LINEAR ALGEBRA AND ITS APPLICATIONS
UE19MA251

Orthogonal Vectors & Subspaces

Definition :

Two subspaces S and T of a vector space V are ***orthogonal*** if every vector x in S is orthogonal to every vector y in T. Thus,

$$x^T y = 0$$

for all x ∈ S and y ∈ T.

Orthogonal Vectors & Subspaces

Examples

1. $Z = \{0\}$ is orthogonal to all subspaces.

2. In R^2, a line can be orthogonal to another line.

3. In R^3, a line can be orthogonal to another line or a plane. But, a plane cannot be orthogonal to another plane.

Note :

If S and T are orthogonal in V then

$$\dim S + \dim T \leq \dim V$$

Fundamental Theorem of Orthogonality

The row space is orthogonal to null space in R^n and the column space is orthogonal to left null space in R^m.

Definition :

Given a subspace V of R^n, the space of all vectors orthogonal to V is called the ***orthogonal complement*** of V written as $V^{\perp}$ and read as " V perp ".

Note : The orthogonal complement of a subspace V is unique.

Fundamental Theorem of Linear Algebra- Part-II

The null space is the orthogonal complement of the row space in R^n and the column space is the orthogonal complement of the left null space in R^m.

Note :

1. If S and T are orthogonal complements in R^n then it is always true that

$$\dim S + \dim T = n$$

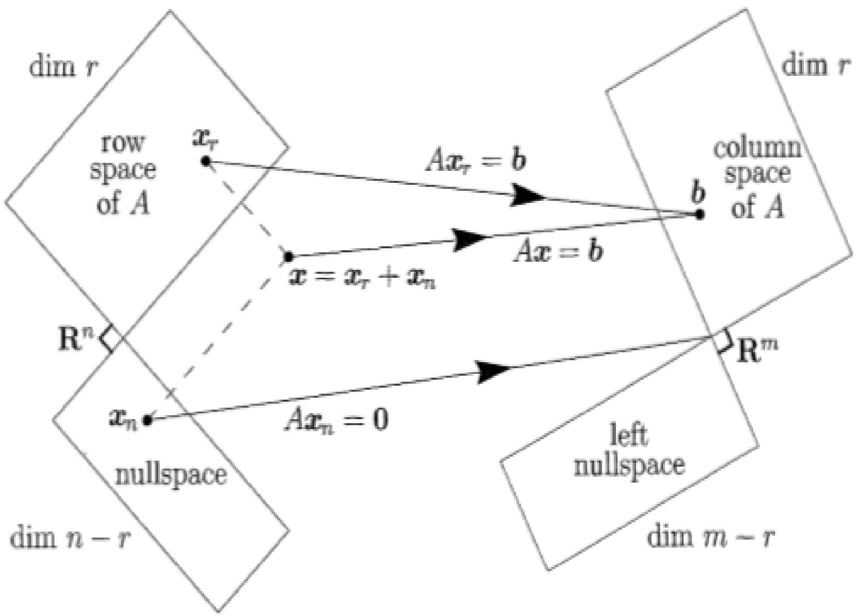
The Matrix And The Subspace

Splitting R^n into orthogonal parts V and W will split every vector into x = v + w.

The vector v is the projection onto the subspace V and the orthogonal component w is the projection of x onto W.

The true effect of matrix multiplication is that every Ax is in C(A). The null space goes to zero. The row space component goes to C(A). Nothing is carried to the left null space.

The Matrix And The Subspace



THANK YOU