An **argument** in propositional logic is a sequence of propositions. All but the final proposition in the argument are called premises and the final proposition is called conclusion. An argument is valid if the truth of all its premises implies that the conclusion is true.

The argument form with premises $p_1, p_2, \ldots, p_n$ and conclusion q is valid, when $(p_1 \land p_2 \land \ldots \land p_n) \rightarrow q$ is a tautology.

p_1
p_2
$\ldots$
p_n

$\therefore q$

An **Argument** form with premises $\mathbf{p_1, p_2, \ldots, p_n}$ and conclusion **q** is valid, when $(p_1 \wedge p_2 \wedge \ldots \wedge p_n) \rightarrow q$ is a tautology.

An argument form is represented as:

p_1

p_2

.

.

p_n

$\therefore q$

Eg:

"If you are the topper the class, you will find a job"

"You are the topper the class"

∴ "You will find a job"

It is a valid argument because it's of the form

$p \to q$

p

∴ q

and $\mathbf{((p \to q) \land p) \to q}$ is a tautology.

Rules of Inference:

- We can use a truth table to show that an argument form is valid.

- But when we have more propositional variables involved in the argument, solving by truth table is too laborious (having just **30 variables** will require over **a billion rows** in the truth table).

- That’s when we need well-known argument forms to simplify a complex argument form at hand.

- These well-known valid argument forms are called **Rules of Inference**.

Rule of Inference	Tautology	Name
p $p \rightarrow q$ $\therefore q$	$[p \wedge (p \rightarrow q)] \rightarrow q$	Modus Ponnens
$\neg q$ $p \rightarrow q$ $\therefore \neg p$	$[\neg q \wedge (p \rightarrow q)] \rightarrow \neg p$	Modus Tollens
$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$	Hypothetical Syllogism
$p \vee q$ $\neg p$ $\therefore q$	$[(p \vee q) \wedge \neg p] \rightarrow q$	Disjunctive Syllogism
p $p \vee q$	$(p) \rightarrow (p \vee q)$	Addition
$p \wedge q$ p	$(p \wedge q) \rightarrow p$	Simplification
p q $p \wedge q$	$[(p) \wedge (q)] \rightarrow (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	$[(p \vee q) \wedge (\neg p \vee r)] \rightarrow (q \vee r)$	Resolution

Modus Ponens

means, the way that affirms by affirming

aka Rule of Detachment

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$$

Example:

“If you are the topper the class, then you will find a job”

“You are the topper of the class”

Therefore, “You will find a job”

Example:

“If I lose the election, then will drink poison”

“I lost the election”

Therefore, “I will drink poison”

Modus Tollens

means, the way that denies by denying

aka Denying the Consequent

$$\begin{array}{l} p \rightarrow q \\ \neg q \\ \text{---------} \\ \therefore \neg p \end{array}$$

Example:

“If you are the topper the class, then you will find a job”

“You did not find a job”

Therefore, “You were not not topper of the class”

Example:

“When I earn a lot of money, I will buy a bungalow”

“I have not bought a bungalow yet”

Therefore, “I have not earnt a lot of money”

Argument:
“If you are the topper the class, then you will find a job”
“You were not the topper of the class”
Therefore, “You will not a find a job”

Argument:
“If a Government is doing a good job, the economy of the country goes up”
“The economy of our country is going up”
Therefore, “Our Government is doing a good job”

What The Fallacy!

Argument:
“If you are the topper the class, then you will find a job”
“You were not the topper of the class”
Therefore, “You will not find a job”

$$\begin{array}{l} p \rightarrow q \\ \neg p \\ \text{---------} \\ \therefore \neg q \end{array}$$

Argument:
“If a Government is doing a good job, the economy of the country goes up”
“The economy of our country is going up”
Therefore, “Our Government is doing a good job”

$$\begin{array}{l} p \rightarrow q \\ q \\ \text{---------} \\ \therefore p \end{array}$$

$p \rightarrow q$ p ----- $\therefore q$ Modus Ponens	$p \rightarrow q$ q ----- $\therefore p$ Fallacy of affirming the conclusion
Modus Tollens $p \rightarrow q$ $\neg q$ ----- $\therefore \neg p$	Fallacy of denying the hypothesis $p \rightarrow q$ $\neg p$ ----- $\therefore \neg q$

Hypothetical Syllogism

$$
\begin{array}{l}
p \rightarrow q \\
q \rightarrow r \\
\text{-----------} \\
\therefore p \rightarrow r
\end{array}
$$

"If I do not wake up, I can not go to work"
"If I can not go to work, I will be poor"
Therefore, "If I do not wake up, I will be poor"

"If I study well, I understand the subject well"
"If I understand the subject well, I'll get good grades"
Therefore, "If I study well, I'll get good grades"

"When I earn a lot of money, I will buy a bungalow"
"If I buy a bungalow, I will be very happy"
Therefore, "When I earn a lot of money, I will be very happy"

Disjunctive Syllogism

$$\begin{array}{l} p \lor q \\ \neg p \\ \hline \therefore q \end{array}$$

Example:

“Test is on tomorrow or day after”

“Test cannot happen tomorrow”

Therefore, “Test is on day after tomorrow”

Example:

“Either you or I have to pick our son from the school”

“I cannot pick up our son from the school”

Therefore, “You have to pick up our son from the school”

Simplification

$$\begin{array}{l} p \land q \\ \hline \therefore p \end{array}$$

Example:

"You and I passed in the Examination"

Therefore, "I passed in the Examination"

Conjunction

$$\begin{array}{l} p \\ q \\ \hline \therefore p \land q \end{array}$$

Example:

"You passed in the Examination"

"I passed in the Examination"

Therefore, "You and I passed in the Examination"

Addition

$$\frac{p}{\therefore\ p \lor q}$$

"It is below freezing now"
Therefore, "It is below freezing or raining now"

"1+2=3"
Therefore, "1+2=3 or I am going win a lottery today"

"It rained today"
Imagine a predictive system made a statement yesterday
"It's going to be cloudy or rainy tomorrow"
Therefore, the predictive system made true statement.

Resolution

$p \vee q$	$\mathbf{p \vee q}$
$p \rightarrow r$	$\mathbf{\neg p \vee r}$
--------	**--------**
$\therefore q \vee r$	$\mathbf{\therefore q \vee r}$

"Dharmendra loves Seeta or Geeta"
"If Dharmendra loves Seeta, then he'll marry her"
∴ "Dharmendra loves Geeta or he marries Seeta"

"He has an offer from Google or Cisco"
"If he has an offer from Google, he will accept the offer"
∴ "He has an offer from Cisco or he has accepted Google's offer"

Resolution rule

$p \vee q_1 \vee q_2 \vee .. \vee q_n$

$\neg p \vee r_1 \vee r_2 \vee .. \vee r_m$

$\therefore q_1 \vee q_2 \vee .. \vee q_n \vee r_1 \vee r_2 \vee .. \vee r_m$

Modus Ponens and Modus Tollens (as a special case of the Resolution rule)

$\neg p \vee q$
$p \vee F$

$\therefore q \vee F$

$p \rightarrow q$
p

$\therefore q$

$\neg p \vee q$
$\neg q \vee F$

$\therefore \neg p \vee F$

$p \rightarrow q$
$\neg q$

$\therefore \neg p$

Hypothetical and Disjunctive Syllogisms (as a special case of the Resolution rule)

$\neg p \vee q$

$\neg q \vee r$

$\therefore \neg p \vee r$

$p \to q$

$q \to r$

$\therefore p \to r$

$p \vee q$

$\neg p \vee F$

$\therefore q \vee F$

$p \vee q$

$\neg p$

$\therefore q$

Q: What rule of inference is used in the following argument? Amar will do internship this summer. Therefore, this summer Amar will do internship or he will holiday in Goa.

Soln: Let

W : Amar will do internship this summer

B : Amar will holiday in Goa this summer

…

Q: What rule of inference is used in the following argument? Amar will do internship this summer. Therefore, this summer Amar will do internship or he will holiday in Goa.

Soln: Let

W : Amar will do internship this summer

B : Amar will holiday in Goa this summer

The argument is of the form

W

$\therefore$ **W $\vee$ B**

The rule of inference used is: **Addition**.

W	**B**	**W v B**	**W → W v B is tautology**
T (will do internship)	T (will holiday at Goa)	T	T → T ≡ T
F (he will not do internship)	T	T	F → T ≡ T (it is true because the promise isn't being broken because the premise wasn't true in the first place)
T	F (will not holiday at Goa)	T	T → T ≡ T (promise is kept)
F	F	F	F → F ≡ T

Q: What rule of inference is used in the following argument?

Jerry is a mathematics major and a Computer Science major.

Therefore, Jerry is a mathematics major.

Soln: Let

…

Q: What rule of inference is used in the following argument?
Jerry is a mathematics major and a Computer Science major. Therefore, Jerry is a mathematics major.

Soln: Let

M : Jerry is a mathematics major

C : Jerry is a Computer Science major

The argument is of the form

$M \wedge C$

$\therefore M$

The rule of inference used is: **Simplification**.

Q: What rule of inference is used in the following argument?
If it is rainy, then the pool will be closed. It is rainy. Therefore, the pool is closed.

Soln: Let

…

Q: What rule of inference is used in the following argument?
If it is rainy, then the pool will be closed. It is rainy. Therefore, the pool is closed.

Soln: Let

R : it is rainy

P : pool is closed

The argument is of the form

$\mathbf{R} \rightarrow \mathbf{P}$

$\mathbf{R}$

$\therefore \mathbf{P}$

The rule of inference used is: **Modus Ponens**.

Q: What rule of inference is used in the following argument? If there is a strike today, the university will close. The university is not closed today. Therefore, there was no strike today.

Soln: Let

…

Q: What rule of inference is used in the following argument? If there is a strike today, the university will close. The university is not closed today. Therefore, there was no strike today.

Soln: Let

S : there is a strike

C : university closes

The argument is of the form

$\mathbf{S \rightarrow C}$

$\mathbf{\neg C}$

$\mathbf{\therefore \neg S}$

The rule of inference used is: **Modus Tollens**.

Q: What rule of inference is used in the following argument?
If I go swimming, I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore, if I go swimming, then I will sunburn.

Soln: Let

…

Q: What rule of inference is used in the following argument? If I go swimming, I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore, if I go swimming, then I will sunburn.

Soln: Let S1 : I go swim

S2 : I stay in the sun too long

S3 : I get sunburn

The argument is of the form

S1 → S2

S2 → S3

∴ S1 → S3

The rule of inference used is: **Hypothetical Syllogism**.

Q: What rule of inference is used in the following argument? Alice is a mathematics major. Therefore Alice is a mathematics major or a computer science major.

Soln: Let

M : Alice is a mathematics major

C : Alice is a Computer Science major.

…

Q: What rule of inference is used in the following argument?
Alice is a mathematics major. Therefore Alice is a mathematics major or a computer science major.

Soln: Let

M : Alice is a mathematics major

C : Alice is a Computer Science major.

The argument is of the form

M

$\therefore$ **M** $\lor$ **C**

The rule of inference used is: **Addition**.

Q: What rule of inference is used in the following argument? Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.

Soln: Let

…

Q: What rule of inference is used in the following argument? Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.

Soln: Let

A : Kangaroos live in Australia

M : Kangaroos are marsupials

The argument is of the form

$\mathbf{A \wedge M}$

$\mathbf{\therefore M}$

The rule of inference used is: **Simplification**.

Q: What rule of inference is used in the following argument? It is either hotter than 40 degrees today or the pollution is dangerous. It is lesser than 40 degrees outside today. Therefore, the pollution is dangerous.

Soln: Let

…

Q: What rule of inference is used in the following argument? It is either hotter than 40 degrees today or the pollution is dangerous. It is lesser than 40 degrees outside today. Therefore, the pollution is dangerous.

Soln: Let

H : hotter than 40 degrees

P : pollution is dangerous

The argument is of the form

$\mathbf{H \lor P}$

$\mathbf{\neg H}$

$\mathbf{\therefore P}$

The rule of inference used is: **Disjunctive Syllogism**.

Q: What rule of inference is used in the following argument? Linda is an excellent swimmer. If Linda is an excellent swimmer, she can work as a lifeguard. Therefore, Linda can work as a lifeguard.

Soln: Let

…

Q: What rule of inference is used in the following argument? Linda is an excellent swimmer. If Linda is an excellent swimmer, she can work as a lifeguard. Therefore, Linda can work as a lifeguard.

Soln: Let

S : Linda is an excellent swimmer

L : Linda can work as a lifeguard

The argument is of the form

S

S $\rightarrow$ **L**

$\therefore$ **L**

The rule of inference used is: **Modus Ponens**.

Q: What rule of inference is used in the following argument?
If I work all night on this homework, then I can answer all the exercises. If I can answer all the exercises, I will understand the material. Therefore, if I work all night on this assignment, I will understand the material.

Soln:
…

Q: What rule of inference is used in the following argument? If I work all night on this homework, then I can answer all the exercises. If I can answer all the exercises, I will understand the material. Therefore, if I work all night on this assignment, I will understand the material.

Soln:

The argument is of the form

Work → **Answer**

Answer → **Understand**

∴ **Work** → **Understand**

The rule of inference used is: **Hypothetical Syllogism**.

Rules of Inference for Quantified Statements

Universal Instantiation

$\forall x P(x)$

$\therefore$ P(c) for any element ‘c’ in the domain

Example:

“All students of this class are smart”

Therefore, “Aditya, who is a student of this class, is smart”

Universal Generalization

P(c) for an arbitrary ‘c’

∴ $\forall x P(x)$

Used to conclude $\forall x P(x)$ is true given P(c) is true for an arbitrary element ‘c’ in the domain.

Example:

For an arbitrary integer c, $c^2 \geq 0$. Because c is an arbitrary integer, we can generalize it for the domain of integers as $\forall x\ (x^2 \geq 0)$.

Logical Equivalences involving quantifiers

Q: Show that $\boldsymbol{\forall x(P(x) \land Q(x)) \equiv \forall x P(x) \land \forall x Q(x)}$ where the same domain is used throughout.
(That is, a universal quantifier can be distributed over a conjunction)

Proof: $\forall x(P(x) \land Q(x)) \equiv \forall x P(x) \land \forall x Q(x)$ means
$\forall x(P(x) \land Q(x)) \leftrightarrow \forall x P(x) \land \forall x Q(x)$ is a tautology.

That is, to prove
$\forall x(P(x) \land Q(x)) \rightarrow \forall x P(x) \land \forall x Q(x)$ is a tautology **and**
$\forall x P(x) \land \forall x Q(x) \rightarrow \forall x(P(x) \land Q(x))$ is a tautology.

To prove: $\forall x(P(x) \land Q(x)) \rightarrow \forall x P(x) \land \forall x Q(x)$ is a T.

Suppose, $\forall x(P(x) \land Q(x))$ is true.

$P(a) \land Q(a)$ is true for any 'a' in the domain.

(By Universal Instantiation)

$P(a)$ is true. $Q(a)$ is true. **(By Simplification)**

Because 'a' is an arbitrary element in the domain and $P(a)$ is true, we can conclude that $\forall x P(x)$ is true.

(By Universal Generalization)

Similarly, $\forall x Q(x)$ is true.

That means $\forall x P(x) \land \forall x Q(x)$ is true. **(By Conjunction)**

$\therefore \forall x(P(x) \land Q(x)) \rightarrow \forall x P(x) \land \forall x Q(x)$ is a tautology ..①

To prove: $\forall x P(x) \land \forall x Q(x) \rightarrow \forall x(P(x) \land Q(x))$ is a T.

Suppose, $\forall x P(x) \land \forall x Q(x)$ is true.

i.e., $\forall x P(x)$ is true and $\forall x Q(x)$ is true.**(By Simplification)**

P(a) is true and Q(a) is true for any ‘a’ in the domain.

(By Universal Instantiation)

Because P(a) and Q(a) true for all values in the domain,

for a given ‘a’, $P(a) \land Q(a)$. **(By Conjunction)**

Because ‘a’ is an arbitrary element in the domain,

$\forall x(P(x) \land Q(x))$ is true **(By Universal Generalization)**

$\therefore \forall x P(x) \land \forall x Q(x) \rightarrow \forall x(P(x) \land Q(x))$ is a tautology. ---②

By ① and ②,

$\forall x(P(x) \land Q(x)) \leftrightarrow \forall x P(x) \land \forall x Q(x)$ is a tautology.

$\therefore \forall x(P(x) \land Q(x)) \equiv \forall x P(x) \land \forall x Q(x)$

Existential Instantiation

$\exists x P(x)$

$\therefore$ P(c) for some element ‘c’

Used to conclude P(c) is true for some (not any) c given $\exists x P(x)$ is true.

Existential Generalization

P(c) for some element ‘c’

$\therefore \exists x P(x)$

Used to conclude $\exists x P(x)$ is true when P(c) is true for a particular element c.

Existential Generalization!

P(Salman Khan, Aishwarya Rai):

Salman Khan: Will you marry me?

Aishwarya Rai: No.

Salman Khan lived happily ever after!

∴∃ a boy and a girl **P(boy, girl):**

Boy: Will you marry me?

Girl: No.

Boy lived happily ever after!

Universal Modus Ponens:

Applying Universal Instantiation and then Modus Ponens is frequently used and hence given a name for it.

$\forall x(P(x) \rightarrow Q(x))$

P(a) where a is a particular element in the domain

$\therefore$ Q(a)

Eg: $\forall x\ (P(x) \rightarrow Q(x))$ Premise

$P(a) \rightarrow Q(a)$ Universal instantiation

P(a) Premise

Q(a) Modus ponens

Universal Modus Tollens:

Applying Universal Instantiation and then Modus Tollens is frequently used and hence given a name for it.

$\forall x(P(x) \rightarrow Q(x))$

$\neg Q(a)$ where a is a particular element in the domain

$\therefore \neg P(a)$

Eg: $\forall x\ (P(x) \rightarrow Q(x))$ Premise

$P(a) \rightarrow Q(a)$ Universal instantiation

$\neg Q(a)$ Premise

$\neg P(a)$ Modus tollens

Q: Use rules of inference to check whether the argument is valid.

“Somebody in my friends circle enjoys whale watching”

“Every person who enjoys whale watching cares about ocean pollution”

Therefore, “There is a person in my friends circle who cares about ocean pollution”.

Soln: Let the domain be the set of all people.

$C(x)$: x is in my friends circle

$W(x)$: x enjoys whale watching

$P(x)$: x cares about ocean pollution

1. $\exists x\,(C(x) \land W(x))$ Premise

2. $C(a) \land W(a)$ Existential Instantiation

3. $W(a)$ Simplification

4. $\forall x\,(W(x) \rightarrow P(x))$ Premise

5. $P(a)$ Universal Modus Ponens

6. $C(a)$ Simplification of 2

7. $C(a) \land P(a)$ Conjunction

8. $\exists x\,(C(x) \land P(x))$ Existential Generalization

That is, “There is a person who is in my friends circle and he/she cares about ocean pollution”.

Therefore, the argument is valid.

Eg: Show that the premises
“A student in this class has not read the book” and
“Everyone in this class passed the exam”
imply the conclusion “Someone who passed the exam has not read the book”.

Let the domain of x be all the people.
$C(x)$: x is a student in this class
$B(x)$: x has read the book
$P(x)$: x passed the exam

Premise: $\exists x\,(C(x) \land \neg B(x))$
Premise: $\forall x\,(C(x) \rightarrow P(x))$
Conclusion: $\exists x\,(P(x) \land \neg B(x))$

Premise: $\exists x\ (C(x) \land \neg B(x))$
Premise: $\forall x\ (C(x) \rightarrow P(x))$
Conclusion: $\exists x\ (P(x) \land \neg B(x))$

1. $\exists x\ (C(x) \land \neg B(x))$ Premise
2. $C(a) \land \neg B(a)$ Existential instantiation
3. $C(a)$ Simplification
4. $\forall x\ (C(x) \rightarrow P(x))$ Premise
5. $C(a) \rightarrow P(a)$ Universal instantiation
6. $P(a)$ Modus ponens of 3,5
7. $\neg B(a)$ Simplification of 2
8. $P(a) \land \neg B(a)$ Conjunction of 6,7
9. $\exists x\ (P(x) \land \neg B(x))$ Existential generalization

Premise: $\exists x\,(C(x) \wedge \neg B(x))$
Premise: $\forall x\,(C(x) \rightarrow P(x))$
Conclusion: $\exists x\,(P(x) \wedge \neg B(x))$

1. $\exists x\,(C(x) \wedge \neg B(x))$ Premise
2. $C(a) \wedge \neg B(a)$ Existential instantiation
3. $C(a)$ Simplification
4. $\forall x\,(C(x) \rightarrow P(x))$ Premise
5. $P(a)$ Universal Modus ponens of 3,4
6. $\neg B(a)$ Simplification of 2
7. $P(a) \wedge \neg B(a)$ Conjunction of 6,7
8. $\exists x\,(P(x) \wedge \neg B(x))$ Existential generalization

Q: Use rules of inference to check whether the argument is valid.

"Siri, a student in this batch, is an Algo guru"

"Everyone who is an Algo guru gets a high paying job"

∴ "Someone in this batch gets a high paying job"

Soln: Let the domain be the set of all people.

B(x): x is in this batch

A(x): x is an Algo guru

H(x): x gets a high paying job

1. $B(Siri) \land A(Siri)$ Premise

2. $A(Siri)$ Simplification

3. $\forall x\ (A(x) \rightarrow H(x))$ Premise

4. $H(Siri)$ Universal Modus Ponens

5. $B(Siri)$ Simplification of 1

6. $B(Siri) \land H(Siri)$ Conjunction of 4,5

7. $\exists x\ (B(x) \land H(x))$ Existential Generalization

That is, “Someone in this batch gets a high paying job”

Therefore, the argument is valid.

Q: Determine whether the argument is valid.

1. “If Superman were **able** and **willing** to prevent evil, then he would **prevent** evil”
2. “If Superman were unable to prevent evil, then he would be **p**ower**less**”
3. “If Superman were unwilling to prevent evil, then he would be **evil-m**inded”
4. “Superman does not prevent evil”
5. “If Superman **exists**, then he is neither powerless nor evil-minded”

Therefore, “Superman does not exists”.

Soln: Let

able: Superman is **able** to prevent evil

willing: Superman is **willing** to prevent evil

prev: Superman **prev**ents evil

pless: Superman is **p**ower**less**

evilm: Superman is **evil**-**m**inded

exists: Superman **exists**

Soln: Let

able: Superman is **able** to prevent evil

willing: Superman is **willing** to prevent evil

prev: Superman **prev**ents evil

pless: Superman is **p**ower**less**

evilm: Superman is **evil**-**m**inded

exists: Superman **exists**

1. (able ∧ willing) → prev Premise
2. ¬able → pless Premise
3. ¬willing → evilm Premise
4. ¬prev Premise
5. exists → ¬(pless ∨ evilm) Premise

1. $(\text{able} \land \text{willing}) \rightarrow \text{prev}$ — Premise
2. $\neg\text{able} \rightarrow \text{pless}$ — Premise
3. $\neg\text{willing} \rightarrow \text{evilm}$ — Premise
4. $\neg\text{prev}$ — Premise
5. $\text{exists} \rightarrow \neg(\text{pless} \lor \text{evilm})$ — Premise
6. $\neg(\text{able} \land \text{willing})$ — Modus Tollens of 1,4
7. $\neg\text{able} \lor \neg\text{willing}$ — De Morgan’s law
8. $\text{able} \lor \text{pless}$ — Equivalent to 2
9. $\neg\text{willing} \lor \text{pless}$ — Resolution of 7,8
10. $\text{willing} \lor \text{evilm}$ — Equivalent to 3
11. $\text{pless} \lor \text{evilm}$ — Resolution of 9,10
12. $\neg\text{exists}$ — Modus Tollens of 5,11

- the argument is valid

Q. Determine the truth value of each of these statements if the domain for all variables consists of all integers.

- $\exists x \forall y \, (x < y^2)$
- $\forall x \forall y \, (x + y = 0)$
- $\exists x \exists y \, (x^2 + y^2 = 6)$
- $\forall x \forall y \exists z \, (z = (x + y) / 2)$

Q. Determine the truth value of each of these statements if the domain for all variables consists of all integers.

- $\exists x \forall y \, (x < y^2)$
  - **TRUE**
- $\forall x \forall y \, (x + y = 0)$
  - **FALSE**
- $\exists x \exists y \, (x^2 + y^2 = 6)$
  - **FALSE**
- $\forall x \forall y \exists z \, (z = (x + y) / 2)$
  - **FALSE**

Identify the rule of inference used in each of these arguments.
(i) You can enter the campus only if you wear your id card. You did not bring your id card. Therefore, you cannot enter the campus.
(ii) She is happy whenever it rains. She takes a selfie whenever she is happy. Therefore, she takes a selfie whenever it rains.
(iii) You have watched “3 Idiots” or you are an idiot. You like Rancho or you have not watched “3 Idiots”. Therefore, you are an idiot or you like Rancho.

Answers:
(i) …

Identify the rule of inference used in each of these arguments.

(i) You can enter the campus only if you wear your id card. You did not bring your id card. Therefore, you cannot enter the campus.

(ii) She is happy whenever it rains. She takes a selfie whenever she is happy. Therefore, she takes a selfie whenever it rains.

(iii) You have watched “3 Idiots” or you are an idiot. You like Rancho or you have not watched “3 Idiots”. Therefore, you are an idiot or you like Rancho.

Answers:

(i) Modus Tollens

(ii) Hypothetical Syllogism

(iii) Resolution

$$p \lor q$$
$$\neg p \lor r$$
$$\overline{q \lor r}$$

Rule of Inference is used.

[Note: I don't like Rancho much now. He wants to leave India.]

Identify the rule of inference used in each of these arguments.
(iii) You have watched "3 Idiots" or you are an idiot. You like Rancho or you have not watched "3 Idiots". Therefore, you are an idiot or you like Rancho.
Answer: (iii) Resolution

Aamir Khan: "(Wife) Kiran and I have lived all our lives in India. For the first time, she said, should we move out of India? That's a disastrous and big statement for Kiran to make to me. She fears for her child. She fears about what the atmosphere around us will be. She feels scared to open the news papers everyday. That does indicate that there is a sense of growing disquiet"

Q. What rules inference are used in this argument?
"No man is an island. Manhattan is an island. Therefore, Manhattan is not a man."

Q. Use rules of inference to find out for the hypotheses
"Randy works hard,"
"If Randy works hard, then he is a dull boy," and
"If Randy is a dull boy, then he will not get the job",
which one of the following two statements can be implied

1. "Randy will get the job"
2. "Randy will not get the job"

Q. What rules of inference are used in this famous argument?
"All men are mortal. Socrates is a man. Therefore, Socrates is mortal."

Q. Use rules of inference to show that the hypotheses "If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on," "If the sailing race is held, then the trophy will be awarded," and "The trophy was not awarded" imply the conclusion "It rained".

Q. In the domain of a selected group of men, we know that

- “Every man is married or happy”,
- “Every man is rich or not happy”,
- “If a man is lazy then he is not rich” and
- “There is a man, who is not married”.

Use rules of inference (including quantifiers) to show that the above hypotheses imply the conclusion **“There is a man, who is not lazy”**.

Q. Use rules of inference to show that the premises
“If he gets a bonus, then he will buy a car”,
“If he gets a salary-hike, then he will buy a house”,
“His wife will be sad only if he neither buys a car nor buys a house” and
“He will get a bonus or a salary-hike”
imply the conclusion
“His wife will not be sad”.

Q: There are three people A, B and C. A is looking at B and B is looking at C. A is married and C is not. Does it follow that a married person is looking at an unmarried one? Justify.

Unrelated fact: Anterior Cingulate Cortex is a part of the brain, which monitors step-by-step process of problem solving. So, it’s the control center of logic. It coordinates all the information stored in your memory to think of creative solutions to the puzzles.

Puzzle: Each student of our department always tells the truth or always lies. A student gives only a “Yes” or a “No” response to a question a visitor asks. Suppose a visitor arrives for visiting the chairperson of the department and has come to a fork where one way leads to the chairperson’s room and the other way does not. One of our students is standing at the fork. What one question can the visitor ask the student to determine which way leads to the chairperson’s room?