PES UNIVERSITY, Bangalore

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Department of Computer Science & Engineering

Automata Formal Languages & Logic

Homework

1. Let Σ = {a,b} and let D = {w|w contains an equal number of occurrences of the substring 01 and 10} .Thus 101 $\in$ D because 101 contains a single 01 and a single 10, but 1010 $\notin$D because 1010 contains two 10s and only one 01. Show that D is a regular language.
2. Determine whether each of the following languages is regular.

- $\{a^n a^n a^n \mid n>0\}$
- $\{www \mid w \in \{x,y,z\}^*, |w|<10^{100}\}$
- $\{vw \mid v,w \in \{a,b\}^*\}$
- $\{ww \mid w \in \{a\}^*\}$

3. Let $L = \{0^n 1^n \mid n >= 0\}$. Is the complement of L a regular language?
4. Consider the following statement: "If A is a non regular language and B is a language such that B$\subseteq$A, then B must be non regular." If the statement is true, give a proof.If it is not true, give a counterexample showing that the statement doesn't always hold.
5. Prove that if we add a finite set of strings to a regular language, the result is regular language.
6. Prove that if we remove a finite set of strings from a regular language, the result is a regular language.
7. Consider the following statement: "If A is a non regular language andBis a language such thatB$\subseteq$A, then B must be non regular." If the statement is true, give a proof.If it is not true, give a counterexample showing that the statement doesn't always hold.
8. Show how to perform tha product construction on NFA's (without lambda transition)
9. Show how to perform tha product construction on lambda NFA's .
10. Show how to modify the product construction so the resulting DFA accepts the difference of the languages of the two given DFA's
11. Show how to modify the product construction so the resulting DFA accepts the union of the languages of the two given DFA's.
12. Give an algorithm to tell whether a regular language L contains at least 1000 strings.
13. Give an algorithm to tell whether two regular languages L_1 and L_2 have at least one string in common.
14. Give an algorithm to tell, for two regular languages L_1 and L_2 over the same alphabet Σ , whether there is any string Σ*that is neither L_1 nor $L_{2.}$
15. The language of regular expression (0+10)* is the set of all strings of 0's and 1's such that every 1 is immediately followed by a 0. Describe the complement of this language over the same alphabet.
16. For each claim below, state whether it is true or false, and prove your answer.

    (a) A regular expression denotes an infinite language if and only if it contains the * operation.

(b) For all languages A and B, $(A^* \cap B^*)^* = (A \cap B)^*$.

(c) For all languages A and B, $(A^* \cup B^*)^* = (A \cup B)^*$.

17. For each statement, indicate whether it is true or false. Prove that your answer is correct.

    (a) If languages A and B are non-regular, than their intersection $A \cap B$ is non-regular.

    (b) If each of the languages $L0$, $L1$, $L2$, … is regular, then their union $\cup i \in \mathbb{N}\, Li$ is Regular

18. Define the following two languages:

    La = {w {a, b}* : in each prefix x of w, #a(x) #b(x)}.

    Lb = {w {a, b}* : in each prefix x of w, #b(x) #a(x)}.

    a) Let L1 = La ∩ Lb. Is L1 regular? Prove your answer.

    b) L2 = La ∪ Lb. Is L2 regular? Prove your answer.

19. If L is regular language,prove that L_1=(uv:u ∈ L,|v| = 3} is also regular.
20. Show that the question, Given a FA M over Σ, does M accept a string of length < 2? is decidable
21. Show that the question: Does L = Σ*? for regular language L is decidable.
22. Suppose h is the homomorphism from {0,1,2} to {a,b} defined by h(0)=a,h(1)=ab and h(2) = ba
    a. What is h(21120)?
    b. If L = 01*2, what is h(L)?
23. This problem is to illustrate proofs of (the many) closure properties of regular languages.

    (a) For a language L let FUNKY(L) = {w | w ∈ L but no proper prefix of w is in L}.

    Prove that if L is regular then FUNKY(L) is also regular using the following technique. Let M = (Q, Σ, δ,s,A) be a DFA accepting L. Describe a NFA N in terms of M that accepts FUNKY(L). Explain the construction of your NFA.

    (b) In Lab 3 we saw that insert1(L) is regular whenever L is regular. Here we consider a different proof technique. Let r be a regular expression. We would like to show that there is another regular expression r 0 such that L(r 0) = insert1(L(r)).

    i. For each of the base cases of regular expressions ;, ε and {a}, a ∈ Σ describe a regular expression for insert1(L(r)).

Ii. Suppose r1 and r2 are regular expressions, and r 0 1 and r 0 2 are regular expressions for the languages insert1(L(r1)) and insert1(L(r2)) respectively. Describe a regular expression for the language insert1(L(r1 + r2)) using r1 ,r2 ,r 0 1 ,r 0 2 .

iii. Same as the previous part but now consider L(r1 r2).

iv. Same as the previous part but now consider L((r1) ∗).

24. We know that if M is a DFA that recognizes language A, swapping the accepting and non accepting states yields a new DFA recognizing the complement of A. We then concluded that the class of regular languages is closed under complement.

a. Show, by giving an example, that if N is an NFA that recognizes language B, swapping the accepting and non accepting states in N does not necessarily yield a new NFA that recognizes the complement of B.

b. Is the class of languages recognized by an NFA closed under complement? Explain your answer

25. Let L1 and L2 be regular languages. Show that the following languages are also regular.

a) The difference L1 -L2 = {w ∈ L1 : w ∉ L2}.

b) The symmetric difference L1 ⊕ L2 = (L1 -L2) ∪ (L2 - L1).

c) The reversal $L^R 1$ = w^R : w ∈ L1 .

Hint: You can use the closure properties presented in class for union, intersection, ?, concatenation, or the fact that every regular language has a DFA/NFA. Try to find the shortest answer

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