PES UNIVERSITY

2020

Unit II: Linear Algebra

Question Bank

1 Check whether the following are linear transformations/operators where v denotes the vector (v_1, v_2, v_3, v_4) belonging to R^4.

(a) $T(v) = 3v + S(v)$ where S is a linear operator on R^4

(b) $T(v) = (v_1+v_2, v_3+2, v_4)$

(c) $T(v) = v \cdot v_0$ (inner product) where v_0 is a fixed vector in R^4.

(d) $T(v) = S_1(S_2(v))$ where $S_2 : R^4 \dashrightarrow V$ and $S_1 : V \dashrightarrow W$ are linear transformations, with V and W, vector spaces.

2 Let $T(x_1, x_2, x_3, x_4) = (2x_1+x_3+x_4, x_1+2x_2+x_4, x_1+x_2+2x_3, x_2+x_3+2x_4)$ w.r.t. the basis $B_1 = \{(1,0,0,0), (2,1,0,0), (1,2,1,0), (1,1,2,1)\}$ for domain and $B_2 = \{(0,0,0,1),(0,0,1,2),(0,1,2,1),(1,2,1,1)\}$ for co-domain.

3 Find the matrix of $T(x, y, z) = (x, x + y, y + z)$ w.r.t., the standard ordered basis for the domain and the basis $\{(1,2,1),(2,1,2),(3,3,4)\}$ for the co-domain.

4 Let S be rotation by 90 degrees and T the reflection across the line y = x.

a For v = (2,5) , calculate TS(v) and ST(v).

b Can you find a non-zero vector w such that TS(w) = ST(w)?

5 Let S be the reflection across the y-axis and T the projection on the x-axis.

a Is ST = TS?

b What if S is instead the reflection across the line y = -x?

6 Find the range, rank, kernel, nullity of T in the following cases

a T is the linear transformation given in Question 2 above

b $T(x_1, x_2, x_3, x_4) = (x_1+x_4, x_1+x_2, x_2+x_3, x_3+x_4)$

7 Find a set of vectors which span $S^{\perp}$ for the following cases of S.

a $S = \{(1,4,-2)\}$

b $S = \{(1,4,3,5), (-6,2,1,4)\}$

c $S = \{(3,1,4), (1,2,3), (5,2,3)\}$

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8 Find the plane through the origin in R^3 that is orthogonal to the intersection of the planes x–2y=0 and y–3z=0.

9 Find the 2-d plane through the origin in R^4 that is orthogonal to the plane by the system x – y + z + t = 0, 3x + 5z – t = 0.

10 Find the matrix for the projection onto

a the vector (-3,1,-2).

b the subspace spanned by (5,2,7) and (6,4,6).

c the x_2x_4-plane in R^4 whose vectors have coordinates (x_1,x_2,x_3,x_4)

d the vector (9,8,7,6).

11 Find the projection matrix:

a If B is the projection matrix for projection onto a subspace V of R^n, what is the matrix for the projection onto $V^{\perp}$?

b Calculate the projection matrix for projection onto the left-null space of a matrix A, using the projection matrix for the same onto C(A).

c What are the matrices for the projections onto the row space and the null space of A?

12 Find the straight line that best fits the following data:

x	0	3	5	8	10
y	2	4	7	12	17

a Find the straight line that best fits the data given above.

b Can you find the quadratic $y = ax^2 + bx + c$ that best fits the above data? (HINT: Write matrix A with all 1's in the first column, x values in the second and x^2 values in the third).

c What are the dimensions of the matrix A in the above hint, if we look for a model of the form y = p(x) where p is a polynomial of degree k?