Digital Design and Computer Organization

Logic Minimization, K-Maps - 4

Reetinder Sidhu

Department of Computer Science and Engineering

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Course Outline

- Digital Design
  - Combinational logic design
    - **Logic Minimization, K-Maps - 4**
  - Sequential logic design
- Computer Organization
  - Architecture (microprocessor instruction set)
  - Microarchitecure (microprocessor operation)

Concepts covered

- K-Map for Four Inputs
- K-Map for Two inputs
- Truth Tables with Don't Care Symbols
- K-Maps with Don't Care Symbols

Logic Minimization, K-Maps - 4

K-Map Structure (four inputs)

K-Map Structure (four input Boolean function)

a	b	c	d	y	minterm
0	0	0	0	1	$\overline{a}\,\overline{b}\,\overline{c}\,\overline{d}$
0	0	0	0	0	$\overline{a}\,\overline{b}\,\overline{c}d$
0	0	1	0	1	$\overline{a}\,\overline{b}c\overline{d}$
0	0	1	0	0	$\overline{a}\,\overline{b}cd$
0	1	0	0	0	$\overline{a}b\overline{c}\,\overline{d}$
0	1	0	0	1	$\overline{a}b\overline{c}d$
0	1	1	1	0	$\overline{a}bc\overline{d}$
0	1	1	1	1	$\overline{a}bcd$
1	0	0	1	1	$a\overline{b}\,\overline{c}\,\overline{d}$
1	0	0	1	0	$a\overline{b}\,\overline{c}d$
1	0	1	0	1	$a\overline{b}c\overline{d}$
1	0	1	0	0	$a\overline{b}cd$
1	1	0	1	0	$ab\overline{c}\,\overline{d}$
1	1	0	1	1	$ab\overline{c}d$
1	1	1	1	0	$abc\overline{d}$
1	1	1	1	1	$abcd$

Four Input Truth Table

Logic Minimization, K-Maps - 4

K-Map Structure (four inputs)

K-Map Structure (four input Boolean function)

a	b	c	d	y	minterm
0	0	0	0	1	$\overline{a}\,\overline{b}\,\overline{c}\,\overline{d}$
0	0	0	0	0	$\overline{a}\,\overline{b}\,\overline{c}d$
0	0	1	0	1	$\overline{a}\,\overline{b}c\overline{d}$
0	0	1	0	0	$\overline{a}\,\overline{b}cd$
0	1	0	0	0	$\overline{a}b\overline{c}\,\overline{d}$
0	1	0	0	1	$\overline{a}b\overline{c}d$
0	1	1	1	0	$\overline{a}bc\overline{d}$
0	1	1	1	1	$\overline{a}bcd$
1	0	0	1	1	$a\overline{b}\,\overline{c}\,\overline{d}$
1	0	0	1	0	$a\overline{b}\,\overline{c}d$
1	0	1	0	1	$a\overline{b}c\overline{d}$
1	0	1	0	0	$a\overline{b}cd$
1	1	0	1	0	$ab\overline{c}\,\overline{d}$
1	1	0	1	1	$ab\overline{c}d$
1	1	1	1	0	$abc\overline{d}$
1	1	1	1	1	$abcd$

Four Input Truth Table

ab \ cd	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

- Each square corresponds to a row of the truth table
- Any two adjacent squares differ only in one literal

- Columns as well as rows arranged in binary order 00,01,11,10

Logic Minimization, K-Maps - 4

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0	0	1	0	1	$\bar{a}\bar{b}c\bar{d}$
0	0	1	0	0	$\bar{a}\bar{b}cd$
0	1	0	0	0	$\bar{a}b\bar{c}\bar{d}$
0	1	0	0	1	$\bar{a}b\bar{c}d$
0	1	1	1	0	$\bar{a}bc\bar{d}$
0	1	1	1	1	$\bar{a}bcd$
1	0	0	1	1	$a\bar{b}\bar{c}\bar{d}$
1	0	0	1	0	$a\bar{b}\bar{c}d$
1	0	1	0	1	$a\bar{b}c\bar{d}$
1	0	1	0	0	$a\bar{b}cd$
1	1	0	1	0	$ab\bar{c}\bar{d}$
1	1	0	1	1	$ab\bar{c}d$
1	1	1	1	0	$abc\bar{d}$
1	1	1	1	1	$abcd$

Four Input Truth Table

ab \ cd	00	01	11	10
00	$\bar{a}\bar{b}\bar{c}\bar{d}$ 0	$\bar{a}\bar{b}\bar{c}d$ 1	$\bar{a}\bar{b}cd$ 3	$\bar{a}\bar{b}c\bar{d}$ 2
01	$\bar{a}b\bar{c}\bar{d}$ 4	$\bar{a}b\bar{c}d$ 5	$\bar{a}bcd$ 7	$\bar{a}bc\bar{d}$ 6
11	$ab\bar{c}\bar{d}$ 12	$ab\bar{c}d$ 13	$abcd$ 15	$abc\bar{d}$ 14
10	$a\bar{b}\bar{c}\bar{d}$ 8	$a\bar{b}\bar{c}d$ 9	$a\bar{b}cd$ 11	$a\bar{b}c\bar{d}$ 10

- Each square corresponds to a row of the truth table
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Logic Minimization, K-Maps - 4

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0	0	1	0	0	$\overline{a}\,\overline{b}\,c\,d$
0	1	0	0	0	$\overline{a}\,b\,\overline{c}\,\overline{d}$
0	1	0	0	1	$\overline{a}\,b\,\overline{c}\,d$
0	1	1	1	0	$\overline{a}\,b\,c\,\overline{d}$
0	1	1	1	1	$\overline{a}\,b\,c\,d$
1	0	0	1	1	$a\,\overline{b}\,\overline{c}\,\overline{d}$
1	0	0	1	0	$a\,\overline{b}\,\overline{c}\,d$
1	0	1	0	1	$a\,\overline{b}\,c\,\overline{d}$
1	0	1	0	0	$a\,\overline{b}\,c\,d$
1	1	0	1	0	$a\,b\,\overline{c}\,\overline{d}$
1	1	0	1	1	$a\,b\,\overline{c}\,d$
1	1	1	1	0	$a\,b\,c\,\overline{d}$
1	1	1	1	1	$abcd$

Four Input Truth Table

ab \ cd	00	01	11	10
00	$\overline{a}\,\overline{b}\,\overline{c}\,\overline{d}$ 0	$\overline{a}\,\overline{b}\,\overline{c}\,d$ 1	$\overline{a}\,\overline{b}\,c\,d$ 3	$\overline{a}\,\overline{b}\,c\,\overline{d}$ 2
01	$\overline{a}\,b\,\overline{c}\,\overline{d}$ 4	$\overline{a}\,b\,\overline{c}\,d$ 5	$\overline{a}\,b\,c\,d$ 7	$\overline{a}\,b\,c\,\overline{d}$ 6
11	$a\,b\,\overline{c}\,\overline{d}$ 12	$a\,b\,\overline{c}\,d$ 13	$abcd$ 15	$a\,b\,c\,\overline{d}$ 14
10	$a\,\overline{b}\,\overline{c}\,\overline{d}$ 8	$a\,\overline{b}\,\overline{c}\,d$ 9	$a\,\overline{b}\,c\,d$ 11	$a\,\overline{b}\,c\,\overline{d}$ 10

- Each square corresponds to a row of the truth table
- Any two adjacent squares differ only in one literal
- Columns as well as rows arranged in binary order 00,01,11,10
- Notion of "wrap-around"
  - Far left and right squares are adjacent
  - Top and bottom squares are also adjacent

Logic Minimization, K-Maps - 4

K-Map Structure (four inputs)

K-Map Structure (four input Boolean function)

a	b	c	d	y	minterm
0	0	0	0	1	$\overline{a}\,\overline{b}\,\overline{c}\,\overline{d}$
0	0	0	0	0	$\overline{a}\,\overline{b}\,\overline{c}d$
0	0	1	0	1	$\overline{a}\,\overline{b}c\overline{d}$
0	0	1	0	0	$\overline{a}\,\overline{b}cd$
0	1	0	0	0	$\overline{a}b\overline{c}\,\overline{d}$
0	1	0	0	1	$\overline{a}b\overline{c}d$
0	1	1	1	0	$\overline{a}bc\overline{d}$
0	1	1	1	1	$\overline{a}bcd$
1	0	0	1	1	$a\overline{b}\,\overline{c}\,\overline{d}$
1	0	0	1	0	$a\overline{b}\,\overline{c}d$
1	0	1	0	1	$a\overline{b}c\overline{d}$
1	0	1	0	0	$a\overline{b}cd$
1	1	0	1	0	$ab\overline{c}\,\overline{d}$
1	1	0	1	1	$ab\overline{c}d$
1	1	1	1	0	$abc\overline{d}$
1	1	1	1	1	$abcd$

Four Input Truth Table

ab \ cd	00	01	11	10
00	1 (0)	0 (1)	0 (3)	1 (2)
01	0 (4)	1 (5)	1 (7)	0 (6)
11	0 (12)	1 (13)	1 (15)	0 (14)
10	1 (8)	0 (9)	0 (11)	1 (10)

- Each square corresponds to a row of the truth table
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- Notion of "wrap-around"
  - Far left and right squares are adjacent
  - Top and bottom squares are also adjacent

Logic Minimization, K-Maps - 4

K-Map Example (four inputs)

K-Map Example (four inputs)

a	*b*	*c*	*d*	*y*
0	0	0	0	1
0	0	0	0	0
0	0	1	0	1
0	0	1	0	0
0	1	0	0	0
0	1	0	0	1
0	1	1	1	0
0	1	1	1	1
1	0	0	1	1
1	0	0	1	0
1	0	1	0	1
1	0	1	0	0
1	1	0	1	0
1	1	0	1	1
1	1	1	1	0
1	1	1	1	1

Four Input Truth Table

ab \ cd	00	01	11	10
00	1_{0}	0_{1}	0_{3}	1_{2}
01	0_{4}	1_{5}	1_{7}	0_{6}
11	0_{12}	1_{13}	1_{15}	0_{14}
10	1_{8}	0_{9}	0_{11}	1_{10}

Logic Minimization, K-Maps - 4

K-Map Example (four inputs)

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a	*b*	*c*	*d*	*y*
0	0	0	0	1
0	0	0	0	0
0	0	1	0	1
0	0	1	0	0
0	1	0	0	0
0	1	0	0	1
0	1	1	1	0
0	1	1	1	1
1	0	0	1	1
1	0	0	1	0
1	0	1	0	1
1	0	1	0	0
1	1	0	1	0
1	1	0	1	1
1	1	1	1	0
1	1	1	1	1

Four Input Truth Table

ab \ cd	00	01	11	10
00	1 [0]	0 [1]	0 [3]	1 [2]
01	0 [4]	1 [5]	1 [7]	0 [6]
11	0 [12]	1 [13]	1 [15]	0 [14]
10	1 [8]	0 [9]	0 [11]	1 [10]

K-Map Example (four inputs)

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a	*b*	*c*	*d*	*y*
0	0	0	0	1
0	0	0	0	0
0	0	1	0	1
0	0	1	0	0
0	1	0	0	0
0	1	0	0	1
0	1	1	1	0
0	1	1	1	1
1	0	0	1	1
1	0	0	1	0
1	0	1	0	1
1	0	1	0	0
1	1	0	1	0
1	1	0	1	1
1	1	1	1	0
1	1	1	1	1

Four Input Truth Table

ab \ cd	00	01	11	10
00	1 $_{0}$	0 $_{1}$	0 $_{3}$	1 $_{2}$
01	0 $_{4}$	1 $_{5}$	1 $_{7}$	0 $_{6}$
11	0 $_{12}$	1 $_{13}$	1 $_{15}$	0 $_{14}$
10	1 $_{8}$	0 $_{9}$	0 $_{11}$	1 $_{10}$

- Two prime implicants

Logic Minimization, K-Maps - 4

K-Map Example (four inputs)

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a	*b*	*c*	*d*	*y*
0	0	0	0	1
0	0	0	0	0
0	0	1	0	1
0	0	1	0	0
0	1	0	0	0
0	1	0	0	1
0	1	1	1	0
0	1	1	1	1
1	0	0	1	1
1	0	0	1	0
1	0	1	0	1
1	0	1	0	0
1	1	0	1	0
1	1	0	1	1
1	1	1	1	0
1	1	1	1	1

Four Input Truth Table

ab \ cd	00	01	11	10
00	1 (0)	0 (1)	0 (3)	1 (2)
01	0 (4)	1 (5)	1 (7)	0 (6)
11	0 (12)	1 (13)	1 (15)	0 (14)
10	1 (8)	0 (9)	0 (11)	1 (10)

- Two prime implicants

K-Map Example (four inputs)

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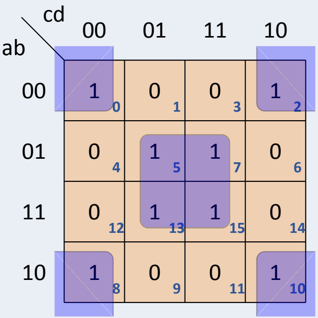
a	*b*	*c*	*d*	*y*
0	0	0	0	1
0	0	0	0	0
0	0	1	0	1
0	0	1	0	0
0	1	0	0	0
0	1	0	0	1
0	1	1	1	0
0	1	1	1	1
1	0	0	1	1
1	0	0	1	0
1	0	1	0	1
1	0	1	0	0
1	1	0	1	0
1	1	0	1	1
1	1	1	1	0
1	1	1	1	1

Four Input Truth Table



- Two prime implicants
- Two essential prime implicants

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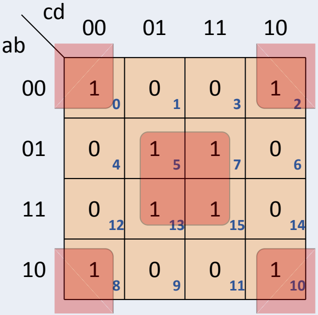
K-Map Example (four inputs)

K-Map Example (four inputs)

a	*b*	*c*	*d*	*y*
0	0	0	0	1
0	0	0	0	0
0	0	1	0	1
0	0	1	0	0
0	1	0	0	0
0	1	0	0	1
0	1	1	1	0
0	1	1	1	1
1	0	0	1	1
1	0	0	1	0
1	0	1	0	1
1	0	1	0	0
1	1	0	1	0
1	1	0	1	1
1	1	1	1	0
1	1	1	1	1

Four Input Truth Table



- Two prime implicants
- Two essential prime implicants

K-Map Example (four inputs)

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K-Map Example (four inputs)

a	*b*	*c*	*d*	*y*
0	0	0	0	1
0	0	0	0	0
0	0	1	0	1
0	0	1	0	0
0	1	0	0	0
0	1	0	0	1
0	1	1	1	0
0	1	1	1	1
1	0	0	1	1
1	0	0	1	0
1	0	1	0	1
1	0	1	0	0
1	1	0	1	0
1	1	0	1	1
1	1	1	1	0
1	1	1	1	1

Four Input Truth Table

ab \ cd	00	01	11	10
00	1 (0)	0 (1)	0 (3)	1 (2)
01	0 (4)	1 (5)	1 (7)	0 (6)
11	0 (12)	1 (13)	1 (15)	0 (14)
10	1 (8)	0 (9)	0 (11)	1 (10)

- Two prime implicants
- Two essential prime implicants
- Two required prime implicants: bd, $\overline{bd}$

Logic Minimization, K-Maps - 4

K-Map Example (four inputs)

K-Map Example (four inputs)

a	b	c	d	y
0	0	0	0	1
0	0	0	0	0
0	0	1	0	1
0	0	1	0	0
0	1	0	0	0
0	1	0	0	1
0	1	1	1	0
0	1	1	1	1
1	0	0	1	1
1	0	0	1	0
1	0	1	0	1
1	0	1	0	0
1	1	0	1	0
1	1	0	1	1
1	1	1	1	0
1	1	1	1	1

Four Input Truth Table

ab \ cd	00	01	11	10
00	1 (0)	0 (1)	0 (3)	1 (2)
01	0 (4)	1 (5)	1 (7)	0 (6)
11	0 (12)	1 (13)	1 (15)	0 (14)
10	1 (8)	0 (9)	0 (11)	1 (10)

- Two prime implicants
- Two essential prime implicants
- Two required prime implicants: bd, $\overline{bd}$
- Minimized Boolean formula: $f(a, b, c, d) = bd + \overline{b}\,\overline{d}$

Logic Minimization, K-Maps - 4

K-Map Example (two inputs)

K-Map Example (two inputs)

a	*b*	*y*
0	0	1
0	1	0
1	0	1
1	1	1

Two Input Truth Table

a \ b	0	1
0	0	1
1	2	3

Logic Minimization, K-Maps - 4

K-Map Example (two inputs)

K-Map Example (two inputs)

a	*b*	*y*
0	0	1
0	1	0
1	0	1
1	1	1

Two Input Truth Table

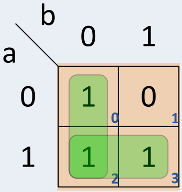
a \ b	0	1
0	1 (0)	0 (1)
1	1 (2)	1 (3)

K-Map Example (two inputs)

K-Map Example (two inputs)

a	*b*	*y*
0	0	1
0	1	0
1	0	1
1	1	1

Two Input Truth Table

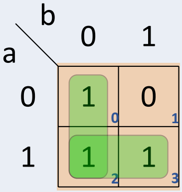


K-Map Example (two inputs)

K-Map Example (two inputs)

a	*b*	*y*
0	0	1
0	1	0
1	0	1
1	1	1

Two Input Truth Table



- Two prime implicants

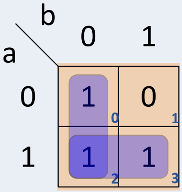
Logic Minimization, K-Maps - 4

K-Map Example (two inputs)

K-Map Example (two inputs)

a	*b*	*y*
0	0	1
0	1	0
1	0	1
1	1	1

Two Input Truth Table



- Two prime implicants

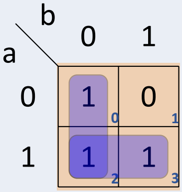
Logic Minimization, K-Maps - 4

K-Map Example (two inputs)

K-Map Example (two inputs)

a	b	y
0	0	1
0	1	0
1	0	1
1	1	1

Two Input Truth Table



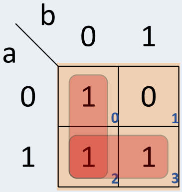
- Two prime implicants
- Two essential prime implicants

K-Map Example (two inputs)

K-Map Example (two inputs)

a	*b*	*y*
0	0	1
0	1	0
1	0	1
1	1	1

Two Input Truth Table



- Two prime implicants
- Two essential prime implicants

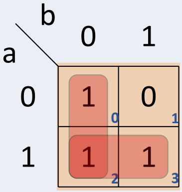
Logic Minimization, K-Maps - 4

K-Map Example (two inputs)

K-Map Example (two inputs)

a	b	y
0	0	1
0	1	0
1	0	1
1	1	1

Two Input Truth Table



- Two prime implicants
- Two essential prime implicants
- Two required prime implicants

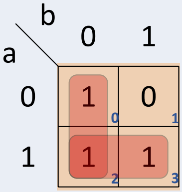
Logic Minimization, K-Maps - 4

K-Map Example (two inputs)

K-Map Example (two inputs)

a	b	y
0	0	1
0	1	0
1	0	1
1	1	1

Two Input Truth Table

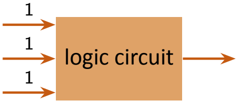


- Two prime implicants
- Two essential prime implicants
- Two required prime implicants
- Minimized Boolean formula: $f(a, b, c, d) = \overline{b} + a$

Logic Minimization, K-Maps - 4

Don’t Cares

- Suppose we are asked to design a three input logic circuit all whose inputs can never be 1 simultaneously

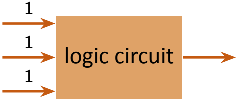


guaranteed can’t happen

Logic Minimization, K-Maps - 4

Don't Cares

- Suppose we are asked to design a three input logic circuit all whose inputs can never be 1 simultaneously



guaranteed can't happen

a	b	c	y
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	?

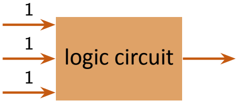
Truth table

a \ bc	00	01	11	10
0	1	1	0	0
1	1	1	?	1

Logic Minimization, K-Maps - 4

Don't Cares

- Suppose we are asked to design a three input logic circuit all whose inputs can never be 1 simultaneously



guaranteed can't happen

a	*b*	*c*	*y*
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

Truth table

a \ bc	00	01	11	10
0	1	1	0	0
1	1	1	0	1

Logic Minimization, K-Maps - 4

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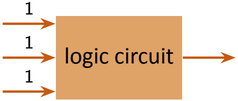

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0	0	1	1
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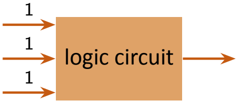

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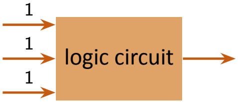

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0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
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- Case 0: $y = \overline{b} + \overline{c}a$

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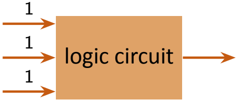

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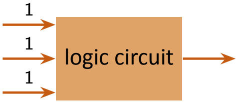

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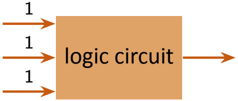

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0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

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Logic Minimization, K-Maps - 4

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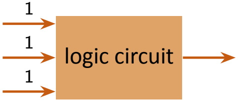

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0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Truth table

a \ bc	00	01	11	10
0	1	1	0	0
1	1	1	1	1

- Case 0: $y = \overline{b} + \overline{c}a$
- Case 1: $y = \overline{b} + a$

LOGIC MINIMIZATION, K-MAPS - 4

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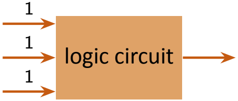

guaranteed can't happen

a	b	c	y
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Truth table

a \ bc	00	01	11	10
0	1	1	0	0
1	1	1	1	1

- Case 0: $y = \overline{b} + \overline{c}a$
- Case 1: $y = \overline{b} + a$
- Either 0 or 1 can result in smaller formula

Logic Minimization, K-Maps - 4

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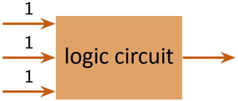


guaranteed can't happen

a	b	c	y
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	X

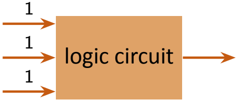
Truth table

a \ bc	00	01	11	10
0	1	1	0	0
1	1	1	X	1

- Case 0: $y = \overline{b} + \overline{c}a$
- Case 1: $y = \overline{b} + a$
- Either 0 or 1 can result in smaller formula
- So initially write X in truth table and K-Map

Don't Cares

- Suppose we are asked to design a three input logic circuit all whose inputs can never be 1 simultaneously



guaranteed can't happen

a	b	c	y
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	X

Truth table

a \ bc	00	01	11	10
0	1	1	0	0
1	1	1	X	1

- Case 0: $y = \overline{b} + \overline{c}a$
- Case 1: $y = \overline{b} + a$
- Either 0 or 1 can result in smaller formula
- So initially write X in truth table and K-Map

Don't Care

Output(s) we **don't care** about are denoted by X, can be treated as either 0 or 1

Logic Minimization, K-Maps - 4

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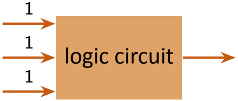

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a	b	c	y
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	X

Truth table

a \ bc	00	01	11	10
0	1	1	0	0
1	1	1	X	1

- Case 0: $y = \overline{b} + \overline{c}a$
- Case 1: $y = \overline{b} + a$
- Either 0 or 1 can result in smaller formula
- So initially write X in truth table and K-Map

Don't Care

Output(s) we **don't care** about are denoted by X, can be treated as either 0 or 1

- Don't Care / X has other use cases, discussed later

Logic Minimization, K-Maps - 4

Think About It

Minimize using K-Maps

- $f(a,b) = \Sigma(0,1,2,3)$
- $f(a,b,c,d) = \Sigma(0,1,5,7,15,14,10)$

Minimize using K-Maps

a	b	c	d	y
0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	1	0	0
0	1	0	0	0
0	1	0	0	0
0	1	1	1	0
0	1	1	1	0
1	0	0	1	1
1	0	0	1	1
1	0	1	0	1
1	0	1	0	X
1	1	0	1	0
1	1	0	1	0
1	1	1	1	0
1	1	1	1	X