LINEAR ALGEBRA AND ITS APPLICATIONS
UE19MA251

Unit 3. Linear Transformations and Orthogonality

Cosines And projections Onto Lines

Definition :

If $a = (a_1, a_2)$, $b = (b_1, b_2)$ include an angle θ between them the ***cosine formula*** states that

$$\cos\theta = \frac{a_1 b_1 + a_2 b_2}{\|a\| \, \|b\|}$$

The same is true for all a, b in R^n.

Projections Onto A Line

The same is true for all a, b in R^n. To find the projection of b onto the line through a given vector ' a ', we find the point p on the line that is closest to b. This point must be some multiple of ' a ' say $p = \hat{x} a$.

Now, the line from b to the closest point p is perpendicular to the vector a and hence

$$\hat{x} = \frac{a^T b}{a^T a}$$

The point of projection is $p = \hat{x} a$

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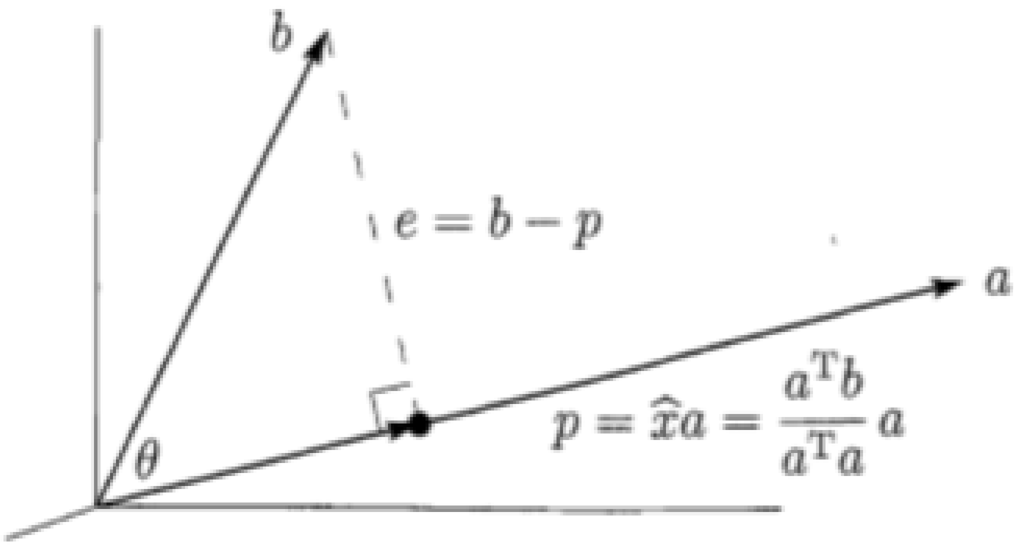
Projections Onto A Line



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Schwarz Inequality

All vectors a and b in R^n satisfy the ***Schwarz Inequality*** which is

$$|a^T b| \leq \|a\| \, \|b\|$$

Note that equality holds if and only if a and b are dependent vectors. The angle is θ = 0o or 180o. In this case, b is identical with its projection p and the distance between b and p is zero.

Projection Matrix of Rank 1

Projections onto a line through a given vector ‘ a ‘ is carried out by a ***Projection Matrix*** given by

$$P = \frac{a\,a^T}{a^T a}$$

This matrix multiplies b and produces p.

That is,

$$Pb = \frac{a\,a^T}{a^T a} b = a \frac{a^T b}{a^T a} = a\,\hat{x} = p$$

Projection Matrix of Rank 1

Note :

1. P is a symmetric matrix.
2. $P^n = P$ for n = 1, 2, 3, ……
3. The rank of P is one.
4. The trace of P is one.
5. If ‘ a’ is a n- dimensional vector then P is a square matrix of order n.
6. If ‘a ‘ is a unit vector then $P = a\,a^T$.

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THANK YOU