PES University, Bangalore

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UE19CS203 – STATISTICS FOR DATA SCIENCE

Unit - 3 - Probability Distributions

QB SOLVED

Principles of Point Estimation - Maximum Likelihood Estimation

Exercises for Section 4.9

1. Let $X_1 \ldots . X_n$ be a random sample from a $N(0, \sigma^2)$ population. Find the MLE of σ

   [Text Book Exercise – Section 4.9 – Q. No. 9 – Pg. No. 285]
   Solution:

   The probability density function is,

$$f(x_i; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_i - \mu)^2}{2\sigma^2}}$$

   The joint probability function of $X_1, \ldots, X_n$ is,

$$f(x_1, \ldots\ldots, x_n; \mu, \sigma) = \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_i - \mu)^2}{2\sigma^2}}$$

   The likelihood function is,

$$f(x_1, \ldots\ldots, x_n; \mu, \sigma) = \frac{1}{\sigma^n (2\pi)^{n/2}} e^{-\sum_{i=1}^{n} \frac{x_i^2}{2\sigma^2}}$$

   The logarithm of likelihood function is,

$$\ln f(x_1, \ldots, x_n : \mu, \sigma) = -n \ln \sigma - \frac{n}{2} \ln 2\pi - \sum_{i=1}^{n} \frac{x_i^2}{2\sigma^2}$$

   Taking derivative with respect to σ and setting it equal to 0,

$$\frac{d}{d\sigma} \ln f(x_1, \ldots, x_n : \mu, \sigma) = \frac{-n}{\sigma} + \frac{\sum_{i=1}^{n} x_i^2}{\sigma^3}$$

$$= \frac{-n\sigma^2 + \sum_{i=1}^{n} x_i^2}{\sigma^3}$$

$$\frac{-n\sigma^2 + \sum_{i=1}^{n} x_i^2}{\sigma^3} = 0$$

$$-n\sigma^2 + \sum_{i=1}^{n} x_i^2 = 0$$

$$\sigma^2 = \frac{\sum_{i=1}^{n} x_i^2}{n}$$

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} x_i^2}{n}}$$

The MLE of σ is $\hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{n} x_i^2}{n}}$