STATISTICS FOR DATA SCIENCE

HYPOTHESIS and INFERENCE

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UNIT-4 HYPOTHESIS and INFERENCE
Session-11
Fixed Level Testing

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Fixed Level Testing

- A hypothesis test measures the plausibility of the null hypothesis by producing a P-value.

- The smaller the P –value, the less plausible the null. We have pointed out that there is no scientifically valid dividing line between plausibility and implausibility, Sometimes, however, a decision has to be made.

Source: Text Book Chapter6

Fixed Level Testing

So it is impossible to specify a “correct” P-value below which we should reject H0. When possible, it is best simply to report the P-value, and not to make a firm decision whether or not to reject. Sometimes, however, a decision has to be made. For example, if items are sampled from an assembly line to test whether the mean diameter is within tolerance, a decision must be made whether to recalibrate the process. If a sample of parts is drawn from a shipment and checked for defects, a decision must be made whether to accept or to return the shipment.

Source: Text Book Chapter6

Fixed Level Testing

If a decision is going to be made on the basis of a hypothesis test, there is no choice but to pick a cutoff point for the P-value. When this is done, the test is referred to as a xed-le vel test.

Fixed-level testing is just like the hypothesis testing we have been discussing so far, except that a firm rule is set ahead of time for rejecting the null hypothesis. A value α, where $0 < \alpha < 1$, is chosen. Then the $P - value$ is computed. If $P \leq \alpha$, the null hypothesis is rejected and the alternate hypothesis is taken as truth. If $P > \alpha$, then the null hypothesis is considered to be plausible. The value of α is called the significance

Source: Text Book Chapter6

level, or, more simply, the level, of the test. Recall from Section 6.2 that if a test results in a $P - value$ less than or equal to α, we say that the null hypothesis is rejected at level $\alpha\ (or\ 100\alpha\%)$, or that the result is statistically significant at level $\alpha\ (or\ 100\alpha\%)$. As we have mentioned, a common choice for α is 0.05.

Fixed Level Testing

- **Fixed-level testing is just like the hypothesis testing we have been discussing so far, except that a firm rule is set ahead of time for rejecting the null hypothesis.**

To conduct a fixed-level test:

- Choose a number α, where $0 < \alpha < 1$. This is called the significance level, or the level, of the test.

- Compute the P-value in the usual way.

- If $P \leq \alpha$, reject H_0. If $P > \alpha$, do not reject H_0.

Example:

- The mean wear in a sample of 45 steel balls was $X = 673.2\mu m$, and the standard deviation was $s = 14.9\mu m$.
- Let μ denote the population mean wear. A test of $H_1: \mu \geq 675$ versus $H_0: \mu < 675$ yielded a *P*-value of 0.209.
- Can we reject H_0 at the 25% level? Can we reject H_0 at the 5% level?

Solution:

- The P −value of 0.209 is less than 0.25
- So if we had chosen a significance level of $\alpha = 0.25$,we would reject H_0.
- Thus we reject H_0 at the 25% level.
- Since $0.209 > 0.05$, we do not reject H_0 at the 5% level.

Example:

A process for a certain type of ore is designed to reduce the concentration of impurities to less than 2%.

- It is known that the standard deviation of impurities for processed ore is 0.6%.

- Let μ represent the mean impurity level, in percent, for ore specimens treated by this process.

Example:

- The impurity of 80 ore specimens is measured, and a test of the hypothesis $H_0: \mu \geq 2$ versus $H_1: \mu < 2$ will be performed.

a. If the test is made at the 5% level, what is the rejection region?

b. If the sample mean impurity level is 1.85, will H_0 be rejected at the 10% level?

Solution 2 (a)

- $H_0: \mu \geq 2$ versus $H_1: \mu < 2$
- $\sigma = 0.6, \quad n = 80$
- $\frac{\sigma}{\sqrt{n}} = 0.06708$
- Null distribution of $\bar{X}$:
$$\bar{X} \sim N(2, 0.6^2/80)$$
- $\alpha = 0.05$

Solution 2 (a)

- $Z = -1.645$ (Critical value for 5%)

The rejection region is

$$=> \bar{X} = \frac{z * s}{\sqrt{n}} + 2$$

$$=> \bar{X} = \frac{-1.645 * 0.6}{\sqrt{80}} + 2$$

$$=> \bar{X} = 1.89$$

Solution 2 (a)

- Hence, H_0 will be rejected if $\bar{X} \leq 1.890$.

- The rejection region

consists of all values of $\bar{X}$ less than or equal to 1.890.

Solution Using Rejection Region Approach:

$H_0: \mu \geq 2$ versus $H_1: \mu < 2$

- Null distribution of $\overline{X}$:

$$\overline{X} \sim N(2, 0.6^2/80)$$

- $\alpha = 0.10$

- $Z = -1.28$ (Critical value)

Solution Using Rejection Region Approach:

$$=> \bar{X} = \frac{z * s}{\sqrt{n}} + 2$$

$$=> \bar{X} = \frac{-1.28 * 0.6}{\sqrt{80}} + 2$$

$$=> \bar{X} = 1.9141$$

Since $1.85 < 1.9141$

$=> H_0$ will be rejected at the 10% level

Solution Using P Value Approach:

$H_0: \mu \geq 2$ versus $H_1: \mu < 2$

- Null distribution of $\bar{X}$:

$$\bar{X} \sim N(2, 0.6^2/80)$$

- $\alpha = 0.10$

$$\bar{X} = 1.85,$$

- Finding z-score for 1.85
- $z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$

Solution Using P Value Approach:

$$Z = (1.85 - 2)/(0.6/\sqrt{80}) = -2.24$$

$$=> P = P(Z < -2.24) = 0.0125$$

$=> P < \alpha => H_0$ will be rejected at the 10% level

Example:

Small Sample test for population mean:

- Before a substance can be deemed safe for landfilling, its chemical properties must be characterized. The article "Landfilling Ash/Sludge Mixtures" (J. Benoit, T. Eighmy, and B. Crannell, *Journal of Geotechnical and Geoenvironmental Engineering*, 1999: 877–888) reports that in a sample of six replicates of sludge from a New Hampshire wastewater treatment plant, the mean pH was 6.68 with a standard deviation of0.20.
- Can we conclude that the mean pH is less than 7.0?

Solution:

Small Sample test for population mean:

$$n = 6$$

$$H_0 : \mu \geq 7.0 \; versus \; H_1 : \mu < 7.0$$

Under H_0, the test statistic

$$t = \frac{\bar{X} - 7.0}{s/\sqrt{n}}$$

has a Student's t distribution with n-1 degrees of freedom.

Solution:

Small Sample test for population mean:

- Has a Student's *t* distribution with five degrees of freedom. Substituting $\overline{X} = 6.68, s = 0.20, and\ n = 6$, the value of the test statistic is

- $t = \frac{6.68 - 7.0}{0.2/\sqrt{6}} = -3.910$

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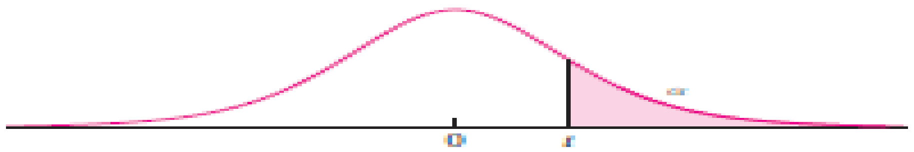
Fixed Level Testing

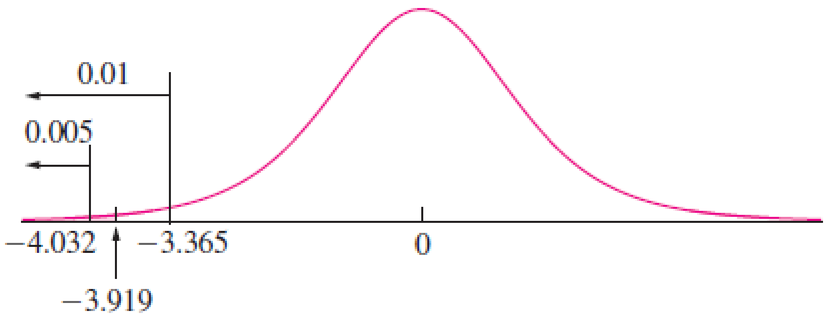
TABLE A.3 Upper percentage points for the Student's t distribution



	α								
ν	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	318.309	636.619
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.385	3.646
35	0.255	0.682	1.306	1.690	2.030	2.438	2.724	3.340	3.591
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	0.254	0.677	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Solution:

Small Sample test for population mean:



- The null distribution is Student's t with five degrees of freedom. The observed value of t is –3.919.
- If H_0 is true, the probability that t takes on a value as extreme as or more extreme than that observed is between 0.005 and 0.01.

Solution:

Small Sample test for population mean:

- Consulting the t table, we find that the value $t = -3.365$ cuts off an area of 0.01 in the left-hand tail, and the value $t = -4.033$ cuts off an area of 0.005 .

- We conclude that the P-value is between 0.005 and 0.01. There is strong evidence that the mean pH is less than 7.0.

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