PES University, Bangalore
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UE19CS203 – STATISTICS FOR DATA SCIENCE

Unit-5 - Power of Test and Simple Linear Regression

QUESTION BANK

Power of a Test

Exercises for section 6.13: [Text Book Exercise 6.13 – Pg. No. [485 – 487]]

1. A test has power 0.90 when $\mu = 15$. True or false:

a. The probability of rejecting H_0 when $\mu = 15$ is 0.90.
b. The probability of making a correct decision when $\mu = 15$ is 0.90.
c. The probability of making a correct decision when $\mu = 15$ is 0.10.
d. The probability that H_0 is true when $\mu = 15$ is 0.10.

2. A test has power 0.80 when $\mu = 3.5$. True or false:

a. The probability of rejecting $H_0$when $\mu = 3.5$ is 0.80.
b. The probability of making a type I error when $\mu = 3.5$ is 0.80.
c. The probability of making a type I error when $\mu = 3.5$ is 0.20.
d. The probability of making a type II error when $\mu = 3.5$ is 0.80.
e. The probability of making a type II error when $\mu = 3.5$ is 0.20.
f. The probability that $H_0$is false when $\mu = 3.5$ is 0.80.

3. If the sample size remains the same, and the level α increases, then the power will ---------. Options: increase, decrease.

4. If the level α remains the same, and the sample size increases, then the power will --------. Options: increase, decrease

5. A tire company claims that the lifetimes of its tires average 50,000 miles. The standard deviation of tire lifetimes is known to be 5000 miles. You sample 100 tires and will test the hypothesis that the mean tire lifetime is at least 50,000 miles against the alternative that it is less. Assume, in fact, that the true mean lifetime is 49,500 miles.

   a. State the null and alternate hypotheses. Which hypothesis is true?
   b. It is decided to reject H_0 if the sample mean is less than 49,400. Find the level and power of this test.
   c. If the test is made at the 5% level, what is the power?
   d. At what level should the test be conducted so that the power is 0.80?
   e. You are given the opportunity to sample more tires. How many tires should be sampled in total so that the power is 0.80 if the test is made at the 5% level?

6. A copper smelting process is supposed to reduce the arsenic content of the copper to less than 1000 ppm. Let μ denote the mean arsenic content for copper treated by this process, and assume that the standard deviation of arsenic content is σ = 100 ppm. The sample mean arsenic content X of 75 copper specimens will be computed, and the null hypothesis $H_0 : \geq 1000$ will be tested against the alternate $H_1 : \mu < 1000$.

   a. A decision is made to reject H_0if $X \leq 980$. Find the level of this test.
   b. Find the power of the test in part (a) if the true mean content is 965 ppm.
   c. For what values of X should H_0 be rejected so that the power of the test will be 0.95 when the true mean content is 965?
   d. For what values of X should H_0be rejected so that the level of the test will be 5%?
   e. What is the power of a 5% level test if the true mean content is 965 ppm?
   f. How large a sample is needed so that a 5% level test has power 0.95 when the true mean content is 965 ppm?

7. A power calculation has shown that if $\mu = 8$, the power of a test of $H_1 : \mu \geq 10$ versus $H_1 : \mu < 10$ is 0.90. If instead $\mu = 7$, which one of the following statements is true?
   i. The power of the test will be less than 0.90
   ii. The power of the test will be greater than 0.90.
   iii. We cannot determine the power of the test without knowing the population standard deviation σ.

8. A new process for producing silicon wafers for integrated circuits is supposed to reduce the proportion of defectives to 10%. A sample of 250 wafers will be tested. Let X represent the number of defectives in the sample. Let p represent the population proportion of defectives produced by the new process. A test will be made of $H_0 : p \geq 0.10$ versus $H_1 : p < 0.10$. Assume the true value of p is actually 0.06.

   a. It is decided to reject H_0 if $X \leq 18$. Find the level of this test.
   b. It is decided to reject H_0 if $X \leq 18$. Find the power of this test.
   c. Should you use the same standard deviation for X to compute both the power and the level? Explain.
   d. How many wafers should be sampled so that the power is 0.90 if the test is made at the 5% level?

9. The following MINITAB output presents the results of a power calculation for a test concerning a population proportion p.

```
Power and Sample Size

Test for One Proportion

Testing proportion = 0.5
(versus not = 0.5)
Alpha = 0.05

Alternative   Sample
 Proportion     Size      Power
        0.4      150   0.691332
```

   a. Is the power calculated for a one-tailed or two-tailed test?
   b. What is the null hypothesis for which the power is calculated?

c. For what alternative value of p is the power calculated?
d. If the sample size were 100, would the power be less than 0.7, greater than 0.7, or is it impossible to tell from the output? Explain.
e. If the sample size were 200, would the power be less than 0.6, greater than 0.6, or is it impossible to tell from the output? Explain.
f. For a sample size of 150, is the power against the alternative p = 0.3 less than 0.65, greater than 0.65, or is it impossible to tell from the output? Explain.
g. For a sample size of 150, is the power against the alternative p = 0.45 less than 0.65, greater than 0.65, or is it impossible to tell from the output? Explain.

10. The following MINITAB output presents the results of a power calculation for a test concerning a population mean μ.

```
Power and Sample Size

1-Sample t Test

Testing mean = null (versus > null)
Calculating power for mean = null + difference
Alpha = 0.05 Assumed standard deviation = 1.5

               Sample   Target
Difference       Size    Power   Actual Power
         1         18     0.85       0.857299
```

a. Is the power calculated for a one-tailed or two-tailed test?
b. Assume that the value of μ used for the null hypothesis is $\mu = 3$. For what alternate value of μ is the power calculated?
c. If the sample size were 25, would the power be less than 0.85, greater than 0.85, or is it impossible to tell from the output? Explain.
d. If the difference were 0.5, would the power be less than 0.90, greater than 0.90, or is it impossible to tell from the output? Explain.
e. If the sample size were 17, would the power be less than 0.85, greater than 0.85, or is it impossible to tell from the output? Explain.

11. The following MINITAB output presents the results of a power calculation for a test of the difference between two means $\mu_1 - \mu_2$.

```
Power and Sample Size

2-Sample t Test

Testing mean 1 = mean 2 (versus not =)
Calculating power for mean 1 = mean 2 + difference
Alpha = 0.05 Assumed standard deviation = 5

              Sample    Target
Difference      Size     Power       Actual Power
         3        60       0.9           0.903115
The sample size is for each group.
```

a. Is the power calculated for a one-tailed or two-tailed test?
b. If the sample sizes were 50 in each group, would the power be less than 0.9, greater than 0.9, or is it impossible to tell from the output? Explain.
c. If the difference were 4, would the power be less than 0.9, greater than 0.9, or is it impossible to tell from the output? Explain.