STATISTICS FOR DATA SCIENCE
HYPOTHESIS and INFERENCE

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UNIT-4 HYPOTHESIS and INFERENCE

Session-7

Large - Sample tests for Difference between two means

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Example

Tennis Elbow

To operate or not to operate?

Mean % of the people cured by Surgery

$\mu_X = 72, \sigma_X = 8, n_X = 32$

Mean % of the people cured by Physiotherapy

$\mu_Y = 75, \sigma_Y = 6, n_Y = 36$

Example:

Can we conduct a Hypothesis test ?

Can we say that Surgery is better than Physiotherapy ?

Large - Sample tests for Difference between two means

- To determine whether the means of two populations are equal.
- The data will consist of two samples, one from each population.
- Let $X_1, \ldots X_{n_X}$ *and* $Y_1, \ldots . Y_{n_Y} (n_X > 30, n_Y > 30)$ be large sample from a population with means $\mu_X and\ \mu_Y$, and standard deviations $\sigma_X\ and\ \sigma_Y$.

Large - Sample tests for Difference between two means

- We will compute the difference of the sample means.

$$H_0\colon \mu_X - \mu_Y = \Delta_0,$$

$$H_0\colon \mu_X - \mu_Y > \Delta_0,$$

$$H_0\colon \mu_X - \mu_Y < \Delta_0$$

Large - Sample tests for Difference between two means

- Compute the z-score,

$$z = \frac{(\bar{X}-\bar{Y})-\Delta_0}{\sqrt{\sigma^2{}_X/n_X+\sigma^2{}_Y/n_Y}}$$

Type equation here.

- $\bar{X} - \bar{Y} \sim N(\Delta_0, \sigma^2{}_X/n_X + \sigma^2{}_Y/n_Y)$

- If σ_X and σ_Y are unknown they may be approximated with s_X *and* s_Y respectively.

- Compute the P-value
- The P-value is an area under the normal curve, which depends on the alternate hypothesis as shown in the table:

Large - Sample tests for Difference between two means

Alternate Hypothesis	*P*-value
$H_1: \mu_X - \mu_Y > \Delta_0$	$Area\ to\ the\ right\ of\ z$
$H_1: \mu_X - \mu_Y < \Delta_0$	$Area\ to\ the\ left\ of\ z$
$H_1: \mu_X - \mu_Y \neq \Delta_0$	$Sum\ of\ the\ areas\ in\ the\ tails\ cut\ off\ by\ z\ and\ -z$

Example:

The article "Effect of Welding Procedure on Flux Cored Steel Wire Deposits" (N. Ramini de Rissone, I. de S. Bott, et al., *Science and Technology of Welding and Joining*, 2003:113–122) compares properties of welds made using carbon dioxide as a shielding gas with those of welds made using a mixture of argon and carbon dioxide.

Example:

- One property studied was the diameter of inclusions, which are particles embedded in the weld.
- A sample of 544 inclusions in welds made using argon shielding averaged 0.37 μm in diameter, with a standard deviation of 0.25 μm.
- A sample of 581 inclusions in welds made using carbon dioxide shielding averaged 0.40 μm in diameter, with a standard deviation of 0.26 μm. (Standard deviations were estimated from a graph.)
- Can you conclude that the mean diameters of inclusions differ between the two shielding gases?

Solution:

$$H_0:\ \mu_X - \mu_Y = 0$$

$$H_1: \mu_X - \mu_Y \neq 0$$

$$\bar{X} = 0.37\ \ , \bar{Y} = 0.40$$

$$s_X = 0.25, S_Y = 0.26, n_X = 544, n_Y = 581$$

$$\bar{X} - \bar{Y} \sim N(0.0.01521^2)$$

Solution:

$$z = \frac{(\bar{X} - \bar{Y}) - \Delta_0}{\sqrt{\sigma^2{}_X/n_X + \sigma^2{}_Y/n_Y}} = -1.97$$

Solution:

- This is a two-tailed test, and the P-value is 0.0488.
- A follower of the 5% rule would reject the null hypothesis.
- It is certainly reasonable to be skeptical about the truth of H_0

Example:

- In a random sample of 100 tube lights produced by company A, the mean lifetime (mlt) of tube light is 1190 hours with standard deviation of 90 hours.
- Also in a random sample of 75 tube lights from company B the mean lifetime is 1230 hours with standard deviation of 120 hours.
- Is there a difference between the mean lifetime of the two brands of tube lights at a significance level of 0.05 ?

Solution:

- Let X_A , X_B denote the lifetime(in hours) of tube lights produced by company A and B respectively.

- It is given that the mean lifetime of tube lights of company A is $\overline{X_A}$=1190, standard deviation for tube lights of A is $s_A = 90$.

- Similarly $\overline{X_B}$ =1230, s_B= 120, n_A = sample size of tube lights from A= 100, n_B = sample size from B = 75

Solution:

$H_0: \mu_A - \mu_B = 0$ i.e., no difference.

Alternate hypothesis: $H_1: \mu_A - \mu_B \neq 0$ i.e., there is difference.

$$\mu_{\bar{X}_A - \bar{X}_B = \Delta_0 = 0}$$

$$\sigma_{\bar{X}_A - \bar{X}_B} = \sqrt{\sigma_{\bar{X}_A}{}^2 + \sigma_{\bar{X}_B}{}^2} = \sqrt{\frac{s_A{}^2}{n_A} + \frac{s_B{}^2}{n_B}}$$

$$= \sqrt{\frac{(90)^2}{100} + \frac{(120)^2}{75}} = 16.5227$$

Solution:

Test statistic:

$$z = \frac{(\bar{X}_A - \bar{X}_B) - \Delta_0}{\sqrt{\frac{s_A{}^2}{n_A} + \frac{s_B{}^2}{n_B}}} = \frac{1190 - 1230}{16.5227} = -2.421$$

For $\alpha = 0.05$.

Reject N.H. since $\mathrm{P-Value} 0.0156 < 0.05$

i.e., yes, there is difference between the mean lifetimes of the tube lights produced by A and B.

Solution:

For $\alpha = 0.01$

Accept N.H. since Reject N.H. since $\mathrm{P-Value} 0.0156 > 0.05$

So there is no difference between $\bar{X}_A$ and $\bar{X}_B$.

Example:

- To test the effects a new pesticide on rice production, a farm land was divided into 60 units of equal areas, all portions having identical qualities as to soil, exposure to sunlight etc.
- The new pesticide is applied to 30 units while old pesticide to the remaining 30.
- Is there reason to believe that the new pesticide is better than the old pesticide if the mean number of kg. of rice harvested/unit using new pesticide (N.P)is 496.31with s.d of 17.18 kgs while for old pesticide (O.P) is 485.41kgs and 14.73kgs.Test at a level of significance (a) $\alpha = 0.005$

Example:

$$H_0: \mu_X - \mu_Y \leq 0$$

$H_1: \mu_X - \mu_Y > 0$ i.e., new pesticide is superior to (better than) old pesticide.

$$\bar{X} = 496.31, \bar{Y} = 485.41, s_X = 17.18, s_Y = 14.73, n_X = 30, n_Y = 30$$

Test statistic is

$$z = \frac{(\bar{X}-\bar{Y})-\Delta_0}{\sqrt{\sigma^2{}_X/n_X + \sigma^2{}_Y/n_{\bar{Y}}}} = \frac{(496.31-485.41)-0}{\sqrt{\frac{(17.18)^2}{30}+\frac{(14.73)^2}{30}}} = 2.63814$$

Example:

Decision

$\boldsymbol{\alpha = 0.05}$

Reject N.H. since $\mathbf{P - Value 0.041 < 0.05}$ i.e., accept A.H. or new pesticide is superior to old pesticide.

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