



AUTOMATA FORMAL LANGUAGES AND LOGIC

Dr Pooja Agarwal

Professor

Department of Computer Science & Engineering

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First Order Logic

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Outline

- Models for First Order Logic
- Symbols and Interpretations
- Terms
- Atomic Sentences
- Complex Sentences

Procedural approach vs Declarative approach

Propositional Logic has its limitations that you can not deal properly with general statements of the form

- “All men are mortal”
- ”Socrates is a man”
- SOCRATES IS MORTAL

Example:

P = All men are mortal

Q = Socrates is a Man

R = Socrates is mortal

Then $(P \ \& \ Q) \rightarrow R$ is not valid

- The language of **first-order logic**, whose syntax and semantics is built around objects and relations.
- First-order logic can also express facts about *some* or *all* of the objects in the universe.
- This enables one to represent general laws or rules, such as the statement “Squares neighboring the wumpus are smelly.”

Objects: people, houses, numbers, theories, Ronald McDonald, colors, baseball games, wars, centuries . . .

Tuples: A tuple is a collection of objects arranged in fixed order and is written with angle brackets.

Relations: these can be unary relations or **properties** such as red, round, bogus, prime, multistoried . . ., or more general n-ary relations such as brother of, bigger than, inside, part of, has color, occurred after, owns, comes between, . . .

Functions: Function is generally used at the place where it is uniquely related to that object. **For example: Mother(Sita)**
father of, best friend, third inning of, one more than, beginning of . . .

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SYNTAX AND SEMANTICS OF FIRST-ORDER LOGIC



- **Domain** of a model is the set of objects or **domain elements** it contains.
- The domain is required to be *nonempty*—every possible world must contain at least one object.
- Mathematically speaking, it doesn't matter
 - *what* these objects are—all that matters is *how many* there are in each particular model

The basic syntactic elements of first-order logic are the **symbols** that stand for objects, relations, and functions.

- The symbols, come in three kinds:
 - **Constant symbols**, which stand for objects
e.g. **Richard, John**
 - **Predicate symbols**, which stand for relations
e.g. **Brother(Richard, John)**
 - **Function symbols**, which stand for functions.
e.g. **Father(Richard)**

Connectives

- Negation :
- AND or Conjunction:
- OR or Disjunction
- Implication
- Bidirectional or “IF and Only If”

A “TERM” is a logical expression that refers to an object.

Constant symbols are terms, but every time every object can't be represented by a distinct symbol.

For Example: Let

Term = $f(t_1, t_2, t_3, \dots, t_n)$

Here f is a function symbol, that refers to some function in the model.

$t_1, t_2, t_3, \dots, t_n$ are objects in the domain

An **Atomic sentence** is formed from predicate symbol, optionally followed by parenthesized list of terms.

For Example:

Brother (Richard, Hanes)

P(X, Y)

Married(Father(Richard), Mother(Hanes))”

Complex sentence is formed using logical connectives, with the same syntax and semantics as in propositional calculus.

For Example:

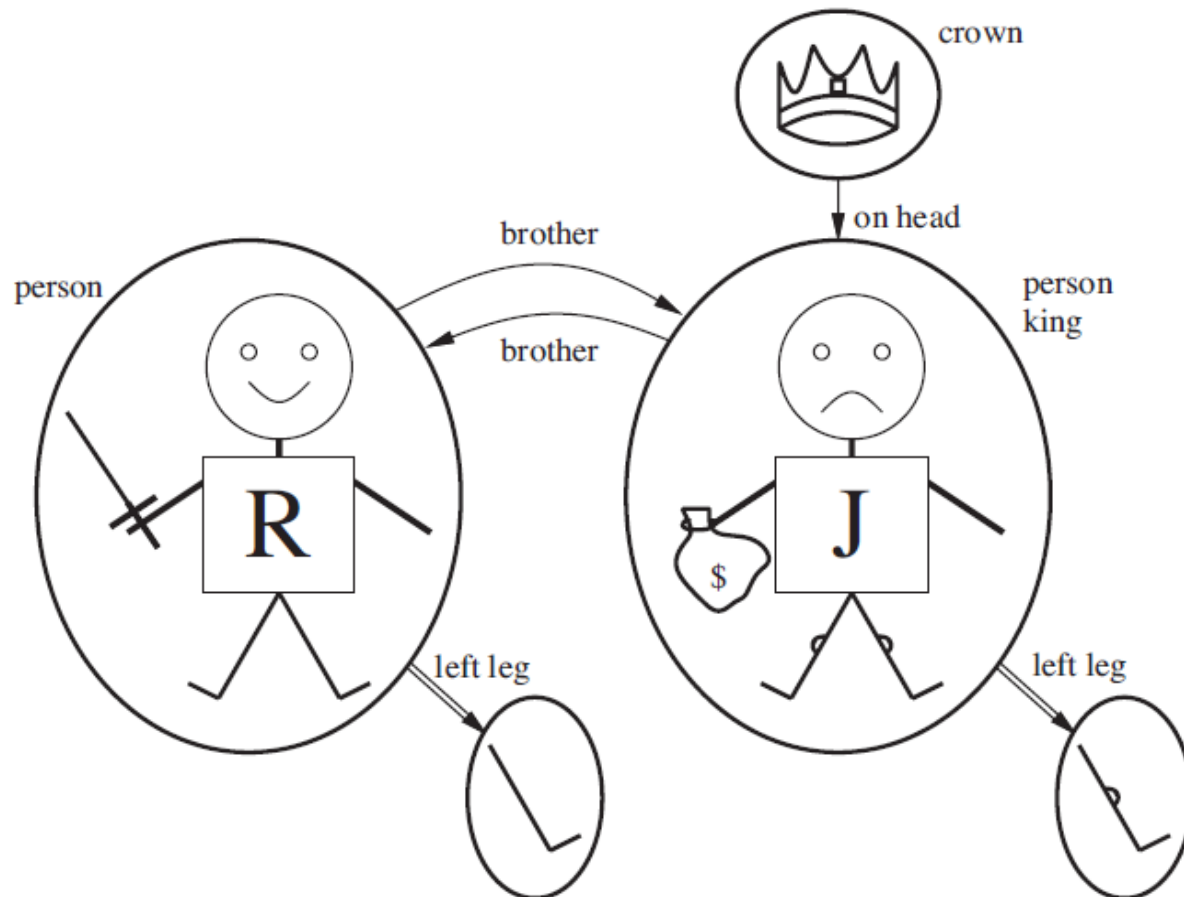
– **Brother (Hand(Richard), John)**

Brother(Richard, John) \wedge Brother (John, Richard)

– **King(Richard) \Rightarrow King (John)**

Uncle(Father(Ravi), Rani)

A model containing five objects



Brotherhood Relation

{ Richard the Lionheart, King John } & { King John, Richard the Lionheart }

Leftleg Function

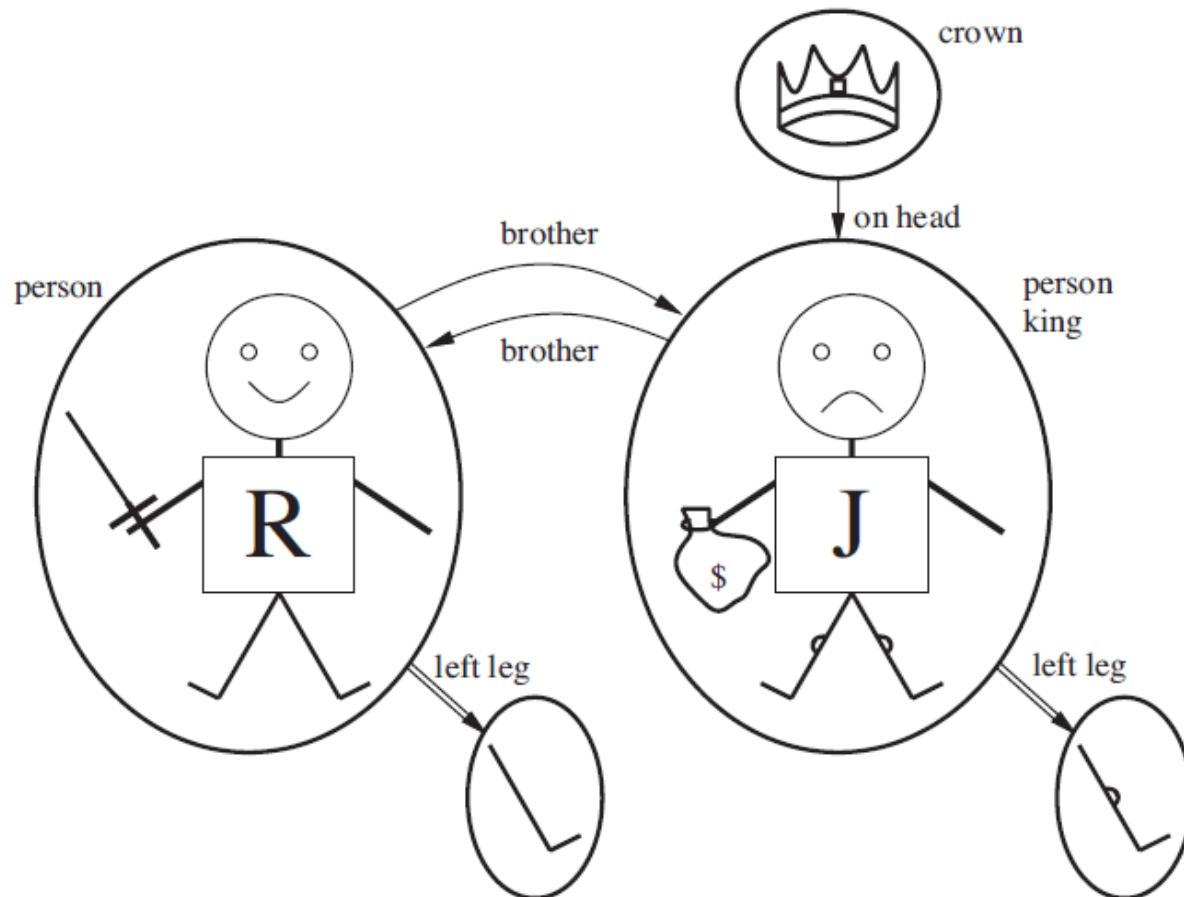
Richard the Lionheart \rightarrow Richard's left leg

King John \rightarrow John's left leg

Onhead

Onhead(crown, John)

A model containing five objects





THANK YOU

Pooja Agarwal

Department of Computer Science & Engineering

poojaagarwal@pes.edu