∴ What are the advantages of Fourier series

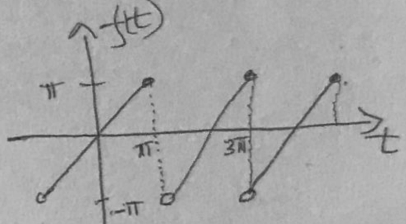
i) Discontinuous function can be represented by Fourier series. (This is not true for Taylor series)

ii) The Fourier series is useful in expanding the periodic functions since outside the closed interval, there exists a periodic extension of function.

iii) Linear combination is easy to handle. (so preferred over LT etc).

<u>How Fourier Series works:</u>

* The sawtooth waveform with period 2π as shown in the figure is given by particular combination i.e



$$f(t) = 2\left(\sin t - \frac{1}{2}\sin 2t + \frac{1}{3}\sin 3t - \frac{1}{4}\sin 4t + \frac{1}{5}\sin 5t \ldots\right)$$ Sawtooth waveform $\rightarrow$ (1)

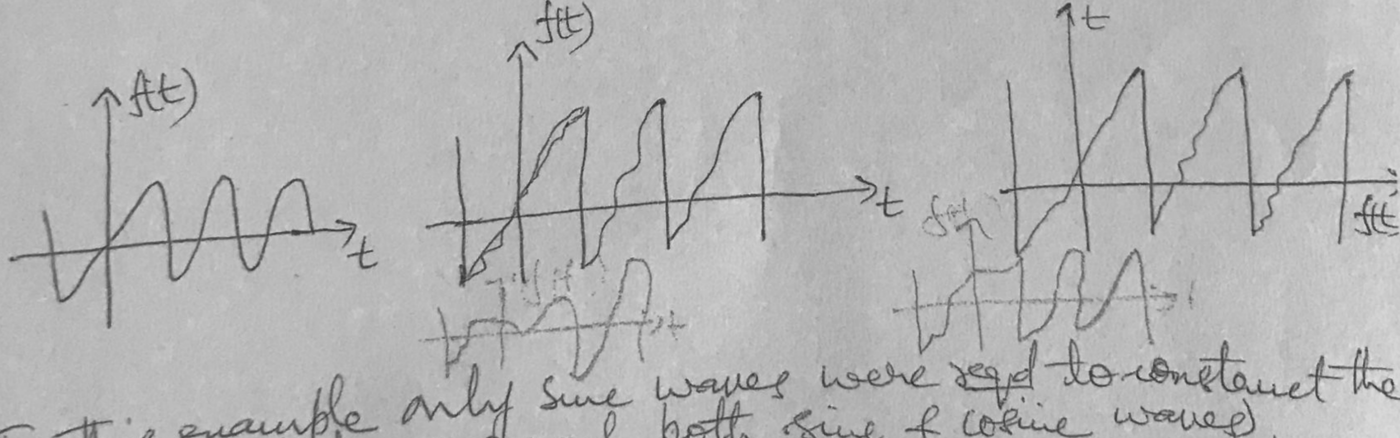
* Following (a), (b) & (c) graphs show the effect of including more & more terms in the series. As more terms are taken we see that the series approaches the desired sawtooth waveform.

$f(t)=1$ (I term of series)

a) $f(t) = 2\sin t$

b) $f(t) = 2\left(\sin t - \frac{1}{2}\sin 2t + \frac{1}{3}\sin 3t\right)$

c) $f(t) = 2\left(\sin t - \frac{1}{2}\sin 2t + \frac{1}{3}\sin 3t - \frac{1}{4}\sin 4t + \frac{1}{5}\sin 5t\right)$



* In this example only sine waves were used to construct the function. (but we need both sine & cosine waves).

where did this series come from i.e series given by (1)

Helpful - Revision

Properties of Sine and Cosine functions:

$f(x) = \cos x$ even

$\cos(-x) = \cos x$ & $\int_{-\pi}^{\pi} \cos\theta \, d\theta = 0.$

$f(x) = \sin x$, odd $\because \sin(-x) = -\sin x$

and $\int_{-\pi}^{\pi} \sin\theta \, d\theta = 0.$

$$\left.\begin{array}{l} \sin n\pi = 0 \\ \sin \frac{(2n-1)\pi}{2} = (-1)^{n+1} \end{array}\right\} \text{ for } n = 0, 1, 2, 3 \ldots$$

$\cos(n\pi) = (-1)^n$ for $n = 0, 1, 2, 3 \ldots$

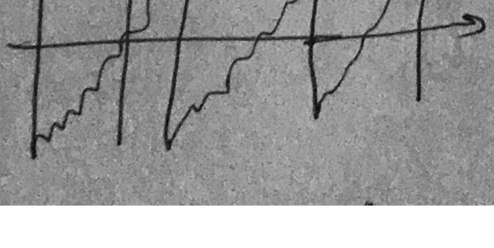
Periodic fn: $f(t+p) = f(t)$ where p is the Primitive period or the smallest Positive value of P which makes $f(t)$ periodic..

Useful Integrals

$$\int t \sin nt \, dt = \frac{1}{n^2}\left[\sin nt - nt \cos nt\right]$$

$$\int t \cos nt \, dt = \frac{1}{n^2}\left[\cos nt + nt \sin nt\right]$$

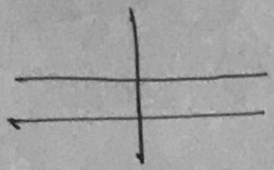
How fourier Series works:



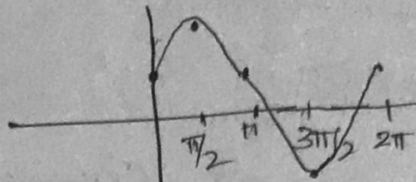
$f(t) = 1 + 2\sin t - \sin 2t + \frac{2}{3}\sin 3t - \frac{1}{2}\sin 4t + \frac{2}{5}\sin 5t + \ldots$

... of ... f(x) is defined in interval (0, 2l

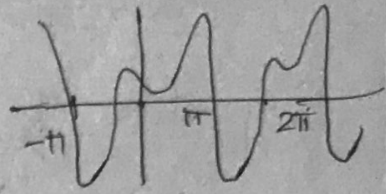
$f(t) = 1$



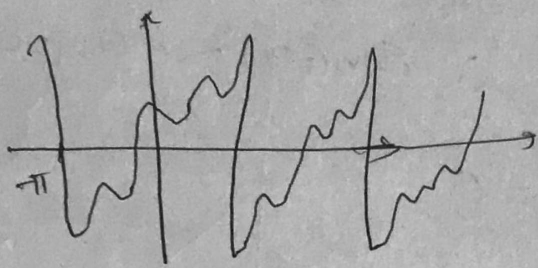
$f(t) = 1 + 2\sin t$



$f(t) = 1 + 2\sin t - \sin 2t$



$f(t) = 1 + 2\sin t - \sin 2t + \frac{2}{3}\sin 3t$



Full Range Fourier Series — **Euler's formula**

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$$

Where a_n and b_n are the Fourier Coefficients, where $\frac{a_0}{2}$ is the mean value, sometimes called as the dc level.

* ~~Any~~ Expressing any periodic piece-wise continuous fn as an infinite series of fundamental and Harmonics is called Fourier Series.

even · even = even
odd · odd = even

$$\int_{-c}^{c} f(x)\,dx = 2\int_{0}^{c} f(x)\,dx$$

even · odd = odd.

$$\int_{-c}^{c} f(x)\,dx = 0.$$

1) Find the FS of $f(x) = x + x^2$ in $(-\pi, \pi)$ & hence deduce $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$

in $(-\pi, \pi)$ non-periodic

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neither even nor odd

Sol:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx.$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} (x + x^2)\, dx = \frac{x^2}{2} + \frac{x^3}{3} \Big]_{-\pi}^{\pi} = \left[\cancel{\frac{\pi^2}{2}} + \frac{\pi^3}{3}\right] - \left[\cancel{\frac{\pi^2}{2}} - \frac{\pi^3}{3}\right]$$

$$= \frac{2\pi^2}{3}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (x + x^2) \cos nx\, dx$$

$$= \frac{1}{\pi} \left[(x + x^2)\left(\cancel{\frac{\sin nx}{n}}\right) - (1 + 2x)\left(-\frac{\cos nx}{n^2}\right) + (2)\left(\cancel{\frac{-\sin nx}{n^3}}\right) \right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi} \left\{ (1 + 2\pi)\frac{(-1)^n}{n^2} - (1 - 2\pi)\frac{(-1)^n}{n^2} \right\}$$

$$= \frac{(-1)^n}{\pi n^2} \left[\cancel{1} + 2\pi - \cancel{1} + 2\pi \right] = \frac{4(-1)^n}{n^2}$$

$$\frac{-2(-1)^n}{n}$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (x + x^2) \sin nx\, dx$$

$$= \frac{1}{\pi} \left\{ (x + x^2)\left(-\frac{\cos nx}{n}\right) - (1 + 2x)\left(-\cancel{\frac{\sin nx}{n^2}}\right) + (2)\left(\frac{\cos nx}{n^3}\right) \right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi} \left\{ \left[-(\pi + \pi^2)\frac{(-1)^n}{n} + \cancel{(\pi + \pi^2)\frac{(-1)^n}{n^2}} \right] + \cancel{\frac{2(-1)^n}{n^3}} \right] - \left[(-\pi + \pi^2)\frac{(-1)^n}{n} + \cancel{\frac{2(-1)^n}{n^3}} \right] \right\}$$

$$= \frac{(-1)^n}{\pi} \left\{ \cancel{\frac{-\pi}{n}} - \cancel{\frac{\pi^2}{n}} - \cancel{\frac{\pi}{n}} + \cancel{\frac{\pi^2}{n}} \right] = \frac{(-1)^n}{\cancel{\pi}} \left[-\frac{2\cancel{\pi}}{n} - \cancel{\frac{\pi^2}{n}} + \cancel{\frac{\pi^2}{n}} \right] = \frac{-2(-1)^n}{n}$$

$$x + x^2 = f(x) = \cancel{\frac{1}{2}} \times \cancel{2}\frac{\pi^3}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^n}{n^2} \cos nx + \sum_{n=1}^{\infty} \frac{-2(-1)^n}{n} \sin nx$$

$$x+x^2=\frac{\pi^2}{3}+4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos nx+2\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin nx \quad (1)$$

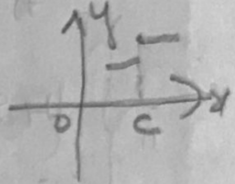
Put $x=\pi \Rightarrow \frac{1}{2}\left[f(-\pi)+f(\pi)\right]=\frac{1}{2}\left[\cancel{-\pi}+\pi^2+\cancel{\pi}+\pi^2\right]=\pi^2$

$\therefore$ (1) gives

$$\pi^2-\frac{\pi^2}{3}=4\sum_{n=1}^{\infty}\frac{(-1)^{2n}}{n^2}+0$$

$$\frac{2\pi^2}{3\times 4}=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2} \quad \text{Put } n=1,2,3 \ldots$$

$$\boxed{\frac{\pi^2}{6}=\sum_{n=1}^{\infty}\frac{1}{n^2}}$$



$f(x)=\phi(x) \quad \alpha<x<c$
$\quad\;\;=\psi(x) \quad c<x<\alpha+2\pi$
$f(x)=\frac{1}{2}\left[f(c^-)+f(c^+)\right]$
at c limit exists
~~the limit~~ $\therefore$ average

3)

$$f(x)=\begin{cases}-\pi & -\pi<x<0\\ x & 0<x<\pi\end{cases}$$

FS of $f(x)$ with period 2π is

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nx+\sum_{n=1}^{\infty}b_n\sin nx \quad (1)$$

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\,dx=\frac{1}{\pi}\left[\int_{-\pi}^{0}-\pi\,dx+\int_0^{\pi}x\,dx\right]=\frac{1}{\pi}\left[-\pi x\right]_{-\pi}^{0}+\left[\frac{x^2}{2}\right]_0^{\pi}$$

$$=\frac{1}{\pi}\left\{-\pi\left[0+\pi\right]+\frac{1}{2}\left[\pi^2-0\right]\right\}=\frac{1}{\pi}\left\{-\pi^2+\frac{\pi^2}{2}\right\}=\frac{1}{\cancel{\pi}}\left[\frac{-\cancel{\pi^2}}{2}\right]=-\frac{\pi}{2}$$

$$a_n=\frac{1}{\pi}\left\{\int_{-\pi}^{0}-\pi\cos nx\,dx+\int_0^{\pi}x\cos nx\,dx\right\}$$

$$=\frac{1}{\pi}\left\{-\pi\left[\frac{\cancel{\sin nx}}{n}\right]_{-\pi}^{0}+\left[x\left(\frac{\cancel{\sin nx}}{n}\right)-(1)\left(-\frac{\cos nx}{n^2}\right)\right]_0^{\pi}\right.$$

$$= \frac{1}{\pi n^2}\left[\frac{(-1)^n}{\cancel{n^2}} - \frac{1}{\cancel{n^2}} \right]$$

$$b_n = \frac{1}{\pi}\left\{ \int_{-\pi}^{0} -\pi \sin nx\,dx + \int_0^{\pi} x \sin nx\,dx \right\}$$

$$= \frac{1}{\pi}\left\{ -\pi\left(-\frac{\cos nx}{n}\right)\Big]_{-\pi}^{0} + \left[x\left(-\frac{\cos nx}{n}\right) - (1)\left(-\frac{\sin nx}{n^2}\right) \right]_0^{\pi} \right\}$$

$$= \frac{1}{\cancel{\pi}}\left\{ \frac{\pi}{n}\left[1 - \frac{(-1)^n}{\cancel{n^2}}\right] + \left[-\frac{\pi}{n}(-1)^n + 0\right] - \left[0 + 0\right] \right\}$$

$$= \frac{1}{n}\left[1 - 2(-1)^n\right]$$

$$\boxed{\therefore f(x) = \frac{1}{2}\left[\frac{-\pi}{2}\right] + \sum_{n\ge1}^{\infty} \frac{1}{\pi n^2}\left[(-1)^n - 1\right]\cos nx + \sum_{n\ge1} \frac{1}{n}\left[1-2(-1)^n\right]\sin nx}$$

put $x = 0 \Rightarrow \frac{1}{2}\left[f(0^+) + f(0^-)\right] =$

in $(-\pi, 0)$. $f(x) = -\pi \Rightarrow f(0^-) = -\pi$

in $(0, \pi)$ $f(x) = x \Rightarrow f(0^+) = 0$ $\quad \therefore \frac{1}{2}\left[0 - \pi\right] = \frac{-\pi}{2}$

$$\frac{-\pi}{2} = \frac{-\pi}{4} + \sum_{n\ge1}^{\infty} \frac{1}{\pi n^2}\left[(-1)^n - 1\right]$$

$$\frac{\pi}{2} = \frac{\pi}{4} + \sum_{n\ge1}^{\infty} \frac{1}{\pi n^2}\left[1 - (-1)^n\right]$$

$$\frac{\pi}{4} = \sum_{n\ge1}^{\infty} \frac{1}{\pi n^2}\left[1 - (-1)^n\right] \Rightarrow \frac{\pi^2}{4} = \sum_{n\ge1}^{\infty} \frac{1}{n^2}\left[1-(-1)^n\right]$$

n is odd $\Rightarrow 2$

n is even $\Rightarrow 0$

$$\frac{\pi^2}{4} = \frac{2}{1^2} + \frac{2}{3^2} + \frac{2}{5^2} + \cdots \Rightarrow \frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$$

2) Obtain FS to represent e^{-ax} from $x=-\pi$ to $x=\pi$

Hence derive series for $\frac{\pi}{\sin n\pi}$.

Sol: Reqd FS is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx.$$

Here e^{-ax} is neither even nor odd

$$\therefore a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} e^{-ax}\,dx = \frac{1}{\pi}\cdot\left[\frac{e^{-ax}}{-a}\right]_{-\pi}^{\pi} = -\frac{1}{\pi a}\left[e^{-a\pi}-e^{a\pi}\right]$$

$$= \frac{2}{\pi a}\left[\frac{e^{a\pi}-e^{-a\pi}}{2}\right]$$

$$= \frac{2}{\pi a}\sinh a\pi$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} e^{-ax}\cos nx\,dx = \frac{1}{\pi}\left\{\frac{e^{-ax}}{a^2+n^2}\left(-a\cos nx + n\sin nx\right)\right\}_{-\pi}^{\pi}$$

$$a_n = \frac{1}{\pi(a^2+n^2)}\left\{e^{-a\pi}\left[-a(-1)^n+0\right] - e^{a\pi}\left(-a(-1)^n+0\right)\right\}$$

$$= \frac{a(-1)^n}{\pi(a^2+n^2)}\left(-e^{-a\pi}+e^{a\pi}\right) = \frac{2a(-1)^n}{\pi(a^2+n^2)}\sinh a\pi \quad \cancel{\cosh a\pi}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} e^{-ax}\sin nx\,dx = \frac{1}{\pi}\left\{\frac{e^{-ax}}{a^2+n^2}\left(-a\sin nx - n\cos nx\right)\right\}_{-\pi}^{\pi}$$

$$b_n = \frac{1}{\pi(a^2+n^2)}\left[e^{-a\pi}\left[0-n(-1)^n\right] - e^{a\pi}\left[0-n(-1)^n\right]\right\}$$

$$= \frac{n(-1)^n}{\pi(a^2+n^2)}\left[-e^{-a\pi}+e^{a\pi}\right] = \frac{2n(-1)^n}{\pi(a^2+n^2)}\left(\frac{e^{a\pi}-e^{-a\pi}}{2}\right)$$

$$= \frac{2n(-1)^n}{\pi(a^2+n^2)}\sinh a\pi$$

put $a=1$, & then put $x=0$

$$e^{ax} = \frac{\sinh a\pi}{a\pi}\left\{1+\sum_{n=1}^{\infty}\frac{2a^2(-1)^n}{a^2+n^2}\cos nx+\sum_{1}^{\infty}\frac{2an(-1)^n}{a^2+n^2}\sin nx\right\}$$

Put $a=1$ & $x=0$

$$e^0 = \frac{\sinh\pi}{\pi}\left\{1+\sum_{n=1}^{\infty}\frac{2(-1)^n}{1+n^2}\right\}$$

$$\frac{\pi}{\sinh\pi} = 1+2\left[\frac{-1}{1+1^2}+\frac{1}{1+2^2}-\frac{1}{1+3^2}+\frac{1}{1+4^2}\cdots\right]$$

$$= 1+\cancel{2}\left(-\frac{1}{\cancel{2}}\right)+2\left[\frac{1}{1+2^2}-\frac{1}{1+3^2}+\frac{1}{1+4^2}\cdots\right]$$

$$\Rightarrow \frac{\pi}{\sinh\pi} = 2\left[\frac{1}{1+2^2}-\frac{1}{1+3^2}+\frac{1}{1+4^2}-\cdots\right]$$

4) Express $f(x)=(\pi-x)^2$ as a FS of period 2π in the interval $0<x<2\pi$. Hence deduce the sum of the series $1+\frac{1}{2^2}+\frac{1}{3^2}+\cdots$

$$f(x)=(\pi-x)^2$$

$$f(2\pi-x) = \left[\pi-(2\pi-x)\right]^2 = \left[-\pi+x\right]^2 = (-1)^2\left[\pi-x\right]^2 = f(x)$$

$f(x)$ is even $\therefore b_n=0$

$$a_0 = \frac{2}{\pi}\int_0^{\pi}(\pi-x)^2\,dx = \frac{2}{\pi}\left(\frac{(\pi-x)^3}{3(-1)}\right)_0^{\pi} = -\frac{2}{3\pi}\left[-\pi^3\right] = \frac{2\pi^2}{3}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi}(\pi-x)^2\cos nx\,dx$$

$$a_n = \frac{2}{\pi}\left\{\left[(\pi-x)^2\left(\frac{\sin nx}{n}\right)-2(\pi-x)(-1)\left(-\frac{\cos nx}{n^2}\right)\right]_0^{\pi}\right.$$

$$a_n = \frac{2}{\pi}\left\{\left[0-2(\pi)\right]\right. = \frac{4}{n^2} = \frac{4}{n^2}$$

(3)

Scanned with CamScanner

$$\cancel{(\pi-x)^2 = \frac{1}{\cancel{2}}\left[\frac{\cancel{8}\pi^2}{9}\right] + \sum_{n=1}^{\infty} \frac{8}{n^2}(-1)^n \cos nx}$$

$$\boxed{(\pi-x)^2 = \frac{1}{\cancel{2}}\left[\frac{\cancel{2}\pi^2}{3}\right] + \sum_{n=1}^{\infty} \frac{4}{n^2}\cos nx}$$

Put $x=0$

$$\pi^2 - \frac{\pi^2}{3} = 4\sum_{n=1}^{\infty}\frac{1}{n^2} \Rightarrow \frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$$

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$i = \begin{cases} I_0 \sin x & 0 \le x \le \pi \\ 0 & \pi \le x \le 2\pi \end{cases}$ — ①

The FS of period 2π is $I = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$

$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$, $a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx$, $b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx$

$a_0 = \frac{I_0}{\pi}\int_0^{\pi} \sin x\,dx = \frac{I_0}{\pi}\left[-\cos x\right]_0^{\pi} = -\frac{I_0}{\pi}\left[-1-1\right] = \frac{2I_0}{\pi}$

$a_n = \frac{I_0}{\pi}\int_0^{\pi} \sin x \cos nx\,dx = \frac{I_0}{\pi}\int_0^{\pi} \cos nx \sin x\,dx$ $\quad \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$

$a_n = \frac{I_0}{2\pi}\left\{\int_0^{\pi}\left[\sin(n+1)x - \sin(n-1)x\right]dx\right\}$

$= \frac{I_0}{2\pi}\left\{\left[-\frac{\cos(n+1)x}{(n+1)} + \frac{\cos(n-1)x}{(n-1)}\right]_0^{\pi}\right\}$

$= \frac{I_0}{2\pi}\left\{\left[-\frac{(-1)^{n+1}}{n+1} + \frac{(-1)^{n-1}}{n-1}\right] - \left[\frac{-1}{n+1} + \frac{1}{n-1}\right]\right\}$

$= \frac{I_0}{2\pi}\left\{(-1)^n\left[\frac{1}{n+1} - \frac{1}{n-1}\right] - \left[\frac{\cancel{-n}+1\cancel{+n}+1}{(n^2-1)}\right]\right\}$ $n \neq 1$

$= \frac{I_0}{2\pi}\left\{(-1)^n\left[\frac{\cancel{n}-1\cancel{-n}-1}{(n^2-1)}\right] - \left[\frac{2}{n^2-1}\right]\right\}$

$= \frac{I_0}{\pi}\left\{\frac{-(-1)^n}{n^2-1} - \frac{1}{n^2-1}\right\} = \frac{-I_0}{(n^2-1)\pi}\left[(-1)^n + 1\right]$

$a_1 = \frac{1}{\pi}\int_0^{\pi} \sin x \cos x\,dx = \frac{1}{2\pi}\int_0^{\pi} \sin 2x\,dx = \frac{1}{2\pi}\left[\frac{-\cos 2x}{2}\right]_0^{\pi}$

$(x) - x^3$ in $(-\pi, \pi)$

$$b_n = \frac{1}{\pi}\int_0^\pi \sin x \sin nx \, dx = \frac{1}{\pi}\int_0^\pi \sin nx \sin x \, dx$$

$$\sin A \sin B = \frac{-1}{2}\left[\cancel{\sin}\cos(A-B) - \cos(A+B)\right]$$

$$b_n = \frac{-1}{2\pi}\int_0^\pi \left[\cos(n-1)x - \cos(n+1)x\right] dx$$

$$b_n = -\frac{1}{2\pi}\left[\frac{\sin(n-1)x}{n-1} - \frac{\sin(n+1)x}{n+1}\right]_0^\pi$$

$$= -\frac{1}{2\pi}\left[0\right]$$

$$b_1 = \frac{I_0}{\pi}\int_0^\pi \sin x \sin x \, dx = \frac{I_0}{2\pi}\int_0^\pi (1 - \cos 2x) \, dx$$

$$b_1 = \frac{I_0}{2\pi}\left[x - \frac{\sin 2x}{2}\right]_0^\pi = \frac{I_0}{2\pi}\left[\pi - 0\right] - 0 \Rightarrow \boxed{\frac{I_0}{2} = b_1}$$

Subst in (1)

$$i = f(x) = I = \frac{I_0}{\pi} + \sum_{n=2}^{\infty}\left[\frac{-I_0}{(n^2-1)\pi}\right]\left[(-1)^n+1\right]\cos nx + \frac{I_0}{2}\sin$$

$\downarrow \left[(-1)^n+1\right] = 2$ if n is e
$= 0$ if n is o

(7)

6) Find the F.S expansion of $f(x)=x(1-x)(2-x)$ in $(0,2)$

Deduce sum of the series $\frac{1}{1^3}-\frac{1}{3^3}+\frac{1}{5^3}-\frac{1}{7^3}+\dots\infty$

$2l=2$
$l=1$

Sol:- $f(2-x)=(2-x)[1-(2-x)][2-(2-x)]$

$=(2-x)[-1+x][x]$

$= -[(2-x)(1-x)x] = -f(x)$

$\Rightarrow$ odd fn. $\Rightarrow a_0=0, a_n=0$

$$b_n=\frac{2}{l}\int_0^l f(x)\sin\left(\frac{n\pi x}{l}\right)dx = \frac{2}{1}\int_0^1 (2x-3x^2+x^3)\sin\left(\frac{n\pi}{1}\right)x\,dx$$

$$b_n=2\left[(2x-3x^2+x^3)\left(-\frac{\cos n\pi x}{n\pi}\right)-(2-6x+3x^2)\cancel{\left(-\frac{\sin n\pi x}{n^2\pi^2}\right)}^{0}\right.$$
$$\left.+(-6+6x)\left(\frac{\cos n\pi x}{n^3\pi^3}\right)-(6)\cancel{\left(\frac{\sin n\pi x}{n^4\pi^4}\right)}^{0}\right]_0^1$$

$$b_n=2\left\{[0+0]-\left[0+\frac{-6}{n^3\pi^3}\right]\right\} = \frac{12}{n^3\pi^3}$$

$$\boxed{f(x)=\sum_{n=1}^{\infty}\frac{12}{n^3\pi^3}\sin\left(\frac{n\pi}{l}\right)x}$$

$\Rightarrow 2x-3x^2+x^3=\sum_{n=1}^{\infty}\frac{12}{n^3\pi^3}\sin\left(\frac{n\pi}{l}\right)x$

$$\frac{\pi^3 \cancel{3}}{\cancel{12}\,8} = \sum_{n=1}^{\infty}\sin\left(\frac{n\pi}{2}\right) \Rightarrow \frac{\pi^3}{32}=1-\frac{1}{3^3}+\frac{1}{5^3}\dots$$

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put $x=1/2$