STATISTICS FOR DATA SCIENCE

POWER OF TEST AND SIMPLE LINEAR REGRESSION

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STATISTICS FOR DATA SCIENCE

Unit 5 : Power of test and Simple linear regression

Session : 1

Sub Topic : Power of test

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- Power computation

Power of a Hypothesis Test :

The power of a test is the probability of rejecting H_0 when it is false.

Recall- type I error and type II error

Hypothesis testing : $\boldsymbol{H_0}$ vs $\boldsymbol{H_1}$

Statistical Conclusion	**Actual State of Reality**	
Researcher Decision	H_0 is true	H_0 is false
Reject H_0	Type I error (α)	Correct Decision (1 – β)
Fail to reject H_0	Correct Decision (1 – α)	Type II error (β)

Power of a Hypothesis Test :

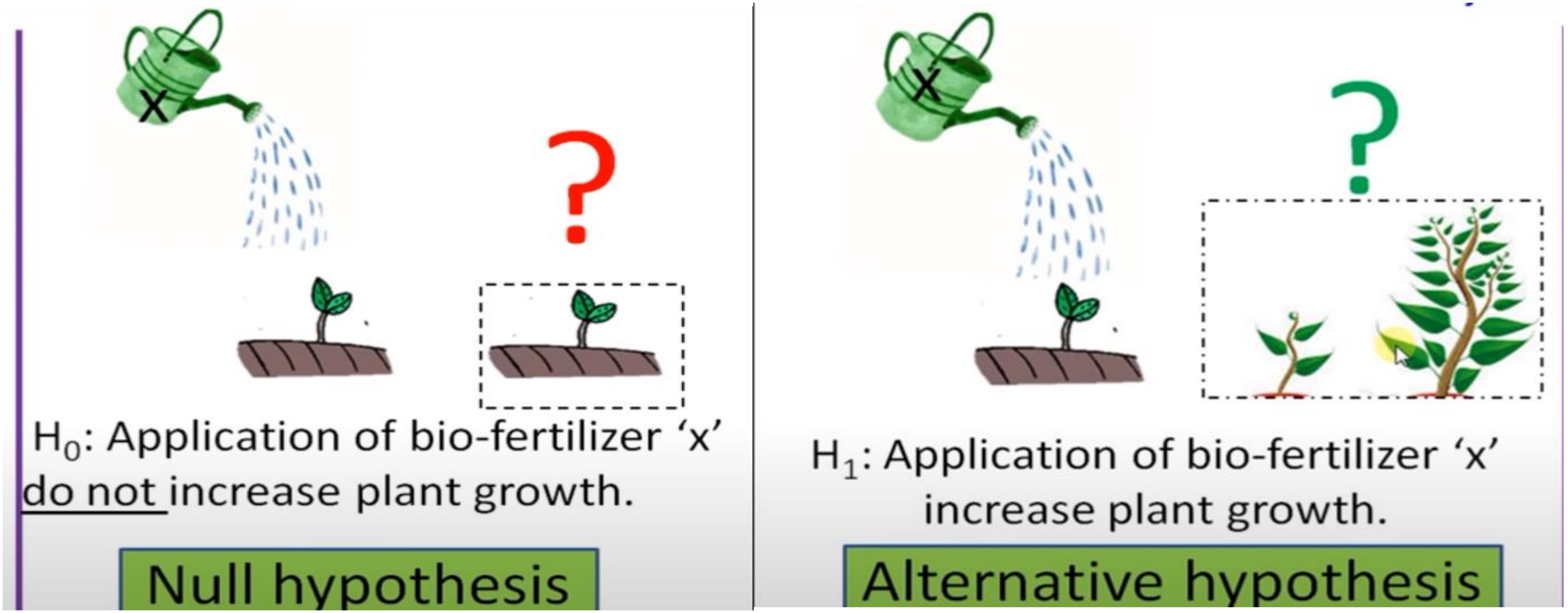
Power = 1 - P(type II error)

= $1 - \beta$.

80% power means you have 80% chance of getting a significant results when the effect is real.

Power of test

Effect of bio-fertilizer 'x' on plant growth



Power is 0.80(or 80%) → there is an 80% chance of rejecting the null hypothesis(false) when conducting the study.

Source: www.majordifferences.com

Why is Power Important?

Power calculations are important to ensure that the experiments have the potential to provide useful calculations.

As researchers, we put a lot of effort into designing and conducting our research. This effort may be wasted if we do not have sufficient power in our studies to find the effect of interest.

Power of test

How large the power must be for a test ?

As with P-values, there is no scientifically valid dividing line between sufficient and insufficient power.

In general, tests with power greater than 0.80 or perhaps 0.90 are considered acceptable, but there are no well-established rules of thumb.

Power of test

Analysis of power is performed:

1) Before gathering data

To determine the **minimal sample size** needed to have desired power in statistical testing (to detect a particular effect size).

2) After gathering data

To determine the **magnitude of power** that your statistical test will have given the sample parameters (**n** and **s**) and the magnitude of the effect that you want to detect.

Note: *Statistical power has relevance only when the null is false.*

Power of test

Power calculations are generally done before data are collected.

The purpose of a power calculation is to determine whether or not a hypothesis test, when performed, is likely to reject H_0 in the event that H_0 is false.

Power of test

Computing the power involves two steps:

1. Compute the rejection region.

Find the a) Null distribution

b) Critical Value

c) Rejection region

2. Compute the probability that the test statistic falls in the rejection region if the alternate hypothesis is true.

Find the a) alternate distribution

b) Z-score under H_1 for the critical point

c) P(reject H_0 | H_1 true)

This is the power.

Example of a power calculation

Assume that a new chemical process has been developed that may increase the yield over that of the current process. The current process is known to have a mean yield of 80 and a standard deviation of 5, where the units are the percentage of a theoretical maximum. If the mean yield of the new process is shown to be greater than 80, the new process will be put into production.

Let μ denote the mean yield of the new process. It is proposed to run the new process 50 times and then to test the hypothesis

$H_0: \mu \leq 80$ versus $H_1 : \mu > 80$ at a significance level of 5%.

Calculation of Power

Problem 1 :

Find the power of the 5% level test of

$\boldsymbol{H_0}$ **: μ ≤ 80 versus** $\boldsymbol{H_1}$ **: μ > 80**

for the mean yield of the new process under the

alternative μ = 81, assuming n = 50 and σ = 5.

Solution:

Null distribution of $\bar{X}$:

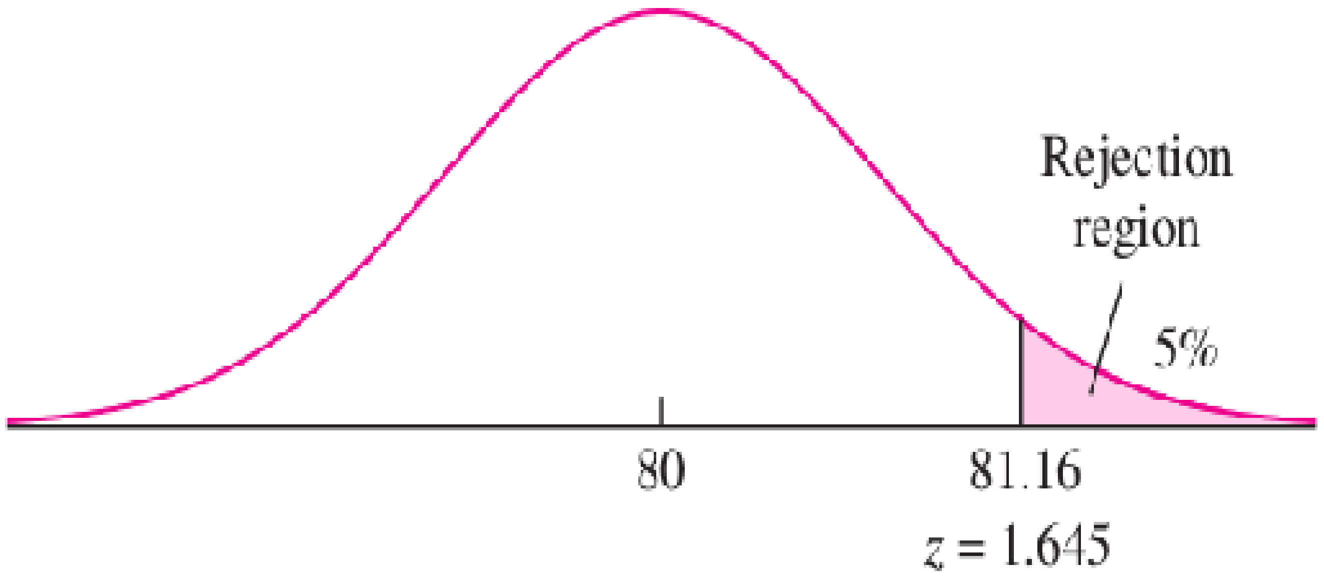
$$\bar{X} \sim N\left(\mu, \sigma_{\bar{X}}^2\right) where\ \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

Power of test

Null distribution of $\bar{X}$:

$$\bar{X} \sim N(80, 0.707^2)$$

The **critical point** has a z-score of 1.645, so its value is
$\bar{X}$ = 80 + (1.645)(0.707) = 81.16.
The rejection region consists of all values of $\bar{X} \geqslant$ 81.16

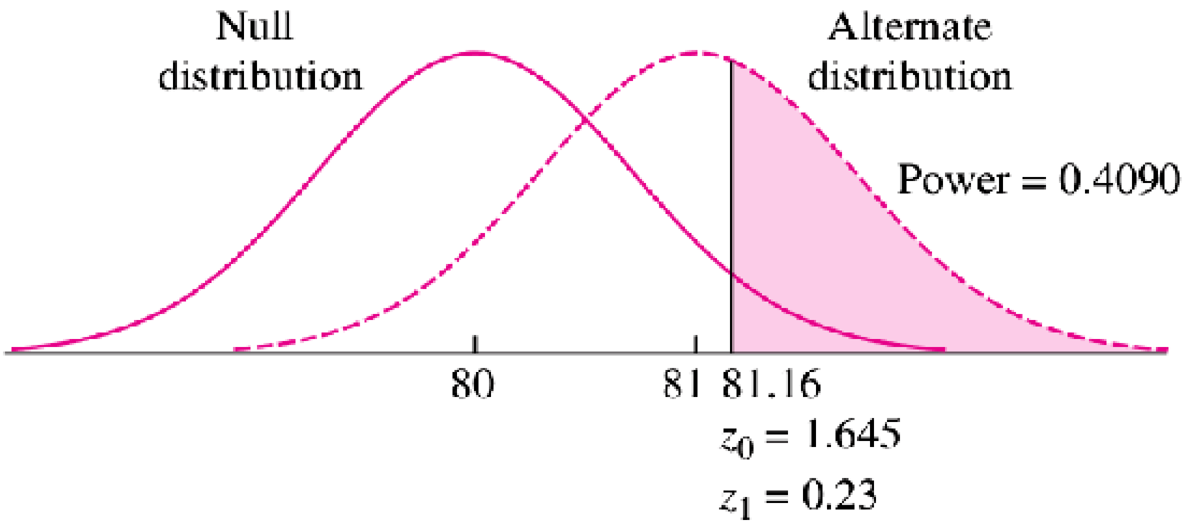


Alternate distribution of $\overline{X}$:

$$\bar{X} \sim N(81, 0.707^2)$$

(The alternate distribution is obtained by shifting the null distribution to chosen value of μ.)

Power is the probability that $\bar{X}$ will fall into the rejection region if the alternate hypothesis μ = 81 is true.



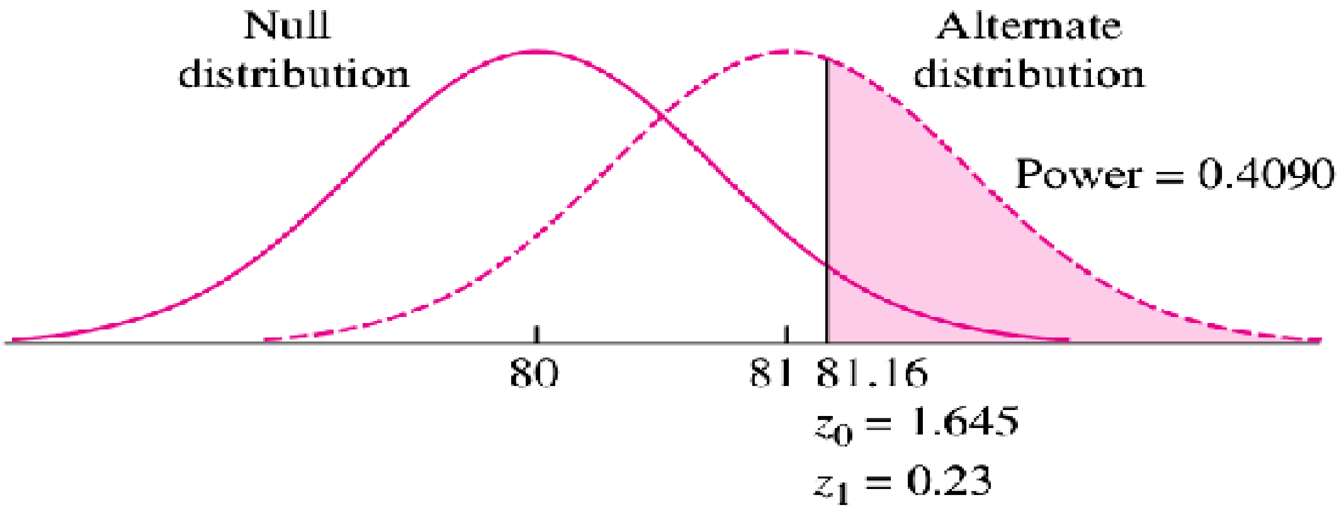
Computing the power

z -Score under H_1 for the critical point 81.16 is

$$z = \frac{\bar{X} - \mu}{\sigma} = \frac{81.16 - 81}{0.707} = 0.23$$

The area to the right of $z = 0.23$ is **0.4090.**

This is the **power** of the test.



Power of test

Conclusion:

A power of 0.4090 is very low.

- It means that if the mean yield of new process is actually equal to 81, there is only a 41% chance that the proposed experiment will detect the improvement over the old process and allow the new process to be put into production.
- It would be unwise to invest time and money to run this experiment, since it has a large chance to fail.

Calculation of Power

Problem 2 :

Find the power of the 5% level test of

H_0 **: μ ≤ 80 versus** H_1 **: μ > 80**

for the mean yield of the new process under the alternative μ = 82, assuming n = 50 and σ = 5.

Solution:

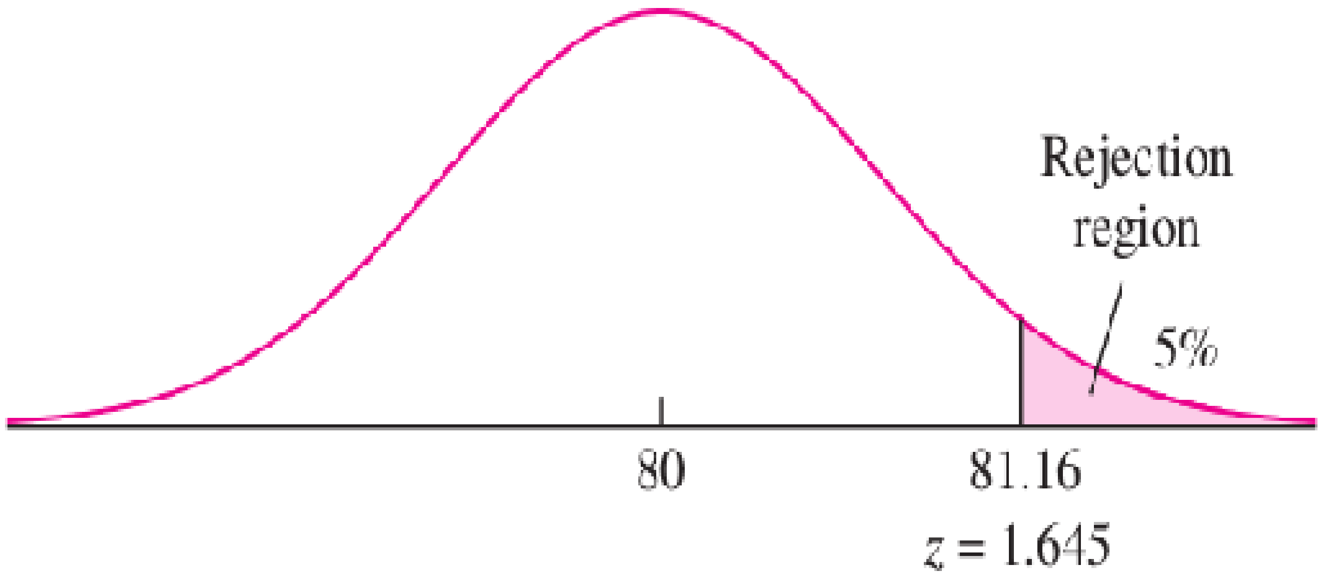
Null distribution of $\bar{X}$:

$$\bar{X} \sim N\left(\mu, \sigma_{\bar{X}}^2\right) \, where \; \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

Null distribution of $\bar{X}$:

$$\bar{X} \sim N(80, 0.707^2)$$

The **critical point** has a z-score of 1.645, so its value is
$\bar{X}$ = 80 + (1.645)(0.707) = 81.16.
The rejection region consists of all values of $\bar{X} \geqslant 81.16$



Power of test

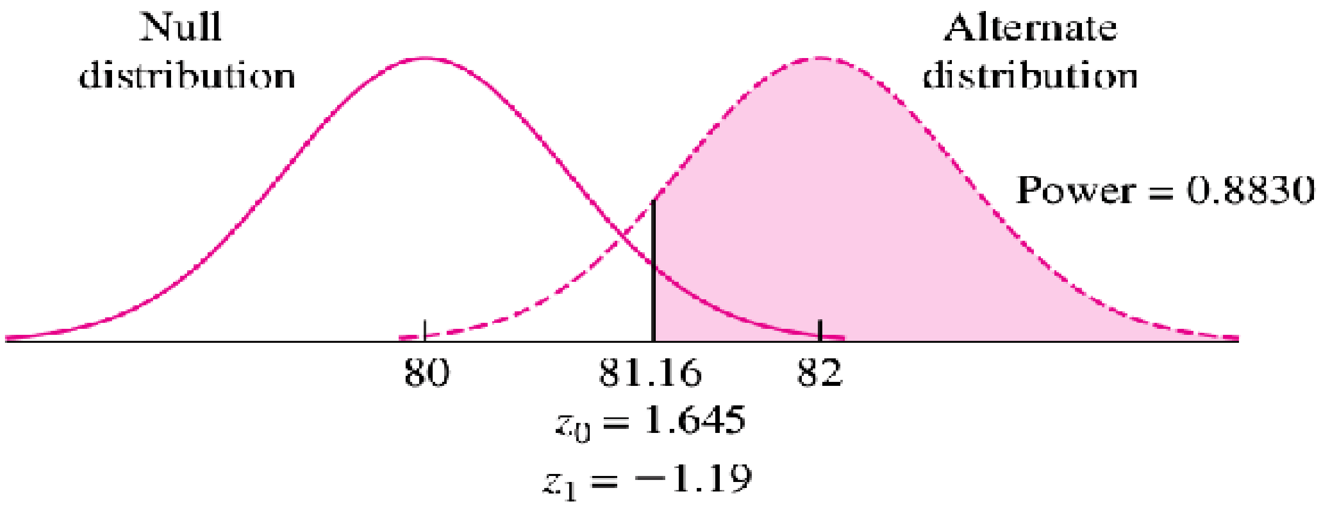
Alternate distribution of $\overline{X}$:

$$\bar{X} \sim N(82, 0.707^2)$$

(**The alternate distribution is obtained by shifting the null distribution to chosen value of μ.)**

Power is the probability that $\bar{X}$ will fall into the

rejection region if the alternate hypothesis μ = 82 is true.



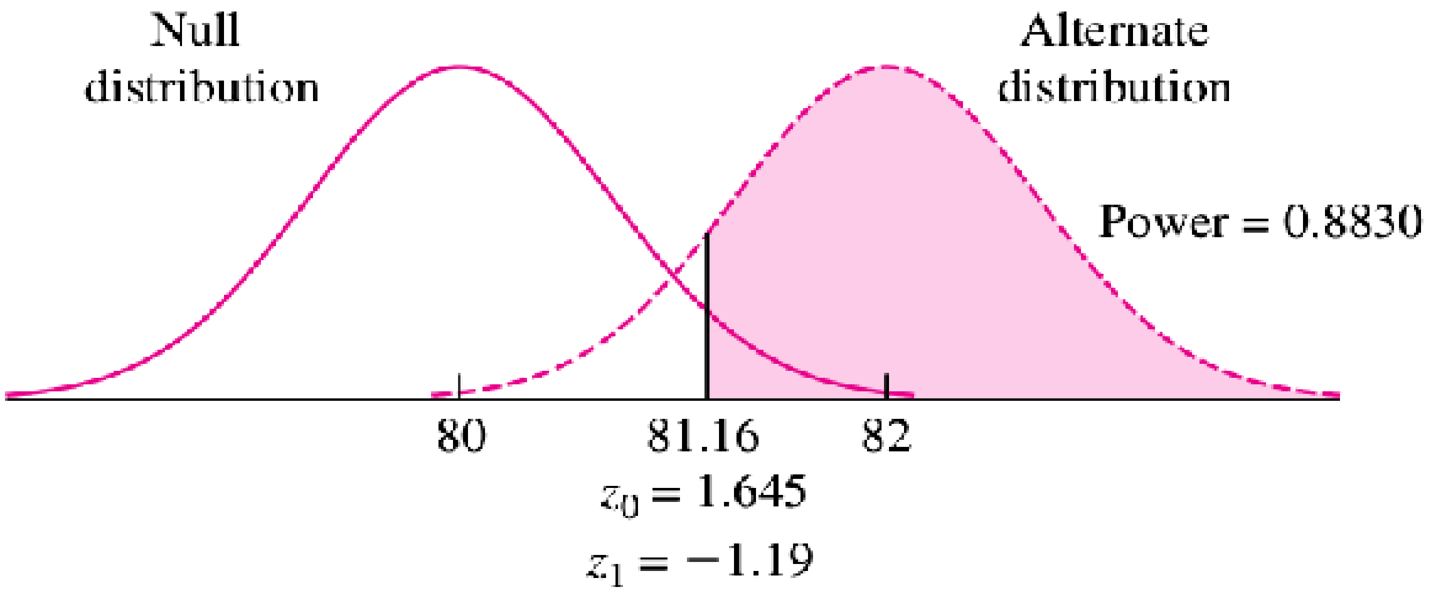
Computing the power

z -Score under $\boldsymbol{H_1}$ for the critical point 81.16 is

$$z = \frac{\bar{X} - \mu}{\sigma} = \frac{81.16 - 82}{0.707} = -1.19$$

The area to the right of $z = -1.19$ is **0.8830.**

This is the **power** of the test.



Conclusion

A power of **0.8830** is high.

- It means that if the mean yield of new process is actually equal to 82, there is a 88.30% chance that the proposed experiment will detect the improvement over the old process and allow the new process to be put into production.
- It would be a wise decision to invest time and money to run this experiment, since it has a large chance to succeed.

Computing Power

In order to compute the power, it is necessary to specify a particular value of μ, because:

power is different for different values of μ

- **if μ is close to H_0** : the power will be small. (If the alternate mean is chosen **very close to the null mean**, the alternate curve will be almost identical with the null, and the power will be very **close to α.)**
- **if μ is far from H_0** : the power will be large (If the alternate mean is **far from the null**, almost all the area under the alternate curve will lie in the rejection region, and the power will be **close to 1).**

Power of test

When power is not large enough, it can be increased by increasing the sample size.

When planning an experiment, one can determine the sample size necessary to achieve a desired power.

Knowing the significance level and the required power allows a researcher to determine a minimum sample size needed for the study.

Power of test

Problem 3 :

In testing the hypothesis H_0 **: μ ≤ 80 versus** H_1 **: μ > 80**

regarding the mean yield of the new process, how many times must the new process be run so that a test conducted at a significance level of 5% will have power 0.90 against the alternative μ = 81, if it is assumed that σ = 5?

Solution:

Let n represent the necessary sample size.

Power of test

Null distribution of $\bar{X}$:

$\bar{X} \sim N\left(\mu, \sigma_{\bar{X}}^2\right) where\ \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$

Critical point : $\mathbf{80 + 1.645\left(\frac{5}{\sqrt{n}}\right)}$

Consider the **alternate distribution of $\bar{X}$.**

Given Power is 0.90. The power of the test is the area of the rejection region under the alternate curve. This area must be 0.90.

Therefore, Z-score is -1.28.

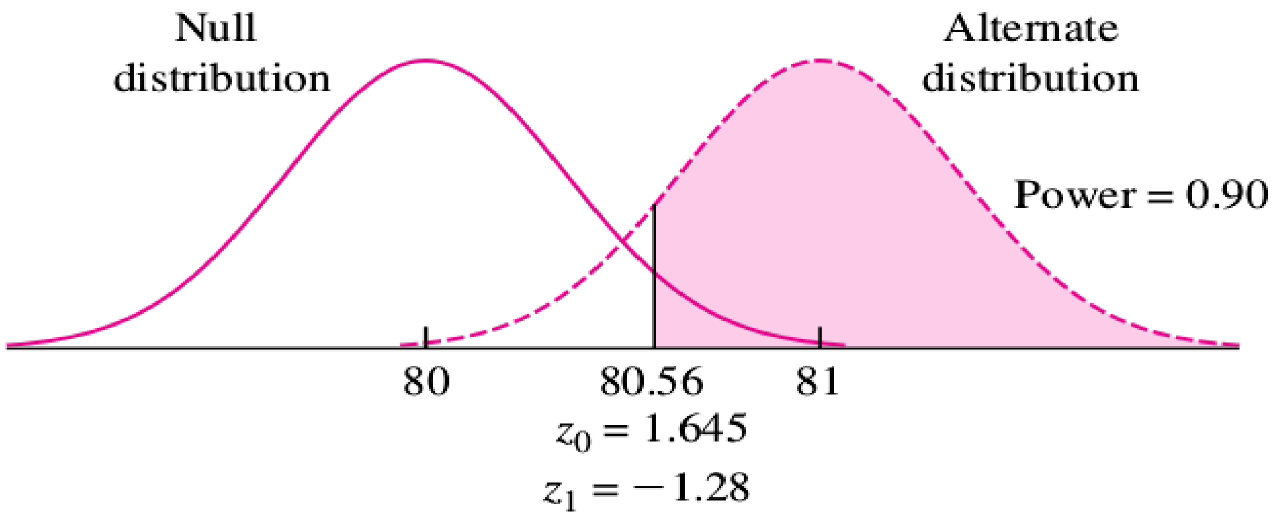
Critical point : $\mathbf{81 - 1.28\left(\frac{5}{\sqrt{n}}\right)}$

We now have two different expression for the critical point. Since there is only one critical point, these two expressions are equal.

Power of test

Set them equal and solve for n

$$\mathbf{80 + 1.645\left(\frac{5}{\sqrt{n}}\right) = 81 - 1.28\left(\frac{5}{\sqrt{n}}\right)}$$
$$\rightarrow \boldsymbol{n \approx 214}.$$



The critical point is 80.56 (The critical point can by computed by substituting this value for n into either side of the equation).

THANK YOU

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