

# ViennaMath 1.0.0

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User Manual



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# Introduction

The beginning of the computing era is inherently linked with the use of computing devices for numerical algorithms. While the first implementations had been carried out in assembly language, the FORTRAN language then provided a means for efficiently abstracting the underlying hardware. This approach to programming is still common for the implementation of numerical algorithms today: Mathematical primitives such as polynomials or functions are in one way or another represented as an array of numbers already in user code. The mathematical meaning of these numbers is, if at all, only implicitly deducible for a programmer.

Abstraction facilities provided by modern programming languages such as C++ are much more powerful and mature compared to an abstraction at an array level. However, implementation guidelines in numerical textbooks written in the past cannot reflect the current state-of-the-art in programming, thus algorithms are often still implemented without making use of additional abstraction mechanisms. One of the reasons is that first attempts of using object-oriented programming have reported poor performance due to additional indirections at runtime. However, this issue (and several others) have been addressed by language improvements and/or programming techniques.

The aim of `ViennaMath` is to show by example that preserving the mathematical abstraction by means of a symbolic math engine can make code much more readable without sacrificing performance. Even though `ViennaMath` makes use of object-oriented programming, certain calculations and optimizations can already be carried out already at compile time, thus eliminating many expensive indirections at runtime. Consequently, the process of hand-tuning code, i.e. the use of information already available at compile time, is shifted from the user to the compiler. This increases productivity without reducing performance.

# Chapter 1

## Installation

This chapter shows how `ViennaMath` can be integrated into a project and how the examples are built. The necessary steps are outlined for several different platforms, but we could not check every possible combination of hardware, operating system, and compiler. If you experience any trouble, please write to the mailing list at

`viennamath-support@lists.sourceforge.net`

### 1.1 Dependencies

- A recent C++ compiler (e.g. GCC version 4.2.x or above and Visual C++ 2008 are known to work)
- CMake [1] as build system (optional, but recommended for building the examples)

### 1.2 Generic Installation of ViennaMath

Since `ViennaMath` is a header-only library, it is sufficient to copy the `viennamath/` folder either into your project folder or to your global system include path. On Unix based systems, this is often `/usr/include/` or `/usr/local/include/`.

On Windows, the situation strongly depends on your development environment. Please consult the documentation of your compiler or development environment on how to set the include path correctly. The include paths in Visual Studio are usually something like `C:\Program Files\Microsoft Visual Studio 9.0\VC\include` and can be set in `Tools -> Options -> Projects and Solutions -> VC++-Directories`.

### 1.3 Building the Examples and Tutorials

An overview of available examples and their purpose is given Tab. 1.1. For building the examples, we suppose that CMake is properly set up on your system. In the following, instructions on how to build the examples on different platforms are given.

File	Purpose
<code>basic.cpp</code>	Basic handling of ViennaMath expressions
<code>latex_output.cpp</code>	How to use and customize the $\text{\LaTeX}$ translator
<code>model_benchmark.cpp</code>	An example of how ViennaMath can eliminate dependencies in an expression
<code>newton_solve.cpp</code>	A Newton solver using ViennaMath expressions
<code>traversal.cpp</code>	How to traverse a ViennaMath expressions
<code>substitute.cpp</code>	Substitute terms in a ViennaMath expressions
<code>vector_expr.cpp</code>	Explains the use of vector expressions

Table 1.1: Overview of the examples in the `examples/` folder

### 1.3.1 Linux

To build the examples, open a terminal and change to:

```
$> cd /your-ViennaMath-path/build/
```

Execute

```
$> cmake ..
```

to obtain a Makefile and type

```
$> make
```

to build the examples. If desired, one can build each example separately instead:

```
$> make basic           #builds the 'basic' tutorial
$> make substitute      #builds the 'substitute' tutorial
```

Speed up the building process by using jobs, e.g. `make -j4`.



### 1.3.2 Mac OS X

The tools mentioned in Section 1.1 are available on Macintosh platforms too. For the GCC compiler the Xcode [2] package has to be installed. To install CMake, external portation tools such as Fink [3], DarwinPorts [4], or MacPorts [5] have to be used.

The build process of ViennaMath is similar to Linux.

### 1.3.3 Windows

In the following the procedure is outlined for Visual Studio: Assuming that an OpenCL SDK and CMake is already installed, Visual Studio solution and project files can be created using CMake:

- Open the CMake GUI.
- Set the ViennaMath base directory as source directory.

- Set the `build/` directory as build directory.
- Click on 'Configure' and select the appropriate generator (e.g. Visual Studio 9 2008).
- Click on 'Configure' again.
- Click on 'Generate' in order to let CMake generate the project files for you.
- The project files can now be found in the `ViennaMath` build directory, where they can be opened and compiled with Visual Studio (provided that the include and library paths are set correctly, see Sec. 1.2).

# Chapter 2

## Basic Types

Since C++ is a statically typed language [6], the basic mathematical building blocks such as constants or variables are represented as types. The possibility of manipulations at compiletime or runtime is accomplished by essentially two different implementations of these primitives. Basic types for runtime evaluations are discussed first, since their interface and handling is potentially more familiar to average C++ programmers. Sec. 2.2 then provides an overview of the basic types used for compiletime manipulations.

The main include file for ViennaMath is `viennamath/expression.hpp` and includes all the types discussed in the remainder of this chapter.

Include `viennamath/expression.hpp` to make all ViennaMath types available.



Note that all types reside in namespace `viennamath`. The namespace is not written explicitly in the following, thus either `viennamath::` prefixes or certain `using` declarations need to be added by the user in order to make the code valid.



### 2.1 Types Evaluated at Runtime

Common to all types represented at runtime is that they inherit from the same abstract base class and can thus be accessed and manipulated using a pointer to that interface. The interface is not fixed a-priori and can be adjusted via a template parameter, which is in the following called `InterfaceType`. Library users should use the expression wrapper objects discussed next, because it provides an automatic memory management and does not involve complicated pointer manipulation.

#### 2.1.1 Expression Wrapper `expr`

The main expression wrapper type in ViennaMath is `rt_expr<InterfaceType>`. The prefix `rt` refers to *runtime* and aids in distinguishing between types processed at runtime, and types processed at compiletime. In most cases, the default parameter for the runtime interface `InterfaceType` is used, in which case users would have to write

```
rt_expr<> my_expression = /* any expression here */;
```



for instantiating an expression wrapper object `my_expression`. In order to avoid users from having to write the `rt_` and the lower-than and greater-than signs, there is a convenience shortcut `expr` provided. The previous code line thus becomes

```
expr my_expression = /* any expression here */;
```

The `expr`-type can be evaluated and manipulated using operator overloads. For example, the addition of two expressions is accomplished by

```
expr ex1 = /* any expression here */;
expr ex2 = /* any expression here */;
expr result = ex1 + ex2;
```

The initialization of expression objects is accomplished by any of the fundamental types discussed in the next subsections. Note that objects of type `expr` are default-constructible, yet they can only be used after an expression has been assigned to them.

### 2.1.2 Constant

Constants in C++ have their own types `double`, `long`, etc. These types can be used with `ViennaMath` directly. In order to also represent constants using a pointer to the runtime interface, a separate class `rt_constant<NumericT, InterfaceType>` is provided. The template parameter `NumericT` denotes the underlying numerical type such as `double`, `long`, or high precision types. There is again a convenience shortcut `constant` provided for the case of the commonly used `rt_constant<double>`, hence a user can write code such as

```
constant pi = 3.1415;
constant pi_squared = pi * pi;
```

An exemplary use with the expression wrapper `expr` is

```
constant pi = 3.1415;
expr pi_squared = pi * pi;
expr result = pi + pi_squared;
```

### 2.1.3 Variable

A mathematical variable in `ViennaMath` is modeled by `rt_variable<InterfaceType>`. and refers to the mapping

$$(x_0, x_1, \dots, x_{N-1}) \mapsto x_j,$$

where the value of  $j$  is provided to the constructor of the variable. By default, the index  $j = 0$  is used. Any vector type offering access to its values using `operator[]` such as `std::vector<T>` can be used for an evaluation of the variable or a compounded expression.

A simple example leading to the mapping  $(x, y) \mapsto x(y + \pi)$  using the types introduced so far is as follows:

```
constant pi = 3.1415;
variable x(0);
variable y(1);
expr f = x * (y + pi);
```

Name	ViennaMath Function	Name	ViennaMath Function
Exponential	<code>exp()</code>	Modulus	<code>fabs()</code>
Sine	<code>sin()</code>	Square Root	<code>sqrt()</code>
Cosine	<code>cos()</code>	Natural Logarithm	<code>log()</code>
Tangent	<code>tan()</code>	Logarithm, Base 10	<code>log10()</code>

Table 2.1: Overview of unary functions defined in ViennaMath.

An evaluation of  $f$  at  $(1,2)$  can be accomplished by using evaluation overload the parenthesis operator and the ViennaMath helper function `make_vector()`, which conveniently creates a suitable vector for evaluation.

```
std::cout << f( make_vector(1,2) ) << std::endl; //prints 5.1415
```

### 2.1.4 Unary Expression

Mappings of the form  $x \mapsto \sin(x)$  are modeled by the `rt_unary_expr<InterfaceType>` class. Thus, they represent a unary function acting on a constant, a variable or an expression. An overview of the unary functions provided with ViennaMath is given in Tab. 2.1.

Function names in Tab. 2.1 are intentionally chosen such that they coincide with the standard functions for floating point types. When calling these functions with floating point types, compilation might fail due to ambiguity. In such case the namespace should be specified explicitly.



Typically, unary expressions are not instantiated explicitly by the library user. Instead, they are generated implicitly by one of the unary functions and then assigned to an object of type `expr` as in the following example:

```
variable x;
expr g = sin(2.0 * x); // wraps a unary expression into 'g'
```

### 2.1.5 Binary Expression

Similar to unary expressions, binary expressions at runtime are mostly handled in the background only. They are created whenever one of the operator overloads for addition, subtraction, multiplication, or division is triggered. In particular, the argument `2.0 * x` to `sin()` in

```
expr g = sin(2.0 * x);
```

is a binary expression. Binary expressions are central for compile time evaluations in Sec. 2.2.

### 2.1.6 Expression Vector

For the cases where a vector-valued expression is required, a user can either instantiate a vector of `expr`, which allows for storing multiple scalar-valued function only, or use the

`rt_vector_expr<InterfaceType>` class provided by `ViennaMath`. A convenience shortcut `vector_expr` is provided. The benefit of using the `vector_expr` class is that it provides the usual operator overloads directly:

```
variable x(0), y(1);
vector_expr vec(3); vec[0] = x; vec[1] = y; vec[2] = x + y;
vector_expr vec2 = x * vec + y * vec;
```

The dot-product of two vector-valued expressions is provided as well:

```
expr h = vec * vec2;
```

## 2.2 Types Evaluated at Compiletime

The runtime types discussed in the previous section enable a convenient handling of expressions. However, there are numerous runtime dispatches required when evaluating such runtime expressions, which are too costly in a high performance setting. The compiletime types discussed in this section avoid any additional runtime dispatches and their use thus result in faster code in general. This gain in performance comes at the price of a few additional restrictions: Since the expression is entirely encoded in the type, there is no equivalent to `expr` in order to assign an expression to a another object<sup>1</sup>. Furthermore, compilation times increase due to the additional work to be done for the compiler. Excessive use of compiletime evaluations and manipulations can even result in minutes to hours of compilation time, even though this is rarely encountered in practice. Another complication stems from the fact that no floating point template arguments are allowed, thus reducing any compiletime calculations to integer calculations. Fractional numbers can be emulated this way, but they cannot resolve all problems.

### 2.2.1 Constant

Since no floating point type is allowed as template argument, only integer values `val` are represented by the class `ct_constant<val>`. Operators are overloaded in the same way as for the runtime evaluation types in Sec. 2.1. One example of a compiletime calculation is given as follows:

```
ct_constant<2> c2;    //the constant '2'
ct_constant<5> c5;    //the constant '5'
std::cout << c2 + c5 << std::endl; //prints '7' (computed at compiletime)
```

Note that `ct_constant<>` can in principle also be mixed with ordinary constants such as

```
std::cout << 2 + c5 << std::endl; //prints '7'
```

However, depending on the optimization capabilities of the C++ compiler used, ordinary constants may or may not be used for compiletime computations, while the compiler is forced to do it in the introductory snippet.

---

<sup>1</sup>The new C++11 standard addresses this issue and provides the `auto` keyword for automatic type deduction. However, `ViennaMath` intentionally does not use any C++11 features yet.



A general guideline is to use `ct_constant<val>` for encoding an integer `val` already known at compile time rather than writing the value explicitly in code.

## 2.2.2 Variable

A mathematical variable for compiletime manipulations is represented by `ct_variable<id>`, where `id` refers to the coordinate entry in the evaluation vector. The meaning of `id` is identical to the constructor argument of a `variable` in the runtime case.

Operators are again overloaded as usual. For example, consider

```
ct_variable<0> x;  
ct_variable<1> y;  
std::cout << x * y << std::endl;
```

## 2.2.3 Unary Expression

The unary functions in Tab. 2.1 can also be called with compiletime types. The corresponding type for the compiletime representation is provided by `ct_unary_expr<E, OP>`, where `E` refers to the expression on which the unary function encoded by the tag `OP` acts. Unary operation tags start with `op_` and are defined in `viennamath/compiletime/unary_op_tags.hpp`. Their type name can be deduced from the function names in Tab. 2.1 by adding the prefix. Note that all unary functions are evaluated at runtime, because the underlying C-functions are called for evaluation. For example, the type `T` of the compiletime unary expression

```
ct_variable<0> x;  
T t = sin(x);
```

is `ct_unary_expr< ct_variable<0>, op_sin<NumericT> >`, where `NumericT` is the floating point type used for the evaluation at runtime (typically `double`).

## 2.2.4 Binary Expression

The binary expression `ct_binary_expr<L, OP, R>` with left hand side expression `L`, operation tag `OP` and right hand side expression `R` are the main types for building more complex expressions. Currently, four binary operations are supported: addition (with tag `op_plus<NumericT>`), subtraction (`op_minus<NumericT>`), multiplication (`op_mult<NumericT>`), and division (`op_div<NumericT>`). Similar to unary expressions, binary expressions are seldomly set up by hand. Two examples of binary expressions are as follows:

```
ct_binary_expr< ct_variable<0>,      // x  
               op_plus<double>,      // +  
               ct_variable<1> >      // y  
  
ct_binary_expr< ct_constant<1>,      // 1  
               op_div<double>,        // /  
               ct_variable<0> >      // x
```

Typical uses of binary expressions are within the manipulation of compiletime expressions in metafunctions. As an example, outputting the first term of a polynomial is considered:

```
ct_variable<0> x;  
ct_variable<1> y;  
print_first( x*y + x*x*y - y*y );
```

Only two versions of the `print_first` function are required. The first one recursively traverses the binary expression along the left hand side argument:

```
template <typename L, typename OP, typename R>  
print_first(ct_binary_expr<L, OP, R> const & b)  
{ print_first(b.lhs()); } //recursion along left hand side
```

The recursion terminates with a general implementation for printing the left-most entry:

```
template <typename T>  
print_first(T const & t)  
{ std::cout << t << std::endl; }
```

If a binary operation consists of one object for compiletime and one for runtime evaluation, the compiletime object is converted to a runtime object and then processed as usual in the runtime setting.



## Chapter 3

# Expression Manipulation

The basic description of the types in Chap. 2 allows for defining expressions and evaluating them. However, for most algorithms expressions need to be manipulated in one way or another, which is the topic of this chapter. Unless otherwise noted, all manipulations considered in the following can be used for both compiletime and runtime expressions using the same interface.

Manipulation functionality resides in folder `viennamath/manipulation/`. The respective header files are not included automatically with `viennamath/expression.hpp` and need to be included as required.



### 3.1 Evaluation

All `ViennaMath` expressions can be evaluated to a floating point number using the parenthesis operator. A vector of values needs to be passed for the evaluation. In the special case that only the first entry of a vector is required for evaluation, it suffices to pass the value directly.

Using `operator()`, however, is possibly not an option for a generic interface with non-`ViennaMath` types. For this reason, `viennamath::eval()` provides a generic evaluation interface. The first argument is the expression to be evaluated, and the second argument is the tuple with the values to be substituted for the variables. For example, the expression  $x^2$  is defined and evaluated at  $x = 2$  as follows:

```
ct_constant<2> c2;  
ct_variable<0> x;  
eval( x*x, 2.0 ); // runtime evaluation  
eval( x*x, c2 ); // compiletime evaluation
```

Note that compiletime evaluation is only performed when both arguments are fully compiletime compatible. As soon as one part of the expression cannot be handled at compile time, a fallback to runtime evaluation is carried out. A hybrid evaluation in such cases is postponed to future releases of `ViennaMath`.

A vector of values needs to be passed as second argument, if a variable formally refers to any other than the first coordinate in a vector. Let us consider several use-cases of `eval()` consisting of various combinations of compiletime and runtime expressions:

```

ct_constant<3> c3;
ct_variable<1> y;
expr f = x*x + y;      // conversion to runtime expression
eval( f, 2.0 );        // runtime exception: insufficient values provided
eval( x*x + y, c2 );   // compilation error: insufficient values provided
eval( x*x + y,
      make_vector(2.0, 3.0) ); // runtime evaluation
eval( x*x + y,
      make_vector(c2, c3) ); // compiletime evaluation
eval( 2.0, 0.0 );      // possible thanks to overloading

```

Since the runtime wrapper `expr` hides information from the compiler, an exception is thrown at runtime if insufficient values are provided for evaluation. For a full compile-time evaluation, insufficient parameters are already detected at an earlier stage. The helper function `make_vector()` generates a suitable vector type both for the runtime and the compiletime case. Instead of using `make_vector()`, a STL vector (`std::vector<double>`) or any compatible type can also be passed. Also note that the last line in the code snippet shows the benefit of using `eval()` instead of `operator()`: Scalars can also be 'evaluated' and are thus reinterpreted as constant functions.

## 3.2 Substitution

Formally, the evaluation of an expression can be seen as a substitution of the variables with values. A generalization is to replace arbitrary expressions with another expression in third expression. This is accomplished by the function `substitute()` defined in `viennamath/manipulation/substitute.hpp`:

```

expr f = (x + y) * (x - y);
substitute(x, y, f);      // returns (y + y) * (y - y) = 0
substitute(x, 1, f);      // returns (1 + y) * (1 - y)
substitute(x + y, x - y, f); // returns (x - y) * (x - y)

```

As with `eval`, substitutions are carried out at compiletime if all parameters are compile-time expressions.

## 3.3 Expansion

It is often desired to expand an expression given as a product of other terms. For example, instead of  $2(x + y)$  one may want to have  $2x - 2y$ . Such a functionality is provided by the function `expand()` defined in `viennamath/manipulation/expand.hpp`:

```

expand( c2 * (x+y) );

```

where `c2` is a compiletime constant and `x, y` are compiletime variables. Note that ViennaMath 1.0.0 does not support the expansion of runtime expressions yet.

ViennaMath 1.0.0 supports compiletime expansion only.



### 3.4 Simplification

In the course of manipulating expressions, simple operations such as  $x + 0$  or  $x/1$  may appear. However, such terms constitute unnecessary overhead for later evaluations, thus it is desirable to have these operations dropped. Such a simplification of the expression can be achieved with the function `simplify()` defined in `viennamath/manipulation/simplify.hpp`:

```
simplify( x + 1.0 * y - 0.0 ); // returns x
simplify( x * (2.0 + 3.0) + (y * 0) / (x * 1) ); // returns 5x
```

`simplify()` is available both for runtime and compiletime manipulation.

### 3.5 Differentiation

The differentiation of an expression with respect to one or several variables is central to many algorithms, among which the Newton scheme is presumably the most widely used. Differentiation routines in ViennaMath reside in `viennamath/manipulation/diff.hpp` and are used in a canonical way by passing the expression to be differentiated as first argument and the differentiation variable as second argument:

```
diff( x + y, x ); // returns 1
diff( (2.0 - x)*(3.0 + y), y ); // returns 2.0 - x
```

As usually, compiletime expressions are differentiated at compiletime, while runtime expressions are differentiated at runtime. An exemplary Newton-solver demonstrating the use of differentiation routines can be found in `examples/newton_solve.cpp`.

### 3.6 Integration

The integration of an expression over an interval can be accomplished in two ways: The first option is by analytical integration, provided that an antiderivative of the integrand can be found easily. The second option is by numerical quadrature using a suitable quadrature rule.

Analytical integration is in ViennaMath 1.0.0 available for compiletime types and polynomials as integrands only. The file `viennamath/manipulation/integrate.hpp` provides the function `integrate()`, which takes the integration interval as first argument, the integrand as second argument and the integration variable as third argument. For example, the integral

$$\int_0^1 x^2 dx \tag{3.1}$$

is computed analytically at compile time using the lines

```
integrate( make_interval(c0, c1),
           x * x,
           x ); //returns the expression 1/3
```



Name	ViennaMath Type	Shortcut	Accuracy
1-point Gauss	rt_gauss_quad_1	gauss_quad_1	1

Table 3.1: Overview of numerical quadrature rules in ViennaMath.

where `c0` and `c1` denote the compiletime constants 0 and 1 and are passed to the helper function `make_interval()` to generate a suitable compiletime interval with the provided lower and upper bound. Note that analytic integration can also be nested and use polynomial lower and upper bounds. For example, integration of  $x^2$  over the unit triangle with vertices (0,0), (1,0) and (0,1) is achieved via

```
integrate( make_interval(c0, c1),
           integrate( make_interval(c0, c1 - x),
                     x * x,
                     y),
           x);           //returns 1/12
```

Analytic integration in ViennaMath 1.0.0 is available for polynomial integrands at compiletime only.



In order to compute an integral numerically using a quadrature rule, the respective rule from Tab. 3.1 needs to be instantiated first. For a 1-point Gauss rule, this is accomplished by

```
rt_numerical_quadrature<InterfaceType>
integrator(new rt_gauss_quad_1<InterfaceType>());
```

for a suitable runtime interface type. If the default interface type is to be used, the shortcut types can be used for convenience:

```
numerical_quadrature integrator(new gauss_quad_1());
```

To carry out the numerical quadrature, two options exist: The first is to pass the integration interval, the integrand, and the integration variable as separate arguments to `operator()` of the integrator object:

```
integrator( make_interval(0.0, 1.0),
           x * x,
           x);           // returns the value 0.25
```

The second option for numerical quadrature is to encode the integral directly in the expression:

```
expr my_integral = integral( make_interval(0, 1), x * x, x );
```

which encodes  $\int_0^1 x^2 dx$  directly in an expression. For numerical quadrature, only the encoded form needs to be passed to the quadrature rule then

```
integrator(my_integral);           // returns the value 0.25 (1-point Gauss)
```

### 3.7 Extract Coefficient

Given a polynomial  $p(x, y) = 1 + x + y + xy$  it can be of interest to extract individual parts of the polynomial. For example, one wishes to extract the coefficient of  $x$ , which is  $(1 + y)$ . In such a case, the function `coefficient()` defined in `viennamath/manipulation/coefficient.hpp` can be used. The first parameter is the variable or expression for which the coefficient should be returned, and the second argument is the expression from which the coefficient is to be extracted:

```
coefficient(x, c1 + x + y + x*y); //returns 1+y
```

Note that higher-order terms in the variable are also returned. For example, the coefficient of  $x$  in  $x + x^2$  is obtained as  $(1 + x)$ .

`coefficient()` is in ViennaMath 1.0.0 available for compiletime types only.



### 3.8 Drop Dependent Terms

In order to drop all terms in an expression which depend on a certain expression type (not necessarily a variable), the convenience function `drop_dependent_terms()` from `viennamath/manipulation/drop_dependent_terms.hpp` can be used. As the name suggests, all terms with a dependency on the expression passed as first parameter are dropped in the expression passed as second variable. For example, all terms depending on  $x$  in  $1 + x + y + xy$  are dropped using the line

```
drop_dependent_terms(x, c1 + x + y + x*y); //returns 1+y
```

in order to obtain  $1 + y$ .

`drop_dependent_terms()` is in ViennaMath 1.0.0 available for compiletime types only.



## Chapter 4

# Additional Features

Various features of `ViennaMath`, which are not necessarily standard features of a symbolic math library, are covered in this chapter. Additional feature requests should be sent to

`viennamath-support@lists.sourceforge.net`

### 4.1 L<sup>A</sup>T<sub>E</sub>X Output

Since `ViennaMath` encourages a high-level description and manipulation of the underlying mathematical problem formulation in source code, it is natural to generate L<sup>A</sup>T<sub>E</sub>X code from `ViennaMath` expressions for debugging purposes. The generated code can be copy&paste'd to LaTeX rendering webpages or used for the automatic generation of program log files in the form of a L<sup>A</sup>T<sub>E</sub>X document.

All conversion is carried out by a separate converter object of type `rt_latex_translator` <InterfaceType> as defined in `viennamath/manipulation/latex.hpp`. A convenience shortcut `latex_translator` is available for the default runtime expression interface. Conversion is triggered by providing the expression to be converted to the functor:

```
latex_translator to_latex;

expr f = sqrt( x + y );
to_latex( f );           //returns the string '\sqrt{x_{0}+x_{1}}'
```

By default, variables are printed as  $x_0$ ,  $x_1$ , etc. This and other output routines can be customized by using the `customize()` member function of the converter. For example, to print 'x' and 'y' instead of 'x\_{0}' and 'x\_{1}', the code

```
to_latex.customize(x, "x");
to_latex.customize(y, "y");
```

is sufficient. Similar customizations can be applied for the output of types and features described in the remainder of this chapter.

The L<sup>A</sup>T<sub>E</sub>X generator works with runtime expression types only. Thus, compiletime expression types need to be converted to runtime expression types first.



## 4.2 Differential Operators

For enabling dimension-independent programming, dimension-independent mathematical differential operations are also provided with ViennaMath. In ViennaMath 1.0.0, the gradient and the divergence operators are provided by the functions `grad()` and `div()` respectively:

```
expr u = grad(x+y);
expr v = grad(x-y);
expr w = div(grad(x*x + y*y));
```

Note that expression containing differential operators cannot be evaluated directly, since a coordinate system needs to be specified first.

A coordinate system is applied to the previous expressions by using the free function `apply_coordinate_system()`, which is defined in the header file `viennamath/manipulation/apply_coordinate_system.hpp`. The first function argument is a tag identifying the coordinate system (either of type `cartesian<1>`, `cartesian<2>`, or `cartesian<3>` for a Cartesian coordinate system in one, two or three dimensions). The second function argument is the expression to which the coordinate system should be applied:

```
apply_coordinate_system(cartesian<1>(), u); //returns 1 + y
apply_coordinate_system(cartesian<2>(), v); //returns (1, -1)
apply_coordinate_system(cartesian<3>(), w); //returns 4
```

Differential operators are very handy in combination with function symbols explained in Sec. 4.4.

## 4.3 Integration Symbols

In certain cases the form of an integral expression is known, but the actual integration domain is determined at some later stage. Here, a symbolic integration domain of type `symbolic_interval` identified by an ID can be specified and substituted by the final integration interval later on. For example, the integral  $\int_{\Omega} x^2 d\Omega$ , where  $\Omega$  is specified at some later point, is specified using ViennaMath as

```
expr my_integral = integral(symbolic_interval(), x*x);
```

The interface is again such that it can be used with both runtime and compiletime types. For simplicity, the resulting integral expression is here assigned to a runtime expression `my_integral`. An ID can be provided to the constructor of `symbolic_interval` for distinguishing between different symbolic intervals. By default, an ID of 0 is used.

In order to substitute the symbolic interval with the actual integration interval, the function `substitute()` as explained in Sec. 3.2 is used. Since the replacement consists of both the integration interval and the variable over which integration is to be carried out, the replacement argument is packed into a `pair` as defined in the C++ STL. Thus, in order to specify  $\Omega$  as the interval  $[0, 1]$  with an integration over the  $x$ -variable, the code

```
expr my_integral2 = substitute( symbolic_interval(),
                               std::make_pair(interval(0, 1), x),
                               my_integral);
```

is sufficient. Note that in the current release of `ViennaMath` only the substitution with a one-dimensional integration domain is supported, but no nested integrals are possible yet.

Note that `std::make_pair()` is defined in the header `<utility>`.



## 4.4 Function Symbols

For discretization schemes based on weak formulations of partial differential equations it is appropriate to work with abstract functions rather than with concrete expressions. For example, the weak form of the Poisson equation,

$$\int_{\Omega} \nabla u \cdot \nabla v \, dx = \int_{\Omega} f v \, dx \quad (4.1)$$

for all test functions in a certain test space  $\mathcal{V}$ , is formulated for functions  $u$  and  $v$ , which are during the discretization replaced by certain trial and test functions, which finally yields a system of linear equations. Such function labels (*function symbols*) are modeled in `ViennaMath` by the type `rt_function_symbol<InterfaceType>` at runtime (with convenience shortcut `function_symbol` for the default runtime interface type) and by `ct_function_symbol<T>` at compiletime, where  $T$  is a tag identifying the function symbol.

As a simple example, the expression  $uv$  is considered, where  $u$  is then substituted with the expression  $(1+x)$  and  $v$  is replaced by  $(1-x)$ :

```
function_symbol u(0);
function_symbol v(1);
expr f = u * v;
expr g = substitute(u, 1.0 + x,
                   substitute(v, 1.0 - x, f)
                   ); // g becomes (1+x)*(1-x)
```

The constructor arguments denote the function symbol ID used for distinguishing the individual function symbols.

Reconsidering the weak form (4.1), function symbols at compiletime can use any arbitrary tag class for identification. `ViennaMath` provides the predefined tags `unknown_tag<id>` and `test_tag<id>`, where the integer template parameter `tag` is used for distinguishing between several function symbols of the same tag. The previous code snippet rewritten for compiletime manipulation thus becomes

```
ct_function_symbol< unknown_tag<0> > u;
ct_function_symbol< test_tag<0> > v;
substitute(u, 1.0 + x,
          substitute(v, 1.0 - x, u*v)
          ); // returns (1+x)*(1-x)
```

Note that the result type of `substitute()` encodes the result  $(1+x) * (1-x)$ , thus the result is usually directly passed to another function (e.g. `eval()`) in order to avoid writing the return type explicitly.

As a final example, the weak form (4.1) (with  $f \equiv 1$  for simplicity) is specified directly as a `ViennaMath` runtime expression and converted to  $\text{\LaTeX}$ -code using the functionality presented in this chapter:

```
function_symbol u(0);
function_symbol v(1);
equation weak_form = make_equation( integral( symbolic_interval(),
                                             grad(u) * grad(v) ),
                                   integral( symbolic_interval(), v ) );

latex_translator to_latex;
std::cout << to_latex(weak_form) << std::endl;
```

**Have a look at ViennaFEM (<http://viennafem.sourceforge.net/>) if you are interested in a software package using ViennaMath for the finite element method.**



# Change Logs

## **Version 1.0.0**

First release

# License

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