STATISTICS FOR DATA SCIENCE

Power Test &
Simple Linear Regression

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STATISTICS FOR DATA SCIENCE

Unit 5 : Power Test & Simple Linear Regression

Session : 5

Sub Topic : Correlation & Regression Analysis

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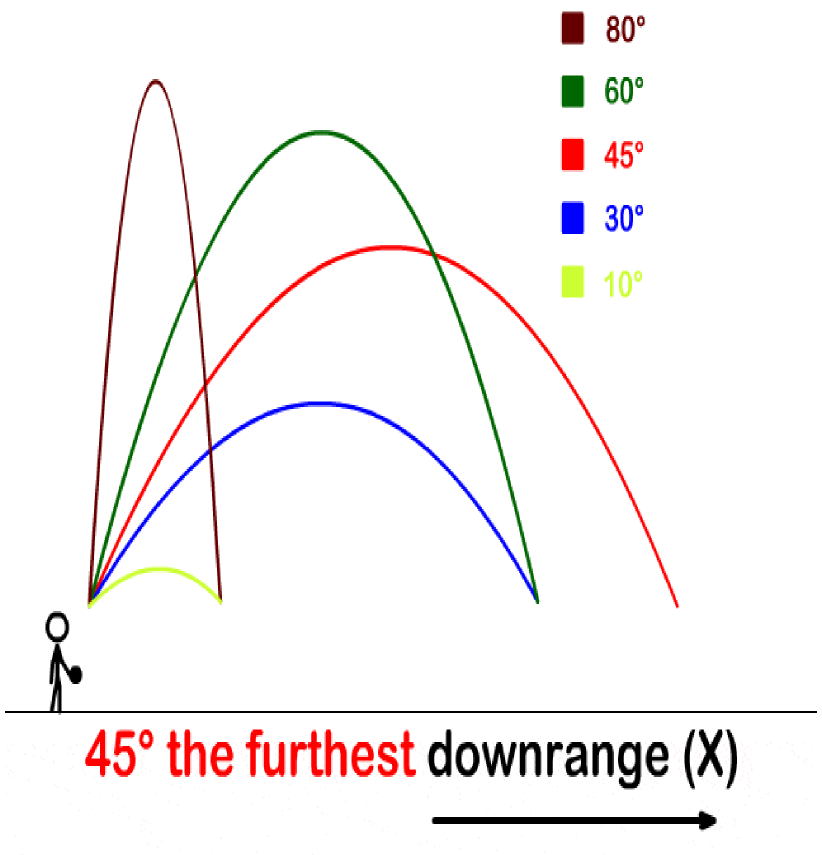
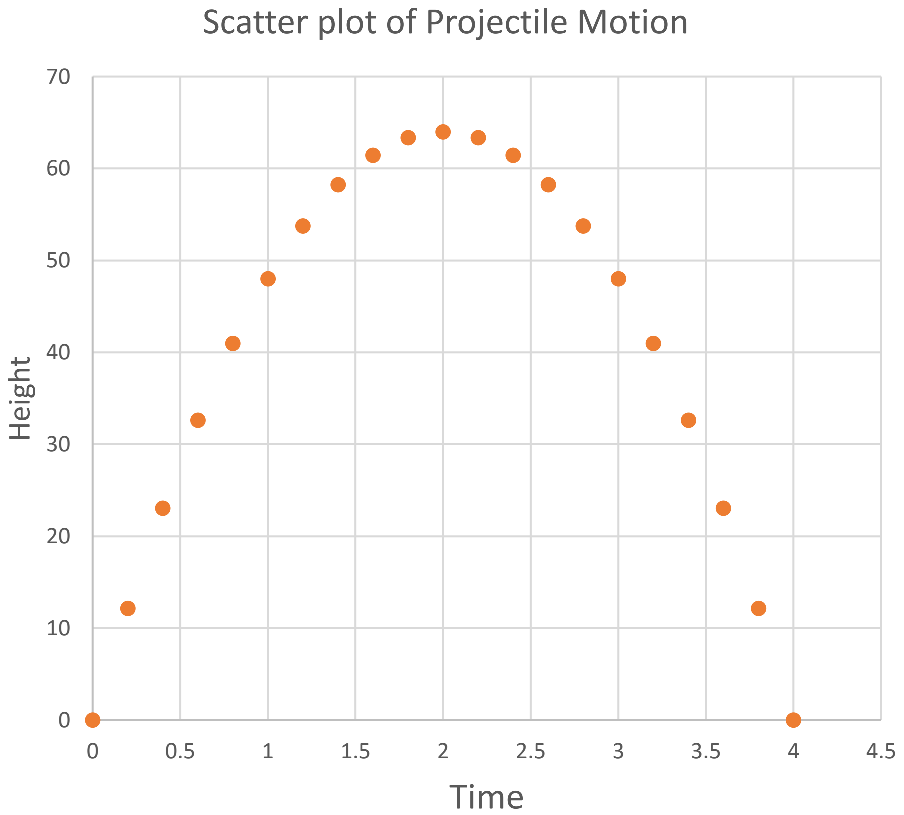
- More about Correlation coefficient!
- What is Regression Analysis ?
- How to obtain the Least Squares line for a given set of Bivariate data?

Correlation Coefficient measures only Linear Association

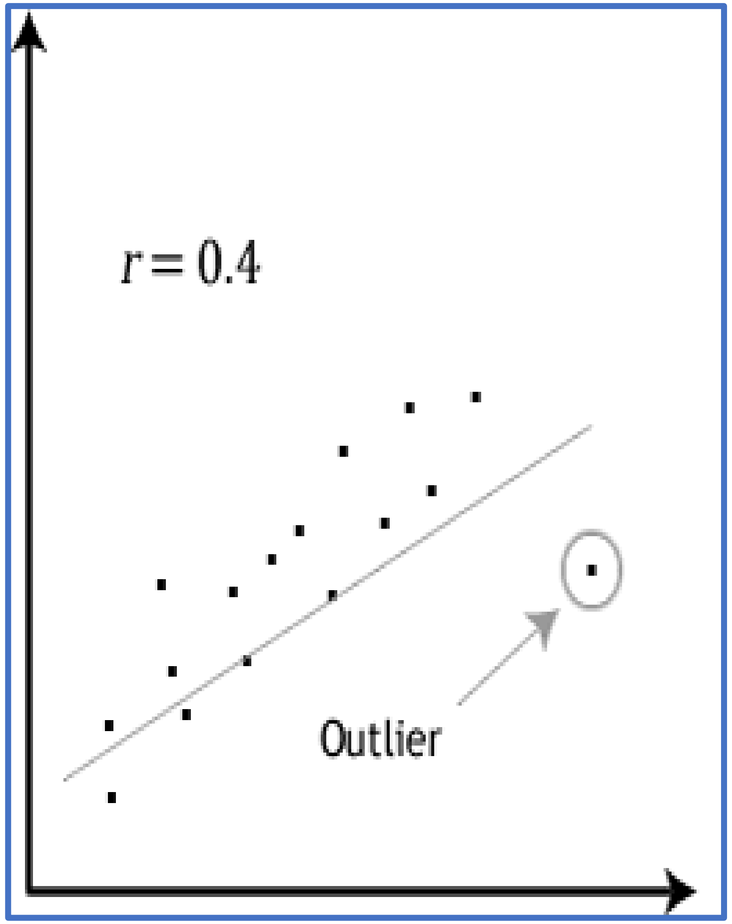


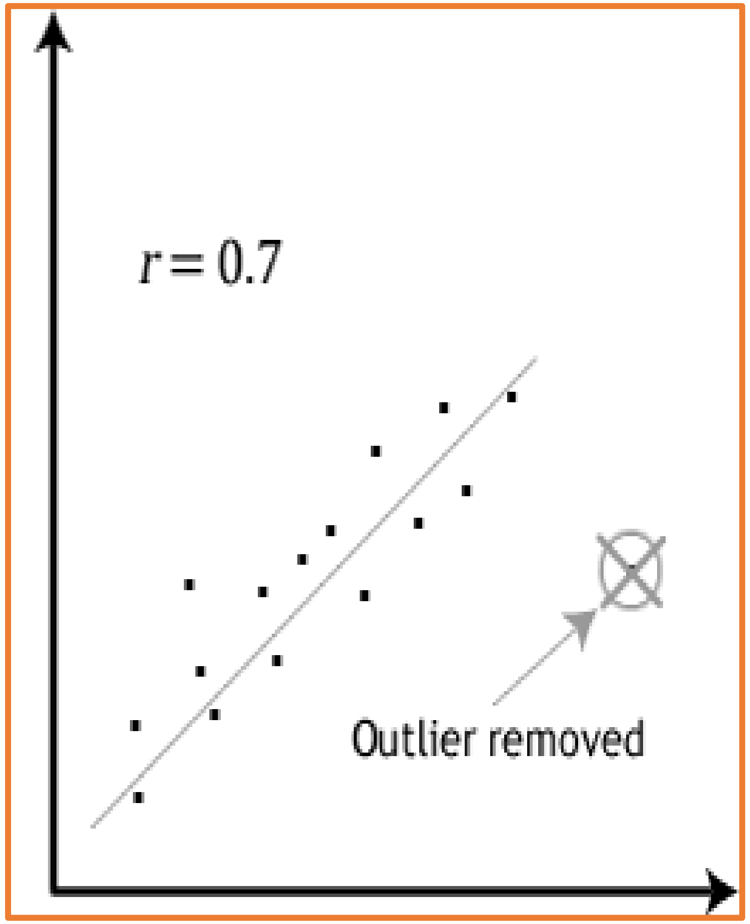
Source : stickmanphysics.com

- Initial velocity $= 64ft/s$
- Equation : $y = 64x - 16x^2$
- Correlation coefficient $r = 0$



Correlation Coefficient - Misleading when outliers are present





Source : medium.com

Anscombe’s Quartet

- Source : slideshare.net

#1 X	#1 Y	#2 X	#2 Y	#3 X	#3 Y	#4 X	#4 Y
10.0	8.04	10.0	9.14	10.0	7.46	8.0	6.58
8.0	6.95	8.0	8.14	8.0	6.77	8.0	5.76
13.0	7.58	13.0	8.74	13.0	12.74	8.0	7.71
9.0	8.81	9.0	8.77	9.0	7.11	8.0	8.84
11.0	8.33	11.0	9.26	11.0	7.81	8.0	8.47
14.0	9.96	14.0	8.10	14.0	8.84	8.0	7.04
6.0	7.24	6.0	6.13	6.0	6.08	8.0	5.25
4.0	4.26	4.0	3.10	4.0	5.39	19.0	12.50
12.0	10.84	12.0	9.13	12.0	8.15	8.0	5.56
7.0	4.82	7.0	7.26	7.0	6.42	8.0	7.91
5.0	5.68	5.0	4.74	5.0	5.73	8.0	6.89

Source : slideshare.net

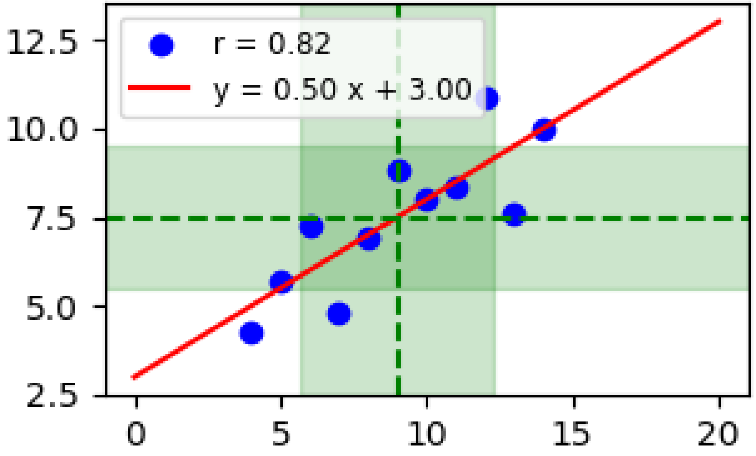
Anscombe's Quartet Summary Statistics

Property	Value
Mean of X	11.0
Variance of X	10.0
Mean of Y	7.5
Variance of Y	3.75
Correlation between X and Y	0.816
Linear regression	y = 3.0 +0.5x

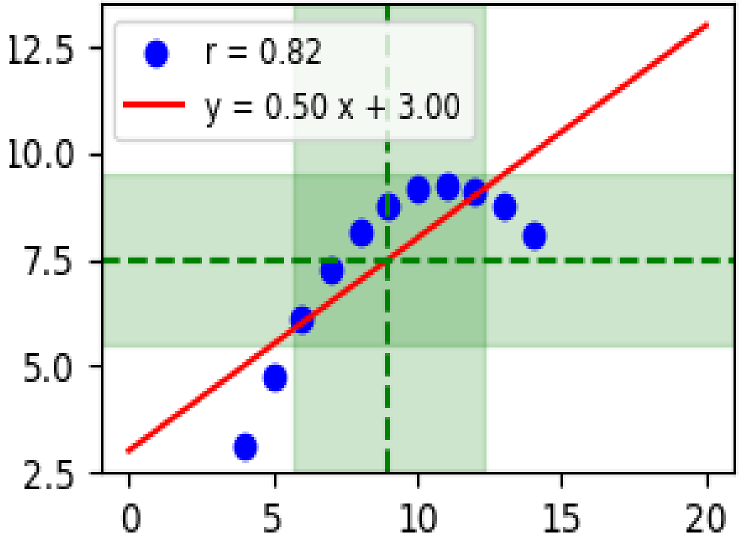
Identical statistics!

Source : slideshare.net

Visual representation of the Anscombe's Quartet



- The first scatter plot appears to be a simple linear relationship, corresponding to two variables



- The second graph is not distributed normally.
- Though a relationship between the two variables is obvious, it is not linear.
- The Pearson correlation coefficient is not relevant.

Source : informatique-python.readthedocs.io

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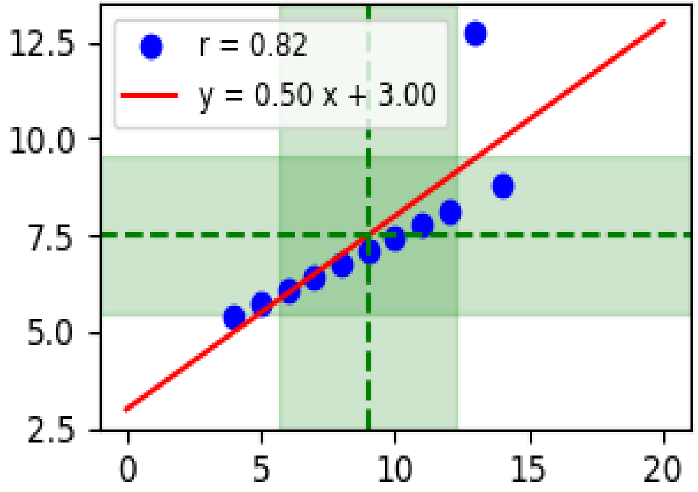
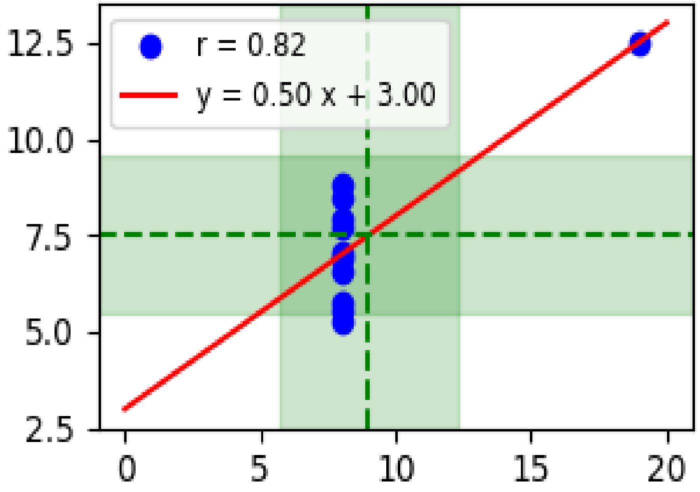
Visual representation of the Anscombe's Quartet



- In the third graph the linear relationship is perfect.
- But one outlier which exerts enough influence to lower the correlation coefficient from 1 to 0.816.



- The fourth example shows another example when one outlier is enough to produce a high correlation coefficient, even though the relationship between the two variables is not linear.

Source : informatique-python.readthedocs.io

Conclusions from the Anscombe’s Quartet

- The presence of outliers in a dataset has an impact on the evaluated value of the correlation coefficient.
- The basic statistic properties are inadequate for describing realistic datasets.
- It is important to look at a set of data visually before starting any kind of analysis.

Source : iconfinder.com

Remark :

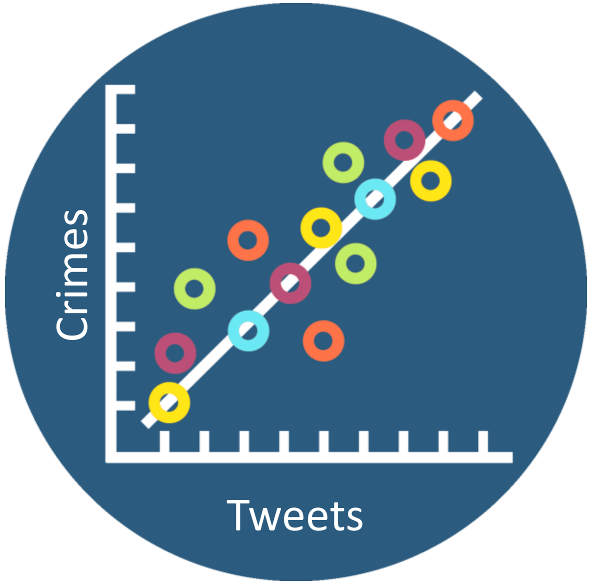
Correlation between X & Y

- *X causes Y*
- *Y causes X*
- *X causes Y & Y causes X*
- *Some third variable Z causes X and Y*
- *It is just a coincidence and there is no casual relationship between X and Y*

Correlation is not causation!!

Example : Relationship between drug related tweets and crime rates (Strong Positive Correlation).

- A strong Positive relationship between tweets and crime has been found.
- But there is no evidence to suggest that *tweets are causing more crime* and *tweets about crime do not necessarily reflect the crime rate*.

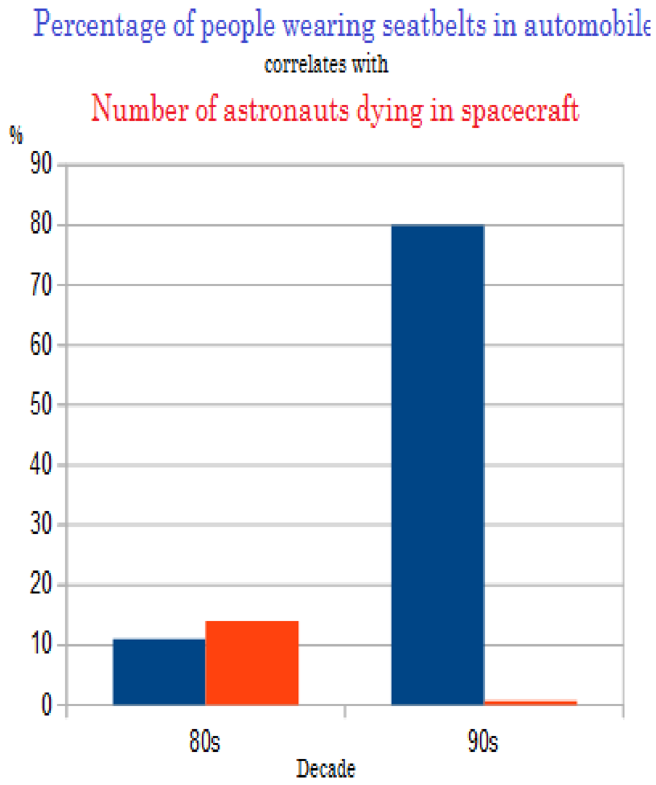


Reference : The Relationship Between Social Media Data and Crime Rates in the United States Yan Wang1 , Wenchao Yu1 , Sam Liu2 , and Sean D. Y Social Media + Society January-March 2019: 1–9 ©

Source :learningspaces.dundee.ac.uk

Correlation is not causation!!

Example 2. : Relationship between wearing seat belt and astronaut deaths (Strong Positive Correlation).



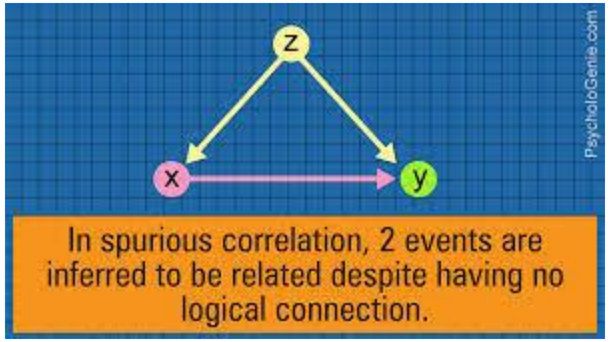
Use your seatbelt and save an astronaut life!

- The graph shows that an increase in wearing car seat belt results in a lower number of astronaut deaths.
- Obviously there isn't a real correlation here: putting your seat belt on in a car has nothing to do with the odds of an accident in space.

Sources :statisticshowto.com, pinterest.com, freeart.com

Confounding Variable

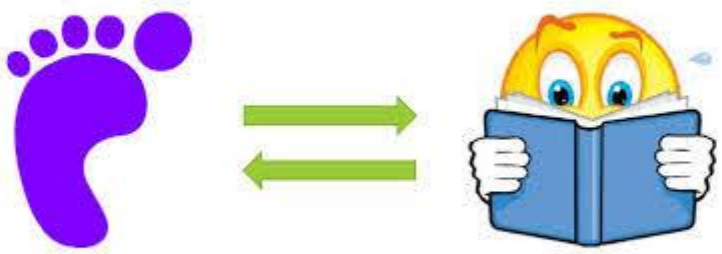
- Confounding Variable is a variable that influences both the independent variable as well as the dependent variable causing a spurious correlation.
- This may interfere in your analysis and ruin your experiment by giving useless results.
- Confounding variables can cause two major problems:
  - Increase variance
  - Introduce bias.
- A confounding variable are like extra independent variables that are having a hidden effect on your dependent variables.
- A confounding variable can be what the actual cause of a correlation is, hence any studies must take these into account and find ways of dealing with them.



Source : psychologenie.com

Confounding Variable!!

Example 1. : Relationship between reading ability and shoe size. (Strong Positive Correlation).



- You collect data on reading ability and shoe size
- You find that bigger the shoe size the better is the reading ability.
- Does that mean bigger shoe size leads to better reading abilities?
- Should children be hence fed growth hormones so that the reading abilities improve?
- Should children start focusing on their reading abilities to increase their shoe size?

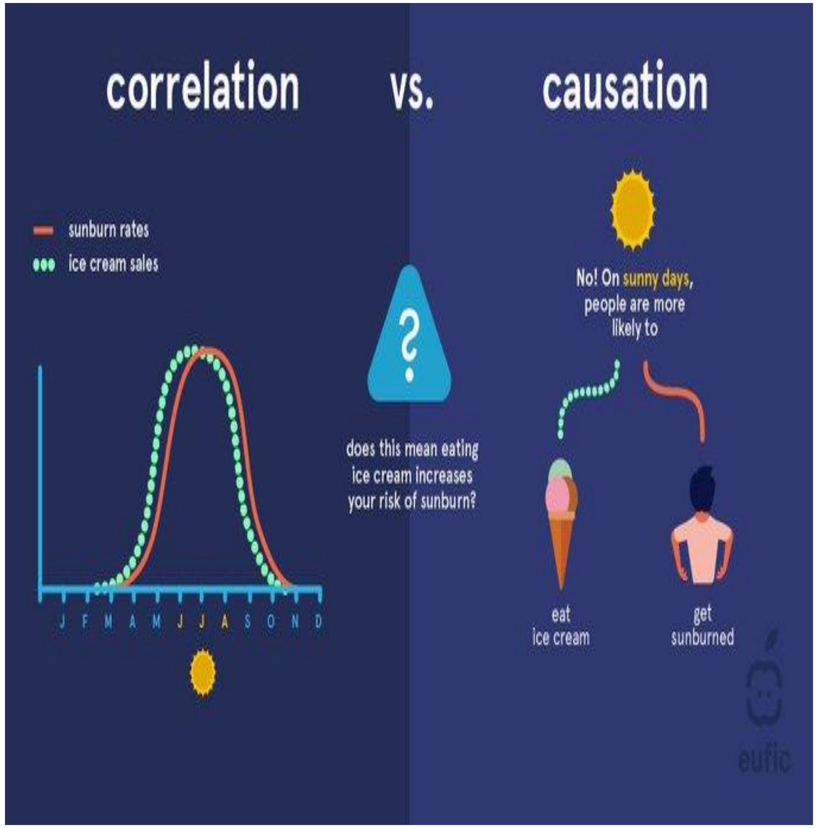
Confounding Variable : There is a third variable—a confounding variable—which causes the increase in both reading ability and shoe size.

Age : As the child's age increase, the foot size increases and also the reading ability increases since the child goes to higher classes.

Source : outsmartstatistics.weebly.com

Confounding Variable!!

Example 2. : Relationship between sun burns and ice – cream consumption. (Strong Positive Correlation).



- ❖ You collect data on sunburns and ice cream consumption.
- ❖ You find that higher ice cream consumption is associated with a higher probability of sunburn.
- ❖ Does that mean,

Possibility #1: Sun burns cause consumption of ice cream.

Possibility #2: Eating ice cream causes sun burns.

Possibility #3: There is a third variable—a confounding variable—which causes the increase in both ice cream sales and sun burn.

Confounding Variable : Hot temperatures

Source : twitter.com

Confounding Variable!!

Example 3. : Relationship between the force you apply to a ball and the distance the ball travels.

- Naturally, you predict that the more force you apply, the further the ball will travel.
- After you run your experiment, you observe that the ball travels further in Condition 2 than it does in Condition 1.
- In other words, you find that the less force you apply, the further the ball travels.
- Confounding variable : the angle of the slope.

Source : psychologyinaction.org

Example Problem 1.

- An environmental scientist is studying the rate of absorption of a certain chemical into skin.
- She places differing volumes of the chemical on different pieces of skin and allows the skin to remain in contact with the chemical for varying lengths of time.
- She then measures the volume of chemical absorbed into each piece of skin.
- She obtains the results shown in the following table.

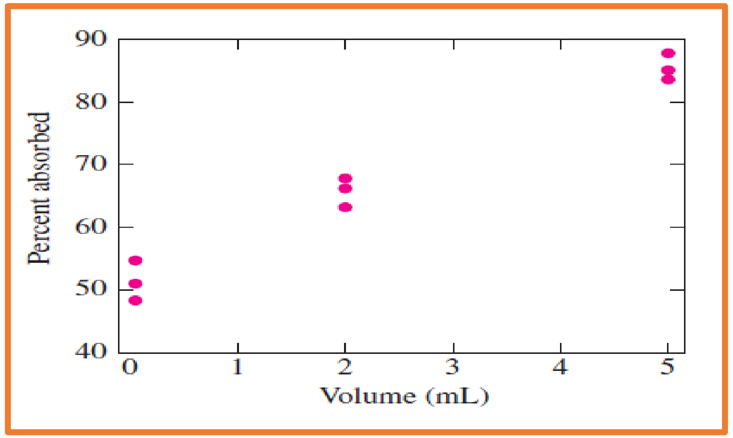
Volume (ml)	Time (h)	Percent Absorbed
0.05	2	48.3
0.05	2	51.0
0.05	2	54.7
2.00	10	63.2
2.00	10	67.8
2.00	10	66.2
5.00	24	83.6
5.00	24	85.1
5.00	24	87.8

Source : Internet

Example Problem

Correlation between Volume & Percent Absorbed

❖ Scatter Plot :



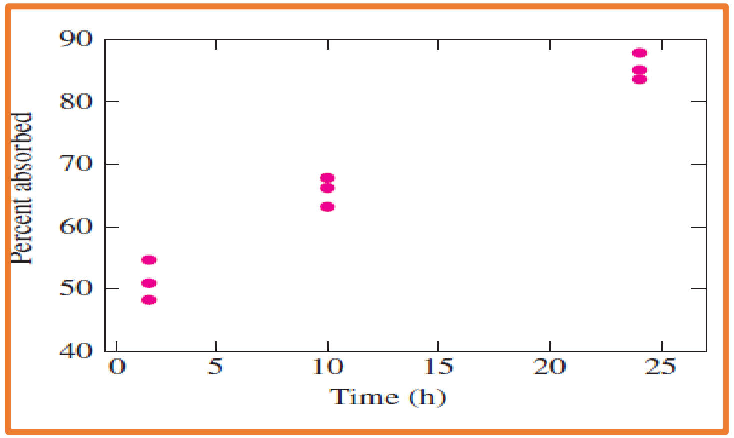
❖ Correlation , $r = 0.988$

❖ Positive Correlation

❖ Increasing the volume causes the percentage absorbed to increase.

Correlation between Time & Percent Absorbed

❖ Scatter Plot :



❖ Correlation , $r = 0.987$

❖ Positive Correlation

❖ Increasing the time that the skin is in contact with chemical causes the percentage absorbed to increase.

Are these conclusions Justified???

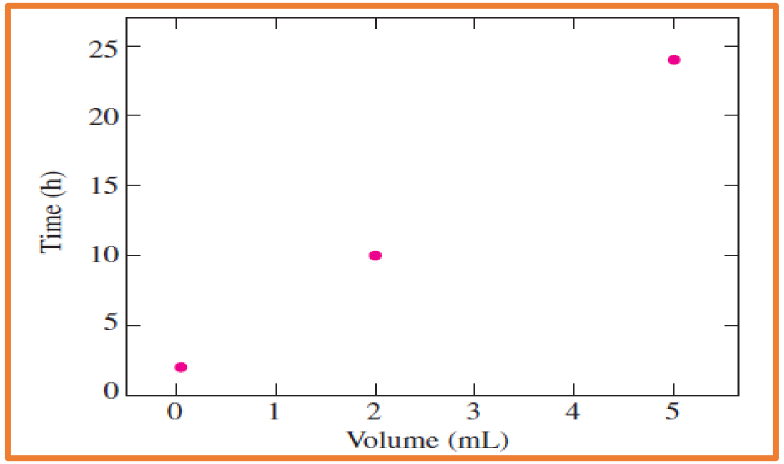
Sources : slideteam.net,
Statistics for Engineers and Scientists, William Navidi

Example Problem

No! The conclusions are not justified!

Suggested Solution :

- The correlation between time & volume has to be explored.
- The Scatter plot :



- The correlation, $r = 0.999$
- Conclusion : These 2 variables are completely confounded.

Example Problem 2.

- The Scientist in Example Problem 1 has repeated the experiment, this time with a new design.

- The results are presented in the table.

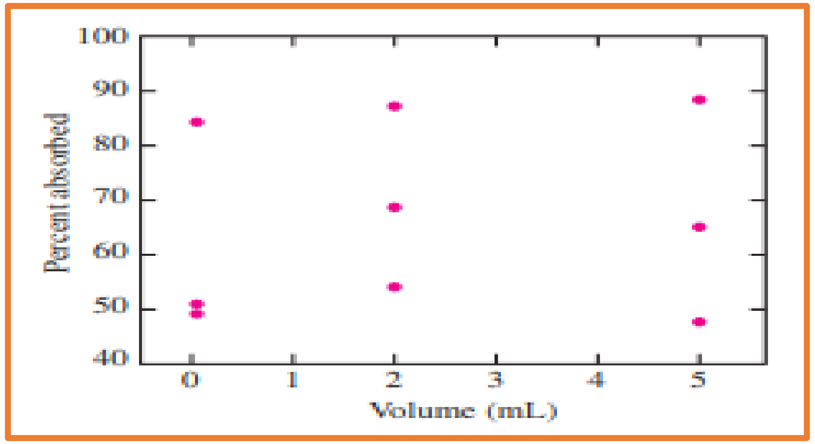
Volume (ml)	Time (h)	Percent Absorbed
0.05	2	49.2
0.05	10	51.0
0.05	24	84.3
2.00	2	54.1
2.00	10	68.7
2.00	24	87.2
5.00	2	47.7
5.00	10	65.1
5.00	24	88.4

Source : Internet

Example Problem 2.

Correlation between Volume & Percent Absorbed

❖ Scatter Plot :



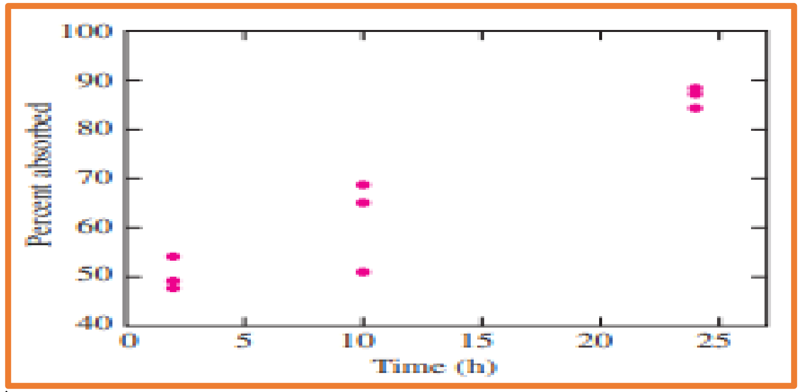
❖ Correlation, $r = 0.121$

❖ Weak Positive Correlation

❖ Hence increase of volume has little or no effect on the percentage absorbed.

Correlation between Time & Percent Absorbed

❖ Scatter Plot :



❖ Correlation , $r = 0.952$

❖ Strong Positive Correlation

❖ Increasing the time that the skin is in contact with the chemical will cause the percentage absorbed to increase.

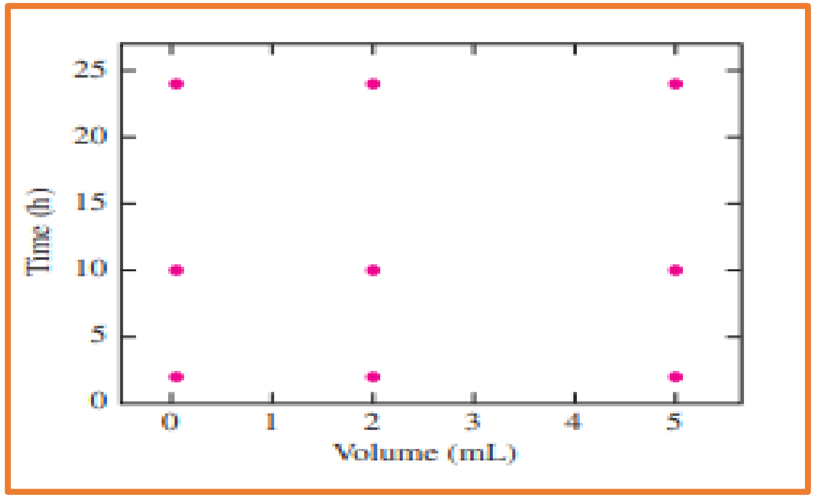
Are these conclusions Justified???

Sources : slideteam.net,
Statistics for Engineers and Scientists, William Navidi

Example Problem 2.

Correlation between Volume & Time

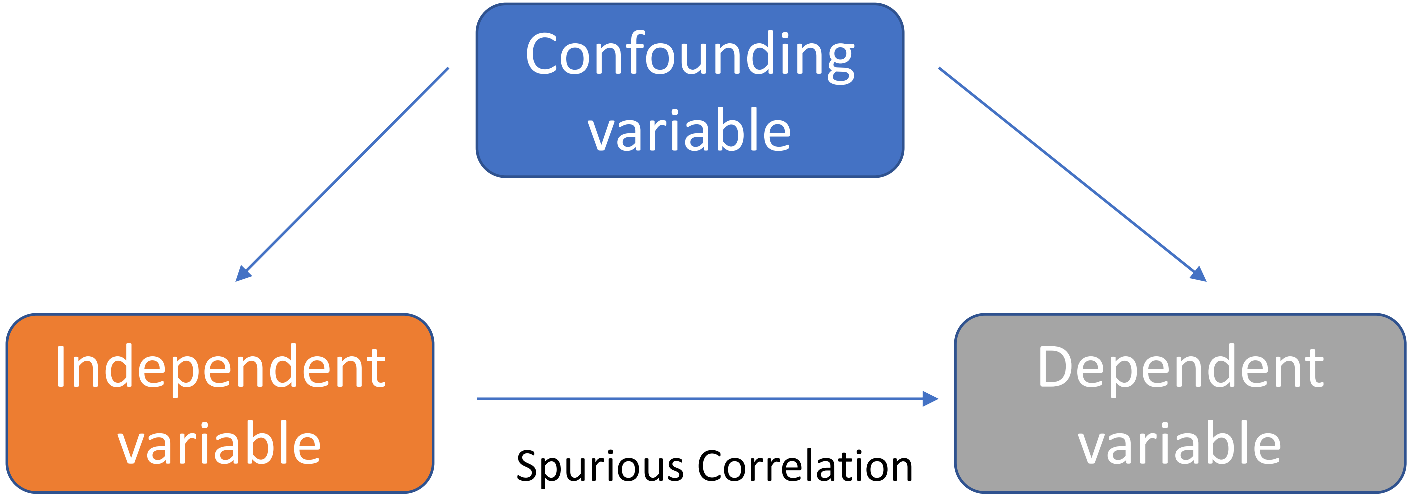
❖ Scatter Plot :



❖ *Time & Volume are not correlated in this case*

Source : Statistics for Engineers and Scientists, William Navidi

Controlled Experiments reduce the risk of Confounding



- ❖One of the ways by which confounding can be avoided in controlled experiments by choosing values for certain factors in such a way that there exists no correlation between those factors.
- ❖ For instance in Example Problem 1. & 2. the environmental scientist reduced confounding by assigning values to volume and time such that they were uncorrelated.
- ❖But this is not possible in all cases.

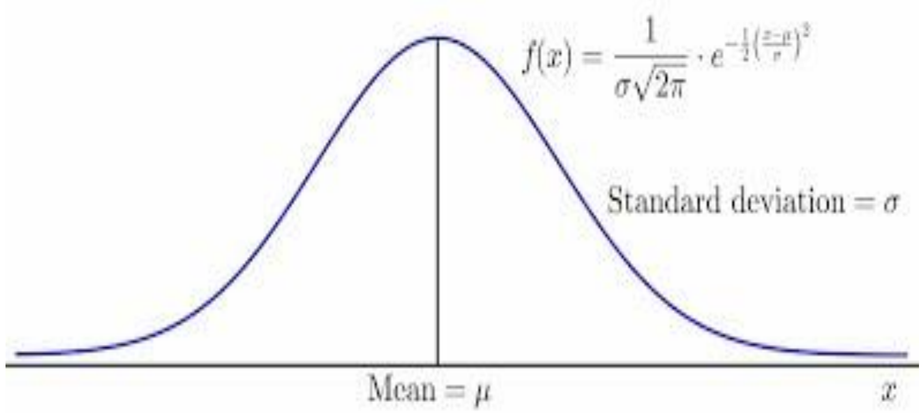
Controlled Experiments reduce the risk of Confounding

- ❖The values of factors cannot be chosen by the observer in case of observational studies/ experiments.
- ❖For instance, in studies involving public health issues like impact of environmental pollutants on human health, the observer cannot assign values to any of the factors.
- ❖Hence it becomes difficult to avoid confounding.
- ❖Example : People who live in areas with higher level pollutants may tend to have lower socio-economic status, which may affect their health.
- ❖ In observational studies to avoid or reduce confounding the study must be repeated a number of times under a variety of conditions before drawing reliable conclusions !!!

Then how can confounding be avoided in such cases ???

Bivariate Normal Distribution

- The “regular” normal distribution has one random variable.

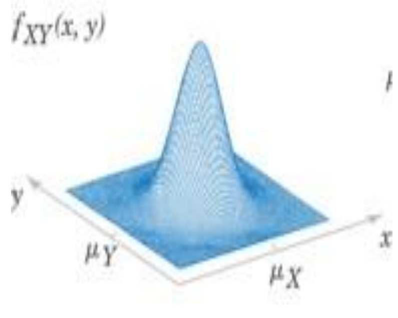


- **A bivariate normal distribution is made up of two independent random variables.**
- The two variables in a bivariate normal are both normally distributed, and they have a normal distribution when both are added together.
- Two random variables $X \ and \ Y$ are said to be bivariate normal, or jointly normal, if $aX + bY$ has a normal distribution for all $a, b \ \in \ R$

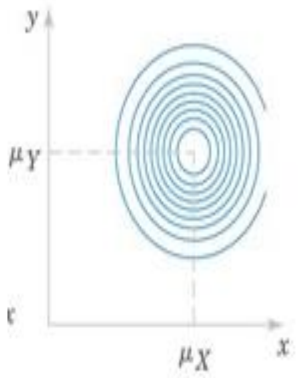
Source : Internet

Bivariate Normal Distribution

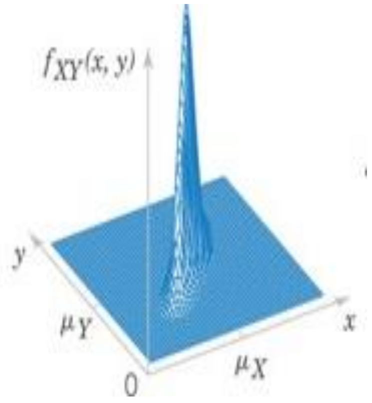
Visually, the bivariate normal distribution is a three-dimensional bell curve.



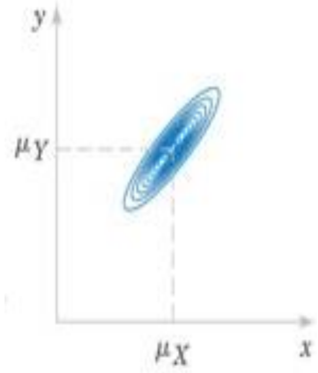
This is the Bivariate Normal distribution of 2 independent random variables X & Y $(\rho = 0)$



By cutting Bivariate Normal distribution horizontally we obtain the contour circles with center at the point of means (μ_x, μ_y)



This is the Bivariate Normal distribution of 2 dependent random variables X & Y $(\rho > 0)$

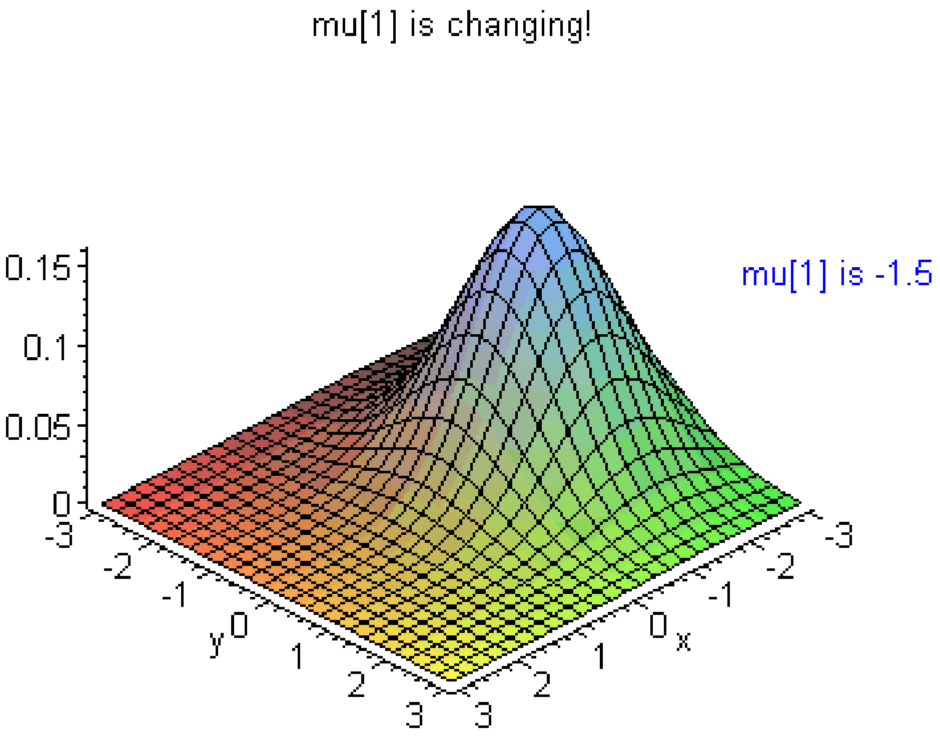


By cutting Bivariate Normal distribution horizontally we obtain the contour ellipses with center at the point of means (μ_x, μ_y)

Source : Internet

Bivariate Normal Distribution

Visually, the bivariate normal distribution is a three-dimensional bell curve.



This animated graph helps you to visualize the Bivariate normal distribution when the centre changes. Here one of the means is varied resulting in a change in the center.

Source : statisticshowto.com

Inference on the population Correlation

If the random variables X and Y have a certain joint distribution called a Bivariate normal distribution, then the sample correlation coefficient can be used

1. To construct a confidence interval on the population correlation coefficient ρ.
2. To test the null hypothesis on the population correlation ρ.

Inference on the Population Correlation

Let,

- X & Y : Random variables with the bivariate normal distribution
- $(x_1, y_1), \ldots, (x_n, y_n)$: Random sample from the joint distribution of X & Y.
- r : Sample correlation of the n points.
- ρ : Population correlation between X & Y.

The Fisher transformation

$$W = \frac{1}{2} ln\left(\frac{1+r}{1-r}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$W \sim N(\mu_w, {\sigma^2}_w)$$

where the mean,

$$\mu_w = \frac{1}{2} ln\left(\frac{1+\rho}{1-\rho}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

and variance,

$$ {\sigma^2}_w = \frac{1}{n-3} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Confidence Interval

Computing confidence interval for ρ

- Obtain the Confidence Interval for μ_W as, $W \pm z\sigma_W$
- Use upper and lower confidence bounds of μ_W to construct

  the confidence interval for ρ using $\rho = \frac{e^{2\mu_W}-1}{e^{2\mu_W}+1}$ which is

  obtained from $\mu_w = \frac{1}{2} ln\left(\frac{1+\rho}{1-\rho}\right)$

Example Problem 1.

- In a study of reaction times, the time to respond to a visual stimulus (x) and the time to respond to an auditory stimulus (y) were recorded for each of 10 subjects.
- Times were measured in ms.
- The results are presented in the following table.

x	161	203	235	176	201	188	228	211	191	178
y	159	206	241	163	197	193	209	189	169	201

- Find a 95% confidence interval for the correlation between the two reaction times.

Example Problem 1.

Solution : We need to obtain the following :

1. Compute the Sample correlation $r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \overline{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \overline{y})^2}}$
2. Obtain W using the Fischer's formula $W = \frac{1}{2} ln\left(\frac{1+r}{1-r}\right)$.

   Then find $W \sim N(\mu_w, \sigma^2{}_w)$ where $\mu_w = \frac{1}{2} ln\left(\frac{1+\rho}{1-\rho}\right)$ and $\sigma^2{}_w = \frac{1}{n-3}$
3. Compute the confidence interval for μ_w using W
4. Finally, convert the confidence interval back to 'ρ' using the relation $\rho = \frac{e^{2\mu_w - 1}}{e^{2\mu_w + 1}}$ which is obtained from $\mu_w = \frac{1}{2} ln\left(\frac{1+\rho}{1-\rho}\right)$

Solution :

x	y	$X = x - \bar{x}$	$Y = y - \bar{y}$	X^2	Y^2	XY
161	159	-36.2	-33.7	1310.44	1135.69	1219.94
203	206	5.8	13.3	33.64	176.89	77.14
235	241	37.8	48.3	1428.84	2332.89	1825.74
176	163	-21.2	-29.7	449.44	882.09	629.64
201	197	3.8	4.3	14.44	18.49	16.34
188	193	-9.2	0.3	84.64	0.09	-2.76
228	209	30.8	16.3	948.64	265.69	502.04
211	189	13.8	-3.7	190.44	13.69	-51.06
191	169	-6.2	-23.7	38.44	561.69	146.94
178	201	-19.2	8.3	368.64	68.89	-159.36
197.2	192.7	0.00	0.00	4867.6	5456.1	4204.6

1. $r = \frac{\sum XY}{\sqrt{\sum X^2}\sqrt{\sum Y^2}}$

$$= \frac{4204.6}{\sqrt{4867.6}\ \sqrt{5456.1}}$$

$$= 0\ .8159$$

2. $W = \frac{1}{2} ln\left(\frac{1+r}{1-r}\right) = \frac{1}{2} ln\left(\frac{1+0.8159}{1-0.8159}\right)$

$$= 1.1444$$

- W is normally distributed with standard deviation$\sigma_W = \sqrt{\frac{1}{n-3}} = \sqrt{\frac{1}{10-3}} = 0.3780$

Solution :

3. A 95% confidence interval for μ_w is given by $W - z\sigma_w < \mu_w < W + z\sigma_w$

That is, $1.1444 - 1.96(0.3780) < \mu_w \; 1.1444 + 1.96(0.3780)$

{We know that for 95% confidence interval for μ_w would be -1.96 & $1.96.$}

$$\Rightarrow \; 0.4036 < \mu_w < 1.8852$$

4. Now to obtain the 95% confidence interval for ρ, we consider

$$\rho = \frac{e^{2\mu_w - 1}}{e^{2\mu_w + 1}}$$

$$\frac{e^{2(0.4036) - 1}}{e^{2(0.4036) + 1}} < \frac{e^{2\mu_w - 1}}{e^{2\mu_w + 1}} < \frac{e^{2(1.8852)} - 1}{e^{2(1.8852)} + 1}$$

$$\Rightarrow \; 0.383 < \rho < 0.955$$

Hypothesis testing

- If $\rho = \rho_0, \rho \leq \rho_0$ and $\rho \geq \rho_0$ where $\rho_0 \neq 0$ is a constant, for performing the Hypothesis testing, we can use the same transformation W.

- If $\rho = 0, \rho \leq 0$ and $\rho \geq 0,$ for performing the Hypothesis testing, we use the test statistic $U = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$ which follows a student t distribution with $n - 2$ degrees of freedom.

Example Problem 2.

- In a study of reaction times, the time to respond to a visual stimulus (x) and the time to respond to an auditory stimulus (y) were recorded for each of 10 subjects.
- Times were measured in ms.
- The results are presented in the following table.

x	161	203	235	176	201	188	228	211	191	178
y	159	206	241	163	197	193	209	189	169	201

i. Find the P – value for testing $H_0\colon\ \rho \leq 0.3$ versus $H_1\colon\ \rho > 0.3$

ii. Test the hypothesis $H_0\colon\ \rho \leq 0$ versus $H_1\colon\ \rho > 0$

Solution :

i. Under H_0, we take $\rho = 0.3$

- We know that for W,

  the mean $\mu_W = \frac{1}{2} ln\left(\frac{1+\rho}{1-\rho}\right) = \frac{1}{2} ln\left(\frac{1+0.3}{1-0.3}\right) = 0.3095$

  and the standard deviation $\sigma = \sqrt{\frac{1}{10-3}} = 0.3780$
- Therefore, $W \sim N(\mu_W, \sigma^2{}_W) \Rightarrow W \sim N(0.3095, 0.3780^2)$
- From the Example Problem 1. the observed value of $W = 1.1444$
- Therefore, $z - score$ is, $z = \frac{1.1444-0.3095}{0.3780} = 2.21$
- From the Normal table, we have the P – value as 0.0136.
- Since the P – value is less than the significance level, we reject the null hypothesis.
- Hence we conclude that $\rho > 0.3$

Solution :

ii. Under H_0, we take $\rho = 0$

- So we need to take the test statistic U given by $U = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$
- From the Example Problem 1. the value of $r = 0.8159$
- Therefore, $U = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} = \frac{0.8159\sqrt{10-2}}{\sqrt{1-0.8159^2}} = 3.991$
- Since U follows the Student's t distribution with $n-2$ degrees of freedom, using the Student's t distribution table for 8 degrees of freedom we find that the P – value is between 0.001 and 0.8159.
- Since the P – value is less than the significance level α, we reject the null hypothesis.
- Hence we conclude that $\rho > 0$

THANK YOU

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