



PES University, Bangalore

(Established under Karnataka Act No. 16 of 2013)

UE19CS203 – STATISTICS FOR DATA SCIENCE

Unit-3 – Probability Distributions

Question Bank - SOLVED

Small Sample Confidence Intervals for Population Mean

Exercises for Section 5.3

1. Find the level of a two-sided confidence interval that is based on the given value of $t_{n-1, \alpha/2}$ and the given sample size.
 - a) $t = 2.776$, sample size 5.
 - b) $t = 2.718$, sample size 12.
 - c) $t = 5.841$, sample size 4.
 - d) $t = 1.325$, sample size 21.
 - e) $t = 1.746$, sample size 17.

[Text Book Exercise – Section 5.3 – Q. No. 3 – Pg. No. 352]

Solution:

- a) $t = 2.776$, sample size 5.

$$df=4$$

In t table row_heading = 4 and look for corresponding col_heading

where row value = 2.776

$$\Rightarrow \text{col_heading} = 0.025 = \alpha/2 \Rightarrow \alpha = 0.050$$

$$\Rightarrow \text{confidence level} = 95\%$$

- b) $t = 2.718$, sample size 12.

$$df=11$$

In t table row_heading = 11 and look for corresponding col_heading

where row value = 2.718

$$\Rightarrow \text{col_heading} = 0.01 = \alpha/2 \Rightarrow \alpha = 0.02$$

$$\Rightarrow \text{confidence level} = 98\%$$

c) $t = 5.841$, sample size 4.
 $df=3$

In t table row_heading = 3 and look for corresponding col_heading

where row value = 5.841

=> col_heading = 0.005 = $\alpha/2$ => $\alpha = 0.01$

=> confidence level = 99%

d) $t = 1.325$, sample size 21.
 $df=20$

In t table row_heading = 20 and look for corresponding col_heading

where row value = 1.325

=> col_heading = 0.10 = $\alpha/2$ => $\alpha = 0.20$

=> confidence level = 80%

e) $t = 1.746$, sample size 17.
 $df=16$

In t table row_heading = 16 and look for corresponding col_heading

where row value = 1.746

=> col_heading = 0.05 = $\alpha/2$ => $\alpha = 0.10$

=> confidence level = 90%

2. The article “An Automatic Visual System for Marble Tile Classification” (L. Carrino, W. Polini, and S. Turchetta, *Journal of Engineering Manufacture*, 2002:1095–1108) describes a measure for the shade of marble tile in which the amount of light reflected by the tile is measured on a scale of 0–255. A perfectly black tile would reflect no light and measure 0, and a perfectly white tile would measure 255. A sample of nine Mezza Perla tiles were measured, with the following results:

204.999	206.149	202.102	207.048	203.496	206.343
203.496	206.676	205.831			

Is it appropriate to use the Student's t statistic to construct a 95% confidence interval for the mean shade of Mezza Perla tile? If so, construct the confidence interval. If not, explain why not.

[Text Book Exercise – Section 5.3 – Q. No. 7 – Pg. No. 353]

Solution:

Yes it is appropriate, since there are no outliers.

By taking an average of given 9 samples, we will get

$$\bar{X} = 205.1267,$$

$$s = 1.7174, n = 9, t_{9-1} = 8$$

$$0.025 = 2.306.$$

The confidence interval is $205.1267 \pm 2.306(1.7174 / \sqrt{9})$, or (203.81, 206.45).

3. The following MINITAB output presents a confidence interval for a population mean, but some of the numbers got smudged and are now illegible. Fill in the missing numbers for (a), (b), and (c).

One-Sample T: X

Variable	N	Mean	StDev	SE Mean	95% CI
X	20	2.39374	(a)	0.52640	(b,c)

[Text Book Exercise – Section 5.3 – Q. No. 15 – Pg. No. 354]

Solution:

- (a) SE Mean is StDev / \sqrt{N} ,
so $0.52640 = \text{StDev} / \sqrt{20}$, so $\text{StDev} = 2.3541$.
- (b) $\bar{X} = 2.39374$, $s = 2.3541$, $n = 20$, $t_{20-1, .005} = 2.861$.
The lower limit of the 99% confidence interval is $2.39374 - 2.861(2.3541 / \sqrt{20}) = 0.888$.
Alternatively, one may compute $2.39374 - 2.861(0.52640)$.
- (c) $\bar{X} = 2.39374$, $s = 2.3541$, $n = 20$, $t_{20-1, .005} = 2.861$.
The upper limit of the 99% confidence interval is $2.39374 + 2.861(2.3541 / \sqrt{20}) = 3.900$.
Alternatively, one may compute $2.39374 + 2.861(0.52640)$.