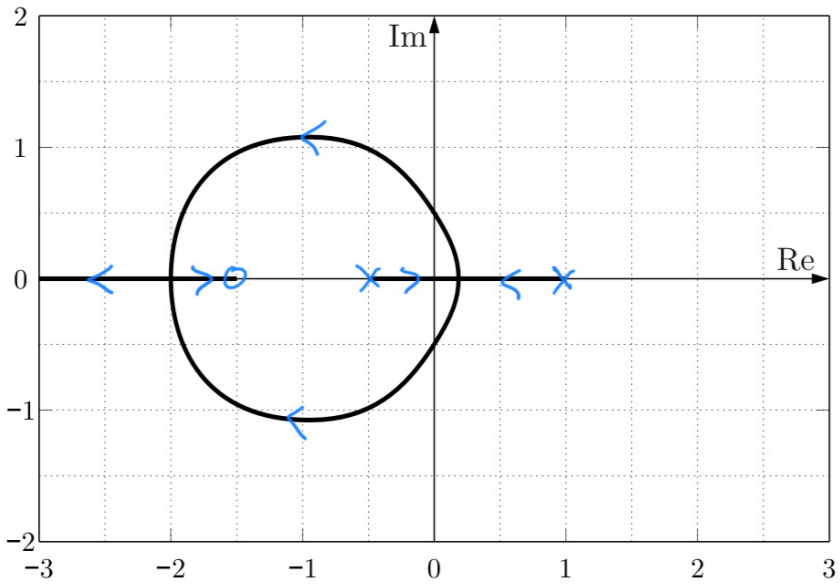
Exam 02-Control Engineering

Monday, 10. February 2020 15:45

1.

a



b From figure,

$s_{N_1} = -1.5$

$s_{p_1} = -0.5 \qquad s_{p_2} = 1$

Now,

$$G_0(s) = 1 \cdot \frac{(s+1.5)}{(s+0.5)(s-1)}$$

$$= \frac{1}{2}\left\{\frac{2s+3}{(s+0.5)(s-1)}\right\}$$

$$= \frac{1}{2}\frac{(s+0.5)+(s-1)+3.5}{(s+0.5)(s-1)}$$

$$= \frac{1}{2}\left\{\frac{1}{s-1} + \frac{1}{s+0.5} + \frac{3.5}{(s+0.5)(s-1)}\right\}$$

$$= \frac{1}{2}\left\{\frac{-1}{1-s} + \frac{2}{1+2s} + \frac{-7}{(1+2s)(1-s)}\right\}$$

$$g(t) = \mathcal{L}^{-1}\{G_0(s)\}(t)$$

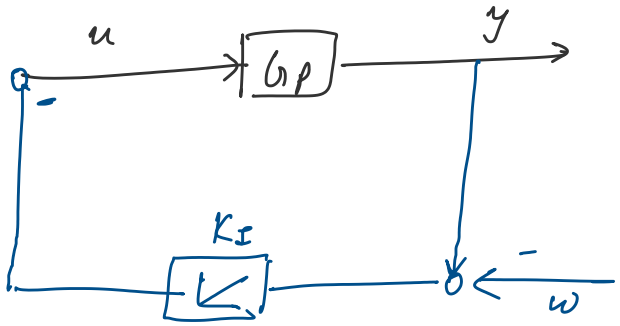
$$= \frac{1}{2}\left\{+1\,e^{t} + \frac{2}{2}e^{-\frac{t}{2}} + \frac{-7}{3}\left(e^{-\frac{t}{2}} - e^{t}\right)\right\}$$

$$= \frac{1}{2}\left\{ e^{1}\left(\frac{10}{3}\right) + e^{-2}\left(\frac{-4}{3}\right)\right\}$$

$$g(t) = \frac{5}{3}e^{t} - \frac{2}{3}e^{-\frac{t}{2}}$$

2.

a



b The $G_I(j\omega)$ controller always show integrating behaviour. ∴ It is preferable for stationary accuracy

From Bode plot :

$$\alpha_{PD} > \alpha_I$$

∴ $G_{PD}(j\omega)$ is preferable w.r.t phase margin

c) ∵ General form of $G_p = \dfrac{1}{T_1 T_2 s^2 + (T_2 + T_1)s + 1}$

with slope changes of -1 at $\frac{1}{T_1}$ and $\frac{1}{T_2}$

From BODE Plot and comparing it w/ given $G_p(j\omega)$

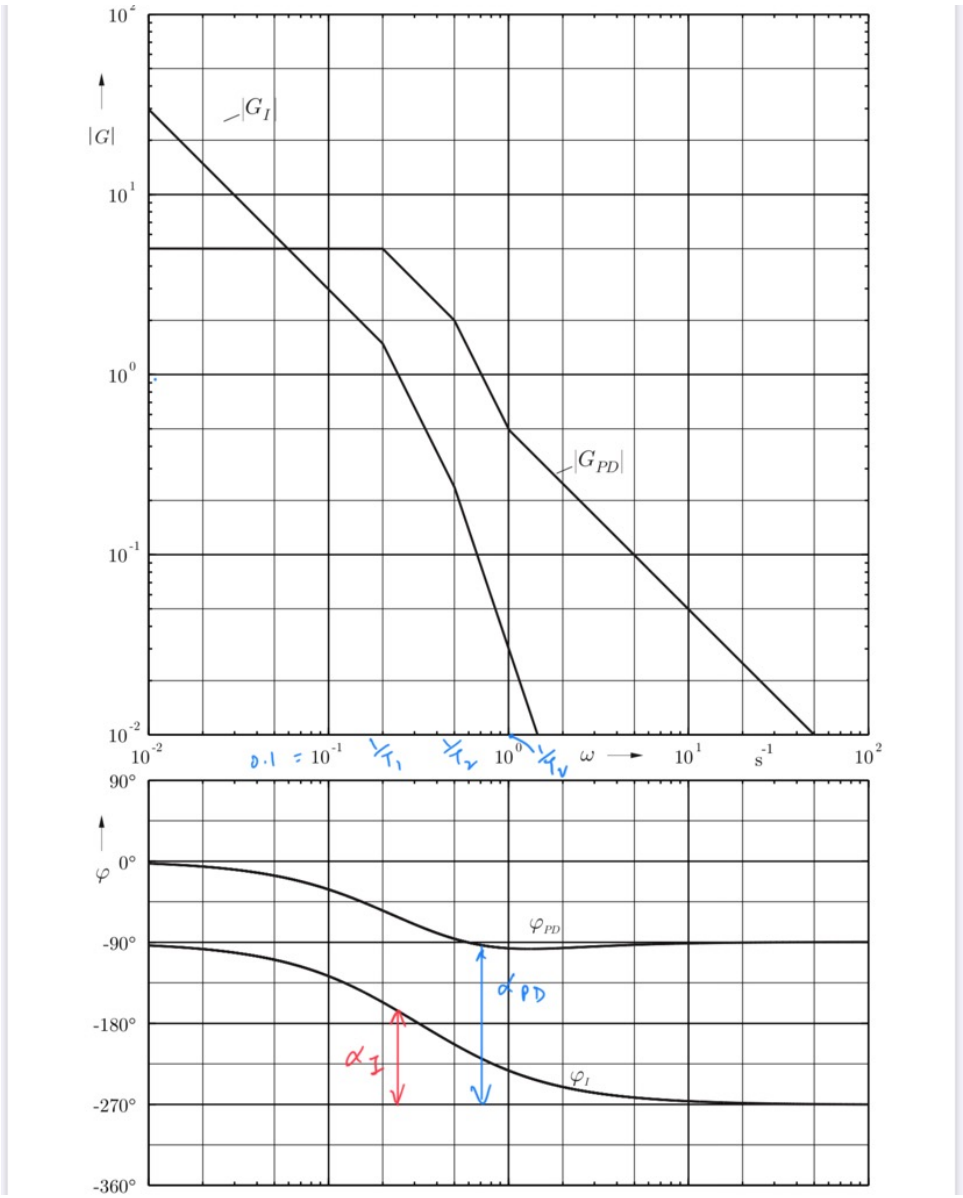
$\frac{1}{T_1} = 0.2\ sec^{-1} \Rightarrow T_1 = 5\ sec$

$\frac{1}{T_2} = 0.5\ sec^{-1} \Rightarrow T_2 = 2\ sec$

∴ $a_2 = T_1 T_2 = 10\ sec^2$ ✓

✓

$a_1 - (1_1 + 1_2) - 7$ see



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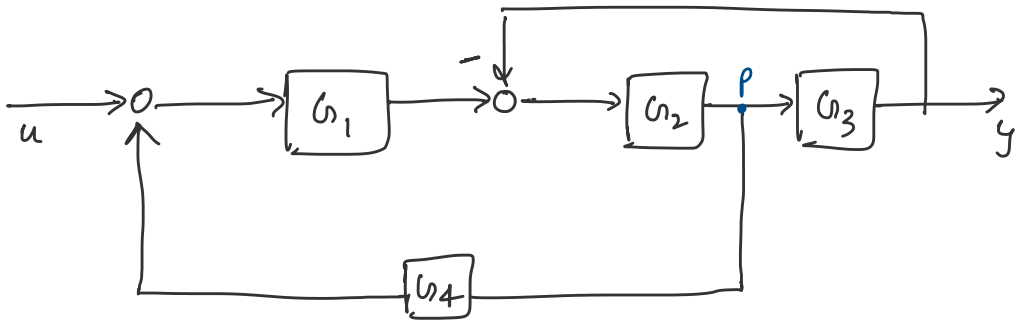


$\{(U + G_4 P) G_1 - Y\} G_2 G_3 = Y$

also, $Y = G_3 P \Rightarrow P = \frac{Y}{G_3}$

Now, $\{(U + \frac{G_4 Y}{G_3}) G_1 - Y\} G_2 G_3 = Y$

$$Y(1 + G_2G_3 - G_1G_2G_4) = U\,G_2G_3$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{G_2G_3}{1 + G_2G_3 - G_1G_2G_4}$$

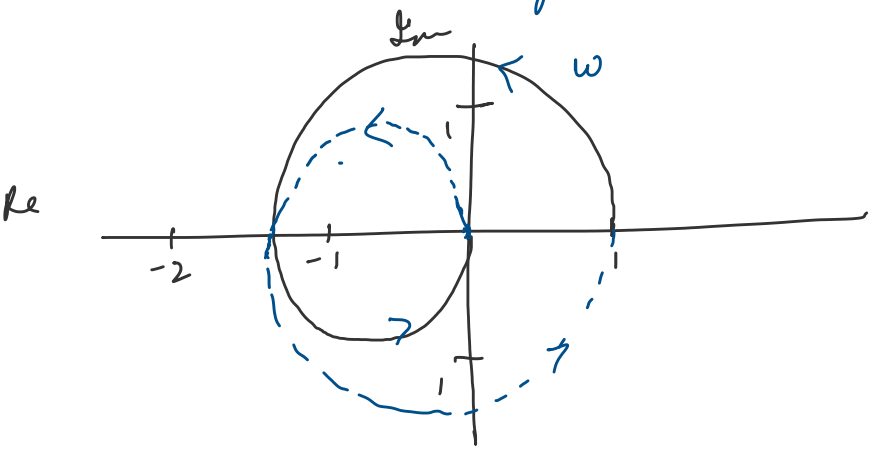
b. $$G(s) = \frac{G_2G_3}{1 + G_2G_3}$$

here, $G_0(s) = G_2G_3$. (say)

Now,

∵ G_2 and G_3 both have one pole in right half plane.

∴ Total pole in right half plane $= 2 = p$



∵ 2 rounds anticlockwise about -1

∴ $n = -2$, no. of revolutions of C'' around -1 (opposite mathematically +ve direction i.e clockwise)

Now, $m = n - p$

$n = m + p = -2 + 2 = 0$

∴ there are 0 zeros in $N(s)$ inside C (right s halfplane)

or 0 no. of poles of $G_2(s)$ in right s half plane. or C''

and, $G_2(s)$ is stable if $n \to 0$

$\therefore$ In our case $G(s)$ is stable

4)

a) $|sI - A| = 0$ For characteristic eq^n

$$\Rightarrow \begin{vmatrix} s+1 & 5 \\ -1 & s-3 \end{vmatrix} = 0$$

$$\Rightarrow s^2 + s - 3s - 3 + 5 = 0$$

$$s^2 - 2s + 2 = 0$$

$$(s-1)^2 = 0$$

$$s = 1$$

$\therefore$ poles lie in left half plane

$\therefore$ system is stable.

b) $s_1 = -1 + j \qquad s_2 = -1 - j$

characteristic eq^n

$$(s - s_1)(s - s_2) = 0$$

$$\{s - (-1+j)\}\{s + 1 + j\} = 0$$

$$\{s + 1 - j\}\{s + 1 + j\} = 0$$

$$(s+1)^2 - j^2 = 0$$

$$s^2 + 1 + 2s + 1 = 0$$

$$s^2 + 2s + 2 = 0 \qquad \text{——————} (1)$$

Now, For given value of $U = -KX$

$$\dot{X} = AX + B - KX$$

$$= (A - BK)X$$

$$= \left\{ \begin{pmatrix} -1 & -5 \\ 1 & 3 \end{pmatrix} - \begin{pmatrix} k_1 & k_2 \\ 0 & 0 \end{pmatrix} \right\} X$$

$$= \underbrace{\begin{pmatrix} -1-k_1 & -5-k_2 \\ 1 & 3 \end{pmatrix}}_{A_k} X$$

Now, Characteristic eqn

$$|sI - A_k| = 0$$

$$\begin{vmatrix} s+1+k_1 & +5+k_2 \\ -1 & s-3 \end{vmatrix} = 0$$

$$s^2 + s(1+k_1) - 3s - 3(1+k_1) + (5+k_2) = 0$$

$$s^2 + s(k_1 - 2) + (2 - 3k_1 + k_2) = 0 \quad \text{---} (2)$$

Comparing eq (1) and (2)

$$2 = k_1 - 2$$

$$\boxed{4 = k_1}$$

$$2 - 3 \cdot 4 + k_2 = 2$$

$$\boxed{k_2 = 12}$$