PES University, Bangalore

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UE19CS203 – STATISTICS FOR DATA SCIENCE

Unit - 3 - Probability Distributions

QUESTION BANK

Confidence Intervals with Paired Data

Exercises for Section 5.7

[Text Book Exercise – Section 5.7 – Q. No. [1 – 10] – Pg. No. [372-374]]

1. The article "Simulation of the Hot Carbonate Process for Removal of CO2 and H2S from Medium Btu Gas" (K. Park and T. Edgar, *Energy Progress*, 1984:174–180) presents an equation used to estimate the equilibrium vapor pressure of CO2 in a potassium carbonate solution. The actual equilibrium pressure (in kPa) was measured in nine different reactions and compared with the value estimated from the equation. The results are presented in the following table:

Reaction	Estimated	Experimental	Difference
1.	45.10	42.95	2.15
2.	85.77	79.98	5.79
3.	151.84	146.17	5.67
4.	244.30	228.22	16.08
5.	257.67	240.63	17.04
6.	44.32	41.99	2.33
7.	84.41	82.05	2.36
8.	150.47	149.62	0.85
9.	253.81	245.45	8.36

Find a 95% confidence interval for the mean difference between the estimated and actual pressures.

2. The article "Effect of Refrigeration on the Potassium Bitartrate Stability and Composition of ItalianWines" (A. Versari, D. Barbanti, et al., *Italian Journal of Food Science,* 2002:45–52) reports a study in which eight types of white wine had their tartaric acid

concentration (in g/L) measured both before and after a cold stabilization process. The results are presented in the following table:

Wine Type	Before	After	Difference
1.	2.86	2.59	0.27
2.	2.85	2.47	0.38
3.	1.84	1.58	0.26
4.	1.60	1.56	0.04
5.	0.80	0.78	0.02
6.	0.89	0.66	0.23
7.	2.03	1.87	0.16
8.	1.90	1.71	0.19

Find a 95% confidence interval for the mean difference between the tartaric acid concentrations before and after the cold stabilization process.

3. Transepidermal water loss (TEWL) is a measure of the rate that water crosses the skin through diffusion and evaporation. In general, damaged skin has a higher TEWL than nondamaged skin. A report submitted to the U.S. Food and Drug Administration (*Therapeutic Equivalence of Topical Products,* A. Bunge, B. N'Dri-Stempfer, et al., 2007) described an experiment in which the outer layer of skin on a small area of the forearm was partially removed in order to measure the concentration of a certain drug. TEWL (in g/m2 per hour) was measured both before and after skin removal. The results for 10 individuals were as follows.

Subject	Before	After
1	18	27
2	12	9
3	14	19
4	11	20
5	12	22
6	17	26
7	16	18
8	18	26
9	14	22
10	14	24

Find a 98% confidence for the increase in TEWL.

4. Breathing rates, in breaths per minute, were measured for a group of 10 subjects at rest, and then during moderate exercise. The results were as follows:

Subject	Rest	Exercise
1	15	30
2	16	37
3	21	39
4	17	37
5	18	40
6	15	39
7	19	34
8	21	40
9	18	38
10	14	34

Find a 95% confidence interval for the increase in breathing rate due to exercise.

5. A group of five individuals with high blood pressure were given a new drug that was designed to lower blood pressure. Systolic blood pressure was measured before and after treatment for each individual, with the following results:

Subject	Before	After
1	170	145
2	164	132
3	168	129
4	158	135
5	183	145

Find a 90% confidence for the mean reduction in systolic blood pressure.

6. A sample of 10 diesel trucks were run both hot and cold to estimate the difference in fuel economy. The results, in mpg, are presented in the following table. (From "In-use Emissions from Heavy-Duty Diesel Vehicles," J. Yanowitz, Ph.D. thesis, Colorado School of Mines, 2001.)

Truck	Hot	Cold
1	4.56	4.26
2	4.46	4.08
3	6.49	5.83
4	5.37	4.96

5	6.25	5.87
6	5.90	5.32
7	4.12	3.92
8	3.85	3.69
9	4.15	3.74
10	4.69	4.19

Find a 98% confidence interval for the difference in mean fuel mileage between hot and cold engines.

7. For a sample of nine automobiles, the mileage (in 1000s of miles) at which the original front brake pads Were worn to 10% of their original thickness was measured, as was the mileage at which the original rear brake pads were worn to 10% of their original thickness. The results are given in the following table.

Automobile	Front	Rear
1	32.8	41.2
2	26.6	35.2
3	35.6	46.1
4	36.4	46.0
5	29.2	39.9
6	40.9	51.7
7	40.9	51.6
8	34.8	46.1
9	36.6	47.3

Find a 95% confidence interval for the difference in mean lifetime between the front and rear brake pads.

8. Refer to Exercise 7. Someone suggests that the paired design be replaced with a design in which 18 cars are sampled, the lifetime of the front brakes is measured on 9 of them, and the lifetime of the rear brakes is measured on the other 9. A confidence interval for the difference between the means would then be constructed by using expression (5.21) (in Section 5.6). He claims that this design will produce a more precise confidence interval, since 18 cars will be used instead of 9.

    a) Will the new design produce a valid confidence interval? Explain.
    b) Is it likely that the confidence interval produced by the new design will be more precise than, less precise than, or about equally precise as the confidence interval produced by the paired design? Explain. (*Hint:* Look at Figure 5.15.)

9. A tire manufacturer is interested in testing the fuel economy for two different tread patterns. Tires of each tread type are driven for 1000 miles on each of 18 different cars. The mileages, in mpg, are presented in the following table.

Car	Tread A	Tread B
1	24.1	20.3
2	22.3	19.7
3	24.5	22.5
4	26.1	23.2
5	22.6	20.4
6	23.3	23.5
7	22.4	21.9
8	19.9	18.6
9	27.1	25.8
10	23.5	21.4
11	25.4	20.6
12	24.9	23.4
13	23.7	20.3
14	23.9	22.5
15	24.6	23.5
16	26.4	24.5
17	21.5	22.4
18	24.6	24.9

a) Find a 99% confidence interval for the mean difference in fuel economy.
b) A confidence interval based on the data in the table has width±0.5 mpg. Is the level of this confidence interval closest to 80%, 90%, or 95%?

10. Refer to Exercise 9. In a separate experiment, 18 cars were outfitted with tires with tread type A, and another 18 were outfitted with tires with tread type B. Each car was driven 1000 miles. The cars with tread type A averaged 23.93 mpg, with a standard deviation of 1.79 mpg. The cars with tread type B averaged 22.19 mpg, with a standard deviation of 1.95 mpg.

a) Which method should be used to find a confidence interval for the difference between the mean mileages of the two tread types: expression (5.24) (in this section) or expression (5.21) (in Section 5.6)?
b) Using the appropriate method, find a 99% confidence interval for the difference between the mean mileages of the two tread types.

c) Is the confidence interval found in part (b) wider than the one found in Exercise 9? Why is this so?