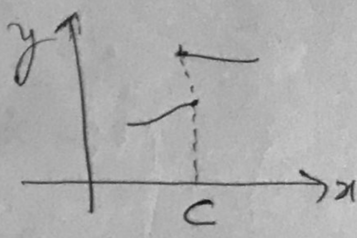
Note:- <u>Functions having points of discontinuity</u> I

If $f(x) = \phi(x)$, $\alpha < x < c$

$= \psi(x)$, $c < x < \alpha + 2\pi$

then c is the point of discontinuity.



$\therefore$ At a point of discontinuity $x = c$, there is a jump in the graph of the function, Both the limits on the left $f(c^-)$ & on the right $f(c^+)$ exist & are different. At such a pt the value of $f(x)$ is given by, ~~f(x)=~~ i.e at $x = c$, $f(x) = \frac{1}{2}[f(c^-) + f(c^+)]$

(9)

$$f(x) = \begin{cases} 2-x & ; 0 < x \leq 4 \\ x-6 & ; 4 \leq x \leq 8 \end{cases}$$

deduce $\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$

$f(x) = \frac{-\pi}{\ldots} + \sum^{\infty} \ldots$ $\left[1-2(-1)^n\right]$

Sol Period of $f(x) = 8-0=8$

$\therefore 2l = 8 \Rightarrow \boxed{l=4}$

The reqd FS is $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{4} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{4}$

Here $\phi(x) = 2-x$ & $\psi(x) = x-6$

$\therefore \phi(2l-x) = \phi(8-x) = 2-(8-x) = -6+x = x-6 = \psi(x)$

$\Rightarrow f(x)$ is even $\therefore b_n = 0$

$a_0 = \frac{2}{4}\int_0^4 f(x)\,dx = \frac{1}{2}\int_0^4 (2-x)\,dx = \frac{1}{2}\left[2x - \frac{x^2}{2}\right]_0^4$

$= \frac{1}{2}\left[8-8\right] = \boxed{0 = a_0}$

$a_n = \frac{2}{4}\int_0^4 f(x) \cos nx\,dx = \frac{1}{2}\int_0^4 (2-x)\cos\frac{n\pi x}{4}\,dx$

$a_n = \frac{1}{2}\left[(2-x)\left(\frac{\sin\frac{n\pi x}{4}}{n\pi/4}\right) - (-1)\left(\frac{-\cos\frac{n\pi x}{4}}{n^2\pi^2/16}\right)\right]_0^4$

$= \frac{1}{2}\left\{-\frac{16}{n^2\pi^2}\left[(-1)^n - 1\right]\right\} = \frac{8}{n^2\pi^2}\left[1-(-1)^n\right]$

$\boxed{= \frac{16}{n^2\pi^2} \text{ if } n = 1, 3, 5 \ldots \quad \& \ 0 \text{ if } n \text{ is even}}$

$\therefore f(x) = \sum_{n=1,3,5}^{\infty} \frac{16}{n^2\pi^2} \cos\left(\frac{n\pi x}{4}\right)$

put $x = 0$, $\therefore f(x) = 2 - 0 = 2$

$\therefore 2 = \frac{16}{\pi^2} \sum_{n=1,3,5\ldots}^{\infty} \frac{1}{n^2}\cdot 1$

$\Rightarrow \frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$ or $= \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots$