AUTOMATA FORMAL LANGUAGES AND LOGIC

First Order Logic – Numbers, Sets and Lists

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NATURAL NUMBERS AND PEANO AXIOMS

- We need a predicate **NatNum** that will be true of natural numbers

PEANO AXIOMS

- We need one constant symbol, 0
- We need one function symbol, S (successor).
- The Peano axioms define natural numbers and addition.

NatNum(0) .

$\forall$ n NatNum(n) $\Rightarrow$ NatNum(S(n))

For example: 0, S(0), S(S(0))

NATURAL NUMBERS

Successor function

$$\forall n\ 0 \neq S(n)\ .$$

$$\forall m, n\ m \neq n \Rightarrow S(m) \neq S(n)$$

Define addition in terms of the successor function:

$$\forall m\ NatNum(m) \Rightarrow +(0,m) = m\ .$$

$$\forall m, n\ NatNum(m) \wedge NatNum(n) \Rightarrow +(S(m), n) = S(+(m, n))\ .$$

NATURAL NUMBERS -INFIX

We can also write S(n) as n+ 1, so the second axiom becomes

$$\forall m, n \; NatNum(m) \land NatNum(n) \Rightarrow (m + 1) + n = (m + n) + 1 \,.$$

SET

- The empty set is a constant written as {}.
- There is one unary predicate, **Set**, which is true of sets.
- The binary predicates are ***x∈s*** (x is a member of set s) and ***s1 ⊆ s2*** (set s1 is a subset, not necessarily proper, of set s2).
- The binary functions are s1 ∩ s2 (the intersection of two sets), ***s1 ∪ s2*** (the union of two sets), and {x|s} (the set resulting from adjoining element x to set s).

Axioms

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One possible set of axioms is as follows:

1. The only sets are the empty set and those made by adjoining something to a set:

$$\forall s: Set(s) \Leftrightarrow (s=\{\}) \vee (\exists x, s2\ Set(s2) \wedge s=\{x|s2\})$$

2. The empty set has no elements adjoined into it. In other words, there is no way to decompose {} into a smaller set and an element:

$$\neg\exists x, s: \{x|s\}=\{\}$$

3. Adjoining an element already in the set has no effect:

$$\forall x, s:\ x\in s \Leftrightarrow s=\{x|s\}\ .$$

Axioms

4. x is a member of s if and only if s is equal to some set s2 adjoined with some element y, where either y is the same as x or x is a member of s2:

$$\forall x, s: x \in s \Leftrightarrow \exists y, s2\ (s=\{y|s2\} \wedge (x=y \vee x \in s2)) .$$

5. A set is a subset of another set if and only if all of the first set's members are members of the second set:

$$\forall s1, s2: s1 \subseteq s2 \Leftrightarrow (\forall x\ x \in s1 \Rightarrow x \in s2) .$$

6. Two sets are equal if and only if each is a subset of the other:

$$\forall s1, s2: (s1 = s2) \Leftrightarrow (s1 \subseteq s2 \wedge s2 \subseteq s1) .$$

Axioms

7. An object is in the intersection of two sets if and only if it is a member of both sets:

$$\forall\, x, s1, s2 :\ x \in (s1 \cap s2) \Leftrightarrow (x \in s1 \land x \in s2)\ .$$

8. An object is in the union of two sets if and only if it is a member of either set:

$$\forall\, x, s1, s2 :\ x \in (s1 \cup s2) \Leftrightarrow (x \in s1 \lor x \in s2)\ .$$

THANK YOU

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