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MODULE 5

Propositional Logic

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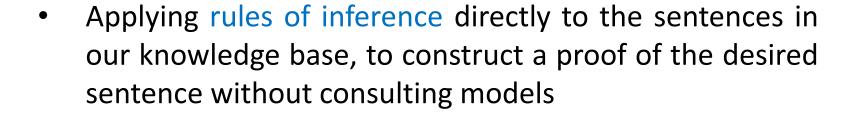


Outline

- Inference and proofs
 - Theorem proving
 - Logical equivalence
 - Validity
 - Satisfiability
 - Inference and Proofs
 - Modus Ponens
 - And-Elimination

Inference and Proofs

Theorem proving





Inference and Proofs



Logical equivalence
Validity
Satisfiability

Inference and Proofs



Logical equivalence

- Two sentences α and β are logically equivalent if they are true in the same set of models.
- We write this as $\alpha \equiv \beta$.
- For example, $P \land Q$ and $Q \land P$ are logically equivalent

Inference and Proofs



• Two sentences are logically equivalent iff true in same models: $\alpha \equiv \beta$ iff $\alpha \models \beta$ and $\beta \models \alpha$

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(\alpha \wedge \beta) \equiv (\beta \wedge \alpha) commutativity of \wedge
           (\alpha \vee \beta) \equiv (\beta \vee \alpha) commutativity of \vee
((\alpha \land \beta) \land \gamma) \equiv (\alpha \land (\beta \land \gamma)) associativity of \land
((\alpha \vee \beta) \vee \gamma) \equiv (\alpha \vee (\beta \vee \gamma)) associativity of \vee
            \neg(\neg \alpha) \equiv \alpha double-negation elimination
       (\alpha \Rightarrow \beta) \equiv (\neg \beta \Rightarrow \neg \alpha) contraposition
       (\alpha \Rightarrow \beta) \equiv (\neg \alpha \lor \beta) implication elimination
      (\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha)) biconditional elimination
       \neg(\alpha \land \beta) \equiv (\neg \alpha \lor \neg \beta) de Morgan
       \neg(\alpha \lor \beta) \equiv (\neg \alpha \land \neg \beta) de Morgan
(\alpha \wedge (\beta \vee \gamma)) \equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) distributivity of \wedge over \vee
(\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) distributivity of \vee over \wedge
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Inference and Proofs



Validity

- A sentence is valid if it is true in all models.
- For example, the sentence **P V ¬P** is valid
- Validity is connected to inference via deduction theorem
- For any sentences α and β , $\alpha \mid = \beta$ if and only if the sentence ($\alpha \Rightarrow \beta$) is valid.

Inference and Proofs



Satisfiability

- A sentence is satisfiable if it is true in, or satisfied by, some model
- Satisfiability can be checked by enumerating the possible models *until one is found* that satisfies the sentence.

Inference and Proofs



Examples

 Decide whether each of the following sentences is valid, unsatisfiable, or satisfiable

- a. Smoke \Rightarrow Smoke Valid
- b. Smoke ⇒ Fire Satisfiable
- c. ((Smoke \land Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \land (Heat \Rightarrow Fire)) Valid

Inference and Proofs

Inference Rules

- > Modus Ponens
- > And-Elimination



Inference and Proofs

Inference Rules

Modus Ponens

$$\alpha \Rightarrow \beta, \alpha$$
 β

- Whenever any sentences of the form $\alpha \Rightarrow \beta$ and α are given, then the sentence β can be inferred
- For example, if (WumpusAhead ∧WumpusAlive) ⇒ Shoot and (WumpusAhead ∧ WumpusAlive) are given, then Shoot can be inferred
- If today is Sunday (p) then it is a holiday(q).
- Today is Sunday(p).
 It infers, It is a holiday



Inference and Proofs

Inference Rules



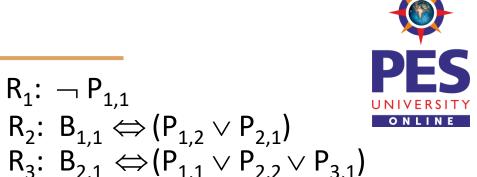
And-Elimination

- from a conjunction, any of the conjuncts can be inferred $\frac{\alpha \wedge \beta}{\alpha}$
- For example, from (WumpusAhead A WumpusAlive),
 WumpusAlive can be inferred.

Inference and Proofs



• We have KB = R1 \wedge R2 \wedge R3 \wedge R4 \wedge R5. We want to prove \neg P1,2



R4: $\neg B_{1,1}$

R5:

 $\mathsf{B}_{\mathsf{2.1}}$

$$R_6: (B_{1,1} \Rightarrow (P_{1,2} \lor P_{2,1})) \land ((P_{1,2} \lor P_{2,1}) \Rightarrow B_{1,1})$$
 by bicond. elim R_2

 $R_7: ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$ by And-Elimination to R_6

 $R_8: (\neg B_{1,1} \Rightarrow \neg (P_{1,2} \lor P_{2,1}))$ by contrapositives

 R_9 : $\neg(P_{1,2} \lor P_{2,1})$ by Modus Ponens with R_8 and R_4

 R_{10} : $\neg P_{1,2} \land \neg P_{2,1}$ by De Morgan's rule

That is: Neither [1,2] nor [2,1] contains a pit.



THANK YOU

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