PES University, Bangalore

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UE19CS203 – STATISTICS FOR DATA SCIENCE

Unit - 3 - Probability Distributions

QUESTION BANK

Principles of Point Estimation (Mean Squared Error))

Exercises for Section 4.9

[Text Book Exercise – Section 4.9 – Q. No. [1 – 4] – Pg. No. [284 - 285]]

1. Choose the best answer to fill in the blank. If an estimator is unbiased, then

   a) The estimator is equal to the true value.
   b) The estimator is usually close to the true value.
   c) The mean of the estimator is equal to the true value.
   d) The mean of the estimator is usually close to the true value.

2. Choose the best answer to fill in the blank. The variance of an estimator measures

   a) How close the estimator is to the true value.
   b) How close repeated values of the estimator are to each other.
   c) How close the mean of the estimator is to the true value.
   d) How close repeated values of the mean of the estimator are to each other.

3. Let X_1 and X_2be independent, each with unknown mean μ and known variance $\sigma^2 = 1$.

   a) Let $\widehat{\mu_1} = \frac{X_1+X_2}{2}$. Find the bias, variance and mean squared error of $\widehat{\mu_1}$.
   b) Let $\widehat{\mu_2} = \frac{X_1+2X_2}{3}$. Find the bias, variance and mean squared error of $\widehat{\mu_2}$.
   c) Let $\widehat{\mu_3} = \frac{X_1+X_2}{4}$. Find the bias, variance and mean squared error of $\widehat{\mu_3}$.
   d) For what values of μ does $\widehat{\mu_3}$ have smaller mean squared error than $\widehat{\mu_1}$?
   e) For what values of μ does $\widehat{\mu_3}$ have smaller mean squared error than $\widehat{\mu_2}$?

4. Let $X_1 \ldots.. X_n$ be a random sample from $N(\mu, \sigma^2)$ population. For any constant k > 0, define $\widehat{\sigma_k} = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{k}$. Consider $\hat{\sigma}_k{}^2$ as an estimator of σ^2.

a) Compute bias of ${\hat{\sigma}_k}^2$ in terms of k. [Hint: The sample variance s^2 is unbiased and ${\hat{\sigma}_k}^2 = (n-1)s^2/k$].

b) Compute the variance of ${\hat{\sigma}_k}^2$ in terms of k. [Hint: $\sigma_{s^2}^2 = 2\sigma^4/(n-1)$, and ${\hat{\sigma}_k}^2 = (n-1)s^2/k$].

c) Compute mean squared error of ${\hat{\sigma}_k}^2$ in terms of k.

d) For what value of k is the mean squared error of ${\hat{\sigma}_k}^2$ minimized?