Fourier Series : Gist

i) Fourier Series of period 2π & Euler's formulae for the Fourier Co-efficients. $(c \le x \le c+2\pi)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$a_0 = \frac{1}{\pi}\int_c^{c+2\pi} f(x)\,dx, \quad a_n = \frac{1}{\pi}\int_c^{c+2\pi} f(x)\cos nx\,dx, \quad b_n = \frac{1}{\pi}\int_c^{c+2\pi} f(x)\sin nx\,dx.$$

ii) F S of arbitrary period $2l$ & Euler's formulae! $(c \le x \le c+2l)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$$

$$a_0 = \frac{1}{l}\int_c^{c+2l} f(x)\,dx, \quad a_n = \frac{1}{l}\int_c^{c+2l} f(x)\cos\frac{n\pi x}{l}\,dx, \quad b_n = \frac{1}{l}\int_c^{c+2l} f(x)\sin\frac{n\pi x}{l}$$

iii) Even in $(-\pi,\pi)$ or $(-l,l)$ is $f(-x) = f(x)$: $(0,2\pi)$: $f(2\pi-x) = f(x)$

odd in $(-\pi,\pi)$ or $(-l,l)$ i.e. $f(-x) = -f(x)$: $f(2\pi-x) = -f(x)$

$(0,2l)$ OR $f(2l-x) = f(x)$

& $f(2l-x) = -f(x)$

IV) Fourier co-efficients in the case of even & odd nature of

Interval	$f(x)$ is even: a_0	a_n	b_n	$f(x)$ is odd: a_0	a_n	b_n
$(-\pi,\pi)$ or $(0,2\pi)$	$\frac{2}{\pi}\int_0^{\pi} f(x)\,dx$	$\frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx$	0	0	0	$\frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$
$(-l,l)$ or $(0,2l)$	$\frac{2}{l}\int_0^{l} f(x)\,dx$	$\frac{2}{l}\int_0^{l} f(x)\cos\frac{n\pi x}{l}\,dx$	0	0	0	$\frac{2}{l}\int_0^{l} f(x)\sin\frac{n\pi x}{l}$

V) Half range Fourier series: (Cosine & Sine series)

f(x) in		Series	Fourier co-efficients
$(0, \pi)$	Cosine series	$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$	$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$, $a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx$
$(0, \pi)$	Sine series	$\sum_{n=1}^{\infty} b_n \sin nx$	$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$.
$(0, l)$	Cosine series	$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l}$	$a_0 = \frac{2}{l}\int_0^{l} f(x)\,dx$, $a_n = \frac{2}{l}\int_0^{l} f(x)\cos\frac{n\pi x}{l}\,dx$
$(0, l)$	Sine series	$\sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$	$b_n = \frac{2}{l}\int_0^{l} f(x)\sin\frac{n\pi x}{l}\,dx$

VI Harmonic Analysis:

i) period: 2π. Interval $c \le x < c+2\pi$. Euler's formulae are

$$a_0 = \frac{2}{N}\Sigma y, \quad a_n = \frac{2}{N}\Sigma y\cos nx, \quad b_n = \frac{2}{N}\Sigma y\sin nx$$

ii) period $2l$: Interval: $c \le x < c+2l$, $\theta = (\pi x / l)$

$$a_0 = \frac{2}{N}\Sigma y, \quad a_n = \frac{2}{N}\Sigma y\cos n\theta, \quad b_n = \frac{2}{N}\Sigma y\sin n\theta.$$

VII Parsevals Identity:

$$\int_{-l}^{l}[f(x)]^2\,dx = \left[\left(\frac{a_0}{2}\right)^2 + \sum_{n=1}^{\infty}(a_n^2 + b_n^2)\right]$$

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