Automata Formal Languages & Logic

Q&A-Context Free Grammar

1. Describe the language generated by G=({S,A}, {a,b}, P, S). The set of productions P is given as:
   S->aA|bA
   A->aAa|bAb|aAb|bAa| λ
   Solution:
   This language rejects the empty string, but can start with either a or b. The next state the system has, A, is independent of whether we started with an a or b. Next, also observe that after this, the string can take the form of all possible strings of even length (since all 4 combinations: aa, ab ,ba or bb are allowed, along with the empty string). Hence, this language accepts all strings which have an odd length. w | w is a string over {a, } with an odd length}
   L= { w | w is a string over {a,b}* with an odd length}.

2. Construct the CFG for the language L= $\{a^n b^m c^k \mid n \neq m \text{ or } m \neq k\}$.
   Solution:
   There are four 'cases' to consider:
   1. More a's than b's (w/ any number of c's).
   2. More b's than a's (w/ any number of c's).
   3. More b's than c's (w/ any number of a's).
   4. More c's than b's (w/ any number of a's).
   Production Rules: $S \to S_1S_3 \mid S_2S_3 \mid S_4S_5 \mid S_4S_6$
   Each of the four 'cases' are accounted for (from left to right in the above production).
   $S_1 \to aS_1b \mid aS_1 \mid a$
   $S_2 \to aS_2b \mid S_2b \mid b$
   $S_3 \to S_3c \mid \lambda$
   $S_4 \to aS_4 \mid \lambda$
   $S_5 \to bS_5c \mid bS_6 \mid b$
   $S_6 \to bS_6c \mid S_6c \mid c$

3. For the regular expression (011+1)*(01)* obtain a context free grammar.

Solution:

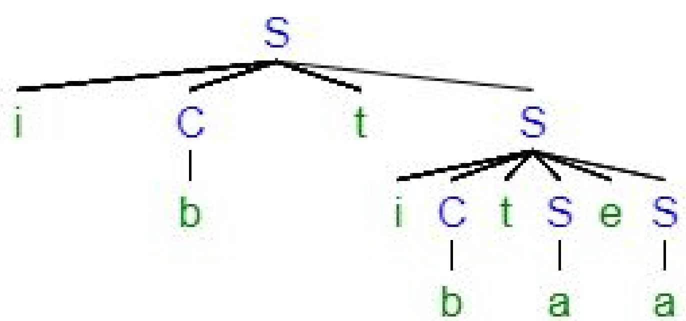
The regular expression is (011+1)*(01)* of the form A*B* where A can be 011 or 1 and B is 01.The regular expression A*B* means that any number of a's(possibly none) are followed by any number of b's(possibly none).Any number of a's can be generated using the productions.

G=(V,T,P,S) where V= {S,A,B} ,T= {0,1} ,S is the start symbol.

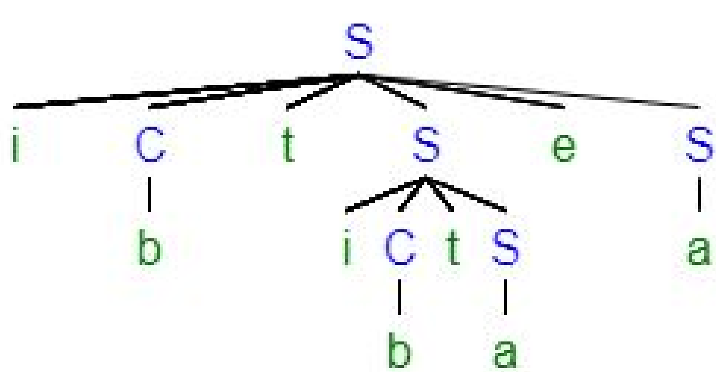
P= {
S->AB
A->011A|1A| λ
B->01B| λ
}

4. Is the following grammar ambiguous?
   S->iCtS | iCtSeS | a
   C->b
   Solution:

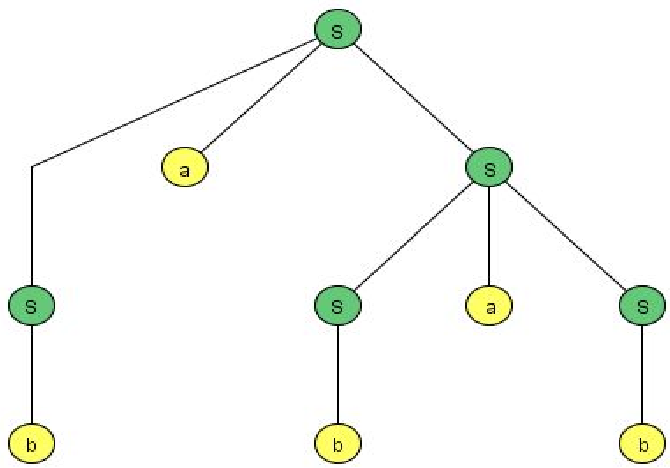
   Leftmost derivation1
   S⇒iCtS⇒ibtS⇒ibtiCtSeS⇒ibtibtSeS⇒ibtibtaeS⇒ibtibtaea

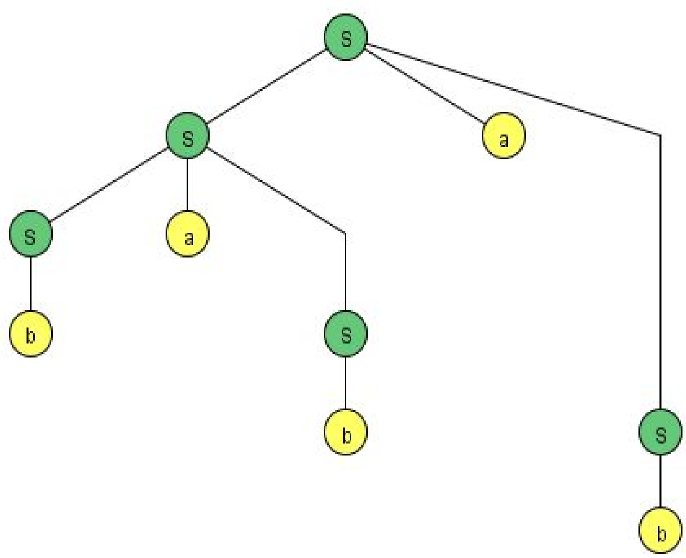


   Leftmost derivation2
   S⇒iCtSeS⇒ibtSeS⇒ibtiCtSeS⇒ibtibtSeS⇒ibtibtaeS⇒ibtibtaea



5. Show that $S \rightarrow SaS \mid b$ is ambiguous. Construct an unambiguous equivalent of the grammar.
Solution: Consider the string *babab*. There are two different leftmost derivations for this string as shown in the two parse trees below. Hence it is ambiguous.





An unambiguous equivalent is the regular grammar: $S \rightarrow baS \mid b$

6. Using the grammar G=(V,T,P,S),with V={S} P={S->S∪S | SS | S* | (S) | 0 | 1 | λ } ,give the left derivation and the corresponding parse tree for the strong(0∪ (10)*1)*.

Solution:

A derivation for (0 ∪ (10)∗1)∗ is

S ⇒ S∗ ⇒ (S)∗ ⇒ (S ∪ S)∗ ⇒ (0 ∪ S)∗ ⇒ (0 ∪ SS)∗ ⇒ (0 ∪ S∗S)∗ ⇒ (0 ∪ (S)∗S)∗ ⇒ (0 ∪ (SS)∗S)∗ ⇒ (0 ∪ (1S)∗S)∗ ⇒ (0 ∪ (10)∗S)∗ ⇒ (0 ∪ (10)∗1)∗

The corresponding parse tree is,