UE19CS203 – STATISTICS DATA SCIENCE

Unit-3 – Probability Distribution

Question Bank - SOLVED

Confidence Intervals with Paired Data

Exercises for Section 5.7

1. Breathing rates, in breaths per minute, were measured for a group of 10 subjects at rest, and then during moderate exercise. The results were as follows:

Subject	Rest	Exercise
1	15	30
2	16	37
3	21	39
4	17	37
5	18	40
6	15	39
7	19	34
8	21	40
9	18	38
10	14	34

Find a 95% confidence interval for the increase in breathing rate due to exercise.

[Text Book Exercise – Section 5.7 – Q. No. 4 – Pg. No. 373]
Solution:

The differences are: 15, 21, 18, 20, 22, 24, 15, 19, 20, 20.
D = 19.4, sD = 2.836273, n = 10, t10−1,.025 = 2.262.
The confidence interval is 19.4±2.262(2.836273/ √ 10), or (17.3712, 21.4288).

2. A tire manufacturer is interested in testing the fuel economy for two different tread patterns. Tires of each tread type are driven for 1000 miles on each of 18 different cars. The mileages, in mpg, are presented in the following table.

Car	Tread A	Tread B
1	24.1	20.3
2	22.3	19.7

3	24.5	22.5
4	26.1	23.2
5	22.6	20.4
6	23.3	23.5
7	22.4	21.9
8	19.9	18.6
9	27.1	25.8
10	23.5	21.4
11	25.4	20.6
12	24.9	23.4
13	23.7	20.3
14	23.9	22.5
15	24.6	23.5
16	26.4	24.5
17	21.5	22.4
18	24.6	24.9

a) Find a 99% confidence interval for the mean difference in fuel economy.
b) A confidence interval based on the data in the table has width±0.5 mpg. Is the level of this confidence interval closest to 80%, 90%, or 95%?

[Text Book Exercise – Section 5.7 – Q. No. 9 – Pg. No. 374]
Solution:

(a) The differences are:
3.8,2.6,2.0,2.9,2.2,−0.2,0.5,1.3,1.3,2.1,4.8,1.5,3.4,1.4,1.1,1.9,−0.9,−0.3.
D = 1.74444, sD = 1.46095, n = 18, $t_{18-1,.005}$ = 2.898.
The confidence interval is 1.74444±2.898(1.46095/ $\sqrt{18}$), or (0.747, 2.742).

(b)The level 100(1−α)% can be determined from the equation $t_{17,\alpha/2}$ (1.46095/ $\sqrt{18}$) = 0.5. From this equation, $t_{17,\alpha/2}$ = 1.452.
The t table indicates that the value of α/2 is between 0.05 and 0.10, and closer to 0.10.
Therefore the level 100(1−α)% is closest to 80%.