STATISTICS FOR DATA SCIENCE

HYPOTHESIS and INFERENCE

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UNIT-4 HYPOTHESIS and INFERENCE

Session-5

Drawing Conclusions from the Results of Hypothesis Tests

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Choose H_0 to Answer the Right Question:

- When performing a hypothesis test, it is important to choose H_0 and H_1 appropriately so that the result of the test can be useful in forming a conclusion.

Choose H_0 to Answer the Right Question:

Example:

- **Specifications for a water pipe call for a mean breaking strength μ of more than $2000\ lb$ per linear foot.**
- **Engineers will perform a hypothesis test to decide whether or not to use a certain kind of pipe.**
- **They will select a random sample of $1\ ft$ sections of pipe, measure their breaking strengths, and perform a hypothesis test.**

Source: Text Book Chapter6

Choose H_0 to Answer the Right Question:

Example:

- The pipe will not be used unless the engineers can conclude that $\mu > 2000$.
- Assume they test $H_0: \mu \leq 2000$ *versus* $H_1: \mu > 2000$.
- Will the engineers decide to use the pipe if H_0 is rejected? What if H_0 is not rejected?

Source: Text Book Chapter6

Choose H_0 to Answer the Right Question:

Solution:

- If H_0 is rejected, the engineers will conclude that $\mu > 2000$, and they will use the pipe.

- If H_0 is not rejected, the engineers will conclude that μ *might* be less than or equal to 2000, and they will not use the pipe
- .

Source: Text Book Chapter6

Choose H_0 to Answer the Right Question:

Example:

- Assume the engineers test $H_0: \mu \geq 2000 \; versus \; H_1: \mu < 2000.$

- Will the engineers decide to use the pipe if H_0 is rejected? What if H_0 is not rejected?

Source: Text Book Chapter6

Choose H_0 to Answer the Right Question:

Solution:

- If H_0 is rejected, the engineers will conclude that $\mu < 2000$, and they will not use the pipe.

- If H_0 is not rejected, the engineers will conclude that μ *might* be greater than or equal to 2000, but that it also might not be. So again, they won't use the pipe.

Source: Text Book Chapter6

Statistical Significance Is Not the Same as Practical Significance:

When a result has a small $P - value$, we say that it is “statistically significant.” In common usage, the word signi cant means “important.” It is therefore tempting to think that statistically significant results must always be important. This is not the case. Sometimes statistically significant results do not have any scientific or practical importance. We will illustrate this with an example. Assume that a process used to manufacture synthetic fibers is known to produce fibers with a mean breaking strength of $50\ N$. A new process, which would require considerable retooling to implement, has been developed. In a sample of 1000 fibers produced by this new method, the average breaking strength was $50.1\ N$, and the standard deviation was $1\ N$. Can we conclude that the new process produces fibers with greater mean breaking strength?

Source: Text Book Chapter6

Statistical Significance Is Not the Same as Practical Significance:

- To answer this question, let μ be the mean breaking strength of fibers produced by the new process. We need to test $H_0: \mu \leq 50$ versus $H_1: \mu > 50$. In this way, if we reject H_0, we will conclude that the new process is better. Under H_0, the sample mean X has a normal distribution with mean 50 and standard deviation $\frac{1}{\sqrt{10000}} = 0.0316$. The z-score is

$$z = \frac{50.1-50}{0.0316} = 3.16$$

Statistical Significance Is Not the Same as Practical Significance:

- The P-value is 0.0008. This is very strong evidence against H0. The new process produces fibers with a greater mean breaking strength.
- What practical conclusion should be drawn from this result? On the basis of the hypothesis test, we are quite sure that the new process is better. Would it be worthwhile to implement the new process? Probably not. The reason is that the difference between the old and new processes, although highly statistically significant, amounts to only 0.1 N. It is unlikely that this is difference is large enough to matter.
- The lesson here is that a result can be statistically significant without being large enough to be of practical importance. How can this happen? A difference is statistically significant when it is large compared to its standard deviation.

Source: Text Book Chapter6

Statistical Significance Is Not the Same as Practical Significance:

In the example, a difference of 0.1 N was statistically significant because the standard deviation was only 0.0316 N. When the standard deviation is very small, even a small difference can be statistically significant.

Source: Text Book Chapter6

Statistical Significance Is Not the Same as Practical Significance:

- **The P-value does not measure practical significance.**
- **What it does measure is the degree of confidence we can have that the true value is really different from the value specified by the null hypothesis.**

Source: Text Book Chapter6

Statistical Significance Is Not the Same as Practical Significance:

- **When the *P*-value is small, then we can be confident that the true value is really different.**
- **This does not necessarily imply that the difference is large enough to be of practical importance.**

The Relationship Between Hypothesis Tests and Confidence Intervals:

- **Both confidence intervals and hypothesis tests are concerned with determining plausible values for a quantity such as a population mean μ. In a hypothesis test for a population mean μ, we specify a particular value of μ (the null hypothesis) and determine whether that value is plausible. In contrast, a confidence interval for a population mean μ can be thought of as the collection of all values for μ that meet a certain criterion of plausibility, specified by the confidence level $100(1-\alpha)\%$. In fact, the relationship between confidence intervals and hypothesis tests is very close.**

Source: Text Book Chapter6

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The Relationship Between Hypothesis Tests and Confidence Intervals:

- **The values contained within a two-sided level $100(1-\alpha)\%$confidence interval for a population mean μ are precisely those values for which the P-value of a two-tailed hypothesis test will be greater than α.**

- **Example: the 95% confidence interval consists of precisely those values of μ whose P-values are greater than 0.05 in a hypothesis test.**

Source: Text Book Chapter6

The Relationship Between Hypothesis Tests and Confidence Intervals:

- **A one-sided level $100(1 - \alpha)\%$ confidence interval consists of all the values for which the *P*-value in a one-tailed test would be greater than α.**

- The confidence level is equivalent to (1 – α) level. So, if your significance level is 0.05, the corresponding confidence level is 95%.

Source: Text Book Chapter6

Rejection Region approach for Hypothesis Test

Critical Point & Rejection Region

- A critical point is a value of the test statistic that produces a P-value exactly equal to α.
- The region on the side of the critical point that leads to rejection is called the rejection region.
- The critical point itself is also in the rejection region.

Source: Text Book Chapter6

Example:

- A machine runs on an average of 125 hours/year.
- A random sample of 49 machines has an annual average use of 126.9 hours with standard deviation 8.4 hours.
- Does this suggest to believe that machines are used on the average more than 125 hours annually at 0.05 level of significance.

Solution:

$\mu =$ average number of hours a machine runs in an year.

$H_0 . \mu \leq 125$ hours/year , $H_1 . \mu > 125$

L.O.S.: $\propto$ = 0.05

Calculation: $Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{126.9-125}{8.4/\sqrt{49}}$ = 1.58

P- value is $.0571 > 0.05$

So we need to accept H_o.

We can not believe that machine works more than 125 hours in an year.

Example:

- A manufacture of tyres guarantees that the average lifetime of its tyres is more than 28000 miles.

- If 40 tyres of this company tested, yields a mean lifetime of 28463 miles with s.d. of 1348 miles.

- Can the guarantee be accepted at 0.01 L.O.S.?

Solution:

Significance level $\alpha = 0.01$

$$H_0: \mu \leq 28000, \qquad H_1: \mu > 28000$$

$$\bar{X} = 28463\, miles, n = 40 \;\; \sigma \rightarrow s = 1348\, miles$$

z$= \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{28463-28000}{1348/\sqrt{40}}$=2.17

P value: $0.015 > 0.01$

We need to Reject the null hypothesis.

Example:

- Can it be concluded that the average lifespan of Indian is more than 70 years if a random sample of 100 Indians has an average lifespan of 71.8years with a s.d of 8.9 years

Solution:

Significance level $\alpha = 0.05$

$H_0: \mu \leq 70\ years,\ H_1: \mu > 70$years

$\bar{X} = 71.8\ years, \quad n = 100\ \ \sigma \to s = 8.9\ years$

z$= \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{71.8-70}{8.9/\sqrt{100}}$=2.02

P value: $0.015 > 0.01$

We need to Reject the null hypothesis.

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