PES University, Bangalore

(Established under Karnataka Act No. 16 of 2013)

UE19CS203 – STATISTICS FOR DATA SCIENCE

Unit-2 - Random Variables

QB SOLVED

Binomial Distribution

Exercises for Section 4.2

1. **Let X ~ Bin(9, 0.4). Find**

   a) **P(X > 6)**
   b) **P(X ≥ 2)**
   c) **P(2 ≤ X < 5)**
   d) **P(2 < X ≤ 5)**
   e) **P(X = 0)**
   f) **P(X = 7)**
   g) $\boldsymbol{\mu_X}$
   h) $\boldsymbol{\sigma_X^2}$

   [Text Book Exercise – Section 4.2 – Q. No.2 – Pg. No. 212]

   Solution

   a) **P(X > 6)**

   $X \sim Bin\ (9, 0.4)$, n = 9, p = 0.4.

   To find **P(X > 6)**

   The formula to be used,

$$P\ (X = x) = \begin{cases} \dfrac{n!}{x!\,(n-x)!}\ p^x\ (1-p)^{n-x} & x = 0,1,\dots,n \\ 0 & otherwise \end{cases}$$

$$P\ (X > 6) = P(X = 7) + P\ (X = 8) + P\ (X = 9)$$

$$= \frac{9!}{7!\,(9-7)!}\,(0.4)^7\,(1-0.4)^{9-7} + \frac{9!}{8!\,(9-8)!}\,(0.4)^8\,(1-0.4)^{9-8} + \frac{9!}{9!\,(9-9)!}\,(0.4)^9\,(1-0.4)^{9-9}$$

$$= 0.0212 + 0.0035 + 0.0003$$

$$= 0.0250$$

b) P(X ≥ 2)

$X \sim Bin\,(9, 0.4)$, n = 9, p = 0.4.

To find **P(X ≥ 2)**

The formula to be used,

$$P\,(X = x) = \begin{cases} \frac{n!}{x!\,(n-x)!}\,p^x\,(1-p)^{n-x} & x = 0,1,\ldots,n \\ 0 & otherwise \end{cases}$$

$$P\,(X \geq 2) = 1 - P(X \leq 1)$$

$$= 1 - P(X = 0) - P(X = 1)$$

$$= 1 - \frac{9!}{0!\,(9-0)!}\,(0.4)^0\,(1-0.4)^{9-0} - \frac{9!}{1!\,(9-1)!}\,(0.4)^1\,(1-0.4)^{9-1}$$

$$= 1 - 0.0101 - 0.0605$$

$$= 0.9295$$

c) P(2 ≤ X < 5)

$$P\,(2 \leq X < 5) = P(X = 2) + P\,(X = 3) + P\,(X = 4)$$

$$= \frac{9!}{2!\,(9-2)!}\,(0.4)^2\,(1-0.4)^{9-2} + \frac{9!}{3!\,(9-3)!}\,(0.4)^3\,(1-0.4)^{9-3} + \frac{9!}{4!\,(9-4)!}\,(0.4)^4\,(1-0.4)^{9-4}$$

$= 0.1612 + 0.2508 + 0.2508$

$= 0.6659$

d) **P(2 < X ≤ 5)**

$P\,(2 < X \leq 5) = P(X = 3) + P\,(X = 4) + P\,(X = 5)$

$$= \frac{9!}{3!\,(9-3)!}\,(0.4)^3\,(1-0.4)^{9-3} + \frac{9!}{4!\,(9-4)!}\,(0.4)^4\,(1-0.4)^{9-4} + \frac{9!}{5!\,(9-5)!}\,(0.4)^5\,(1-0.4)^{9-5}$$

$= 0.2508 + 0.2508 + 0.1672$

$= 0.6689$

e) **P(X = 0)**

$$P\,(X = 0) = \frac{9!}{0!\,(9-0)!}\,(0.4)^0\,(1-0.4)^{9-0}$$

$= 0.0101$

f) **P(X = 7)**

$$P\,(X = 7) = \frac{9!}{7!\,(9-7)!}\,(0.4)^7\,(1-0.4)^{9-7}$$

$= 0.0212$

g) $\boldsymbol{\mu_X}$

The mean can be found by the formula,

$\mu = np$

$\mu = (9)\,(0.4)$

$\mu = 3.6$

h) $\boldsymbol{\sigma_X^2}$

The variance can be found by the formula,

$\sigma_X^2 = np(1 - p)$

$\mu = (9)(0.4)(1 - 0.4)$

$\sigma_X^2 = 2.16$

2. **A quality engineer takes a random sample of 100 steel rods from a day's production, and finds that 92 of them meet specifications.**

    a) **Estimate the proportion of that day's production that meets specifications, and find the uncertainty in the estimate.**
    b) **Estimate the number of rods that must be sampled to reduce the uncertainty to 1%.**

[Text Book Exercise – Section 4.2 – Q. No.10 – Pg. No. 213]

<u>Solution</u>

Let X be the number of rods that meets the specifications.

Let p be the proportion of the day's production that meets specifications.

n = 100, The observed value of X = 92.

Then, X ~ Bin (100, p).

a) **Estimate the proportion of that day's production that meets specifications, and find the uncertainty in the estimate.**

$$Sample\ Proportion \quad \hat{p} = \frac{92}{100} = 0.92$$

To find the uncertainty, substitute $\hat{p}$ in the formula,

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

$$\sigma_{\hat{p}} = \sqrt{\frac{0.92(1-0.92)}{100}} = 0.27$$

b) Estimate the number of rods that must be sampled to reduce the uncertainty to 1%.

To find the value of n,

$$\sigma_{\hat{p}} = \sqrt{\frac{0.92(1-0.92)}{n}} = 0.01$$

$$= 0.92 * 0.08 = 0.0001 * n$$

$$n = \frac{0.92 * 0.08}{0.0001} = 736$$

n = 736