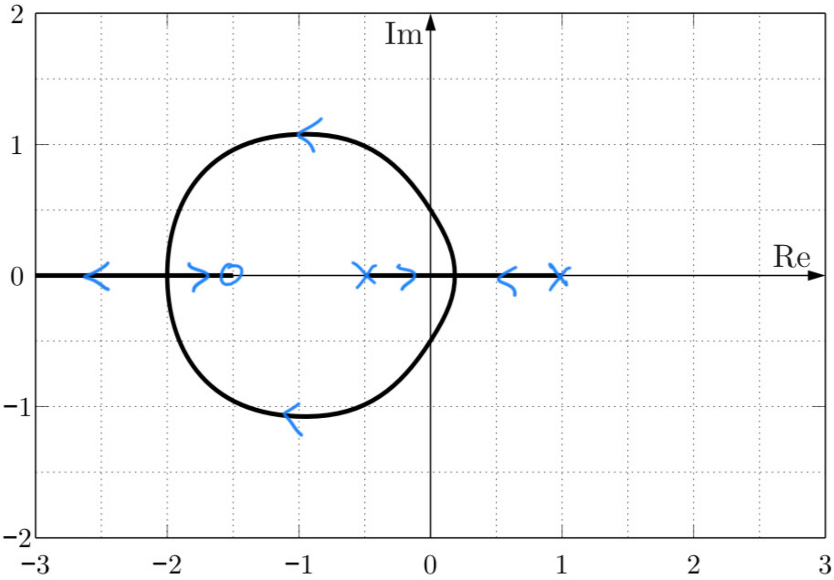
Exam 02-Control Engineering

Monday, 10. February 2020 15:45

1.

a



b From figure,

$s_{N_1} = -1.5$

$s_{P_1} = -0.5$ $\quad s_{P_2} = 1$

Now,

$$G_0(s) = 1 \cdot \frac{(s+1.5)}{(s+0.5)(s-1)}$$

$$= \frac{1}{2}\left\{\frac{2s+3}{(s+0.5)(s-1)}\right\}$$

$$= \frac{1}{2} \frac{(s+0.5) + (s-1) + 3.5}{(s+0.5)(s-1)}$$

$$= \frac{1}{2}\left\{\frac{1}{s-1} + \frac{1}{s+0.5} + \frac{3.5}{(s+0.5)(s-1)}\right\}$$

$$= \frac{1}{2}\left\{\frac{-1}{1-s} + \frac{2}{1+2s} + \frac{-7}{(1+2s)(1-s)}\right\}$$

$$g(t) = \mathcal{L}^{-1}\{G_0(s)\}(t)$$

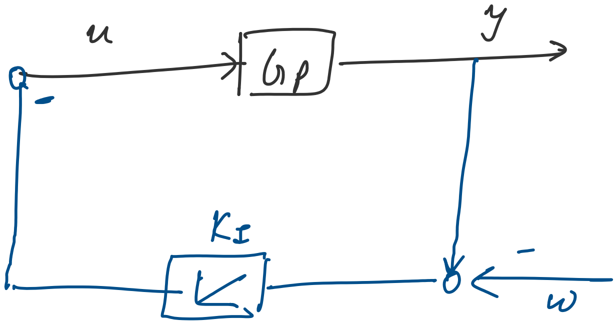
$$= \frac{1}{2}\left\{+1e^{t} + \frac{2}{2}e^{-\frac{t}{2}} + \frac{-7}{3}\left(e^{-\frac{t}{2}} - e^{t}\right)\right\}$$

$$= \frac{1}{2}\left\{ e^{t}\left(\frac{10}{3}\right) + e^{-2}\left(\frac{-4}{3}\right) \right\}$$

$$g(t) = \frac{5}{3} e^{t} - \frac{2}{3} e^{-t/2}$$

2.

a



b The $G_I(j\omega)$ controller always show integrating behaviour. ∴ It is preferable for stationary accuracy

From Bode plot :

$$\alpha_{PD} > \alpha_I$$

∴ $G_{PD}(j\omega)$ is preferable w.r.t phase margin

c) ∵ General form of $G_p = \dfrac{1}{T_1 T_2 s^2 + (T_2 + T_1)s + 1}$

with slope changes of -1 at $\frac{1}{T_1}$ and $\frac{1}{T_2}$

From BODE Plot and comparing it w/ given $G_p(j\omega)$

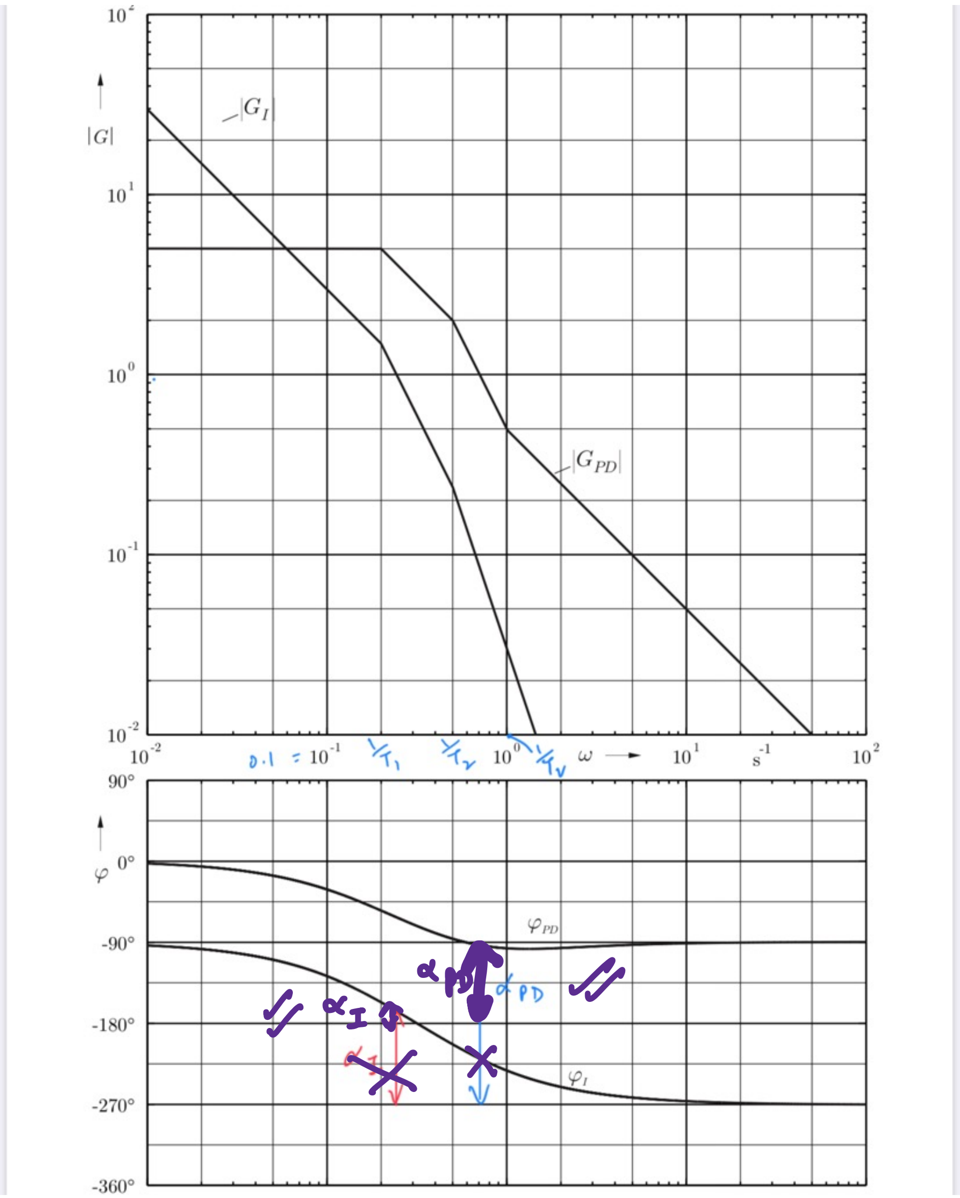
$\frac{1}{T_1} = 0.2\ sec^{-1} \Rightarrow T_1 = 5$ sec

$\frac{1}{T_2} = 0.5\ sec^{-1} \Rightarrow T_2 = 2$ sec

∴ $a_2 = T_1 T_2 = 10\ sec^2$ ✓

$a_1 = (T_1 + T_2) = 7$ sec ✓

$a_1 - (T_1 + T_2) \quad - \quad 1 \quad$ see



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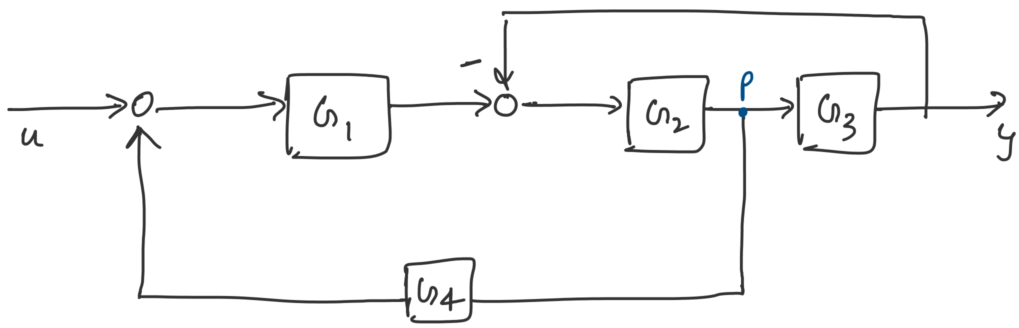


$$\{(U + G_4 P)\, G_1 - Y\}\, G_2 G_3 = Y$$

Also, $Y = G_3 P \Rightarrow P = \dfrac{Y}{G_3}$

Now, $\left\{\left(U + \dfrac{G_4}{\ } Y\right) G_1 - Y\right\} G_2 G_3 = Y$

$((\quad G_3 \quad / \quad)$

$$Y(1 + G_2 G_3 - G_1 G_2 G_4) = U\, G_2 G_3 G_1$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{G_2\, G_3\, G_1}{1 + G_2 G_3 - G_1 G_2 G_4}$$

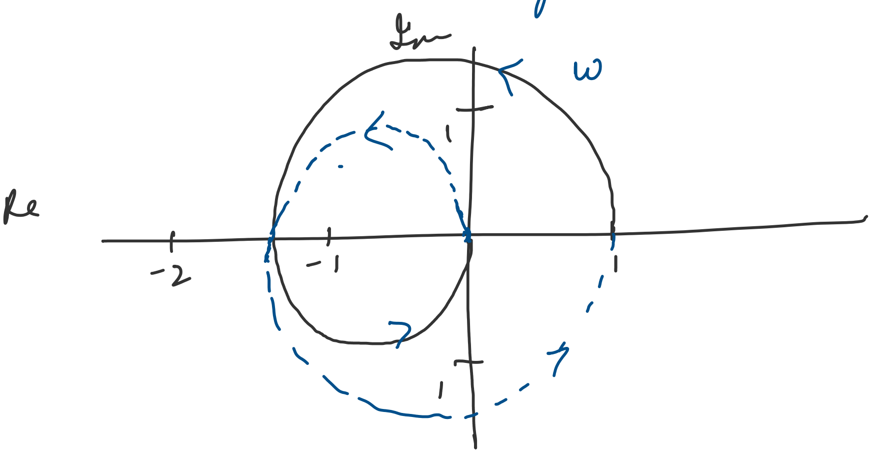
b. $$G(s) = \frac{G_2 G_3}{1 + G_2 G_3}$$

here, $G_0(s) = G_2\, G_3$. (say)

Now,

$\because$ G_2 and G_3 both have one pole in right half plane.

$\therefore$ Total pole in right half plane $= 2 = p$



$\because$ 2 rounds anticlockwise about -1

$\therefore$ $m = -2$, no. of revolutions of C'' around -1 (opposite mathematically +ive direction i.e clockwise)

Now, $m = n - p$

$n = m + p = -2 + 2 = 0$

$\therefore$ there are 0 zeros in $N(s)$ inside C (right s halfplane)

or 0 no. of poles of $G_2(s)$ in right s half plane. or C''

and, $G_2(s)$ is stable if $n \to 0$

$\therefore$ In our case $G(s)$ is stable

4)

a) $|sI - A| = 0$ For characteristic eq^n

$$\Rightarrow \begin{vmatrix} s+1 & 5 \\ -1 & s-3 \end{vmatrix} = 0$$

$$\Rightarrow s^2 + s - 3s - 3 + 5 = 0$$

$$s^2 - 2s + 2 = 0$$

$$\cancel{(s-1)^2 = 0} \qquad s = \frac{+2 \pm \sqrt{4-8}}{2} = +1 \pm i1$$

$$\cancel{s = 1}$$

$\because$ poles lie in left half plane

$\therefore$ system is stable.

b) $s_1 = -1 + j \qquad s_2 = -1 - j$

characteristic eq^n

$$(s - s_1)(s - s_2) = 0$$

$$\{s - (-1 + j)\}\{s + 1 + j\} = 0$$

$$\{s + 1 - j\}\{s + 1 + j\} = 0$$

$$(s+1)^2 - j^2 = 0$$

$$s^2 + 1 + 2s + 1 = 0$$

$$s^2 + 2s + 2 = 0 \quad \text{——————} \quad (1)$$

Now, For given value of $U = -KX$

$$\dot{X} = AX + B\,{-K}X$$

$$= (A - BK)X$$

$$= \left\{ \begin{pmatrix} -1 & -5 \\ 1 & 3 \end{pmatrix} - \begin{pmatrix} k_1 & k_2 \\ 0 & 0 \end{pmatrix} \right\} X$$

$$= \underbrace{\begin{pmatrix} -1-k_1 & -5-k_2 \\ 1 & 3 \end{pmatrix}}_{A_K} X$$

Now, characteristic eq^n

$$|sI - A_K| = 0$$

$$\begin{vmatrix} s+1+k_1 & +5+k_2 \\ -1 & s-3 \end{vmatrix} = 0$$

$$s^2 + s(1+k_1) - 3s - 3(1+k_1) + (5+k_2) = 0$$

$$s^2 + s(k_1 - 2) + (2 - 3k_1 + k_2) = 0 \quad \text{(2)}$$

Comparing eq (1) and (2)

$$2 = k_1 - 2$$

$$\boxed{4 = k_1}$$

$$2 - 3 \cdot 4 + k_2 = 2$$

$$\boxed{k_2 = 12}$$