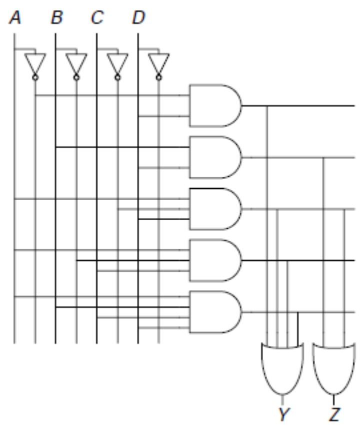
PES University

Department of Computer science and Engineering

Digital Design and Computer Organization

Logic Minimization and K-maps

1. Simplify the following Boolean equations using Boolean theorems. Check for correctness using a truth table or K-map.
   a) Y=AC+A’B’C b)Y= A’B’+A’BC’+(A+C’)’ c) Y=A’B’C’D’+AB’C’+ AB’CD’+ABD+A’B’CD’+BC’D+A’
2. Simplify the following Boolean equations using Boolean theorems. Check for correctness using a truth table or K-map.
   (a) Y = A’BC+A’BC’
   (b) Y = A+(A’B+A’ B’)’ +(A+B’)
   (c) Y = ABC+ABD+ABE+ACD+ACE+ (A+D+ E)’+ B’ C’D+ B’C’E+ B’D’E’+C’D’E’
3. Simplify each of the following Boolean equations. Sketch a reasonably simple combinational circuit implementing the simplified equation.
   (a) Y = A’BC+(BC’)’+BC
   (b) Y = (A+ B+C)’D+AD+ B
   (c) Y = ABCD+A’BC’D+(B’+D)’E
4. Write Boolean equations for the circuit in Figure given below. You need not minimize the equations



5. Find a minimal Boolean equation for the function in truth table given below. Remember to take advantage of the don’t care entries.

Truth Table

A	B	C	D	Y
0	0	0	0	X
0	0	0	1	X
0	0	1	0	X
0	0	1	1	0

0	1	0	0	0
0	1	0	1	X
0	1	1	0	0
0	1	1	1	X
1	0	0	0	1
1	0	0	1	0
1	0	1	0	X
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	X
1	1	1	1	1

6. Find a minimal Boolean equation for the function in Figure below Remember to take advantage of the don't care entries.

A	B	C	D	Y
0	0	0	0	0
0	0	0	1	1
0	0	1	0	X
0	0	1	1	X
0	1	0	0	0
0	1	0	1	X
0	1	1	0	X
0	1	1	1	X
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	X
1	1	1	1	1

7. Complete the design of the seven-segment decoder segments *Sc* through *Sg* .
   (a) Derive Boolean equations for the outputs *Sc* through *Sg* assuming that inputs greater than 9 must produce blank (0) outputs.
   (b) Derive Boolean equations for the outputs *Sc* through *Sg* assuming that inputs greater than 9 are don't cares.
   (c) Sketch a reasonably simple gate-level implementation of part (b). Multiple outputs can share gates where appropriate.

8. A circuit has four inputs and two outputs. The inputs, *A*3:0, represent a number from 0 to 15. Output *P* should be TRUE if the number is prime (0 and 1 are not prime, but 2, 3, 5, and so on, are prime). Output *D* should be TRUE if the number is divisible by 3. Give simplified Boolean equations for each output and sketch a circuit

9. Design a circuit that will tell whether a given month has 31 days in it. The month is specified by a 4-bit input, *A*3:0. For example, if the inputs are 0001, the month is January, and if the inputs are 1100, the month is December. The circuit output, *Y*, should be HIGH only when the month specified by the inputs has 31 days in it. Write the simplified equation, and draw the circuit diagram using a minimum number of gates. (Hint: Remember to take advantage of don't cares.).

10. Simplify the Boolean function: $F(x, y, z) = \Sigma(2, 3, 4, 5)$
11. Simplify the Boolean function: $F(x, y, z) = \Sigma(0, 2, 4, 5, 6)$

12. Simplify the following Boolean function into (a) sum-of-products form and (b) product-of-sums form:

    $F(A, B, C, D) = \Sigma(0, 1, 2, 5, 8, 9, 10)$
13. Simplify the Boolean function $F(w, x, y, z) = \Sigma(1, 3, 7, 11, 15)$ which has the don't-care conditions

    $d(w, x, y, z) = \Sigma(0, 2, 5)$
14. Simplify the Boolean function by finding all its prime implicants and essential prime implicants: $F(A, B, C, D) = \Sigma(0, 2, 3, 5, 7, 8, 9, 10, 11, 13, 15)$
15. Simplify the following Boolean expressions, using four-variable maps:

    (a) A'B'C'D'+ AC'D'+ B'CD'+ A'BCD + BC'D (b) x'z + w'xy'+ w(x'y + xy')

    (c) w'z + xz + x'y + wx'z (d) AD'+ B'C'D + BCD' + BC'D

16. Simplify the following Boolean functions by first finding the prime and essential prime implicants:

    (a) $F(w, x, y, z) = \Sigma(0, 2, 5, 7, 8, 10, 12, 13, 14, 15)$

    (b) $F(A, B, C, D) = \Sigma(0, 2, 3, 5, 7, 8, 10, 11, 14, 15)$

    (c) $F(A, B, C, D) = \Sigma(1, 3, 4, 5, 10, 11, 12, 13, 14, 15)$

    (d) $F(w, x, y, z) = \Sigma(0, 1, 4, 5, 6, 7, 9, 11, 14, 15)$