PES University, Bangalore

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UE19CS203 – STATISTICS FOR DATA SCIENCE

Unit-4 - Hypothesis and Inference

QUESTION BANK – SOLVED

Large-small tests for Difference between two means :

Exercises for section 6.5: [Text Book Exercise 6.5– Pg. No. [427 – 429]]

1. Fifty specimens of a new computer chip were tested for speed in a certain application, along with 50 specimens of chips with the old design. The average speed, in MHz, for the new chips was 495.6, and the standard deviation was 19.4. The average speed for the old chips was 481.2, and the standard deviation was 14.3.
   a. Can you conclude that the mean speed for the new chips is greater than that of the old chips? State the appropriate null and alternate hypotheses, and then find the *P*-value.
   b. A sample of 60 even older chips had an average speed of 391.2 MHz with a standard deviation of 17.2 MHz. Someone claims that the new chips average more than 100MHzfaster than these very old ones. Do the data provide convincing evidence for this claim? State the appropriate null and alternate hypotheses, and then find the *P*-value.

   [Text Book Exercise – Section 6.5 – Q. No.8 – Pg. No. 428]

   Solution:

   a) Given: $n_X = 50,\ \bar{X} = 495.6,\ \sigma_X = 19.4$
   $n_Y = 50,\ \bar{Y} = 481.2,\ \sigma_Y = 14.3$

   Let us assume α=0.05

Given claim: $\mu_X - \mu_Y > 0$
The null and alternative hypotheses are

$$H_0: \mu_X - \mu_Y \leq 0$$
$$H_1: \mu_X - \mu_Y > 0$$

Determine the value of the test statistic:

$$z = \frac{(\bar{X} - \bar{Y}) - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_X^2}{n_X} + \frac{\sigma_Y^2}{n_Y}}}$$
$$= \frac{(495.6 - 481.2) - (0)}{\sqrt{\frac{19.4^2}{50} + \frac{14.3^2}{50}}} = 4.22$$

The P-value is the probability of obtaining a value more extreme or equal to the standardized test statistic z, assuming that the null hypothesis is true.

Determine the probability using the normal probability table

$$P(Z > 4.22) = 1 - P(Z < 4.22) \approx 1 - 1 = 0.$$

If the P-vale is smaller than the significance level α , then the null hypothesis is rejected, P< 0.05 => Reject H_0
There is sufficient evidence to support the claim that the mean speed for the new chips is greater than that of the old chips.

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b) Given: $n_X = 50,\ \bar{X} = 495.6,\ \sigma_X = 19.4$
$n_Y = 60,\ \bar{Y} = 391.2,\ \sigma_Y = 17.2$

Let us assume α=0.05

Given claim: $\mu_X - \mu_Y > 100$
The null and alternative hypotheses are

$$H_0: \mu_X - \mu_Y \leq 100$$
$$H_1: \mu_X - \mu_Y > 100$$

Determine the value of the test statistic:

$$z = \frac{(\bar{X} - \bar{Y}) - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_X^2}{n_X} + \frac{\sigma_Y^2}{n_Y}}}$$

$$= \frac{(495.6 - 391.2) - (100)}{\sqrt{\frac{19.4^2}{50} + \frac{17.2^2}{60}}} \approx 1.25$$

The P-value is the probability of obtaining a value more extreme or equal to the standardized test statistic z, assuming that the null hypothesis is true.

Determine the probability using the normal probability table

$$P(Z > 1.25) = 1 - P(Z < 1.25) = 1 - 0.8944 = 0.1056$$

If the P-vale is smaller than the significance level α , then the null hypothesis is rejected, P> 0.05 => Fail to Reject H_0

There is not sufficient evidence to support the claim that the mean speed for the new chips is more than 100 Mhz greater than that of the old chips.