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Automata Formal Languages & Logic

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Unit 3

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A context free grammar is in chomsky normal form (CNF) if all the productions rules satisfy one of the following condition

1) A non - terminal generating terminal
   Ex: X->x
2) A non - terminal generating two non terminals
   Ex: X-> XY
3) Start symbol generating λ
   Ex: S->λ

To convert the CFG to CNF the CFG should be clean, that is it must not have any

1) Lambda production
2) Unit Productions
3) Useless variables:

Steps to Convert CFG to CNF

Step 1: Eliminate lambda productions

Step2: Eliminate Unit production

Step 3: Eliminating Useless productions and Symbols

Example 1:

S -> aX | Yb
X ->S | λ
Y -> bY | b

Solution :

Step 1:

Eliminate λ productions

S -> aX | a | Yb
X-> S
Y-> bY | b

Example 1:

S -> aX | Yb
X ->S | λ
Y -> bY | b

Solution :

Step 2: Eliminate Unit Productions

S -> aX | a | Yb
X -> aX | a | Yb
Y -> bY | b

Example 1:

S -> aX | Yb

X ->S | λ

Y -> bY | b

Solution :

Step 3: There are no useless productions

Step 4: Conversion to CNF:

S-> AX | YB | a

X -> AX | YB | a

Y -> BY | b

A -> a

B -> b

Example 2:

$S \rightarrow aSa \mid bSb \mid A \mid \lambda$

$A \rightarrow a \mid b \mid \lambda$

Solution :

Step 1: Remove λ production

$S \rightarrow aSa \mid aa \mid bSb \mid bb \mid A$

$A \rightarrow a \mid b$

Example 2:

$S \rightarrow aSa \mid bSb \mid A \mid \lambda$

$A \rightarrow a \mid b \mid \lambda$

Solution :

Step 2: Remove unit production ($S \rightarrow A$)

$S \rightarrow aSa \mid aa \mid bSb \mid bb \mid a \mid b$

$A \rightarrow a \mid b$

Example 2:

$S \rightarrow aSa \mid bSb \mid A \mid \lambda$

$A \rightarrow a \mid b \mid \lambda$

Solution :

Step 3: Remove useless production(A)

$S \rightarrow aSa \mid aa \mid bSb \mid bb \mid a \mid b$

Now the CFG is

$A \rightarrow a$

$B \rightarrow b$

$S \rightarrow ASA \mid BSB \mid AA \mid BB \mid a \mid b$

Example 2:

$S \rightarrow aSa \mid bSb \mid A \mid \lambda$

$A \rightarrow a \mid b \mid \lambda$

Solution :

Step 4: To CNF

$A \rightarrow a$

$B \rightarrow b$

$C \rightarrow AS$

$D \rightarrow BS$

$S \rightarrow CA \mid DB \mid AA \mid Bb \mid a \mid b$

Example 3:

$S \rightarrow BAB$

$B \rightarrow bba$

$A \rightarrow Bc$

Solution :

Step 1: There are no λ production

Step 2 There are no unit productions

Step 3: There are no useless production

Example 3:

$S \rightarrow BAB$

$B \rightarrow bba$

$A \rightarrow Bc$

Solution :

Step 4: To CNF

$C \rightarrow a$

$D \rightarrow b$

$E \rightarrow c$

$F \rightarrow BA$

$G \rightarrow DD$

$S \rightarrow FB$

$B \rightarrow GC$

$A \rightarrow BE$

Example 4:

$S \rightarrow Aa \mid B \mid Ca$

$B \rightarrow aB \mid b$

$C \rightarrow Db \mid D$

$D \rightarrow E \mid d$

$E \rightarrow ab$

Solution :

Step 1: There are no λ production

Step 2: Remove unit production

$S \rightarrow Aa \mid aB \mid b \mid Ca$

$B \rightarrow aB \mid b$

$C \rightarrow Db \mid ab \mid d$

$D \rightarrow ab \mid d$

$E \rightarrow ab$

Example 4:

S →Aa | B | Ca

B → aB | b

C → Db | D

D → E | d

E → ab

Solution :

Step 3: Remove useless production

E is useless production and Aa is useless as there is no variable A

Step 4: To CNF

X → a

Y → b

S → XB | b | CX

B → XB | b

C → DY | XY | d

D → XY | d

Example 5:

$S \rightarrow aAa \mid bBb \mid BB$

$A \rightarrow C$

$B \rightarrow S \mid A$

$C \rightarrow S \mid \lambda$

Solution :

Step 1: Remove λ production

$S \rightarrow aAa \mid aa \mid bBb \mid bb \mid BB$

$A \rightarrow C$

$B \rightarrow S \mid A$

$C \rightarrow S$

Example 5:

$S \rightarrow aAa \mid bBb \mid BB$

$A \rightarrow C$

$B \rightarrow S \mid A$

$C \rightarrow S \mid \lambda$

Solution :

step 2: Remove unit productions

$S \rightarrow aAa \mid aa \mid bBb \mid bb \mid BB$

$A \rightarrow aAa \mid aa \mid bBb \mid bb \mid BB$

$B \rightarrow aAa \mid aa \mid bBb \mid bb \mid BB$

$C \rightarrow aAa \mid aa \mid bBb \mid bb \mid BB$

Example 5:

$S \rightarrow aAa \mid bBb \mid BB$

$A \rightarrow C$

$B \rightarrow S \mid A$

$C \rightarrow S \mid \lambda$

Solution :

Step 3: Remove useless production

C is useless production

$S \rightarrow aAa \mid aa \mid bBb \mid bb \mid BB$

$A \rightarrow aAa \mid aa \mid bBb \mid bb \mid BB$

$B \rightarrow aAa \mid aa \mid bBb \mid bb \mid BB$

Example 5:

$S \rightarrow aAa \mid bBb \mid BB$

$A \rightarrow C$

$B \rightarrow S \mid A$

$C \rightarrow S \mid \lambda$

Solution :

Step 4: To CNF

$X \rightarrow a$

$Y \rightarrow b$

$P \rightarrow XA$

$Q \rightarrow YB$

$S \rightarrow PX \mid XX \mid QY \mid YY \mid BB$

$A \rightarrow PX \mid XX \mid QY \mid YY \mid BB$

$B \rightarrow PX \mid XX \mid QY \mid YY \mid BB$

Example 6:

$E \rightarrow E + T \mid T$

$T \rightarrow T * F \mid F$

$F \rightarrow num \mid id$

Solution :

Step 1: There are no λ production

Example 6:

$E \rightarrow E + T \mid T$

$T \rightarrow T * F \mid F$

$F \rightarrow num \mid id$

Solution :

Step 2: Remove unit production ($E \rightarrow T$) and($T \rightarrow F$)

$E \rightarrow E + T \mid T * F \mid F$

$T \rightarrow T * F \mid num \mid id$

$F \rightarrow num \mid id$

This results in $E \rightarrow F$, remove this

$E \rightarrow E + T \mid T * F \mid num \mid id$

$T \rightarrow T * F \mid num \mid id$

$F \rightarrow num \mid id$

Example 6:

$E \rightarrow E + T \mid T$

$T \rightarrow T * F \mid F$

$F \rightarrow num \mid id$

Solution :

Step 3: There are no use less productions

Now the CFG is:

$A \rightarrow +$

$B \rightarrow *$

$E \rightarrow EAT \mid TBF \mid num \mid id$

$T \rightarrow TBT \mid num \mid id$

$F \rightarrow num \mid id$

Example 6:

$E \rightarrow E + T \mid T$

$T \rightarrow T * F \mid F$

$F \rightarrow num \mid id$

Solution :

Step 4 : To CFG

$A \rightarrow +$

$B \rightarrow *$

$C \rightarrow EA$

$D \rightarrow TB$

$E \rightarrow CT \mid DF \mid num \mid id$

$T \rightarrow DT \mid num \mid id$

$F \rightarrow num \mid id$

THANK YOU

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