PES University, Bangalore

(Established under Karnataka Act No. 16 of 2013)

UE19CS203 – STATISTICS FOR DATA SCIENCE

Unit-2 - Random Variables

QB SOLVED

Poisson Distribution

Exercises for Section 4.3

1. **The number of flaws in a given area of aluminum foil follows a Poisson distribution with a mean of 3 per m^2. Let X represent the number of flaws in a 1 m^2 sample of foil.**

   a) **P(X = 5)**
   b) **P(X = 0)**
   c) **P(X < 2)**
   d) **P(X > 1)**
   e) μ_X
   f) σ_X

   [Text Book Exercise – Section 4.3 – Q. No. 2 – Pg. No. 227]

   Solution

   Based on the given data, X ~ Poisson (3)

   a) **P(X = 5)**

   Using the formula for Poisson probability function,

$$P\ (X = x) = \ e^{-\lambda}\frac{\lambda^x}{x!}$$

$$\mathrm{P}\ (\mathrm{X} = 5) = \ \mathrm{e}^{-3}\ \frac{3^5}{5!} = 0.1008$$

   b) **P(X = 0)**

$$\mathrm{P}\ (\mathrm{X} = 0) = \ \mathrm{e}^{-3}\ \frac{3^0}{0!} = 0.0498$$

c) P(X < 2)

$$P(X < 2) = P(X = 0) + P(X = 1)$$

$$= e^{-3}\frac{3^0}{0!} + e^{-3}\frac{3^1}{1!}$$

$$= 0.0497 + 0.1494$$

$$= 0.1991$$

d) P(X > 1)

$$P(X > 1) = 1 - P(X = 0) - P(X = 1)$$

$$= 1 - e^{-3}\frac{3^0}{0!} - e^{-3}\frac{3^1}{1!}$$

$$= 1 - 0.0497 - 0.1494$$

$$= 0.8009$$

e) μ_X

Based on the given data, X ~ Poisson (3)

$$\mu_X = 3$$

f) σ_X

The standard deviation is given by,

$$\sigma_X = \sqrt{\lambda} = \sqrt{3} = 1.732$$

2. A chemist wishes to estimate the concentration of particles in a certain suspension. She withdraws 3mL of the suspension and counts 48 particles. Estimate the concentration in particles per mL and find the uncertainty in the estimate.

[Text Book Exercise – Section 4.3 – Q. No. 10 – Pg. No. 228]

Solution

Let X represents the number of particles observed in 3 ml.

λ denotes the mean particles per ml. It can be estimated as

$$\hat{\lambda} = \frac{X}{t}$$

Here X = 48, t= 3, the concentration in particles per mL,

$$\hat{\lambda} = \frac{X}{t} = \frac{48}{3} = 16$$

To find uncertainty,

$$\sigma_{\hat{\lambda}} = \sqrt{\frac{\lambda}{t}} = \frac{16}{3} = 2.3094$$