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Exhaustive Search: Travelling Salesman Problem

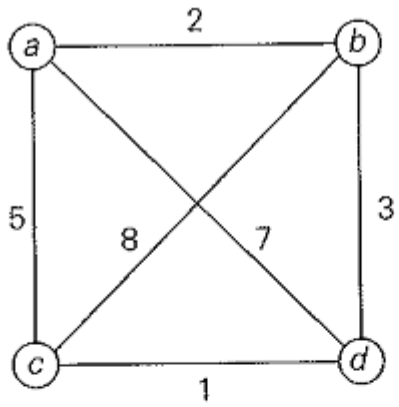
Dr. Shylaja S S

Many important problems require finding an element with a special property in a domain that grows exponentially (or faster) with an instance size. Typically, such problems arise in situations that involve-explicitly or implicitly-combinatorial objects such as permutations, combinations, and subsets of a given set. Many such problems are optimization problems: they ask to find an element that maximizes or minimizes some desired characteristic such as a path's length or an assignment's cost.

Exhaustive search is simply a brute-force approach to combinatorial problems. It suggests generating each and every element of the problem's domain, selecting those of them that satisfy all the constraints, and then finding a desired element (e.g., the one that optimizes some objective function). Note that though the idea of exhaustive search is quite straightforward, its implementation typically requires an algorithm for generating certain combinatorial objects. We assume here that they exist. We illustrate exhaustive search by applying it to three important problems: the traveling salesman problem, the knapsack problem, and the assignment problem. In this section, we shall discuss the traveling salesman problem.

The traveling salesman problem (TSP) has been intriguing researchers for the last 150 years by its seemingly simple formulation, important applications, and interesting connections to other combinatorial problems. In layman's terms, the problem asks to find the shortest tour through a given set of n cities that visits each city exactly once before returning to the city where it started. The problem can be conveniently modeled by a weighted graph, with the graph's vertices representing the cities and the edge weights specifying the distances. Then the problem can be stated as the problem of finding the shortest Hamiltonian circuit of the graph. (A Hamiltonian circuit is defined as a cycle that passes through all the vertices of the graph exactly once. It is named after the Irish mathematician Sir William Rowan Hamilton (1805-1865), who became interested in such cycles as an application of his algebraic discoveries.) It is easy to see that a Hamiltonian circuit can be also defined as a sequence of $n + 1$ adjacent vertices $v_{i0}, v_{i1}, \ldots, V_{in-1}, v_{i0}$, where the first vertex of the sequence is the same as the last one while all the other $n - 1$ vertices are distinct. Further, we can assume, with no loss of generality, that all circuits start and end at one particular vertex (they are cycles after all, are they

not?). Thus, we can get all the tours by generating all the permutations of n - 1 intermediate cities, compute the tour lengths, and find the shortest among them. Fig. 1 presents a small instance of the problem and its solution by this method.



Tour	Length	
$a \rightarrow b \rightarrow c \rightarrow d \rightarrow a$	$l = 2 + 8 + 1 + 7 = 18$	
$a \rightarrow b \rightarrow d \rightarrow c \rightarrow a$	$l = 2 + 3 + 1 + 5 = 11$	optimal
$a \rightarrow c \rightarrow b \rightarrow d \rightarrow a$	$l = 5 + 8 + 3 + 7 = 23$	
$a \rightarrow c \rightarrow d \rightarrow b \rightarrow a$	$l = 5 + 1 + 3 + 2 = 11$	optimal
$a \rightarrow d \rightarrow b \rightarrow c \rightarrow a$	$l = 7 + 3 + 8 + 5 = 23$	
$a \rightarrow d \rightarrow c \rightarrow b \rightarrow a$	$l = 7 + 1 + 8 + 2 = 18$	

Fig. 1: Solution to a small instance of the traveling salesman problem by exhaustive search

An inspection of Fig. 1 reveals three pairs of tours that differ only by their direction. Hence, we could cut the number of vertex permutations by half. We could, for example, choose any two intermediate vertices, say, B and C, and then consider only permutations in which B precedes C. (This trick implicitly defines a tour's direction.)

This improvement cannot brighten the efficiency picture much, however. The

total number of permutations needed will still be (n- 1)!/2, which makes the exhaustive-search approach impractical for all but very small values of n. On the other hand, if you always see your glass as half-full, you can claim that cutting the work by half is nothing to sneeze at, even if you solve a small instance of the problem, especially by hand. Also note that had we not limited our investigation to the circuits starting at the same vertex, the number of permutations would have been even larger, by a factor of n.

The Exhaustive Search solution to the Travelling Salesman problem can be obtained by keeping the origin city constant and generating permutations of all the other n – 1 cities. Thus, the total number of permutations needed will be (n – 1)!