Unit - IV

Ordinary Differential Equations

Solve the following differential equations

1. $(\sin x \sin y - x e^{y}) dy = (e^{y} + \cos x \cos y) dx$

$(e^{y} + \cos x \cos y) dx - (\sin x \sin y - x e^{y}) dy = 0$

$$\frac{\partial M}{\partial y} = e^{y} - \cos x \sin y$$

$$\frac{\partial N}{\partial x} = -\cos x \sin y + e^{y} = \frac{\partial M}{\partial y}$$

$\therefore$ The given DE is exact

General solution is

$$\int M\,dx_{(y = const)} + \int (N \text{ terms independent of } x) dy = c$$

$$\int e^{y} + \cos x \cos y \, dx + \int 0 \, dy = c$$

$$x e^{y} + \sin x \cos y = c$$

2. $(y - x^{3}) dx + (x + y^{3}) dy = 0$

It is in the form of $M dx + N dy = 0$

$$\frac{\partial M}{\partial y} = 1$$

$$\frac{\partial N}{\partial x} = 1 = \frac{\partial M}{\partial y}$$

$\therefore$ The given D.E is exact

General solution is

$\int M\,dx_{(y=\text{const})} + \int (N \text{ terms independent of } x)\,dy = c$

$\int y - x^3\,dx + \int y^3\,dy = c$

$xy - \frac{x^4}{4} + \frac{y^4}{4} = c$

$4xy - x^4 + y^4 = 4c$

3. $2xy\,dy - (x^2+y^2+1)\,dx = 0$

$(x^2+y^2+1)\,dx - 2xy\,dy = 0.$

It is in the form of $M\,dx + N\,dy = 0.$

$\frac{\partial M}{\partial y} = 2y.$

$\frac{\partial N}{\partial x} = -2y \neq \frac{\partial M}{\partial y}$ => The DE is not exact

$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{2y+2y}{-2xy} = \frac{-2}{x} = f(x)$

$I.F = e^{\int f(x)} = e^{\int -2/x\,dx} = \frac{1}{x^2}$

Multiplying I.F with the given DE

$\left(1 + \frac{y^2}{x^2} + \frac{1}{x^2}\right)dx - \frac{2y}{x}\,dy = 0.$

$\frac{\partial M}{\partial y} = \frac{2y}{x^2}$

$\frac{\partial N}{\partial x} = \frac{2y}{x^2} = \frac{\partial M}{\partial y}$

$\therefore$ The DE is now exact

General solution is

$$\int M\,dx_{(y = \text{const})} + \int (N \text{ terms independent of } x)\,dy = c$$

$$\int 1 + \frac{y^2}{x^2} + \frac{1}{x^2}\,dx + \int 0\,dy = c.$$

$$x - \frac{y^2}{x} - \frac{1}{x} = c$$

$$x^2 - y^2 - 1 = cx$$

4. $y(x+y)dx + (x+2y-1)dy = 0$

$$(xy+y^2)dx + (x+2y-1)dy = 0.$$

$$\frac{\partial M}{\partial y} = x + 2y$$

$$\frac{\partial N}{\partial x} = 1 \neq \frac{\partial M}{\partial y} \Rightarrow \text{The DE is not exact}$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{x+2y-1}{x+2y-1} = 1 = f(x)$$

$$I.F = e^{\int f(x)} = e^{\int 1\,dx} = e^x$$

Multiplying I.F with the given DE

$$(xy+y^2)e^x dx + (x+2y-1)e^x dy = 0$$

$$\frac{\partial M}{\partial y} = e^x(x+2y)$$

$$\frac{\partial N}{\partial x} = e^x(x+2y-1) + e^x = e^x(x+2y) = \frac{\partial M}{\partial y}$$

$\therefore$ The DE is now exact

General solution is

$\int M dx_{(y=const)} + \int (N \text{ terms independent of } x) dy = c$

$\int x e^x (y) + y^2 e^x \, dx + \int 0 = c$

$y e^x (x-1) + y^2 e^x = c$

$y(x+y-1) = c e^{-x}$

5. $(y^4 + 2y) dx + (xy^3 + 2y^4 - 4x) dy = 0$

$\frac{\partial M}{\partial y} = 4y^3 + 2$

$\frac{\partial N}{\partial x} = y^3 - 4 \neq \frac{\partial M}{\partial y} \Rightarrow$ The DE is not exact

$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \frac{y^3 - 4 - 4y^3 - 2}{y^4 + 2y} = \frac{-(3y^3+6)}{y^4+2y} = \frac{-3}{y} = f(y)$

$I.F = e^{\int f(y) dy} = e^{\int -3/y \, dy} = \frac{1}{y^3}$

Multiplying the given DE with I.F.

$\left(y + \frac{2}{y^2}\right) dx + \left(x + 2y - \frac{4x}{y^3}\right) dy = 0.$

$\frac{\partial M}{\partial y} = 1 - \frac{4}{y^3}$

$\frac{\partial N}{\partial x} = 1 - \frac{4}{y^3} = \frac{\partial M}{\partial y}.$

$\therefore$ The DE is now exact

General solution is

$$\int M\,dx_{(y=\text{const})} + \int (N \text{ terms independent of } x)\,dy = c$$

$$\int y + \frac{2}{y^2}\,dx + \int 2y\,dy = c$$

$$\left(y + \frac{2}{y^2}\right)x + y^2 = c$$

$$(y^3 + 2)x + y^4 = cy^2$$

6. $(x^4 + y^4)dx - xy^3dy = 0$

$$\frac{\partial M}{\partial y} = 4y^3$$

$$\frac{\partial N}{\partial x} = -y^3 \neq \frac{\partial M}{\partial y} \Rightarrow \text{The DE is not exact}$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{4y^3 + y^3}{-xy^3} = \frac{-5}{x} = f(x)$$

$$I.F = e^{\int f(x)} = e^{\int -5/x\,dx} = \frac{1}{x^5}$$

Multiplying I.F with DE

$$\left(\frac{1}{x} + \frac{y^4}{x^5}\right)dx - \frac{y^3}{x^4}dy = 0.$$

$$\frac{\partial M}{\partial y} = \frac{4y^3}{x^5}$$

$$\frac{\partial N}{\partial x} = \frac{4y^3}{x^5} = \frac{\partial M}{\partial y} \Rightarrow \text{The DE is exact}$$

General solution is

$$\int M\,dx_{(y=\text{const})} + \int (N \text{ terms independent of } x)\,dy = c$$

$$\int \frac{1}{x} + \frac{y^4}{x^5}\,dx + \int 0\,dy = c$$

$$\log x - \frac{y^4}{4x^4} = c$$

$$4x^4 \log x - y^4 = 4cx^4$$

7. $(xy\sin(xy) + \cos(xy))y\,dx + (xy\sin(xy) - \cos(xy))x\,dy = 0$

$$\frac{\partial M}{\partial y} = xy\sin(xy) + \cos(xy) + y(x\sin xy + x^2y\cos(xy) - x\sin(xy))$$

$$\frac{\partial N}{\partial x} = xy\sin(xy) - \cos(xy) + x(y\sin(xy) + xy^2\cos(xy) + y\sin(xy))$$

$$\frac{\partial N}{\partial x} \neq \frac{\partial M}{\partial y} \Rightarrow \text{D.E is not exact}$$

But DE is in the form $My\,dx + Nx\,dy = 0$

$$I.F = \frac{1}{Mx - Ny} = \frac{1}{(xy\sin(xy) + \cos(xy))xy - (xy\sin(xy) - \cos(xy))xy}$$

$$I.F = \frac{1}{2xy\cos(xy)}$$

Multiplying DE with IF

$$\left(\frac{1}{2}y\tan(xy) + \frac{1}{2x}\right)dx + \left(\frac{1}{2}x\tan(xy) - \frac{1}{2y}\right)dy = 0$$

General solution is

$$\int M\,dx_{(y=\text{const})} + \int (N \text{ terms independent of } x)\,dy = C$$

$$\int \frac{1}{2} y \tan(xy) + \frac{1}{2x}\,dx + \int \frac{-1}{2y}\,dy = c$$

$$\log(\sec(xy)) + \log(x) - \log(y) = 2c$$

$$x\sec(xy) = y e^{2c}$$

8. $y(1+xy)dx + (1-xy)x\,dy = 0$

$$\frac{\partial M}{\partial y} = 1 + 2xy$$

$$\frac{\partial N}{\partial x} = 1 - 2xy \neq \frac{\partial M}{\partial y} \Rightarrow \text{The DE is not exact}$$

DE is of the form $My\,dx + Nx\,dy = 0$

$$\Rightarrow I.F = \frac{1}{Mx - Ny} = \frac{1}{xy(1+xy) - xy(1-xy)} = \frac{1}{x^2y^2}$$

Multiplying D.E with I.F

$$\left(\frac{1}{x^2y} + \frac{1}{x}\right)dx + \left(\frac{1}{-xy^2} - \frac{1}{y}\right)dy = 0$$

General solution is

$$\int M\,dx_{(y=\text{const})} + \int (N \text{ terms independent of } x)\,dy = c$$

$$\int \frac{1}{x^2y} + \frac{1}{x}\,dx + \int \frac{-1}{y}\,dy = c$$

$$\frac{1}{y}\left(\frac{-1}{x}\right) + \log x - \log y = c$$

$$\log\frac{x}{y} - \frac{1}{xy} = c$$

9. $\frac{dy}{dx} + y = e^{e^x}$

~~Let~~ The given D.E is linear

$I.F = e^{\int 1 dx} = e^x$

General solution

$$y e^x = \int e^{e^x} e^x \, dx + c$$

$$y e^x = e^{e^x} + c$$

10 $y\,dx + (3x - xy + 2)\,dy = 0$

$$\frac{dx}{dy} + \frac{3x}{y} - x + \frac{2}{y} = 0$$

$$\frac{dx}{dy} + x\left(\frac{3}{y} - 1\right) = -\frac{2}{y}$$

The given DE is linear

$I.F = e^{\int 3/y - 1 \, dy} = y^3 e^{-y}$

General solution

$$x y^3 e^{-y} = \int -\frac{2}{y} y^3 e^{-y} + c$$

$$x y^3 e^{-y} = -2 \int y^2 e^{-y} + c$$

$$x y^3 e^{-y} = -2(-y^2 - 2y - 2)e^{-y} + c$$

$$x y^3 = 2y^2 + 4y + 4 + c e^{y}$$

11. $y\,dx - (x + 2y^3)\,dy = 0$

$$\frac{dx}{dy} - \frac{x}{y} - 2y^2 = 0$$

$$\frac{dx}{dy} - \frac{x}{y} = 2y^2$$

The given D.E is linear

$$I.F = e^{\int -1/y\, dy} = \frac{1}{y}$$

General solution

$$x\left(\frac{1}{y}\right) = \int 2y^2\left(\frac{1}{y}\right) + c$$

$$\frac{x}{y} = y^2 + c$$

$$x = y^3 + cy$$

12. $dx - (x^2y^3 + xy)dy = 0$

$$\frac{dx}{dy} - x^2y^3 - xy = 0$$

$$\frac{1}{x^2}\frac{dx}{dy} - \frac{y}{x} = y^3$$

Let $\frac{-1}{x} = t$

$$\frac{1}{x^2}\, dx = dt$$

$$\frac{dt}{dy} + ty = y^3$$

$$I.F = e^{\int y\, dy} = e^{y^2/2}$$

General solution

$$t\, e^{y^2/2} = \int y^3 e^{y^2/2} + c$$

$$\frac{-1}{x} e^{y^2/2} = (y^2 - 2)e^{y^2/2} + c$$

$$-1 = x(y^2 - 2) + cxe^{-y^2/2}$$

$$\frac{dx}{dy} - \frac{x}{y} = 2y^2$$

The given D.E is linear.

$$I.F = e^{\int -1/y \, dy} = \frac{1}{y}.$$

General solution

$$x\left(\frac{1}{y}\right) = \int 2y^2\left(\frac{1}{y}\right) + c$$

$$\frac{x}{y} = y^2 + c$$

$$x = y^3 + cy$$

12. $dx - (x^2y^3 + xy)dy = 0$

$$\frac{dx}{dy} - x^2y^3 - xy = 0$$

$$\frac{1}{x^2}\frac{dx}{dy} - \frac{y}{x} = y^3.$$

Let $-\frac{1}{x} = t$

$$\frac{1}{x^2}\, dx = dt$$

$$\frac{dt}{dy} + ty = y^3.$$

$$I.F = e^{\int y \, dy} = e^{y^2/2}$$

General solution

$$t\, e^{y^2/2} = \int y^3 e^{y^2/2} + c$$

$$-\frac{1}{x} e^{y^2/2} = (y^2 - 2)e^{y^2/2} + c$$

$$-1 = x(y^2 - 2) + cxe^{-y^2/2}$$

Find the Orthogonal Trajectories of family of following curves

1. $x^2+y^2=c^2$

Converting to polar

$x = r\cos\theta$

$y = r\sin\theta$

$r = c$

Differentiating w.r.t θ

$$\frac{dr}{d\theta} = 0$$

Substituting $\frac{dr}{d\theta} = -r^2\frac{d\theta}{dr}$

$$-r^2\frac{d\theta}{dr} = 0.$$

$$d\theta = 0.$$

Integrating.

$$\theta = k$$

Converting to cartesian

$$\tan^{-1} y/x = k$$

$$y = x\tan k$$

$$y = c'x$$

2. $y = cx^2$

Differentiating w.r.t x

$$\frac{dy}{dx} = 2cx = 2x\left(\frac{y}{x^2}\right) = \frac{2y}{x}$$

Substituting $\frac{dy}{dx} = -\frac{dx}{dy}$

$$-\frac{dx}{dy} = \frac{2y}{x}$$

$$2y\,dy + x\,dx = 0$$

$$y^2 + \frac{x^2}{2} = c$$

3. $r = a(1+\sin^2\theta)$

Differentiating w.r.t θ

$$\frac{dr}{d\theta} = a(2\sin\theta\cos\theta) = 2\sin\theta\cos\theta\left(\frac{r}{1+\sin^2\theta}\right)$$

Substituting $\frac{dr}{d\theta} = -r^2\frac{d\theta}{dr}$

$$-r^2\frac{d\theta}{dr} = \cancel{r}\left(\frac{2\sin\theta\cos\theta}{1+\sin^2\theta}\right)$$

$$d\theta\left(\frac{1+\sin^2\theta}{2\sin\theta\cos\theta}\right) + \frac{dr}{r} = 0$$

Integrating

$$\frac{1}{2}\left(\log\sin\theta - \log(\sin^2\theta - 1)\right) + \log r = c$$

$$\left(\frac{\sin\theta}{\sin^2\theta - 1}\right) r^2 = e^{2c}$$

$$r^2 = b\cos\theta\cot\theta \qquad (b = -e^{2c})$$

4. $r = 4a\sec\theta\tan\theta$

Differentiating w.r.t θ

$$\frac{dr}{d\theta} = 4a\left(\sec\theta\tan^2\theta + \sec^3\theta\right)$$

$$\frac{dr}{d\theta} = 4\,\mathrm{Sec}\,\theta\,(\mathrm{Sec}^2\theta + \mathrm{Tan}^2\theta)\left(\frac{r}{4\,\mathrm{Sec}\,\theta\,\mathrm{Tan}\,\theta}\right)$$

$$\frac{dr}{d\theta} = r\left(\frac{1/\mathrm{Cos}^2\theta}{\mathrm{Sin}\,\theta/\mathrm{Cos}\,\theta} + \frac{\mathrm{Sin}\,\theta}{\mathrm{Cos}\,\theta}\right)$$

$$\frac{dr}{d\theta} = r\left(\frac{1}{\mathrm{Sin}\,\theta\,\mathrm{Cos}\,\theta} + \frac{\mathrm{Sin}\,\theta}{\mathrm{Cos}\,\theta}\right) = r\left(\frac{1+\mathrm{Sin}^2\theta}{\mathrm{Sin}\,\theta\,\mathrm{Cos}\,\theta}\right)$$

Substituting $\frac{dr}{d\theta} = -r^2\frac{d\theta}{dr}$

$$-r^2\frac{d\theta}{dr} = r\left(\frac{1+\mathrm{Sin}^2\theta}{\mathrm{Sin}\,\theta\,\mathrm{Cos}\,\theta}\right)$$

$$d\theta\left(\frac{\mathrm{Sin}\,\theta\,\mathrm{Cos}\,\theta}{1+\mathrm{Sin}^2\theta}\right) + \frac{dr}{r} = 0$$

$$\frac{1}{2}\log(1+\mathrm{Sin}^2\theta) + \log r = c$$

$$r^2(1+\mathrm{Sin}^2\theta) = e^{2c} = b^2 \quad (b = a')$$

$\cancel{r^2}$

5. Find constant a such that $y^3 = c_1 x$ and $x^2 + a y^2 = c_2$

Let

$y^3 = c_1 x$ — (1)

$x^2 + a y^2 = c_2$ — (2)

Differentiating (1) w.r.t x

$$3y^2\frac{dy}{dx} = c_1 = \frac{y^3}{x}$$

$$\frac{dy}{dx} = \frac{y}{3x}$$

Substituting $\frac{dy}{dx} = -\frac{dx}{dy}$

$$\cancel{\frac{dy}{dx} = \frac{y}{}} \quad -\frac{dx}{dy} = \frac{y}{3x}$$

$$y\,dy + 3x\,dx = 0.$$

$$\frac{3}{2}x^2 + \frac{1}{2}y^2 = c$$

$$x^2 + \frac{1}{3}y^2 = \frac{2c}{3} \quad -(3)$$

Comparing (2) & (3).

$$e = 1/3.$$

6. Show that the family of parabolas $y^2 = 2cx + c^2$ is self orthogonal

Differentiating w.r.t x

$$2y\frac{dy}{dx} = 2c.$$

$$c = y\frac{dy}{dx}$$

Substituting in the given equation

$$y^2 = 2xy\frac{dy}{dx} + y^2\left(\frac{dy}{dx}\right)^2.$$

$$y = 2x\frac{dy}{dx} + y^2\left(\frac{dy}{dx}\right)^2 \quad -(1)$$

Substituting $\frac{dy}{dx} = -\frac{dx}{dy}$

$$y = -2x\frac{dx}{dy} + y\left(\frac{dx}{dy}\right)^2$$

Multiplying $\,y\left(\frac{dy}{dx}\right)^2$

$$y\left(\frac{dy}{dx}\right)^2 = -2x\left(\frac{dy}{dx}\right) + y$$

$$y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) = y \quad -(2)$$

(1) and (2) are the same

=> the given curve is self orthogonal

Solve the following equations

1. $p^2 - 5p + 6 = 0$

$(p-2)(p-3) = 0$

$p = 2 \quad ; \quad p = 3$

$\frac{dy}{dx} = 2 \quad ; \quad \frac{dy}{dx} = 3$

$dy = 2dx \quad ; \quad dy = 3dx$

Integrating

$y = 2x + c \quad ; \quad y = 3x + c$

$y - 2x - c = 0. \; ; \quad y - 3x - c = 0.$

General solution is

$(y - 2x - c)(y - 3x - c) = 0$

2. $yp^2 + (x - y)p - x = 0$

$$p^2 + \left(\frac{x}{y} - 1\right)p + (-1)\left(\frac{x}{y}\right) = 0$$

$$(p-1)\left(p + \frac{x}{y}\right) = 0$$

$P = 1$; $P = \frac{-x}{y}$

$\frac{dy}{dx} = 1$; $\frac{dy}{dx} = \frac{-x}{y}$

$dy = dx$; $ydy + xdx = 0$

Integrating

$y = x + c$; $\frac{y^2}{2} + \frac{x^2}{2} = c$

$y - x - c = 0$; $y^2 + x^2 - 2c = 0$

General Solution

$(y - x - c)(y^2 + x^2 - 2c) = 0$

3. $p^2 x^4 - px - y = 0$

$y = p^2 x^4 - px$

Differentiating w.r.t x

$$P = 4P^2 x^3 + 2Px^4 \frac{dP}{dx} - P - x\frac{dP}{dx}$$

$$x\frac{dP}{dx}(2Px^3 - 1) + 2P(2Px^3 - 1) = 0$$

$$\left(x\frac{dP}{dx} + 2P\right)(2Px^3 - 1) = 0.$$

unvalid

$$x\frac{dP}{dx} + 2P = 0.$$

$$\frac{dP}{2P} + \frac{dx}{x} = 0.$$

Integrating

$$\frac{1}{2}\log P + \log x = k$$

$$Px^2 = \cancel{k^2}\, e^{2k} = c$$

$p = \frac{c}{x^2}$

Substituting in the given equation

$y = x^4 \cdot \frac{c^2}{x^4} - x \times \frac{c}{x^2}$

$xy = c^2 x - c$

4. $p \tan p - y + \log(\cos p) = 0$

$y = p \tan p + \log(\cos p)$

Differentiating w.r.t x

$p = (\tan p + p \sec^2 p - \tan p) \frac{dp}{dx}$

$p = p \sec^2 p \frac{dp}{dx}$

$dx = \sec^2 p \, dp$

Integrating

$x = \tan p + c.$

$y = p \tan p + \log(\cos p)$

5. $6p^2y^2 - y + 3px = 0$

$x = \frac{y - 6p^2y^2}{3p} = \frac{y}{3p} - 2py^2$

Differentiating w.r.t y

$\frac{1}{p} = \frac{1}{3}\frac{\left(p - y\frac{dp}{dy}\right)}{p^2} - 4py - 2y^2\frac{dp}{dy}$

$$3P = P - y\frac{dP}{dy} - 12P^3y - 6y^2P^2\frac{dP}{dy}$$

$$6y^2P^2\frac{dP}{dy} + y\frac{dP}{dy} + 12P^3y + 2P = 0.$$

$$y\frac{dP}{dy}(6yP^2+1) + 2P(6yP^2+1) = 0$$

$$\left(y\frac{dP}{dy} + 2P\right)(6yP^2+1) = 0$$

$\rightarrow$ invalid

$$y\frac{dP}{dy} + 2P = 0$$

$$\frac{dP}{2yP} + \frac{dy}{y} = 0.$$

Integrating

$$\log\sqrt{P} + \log y = k$$

$$\sqrt{P}\,y = e^k$$

$$Py^2 = c \Rightarrow P = \frac{c}{y^2}$$

Substituting in the given equation.

$$6y^2\left(\frac{c^2}{y^4}\right) - y + 3x\left(\frac{c}{y^2}\right) = 0.$$

$$6c^2 - y^3 + 3cx = 0.$$

$$y^3 = 3cx + 6c^2$$

6. $y = 2px + p^n$

Differentiating w.r.t x

$$P = 2P + 2x\frac{dP}{dx} + nP^{n-1}\frac{dP}{dx}$$

$$-p = \frac{dp}{dx}\left(2x + np^{n-1}\right)$$

$$\frac{dx}{dp} = -\frac{2x}{p} - np^{n-2}$$

$$\frac{dx}{dp} + \left(\frac{2}{p}\right)x = -np^{n-2}$$

This is a linear equation

$$I.F = e^{\int 2/p \, dp} = p^2$$

$$x p^2 = -n\int p^n + c$$

$$x p^2 = \frac{-n p^{n+1}}{n+1} + c$$

$$x = \frac{-n p^{n-1}}{n+1} + \frac{c}{p^2}$$

$$y = 2p\left(\frac{-n p^{n-1}}{n+1} + \frac{c}{p^2}\right) + p^n$$

$$y = \frac{-2np^n}{n+1} + \frac{2c}{p} + p^n$$

$$y = \frac{2c}{p} + \frac{1-n}{1+n} p^n$$

Application Problems

1. Water at 100°C cools in 10 min to 80°C in a room temperature 25°C
   a) Find the temperature of water after 20 min.
   b) Find the time at which temperature drops to 40°C
   c) 26°C

For cooling

$T - T_A = Ce^{-kt}$

At $t = 0$, $T = 100$, $T_A = 25$.

$100 - 25 = Ce^{0}$

$C = 75$.

At $t = 10$, $T = 80$, $T_A = 25$.

$80 - 25 = 75e^{-10k} = 55$

$e^{10k} = \frac{75}{55}$.

$k = 0.031015$

a) $t = 20$, $T_A = 25$

$T - 25 = 75e^{-20(0.031015)}$

$T = 65.3^{\circ}C$

b) $T = 40$, $T_A = 25$

$40 - 25 = 75e^{-kt}$

$e^{kt} = \frac{75}{15} = 5$

$t = \frac{\log_e 5}{k} = 52$.

c) $T = 26^{\circ}$, $T_A = 25$

$26 - 25 = 75e^{-kt}$

$e^{kt} = 75$

$t = \frac{\log_e 75}{k} = 139$

2. A body is heated to 110°c and placed in air at 10°C. After 1 hr its temperature is 60°C. How much additional time is required for it to cool to 30°C

for cooling

$T - T_A = c e^{-k\theta}$

At $t = 0$, $T = 110$, $T_A = 10$

$110 - 10 = c e^{-k(0)}$

$c = 100$

At $t = 1$, $T = 60$, $T_A = 10$

$60 - 10 = 100 e^{-k}$

$e^{k} = 2$

$k = \log_e 2 = 0.693147$

If $T = 30°c$, $T_A = 10$

$30 - 10 = 100 e^{-kt}$

$e^{kt} = 5$

$t = \frac{\log_e 5}{k} = 2.3223$

Additional time $= 1.3223$.

4. A body initially at 80°c cools down to 60°C in 20 mins, the temperature of the air being 40°C. What will be the temperature of the body after 40 min. From the original

For cooling

$T - T_A = c\, e^{-kt}$

at $t = 0$, $T = 80$, $T_A = 40$.

$80 - 40 = c\, e^{-k(0)}$

$c = 40$

at $t = 20$, $T = 60$, $T_A = 40$

$60 - 40 = 40\, e^{-k(20)}$

$e^{20k} = 2$

$k = \frac{\log_e 2}{20} = 0.034657$

at $t = 40$, $T_A = 40$.

$T - 40 = 40\, e^{-40k}$

$T = 50$ °C