PES University, Bangalore

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UE19CS203 – STATISTICS FOR DATA SCIENCE

Unit - 3 - Probability Distributions

QUESTION BANK

Large Sample Confidence Intervals for Population Mean

Exercises for Section 5.1

[Text Book Exercise 5.1 – Section 5.1 – Q. No. [1 – 26] – Pg. No. [335 - 338]]

1. Find the value of $z\alpha/2$ to use in expression (5.1) to construct a confidence interval with level
    a) 95%
    b) 98%
    c) 99%
    d) 80%

2. Find the levels of the confidence intervals that have the following values of $z\alpha/2$:

    a) $z\alpha/2 = 1.96$
    b) $z\alpha/2 = 2.17$
    c) $z\alpha/2 = 1.28$
    d) $z\alpha/2 = 3.28$

3. As the confidence level goes up, the reliability goes ----------and the precision goes ----------.

    Options: up, down

4. The article "Modeling Arterial Signal Optimization with Enhanced Cell Transmission Formulations"(Z. Li, *Journal of Transportation Engineering* 2011:445–454) presents a new method for timing traffic signals in heavily traveled intersections. The effectiveness of the new method was evaluated in a simulation study. In 50 simulations, the mean improvement in traffic flow in a particular intersection was 654.1 vehicles per hour, with a standard deviation of 311.7 vehicles per hour.

a) Find a 95% confidence interval for the improvement in traffic flow due to the new system.
b) Find a 98% confidence interval for the improvement in traffic flow due to the new system.
c) A traffic engineer states that the mean improvement is between 581.6 and 726.6 vehicles per hour. With what level of confidence can this statement be made?
d) Approximately what sample size is needed so that a 95% confidence interval will specify the mean to within ±50 vehicles per hour?
e) Approximately what sample size is needed so that a 98% confidence interval will specify the mean to within ±50 vehicles per hour?

5. In a sample of 100 steel wires the average breaking strength is 50 kN, with a standard deviation of 2 kN.

a) Find a 95% confidence interval for the mean breaking strength of this type of wire.
b) Find a 99% confidence interval for the mean breaking strength of this type of wire.
c) An engineer claims that the mean breaking strength is between 49.7 kN and 50.3 kN. With what level of confidence can this statement be made?
d) How many wires must be sampled so that a 95% confidence interval specifies the mean breaking strength to within ±0.3 kN?
e) How many wires must be sampled so that a 99% confidence interval specifies the mean breaking strength to within ±0.3 kN?

6. The article "Application of Surgical Navigation to Total Hip Arthroplasty" (T. Ecker and S. Murphy,*Journal of Engineering in Medicine*, 2007:699–712) reports that in a sample of 123 hip surgeries of a certain type, the average surgery timewas 136.9 minutes with a standard deviation of 22.6 minutes.

a) Find a 95% confidence interval for the mean surgery time for this procedure.
b) Find a 99.5% confidence interval for the mean surgery time for this procedure.
c) A surgeon claims that the mean surgery time is between 133.9 and 139.9 minutes. With what level of confidence can this statement be made?
d) Approximately howmany surgeries must be sampled so that a95%confidence interval will specify the mean to within ±3 minutes?
e) Approximately howmany surgeries must be sampled so that a99%confidence interval will specify the mean to within ±3 minutes?

7. The capacities (in ampere-hours) were measured for a sample of 120 batteries. The average was 178 and the standard deviation was 14.

    a) Find a 95% confidence interval for the mean capacity of batteries produced by this method.
    b) Find a 99% confidence interval for the mean capacity of batteries produced by this method.
    c) An engineer claims that the mean capacity is between 176 and 180 ampere-hours. With what level of confidence can this statement be made?
    d) Approximately how many batteries must be sampled so that a95%confidence interval will specify the mean to within ±2 ampere-hours?
    e) Approximately how many batteries must be sampled so that a99%confidence interval will specify the mean to within ±2 ampere-hours?

8. Oven thermostats were tested by setting them to 350◦F and measuring the actual temperature of the oven. In a sample of 67 thermostats, the average temperature was 348.2◦F and the standard deviation was 5.1◦F.

    a) Find a 90% confidence interval for the mean oven temperature.
    b) Find a 95% confidence interval for the mean oven temperature.
    c) What is the confidence level of the interval *(*347.5, 348.9*)*?
    d) How many thermostats must be sampled so that a 90% confidence interval specifies the mean to within ±0.8◦F?
    e) How many thermostats must be sampled so that a 95% confidence interval specifies the mean to within ±0.8◦F?

9. In a sample of 80 ten-penny nails, the average weight was 1.56 g and the standard deviation was 0.1 g.

    a) Find a 95% confidence interval for the mean weight of this type of nail.
    b) Find a 98% confidence interval for the mean weight of this type of nail.
    c) What is the confidence level of the interval *(*1.54, 1.58*)*?
    d) How many nails must be sampled so that a 95% confidence interval specifies the mean to within ±0.01 g?
    e) Approximately how many nails must be sampled so that a 98% confidence interval will specify the mean to within ±0.01 g?

10. In a sample of 60 electric motors, the average efficiency (in percent) was 85 and the standard deviation was 2.

a) Find a 95% confidence interval for the mean efficiency.
b) Find a 99.5% confidence interval for the mean efficiency.
c) What is the confidence level of the interval *(*84.63, 85.37*)*?
d) How many thermostats must be sampled so that a 95% confidence interval specifies the mean to within ±0.35?
e) How many thermostats must be sampled so that a 99.5% confidence interval specifies the mean to within ±0.35?

11. The sugar content in a one-cup serving of a certain breakfast cereal was measured for a sample of 140 servings. The average was 11.9 g and the standard deviation was 1.1 g.

a) Find a 95%confidence interval for the mean sugar content.
b) Find a 99%confidence interval for the mean sugar content.
c) What is the confidence level of the interval *(*11.81, 11.99*)*?
d) How large a sample is needed so that a 95% confidence interval specifies the mean to within ±0.1?
e) How large a sample is needed so that a 99% confidence interval specifies the mean to within ±0.1?

12. Refer to Exercise 5.

a) Find a 95% lower confidence bound for the mean strength.
b) Someone says that the mean strength is less than 50.4 kN. With what level of confidence can this statement be made?

13. Refer to Exercise 6.

a) Find a 98% lower confidence bound for the mean time.
b) Someone says that the mean time is greater than 134.3 minutes.With what level of confidence can this statement be made?

14. Refer to Exercise 7.

a) Find a 95% lower confidence bound for the mean capacity of this type of battery.
b) An engineer claims that the mean capacity is greater than 175 ampere-hours. With what level of confidence can this statement be made?

15. Refer to Exercise 8.

a) Find a 99% upper confidence bound for the mean temperature.

b) The claim is made that the mean temperature is less than 349.5◦F. With what level of confidence can this statement be made?

16. Refer to Exercise 9.

a) Find a 90% upper confidence bound for the mean weight.
b) Someone says that the mean weight is less than 1.585 g. With what level of confidence can this statement be made?

17. Refer to Exercise 10.

a) Find a 98% lower confidence bound for the mean efficiency.
b) The claim is made that the mean efficiency is greater than 84.6%.With what level of confidence can this statement be made?

18. Refer to Exercise 11.

a) Find a 95% upper confidence bound for the mean sugar content.
b) The claim is made that the mean sugar content is greater than 11.7 g.With what level of confidence can this statement be made?

19. An investigator computes a 95% confidence interval for a population mean on the basis of a sample of size 70. If she wishes to compute a 95% confidence interval that is half as wide, how large a sample does she need?

20. A 95% confidence interval for a population mean is computed from a sample of size 400. Another 95% confidence interval will be computed from a sample of size 100 drawn from the same population.

Choose the best answer to fill in the blank: The interval from the sample of size 400 will be approximately as the interval from the sample of size 100.

a. One-eighth as wide
b. One-fourth as wide
c. One-half as wide
d. The same width
e. Twice as wide
f. Four times as wide
g. Eight times as wide

21. Based on a large sample of capacitors of a certain type, a 95% confidence interval for the mean capacitance, in μF, was computed to be $(0.213, 0.241)$.Find a 90% confidence interval for the mean capacitance of this type of capacitor.

22. Sixty-four independent measurements were made of the speed of light. They averaged 299,795 km/s and had a standard deviation of 8 km/s. True or false:

    a) A 95% confidence interval for the speed of light is 299,795 ± 1.96 km/s.
    b) The probability is 95% that the speed of light is in the interval 299,795 ± 1.96.
    c) If a 65th measurement is made, the probability is 95% that it would fall in the interval 299,795 ± 1.96.

23. A large box contains 10,000 ball bearings. A random sample of 120 is chosen. The sample mean diameter is 10 mm, and the standard deviation is 0.24 mm. True or false:

    a) A 95% confidence interval for the mean diameter of the 120 bearings in the sample is $10 \pm (1.96)(0.24)/\sqrt{120}$.
    b) A 95% confidence interval for the mean diameter of the 10,000 bearings in the box is $10 \pm (1.96)(0.24)/\sqrt{120}$.
    c) A 95% confidence interval for the mean diameter of the 10,000 bearings in the box is $10 \pm (1.96)(0.24)/\sqrt{10{,}000}$.

24. Each day a quality engineer selects a random sample of 50 power supplies from the day's production, measures their output voltages, and computes a 90% confidence interval for the mean output voltage of all the power supplies manufactured that day. What is the probability that more than 15 of the confidence intervals constructed in the next 200 days will fail to cover the true mean?

25. Based on a sample of repair records, an engineer calculates a 95% confidence interval for the mean cost to repair a fiber-optic component to be $(\$140, \$160)$. A supervisor summarizes this result in a report, saying, "We are 95% confident that the mean cost of repairs is less than \$160." Is the supervisor underestimating the confidence, overestimating it, or getting it right? Explain.

26. A meteorologist measures the temperature in downtown Denver at noon on each day for one year. The 365 readings average 57◦F and have standard deviation 20◦F. The meteorologist computes a 95% confidence interval for the mean temperature at noon to be $57^\circ \pm (1.96)(20)/\sqrt{365}$. Is this correct? Why or why not?