PES University, Bangalore

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UE19CS203 – STATISTICS FOR DATA SCIENCE

Unit-2 - Random Variables

QUESTION BANK

Poisson Distribution

Exercises for Section 4.3

[Text Book Exercise – Section 4.3 – Q. No. [1 – 21] – Pg. No. [227 - 230]]

1. Let X ~ Poisson(4). Find

    a) P(X = 1)
    b) P(X = 0)
    c) P(X < 2)
    d) P(X > 1)
    e) μ_X
    f) σ_X

2. The number of flaws in a given area of aluminum foil follows a Poisson distribution with a mean of 3 per m^2. Let X represent the number of flaws in a 1 m^2 sample of foil.

    a) P(X = 5)
    b) P(X = 0)
    c) P(X < 2)
    d) P(X > 1)
    e) μ_X
    f) σ_X

3. In a certain city, the number of potholes on a major street follows a Poisson distribution with a rate of 3 per mile. Let *X* represent the number of potholes in a two-mile stretch of road. Find

    a) P(X = 4)
    b) P(X ≤ 1)
    c) P(5 ≤ X < 8)

d) μ_X
e) σ_X

4. Geologists estimate the time since the most recent cooling of a mineral by counting the number of uranium fission tracks on the surface of the mineral. A certain mineral specimen is of such an age that there should be an average of 6 tracks per cm2 of surface area. Assume the number of tracks in an area follows a Poisson distribution. Let X represent the number of tracks counted in 1 cm2 of surface area. Find

   a) $P(X = 7)$
   b) $P(X \geq 3)$
   c) $P(2 < X < 7)$
   d) μ_X
   e) σ_X

5. A data center contains 1000 computer servers. Each server has probability 0.003 of failing on a given day.

   a) What is the probability that exactly two servers fail?
   b) What is the probability that fewer than 998 servers function?
   c) What is the mean number of servers that fail?
   d) What is the standard deviation of the number of servers that fail?

6. One out of every 5000 individuals in a population carries a certain defective gene. A random sample of 1000 individuals is studied.

   a) What is the probability that exactly one of the sample individuals carries the gene?
   b) What is the probability that none of the sample individuals carries the gene?
   c) What is the probability that more than two of the sample individuals carry the gene?
   d) What is the mean of the number of sample individuals that carry the gene?
   e) What is the standard deviation of the number of sample individuals that carry the gene?

7. The number of hits on a certain website follows a Poisson distribution with a mean rate of 4 per minute.

   a) What is the probability that 5 messages are received in a given minute?
   b) What is the probability that 9 messages are received in 1.5 minutes?

c) What is the probability that fewer than 3 messages are received in a period of 30 seconds?

8. The number of cars arriving at a given intersection follows a Poisson distribution with a mean rate of 4 per second.

   a) What is the probability that 3 cars arrive in a given second?
   b) What is the probability that 8 cars arrive in three seconds?
   c) What is the probability that more than 3 cars arrive in a period of two seconds?

9. A random variable X has a binomial distribution, and a random variable Y has a Poisson distribution. Both X and Y have means equal to 3. Is it possible to determine which random variable has the larger variance? Choose one of the following answers:

   a) Yes, X has the larger variance.
   b) Yes, Y has the larger variance.
   c) No, we need to know the number of trials, n, for X.
   d) No, we need to know the success probability, p, for X.
   e) No, we need to know the value of λ for Y.

10. A chemist wishes to estimate the concentration of particles in a certain suspension. She withdraws 3mL of the suspension and counts 48 particles. Estimate the concentration in particles per mL and find the uncertainty in the estimate.

11. A microbiologist wants to estimate the concentration of a certain type of bacterium in a wastewater sample. She puts a 0.5 mL sample of the wastewater on a microscope slide and counts 39 bacteria. Estimate the concentration of bacteria, per mL, in this wastewater, and find the uncertainty in the estimate.

12. Two-dimensional Poisson process. The number of plants of a certain species in a certain forest has a Poisson distribution with mean 10 plants per acre. The number of plants in T acres therefore has a Poisson distribution with mean 10T .

    a) What is the probability that there will be exactly 18 plants in a two-acre region?
    b) What is the probability that there will be exactly 12 plants in a circle with radius 100 ft? (1 acre = 43,560 ft2.)
    c) The number of plants of a different type follows a Poisson distribution with mean λ plants per acre, where λ is unknown. A total of 5 plants are counted in a 0.1 acre area. Estimate λ, and find the uncertainty in the estimate.

13. The number of defective components produced by a certain process in one day has a Poisson distribution with mean 20. Each defective component has probability 0.60 of being repairable.

    a) Find the probability that exactly 15 defective components are produced.
    b) Given that exactly 15 defective components are produced, find the probability that exactly 10 of them are repairable.
    c) Let N be the number of defective components produced, and let X be the number of them that are repairable. Given the value of N, what is the distribution of X?
    d) Find the probability that exactly 15 defective components are produced, with exactly 10 of them being repairable.

14. The probability that a certain radioactive mass emits no particles in a one-minute time period is 0.1353. What is the mean number of particles emitted per minute?

15. The number of flaws in a certain type of lumber follows a Poisson distribution with a rate of 0.45 per linear meter.

    a) What is the probability that a board 3 meters in length has no flaws?
    b) How long must a board be so that the probability it has no flaw is 0.5?

16. Grandma is trying out a new recipe for raisin bread. Each batch of bread dough makes three loaves, and each loaf contains 20 slices of bread.

    a) If she puts 100 raisins into a batch of dough, what is the probability that a randomly chosen slice of bread contains no raisins?
    b) If she puts 200 raisins into a batch of dough, what is the probability that a randomly chosen slice of bread contains 5 raisins?
    c) How many raisins must she put in so that the probability that a randomly chosen slice will have no raisins is 0.01?

17. Mom and Grandma are each baking chocolate chip cookies. Each gives you two cookies. One of Mom's cookies has 14 chips in it and the other has 11. Grandma's cookies have 6 and 8 chips.

    a) Estimate the mean number of chips in one of Mom's cookies.
    b) Estimate the mean number of chips in one of Grandma's cookies.

c) Find the uncertainty in the estimate for Mom's cookies.
d) Find the uncertainty in the estimate for Grandma's cookies.
e) Estimate how many more chips there are on the average in one of Mom's cookies than in one of Grandma's. Find the uncertainty in this estimate.

18. You have received a radioactive mass that is claimed to have a mean decay rate of at least 1 particle per second. If the mean decay rate is less than 1 per second, you may return the product for a refund. Let X be the number of decay events counted in 10 seconds.

    a) If the mean decay rate is exactly 1 per second (so that the claim is true, but just barely), what is $P(X \leq 1)$?
    b) Based on the answer to part (a), if the mean decay rate is 1 particle per second, would one event in 10 seconds be an unusually small number?
    c) If you counted one decay event in 10 seconds, would this be convincing evidence that the product should be returned? Explain.
    d) If the mean decay rate is exactly 1 per second, what is $P(X \leq 8)$?
    e) Based on the answer to part (d), if the mean decay rate is 1 particle per second, would eight events in 10 seconds be an unusually small number?
    f) If you counted eight decay events in 10 seconds, would this be convincing evidence that the product should be returned? Explain.

19. Someone claims that a certain suspension contains at least seven particles per mL. You sample 1 mL of solution. Let X be the number of particles in the sample.

    a) If the mean number of particles is exactly seven per mL (so that the claim is true, but just barely), what is $P(X \leq 1)$?
    b) Based on the answer to part (a), if the suspension contains seven particles per mL, would one particle in a 1 mL sample be an unusually small number?
    c) If you counted one particle in the sample, would this be convincing evidence that the claim is false? Explain.
    d) If the mean number of particles is exactly 7 per mL, what is $P(X \leq 6)$?
    e) Based on the answer to part (d), if the suspension contains seven particles per mL, would six particles in a 1 mL sample be an unusually small number?
    f) If you counted six particles in the sample, would this be convincing evidence that the claim is false? Explain.

20. A physicist wants to estimate the rate of emissions of alpha particles from a certain source. He makes two counts. First, he measures the background rate by counting the number of particles in 100 seconds in the absence of the source. He counts 36

background emissions. Then, with the source present, he counts 324 emissions in 100 seconds. This represents the sum of source emissions plus background emissions.

a) Estimate the background rate, in emissions per second, and find the uncertainty in the estimate.
b) Estimate the sum of the source plus background rate, in emissions per second, and find the uncertainty in the estimate.
c) Estimate the rate of source emissions in particles per second, and find the uncertainty in the estimate.
d) Which will provide the smaller uncertainty in estimating the rate of emissions from the source:
   (1) counting the background only for 150 seconds and the background plus the source for 150 seconds, or
   (2) counting the background for 100 seconds and the source plus the background for 200 seconds? Compute the uncertainty in each case.
e) Is it possible to reduce the uncertainty to 0.03 particles per second if the background rate is measured for only 100 seconds? If so, for how long must the source plus background be measured? If not, explain why not.

21. (Requires material from Section 3.3.) Refer to Example 4.27. Estimate the probability that a 1 m2 sheet of aluminum has exactly one flaw, and find the uncertainty in this estimate. **(Exclude)**