Digital Design and Computer Organization

Carry-lookahead and Prefix adders - 4

Reetinder Sidhu

Department of Computer Science and Engineering

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Course Outline

- Digital Design
  - Combinational logic design
  - Sequential logic design
    - ★ **Carry-lookahead and Prefix adders - 4**
- Computer Organization
  - Architecture (microprocessor instruction set)
  - Microarchitecure (microprocessor operation)

Concepts covered

- Parallel Prefix Incrementer

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Parallel Prefix Applicability

- Parallel prefix technique worked for AND gate
- Can it work for any gate (or Boolean function)?

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- Parallel prefix technique works only for **associative** functions

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Parallel Prefix Applicability

- Parallel prefix technique worked for AND gate
- Can it work for any gate (or Boolean function)?
- Parallel prefix technique works only for **associative** functions
  - Ex: $a \cdot (b \cdot c) = (a \cdot b) \cdot c$

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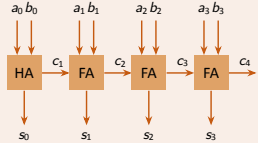
Associative Ripple Carry?

Ripple Carry Adder



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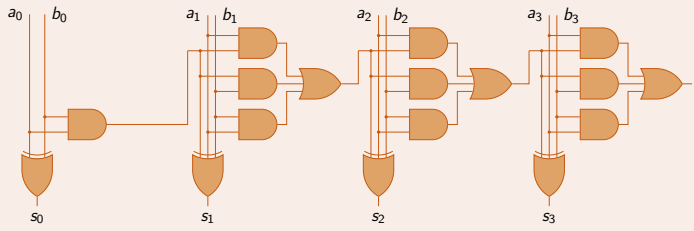

- $c_{i+1} = ab + bc_i + c_i a$

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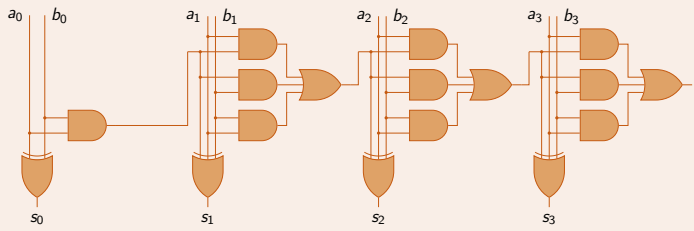

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- Generate and Propagate:

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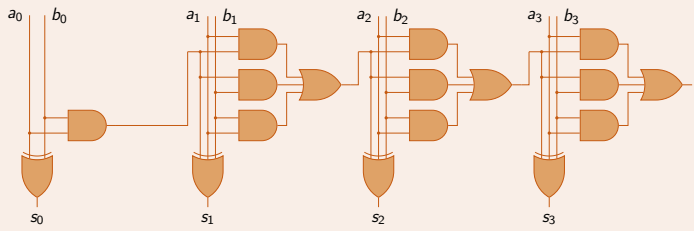

- $c_{i+1} = ab + bc_i + c_i a$
- Generate and Propagate:
  - $\mathbf{g_i}$ carry generated in position i

- $g_i = a_i b_i$

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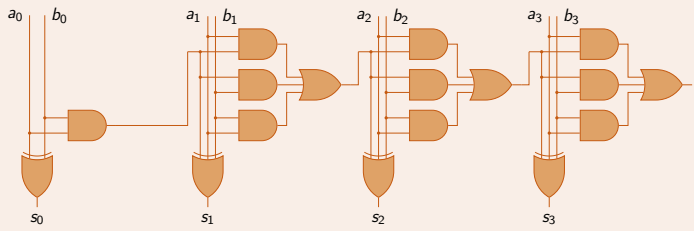

- $c_{i+1} = ab + bc_i + c_i a$
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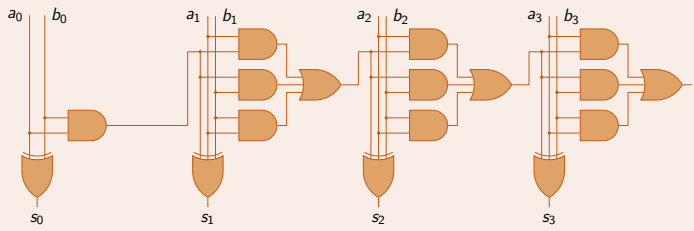

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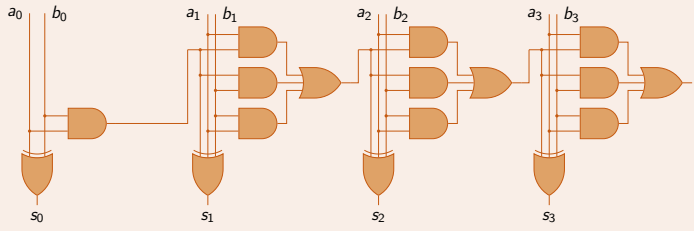

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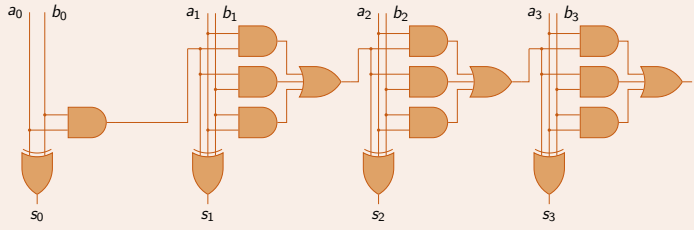

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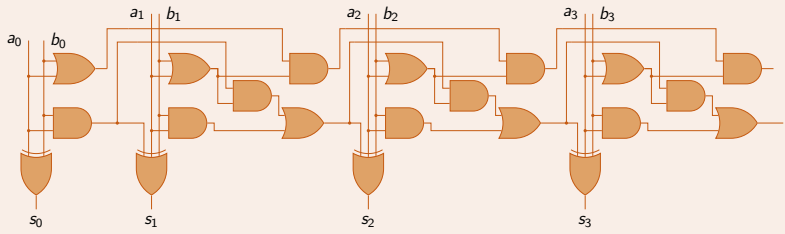

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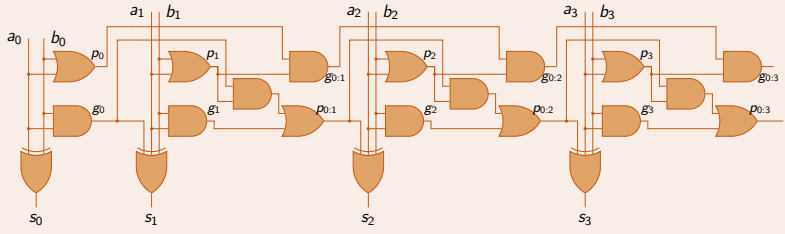

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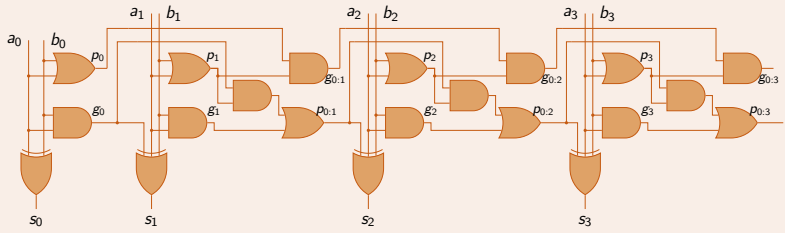

- Define an operation $\otimes$:
  $(p_i, g_i) \otimes (p_j, g_j) = (g_i + p_i g_j, p_i p_j)$

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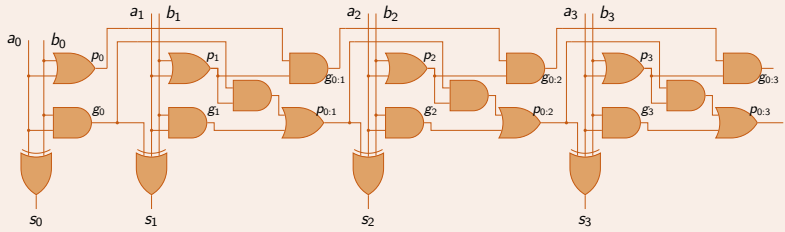

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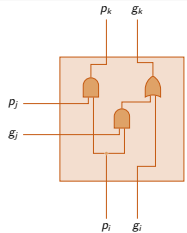

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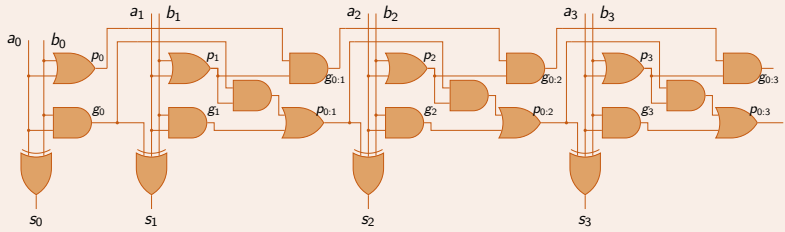

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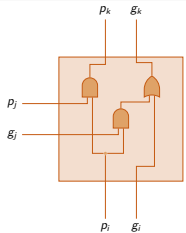

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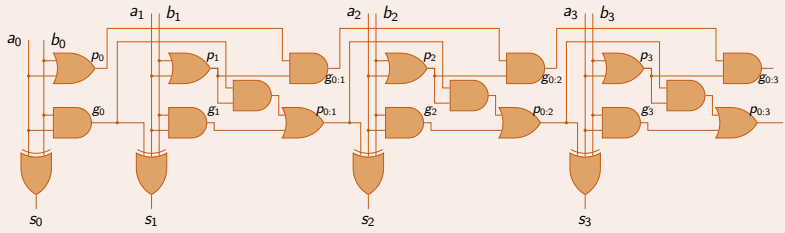
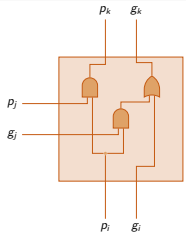


- Define an operation $\otimes$:
  $(p_i, g_i) \otimes (p_j, g_j) = (g_i + p_i g_j, p_i p_j)$
- $\otimes$ is **associative**:
  - $(p_i, g_i) \otimes ((p_j, g_j) \otimes (p_k, g_k)) =$
    $((p_i, g_i) \otimes (p_j, g_j)) \otimes (p_k, g_k)$



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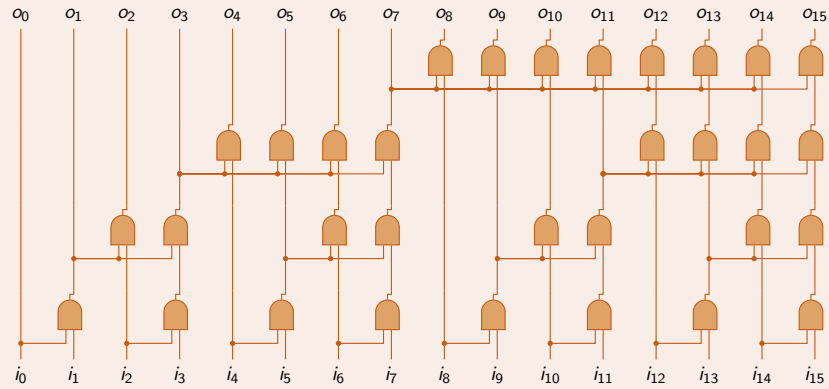
Parallel Prefix for AND

Parallel Prefix Computation for AND



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Parallel Prefix for $\otimes$ Operation

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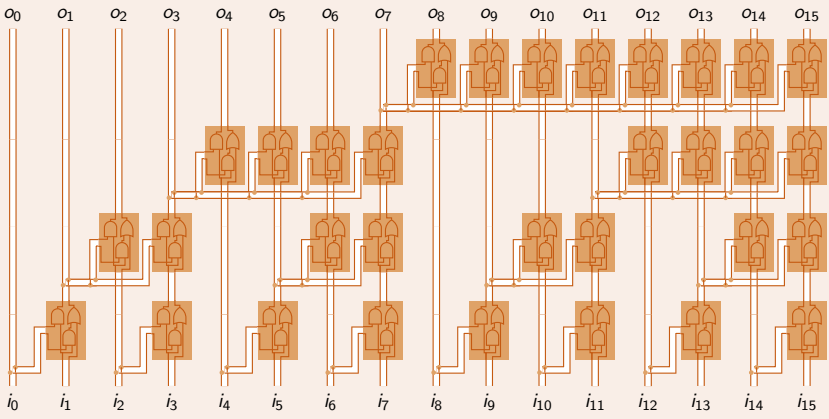
Parallel Prefix Computation for $\otimes$ Operation



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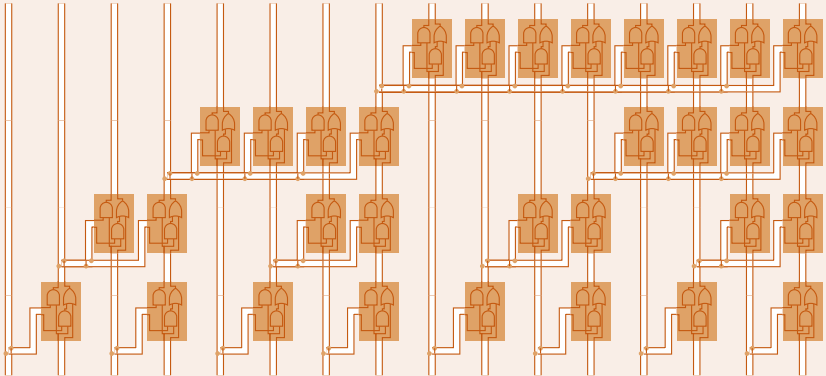

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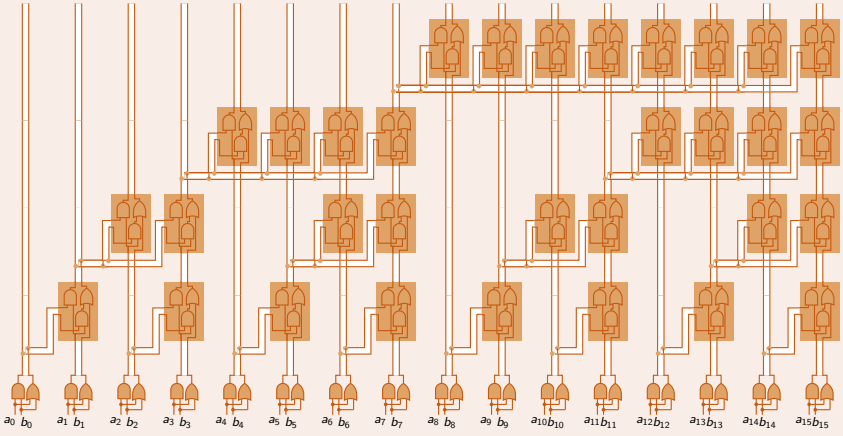

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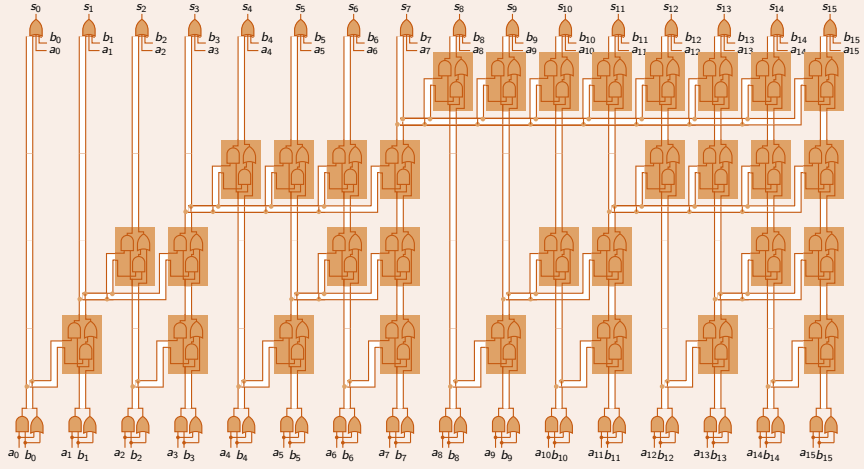

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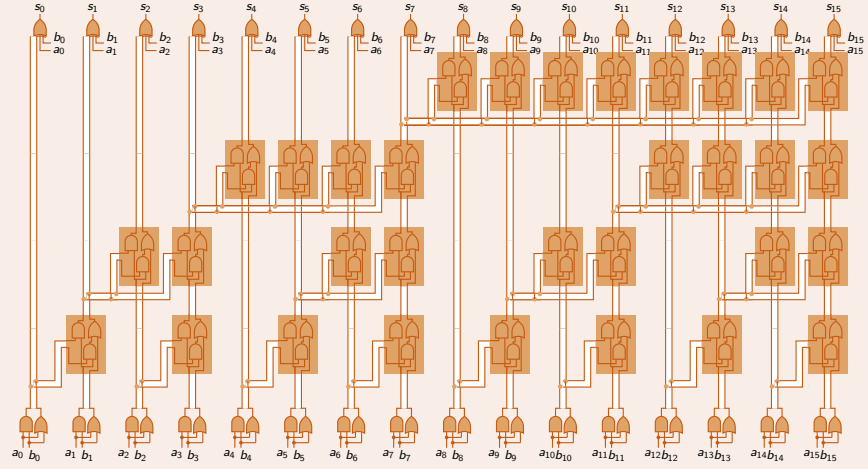

- Critical Path Delay:

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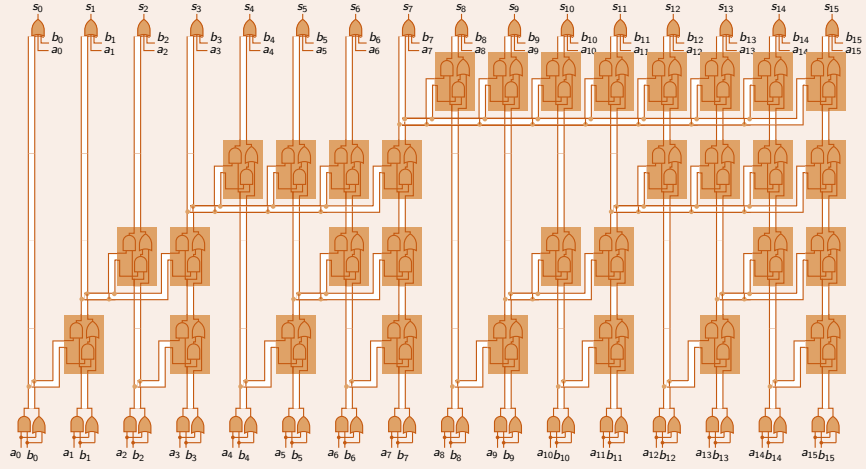

- Critical Path Delay:
  - p_i and g_i computation: t_{pg}

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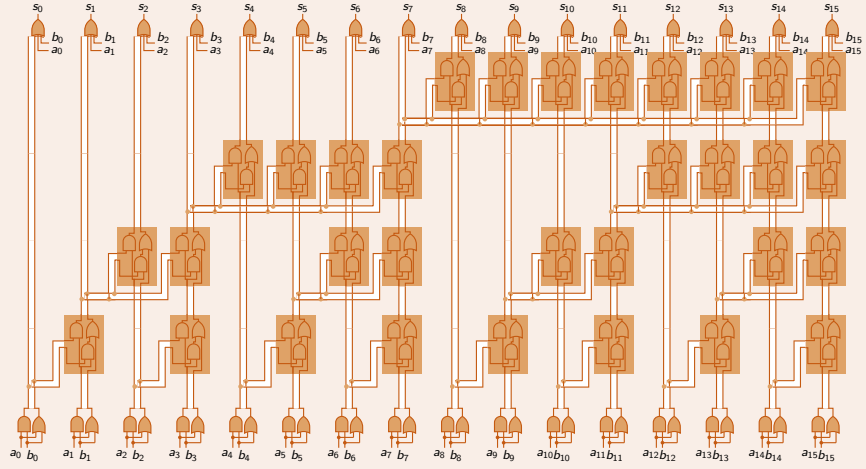

- Critical Path Delay:
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  - $p_{i:j}$ and $g_{i:j}$ computation: t_{pg_prefix}

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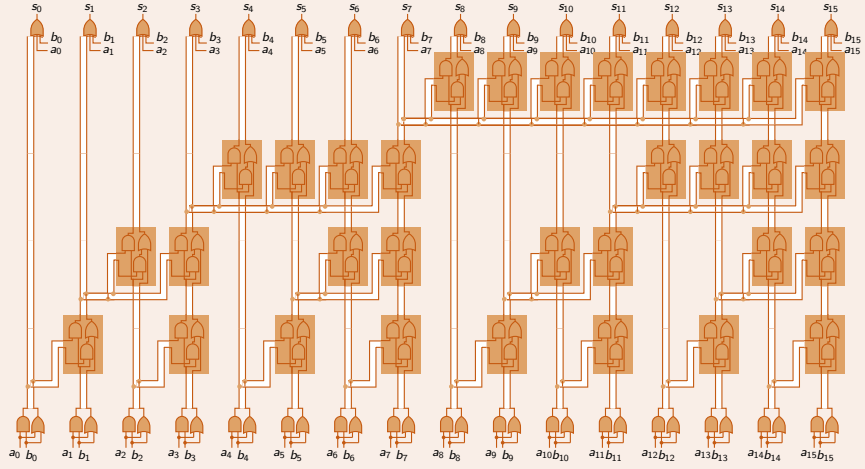

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  - s_i computation: t_{XOR}

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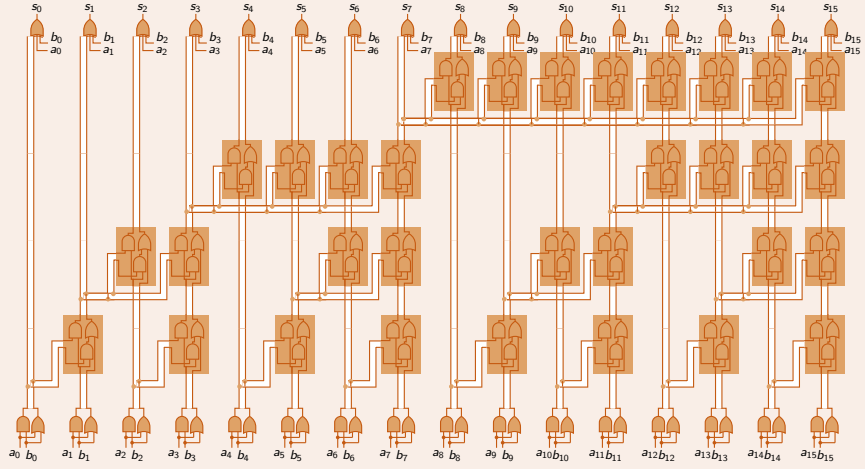

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  - p_i and g_i computation: t_{pg}
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  - s_i computation: t_{XOR}
- $t_{PA} = t_{pg} + (\log_2 N) t_{pg_prefix} + t_{XOR}$