PES University, Bangalore

(Established under Karnataka Act No. 16 of 2013)

UE19CS203 – STATISTICS FOR DATA SCIENCE

Unit-2 - Random Variables

QB SOLVED

Continuous Random Variables

1. **Elongation (in percent) of steel plates treated with aluminum are random with probability density function**

$$f(x) = \begin{cases} \dfrac{x}{250} & 20 < x < 30 \\ 0 & otherwise \end{cases}$$

a) **What proportion of steel plates has elongations greater than 25%?**
b) **Find the mean elongation.**
c) **Find the variance of the elongations.**
d) **Find the standard deviation of the elongations.**
e) **Find the cumulative distribution function of the elongations.**
f) **A particular plate elongates 28%. What proportion of plates elongate more than this?**

[Text Book Exercise – Section 2.4 – Q. No.14 – Pg. No. 114]

Solution:

a) **What proportion of steel plates has elongations greater than 25%?**

Compute P (X > 25)

$$P\ (X > 25) = \int_{25}^{\infty} f(x)\ dx$$

$$= \int_{25}^{30} 250\, dx + \int_{30}^{\infty} 0\ dx$$

$$= \frac{1}{250}\left(\frac{x^2}{2}\right) \Big|_{25}^{30}$$

$= 0.55$

b) Compute mean elongation.

The formula to compute mean elongation is,

$$\mu_X = \int_{-\infty}^{\infty} x\, f(x)dx$$

$$= \int_{-\infty}^{20} 0\, dx + \int_{20}^{30} x\left(\frac{x}{250}\right) dx + \int_{30}^{\infty} 0\, dx$$

$$= 0 + \frac{1}{250}\left(\frac{x^3}{3}\right) \Big|_{20}^{30} + 0$$

$$= \left(\frac{x^3}{750}\right)\Bigg|_{20}^{30}$$

$$= 25.33$$

c) Compute the variance of the elongations.

The formula to compute variance of the elongation is,

$$\sigma_X^2 = \int_{-\infty}^{\infty} x^2\ f(x)dx - \mu_X^2$$

$$= \int_{-\infty}^{20} 0x^2\, dx + \int_{20}^{30} x^2\left(\frac{1}{250}\right) dx + \int_{30}^{\infty} 0\, x^2 dx - (25.33)^2$$

$$= \frac{1}{250}\left(\frac{x^4}{4}\right) \Big|_{20}^{30} - (25.33)^2$$

$$= 8.3911$$

d) Find the standard deviation of the elongations.

Compute standard deviation.

$$\sigma_X = \sqrt{\sigma_X^2}$$

$$= \sqrt{8.3911} = 2.9$$

e) Find the cumulative distribution function of the elongations.

The cumulative distribution function is given by,

$$F\ (x) = \int_{-\infty}^{x} f(x)\, dt$$

The computation of the cumulative distribution function involves the following cases.

(i) If X ≤ 20,

$$F\ (x) = \int_{-\infty}^{20} 0\ dt = 0$$

(ii) If 20 < x < 30,

$$F(x) = \int_{-\infty}^{20} 0\ dt + \int_{20}^{x} \frac{t}{250} dt$$

$$= 0 + \frac{1}{250}\left(\frac{t^2}{2}\right) \mid \begin{matrix} x \\ 20 \end{matrix}$$

$$= \frac{1}{500}\ (x^2 - \ 400)$$

(iii) If X ≥ 30,

$$F(x) = \int_{-\infty}^{20} 0\ dt + \int_{20}^{30} \frac{t}{250} dt + \int_{30}^{x} 0\ \ dt$$

$$= 0 + \frac{1}{250}\left(\frac{t^2}{2}\right) \mid \begin{matrix} x \\ 20 \end{matrix} + \ 0$$

$$= 1$$

f) A particular plate elongates 28%. What proportion of plates elongate more than this?

Compute P (X > 28)

$$P\ (X > 28) = \int_{25}^{30} \frac{x}{250}\ dx$$

$$= \frac{1}{250}\left(\frac{x^2}{2}\right) \mid \begin{matrix} 30 \\ 28 \end{matrix}$$

$$= 0.232$$

2. The diameter of a rivet (in mm) is a random variable with probability density function

$$\boldsymbol{f(x)} = \begin{cases} \boldsymbol{6\ (x - \ 12)(13 - x)} & 12 < \ x \leq 13 \\ \boldsymbol{0} & otherwise \end{cases}$$

a) What is the probability that the diameter is less than 12.5 mm?
b) Find the mean diameter.
c) Find the standard deviation of the diameters.
d) Find the cumulative distribution function of the diameter.
e) The specification for the diameter is 12.3 to 12.7 mm. What is the probability that the specification is met?

[Text Book Exercise – Section 2.4 – Q. No.26 – Pg. No. 116]

<u>Solution</u>

a) What is the probability that the diameter is less than 12.5 mm?

Compute P (X < 12.5)

$$P\ (X < 12.5) = \int_{\infty}^{12.5} f(x)\ dx$$

$$= \int_{-\infty}^{12} 0\ dx + \int_{12}^{12.5} 6(x - 12)(13 - x)\ dx$$

$$= \ 0 + \int_{12}^{12.5} 6(-x^2 + 25x - 156)\ dx$$

$$= 6\left(-\frac{x^3}{3} + \frac{25x^2}{2} - \ 156x\right) \mid \begin{matrix} 12.5 \\ 12 \end{matrix}$$

$$= -2x^3 + \ 75x^2 - 936x \mid \begin{matrix} 12.5 \\ 12 \end{matrix}$$

$$= 0.5$$

b) Find the mean diameter.

The formula to compute mean elongation is,

$$\mu_X = \int_{-\infty}^{\infty} x\, f(x) dx$$

$$= \int_{-\infty}^{12} 0x\, dx + \int_{12}^{13} 6x(x-12)(13-x)\ dx + \int_{13}^{\infty} 0x\ dx$$

$$= 0 + \int_{12}^{13} 6(-x^3 + 25x^2 - 156x)\ dx + 0$$

$$= 6\left(-\frac{x^4}{4} + \frac{25x^3}{3} - 78x^2\right) |_{12}^{13}$$

$$= \left(-\frac{3x^4}{2} + 50x^3 - 468x^2\right) |_{12}^{13}$$

$$= 12.5$$

c) Find the standard deviation of the diameters.

The formula to compute variance is,

$$\sigma_X^2 = \int_{-\infty}^{\infty} x^2\ f(x)dx - \mu_X^2$$

$$= \int_{-\infty}^{12} 0x^2\, dx + \int_{12}^{13} 6x^2(x-12)(13-x)\ dx + \int_{13}^{\infty} 0x^2\ dx - (12.5)^2$$

$$= 0 + 6\left(-\frac{x^5}{5} + \frac{25x^4}{4} - 52x^3\right) |_{12}^{13} + 0 - 156.25$$

$$= \left(-\frac{6x^5}{5} + \frac{75x^4}{2} - 312x^3\right) |_{12}^{13} - 156.25$$

$$= 0.05$$

The standard deviation is,

$\sigma = \sqrt{0.05} = 0.2236$

d) Find the cumulative distribution function of the diameter.

The cumulative distribution function is given by,

$$F\,(x) = \int_{-\infty}^{x} f(x)\,dt$$

The computation of the cumulative distribution function involves the following cases.

(i) If X ≤ 12,

$$F\,(x) = \int_{-\infty}^{x} 0\,dt = 0$$

(ii) If 12 < x < 13,

$$F(x) = \int_{-\infty}^{12} 0\,dt + \int_{12}^{x} 6(t-12)(13-t)\,dt$$

$$= 0 + \int_{12}^{x} 6(-t^2 + 25t - 165)dt$$

$$= 6\left(\frac{-t^3}{3} + \frac{25t^2}{2} - 156t\right)\Big|_{12}^{x}$$

$$= -2x^3 + 75x^2 - 936x + 3888$$

(iii) If X ≥ 13,

$$F(x) = \int_{-\infty}^{12} 0\,dt + \int_{12}^{13} 6(t-12)(13-t)\,dt + \int_{13}^{\infty} 0dt$$

$$= 0 + \int_{12}^{13} 6(-t^2 + 25t - 165)dt + 0$$

$$= 6\left(\frac{-t^3}{3} + \frac{25t^2}{2} - 156t\right)\Big|_{12}^{13}$$

$$= 1$$

e) The specification for the diameter is 12.3 to 12.7 mm. What is the probability that the specification is met?

Compute P (12.3 < X < 12.7)

$$= \int_{12.3}^{12.7} f(x)dx$$

$$= \int_{12.3}^{12.7} 6(x-12)(13-x)dx$$

$$= \int_{12.3}^{12.7} 6(-x^2 + 25x - 156)dx$$

$$= 6\left(\frac{-x^3}{3} + \frac{25x^2}{2} - 156x\right)\Big|_{12.3}^{12.7}$$

$$= 0.568$$