

Renna Sultana

Department of Science and Humanities



MATRICES AND GAUSSIAN ELIMINATION

Renna Sultana

Department of Science and Humanities

GAUSSIAN ELIMINATION:

1. Check for consistency and solve the following system of equations if consistent:

(i)
$$x_1 + x_2 - 2x_3 + 3x_4 = 4$$

 $2x_1 + 3x_2 + 3x_3 - x_4 = 3[A:b] = \begin{pmatrix} 1 & 1 & -2 & 3:4 \\ 2 & 3 & 3 & -1:3 \\ 5 & 7 & 4 & 1:5 \end{pmatrix} \xrightarrow{R_2 - 2R_1} \xrightarrow{R_3 - 5R_1}$

This gives 0=-5 which is not possible.

Also
$$r(A)=2$$
 and $r[A:b]=3$

System is inconsistent and has no solution



GAUSSIAN ELIMINATION:

(ii)
$$x_1 + 2x_1 - 3x_1 - 3x_1 - 3x_1$$

$$R_2 - 2R_1$$

$$R_3 - 3R_1$$

(ii)
$$x_1 + 2x_2 + x_3 = 3$$

 $2x_1 + 5x_2 - x_3 = -4$
 $3x_1 - 2x_2 - x_3 = 5$
 $\begin{bmatrix} 1 & 2 & 1: & 3 \\ 2 & 5 & -1: -4 \\ 3 & -2 & -1:5 \end{bmatrix}$

$$[A:b] = \begin{pmatrix} 1 & 2 & 1 & 3 \\ 2 & 5 & -1 & -4 \\ 3 & -2 & -1 & 5 \end{pmatrix}$$

$$\Rightarrow \begin{cases} x_1 + 2x_2 + x_3 = 3 \\ x_2 - 3x_3 = -10 \\ -28x_3 = -84 \end{cases}$$
 r(A)=r[A:b]=3=n. System is **consistent** and has a unique solution.
$$(x_1, x_2, x_3) = (2, -1, 3)$$

$$(x_1, x_2, x_3) = (2, -1, 3)$$



GAUSSIAN ELIMINATION:



(iii)
$$2x-3y+2z=1$$
$$5x-8y+7z=1$$
$$y-4z=3$$

$$\begin{pmatrix}
2 & -3 & 2:1 \\
5 & -8 & 7:1 \\
0 & 1 & -4:3
\end{pmatrix}
\xrightarrow{R_2 - \left(\frac{5}{2}\right)R_1}$$

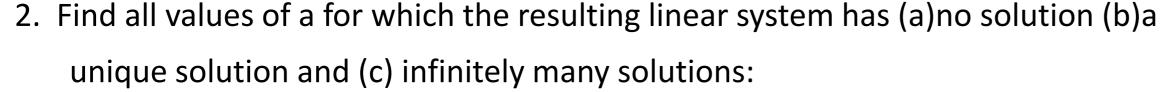
$$\begin{pmatrix}
2 & -3 & 2:1 \\
0 & -1/2 & 2:-3/2 \\
0 & 1 & -4:3
\end{pmatrix}
\xrightarrow{R_3 + 2R_2}$$

$$\begin{pmatrix}
2 & -3 & 2:1 \\
0 & -1/2 & 2:-3/2 \\
0 & 0:0
\end{pmatrix}$$

$$\Rightarrow \begin{cases} 2x - 3y + 2z = 1 \\ -(1/2)y + 2z = -3/2 \end{cases}$$

r(A)=r(A:b)=2< n(=3) hence system is **consistent** and has **infinite number of solutions**. i.e (x, y, z)=(5k+5, 4k-+3, k)

GAUSSIAN ELIMINATION:



$$x + y - z = 2$$

$$x + 2y + z = 3$$

$$x + y + (a^{2} - 5)z = a$$

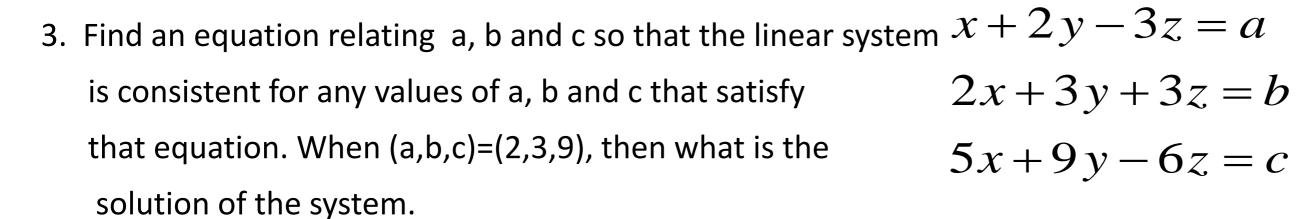
$$\begin{bmatrix} 1 & 1 & -1:2 \\ 1 & 2 & 1:3 \end{bmatrix}$$

$$[A:b] = \begin{pmatrix} 1 & 1 & -1:2 \\ 1 & 2 & 1:-3 \\ 1 & 1 & a^2-5:a \end{pmatrix} \xrightarrow{R_2-R_1} \begin{pmatrix} 1 & 1 & -1:2 \\ 0 & 1 & 2:-5 \\ 0 & 0 & a^2-4:a-2 \end{pmatrix}$$

- (a) System has no solution if $\alpha = -2$ (when r(A) \neq t(A:b))
- (b) System has a unique solution if $\alpha \neq \pm 2$ (when r(A) = r(A:b)=3=n)
- (c) System has infinitely many solutions if $\alpha = 2$ (when r(A) = r(A:b)=2<n)



GAUSSIAN ELIMINATION:



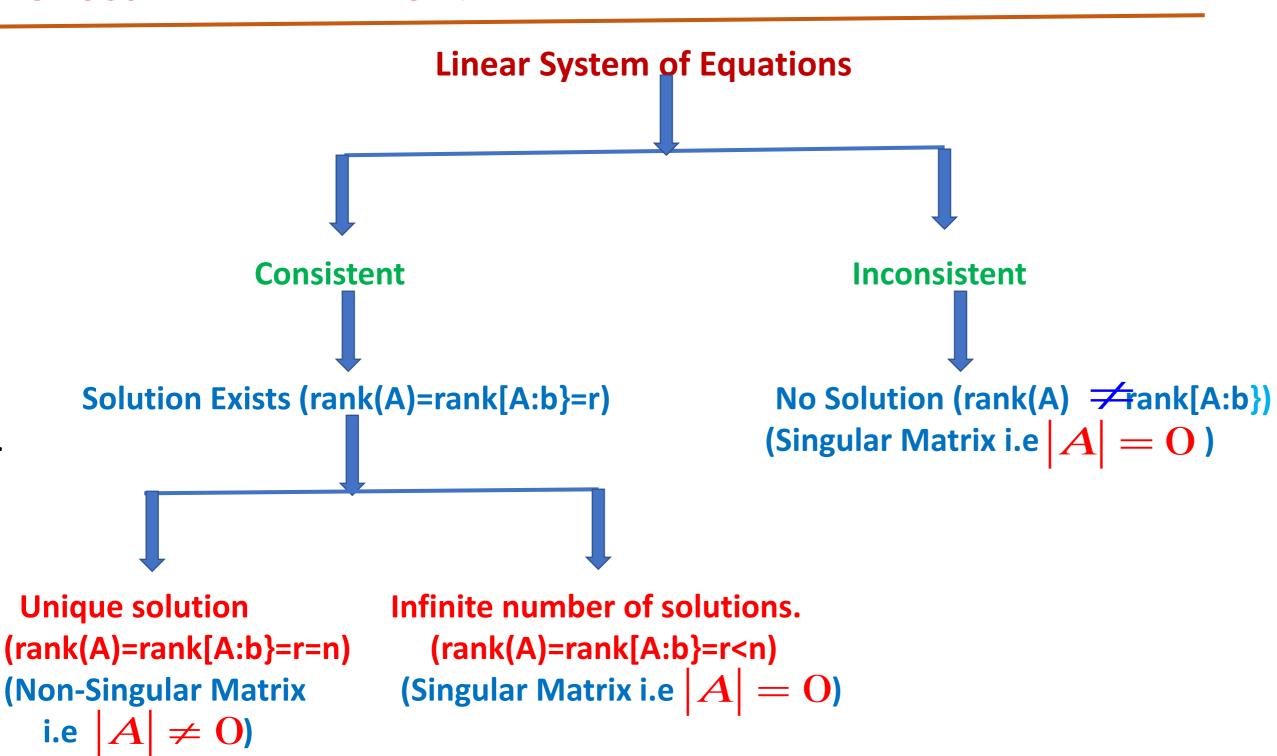
$$\begin{pmatrix}
1 & 2 & -3: a \\
2 & 3 & 3: b \\
5 & 9 & -6: c
\end{pmatrix}
\xrightarrow{R_2 - 2R_1}
\xrightarrow{R_3 - 5R_1}
\begin{pmatrix}
1 & 2 & -3: a \\
0 & -1 & 9: b - 2a \\
0 & -1 & 9: c - 5a
\end{pmatrix}$$

The given linear system will be **consistent** if a,b,c satisfy the relation c-b-3a=0. When (a, b, c)=(2, 3, 9), then the solution of the system is (x, y, z)=(-15k, 9k+1, k)

PES UNIVERSITY ONLINE

.

GAUSSIAN ELIMINATION:







THANK YOU

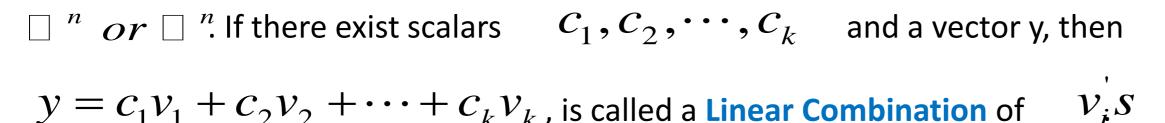
Renna Sultana

Department of Science and Humanities

rennasultana@pes.edu

INTRODUCTION:





Note:
$$\Box^n = \{(x_1, x_2, ... x_n) / x_i \in \Box, i = 1, 2, ..., n\}$$
 is Set of all n-tuples. $\Box^n = \{(x_1, x_2, ... x_n) / x_i \in \Box, i = 1, 2, ..., n\}$

- \Leftrightarrow If $c_1v_1+c_2v_2+\cdots+c_kv_k=0$ for all $c_i=0$, then v_is are said to be Linearly Independent.

Exs: $\{(1,2), (0,2), (3,4)\}$ is a L.D set since (3,4)=3(1,2)-1(0,2) or $c_1(1,2)+c_2(0,2)+c_3(3,4)=(0,0)$ $\{(2,1,0), (1,0,2), (0,1,2)\}$ is a L.I set since $c_1=c_2=c_3=0$.

