

Date :- Aug 19, 2020

Cross product ←

Part 1 and Part 2 final state
or

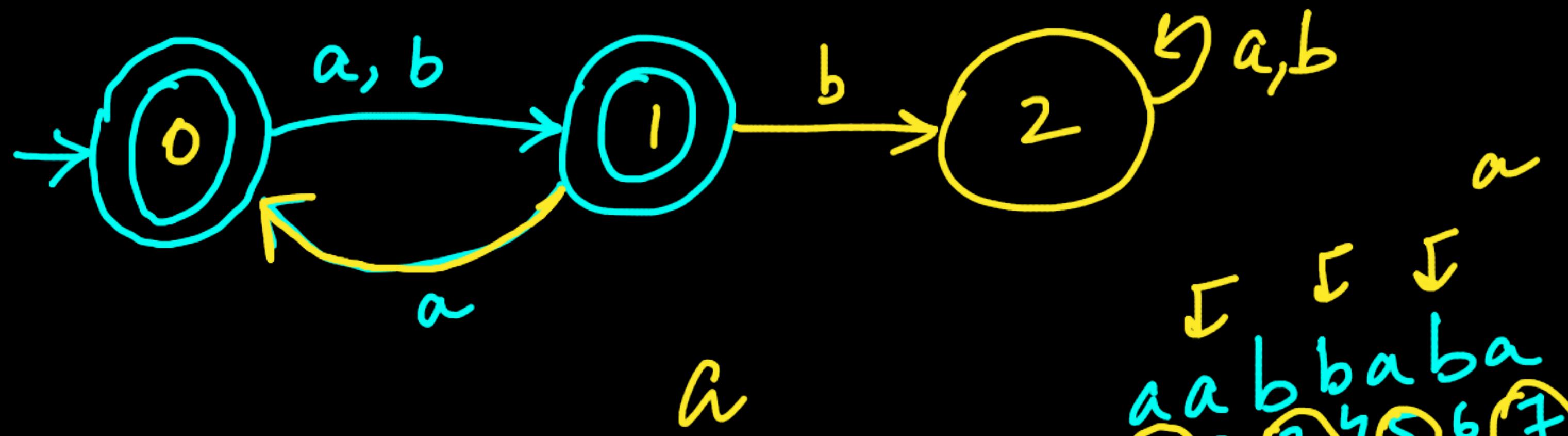
All even numbered symbols $\Rightarrow a$
(posn)

or
and

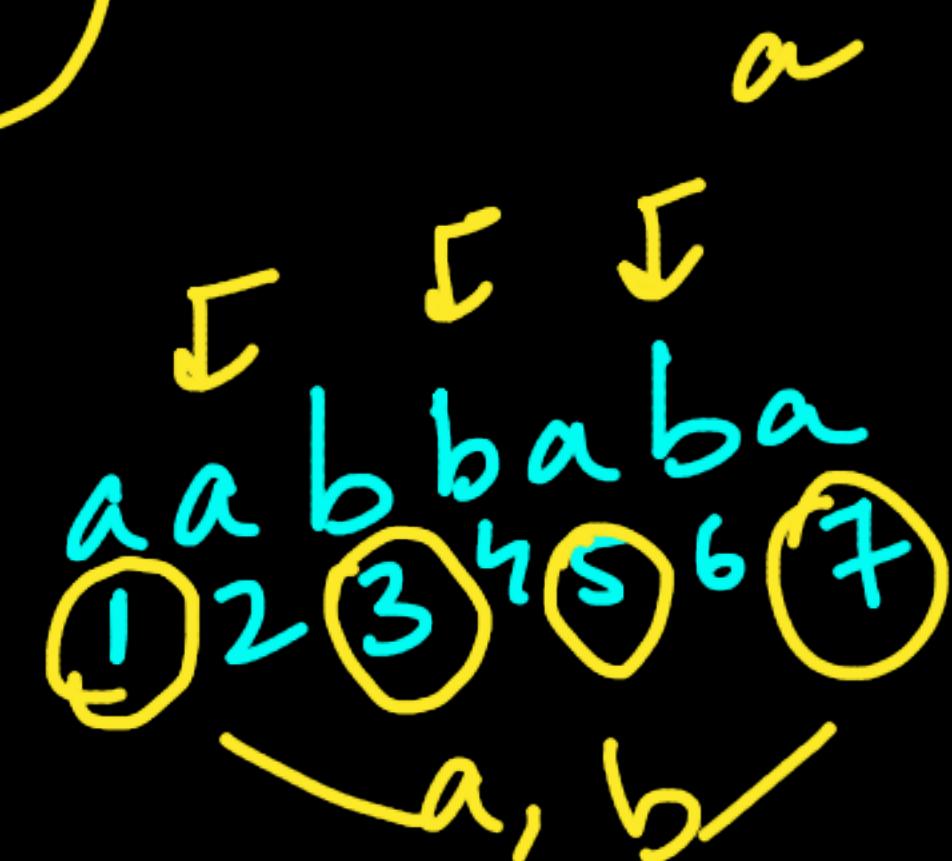
All odd numbered symbols $\Rightarrow b$
(posn)

i) Even numbered \Rightarrow 'a'

$L = \{ \lambda, a, b, \underline{\underline{aa}}, \underline{\underline{ab}}, \underline{\underline{ba}}, \underline{\underline{aaa}}, \underline{\underline{aab}}$



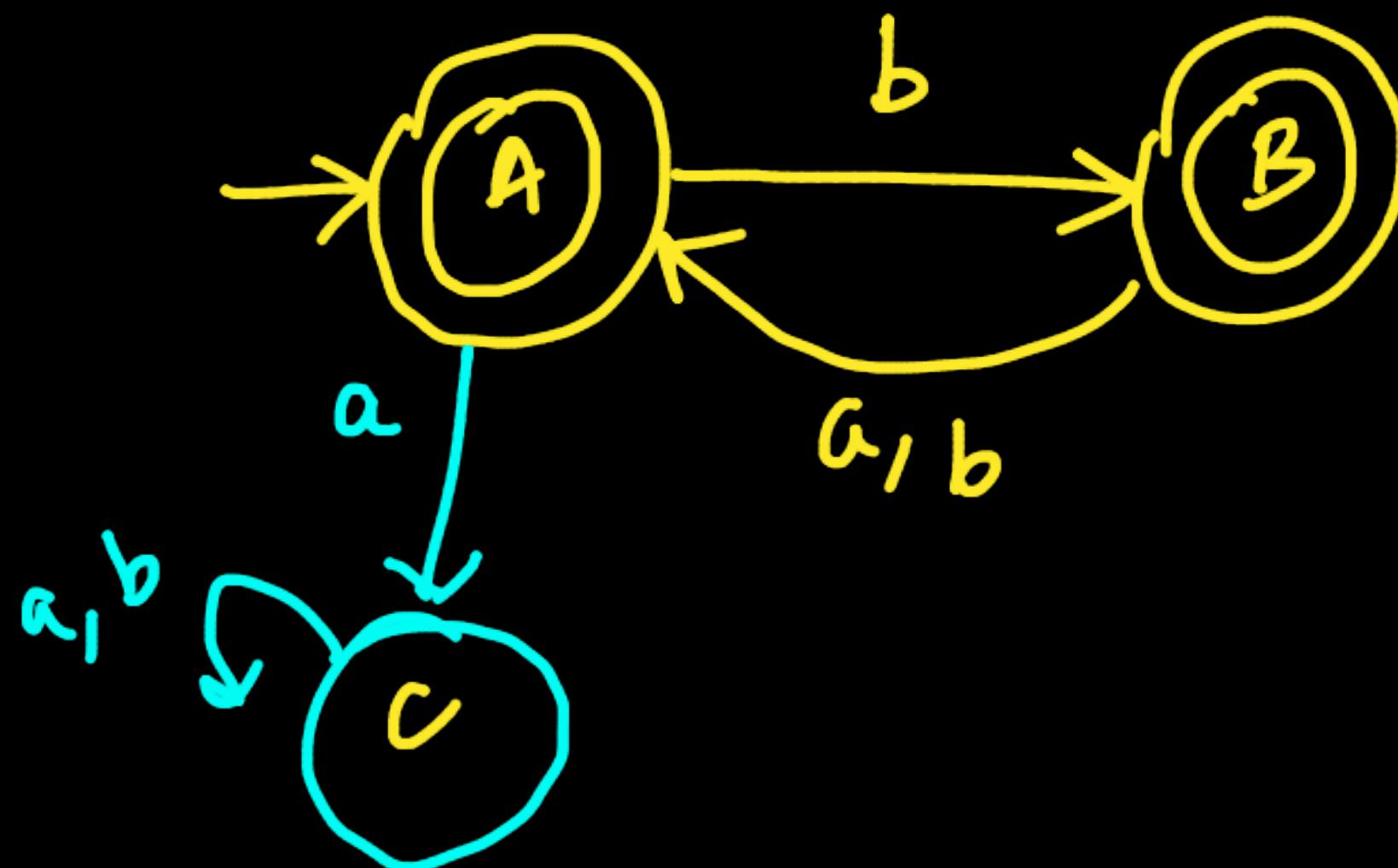
a



a

2) Odd numbered posn \Rightarrow "b"

$L = \{ \lambda, b, b^a, bab, bbb \}$



and

(Intersection)

$\xrightarrow{\text{union}}$

$$L_1 = \{ \lambda, a, b, aa, \underline{ba}, \dots \}$$

$$L_2 = \{ \lambda, b, \overset{\text{odd} \Rightarrow b}{ba}, bb, \dots \}$$

\neq or
(Union)

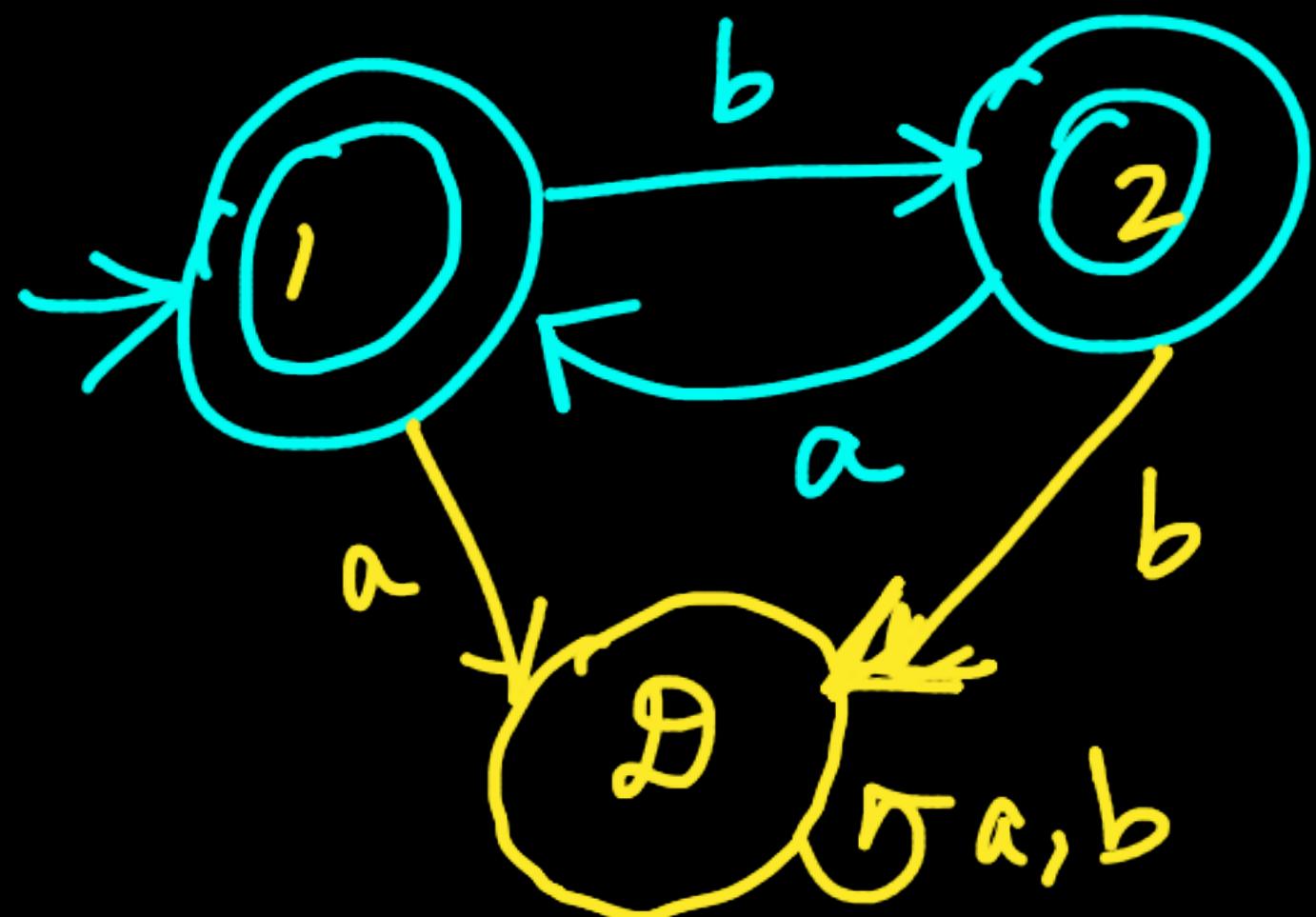
$$L_1 \cup L_2$$

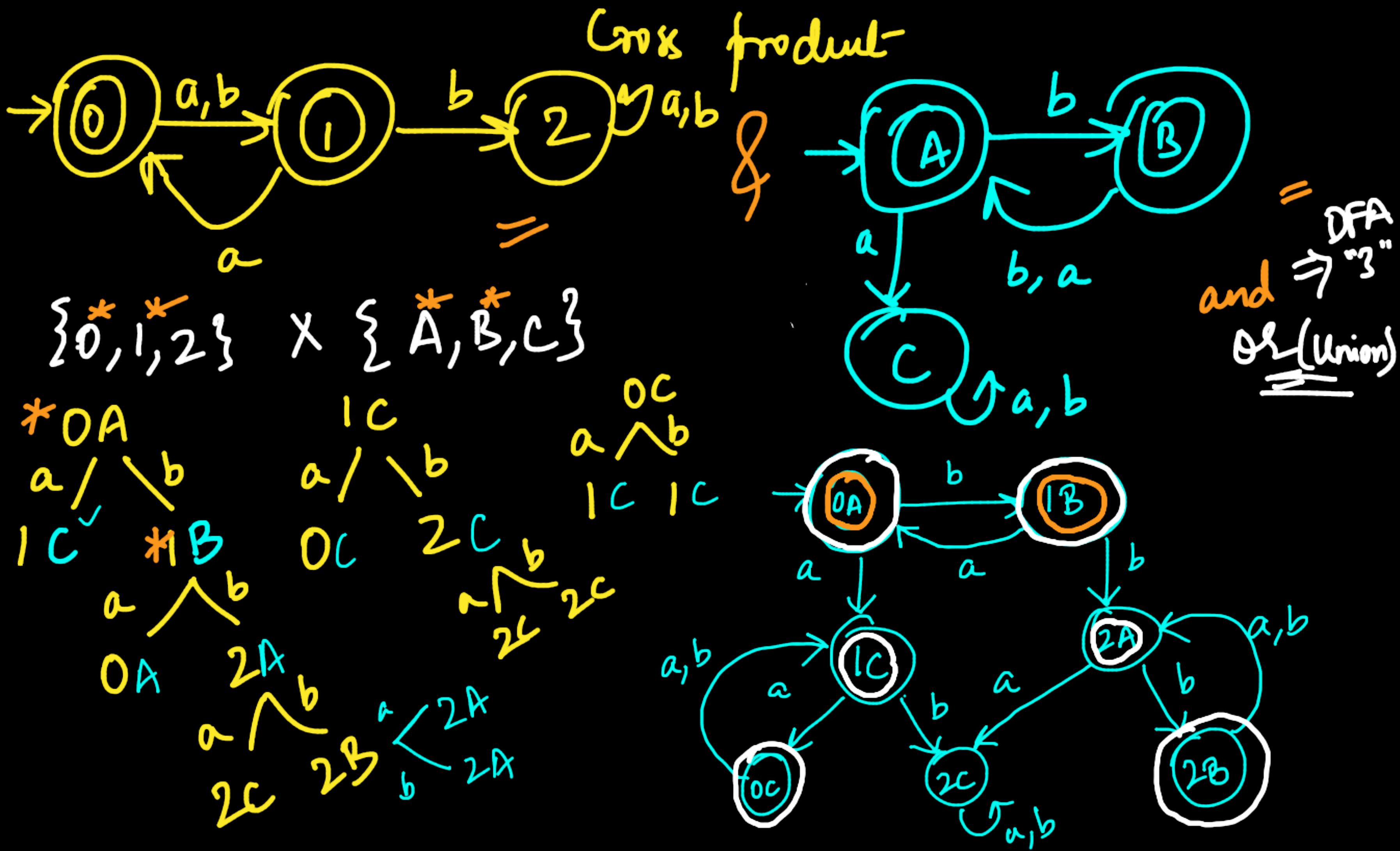
$$= \{ \lambda, a, b, \underline{aa}, \underline{ba}, \underline{bb} \dots \dots \dots \}$$

$$L_1 \cap L_2 = \{ \lambda, b, ba, \dots \}$$

Intersection
 \downarrow
 directly
 $\leftarrow \downarrow$
 odd $\Rightarrow b$ & even $\Rightarrow a$
 pos

$$\begin{aligned}
 L = \{ & \lambda, \underline{\underline{b}}, \underline{\underline{ba}}, \underline{\underline{bab}}, \\
 & \underline{\underline{babab}}, \underline{\underline{bababab}} \dots \\
 & \lambda + (\underline{\underline{ba}})^*(\underline{\underline{b}})
 \end{aligned}$$



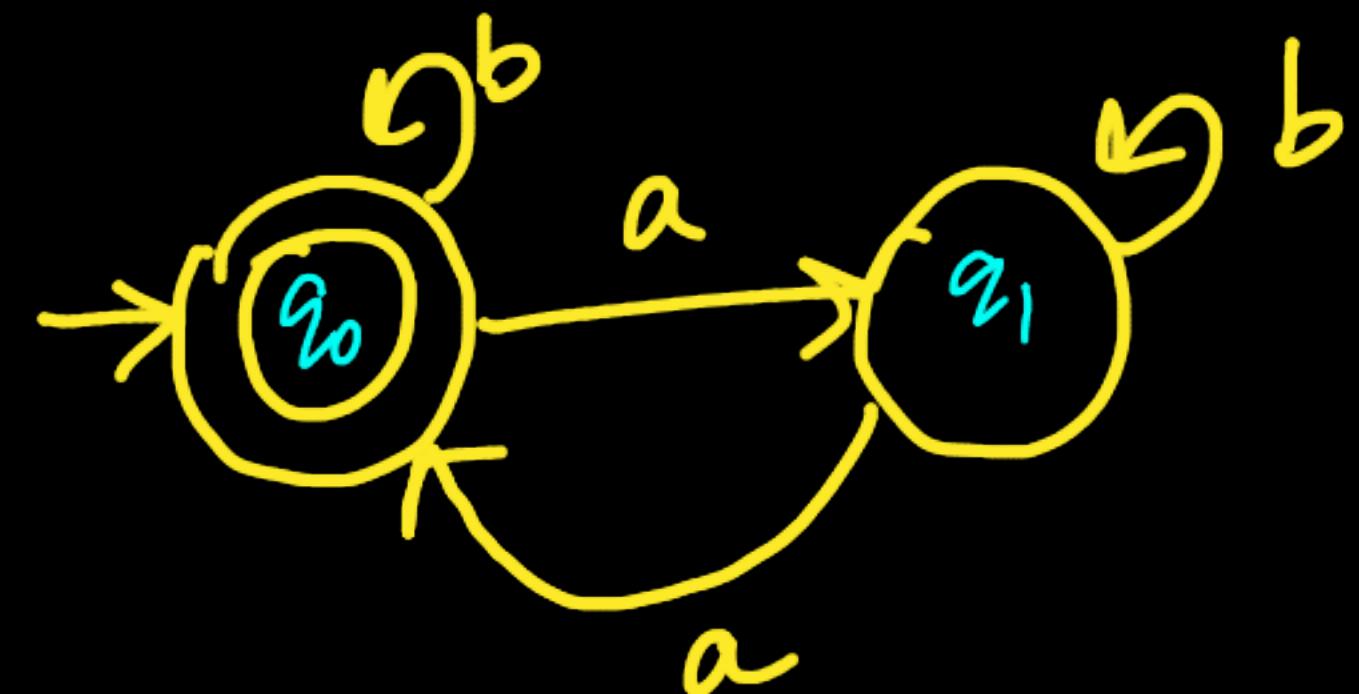


Q:- $L = \{ w : n_{a(w)} \bmod 2 = 0, w \in \{a, b\}^*\}$

no. of 'a's in the string 'w' must be even

$L = \{\lambda, b, bb, bbb, \dots, \underbrace{a, aa, \dots}_{b's}, \underbrace{b, bb, \dots}_{b's}\}$

$\begin{matrix} | & a & | & a & | & a & | & a & | \\ b & | & b & | & b & | & b & | & b \end{matrix}$



$$\frac{\%}{2} \equiv (\bmod)$$

$\% \equiv$ remainder

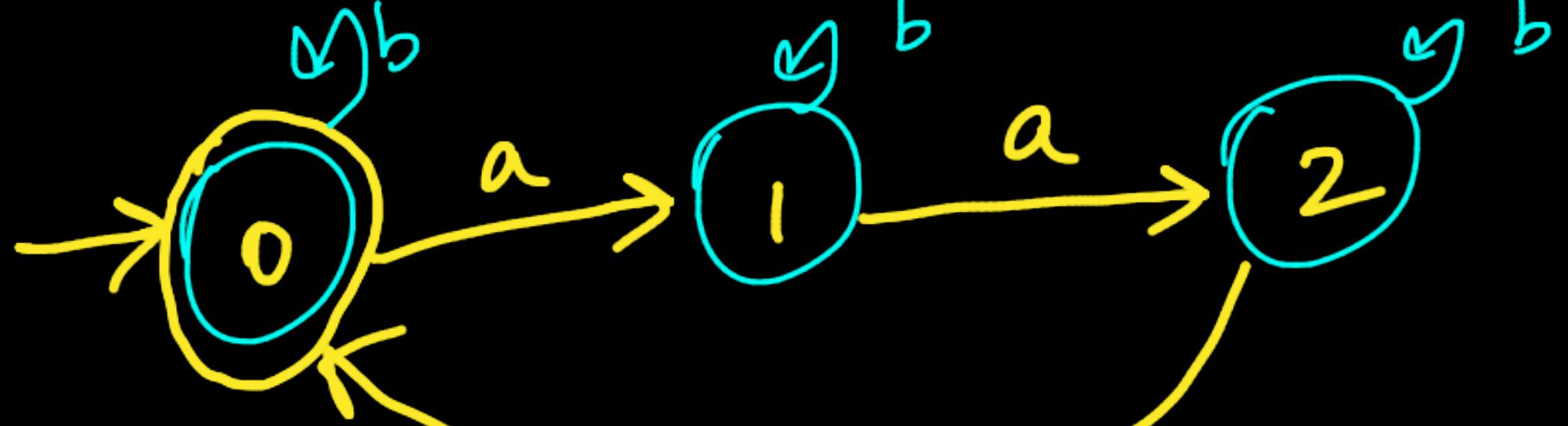
$$\begin{array}{c} / \\ 0 \\ \backslash \\ 1 \end{array}$$

$$\% (n)$$

States: $\Rightarrow 0, 1, 2, \dots, n-1$

$$Q: L = \{ w : Na(w) \bmod 3 = 0, w \in \{a, b\}^*\}$$

$$L = \{\lambda, b, bb, \dots, \underbrace{aaa}_{1}, \underbrace{aaa, aa, a}_{1, 1, 1}, \dots; i \% 3 = 1\}$$



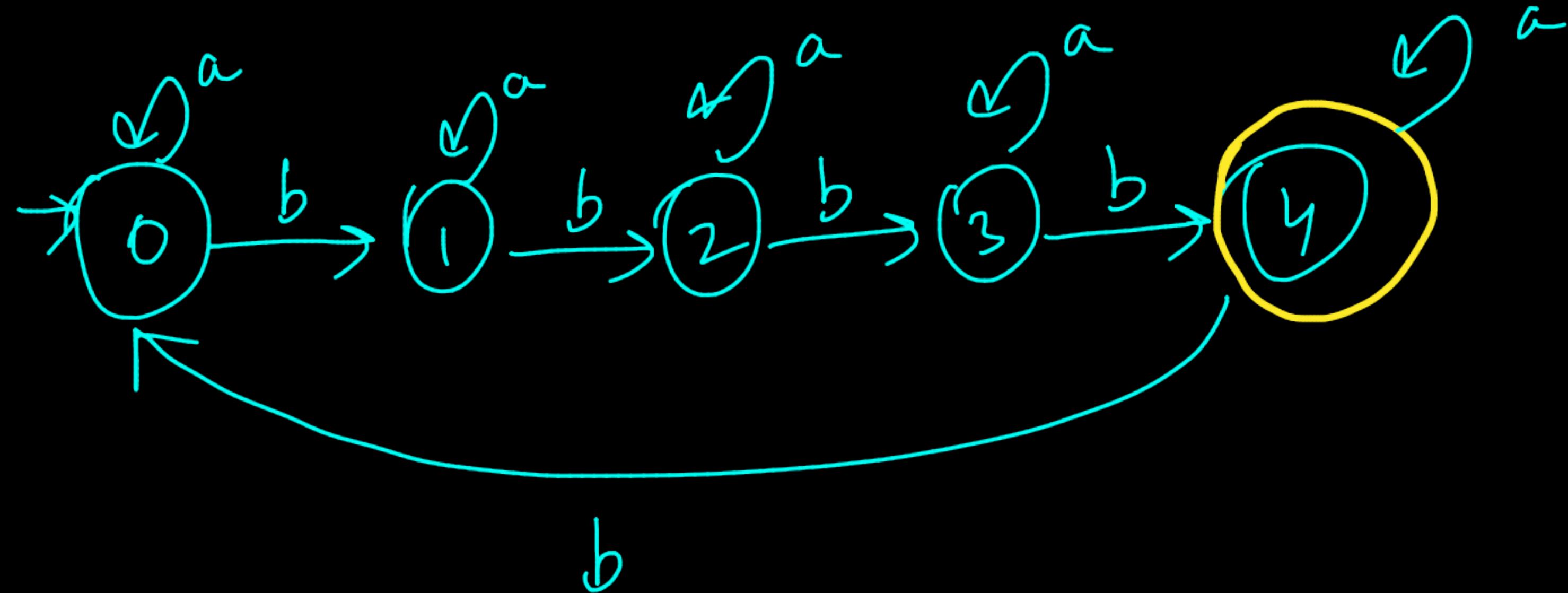
$$\boxed{aaaaaa} \bmod 3 = 2$$

$$Na(w) \bmod 3 = \boxed{2} \quad \wedge \text{final state}$$

$$\begin{aligned} 1 \% 3 &= 1 \\ 2 \% 3 &= 2 \\ 3 \% 3 &= 0 \\ 4 \% 3 &= 1 \end{aligned}$$

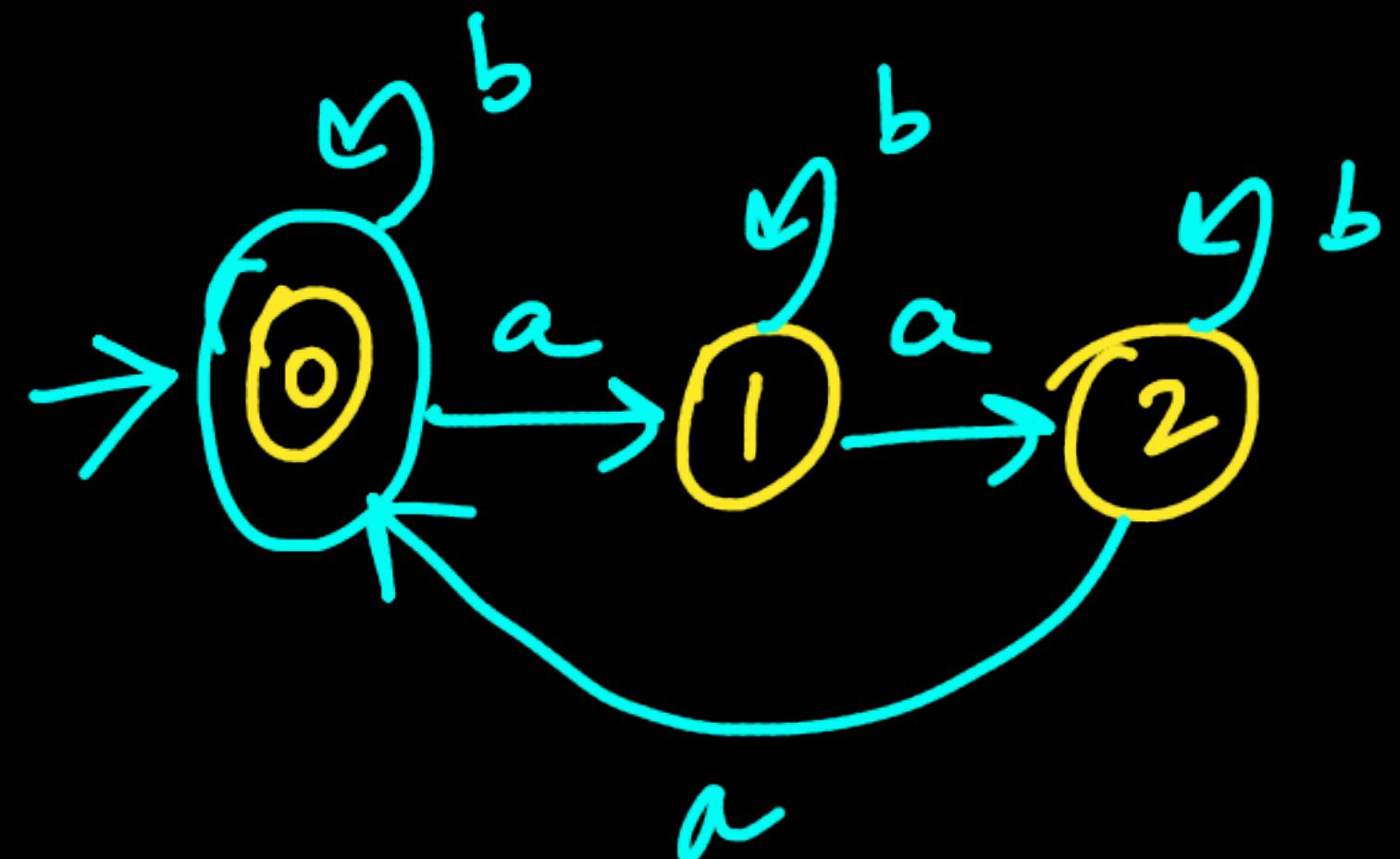
state

$$L = \{ w : n_b(w) \bmod 5 = 4, w \in \{a, b\}^*\}$$



Class 2 - Aug 19, 2020

Mod



$w \in \{a, b\}^*$

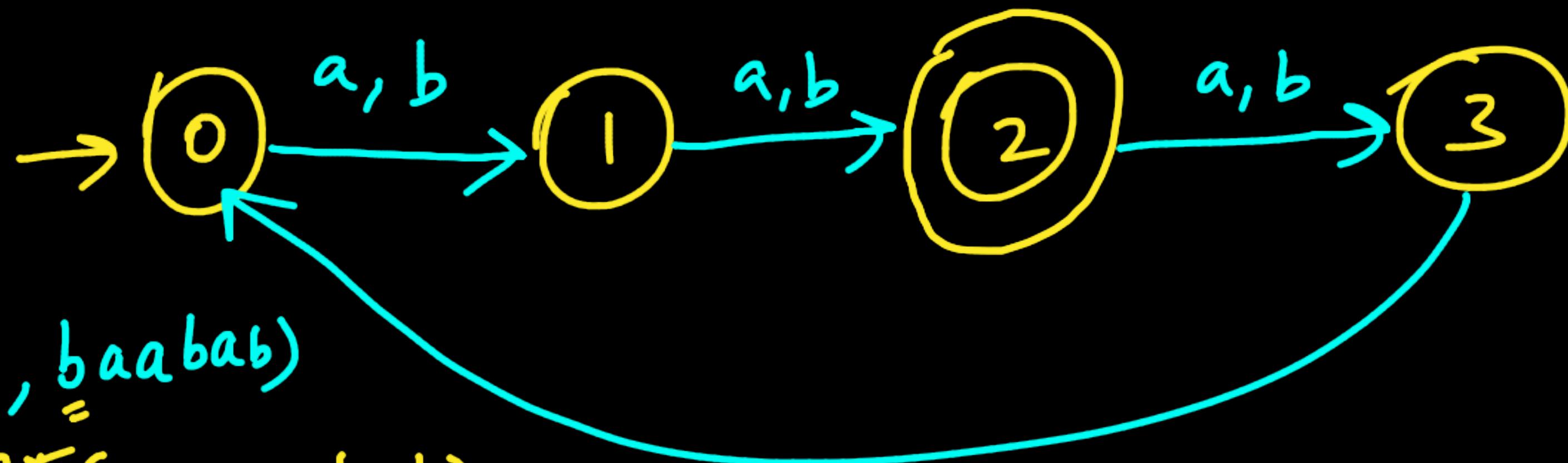
$n_a(w) \bmod 3 = \boxed{0}$

$$7 \% 3 = 1$$

$$8 \% 3 = 2$$

$$9 \% 3 = 0$$

$$L = \{ w ; |w| \bmod 4 = \boxed{2}, w \in \{a,b\}^* \}$$



$$\delta^*(0, \underline{\underline{baabab}})$$

$$= \delta^*(1, \underline{\underline{aabab}})$$

$$= \delta^*(2, \underline{\underline{bab}})$$

$$= \delta^*(3, \underline{\underline{bab}})$$

$$a, b$$

$$\delta^*(0, \underline{\underline{ab}})$$

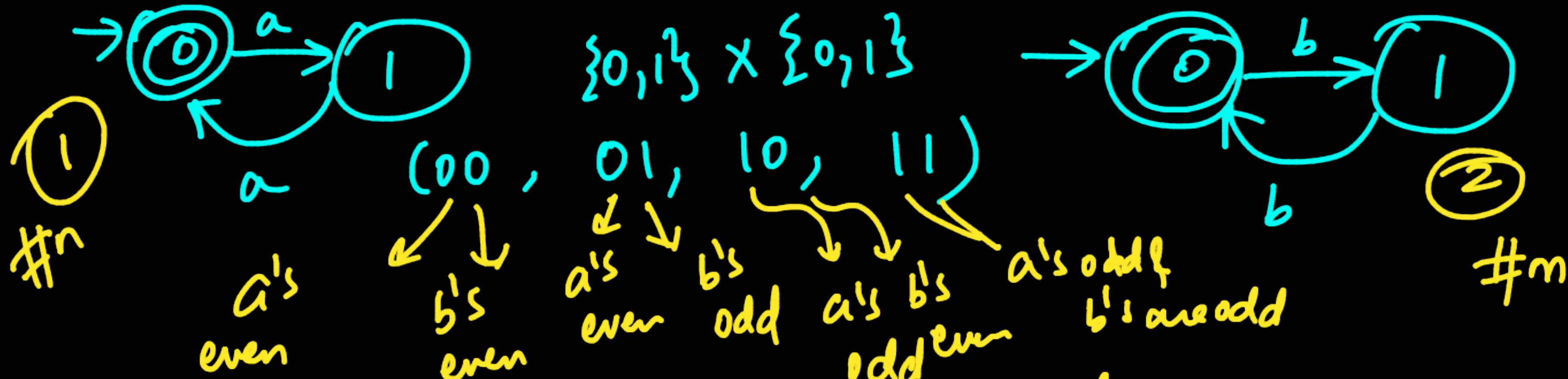
$$baabab$$

$$6 \% 4 = 2$$

$$\delta^*(1, b) = 2 \Rightarrow \text{final state} \checkmark$$

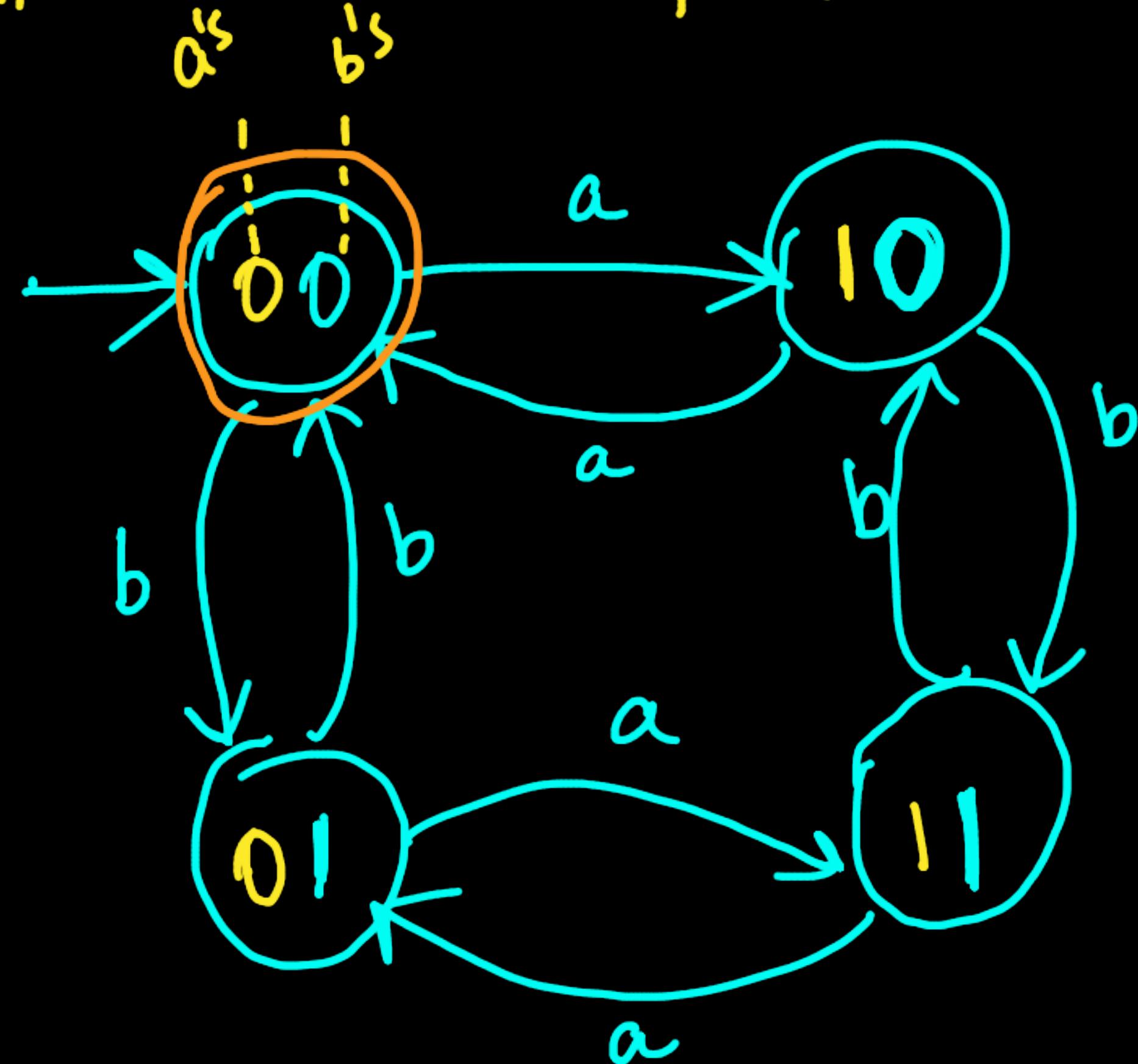
$L = \{ w : n_a(w) \equiv 0 \pmod{2} \text{ and } n_b(w) \equiv 0 \pmod{2} \}$
 $\Rightarrow (n_0 \cdot \text{# of } a's - b's) \text{ in the string must be even}$
 $\Rightarrow (\text{even no. of } a's \text{ & even no. of } b's)$

$L = \{ \lambda, aa, bb, \underline{abab}, \underline{\underline{baab}} \}$



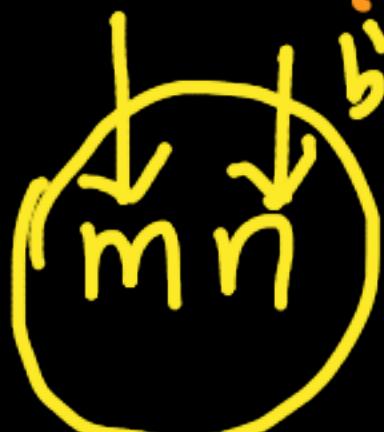
$\# \text{ states in final DFA} = n \times m = 2 \times 2 = 4$
 exactly 4 states

#a's even & b's are even \Rightarrow directly



final state $\hookrightarrow 0^{\text{even}}$ $\Rightarrow 1^{\text{odd}}$

0	1
1	0
0	1
1	0

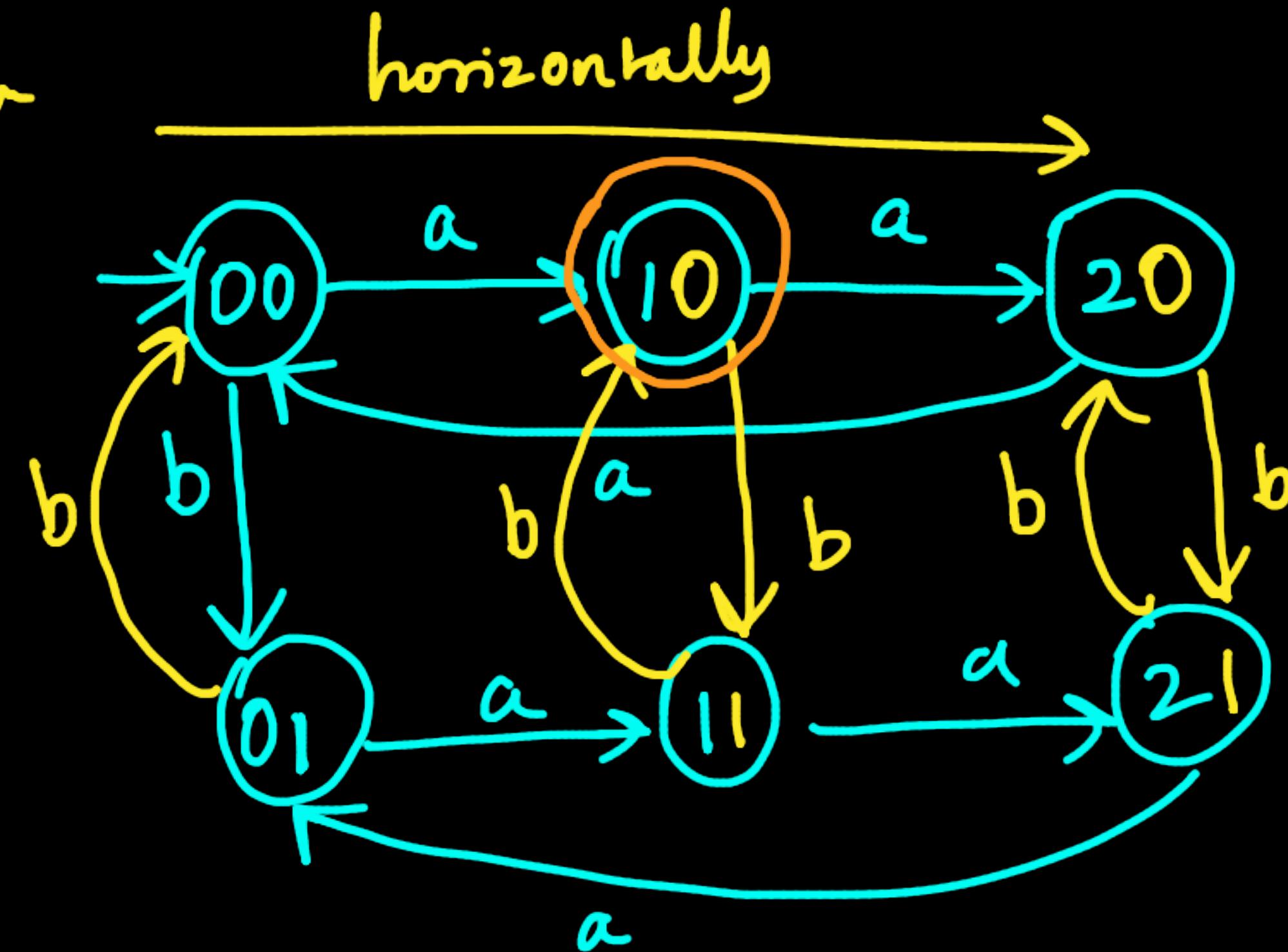
$Q_0 = a^s$ $n_a(w) \bmod 3 = 1$ and $n_b(w) \bmod 2 = 0$


 # 3 * #2 = 6 states

a's counter

b's counter

Vertically



00

find state
10

11

00

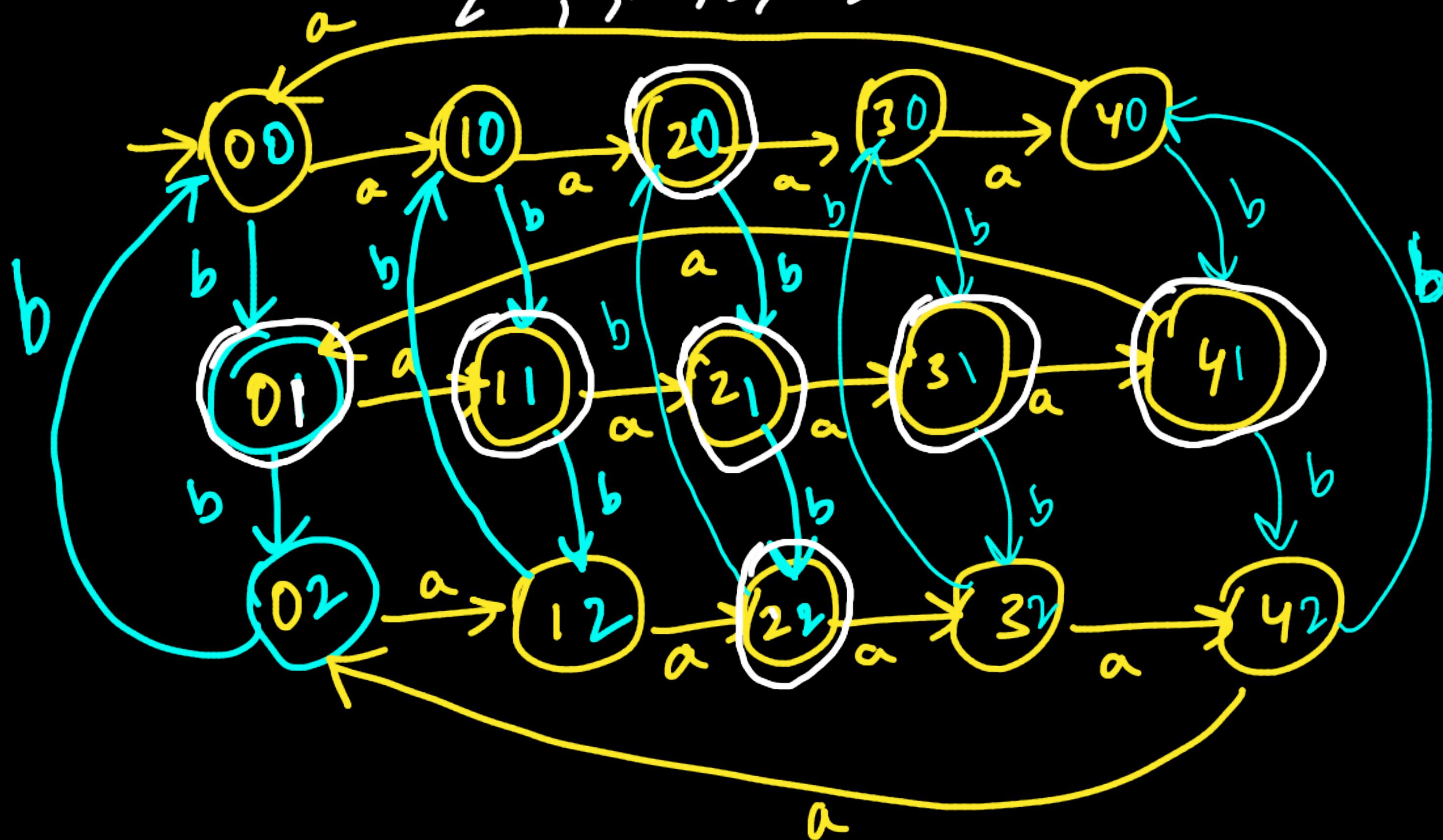
10

20

Q:- $n_a(w) \bmod 5 = 2$ or $n_b(w) \bmod 3 = 1$

States = $5 * 3 = 15$ states

$\{0, 1, 2, 3, 4\} \times \{0, 1, 2\}$



Homework

$$\Sigma = \{a, b, c\}$$

$$n_a(w) \bmod 3 = 2 \quad \& \quad n_b(w) \bmod 2 = 0 \\ (0, 1, 2) \qquad \qquad \qquad \times \qquad \qquad \qquad (0, 1)$$

$$\& \quad n_c(w) \bmod 2 = 1 \\ \times \qquad \qquad \qquad (0, 1)$$

Q: $L = \{ a^n b^m : n, m \geq 0 \}$ ✓

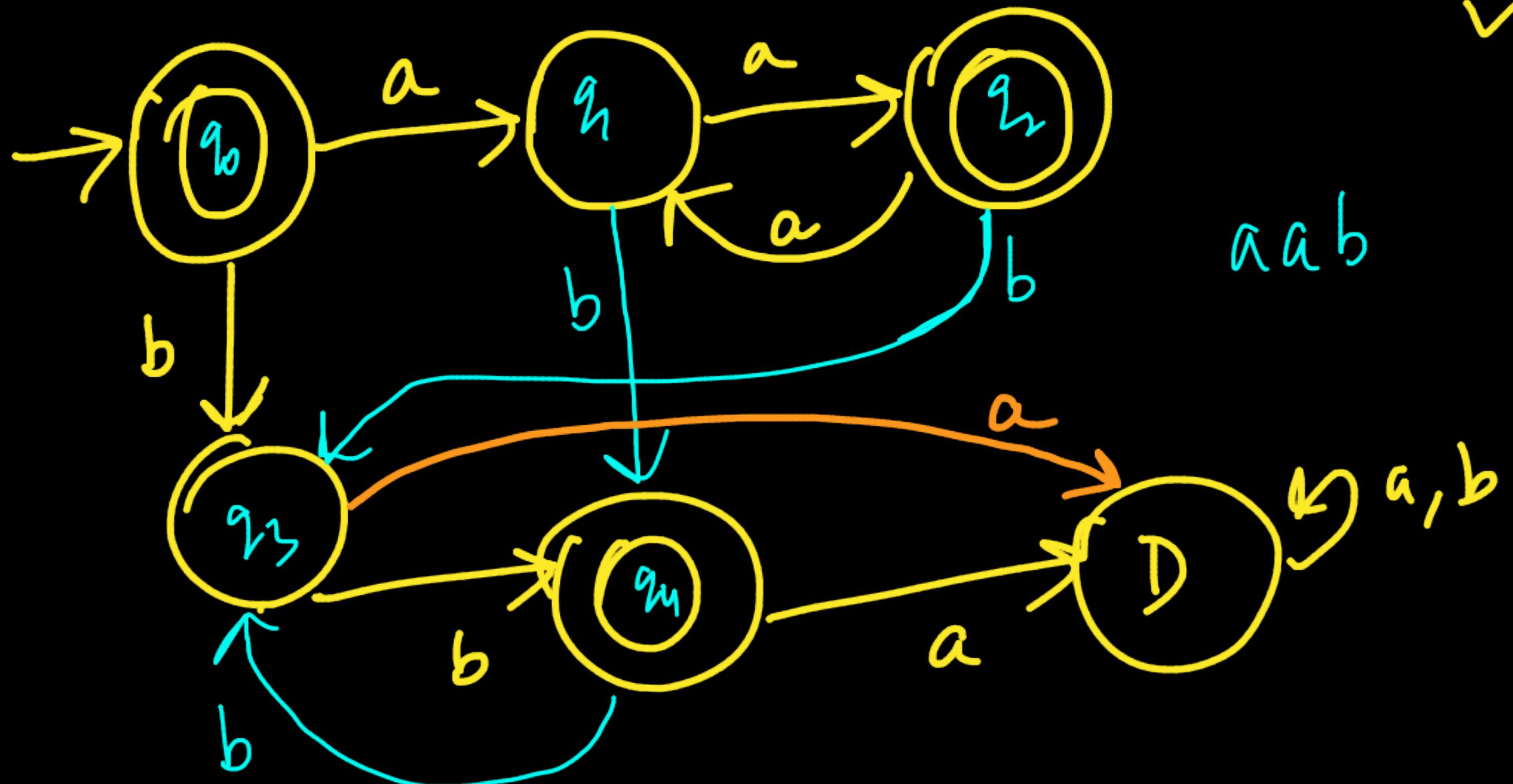
Def
 $L = \{ \underline{\underline{a^n b^m}} : (n+m) \bmod 2 = 0 \}$
(length of string) mod 2 = 0

ds b's
n m
even 0 even $|w| = \text{even}$
odd odd λ
even even even $\Rightarrow \{ \underline{\underline{ab}}, \underline{\underline{aaabb}}, \underline{\underline{abb}} \}$
even even even $\{ aabb, aaabb, aabbba \}$

$$f = \{ a^n b^m \mid (n+m) \bmod 2 = 0 \}$$

$$= \{ \lambda, \frac{ab}{aa}, \frac{aab}{abb}, \frac{abbb}{aabb}, \frac{aaab}{aab} \dots \}$$

#a's #b's
even even zero
odd odd
even even.



Another DFA

$$L = \{ a^n b^m \mid (n+m) \bmod 2 = 0 \}$$

