PES UNIVERSITY, Bangalore

(Established under Karnataka Act No. 16 of 2013)

Department of Computer Science & Engineering

Automata Formal Languages & Logic

Homework – Context Free Grammar

1. Strings over {*a, b*} with either more *a* s than *b* s or less *a* s than *b* s. Is this language regular or context-free? Explain.

2. Describe the language of the following context-free grammars as concisely as possible:
   a. $S \rightarrow pA \mid qB \mid rC, A \rightarrow Sp, B \rightarrow Sq, C \rightarrow \lambda$
   b. $S \rightarrow AB \mid \lambda, A \rightarrow aB, B \rightarrow Bb \mid b$
   c. $S \rightarrow aS \mid bS \mid SSS \mid \lambda$
   d. $S \rightarrow a \mid bB \mid ccC, B \rightarrow bB \mid \lambda, C \rightarrow cC \mid \lambda, D \rightarrow d$

3. Consider the grammar G = (V, T, P, S), where
   V = {a, b, S, A},
   T = {a, b},
   P= { $S \rightarrow AA$,
   $A \rightarrow AAA$,
   $A \rightarrow a$,
   $A \rightarrow bA$,
   $A \rightarrow Ab$ }.
   (a) Which strings of L(G) can be produced by derivations of four or fewer steps?
   (b) Give at least four distinct derivations for the string "babbab".
   (c) For any m, n, p > 0, describe a derivation in G of the string $b^m ab^n ab^p$.

4. Construct a context free grammar for L={ w#x : w^R is a substring of x for w,x∈{0,1}*}.

5. Give context-free grammars for each of the following languages.
   L = {ww|w ∈{0,1}∗}
   L = {w | Number of 0's is exactly twice the number of 1's}
   L = {wz | w,z ∈{0,1}∗,|w| = |z|,w ≠ z}.

6. For the regular expression a(b+c*) obtain a context free grammar.

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7. Answer each part of the following context free grammar G.

   R□XRX|S
   S□aTb|bTa
   T□XTX|X| λ
   X□a|b

   a. Give three strings in L(G).
   b. Give three strings not in L(G).
   c. Give description in English of L(G).

8. L={xy|x,y ∈ {0,1}* and |x|=|y| but x≠y}.Show that L is a context free language.

9. Give a context free language for the language L={$a^i b^j c^k$|I,j,k>=0,k=2i-j}.
   For example, abc , abb , aabccc ∈L but aaabc,bcc ∉ L

10. Construct a context free grammar for the language L={$1^{2m}0^{3m}$|m>=0}.

11. Given the grammar G:
    E□ E+T|T
    T□T*F|F
    F□(E)|a
    Given the parse tree and derivation for each string.

    a. a
    b. a+a+a
    c. ((a))

12. Given a CFG that generates the language L={$a^i b^j c^k$ | i=j or j=k where i,j,k>=0}.
    Is your grammar ambiguous? why or why not?

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13. Let G=(V,T,P,S) be the following grammar.

    <STMT> □ <ASSIGN> | <IF-THEN> | <IF-THEN-ELSE>
    < IF-THEN> □if condition then <STMT>
    < IF-THEN-ELSE > □ if condition then < STMT > < STMT >
    <ASSIGN> □ a:=1

    G is a grammar for a programming language, but G is ambiguous.
    a. Show that G is ambiguous.
    b. Give a new unambiguous grammar for the same language.

14. Obtain the left most derivation and parse tree for the string "aaabbabbba" using the following grammar. Is it possible to obtain the same string again by applying left most derivation but by selecting different productions?

    S□aB|bA
    A□aS|bAA|a
    B□bS|aBB|b

15. G denotes the context-free grammar defined by the following rules.

    S□ASB/ab/SS
    A□aA/A
    B□bB/A

    (i)Give a leftmost derivation of "aaabb" in G. Draw the associated parse tree.

    (ii)Give the rightmost derivation of "aaabb" in G. Draw the associated parse tree.

    (iii) Show that G is ambiguous. Explain with steps.

    (iv) Construct an unambiguous grammar equivalent to G.

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16. Construct the parse tree for the string "00101"for the following CFG using both leftmost and rightmost derivation. The CFG is

    $S \rightarrow A1B$

    $A \rightarrow 0A \mid \lambda$

    $B \rightarrow 0B \mid 1B \mid \lambda$

17. Given the grammar G,

    $S \rightarrow AB \mid C$

    $A \rightarrow aAb \mid ab$

    $B \rightarrow cBd \mid cd$

    $C \rightarrow aCd \mid aDd$

    $D \rightarrow bDc \mid bc.$

    a. Obtain the left most derivation and parse tree for the string "aabbccdd".
    b. Give description in English of L(G).
    c. Construct an unambiguous grammar equivalent to G.If not possible , mention the reason.

18. Show that the given grammar is ambiguous. Also, find an equivalent unambiguous grammar.

    $S \rightarrow ABA$

    $A \rightarrow aA \mid \lambda$

    $B \rightarrow bB \mid \lambda$

19. Consider the language $L = \{w \in \{a, b\}^* : w \text{ contains equal numbers of a's and b's}\}$

    a. Write a context-free grammar G for L.
    b. Show two derivations (if possible) for the string "aabbab" using G. Show at least one leftmost derivation.
    c. If G is ambiguous (i.e., you found multiple parse trees), remove the ambiguity. (Hint: look out for two recursive occurrences of the same nonterminal in the right side of a rule, e.g, $X \rightarrow XX$).

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20. Consider the following grammar, which generates all the non-empty strings of balanced parentheses

    P□()|(P)|PP

    a. Prove that this grammar is ambiguous
    b. Modify the grammar so to obtain a non-ambiguous grammar generating the same language.