Digital Design and Computer Organization

Boolean Algebra, Identities - 2

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Department of Computer Science and Engineering

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Course Outline

- Digital Design
  - Combinational logic design
    - **Boolean Algebras, Identities**
  - Sequential logic design
- Computer Organization
  - Architecture (microprocessor instruction set)
  - Microarchitecure (microprocessor operation)

Concepts covered

- Bubbled Input Gates
- Multi Input Gates
- Boolean Algebra
- Boolean Identities

How to Represent Boolean Functions?

- Truth tables, combinational logic circuits and Boolean formulas are just different ways of representing Boolean functions
- The same Boolean function can be expressed in any of the three ways, depending on the situation
- There exist multiple Boolean formulas / logic circuits for each Boolean function but only one truth table
  - Ex: $a\overline{b} + \overline{a}b$ and $(a + b)(\overline{a} + \overline{b})$ represent the same Boolean function

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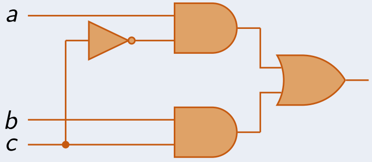
Example

The three representations below are different ways of specifying **the same Boolean function:**

$$a \cdot \overline{c} + b \cdot c$$



a	b	c	y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Logic Gates with Inverted Inputs

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Bubble

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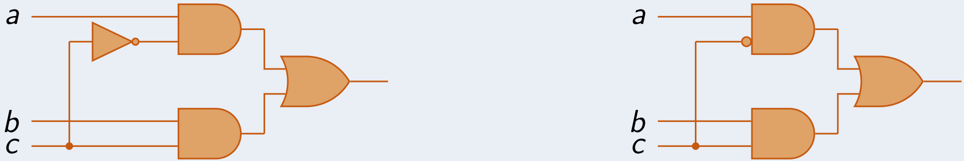
Bubble Example



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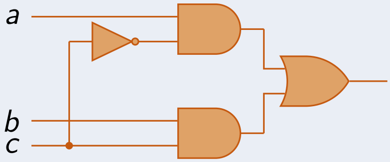
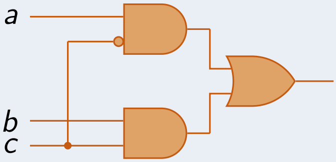
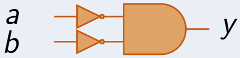
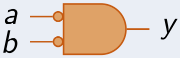




Bubbled AND Example





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Using Boolean Formulas

- Multiple Boolean formulas can represent the same Boolean function
- Consider the following two Boolean formulas:
  - $a\overline{b} + \overline{a}b$
  - $(a + b)(\overline{a} + \overline{b})$
- Do they represent the same Boolean function?
- What is the smallest Boolean formula for a given Boolean function?
- To answer above questions, we need **Boolean Algebra**

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Algebra

In mathematics, an Algebra is composed of four things: a set of elements, operations on those elements, identity elements and laws/identities

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Standard Algebra

1. **Set** Real numbers
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4. **Laws/Identities** Commutative, associative, distributive, . . .

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Boolean Algebra

1. **Set** $\{0, 1\}$
2. **Operations** AND, OR, NOT
3. **Identity elements** 0 (for AND), 1 (for OR)
4. **Laws/Identities** Commutative, associative, distributive, . . .

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Boolean Identities / Laws

Name	Law	Dual Law
Commutative	$a \cdot b = b \cdot a$	$a + b = b + a$
Associative	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$	$(a + b) + c = a + (b + c)$
Distributive	$a \cdot (b + c) = a \cdot b + a \cdot c$	$a + (b \cdot c) = (a + b) \cdot (a + c)$
DeMorgan's	$\overline{(a + b)} = \overline{a} \cdot \overline{b}$	$\overline{(a \cdot b)} = \overline{a} + \overline{b}$

Principle of Duality

Bool equation remains true if $+$ and $\cdot$ are exchanged, and also 0 and 1 are exchanged

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Boolean Identities / Laws

Name	Law	Dual Law
Idempotency	$a \cdot a = a$	$a + a = a$
Identity	$a \cdot 1 = a$	$a + 0 = a$
Boundedness	$a \cdot 0 = 0$	$a + 1 = 1$
Complement	$a \cdot \overline{a} = 0$	$a + \overline{a} = 1$
Absorption	$a + a \cdot b = a$	$a \cdot (a + b) = a$
Involution	$\overline{\overline{a}} = a$	
Useful Identity	$a + \overline{a} \cdot b = a + b$	$a \cdot (\overline{a} + b) = a \cdot b$

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Useful Identity	$a + \overline{a} \cdot b = a + b$	$a \cdot (\overline{a} + b) = a \cdot b$

Boolean identities will be used for proving Boolean formula equivalence and logic minimization

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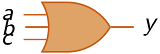
Gates with Multiple Inputs

- In subsequent logic circuits, there will be gates such as: 

- What Boolean function (if any) does the three input OR gate represent?
- Based on Boolean formula syntax rules, OR function can be applied to three inputs in three ways:
  - $a + (b + c)$
  - $b + (c + a)$
  - $c + (a + b)$
- But according to the associative law of Boolean algebra, all three Boolean formulas are equal
- So we can drop the brackets and simply write:

$$a + (b + c) = b + (c + a) = c + (a + b) = a + b + c$$

- The three input OR gate shown corresponds to the above Boolean formula
- In this manner one obtains multiple input AND, OR and XOR gates

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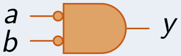
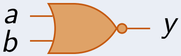
- Consider the following two Boolean formulas:
  - $a\overline{b} + \overline{a}b$
  - $(a + b)(\overline{a} + \overline{b})$
- Use the Boolean identities to prove (or disprove) that the above two formulas are equivalent (represent the same Boolean functions)
- Consider the combinational logic circuits below:





  - Above logic circuits represent LHS and RHS of which Boolean identity?