LINEAR ALGEBRA AND ITS APPLICATIONS

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MATRICES AND GAUSSIAN ELIMINATION

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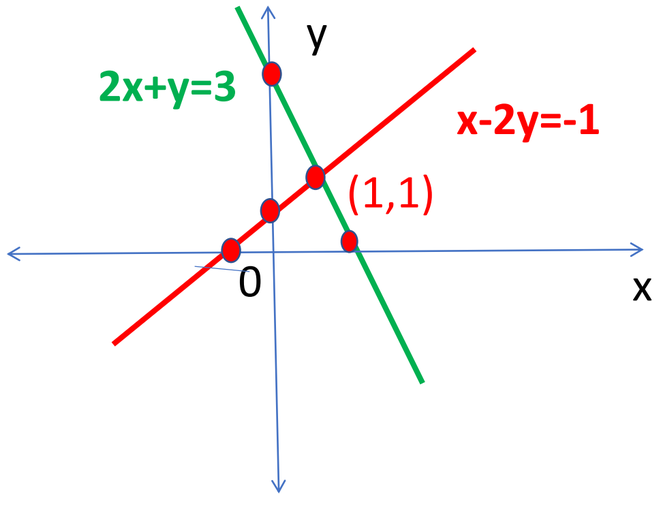
THE GEOMETRY OF LINEAR EQUATIONS:

Course Content: The Geometry of Linear Equations

1. Solve the system of equations 2x+y=3; x-2y=-1 and draw the Row picture and Column picture.

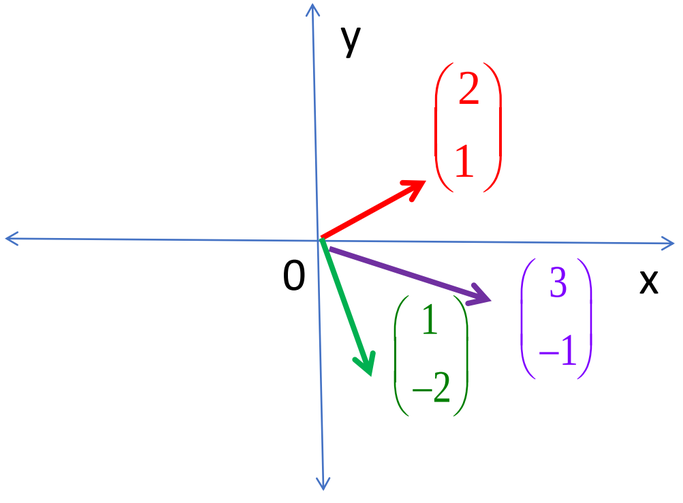
Row picture



Intersecting lines
Unique solution (1,1)

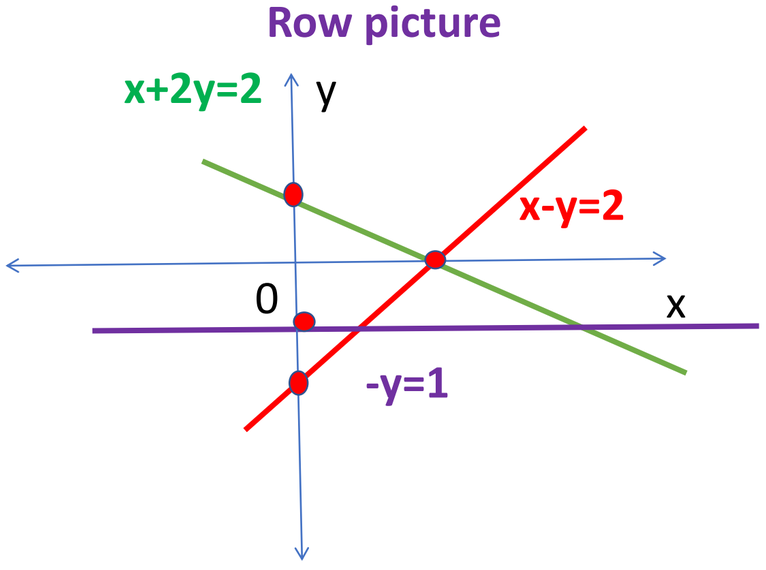
Column picture

$$x\begin{pmatrix}2\\1\end{pmatrix}+y\begin{pmatrix}1\\-2\end{pmatrix}=\begin{pmatrix}3\\-1\end{pmatrix} \Rightarrow 1\begin{pmatrix}2\\1\end{pmatrix}+1\begin{pmatrix}1\\-2\end{pmatrix}=\begin{pmatrix}3\\-1\end{pmatrix}$$



THE GEOMETRY OF LINEAR EQUATIONS:

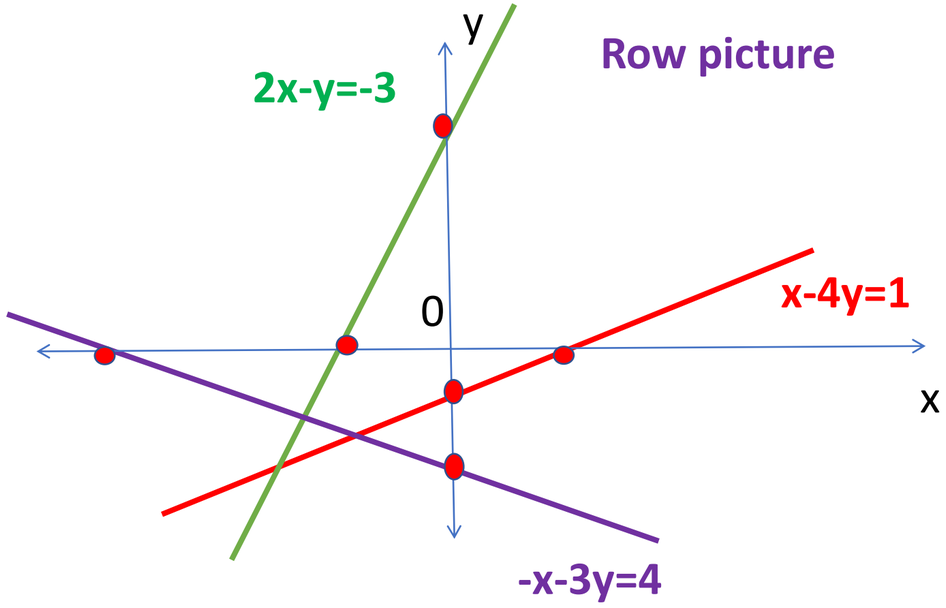
2. Sketch the three lines x+2y=2; x-y=2; -y=1(row picture only) and decide if the three equations are solvable. What happens if all right hand sides are zero? Is there any non-zero choice of right hand sides that allow the three lines to intersect at a common point of intersection?



The given three equations are not solvable as they do not have a common point of intersection. If all right hand sides are zero we get a trivial(zero) solution i.e x=0, y=0. If RHS of first equation is -1 then the three lines x+2y=-1; x-y=2; -y=1 intersect at a common point of intersection (1,-1)

THE GEOMETRY OF LINEAR EQUATIONS:

3. Sketch the three lines **x - 4y = 1; 2x - y = -3; -x - 3y = 4** and decide if the three equations are solvable. Do they have a common point of intersection? Explain.



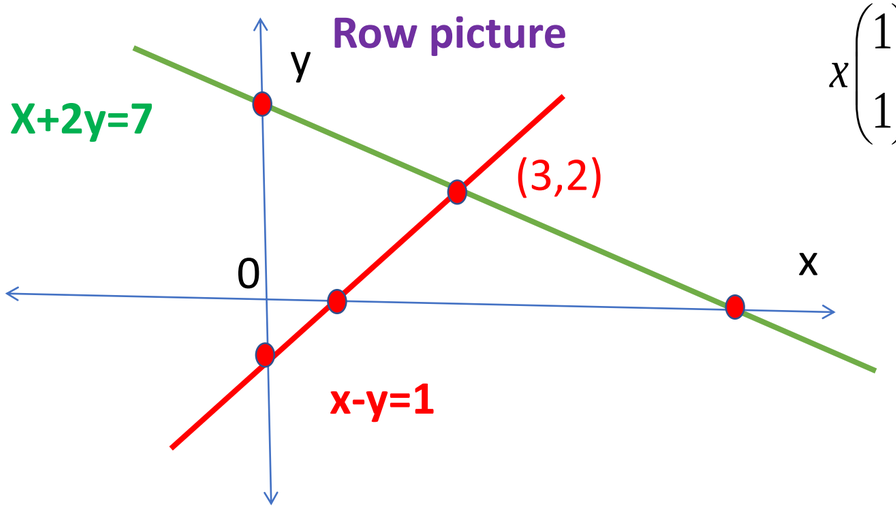
Given three equations are not solvable as they have no common point of intersection.

THE GEOMETRY OF LINEAR EQUATIONS:

4. Draw the row picture and column picture for the following system of equations and discuss its consistency, singularity, and existence of the solution:

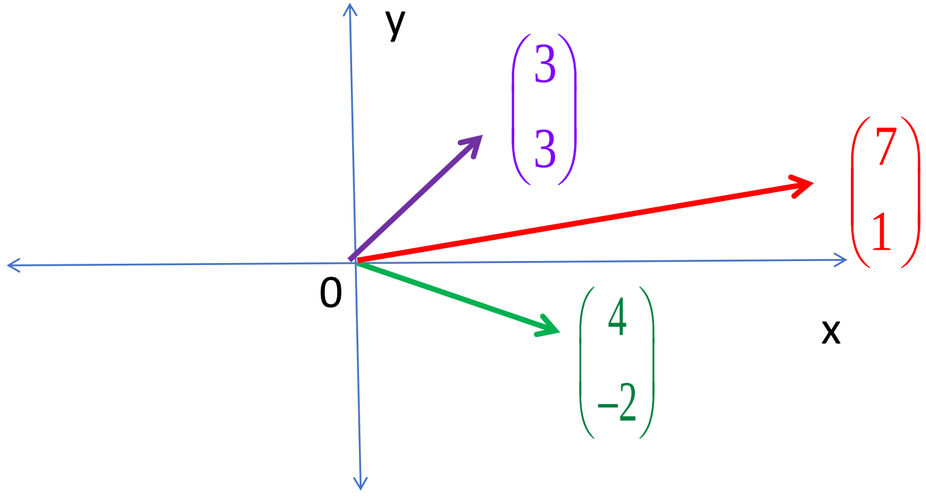
(i) x + 2y = 7; x - y = 1

Row picture



Column picture

$$x\begin{pmatrix}1\\1\end{pmatrix}+y\begin{pmatrix}2\\-1\end{pmatrix}=\begin{pmatrix}7\\1\end{pmatrix} \Rightarrow 3\begin{pmatrix}1\\1\end{pmatrix}+2\begin{pmatrix}2\\-1\end{pmatrix}=\begin{pmatrix}7\\1\end{pmatrix}$$



(ii) 2x + 3y = 6; 4x + 6y = 12 (iii) x - 3y = 5; x - 3y = -5

INTRODUCTION:

Example:

$$A \rightarrow \left(\begin{array}{cccc:c} 1 & 1 & -2 & 3 & 4 \\ 2 & 3 & 3 & -1 & 3 \\ 5 & 7 & 4 & -1 & 5 \end{array}\right) \xrightarrow[R_3-5R_1]{R_2-2R_1} \left(\begin{array}{cccc:c} 1 & 1 & -2 & 3 & 4 \\ 0 & 1 & 7 & -7 & -5 \\ 0 & 2 & 14 & -16 & -15 \end{array}\right) \xrightarrow{R_3-2R_2}$$

$$\left(\begin{array}{cccc:c} 1 & 1 & -2 & 3 & 4 \\ 0 & 1 & 7 & -7 & -5 \\ 0 & 0 & 0 & -2 & -5 \end{array}\right) = U \xrightarrow{R_3/(-2)} \left(\begin{array}{cccc:c} 1 & 1 & -2 & 3 & 4 \\ 0 & 1 & 7 & -7 & -5 \\ 0 & 0 & 0 & 1 & 5/2 \end{array}\right) \xrightarrow[R_2+7R_3]{R_1-3R_3}$$

$$\left(\begin{array}{cccc:c} 1 & 1 & -2 & 0 & -7/2 \\ 0 & 1 & 7 & 0 & 25/2 \\ 0 & 0 & 0 & 1 & 5/2 \end{array}\right) \xrightarrow{R_1-R_2} \left(\begin{array}{cccc:c} 1 & 0 & -9 & 0 & -16 \\ 0 & 1 & 7 & 0 & 25/2 \\ 0 & 0 & 0 & 1 & 5/2 \end{array}\right) = R$$

References/Links:

https://upload.wikimedia.org/wikipedia/commons/c/c0/Intersecting_Lines.svg

Google search: Graphs of row and column pictiure for a system of linear equations

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THANK YOU

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