+UE19MA251 LINEAR ALGEBRA AND ITS APPLICATIONS

.Unit 4: Orthogonalization , Eigen Values and Eigen Vectors

Orthogonal Bases, The Gram- Schmidt Orthogonalization, Introduction to Eigenvalues and Eigenvectors, Properties of Eigenvalues and Eigenvectors, Symmetric Matrices, Diagonalization of a Matrix.

Class No.	Portions to be covered
42	Orthogonal Bases- Orthogonal Matrices, Properties
43	Rectangular Matrices with orthonormal columns
44	The Gram- Schmidt Orthogonalization
45	A = QR Factorization
46	**Scilab Class Number 7- The Gram- Schmidt process**
47	Introduction to Eigen values and Eigenvectors
48	Properties of eigenvalues and eigenvectors, Cayley-Hamilton theorem
49	**Scilab Class Number 8&9- Eigen Values and Eigen Vectors**
50	Problems on Properties of Eigen values and Eigen vectors
51	Symmetric Matrices, Diagonalization of a Matrix
52	Problems on Diagonalization of a Matrix
53	Powers and Products of Matrices
54	Supplementary Problems

Classwork problems:

1.	Given the orthonomal basis $S=\left\{\left(\frac{1}{\sqrt{2}},0,\frac{-1}{\sqrt{2}}\right),(0,1,0),\left(\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right)\right\}$ for R^3. Express the vector (1,2,3) as a linear combination of the vectors in S. Answer: (1,2,3)= $-\sqrt{2}v_1+2v_2+2\sqrt{2}v_3$
2.	Let W = { (a, b, b) / a , b are real } and let v = (3, 2, 6). (i) Find an orthonormal basis for W (i) Find the projection of v onto W , say v_1 (ii) Decompose v into a sum of two vectors v_1+ v_2 where v_2 is projection of v onto $W^{\perp}$. Answer : v = (3, 4, 4) + (0, -2 , 2)

3.	Find a third column so that the matrix $Q=\begin{pmatrix} 1/\sqrt{3} & 1/\sqrt{14} & -- \\ 1/\sqrt{3} & 2/\sqrt{14} & -- \\ -1/\sqrt{3} & 3/\sqrt{14} & -- \end{pmatrix}$ is orthogonal. Answer : $\left(5/\sqrt{42}, -4/\sqrt{42}, 1/\sqrt{42}\right)$
4.	If W is a subspace spanned by the orthogonal vectors (2, 5, -1) and (-2 , 1, 1) find the point in W that is closest to (1, 2, 3). Answer : (-2/5, 2, 1/5)
5.	What multiple of $a_1=\begin{pmatrix} 2 \\ 2 \end{pmatrix}$ should be subtracted from $a_2=\begin{pmatrix} 4 \\ 0 \end{pmatrix}$ to make the result orthogonal to a_1? Factor A =QR with orthonormal vectors in Q. Answer : 1
6.	Find an orthonormal set q_1, q_2, q_3 for which q_1 and q_2 span the column space of $A=\begin{pmatrix} 1 & 1 \\ 2 & 3 \\ 2 & 1 \end{pmatrix}$ Which fundamental subspace contains q_3 ? What is the least squares solution of Ax = b if b = (0, 3, 0) ? Answer : $q_3 = \left(-4/\sqrt{18}, 1/\sqrt{18}, 1/\sqrt{18}\right)$. x = (-5/6.3.2)
7.	Use the Gram – Schmidt process to find a set of orthonormal vectors from the independent vectors a_1 = (1, 0, 1) , a_2 = (1, 0, 0) and a_3 = (2, 1, 0). Also find A = QR factorization where A = [a_1 a_2 a_3] Answer : $\left(1/\sqrt{2}, 0, 1/\sqrt{2}\right), \left(1/\sqrt{2}, 0, -1/\sqrt{2}\right) (0,1,0)$
8.	Find orthogonal vectors A , B, C by Gram- Schmidt method from a = (0, 1 1, 1) , b = (1, 1, -1 , 0) and c = (1, 0, 2, -1) Answer : $A=\left(0, 1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}\right), B=\left(1/\sqrt{3}, 1/\sqrt{3}, -1/\sqrt{3}, 0\right)\ C=\left(4/3, 0, 4/3, -4/3\right)$
9.	Find the eigenvalues and the corresponding eigenvectors of $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix} \quad \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix} \quad \begin{pmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ -1 & 1 & -1 \end{pmatrix}$ Answer :Eigen values are (i) -2, 3, 6 (ii) 1, 1, 5 (iii) 1, i, -i
10.	Find the eigenvalues and eigenvectors of $A=\begin{pmatrix} -6 & -1 \\ 2 & -3 \end{pmatrix}$. Verify that the trace equals the sum of eigenvalues and the determinant equals their product. If we shift A to A – 7 I what are the eigenvalues and eigenvectors and how are they related to those of A ? Also find the eigenvalues and eigenvectors of A , A^2 , A^{-1} and A + 4I Answer: -4, -5; (1, -2),(-1,1); -11, -12

11.	Find the characteristic equation and hence find the inverse of $\begin{pmatrix} 1 & 3 & 1 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$ using the Cayley – Hamilton's theorem.
12.	Find the matrix A whose eigen values are 2 and 5, and whose eigen vectors are $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ respectively using $S\Delta S^{-1}$. Answer: $A = \begin{pmatrix} 2 & 3 \\ 0 & 5 \end{pmatrix}$
13.	Factor $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ into $S\Delta S^{-1}$ and hence compute A^{85}. Answer: Eigen values are 1,3 and Eugen vectors are $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
14.	Find all eigenvalues and eigenvectors of A = $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ and write two different diagonalizing matrices S. Answer : eigenvalues are 0, 0, 3 with eigenvectors (-1 , 1, 0) , (-1, 0, 1), (1, 1, 1)
15.	Find the matrices S and S^{-1} to diagonalize $A = \begin{pmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \end{pmatrix}$ What are limits of Δ^k and $S\Delta^k S^{-1}$ as k $\rightarrow \infty$? Answer : eigenvalues of A are 1 and 0.2 with eigenvectors (1 , 1) , (1 , -1). And $\Delta^k \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $S\Delta^k S^{-1} \rightarrow \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$