SRN

PES University, Bangalore

(Established under Karnataka Act No. 16 of 2013)

UE15MA251

MAY 2017: END SEMESTER ASSESSMENT (ESA) B.TECH. IV SEMESTER

UE15MA251- LINEAR ALGEBRA AND ITS APPLICATIONS

Time: 3 Hrs — Max Marks: 100

Answer All Questions

1.	a)	Use the method of Gaussian Elimination to determine if the system of equations $2x-3y+4z=7$, $x-y+2z=10$, $8x-11y+16z=41$ is consistent or not. Solve the system completely if it is consistent. What is the solution if the coefficient of z in the second equation is replaced by 3 ?	7
	b)	Establish conditions in terms of a and b satisfying which $A=\begin{bmatrix} a & b & b \\ a & a & b \\ a & a & a \end{bmatrix}$ is invertible. Assuming these conditions, obtain A^{-1} by Gauss- Jordan method. Choosing appropriate values for a and b in the given A, solve the system $2u+v+w=4$, $2u+2v+w=3$, $2u+2v+2w=4$.	7
	c)	Let $l_{21}=3$, $l_{31}=2$, $l_{41}=-1$ be the first set of multipliers obtained while reducing a 4×4 matrix A into U , using Gaussian Elimination. If the permutation matrix P_{23} is used to produce the next set of multipliers $l_{32}=0$, $l_{42}=1$ and if the permutation matrix P_{34} puts A finally into its echelon form $U=\begin{bmatrix} 3 & 0 & 2 & 2 \\ 0 & 5 & -8 & 3 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & -4 \end{bmatrix}$, find the matrix A.	6
2.	a)	Identify the independent and dependent vectors from the set of vectors (-3 , 2 , 4) , (3 , -2 , -4) , (-3 , 1 , 4). Express the dependent vector in terms of the independent ones.	6
	b)	Find a basis and the dimension of the subspace $S=\{(x,y,z)/x-2y+3z=0, 2x-5z=0\}$. Also, check if (0, 3, 2) belongs to S.	6
	c)	Find a basis and the dimension of C (A) , N (A) and N (A^T) of $A=\begin{bmatrix} 0 & -1 & 1 & 2 & -4 \\ 2 & -2 & 3 & 0 & 7 \\ 6 & -10 & 13 & 8 & 5 \\ 2 & -8 & 9 & 12 & -17 \end{bmatrix}$	8
3.	a)	If S is the subspace spanned by (0 , 1 , -1) , (1 , 0, -2) find the matrices that project every vector in R^3 onto S and $S^{\perp}$. Also, express the vector (2 , -2 , 4) as a sum of a vector in S and a vector in $S^{\perp}$.	7
	b)	Find the best straight line fit to the measurements b = 2 , 0, -3 , -5 at t = -1 , 0, 1 , 2 using the principle of least squares.	7
	c)	Find the matrix A that rotates every vector in R^2 about origin in the counter clockwise direction through 60° and then reflects it on the line x + y = 0. Also find the matrix B that reflects every vector in R^2 on the line x + y = 0 and then rotates it about the origin through 60° in the counter clockwise direction. Are A and B equal?	6

Please Turn Over

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4.	a)	Apply the Gram – Schmidt process on the vectors a = (0, 0, 1), b = (0, 1, 1) and c = (1, 1, 1) to produce an orthonormal set of vectors and write the result in the form A = QR.	7
	b)	Find the eigen values and the eigen vectors of the matrix $\begin{bmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$.	7
	c)	Diagonalize A = $\begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$ and find one of its square roots – a matrix R such that $R^2 = A$.	6
5.	a)	A company manufactures printers and keyboards. The contribution margins of the printer and keyboard are \$30 and \$20 respectively. Two types of skilled labour are required to manufacture these products: soldering and assembling. A printer requires 2 hours of soldering and one hour of assembling. A keyboard requires 1 hour of soldering and 1 hour of assembling. The company has 1,000 soldering hours and 800 assembling hours available per week. There are no constraints on the supply of raw materials. Demand for keyboards is unlimited, but at most 350 printers are sold each week. The company wishes to maximize its weekly total contribution margin. Formulate a LPP and provide a graphical solution. What is the maximum weekly total contribution margin ? **(Sketch neatly on the graph sheet)**	10
	b)	Solve using the Simplex method the LPP : Maximize $Z = 3x_1 + 2x_2 + 5x_3$ subject to $x_1 + 2x_2 + x_3 \leq 430$, $3x_1 + 2x_3 \leq 460$, $x_1 + 4x_2 \leq 420$ where $x_1, x_2, x_3 \geq 0$. Write down the optimal solution and the maximum value of Z.	10