PES UNIVERSITY, Bangalore

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Department of Computer Science & Engineering

Automata Formal Languages & Logic

Homework

1. Determine whether A = $\{a^{2n} | n>=0\}$ is regular.
2. Let Σ = {a,b} and let D = {w|w contains an equal number of occurrences of the substring 01 and 10} .Thus 101 $\in$ D because 101 contains a single 01 and a single 10, but 1010 $\notin$D because 1010 contains two 10s and only one 01. Show that D is a regular language.
3. Determine whether each of the following languages is regular.
   a. $\{a^n a^n a^n \mid n>0\}$
   b. $\{www \mid w \in \{x,y,z\}^*, |w|<10^{100}\}$
   c. $\{vw \mid v,w \in \{a,b\}^*\}$
   d. $\{ww \mid w \in \{a\}^*\}$

4. The pumping lemma says that every regular language has a pumping length p, such that every string in the language can be pumped if it has length p or more. If p is a pumping length for the language A,so is any length p` >= p.The minimum pumping length for A is the smallest p that is a pumping length for A.For example, if A = 01^*, the minimum pumping length is 2 . The reason is that the string s=0 is in A and has length 1 yet s cannot be pumped , but any string in A of length 2 or more contain a 1 and hence can be pumped by dividing it so that x = 0 ,y=1 and z is the rest. For each of the following languages,give the minimum pumping length and justify your answer.
   a. ε
   b. Σ^*
   c. 10(11*0)*0

5. Are the following languages regular? Prove your results.
   (a) same number of 0s and 1s
   (b) same number of 01s and 10s
   (c) same number of 00s and 11s
6. When using the pumping lemma, for which of the following we can make our own choices (\exists), and for which of the following we have to consider *all* cases (\forall)?
   (a) pumping length p
   (b) string s
   (c) the decomposition s = xyz

(d) the pumping factor i

7. Use the pumping lemma to prove that they are not regular. (In (a) and (c), the alphabet is $\{a, b\}$.)
   (a) $\{ a^l b^m \mid l \leq m \leq 2l \}$.
   (b) $\{ ww \mid w \in \{0, 1\}^* \}$.
   (c) $\{ a^{kl} b^l \mid$ k and l are non-negative integers $\}$.
8. . Use the pumping lemma to show that the following languages are not regular.

a) $A = \{0^n 1^m 0^{n+m} : n,m >= 0\}$

b) $B = \{0^a 1^b 0^{a-b} : a,b >= 0\}$