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BEAMS(1/2)

- At the end of this session students will be able to:
- Understand the external effects acting on beams.
- Identify the types of supports acting on beams
- Calculate the loads acting
- Differentiate various types of beams

BEAMS(2/2)

- *Beams* are structural members which offer resistance to bending due to applied loads.

- Most beams are long prismatic bars

- The loads are usually applied normal to the axes of the bars.

Types of Beams(1/2)

- Beams supported so that their external support reactions can be calculated by the methods of statics alone are called *statically determinate beams*.

- A beam which has more supports than needed to provide equilibrium is *statically indeterminate*.

Types of Beams(2/2)

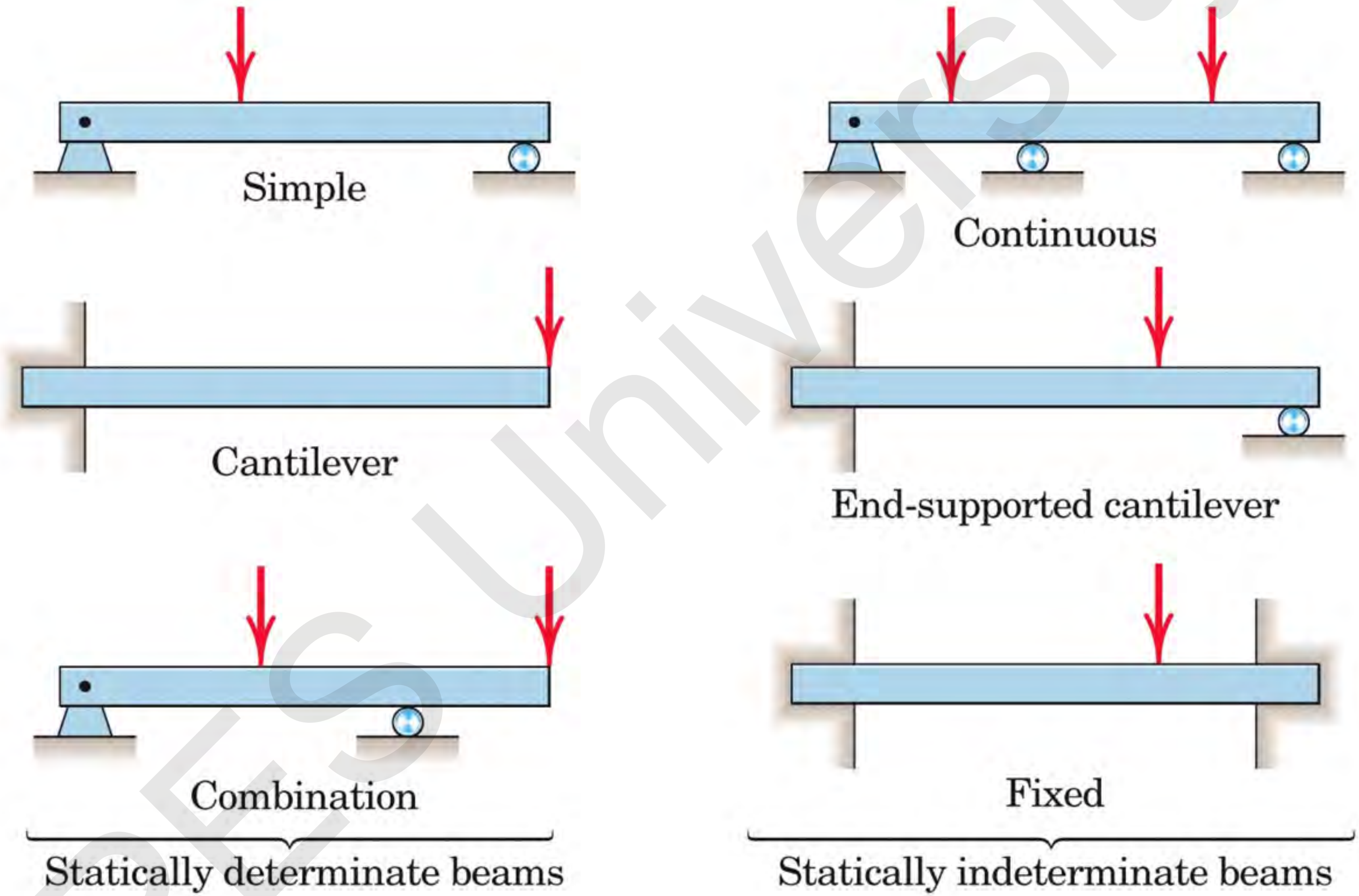


Figure 5-18

Distributed Loads(1/3)

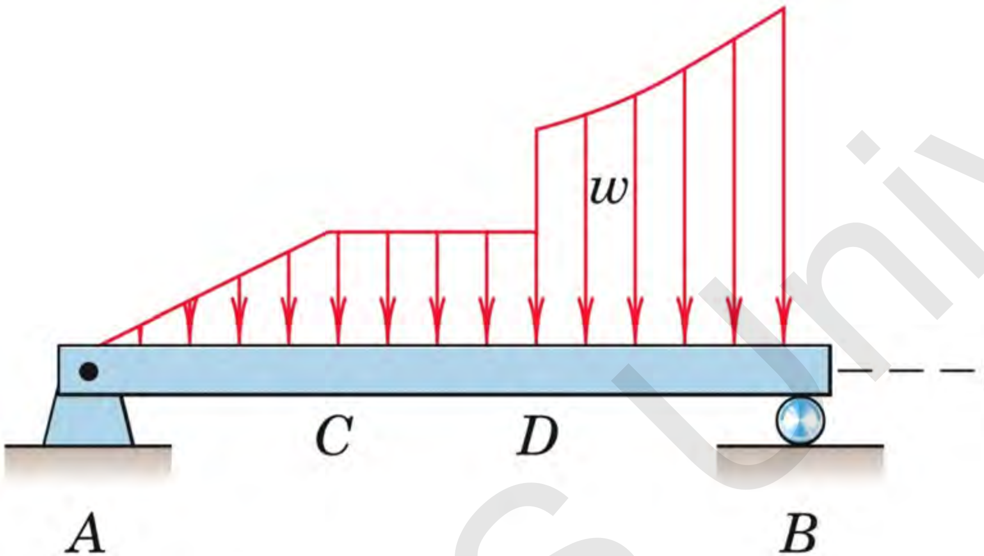


Figure 5-19

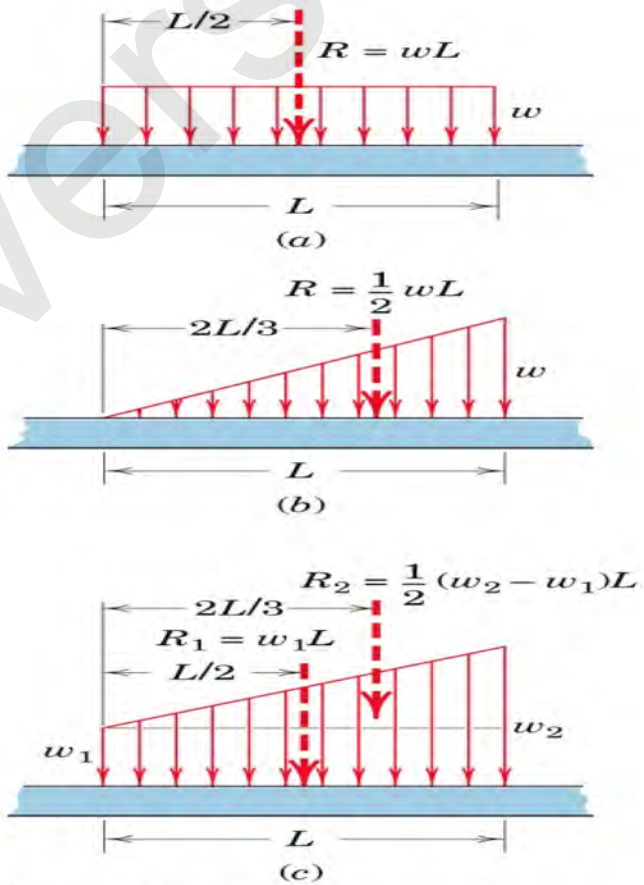


Figure 5-20

Distributed Loads(2/3)

- For a more general load distribution, we must start with a
- differential increment of force

$$dR = w\,dx.$$

- The total load *R* is then the sum of the differential forces

$$R = \int w\,dx$$

- the resultant *R* is located at the centroid of the area under consideration
- The *x*-coordinate of this centroid is found by the principle of moments

$$R\bar{x} = \int xw\,dx,$$

(or)

$$\bar{x} = \frac{\int xw\,dx}{R}$$

Distributed Loads(3/3)

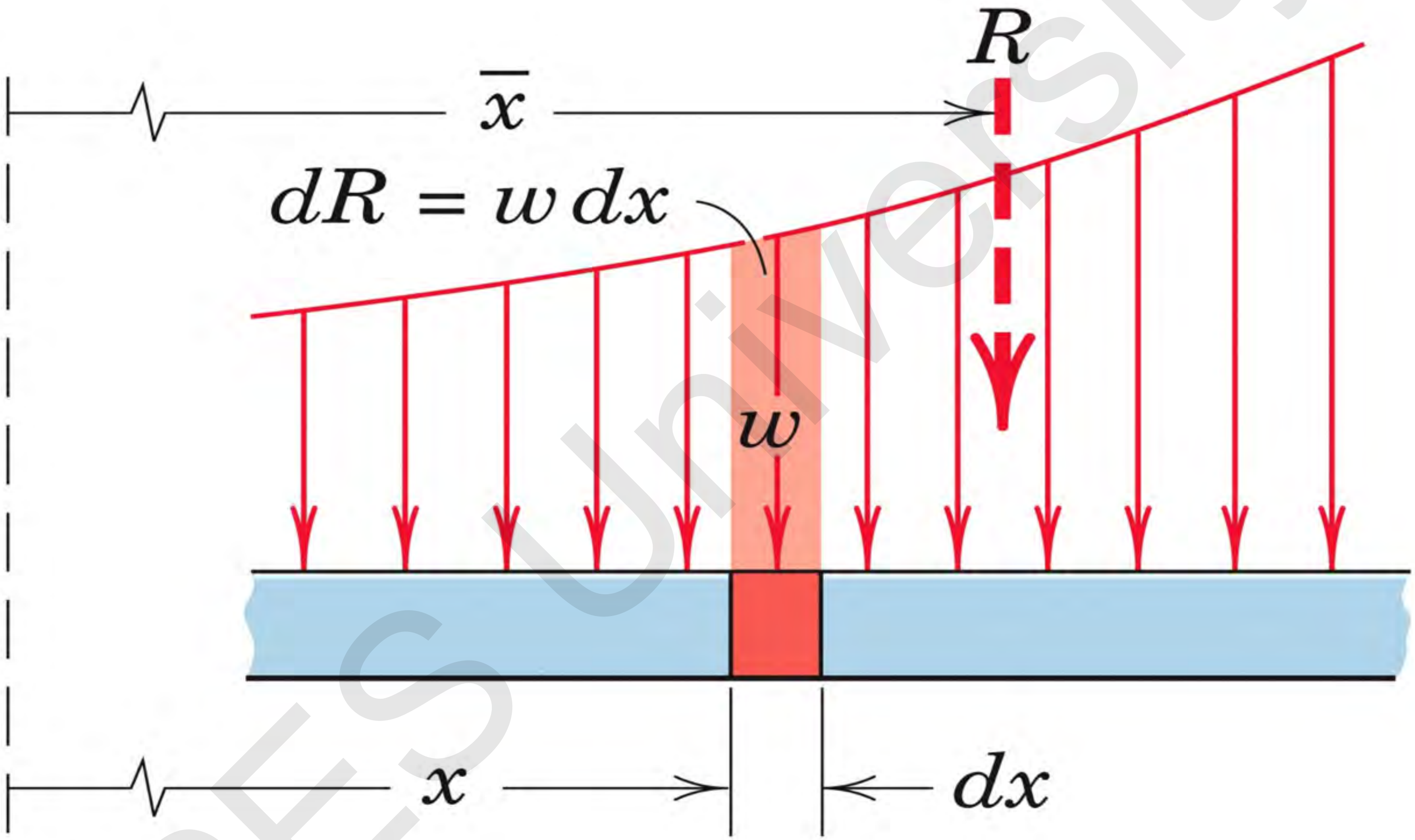
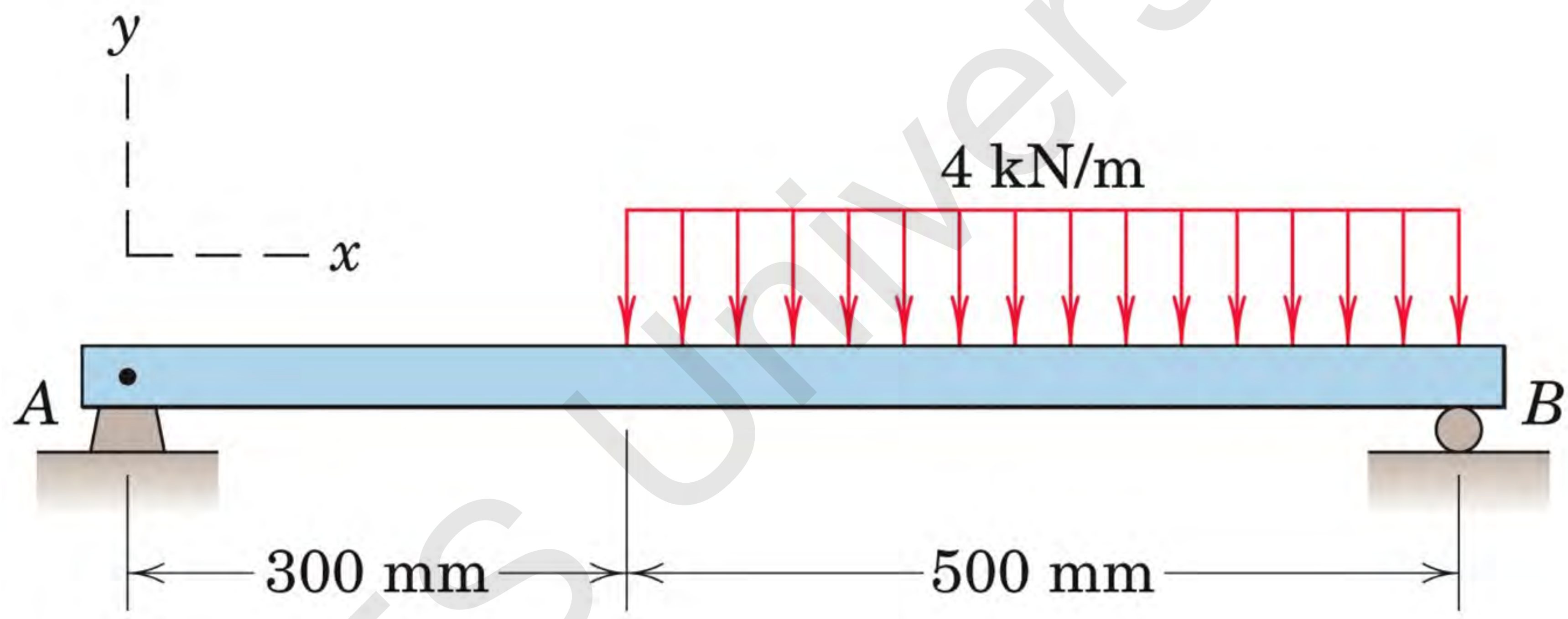


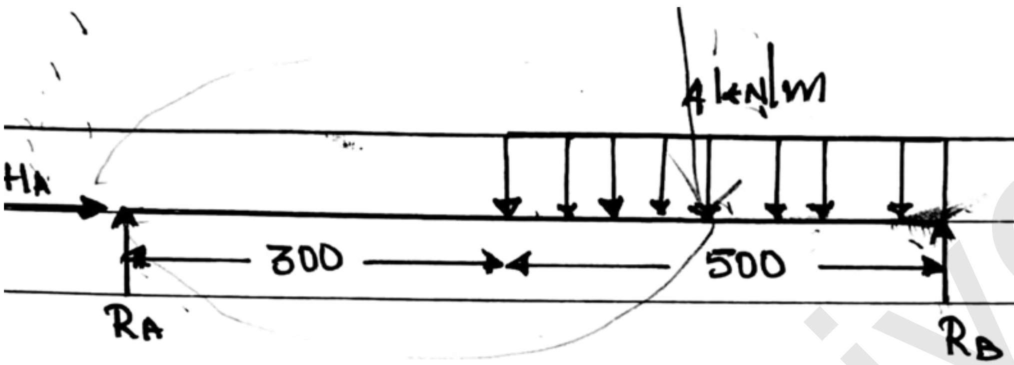
Figure 5-21

Determine the reactions at *A* and *B* for the beam
subjected to the uniform load distribution.



Problem 5-101

solution



$\Sigma M_A = 0 \Rightarrow (-R_B \times 800) + 4 \times 500 \times \left(\frac{500}{2} + 300\right) = 0$

$\Rightarrow 800\, R_B = 1100000$

$\Rightarrow R_B = 1875 \; N$

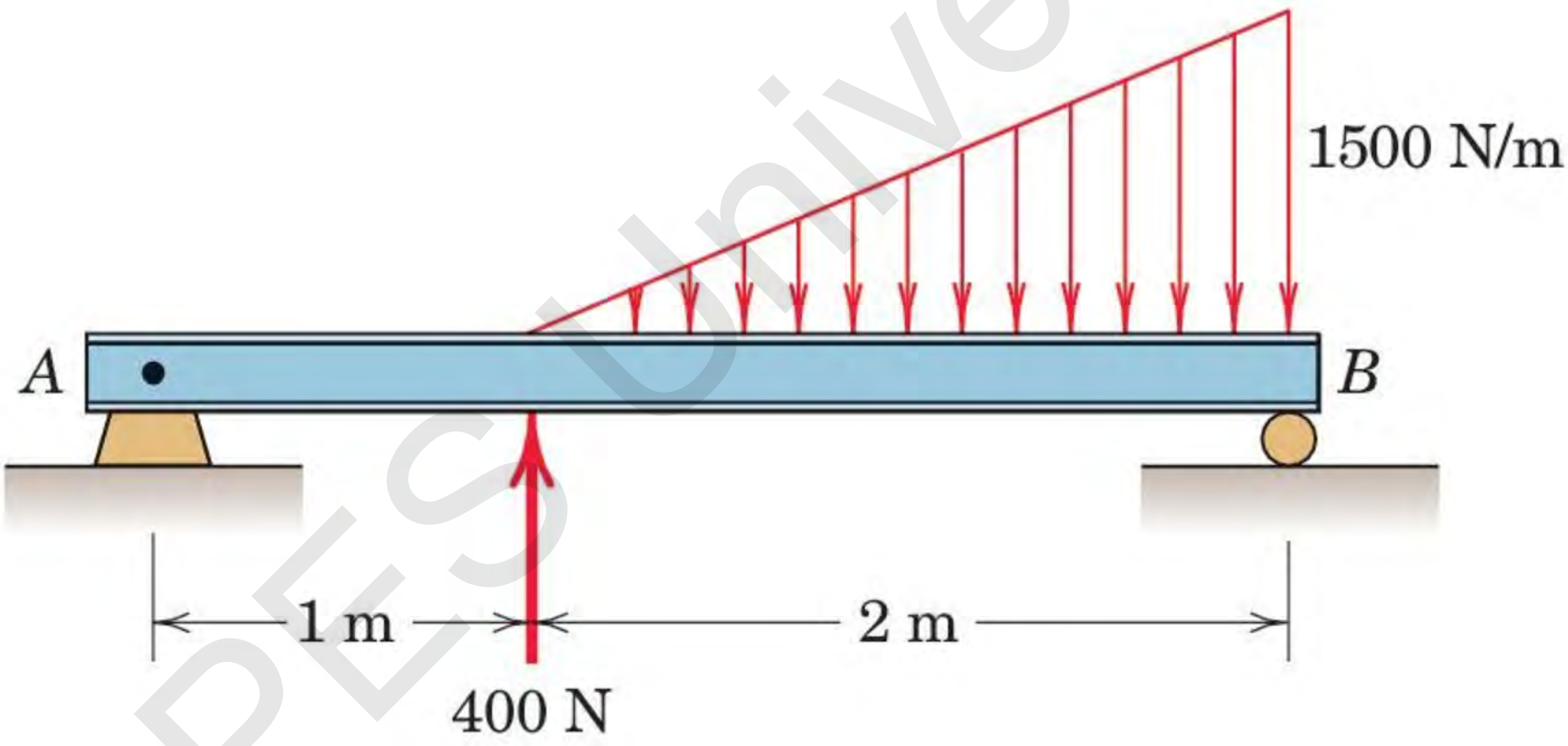
$\Sigma F_y = 0 \Rightarrow R_A + R_B - 4 \times 500 = 0$

$\Rightarrow R_A + R_B = 2000$

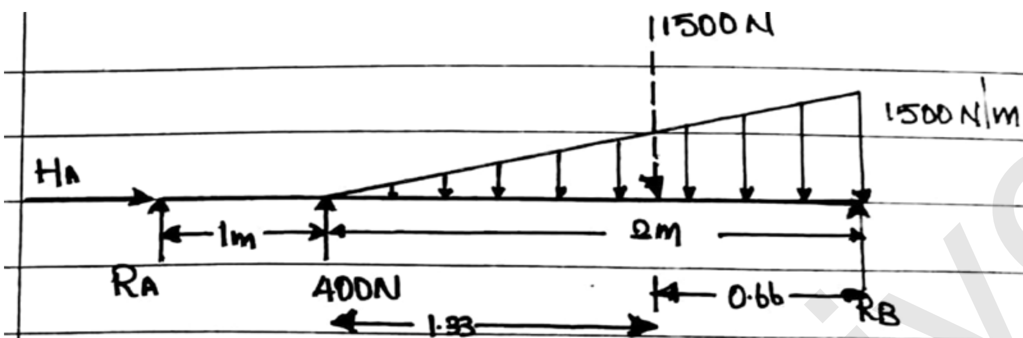
$\Rightarrow R_A = 625 \; N$

$\Sigma F_x = 0 \Rightarrow H_A = 0$

Calculate the support reactions at *A* and *B* for the loaded beam.



solution



$R = \frac{1}{2} \times 2 \times 1500 = 1500\,N$ located @ $\bar{x} = \frac{b}{3} = \frac{2}{3}$ from B

$= 0.666\,m$

$\Sigma M_A = 0 \Rightarrow -R_B \times 3 + 1500 \times 2.3334 - 400 \times 1 = 0$

$\Rightarrow 3R_B = 3100.1$

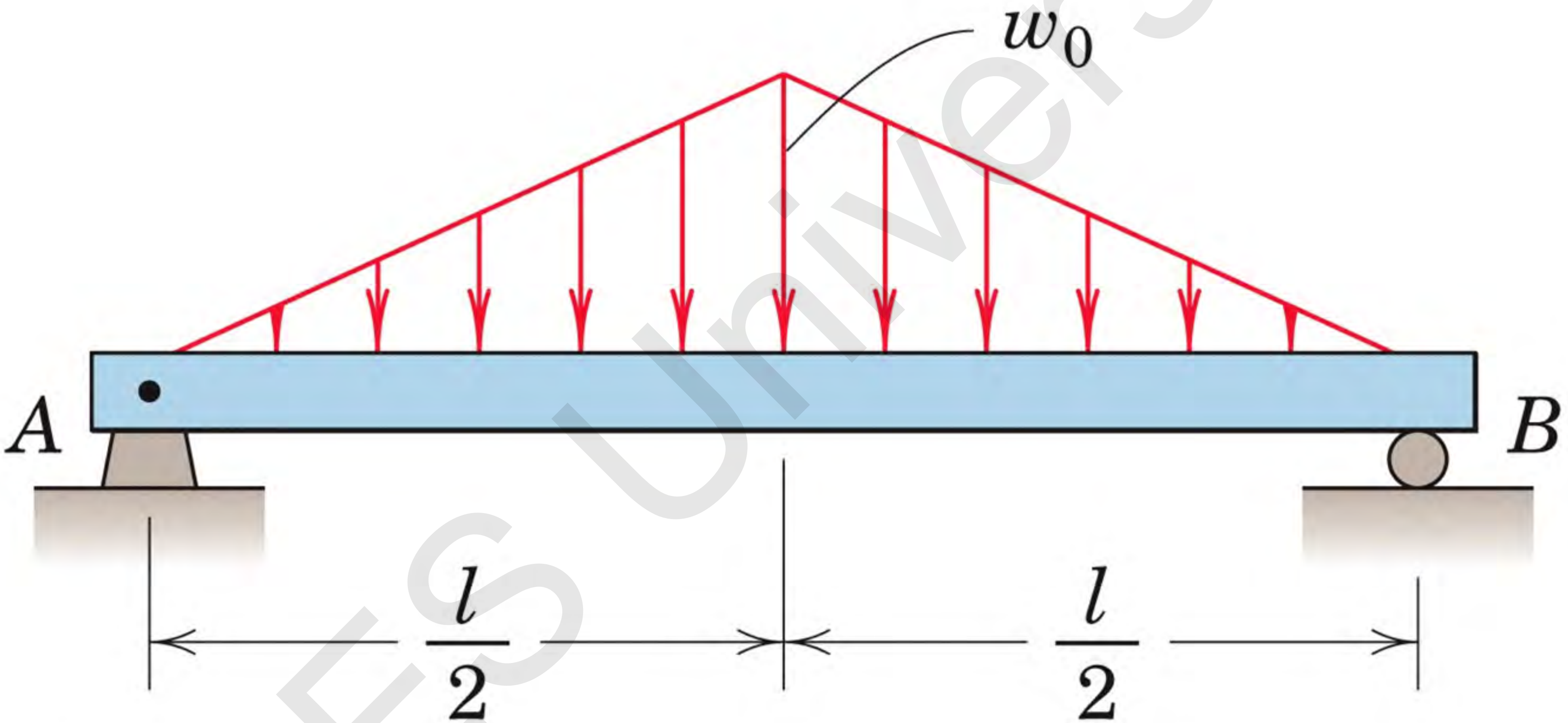
$\Rightarrow R_B = 1033.36\,N$

$\Sigma F_y = 0 \Rightarrow R_A + R_B + 400 - 1500 = 0$

$\Rightarrow R_A + R_B = 1100$

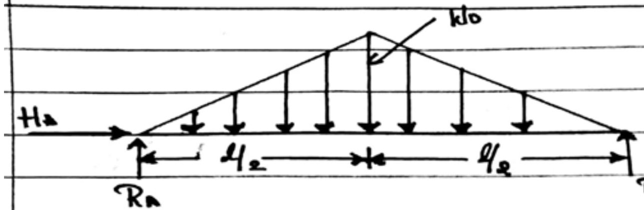
$\Rightarrow R_A = 66.7\,N$

Determine the reactions at the supports *A* and *B* for the beam loaded as shown.



Problem 5-107

solution



Resultant, $R = \left[\frac{1}{2} \times \frac{l}{2} \times w_0\right] 2 = \frac{1}{2} w_0 l$ @ $\bar{x} = l/2$

$$\Sigma M_A = 0 \Rightarrow (R_B \times l) - \frac{1}{2} w_0 l \times \frac{l}{2} = 0$$

$$\Rightarrow R_B \times l = \frac{w_0 l^2}{4}$$

$$\Rightarrow R_B = \frac{w_0 l}{4}$$

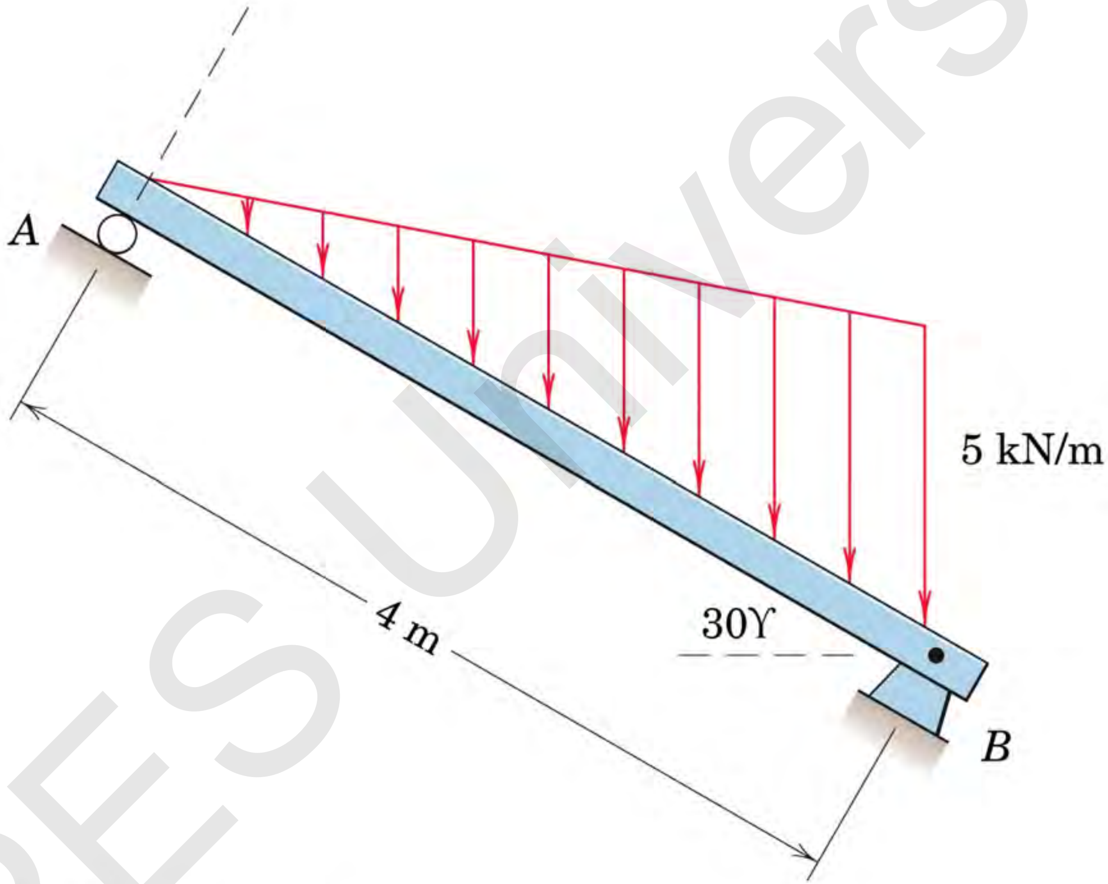
$$\Sigma F_y = 0 \Rightarrow \frac{w_0 l}{4} - \frac{w_0 l}{2} + R_A = 0$$

$$\Rightarrow R_A = \frac{w_0 l}{2} - \frac{w_0 l}{4}$$

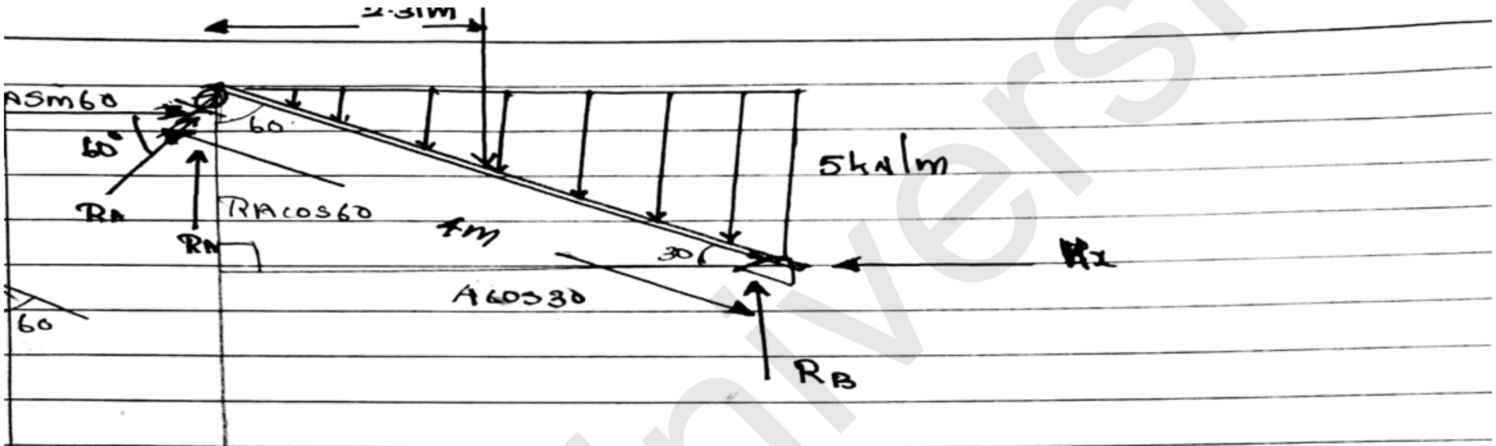
$$= \frac{2w_0 l - w_0 l}{4}$$

$$R_A = \frac{w_0 l}{4}$$

Determine the reactions at points *A* and *B* of the inclined beam subjected to the vertical load distribution shown. The value of the load distribution at the right end of the beam is 5 kN per *horizontal* m



solution



$$\therefore\ R = \frac{1}{2} \times b \times h = \frac{1}{2} \times 4\cos 30 \times 5 = 8.66\ kN$$

$$\bar{x} = \frac{2}{3} \times 4\cos 30 = 2.31\ m$$

$$\Sigma F_x = 0 \Rightarrow R_A \sin 60 - H_x = 0 \quad \text{——} \quad (1)$$

$$\Sigma M_B = 0 \Rightarrow R_A \times 4 - R[4\cos 30 - 2.31] = 0$$

$$\Rightarrow 4R_A = 8.66[4\cos 30 - 2.31] = 0$$

$$\Rightarrow R_A = 2.5\ kN$$

$$\Sigma F_y = 0 \Rightarrow R_A \cos 60 - R - R_B = 0$$

$$\Rightarrow 2.5\cos 60 - 8.66 - R_B = 0$$

$$\Rightarrow R_B = 6.50\ kN$$

$$\Rightarrow H_x = 1.25\ kN$$

Two-Force Members

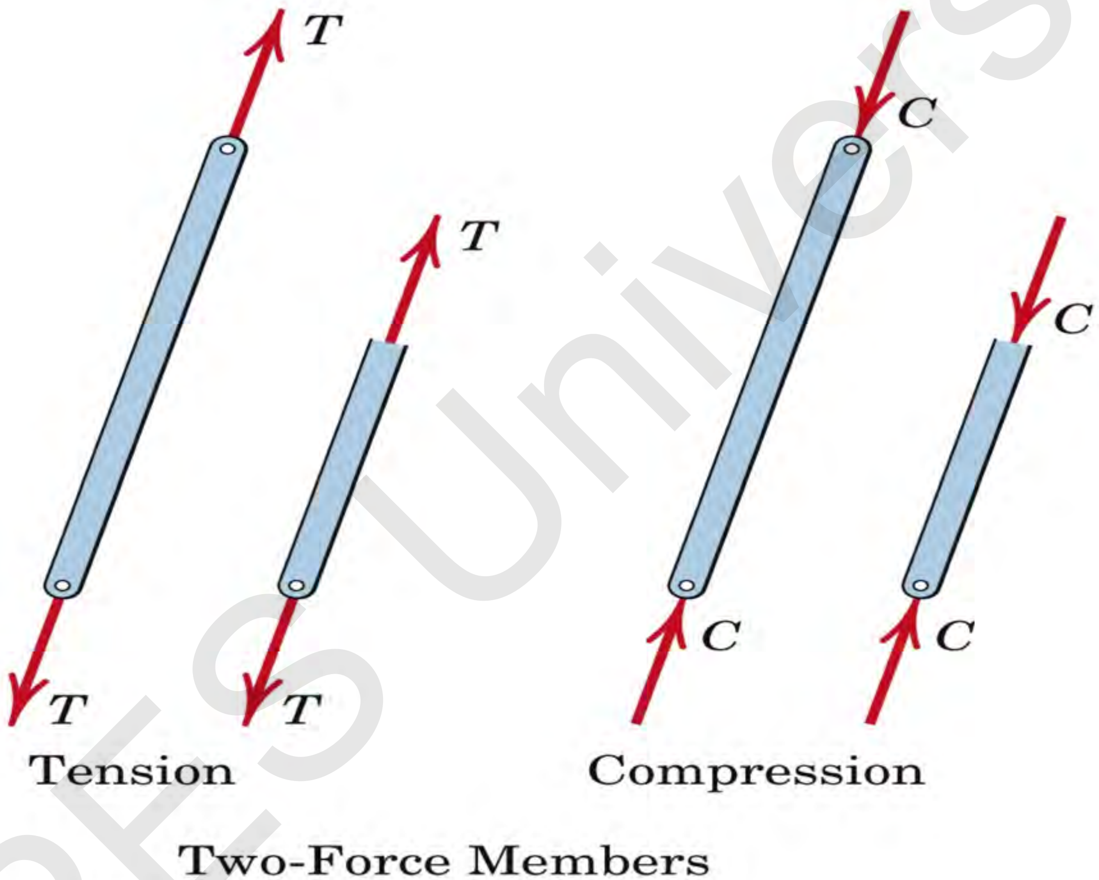


Figure 4-4

Method of Joints(1/2)

- This method for finding the forces in the members of a truss consists of satisfying the conditions of equilibrium for the forces acting on the connecting pin of each joint.
- The method therefore deals with the equilibrium of concurrent forces, and only two independent equilibrium equations are involved.

Method of Joints(2/2)

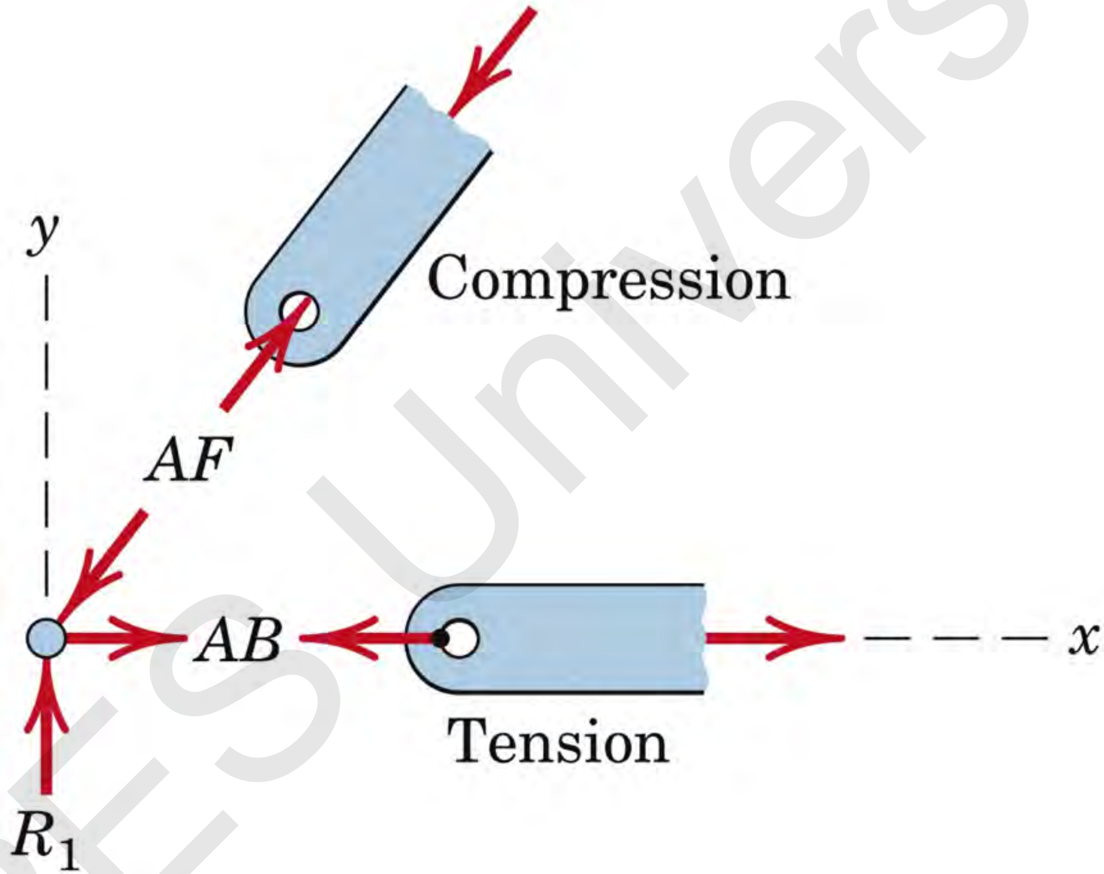


Figure 4-7

Indicating Tension T & Compression C of Various Members on the Original Truss Diagram

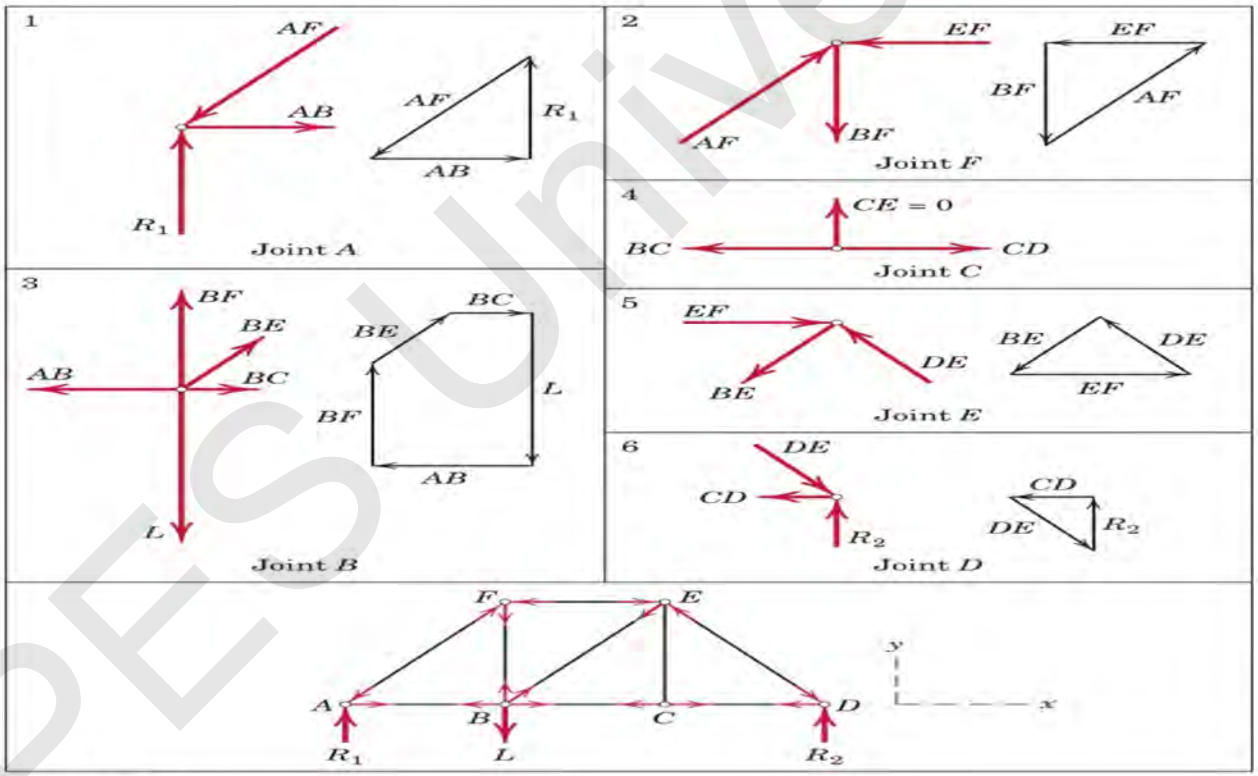


Figure 4-8

Internal and External Redundancy

- If a plane truss has more external supports than are necessary to ensure a stable equilibrium configuration, the truss as a whole is statically indeterminate, and the extra supports constitute *external* redundancy.
- If a truss has more internal members than are necessary to prevent collapse when the truss is removed from its supports, then the extra members constitute *internal* redundancy and the truss is again statically indeterminate.

STRUCTURES

- By the end of this session students will be able to:
- Understand and define trusses
- Differentiate plane trusses
- Analyse the trusses using method of joints
- Identify the nature of forces acting in the internal member of trusses
- Differentiate between member and joint
- Understand internal indeterminacy

STRUCTURES - Introduction (1/2)

- In engineering, a **truss** is a structure that "consists of two-force members only, where the members are organized so that the assemblage as a whole behaves as a single object".
- A "two-force member" is a structural component where force is applied to only two points.

Truss bridge for a single-track railway, converted to pedestrian use and pipeline support

PLANE TRUSS

- A **plane truss** is **defined** as a two- dimensional framework of straight prismatic members connected at their ends by frictionless hinged joints, and subjected to loads and reactions that act only at the joints

- When the members of the truss lie essentially in a single plane, the truss is called a *plane truss*.

- For bridges and similar structures, plane trusses are commonly utilized in pairs with one truss assembly placed on each side of the structure.

A Section of a Typical Bridge Structure

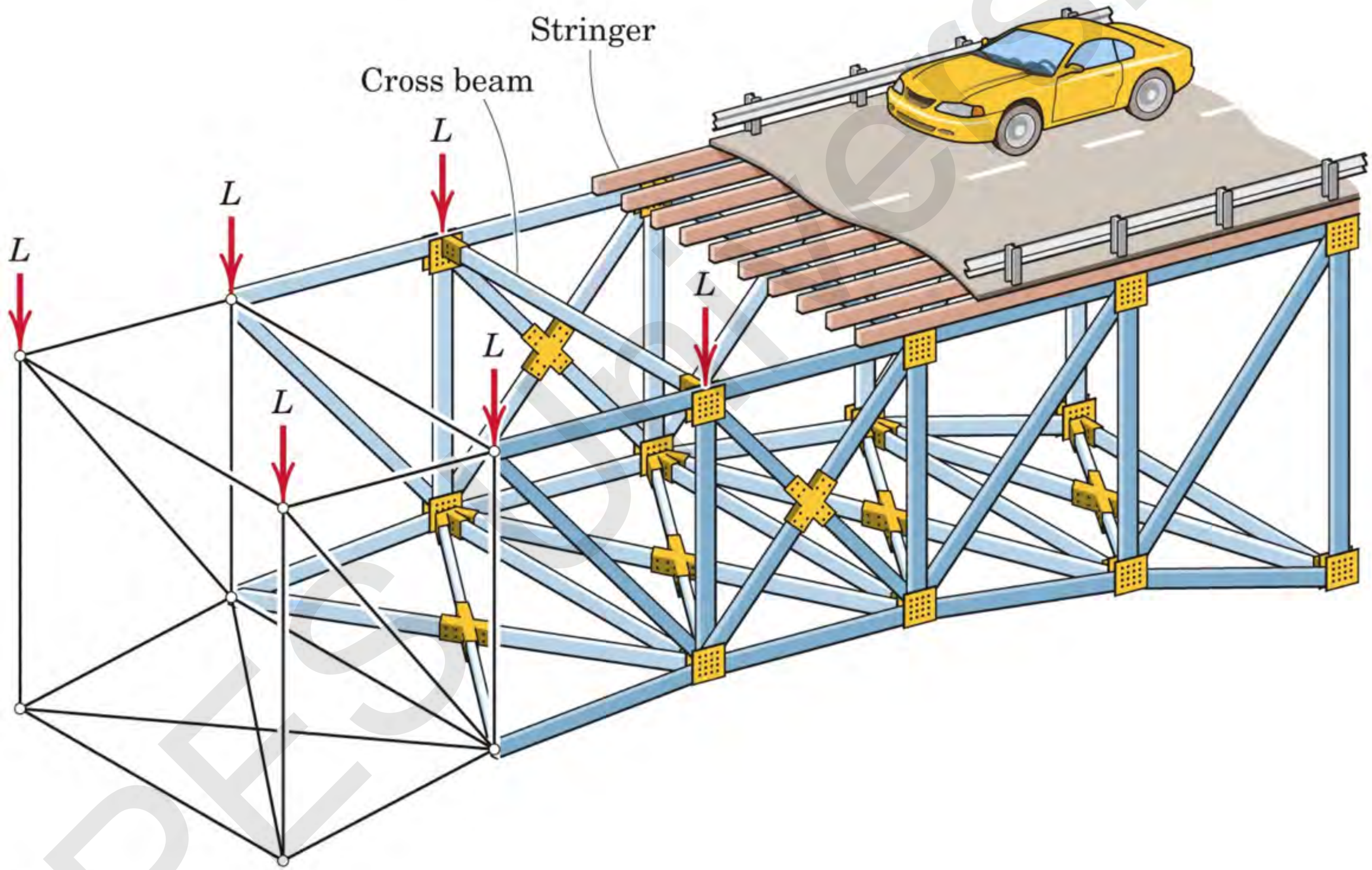
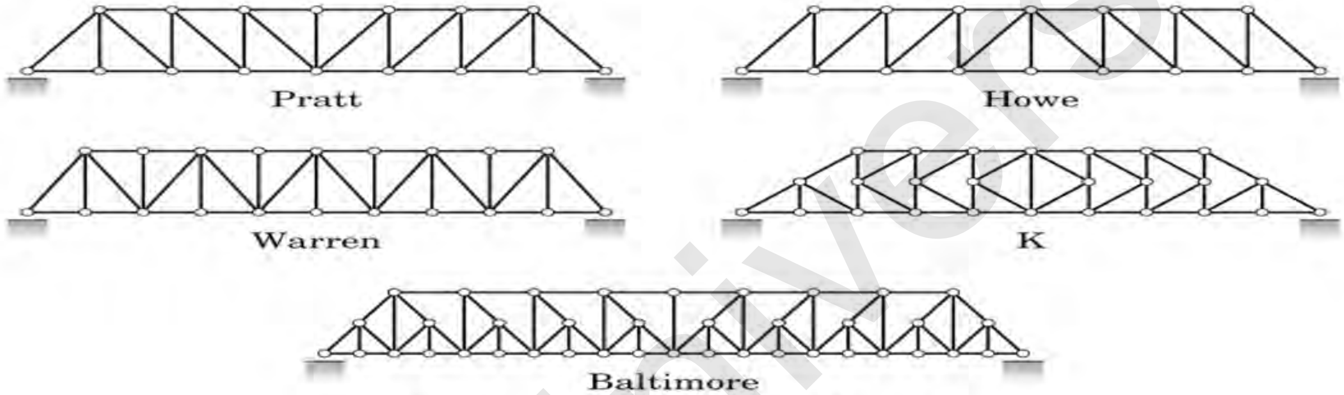
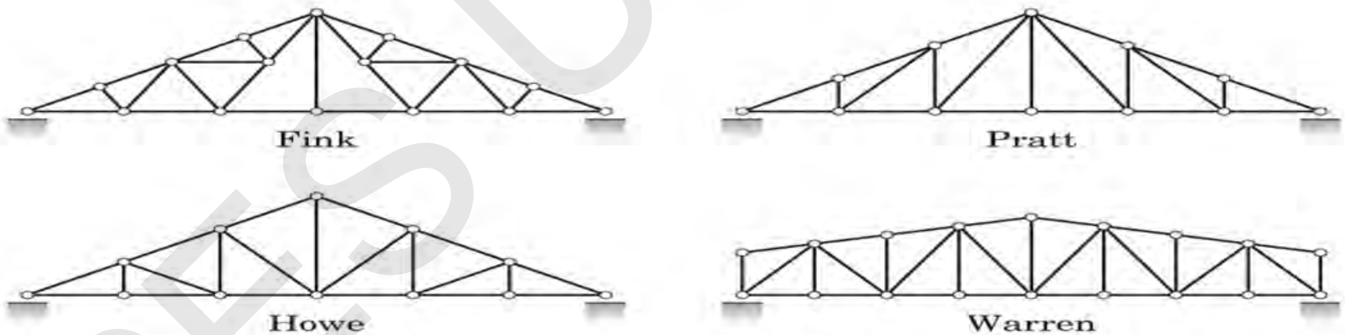


Figure 4-1

Examples of commonly used trusses which can be analysed as plane trusses



Commonly Used Bridge Trusses



Commonly Used Roof Trusses

Figure 4-2

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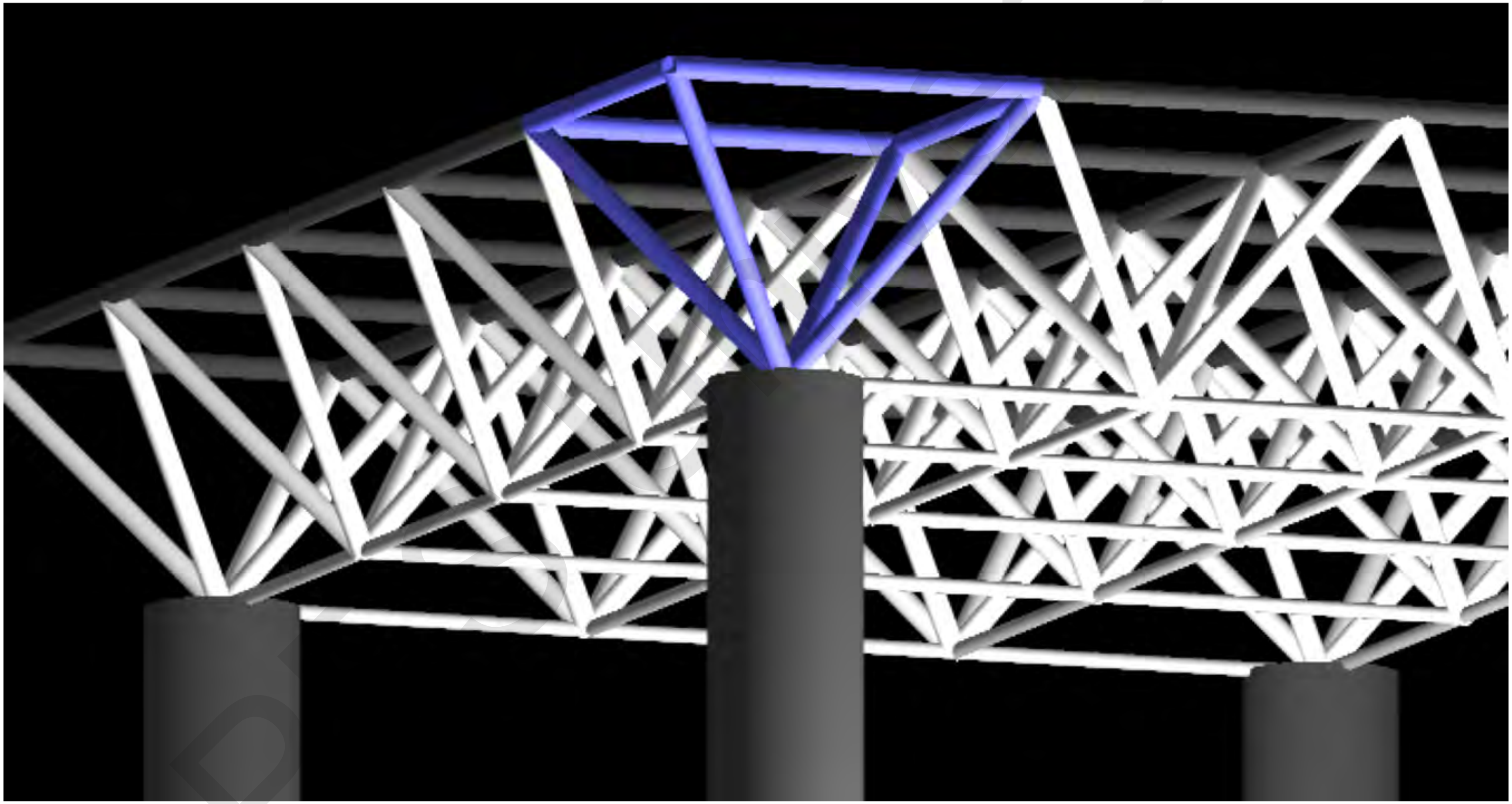
- Planar roof truss

- The roof trusses of the Basilica di Santa Croce in Florence

- Simple tetrahedron

- This electrical pylon is a three-dimensional truss structure

Diagram of a planar space frame such as used for a roof



Rigid & Non Rigid Frames

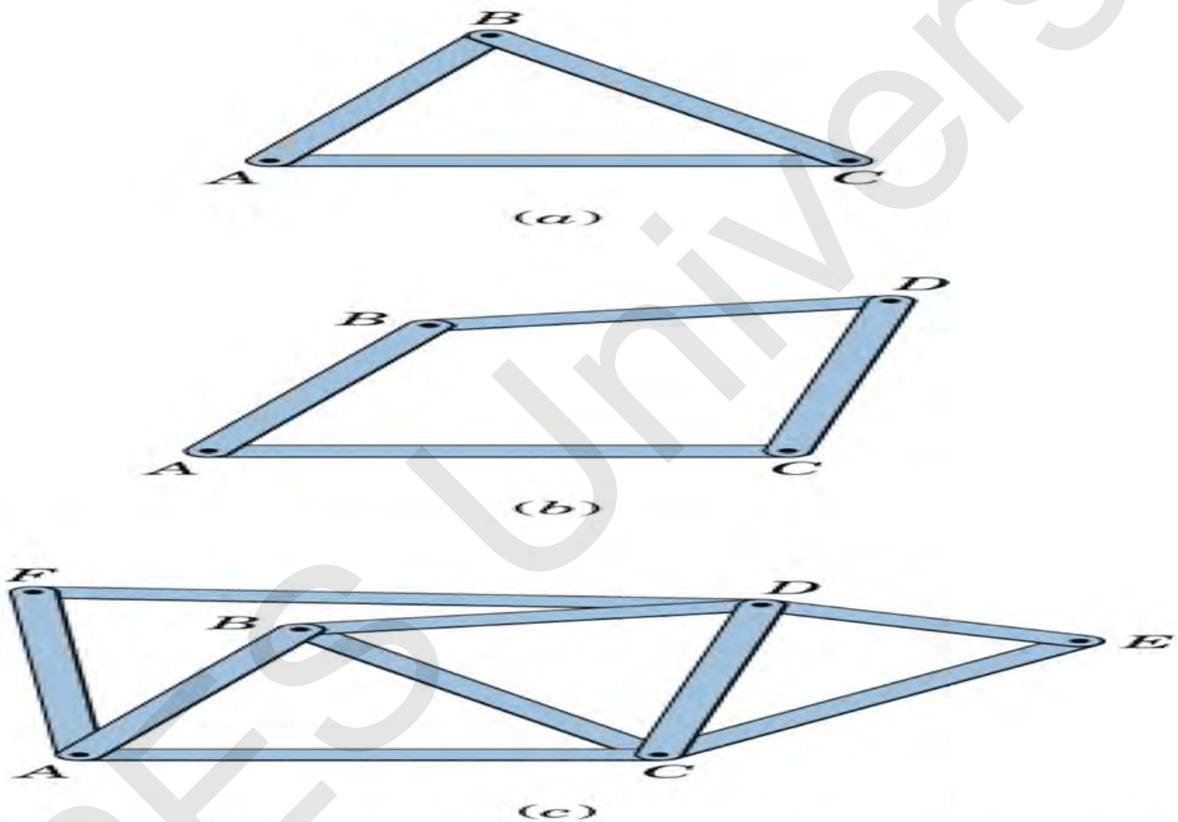
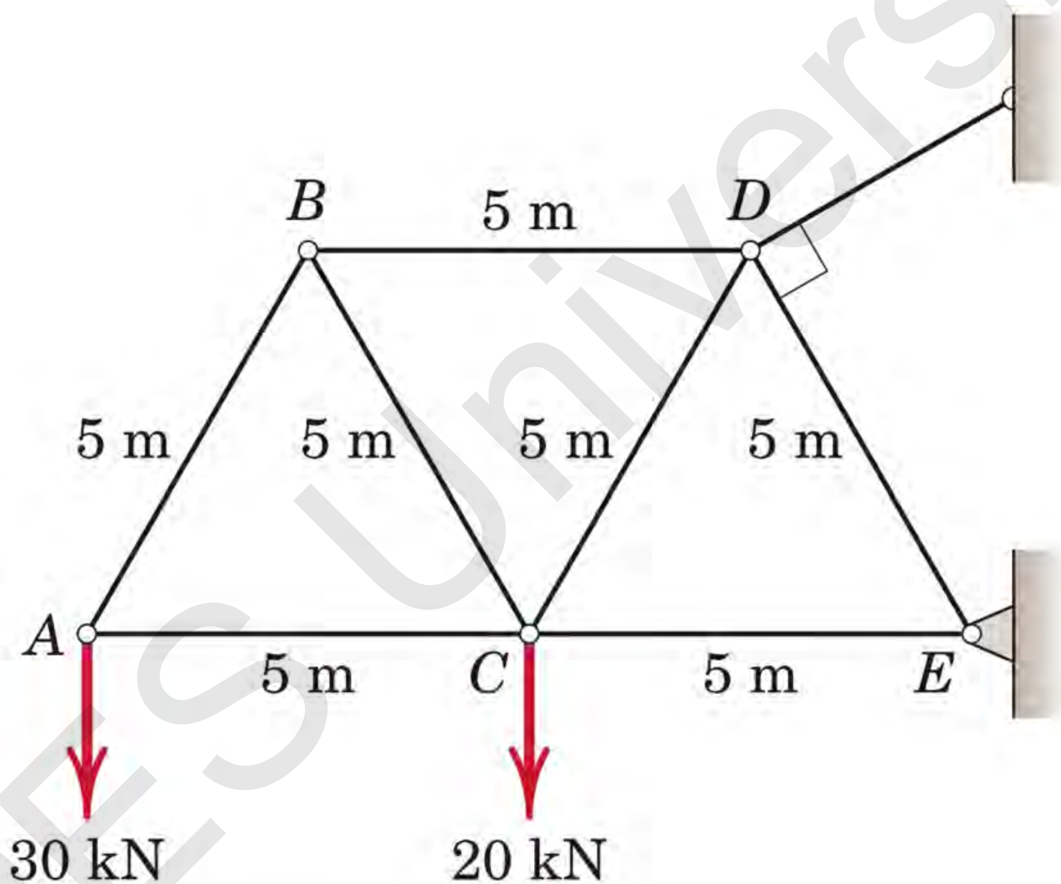


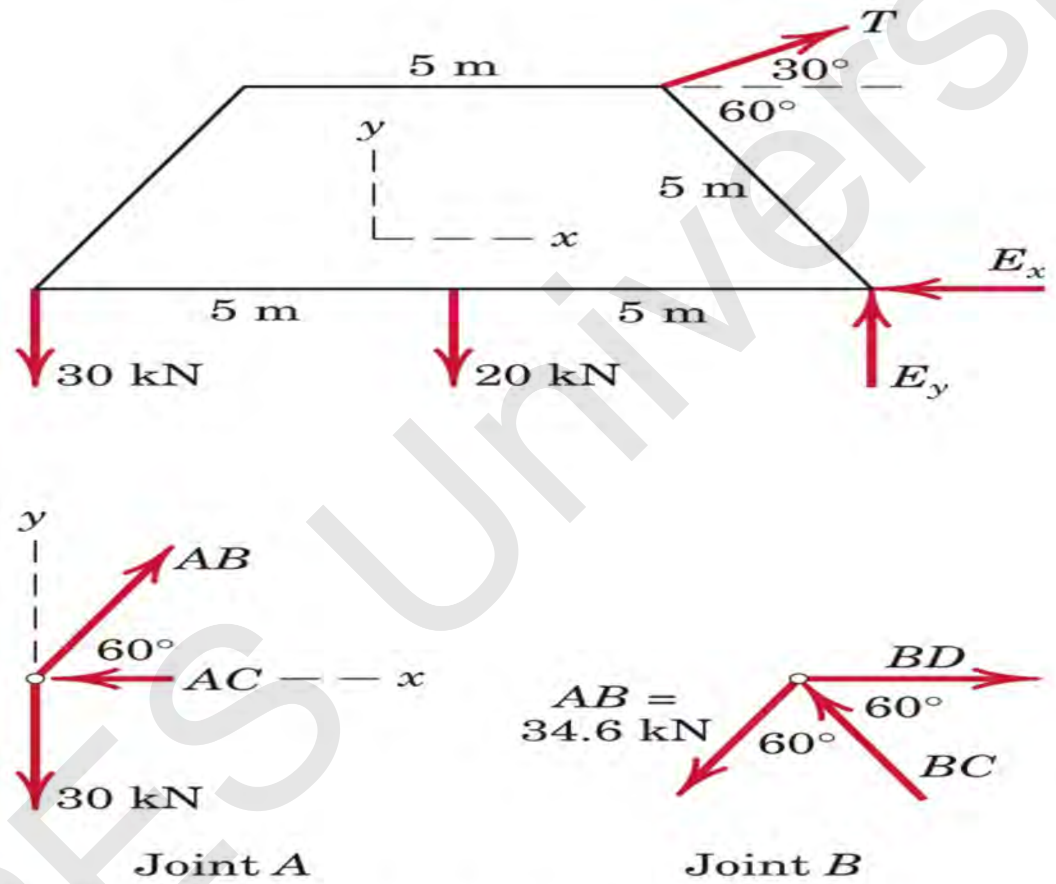
Figure 4-3

Compute the force in each member of the loaded cantilever truss by the method of joints.

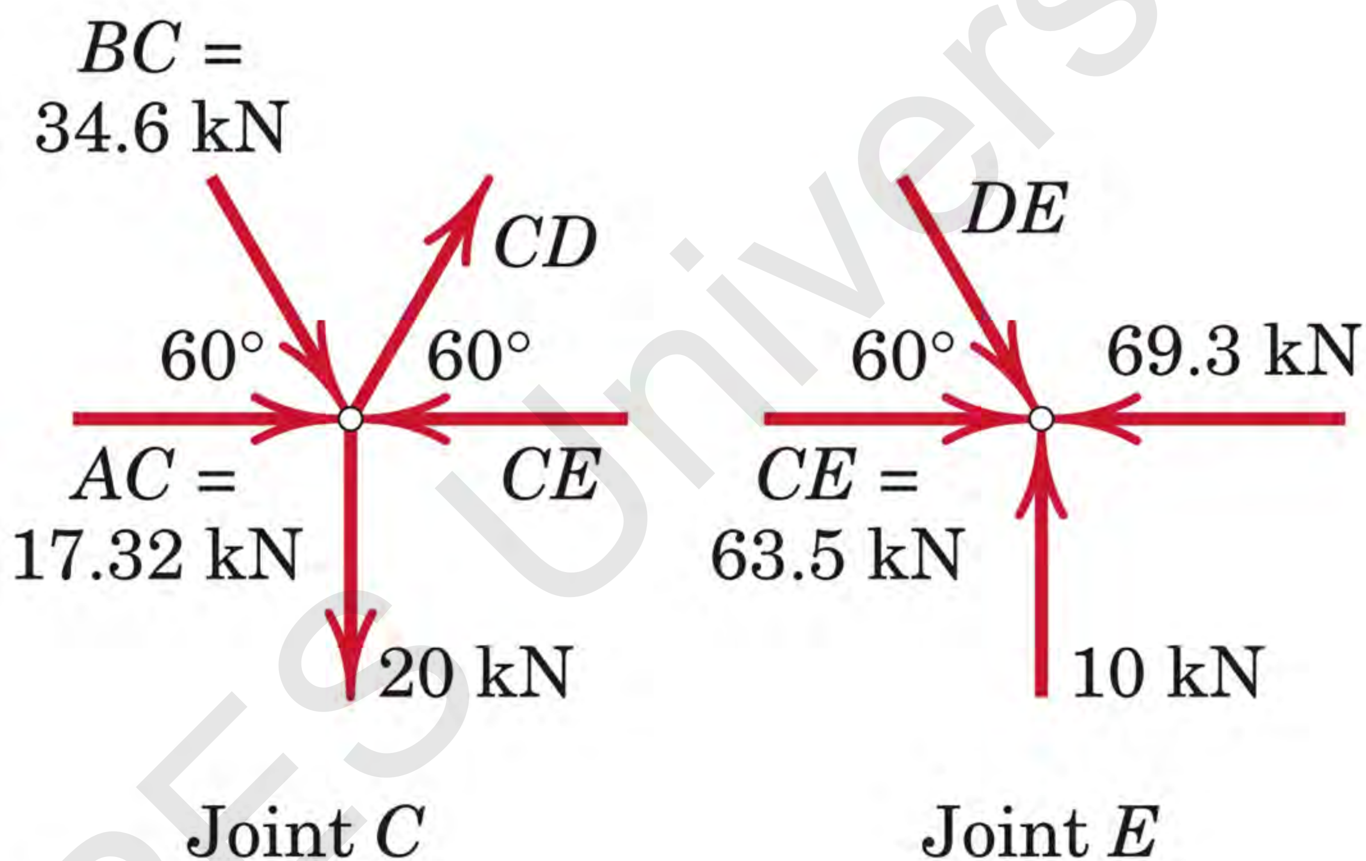


Sample Problem 4.1a

Solution



Sample Problem 4.1b



Sample Problem 4.1c

Solution. If it were not desired to calculate the external reactions at D and E, the analysis for a cantilever truss could begin with the joint at the loaded end. However, this truss will be analyzed completely, so the first step will be to compute the external forces at D and E from the free-body diagram of the truss as a whole. The equations of equilibrium give

$[\Sigma M_E = 0]$	$5T - 20(5) - 30(10) = 0$	$T = 80$ kN
$[\Sigma F_x = 0]$	$80 \cos 30° - E_x = 0$	$E_x = 69.3$ kN
$[\Sigma F_y = 0]$	$80 \sin 30° + E_y - 20 - 30 = 0$	$E_y = 10$ kN

Next we draw free-body diagrams showing the forces acting on each of the connecting pins. The correctness of the assigned directions of the forces is verified when each joint is considered in sequence. There should be no question about the correct direction of the forces on joint A. Equilibrium requires

$[\Sigma F_y = 0]$	$0.866AB - 30 = 0$	$AB = 34.6$ kN T	*Ans.*
$[\Sigma F_x = 0]$	$AC - 0.5(34.6) = 0$	$AC = 17.32$ kN C	*Ans.*

where T stands for tension and C stands for compression.

Joint B must be analyzed next, since there are more than two unknown forces on joint C. The force BC must provide an upward component, in which case BD must balance the force to the left. Again the forces are obtained from

$$[\Sigma F_y = 0] \qquad 0.866BC - 0.866(34.6) = 0 \qquad BC = 34.6 \text{ kN } C \qquad \textit{Ans.}$$

$$[\Sigma F_x = 0] \qquad BD - 2(0.5)(34.6) = 0 \qquad BD = 34.6 \text{ kN } T \qquad \textit{Ans.}$$

Joint C now contains only two unknowns, and these are found in the same way as before:

$$[\Sigma F_y = 0] \qquad 0.866CD - 0.866(34.6) - 20 = 0$$

$$CD = 57.7 \text{ kN } T \qquad \textit{Ans.}$$

$$[\Sigma F_x = 0] \qquad CE - 17.32 - 0.5(34.6) - 0.5(57.7) = 0$$

$$CE = 63.5 \text{ kN } C \qquad \textit{Ans.}$$

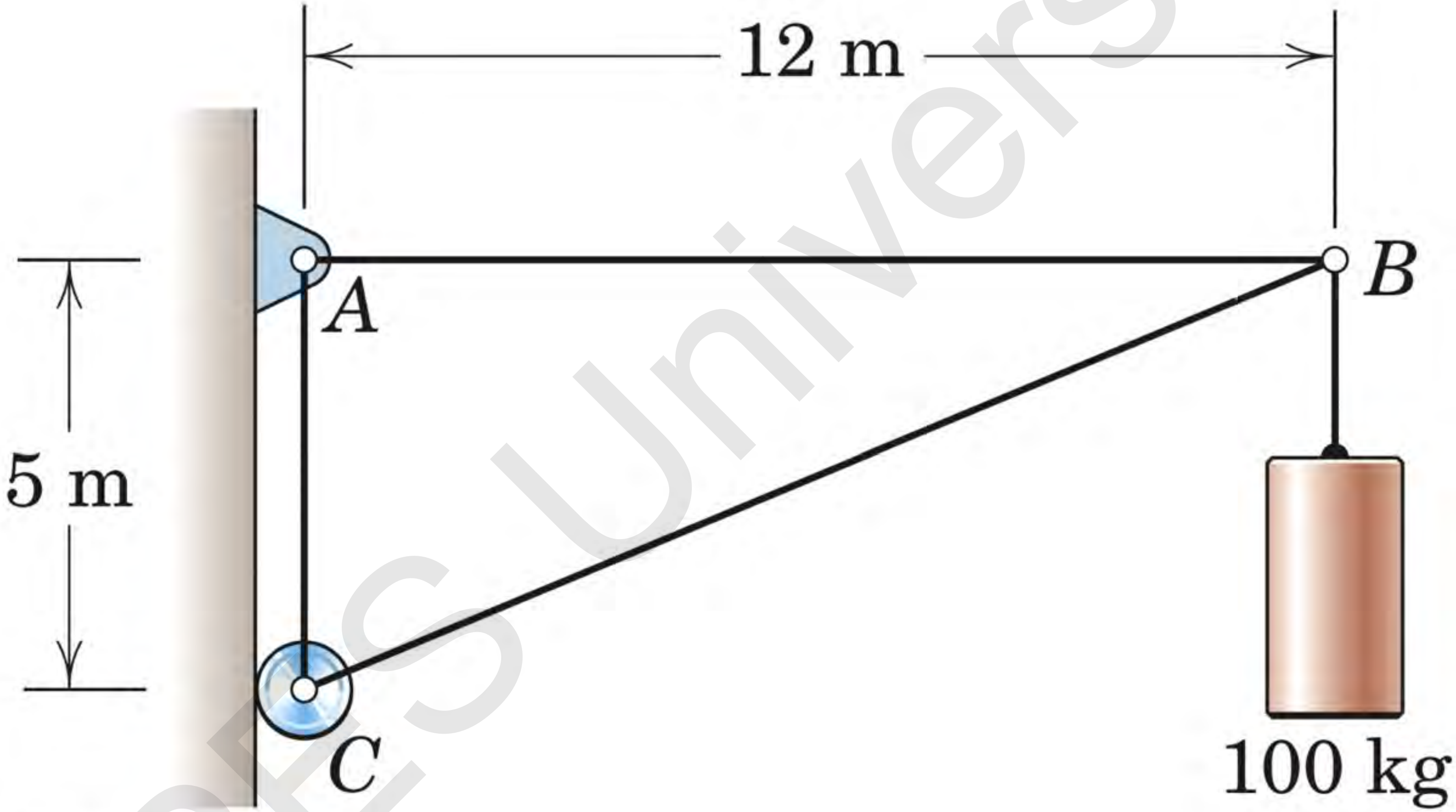
Finally, from joint E there results

$$[\Sigma F_y = 0] \qquad 0.866DE = 10 \qquad DE = 11.55 \text{ kN } C \qquad \textit{Ans.}$$

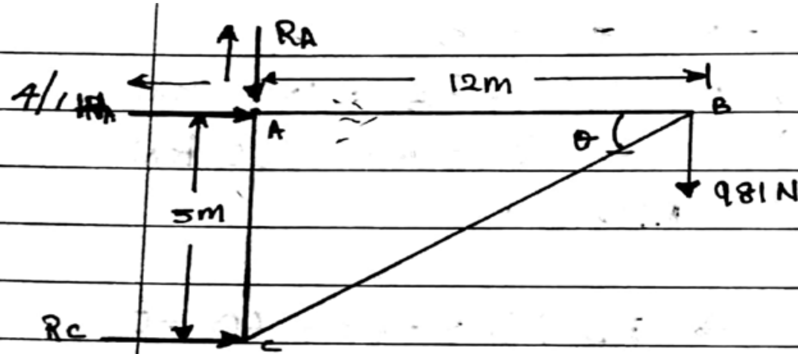
and the equation $\Sigma F_x = 0$ checks.

Note that the weights of the truss members have been neglected in comparison with the external loads.

Determine the force in each member of the loaded truss.



Problem 4-1



$\tan\theta = 5/12 \Rightarrow \theta = 22.66$

$\Sigma V = 0 \Rightarrow -R_A - 981 = 0$

$\Rightarrow R_A = -981\ N$

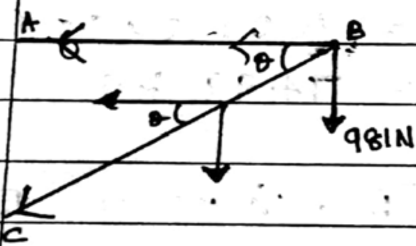
$\Sigma H = 0 \Rightarrow H_A + R_C = 0$

$\Sigma M_A = 0 \Rightarrow -5R_C + 981 \times 12 = 0$

$\Rightarrow R_C = 2354\ N$

$\therefore H_A = -2354\ N$

Joint B



$\Sigma V = 0 \Rightarrow -981 - BC \sin 22.66 = 0$

$\Rightarrow -981 = BC \sin 22.66$

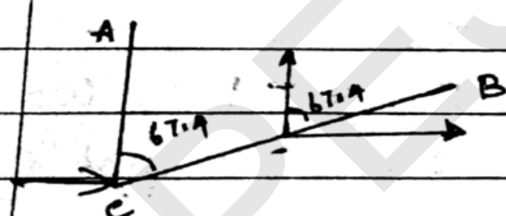
$\Rightarrow BC = -2552.7N$ C

$\Sigma H = 0 \Rightarrow -AB - BC \cos 22.66 = 0$

$\Rightarrow AB = -BC \cos 22.66$

$\Rightarrow AB = 2356.6N$ T

Joint C



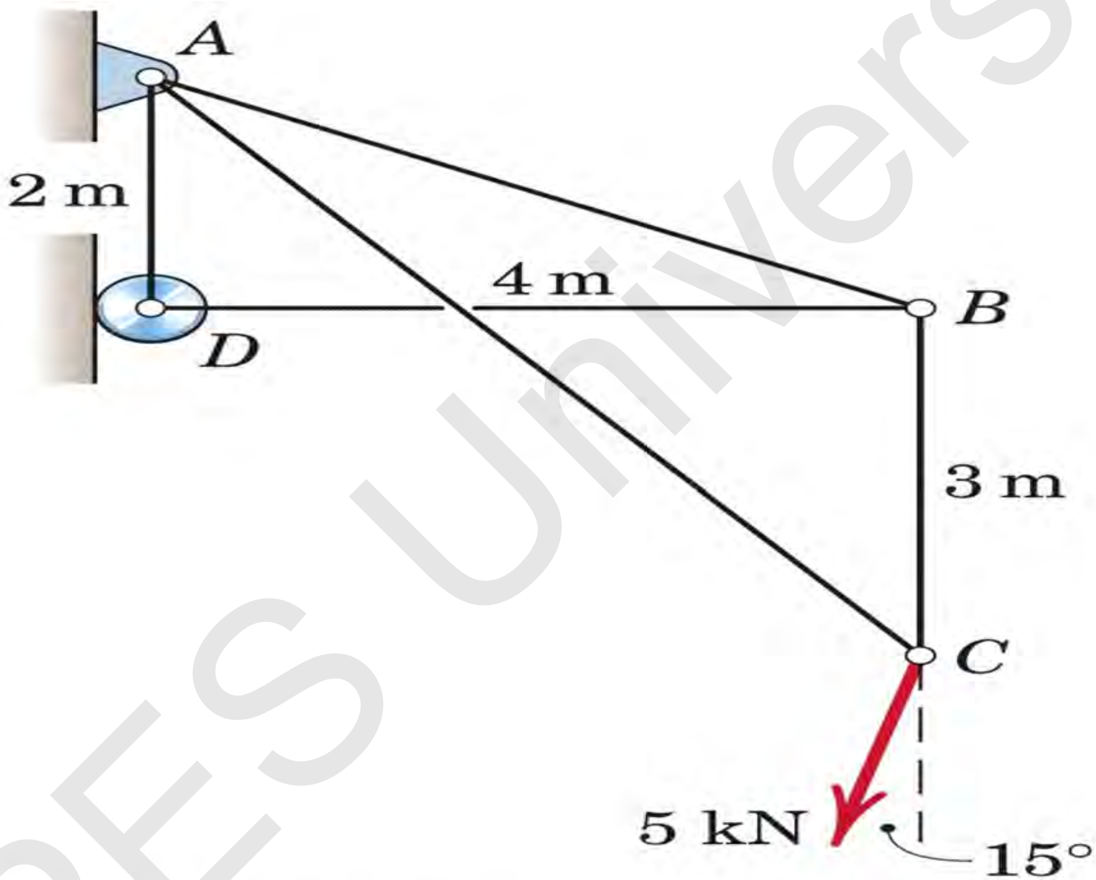
$\Sigma V = 0 \Rightarrow CA + CB \cos 67.4 = 0$

$\Rightarrow CA = -CB \cos 67.4$

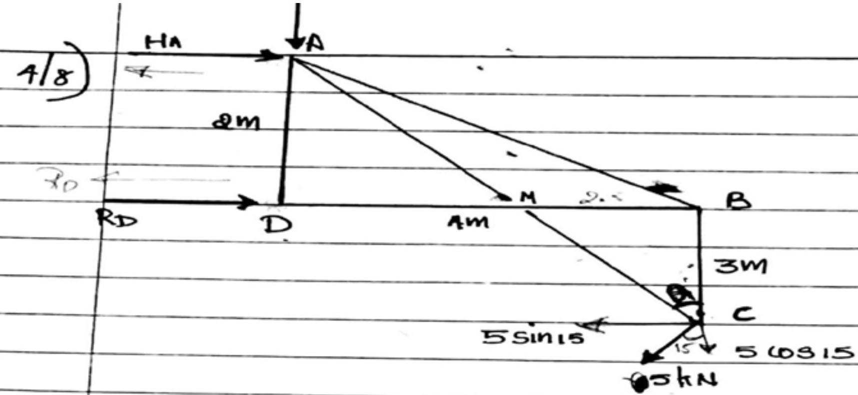
$\Rightarrow CA = 980\ N$ T

$\Sigma H = 0$

Determine the force in each member of the loaded truss.



Problem 4-8



Similar triangles

$\frac{BM}{DM} = \frac{BC}{AD}$

$\frac{BM}{DM} = \frac{3}{2} \rightarrow 2BM = 3DM$

$\Rightarrow$ DM = 0.66 BM ~~BM = 0.6 DM~~

$\Rightarrow DM + BM = 4$

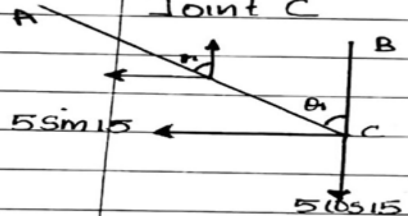
$\Rightarrow 0.66 BM + BM = 4$

$\Rightarrow 1.66 BM = 4$

$\Rightarrow 2.40 = BM$

$\theta_1 = \tan^{-1}\left(\frac{2.4}{3}\right) = 38.65°$

Joint C



$\Sigma H = 0 \Rightarrow -5\sin 15 - AC\sin 38.65 = 0$

$\Rightarrow AC \sin 38.65 = -5\sin 15$

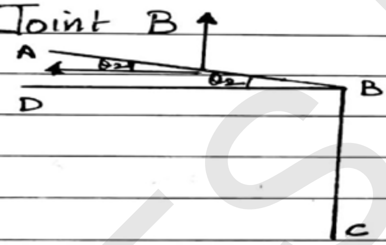
$\Rightarrow \underline{AC = -2.07 \text{ kN} \quad C}$

$\Sigma V = 0 \Rightarrow CB - 5\cos 15 + AC\cos 38.65 = 0$

$\Rightarrow CB = 5\cos 15 - AC\cos 38.65$

$\Rightarrow \underline{CB = 6.44 \text{ kN} \quad T}$

Joint B



$\theta_2 = \tan^{-1}\left(\frac{2}{4}\right) = 26.56$

$\Sigma H = 0 \Rightarrow -BD - BA\cos 26.56 = 0$

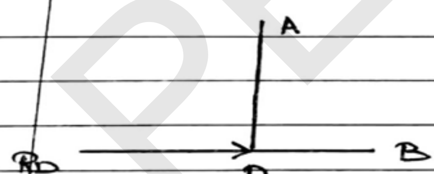
$\Sigma V = 0 \Rightarrow BA\sin 26.56 - BC = 0$

$\Rightarrow \underline{BA = 14.40 \text{ kN} \quad T}$

$\Rightarrow BD = -BA\cos 26.56$

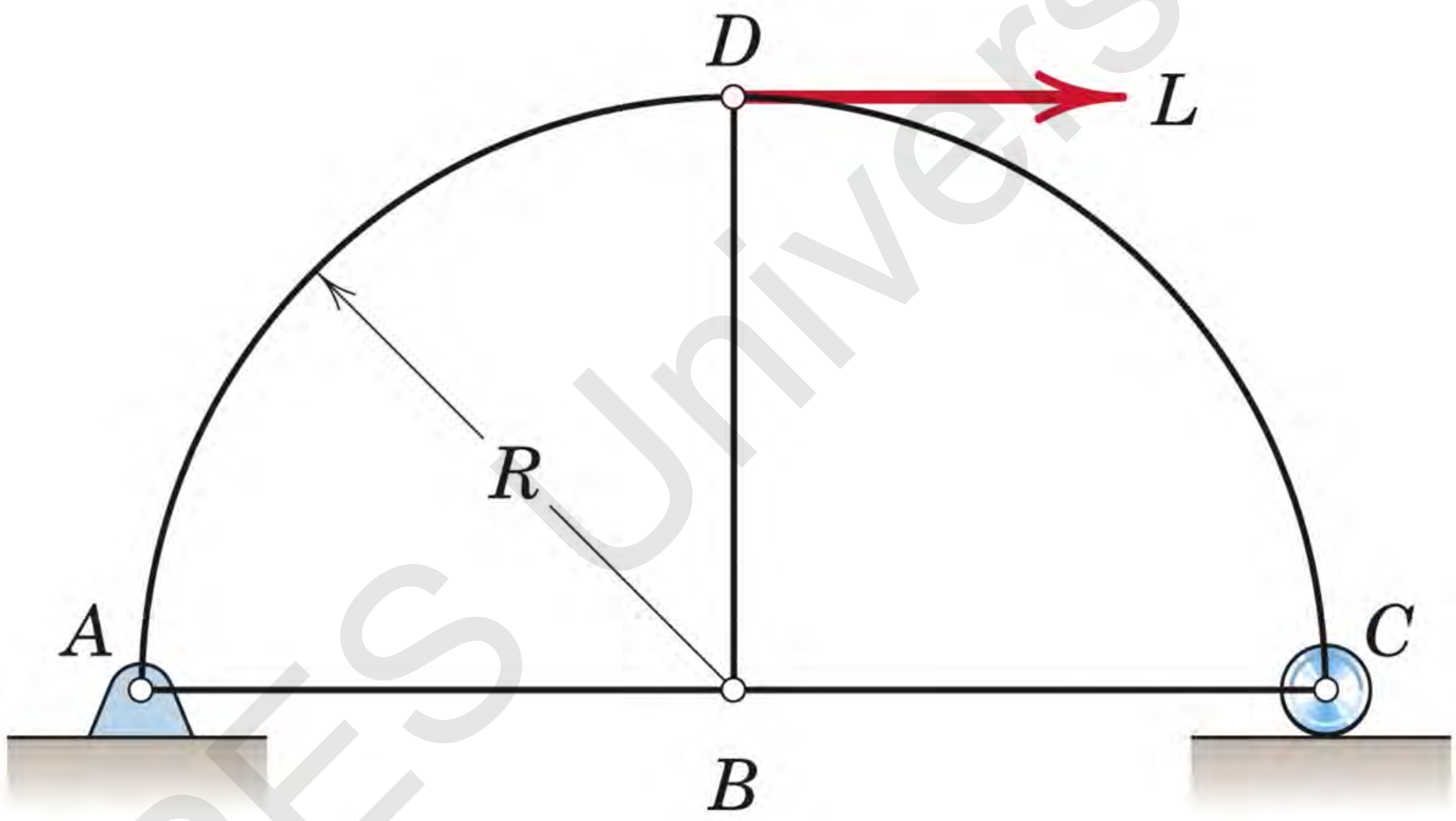
$\Rightarrow \underline{BD = -12.8 \text{ kN} \quad C}$

Joint D

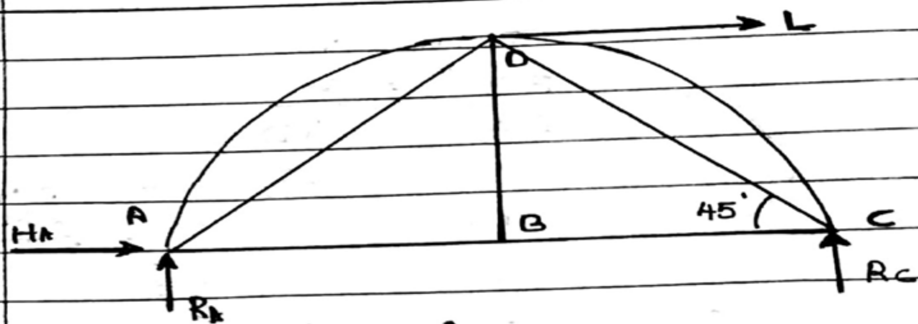


$\Sigma V = 0 \Rightarrow AD = 0 \text{ kN}$

Determine the forces in members *AB*, *BC*, and *BD* of the loaded truss.



Problem 4-11



$\Sigma H = 0 \Rightarrow H_A + L = 0$

$\Rightarrow H_A = -L$

$\Sigma M_A = 0 \Rightarrow (-R_C \times 2R) + (L \times R) = 0$

$\Rightarrow +2R \cdot R_C = +L \cdot R$

$\Rightarrow R_C = +\frac{L R}{2R}$

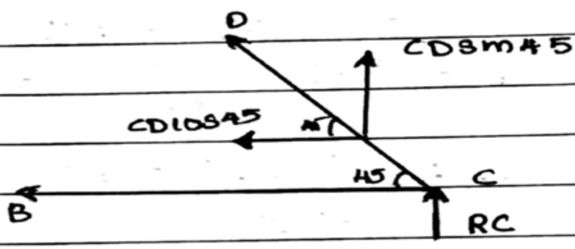
$R_C = \frac{L}{2}$

$\Sigma V = 0 \Rightarrow R_A + R_C = 0$

$\Rightarrow R_A = -R_C$

$\Rightarrow R_A = -\frac{L}{2}$

Joint B



$\Sigma V = 0 \Rightarrow RC + CD \sin 45 = 0$

$\Rightarrow CD \sin 45 = -RC$

$\Rightarrow CD = -RC / \sin 45$

$= -\frac{L}{2} \times \frac{1}{.707}$

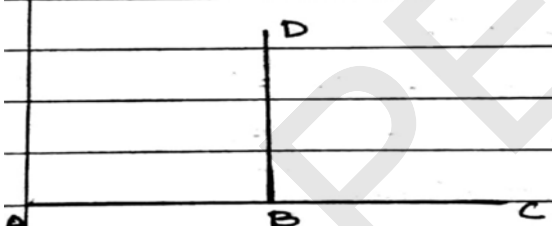
$CD = -L/1.4 = -L/\sqrt{2}$

$\Sigma H = 0 \Rightarrow -CB - CD \cos 45 = 0$

$\Rightarrow CB = +\frac{L}{\sqrt{2}} \cos 45$

$\Rightarrow CB = 0.5\,L$

Joint B



$\Sigma V = 0 \Rightarrow BD = 0$

$\Sigma H = 0 \Rightarrow BC - AB = 0$

$\Rightarrow BC = AB$

$\Rightarrow AB = 0.5\,L \quad T$