UE19MA251 LINEAR ALGEBRA AND ITS APPLICATIONS

.**Unit 3: Linear Transformations and Orthogonality**

Linear Transformations , Inner Products and Cosines, Orthogonal Vectors and Subspaces, Cosines and Projections onto Lines, Projections and Least Squares.

Self Learning Component: Examples of Vector Spaces and Subspaces

Class No.	Portions to be covered
30-31	Linear Transformations , Examples
32-33	Transformations Represented by Matrices
34	Rotations, Reflections
35	Projections
36	Orthogonal Vectors and Subspaces
37	Inner Products
38	Cosines and Projections onto Lines
39	Projections and Least Squares
40	Problems
41	**Scilab Class Number 5 –Projections by Least Squares**

Classwork problems:

1.	Which of these transformations are not linear? Give reasons. $(i) T(v) = \frac{v}{\lVert v \rVert}$, $(ii) T(v) = v_1 + v_2 + v_3$, $(iii) T(v) = (v_1, 2v_2, 3v_3)$, $(iv) T(v) = v$ except that $T(0, v_2) = (0,0)$
2.	Find the image of these points after applying the transformation given: (i)Reflect (2,3) across 45° line and then project on y-axis. (ii)Project (3,4) on x-axis and then rotate by 30° in counter clockwise direction,
3.	Suppose T is reflection across the 45° line and S is reflection across the y-axis. If v=(2,1) and T(v)=(1,2). Find S(T(v)) and T(S(v)) Show that ST $\neq$ TS
4.	From the cubics P_3 to P_4 what matrix represents multiplication by 5-4t?
5.	For each of the following linear transformations T, find a basis and the dimension of the range and kernel of T: (i) $T : \sim^3 \to \sim^2$ defined by T(x, y, z)=(x+y, y-z) (ii) $T : \sim^2 \to \sim^2$ defined by T(x, y)=(x+2y, 2x-y)

6.	Find the matrix of the linear transformation T on $\sim^{3}$defined by T(x,y,z)=(x+2y+z, 2x-y, 2y+z) with respect to (i)the standard basis(1,0,0),(0,1,0), ((0,0,1) (ii)the basis(1,0,1),(0,1,1), ((0,0,1)
7.	If V is a subspace of $\sim^{5}$spanned by (1,2,3,-1,2) and (2,4,7,2,-1). Find a basis of the orthogonal complement $V^{\perp}$of V.
8.	Find all vectors in $\sim^{4}$that are orthogonal to (1,2,4,4,1) and (2,9,8,2). Produce an orthonormal basis from these mutually orthogonal vectors.
9.	Let P be the plane in $\sim^{4}$ with equation x-2y+3z-t=0. Find a vector perpendicular to P. what matrix has the plane P as its null space? What is a basis for P?
10.	Find the matrix P that projects every point in $\sim^{3}$onto the line of intersection of the planes x+y+t=0 and x-t=0. What are the column space and row space of this matrix.
11.	Let A=[3 1 -1] and let V be the nullspace of A. Find (i)a basis for V and a basis for $V^{\perp}$(ii)a projection matrix P_1 onto $V^{\perp}$ (iii)the projection matrix P_2 onto V.
12.	Project b=(1,2,2) onto the line through a=(2,-2,1). Check if e is perpendicular to a.
13.	What point on the plane x + y - z=0 is closest to b=(2,1,0)?
14.	Find the projection of b=(3,3,3) onto the column space of A spanned by (1,0,2) and (1,1,4).Split b into p+q with p in $C(A^T)$and q in N(A).
15.	Find $\|E\|^2=\|Ax-b\|^2$ and solve the normal equations $A^TA\hat{x}=A^Tb$.Find the solution $\hat{x}$and the projection p=A$\hat{x}$. Why is p=b? (Use Least squares method) Given $A=\begin{pmatrix}1&0\\0&1\\1&1\end{pmatrix}, x=\begin{pmatrix}u\\v\end{pmatrix}, b=\begin{pmatrix}1\\3\\4\end{pmatrix}$