



LINEAR ALGEBRA AND ITS APPLICATIONS

UE19MA251

Unit 3. Linear Transformations and Orthogonality

Orthogonal Vectors & Subspaces



Definition:

The **norm or length** of a n-dimensional vector $x = (x_1, x_2, \dots, x_n)$ is written as $\|x\|$ and is defined as

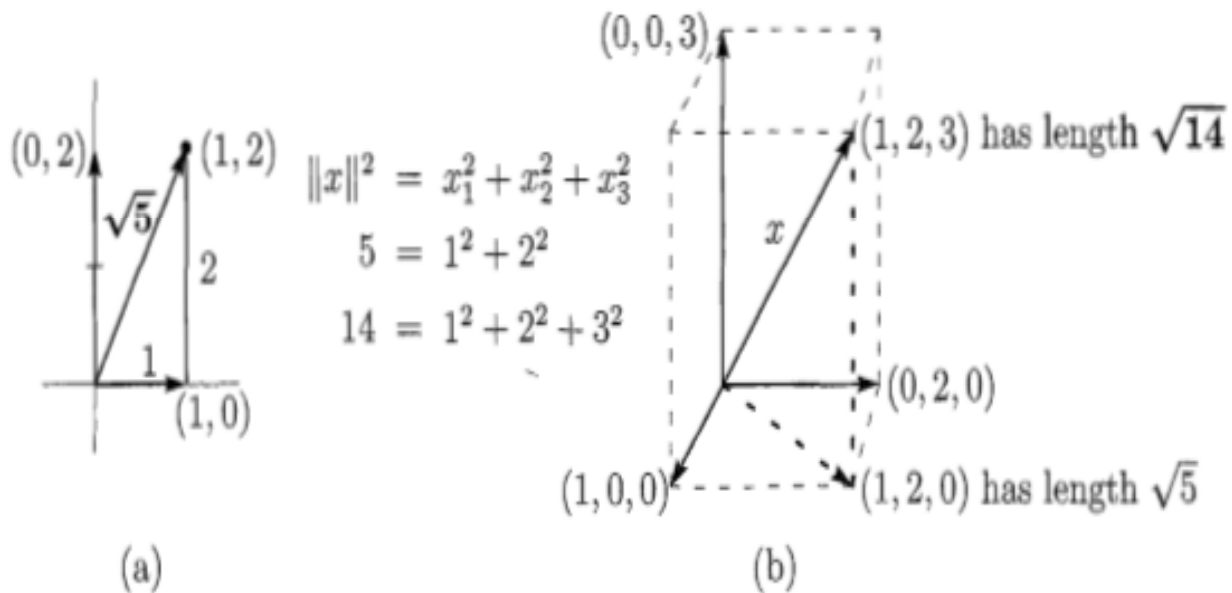
$$\|x\| = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$$

We can also write $\|x\|^2 = x^T x$

Note : Zero is the only vector whose norm is 0.

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Definition:

The **inner product** or dot product or scalar product of two vectors $x = (x_1, x_2, \dots, x_n)$ and $y = (y_1, y_2, \dots, y_n)$ is denoted by

$$x^T y \text{ or } x \circ y \text{ or } \langle x, y \rangle$$

and is defined by

$$x^T y = x_1 y_1 + x_2 y_2 + \dots + x_n y_n$$

Note that

$$x^T y = y^T x$$

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Definition :

Two vectors $x = (x_1, x_2, \dots, x_n)$ and $y = (y_1, y_2, \dots, y_n)$ are said to be orthogonal if

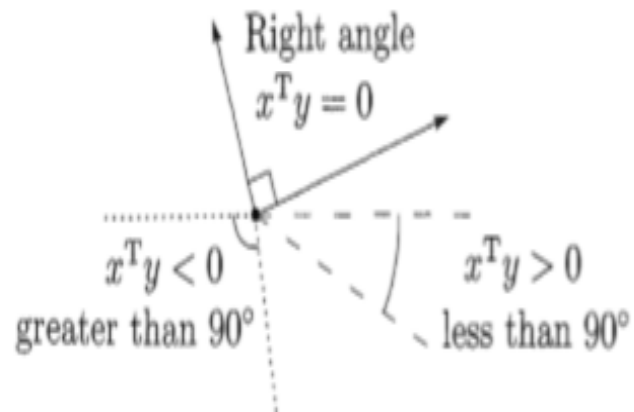
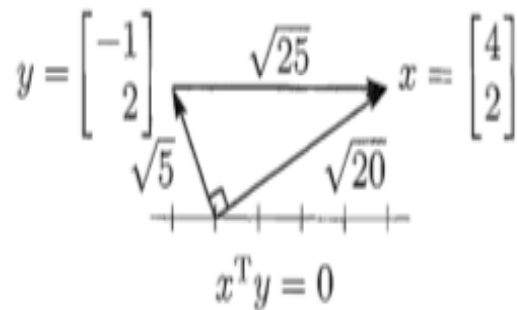
$$x^T y = y^T x = 0$$

Note :

1. Zero is the only vector that is orthogonal to itself.
2. Zero is the only vector that is orthogonal to every other vector.

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Examples

1. The coordinate vectors $(1, 0, \dots, 0)$, $(0, 1, 0, \dots, 0)$, \dots , $(0, 0, \dots, 0, 1)$ are mutually orthogonal in \mathbb{R}^n .
2. The vectors (c, s) , $(-s, c)$ are orthogonal in \mathbb{R}^2 .
3. The vectors $(2, 1, 0)$, $(-1, 2, 0)$ are orthogonal in \mathbb{R}^3 .



THANK YOU
