LINEAR ALGEBRA AND ITS APPLICATIONS

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MATRICES AND
GAUSSIAN ELIMINATION

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GAUSSIAN ELIMINATION:

1.Check for consistency and solve the following system of equations if consistent:

(i)
$$x_1 + x_2 - 2x_3 + 3x_4 = 4$$
$$2x_1 + 3x_2 + 3x_3 - x_4 = 3$$
$$5x_1 + 7x_2 + 4x_3 + x_4 = 5$$

$$[A:b] = \begin{pmatrix} 1 & 1 & -2 & 3:4 \\ 2 & 3 & 3 & -1:3 \\ 5 & 7 & 4 & 1:5 \end{pmatrix} \xrightarrow[R_3-5R_1]{R_2-2R_1}$$

$$\begin{bmatrix} 1 & 1 & -2 & 3:4 \\ 0 & 1 & 7 & -7:-5 \\ 0 & 2 & 14 & -14:-15 \end{bmatrix} \xrightarrow{R_3-2R_2} \begin{bmatrix} 1 & 1 & -2 & 3:4 \\ 0 & 1 & 7 & -7:-5 \\ 0 & 0 & 0 & 0:-5 \end{bmatrix}$$

This gives 0=-5 which is not possible.
Also r(A)=2 and r[A:b]=3
System is **inconsistent** and has **no solution**

GAUSSIAN ELIMINATION:

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(ii)
$$\begin{aligned} x_1 + 2x_2 + x_3 &= 3 \\ 2x_1 + 5x_2 - x_3 &= -4 \\ 3x_1 - 2x_2 - x_3 &= 5 \end{aligned}$$

$$[A:b] = \begin{pmatrix} 1 & 2 & 1: & 3 \\ 2 & 5 & -1: & -4 \\ 3 & -2 & -1: & 5 \end{pmatrix}$$

$$\xrightarrow[R_3-3R_1]{R_2-2R_1} \begin{bmatrix} 1 & 2 & 1: & 3 \\ 0 & 1 & -3: & -10 \\ 0 & -8 & -4: & -4 \end{bmatrix} \xrightarrow{R_3+8R_2} \begin{bmatrix} 1 & 2 & 1: & 3 \\ 0 & 1 & -3: & -10 \\ 0 & 0 & -28: & -84 \end{bmatrix}$$

$$\Rightarrow \begin{cases} x_1 + 2x_2 + x_3 = 3 \\ \qquad x_2 - 3x_3 = -10 \\ \qquad -28x_3 = -84 \end{cases}$$

r(A)=r[A:b]=3=n. System is **consistent** and has **a unique solution**.

$$(x_1, x_2, x_3) = (2, -1, 3)$$

GAUSSIAN ELIMINATION:

(iii)

$$2x-3y+2z=1$$
$$5x-8y+7z=1$$
$$y-4z=3$$

$$\begin{pmatrix} 2 & -3 & 2:1 \\ 5 & -8 & 7:1 \\ 0 & 1 & -4:3 \end{pmatrix} \xrightarrow{R_2-\left(\frac{5}{2}\right)R_1} \begin{pmatrix} 2 & -3 & 2:\ 1 \\ 0 & -1/2 & 2:-3/2 \\ 0 & 1 & -4:\ 3 \end{pmatrix} \xrightarrow{R_3+2R_2} \begin{pmatrix} 2 & -3 & 2:\ 1 \\ 0 & -1/2 & 2:-3/2 \\ 0 & 0 & 0:\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{cases} 2x-3y+2z=1 \\ -(1/2)y+2z=-3/2 \end{cases}$$

r(A)=r(A:b)=2<n(=3) hence system is **consistent** and has **infinite number of solutions**.

i.e **(x, y, z)=(5k+5, 4k-+3, k)**

GAUSSIAN ELIMINATION:

2. Find all values of a for which the resulting linear system has (a)no solution (b)a unique solution and (c) infinitely many solutions:

$$x + y - z = 2$$
$$x + 2y + z = 3$$
$$x + y + \left(a^2 - 5\right) z = a$$

$$[A:b] = \begin{pmatrix} 1 & 1 & -1 : 2 \\ 1 & 2 & 1 \ : 3 \\ 1 & 1 & a^2 - 5 : a \end{pmatrix} \xrightarrow[R_3 - R_1]{R_2 - R_1} \begin{pmatrix} 1 & 1 & -1 : \ 2 \\ 0 & 1 & 2 \ : 5 \\ 0 & 0 & a^2 - 4 : a - 2 \end{pmatrix}$$

(a) System has **no solution** if $a = -2$ (when r(A) $\neq$ r(A:b))

(b) System has **a unique solution** if $a \neq \pm 2$(when r(A) = r(A:b)=3=n)

(c) System has **infinitely many solutions** if $a = 2$ (when r(A) = r(A:b)=2<n)

GAUSSIAN ELIMINATION:

3. Find an equation relating a, b and c so that the linear system is consistent for any values of a, b and c that satisfy that equation. When (a,b,c)=(2,3,9), then what is the solution of the system.

$$x+2y-3z=a$$
$$2x+3y+3z=b$$
$$5x+9y-6z=c$$

$$\begin{pmatrix} 1 & 2 & -3 : a \\ 2 & 3 & 3 : b \\ 5 & 9 & -6 : c \end{pmatrix} \xrightarrow[R_3-5R_1]{R_2-2R_1} \begin{pmatrix} 1 & 2 & -3 : a \\ 0 & -1 & 9 : b-2a \\ 0 & -1 & 9 : c-5a \end{pmatrix}$$

$$\xrightarrow{R_3-R_2} \begin{pmatrix} 1 & 2 & -3 : a \\ 0 & -1 & 9 : b-2a \\ 0 & 0 & 0 : c-b-3a \end{pmatrix}$$

The given linear system will be **consistent** if a,b,c satisfy the relation c-b-3a=0.

When (a, b, c)=(2, 3, 9), then the solution of the system is **(x, y, z)=(-15k, 9k+1, k)**

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GAUSSIAN ELIMINATION:

Linear System of Equations

Consistent

Solution Exists (rank(A)=rank[A:b}=r)

Unique solution
(rank(A)=rank[A:b}=r=n)
(Non-Singular Matrix i.e $|A| \neq 0$)
$\left(A^{-1} exists\right)$

Infinite number of solutions.
(rank(A)=rank[A:b}=r<n)
(Singular Matrix i.e $|A| = 0$)

Inconsistent

No Solution (rank(A) $\neq$ rank[A:b})
(Singular Matrix i.e $|A| = 0$)

THANK YOU

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INTRODUCTION:

- **Linear Dependence and Independence:** Let $v_1, v_2, \cdots, v_k$ be elements of $\square^n$ *or* $\square^n$. If there exist scalars $c_1, c_2, \cdots, c_k$ and a vector y, then

  $y = c_1v_1 + c_2v_2 + \cdots + c_kv_k$, is called a **Linear Combination** of $v_i's$

Note: $\square^n = \{(x_1, x_2, \ldots x_n) / x_i \in \square, i = 1, 2, \cdots, n\}$ is Set of all n-tuples.

$\square^n = \{(x_1, x_2, \ldots x_n) / x_i \in \square, i = 1, 2, \cdots, n\}$

- Let $v_1, v_2, \cdots, v_k$ be elements of $\square^n$ *or* $\square^n$. Then these elements are said to be **Linearly Dependent** if there exist scalars $c_1, c_2, \cdots, c_k$ not all zero such that $c_1v_1 + c_2v_2 + \cdots + c_kv_k = 0$ i.e if there exist at least one $c_i \neq 0$ then $v_i's$ are Linearly Dependent.

- If $c_1v_1 + c_2v_2 + \cdots + c_kv_k = 0$ for all $c_i = 0$, then $v_i's$ are said to be **Linearly Independent.**

Exs: {(1,2), (0,2), (3,4)} is a L.D set since (3,4)=3(1,2)-1(0,2) or $c_1(1,2)+c_2(0,2)+c_3(3,4)=(0,0)$

{(2,1,0), (1,0,2), (0,1,2)} is a L.I set since $c_1=c_2=c_3=0$.