

ENGINEERING PHYSICS

UNIT-5

DIELECTRIC & MAGNETIC MATERIALS

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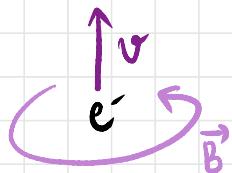
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(I)

MAGNETISM

Origin of Magnetism

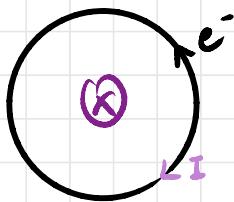
- Charges in motion responsible for magnetism



magnetism due to relativistic effects

Magnetic Dipole Moment

e^- in orbit



$$\begin{aligned} \mu &= IA \\ &= -\frac{e}{T} \pi r^2 \end{aligned}$$

$$\mu = -\frac{e}{2\pi} v \pi r^2 = -\frac{evr}{2}$$

$$\mu = -\frac{e}{2m} L$$

$$\vec{\mu} = -\frac{e}{2m} \vec{L}$$

$$\vec{\mu} = \gamma \vec{L}$$

$$\gamma = \frac{\mu}{L}$$

↑ gyromagnetic ratio

Quantum Picture

- \vec{L} is quantised

$$L = n\hbar$$

$$L = \sqrt{l(l+1)} \hbar$$

$$\mu = \frac{-e}{2m} n\hbar$$

$n=1$

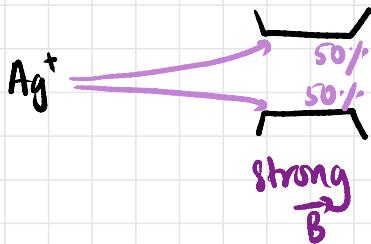
$$\mu_B = \frac{-e}{2m} \hbar \quad \begin{matrix} \text{Bohr magneton} \\ \text{quanta of } \mu \end{matrix}$$
$$= 9.214 \times 10^{-24} \text{ JT}^{-1}$$

- Spin from e^- also contributes to angular momentum — spin angular momentum

$$M_s = \frac{e\sqrt{s(s+1)}}{2m} \hbar$$

SWF of H-atom

Stern-Gerlach Experiment



something other than charge contributes

spin; either up or down
tiny magnet

$s = 1/2$ (spin quantum number - $m_s = \pm 1/2$)

- Two dipole moments interact — resultant

$$\vec{J} = \vec{L} + \vec{S}$$

$$J = \sqrt{j(j+1)} \hbar$$

total angular momentum

2j+1 orientations

Landé g-factor

- Energy of system: interaction of $\vec{\mu}$ with \vec{B}

$$g_e = \frac{1 + j(j+1) + s(s+1) - l(l+1)}{2j(j+1)}$$

$$\mu = \frac{e}{2m} g_e \hbar$$

Pure Orbital

$$g_e \text{ for } s=0, \quad \vec{J} = \vec{L} \Rightarrow j=l$$

$$g_e = 1 \quad (\text{no spin angular momentum})$$

$$\mu_{orb} = \frac{e \hbar}{2m}$$

Pure Spin

$$l=0, \quad \vec{J} = \vec{s} \Rightarrow j=s$$

$$g_e = 1+1 = 2$$

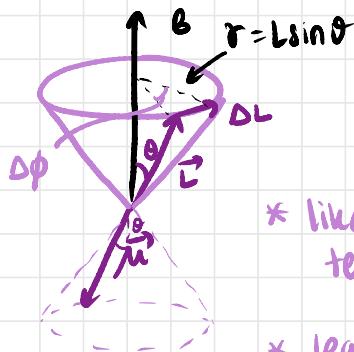
$$\mu_s = \left(\frac{e}{2m} \hbar \right) \times 2$$

- Therefore, spin dipole moment > orbital dipole moment
- Significant contribution from spin
- All magnetic properties from spin (μ_s significant)

for any system

$$\mu = \frac{e}{2m} \tau g e$$

Behaviour of Dipole in External Magnetic Field



$\vec{\tau}$ changes with time

* like cycle wheel with tension \uparrow

* learn about μ through L

• Torque on $\vec{\tau}$

• $\vec{\tau}$ starts moving — gyroscopic precession

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

$$\vec{\mu} \times \vec{B} = \vec{\tau}$$

$$\mu B \sin \theta = \frac{dL}{dt}$$

$$\Delta L = L \sin \theta \Delta \phi$$

$$\frac{\Delta L}{\Delta t} = L \sin \theta \frac{\Delta \phi}{\Delta t}$$

$$\lim_{\Delta t \rightarrow 0} \frac{\Delta L}{\Delta t} = \frac{dL}{dt} = L \sin \theta \frac{d\phi}{dt} \leftarrow \text{frequency } \omega_L$$

$$\mu B \sin \theta = \frac{dL}{dt} = L \sin \theta \omega_L \leftarrow \text{Larmor frequency}$$

$$MB = L \omega_L$$

\downarrow
gyromagnetic
ratio

$$\omega_L = \frac{\mu}{L} B = \gamma B$$

$$\omega_L = \frac{e}{2m} B$$

- Dipole moment in \vec{B} does not align, but precesses about it at a frequency

for $B = 1T$

$$\omega_L = 8.79 \times 10^{10} \text{ rad T}^{-1} \text{ s}^{-1}$$

$$f_L = \frac{e}{4\pi m} B$$

14 GHz T⁻¹

for $B = 1T$

$$f_L = 1.4 \times 10^{10} \text{ Hz T}^{-1}$$

- Analysis can be repeated on nucleus - nuclear dipole moment (proton)
- Relative strength of dipole moment smaller

$$\mu_{\text{nu}} = \frac{e}{2m_p} B$$

- Does not contribute much to dipole moment
- f_L for protons = $\frac{e}{4\pi m_p} B$

$$\omega_L = 4.78 \times 10^7 \text{ rad s}^{-1} T^{-1}$$

$$f_L = 7.8 \times 10^6 \text{ Hz T}^{-1}$$

$$= 7.8 \text{ MHz T}^{-1} \quad (\underbrace{\sim \frac{1}{1840}}_{\text{times}} \text{ times})$$

- Basis of nuclear magnetic resonance (NMR) used in medical imaging
- e^- and protons accelerated \Rightarrow emit radiation
- 1 in kHz and 1 in MHz
- Can find radiation, find g_e and determine the kinds of atoms

Terms

1. Magnetisation (M)

- dipole moment per unit volume

2. Magnetic Flux Density (B) Tesla = Wb/m²

- magnetic field developed inside material

3. Magnetic Field (H)

- magnetic field applied to material

4. Magnetic Susceptibility (χ)

$$\chi_m = \frac{M}{H}$$

$$B = \mu_0 (H + M) = \mu_0 H (1 + \chi)$$

$$B = \mu_0 (1 + \chi) H$$

$$B = \mu_0 \mu_r H$$

$$B = \mu H$$

$$\mu_r = 1 + \chi$$

relative permeability

Diagmagnetic Behaviour of Atomic Dipoles in Weak \vec{B}_{ext}

- First response of a material placed in weak \vec{B} ' not to get magnetised
- Induced magnetic moment which opposes magnetic field through circulating currents
- Diamagnetic response of all materials, irrespective of type
- Due to precessing charge,

$$\begin{aligned} M_{\text{ind}} &= -\frac{e}{2} \omega r^2 \\ &= -\frac{e}{2} \left(\frac{e}{2am} \right) Br^2 \end{aligned}$$

$$\mu = \frac{-e^2}{4m} r^2 B$$

If there are N_a atoms / volume and $2 e^-$ per atom,

$$M = N_a Z \mu_{\text{ind}} = -N_a Z \frac{e^2}{4m} r^2 B = -N_a Z \frac{e^2}{4m} r^2 \mu_0 H$$

$$\chi_m = \frac{M}{H} = -N_a Z \frac{e^2}{4m} r^2 \mu_0$$

$$\chi_m = -\frac{N_a Z \mu_0 e^2 r^2}{4m}$$

negative, order of 10^{-5}

Types of Magnetic Materials

1. Diamagnetic
2. Paramagnetic
3. Ferromagnetic
4. Anti-ferromagnetic
5. Ferrimagnetic

DIAMAGNETIC MATERIALS

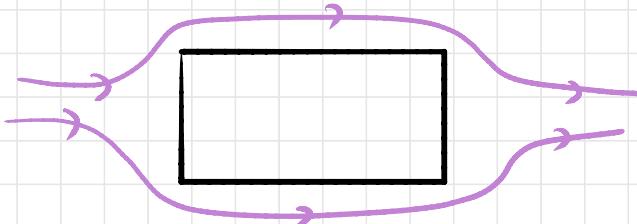
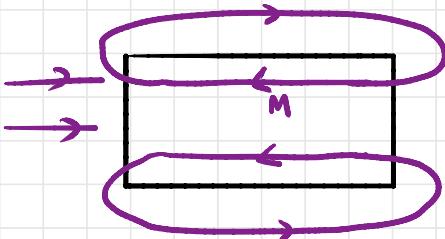
Lenz's Law at atomic level

- Materials with diamagnetic response even for high \vec{B} (negative susceptibility $\sim -10^{-5}$)
- Classical $X_{\text{dia}} = -Z \frac{Na e^2 \mu_0}{6m} \langle r^2 \rangle$

Na = no. of atoms / volume

$\langle r^2 \rangle$ = mean square value of radius

- X_{dia} independent of temperature
- e^- shells / subshells are closed
- Superconductors - perfect diamagnetic
- \vec{M} opposes \vec{H} (magnetisation opposes applied field)



add up outside, cancel inside

Examples of Diamagnetic Materials

- water, wood, NaCl
- organic compounds (petroleum), plastics
- Cu, Hg, Au, Bi, Sb

highest χ in diamagnetic (-1.66×10^{-4})

PARAMAGNETIC MATERIALS

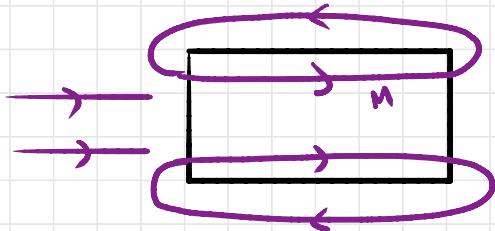
- Unquenched spin/ angular momentum
- Atoms do not interact with each other

when $\vec{H}' = 0$



randomly oriented
dipoles

- Thermal energy \gg energy of interaction between dipoles ($\mu_B \ll kT$) either in \vec{H}' or no \vec{H}'
- In the presence of \vec{H}'



cancels outside, adds up inside

- Due to unpaired electrons (net μ)

$$\mu = g_e \mu_B m_j$$

N atoms per unit volume,

$$M = N g_e \mu_B m_j \text{ (Probability factor)}$$

Temperature Dependence

- Two competing energies: temp. and interaction energy

$$\begin{aligned} 1. & \quad \Delta E = \mu B = -g_e \mu_B m_j \cdot B \\ & \text{interaction energy} \\ 2. & \quad kT \end{aligned}$$

Langevin - classical

- Maxwell-Boltzmann distribution (quantised - M_j)

$$M = \frac{\sum N g_e \mu_B m_j e^{(g_e \mu_B m_j \mu_0 H) / kT}}{\sum_{-j} e^{(g_e \mu_B m_j \mu_0 H) / kT}}$$

Case I: $\mu_B < kT$

Case II: $\mu_B > kT$

Case I: $\mu_B < kT$

exponent < 1 ($x < 1$)

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$M = \frac{N g_e \mu_B \sum_j m_j \left(1 + \frac{m_j g_e \mu_B \mu_0 H}{kT} \right)}{\sum_j 1 + \frac{m_j g_e \mu_B \mu_0 H}{kT}}$$

\downarrow
2j+1 orientations ($-j \dots 0 \dots j$)

$$= \frac{N g_e \mu_B \sum_{-j}^j m_j + (m_j)^2 \frac{g_e \mu_B \mu_0 H}{kT}}{\sum_{-j}^j 1 + \frac{m_j g_e \mu_B \mu_0 H}{kT}}$$

$$\sum_j m_j = 0 = j + (j-1) + \dots + 0 + \dots - (j-1) - j$$

$$\sum_{-j}^j (m_j)^2 = j^2 + (j-1)^2 + \dots + (j-1)^2 + j^2$$

$$= 2 \sum_1^j (m_j)^2 = 2 \frac{(j)(j+1)(2j+1)}{6}$$

$$= \frac{j(j+1)(2j+1)}{3}$$

$$\sum_j 1 = 2j+1$$

$$M = \frac{Ng_e \mu_B (j)(j+1) \cancel{(2j+1)} \cancel{\left(\frac{g_e \mu_B \mu_0 H}{kT}\right)}}{3}$$

$$= \frac{N(g_e)^2 (\mu_B)^2 \mu_0 H}{kT} \frac{j(j+1)}{3}$$

$$= \frac{N(g_e)^2 (\mu_B)^2 \mu_0 H}{kT} \frac{j^2}{3\hbar^2}$$

total angular momentum
not either spin or orbital

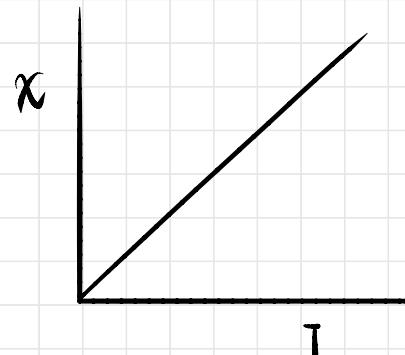
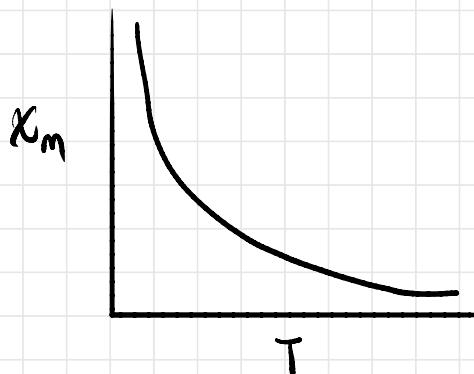
$$M = \frac{C}{T} H$$

$$\chi_m = \frac{M}{H} = \frac{C}{T}$$

$$\chi_m \propto \frac{1}{T}$$

susceptibility inversely proportional to T

Curie's Law
for Paramagnetic materials



Case II: $\mu_B > kT$

- Dipoles can interact with each other
- Magnetic interaction energy much greater than thermal energy

$$M = \frac{Nge/\mu_B}{\sum_j m_j e^{m_j g_e \mu_0 \mu_B H / kT}} \frac{\sum_j m_j e^{m_j g_e \mu_0 \mu_B H / kT}}{\sum_j e^{m_j g_e \mu_0 \mu_B H / kT}}$$

low temp OR
high μ_B
(2 diff.
scenarios)

$$x = g_e \mu_0 \mu_B H / kT$$

$$M = \frac{Nge/\mu_B \sum_j m_j e^{m_j x}}{\sum_j e^{m_j x}}$$

(Similar to BBR,
page 31 Unit 1)

$$M = Nge/\mu_B \left(\frac{d}{dx} \left(\ln \left(\sum_j e^{m_j x} \right) \right) \right)$$

derivative
of \ln

$$= Nge/\mu_B \left(\frac{d}{dx} \left(\ln \left[e^{jx} + e^{(j-1)x} + e^{(j-2)x} + \dots + e^{-jx} \right] \right) \right)$$

$$= N g_e \mu_B \left(\frac{d}{dx} \left(\ln \left(e^{jx} \underbrace{\left[1 + e^{-x} + e^{-2x} + \dots + e^{-2jx} \right] \right) \right) \right)$$

$$= N g_e \mu_B \frac{d}{dx} \left(\ln \left(e^{jx} \left(\frac{1 - e^{-(2j+1)x}}{1 - e^{-x}} \right) \right) \right)$$

GP $\frac{a(1-r^n)}{1-r}$

$$= N g_e \mu_B \frac{d}{dx} \left[\ln \left(\frac{e^{jx} - e^{-(j+1)x}}{1 - e^{-x}} \right) \right]$$

$$= N g_e \mu_B \frac{d}{dx} \left[\ln \left(\left(\frac{e^{jx} - e^{-(j+1)x}}{1 - e^{-x}} \right) \frac{e^{x/2}}{e^{x/2}} \right) \right]$$

$$= N g_e \mu_B \frac{d}{dx} \left[\ln \left(\frac{e^{(j+1/2)x} - e^{-(j+1/2)x}}{e^{x/2} - e^{-x/2}} \right) \right]$$

$$= N g_e \mu_B \frac{d}{dx} \left[\ln \left(\frac{\sinh \left(\frac{(2j+1)}{2} x \right)}{\sinh(x/2)} \right) \right]$$

$$= N g_e \mu_B \frac{d}{dx} \left[\ln \left(\sinh \left(\frac{(2j+1)}{2} x \right) \right) - \ln \left(\sinh \frac{x}{2} \right) \right]$$

$$= N g_e \mu_B \left[\frac{\cosh \left(\frac{(2j+1)}{2} x \right)}{\sinh \left(\frac{(2j+1)}{2} x \right)} \cdot \left(\frac{2j+1}{2} \right) - \frac{\cosh(x/2)}{\sinh(x/2)} \left(\frac{1}{2} \right) \right]$$

$$= \frac{N g_e \mu_B}{2} \left[(2j+1) \coth \left(\frac{2j+1}{2} x \right) - \coth \left(\frac{x}{2} \right) \right]$$

$$\text{Let } x = \frac{a}{j}$$

$$M = \frac{Ng_e \mu_B}{2} \left[(2j+1) \coth\left(\frac{2j+1}{2j}\right)a - \coth\left(\frac{a}{2j}\right) \right]$$

$$M = \underbrace{Ng_e \mu_B}_\text{constant for a system} \left[\frac{2j+1}{2j} \coth\left(\frac{2j+1}{2j}\right)a - \frac{1}{2j} \coth\left(\frac{a}{2j}\right) \right]$$

$$B_j(a) = \frac{2j+1}{2j} \coth\left(\frac{2j+1}{2j}\right)a - \frac{1}{2j} \coth\left(\frac{a}{2j}\right)$$

Brillouin function

- Maximum magnetisation a material can have (when all are aligned) — saturation magnetisation
- $M = M_s B_j(a)$

$$B_j(a) = \frac{M}{M_s}$$

Max = 1

$B_j(a)$ can have a max. value of 1

Brillouin Function $B_j(a)$

Case I: $j \rightarrow \infty$

- if j is very large, $2j \gg 1$

$$B_j(a) = \coth(a) - \frac{1}{a} \frac{\cosh(a/2j)}{\sinh(a/2j)}$$

$$= \frac{1}{a} \frac{\cosh(a/2j)}{\sinh(a/2j)}.$$

$$B_j(a) = \coth(a) - \frac{1}{a}$$

Langevin's
function
(classical)

- Langevin's method: dipoles orient in all possible orientations ($-\pi$ to $+\pi$)
- Implication of j : for $j > m_j$: $-j$ to j ($2j+1$) orientations
- As $j \gg$ large, $2j+1 \rightarrow$ large \rightarrow Langevin's function
- Langevin's function

$$L(a) = \frac{a}{3} - \frac{a^3}{45} + \frac{2a^5}{945} - \dots$$

$$a = x_j$$

$$\chi = \frac{g_e \mu_0 / \mu_B H}{kT}$$

$$\therefore a = \frac{g_e \mu_0 / \mu_B H_j}{kT}$$

small a / small χ
means high
temperature

\therefore higher order terms can be neglected

$$\therefore L(a) = \frac{a}{3}$$

• For large values of j and small values of a ,

$$B_j(a) = \frac{a}{3} = \frac{M}{M_s}$$

$$M = M_s \frac{a}{3} \quad ; \quad M_s = N g_e \mu_B j$$

$$M = N g_e \mu_B j \frac{g_e \mu_0 / \mu_B H_j}{3 kT}$$

$$M = \frac{N g_e^2 \mu_B^2 \mu_0 H j^2}{3 kT}$$

Similar to
Curie's Law

Curie's Law

$$M = \frac{N g e^2 \mu_B^2 M_0 H}{kT} \frac{j(j+1)}{3}$$

At high values of j , $j(j+1) \approx j^2$

- For any material, at large values of j and high temperatures, paramagnetic behaviour still persists

$$\chi = \frac{N g e^2 \mu_B^2 M_0}{kT} \frac{j^2}{3} \quad (\text{positive value})$$

Case II : $j = 1/2$

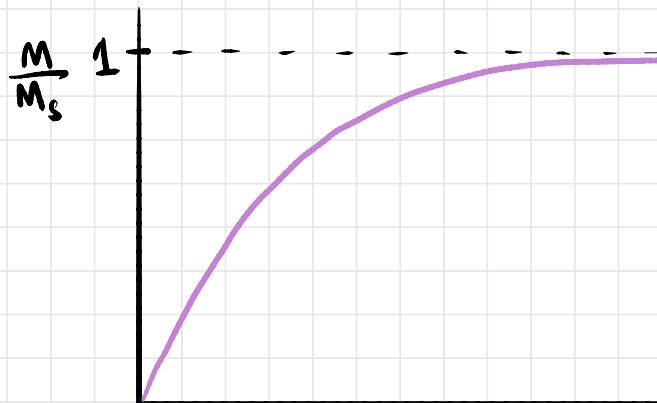
- j is purely spin ; no orbital

$$B_j(a) = \frac{2j+1}{2j} \coth\left(\frac{2j+1}{2j}a\right) - \frac{1}{2j} \coth\left(\frac{a}{2j}\right)$$

$$\begin{aligned} B_{1/2}(a) &= 2 \coth(2a) - \coth a \\ &= \frac{2(1 + \tanh^2 a)}{2 \tanh a} - \frac{2}{2 \tanh a} \end{aligned}$$

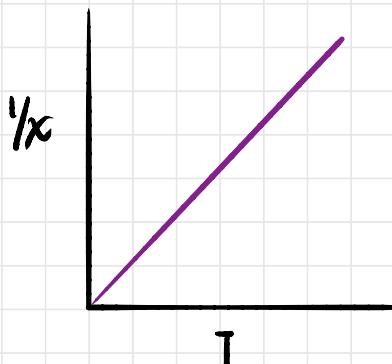
$$B_{1/2}(a) = \tanh a$$

$$M = M_s \tanh a$$

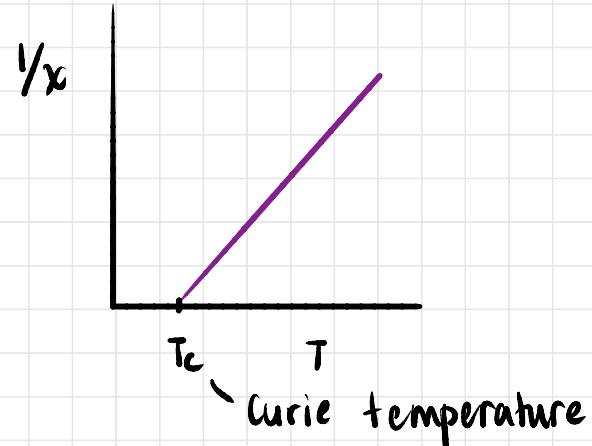


$$\chi = \frac{g \mu_B \mu_0 H_0}{kT}$$

$1/\chi$ vs T



pure paramagnetic



Not pure paramagnetic

Weiss Molecular Field Theory

- Dipole creates magnetic field surrounding it
- Influences neighbouring dipoles

$$\chi_{\text{actual}} = \frac{M}{H_{\text{tot}}} = \frac{\text{magnetisation}}{\text{external + internal field}}$$

$$\chi_{\text{actual}} = \frac{M}{H + \lambda M} = \frac{C}{T}$$

net contribution
of neighbours

$$C(H + \lambda M) = MT$$

$$M(T - \lambda C) = CH$$

$$\chi_m = \frac{M}{H} = \frac{C}{T - \lambda C} \quad \text{λC must have units of temp}$$

measured
experimentally

$$\chi_m = \frac{C}{T - T_c}$$

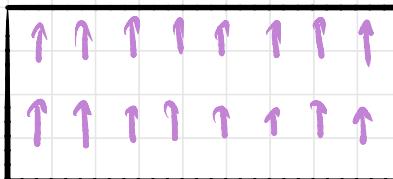
- Materials have paramagnetic behaviour above a temperature T_c (Curie temperature)
- For purely paramagnetic substances, $T_c = 0$ and graph passes through origin

- Ferromagnetic substances at high temperatures: thermal energy \approx field energy and leads to random orientations of dipole

FERROMAGNETIC MATERIALS

- $\chi_m = +ve$, 1 to $\sim 10^5$
- For $T < T_c$, ordered dipoles

strong interaction

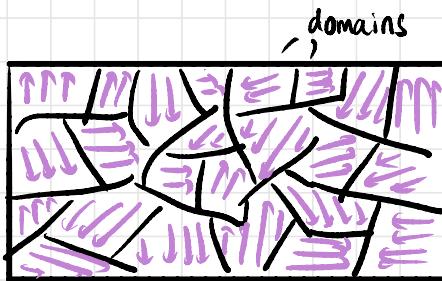


pure single crystal

(too expensive)

- Even for $H=0$, M is nonzero ($M>0$)
- Adjacent dipoles orient in the same direction

density close to crystalline density

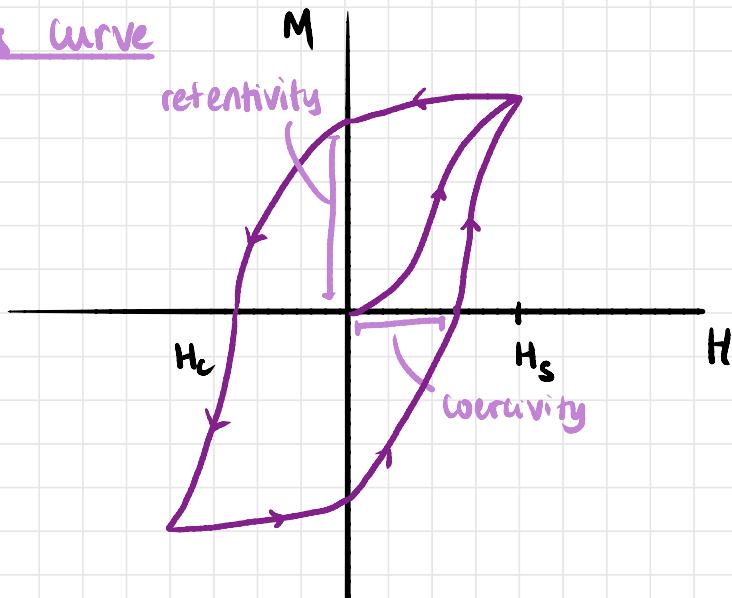


sintered polycrystalline material

(high temp, pressure)

- For $H=0$, $M=0$ (net $M=0$)
- Need to be magnetised at high fields

Hysteresis Curve



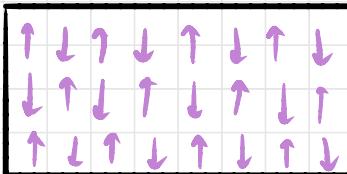
- Area under curve — energy lost
- Hysteresis — memory
- Hard discs — ferromagnetic materials (thin films)
magnetic memory
- Floppy drive \longrightarrow 1 TB hard disc
- Domain sizes — 50-100 nm \Rightarrow can individually manipulate dipoles
- Formatting discs: $-H_c$ or $+H_c$ used
- High T_c materials preferred
- Partly filled 3d-4d shells filled using Hund's Rule
- Fe, Ni, Co, Gd, Dysporium, Samarium, Neodymium w/ Co

ANTIFERROMAGNETIC MATERIALS

- for $T < T_N$ ↓ Neel temperature

- also ordered dipoles, magnetisation is 0

antiparallel
dipoles
(equal)



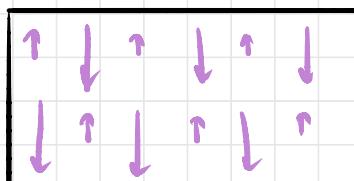
highly strong coupling
between neighbours

- magnetic insulators (can be used on ferromagnetic) due to high inertia
- also used in magnetic memory (in read-write head)
- MnO, NiO, CoO

FERRIMAGNETIC MATERIALS

- for $T < T_c$
- Ordered dipoles, still destructive
- Domains can be flipped at high frequency
- Antiparallel but unequal magnitudes

net dipole
alignment



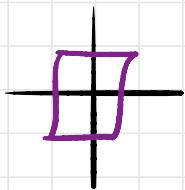
- Sub-lattices have different magnetic moments
- Cations of 2 or more types, diff magnetic moments
- NiFe_2O_4 , CoFe_3O_4 , $\text{BaFe}_{12}\text{O}_{19}$

* refer pdf for nature of graph

Hard & Soft Magnetic Materials

- Ferromagnetic Materials, $T < T_c$
- Based on nature of hysteresis curve

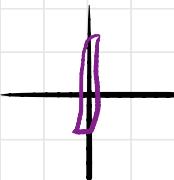
Hard Magnetic Materials



- More squarish $\rightarrow H_s \approx H_c$
- Hard discs use — better permanent magnets
- Difficult to magnetise and demagnetise (large area under curve)
- Ideal for memory devices

Soft Magnetic Materials

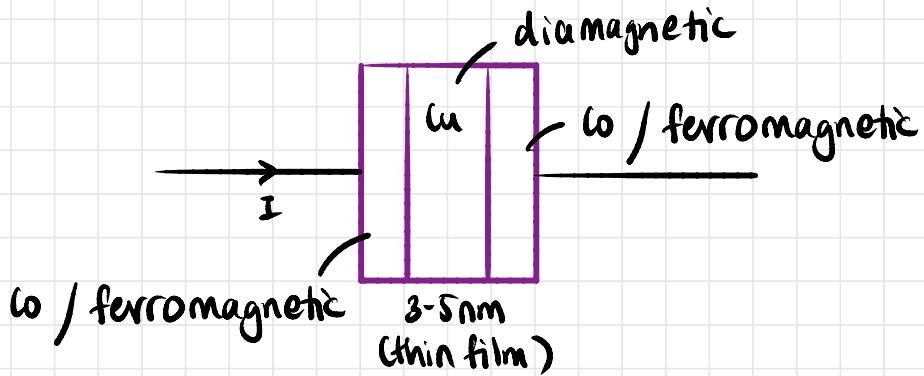
- Thin hysteresis curve



- Easier to magnetise and demagnetise (small coercivity)
- Good candidate for electromagnets, transformers, motors, generators
- High frequency switching (AC - 50 Hz)
- Low losses (low area under curve) and low heat losses
- Ferrites - used in high frequency switching

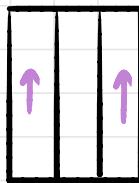
Magneto Resistance

- Change in resistance in the presence of an external magnetic field (small change)
- Resistance depends on spin orientation
- Device - Giant Magneto Resistance Device



$$R = \frac{V}{I}$$

- Resistance very sensitive to magnetisation state
- Parallel resistance: both dipole moments in same orientation (low resistance)



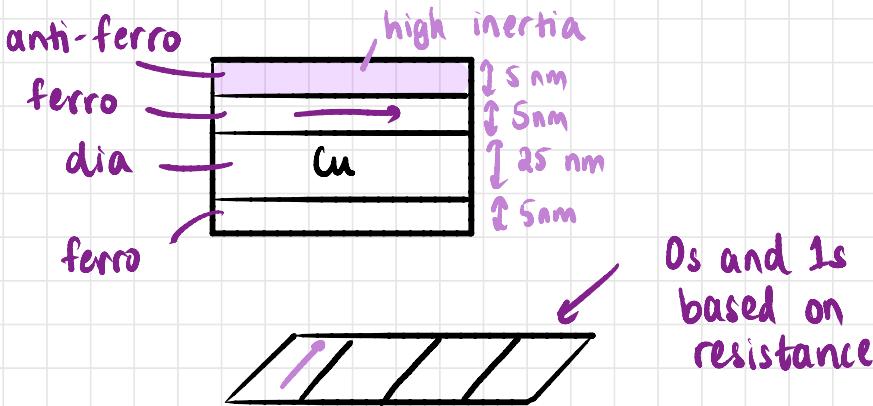
- Antiparallel resistance: dipole moments in opposite directions (high resistance)



- By switching the orientation of one of the layers, the resistance can be controlled
- GMR coefficient

$$GMR = \frac{R_{\uparrow\downarrow} - R_{\uparrow\uparrow}}{R_{\uparrow\uparrow}} \rightarrow \text{up to } 50\%$$

- Modified construction: 4 layers



- A pin layer (antiferromagnetic) layer is placed over one ferromagnetic layer to maintain its orientation (insulator)
- A thick diamagnetic layer is placed between the two ferromagnetic layers
- The other ferromagnetic layer is the free layer

(II)

DIELECTRICS

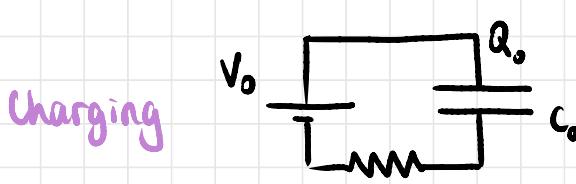
- Charges cannot be continuously displaced in such media
- Insulators are similar
- Dielectrics: charges may be displaced by small distances
- Used in capacitors

$$C_0 = \frac{\epsilon_0 A}{d}$$

$$\epsilon_r = \frac{C}{C_0}$$

$$C = \epsilon_r \frac{\epsilon_0 A}{d}$$

$$C = \epsilon_r C_0$$



$$Q_0 = C_0 V_0$$

$$E_0 = \frac{V_0}{d}$$

Dielectric without power supply

$$C = \epsilon_r C_0$$

$$Q_0 = CV$$

$$\therefore V = \frac{V_0}{\epsilon_r}$$

$$E = \frac{V}{d} = \frac{V_0}{\epsilon_r d}$$

$$E = \frac{E_0}{\epsilon_r}$$

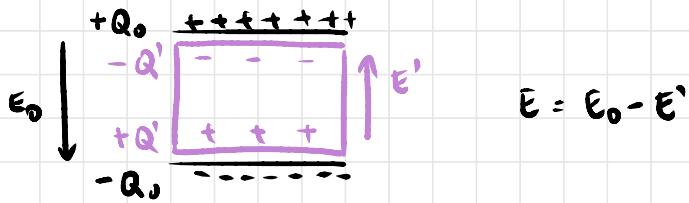
Dielectric with power supply

$$C = \epsilon_r C_0$$

$$V = V_0$$

$$E = E_0$$

$$Q = \epsilon_r Q_0$$



Using Gauss' Law

metal plate $+Q_0$ pillbox A $E_0 = \frac{Q_0}{A\epsilon_0} = \frac{\sigma_0}{\epsilon_0}$

dielectric medium
(assume uniform \vec{E}) $+Q'$ $\downarrow \uparrow \uparrow \uparrow$ A $E' = \frac{Q'}{A\epsilon_0} = \frac{\sigma'}{\epsilon_0}$ surface charge density

$$\sigma' = \frac{Q \cdot d}{A \cdot d} = \frac{\text{dipole moment}}{\text{volume}}$$

= polarisation in the material

$$\sigma' = P$$

- Note: in magnetism, flux could increase or decrease with media
- In dielectrics, polarised field always opposes

$$E = E_0 - E' \quad (\text{opposite directions})$$

$$E = E_0 - \frac{P}{\epsilon_0} \quad \text{polarisation}$$

$$\text{We define } \epsilon_r = \frac{E_0}{E} = \frac{c}{c_0} = \frac{V_0}{V}$$

$$\therefore E = \epsilon_r E - \frac{P}{\epsilon_0}$$

$$P = \epsilon_0 (\epsilon_r - 1) E$$

polarisation of material in the presence of an electric field

$$P = \chi_d \epsilon_0 E$$

$$\chi_d = \epsilon_r - 1$$

$$P \propto E$$

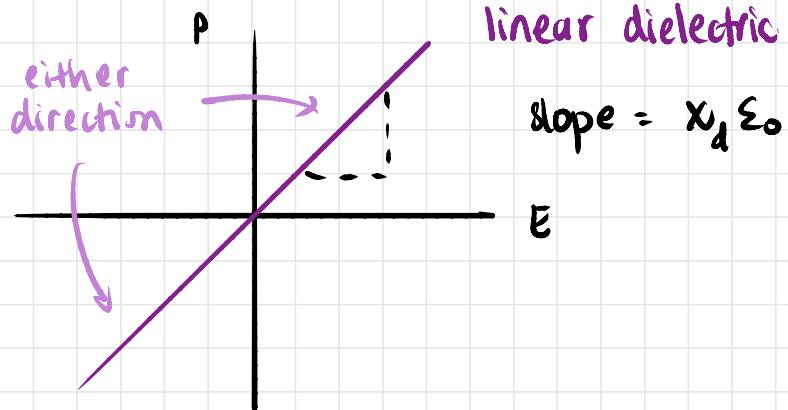
dielectric susceptibility

Similar to Magnetisation,

$$M = \chi_m \mu_0 B$$

$$\chi_m = \mu_r - 1$$

magnetic susceptibility



Atomic Level

- In the absence of an external \vec{E} , atoms are neutral
- Centre of +ve charges = centre of -ve charges = centre of atom (nucleus) \Rightarrow no dipoles exist
- In the presence of an external field, charges get displaced slightly (spherical symmetry lost)
- Small dipoles formed



- Polarisability = dipole moment per atom (α_e) (of atoms)

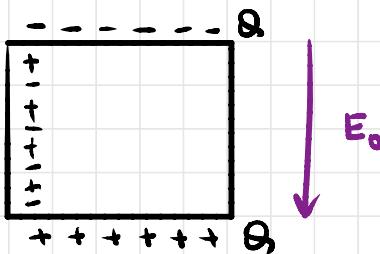
$$P = N \alpha_e E \quad (N = \text{No. of atoms / vol})$$

$$N\alpha_e = \epsilon_0 (\epsilon_r - 1)$$

atomic polarisability \curvearrowleft

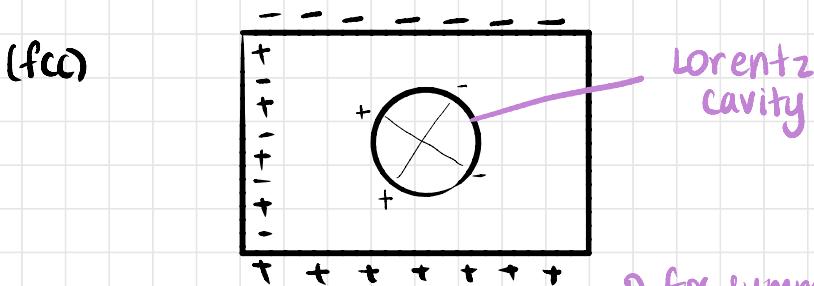
$$\alpha_e = \frac{\epsilon_0 (\epsilon_r - 1)}{N}$$

Dielectrics in External Field



fcc crystal:
remains symmetric
when viewed from
any direction

- Internal dipoles also contribute to net \vec{E}
- Local field: field at a site, slightly diff from \vec{E}_{avg} due to local effects



0 for symmetric
(fcc)

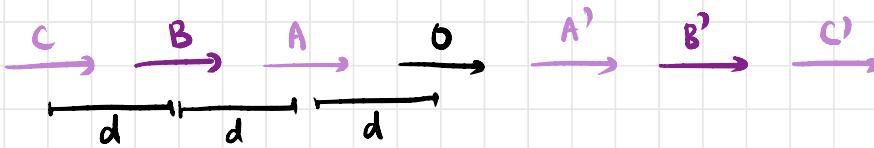
$$E_{loc} = E_0 + E_{surface} + E_{Lorentz\ cavity} + E_{other\ dipoles}$$

(depolarising)

inside cavity

$$E_{loc} = E_0 + E_1 + E_2 + E_3$$

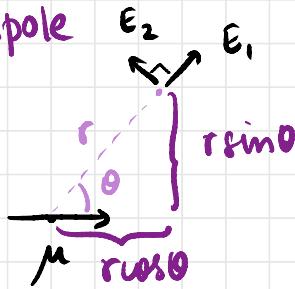
Consider a line of dielectrics (fixed)



Find contributions of A and A' on O

Pairs of dipoles at uniform distances

\vec{E} due to dipole



$$E_1 = \frac{\mu_0 \cos \theta}{4\pi \epsilon_0 r^3}$$

$$E_2 = \frac{\mu_0 \sin \theta}{4\pi \epsilon_0 r^3}$$

Field at O due to A

$$E_A = \frac{\mu_0}{4\pi \epsilon_0 d^3}$$

Field due to A'

$$E_{A'} = \frac{\mu_0}{4\pi \epsilon_0 d^3}$$

Net field at O due to A and A'

$$E_{AA'} = \frac{\mu_0}{\pi \epsilon_0 d^3}$$

The total electric field at O due to pairs of dipoles equidistant from O

$$E_3 = \frac{\mu}{\pi \epsilon_0 d^3} + \frac{\mu}{\pi \epsilon_0 (2d)^3} + \dots$$

$$E_3 = \frac{\mu}{\pi \epsilon_0 d^3}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^3}$$

Apéry's constant

≈ 1.2

constant (geometry)

$$E_3 = \frac{1.2 \mu}{\pi \epsilon_0 d^3}$$

dipole moment = P (polarisation)
volume

$$= \frac{8P}{\epsilon_0} = \frac{P}{3\epsilon_0} \text{ (for cubic material)}$$

The contribution of $E_{AA'}$ to E_3 is the most significant one ($\sim 83\%$)

The contribution of $E_{BB'}$ to E_3 is $1/8^{\text{th}}$ $E_{AA'}$'s ($\sim 10\%$)

The contribution of $E_{CC'}$ to E_3 is $\sim 3\%$

depolarisation

$$\therefore E_{\text{loc}} = \underbrace{E_0 + E_1}_{E} + \frac{P}{3\epsilon_0}$$

$$P = N\alpha_e E_{loc} = \epsilon_0(\epsilon_r - 1) E$$

← polarisability
 ← average
 (bulk)

$$N\alpha_e \left(E + \frac{P}{3\epsilon_0} \right) = \epsilon_0(\epsilon_r - 1) E$$

$$P = \chi_d \epsilon_0 E = \epsilon_0(\epsilon_r - 1) E$$

$$N\alpha_e \left(E + \frac{(\epsilon_r - 1)E}{3} \right) = \epsilon_0(\epsilon_r - 1) E$$

$$\frac{N\alpha_e}{\epsilon_0} \left(\frac{3 + \epsilon_r - 1}{3} \right) = \epsilon_r - 1$$

Clausius-Mosotti Relation

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N\alpha_e}{3\epsilon_0}$$

Relates microscopic and macroscopic parameters (solids)

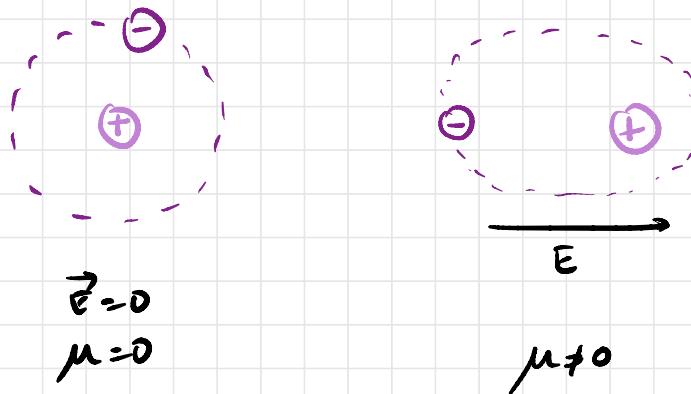
For gases, α is not fixed and we use the conventional relationship (page 34)

$$\frac{N\alpha_e}{\epsilon_0} = \epsilon_r - 1$$

Polarisation Mechanisms

1. Electronic Polarisation

- Atom subjected to electric field : separation of +ve and -ve charge centres
- Dipoles induced



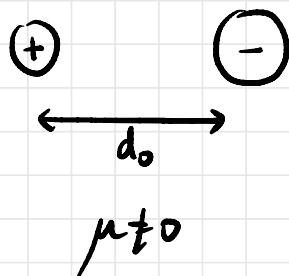
- Limit of field that can be applied: ionisation

$$\alpha_e = 4\pi\epsilon_0 R^3$$

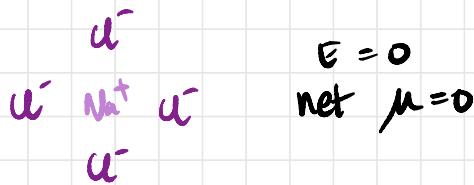
- Mechanism independent of temperature
- Atomic level mechanism

2 Ionic Polarisation

- Materials bonded ionically (permanent dipoles)



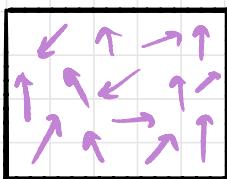
- Crystal structure: fcc



- When \vec{E} applied, dipole induced ($\mu \neq 0$)
- Polarisability due to displacement of ions from equilibrium (Hooke's Law)
- Polarisability α_i is dependent on Young's Modulus of elasticity
- γ is temperature-independent \Rightarrow mechanism is temperature independent

3. Orientation Polarisation

- Applicable to polar materials



$$\mu \neq 0$$

$$E = 0$$

- If $kT \gg$ large, $\mu = 0$ (similar to paramagnetic materials)

$$P = \frac{N\mu^2 E}{3kT}$$

Water Molecules



$$\mu \longrightarrow \epsilon_r = 80$$

cannot be made useful;
we get conductivity and
cannot be capacitor

- Dipoles exist when $E = 0$
- External field can orient dipoles at lower temperatures
- Temperature dependent.

4. Space Charge Limited Polarisation

- To make capacitor, multiple granules of system mixed with low-melting oxides (Cs_2O_2 , Bi_2O_3) glasses
- Low-melting oxides have diff conducting characteristics (phases)
- They do not stay together at low temp.
- Barium Titinate gets thin layer of glasses
- Different conducting phases \Rightarrow phase charge polarization (artificial polarisation)

Non-Linear Dielectrics

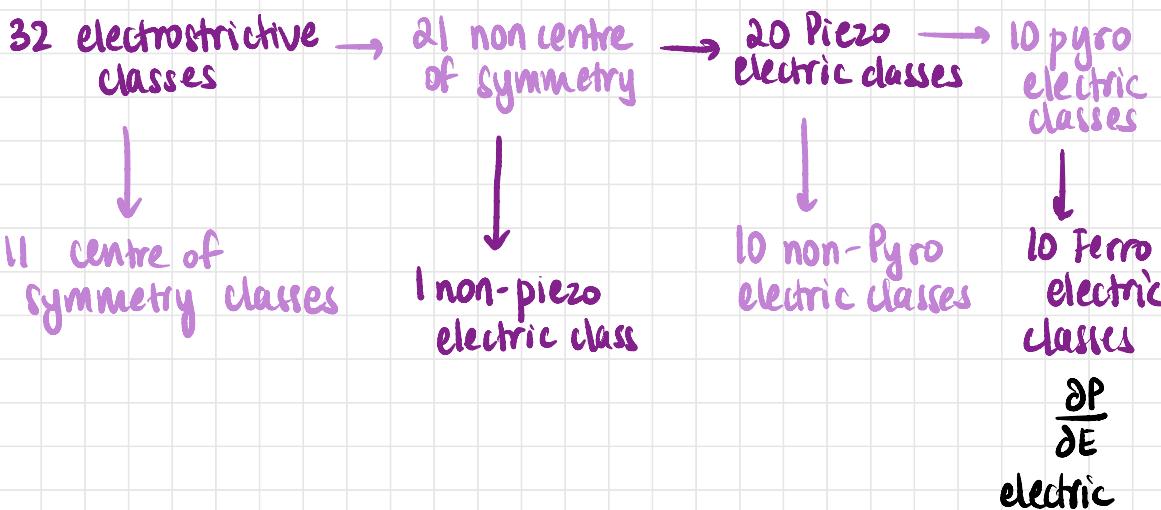
- $\frac{\partial P}{\partial E} \neq \text{constant}$

mech

$$\frac{\partial P}{\partial S}$$

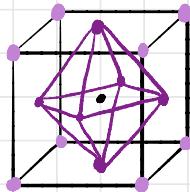
thermal

$$\frac{\partial P}{\partial T}$$



Centro Symmetric System

- BaTiO_3 — standard material
- $T > 120^\circ\text{C}$, system in cubic form



- - Oxygen ($6 \times 4/2 = 3$)
- - Barium ($8 \times 1/8 = 1$)
- - Titanium ($1 \times 1 = 1$)

$$\begin{array}{l} \text{Oxygen: } 3 \times 0^{2-} = 6^- \\ \text{Barium: } 1 \times \text{Ba}^{2+} = 2^+ \\ \text{Titanium: } 1 \times \text{Ti}^{4+} = 4^+ \end{array}$$

- In this case, material is centro symmetric as the +ve and -ve charges coincides
- Material is paraelectric (linear response)

Non-Centro Symmetric System

Case I

- T between 5° and 120°
- Material gets into tetragonal phase (elongated cube)
- Ti^{4+} no longer at centre of crystal
- Charge separation (fraction of \AA)

- $\mu \neq 0$ when $E = 0$
- Can manipulate position of Ti^{4+} using mechanical stress, thermal energy, electric field \rightarrow polarization

Case II

- $T < 5^\circ C$ and $T > -90^\circ C$
- Orthogonal system ($a \neq b \neq c$)

Case III

- $T < -90^\circ$
- Orthorhombic system

FERROELECTRIC MATERIALS

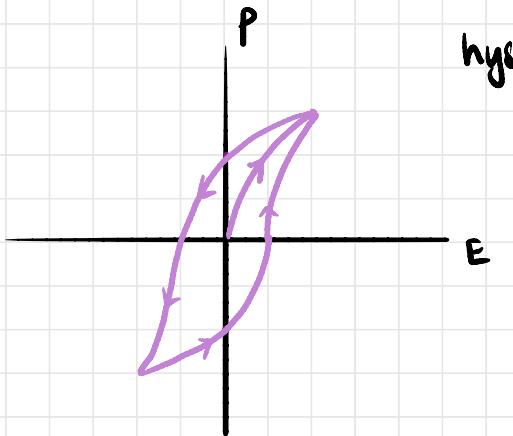
- Characteristics very similar to ferromagnetic materials
- Nothing to do with iron

polycrystalline material



$$E = 0$$

$$\mu = 0$$



hysteresis

- Good for memory
- Dynamic RAM in cellphones
- 1 — polarisation ($+E_s$) \uparrow_{pr}
0 — polarisation ($-E_s$) \downarrow_{pr}
- To erase , E_c (+or-) used

PIEZOELECTRICS

- Non centro-symmetric system
- Response to mechanical stress ($\frac{\partial P}{\partial S}$) — highly directional
- Volume changes \rightarrow polarization (per unit volume) changes
- Mechanical stress creates electric potential difference
- Indirect piezoelectric effect : electrical \rightarrow mechanical (Crystal oscillates due to \vec{E}')

direction of
stress
↓

- If frequency of \vec{E} = natural frequency of crystal, mechanical standing waves formed (resonance)

Tensor

- X is tensor
- not a vector, nor a scalar
- collection of points showing some directionality
- PVDF - plastic film: vibrates in wind and generates microvolt power
- When crystal strained, electric field produced and potential difference developed
- Range of applications
 - sensitive chemical and biological sensors
 - pressure measurement to study explosives, IC engines, etc
 - electric igniters
 - electronic drum pads (transducers)

PYROELECTRICS

- Non centro symmetric crystals
- Response to thermal stress $(\frac{\partial P}{\partial T})$ for $T <$ Curie temp
- Pyroelectric coefficient π_i

$T > T_c$,
 ✓ centro
 symmetric

$$\pi_i = \frac{\partial P}{\partial T}$$

- Heat sensing element - fire alarm
- Device to monitor capacitance

