PES University, Bangalore

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UE19CS203 – STATISTICS FOR DATA SCIENCE

Unit - 3 - Probability Distributions

QUESTION BANK

Central Limit Theorem

Exercises for Section 4.11

[Text Book Exercise – Section 4.11 – Q. No. [1 – 20] – Pg. No. [300 - 302]]

1. Bottles filled by a certain machine are supposed to contain 12 oz of liquid. In fact the fill volume is random with mean 12.01 oz and standard deviation 0.2 oz.
   a) What is the probability that the mean volume of a random sample of 144 bottles is less than 12 oz?
   b) If the population mean fill volume is increased to 12.03 oz, what is the probability that the mean volume of a sample of size 144 will be less than 12 oz?

2. A 500-page book contains 250 sheets of paper. The thickness of the paper used to manufacture the book has mean 0.08 mm and standard deviation 0.01 mm.
   a) What is the probability that a randomly chosen book is more than 20.2 mm thick (not including the covers)?
   b) What is the 10th percentile of book thicknesses?
   c) Someone wants to know the probability that a randomly chosen page is more than 0.1 mm thick. Is enough information given to compute this probability? If so, compute the probability. If not, explain why not.

3. A commuter encounters four traffic lights each day on her way to work. Let X represent the number of these that are red lights. The probability mass function of X is as follows.

x	0	1	2	3	4
P(X = x)	0.1	0.3	0.3	0.2	0.1

   What is the probability that in a period of 100 days, the average number of red lights encountered is more than 2 per day?

4. The amount of warpage in a type of wafer used in the manufacture of integrated circuits has mean 1.3 mm and standard deviation 0.1 mm. A random sample of 200 wafers is drawn.
    a) What is the probability that the sample mean warpage exceeds 1.305 mm?
    b) Find the 25th percentile of the sample mean.
    c) How many wafers must be sampled so that the probability is 0.05 that the sample mean exceeds 1.305?

5. Drums labeled 30 L are filled with a solution from a large vat. The amount of solution put into each drum is random with mean 30.01 L and standard deviation 0.1 L.
    a) What is the probability that the total amount of solution contained in 50 drums is more than 1500 L?
    b) If the total amount of solution in the vat is 2401 L, what is the probability that 80 drums can be filled without running out?
    c) How much solution should the vat contain so that the probability is 0.9 that 80 drums can be filled without running out?

6. The temperature of a solution will be estimated by taking n independent readings and averaging them. Each reading is unbiased, with a standard deviation of $\sigma = 0.5^\circ$C. How many readings must be taken so that the probability is 0.90 that the average is within $\pm 0.1^\circ$C of the actual temperature?

7. Radioactive mass A emits particles at a mean rate of 20 per minute, and radioactive mass B emits particles at a mean rate of 25 per minute.
    a) What is the probability that fewer than 200 particles are emitted by both masses together in a five-minute time period?
    b) What is the probability that mass B emits more particles than mass A in a two-minute time period?

8. The concentration of particles in a suspension is 50 per mL. A 5 mL volume of the suspension is withdrawn.
    a) What is the probability that the number of particles withdrawn will be between 235 and 265?
    b) What is the probability that the average number of particles per mL in the withdrawn sample is between 48 and 52?
    c) If a 10 mL sample is withdrawn, what is the probability that the average number per mL of particles in the withdrawn sample is between 48 and 52?
    d) How large a sample must be withdrawn so that the average number of particles per mL in the sample is between 48 and 52 with probability 95%?

9. A battery manufacturer claims that the lifetime of a certain type of battery has a population mean of 40 hours and a standard deviation of 5 hours. Let $\bar{X}$ represent the mean lifetime of the batteries in a simple random sample of size 100.
    a) If the claim is true, what is P ($\bar{X} \leq 36.7$)?
    b) Based on the answer to part (a), if the claim is true, is a sample mean lifetime of 36.7 hours unusually short?
    c) If the sample mean lifetime of the 100 batteries were 36.7 hours, would you find the manufacturer's claim to be plausible? Explain.
    d) If the claim is true, what is P ($\bar{X} \leq 39.8$)?
    e) Based on the answer to part (d), if the claim is true, is a sample mean lifetime of 39.8 hours unusually short?
    f) If the sample mean lifetime of the 100 batteries were 39.8 hours, would you find the manufacturer's claim to be plausible? Explain.

10. The manufacture of a certain part requires two different machine operations. The time on machine 1 has mean 0.5 hours and standard deviation 0.4 hours. The time on machine 2 has mean 0.6 hours and standard deviation 0.5 hours. The times needed on the machines are independent. Suppose that 100 parts are manufactured.
    a) What is the probability that the total time used by machine 1 is greater than 55 hours?
    b) What is the probability that the total time used by machine 2 is less than 55 hours?
    c) What is the probability that the total time used by both machines together is greater than 115 hours?
    d) What is the probability that the total time used by machine 1 is greater than the total time used by machine 2?

11. Seventy percent of rivets from vendor A meet a certain strength specification, and 80% of rivets from vendor B meet the same specification. If 500 rivets are purchased from each vendor, what is the probability that more than 775 of the rivets meet the specifications?

12. Radiocarbon dating: Carbon-14 is a radioactive isotope of carbon that decays by emitting a beta particle. In the earth's atmosphere, approximately one carbon atom in 1012 is carbon-14. Living organisms exchange carbon with the atmosphere, so this same ratio holds for living tissue. After an organism dies, it stops exchanging carbon with its environment, and its carbon-14 ratio decreases exponentially with time. The rate at which beta particles are emitted from a given mass of carbon is proportional to the carbon-14 ratio, so this rate decreases exponentially with time as well. By measuring the rate of beta emissions in a sample of tissue, the time since the death of the organism can be estimated. Specifically, it is known that t years after death, the number of beta particle emissions

occurring in any given time interval from 1 g of carbon follows a Poisson distribution with rate $\lambda = 15.3e^{-0.0001210t}$ events per minute. The number of years t since the death of an organism can therefore be expressed in terms of λ:

$$t = \frac{\ln 15.3 - \ln \lambda}{0.0001210}$$

An archaeologist finds a small piece of charcoal from an ancient campsite. The charcoal contains 1 g of carbon.

a) Unknown to the archaeologist, the charcoal is 11,000 years old. What is the true value of the emission rate λ?

b) The archaeologist plans to count the number X of emissions in a 25 minute interval. Find the mean and standard deviation of X.

c) The archaeologist then plans to estimate λ with $\hat{\lambda} = X/25$. What is the mean and standard deviation of $\hat{\lambda}$?

d) What value for $\hat{\lambda}$ would result in an age estimate of 10,000 years?

e) What value for $\hat{\lambda}$ would result in an age estimate of 12,000 years?

f) What is the probability that the age estimate is correct to within ± 1000 years?