DESIGN AND ANALYSIS OF ALGORITHMS

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2-3 Trees

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2-3 tree is a tree that can have nodes of two kinds: 2-nodes and 3-nodes.

- A ***2-node*** contains a single key K and has two children:

  the left child serves as the root of a subtree whose keys are less than K, and the right child serves as the root of a subtree whose keys are greater than K.

- A ***3-node*** contains two ordered keys $K1$ and $K2$ $(K1 < K2)$ and has three children.

- The leftmost child serves as the root of a subtree with keys less than $K1$, the middle child serves as the root of a subtree with keys between $K1$ and $K2$, and the rightmost child serves as the root of a subtree with keys greater than $K2$
- The last requirement of the 2-3 tree is that all its leaves must be on the same level. In other words, a 2-3 tree is always perfectly height-balanced: the length of a path from the root to a leaf is the same for every leaf.

Here are the properties of a 2-3 tree:

- each node has either one value or two value
- a node with one value is either a leaf node or has exactly two children (non-null). Values in left subtree < value in node < values in right subtree
- a node with two values is either a leaf node or has exactly three children (non-null). Values in left subtree < first value in node < values in middle subtree < second value in node < value in right subtree.
- all leaf nodes are at the same level of the tree

2-3 Tree

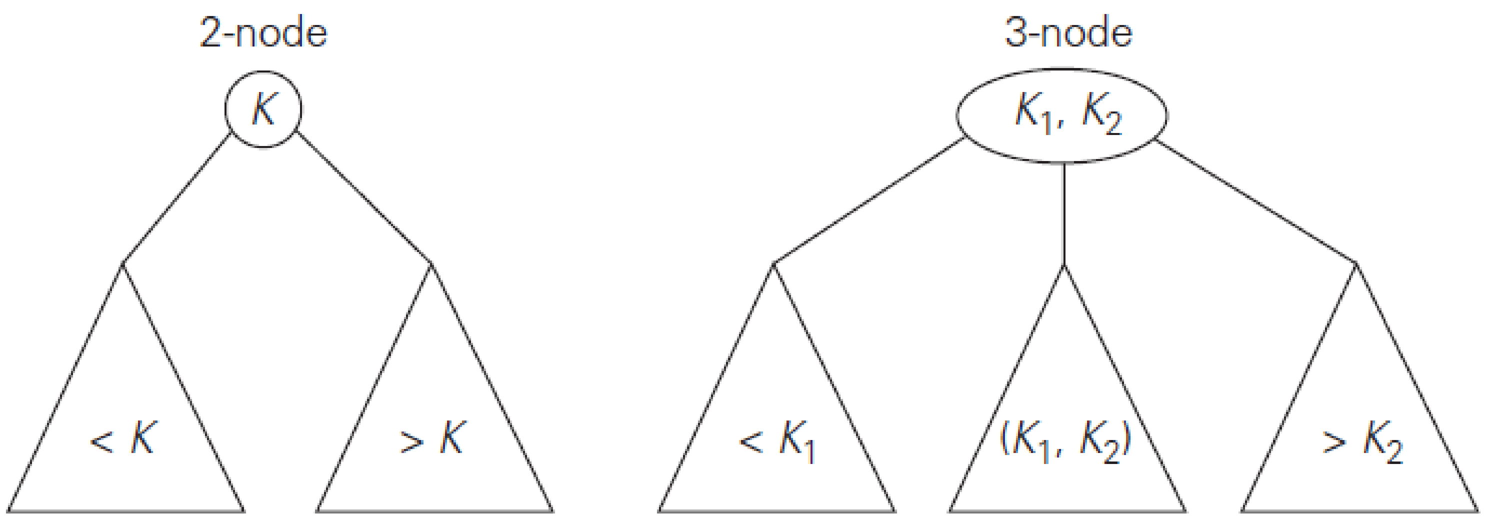


FIGURE 6.7 Two kinds of nodes of a 2-3 tree.

Search: To search a key **K** in given 2-3 tree **T**, we follow the following procedure:

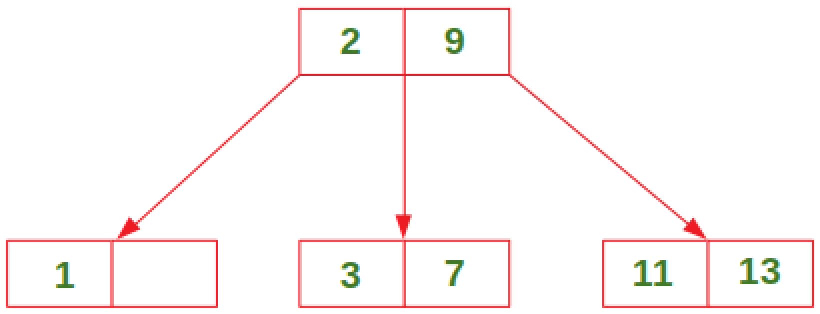
Base cases:

1. If **T** is empty, return False (key cannot be found in the tree).
2. If current node contains data value which is equal to **K**, return True.
3. If we reach the leaf-node and it doesn't contain the required key value **K**, return False.

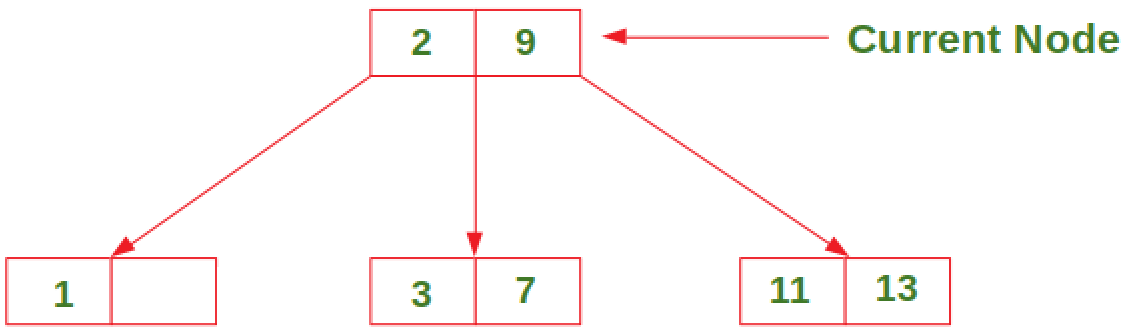
Recursive Calls:

1. If **K** < currentNode.leftVal, we explore the left subtree of the current node.
2. Else if currentNode.leftVal < **K** < currentNode.rightVal, we explore the middle subtree of the current node.
3. Else if **K** > currentNode.rightVal, we explore the right subtree of the current node.

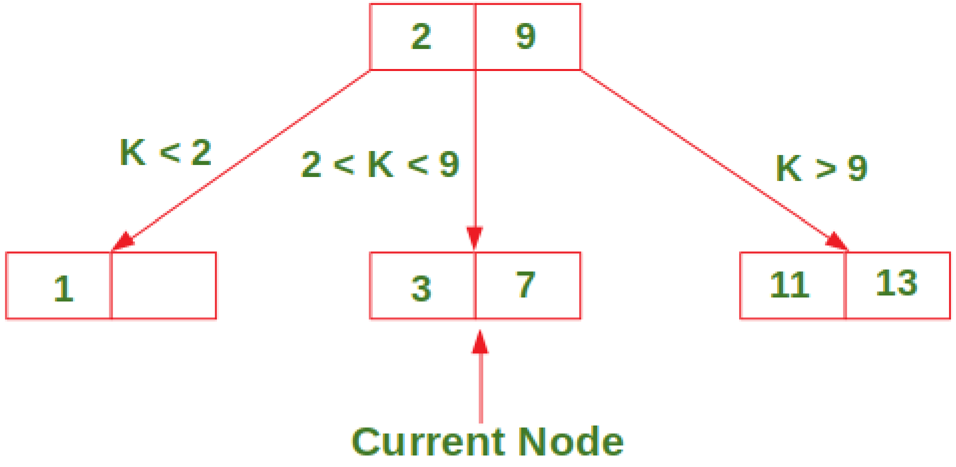
Search 5 in the following 2-3 Tree:



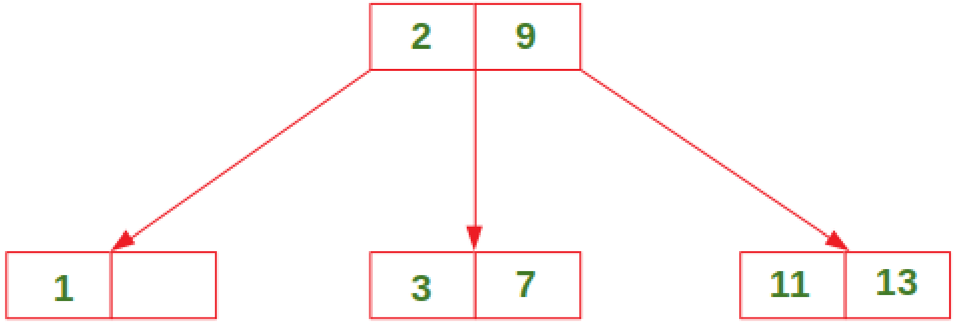
Search 5



Search 5



Search 5



5 Not Found. Return False

Insertion

The insertion algorithm into a two-three tree is quite different from the insertion algorithm into a binary search tree. In a two-three tree, the algorithm will be as follows:

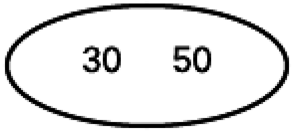
- If the tree is empty, create a node and put value into the node
- Otherwise find the leaf node where the value belongs.
- If the leaf node has only one value, put the new value into the node
- If the leaf node has more than two values, split the node and promote the median of the three values to parent.
- If the parent then has three values, continue to split and promote, forming a new root node if necessary

2-3 Tree

Insert 50

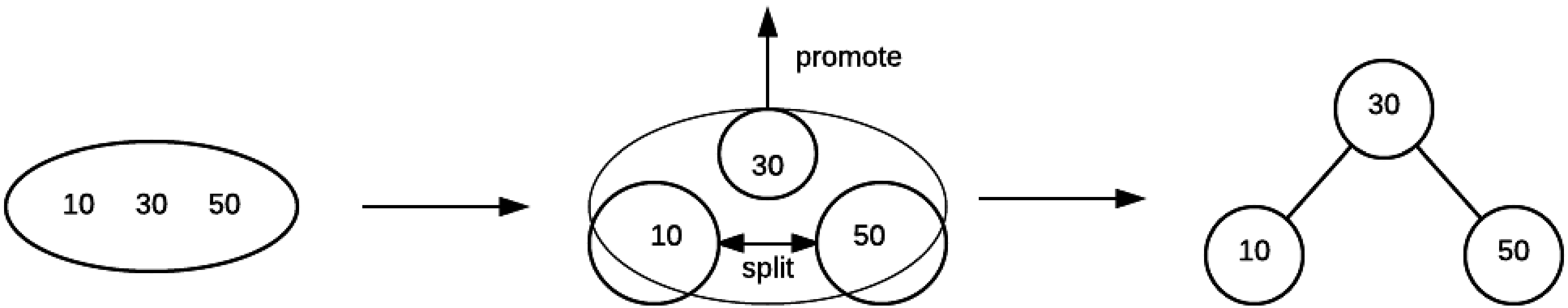
2-3 Tree

Insert 30

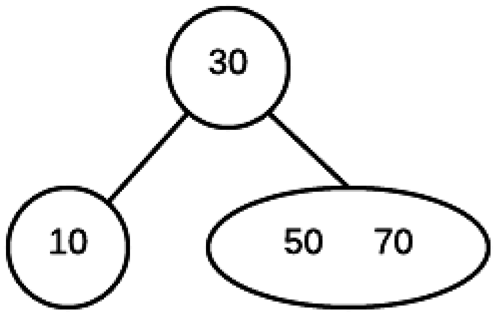


2-3 Tree

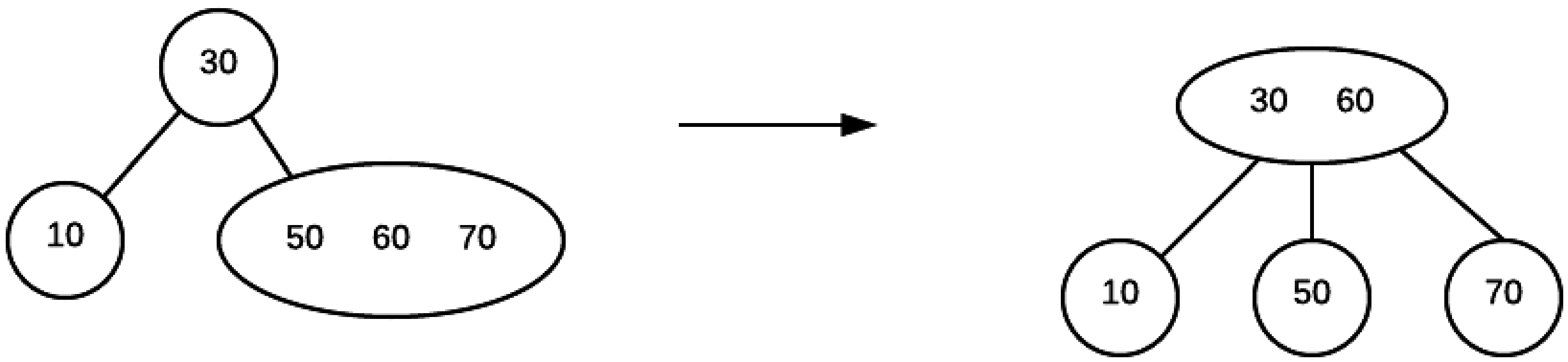
Insert 10



Insert 70



Insert 60



2-3 Tree

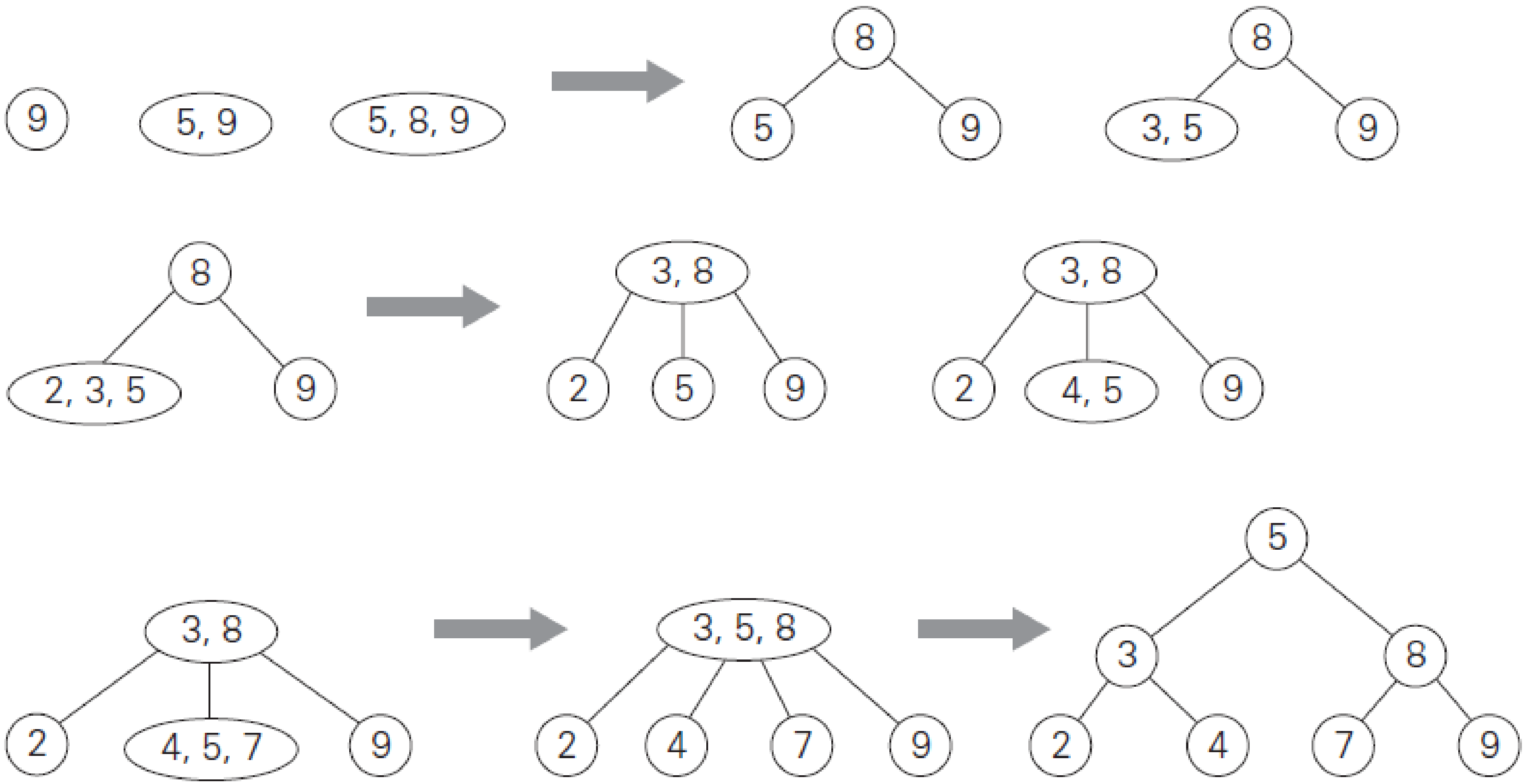


FIGURE 6.8 Construction of a 2-3 tree for the list 9, 5, 8, 3, 2, 4, 7.

Time Complexity

The property of being perfectly balanced, enables the **2-3 Tree** operations of insert, delete and search to have a time complexity of O(log (n)).

THANK YOU

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