PES University, Bangalore

(Established under Karnataka Act No. 16 of 2013)

UE19CS203 – STATISTICS FOR DATA SCIENCE

Unit-2 - Random Variables

QB SOLVED

Chebyshev's inequality

1. **Chebyshev's inequality (Section 2.4) states that for any random variable X with mean μ and variance σ^2, and for any positive number k, $P(|X - \mu| \geq k\sigma) \leq 1/k^2$. Let X ~ N (μ, σ^2). Compute P (|X −μ| ≥ kσ) for the values k = 1, 2, and 3. Are the actual probabilities close to the Chebyshev bound of $1/k^2$, or are they much smaller?**

   [Text Book Exercise – Section 4.5 – Q. No. 26 – Pg. No. 256]

   Solution

   Case: 1

   Consider, k = 1

   About 98% of population is in the interval, $\mu \pm \sigma$

   Using Chebyshev's Inequality $P\left(|X - \mu_X| > k_{\sigma_X}\right) \leq \frac{1}{k^2}$

   $P(|X - \mu_X| > \sigma) = 1 - 0.68$

   $$= 0.32$$
   $$\leq \frac{1}{k^2} = \frac{1}{1^2}$$
   $$= 1$$

   Case: 2

   Consider, k = 2

   About 95% of population is in the interval, $\mu \pm 2\sigma$

   Using Chebyshev's Inequality $P\left(|X - \mu_X| > k_{\sigma_X}\right) \leq \frac{1}{k^2}$

   $P(|X - \mu_X| > 2\sigma) = 1 - 0.95$

   $$= 0.05$$

$\leq \frac{1}{k^2} = \frac{1}{2^2} = \frac{1}{4}$
$= 0.25$

Case: 3

Consider, k = 2

About 99.7% of population is in the interval, $\mu \pm 3\sigma$

Using Chebyshev's Inequality $P\left(|X - \mu_X| > k_{\sigma_X}\right) \leq \frac{1}{k^2}$

$P(|X - \mu_X| > 2\sigma) = 1 - 0.997$

$= 0.003$
$\leq \frac{1}{k^2} = \frac{1}{3^2} = \frac{1}{9}$
$= 0.111$

k	$P\left(|X - \mu_X| > k_{\sigma_X}\right)$	$\frac{1}{k^2}$
1	0.32	1
2	0.05	0.25
3	0.003	0.111

The actual probabilities are much smaller than the Chebyshev bounds.