Optimization Models

EECS 127 / EECS 227AT

Patricia Hidalgo-Gonzalez

EECS department
UC Berkeley

Fall 2018

LECTURE 24

Applications to Energy Systems

> The good news is, we have everything we need now to respond to the challenge of global warming. We have all the technologies we need, more are being developed. But we should not wait, we cannot wait, we must not wait.
>
> Al Gore

Outline

Introduction to energy

Overview

Our society requires energy for:

- Transportation
- Infrastructure (hospitals, buildings, street lights, etc.)
- Production of goods
- Services
- Etc.

Introduction to energy

Overview

Energy sources:

- Fossil fuel (natural gas, coal, petroleum)
- Nuclear energy
- Renewable energy (solar, wind, geothermal, hydro)

Figure: Solar panels and wind turbines.[1]

[1] Source: source: https://inhabitat.com/

Introduction to energy

Overview

Pollution from using energy by type:

- Fossil fuel: CO_2, methane, PM2.5, PM10
- Nuclear: Radioactive waste
- Renewable: Noise pollution from wind turbines

Figure: Coal power plant.[2]

[2] Source: http://www.independent.co.uk/

Introduction to energy

Energy management as an optimization problem

Therefore, as a society we have a set of **constraints** to manage our electricity system:

- Produce enough electricity to supply demand
- Maintain air pollution below permitted levels (if applies)
- Laws of physics: flow of electricity in the transmission system
- Technical characteristics of power plants (max/min capacity, start up time, maximum ramp, down time, dispatchable, baseload, etc.)
- Resource availability (wind, solar, geothermal, hydro)

There are **costs** associated to operating and expanding the electricity system:

- Fuel cost
- Maintenance cost
- Investment cost of each technology per MW
- Investment cost of transmission lines

Objective function: minimize total costs of operation and investment

Types of optimization problems in energy

Energy dispatch as transportation problem in a uninodal network

- We consider a power system with only one node (or bus).
- There are n generators of different technologies connected to the node.
- There is an hourly cost c_i for producing electricity in each generator.
- There is an hourly demand of electricity d_t.
- We consider T hours of simulation.
- Objective: Minimize total cost.
- Constraints: Generation equal to demand at each hour.
- Constraints: Each generator has a capacity limit for the power it can produce ($p_i^{\min}$ and $p_i^{\max}$).

$$
\begin{array}{lll}
\min_p & \sum_t \sum_i c_i p_{i,t} & \\
\text{subject to} & \sum_i p_{i,t} = d_t & \forall t = 1 \dots T \\
 & p_{i,t} \leq p_i^{\max} & \forall i = 1, \dots, n, t = 1, \dots, T \\
 & p_i^{\min} \leq p_{i,t} & \forall i = 1, \dots, n, t = 1, \dots, T
\end{array}
$$

where $p_{i,t}$ (decision variable) is the power produced [MWh] by generator i at time t. This problem is an LP!

Types of optimization problems in energy

Energy dispatch as transportation problem in a multinodal network

- We consider a network with N nodes connected by transmission lines. ℓ is the index for transmission lines. $r(\ell)$ and $s(\ell)$ are the receiving-end node and sending-end node of transmission line ℓ (respectively). $\theta_{i,t}$ is the voltage angle at node i and time t.
- There is one generator per node (can be generalized).
- Constraints: Each transmission line has a capacity limit $F_\ell^{\max}$ and susceptance B_ℓ.

$$
\begin{array}{lll}
\min_{p,p^L,\theta} & \sum_t \sum_i c_i p_{i,t} & \\
\text{subject to} & p_{i,t} - \sum_{\ell|s(\ell)=i} p_{\ell,t}^L + \sum_{\ell|r(\ell)=i} p_{\ell,t}^L = d_{i,t} & \forall i = 1,\ldots,N, t = 1\ldots,T \\
& p_{i,t} \leq p_i^{\max} & \forall i = 1,\ldots,N, t = 1,\ldots,T \\
& p_i^{\min} \leq p_{i,t} & \forall i = 1,\ldots,N, t = 1,\ldots,T \\
& p_{\ell,t}^L = B_\ell(\theta_{s(l),t} - \theta_{r(l),t}) & \forall \ell, t = 1,\ldots,T \\
& p_{\ell,t}^L \leq F_\ell^{\max} & \forall \ell, t = 1,\ldots,T \\
& -F_\ell^{\max} \leq p_{\ell,t}^L & \forall \ell, t = 1,\ldots,T
\end{array}
$$

where $p_{i,t}$ is the power by generator i at time t and $p_{\ell,t}^L$ power flow through transmission line ℓ at time t. This problem is an LP!

Types of optimization problems in energy

Power system capacity expansion and dispatch

The optimization problems introduced can become more interesting when we can decide about where and when to install more power plants and transmission lines (binary decisions). This is called a capacity expansion problem.

An open source capacity expansion model developed at UC Berkeley is the SWITCH model [3].

- Mixed Integer LP (MILP) or LP if relaxed
- Minimizes total cost of investment and operation of generation and transmission
- Area: Western Electricity Coordinating Council divided in 50 demand zones
- 10,000+ potential power plants to be installed in the WECC
- Four investment periods: 2020, 2030, 2040, 2050
- Dispatch for sampled hours simultaneously optimized with investment decisions
- 1,000,000+ decision variables
- Other regions studied: China, Chile, Nicaragua, Mexico, Hawaii, Kenya, etc.

[3] https://github.com/switch-model/

Types of optimization problems in energy

Power system capacity expansion and dispatch

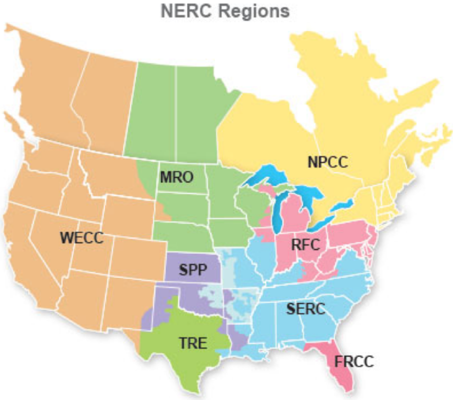


Figure: U.S. NERC regions. The SWITCH model for the US optimizes the WECC region.

Some results and policy impacts

We developed a robust version of the optimization problem for the California Energy Commission. There were three possible futures under climate change (from climate models projections).

The possible futures would impact:

- Hydropower availabilty
- Hourly demands in each node

We modeled this problem as a two-stages optimization problem:

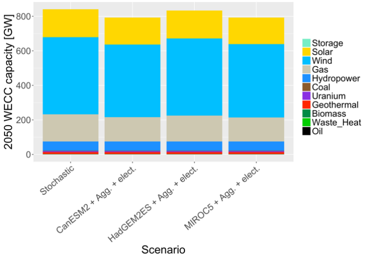
- Three scenarios with equal probabilities
- Investment decisions are equal for the three scenarios (robustness)
- Operation decisions are specific to the scenario
- Objective function: expected value of the cost of the three scenarios

Types of optimization problems in energy

Power system capacity expansion and dispatch

Some results and policy impacts [4, 5]

- Total capacity installed in the stochastic problem in the WECC by 2050 is 4% higher than in the deterministic cases.
- There was a 5.6% more of installed gas in the solution to the stochastic formulation due to a greater need of operational flexibility.



[4] Wei, M.*, Raghavan, S.*, Hidalgo-Gonzalez, P.*, Johnston, J., Henriquez, R., Kammen, D. (LBNL and UC Berkeley) 2018. Building a Healthier and more Robust Future: 2050 Low Carbon Energy Scenarios For California. Californias Fourth Climate Change Assessment, California Energy Commission. Publication.

[5] Hidalgo-Gonzalez, P.L.*, Johnston, J.L., Kammen, D.M., Stochastic power system planning under climate change in western North America (In preparation).

Types of optimization problems in energy

Exact power flow

Background

Transmission lines usually transport power in AC (Alternating Current).

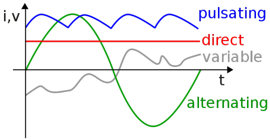


Figure: Alternating Current. Source: Wikipedia

- AC uses sinusoidal waves with fixed frequency (60Hz in the U.S.) for current (I) and voltages (V).
- Voltages and currents can be fully described by a phase angle θ and a magnitude:
  - $V = A_V \cos(\omega t + \theta_V)$
  - $I = A_I \cos(\omega t + \theta_I)$

Types of optimization problems in energy

Exact power flow

Background

- Transmission lines have impedance $Z \in \mathbb{C}$, $Z = R + iX$. The admittance is the reciprocal: $Y = \frac{1}{Z} = G - iB$.
- $V = IZ$
- Voltages, currents, and impedances are represented as complex numbers in rectangular or polar form.
- Complex power, $S = VI^* = P + iQ$, units: VA.
- P is real or active power, units: W.
- Q is reactive power, units: var.
- To make calculations easier, power system engineers use 'per unit' metric. To do this, a base voltage, impedance and complex power are set. All the quantities are normalized according to the base. For example, if $V = 70kV$ and the base is $100kV$, then $V = 0.7$ p.u.

Optimal Power Flow (OPF) formulation[6] (non convex QCQP)

$$
\begin{array}{lll}
\min_{v,p,q} & C(v,p,q) & \\
\text{subject to} & p_{ij} + iq_{ij} = v_i \left(v_i^* - v_j^*\right) y_{ij}^* & \forall (i,j) \in N \times N \\
 & \sum_{j=1}^{N} p_{ij} = p_i & \forall i = 1...N \\
 & \sum_{j=1}^{N} q_{ij} = q_i & \forall i = 1...N \\
 & \underline{p}_i \leq p_i \leq \overline{p}_i & \forall i = 1...N \\
 & \underline{q}_i \leq q_i \leq \overline{q}_i & \forall i = 1...N \\
 & p_{ij}^2 + q_{ij}^2 \leq \overline{s}_{ij}^2 & \forall (i,j) \in N \times N \\
 & \underline{v}_i \leq v_i \leq \overline{v}_i & \forall i = 1...N
\end{array}
$$

where

p_{ij}, q_{ij}: active and reactive power from node i to j,

p_i, q_i: active and reactive power produced or consumed at node i,

v_i: voltage at node i,

$\overline{s}_{ij}$: upper limit of the apparent power from node i to j,

$\underline{p}_i$, $\underline{q}_i$, $\underline{v}_i$, $\overline{p}_i$, $\overline{q}_i$, $\overline{v}_i$: are lower and upper bounds (respectively) for active power, reactive power and voltage.

[6] J. Carpentier, "Contribution to the economic dispatch problem," Bull. Soc. Francoise Electr., vol. 3, no. 8, pp. 431-447, 1962.

Types of optimization problems in energy

Exact power flow

Optimal Power Flow (OPF) formulation (non convex QCQP)

- $C(v, p, q)$ represents a cost function of providing power to the grid. It is generally assumed to be convex.
- This exact representation of the OPF can become intractable when dimensions are high (due to being NP-hard).
- The quadratic equality constraint makes this problem non convex.
- The QCQP representation is known as AC power flow.
- Common algorithm used to solve this problem: Newton-Raphson.
- Related courses at Berkeley: Electric Power Systems (ER 254), Introduction to Power Systems (EE 137A and EE 137B), Control and Optimization for Power Systems (IEOR 290)

Types of optimization problems in energy

Convex relaxations and approximations of OPF

Approximation: An approximation of a optimization problem can be found by posing assumtions about the problem that allow to simplify/approximate certain mathematical expressions of the original problem.

For example, an assumption about the original feasible set can allow to mathematically approximate some expressions in the constraints and/or obj. function (e.g., if we assume $\theta \approx 0$, we can approxiate $sin(\theta) \approx \theta$).

Relaxation: A relaxation of an optimization problem can be found by *relaxing* some constraints in the original problem. To *relax* a constraint refers to substituting the feasible set for a feasible set that contains the original one.

For example, a relaxation of $x_1 + x_2 = 0$ could be $x_1 + x_2 \leq 0$.

Notice that the feasible set of a relaxation of an optimization problem will always contain the feasible set of the original problem. Thus, it provides a lower bound for the optimal value of the original problem.

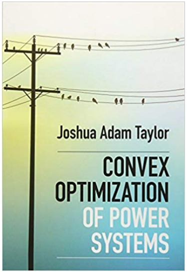
Types of optimization problems in energy

Convex relaxations and approximations of OPF

The most popular relaxations and approximations are:

1. OPF as LP (approximation)
2. OPF as decoupled LP (approximation)
3. OPF as SDP (relaxation)
4. OPF as SOCP (relaxation)

A good resource for this topic is J. A. Taylor, *Convex Optimization of Power Systems*. Cambridge University Press, Cambridge, UK, 2015. A good course that provides mathematical background: **EE 227BT**.



1. OPF as LP (approximation)

The linear approximation of the OPF can be obtained using the polar coordinates representation of the exact OPF (QCQP) and enforcing the following assumptions:

- Susceptances are much larger than conductances ($g_{ij} << b_{ij} \Rightarrow g_{ij} \approx 0$).
- Voltage magnitudes close to 1 per unit ($|v_i|$=1).
- Voltage angles are small enough to approximate $sin(\theta_i - \theta_j)$ as $\theta_i - \theta_j$.
- Reactive power flows q are zero (too small compared to active power p).

Thus, the LP approximation can be written as

$$
\begin{array}{lll}
\min_{v,p,q} & C(v,p,q) & \\
\text{subject to} & \sum_{j=1}^{N} p_{ij} = p_i & \forall i = 1...N \\
 & \underline{p}_i \leq p_i \leq \overline{p}_i & \forall i = 1...N \\
 & |p_{ij}| \leq \overline{s}_{ij} & \forall (i,j) \in N\text{x}N \\
 & p_{ij} = b_{ij}(\theta_i - \theta_j) & \forall (i,j) \in N\text{x}N
\end{array}
$$

This representation is colloquially known as DC power flow.

2. OPF as decoupled LP (approximation)

This decoupled linear approximation [7] can be derived from the exact OPF (QCQP) transforming it to polar coordinates and under the following assumptions:

- Susceptances are much larger than conductances ($g_{ij} << b_{ij} \Rightarrow g_{ij} \approx 0$).
- Voltage magnitudes close to 1 per unit ($|v_i|$=1).
- Voltage angles are small enough to approximate $sin(\theta_i - \theta_j)$ as $\theta_i - \theta_j$ and $cos(\theta_i - \theta_j) \approx 1$.

[7] B. Stott and O. Alsac, "Fast Decoupled Load Flow," in IEEE Transactions on Power Apparatus and Systems, vol. PAS-93, no. 3, pp. 859-869, May 1974.

Types of optimization problems in energy

Convex relaxations and approximations of power flow

2. OPF as decoupled LP (approximation)

Thus, the decoupled LP approximation can be written as

$$
\begin{array}{lll}
\min_{v,p,q} & C(v,p,q) & \\
\text{subject to} & \sum_{j=1}^{N} p_{ij} = p_i & \forall i = 1...N \\
& \underline{p}_i \leq p_i \leq \overline{p}_i & \forall i = 1...N \\
& |p_{ij}| \leq \overline{s}_{ij} & \forall (i,j) \in N \times N \\
& p_{ij} = b_{ij}(\theta_i - \theta_j) & \forall (i,j) \in N \times N \\
& \sum_{j=1}^{N} q_{ij} = q_i & \forall i = 1...N \\
& \underline{q}_i \leq q_i \leq \overline{q}_i & \forall i = 1...N \\
& \underline{v}_i \leq |v_i| \leq \overline{v}_i & \forall i = 1...N \\
& q_{ij} = b_{ij}(|v_i| - |v_j|) & \forall (i,j) \in N \times N
\end{array}
$$

3. OPF as SDP (relaxation)

We start writing a nonconvex SDP of the exact OPF, from which we derive the convex SDP relaxation[8].

$$
\begin{array}{lll}
\min_{V,p,q} & C(V,p,q) & \\
\text{subject to} & p_{ij} + iq_{ij} = (V_{ii} - V_{ij})\, y_{ij}^* & \forall (i,j) \in N \times N \\
 & \underline{v}_i^2 \le V_{ii} \le \overline{v}_i^2 & \forall i = 1 \ldots N \\
 & V \succeq 0 & \\
 & \text{rank } V = 1 &
\end{array}
$$

where $V = vv^*$. $V = vv^*$ can be written as $V \succeq 0$ and rank $V = 1$. From this representation, we relax the problem by dropping the rank 1 constraint and we obtain the convex SDP relaxation of the OPF:

$$
\begin{array}{lll}
\min_{V,p,q} & C(V,p,q) & \\
\text{subject to} & p_{ij} + iq_{ij} = (V_{ii} - V_{ij})\, y_{ij}^* & \forall (i,j) \in N \times N \\
 & \underline{v}_i^2 \le V_{ii} \le \overline{v}_i^2 & \forall i = 1 \ldots N \\
 & V \succeq 0 &
\end{array}
$$

[8] R. A. Jabr, "Radial distribution load flow using conic programming," IEEE Trans. Power Syst., vol. 21, no. 3, pp. 1458-1459, Aug. 2006.

4. OPF as SOCP (relaxation) This relaxation can be derived from the nonconvex SDP of the exact OPF:

$$\begin{array}{lll} \min_{V,p,q} & C(V,p,q) & \\ \text{subject to} & p_{ij} + iq_{ij} = (V_{ii} - V_{ij})\, y_{ij}^* & \forall (i,j) \in N \times N \\ & \underline{v}_i^2 \leq V_{ii} \leq \overline{v}_i^2 & \forall i = 1 \ldots N \\ & V \succeq 0 & \\ & \text{rank } V = 1 & \end{array}$$

To obtain the SOCP relaxation we drop the constraint rank $V = 1$ and relax the constraint $V \succeq 0$ by using a necessary (but not sufficient) condition $V_{ij} V_{ij}^* \leq V_{ii} V_{jj}$ and $V_{ii} \geq 0$. Therefore, the SOCP relaxation can be written as follows

$$\begin{array}{lll} \min_{V,p,q} & C(V,p,q) & \\ \text{subject to} & p_{ij} + iq_{ij} = (V_{ii} - V_{ij})\, y_{ij}^* & \forall (i,j) \in N \times N \\ & \underline{v}_i^2 \leq V_{ii} \leq \overline{v}_i^2 & \forall i = 1 \ldots N \\ & V_{ij} V_{ij}^* \leq V_{ii} V_{jj} & \forall (i,j) \in N \times N \\ & V_{ii} \geq 0 & \forall i = 1 \ldots N \end{array}$$

Approximations and relaxations discussion

Notes about linear approximations:

- Linear approximations have served for many years to solve the OPF problem in a quick an efficient way.
- However, they cannot represent the system where the voltage and angles in the nodes are under stress and differ from 1 per unit and the sine of the angle cannot be approximated by the angle.
- These "stressed" conditions are the case, for example, of high penetration of distributed photovoltaic panels, blackouts, etc.
- Thus, linear approximations do not serve to study the operation details needed in extreme cases.

Approximations and relaxations discussion

Notes about convex relaxations:

- They at least provide a lower bound for the value of the optimal objective function.
- In some cases, the optimal solution of the relaxations could also be feasible in the original problem. If that is the case, the solution found by the relaxation could also be the optimal solution for the original problem.
- Under mild assumptions, exactness occurs in radial networks[9] [10] [11] [12].

[9] J. Lavaei and S. H. Low, "Zero Duality Gap in Optimal Power Flow Problem," in IEEE Transactions on Power Systems, vol. 27, no. 1, pp. 92-107, Feb. 2012.

[10] S. Low, "Convex Relaxation of Optimal Power Flow-Part I: Formulations and Equivalence," in IEEE Transactions on Control of Network Systems, vol. 1, no. 1, pp. 15-27, March 2014.

[11] S. Low, "Convex Relaxation of Optimal Power Flow-Part II: Exactness," in IEEE Transactions on Control of Network Systems, vol. 1, no. 2, pp. 177-189, June 2014.

[12] B. Zhang and D. Tse, "Geometry of injection regions of power networks," in IEEE Transactions on Power Systems, vol. 28, no. 2, pp. 788-797, May 2013.

Types of optimization problems in energy

Uncertainty

There are several parameters subject to uncertainty for all the problems introduced:

- Hourly electricity demand
- Resource availability (solar irradiance, wind speed, hydropower availability)
- Future cost uncertainty
- Etc.

Through robust optimization we can find feasible and optimal solutions for the uncertainty we model. A common approach in power systems is to optimize for the worst scenario (remember your homework? It's used in real life :)).

Frequency control

Background and motivation

In the previous topics frequency is assumed to be constant. However, frequency can deviate from its nominal value.

The **swing equation** models changes in frequency:

$$\Delta\dot{f} = -\frac{f_0}{2HSD}\Delta f + \frac{f_0}{2HS}(p - d + p_{control})$$

where
Δf is the change in grid frequency (Hz)
H **inertia constant (s) = kinetic energy of generator / rated power**
S total power of the generators
D frequency damping of the system load (load sensitivity to changes in frequency)
p power from generators
d demand $p_{control}$ additional power from controller

Inertia constant... With renewable energy, inertia is not constant anymore

- Wind and solar power $= 0$ s
- Thermal power $= 6 - 9$ s
- System inertia $=$ weighted average of inertia of generators currently dispatching

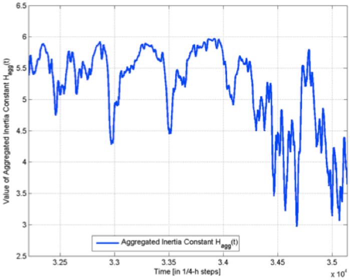


Figure: Changes in inertia in Germany every 15 min [13]

[13] Ulbig et al., Impact f Low Rotational Inertia on Power System Stability and Operation, arXiv, 2014.

Frequency control

Background and motivation

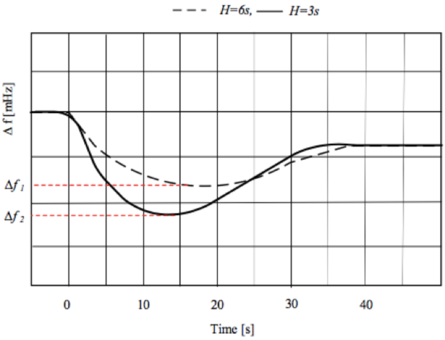


Figure: Frequency deviation for two different inertia constants [14]

[14] Dreidy et al., Inertia response and frequency control techniques for renewable energy sources: A review., 2016, Renewable and Sustainable Energy Reviews

Frequency control

Background and motivation

Research gap

- Previous studies have focused on controllers design [15] [16]
- They study control performance for a few values of inertia (H)
- They do not model the system understanding inertia as a function of time (H(t))

Research questions

- New framework for power dynamics as a hybrid system [17]
- Stable controller design with machine learning [18]

---

[15] Poolla, B. K., S. Bolognani, and F. Drfler. Placing Rotational Inertia in Power Grids. In 2016 American Control Conference (ACC), 231420, 2016.

[16] Ulbig, A., T. Rinke, S. Chatzivasileiadis, and G. Andersson. Predictive Control for Real-Time Frequency Regulation and Rotational Inertia Provision in Power Systems. In 52nd IEEE Conference on Decision and Control, 294653, 2013

[17] P. Hidalgo-Gonzalez, R. Dobbe, R. Henriquez-Auba, D. S. Callaway, and C. J. Tomlin, Frequency regulation in hybrid power dynamics with variable and low inertia due to renewable energy, in Decision and Control (CDC), 2018 IEEE 57th Annual Conference on. IEEE, 2018

[18] P. Hidalgo-Gonzalez, R. Henriquez-Auba, D. S. Callaway, and C. J. Tomlin, Frequency regulation using data-driven controllers in power grids with variable inertia due to renewable energy, (Submitted)

Frequency control

Control design with ML[19]

Power dynamics as hybrid system

$$\begin{bmatrix} \dot{\theta} \\ \dot{\omega} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & I \\ -M_{q(t)}^{-1} L & -M_{q(t)}^{-1} D \end{bmatrix}}_{\hat{A}_{q(t)}} \begin{bmatrix} \theta \\ \omega \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ M_{q(t)}^{-1} \end{bmatrix}}_{\hat{B}_{q(t)}} p_{\text{in}}$$

Generation of training set (x, u) from LQR optimal control ($u = -K_q x$)

$$\begin{aligned} \min_{\boldsymbol{x}, \boldsymbol{u}} \ & \sum_{t=0}^{T} x_t Q x_t + u_t R u_t \\ \text{s.t.} \ & x_0 = x^{(0)} \\ & x_{t+1} = A_{q(t)} x_t + B_{q(t)} u_t, \ t \in \{0, T-1\} \end{aligned}$$

Least Squares to learn fixed controller proportional to states ($u = K_L x$)

$$\min_{K_{\text{L}}} \sum_{k=1}^{N} \sum_{t=1}^{T} \left\| u_t^{(k)} - K_{\text{L}} x_t^{(k)} \right\|_2^2$$

[19] P. Hidalgo-Gonzalez, R. Henriquez-Auba, D. S. Callaway, and C. J. Tomlin, Frequency regulation using data-driven controllers in power grids with variable inertia due to renewable energy, (Submitted)

Frequency control

Control design with ML[20]

- The learned controller ($u = K_L x$) is not stable in lowest inertia mode.
- We add virtual inertia to stabilize.

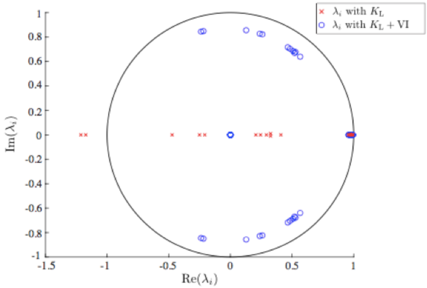


Fig. 2. Eigenvalue placement for the closed loop system in mode q_1 using the learned controller K_L (crosses) and adding virtual inertia control K_L + VI (circles).

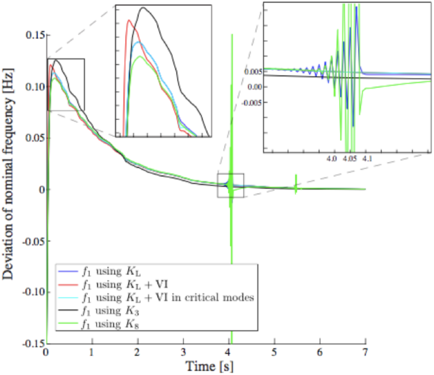


Fig. 3. Frequency deviations for node 1 for 5 different controllers from a hybrid system simulation.

[20] P. Hidalgo-Gonzalez, R. Henriquez-Auba, D. S. Callaway, and C. J. Tomlin, Frequency regulation using data-driven controllers in power grids with variable inertia due to renewable energy, (Submitted)

Conclusions and take-aways

- Convex optimization is at the core of power systems
- Convex relaxations and approximations of power flow have simplified simulations considerably
- Convex optimization is key for optimal control theory
- Now that you've taken convex optimization, you are very well suited to study control theory and work on renewable energy integration :D!