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Unit-4

Orthogonalization, Eigen Values and Eigen Vectors

Topics in the Module:

- Orthogonal Bases, orthogonal matrices and properties
- Rectangular matrices with orthonormal columns
- ❖ The Gram- Schmidt Orthogonalization
- **❖** A=QR factorization
- Introduction to Eigenvalues and Eigenvectors
- Properties of Eigenvalues and Eigenvectors and C-H theorem
- Problems on Eigenvalues and Eigenvectors and C-H theorem
- Symmetric Matrices and Diagonalization of a Matrix
- Problems on diagonalization of a matrix
- Powers and products of matrices





CLASS-1

ORTHOGONAL BASES, ORTHOGONAL MATRICES AND PROPERTIES



In an <u>orthogonal</u> <u>basis</u>, every vector is perpendicular to every other vector. The coordinate axes are mutually orthogonal.

Mutually perpendicular unit vectors are called **Orthonormal** vectors.



- For the vector space R²,
- 1. The set (2, 0), (0, 2) is an orthogonal basis.
- 2. The set (1, -2), (2, 1) is an orthogonal basis.
 - 3. The set (1, 0), (0, 1) is an orthonormal basis.

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- A matrix with Orthonormal columns will be called Q.
- A square matrix with Orthonormal columns is called an <u>Orthogonal matrix</u> denoted by Q.

Ex: Rotation matrix, any permutation matrix.

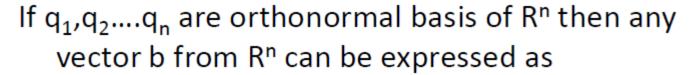
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Properties of Q

- If Q (square or rectangular) has orthonormal columns, then $Q^T Q = I$.
- An orthogonal matrix is a square matrix with orthonormal columns. Then Q^T is Q⁻¹.
- If Q is rectangular then Q^T is left inverse of Q.
- Multiplication by any Q preserves length. The norms of x and Qx are equal.

Properties of Q (Continued....)

 Also, Q preserves inner products and angles, since (Q x)^T (Qy) = x^TQ^TQy= x^Ty.



$$b = x_1q_1 + x_2q_2 + + x_n q_n$$
 Eqn (1)

Multiply both sides by q_1^T . Then $q_1^Tb = x_1$.

Similarly,
$$x_2 = q_2^{T}b$$
,, $x_n = q_n^{T}b$.

Hence, b=
$$(q_1^Tb)q_1 + (q_2^Tb)q_2 + + (q_n^Tb)q_n$$

= sum of one dimensional projections on to q_i's.

The matrix form of equation (1) is Qx = b and the solution of this system of equations is

$$x = Q^{-1}b = Q^{T}b$$



Properties of Q (Continued....)

 The rows of a square matrix are orthonormal whenever the columns are

$$Q = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & 0 & -2/\sqrt{6} \\ 1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6} \end{bmatrix}.$$





THANK YOU

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