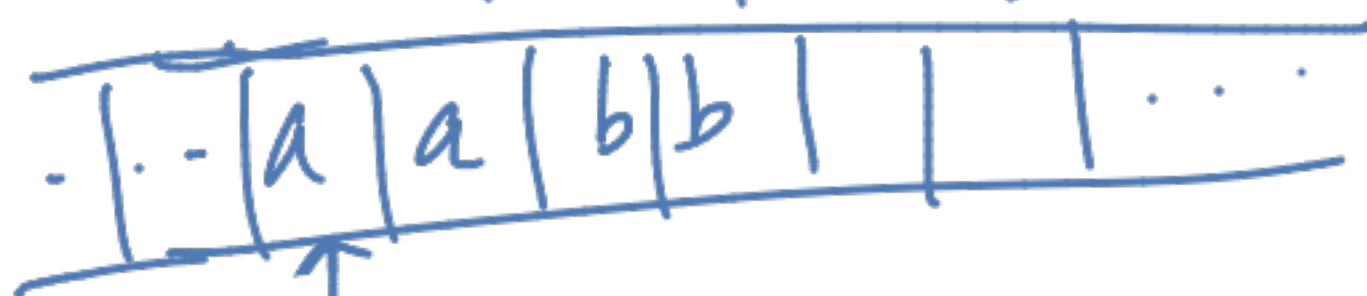1) $L = \{a^n b^n, n \geq 1\}$

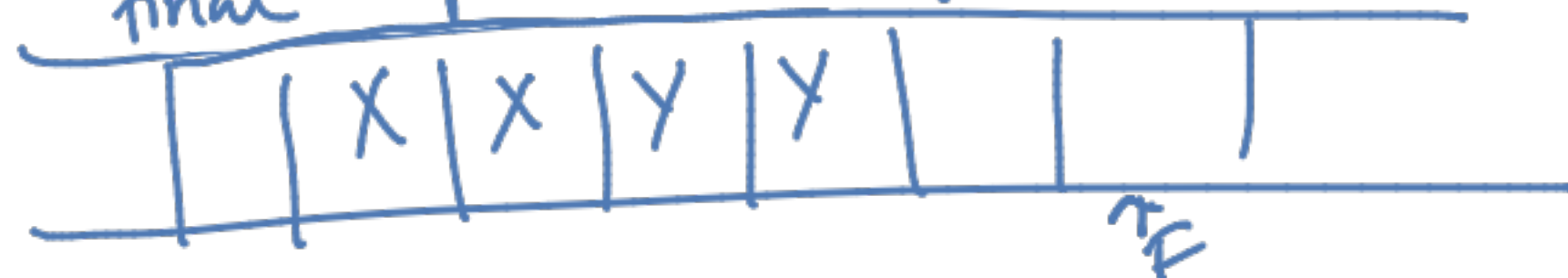## Logic

- Match a 'b' for every 'a'

- Mark the matched symbols

  Encode

  matched 'a' as 'X'

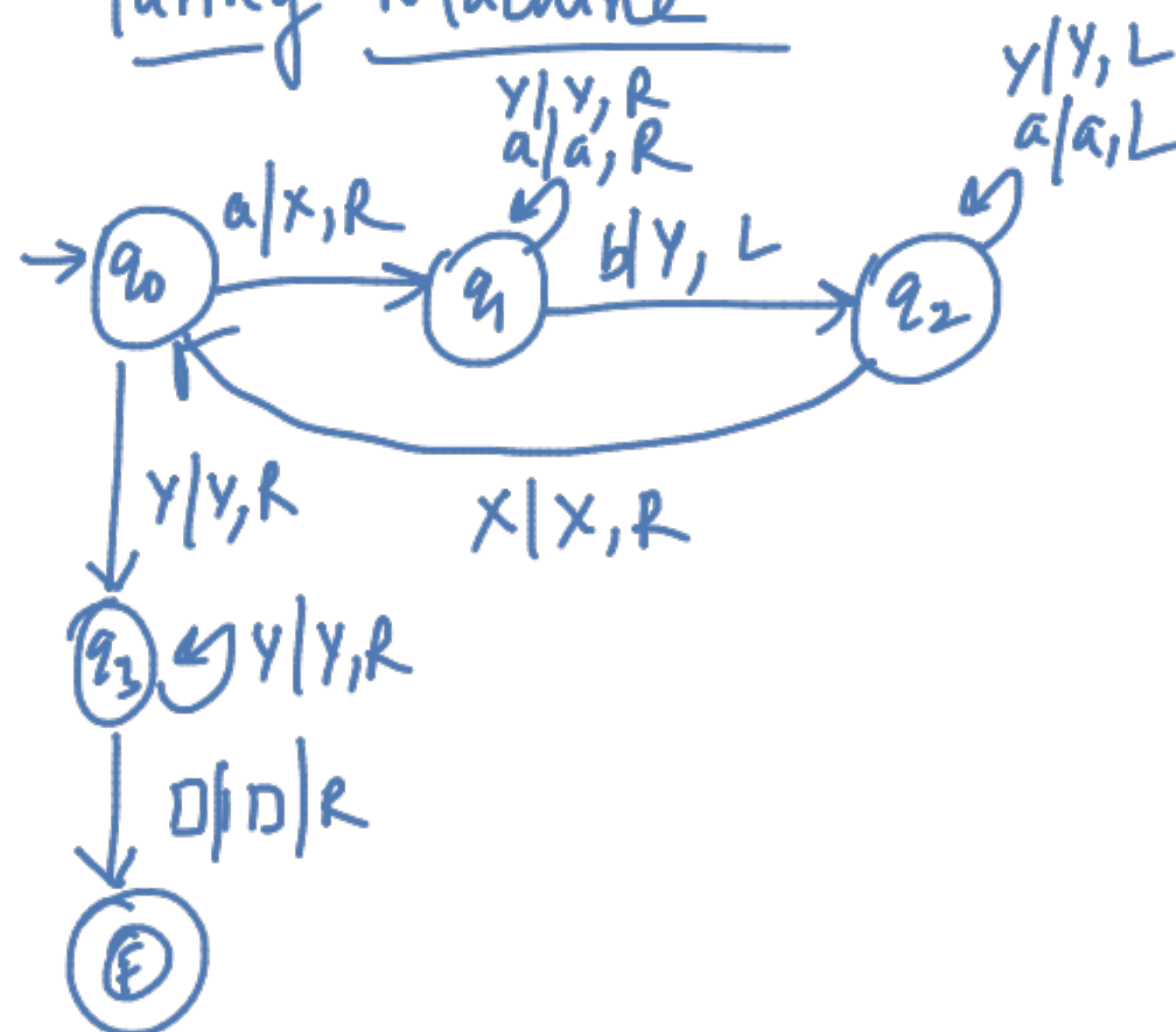 matched 'b' as 'Y'

Turing Machine

$$\xrightarrow{} q_0 \xrightarrow{a|X,R} q_1 \xrightarrow{b|Y,L} q_2$$

State q_1: $Y|Y,R$, $a|a,R$

State q_2: $Y|Y,L$, $a|a,L$

$q_2 \xrightarrow{X|X,R} q_0$

$q_0 \xrightarrow{Y|Y,R} q_3$

q_3: $Y|Y,R$

$q_3 \xrightarrow{\square|\square,R} F$

Initial input tape Conf :-

.	..	a	a	b	b				...

$\uparrow q_0$

final input tape Conf :-

		X	X	Y	Y			

$\uparrow F$

Trace aabb

$q_0\ aabb$

$\vdash X\ q_1\ abb$

$\vdash X\ a\ q_1\ bb$

$\vdash X\ q_2\ aYb$

$\vdash q_2\ XaYb$

$\vdash X\ q_0\ aYb$

$\vdash X\ X\ q_1\ Yb$

$\vdash X\ X\ Y\ q_1\ b$

$\vdash X\ X\ q_2\ YY$

$\vdash X\ q_2\ XYY$

$\vdash X\ X\ q_0\ YY$

$\vdash X\ X\ Y\ q_3\ Y$

$\vdash X\ X\ Y\ Y\ q_3\ \square\square$

$\vdash X\ X\ Y\ Y\ \square\ F\ \square$

2) $L = \{a^n b^n c^n, n \geq 1\}$

Logic

Match a'b and a'c' for every a.

Encode all the marked symbols as:-

a — 'X'
b — 'Y'
c — 'Z'

Input tape

Initial conf: | a | a | b | b | c | c | |

final conf: | X | X | Y | Y | Z | Z | |

Turing machine

$q_0 \xrightarrow{a|X,R} q_1 \xrightarrow{b|Y,R} q_2 \xrightarrow{c|Z,L} q_3$

At q_1: $Y|Y,R$, $a|a,R$

At q_2: $Z|Z,R$, $b|b,R$

At q_3: $a|a,L$, $Y|Y,L$, $b|b,L$, $Z|Z,L$

$q_3 \xrightarrow{X|X,R} q_0$

$q_0 \xrightarrow{Y|Y,R} q_4$

At q_4: $Y|Y,R$, $Z|Z,R$

$q_4 \xrightarrow{0|0,R} q_f$
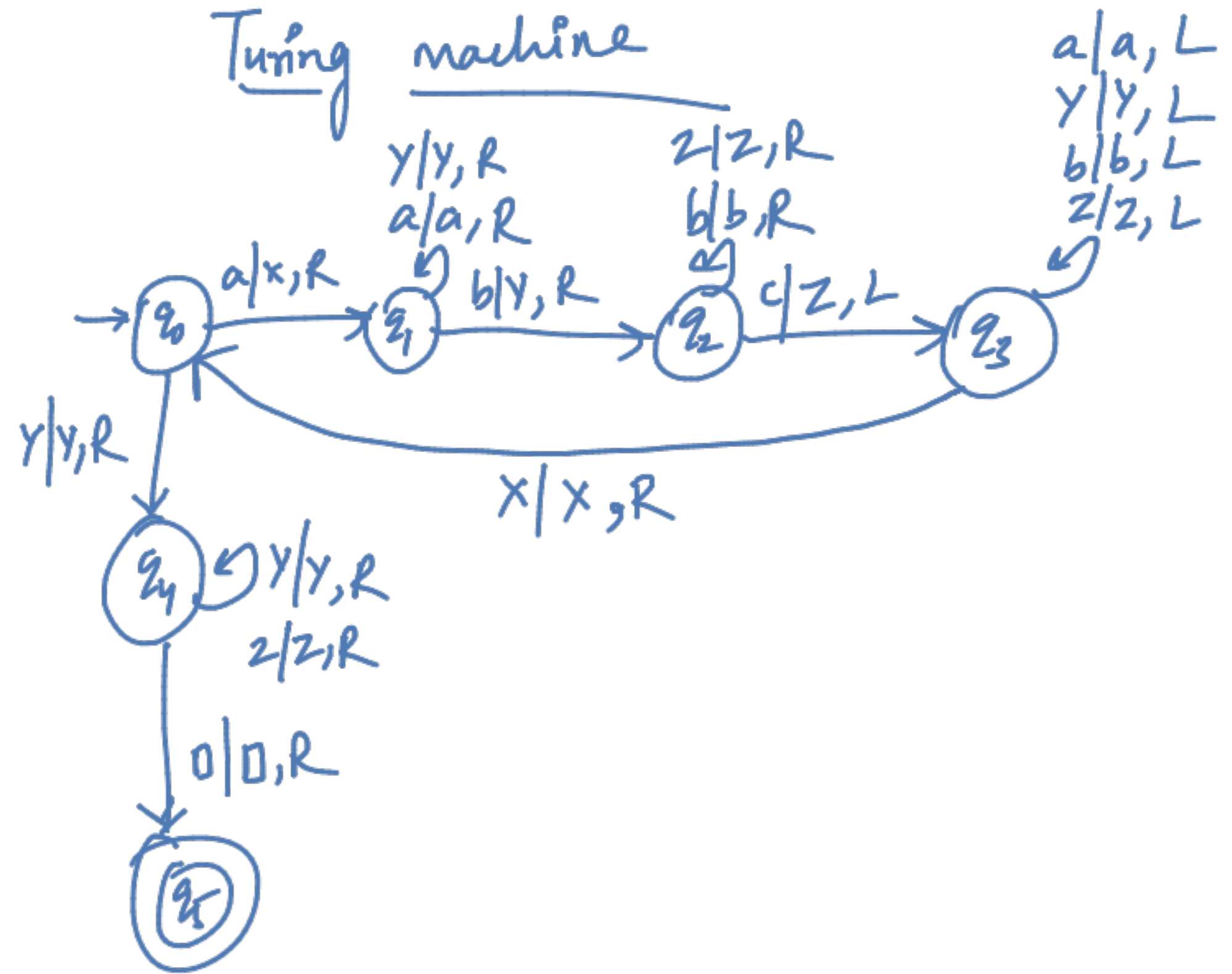
3) Construct a Turing Machine for the language of multiplication

Input tape
Initial conf:

| | 0 | 0 | 1 | 1 | 0 | 0 | 0 | |

final conf:

| | 0 | 0 | 1 | 1 | 0 | 0 | 0 | | 0 | 0 | 0 | 0 | 0 | 0 | |

Given input:

$$0^m 1 0^n$$

Produce output:

$$0^{m*n}$$

Logic :-

Repeat n (m times)

3) Continued ...

Input tape

Time													
0	0	0	1	0	0	0							→ Initial conf.
1	X	0	1	0	0	0							
2	X	0	1	Y	0	0		0					
3	X	0	1	Y	Y	0		0	0				
4	X	0	1	Y	Y	Y		0	0	0			
5	X	0	1	0	0	0		0	0	0			
6	X	X	1	0	0	0		0	0	0			
7	X	X	1	Y	0	0		0	0	0	0		
8	X	X	1	Y	Y	0		0	0	0	0	0	
9	X	X	1	Y	Y	Y		0	0	0	0	0	0
10	X	X	1	0	0	0		0	0	0	0	0	0
11	0	0	1	0	0	0		0	0	0	0	0	0	→ final conf.

3) Continued . . . .

q_0 $\xrightarrow{0|X,R}$ q_1 with self-loop $0|0,R$; $q_1 \xrightarrow{1|1,R} q_2$

$q_2 \xrightarrow{0|Y,R} q_3$ with self-loop $0|0,R$; $q_3 \xrightarrow{\square|\square,R} q_4$

$q_4 \xrightarrow{\square|0,L} q_5$ with self-loops $0|0,L$ and $\square|\square,L$

$q_5 \xrightarrow{Y|Y,R} q_2$

$q_2 \xrightarrow{\square|\square,L} q_6$

$q_0 \xrightarrow{1|1,L} q_9$

$q_9 \xrightarrow{X|0,L} q_{10}$

q_{10} self-loop $X|0,L$; $q_{10} \xrightarrow{\square|0,R} q_{11}$

$q_6 \xrightarrow{Y|0,L} q_7$; q_7 self-loop $Y|0,L$; $q_7 \xrightarrow{1|1,L} q_8$

q_8 self-loop $0|0,L$; $q_8 \xrightarrow{X|X,R} q_0$