PES University, Bangalore

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UE19CS203 – STATISTICS FOR DATA SCIENCE

Unit-2 - Random Variables

QUESTION BANK

Linear Functions of Random Variables

Exercises for Section 2.5

[Text Book Exercise – Section 2.5 – Q. No. [1 – 18] – Pg. No. [124 - 126]]

1. If X and Y are independent random variables with means $\mu_X = 9.5$ and $\mu_Y = 6.8$ and standard deviations $\sigma_X = 0.4$ and $\sigma_Y = 0.1$, find the means and standard deviations of the following:

    a) 3X
    b) $Y - X$
    c) $X + 4Y$

2. The bottom of a cylindrical container has an area of 10 cm^2. The container is filled to a height whose mean is 5 cm, and whose standard deviation is 0.1 cm. Let V denote the volume of fluid in the container.

    a) Find μ_V
    b) Find σ_V

3. The lifetime of a certain transistor in a certain application has mean 900 hours and standard deviation 30 hours. Find the mean and standard deviation of the length of time that four transistors will last.

4. Two batteries, with voltages V1 and V2, are connected in series. The total voltage V is given by V = V1+V2. Assume that V1 has mean 12 V and standard deviation 1 V, and that V2 has mean 6 V and standard deviation 0.5 V.

    a) Find μ_V.
    b) Assuming V1 and V2 to be independent, find σ_V.

5. A laminated item is composed of five layers. The layers are a simple random sample from a population whose thickness has mean 1.2 mm and standard deviation 0.04 mm.

    a) Find the mean thickness of an item.
    b) Find the standard deviation of the thickness of an item.

6. Two independent measurements are made of the lifetime of a charmed strange meson. Each measurement has a standard deviation of $7 * 10^{-15}$ seconds. The lifetime of the meson is estimated by averaging the two measurements. What is the standard deviation of this estimate?

7. The molarity of a solute in solution is defined to be the number of moles of solute per liter of solution (1 mole = 6.02 × 1023 molecules). If X is the molarity of a solution of magnesium chloride ($MgCl_2$), and Y is the molarity of a solution of ferric chloride ($FeCl_3$), the molarity of chloride ion (Cl^-) in a solution made of equal parts of the solutions of $MgCl_2$ and $FeCl_3$ is given by M = X +1.5Y . Assume that X has mean 0.125 and standard deviation 0.05, and that Y has mean 0.350 and standard deviation 0.10.

    a) Find μ_M.
    b) Assuming X and Y to be independent, find σ_M.

8. A machine that fills bottles with a beverage has a fill volume whose mean is 20.01 ounces, with a standard deviation of 0.02 ounces. A case consists of 24 bottles randomly sampled from the output of the machine.

    a) Find the mean of the total volume of the beverage in the case.
    b) Find the standard deviation of the total volume of the beverage in the case.
    c) Find the mean of the average volume per bottle of the beverage in the case.
    d) Find the standard deviation of the volume per bottle of the beverage in the case.
    e) How many bottles must be included in a case for the standard deviation of the average volume per bottle to be 0.0025 ounces?

9. The four sides of a picture frame consist of two pieces selected from a population whose mean length is 30 cm with standard deviation 0.1 cm, and two pieces selected from a population whose mean length is 45 cm with standard deviation 0.3 cm.

    a) Find the mean perimeter.
    b) Assuming the four pieces are chosen independently, find the standard deviation of the perimeter.

10. A gas station earns \$2.60 in revenue for each gallon of regular gas it sells, \$2.75 for each gallon of midgrade gas, and \$2.90 for each gallon of premium gas. Let X_1, X_2, and X_3 denote the numbers of gallons of regular, midgrade, and premium gasoline sold in a day. Assume that X_1, X_2, and X_3 have means $\mu_1 = 1500$, $\mu_2 = 500$, and $\mu_3 = 300$, and standard deviationsσ_1. = 180, $\sigma_2 = 90$, and $\sigma_3 = 40$, respectively.

    a) Find the mean daily revenue.
    b) Assuming X_1, X_2, and X_3 to be independent, find the standard deviation of the daily revenue.

11. A certain commercial jet plane uses a mean of 0.15 gallons of fuel per passenger-mile, with a standard deviation of 0.01 gallons.

    a) Find the mean number of gallons the plane uses to fly 8000 miles if it carries 210 passengers.
    b) Assume the amounts of fuel used are independent for each passenger-mile traveled. Find the standard deviation of the number of gallons of fuel the plane uses to fly 8000 miles while carrying 210 passengers.
    c) The plane used X gallons of fuel to carry 210 passengers 8000 miles. The fuel efficiency is estimated as $X/(210 \times 8000)$. Find the mean of this estimate.
    d) Assuming the amounts of fuel used are independent for each passenger-mile, find the standard deviation of the estimate in part (c).

12. The Needleman-Wunsch method for aligning DNA sequences assigns 1 point whenever a mismatch occurs, and 3 points whenever a gap (insertion or deletion) appears in a sequence. Assume that under certain conditions, the number of mismatches has mean 5 and standard deviation 2, and the number of gaps has mean 2 and standard deviation 1.

    a) Find the mean of the Needleman-Wunsch score.
    b) Assume the number of gaps is independent of the number of mismatches. Find the variance of the Needleman-Wunsch score.

13. In the article "An Investigation of the Ca–CO3–CaF2– K2SiO3–SiO2–Fe Flux System Using the Submerged Arc Welding Process on HSLA-100 and AISI-1018 Steels" (G. Fredrickson, M.S. thesis, Colorado School of Mines, 1992), the carbon equivalent *P* of a weld metal is defined to be a linear combination of the weight percentages of carbon (C), manganese (Mn), copper (Cu), chromium (Cr), silicon (Si), nickel (Ni), molybdenum (Mo), vanadium (V), and boron (B). The carbon equivalent is given by

$$P = C + \frac{Mn + Cu + Cr}{20} + \frac{Si}{30} + \frac{Ni}{60} + \frac{Mo}{15} + \frac{V}{10} + 5B$$

Means and standard deviations of the weight percents of these chemicals were estimated from measurements on 45 weld metals produced on HSLA-100 steel base metal. Assume the means and standard deviations (SD) are as given in the following table.

	Mean	SD
C	0.0695	0.0018
Mn	1.0477	0.0269
Cu	0.8649	0.0225
Cr	0.7356	0.0113
Si	0.2171	0.0185
Ni	2.8146	0.0284
Mo	0.5913	0.0031
V	0.0079	0.0006
B	0.0006	0.0002

a) Find the mean carbon equivalent of weld metals produced from HSLA-100 steel base metal.
b) Assuming the weight percents to be independent, find the standard deviation of the carbon equivalent of weld metals produced from HSLA-100 steel base metal.

14. The oxygen equivalence number of a weld is a number that can be used to predict properties such as hardness, strength, and ductility. The article "Advances in Oxygen Equivalence Equations for Predicting the Properties of Titanium Welds" (D. Harwig, W. Ittiwattana, and H. Castner, The Welding Journal, 2001:126s–136s) presents several equations for computing the oxygen equivalence number of a weld. One equation, designed to predict the hardness of a weld, is X = O + 2N + (2/3) C, where X is the oxygen equivalence, and O, N, and C are the amounts of oxygen, nitrogen, and carbon, respectively, in weight percent, in the weld. Suppose that for welds of a certain type, μ_O = 0.1668, μ_N = 0.0255, μ_C = 0.0247, σ_O = 0.0340, σ_N = 0.0194, and σ_C = 0.0131.

a) Find μ_X.
b) Suppose the weight percents of O, N, and C are independent. Find σ_X .

15. Measurements are made on the length and width (in cm) of a rectangular component. Because of measurement error, the measurements are random variables. Let X denote the

length measurement and let Y denote the width measurement. Assume that the probability density function of X is

$$f(x) = \begin{cases} 10 & 9.95 < x < 10.05 \\ 0 & otherwise \end{cases}$$

and that the probability density function of Y is

$$g(y) = \begin{cases} 5 & 4.9 < y < 5.1 \\ 0 & otherwise \end{cases}$$

Assume that the measurements X and Y are independent.

a) Find P(X < 9.98).
b) Find P(Y > 5.01).
c) Find P(X < 9.98 and Y > 5.01).
d) Find μ_X
e) Find μ_Y

16. The thickness X of a wooden shim (in mm) has probability density function

$$f(x) = \begin{cases} \frac{3}{4} - \frac{3(x-5)^2}{4} & 4 < x < 6 \\ 0 & otherwise \end{cases}$$

a) Find μ_X
b) Find σ_X^2
c) Let *Y* denote the thickness of a shim in inches (1 mm = 0.0394 inches). Find μ_Y and σ_Y^2
d) If three shims are selected independently and stacked one atop another, find the mean and variance of the total thickness.

17. The article "Abyssal Peridotites > 3800 Ma from Southern West Greenland: Field Relationships, Petrography, Geochronology, Whole-Rock and Mineral Chemistry of Dunite and Harzburgite Inclusions in the Itsaq Gneiss Complex" (C. Friend, V. Bennett, and A. Nutman, Contrib Mineral Petrol, 2002:71–92) describes the chemical compositions of certain minerals in the early Archaean mantle. For a certain type of olivine assembly, the silicon dioxide (SiO2) content (in weight percent) in a randomly chosen rock has mean 40.25 and standard deviation 0.36.

a) Find the mean and standard deviation of the sample mean SiO_2 content in a random sample of 10 rocks.
b) How many rocks must be sampled so that the standard deviation of the sample mean SiO_2 content is 0.05?

18. The number of bytes downloaded per second on an information channel has mean 105 and standard deviation 104. Among the factors influencing the rate is congestion, which produces alternating periods of faster and slower transmission. Let X represent the number of bytes downloaded in a randomly chosen five-second period.

a) Is it reasonable to assume that $\mu_X = 5 * 10^5$? Explain.
b) Is it reasonable to assume that $\sigma_X = \sqrt{5} * 10^4$? Explain.