Design and Analysis of Algorithms

Branch and Bound

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Unit 5: Limitations of Algorithmic Power and Coping with the Limitations

- Dynamic Programming
  - Computing a Binomial Coefficient
  - The Knapsack Problem
  - Memory Functions
  - Warshall's and Floyd's Algorithms
  - Optimal Binary Search Trees
- Limitations of Algorithmic Power
  - Lower-Bound Arguments
  - Decision Trees
  - P, NP, and NP-Complete, NP-Hard Problems
- Coping with the Limitations
  - Backtracking
  - **Branch and Bound**

Concepts covered

- Backtracking
  - General Approach
  - Knapsack Problem
  - Assignment Problem
  - Travelling Salesman Problem

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Introduction

- An enhancement of backtracking
- Applicable to optimization problems
- For each node (partial solution) of a state-space tree, computes a bound on the value of the objective function for all descendants of the node (extensions of the partial solution)
- Uses the bound for:
  - ruling out certain nodes as “nonpromising” to prune the tree (if a node’s bound is not better than the best solution seen so far)
  - guiding the search through state-space

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Example: Assignment Problem

- Select one element in each row of the cost matrix C so that:
  - no two selected elements are in the same column
  - the sum is minimized
- Example

	Job 1	Job 2	Job 3	Job 4
Person *a*	9	2	7	8
Person *b*	6	4	3	7
Person *c*	5	8	1	8
Person *d*	7	6	9	4

- Lower bound (sum of smallest elements in each row): $2 + 3 + 1 + 4 = 10$
- Best-first branch-and-bound variation: Generate all the children of the most promising node

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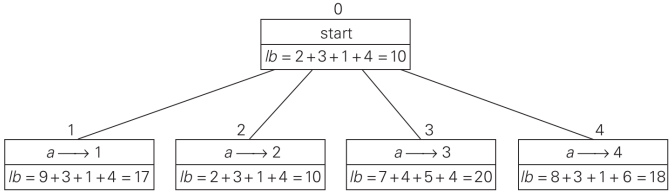
Example: First two levels of the state-space tree



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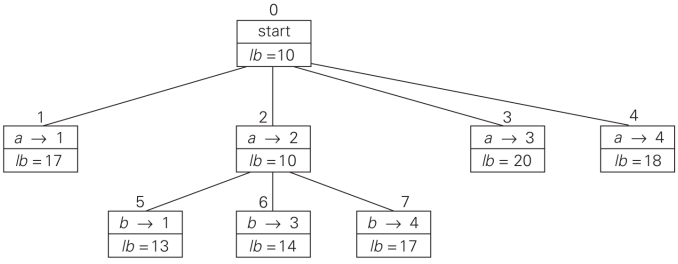
Example: First three levels of the state-space tree



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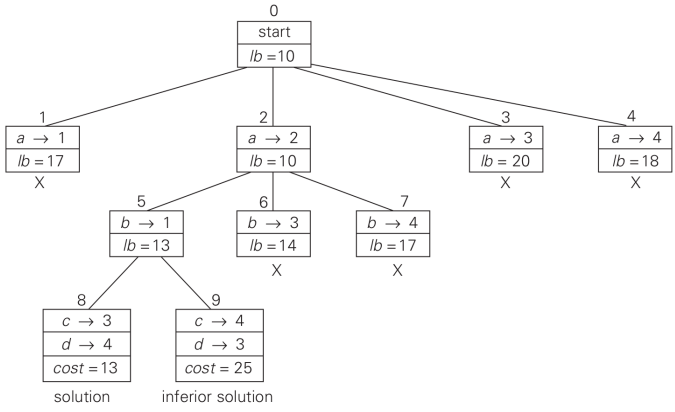
Example: Complete state-space tree



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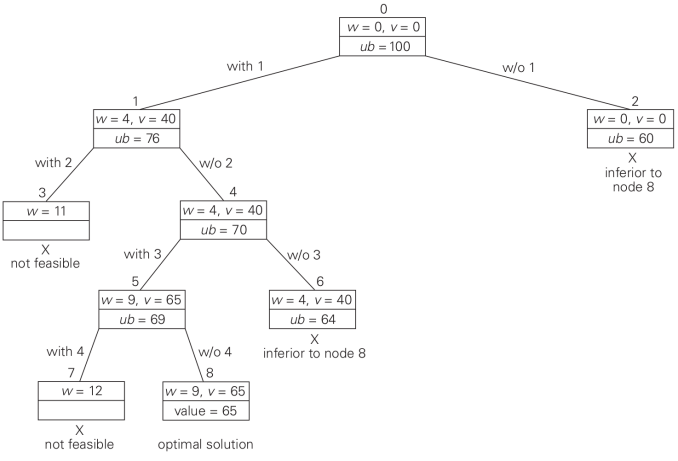
Example: Knapsack Problem



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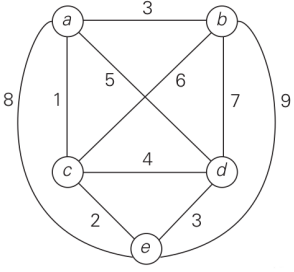
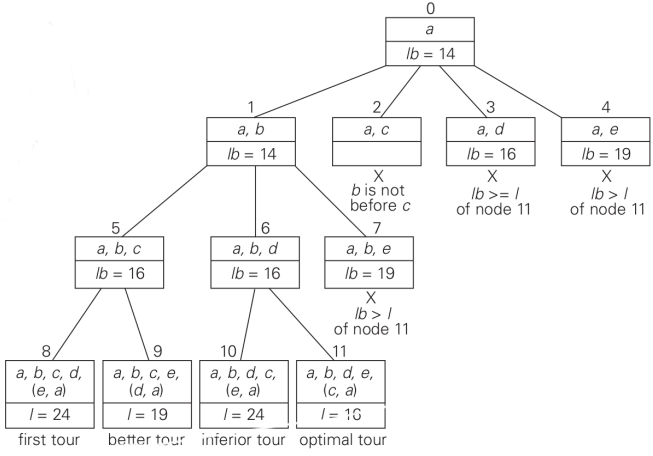
Example: Traveling Salesman Problem





Think About It

- What data structure would you use to keep track of live nodes in a best-first branch-and-bound algorithm?
- Solve the assignment problem by the best-first branch-and-bound algorithm with the bounding function based on matrix columns rather than rows