PES University, Bangalore
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UE19CS203 – STATISTICS FOR DATA SCIENCE

Unit-4 - Hypothesis and Inference

QUESTION BANK

Exercises for section 6.1: [Text Book Exercise 6.1– Pg. No. [403 – 405]]

1. In an experiment to measure the lifetimes of parts manufactured from a certain aluminum alloy, 73 parts were loaded cyclically until failure. The mean number of kilocycles to failure was 783, and the standard deviation was 120. Let μ represent the mean number of kilocycles to failure for parts of this type. A test is made of $H_0 : \mu \leq 750$ versus $H_1 : \mu > 750$.
   a. Find the P-value.
   b. Either the mean number of kilocycles to failure is greater than 750, or the sample is in the most extreme $----$% of its distribution.

2. A simple random sample consists of 65 lengths of piano wire that were tested for the amount of extension under a load of 30 N. The average extension for the 65 lines was 1.102mmand the standard deviation was 0.020 mm. Let μ represent the mean extension for all specimens of this type of piano wire.
   a. Find the P-value for testing $H_0: \mu \leq 1.1$ versus $H_1 : \mu > 1.1$.
   b. Either the mean extension for this type of wire is greater than 1.1 mm, or the sample is in the most extreme------------ % of its distribution.

3. The article "Evaluation of Mobile Mapping Systems for Roadway Data Collection" (H. Karimi, A. Khattak, and J. Hummer, *Journal of Computing in Civil Engineering*, 2000:168–173) describes a system for remotely measuring roadway elements such as the width of lanes and the heights of traffic signs. For a sample of 160 such elements, the average error (in percent) in the measurements was 1.90, with a standard deviation of 21.20. Let μ represent the mean error in this type of measurement.

a. Find the P-value for testing $H_0: \mu = 0$ versus $H_1 : \mu \neq 0$.
b. Either the mean error for this type of measurement is nonzero, or the sample is in the most extreme-------------% of its distribution

4. The pH of an acid solution used to etch aluminum varies somewhat from batch to batch. In a sample of 50 batches the mean pH was 2.6, with a standard deviation of 0.3. Let μ represent the mean pH for batches of this solution.

a. Find the P-value for testing $H_0: \mu \leq 2.5$ versus $H_1: \mu > 2.5$.
b. Either the mean pH is greater than 2.5 mm, or the sample is in the most extreme
………% of its distribution.

5. Recently many companies have been experimenting with telecommuting, allowing employees to work at home on their computers. Among other things, telecommuting is supposed to reduce the number of sick days taken. Suppose that at one firm, it is known that over the past few years employees have taken a mean of 5.4 sick days. This year, the firm introduces telecommuting. Management chooses a simple random sample of 80 employees to follow in detail, and, at the end of the year, these employees average 4.5 sick days with a standard deviation of 2.7 days. Let μ represent the mean number of sick days for all employees of the firm.

a. Find the P-value for testing $H_0: \mu \geq 5.4$ versus $H_1: \mu < 5.4$.
b. Do you believe it is plausible that the mean number of sick days is at least 5.4, or are you convinced that it is less than 5.4? Explain your reasoning.

6. A certain type of stainless steel powder is supposed to have a mean particle diameter of $\mu = 15\,\mu$m. A random sample of 87 particles had a mean diameter of 15.2 μm, with a standard deviation of $1.8\ \mu m$. A test is made of $H_0 : \mu = 15$ versus $H_1 : \mu \neq 15$.

a. Find the P −value.
b. Do you believe it is plausible that the mean diameter is $15\ \mu m$, or are you convinced that it differs from $15\ \mu m$? Explain your reasoning.

7. When it is operating properly, a chemical plant has a mean daily production of at least 740 tons. The output is measured on a simple random sample of

60 days. The sample had a mean of 715 tons/day and a standard deviation of 24 tons/day. Let μ represent the mean daily output of the plant. An engineer tests $H_0: \mu \geq 740$ versus $H_1: \mu < 740$.

a. Find the P-value.
b. Do you believe it is plausible that the plant is operating properly or are you convinced that the plant is not operating properly? Explain your reasoning.

8. Lasers can provide highly accurate measurements of small movements. To determine the accuracy of such a laser, it was used to take 100 measurements of a known quantity. The sample mean error was $25\mu m$ with a standard deviation of $60\mu m$. The laser is properly calibrated if the mean error is $\mu = 0$. A test is made of $H_0: \mu = 0$ versus $H_1 : \mu \neq 0$.

a. Find the P-value.
b. Do you believe it is plausible that the laser is properly calibrated, or are you convinced that it is out of calibration? Explain your reasoning.

9. The article "Predicting Profit Performance for Selecting Candidate International Construction Projects" (S. Han, D. Kim, and H. Kim, *Journal of Construction Engineering and Management Science*, 2007: 425–436) presents an analysis of the profit of international construction projects. In a sample of 126 projects, the average profit margin (in percent) was 8.24 with a standard deviation of 16.33. A test is made of $H_0 : \mu \geq 10$ versus $H_1 : \mu < 10$

a. Find the P-value.
b. Do you believe that it is plausible that the mean profit margin is at least 10%, or are you convinced that it is less than 10%? Explain your reasoning.

10. A new concrete mix is being designed to provide adequate compressive strength for concrete blocks. The specification for a particular application calls for the blocks to have a mean compressive strength μ greater than $1350\ kPa$. A sample of 100 blocks is produced and tested. Their mean compressive strength is $1356\ kPa$ and their standard deviation is $70\ kPa$. A test is made of $H_0: \mu \leq 1350$ versus $H_1: \mu > 1350$.

a. Find the P-value.

b. Do you believe it is plausible that the blocks do not meet the specification, or are you convinced that they do? Explain your reasoning.

11. Fill in the blank: If the null hypothesis is $H_0: \mu \leq 5$,then the mean of $\bar{X}$ under the null distribution is..............

i. 0

ii. 5

iii. Any number less than or equal to 5.

iv. We can't tell unless we know H_1

12. Fill in the blank: In a test of $H_0: \mu \geq 10$ versus $H_1: \mu < 10$, the sample mean was $\bar{X} = 8$ and the P-value was 0.04. This means that if $\mu = 10$, and the experiment were repeated 100 times, we would expect to obtain a value of $\bar{X}$ of 8 or less approximately ------------times.

i. 8

ii. 0.8

iii. 4

iv. 0.04

v. 80

13. An engineer takes a large number of independent measurements of the length of a component and obtains $\bar{X} = 5.2$ mm and $\sigma_X = 0.1\ mm$. Use this information to find the P-value for testing $H_0: \mu = 5.0$ versus $H_1 : \mu \neq 5.0$.

14. The following MINITAB output presents the results of a hypothesis test for a population mean μ.

```
One-Sample Z: X

Test of mu = 73.5 vs not = 73.5
The assumed standard deviation = 2.3634

Variable    N     Mean   StDev  SE Mean        95% CI               Z      P
X         145  73.2461  2.3634   0.1963  (72.8614, 73.6308)  -1.29  0.196
```

a. Is this a one-tailed or two-tailed test?

b. What is the null hypothesis?

c. What is the P-value?

d. Use the output and an appropriate table to compute the P-value for the test of $H_0: \mu \geq 73.6$ versus $H_1: \mu < 73.6$.

e. Use the output and an appropriate table to compute a 99% confidence interval for μ.

15. The following MINITAB output presents the results of a hypothesis test for a population mean μ. Some of the numbers are missing. Fill in the numbers for (a) through (c).

```
One-Sample Z: X

Test of mu = 3.5 vs > 3.5
The assumed standard deviation = 2.00819

                                                    95%
                                                  Lower
Variable      N      Mean      StDev    SE Mean    Bound     Z      P
X            87   4.07114    2.00819      (a)    3.71700   (b)    (c)
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