



**PES UNIVERSITY, Bangalore**  
(Established under Karnataka Act No. 16 of 2013)  
**Department of Computer Science & Engineering**

**Automata Formal Languages & Logic**

**Question Bank - Unit 2**

**Questions from the Prescribed Textbook**

Topic	Exercise No.	Question No's
Pumping lemma	4.3	Q1-Q27

**Extra Questions**

1. Prove that the following languages are not regular. You may use the pumping lemma and the closure of the class of regular languages under union, intersection and complement.
  - a)  $\{0^n 1^m 0^n \mid m, n \geq 0\}$
  - b)  $\{wtw \mid w, t \in \{0,1\}^*\}$
2. Let  $\Sigma = \{0,1,+,=\}$  and  $ADD = \{x=y+z \mid x,y,z \text{ are binary integers and } x \text{ is the sum of } y \text{ and } z\}$ . Show that ADD is not regular.
3. The pumping lemma says that every regular language has a pumping length  $p$ , such that every string in the language can be pumped if it has length  $p$  or more. If  $p$  is a pumping length for the language  $A$ , so is any length  $p' \geq p$ . The minimum pumping length for  $A$  is the smallest  $p$  that is a pumping length for  $A$ . For example, if  $A = 01^*$ , the minimum pumping length is 2. The reason is that the string  $s=0$  is in  $A$  and has length 1 yet  $s$  cannot be pumped, but any string in  $A$  of length 2 or more contains a 1 and hence can be pumped by dividing it so that  $x = 0$ ,  $y=1$  and  $z$  is the rest. For each of the following languages, give the minimum pumping length and justify your answer.
  - a.  $1^*01^*01^*$
  - b.  $(01)^*$
  - c.  $1011$
  - d.  $001 \cup 0^*1^*$
4. To show that Language contains equal numbers of  $a$  and  $b$ , if we select the string  $w$  as follows, what could the adversary do in each case?



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- a.  $w = (ab)^m$
  - b.  $w = a^{m/2}b^{m/2}$
5. Show that the following languages over  $\{a, b\}$  are not regular:
- a.  $\{a^n b a^n : n \geq 1\}$ .
  - b.  $\{a^n : n \text{ is a perfect cube}\}$ .
  - c.  $\{w : \# a(w) < \# b(w)\}$ .
  - d.  $\{wb^n : w \in \{a, b\}^*, \text{Length}(w) = n\}$ .
  - e.  $\{(ab)^m b^n : m > n \geq 0\}$ .
  - f.  $\{a^m b^n : m \leq n, m, n \in \mathbb{N}\}$ .
  - g.  $\{a^m b^n a^k : k \geq m + n\}$ .
  - h.  $\{a^n b^{2n} : n \geq 1\}$ .
  - i.  $\{a^m b^n : 0 < m < n\}$ .
  - j.  $\{a^m b^n a^k : k \neq m + n\}$ .
  - k.  $\{a^m b^n a^k : m = n \text{ or } n \neq k\}$ .
  - l.  $\{a^m b^n : m \leq n\}$ .
  - m.  $\{w \in \{a, b\}^* : \# a(w) \neq \# b(w)\}$ .
  - n.  $\{ww : w \in \{a, b\}^*\}$ .
  - o.  $\{w^R w : w \in \{a, b\}^*\}$ .
  - p.  $\{wwww^R : w \in \{a, b\}^*\}$ .



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- q.  $\{a^m b^n : m > n\} \cup \{a^m b^n : m + 1 \neq n\}$ .
- r.  $\{uww^R v : u, v, w \in \{a, b\}^+\}$ .
- s.  $\{ww^n v : v, w \in \{a, b\}^+, n \geq 1\}$ .
- t.  $\{ww : w \in \{a, b\}^*\}$ , where  $w$  is the string obtained from  $w$  by changing  $a$  to  $b$ , and  $b$  to  $a$  simultaneously.
6. Are the following languages over  $\{a\}$  regular?
- $\{a^n : n = m^2 \text{ for some } m \in \mathbb{N}\}$ .
  - $\{a^n : n = 2m \text{ for some } m \in \mathbb{N}\}$ .
  - $\{a^{p-1} : p \text{ is a prime number}\}$ .
  - $\{a^{mk} : m \text{ and } k \text{ are prime numbers}\}$ .
  - $\{a^n : n \text{ is either a prime or a product of two or more primes}\}$ .
7. Show that the set of balanced parentheses is not a regular language.
8. Let  $L$  be a regular language. Consider the two languages:
- $L_1 = \{w : w^n \in L \text{ for some } n \in \mathbb{N}\}$  and  $L_2 = \{w^n : w \in L \text{ for some } n \in \mathbb{N}\}$ .
- Which one of  $L_1, L_2$  is regular and which is not? Justify.
9. Are the following languages regular?
- $\{uww^R v : u, v, w \in \{a, b\}^+ (u) \geq (v)\}$ .
  - $\{a^m b^n b a^{m+n} : m, n \geq 1\}$ .



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