PES University, Bangalore

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UE19CS203 – STATISTICS FOR DATA SCIENCE

Unit-2 - Random Variables

QB SOLVED

Linear Functions of Random Variables

Exercises for Section 2.5

1. **The oxygen equivalence number of a weld is a number that can be used to predict properties such as hardness, strength, and ductility. The article "Advances in Oxygen Equivalence Equations for Predicting the Properties of Titanium Welds" (D. Harwig, W. Ittiwattana, and H. Castner, The Welding Journal, 2001:126s–136s) presents several equations for computing the oxygen equivalence number of a weld. One equation, designed to predict the hardness of a weld, is X = O + 2N + (2/3) C, where X is the oxygen equivalence, and O, N, and C are the amounts of oxygen, nitrogen, and carbon, respectively, in weight percent, in the weld. Suppose that for welds of a certain type, $\mu_O = 0.1668$, $\mu_N = 0.0255$, $\mu_C = 0.0247$, $\sigma_O = 0.0340$, $\sigma_N = 0.0194$, and $\sigma_C = 0.0131$.**

    a) **Find μ_X.**
    b) **Suppose the weight percents of O, N, and C are independent. Find σ_X.**

[Text Book Exercise – Section 2.5 – Q. No.14 – Pg. No. 126]

Solution

a) Find μ_X.

We will use the formula, $\mu_{C_1X_1+C_2X_2+\dots\dots+C_nX_n} = C_1\mu_{X_1} + C_2\mu_{X_2} + \dots\dots + C_n\mu_{X_n}$

Therefore the mean of the oxygen equivalence number of a weld is given by,

$$\mu_X = \mu_{O+2N+2C/3} = \mu_O + 2\mu_N + \left(\frac{2}{3}\right)\mu_C$$

$$= 0.1668 + (2)(0.0255) + \left(\frac{2}{3}\right)(0.0247)$$

$= 0.1668 + (2)(0.0255) + \left(\frac{2}{3}\right)(0.0247)$

$= 0.2342$

b) Suppose the weight percents of O, N, and C are independent. Find σ_X.

We will use the formula, $\sigma_{C_1X_1+C_2X_2+\ldots\ldots+C_nX_n} = C_1\sigma_{X_1} + C_2\sigma_{X_2} + \ldots\ldots + C_n\sigma_{X_n}$

Therefore the standard deviation of the oxygen equivalence number of a weld is given by,

$$\sigma_X = \sigma_{O+2N+2C/3} = \sqrt{\sigma_O^2 + (2)^2\sigma_N^2 + \left(\frac{2}{3}\right)\sigma_C^2}$$

$$\sqrt{(0.0340)^2 + 4(0.0194)^2 + \left(\frac{2}{3}\right)^2 (0.0131)^2 = 0.05232}$$

2. The thickness X of a wooden shim (in mm) has probability density function

$$f(x) = \begin{cases} \frac{3}{4} - \frac{3(x-5)^2}{4} & 4 < x < 6 \\ 0 & otherwise \end{cases}$$

- **a) Find μ_X**
- **b) Find σ_X^2**
- **c) Let *Y* denote the thickness of a shim in inches (1 mm = 0.0394 inches). Find μ_Y and σ_Y^2**
- d) **If three shims are selected independently and stacked one atop another, find the mean and variance of the total thickness.**

[Text Book Exercise – Section 2.5 – Q. No.16 – Pg. No. 126]

<u>Solution</u>

a) Find μ_X

The formula to compute mean is,

$$\mu_X = \int_{-\infty}^{\infty} x\, f(x)dx$$

$$= \int_{-\infty}^{4} 0 \ dx + \int_{4}^{6} \left(\frac{3}{4} - \frac{3(x-5)^2}{4}\right) x \ dx + \int_{6}^{\infty} 0 \ dx$$

$$= 0 + \int_{4}^{6} \left(\frac{3}{4} - \frac{3(x-5)^2}{4}\right) x \ dx + 0$$

$$= \int_{4}^{6} \left(-\frac{3}{4}x^3 + \frac{15}{2}x^2 - 18x\right) dx$$

$$= \left(-\frac{3}{16}x^4 + \frac{5}{2}x^3 - 9x^2\right) \Big|_{4}^{6}$$

$$= 5$$

b) Find σ_X^2

The formula to compute variance is,

$$\sigma_X^2 = \int_{-\infty}^{\infty} x^2 \ f(x) dx - \mu_X^2$$

$$= \int_{-\infty}^{4} 0x^2 \ dx + \int_{4}^{6} x^2 \left(\frac{3}{4} - \frac{3(x-5)^2}{4}\right) dx + \int_{6}^{\infty} 0x^2 \ dx - (5)^2$$

$$= \int_{4}^{6} \left(-\frac{3}{4}x^4 + \frac{15}{2}x^3 - 18x^2\right) dx - (5)^2$$

$$= \left(-\frac{3}{20}x^5 + \frac{15}{8}x^4 - 6x^3\right) \Big|_{4}^{6} - 25$$

$$= \frac{126}{5} - 25 = \frac{1}{5}$$

$$= 0.2$$

c) Let *Y* denote the thickness of a shim in inches (1 mm = 0.0394 inches). Find μ_Y and σ_Y^2

The thickness of a shim in inches is Y = 0.0394 inches.

$$\mu_Y = 0.0394 \ \mu_X$$

$$= 0.0394 * 5$$

$$= 0.197$$

$$\sigma_Y^2 = (0.0394)\sigma_X^2$$

$$= 0.0394 * 0.2$$

$$= 0.00031$$

d) If three shims are selected independently and stacked one atop another, find the mean and variance of the total thickness.

Let X_1, X_2, X_3 be three thickness in millimeters.
Then $S = X_1 + X_2 + X_3$ is the total thickness.

Where, $\mu = 5$, $\sigma^2 = 0.2$

Then mean and variance of total thickness includes,

$$\mu_{X_1+X_2+X_3} = \mu_{X_1} + \mu_{X_2} + \mu_{X_3}$$

$$= 3\mu = (3)(5)$$

$$= 15$$

$$\sigma^2_{X_1+X_2+X_3} = \sigma^2_{X_1} + \sigma^2_{X_2} + \sigma^2_{X_3}$$

$$= 3\sigma^2 = (3)(0.2)$$

$$= 0.6$$