STATISTICS FOR DATA SCIENCE

HYPOTHESIS AND INFERENCE

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STATISTICS FOR DATA SCIENCE

Unit 4 :HYPOTHESIS AND INFERENCE

Session : 12

Sub Topic :Type I and Type II Errors

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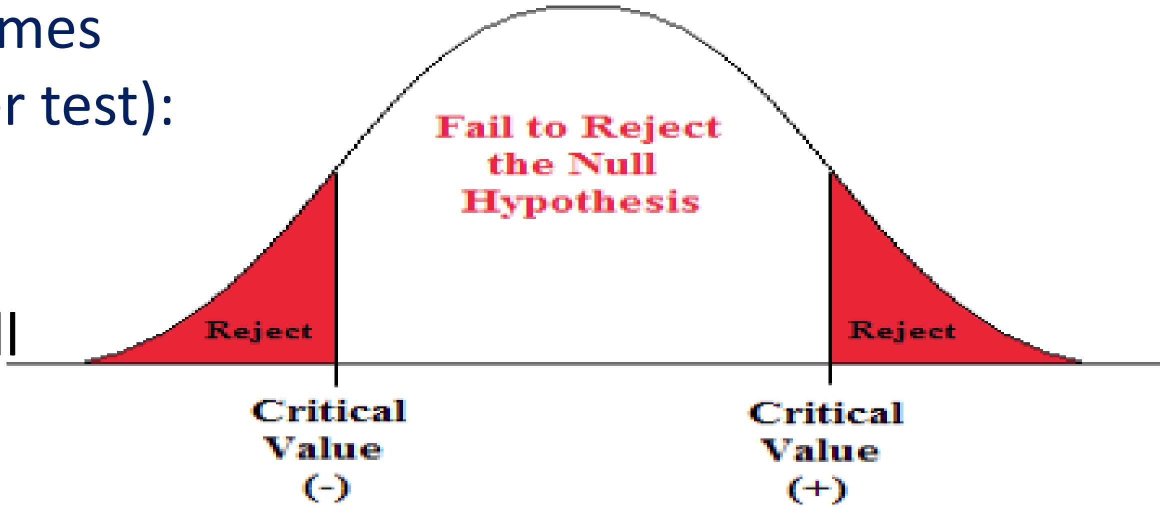
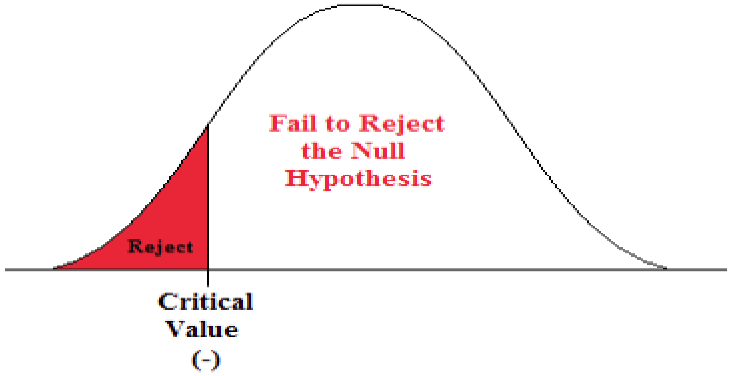
ERRORS IN HYPOTHESIS TESTING

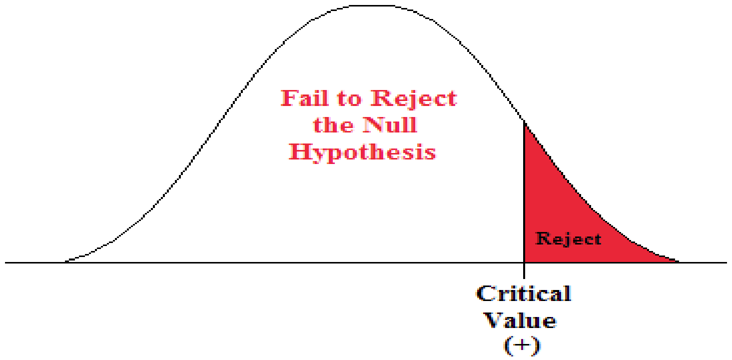
Hypothesis testing : H_0 vs H_1

Two possible outcomes
(only one occurs per test):

- Reject Null
- Fail to Reject Null







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Type I and Type II errors

Because hypothesis tests are based on probability, while we hope to make correct decisions, it is possible to get results that are contrary to reality .

When we get a result that is contrary to the truth, this is known as making **an error in hypothesis testing.**

There are exactly two kinds of errors

Null is true but we reject it
Null is false but we fail to reject it

Type I and Type II errors

Suppose we test $H_0\!: \mu = 15$

$H_1\!: \mu > 15$

And we reject H_0 at $\alpha = 0.05$

One of two things will occur:

1. The null hypothesis is false and we made the correct decision

2. The null hypothesis is true and we made a type I error

Type I and Type II errors

Suppose we test $H_0\!: \mu = 15$

$H_1\!: \mu > 15$

And we do not reject H_0 at $\alpha = 0.05$

One of two things will occur:

1. The null hypothesis is true and we made the correct decision

2. The null hypothesis is false and we made a type II error

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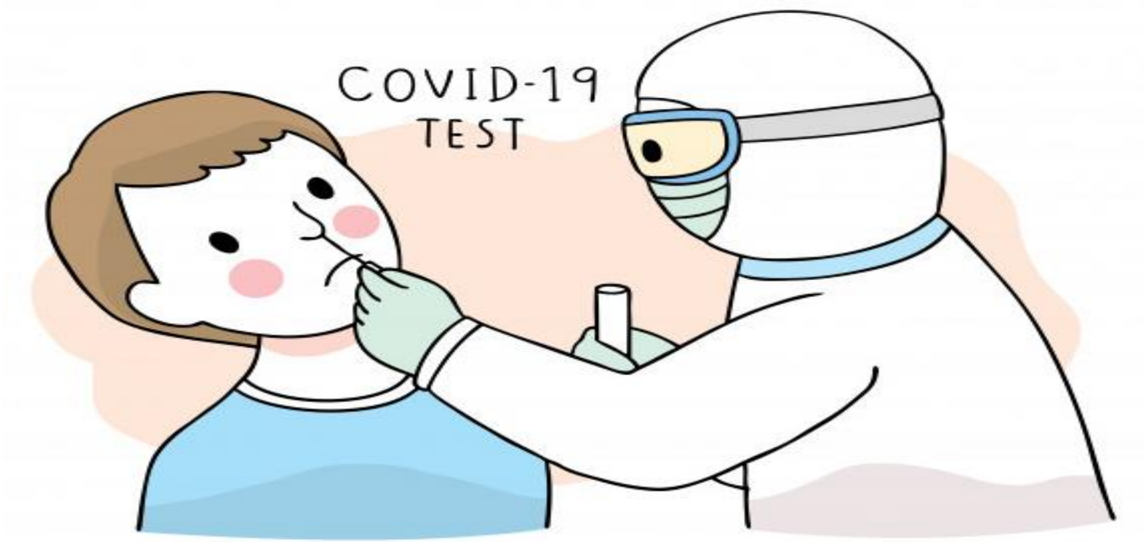
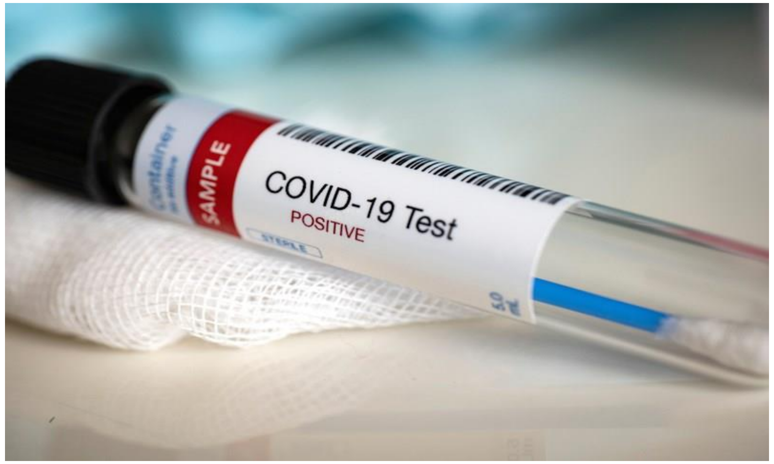
Type I and Type II errors - Example





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H_0 : A person is tested positive for Covid-19

	Actual State of Reality	
Researcher Decision	H_0 is true **Covid +ve**	H_0 is false **Covid -ve**
Reject H_0 **Covid -ve**	Type I error (α) (Erroneously reported that the patient is Covid –ve)	Correct Decision $(1 - \beta)$
Fail to reject H_0 **Covid +ve**	Correct Decision $(1 - \alpha)$	Type II error (β) (Erroneously reported that the patient is Covid +ve)

Type I and Type II errors- Example - A Judicial trial

Presumption of Innocence

H_0 : Assumed to be innocent until proven guilty

Prosecution's claim is

$H_1: The\ person\ is\ guilty$

Source: Internet

Hypothesis testing : H_0 vs H_1

H_0 : Person is not guilty of the crime

	Truth	
Jury Decision	H_0 is true Innocent	H_0 is false Guilty
Reject H_0 Guilty	Type I error (α) -Person is convicted by the court when he actually did not commit the crime(convicting an innocent person)	Correct Decision $(1 - \beta)$
Fail to reject H_0 Innocent	Correct Decision $(1 - \alpha)$	Type II error (β) - Person is acquitted by the court when he actually did commit the crime (letting a guilty person go free)

Type I and Type II errors

Source: Internet

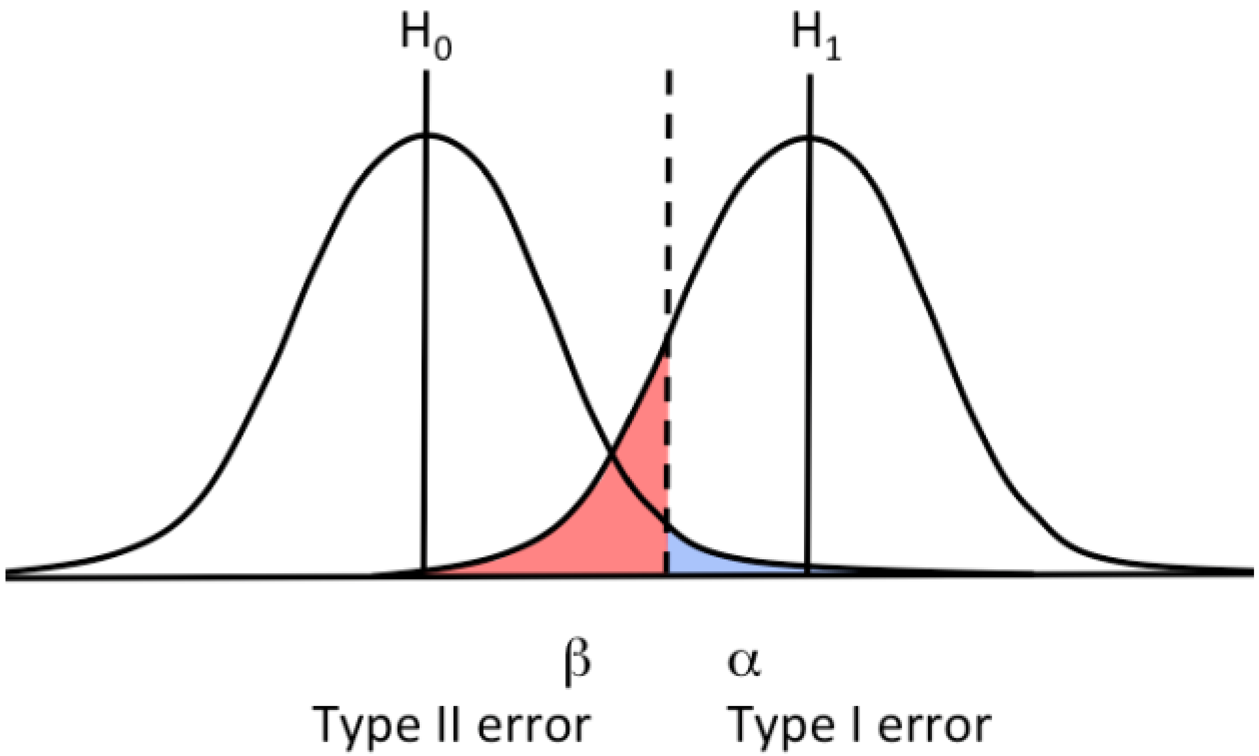
Hypothesis testing : H_0 vs H_1

H_0 : It is not a spam vs H_1 : It is a spam

	Truth	
Decision	H_0 is true It is not a spam	H_0 is false It is a spam
Reject H_0 It is a spam	Type I error (α)- an important message that ends up in your spam folder or, much worse, gets deleted.	Correct Decision $(1 – \beta)$
Fail to reject H_0 It is not a spam	Correct Decision $(1 – \alpha)$	Type II error (β) (spam that ends up in your inbox is annoying.)

Type I and Type II errors

When designing experiments whose data will be analyzed with a fixed-level test, it is important to try to make the probabilities of type I and type II errors reasonably small.



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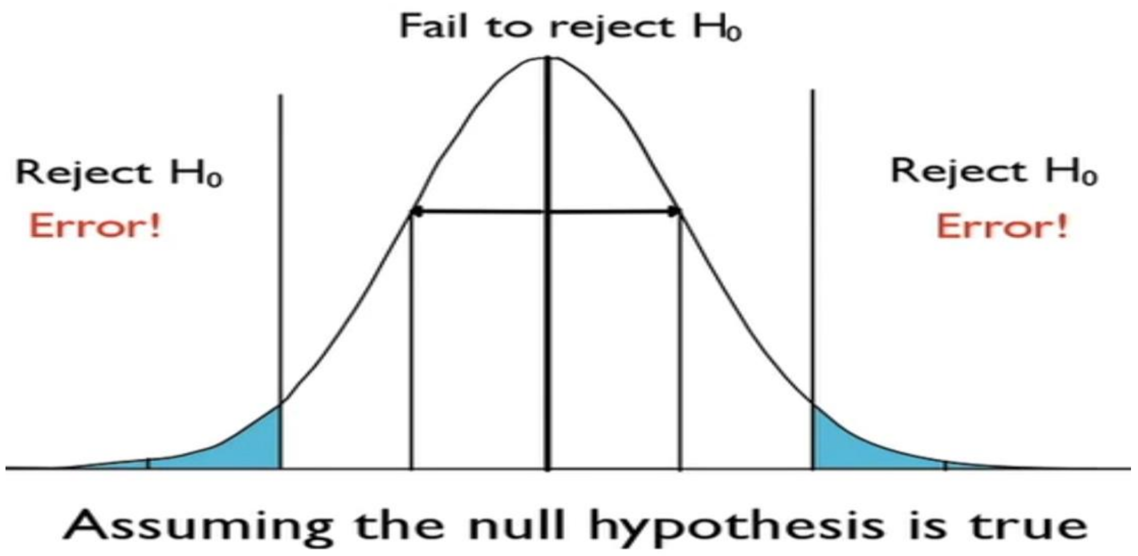
Type I and Type II errors

Type I error:

P(type I error)= P(reject H_0 when H_0 is true)=α



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Type I and Type II errors

Type I error :

- **When the null hypothesis is true and you reject it, you make a type I error.**
- The probability of making a type I error is α, which is the level of significance you set for your hypothesis test.
- An α of 0.05 indicates that you are willing to accept a 5% chance that you are wrong when you reject the null hypothesis.
- To lower this risk, you must use a lower value for α.
- However, using a lower value for alpha means that you will be less likely to detect a true difference if one really exists.

Type I and Type II errors

If α is the significance level that has been chosen for the test, then the probability of a type I error is never greater than α.

Let $X_1, X_2. . . , X_n$ be a large random sample from a population with mean μ and variance σ^2.

$\bar{X} \sim N(\mu, {}^{\sigma^2}/_{n})$

Suppose we test $\boldsymbol{H_0: \mu \leq 0}$ versus $\boldsymbol{H_1: \mu > 0}$

at the fixed level $\boldsymbol{\alpha = 0.05}$

$\Longrightarrow$ **we will reject $\boldsymbol{H_0 \ if \ P \leq 0.05}$**

Type I and Type II errors

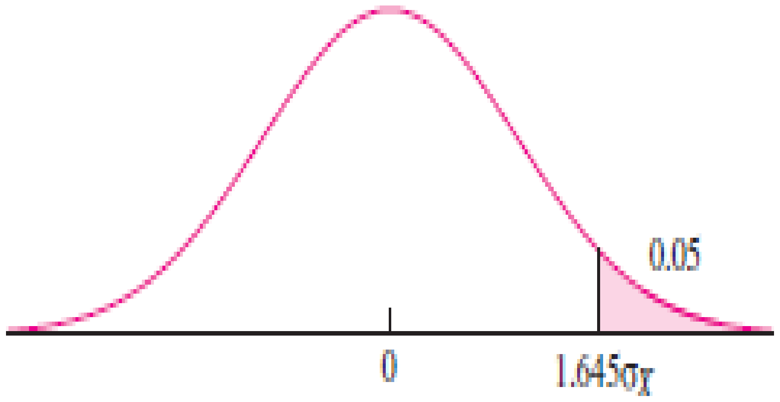


FIGURE 6.25 The null distribution with the rejection region for $H_0 : \mu \leq 0$.

Assume the null hypothesis is true. We will compute the probability of type I error and show that it is no greater than 0.05.

Type I and Type II errors

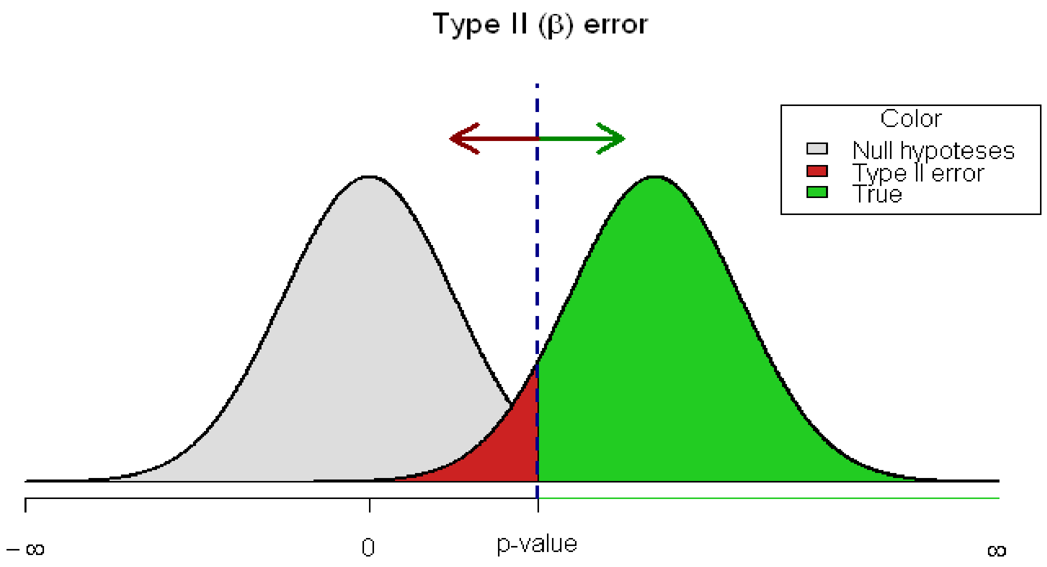
Next, consider the case where $\mu < 0$.
Then the distribution of $\bar{X}$ is obtained by shifting the curve in Figure to the left, so $P(\bar{X} \geq 1.645\sigma_{\bar{X}}) < 0.05$, and the probability of a type I error is less than 0.05.

We could repeat this illustration using any number α in place of 0.05.
We conclude that if $\boldsymbol{H_0}$ is true, the probability of a type I error is never greater than α.
Furthermore, note that if μ is on the boundary of H_0 (μ = 0 in this case), then the probability of a type I error is equal to α.

Type I and Type II errors

Type II error:

P(type II error)= P(Fail to reject H_0 when H_0 is false)=$\boldsymbol{\beta}$



Type II error:

- **When the null hypothesis is false and you fail to reject it, you make a type II error.**
- The probability of making a type II error is β, which depends on the **power of the test**.
- You can decrease your risk of committing a type II error by ensuring your test has enough power.
- You can do this by ensuring your sample size is large enough to detect a practical difference when one truly exists.

Problem 1

A vendor claims that no more than 10% of the parts she supplies are defective. Let p denote the actual proportion of parts that are defective. A test is made of the hypotheses $H_0: p \leq 0.10$ versus $H_1: p > 0.10$.

For each of the following situations, determine whether the decision was correct, a type I error occurred, or a type II error occurred.

Type I and Type II errors

a. The claim is true, and H_0 is rejected.

b. The claim is false, and H_0 is rejected.

c. The claim is true, and H_0 is not rejected.

d. The claim is false, and H_0 is not rejected.

Ans: (a) Type I error (b) Correct decision (c) Correct decision (d) Type II error

Problem 2 :

A hypothesis test is to be performed, and it is decided to reject the null hypothesis.
If $P \leq 0.10$.

If H_0 is in fact true, what is the maximum probability that it will be rejected?

Solution:

The maximum probability of rejecting H_0 when true is the level $\alpha = 0.10$.

Problem 3 :

A test is made of the hypotheses:

H_0 : μ ≤ 10 versus H_1 : μ > 10

For each of the following situations, determine whether the decision was correct, a type I error occurred, or a type II error occurred.

a. μ = 8, H_0 is rejected.
b. μ = 10, H_0 is not rejected.
c. μ = 14, H_0 is not rejected.
d. μ = 12, H_0 is rejected.

Type I and Type II errors- Problem 1

H_0 : μ ≤ 10 versus H_1 : μ > 10

a. μ = 8, H_0 is rejected

Type I error. H_0 is true and was rejected.

b. μ = 10, H_0 is not rejected

Correct decision. H_0 is true and was not rejected.

c. μ = 14, H_0 is not rejected

Type II error. H_0 is false and was not rejected.

d. μ = 12, H_0 is rejected

Correct decision. H_0 is false and was rejected.

Type I and Type II errors

Problem 4:

Null Hypothesis is that the battery for a heart pacemaker has an average life of 300 days, with the alternative hypothesis that the average life is more than 300 days. If you are the quality control manger for the battery manufacturer then

a) Would you rather make a Type I error or a Type II error
b) Based on your answer to part(a), should you use a high or low significance level?

Type I and Type II errors

Solution:

Given H_0 : μ = 300 days versus H_1 : μ > 300days

(a) It is better to make a Type II error (where H_0 is false. That is, average life is actually more than 300 days but wwe accept H_0 and assume that the average life is equal to 300 days).

(b) As we increase the significance level (α) we increase the chances of making type I error. Since here it is better to make a type II error we shall choose a low α.

THANK YOU

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