STATISTICS FOR DATA SCIENCE

POWER OF TEST AND SIMPLE LINEAR REGRESSION

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STATISTICS FOR DATA SCIENCE

Unit 5 : Power of test and Simple linear regression

Session : 2

Sub Topic : Power of test

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Calculating Power
A Two-tailed Z test Example

Power of test

Computing the power involves two steps:

1. Compute the rejection region.

Find the a) null distribution

b) Critical Value

c) Rejection region

2. Compute the probability that the test statistic falls in the rejection region if the alternate hypothesis is true.

Find the a) alternate distribution

b) Z-score under H_1 for the critical point

c) P(reject H_0 | H_1 true)

This is the power.

Power of test

Problem 1:

A pollster will conduct a survey of a random sample of voters in a community to estimate the proportion who support a measure on school bonds. Let p be the proportion of the population who support the measure. The pollster will test H_0 **: p = 0.50 versus** H_1 **: p ≠ 0.50** at the **5% level**. If 200 voters are sampled, **what is the power of the test if the true value of p is 0.55?**

Solution:

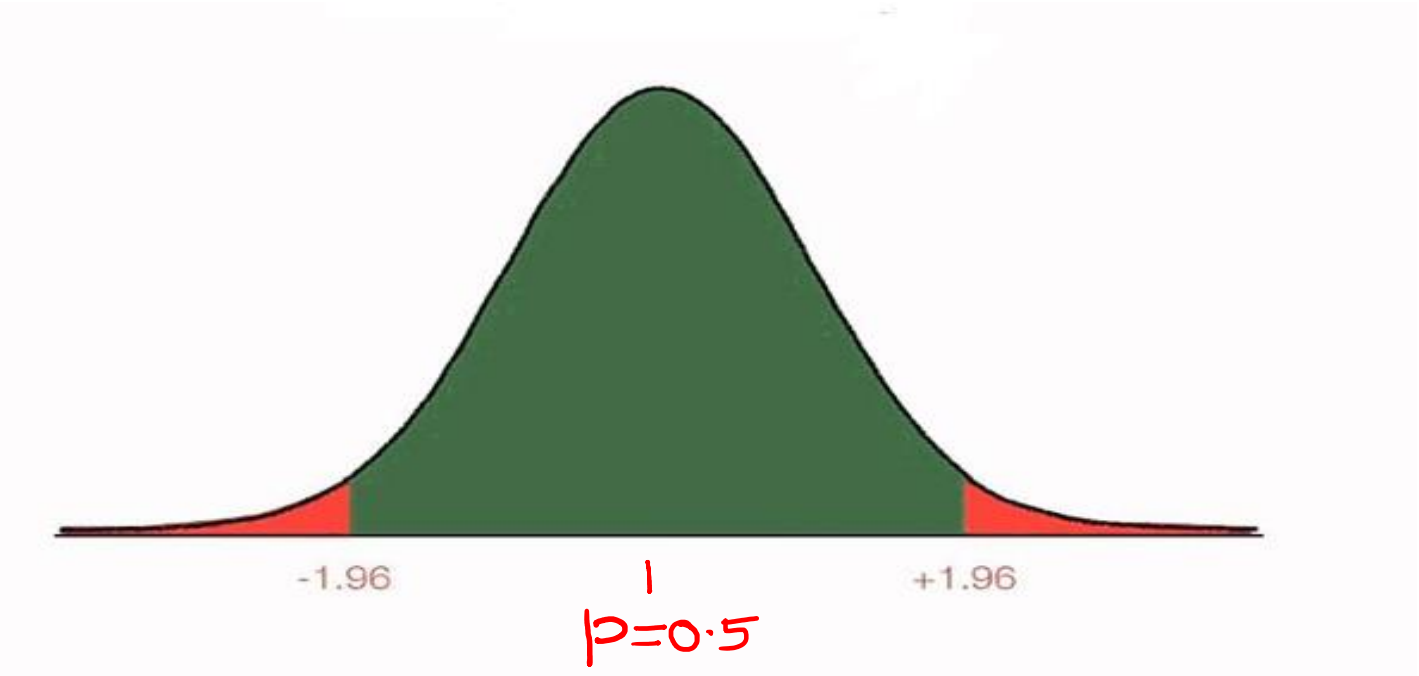
Null distribution of $\hat{p}$:

$$\hat{p} \sim N\left(p_0, \sigma_{\hat{p}}^2\right) \; where \; \sigma_{\hat{p}} = \sqrt{\frac{p_0(1-p_0)}{n}}$$

Power of test

Null distribution of $\hat{p}$: $\hat{p} \sim N\left(0.5, \frac{0.5(1-0.5)}{200}\right)$

$\sim N(0.5, 0.00125)$



0.43 0.569

Power of test

Null distribution of $\hat{p}$:

$$\hat{p} \sim N\left(0.5, \frac{0.5(1-0.5)}{200}\right)$$

$$\sim N(0.5, 0.00125)$$

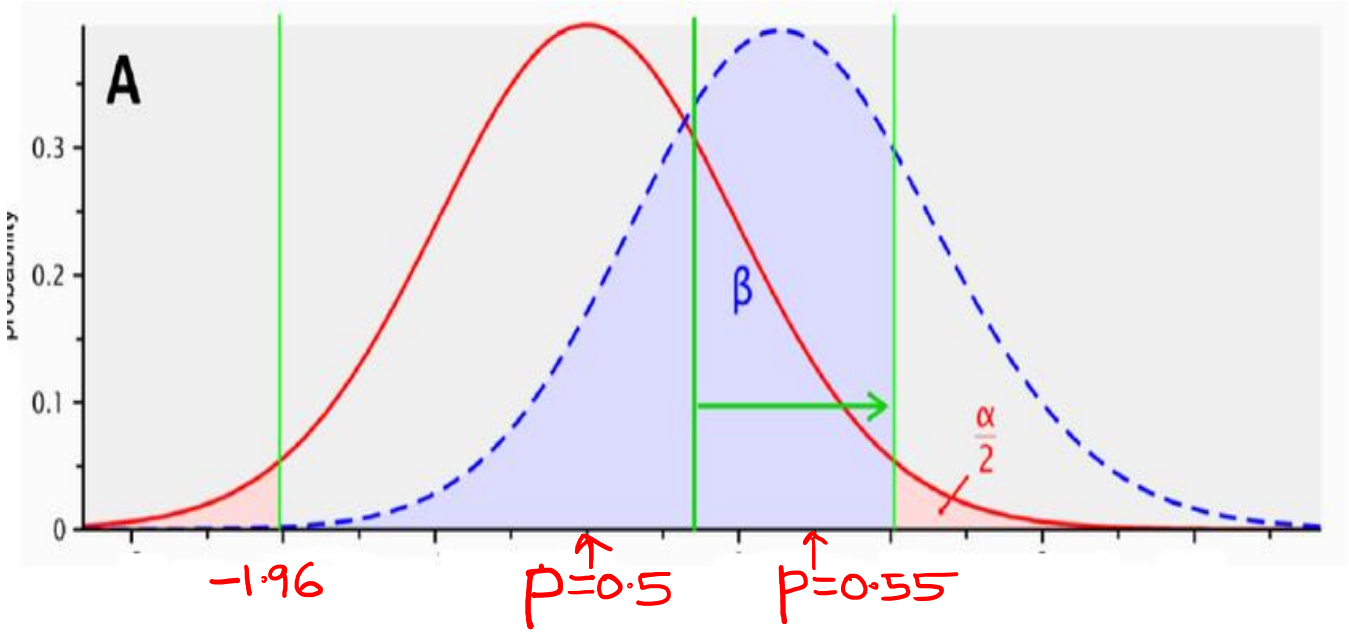
Given $\alpha = 0.05$. Since its two-sided test, the critical value is

For $z \leq -1.96$ $\hat{p} \leq 0.5 + (-1.96)\sqrt{0.00125}$ $\hat{p} \leq 0.43$	For $z \geq -1.96$ $\hat{p} \geq 0.5 + (1.96)\sqrt{0.00125}$ $\hat{p} \geq 0.569$

Power of test

Alternate distribution of $\hat{p}$:

$$\hat{p} \sim N(0.55, 0.00125)$$



Alternate distribution of $\hat{p}$:

$$\hat{p} \sim N(0.55, 0.00125)$$

Z-scores corresponding to the critical points are

$z = \frac{0.43 - 0.55}{\sqrt{0.00125}}$ $z = -3.39$ $P(z < -3.39) = 0.0003$	$z = \frac{0.569 - 0.55}{\sqrt{0.00125}}$ $z = 0.54$ $P(z > 0.54) = 0.2946$

Power = 0.0003 + 0.2946 = 0.2949

Power of test

Problem 2

A pollster will conduct a survey of a random sample of voters in a community to estimate the proportion who support a measure on school bonds.

Let p be the proportion of the population who support the measure. The pollster will test:

H_0 **: p = 0.50 versus** H_1 **: p ≠ 0.50**

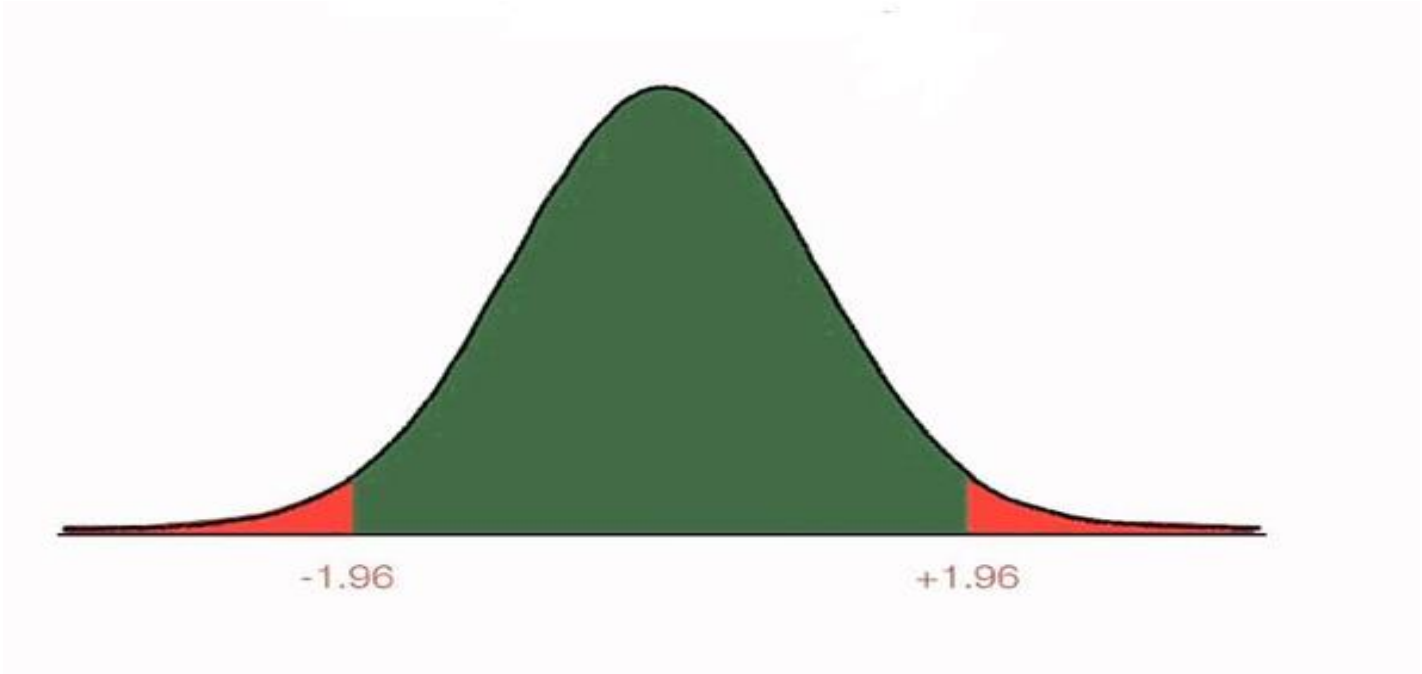
at the **5% level**. **How many voters must be sampled so that the power will be 0.8 when the true value of p = 0.55?**

Power of test

Let n represent the necessary sample size.

Null distribution of $\hat{p}$: $\hat{p} \sim N\left(0.5, \frac{0.5(1-0.5)}{n}\right)$

$$\sim N\left(0.5, \frac{0.25}{n}\right)$$



Power of test

We can generally ignore the miniscule region associated with one of the tails

Critical Value under Null Distribution	
$Z \leq -1.96$ $\hat{p} \leq 0.5 - 1.96 \times \sqrt{\frac{0.25}{n}}$ (Ignored)	$Z = 1.96$ $\hat{p} = 0.5 + 1.96 \times \sqrt{\frac{0.25}{n}}$

Power of test

We can generally ignore the miniscule region associated with one of the tails

Critical Value under Alternate Distribution	
$Z = \frac{(0.43 - 0.55)}{\sqrt{0.25/n}}$ $Z = -3.39$ $P(Z < -3.39) = .0003$ (ignored)	Since Power $= 0.80$ $P(Z > z) = 0.80$ $=> z - score = -0.84$ $\hat{p} = 0.55 - 0.84 \times \sqrt{\frac{0.25}{n}}$

Power of test

Setting the critical values equal

$$0.5 + 1.96 \times \sqrt{\frac{0.25}{n}} = 0.55 - 0.84 \times \sqrt{\frac{0.25}{n}}$$

$$\boldsymbol{n = 784}$$

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THANK YOU

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