PES University, Bangalore
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UE19CS203 – STATISTICS FOR DATA SCIENCE
Unit-4 - Hypothesis and Inference

QUESTION BANK – SOLVED

Fixed Level Testing:

Exercises for section 6.12: [Text Book Exercise 6.12– Pg. No. 478]

1. A process for a certain type of ore is designed to reduce the concentration of impurities to less than 2%. It is known that the standard deviation of impurities for processed ore is 0.6%. Let μ represent the mean impurity level, in percent, for ore specimens treated by this process. The impurity of 80 ore specimens is measured, and a test of the hypothesis
$H_0 : \mu \geq 2$ versus $H_1 : \mu < 2$ will be performed.

   a. If the test is made at the 5% level, what is the rejection region?
   b. If the sample mean impurity level is 1.85, will H_0 be rejected at the 10% level?
   c. If the sample mean pH is 1.85, will H_0 be rejected at the 1% level?
   d. If the value 1.9 is a critical point, what is the level of the test?

[Text Book Exercise – Section 6.12 – Q. No2 – Pg. No. 478]

Solution:

Given

$$\sigma = 0.6, \qquad n = 80$$
$$H_0 : \mu \geq 2$$

$$H_1 : \mu < 2$$

a) α = 5%=0.05

The critical value is the z-value in the normal probability table in the appendix corresponding to a probability of α =0.05 is

Z = -1.645

Then the rejection region contains all values smaller than -1.645.

The sample mean corresponding to a z-score is

$$\bar{x}=\mu + z\frac{\sigma}{\sqrt{n}}=2 - 1.645\frac{0.6}{\sqrt{80}} \approx 1.88965$$

Thus, the rejection region contains all sample means smaller than 1.88965.

b) $\bar{x} = 1.85, \ \alpha = 10\% = 0.10$

The critical value is the z-value in the normal probability table in the appendix corresponding to a probability of , $\alpha = 0.10$ is

Z = -1.28

Then the rejection region contains all values smaller than -1.28.

The sample mean corresponding to a z-score is

$$\bar{x}=\mu + z\frac{\sigma}{\sqrt{n}}=2 - 1.28\frac{0.6}{\sqrt{80}} \approx 1.91435$$

Thus, the rejection region contains all sample means smaller than 1.91435.

Since the sample mean 1.85 is smaller than 1.91435, we reject the null hypothesis.

c) $\overline{x} = 1.85, \alpha = 1\% = 0.01$

The critical value is the z-value in the normal probability table in the appendix corresponding to a probability of , $\alpha = 0.01$ is

Z = -2.33

Then the rejection region contains all values smaller than -2.33.

The sample mean corresponding to a z-score is

$$\bar{x} = \mu + z\frac{\sigma}{\sqrt{n}} = 2 - 2.33\frac{0.6}{\sqrt{80}} \approx 1.843699$$

Thus, the rejection region contains all sample means smaller than 1.843699.

Since the sample mean 1.85 is larger than 1.843699 and thus not in the rejection region, we then fail to reject the null hypothesis.

d) $\bar{x} = 1.9$

The z-value is

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$z = \frac{1.9 - 2}{0.6/\sqrt{80}} \approx -1.49$$

The significance level is the probability of obtaining a value more extreme or equal to the standardized test statistic z, assuming that the null hypothesis is true. Determine the probability using normal probability table in the appendix.

α = P(Z < -1.49) = 0.0681 = 6.81%

2. A hypothesis test is to be performed, and it is decided to reject the null hypothesis if P ≤ 0.10. If H_0 is in fact true, what is the maximum probability that it will be rejected?

[Text Book Exercise – Section 6.12 – Q. No.6 – Pg. No. 478]

Solution:

We reject the null hypothesis if the P-value is at most 0.10.

This then means that the significance level is 0.10 or 10%.

The significance level is the probability of making a type I error and is thus the probability of reject the null hypothesis, when it is in fact true.
Thus, the maximum probability that the null hypothesis is reject (when it is true), is 0.10.