# STATISTICS FOR DATA SCIENCE

## HYPOTHESIS AND INFERENCE

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# STATISTICS FOR DATA SCIENCE

## Unit 4 :HYPOTHESIS AND INFERENCE

## Session : 12

## Sub Topic :Type I and Type II Errors

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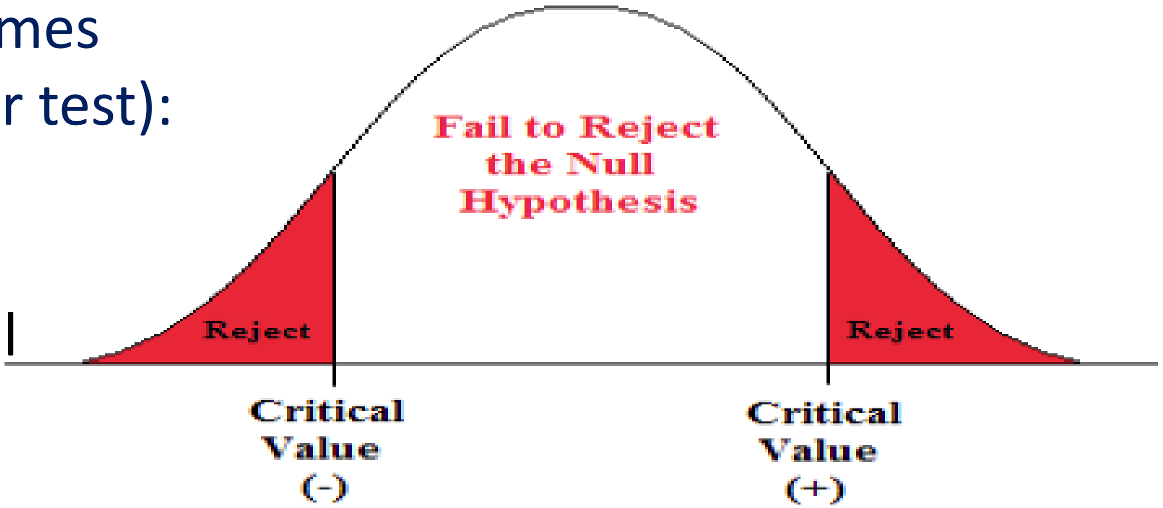
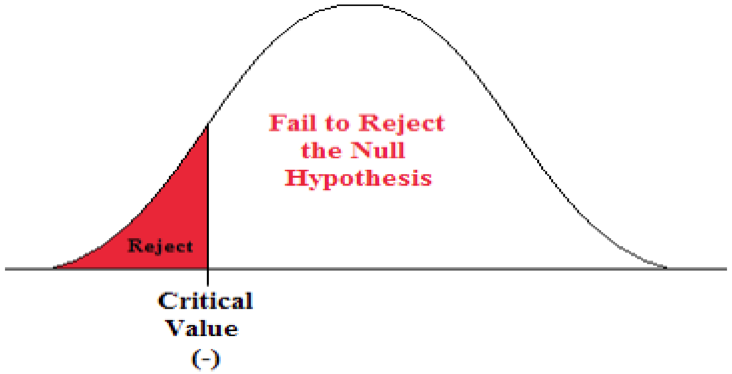
# ERRORS IN HYPOTHESIS TESTING

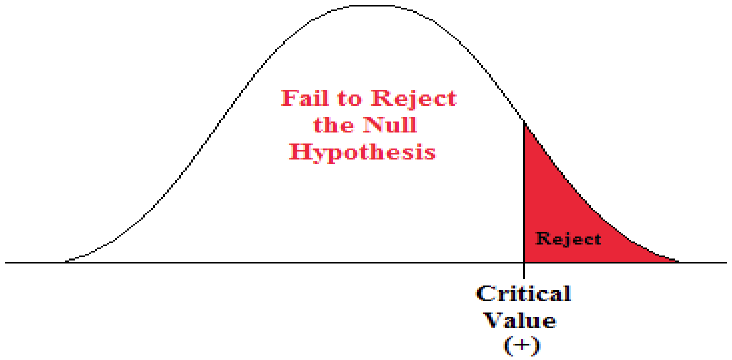
**Hypothesis testing : $H_0$ vs $H_1$**

Two possible outcomes
(only one occurs per test):

- Reject Null
- Fail to Reject Null

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## Type I and Type II errors

When we conduct a hypothesis test, we cannot know **for certain** if the null is true or false (this is why we are conducting the test in the first place).

Because hypothesis tests are based on probability, while we hope to make correct decisions, it is possible to get results that are contrary to reality( such as the null being true in reality but we reject it based on our statistical test results).

## Type I and Type II errors

When we get a result that is contrary to the truth, this is known as making **an error in hypothesis testing.**

There are exactly two kinds of errors

Null is true but we reject it
Null is false but we fail to reject it

## Type I and Type II errors

Suppose we test $H_0: \mu = 15$

$H_1: \mu > 15$

And we reject $H_0$ at $\alpha = 0.05$

One of two things will occur:

1. The null hypothesis is false and we made the correct decision

2. The null hypothesis is true and we made a type I error

## Type I and Type II errors

Suppose we test $H_0: \mu = 15$

$H_1: \mu > 15$

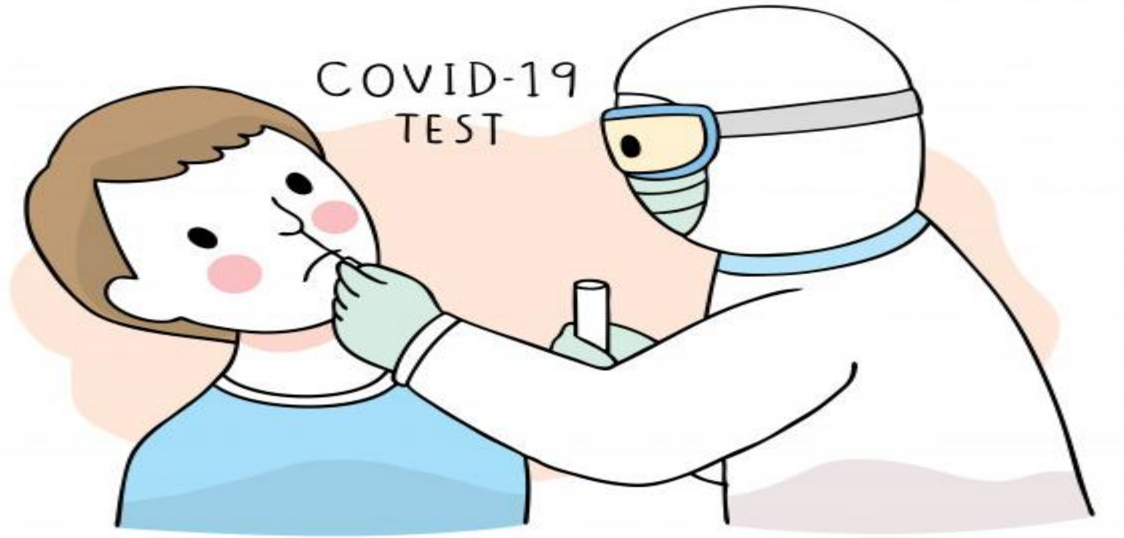
And we do not reject $H_0$ at $\alpha = 0.05$

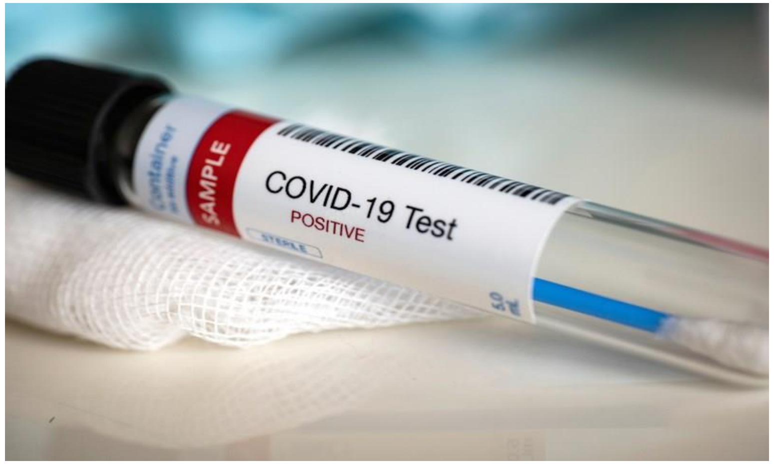
One of two things will occur:

1. The null hypothesis is true and we made the correct decision

2. The null hypothesis is false and we made a type II error

## Type I and Type II errors - Example

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## Hypothesis testing : $H_0$ vs $H_1$

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### $H_0$ : A person is tested positive for Covid-19

| | **Actual State of Reality** | |
|---|---|---|
| **Researcher Decision** | $H_0$ is true<br>**Covid +ve** | $H_0$ is false<br>**Covid -ve** |
| Reject $H_0$<br>**Covid -ve** | Type I error ($\alpha$)<br>(Erroneously reported that the patient is Covid –ve) | Correct Decision<br>(1 – β) |
| Fail to reject $H_0$<br>**Covid +ve** | Correct Decision<br>$(1 - \alpha)$ | Type II error (β)<br>(Erroneously reported that the patient is Covid +ve) |

## Type I and Type II errors- Example - A Judicial trial

Presumption of Innocence

$H_0$ : Assumed to be innocent until proven guilty

Prosecution's claim is

$H_1 : The\ person\ is\ guilty$

Source: Internet

**Consider a criminal trial:**

**We test the hypothesis**

$H_0$: **The defendant did not commit the crime**

$H_1$: **The defendant committed the crime**

**Type I error: Convicting a person who in reality did not commit the crime**

**Type II error : Acquitting a person who in reality , committed a crime**

## Hypothesis testing : $H_0$ vs $H_1$

### $H_0$ : Person is not guilty of the crime

| | **Truth** | |
|---|---|---|
| **Jury Decision** | $H_0$ is true<br>Innocent | $H_0$ is false<br>Guilty |
| Reject $H_0$<br>Guilty | Type I error ($\alpha$) -Person is convicted by the court when he actually did not commit the crime(convicting an innocent person) | Correct Decision<br>$(1 - \beta)$ |
| Fail to reject $H_0$<br>Innocent | Correct Decision<br>$(1 - \alpha)$ | Type II error (β) - Person is acquitted by the court when he actually did commit the crime (letting a guilty person go free) |

## Type I and Type II errors


Source: Internet

POPFile

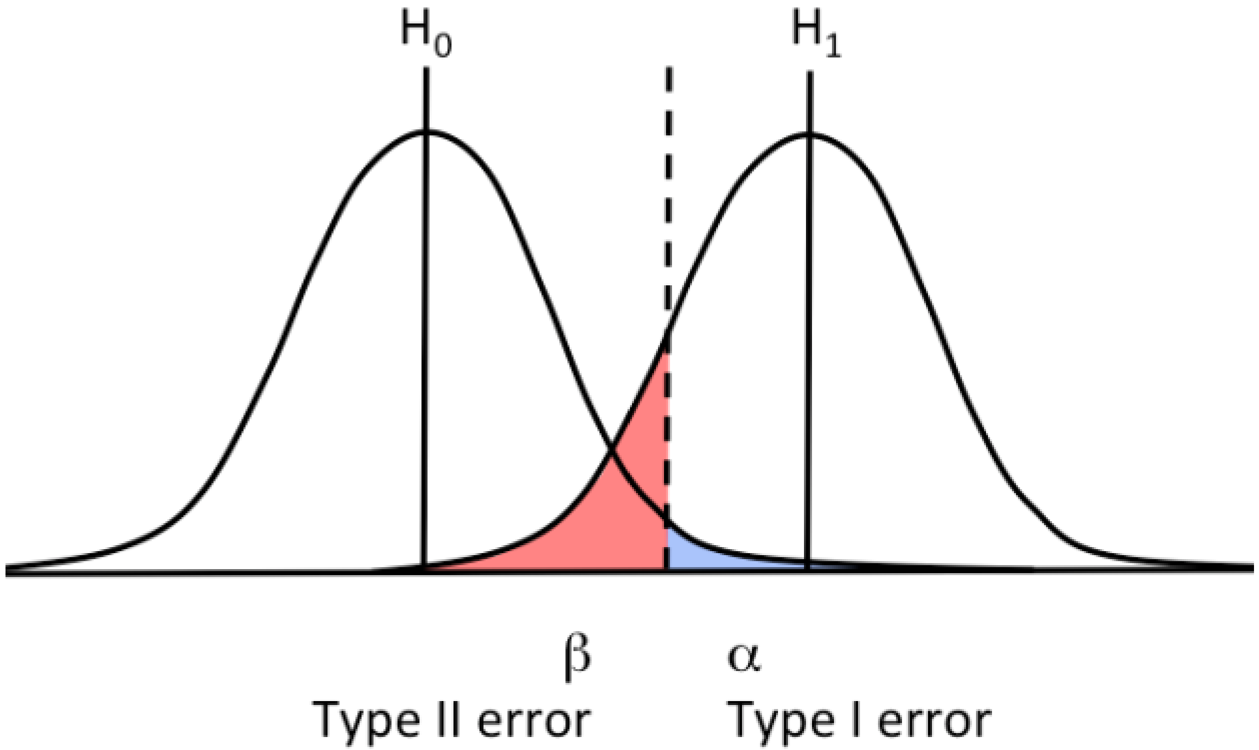
## Hypothesis testing : $H_0$ vs $H_1$

### $H_0$ : It is not a spam vs $H_1$ : It is a spam

| | Truth | |
|---|---|---|
| POPFile Decision | $H_0$ is true<br>It is not a spam | $H_0$ is false<br>It is a spam |
| Reject $H_0$<br>It is a spam | Type I error ($\alpha$)-<br>an important message that ends up in your spam folder or, much worse, gets deleted. | Correct Decision<br>$(1 - \beta)$ |
| Fail to reject $H_0$<br>It is not a spam | Correct Decision<br>$(1 - \alpha)$ | Type II error (β)<br>(spam that ends up in your inbox is annoying.) |

## Type I and Type II errors

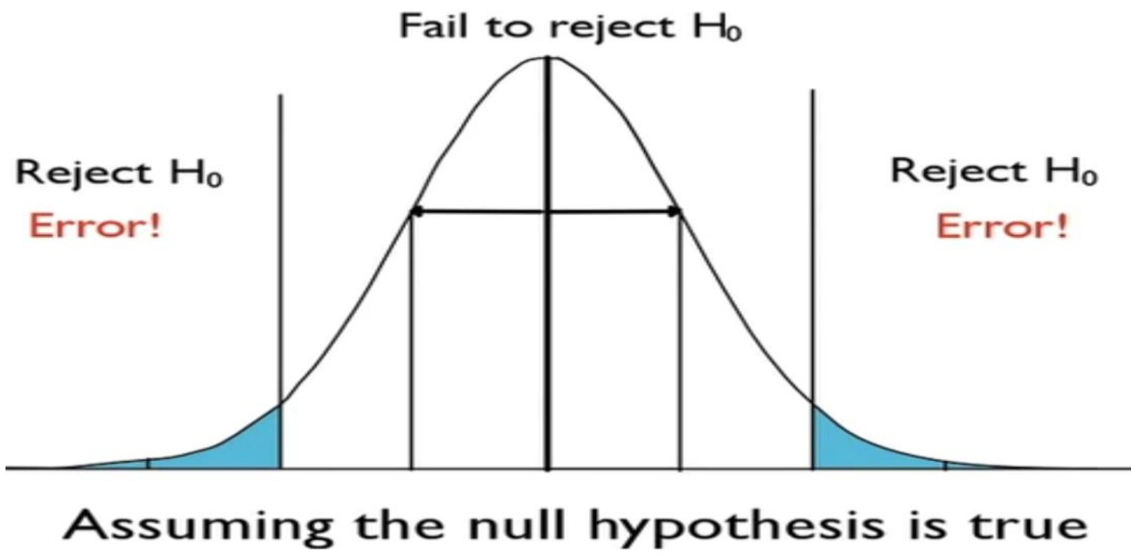
When designing experiments whose data will be analyzed with a fixed-level test, it is important to try to make the probabilities of type I and type II errors reasonably small.



## Type I and Type II errors

### Type I error:

P(type I error)= P(reject $H_0$ when $H_0$ is true)=$\alpha$



## Type I and Type II errors

### Type I error :

- **When the null hypothesis is true and you reject it, you make a type I error.**
- The probability of making a type I error is α, which is the level of significance you set for your hypothesis test.
- An α of 0.05 indicates that you are willing to accept a 5% chance that you are wrong when you reject the null hypothesis.
- To lower this risk, you must use a lower value for α.
- However, using a lower value for alpha means that you will be less likely to detect a true difference if one really exists.

## Type I and Type II errors

**If $\alpha$ is the significance level that has been chosen for the test, then the probability of a type I error is never greater than $\alpha$.**

Let $X_1, X_2. . . , X_n$ be a large random sample from a population with mean $\mu$ and variance $\sigma^2$.

$\bar{X} \sim N(\mu, \sigma^2/n)$

**Suppose we test $H_0\!: \mu \leq 0$ versus $H_1\!: \mu > 0$**

**at the fixed level $\alpha = 0.05$**

➡ **we will reject $H_0$ *if* $P \leq 0.05$**

## Type I and Type II errors

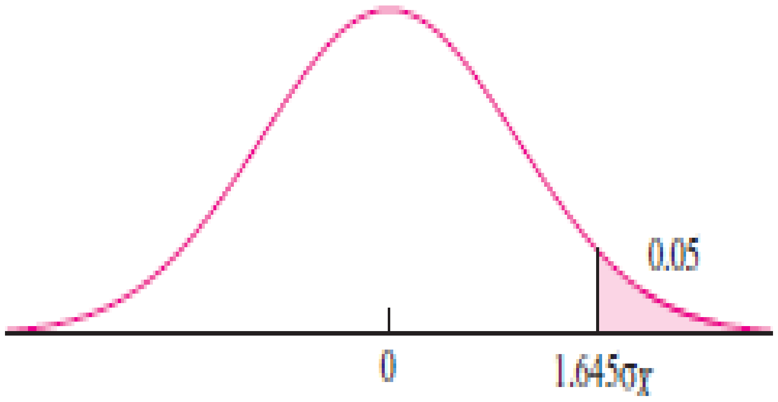



FIGURE 6.25 The null distribution with the rejection region for $H_0: \mu \leq 0$.

Assume the null hypothesis is true. We will compute the probability of type I error and show that it is no greater than 0.05.

## Type I and Type II errors

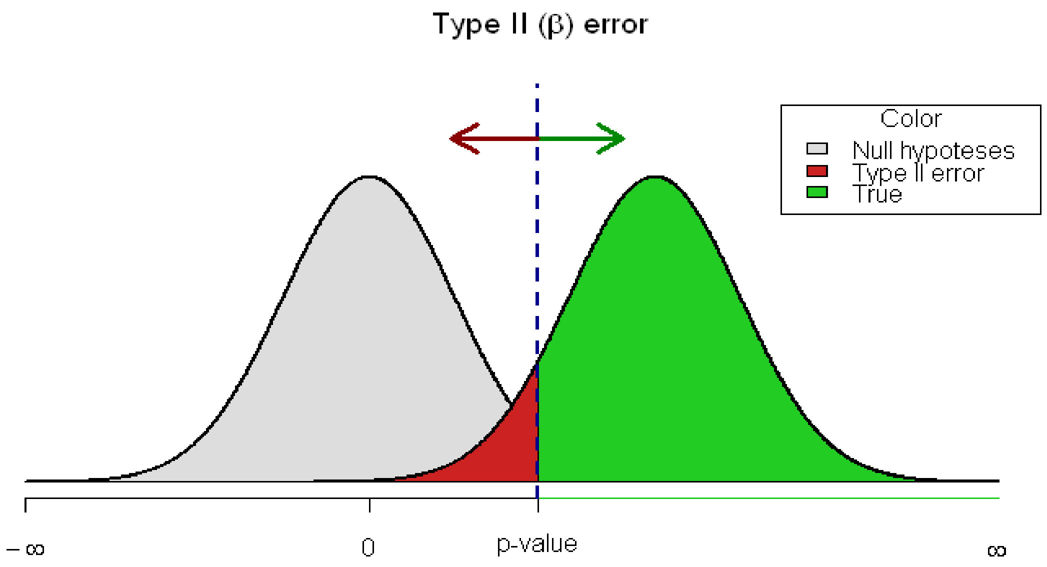
Next, consider the case where $\mu < 0$.
Then the distribution of $\bar{X}$ is obtained by shifting the curve in Figure to the left, so $P(\bar{X} \geq 1.645\sigma_{\bar{X}}) < 0.05$, and the probability of a type I error is less than 0.05.

We could repeat this illustration using any number $\alpha$ in place of 0.05.
We conclude that if $\boldsymbol{H_0}$ is true, the probability of a type I error is never greater than $\alpha$.
**Furthermore, note that if $\mu$ is on the boundary of $H_0$ ($\mu = 0$ in this case), then the probability of a type I error is equal to $\alpha$.**

## Type I and Type II errors

**Type II error:**

P(type II error)= P(Fail to reject $H_0$ when $H_0$ is false)=$\boldsymbol{\beta}$



## Type II error:

● **When the null hypothesis is false and you fail to reject it, you make a type II error.**
● The probability of making a type II error is β, which depends on the **power of the test**.
● You can decrease your risk of committing a type II error by ensuring your test has enough power.
● You can do this by ensuring your sample size is large enough to detect a practical difference when one truly exists.

## Type I and Type II errors

- The smaller we make the probability of a type I error, the larger the probability of a type II error becomes.
- The usual strategy is to begin by choosing a value for $\alpha$ so that the probability of a type I error will be reasonably small.
- A conventional choice for $\alpha$ is 0.05.
- If the probability of a type II error is large, it can be reduced only by redesigning the experiment—for example by increasing the sample size.
- Calculating and controlling the size of the type II error is somewhat more difficult than calculating and controlling the size of the type I error.

**Problem 1**

**A vendor claims that no more than $10\%$ of the parts she supplies are defective. Let $p$ denote the actual proportion of parts that are defective. A test is made of the hypotheses $H_0: p \leq 0.10$ versus $H_1: p > 0.10$.**

**For each of the following situations, determine whether the decision was correct, a type I error occurred, or a type II error occurred.**

a. The claim is true, and $H_0$ is rejected.

b. The claim is false, and $H_0$ is rejected.

c. The claim is true, and $H_0$ is not rejected.

d. The claim is false, and $H_0$ is not rejected.

Ans: (a) Type I error (b) Correct decision (c) Correct decision (d) Type II error

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## Problem 2 :

A hypothesis test is to be performed, and it is decided to reject the null hypothesis.
If $P \leq 0.10$.

If $H_0$ is in fact true, what is the maximum probability that it will be rejected?

**Solution:**

**The maximum probability of rejecting $H_0$ when true is the level $\alpha = 0.10$.**

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**Problem 3 :**

A test is made of the hypotheses:

$H_0$ : μ ≤ 10 versus $H_1$ : μ > 10

For each of the following situations, determine whether the decision was correct, a type I error occurred, or a type II error occurred.

a. μ = 8, $H_0$ is rejected.
b. μ = 10, $H_0$ is not rejected.
c. μ = 14, $H_0$ is not rejected.
d. μ = 12, $H_0$ is rejected.

## Type I and Type II errors- Problem 1

$H_0$ : μ ≤ 10 versus $H_1$ : μ > 10

**a. μ = 8, $H_0$ is rejected**

Type I error. $H_0$ is true and was rejected.

**b. μ = 10, $H_0$ is not rejected**

Correct decision. $H_0$ is true and was not rejected.

**c. μ = 14, $H_0$ is not rejected**

Type II error. $H_0$ is false and was not rejected.

**d. μ = 12, $H_0$ is rejected**

Correct decision. $H_0$ is false and was rejected.

## Type I and Type II errors

Problem 4:

Null Hypothesis is that the battery for a heart pacemaker has an average life of 300 days, with the alternative hypothesis that the average life is more than 300 days. If you are the quality control manger for the battery manufacturer then

a) Would you rather make a Type I error or a Type II error
b) Based on your answer to part(a), should you use a high or low significance level?

$\boldsymbol{H_0}$ **: μ = 300 days versus** $\boldsymbol{H_1}$ **: μ > 300days**

## Type I and Type II errors

**Solution:**

Given $H_0$ : μ = 300 days versus $H_1$ : μ > 300days

(a) It is better to make a Type II error (where $H_0$ is false. That is, average life is actually more than 300 days but wwe accept $H_0$ and assume that the average life is equal to 300 days).

(b) As we increase the significance level ($\alpha$) we increase the chances of making type I error. Since here it is better to make a type II error we shall choose a low $\alpha$.

# THANK YOU

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