b-4 & 0 \\ 0 & 0 & a+3 \end{pmatrix}$$

This system will have only either a trivial solution or infinitely many solutions. It will have

(i) **a trivial solution** if $a \neq -3 \ \& \ b \neq 4$ then r(A)=3=n

(ii) **Infinitely many solutions** if $a=-3$ or $b=4$ *or both* then r(A) will be 2 or 1 respectively.

## GAUSSIAN ELIMINATION:

3. Which number q makes this system singular and which right hand side t gives it infinitely many solutions. Find the solution that has z=1.

$$x+4y-2z=1$$
$$x+7y-6z=6$$
$$3y+qz=t$$

$$\begin{pmatrix} 1 & 4 & -2:1 \\ 1 & 7 & -6:6 \\ 0 & 3 & q:t \end{pmatrix} \xrightarrow{R_2-R_1} \begin{pmatrix} 1 & 4 & -2:1 \\ 0 & 3 & -4:5 \\ 0 & 3 & q:t \end{pmatrix} \xrightarrow{R_3-R_2} \begin{pmatrix} 1 & 4 & -2:1 \\ 0 & 3 & -4:5 \\ 0 & 0 & q+4:t-5 \end{pmatrix}$$

If $q=-4$ the system will be **singular.**

$t=5$ makes the system **consistent** and gives it **infinitely many solutions**.

This gives 3y-4z=5; x+4y-2z=1.

z=1 gives y=3 and x=-9. Hence the solution which has z=1 is **(-9, 3, 1)**.

## GAUSSIAN ELIMINATION:

4. Check for consistency and solve the following system of equations if consistent. Also discuss its rank:

(i)
$$3x + y + 2z = 3$$
$$2x - 3y - z = -3$$
$$x + 2y + z = 4$$

$$\begin{pmatrix} 3 & 1 & 2 : 3 \\ 2 & -3 & -1 : -3 \\ 1 & 2 & 1 : 4 \end{pmatrix} \xrightarrow[R_3 - \left(\frac{1}{3}\right)R_1]{R_2 - \left(\frac{2}{3}\right)R_1} \begin{pmatrix} 3 & 1 & 2 : 3 \\ 0 & -11/3 & -7/3 : -5 \\ 0 & 5/3 & -1/3 : 3 \end{pmatrix}$$

$$\xrightarrow{R_3 + \left(\frac{5}{11}\right)R_2} \begin{pmatrix} 3 & 1 & 2 : 3 \\ 0 & -11/3 & -7/3 : -5 \\ 0 & 0 & -24/33 : 8/11 \end{pmatrix} \Rightarrow \begin{cases} 3x + y + 2z = 3 \\ (-11/3)\, y - 7/3z = -5 \\ (-24/33)\, z = 8/11 \end{cases}$$

r(A)=r(A:b)=3=n hence system is **consistent** and has **a unique solution**.

i.e **(x, y, z)=(1, 2,-1).**

## GAUSSIAN ELIMINATION:

4. Solve Ax=b for x if $A^{-1}=\begin{pmatrix} 1 & 0 & -2 \\ 2 & 1 & 3 \\ 4 & 2 & 5 \end{pmatrix}$ and b= $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$

$$A^{-1}=\left(\begin{array}{ccc:ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 2 & 1 & 3 & 0 & 1 & 0 \\ 4 & 2 & 5 & 0 & 0 & 1 \end{array}\right) \xrightarrow[R_3-4R_1]{R_2-2R_1} \left(\begin{array}{ccc:ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & 7 & -2 & 1 & 0 \\ 0 & 2 & 13 & -4 & 0 & 1 \end{array}\right)$$

$$\xrightarrow{R_3-2R_2} \left(\begin{array}{ccc:ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & 7 & -2 & 1 & 0 \\ 0 & 0 & -1 & 4 & -4 & 1 \end{array}\right) \xrightarrow[R_2+7R_3]{R_1-2R_3} \left(\begin{array}{ccc:ccc} 1 & 0 & 0 & -7 & 8 & -2 \\ 0 & 1 & 0 & 26 & -27 & 7 \\ 0 & 0 & -1 & 4 & -4 & 1 \end{array}\right)$$

$$\xrightarrow{-R_3} \left(\begin{array}{ccc:ccc} 1 & 0 & 0 & -7 & 8 & -2 \\ 0 & 1 & 0 & 26 & -27 & 7 \\ 0 & 0 & 1 & -4 & 4 & -1 \end{array}\right)$$

## GAUSSIAN ELIMINATION:

(ii)
$$\begin{aligned} 2x-3y+2z&=1\\ 5x-8y+7z&=1\\ y-4z&=8 \end{aligned}$$

$$\begin{pmatrix} 2 & -3 & 2:1 \\ 5 & -8 & -7:1 \\ 0 & 1 & -4:8 \end{pmatrix} \xrightarrow{R_2-\left(\frac{5}{2}\right)R_1} \begin{pmatrix} 2 & -3 & 2:\ 1 \\ 0 & -1/2 & 2:-3/2 \\ 0 & 1 & -4:\ 8 \end{pmatrix} \xrightarrow{R_3+2R_2} \begin{pmatrix} 2 & -3 & 2:\ 1 \\ 0 & -1/2 & 2:-3/2 \\ 0 & 0 & 0:\ 5 \end{pmatrix}$$

r(A)=2 and r(A:b)=3 hence system is **inconsistent** and has **no solution**.

## GAUSSIAN ELIMINATION:

(iii)
$$2x-3y+2z=1$$
$$5x-8y+7z=1$$
$$y-4z=3$$

$$\begin{pmatrix} 2 & -3 & 2:1 \\ 5 & -8 & -7:1 \\ 0 & 1 & -4:3 \end{pmatrix} \xrightarrow{R_2-\left(\frac{5}{2}\right)R_1} \begin{pmatrix} 2 & -3 & 2: \ 1 \\ 0 & -1/2 & 2:-3/2 \\ 0 & 1 & -4: \ 3 \end{pmatrix} \xrightarrow{R_3+2R_2} \begin{pmatrix} 2 & -3 & 2: \ 1 \\ 0 & -1/2 & 2:-3/2 \\ 0 & 0 & 0: \ 0 \end{pmatrix}$$

r(A)=r(A:b)=2<n(=3) hence system is **consistent** and has **infinite number of solutions**.

i.e **(x, y, z)=(5-7k, 4k-3, k)**

# THANK YOU

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