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Discussion #1

Exercise 1 (Subpaces and dimensions) Consider the set $\mathcal{S}$ of points such that

$$x_1 + 2x_2 + 3x_3 = 0, \quad 3x_1 + 2x_2 + x_3 = 0.$$

Show that $\mathcal{S}$ is a subspace. Determine its dimension, and find a basis for it.

Exercise 2 (Direct sum) Find $k \in \mathbb{R}$ such that $\mathbb{R}^3 = S \oplus T$, with $S = span\left\{\begin{bmatrix} \text{-3} \\ 4 \\ 1 \end{bmatrix}\right\}$ and $T = \{(x, y, z) \in \mathbb{R}^3 : kx + y - z = 0\}$

Exercise 3 (Extrema of inner product over a ball) Let $y \in \mathbb{R}^n$ be a given non-null vector, and let $\mathcal{X} = \{x \in \mathbb{R}^n : \|x\|_2 \leq r\}$, where r is some given positive number.

1. Determine the optimal value p_1^* and the optimal set (i.e., the set of *all* optimal solutions) of the problem $\min_{x\in\mathcal{X}} |y^\top x|$.
2. Determine the optimal value p_2^* and the optimal set of the problem $\max_{x\in\mathcal{X}} |y^\top x|$.
3. Determine the optimal value p_3^* and the optimal set of the problem $\min_{x\in\mathcal{X}} y^\top x$.
4. Determine the optimal value p_4^* and the optimal set of the problem $\max_{x\in\mathcal{X}} y^\top x$.

Exercise 4 (Inner product) Let $x, y \in \mathbb{R}^n$. Under which condition on $\alpha \in \mathbb{R}^n$ does the function

$$f(x, y) = \sum_{k=1}^{n} \alpha_k x_k y_k$$

define an inner product on $\mathbb{R}^n$?