

# AUTOMATA, FORMAL LANGUAGES AND LOGIC

## MODULE 5

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### Propositional Logic

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### Outline

- ◆ Propositional logic - A very Simple Logic
  - Syntax and Semantics
  - – A Simple Knowledge Base
  - – A Simple Inference Procedure

# AUTOMATA, FORMAL LANGUAGES AND LOGIC


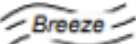




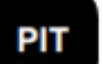
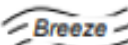

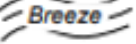
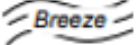
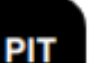
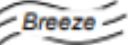
## Propositional Logic (A simple Knowledge Base)

### Knowledge Base

To construct the **knowledge base**, let us focus on immutable aspects of wumpus world

We need the following symbols for each  $[i, j]$  location:

- $P_{i,j}$  be true if there is a pit in  $[i, j]$ .
- $B_{i,j}$  be true if there is a breeze in  $[i, j]$ .
- $W_{i,j}$  be true if there is a wumpus in  $[i, j]$ , dead or alive.
- $S_{i,j}$  be true if agent perceives a stench in  $[i, j]$ .

1,4  Stench	2,4	3,4  Breeze	4,4  PIT
1,3 	2,3  Breeze Stench  Gold	3,3  PIT	4,3  Breeze
1,2  Stench	2,2	3,2  Breeze	4,2
1,1 START	2,1  Breeze	3,1  PIT	4,1  Breeze

1. There is no pit in [1,1],

$$R_1: \neg P_{1,1}$$

2. A square is **breezy** iff there is **pit** in its neighboring squares

$$R_2: B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$

$$R_3: B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$$

3. Let us include the **breeze** percept for the 1<sup>st</sup> two squares, leading to the situation

$$R_4: \neg B_{1,1}$$

$$R_5: B_{2,1}$$

The knowledge base might be considered as **conjunction of one or more statements**

Here the **PROPOSITION SYMBOLS** are :

$$B_{1,1}, B_{2,1}, P_{1,1}, P_{1,2}, P_{2,1}, P_{2,2}, P_{3,1}$$

1,4 Stench	2,4	3,4 Breeze	4,4 PIT
1,3 Wumpus	2,3 Dreeze Stench Gold	3,3 PIT	4,3 Breeze
1,2 Stench	2,2	3,2 Breeze	4,2
1,1 START	2,1 Breeze	3,1 PIT	4,1 Breeze

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## Propositional Logic (A simple Inference Procedure)



**Goal:** If  $\alpha_1 = \neg P_{1,2}$  and  $\alpha_2 = P_{2,2}$   
Whether  $KB \models \alpha_1$  or  $KB \models \alpha_2$   
for some sentence  $\alpha_1$  and  $\alpha_2$ .

Is  $\neg P_{1,2}$  entailed by our Knowledge Base?

Is  $P_{2,2}$  entailed by our Knowledge Base?

### Use Model Checking Approach

1. Enumerate the models
2. Check for  $\alpha$  being true, in every model in which KB is true

For Wumpus world **PROPOSITION SYMBOLS** are :

$B_{1,1}, B_{2,1}, P_{1,1}, P_{1,2}, P_{2,1}, P_{2,2}, P_{3,1}$

So with 7 symbols there are  $2^7 = 128$  possible models

For 3 symbols there are  $2^3 = 8$  models

A	B	C
True	True	True
True	False	False
False	True	False
False	False	True
True	True	False
True	False	True
False	True	True
False	False	False

# AUTOMATA, FORMAL LANGUAGES AND LOGIC

## Propositional Logic (A simple Inference Procedure)

### Truth Table for inference

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	$KB$
false	false	false	false	false	false	false	false
false	false	false	false	false	false	true	false
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
false	true	false	false	false	false	false	false
false	true	false	false	false	false	true	<u>true</u>
false	true	false	false	false	true	false	<u>true</u>
false	true	false	false	false	true	true	<u>true</u>
false	true	false	false	true	false	false	false
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
true	true	true	true	true	true	true	false

Is  $\neg P_{1,2}$  entailed by our KB?

Here  $\neg P_{1,2}$  is true

Hence, There is no pit in [1,2].

# AUTOMATA, FORMAL LANGUAGES AND LOGIC

## Propositional Logic (A simple Inference Procedure)

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	$KB$
false	false	false	false	false	false	false	false
false	false	false	false	false	false	true	false
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	true	false	false	false	false	false	false
false	true	false	false	false	false	true	<u>true</u>
false	true	false	false	false	true	false	<u>true</u>
false	true	false	false	false	true	true	<u>true</u>
false	true	false	false	true	false	false	false
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	true	true	true	true	true	true	false

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

1.  $R_1: \neg P_{1,1}$
2.  $R_2: B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$
3.  $R_3: B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$
4.  $R_4: \neg B_{1,1}$
5.  $R_5: B_{2,1}$

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## Propositional Logic (A simple Inference Procedure)

$$KB = R1 \wedge R2 \wedge R3 \wedge R4 \wedge R5$$

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	$KB$
false	false	false	false	false	false	false	false
false	false	false	false	false	false	true	false
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
false	true	false	false	false	false	false	false
false	true	false	false	false	false	true	<u>true</u>
false	true	false	false	false	true	false	<u>true</u>
false	true	false	false	false	true	true	<u>true</u>
false	true	false	false	true	false	false	false
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
true	true	true	true	true	true	true	false

1.  $R_1: \neg P_{1,1}$
2.  $R_2: B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$
3.  $R_3: B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$
4.  $R_4: \neg B_{1,1}$
5.  $R_5: B_{2,1}$



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## Propositional Logic (A simple Inference Procedure)

### Truth Table for inference

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	$KB$
false	false	false	false	false	false	false	false
false	false	false	false	false	false	true	false
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	true	false	false	false	false	false	false
false	true	false	false	false	false	true	<u>true</u>
false	true	false	false	false	true	false	<u>true</u>
false	true	false	false	false	true	true	<u>true</u>
false	true	false	false	true	false	false	false
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	true	true	true	true	true	true	false

Is  $P_{2,2}$  entailed by our Knowledge Base?

Here  $P_{2,2}$  is true in 2 of the three models and false in one.

**Hence, We can't yet tell whether there is a pit in [2,2].**



# THANK YOU

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