VECTOR SPACES

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CLASS 6 : CONTENT

- Column Space
- Row Space

VECTOR SPACES
COLUMN SPACE

The column Space

Definition: Let A be a m x n matrix. The column space of A is the set of all linear combinations of the columns of A denoted by C(A). Thus,

C(A) = { b Є R m / Ax = b is solvable }

Note : C(A) is a subspace of R^m

CLASS 6 : CONTENT

- In other words, The space spanned by linear combination of linearly independent columns of matrix A spans the Column Space of Matrix A .
- Column Space is denoted by C(A)
- C(A) can lie anywhere in between the zero space and the whole space R^m
- The system of Linear equations Ax=b is solvable iff the vector ‘b’ can be expressed a combination of columns of A ,then ‘b’ is in C(A).

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The column Space

Few examples…. 1. The smallest possible column space comes from the zero matrix A = 0. The only combination of the columns is b = 0.

2. If A is a 5 x 5 identity matrix then C(A) is the whole of R^5 the 5 columns of A can combine to produce any 5 dimensional vector b. In fact, any 5 x 5 nonsingular matrix A will have R^5 as its column space !!

VECTOR SPACES

COLUMN SPACE

Let

$$A = \begin{bmatrix} 1 & 0 \\ 5 & 4 \\ 2 & 3 \end{bmatrix}$$

then C(A) is the subspace of R^3 consisting of vectors b that are linear combinations of the vectors (1, 5, 2) and (0, 4, 3). Geometrically the subspace is a 2- d plane.

Let

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 5 & 4 & 9 \\ 2 & 3 & 5 \end{bmatrix}$$

Then C(B) is the subspace of R^3 consisting of vectors b that are linear combinations of the vectors (1, 5, 2) , (0, 4, 3) and (1, 9, 5).

COLUMN SPACE

Note : The column spaces of A and B are same though the matrices are different. This is because the new column is a linear combination of the other two columns. Hence, appending a dependent column does not alter the column space of a matrix .

THANK YOU

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