AFLL

UNIT - 3

CLASS NOTES

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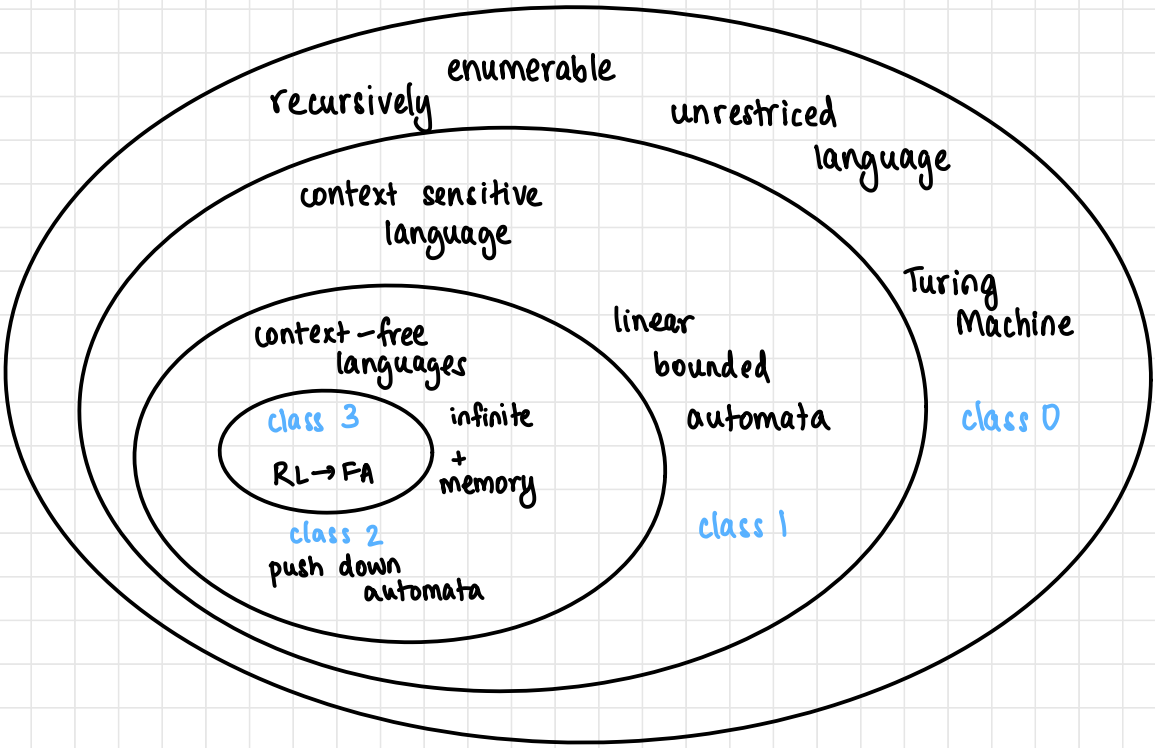
CONTEXT-FREE LANGUAGES

push down automata

- With memory

Chomsky Hierarchy

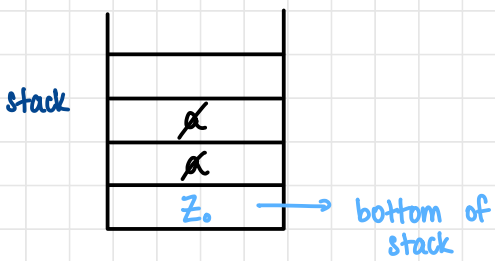


Context-Free Languages

- Compiler design
- CFAs used for syntax analysis
- $a^n b^n \rightarrow$ CFL

aabb ⟶ (( ))

Parenthesis matching



CFL

1) Context-free grammar
2) Push down automata

FA + memory ⟶ PDA

5 tuple

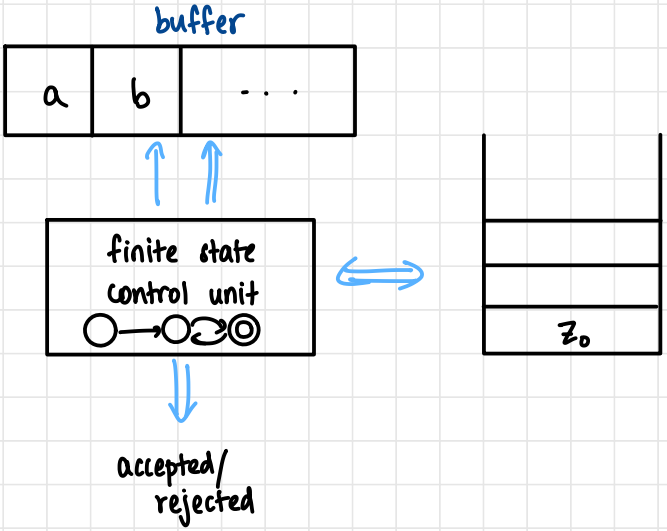
Q
Σ
δ
q_0
F

\+ stack

7 tuple

Q → state
Σ → symbols
(Γ) → stack
δ → transition function
q_0 → start state
Z_0 → bottom of stack
F → final state

Model of PDA



CFGs

1) Linear grammar ⟶ 1 non-terminal, any pos
2) Non-linear grammar ⟶ any no. of non-terminals

$$G = \{ V, T, P, S \}$$

- V: variables — non-terminals (uppercase)
- T: terminals (lowercase)
- P: prod. rules
- S: start variable

LINEAR GRAMMAR

Question 1

$L = \{ a^n b^n \mid n \geq 0 \}$

$S \rightarrow aSb \mid \lambda$

Question 2

$L = \{ ww^R \mid w \in \{a,b\}^* \}$ (even palindrome)

$= \{ \lambda, aa, bb, abba, \ldots \}$

$S \rightarrow aSa \mid bSb \mid \lambda$

Question 3

odd palindrome

$S \rightarrow aSa \mid bSb \mid x$

Question 4

$\{ ww^R, w \in (ab)^* + (ba)^* \}$

$S \rightarrow abSba \mid baSab \mid \lambda$

Question 5

$L = \{ a^n w w^R b^n, w \in (a,b)^* \}$

$S \rightarrow aSb \mid A$
$A \rightarrow \lambda \mid aAa \mid bAb$

Question 6

$L = \{ a^n w w^R b^n \mid w \in \{a,b\}^* \}$

$S \rightarrow aSb \mid A$
$A \rightarrow aAa \mid bAb \mid \lambda$

Question 7

$$L = \{a^n b^{n+1} \mid n \geq 0\} \quad \Sigma = \{a,b\}^*$$

$$S \rightarrow aSb \mid b$$

Question 8

$$L = \{a^n b^{2n}, n \geq 0\}$$

$$S \rightarrow aSbb \mid \lambda$$

Question 9

$$L = \{a^n b^m, n > m\}$$

$$S \rightarrow aSb \mid a \mid aS$$

Question 10

$$L = \{a^n b^{n-3} \mid n \geq 3\}$$

$n=3$	$3a$	$0b$
$n=4$	$4a$	$1b$
$n=5$	$5a$	$2b$

$$S \rightarrow aSb \mid aaa$$

Question 11

$$L = \{a^n b^m,\ a \neq b\}$$

$$S \rightarrow A \mid B$$
$$A \rightarrow aAb \mid a \mid aA$$
$$B \rightarrow aBb \mid b \mid bB$$

Question 12

$$L = \{a^n b^m,\ n = 2 + (m \bmod 3)\}$$

$m = 0$	$n = 2$	aa
$m = 1$	$n = 3$	$aaab$
$m = 2$	$n = 4$	$aaaabb$
$m = 3$	$n = 2$	$aabbb$
$m = 4$	$n = 3$	$aaabbbb$
$m = 5$	$n = 4$	$aaaabbbbb$

$$S \rightarrow aaA \mid aaabA \mid aaaabbA$$
$$A \rightarrow bbbA \mid \lambda$$

Question 13

$$L = \{a^{n+2} b^m \mid m > n,\ n \geq 0\}$$

$$S \rightarrow aSb \mid bS \mid aab$$

Question 14

$$L = \{a^n b^m c^m d^n \mid n, m \geq 1\}$$

$$S \rightarrow aSd \mid aAd$$
$$A \rightarrow bAc \mid bc$$

Question 15

$$L = \{a^n b^m c^k \mid k = n+m,\ n, m, k \geq 0\}$$

$$a^n b^m c^m c^n$$

$$S \rightarrow aSc \mid A$$
$$A \rightarrow bAc \mid \lambda$$

Question 16

$$L = \{a^n b^m c^k,\ k = n+2m,\ n, m, k \geq 0\}$$

$$a^n b^m c^{2m} c^n$$

$$S \rightarrow aSc \mid A$$
$$A \rightarrow bAcc \mid \lambda$$

Question 17

$$L = \{|w| \bmod 3 \neq |w| \bmod 2\} \quad \Sigma = \{a\}^*$$

		mod 2	mod 3	
	λ	0	0	✗
	a	1	1	✗
a^2	aa	0	2	✓
a^3	aaa	1	0	✓

a^4	aaaa	0	1	✓
a^5	aaaaa	1	2	✓
	aaaaaa	0	0	✗

$L = \{a^2, a^3, a^4, a^5, a^8, a^8, a^{10}, a^{11}$

$S \rightarrow aaA \mid aaaA \mid aaaaA \mid aaaaaA$
$A \rightarrow aaaaaaA \mid \lambda$

NON-LINEAR GRAMMAR

context-free

Question 18

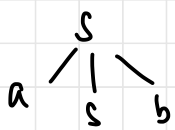
$L = \{uvwv^R, |u| = |w| = 2, v \geq 1, \Sigma = \{a,b\}^*\}$

$S \rightarrow AB$
$A \rightarrow aa \mid bb \mid ab \mid ba$
$B \rightarrow aBa \mid bBb \mid A$

Question 19

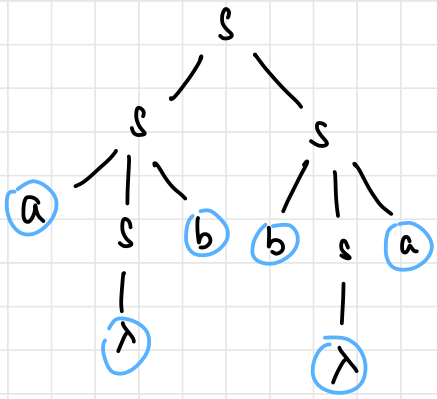
$L = \{n_a(w) = n_b(w), w \in \{a,b\}^*\}$

$S \rightarrow aSb \mid bSa \mid \lambda$ ← does not accept abba



won't accept

$S \rightarrow aSb \mid bSa \mid SS$



abba

Question 20

$L = \{ n_a(w) = n_b(w) + 1 \}$

$S \rightarrow aSb \mid bSa \mid abS \mid baS \mid a$

OR

$S \rightarrow AaA$
$A \rightarrow aAb \mid bAa \mid AA \mid \lambda$

Question 21

$L = \{ n_a(w) = 2 * n_b(w) \}$

$S \rightarrow bSaSa \mid aSaSb \mid aSbSa \mid SS \mid \lambda$

Question 22

$$L = \{ n_a(w) > n_b(w) \mid w \in \{a,b\}^* \}$$

$$S \to AaA$$
$$A \to aAb \mid bAa \mid aA \mid Aa \mid AA \mid \lambda$$

Question 23

$$L = \{ n_a(w) \neq n_b(w), \; w \in \{a,b\}^* \}$$

$$S \to AaA \mid BbB$$
$$A \to aAb \mid bAa \mid aA \mid Aa \mid \lambda$$
$$B \to aBb \mid bBa \mid bB \mid Bb \mid \lambda$$

Question 24

$$L = \{ a^n b^n \cup a^n b^{2n} \}$$

$$S \to S_1 \mid S_2$$
$$S_1 \to aS_1b \mid \lambda$$
$$S_2 \to aS_2bb \mid \lambda$$

COMPILER DESIGN (C LANG)

① Proper Nesting of Parentheses

1) Simple nesting ⟶ ((()))

$$S \rightarrow (S) \mid \lambda$$

2) Proper nesting → (()()(()))

$$S \rightarrow (S) \mid SS \mid \lambda$$

3) Multiple kinds of brackets

$$S \rightarrow (S) \mid [S] \mid \{S\} \mid SS \mid \lambda$$

② Arithmatic Expressions

literal, variable

$$\Sigma = \{+, *, /, -, (), num, \%, identifier, \hat{} \}$$

$$E \rightarrow E+E \mid E-E \mid E*E \mid E/E \mid E\%E \mid (E) \mid E\hat{}E \mid id \mid num$$

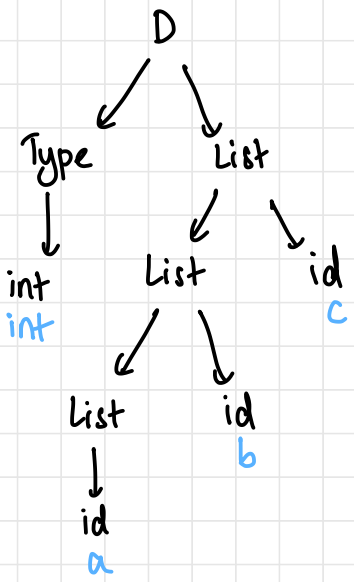
③ Variable declaration

int x; int (a, b, c);

type, list, type, list

$$D \rightarrow Type\ List$$
$$List \rightarrow List, id \mid id$$
$$Type \rightarrow int \mid float \mid double \mid char$$

parse tree
for int a, b, c;



④ Nested if-else

```
if (cond) {
    statement
}
else {
    if (cont) {
        statement
    }
    statement
}
```

$\Sigma = \{ \text{if, cond, statement, else, } \{\} \}$

$S \rightarrow \text{if cond } S \mid \text{if cond } S \text{ else } S \mid \{\text{statement}\}$

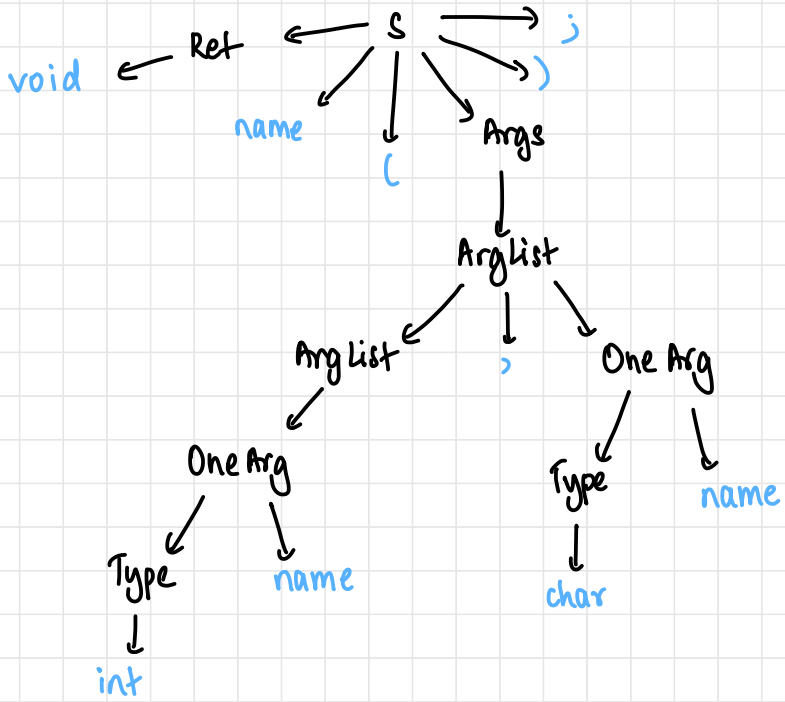
⑤ Function Prototype

```
return_type  name (type, type);
return-type  name (type name);
return_type  name ();
```

S → Ret name (Args);
Ret → void | Type
Type → char | float | int | double
Args → λ | ArgList | void
ArgList → ArgList, OneArg | OneArg
OneArg → Type name | Type

Parse tree for

void name (int name, char name);



Derivations for Non-Linear Grammars

1) Leftmost Derivation (LMD)
   - always expand leftmost sentencial form

2) Rightmost Derivation (RMD)
   - always expand rightmost sentencial form

note:
cannot mix!
MUST do
LMD or
RMD

Question 25

$S \to aSb \mid bSa \mid SS \mid \lambda$ (expand LMD & RMD)

$w = abba$

LMD

$S \overset{LM}{\Rightarrow} SS$ — using $S \to SS$

$\overset{LM}{\Rightarrow} aSbS$ — $S \to aSb$

$\overset{LM}{\Rightarrow} abS$ — $S \to \lambda$

$\overset{LM}{\Rightarrow} abbSa$ — $S \to bSa$

$\overset{LM}{\Rightarrow} abba$ — $S \to \lambda$

each of these is a sentential form

sentence

RMD

$S \overset{RM}{\Rightarrow} SS$ — $S \to SS$

$\overset{RM}{\Rightarrow} SbSa$ — $S \to bSa$

$\overset{RM}{\Rightarrow} Sba$ — $S \to \lambda$

$\overset{RM}{\Rightarrow} aSbba$ — $S \to aSb$

$\overset{RM}{\Rightarrow} abba$ — $S \to \lambda$

sentential forms $\in (V \cup T)^*$

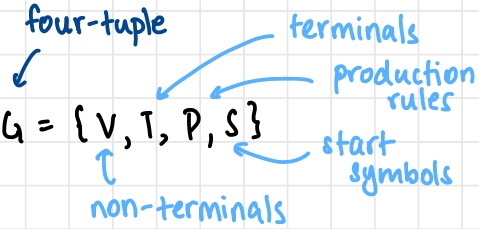
non terminals

terminals

parsing & ambiguity

Ambiguous Grammar

- A grammar is said to be ambiguous iff there exists a string w that belongs to the grammar and there exist 2 diff. LMDs or 2 diff RMDs for the string (or parse trees)



Question 26

Is the grammar ambigious?

variable, literal

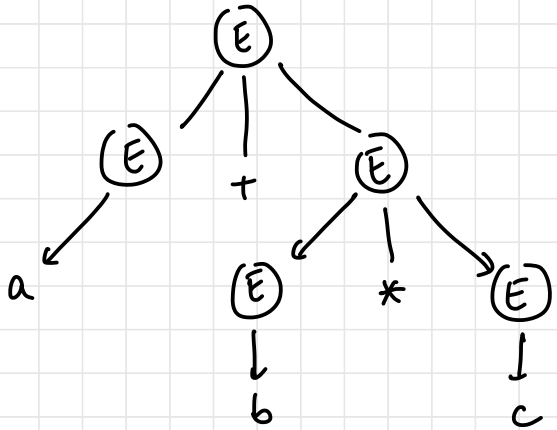
$$\Sigma = \{+, *, /, -, (), \text{var}, \%, \text{constant}, \hat{} \}$$

$$E \rightarrow E+E \mid E-E \mid E*E \mid E/E \mid E\%E \mid (E) \mid E\hat{}E \mid \text{var} \mid \text{constant}$$

$$w = a+b*c$$

Derivation #1

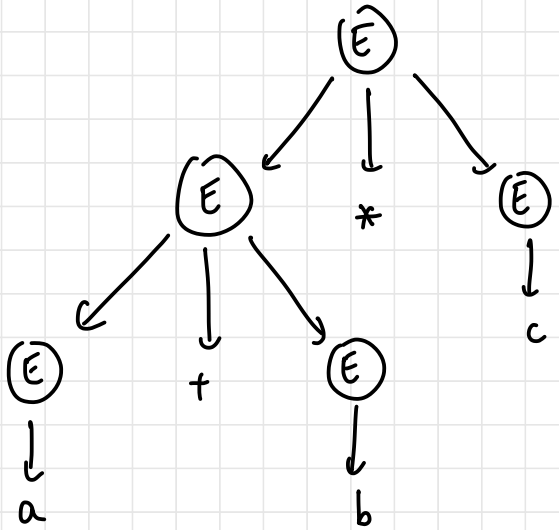
		using
$E \overset{lm}{\Rightarrow}$	$E+E$	$E \rightarrow E+E$
$\overset{lm}{\Rightarrow}$	$a+E$	$E \rightarrow a$
$\overset{lm}{\Rightarrow}$	$a+E*E$	$E \rightarrow E*E$
$\overset{lm}{\Rightarrow}$	$a+b*E$	$E \rightarrow b$
$\overset{lm}{\Rightarrow}$	$a+b*c$	$E \rightarrow c$



evaluation

Derivation #2

$$
\begin{aligned}
E &\overset{lm}{\Rightarrow} E*E && E \to E*E \\
&\overset{lm}{\Rightarrow} E+E*E && E \to E+E \\
&\overset{lm}{\Rightarrow} a+E*E && E \to a \\
&\overset{lm}{\Rightarrow} a+b*E && E \to b \\
&\overset{lm}{\Rightarrow} a+b*c && E \to c
\end{aligned}
$$



evaluation

- This is not desirable (giving compiler a choice)
- Order of operations
- Should not retain such grammars

Note:

- There is no algorithm to prove that a grammar is ambiguous.
- Must take an example and derive (start from minimal length)

Question 27

Find if the grammar is ambigious or not.

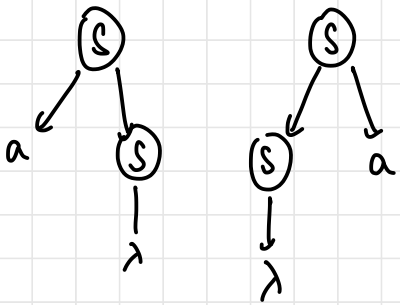
$$S \to aS \mid Sa \mid \lambda$$

1) length = 0

$w = \lambda$; cannot derive further

2) length is 1

$w = 1$



$S \overset{LM}{\Rightarrow} aS$ $\quad S \to aS$

$\Rightarrow a\lambda$ $\quad S \to \lambda$

$S \overset{LM}{\Rightarrow} Sa$ $\quad S \to Sa$

$\Rightarrow \lambda a$ $\quad S \to \lambda$

$\therefore$ the grammar is ambigious

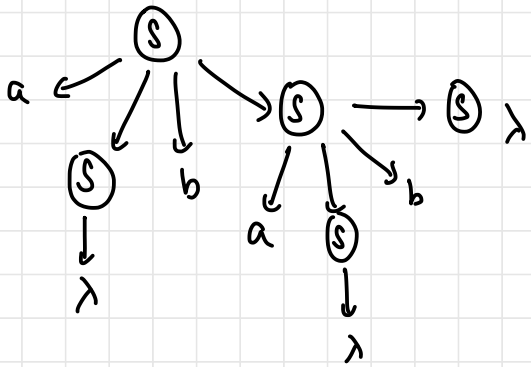
Question 28

$S \rightarrow aSbS \mid bSaS \mid \lambda$

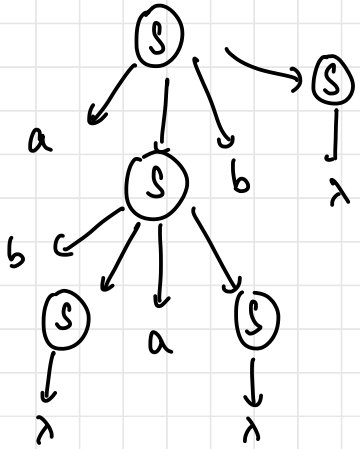
1) length = 0
   $w = \lambda$; unambiguous

2) length = 2
   $w = ab$ or $w = ba$; unambiguous

4) length = 4
   $w = abab$



$S \overset{lm}{\Rightarrow} aSbS$ $S \rightarrow aSbS$
$\Rightarrow abS$ $S \rightarrow \lambda$
$\Rightarrow abaSbS$ $S \rightarrow aSbS$
$\Rightarrow ababS$ $S \rightarrow \lambda$
$\Rightarrow abab$ $S \rightarrow \lambda$



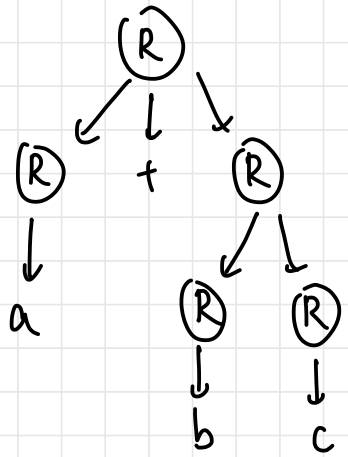
$S \overset{lm}{\Rightarrow} aSbS$ $S \rightarrow aSbS$
$\Rightarrow abSaSbS$ $S \rightarrow bSaS$
$\Rightarrow abaSbS$ $S \rightarrow \lambda$
$\Rightarrow ababS$ $S \rightarrow \lambda$
$\Rightarrow abab$ $S \rightarrow \lambda$

The grammar is ambiguous

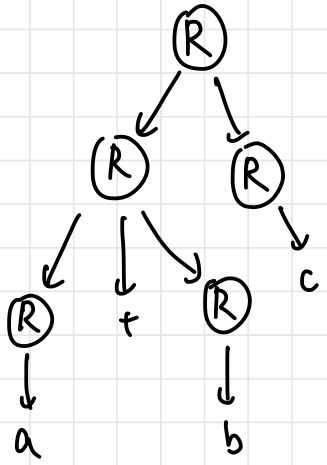
Question 29

$R \rightarrow R+R \mid RR \mid R^* \mid a \mid b \mid c$

$w = a+bc$



$R \overset{lm}{\Rightarrow} R+R \quad R \rightarrow R+R$
$\Rightarrow a+R \quad R \rightarrow a$
$\Rightarrow a+RR \quad R \rightarrow RR$
$\Rightarrow a+bR \quad R \rightarrow b$
$\Rightarrow a+bc \quad R \rightarrow c$



$R \overset{lm}{\Rightarrow} RR \quad R \rightarrow RR$
$\Rightarrow R+RR \quad R \rightarrow R+R$
$\Rightarrow a+RR \quad R \rightarrow a$
$\Rightarrow a+bR \quad R \rightarrow b$
$\Rightarrow a+bc \quad R \rightarrow c$

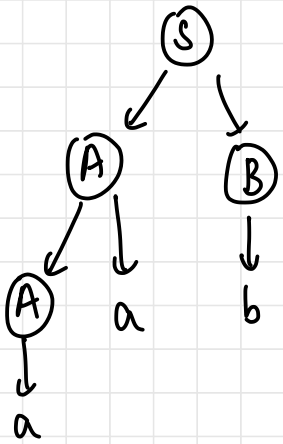
The grammar is ambigious

Question 30

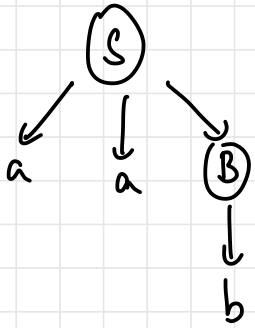
$S \rightarrow AB \mid aaB$
$A \rightarrow a \mid Aa$
$B \rightarrow b$

1) length = 2
ab ; unambiguous

2) length = 3
w = aaB



$S \overset{lm}{\Rightarrow} AB$ $S \rightarrow AB$
$\Rightarrow AaB$ $A \rightarrow Aa$
$\Rightarrow aaB$ $A \rightarrow a$
$\Rightarrow aab$ $B \rightarrow b$



$S \overset{lm}{\Rightarrow} aaB$ $S \rightarrow aaB$
$\Rightarrow aab$ $B \rightarrow b$

Grammar is ambiguous

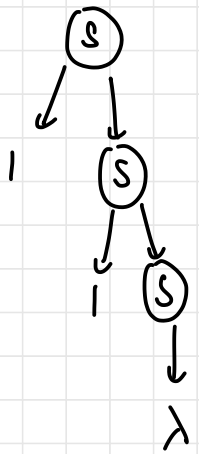
Question 31

$S \rightarrow 1S \mid 11S \mid \lambda$

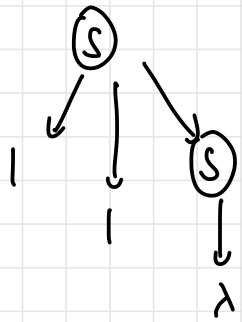
1) length = 0
   unambiguous

2) length = 1
   unambiguous

3) length = 2



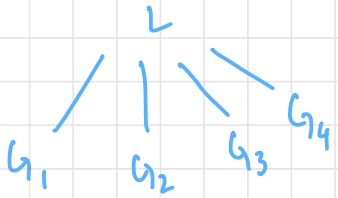
$S \overset{lm}{\Rightarrow} 1S \quad S \rightarrow 1S$
$\Rightarrow 11S \quad S \rightarrow 1S$
$\Rightarrow 11 \quad S \rightarrow \lambda$



$S \overset{lm}{\Rightarrow} 11S \quad S \rightarrow 11S$
$\Rightarrow 11 \quad S \rightarrow \lambda$

Grammar Ambiguous vs Language Ambiguous

- Ambiguous language: all grammars are ambiguous
- Inherently ambiguous → language is ambiguous



Question 32

check for ambiguity

$$\mathcal{L} = \{a^n b^n c^m\} \cup \{a^n b^m c^m\} \quad n, m \geq 0$$

$$\mathcal{L} = L_1 \cup L_2$$

G_1

$S_1 \rightarrow S_1 c \mid A$

$A \rightarrow aAb \mid \lambda$

G_2

$S_2 \rightarrow aS_2 \mid B$

$B \rightarrow bBc \mid \lambda$

$S \rightarrow S_1 \mid S_2$

$S_1 \rightarrow S_1 c \mid A$

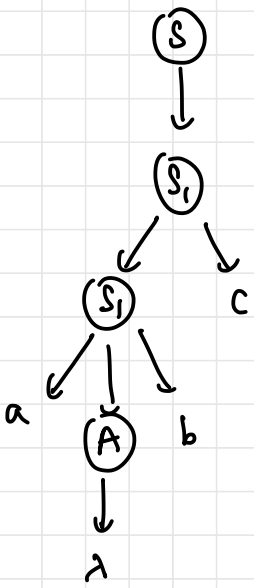
$A \rightarrow aAb \mid \lambda$

$S_2 \rightarrow aS_2 \mid B$

$B \rightarrow bBc \mid \lambda$

$a^n b^n c^n \rightarrow abc$ smallest

$w = abc$

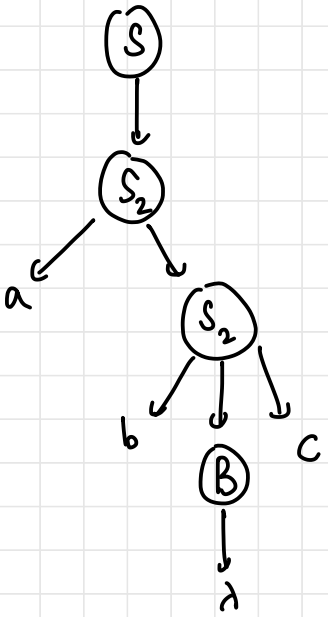


$S \overset{lm}{\Rightarrow} S_1 \quad S \to S_1$

$\Rightarrow S_1 C \quad S_1 \to S_1 C$

$\Rightarrow aAbC \quad S_1 \to aAb$

$\Rightarrow abc \quad A \to \lambda$



$S \overset{lm}{\Rightarrow} S_2 \quad S \to S_2$

$\Rightarrow aS_2 \quad S_2 \to aS_2$

$\Rightarrow abBc \quad S_2 \to bBc$

$\Rightarrow abc \quad B \to \lambda$

L is inherently ambiguous (only one grammar)

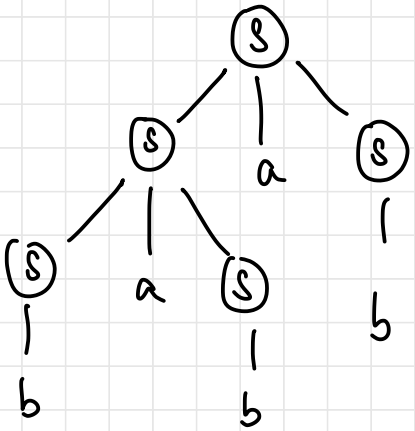
Question 33

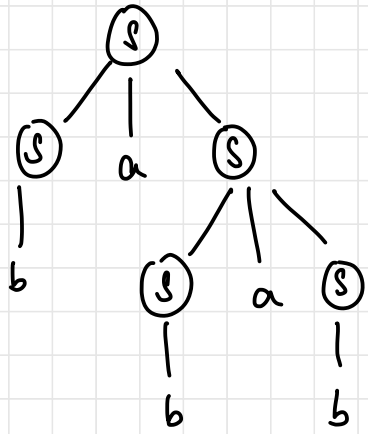
$G = \quad S \to SaS \mid b$

length 1 ✗

length 3 $\to$ bab ✗

length 5 → babab





$S \overset{lm}{\Rightarrow} SaS$ $S \to SaS$
$\Rightarrow SaSaS$ $S \to SaS$
$\Rightarrow baSaS$ $S \to b$
$\Rightarrow babaS$ $S \to b$
$\Rightarrow babab$ $S \to b$

$S \overset{lm}{\Rightarrow} SaS$ $S \to SaS$
$\Rightarrow baS$ $S \to b$
$\Rightarrow baSaS$ $S \to SaS$
$\Rightarrow babaS$ $S \to b$
$\Rightarrow babab$ $S \to b$

G is ambiguous

$L = \{b, bab, babab, bababab \ldots\}$

RE : $b(ab)^*$ or $(ba)^*b$

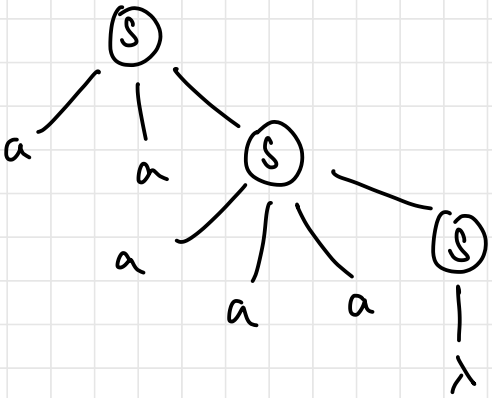
$S \to Ab$
$A \to baA \mid \lambda$] unambigious

Question 34

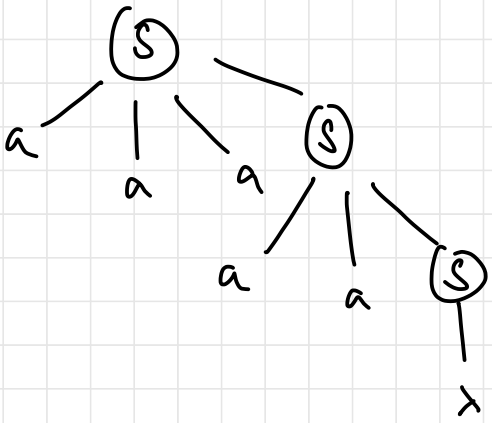
$S \rightarrow aaS \mid aaaS \mid \lambda$

length 0 — x
length 2 — x
length 3 — x
length 4 — x

length 5 — aaaaa



$S \overset{lm}{\Rightarrow} aaS \quad S \rightarrow aaS$
$\Rightarrow aa\,aaaS \quad S \rightarrow aaaS$
$\Rightarrow aaaaa \quad S \rightarrow \lambda$



$S \overset{lm}{\Rightarrow} aaaS \quad S \rightarrow aaaS$
$\Rightarrow aaaaaS \quad S \rightarrow aaS$
$\Rightarrow aaaaa \quad S \rightarrow \lambda$

$L = \{ \lambda, aa, aaa, aaaa, aaaa \ldots \}$

$RE = (aa + aaa)^*$

$= \lambda + aaa^*$ $\quad Ca^n,\ n = 0 \mid n \geq 2$

G_1 $\quad S \rightarrow aaA \mid \lambda$
$A \rightarrow aA \mid \lambda$

ELIMINATING AMBIGUITY

Question 35 Make unambiguous

$E \rightarrow E + E \mid E - E \mid E * E \mid E \hat{} E \mid E / E \mid (E) \mid id \mid num$

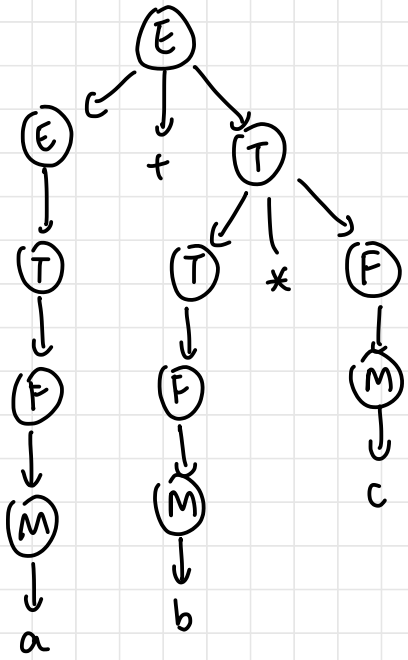
- highest precedence: last production rule (bottom branch)
- left associativity: non-terminal at left (tree grows from left side)

$+, -$ $\rightarrow$ left associativity
$*, /$ $\rightarrow$ left associativity
$\hat{}$ $\rightarrow$ right associativity

$E \rightarrow E + T \mid E - T \mid T$
$T \rightarrow T * F \mid T / F \mid F$
$F \rightarrow M \hat{} F \mid M$
$M \rightarrow (E) \mid id \mid num$

unambiguous grammar

w = a+b*c



$E \overset{lm}{\Rightarrow} E+T$
$\Rightarrow T+T$
$\Rightarrow F+T$
$\Rightarrow M+T$
$\Rightarrow a+T$
$\Rightarrow a+T*F$
$\Rightarrow a+F*F$
$\Rightarrow a+M*F$
$\Rightarrow a+b*M$
$\Rightarrow a+b*c$

Question 36

$R \rightarrow R+R \mid RR \mid R^* \mid a \mid b \mid c$ make unambiguous

precedence:

+	low	left associative
.		left associative
*	high	no associativity

$R \rightarrow R+A \mid A$
$A \rightarrow AB \mid B$
$B \rightarrow B^* \mid a \mid b \mid c$

Question 37

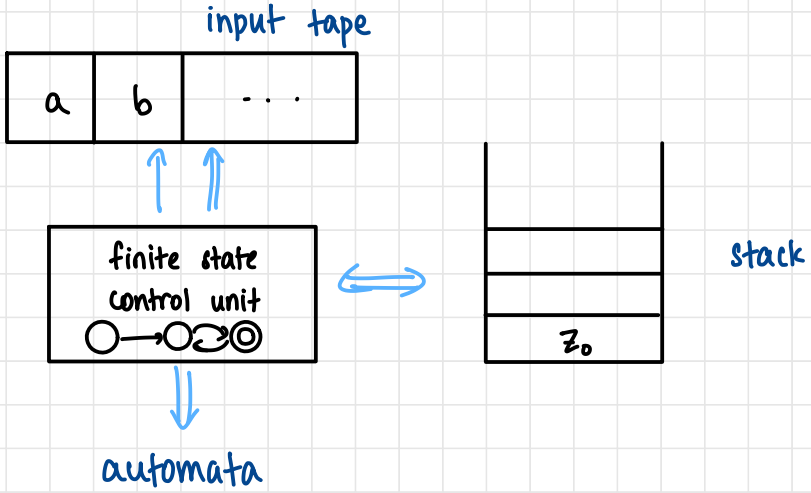
$A \rightarrow A \$ B \mid B$
$B \rightarrow B \# C \mid C$
$C \rightarrow C @ D \mid D$
$D \rightarrow B$

Precedence?
Associativity?

$ — lowest, left
# — left
@ — highest, left

PUSH DOWN AUTOMATA

- FA + stack



PDA

λ-NFA + memory $\longrightarrow$ PDA

$M = (Q, \Sigma, \delta, q_0, F)$ + memory

$M = (Q, \Sigma, \delta, q_0, F, (Z_0, \Gamma))$

tor

Z_0: bottom of stack

Γ: stack symbols

1) Deterministic PDA (DPDA)

- input symbol, current state, top of stack — one move
- end of every string is λ; must show transition
- similar to DFA
- accepts only deterministic CFLs

$$\delta = Q \times (\Sigma \cup \lambda) \times T \longrightarrow Q \times \Gamma^*$$

Q: state; $(\Sigma \cup \lambda)$: input symbol; T: top; Γ^*: pop/push

2) Non-deterministic PDA (NPDA)

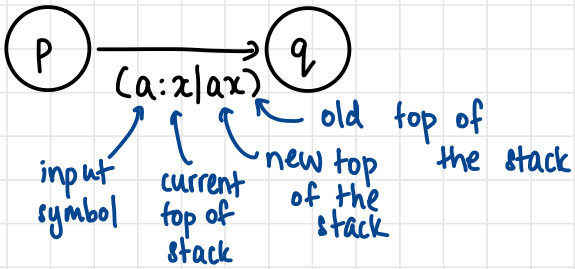
- λ is a symbol
- end of every string is λ; must show transition
- more powerful than DPDA
- accepts any CFL

$$\delta = Q \times (\Sigma \cup \lambda) \times T \longrightarrow 2^{Q \times \Gamma^*}$$

Q: state; $(\Sigma \cup \lambda)$: input symbol; T: top; $Q \times \Gamma^*$: pop/push

Configuration of Machine

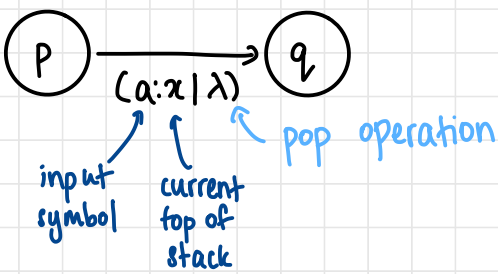
1) Push



Representation

(a:x → ax)
(a; x | ax)

2) Pop



Tracing Operation

- To trace operation of PDA, we keep track of the current state, current stack contents, unread part of the input string
- called a triplet (q, W, U) — instantaneous description of machine

current state of machine
$q \in Q$

set of input symbols remaining on input tape

current stack contents

Acceptance of String

1) Final state acceptance
   - final state when all inputs have been read
   - in $M = (Q, \Sigma, q_0, F, \delta, Z_0, \Gamma)$

2) Empty stack
   - stack empty when all input symbols have been read

$$(q, w, u) = (p, \lambda, \lambda)$$

empty tape (string) — empty stack

- $M = (Q, \Sigma, q_0, \phi, \delta, Z_0, \Gamma)$

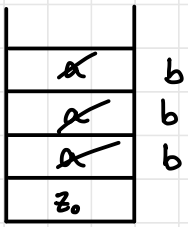
no final state

Question 38

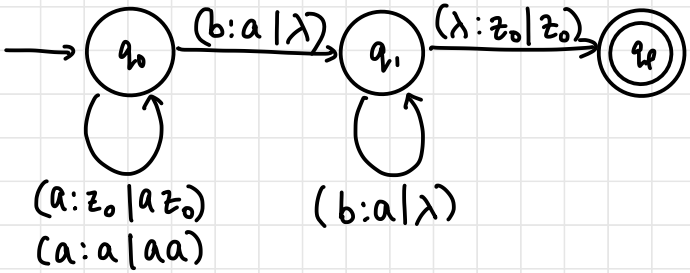
$$L = \{a^n b^n \mid n \geq 1\}$$

$$S \rightarrow aSb \mid ab$$

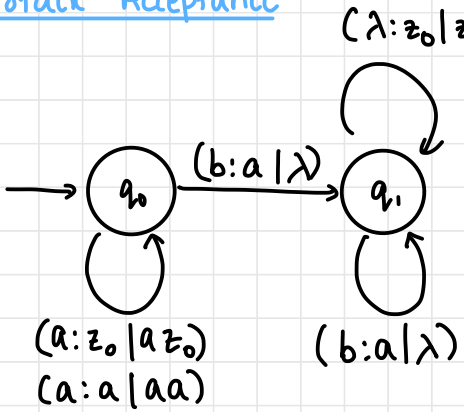
- push a, pop b



Final state Acceptance



Empty Stack Acceptance



do not show extra final state for empty stack acceptance

Transition Function

$w = aabb$

next state

elements of stack (2)

$\delta(q_0, a, z_0) = (q_0, az_0)$

$\delta(q_0, a, a) = (q_0, aa)$

pop

$\delta(q_0, b, a) = (q_1, \lambda)$

$\delta(q_1, b, a) = (q_1, \lambda)$

(a) $\delta(q_1, \lambda, z_0) = (q_f, z_0)$ → final state

(b) $\delta(q_1, \lambda, z_0) = (q_1, z_0)$ → empty stack

Instantaneous Description

(q, w, u)

$\vdash^*$: sequence of moves

$\vdash$: move

$w = aabb$

Turnstile notation

$(q_0, aabb, z_0) \vdash (q_0, -abb, az_0)$

$\vdash (q_0 --bb, aaz_0)$

$\vdash (q_1, ---b, az_0)$

$\vdash (q_1, ----, z_0)$ ↷ same

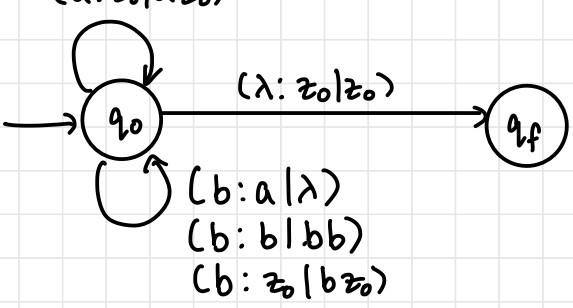
$\vdash (q_1, \lambda, z_0)$

$\vdash (q_f, z_0)$ — final state

Question 39

$n_a(w) = n_b(w)$ (order unimportant)

$(a : b \mid \lambda)$
$(a : a \mid aa)$
$(a : z_0 \mid az_0)$

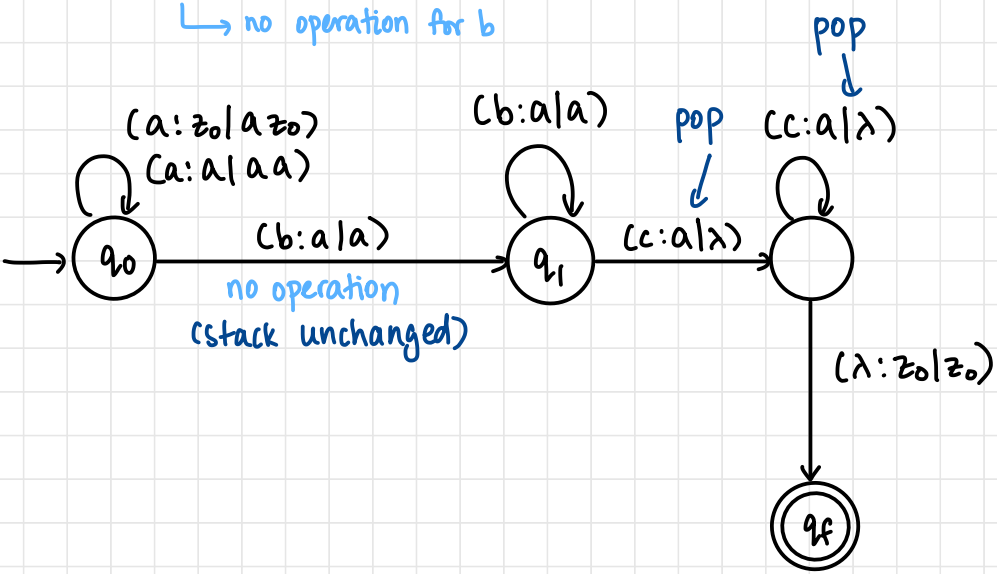


Question 40

match a & c

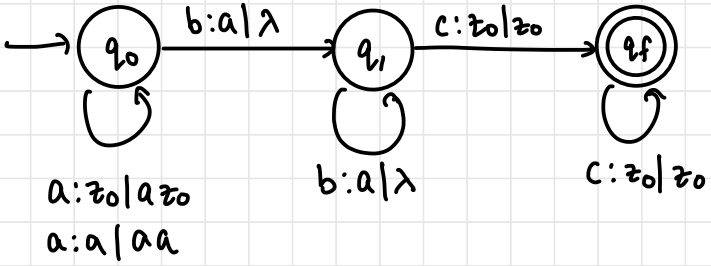
$L = \{a^n b^m c^n \mid n, m \geq 1\}$

↳ no operation for b



Question 41

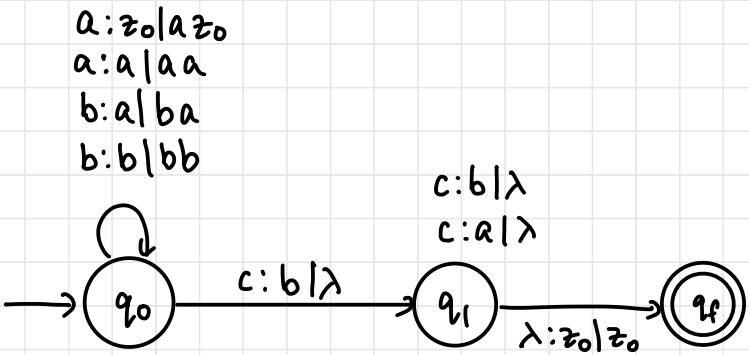
$L = \{a^n b^n c^m \mid n, m \geq 1\}$



Question 42

$L = \{a^n b^m c^{m+n}, n,m \geq 1\}$

if $n,m \geq 0$, q_0 is accepting state

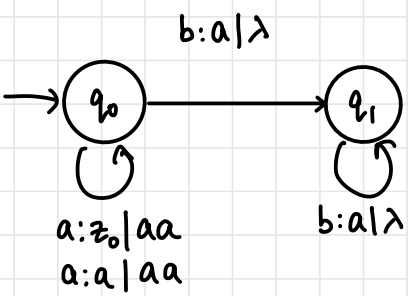


Question 43

$L = \{a^n b^{2n} | n \geq 1\}$ abb, aabbbb

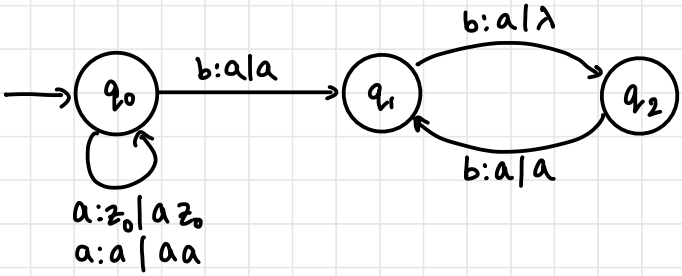
Solution 1

Push 2 a's



Solution 2

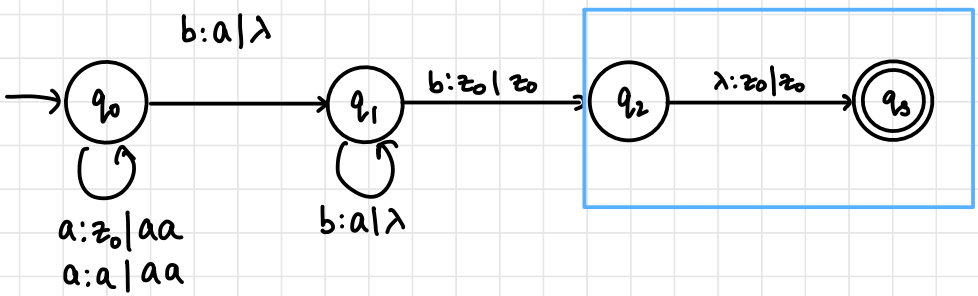
pop at alternate b's



Question 44

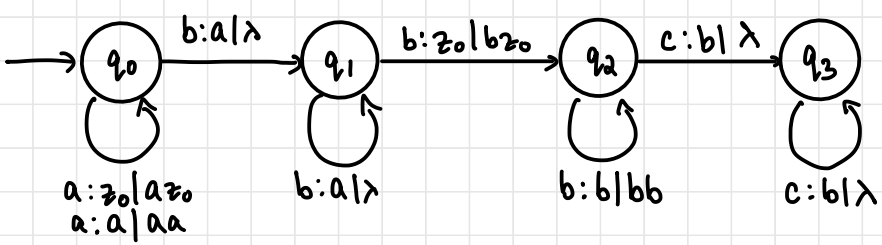
$L = \{a^n b^{2n+1} \mid n \geq 1\}$

final state accepting



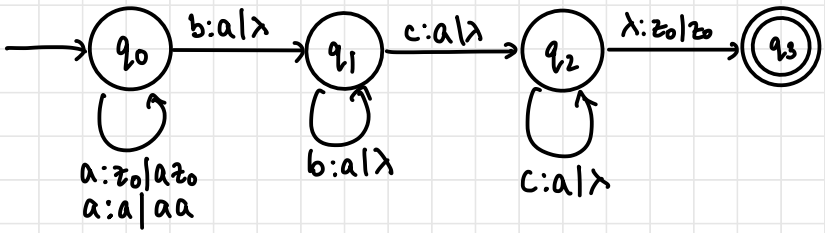
Question 45

$L = \{a^n b^{m+n} c^m\}$



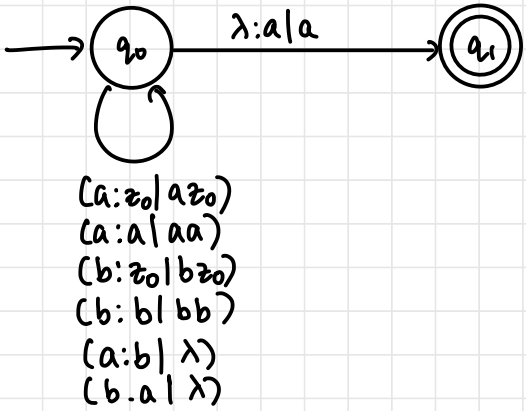
Question 46

$$L = \{ a^{m+n} b^m c^n \mid m, n \geq 1 \}$$



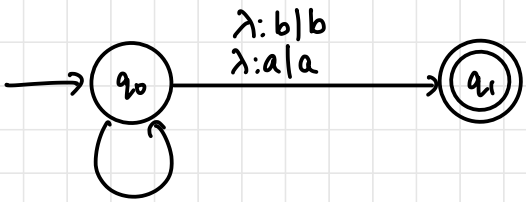
Question 47

$$L = \{ n_a(w) > n_b(w) \}$$



Question 48

$$L = \{ n_a(w) \neq n_b(w) \}$$

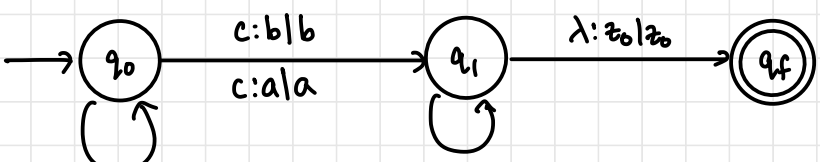


(a: z_0 | a z_0)
(a: a | aa)
(b: z_0 | b z_0)
(b: b | bb)
(a: b | λ)
(b. a | λ)

Question 49

$L = \{wcw^R \mid w \in \{a,b\}^*\}$ odd palindrome

$\underbrace{ab}_{w}c\underbrace{ba}_{w^R}$



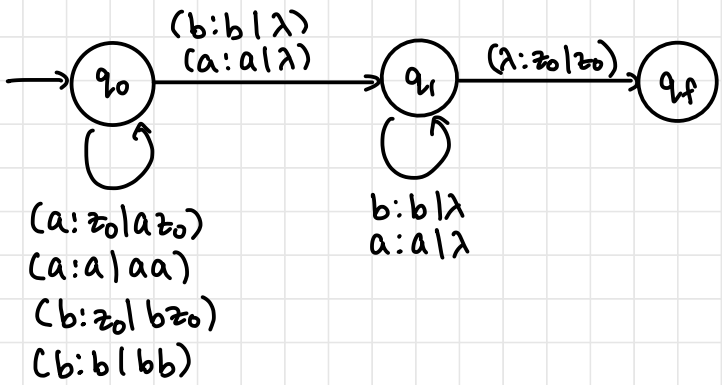
(a: z_0 | a z_0)
(a: a | aa)
(a: b | ab)
(b: z_0 | b z_0)
(b: b | bb)
(b: a | bb)

b: b | λ
a: a | λ

NON-DETERMINISTIC PDA

Question 50

$L = \{ww^R \mid w \in \{a,b\}^*\}$ even palindrome



Instantaneous description

$w = \underbrace{aa}_{w}\underbrace{aa}_{w^R}$

(q, w, u)

$(q_0, aaaa, z_0)$

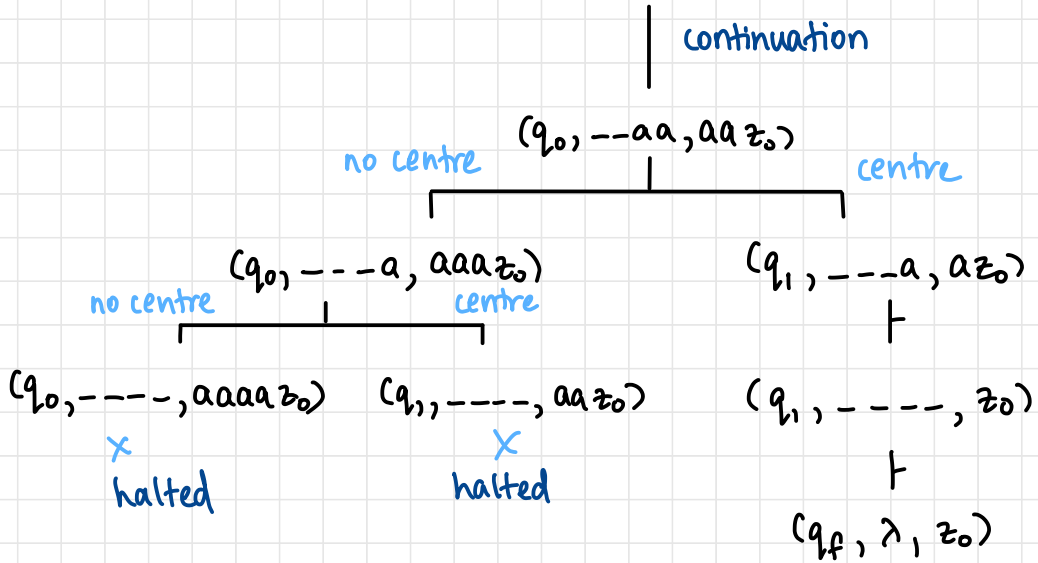
$\vdash$

$(q_0, \_aaa, az_0)$

no centre | centre

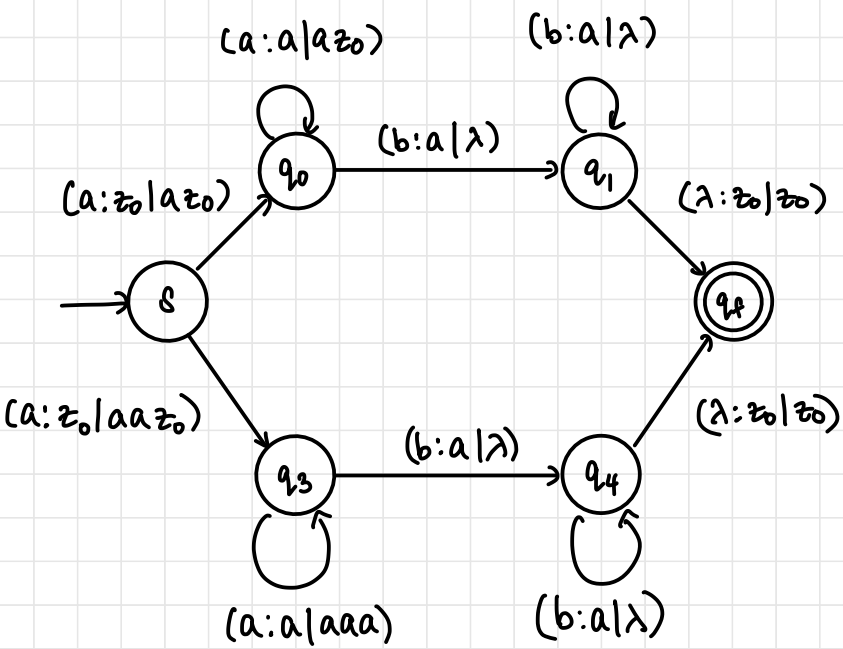
$(q_0, \_\_aa, aaz_0)$

$(q_1, \_\_aa, z_0)$ halted ✗



Question 51

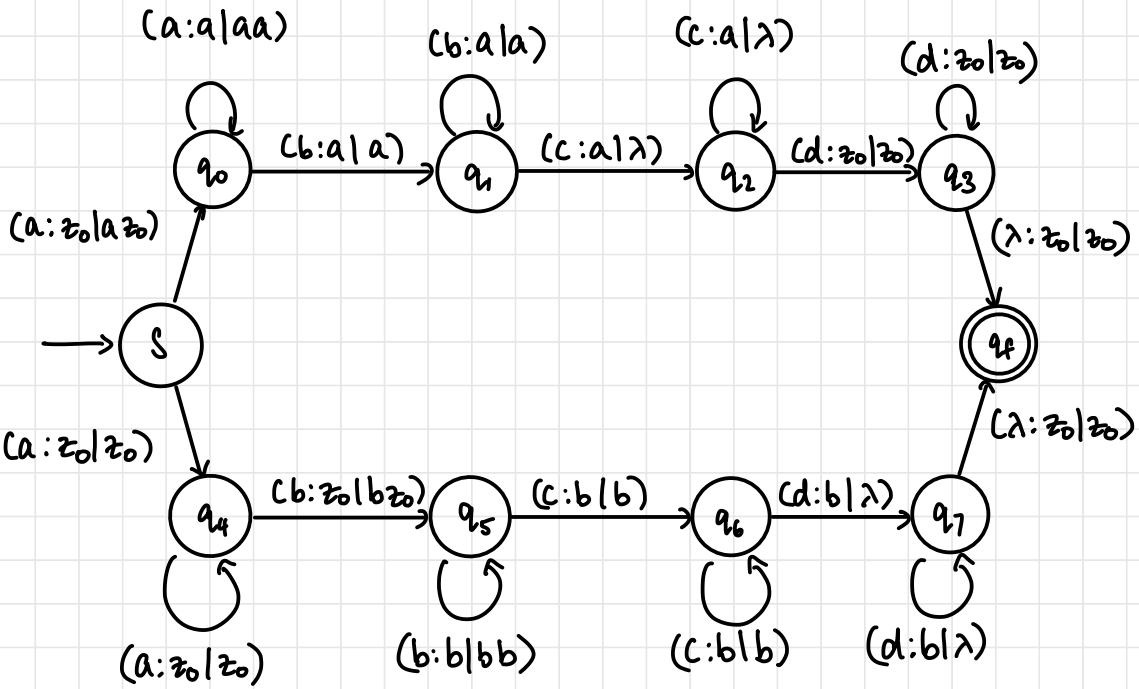
$L = \{a^n b^n \cup a^n b^{2n} \mid n \geq 1\}$ NPDA



Question 52

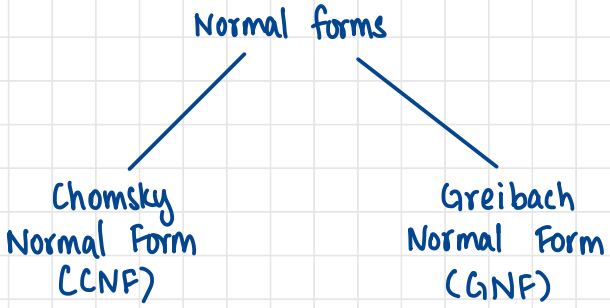
$$L = \{a^i b^j c^k d^l \mid i = k \text{ or } j = l \; ; \; i, j, k, l \geq 1\}$$

$$n_a(w) = n_c(w) \quad \text{or} \quad n_b(w) = n_d(w)$$



normal forms

- Standard rules to right CFG
- For efficient parsing
- Normalised form
- RHS of productions should become useful



- CNF is used by efficient parsing algorithm (CYK algorithm)
- GNF rules state that string of length n requires only n steps

CHOMSKY NORMAL FORM

- Restricts no. of symbols on the right side of a production to be two
- Parse tree for derivation is a binary tree
- Every derivation of a string of n letters has exactly 2n-1 steps
- There can be more than one CNF for a CFG
- All CFGs can be converted to CNF

Rules

1) A non-terminal generating terminal

$$X \rightarrow x$$

2) A non-terminal generating two non-terminals

$$X \rightarrow XY$$

3) Only start symbol can generate λ, only if λ is a part of the language

$$S \rightarrow \lambda$$

Cleaned Grammar

- Before converting to CNF, the grammar must be cleaned (should not have a λ production except for the start symbol)
- If any variable produces λ, it is called a nullable variable
- Should not have unit productions (no useful operation)
- No useless productions (must remove)
  1) Non-generating variables $S \rightarrow aSb|s$ ← never terminates
  2) Unreachable variables $S \rightarrow aSb|a$; $A \rightarrow a|b$ ← unreachable
- The steps must be followed in order

  1) Eliminate λ productions
  2) Eliminate unit productions → each step must increase length of sentential form or no. of terminals
  3) Eliminate useless productions

Question 53

Clean up nullable variables (NOT CNF)

$S \rightarrow ASA \mid aB$
$A \rightarrow B \mid S$
$B \rightarrow b \mid \lambda$

removed $B \rightarrow \lambda$

$S \rightarrow ASA \mid aB \mid a$
$A \rightarrow B \mid \lambda \mid S$
$B \rightarrow b$

removed $A \rightarrow \lambda$

$S \rightarrow ASA \mid AS \mid SA \mid S \mid aB \mid a$
$A \rightarrow B \mid S$
$B \rightarrow b$

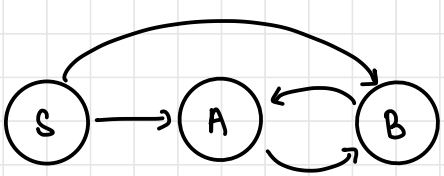
Question 54

Clean up unit productions (NOT CNF) replace with RHS

$S \rightarrow Aa \mid B$
$B \rightarrow A \mid bb$
$A \rightarrow a \mid bc \mid B$

Dependency graph

TODO: verify



$S \rightarrow Aa \mid bb \mid a \mid bc$
$B \rightarrow a \mid bc \mid bb$
$A \rightarrow a \mid bc \mid bb$

Replace B with RHS of B & its dependencies

Question 55

Clean up useless productions

(a) $S \to aSb \mid \lambda \mid A$
$A \to aA$ ← useless (cannot terminate)

$S \to aSb \mid \lambda$

(b) $S \to aS \mid AB \mid \lambda$
$A \to bA$
$B \to AA$

$S \to aS \mid \lambda$

Question 56

$S \to aB \mid bX$
$A \to Bad \mid bSX \mid a$
$B \to aSB \mid bBX$ ← useless (no terminal)
$X \to SBD \mid aBX \mid ad$

removing B

$S \to bX$
$A \to bSX \mid a$ ← not reachable
$X \to ad$

removing A

$S \to bX$
$X \to ad$

Question 57

Convert CFG to CNF

$S \rightarrow aA \mid aBB$
$A \rightarrow aaA \mid \lambda$
$B \rightarrow bB \mid bbC$
$C \rightarrow B$

1) Remove λ productions

$S \rightarrow aA \mid a \mid aBB$ ← account for λ
$A \rightarrow aaA \mid aa$ ← (with & without)
$B \rightarrow bB \mid bbC$
$C \rightarrow B$

2) Remove unit productions

$S \rightarrow aA \mid a \mid aBB$
$A \rightarrow aaA \mid aa$
$B \rightarrow bB \mid bbC$
$C \rightarrow bB \mid bbC$

replace $C \rightarrow B$ with B's RHS

3) Remove useless productions

$S \rightarrow aA \mid a$
$A \rightarrow aaA \mid aa$

remove B & C (non-terminating)

4) Convert to CNF

$S \rightarrow BA \mid a$
$A \rightarrow BBA \mid BB$
$B \rightarrow a$

introduce B (non-terminal)

not yet in CNF

$S \rightarrow BA \mid a$
$A \rightarrow DA \mid BB$
$B \rightarrow a$
$D \rightarrow BB$

Question 58

$S \rightarrow Aa \mid B \mid Ca$
$B \rightarrow aB \mid b$
$C \rightarrow Db \mid D$
$D \rightarrow E \mid d$
$E \rightarrow ab$

1) Unit productions

Dependency graph

no RHS

$S \rightarrow Aa \mid aB \mid b \mid Ca$
$B \rightarrow aB \mid b$
$C \rightarrow Db \mid ab \mid d$
$D \rightarrow ab \mid d$
$E \rightarrow ab$ ← unreachable

2) Useless productions

$S \rightarrow aB \mid b \mid Ca$
$B \rightarrow aB \mid b$
$C \rightarrow Db \mid ab \mid d$
$D \rightarrow ab \mid d$

3) Convert to CNF

$S \rightarrow AB \mid b \mid CA$
$A \rightarrow a$
$B \rightarrow AB \mid b$
$C \rightarrow DE \mid AE \mid d$
$E \rightarrow b$
$D \rightarrow AE \mid d$

Question 59

$S \rightarrow ABa$
$A \rightarrow aab$
$B \rightarrow Ac$

] clean grammar

$X \rightarrow a$
$Y \rightarrow b$
$Z \rightarrow c$
$S \rightarrow ABX$
$A \rightarrow XXY$
$B \rightarrow AZ$

$\longrightarrow$

$X \rightarrow a$
$Y \rightarrow b$
$Z \rightarrow c$
$W \rightarrow AB$
$V \rightarrow XY$
$S \rightarrow WX$
$A \rightarrow XV$
$B \rightarrow AZ$

Question 60

$S \rightarrow aSa \mid bSb \mid A \mid \lambda$
$A \rightarrow a \mid b \mid \lambda$

1) Nullable variables
   - language accepts λ

$S \rightarrow \lambda \mid aSa \mid bSb \mid aa \mid bb \mid A$
$A \rightarrow a \mid b$

2) Unit production

$S \rightarrow \lambda \mid aSa \mid bSb \mid aa \mid bb \mid a \mid b$
$A \rightarrow a \mid b$

3) Useless production

$S \rightarrow \lambda \mid aSa \mid bSb \mid aa \mid bb \mid a \mid b$

4) Convert to CNF

$A \rightarrow a$
$B \rightarrow b$
$S \rightarrow \lambda \mid ASA \mid BSB \mid AA \mid BB \mid a \mid b$

$S \rightarrow \lambda \mid CA \mid DB \mid AA \mid BB \mid a \mid b$
$A \rightarrow a$
$B \rightarrow b$
$C \rightarrow AS$
$D \rightarrow BS$

Question 61

$S \rightarrow a \mid aA \mid B$
$A \rightarrow aBB \mid \lambda$
$B \rightarrow Aa \mid b$

1) Remove λ

$S \rightarrow a \mid aA \mid B$
$A \rightarrow aBB$
$B \rightarrow Aa \mid b \mid a$

2) Remove unit

$S \rightarrow a \mid aA \mid Aa \mid b$
$A \rightarrow aBB$
$B \rightarrow Aa \mid b \mid a$

3) Convert to CNF (no useless)

$S \rightarrow a \mid XA \mid AX \mid b$
$X \rightarrow a$
$A \rightarrow XBB$
$B \rightarrow AX \mid b \mid a$

Let $BB \Rightarrow Y$

$S \rightarrow a \mid XA \mid AX \mid b$
$X \rightarrow a$
$A \rightarrow XY$
$Y \rightarrow BB$
$B \rightarrow AX \mid b \mid a$

Question 62

$S \rightarrow ASA \mid aB$
$A \rightarrow B \mid S$
$B \rightarrow b \mid \lambda$

1) Remove λ

$S \rightarrow ASA \mid aB \mid a$
$A \rightarrow \lambda \mid B \mid S$
$B \rightarrow b$

$S \rightarrow ASA \mid AS \mid SA \mid aB \mid a \mid S$
$A \rightarrow B \mid S$
$B \rightarrow b$

2) Remove unit

$S \rightarrow ASA \mid AS \mid SA \mid aB \mid a$
$A \rightarrow b \mid ASA \mid AS \mid SA \mid aB \mid a$
$B \rightarrow b$

3) Convert

$S \rightarrow XA \mid AS \mid SA \mid YB \mid a$
$X \rightarrow AS$
$Y \rightarrow a$
$A \rightarrow b \mid XA \mid AS \mid SA \mid YB \mid a$
$B \rightarrow b$

Question 63

$S \to aXbX$
$X \to aY \mid bY \mid \lambda$
$Y \to X \mid c$ c terminal c

1) Remove λ

$S \to abX \mid aXb \mid ab \mid aXbX$
$X \to aY \mid bY$
$Y \to \lambda \mid X \mid c$

$S \to abX \mid aXb \mid ab \mid aXbX$
$X \to a \mid aY \mid b \mid bY$
$Y \to X \mid C$

2) Remove unit

$S \to abX \mid aXb \mid ab \mid aXbX$
$X \to a \mid aY \mid b \mid bY$
$Y \to a \mid aY \mid b \mid bY \mid c$

3) Convert

$A \to a$
$B \to b$
$S \to ABX \mid AXB \mid AB \mid AXBX$
$X \to a \mid AY \mid b \mid BY$
$Y \to a \mid AY \mid b \mid BY \mid c$

$E \rightarrow AX$
$D \rightarrow BX$
$A \rightarrow a$
$B \rightarrow b$
$S \rightarrow AD \mid EB \mid AB \mid ED$
$X \rightarrow a \mid AY \mid b \mid BY$
$Y \rightarrow a \mid AY \mid b \mid BY \mid c$

Question 64

$S \rightarrow AbA$
$A \rightarrow Aa \mid \lambda$

1) Remove λ

$S \rightarrow bA \mid Ab \mid AbA \mid b$
$A \rightarrow Aa \mid a$

2) Convert

$B \rightarrow b$
$S \rightarrow BA \mid AB \mid ABA \mid b$
$A \rightarrow AX \mid a$
$X \rightarrow a$

$Y \rightarrow AB$
$B \rightarrow b$
$S \rightarrow BA \mid AB \mid YA \mid b$
$A \rightarrow AX \mid a$
$X \rightarrow a$

Question 65

$S \rightarrow BAB$
$B \rightarrow bba$
$A \rightarrow B$

1) Remove unit

$S \rightarrow BbbaB$
$B \rightarrow bba$
$A \rightarrow bba$ → useless

2) Remove useless

$S \rightarrow BbbaB$
$B \rightarrow bba$

3) Convert

$X \rightarrow a$
$Y \rightarrow B$
$S \rightarrow BYYXB$
$B \rightarrow YYX$

$X \rightarrow a$
$Y \rightarrow B$
$W \rightarrow YY$
$B \rightarrow WX$
$Z \rightarrow XB$
$S \rightarrow VZ$
$V \rightarrow BW$

Question 66

$S \rightarrow aX \mid Yb$
$X \rightarrow S \mid \lambda$
$Y \rightarrow bY \mid b$

1) Remove λ

$S \rightarrow aX \mid a \mid Yb$
$X \rightarrow S$
$Y \rightarrow bY \mid b$

2) Remove unit

$S \rightarrow aX \mid a \mid Yb$
$X \rightarrow aX \mid a \mid Yb$
$Y \rightarrow bY \mid b$

3) Convert

$S \rightarrow AX \mid a \mid YB$
$X \rightarrow AX \mid a \mid YB$
$Y \rightarrow BY \mid b$
$A \rightarrow a$
$B \rightarrow b$

CYK Algorithm

- Cocke, Younger, Kasami
- Also called membership algorithm / parsing algorithm
- bottom-up parsing
- dynamic programming
- only works with CNF CFGs

Question 67

$S \rightarrow AB|BC$
$A \rightarrow BA|a$
$B \rightarrow CC|b$
$C \rightarrow AB|a$

$W = baaba$

Triangle Table

$|W|=5$

(S) AC x_{15}				
$\emptyset$ x_{14}	SCA x_{25}			
$\emptyset$ x_{13}	B x_{24}	B x_{35}		
AS x_{12}	B x_{23}	SC x_{34}	AS x_{45}	
B x_{11}	AC x_{22}	AC x_{33}	B x_{44}	AC x_{55}
b	a	a	b	a

← substring length 5
← substring length 4
← substring length 3
← substring length 2
← substring length 1

can apply cross-product OR use formula (row 2)

b|a|a|b|a
1 2 3 4 5

x_{ij} = substring from position i to position j

Substrings of length 1

b a a b a (5 possibilities)

1 2 3 4 5

Substrings of length 2

b a a b a (4 possibilities)

1 2 3 4 5

Substrings of length 3

b a a b a (3 possibilities)

1 2 3 4 5

Substrings of length 4

b a a b a (2 possibilities)

Substrings of length

b a a b a (1 possibility)

Filling the Table

Square bracket: cross product

$$x_{ij} = [x_{i,i} \times x_{i+1,j}] \cup [x_{i,i+1} \times x_{i+2,j}] \cup$$
$$[x_{i,i+2} \times x_{i+3,j}] \cup [x_{i,i+3} \times x_{i+4,j}] \cup$$
$$\ldots [x_{i,j-1} \times x_{j,j}]$$

- for first row, write all non-terminals that produce the terminal

Using Cross Product

$x_{12} = B \times AC$

$= BA, BC$ ← what non-terminals produce?

$= A, S$

$x_{12} = AS$

$S \rightarrow AB \mid BC$
$A \rightarrow BA \mid a$
$B \rightarrow CC \mid b$
$C \rightarrow AB \mid a$

Using Formula

$x_{12} = [x_{11}, x_{22}]$ $\quad i=1, j=2$

$= B \times AC$
$= A, S$

$x_{23} = AC \times AC$
$= AA, AC, CA, CC$
$= AA, AC, CA, B$
(not produced)

$x_{23} = B$

$x_{34} = x_{33} \times x_{44}$

$= AC \times B$
$= AB, CB$
$= S, C$

$x_{45} = x_{44} \times x_{55}$

$= B \times AC$
$= BA, BC = A, S$

x_{15}				
x_{14}	x_{25}			
x_{13}	x_{24}	x_{35}		
AS x_{12}	B x_{23}	SC x_{34}	AS x_{45}	
B x_{11}	AC x_{22}	AC x_{33}	B x_{44}	AC x_{55}

$x_{13} = [x_{11} \times x_{23}] \cup [x_{12} \times x_{33}]$

$= [B \times B] \cup [AS \times AC]$

$= BB, AA, AC, SA, SC$

not produced $= \emptyset$

$S \rightarrow AB|BC$
$A \rightarrow BA|a$
$B \rightarrow CC|b$
$C \rightarrow AB|a$

$x_{24} = [x_{22} \times x_{34}] \cup [x_{23} \times x_{44}]$

$= [AC \times SC] \cup [B \times B]$

$= AS, AC, CS, CC, BB$

$= B$

x_{15}				
$\emptyset$ x_{14}	SCA x_{25}			
$\emptyset$ x_{13}	B x_{24}	B x_{35}		
AS x_{12}	B x_{23}	SC x_{34}	AS x_{45}	
B x_{11}	AC x_{22}	AC x_{33}	B x_{44}	AC x_{55}

$x_{35} = [x_{33} \times x_{45}] \cup [x_{34} \times x_{55}]$

$= [AC \times AS] \cup [SC \times AC]$

$= AA, AS, CA, CS, SA, SC, CC$

$= B$

$x_{14} = [x_{11} \times x_{24}] \cup [x_{12} \times x_{34}] \cup [x_{13} \times x_{44}]$

$= [B \times B] \cup [AS \times SC] \cup [\emptyset \times B]$

$= BB, AS, SS, AC, SC, \emptyset$

$= \emptyset$

$x_{25} = [x_{22} \times x_{35}] \cup [x_{23} \times x_{45}] \cup [x_{24} \times x_{55}]$

$= [AC \times B] \cup [B \times AS] \cup [B \times AC]$

$= AB, CB, BA, BS, BC = SCA$

S C A S

x_{15} ← if S is present in the cell, string belongs to grammar

$x_{15} = [x_{11} \times x_{25}] \cup [x_{12} \times x_{35}] \cup [x_{13} \times x_{45}] \cup [x_{14} \times x_{55}]$

$x_{15} = [B \times SCA] \cup [AS \times B] \cup [\emptyset \times AS] \cup [\emptyset \times AC]$

$= [BS, BC, BA] \cup [AB, SB]$

$= BS, BC, BA, AB, SB$

S A S C

(S)AC x_{15}				
$\emptyset$ x_{14}	SCA x_{25}			
$\emptyset$ x_{13}	B x_{24}	B x_{35}		
AS x_{12}	B x_{23}	SC x_{34}	AS x_{45}	
B x_{11}	AC x_{22}	AC x_{33}	B x_{44}	AC x_{55}

S → AB|BC
A → BA|a
B → CC|b
C → AB|a

Shortcut Method

S → AB|BC
A → BA|a
B → CC|b
C → AB|a

- first 2 rows, same

x_{13} = draw 2 arrows as shown
take cross of bottoms & tips

x_{15}				
x_{14}	x_{25}			
$\emptyset$ x_{13}	x_{24}	x_{35}		
AS x_{12}	B x_{23}	SC x_{34}	AS x_{45}	
B x_{11}	AC x_{22}	AC x_{33}	B x_{44}	AC x_{55}

$x_{13} = BB, AA, AC, SA, SC = \emptyset$

$S \rightarrow AB|BC$
$A \rightarrow BA|a$
$B \rightarrow CC|b$
$C \rightarrow AB|a$

$x_{24} = AS, AC, CS, CC, BB = B$

B

x_{15}				
x_{14}	x_{25}			
$\emptyset$ x_{13}	B x_{24}	B x_{35}		
AS x_{12}	B x_{23}	SC x_{34}	AS x_{45}	
B x_{11}	AC x_{22}	AC x_{33}	B x_{44}	AC x_{55}

$x_{25} = AA, AS, CA, CS, SA, SC, CA, CC$
$= B$

B

$x_{14} = [x_{11} \times x_{24}] \cup [x_{13} \times x_{44}] \cup [x_{12} \times x_{34}]$

$= [B \times B] \cup [\emptyset \times B] \cup [AS \times SC]$

$= BB, AS, AC, SS, SC$

x_{15}				
$\emptyset$ x_{14}	x_{25}			
$\emptyset$ x_{13}	B x_{24}	B x_{35}		
AS x_{12}	B x_{23}	SC x_{34}	AS x_{45}	
B x_{11}	AC x_{22}	AC x_{33}	B x_{44}	AC x_{55}

$x_{25} = [x_{22} \times x_{35}] \cup [x_{24} \times x_{55}] \cup [x_{23} \times x_{45}]$

$= [AC \times B] \cup [B \times AC] \cup [B \times AS]$

$= AB, CB, BA, BC, BS$

S C A S

x_{15}				
$\emptyset$ x_{14}	SCA x_{25}			
$\emptyset$ x_{13}	B x_{24}	B x_{35}		
AS x_{12}	B x_{23}	SC x_{34}	AS x_{45}	
B x_{11}	AC x_{22}	AC x_{33}	B x_{44}	AC x_{55}

GREIBACH NORMAL FORM

- In GNF, only head of the production should be a terminal
- Any number of non-terminals after the first symbol

$$A \rightarrow a\alpha \quad \alpha \in V^* \text{ (variables)}$$
$$A \rightarrow a$$

- No. of derivations = length of string (|w|)
- Grammar should be cleaned (just like in CNF)

Question 68

Convert to GNF & derive "aababb"

$S \rightarrow aB \mid bA$
$A \rightarrow a \mid aS \mid bAA$
$B \rightarrow b \mid bS \mid aBB$

already in GNF

$S \Rightarrow aB \rightarrow 1$
$\Rightarrow aaBB \rightarrow 2$
$\Rightarrow aabSB \rightarrow 3$
$\Rightarrow aababB \rightarrow 4$
$\Rightarrow aababB \rightarrow 5$
$\Rightarrow aababb \rightarrow 6$

OR

$S \Rightarrow aB \rightarrow 1$
$\Rightarrow aaBB \rightarrow 2$
$\Rightarrow aabB \rightarrow 3$
$\Rightarrow aababB \rightarrow 4$
$\Rightarrow aababB \rightarrow 5$
$\Rightarrow aababb \rightarrow 6$

- length of string = 6

Question 69

$S \rightarrow AB$
$A \rightarrow aA \mid bB \mid b$
$B \rightarrow b$

convert to GNF (already cleaned)

Replace A with RHS

$S \rightarrow aAB \mid bBB \mid bB$
$A \rightarrow aA \mid bB \mid b$
$B \rightarrow b$

Question 70

$S \rightarrow abSb \mid aa$

convert to GNF (using substitution rule)

$A \rightarrow a$
$B \rightarrow b$
$S \rightarrow aBSB \mid aA$

Question 71

$S \rightarrow ABb \mid a$
$A \rightarrow aaA \mid B$
$B \rightarrow bAB$

i) Remove useless

$S \rightarrow ABb \mid a$
$A \rightarrow aaA \mid bAB$
$B \rightarrow bAB$

2) Remove useless

$S \rightarrow a$ → in GNF

Question 72

$S \rightarrow aSb \mid bSa \mid SS \mid \lambda$

$S \rightarrow \lambda \mid aSb \mid ab \mid bSa \mid ba \mid SS \mid S$

$A \rightarrow a$
$B \rightarrow b$
$S \rightarrow \lambda \mid aSB \mid aB \mid bSA \mid bA$

Question 73

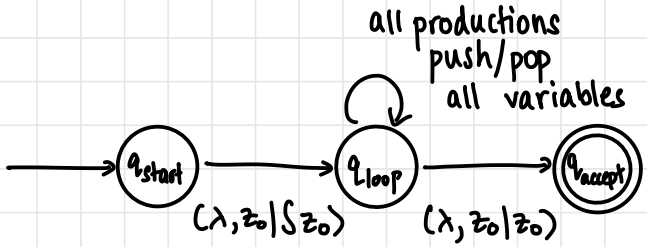
$S \rightarrow aA \mid bB$
$B \rightarrow bB \mid \lambda$
$A \rightarrow aA \mid \lambda$

$S \rightarrow a \mid aA \mid b \mid bB$
$B \rightarrow bB \mid b$
$A \rightarrow a \mid aA$

EQUIVALENCE OF CFG & PDA

Conversion of CFG to PDA

Skeleton of PDA



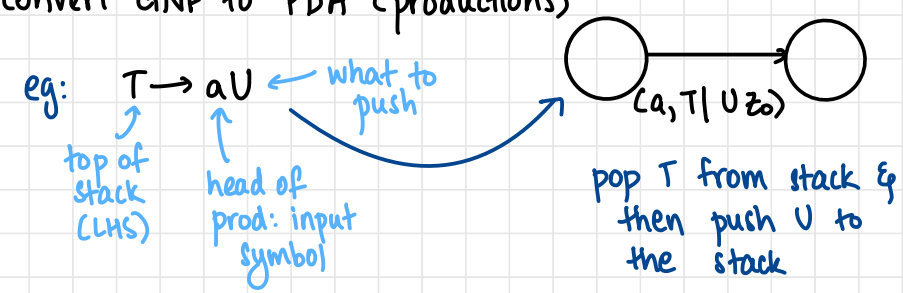
compulsory !!

↑ artificially push S to stack] to make S the bottom of the stack

Algorithm

1) Convert CFG to GNF

2) Convert GNF to PDA (productions)



eg: S → aABC ⟶ $(a, S | ABC Z_0)$

(pop)

eg: T → a ⟶ $(a, T | \lambda)$

(pop)

Question 74

Convert CFG to PDA

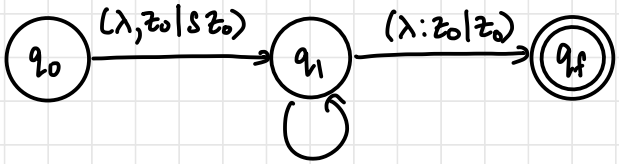
$$L = \{ww^R \mid w \in \{a,b\}^*\}$$ even palindrome

$$S \rightarrow aSa \mid bSb \mid \lambda$$

1) Convert to GNF

$$A \rightarrow a$$
$$B \rightarrow b$$
$$S \rightarrow aSA \mid bSB \mid \lambda$$

2) Convert to PDA



push $(a : S \mid SA)$
push $(b : S \mid SB)$
pop $(\lambda : S \mid \lambda)$
pop $(a : A \mid \lambda)$
pop $(b : B \mid \lambda)$

(NPDA)

w = "abba"

$$S \overset{lm}{\Rightarrow} aSA \Rightarrow abSBA \Rightarrow abBA \Rightarrow abbA \Rightarrow abba$$

using
$S \rightarrow aSA$
$S \rightarrow bSB$
$S \rightarrow \lambda$
$B \rightarrow b$
$A \rightarrow a$

$\delta(q_0, abba, z_0)$

$\vdash \delta(q_1, abba, Sz_0)$

$\vdash \delta(q_1, \_bba, SAz_0)$

$\vdash \delta(q_1, \_\_ba, SBAz_0)$

$\delta(q_1, \_\_\_a, SBBAz_0)$ ✗

$\delta(q_1, \_\_ba, BAz_0)$

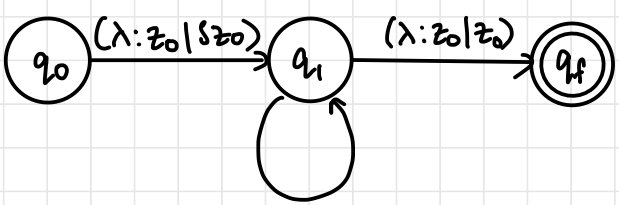
$\vdash \delta(q_1, \_\_\_a, Az_0)$

$\vdash \delta(q_1, \_\_\_\_, z_0)$

$\vdash \delta(q_f, \lambda, z_0)$

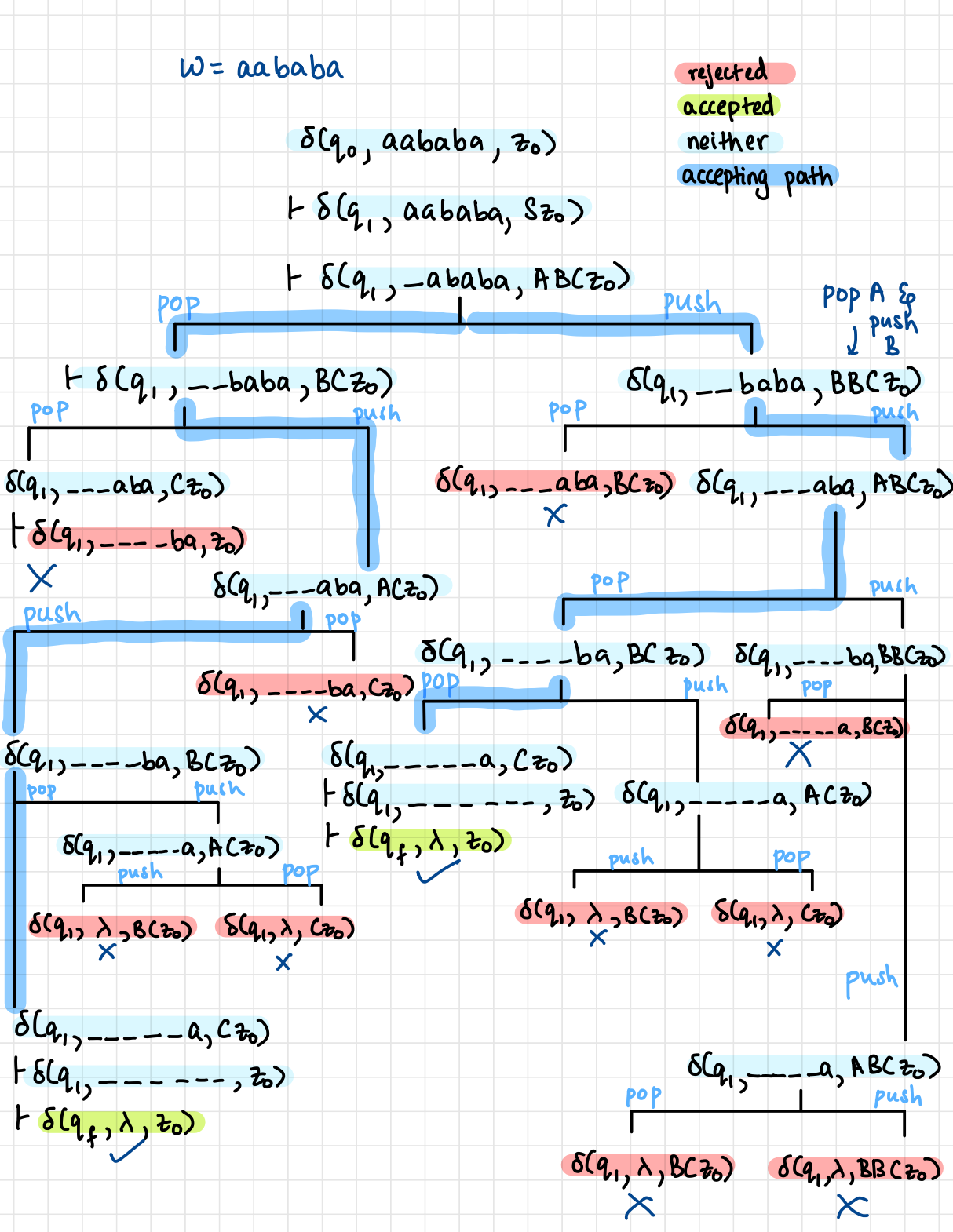
Question 75

$S \rightarrow aABC$
$A \rightarrow aB \mid a$
$B \rightarrow bA \mid b$
$C \rightarrow a$



(NPDA)

(a:S|ABCz0)
(a:A|Bz0)
(a:A|λ)
(b:B:Az0)
(b:B:λ)
(a:C:λ)


w = aababa
rejected
accepted
neither
accepting path
δ(q0, aababa, Z0)
⊢ δ(q1, aababa, SZ0)
⊢ δ(q1, _ababa, ABCZ0)
pop
push
pop A & push B
⊢ δ(q1, __baba, BCZ0)
δ(q1, __baba, BBCZ0)
δ(q1, ___aba, CZ0)
⊢ δ(q1, ____ba, Z0)
δ(q1, ___aba, ACZ0)
δ(q1, ____ba, CZ0)
δ(q1, ____ba, BCZ0)
δ(q1, ____ba, BBCZ0)
δ(q1, _____a, CZ0)
⊢ δ(q1, ______, Z0)
⊢ δ(qf, λ, Z0)
δ(q1, ____a, ACZ0)
δ(q1, λ, BCZ0)
δ(q1, λ, CZ0)
δ(q1, ____a, ABCZ0)
δ(q1, λ, BBCZ0)