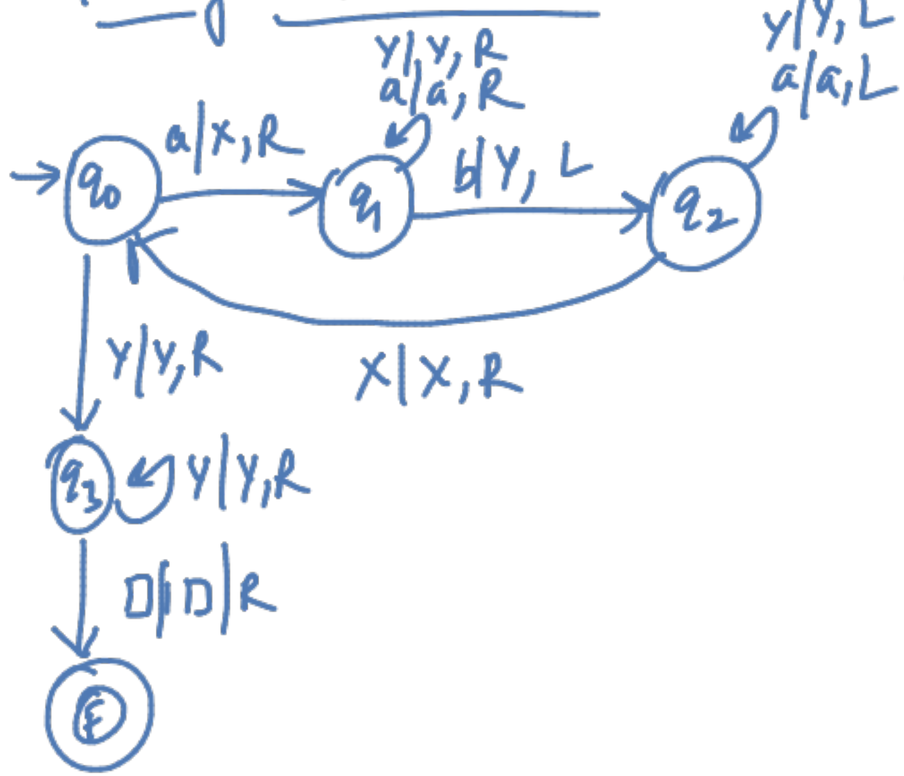
1) $L = \{a^n b^n , n \geq 1\}$

Logic

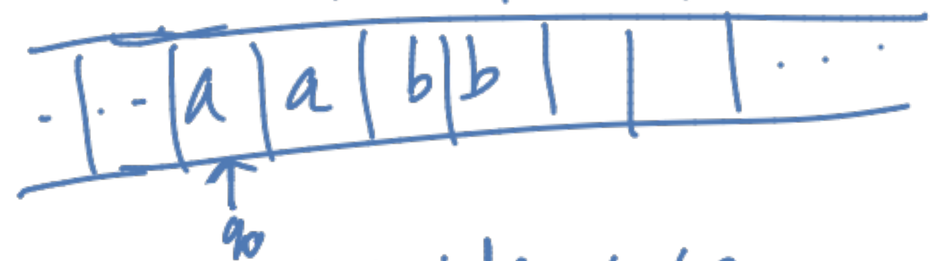
- Match a 'b' for every 'a'
- Mark the matched symbols

Encode
matched 'a' as 'X'
matched 'b' as 'Y'

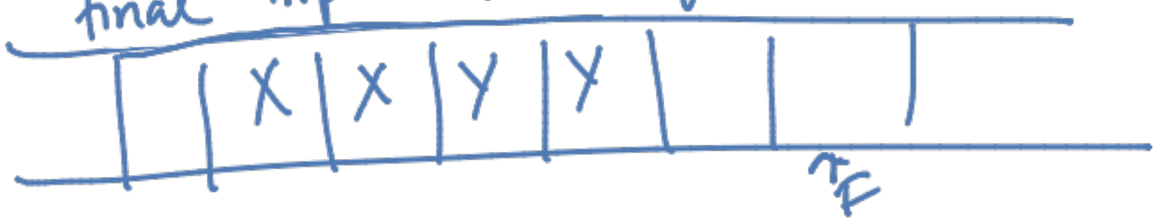
Turing Machine



Initial input tape Conf:-



final input tape Conf:-



Trace aabb

q_0 aabb
$\vdash$ X q_1 abb
$\vdash$ X a q_1 bb
$\vdash$ X q_2 a Y b
$\vdash$ q_2 X a Y b
$\vdash$ X q_0 a Y b
$\vdash$ X X q_1 Y b
$\vdash$ X X Y q_1 b
$\vdash$ X X q_2 Y Y
$\vdash$ X q_2 X Y Y
$\vdash$ X X q_0 Y Y
$\vdash$ X X Y q_3 Y
$\vdash$ X X Y Y q_3 □ □
$\vdash$ X X Y Y □ F □

2) $L = \{a^n b^n c^n , n \geq 1\}$

Logic

Match a 'b' and a 'c' for every a.

Encode all the marked symbols as:-

a - 'X'

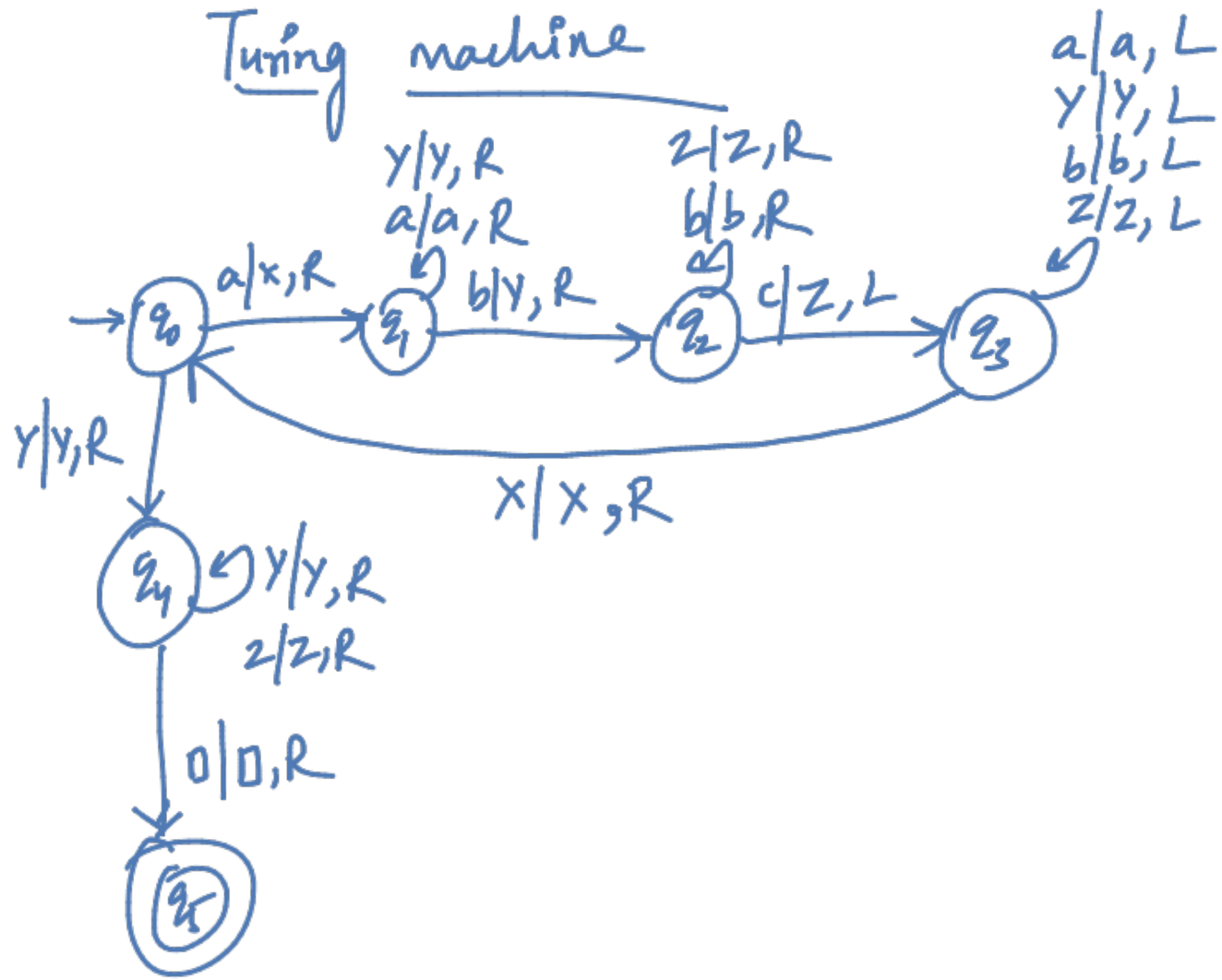
b - 'Y'

c - 'Z'

Input tape

Initial conf: |a|a|b|b|c|c| |

final conf: |X|X|Y|Y|Z|Z| |

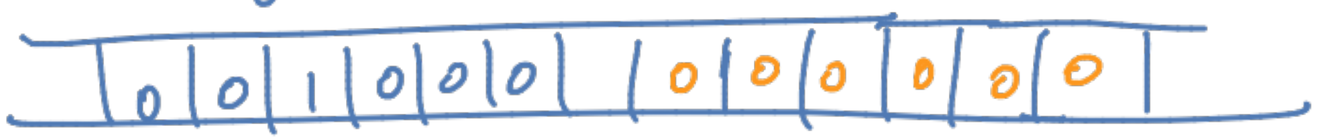


3) Construct a Turing Machine for the language of multiplication

Input tape

Initial conf:

final conf:



Given input:-

$0^m 1 0^n$

produce output:-

0^{m*n}

Logic :-

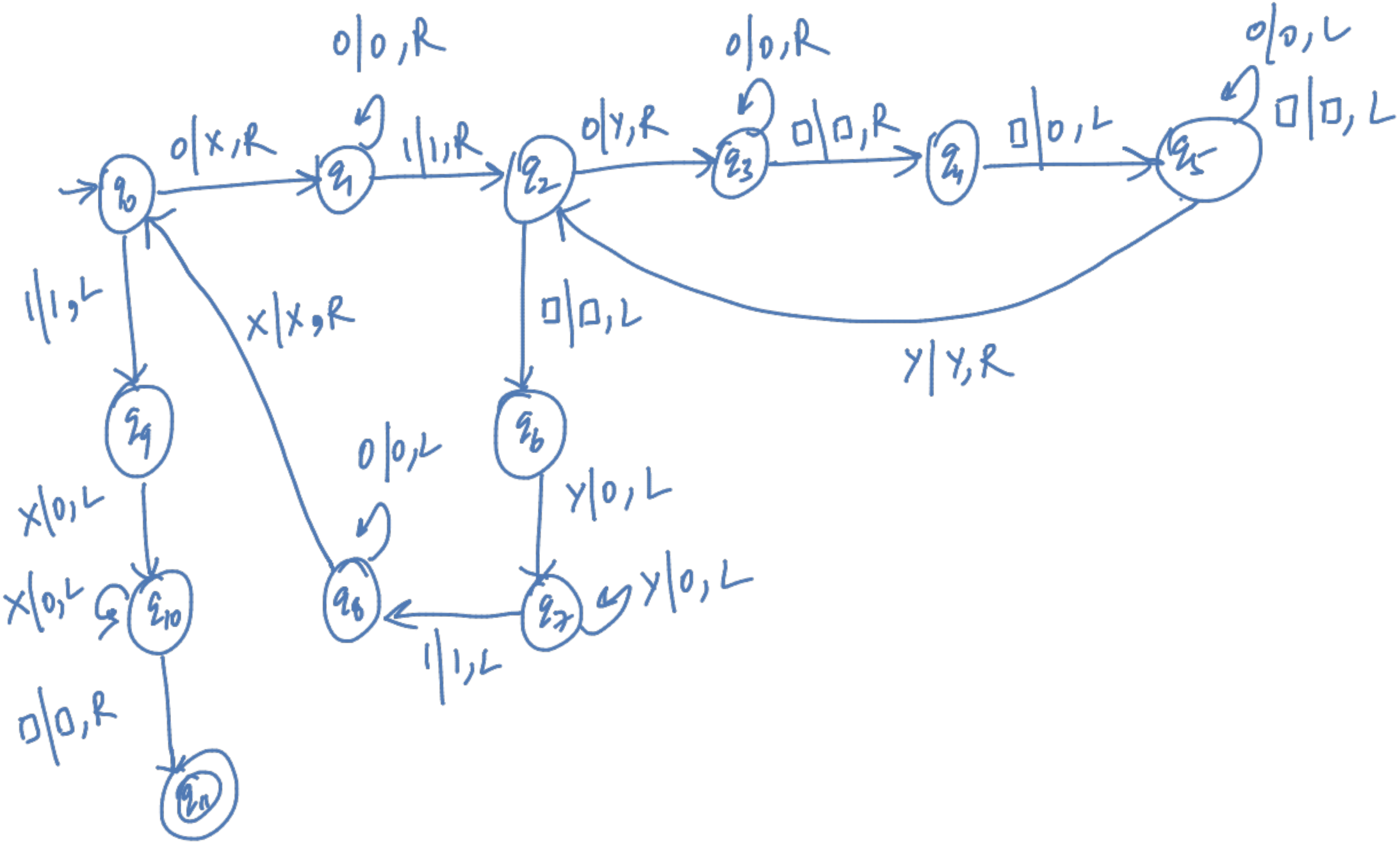
Repeat n (m times)

3) Continued...

Input tape

Time																
0		0	0	1	0	0	0									→ Initial conf.
1		X	0	1	0	0	0									
2		X	0	1	Y	0	0		0							
3		X	0	1	Y	Y	0		0	0						
4		X	0	1	Y	Y	Y		0	0	0					
5		X	0	1	0	0	0		0	0	0					
6		X	X	1	0	0	0		0	0	0					
7		X	X	1	Y	0	0		0	0	0	0				
8		X	X	1	Y	Y	0		0	0	0	0	0			
9		X	X	1	Y	Y	Y		0	0	0	0	0	0		
10		X	X	1	0	0	0		0	0	0	0	0	0		
11		0	0	1	0	0	0		0	0	0	0	0	0		→ final conf.

3) Continued . . .


0|0,R
0|0,R
0|0,L
0|X,R
1|1,R
0|Y,R
□|□,R
□|□,L
□|□,L
1|1,L
X|X,R
□|□,L
Y|Y,R
0|0,L
Y|0,L
X|0,L
X|0,L
Y|0,L
1|1,L
□|0,R