# VECTOR SPACES

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# CLASS 8 : CONTENT

- Left Null Space

# LEFT NULL SPACE

## Left Null Space

- Null Space of $A^T$ is left null space
- Solutions to $A^T y = 0 \Rightarrow y^T A = 0$ spans the left null space

- $N(A^T) \subseteq R^m$ , LEFT NULL IS A SUBSPACE OF $R^m$
- $Dimension\ of\ N(A^T) = m - r$
- LINEAR COMBINATION OF ROWS WHICH GIVES ZERO ROWS FORMS THE BASIS FOR LEFT NULL SPACE

# LEFT NULL SPACE

Obtain the left null space for the following :

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 6 & 3 \\ 0 & 2 & 5 \end{bmatrix}$$

$$A = \left[\begin{array}{ccc|c} 1 & 2 & 1 & b_1 \\ 2 & 6 & 3 & b_2 \\ 0 & 2 & 5 & b_3 \end{array}\right] \xrightarrow{R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 2 & 1 & b_1 \\ 0 & 2 & 1 & b_2 - 2b_1 \\ 0 & 2 & 5 & b_3 \end{array}\right] \xrightarrow{R_3 - R_2} \left[\begin{array}{ccc|c} 1 & 2 & 1 & b_1 \\ 0 & 2 & 1 & b_2 - 2b_1 \\ 0 & 0 & 4 & b_3 - b_2 + 2b_1 \end{array}\right]$$

$\because$ No zero rows ; Left Null Space {zero vector}

Basis $N(A^T) = \left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$

Dim $N(A^T) = 0$

$N(A^T)$ is origin in $R^3$.

# LEFT NULL SPACE

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 2 & 4 & 8 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & b_1 \\ 1 & 2 & 4 & b_2 \\ 2 & 4 & 8 & b_3 \end{array}\right] \xrightarrow[R_3 - 2R_1]{R_2 - R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & b_1 \\ 0 & 1 & 3 & b_2 - b_1 \\ 0 & 2 & 6 & b_3 - 2b_1 \end{array}\right]$$

$$\xrightarrow{R_3 - 2R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & b_1 \\ 0 & 1 & 3 & b_2 - b_1 \\ 0 & 0 & 0 & b_3 - 2b_1 - 2(b_2 - b_1) \end{array}\right]$$

Combination of Rows which gives zero rows is

$(b_3 - 2b_2 + 0 \cdot b_1)$

# LEFT NULL SPACE

- Solutions to $A^T y = 0$ or $y^T A = 0$ gives $N(A^T)$

Basis of $N(A^T) = \left\{ \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix} \right\}$ Dimension $N(A^T) = 1$

$N(A^T)$ line spanned by $(0, -2, 1)$ in $\mathbb{R}^3$.

$\because \exists$ one zero row, $N(A^T)$ Basis has one vector.

# LEFT NULL SPACE

$$A = \begin{bmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 7 \\ -1 & -3 & 3 & 4 \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} 1 & 3 & 3 & 2 & b_1 \\ 2 & 6 & 9 & 7 & b_2 \\ -1 & -3 & 3 & 4 & b_3 \end{array}\right] \xrightarrow[R_3 + R_1]{R_2 - 2R_1} \left[\begin{array}{cccc|c} 1 & 3 & 3 & 2 & b_1 \\ 0 & 0 & 3 & 3 & b_2 - 2b_1 \\ 0 & 0 & 6 & 6 & b_3 + b_1 \end{array}\right]$$

$$\xrightarrow{R_3 - 2R_2} \left[\begin{array}{cccc|c} 1 & 3 & 3 & 2 & b_1 \\ 0 & 0 & 3 & 3 & b_2 - 2b_1 \\ 0 & 0 & 0 & 0 & b_3 + b_1 - 2(b_2 - 2b_1) \end{array}\right]$$

$b_3 + b_1 - 2b_2 + 4b_1$

$\Rightarrow b_3 - 2b_2 + 5b_1$

Combination of rows which produces zero rows is $b_3 - 2b_2 + 5b_1$

# LEFT NULL SPACE

$$A = \begin{bmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 7 \\ -1 & -3 & 3 & 4 \end{bmatrix}$$

Basis $N(A^T) = \left\{ \begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix} \right\}$

Dimension of $N(A^T) = 1$

$N(A^T)$ is a line spanned by $\begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix}$ in $\mathbb{R}^3$

# THANK YOU

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