



# AUTOMATA, FORMAL LANGUAGES AND LOGIC

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**Dr Pooja Agarwal**

Professor,  
Department of Computer Science & Engineering

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## MODULE 5

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### Propositional Logic & First-Order Logic

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## Syntax

- **Syntax** defines the sentences in the language.
- Can be thought of as a “grammar” of representation.
- Consider a stmt: **Buffalo is Black.**
- It can be represented as:
  - 1)  $p = \text{Buffalo is black, or}$
  - 2)  $\text{Black}(\text{Buffalo})$
  - 3)  $\text{for all } x, \text{Buffalo}(x) \rightarrow \text{black}(x)$   
means for all  $x$ , if  $x$  is buffalo,  $x$  is black

} Different  
syntax

## Logic In general

- Logics are formal languages for representing information such that conclusions can be drawn.

- Let

A= Postman delivers post from Monday to Friday

B= Today is Sunday.

C ( conclusion)=Post is not delivered today.

## Semantics

- **Semantics** OR "meaning" of sentences;
  - i.e., define **truth** of a sentence in a possible world.
  - Eg  $2xy=3$  is **true** for whenever  $x=3$ ,  $y=1/2$  in a world but would be **false** for  $x=3$ ,  $y=2$  in a world
  - $X + Y = 4$  is **true** in a word where  $X$  is 2 and  $Y$  is 2. But **false** in a word where  $X$  is 1 and  $Y$  is 1.

## Model

- The word “**Model**” is also used for “possible world”

Models are mathematical abstraction.

*Think of a possible world having*

x men and y women sitting at a table playing chess

The sentence  $x+y = 4$  is true, **when** there are **4** people in total.

## Model

If,  
a sentence  $\alpha$  is true in model  $M$ ,  
we say, that  $M$  satisfies  $\alpha$  OR  
sometimes  $M$  is a model of  $\alpha$ .

We use the notation  $M(\underline{\alpha})$  to represent the set of all models of  $\alpha$ .

## Entailment

- **Entailment** means that one thing **follows from** another:

$$p \models q$$

means **p entails the sentence q**.

- In every model, in which **p is true**, **q is also true**.
- **The truth of q is contained in p**

$$\text{eg, } a + b = 9 \models 9 = a + b$$

Sentence  $x = 0$  entails the sentence  $xy = 0$ .



## Model

$M$  is a model of a sentence  $\alpha$

If  $\alpha$  is true in  $M$  and  $M(\alpha)$  is the set of all models of  $\alpha$

$\alpha \models \beta$  means  $\alpha$  entails the sentence  $\beta$

i.e  $\alpha \models \beta$

iff  $M(\alpha)$  is a subset of  $M(\beta)$ .



# THANK YOU

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**Dr Pooja Agarwal**

Department of Computer Science & Engineering

**[poojaagarwal@pes.edu](mailto:poojaagarwal@pes.edu)**