**PES University, Bangalore**

(Established under Karnataka Act No. 16 of 2013)

**UE19CS203 – STATISTICS FOR DATA SCIENCE**

**Unit-2 - Random Variables**

**QUESTION BANK**

## Binomial Distribution

**Exercises for Section 4.2**
**[Text Book Exercise – Section 4.2 – Q. No. [1 – 26] – Pg. No. [212 - 215]]**

1. Let X ~ Bin(7, 0.3). Find

   a) P(X = 1)
   b) P(X = 2)
   c) P(X < 1)
   d) P(X > 4)
   e) $\mu_X$
   f) $\sigma_X^2$

2. Let X ~ Bin(9, 0.4). Find

   a) P(X > 6)
   b) P(X ≥ 2)
   c) P(2 ≤ X < 5)
   d) P(2 < X ≤ 5)
   e) P(X = 0)
   f) P(X = 7)
   g) $\mu_X$
   h) $\sigma_X^2$

3. Find the following probabilities:

   a) P (X = 2) when X ~ Bin (4, 0.6)
   b) P (X > 2) when X ~ Bin (8, 0.2)
   c) P (X ≤ 2) when X ~ Bin (5, 0.4)
   d) P (3 ≤ X ≤ 5) when X ~ Bin (6, 0.7)

4. At a certain airport, 75% of the flights arrive on time. A sample of 10 flights is studied.

   a) Find the probability that all 10 of the flights were on time.
   b) Find the probability that exactly eight of the flights were on time.
   c) Find the probability that eight or more of the flights were on time.

5. Of all the registered automobiles in a certain state, 10% violate the state emissions standard. Twelve automobiles are selected at random to undergo an emissions test.

   a) Find the probability that exactly three of them violate the standard.
   b) Find the probability that fewer than three of them violate the standard.
   c) Find the probability that none of them violate the standard.

6. A fair die is rolled 8 times.

   a) What is the probability that the die comes up 6 exactly twice?
   b) What is the probability that the die comes up an odd number exactly five times?
   c) Find the mean number of times a 6 comes up.
   d) Find the mean number of times an odd number comes up.
   e) Find the standard deviation of the number of times a 6 comes up.
   f) Find the standard deviation of the number of times an odd number comes up.

7. Of all the weld failures in a certain assembly, 85% of them occur in the weld metal itself, and the remaining 15% occur in the base metal. A sample of 20 weld failures is examined.

   a) What is the probability that exactly five of them are base metal failures?
   b) What is the probability that fewer than four of them are base metal failures?
   c) What is the probability that none of them are base metal failures?
   d) Find the mean number of base metal failures.
   a) Find the standard deviation of the number of base metal failures.

8. A general contracting firm experiences cost overruns on 20% of its contracts. In a company audit, 20 contracts are sampled at random.

   a) What is the probability that exactly four of them experience cost overruns?
   b) What is the probability that fewer than three of them experience cost overruns?
   c) What is the probability that none of them experience cost overruns?
   d) Find the mean number that experience cost overruns.

e) Find the standard deviation of the number that experience cost overruns.

9. Several million lottery tickets are sold, and 60% of the tickets are held by women. Five winning tickets will be drawn at random.

   a) What is the probability that three or fewer of the winners will be women?
   b) What is the probability that three of the winners will be of one gender and two of the winners will be of the other gender?

10. A quality engineer takes a random sample of 100 steel rods from a day's production, and finds that 92 of them meet specifications.

    a) Estimate the proportion of that day's production that meets specifications, and find the uncertainty in the estimate.
    b) Estimate the number of rods that must be sampled to reduce the uncertainty to 1%.

11. In a random sample of 100 parts ordered from vendor A, 12 were defective. In a random sample of 200 parts ordered from vendor B, 10 were defective.

    a) Estimate the proportion of parts from vendor A that are defective, and find the uncertainty in the estimate.
    b) Estimate the proportion of parts from vendor B that are defective, and find the uncertainty in the estimate.
    c) Estimate the difference in the proportions, and find the uncertainty in the estimate.

12. Of the items manufactured by a certain process, 20% are defective. Of the defective items, 60% can be repaired.

    a) Find the probability that a randomly chosen item is defective and cannot be repaired.
    b) Find the probability that exactly 2 of 20 randomly chosen items are defective and cannot be repaired.

13. Of the bolts manufactured for a certain application, 90% meet the length specification and can be used immediately, 6% are too long and can be used after being cut, and 4% are too short and must be scrapped.

a) Find the probability that a randomly selected bolt can be used (either immediately or after being cut).
b) Find the probability that fewer than 9 out of a sample of 10 bolts can be used (either immediately or after being cut).

14. Gears produced by a grinding process are categorized either as conforming (suitable for their intended purpose), owngraded (unsuitable for the intended purpose but usable for another purpose), or scrap (not usable). Suppose that 80% of the gears produced are conforming, 15% are downgraded, and 5% are scrap. Ten gears are selected at random.

a) What is the probability that one or more is scrap?
b) What is the probability that eight or more are not scrap?
c) What is the probability that more than two are either downgraded or scrap?
d) What is the probability that exactly nine are either conforming or downgraded?

15. A commuter must pass through three traffic lights on her way to work. For each light, the probability that it is green when she arrives is 0.6. The lights are independent.

a) What is the probability that all three lights are green?
b) The commuter goes to work five days per week. Let $X$ be the number of times out of the five days in a given week that all three lights are green. Assume the days are independent of one another. What is the distribution of $X$?
c) Find $P(X = 3)$.

16. A distributor receives a large shipment of components. The distributor would like to accept the shipment if 10% or fewer of the components are defective and to return it if more than 10% of the components are defective. She decides to sample 10 components, and to return the shipment if more than 1 of the 10 is defective.

a) If the proportion of defectives in the batch is in fact 10%, what is the probability that she will return the shipment?
b) If the proportion of defectives in the batch is 20%, what is the probability that she will return the shipment?
c) If the proportion of defectives in the batch is 2%, what is the probability that she will return the shipment?
d) The distributor decides that she will accept the shipment only if none of the sampled items are defective. What is the minimum number of items she should sample if she wants to have a probability no greater than 0.01 of accepting the shipment if 20% of the components in the shipment are defective?

17. A k out of n system is one in which there is a group of n components, and the system will function if at least k of the components function. Assume the components function independently of one another.

    a) In a 3 out of 5 system, each component has probability 0.9 of functioning. What is the probability that the system will function?
    b) In a 3 out of n system, in which each component has probability 0.9 of functioning, what is the smallest value of n needed so that the probability that the system functions is at least 0.90?

18. Refer to Exercise 17 for the definition of a *k* out of *n* system. For a certain 4 out of 6 system, assume that on a rainy day each component has probability 0.7 of functioning, and that on a nonrainy day each component has probability 0.9 of functioning.

    a) What is the probability that the system functions on a rainy day?
    b) What is the probability that the system functions on a nonrainy day?
    c) Assume that the probability of rain tomorrow is 0.20. What is the probability that the system will function tomorrow?

19. A certain large shipment comes with a guarantee that it contains no more than 15% defective items. If the proportion of defective items in the shipment is greater than 15%, the shipment may be returned. You draw a random sample of 10 items. Let X be the number of defective items in the sample.

    a) If in fact 15% of the items in the shipment are defective (so that the shipment is good, but just barely), what is $P(X \geq 7)$?
    b) Based on the answer to part (a), if 15% of the items in the shipment are defective, would 7 defectives in a sample of size 10 be an unusually large number?
    c) If you found that 7 of the 10 sample items were defective, would this be convincing evidence that the shipment should be returned? Explain.
    d) If in fact 15% of the items in the shipment are defective, what is $P(X \geq 2)$?
    e) Based on the answer to part (d), if 15% of the items in the shipment are defective, would 2 defectives in a sample of size 10 be an unusually large number?
    f) If you found that 2 of the 10 sample items were defective, would this be convincing evidence that the shipment should be returned? Explain.

20. An insurance company offers a discount to homeowners who install smoke detectors in their homes. A company representative claims that 80% or more of policyholders have smoke detectors. You draw a random sample of eight policyholders. Let *X* be the number of policyholders in the sample who have smoke detectors.

a) If exactly 80% of the policyholders have smoke detectors (so the representative's claim is true, but just barely), what is $P(X \leq 1)$?
b) Based on the answer to part (a), if 80% of the policyholders have smoke detectors, would one policyholder with a smoke detector in a sample of size 8 be an unusually small number?
c) If you found that one of the eight sample policyholders had a smoke detector, would this be convincing evidence that the claim is false? Explain.
d) If exactly 80% of the policyholders have smoke detectors, what is $P(X \leq 6)$?
e) Based on the answer to part (d), if 80% of the policyholders have smoke detectors, would six policyholders with smoke detectors in a sample of size 8 be an unusually small number?
f) If you found that six of the eight sample policyholders had smoke detectors, would this be convincing evidence that the claim is false? Explain.

21. A message consists of a string of bits (0s and 1s). Due to noise in the communications channel, each bit has probability 0.3 of being reversed (i.e., a 1 will be changed to a 0 or a 0 to a 1). To improve the accuracy of the communication, each bit is sent five times, so, for example, 0 is sent as 00000. The receiver assigns the value 0 if three or more of the bits are decoded as 0, and 1 if three or more of the bits are decoded as 1. Assume that errors occur independently.

a) A 0 is sent (as 00000). What is the probability that the receiver assigns the correct value of 0?
b) Assume that each bit is sent *n* times, where *n* is an odd number, and that the receiver assigns the value decoded in the majority of the bits. What is the minimum value of *n* necessary so that the probability that the correct value is assigned is at least 0.90?

22. Let $X \sim \text{Bin}(n, p)$, and let $Y = n - X$. Show that $Y \sim \text{Bin}(n, 1 - p)$.

23. Porcelain figurines are sold for \$10 if flawless, and for \$3 if there are minor cosmetic flaws. Of the figurines made by a certain company, 90% are flawless and 10% have minor cosmetic flaws. In a sample of 100 figurines that are sold, let *Y* be the revenue earned by selling them and let *X* be the number of them that are flawless.

a) Express Y as a function of X.
b) Find $\mu_X$.
c) Find $\sigma_Y$ .

24. One design for a system requires the installation of two identical components. The system will work if at least one of the components works. An alternative design requires four of these components, and the system will work if at least two of the four components work. If the probability that a component works is 0.9, and if the components function independently, which design has the greater probability of functioning?

25. (Requires material from Section 3.3.) Refer to Example 4.14. Estimate the probability that exactly one of the four tires has a flaw, and find the uncertainty in the estimate. **(Exclude)**

26. If p is a success probability, the quantity $p/(1 - p)$ is called the odds. Odds are commonly estimated in medical research. The article "A Study of Twelve Southern California Communities with Differing Levels and Types of Air Pollution" (J. Peters, E. Avol, et al., The American Journal of Respiratory and Critical Care Medicine, 1999:760–767) reports an assessment of respiratory health of southern California children. Assume that 88 boys in a sample of 612 reported being diagnosed with bronchitis during the last 12 months.

    a) Estimate the proportion p of boys who have been diagnosed with bronchitis, and find the uncertainty in the estimate.
    b) (Requires material from Section 3.3.) Estimate the odds, and find the uncertainty in the estimate. **(Exclude)**