



# AUTOMATA FORMAL LANGUAGES AND LOGIC

---

**Dr Pooja Agarwal**

**Professor**

Department of Computer Science & Engineering

# AUTOMATA FORMAL LANGUAGES AND LOGIC

---

## First Order Logic

**Dr Pooja Agarwal**

Department of Computer Science & Engineering

# AUTOMATA FORMAL LANGUAGES AND LOGIC

## First Order Logic

---



### Outline

- Models for First Order Logic
- Symbols and Interpretations
- Terms
- Atomic Sentences
- Complex Sentences

### Procedural approach vs Declarative approach

Propositional Logic has its limitations that you can not deal properly with general statements of the form

- “All men are mortal”
- ”Socrates is a man”
- SOCRATES IS MORTAL

#### Example:

P = All men are mortal

Q = Socrates is a Man

R = Socrates is mortal

Then  $(P \ \& \ Q) \rightarrow R$  is not valid

- The language of **first-order logic**, whose syntax and semantics is built around objects and relations.
- First-order logic can also express facts about *some* or *all* of the objects in the universe.
- This enables one to represent general laws or rules, such as the statement “Squares neighboring the wumpus are smelly.”

**Objects:** people, houses, numbers, theories, Ronald McDonald, colors, baseball games, wars, centuries . . .

**Tuples:** A tuple is a collection of objects arranged in fixed order and is written with angle brackets.

**Relations:** these can be unary relations or **properties** such as red, round, bogus, prime, multistoried . . ., or more general n-ary relations such as brother of, bigger than, inside, part of, has color, occurred after, owns, comes between, . . .

**Functions:** Function is generally used at the place where it is uniquely related to that object. **For example: Mother(Sita)**  
father of, best friend, third inning of, one more than, beginning of . . .

# AUTOMATA FORMAL LANGUAGES AND LOGIC

## SYNTAX AND SEMANTICS OF FIRST-ORDER LOGIC

---



- **Domain** of a model is the set of objects or **domain elements** it contains.
- The domain is required to be *nonempty*—every possible world must contain at least one object.
- Mathematically speaking, it doesn't matter
  - *what* these objects are—all that matters is *how many* there are in each particular model

The basic syntactic elements of first-order logic are the **symbols** that stand for objects, relations, and functions.

- The symbols, come in three kinds:
  - **Constant symbols**, which stand for objects  
e.g. **Richard, John**
  - **Predicate symbols**, which stand for relations  
e.g. **Brother(Richard, John)**
  - **Function symbols**, which stand for functions.  
e.g. **Father(Richard)**



### Connectives

- Negation :
- AND or Conjunction:
- OR or Disjunction
- Implication
- Bidirectional or “IF and Only If”

A “TERM” is a logical expression that refers to an object.

Constant symbols are terms, but every time every object can't be represented by a distinct symbol.

**For Example: Let**

Term =  $f(t_1, t_2, t_3, \dots, t_n)$

Here  $f$  is a function symbol, that refers to some function in the model.

$t_1, t_2, t_3, \dots, t_n$  are objects in the domain

An **Atomic sentence** is formed from predicate symbol, optionally followed by parenthesized list of terms.

For Example:

**Brother (Richard, Hanes)**

**P(X, Y)**

**Married(Father(Richard), Mother(Hanes))"**

**Complex sentence** is formed using logical connectives, with the same syntax and semantics as in propositional calculus.

For Example:

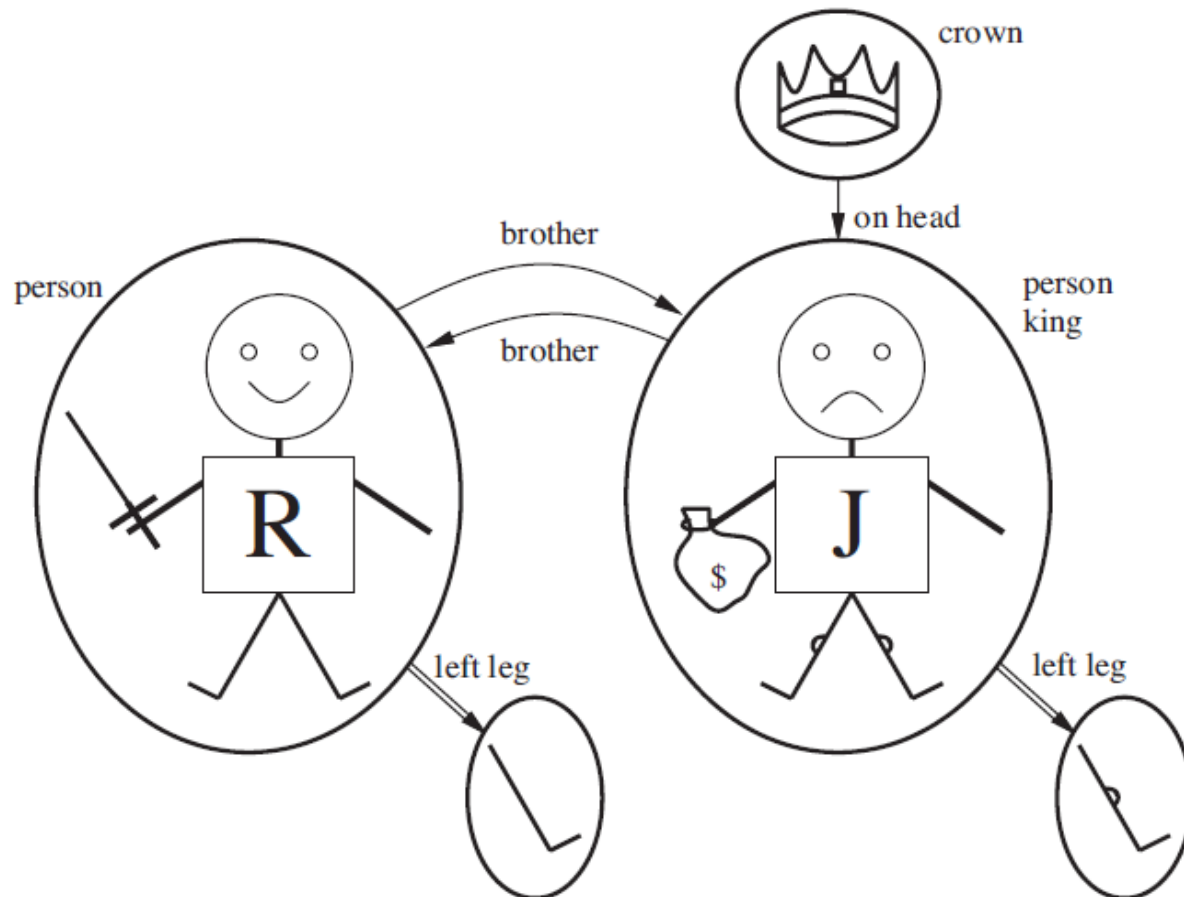
– **Brother (Hand(Richard), John)**

**Brother(Richard, John)  $\wedge$  Brother (John, Richard)**

– **King(Richard)  $\Rightarrow$  King (John)**

**Uncle(Father(Ravi), Rani)**

### *A model containing five objects*



### Brotherhood Relation

$\{ \text{Richard the Lionheart, King John} \} \& \{ \text{King John, Richard the Lionheart} \}$

### Leftleg Function

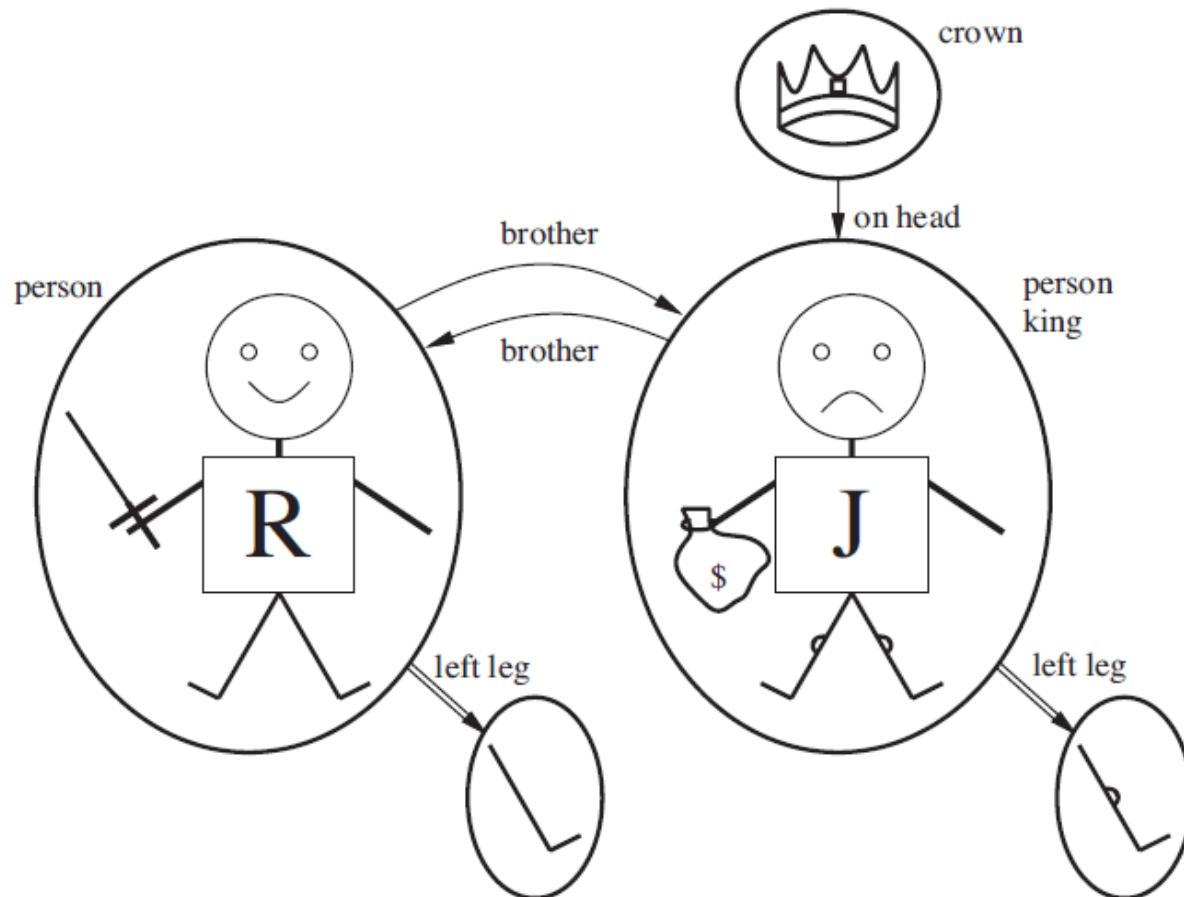
Richard the Lionheart  $\rightarrow$  Richard's left leg

King John  $\rightarrow$  John's left leg

### Onhead

Onhead(crown, John)

### *A model containing five objects*





# THANK YOU

---

**Pooja Agarwal**

Department of Computer Science & Engineering

**[poojaagarwal@pes.edu](mailto:poojaagarwal@pes.edu)**