UE19CS251
Design and Analysis of Algorithms
Unit 5: Limitations of Algorithmic Power and Coping with the Limitations

The Knapsack Problem

PES University

Outline

Concepts covered

- The Knapsack Problem
  - Introduction
  - Recurrence
  - Example

1 Problem Definition

- Given
  - n items of $\begin{array}{llllll} \text{integer weights :} & w_1 & w_2 & \dots & w_n \\ \text{values :} & v_1 & v_2 & \dots & v_n \end{array}$
  - knapsack of capacity W (integer $W > 0$)
- Find the most valuable subset of items such that sum of their weights does not exceed W

2 Knapsack Recurrence

- To design a dynamic programming algorithm, we need to derive a recurrence relation that expresses a solution to an instance of the knapsack problem in terms of solutions to its smaller subinstances

- Consider the smaller knapsack problem where number of items is i ($i \leq n$) and the knapsack capacity is j ($j \leq W$)
- <2-> Then

$$F(i,j) = \begin{cases} \max(F(i-1,j), \quad v_i + F(i-1, j-w_i)) & \text{if } j - w_i \geq 0 \\ F(i-1,j) & \text{if } j - w_i < 0 \end{cases}$$

3 Example

$$F(i,j) = \begin{cases} \max(F(i-1,j), \quad v_i + F(i-1, j-w_i)) & \text{if } j - w_i \geq 0 \\ F(i-1,j) & \text{if } j - w_i < 0 \end{cases}$$

Dynamic Programming Example

item i	weight w_i	value v_i
1	2	12
2	1	10
3	3	20
4	2	15

What is the maximum value that can be stored in a knapsack of capacity 5?

	capacity j				
i	1	2	3	4	5
1	0	12	12	12	12
2	10	12	22	22	22
3	10	12	22	30	32
4	10	15	25	30	37

Given above 6 items, maximum value that can be stored in a knapsack of capacity 5 is **37**

4 Complexity

- Space complexity: $\Theta(nW)$
- Time complexity: $\Theta(nW)$
- Time to compose optimal solution: $O(n)$

5 Think About It

- <2-> Write pseudocode of the bottom-up dynamic programming algorithm for the knapsack problem
- <3-> True or False:

1. <3-> A sequence of values in a row of the dynamic programming table for the knapsack problem is always nondecreasing?
2. <4-> A sequence of values in a column of the dynamic programming table for the knapsack problem is always nondecreasing?