Linear-time Approximations for Dominating Sets and Independent Dominating Sets in Unit Disk Graphs

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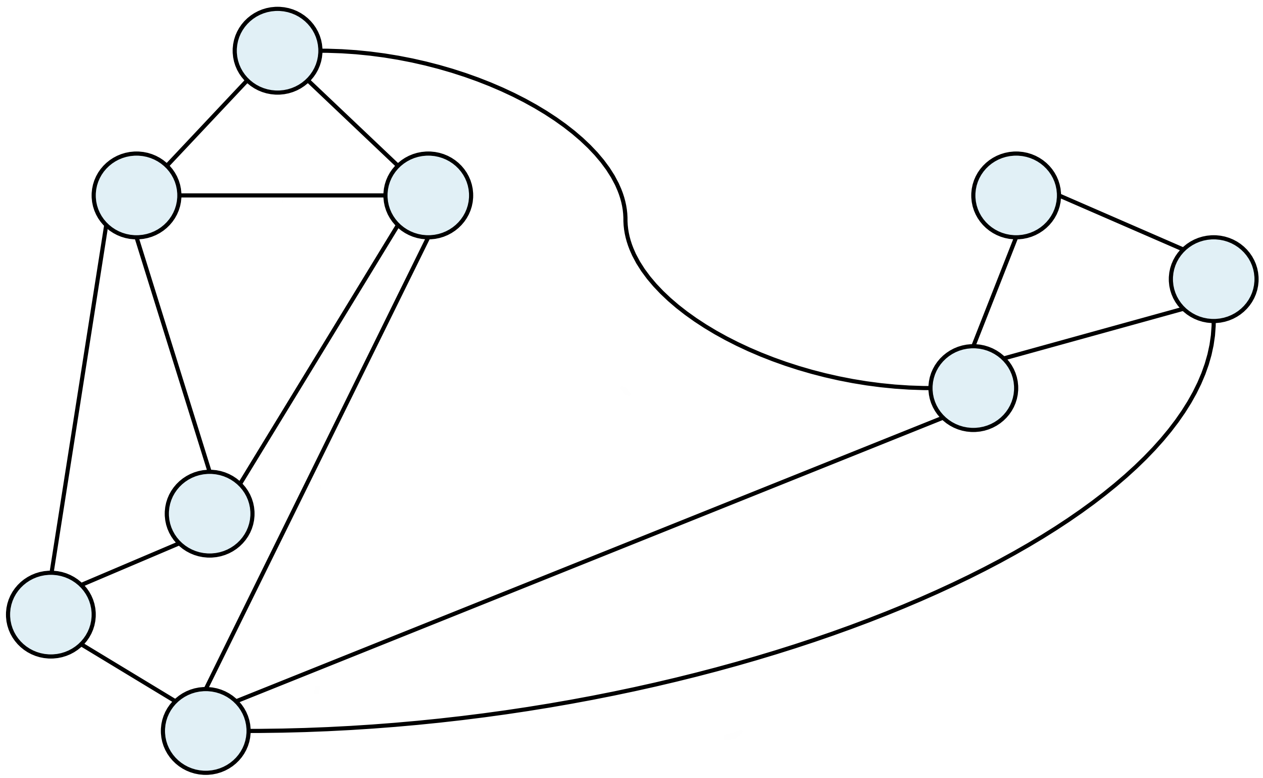
Dominating set

$G\ (V, E)$



$D \subseteq V$

D dominating set $\quad \Longleftrightarrow \quad \forall\, w \in V \setminus D,\ \ \exists\, v \in D \ \mid\ vw \in E$

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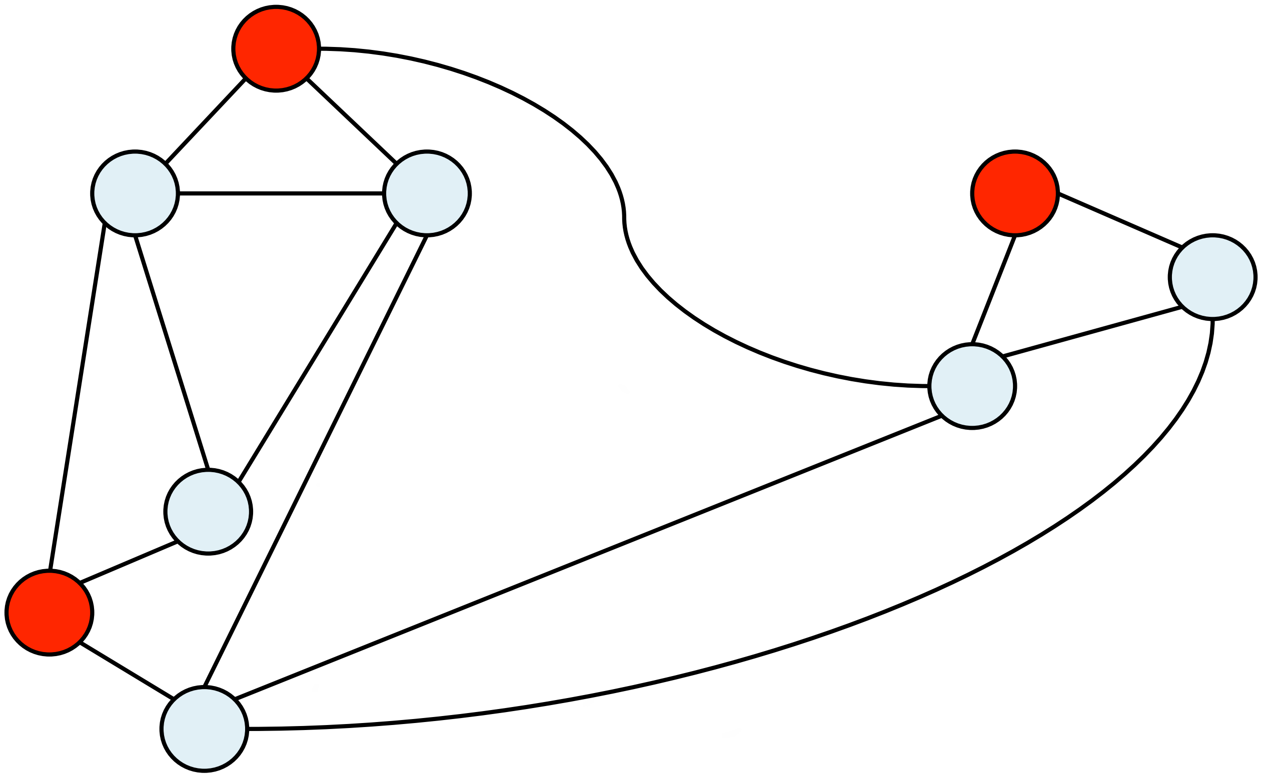

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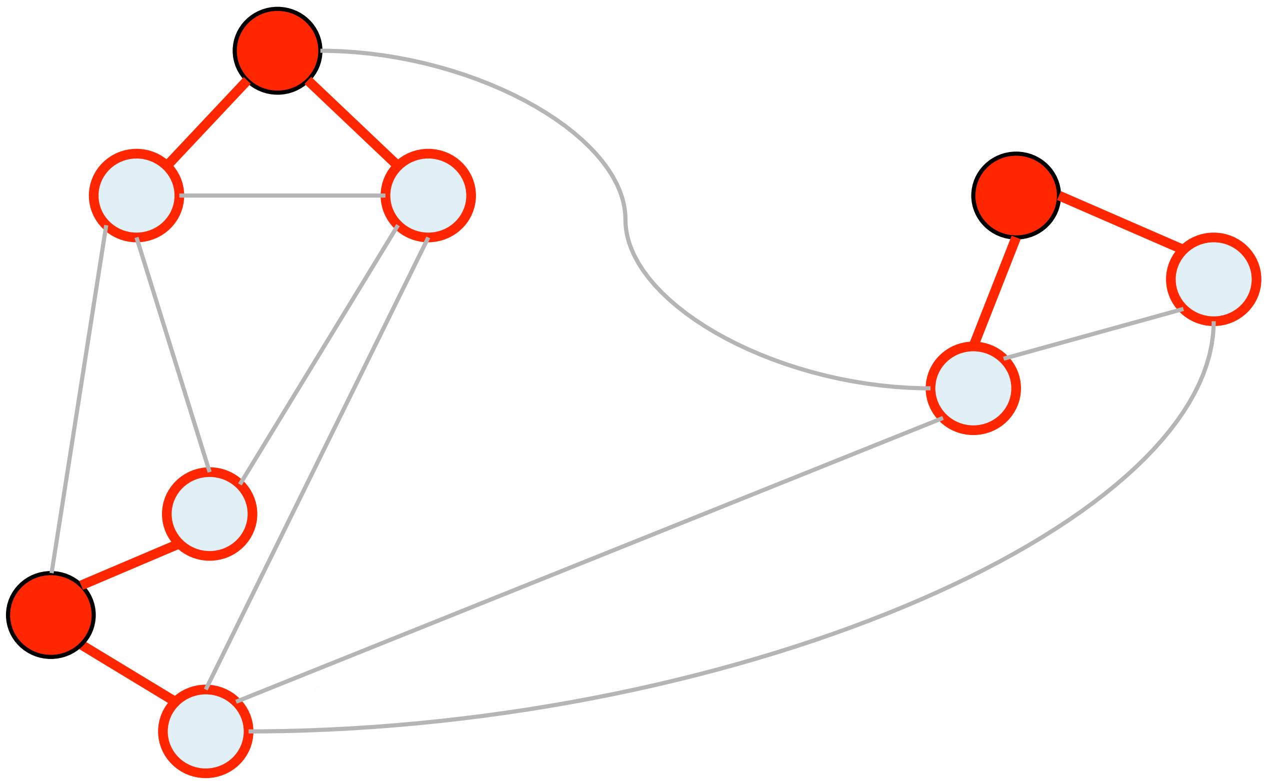

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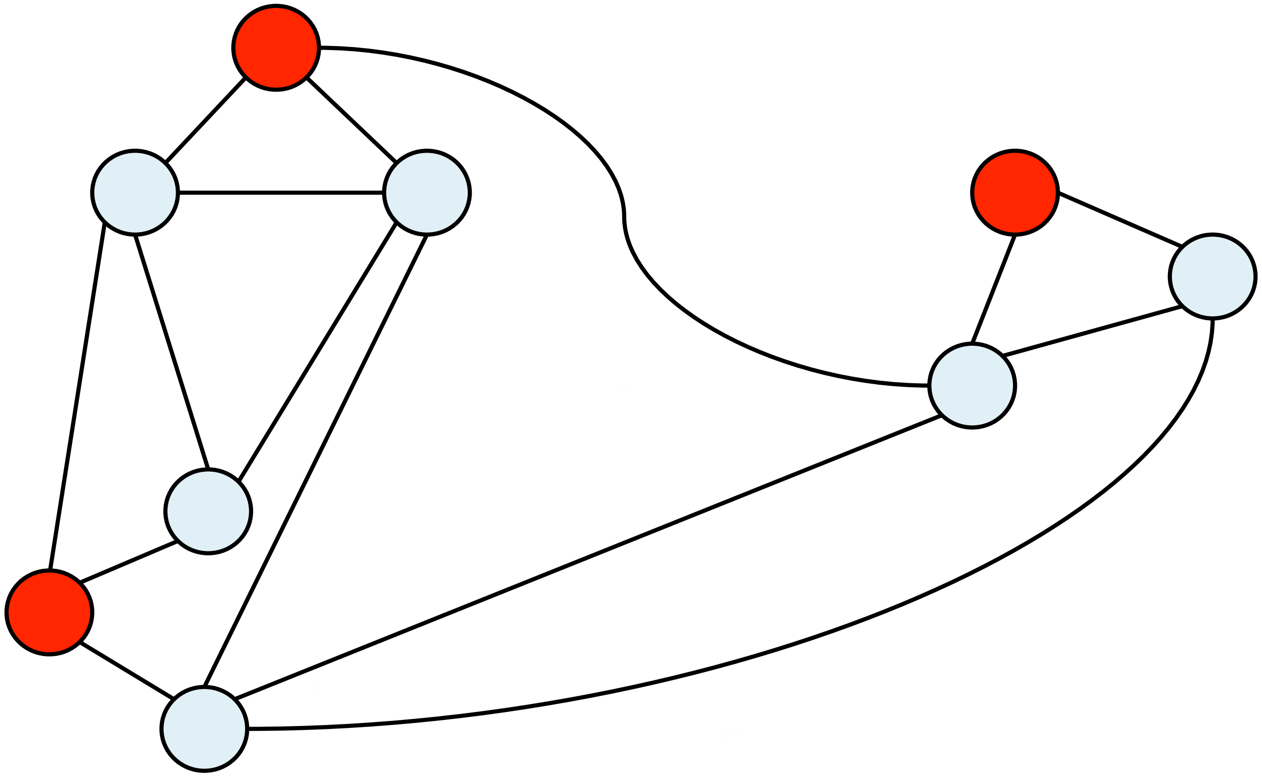

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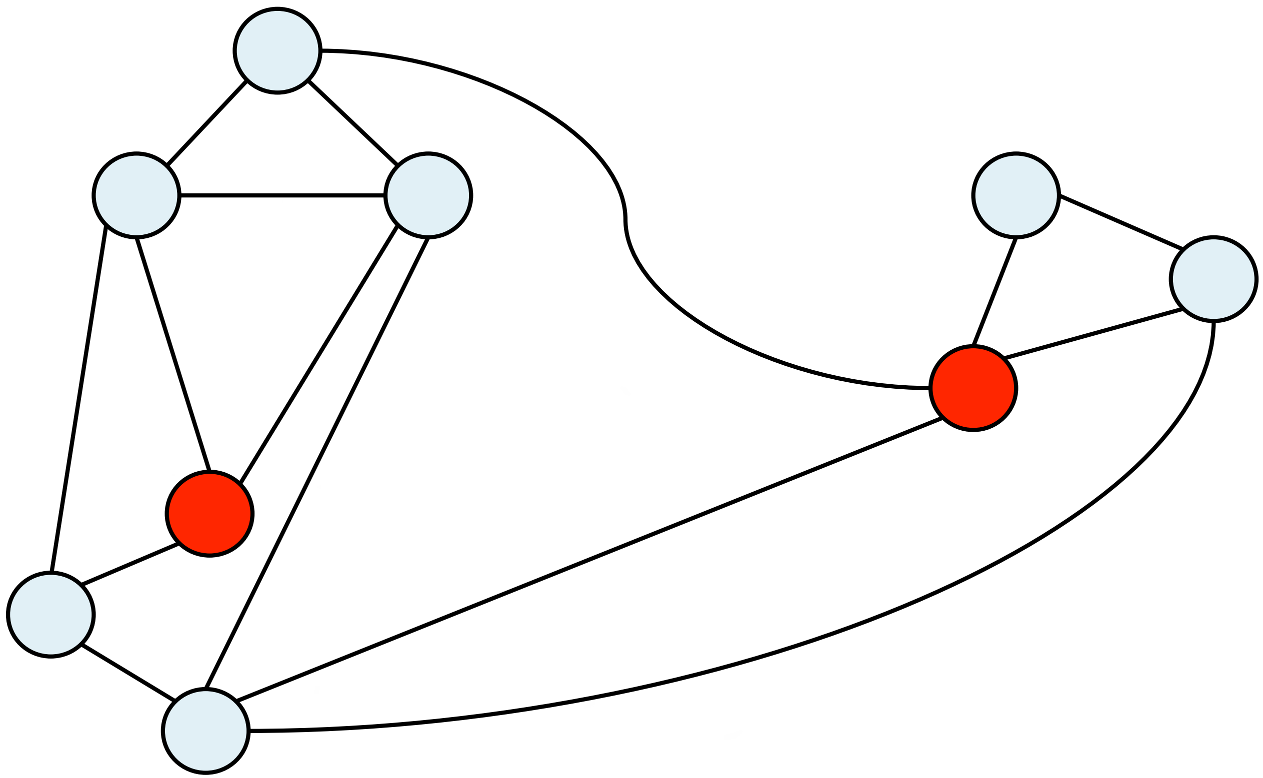

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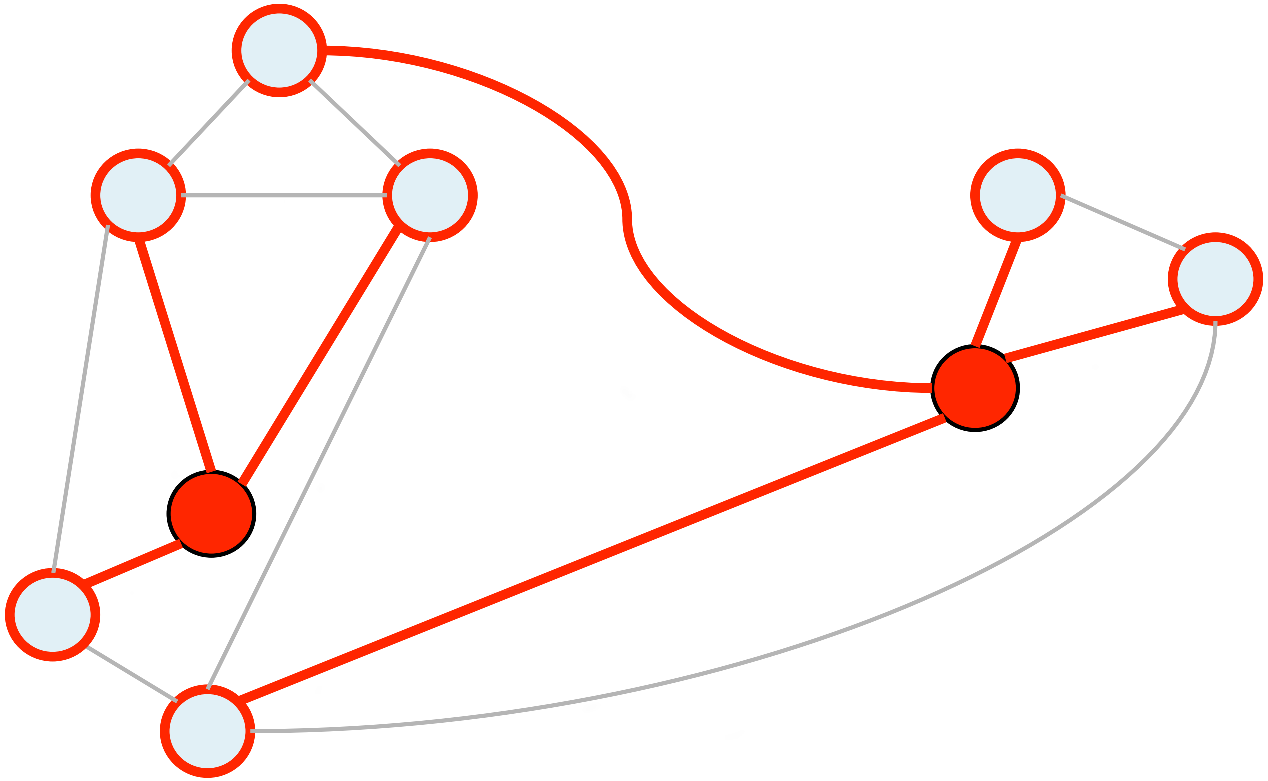

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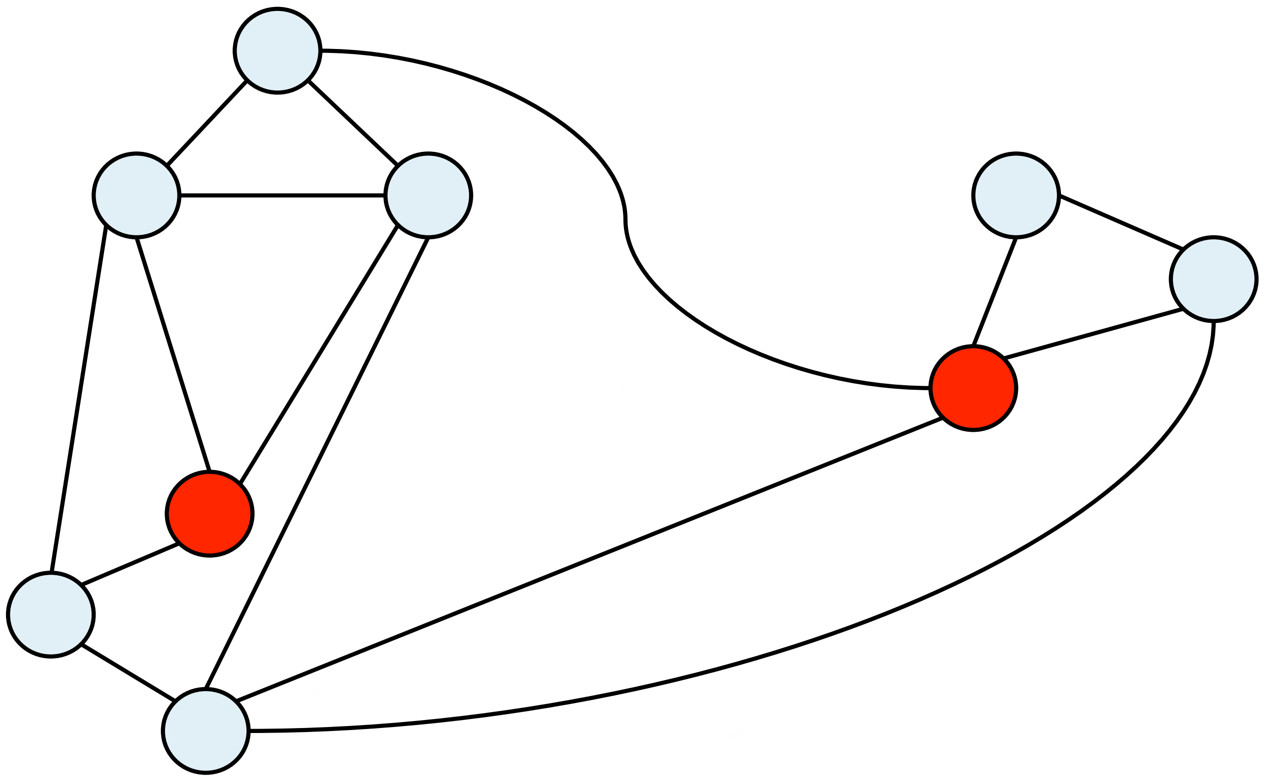

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Minimum dominating set problem

<u>Input</u>: graph G (V, E)
<u>Output</u>: dominating set D of G s.t. $|D|$ is minimum

→ NP-hard

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<u>Decision version</u>
Input: graph G (V, E), integer k
Question: is there a dominating set D of G s.t. $|D| \leq k$?

→ NP-complete
(Garey & Johnson 1979)

Approximation Algorithms

- Not always return an optimum solution
- Close (enough) to optimum
- Guaranteed approximation factor α

Minimization problem: $\alpha > 1$

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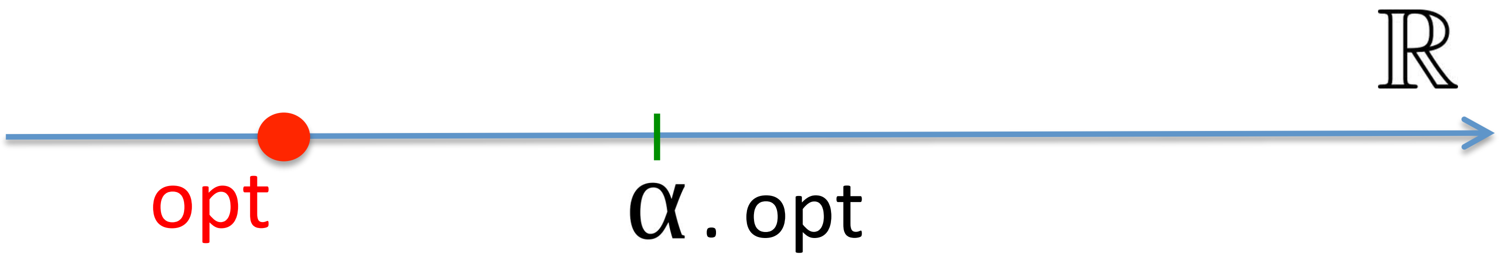

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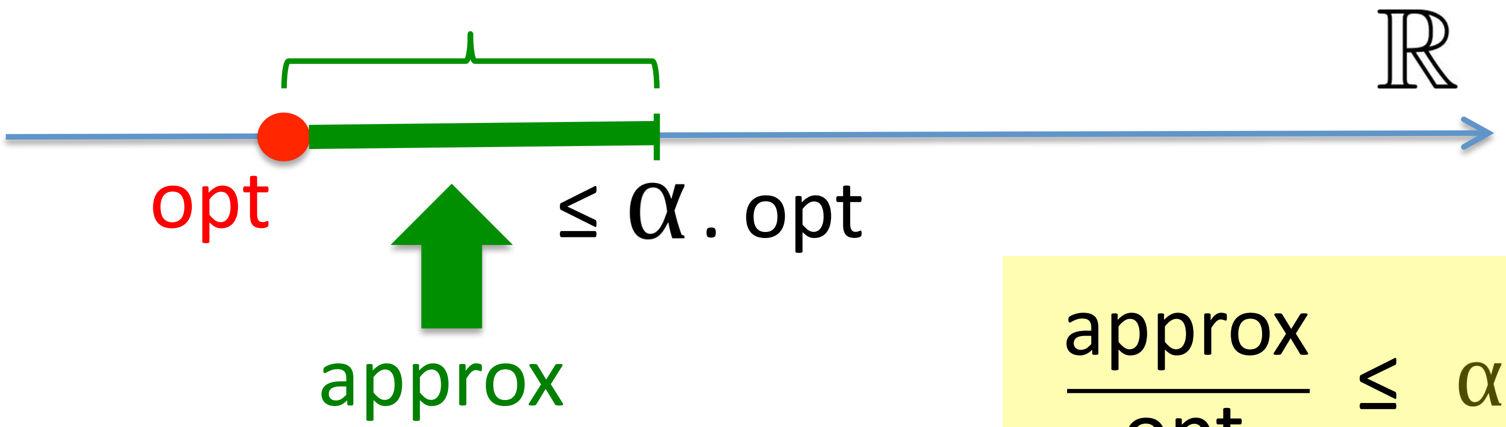

$$\frac{\text{approx}}{\text{opt}} \leq \alpha$$

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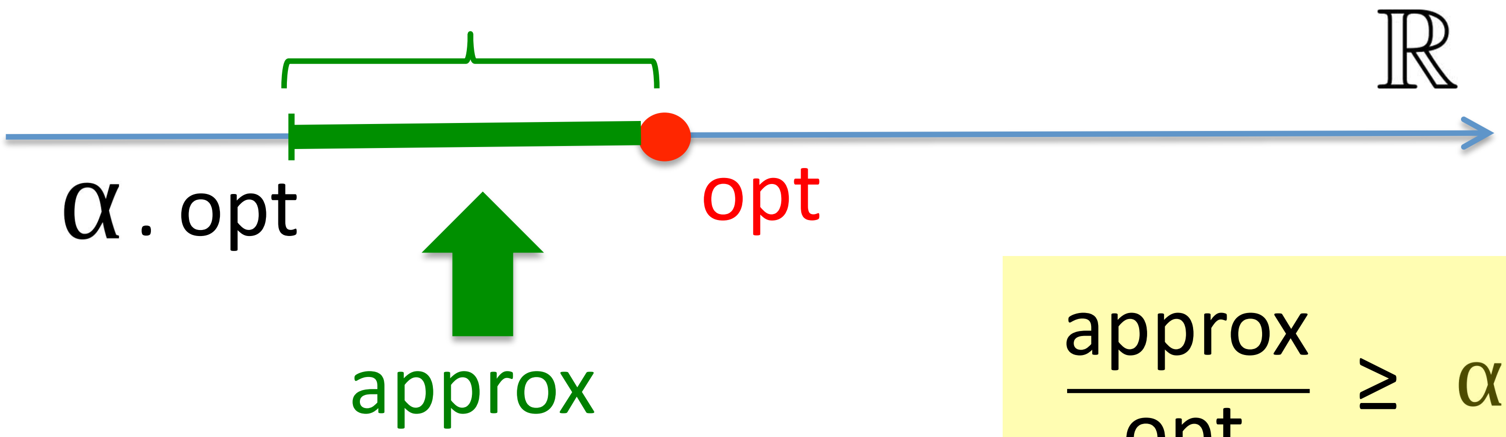
Maximization problem: $0 < \alpha < 1$



$$\frac{\text{approx}}{\text{opt}} \geq \alpha$$

Minimum dominating set problem

Input: graph G (V, E)
Output: dominating set D of G s.t. $|D|$ is minimum

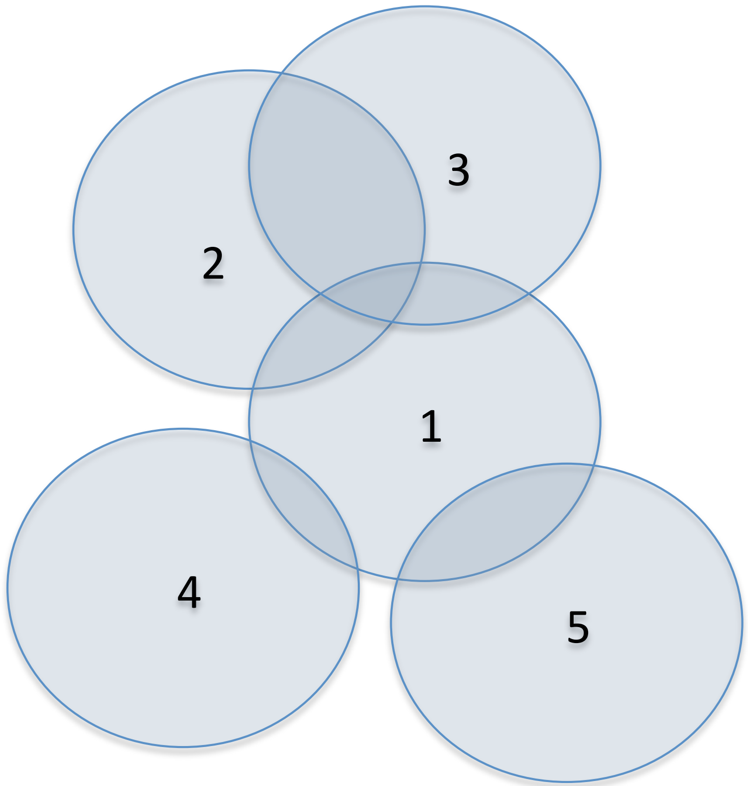
→ NP-hard

→ (1+log n)-approximation algorithm (Johnson 1974)

→ Not approximable within a (c log n) factor, for some $c > 0$
(Raz & Safra 1997)

Unit disk graph

Model of
congruent disks



Graph
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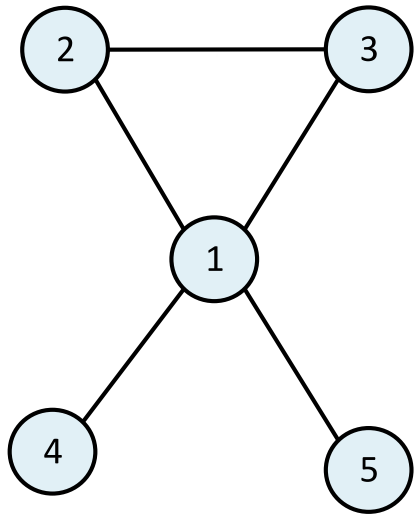


Dominating sets in unit disk graphs

→ Several applications, e.g. ad-hoc wireless networks
(Marathe, Breu, Hunt III, Ravi & Rosenkrantz 1995)

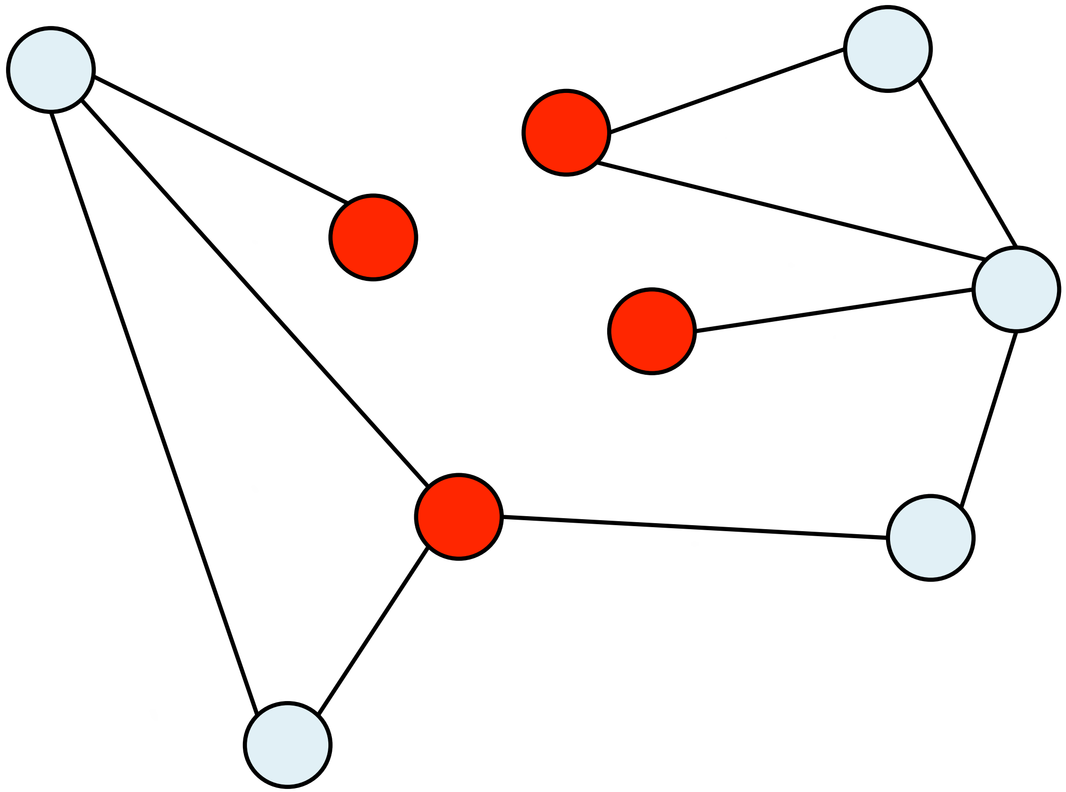
→ NP-hard nonetheless
(Clark, Colbourn & Johnson 1990)

→ Constant factor approximations (breaking the log *n* barrier),
and even PTAS

Two simple facts

1st fact:

Every maximal independent set is a dominating set.



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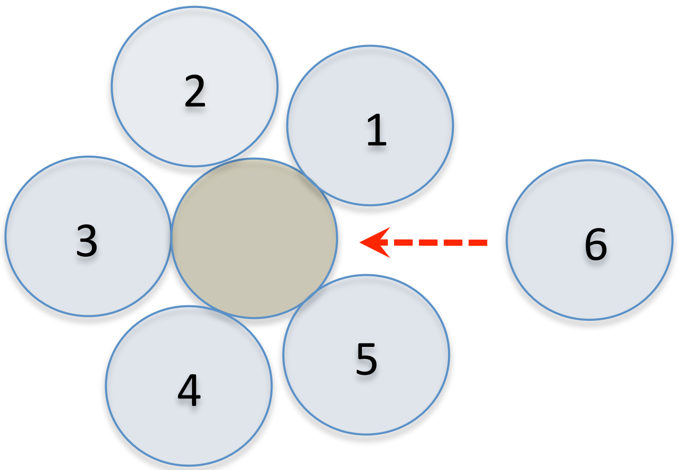
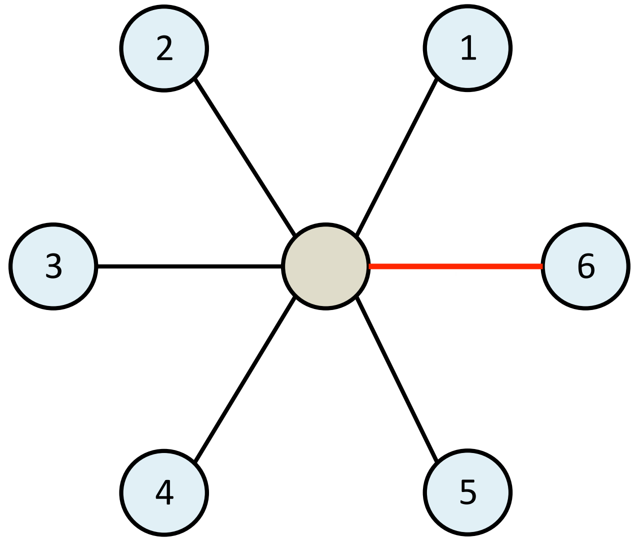
<u>2nd fact</u>:

A unit disk graph contains no $K_{1,6}$ as an induced subgraph.





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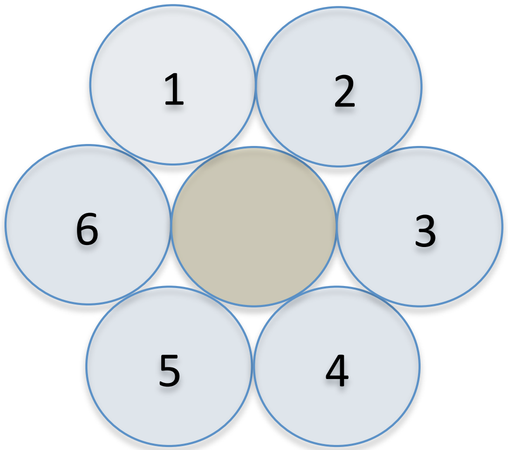

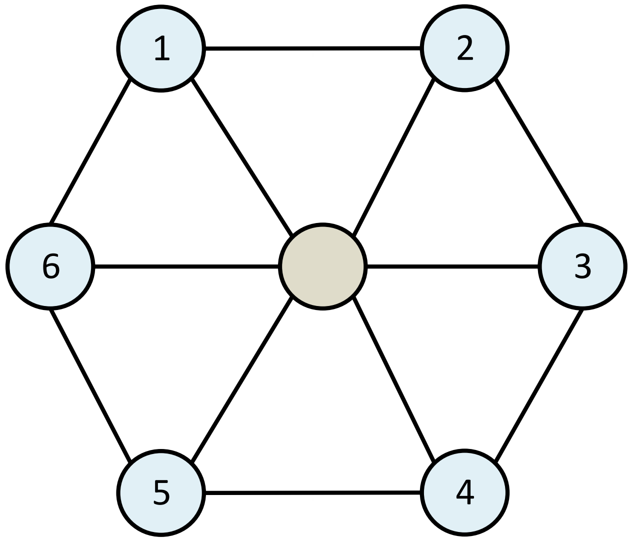

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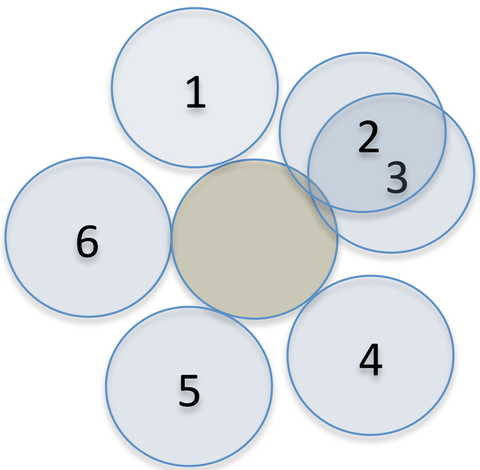

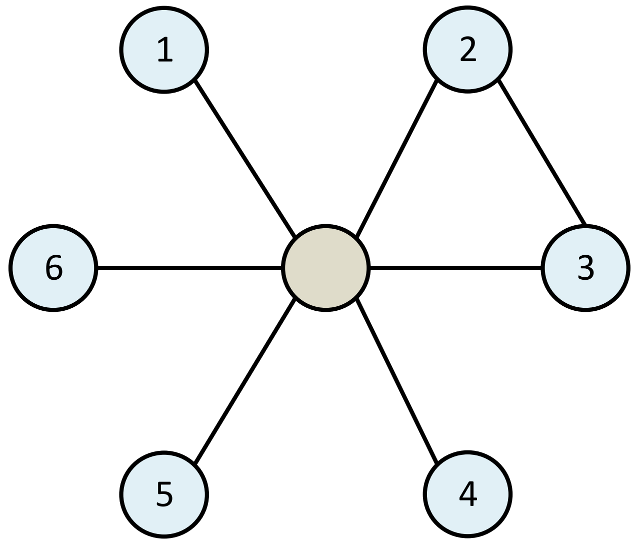

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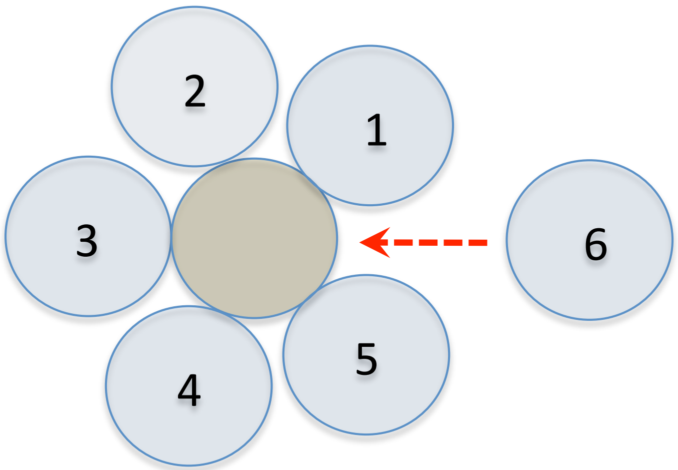

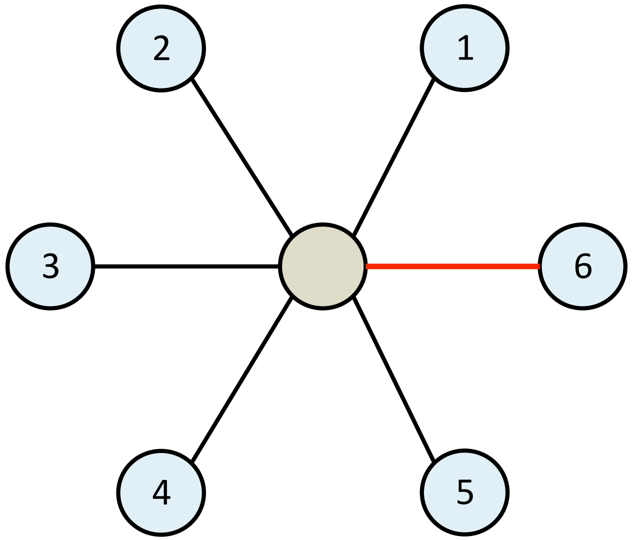

$K_{1,6}$

5-approximation

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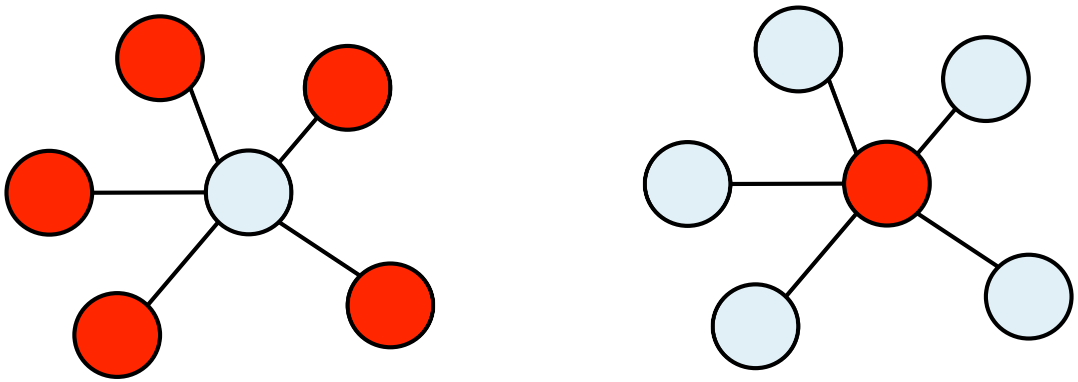
A unit disk graph contains no $K_{1,6}$ as an induced subgraph.

Corollary:

If *G* is a unit disk graph, then
every maximal independent set *S* of *G* is a **5-approximation**
for the minimum (independent) dominating set of *G.*



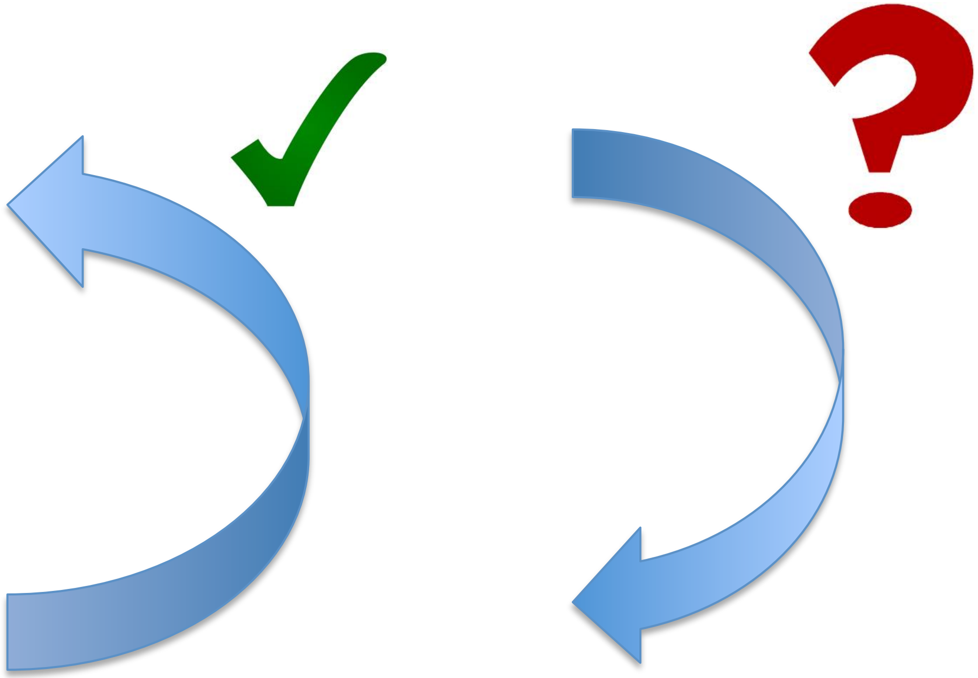
Algorithms for unit disk graphs

→ *Graph-based* algorithms

Input: a graph

→ *Geometric* algorithms

Input: a geometric model



Dominating sets in unit disk graphs

→ Vast literature on approximation algorithms:
(Marathe, Breu, Hunt III, Ravi & Rosenkrantz 1995)

O(n+m) graph-based 5-approximation (MBHRR'95)

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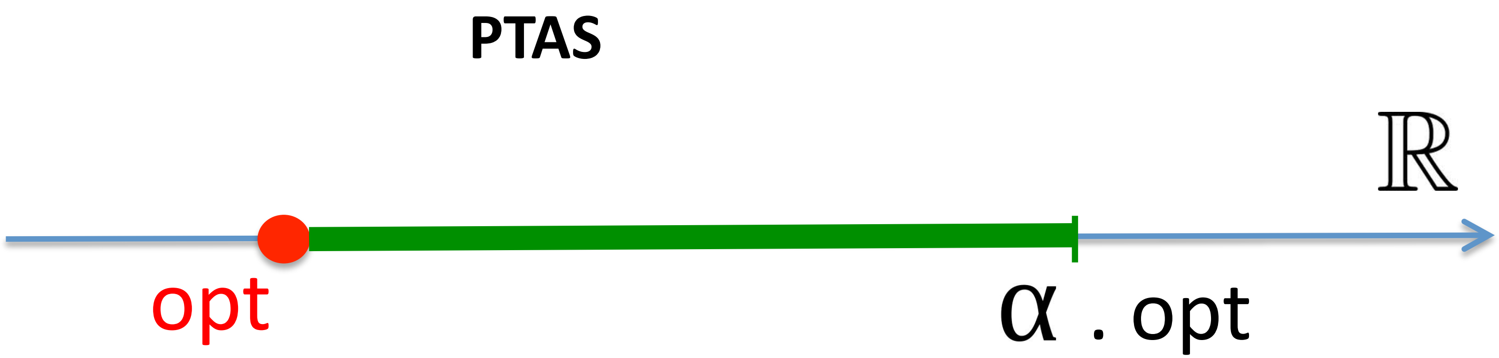
*PTAS

Polynomial time approximation schemes (PTAS)

- Approximation factor $\alpha = 1 \pm \varepsilon$ is as good as you want ($\varepsilon > 0$)



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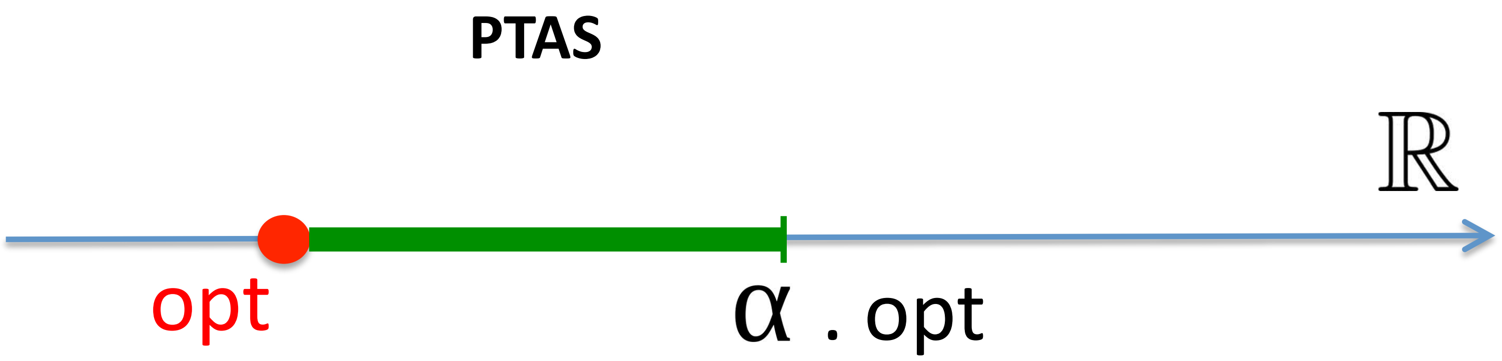

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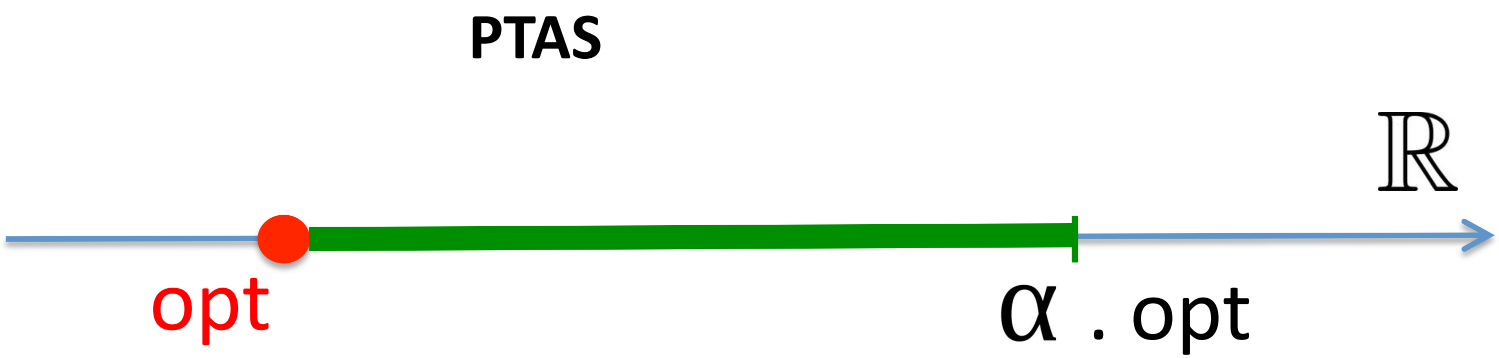
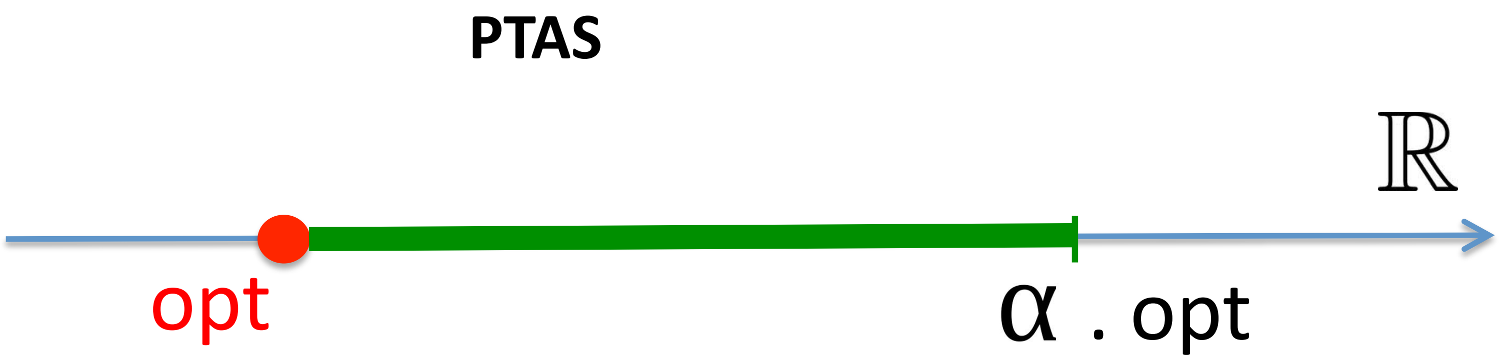


Polynomial time approximation schemes (PTAS)

- Approximation factor $\alpha = 1 \pm \varepsilon$ is as good as you want ($\varepsilon > 0$)
- Running time may grow exponentially with $1/\varepsilon$
- For fixed ε running time is polynomial (on the input size)



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Our contribution:
$O(n+m)$ graph-based 4._____ -approximation
$O(n \log n)$ geometric 4._____ -approximation (FFMS'12)

100 meters world record

1995

Leroy Burrell (USA)
9.85 s

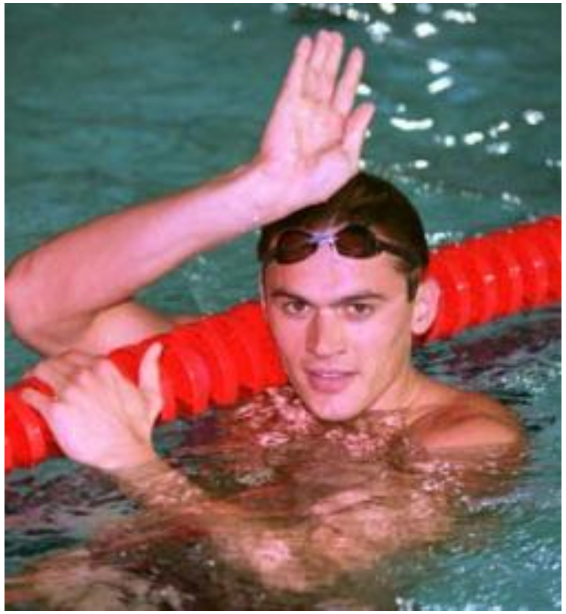
2012

Usain Bolt (JAMAICA)
9.58 s

- 2.7%

100 meters freestyle world record

1995



Alexander Popov (RUSSIA)
48.21 s

2012

César Cielo (BRAZIL)
46.91 s

- 2.6%

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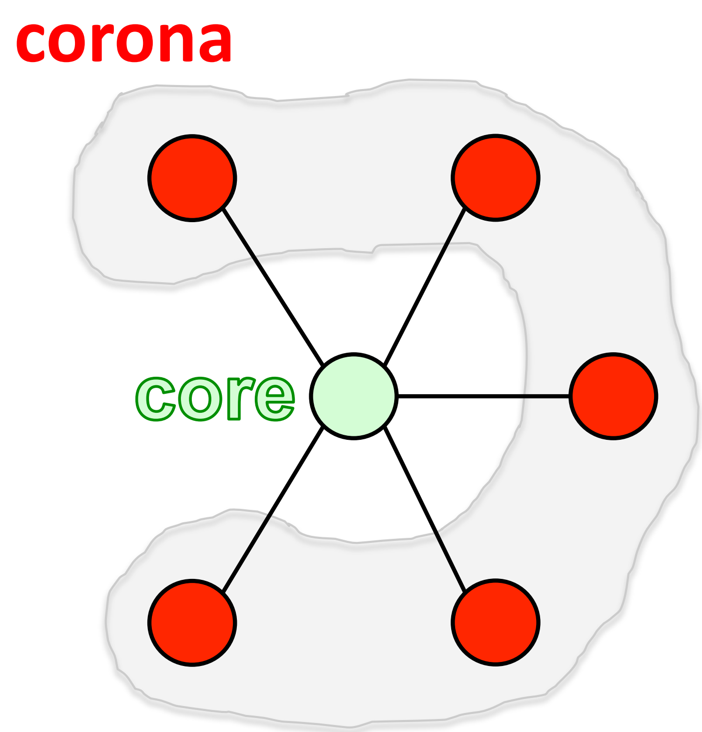
Our contribution: $O(n+m)$ graph-based 4.888... -approxim
$O(n \log n)$ geometric 4.888... -approximation
(FFMS'12)

- 2.2%

Coronas and cores

Let D be a maximal independent set of graph $G(V,E)$.
A **corona** consists of exactly 5 (five) vertices of D presenting a common neighbor in $V \setminus D$, called a **core**.



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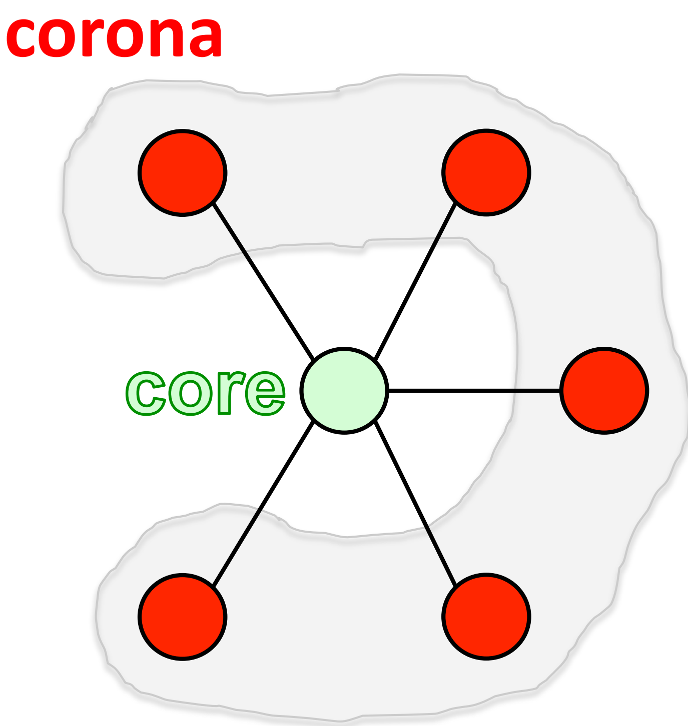
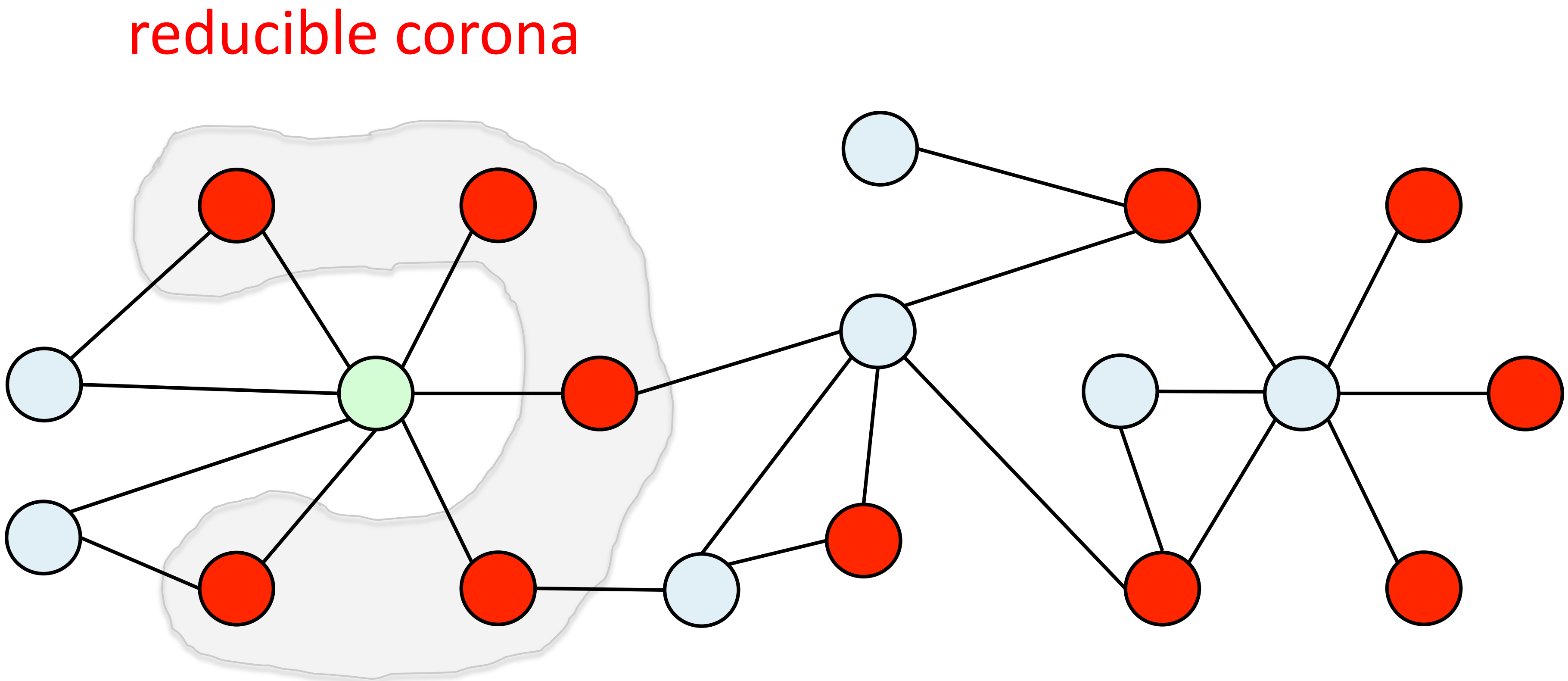
A corona C can be

- reducible,
  if it has a core c s.t.
  $D \setminus C \cup \{c\}$ is still
  a dominating set of G;

  or
- irreducible,
  otherwise.



Reducible and irreducible coronas



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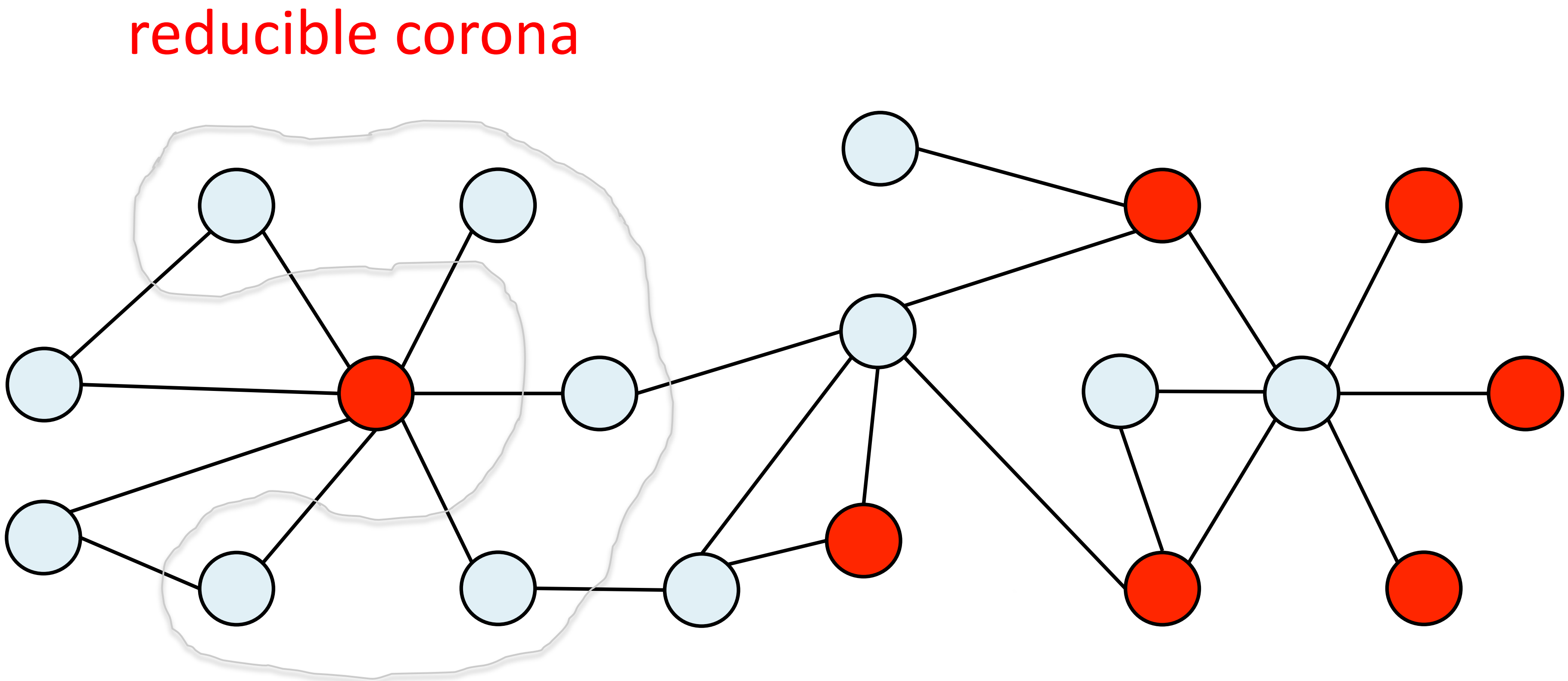

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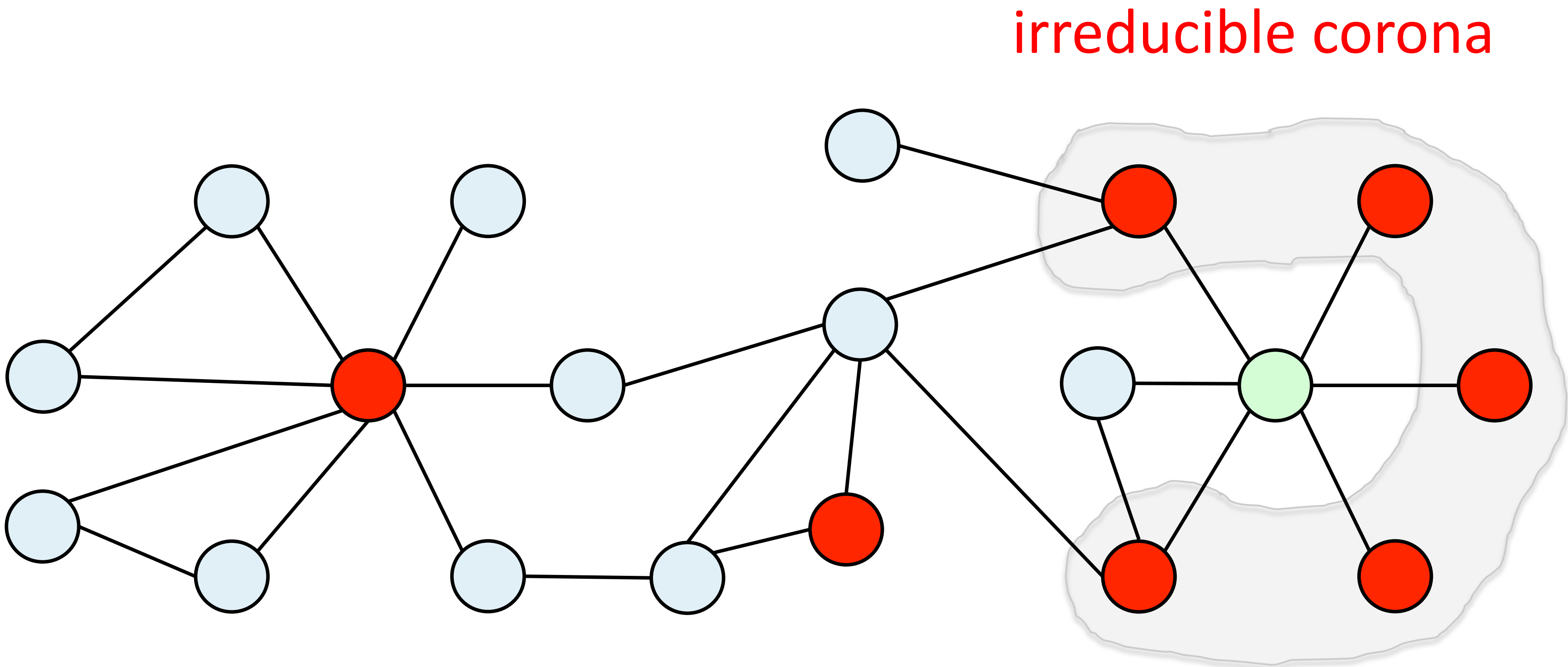
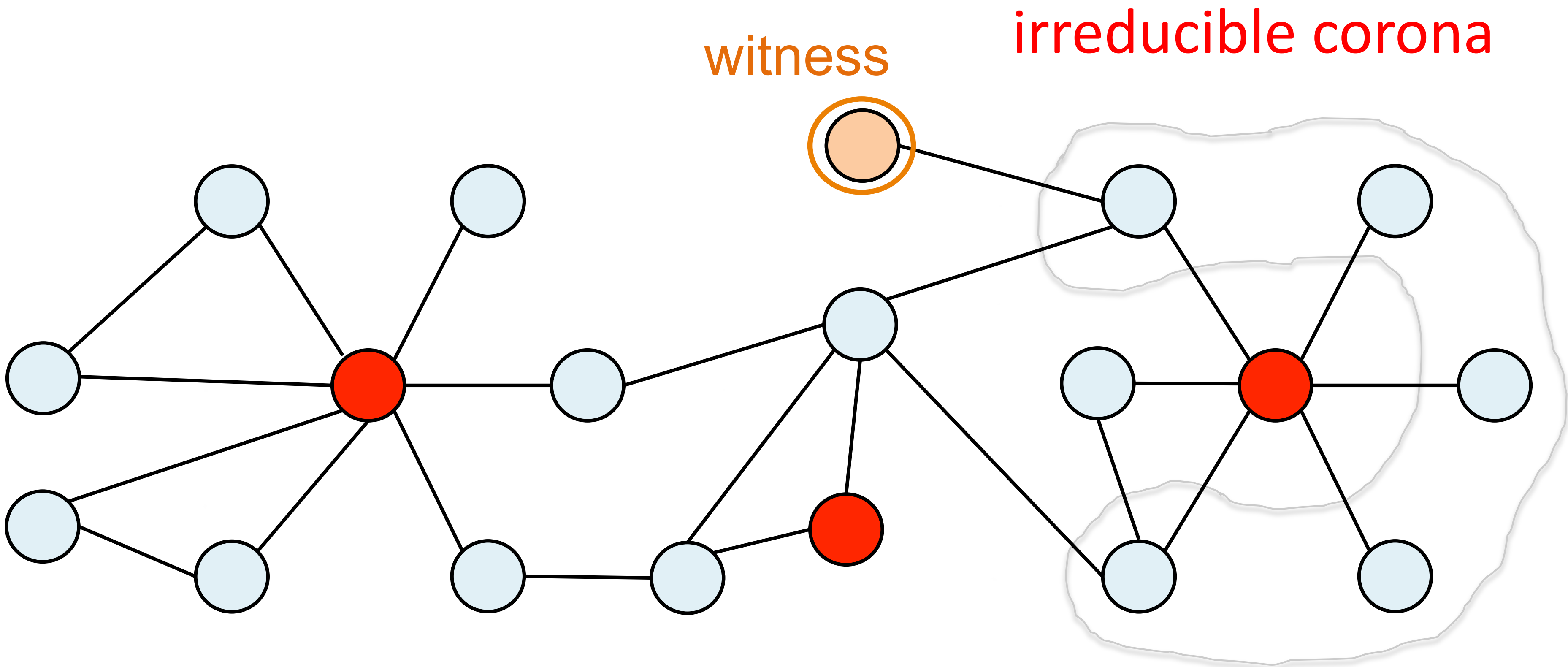


Witnesses



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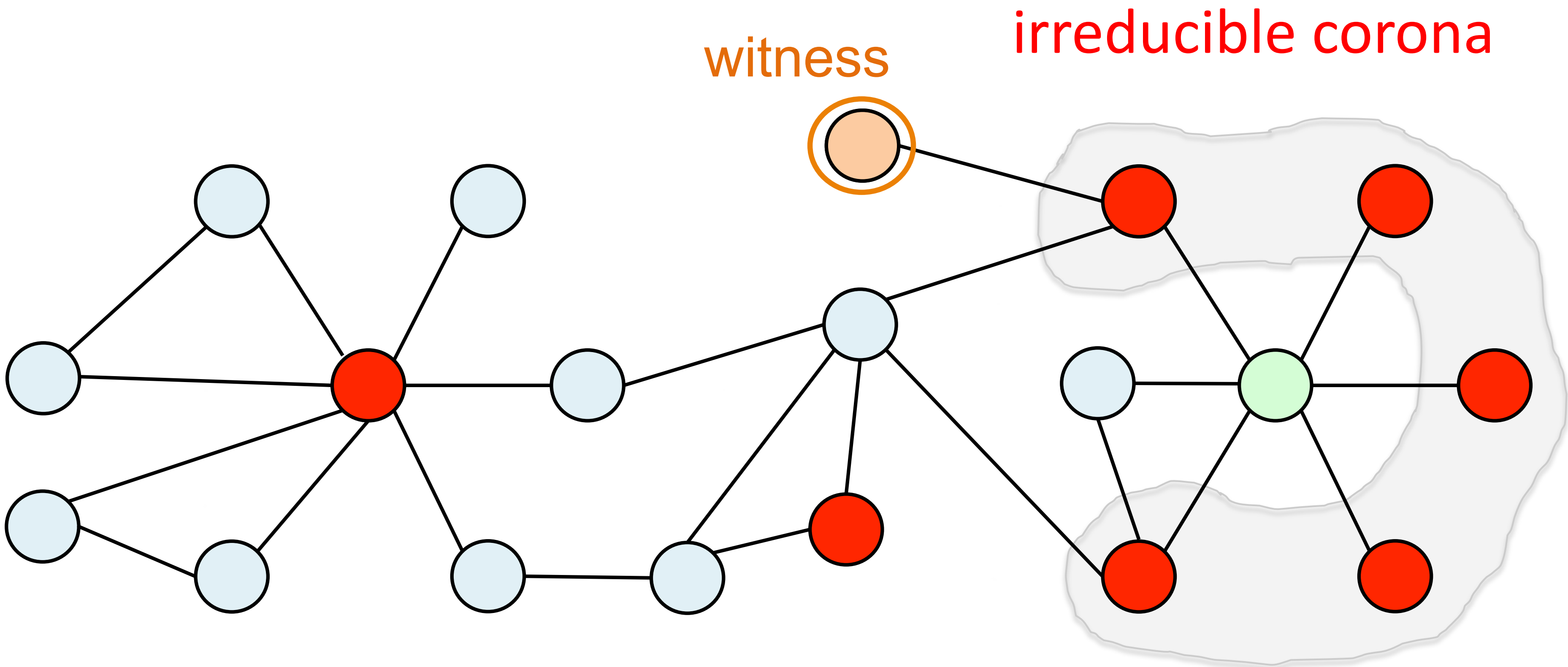
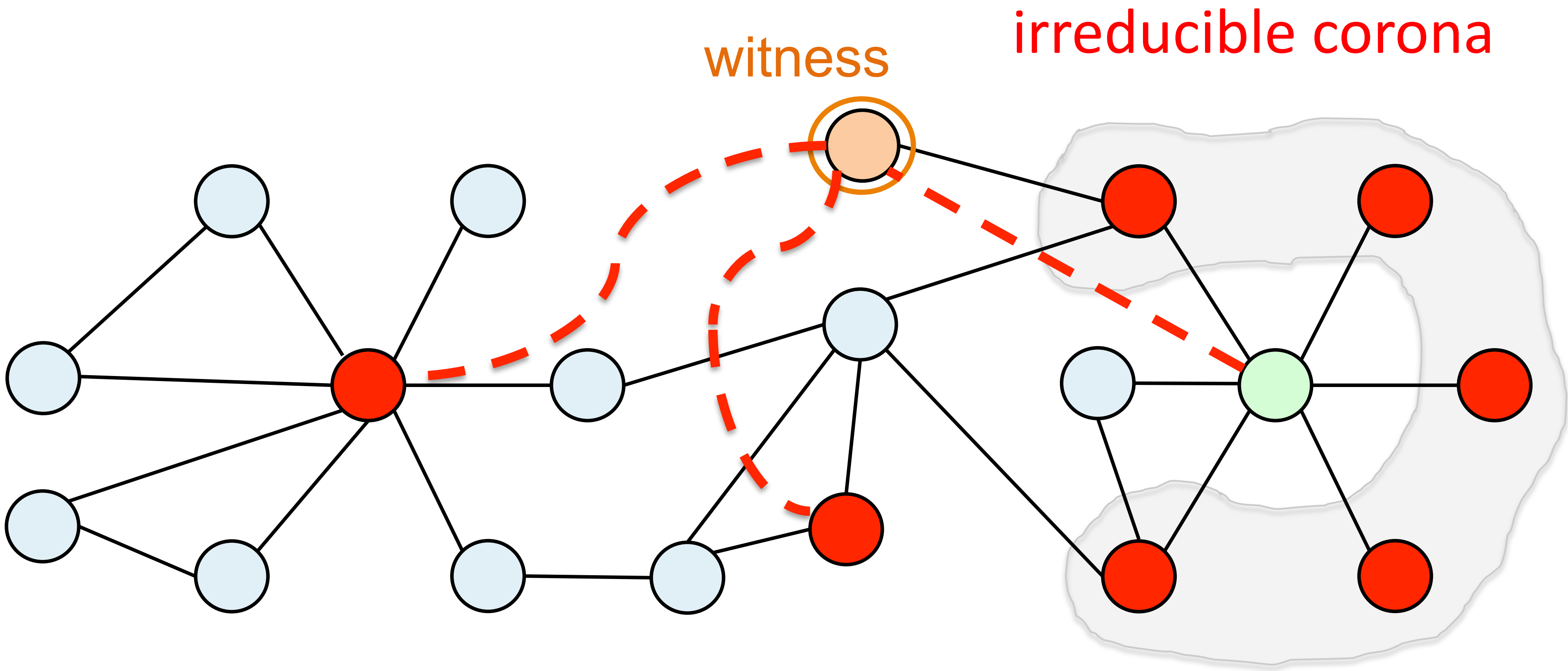


Witnesses

Let C be a corona of graph $G(V,E)$, and let c be a core of C.
A vertex w is a witness of c iff $cw \notin E$, and $N_D[w] \subseteq C$.



4.888...-approximation

1. Obtain a maximal independent set *D*
2. While there is a reducible corona *C* in *D*
3. Update *D* by reducing *C*
4. Return *D*

Input: adjacency lists (graph)
Time: O(n+m)

Input: center coordinates in Real RAM Model
Time: O(n log n)

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<u>Lemma</u>: a maximal independent set *D with no reducible coronas* is a 4.888…-approximation for the minimum (independent) dominating set.

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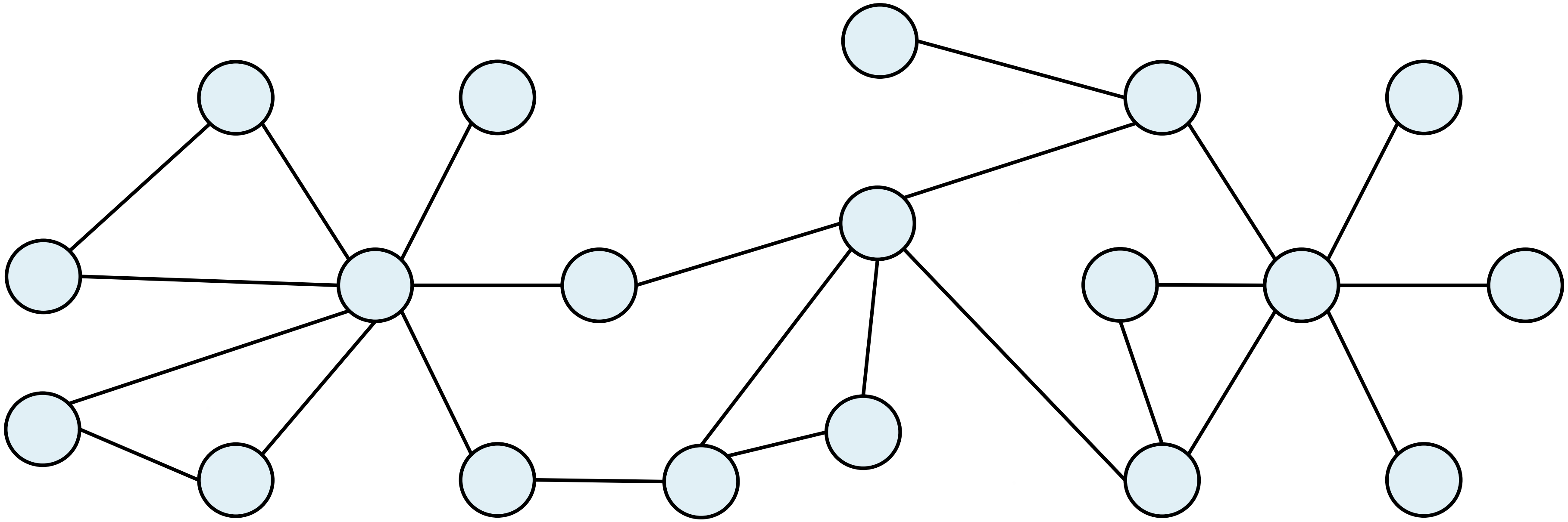

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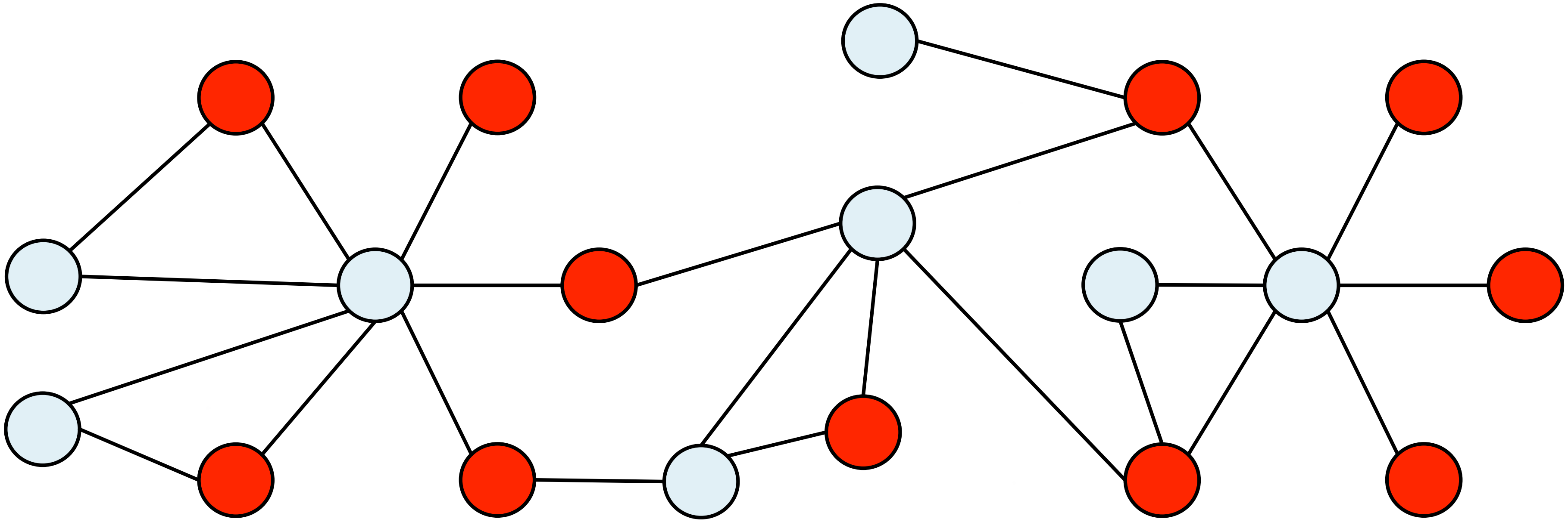
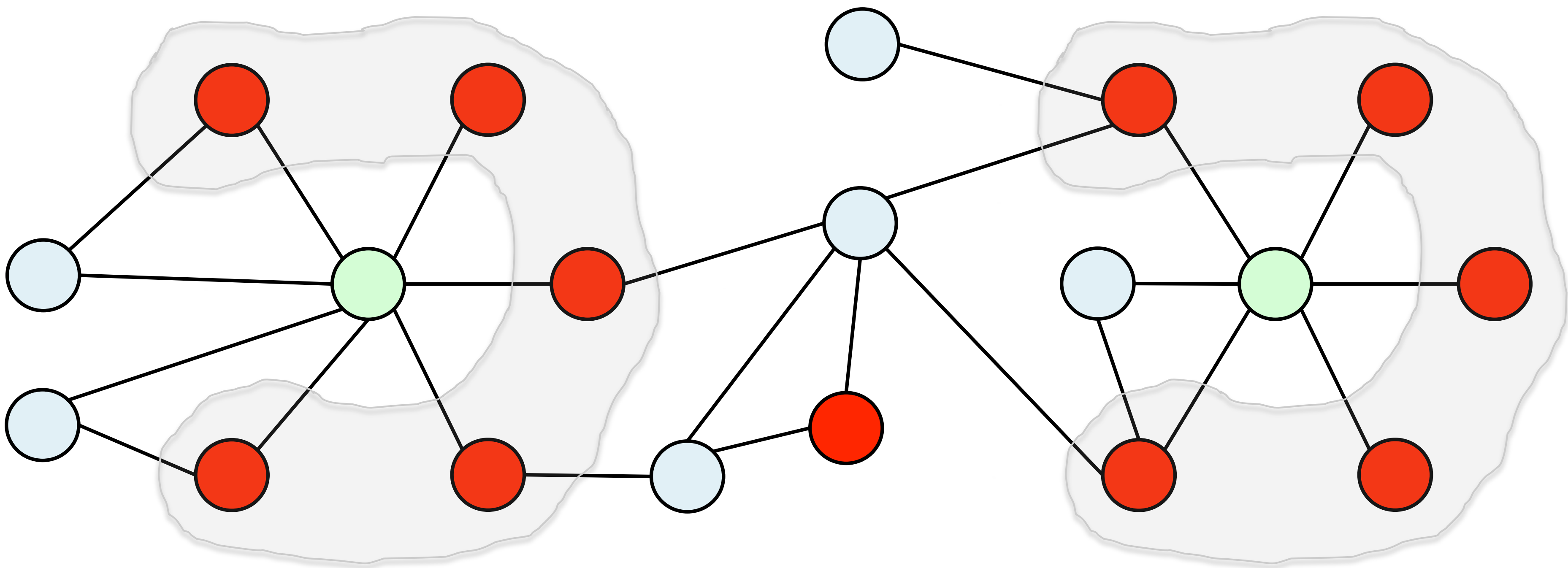


4.888...-approximation

1. Obtain a maximal independent set *D*
2. While there is a reducible corona *C* in *D*
3. Update *D* by reducing *C*
4. Return *D*



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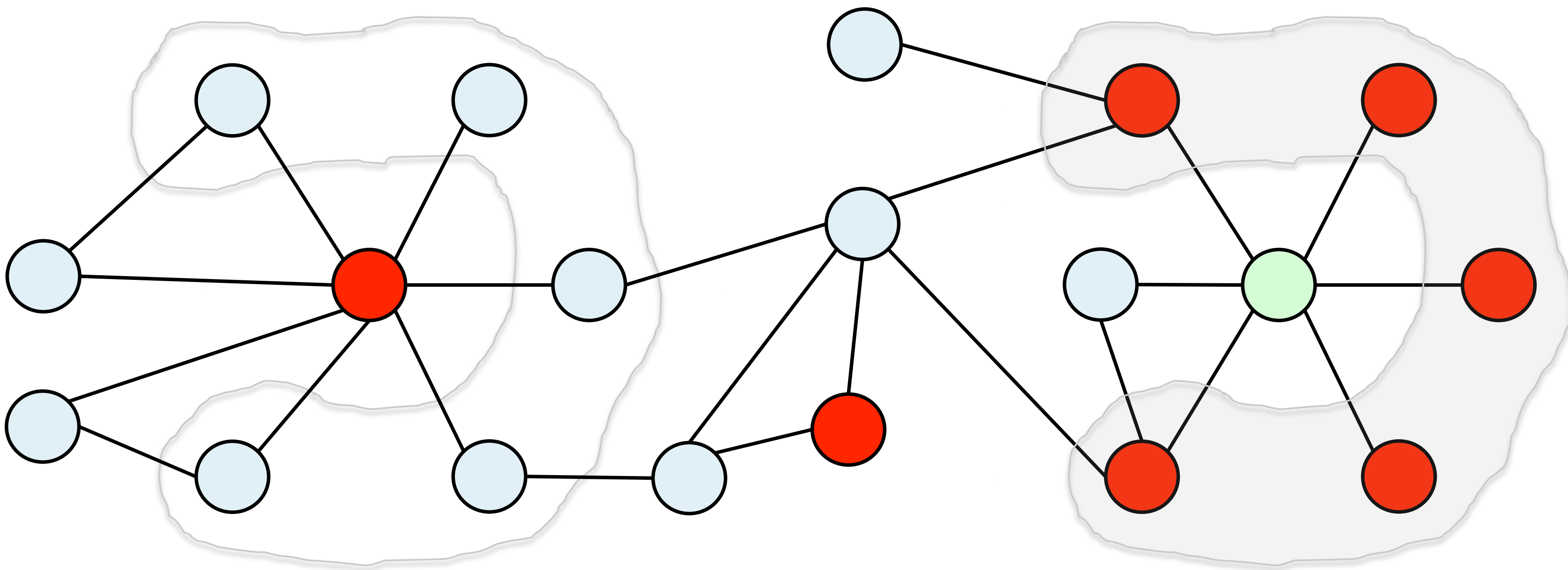
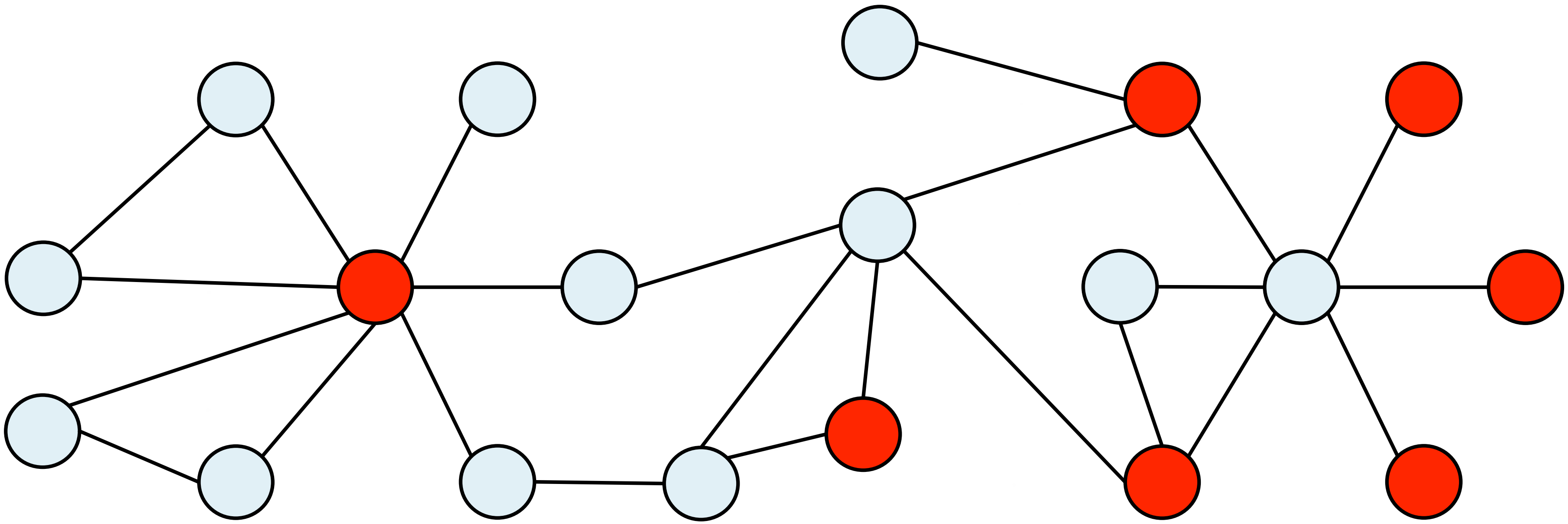


4.888...-approximation

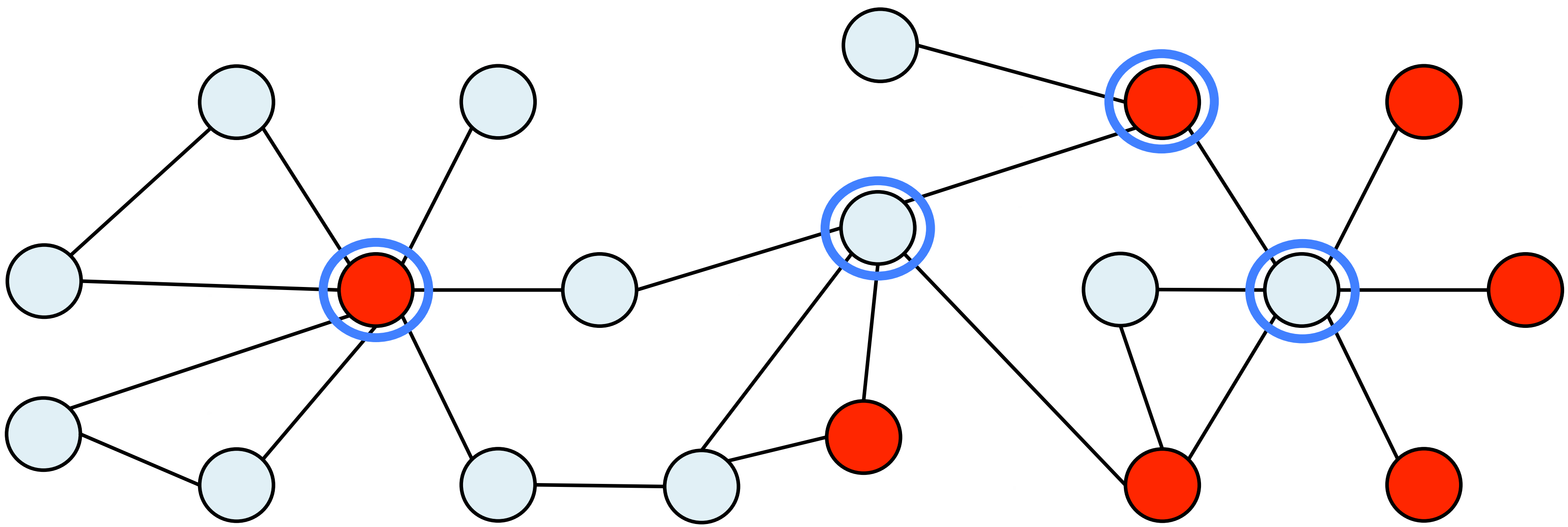
D – a maximal independent set with no reducible coronas (algorithm output)



4.888...-approximation

D – a maximal independent set with no reducible coronas (algorithm output)

*D** – a minimum dominating set (optimum solution)

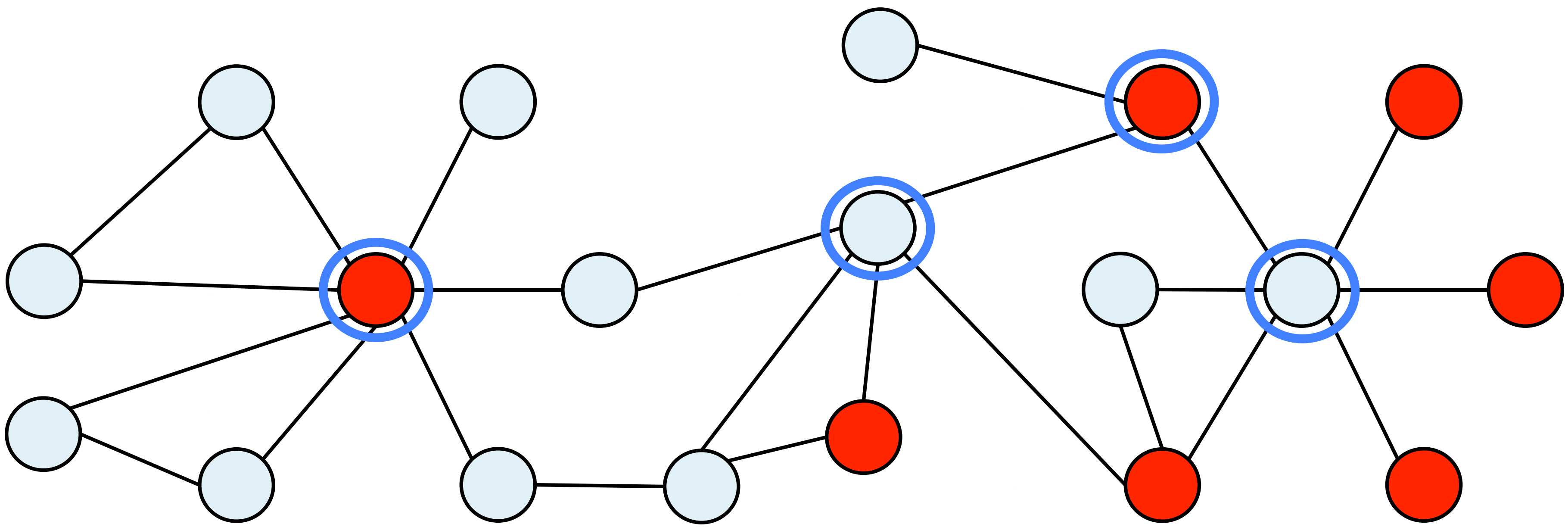


4.888...-approximation

D – a maximal independent set with no reducible coronas (algorithm output)

D^* – a minimum dominating set (optimum solution)

$$\frac{|D|}{|D^*|} \leq \ ??$$



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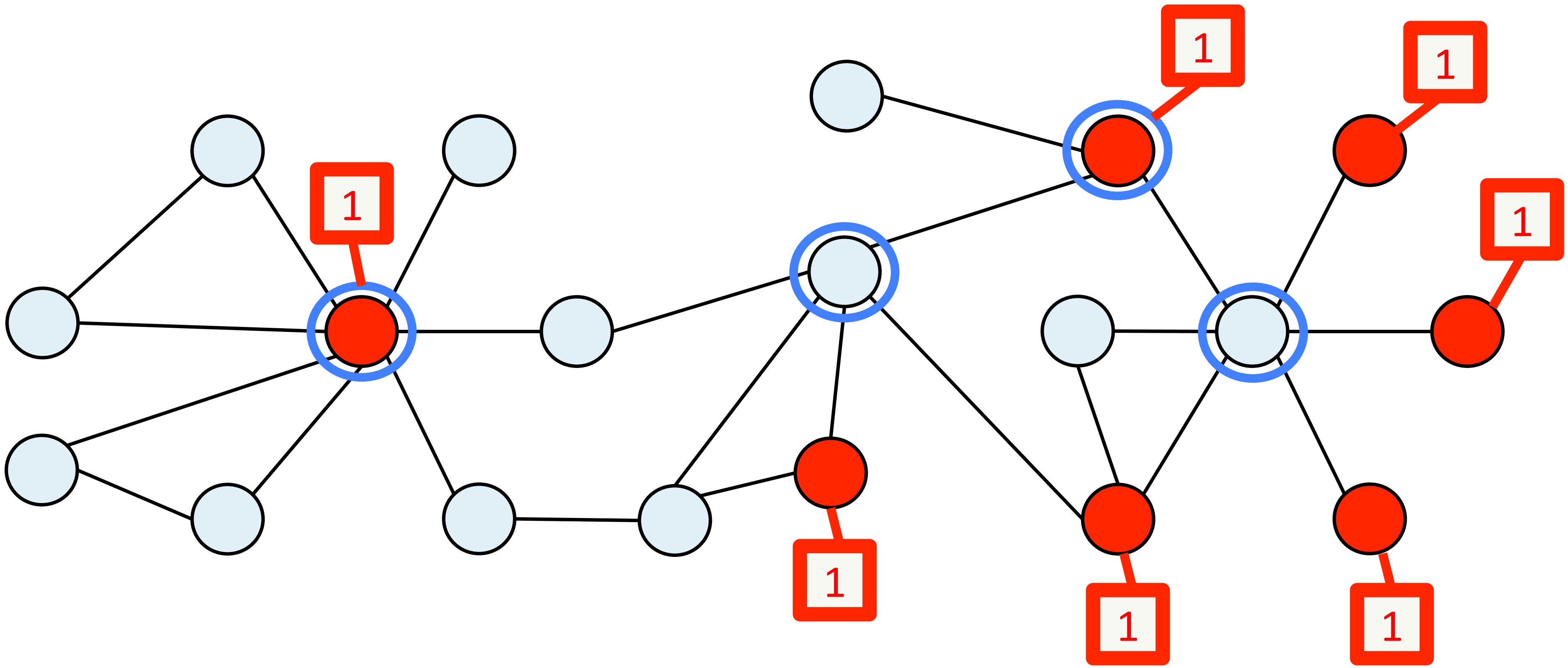

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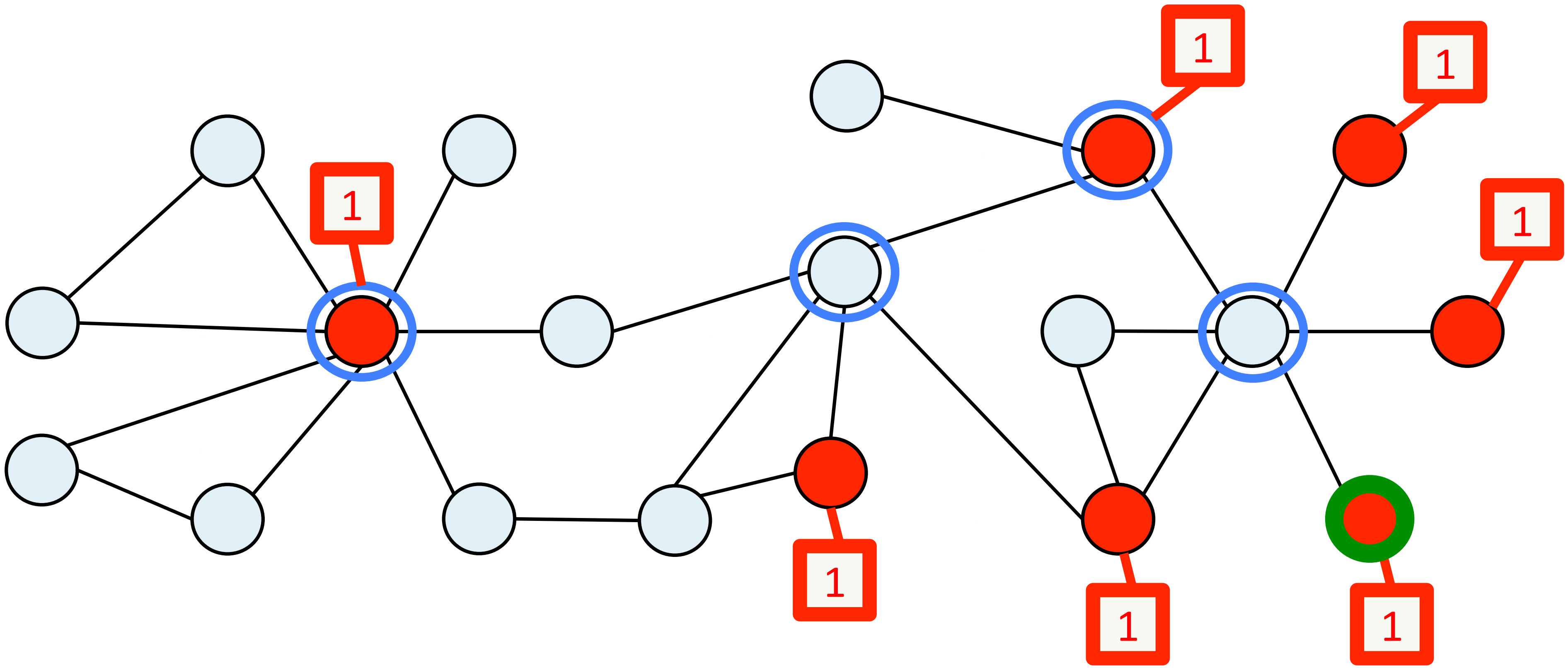

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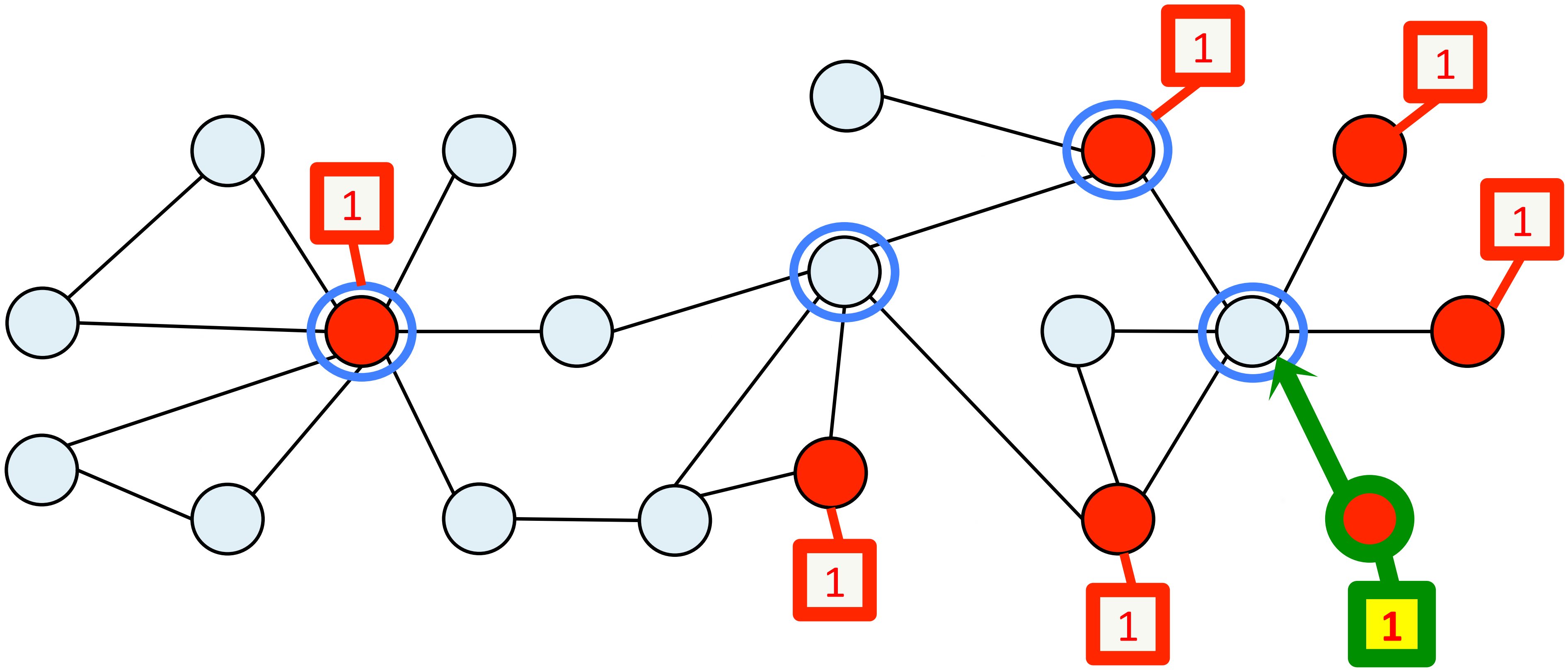

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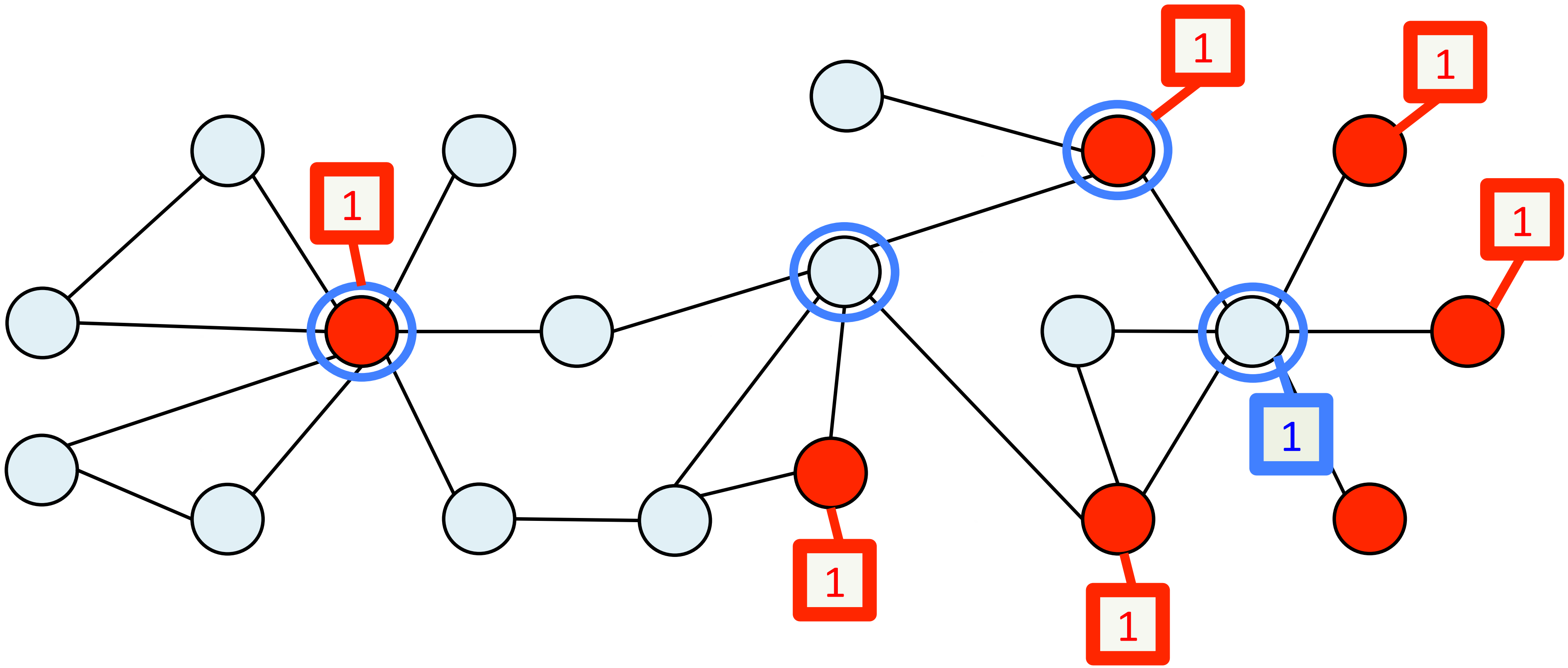

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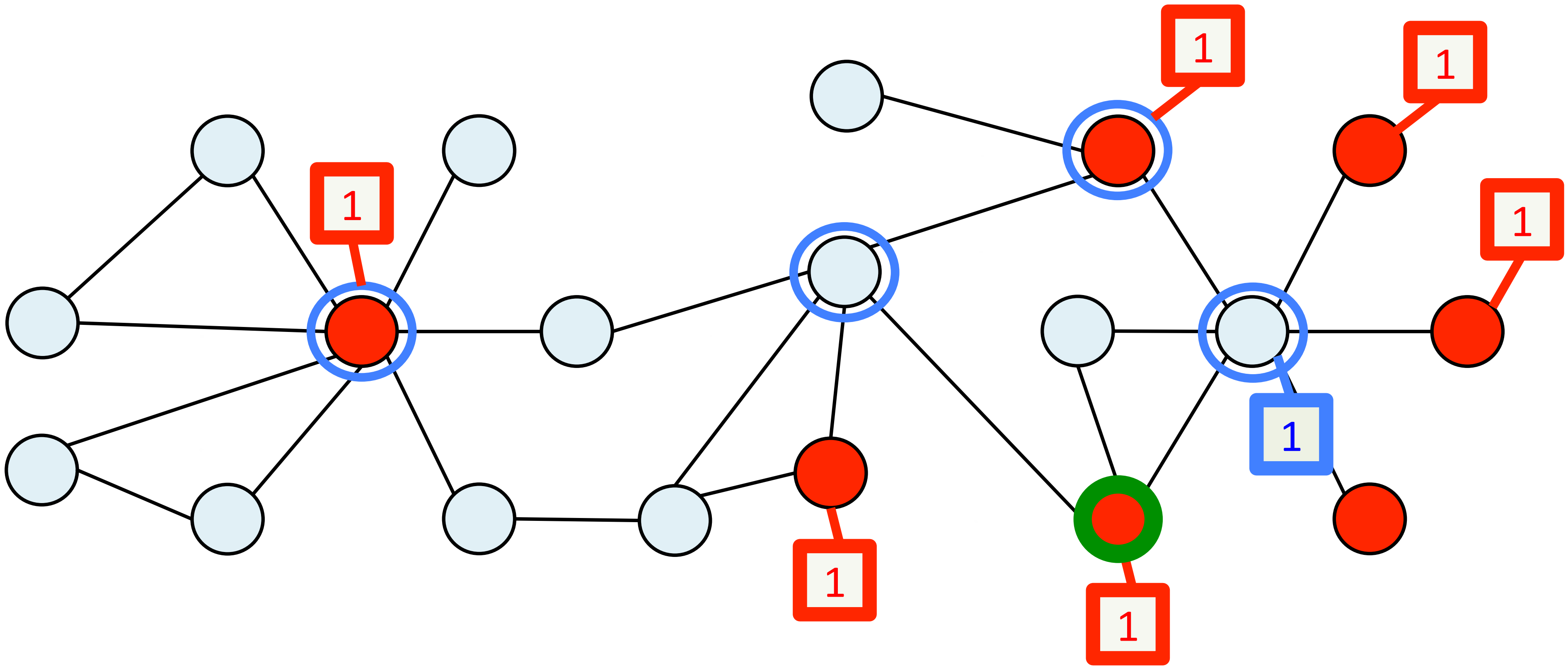

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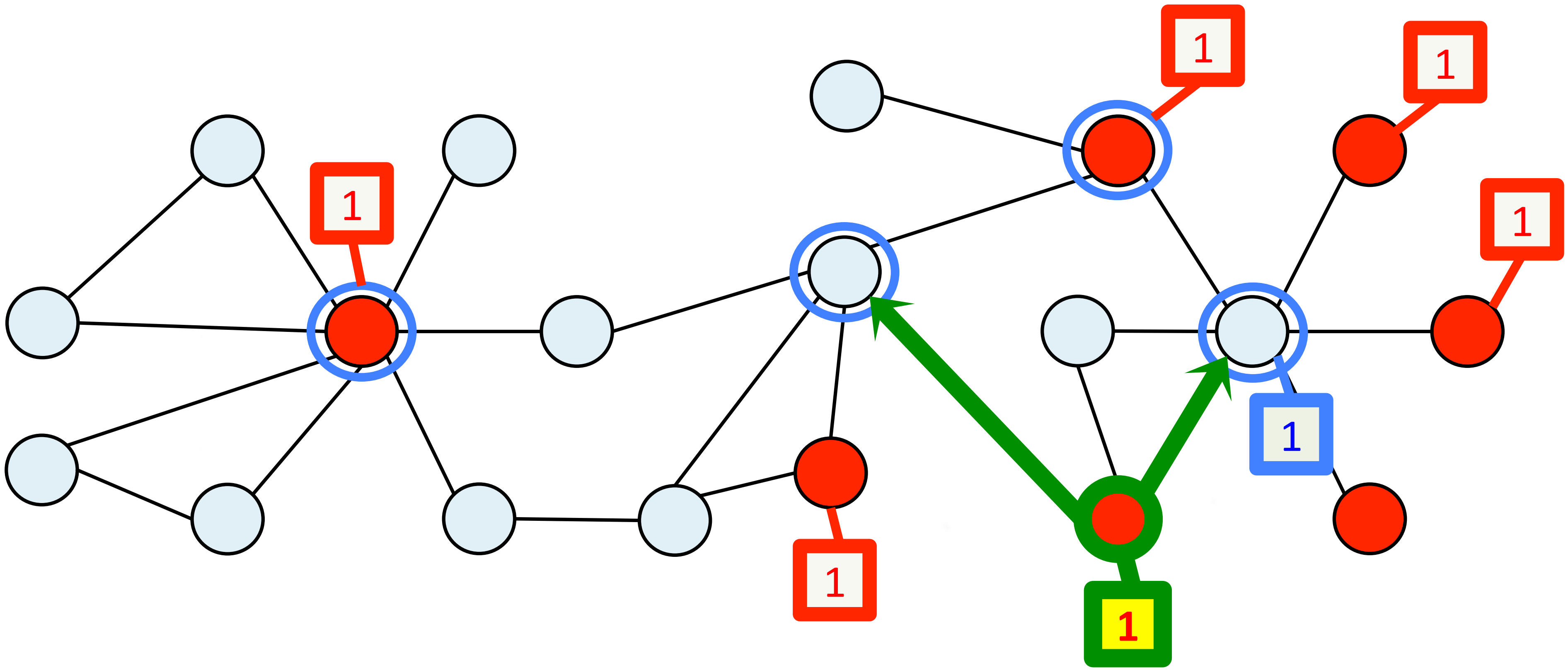

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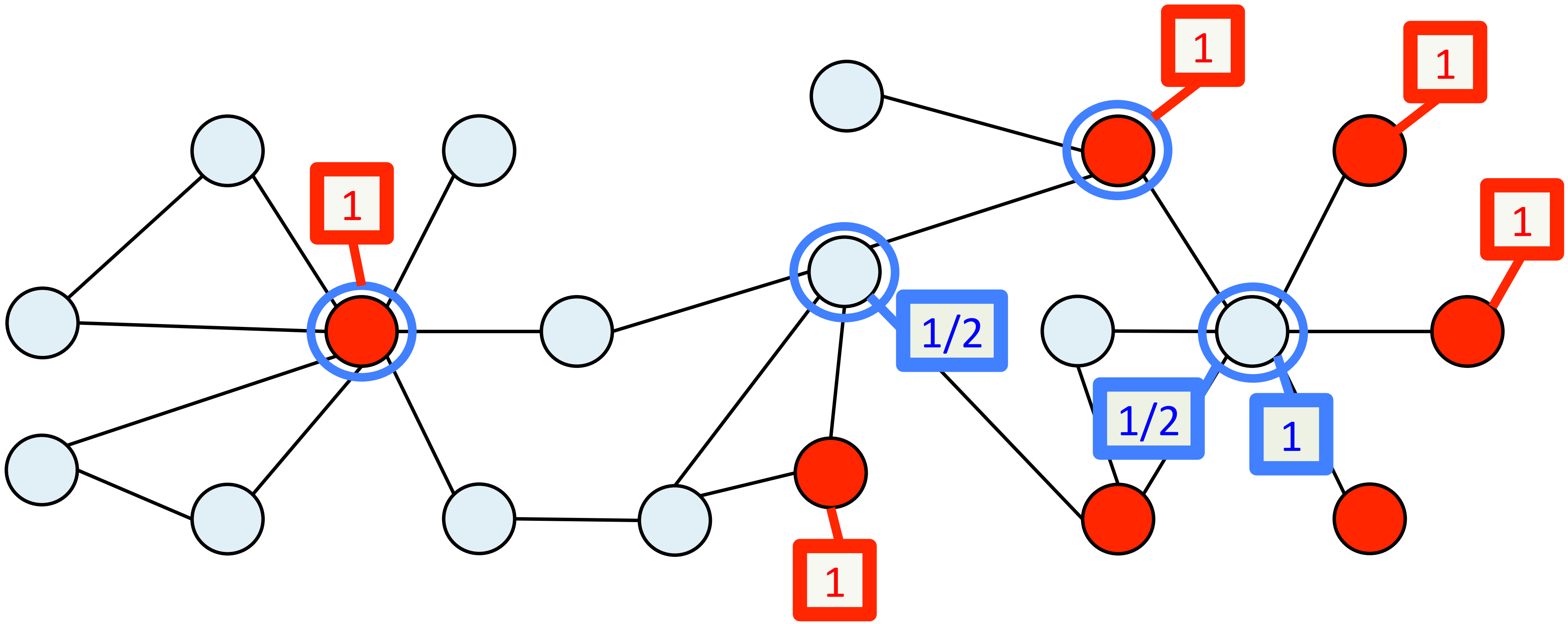

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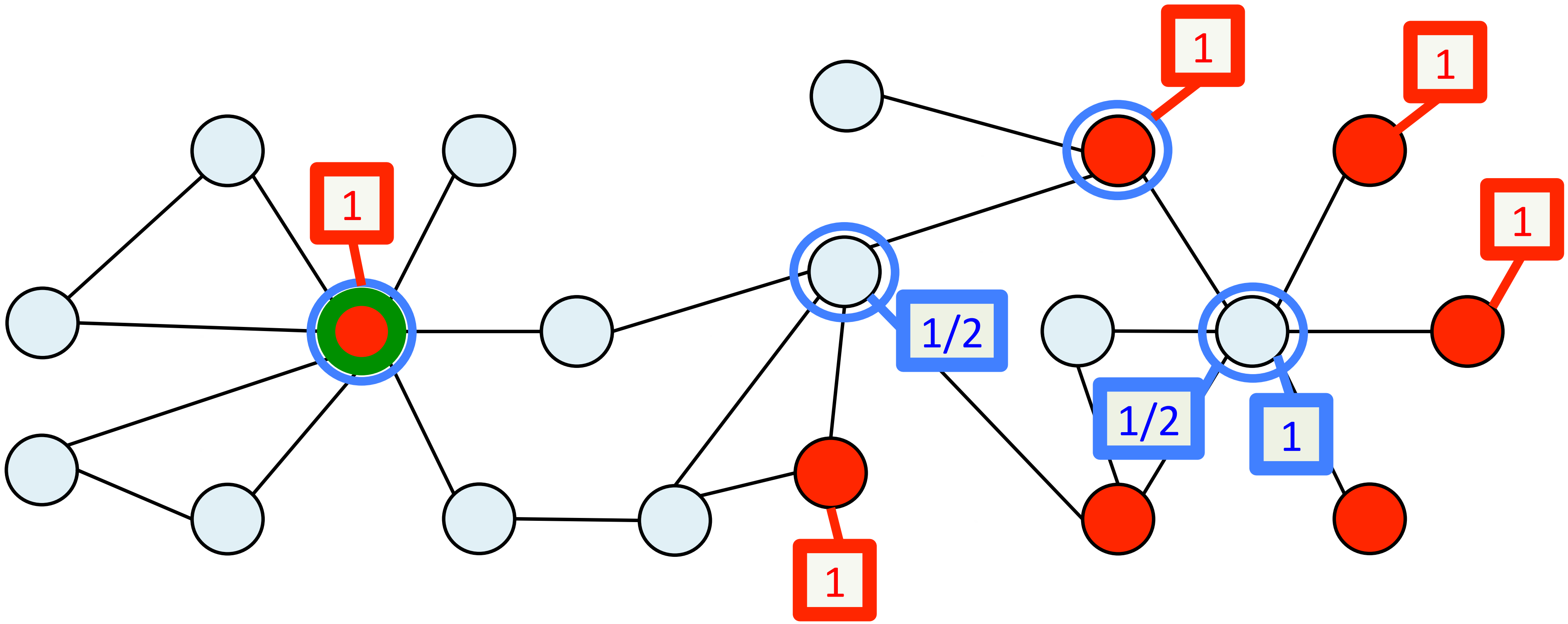

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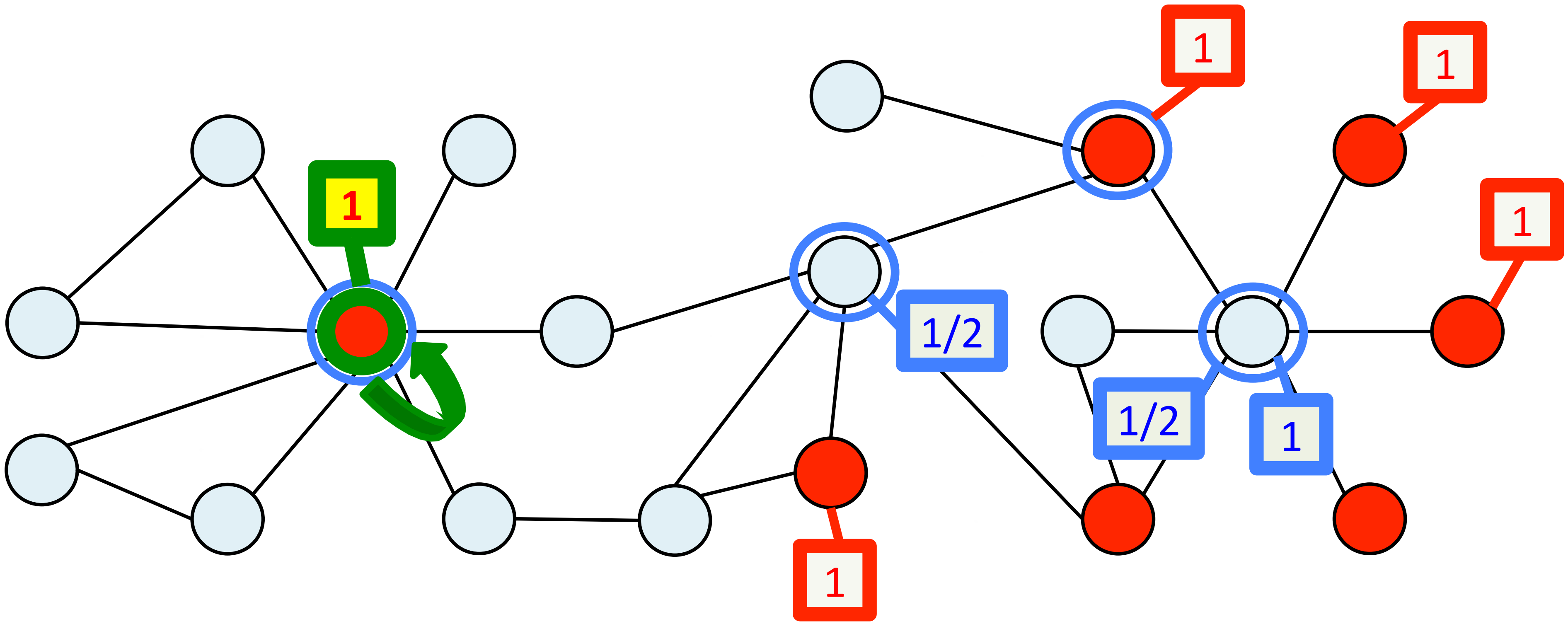

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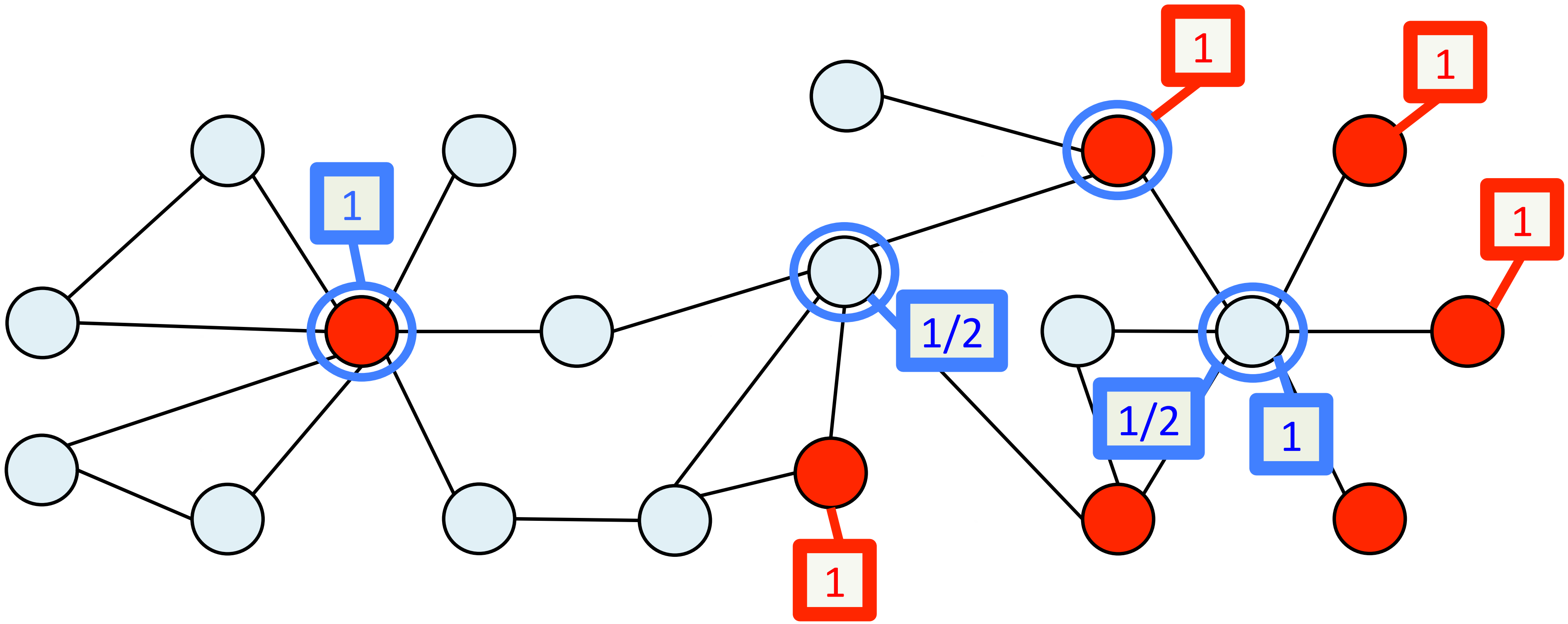

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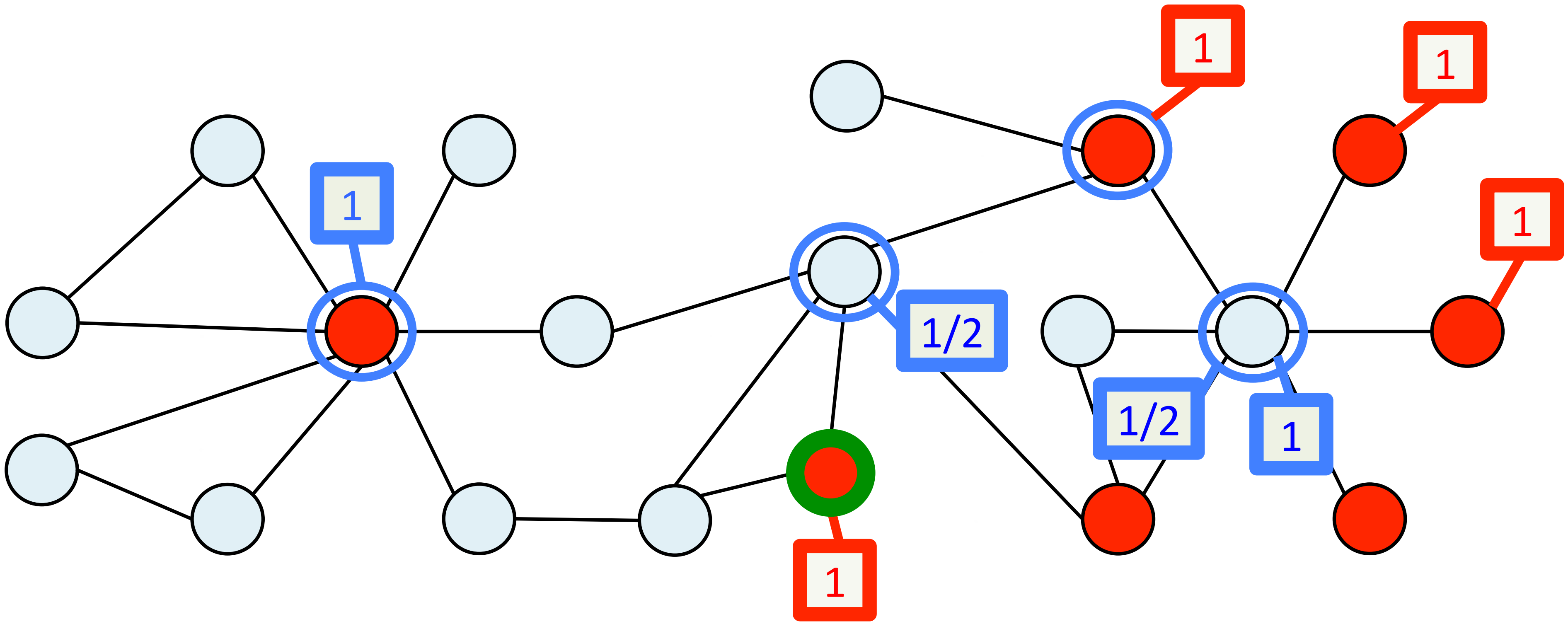

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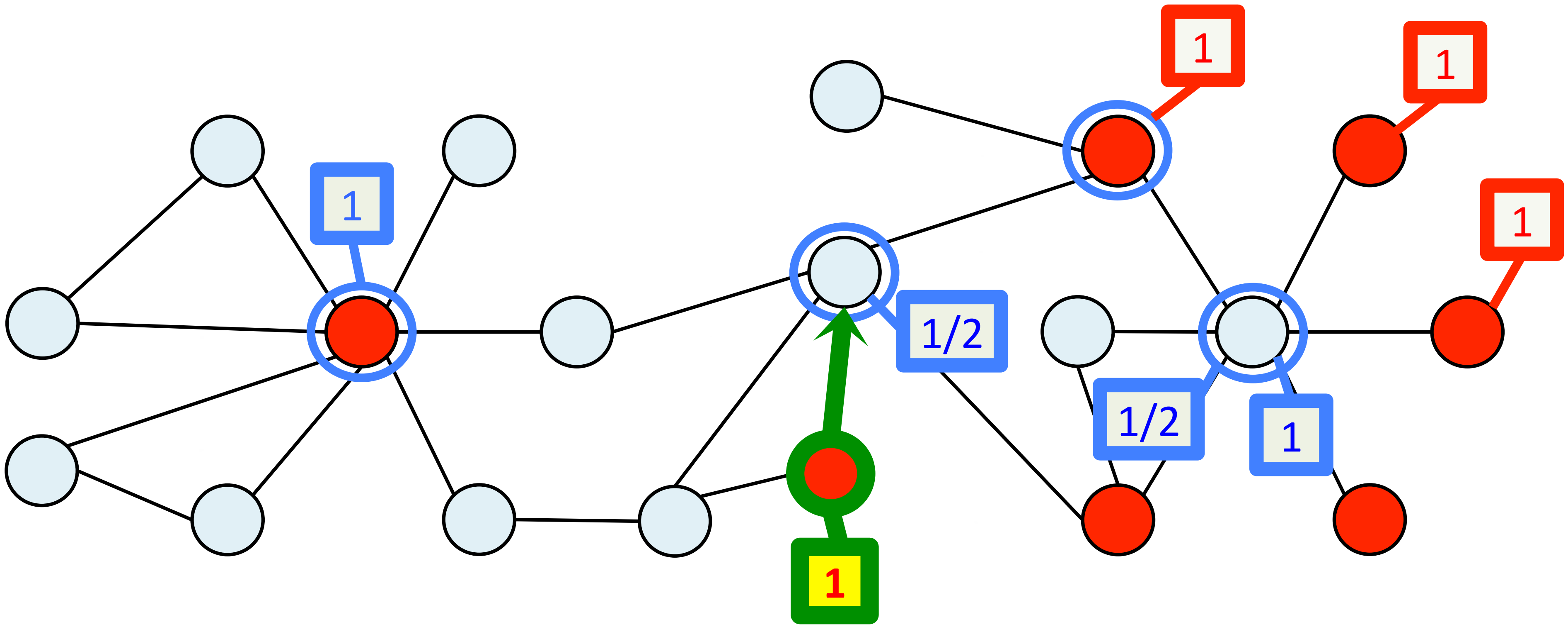

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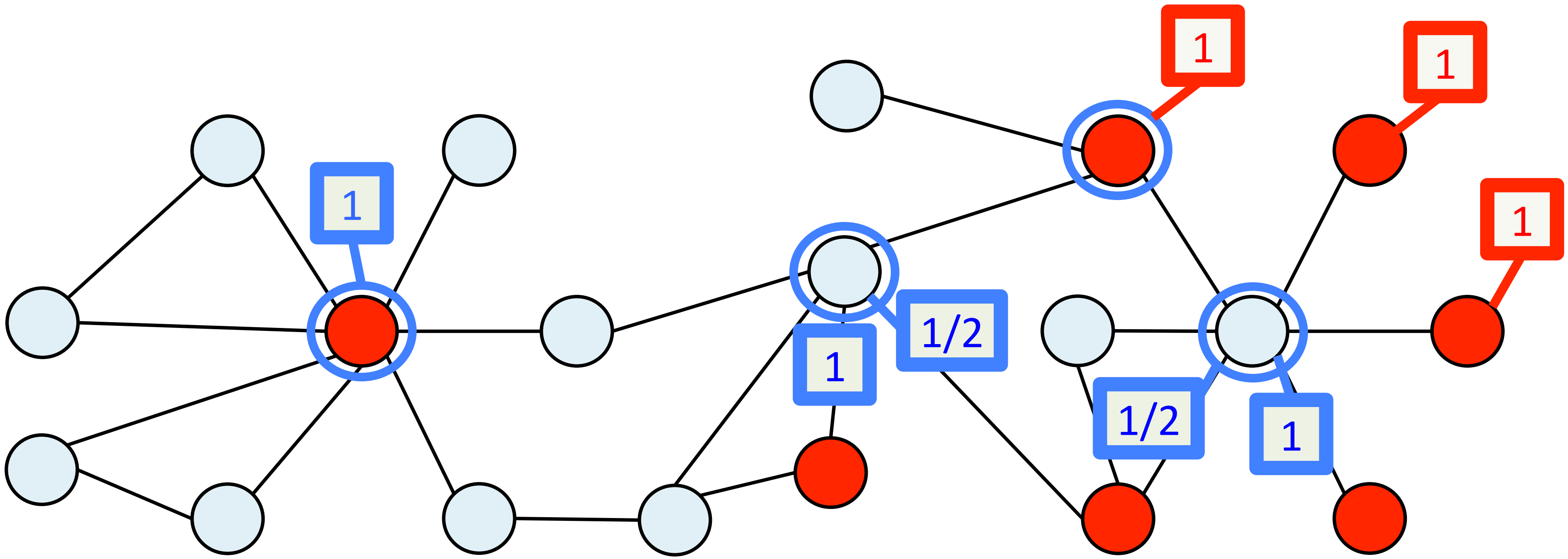

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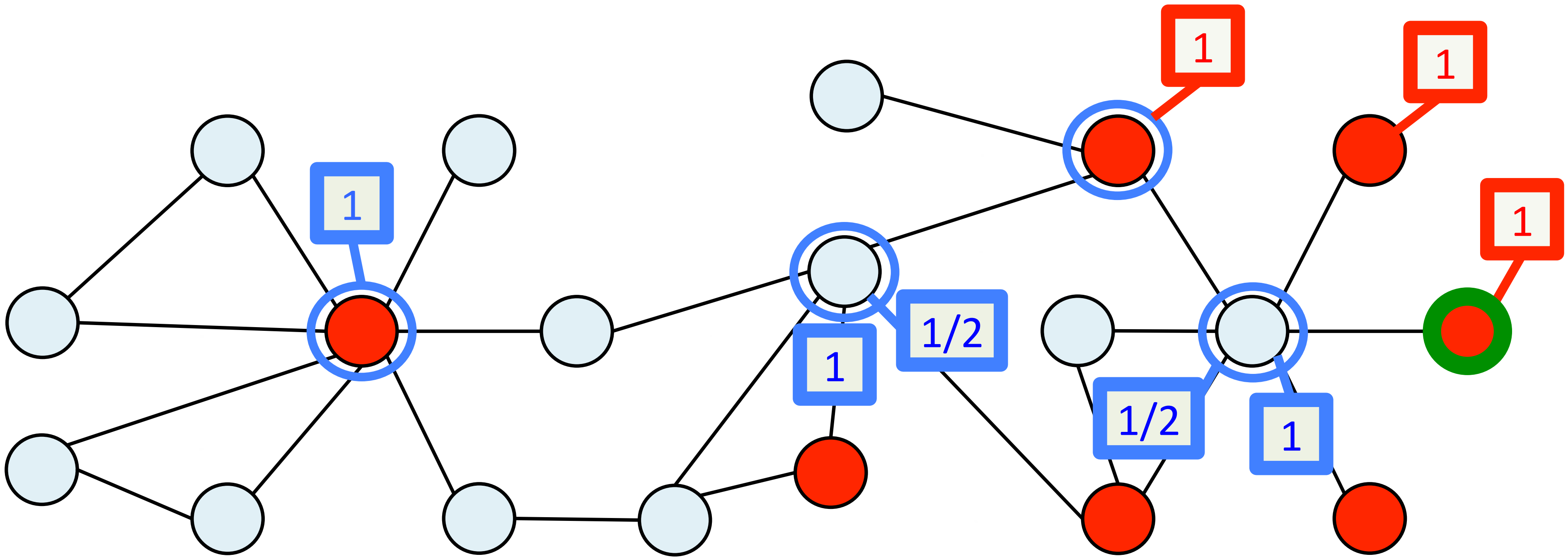

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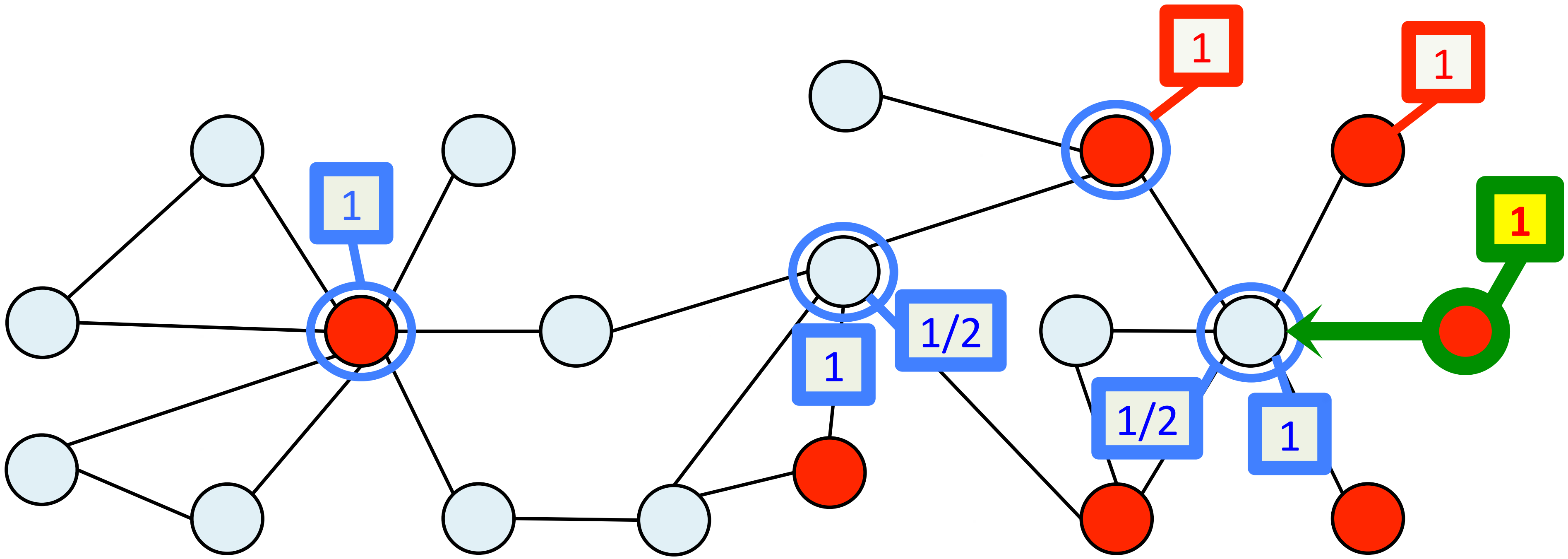

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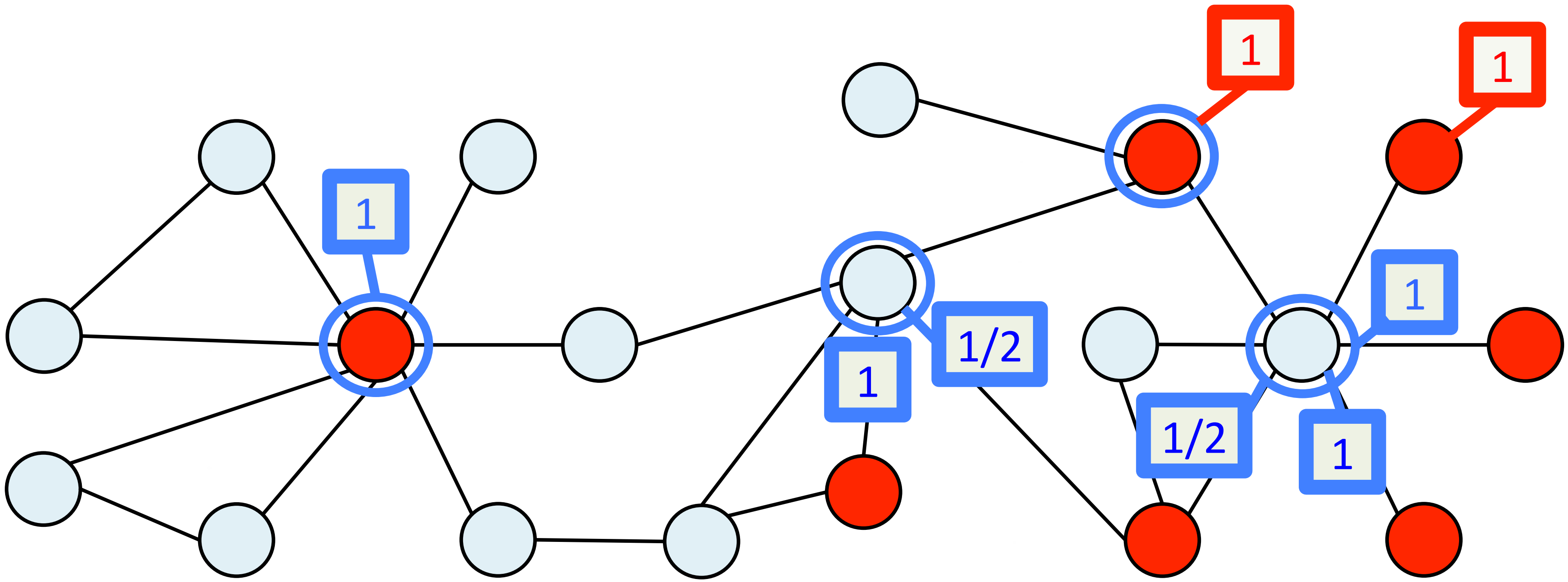

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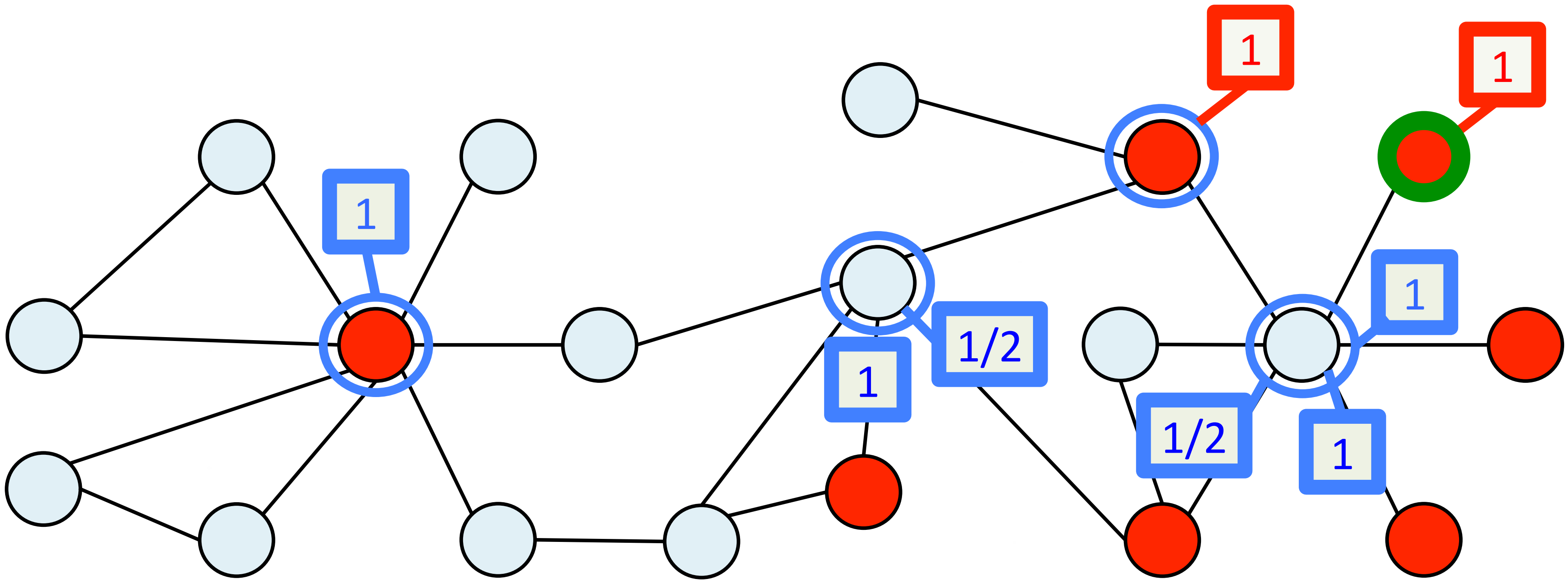

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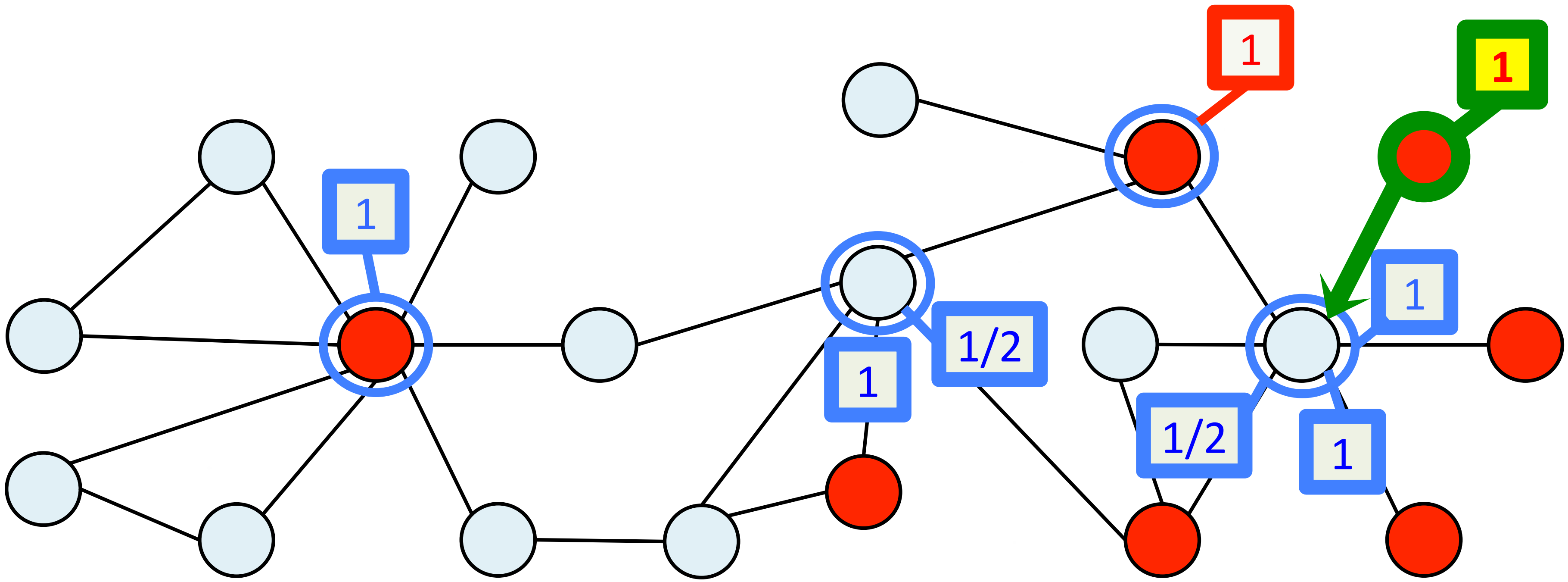

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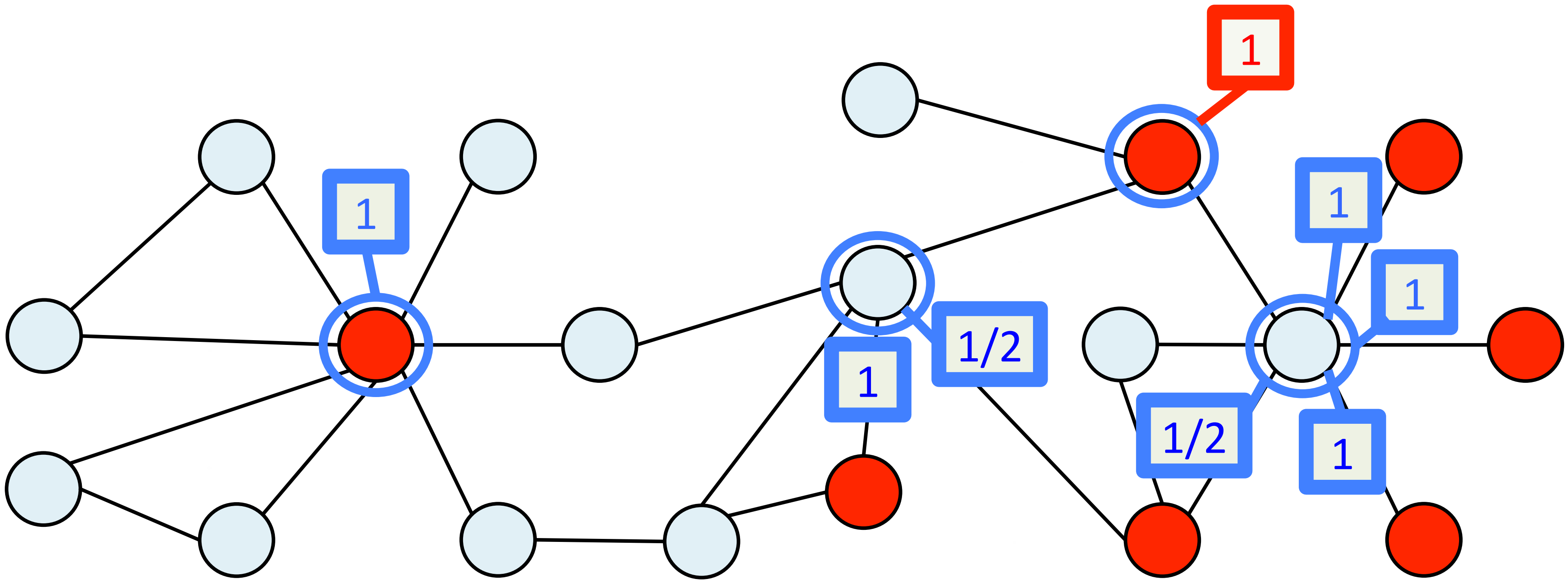

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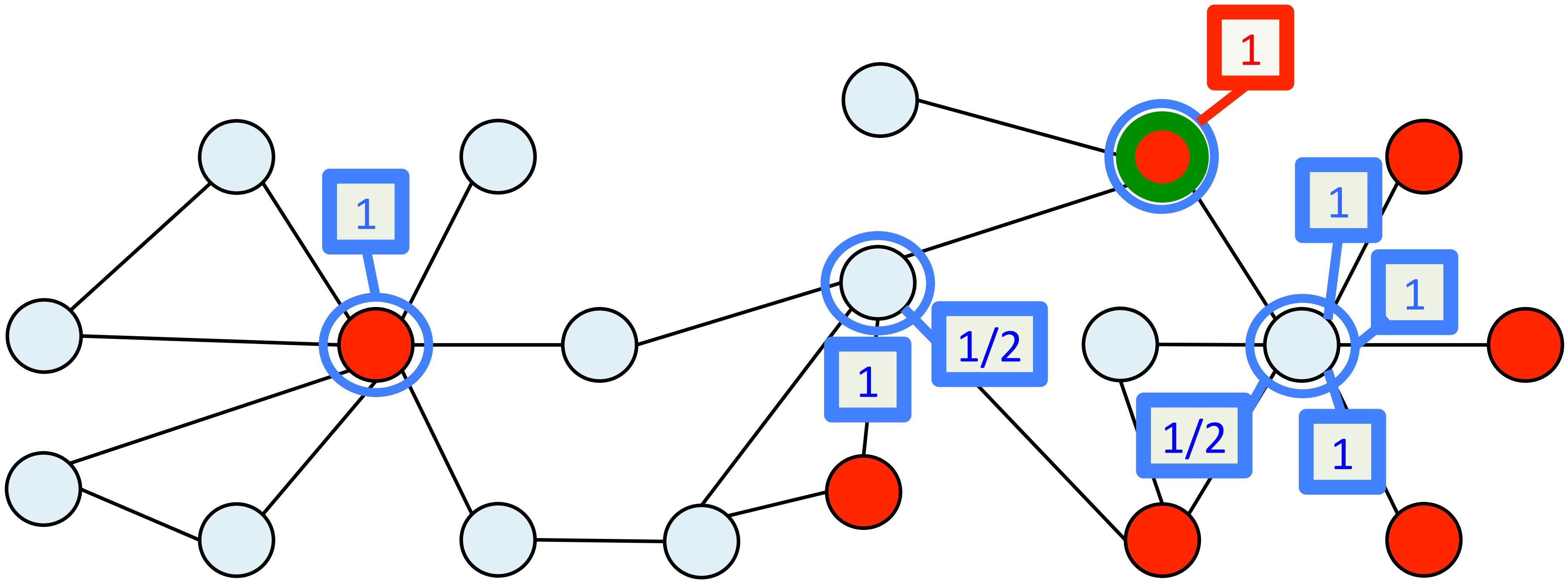

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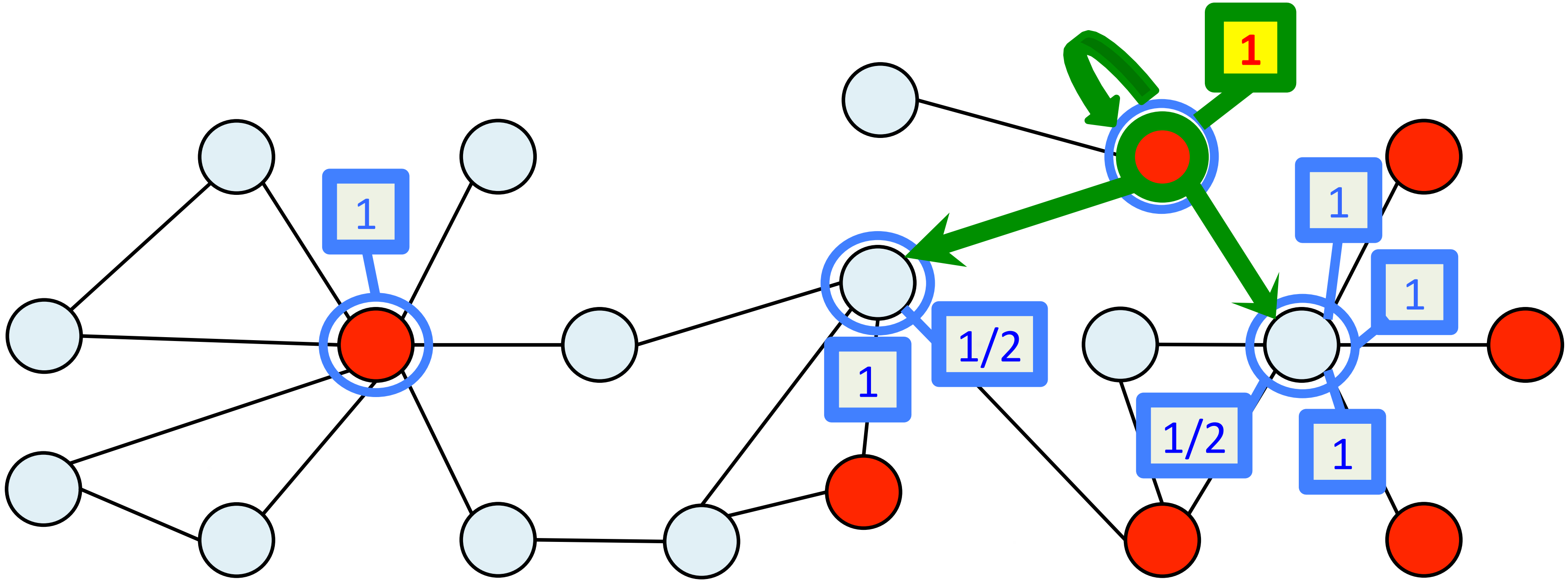

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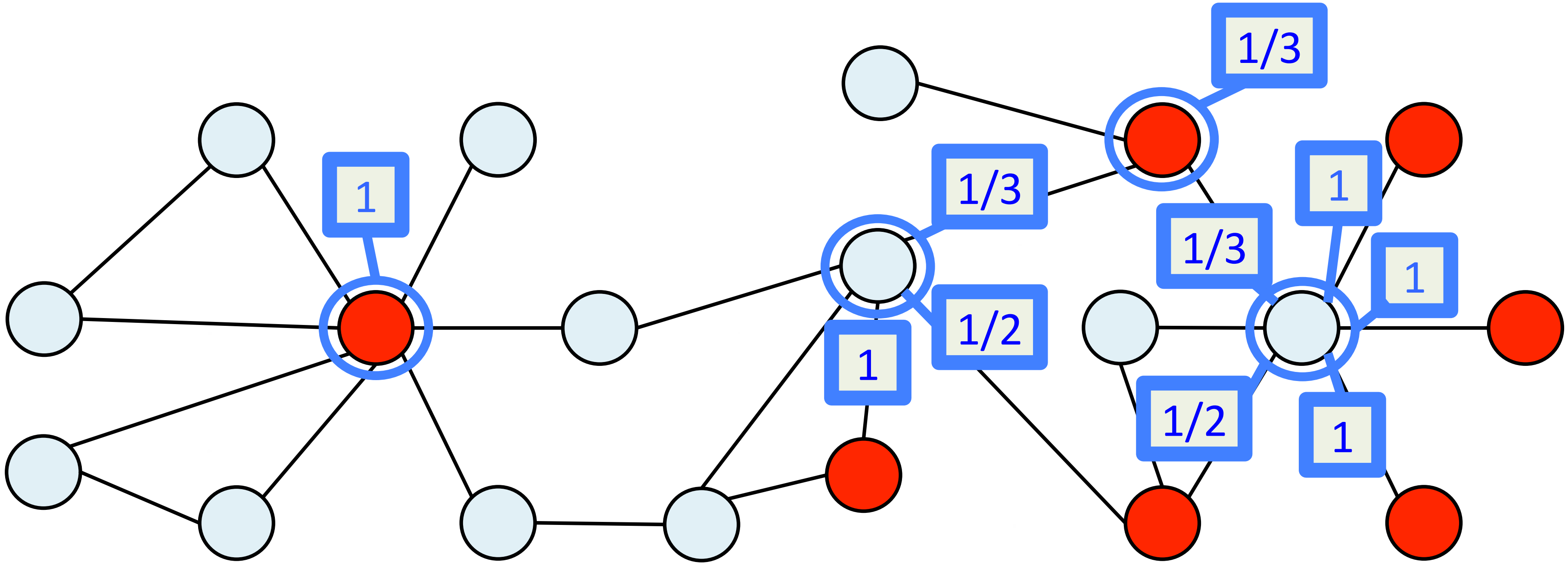

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$$f : D^* \to (0, 5]$$

$$f(v^*) = \sum_{u \in N_D[v^*]} \frac{1}{|N_{D^*}[u]|}$$

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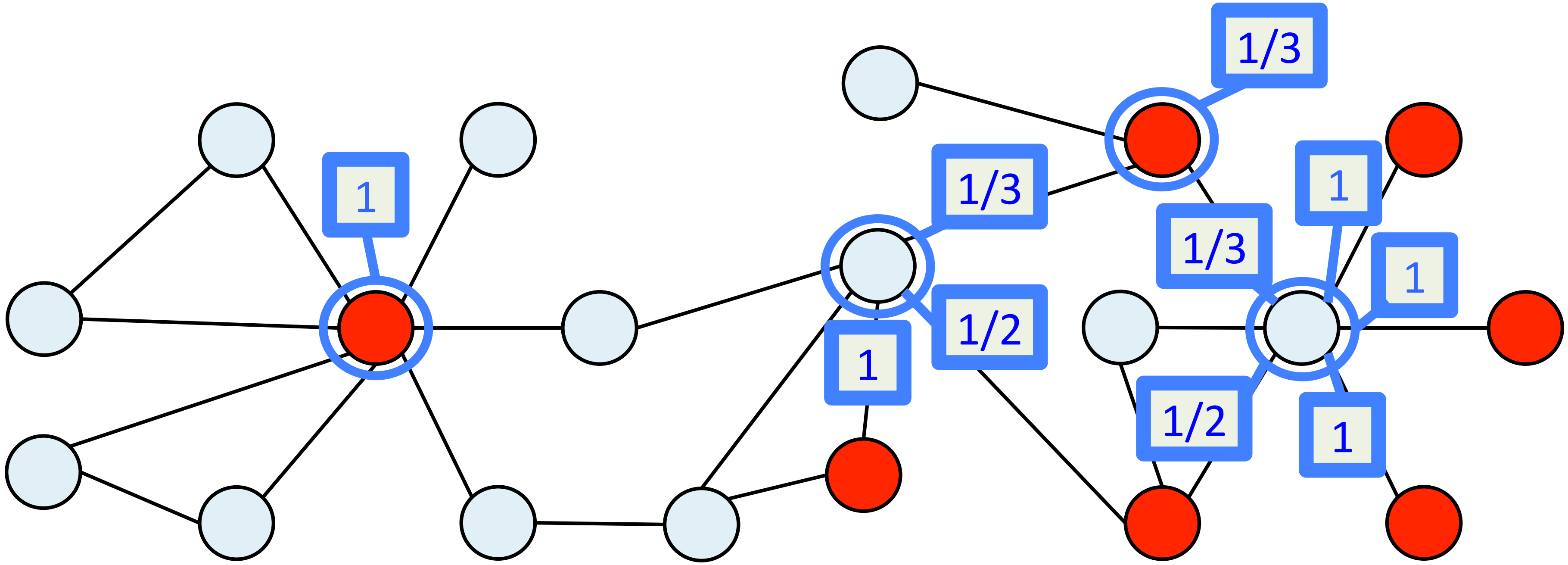

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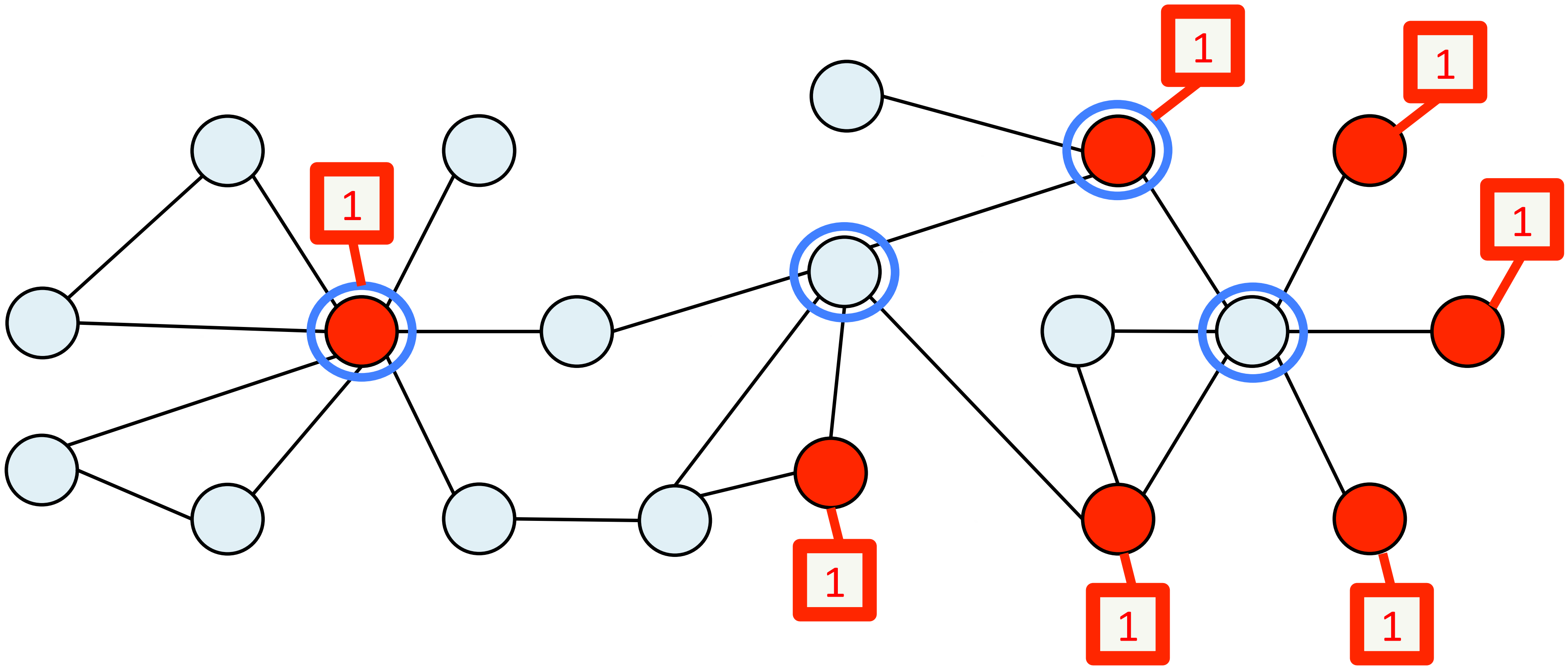

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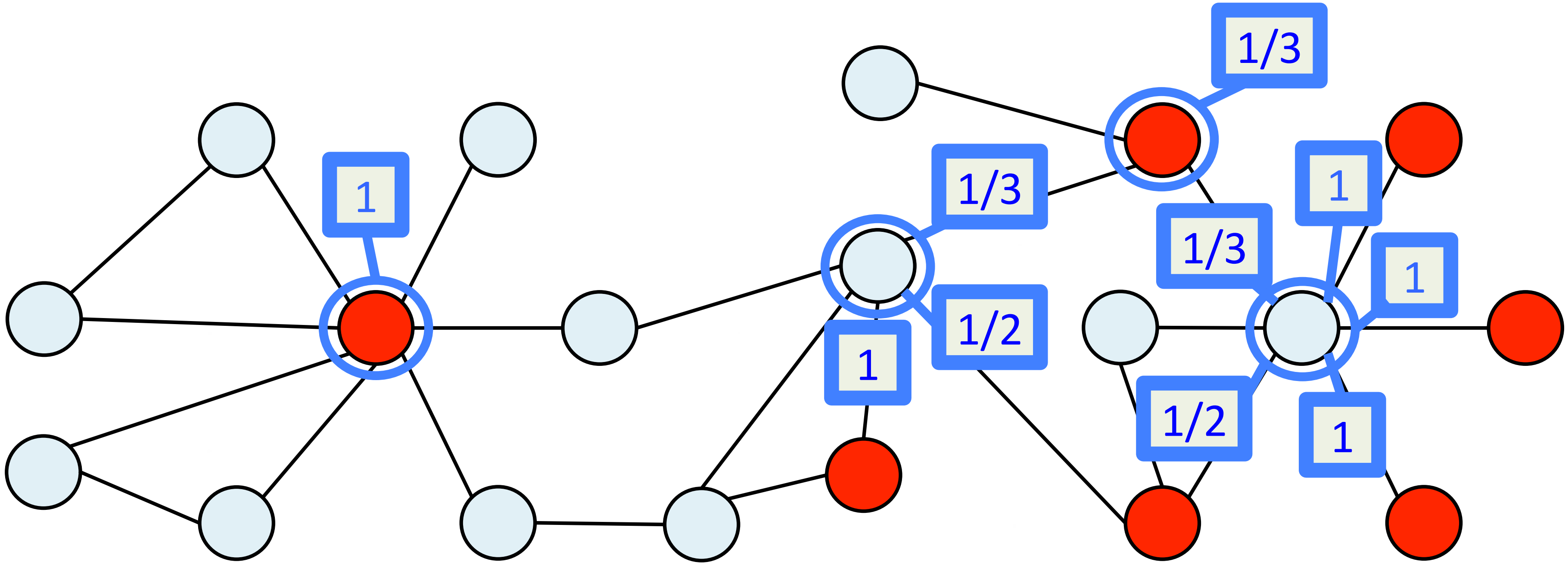

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average of *f*(.)

4.888...-approximation

D $\qquad f : D^* \longrightarrow (0, 5]$

D^* $\qquad \dfrac{|D|}{|D^*|} =$ average of f (.) over D* $\leq$ 4,888...

4.888...-approximation

D $\qquad f : D^* \longrightarrow (0, 5]$

D^* $\qquad \dfrac{|D|}{|D^*|} =$ $\boxed{\text{average of } f\,(.) \text{ over } D^* \leq 4{,}888...}$

c^*

$4{,}888... < f\,(c^*) \leq 5$

4.888...-approximation

D $\quad f : D^* \longrightarrow (0, 5]$

D^* $\quad \dfrac{|D|}{|D^*|} = \boxed{\text{average of } f(.) \text{ over } D^* \leq 4{,}888\ldots}$

$4{,}888\ldots < f(c^*) \leq 5$ $\qquad f(r^*) \leq 4$

4.888...-approximation

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$\Updownarrow$

$f(c^*) = 5$

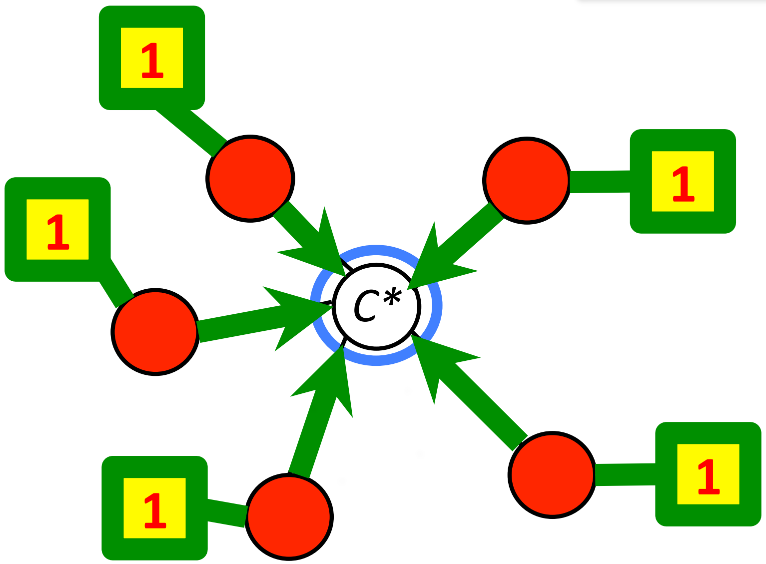
4.888...-approximation

D

$f \; : \; D^* \longrightarrow (0, 5]$

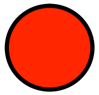
D^*

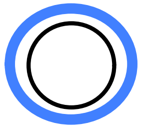
$$\frac{|D|}{|D^*|} = \boxed{\text{average of } f\,(.) \text{ over } D^* \;\leq\; 4{,}888...}$$



$f\,(c^*) = 5$

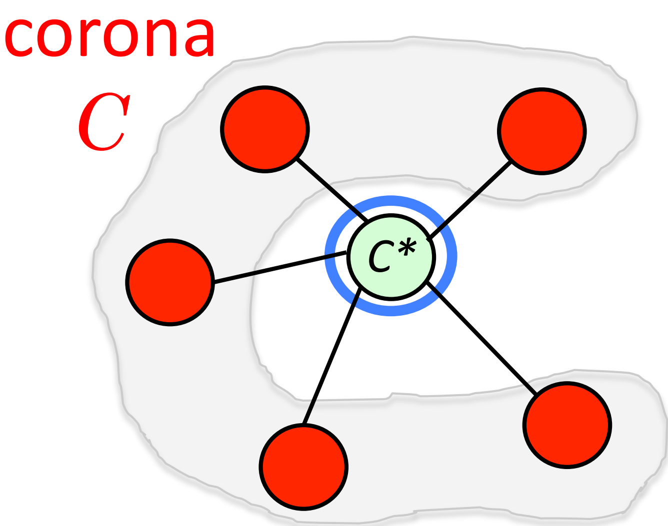
4.888...-approximation

 D

 D^*

$$f : D^* \longrightarrow (0, 5]$$

$$\frac{|D|}{|D^*|} = \boxed{\text{average of } f\,(.) \text{ over } D^* \;\le\; 4{,}888\ldots}$$



$f\,(c^*) = 5$

4.888...-approximation

D

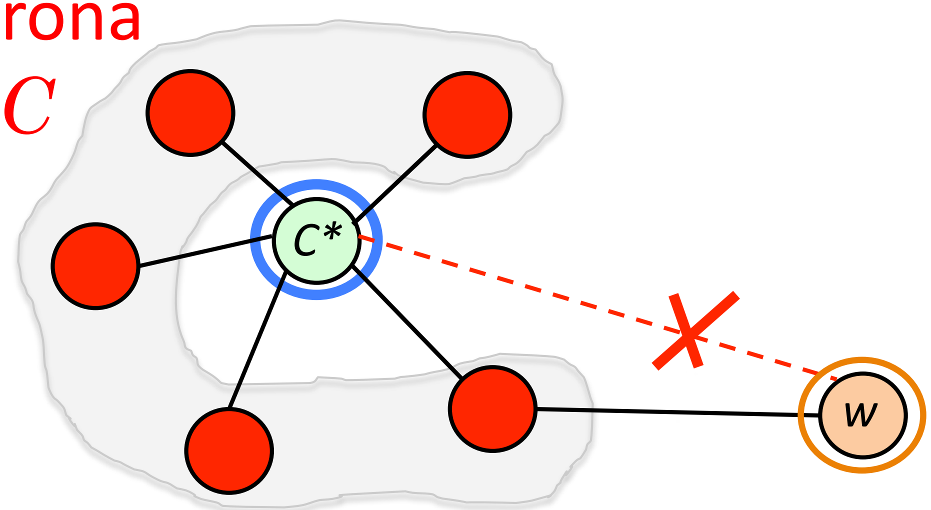
$f : D^* \longrightarrow (0, 5]$

D^*

$$\frac{|D|}{|D^*|} = \boxed{\text{average of } f(.) \text{ over } D^* \leq 4{,}888...}$$

corona

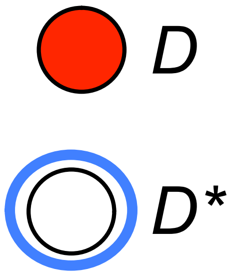
C



$f(c^*) = 5$

$N_D[w] \subseteq C$

4.888...-approximation



$f : D^* \longrightarrow (0, 5]$

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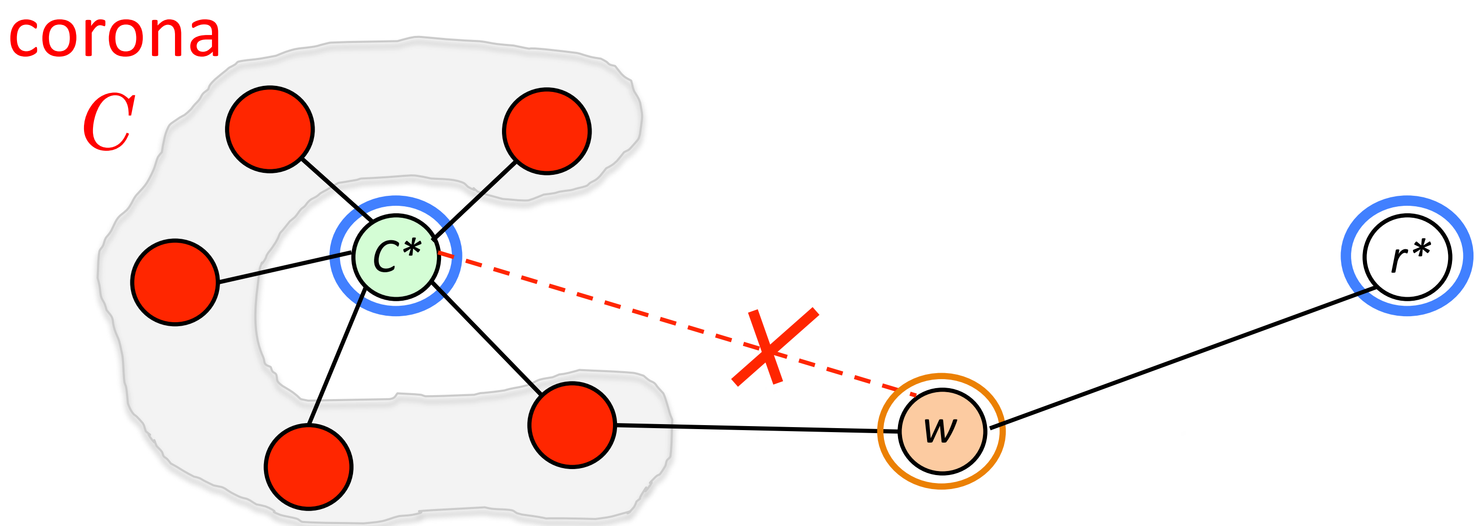

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4.888...-approximation

D

D^*

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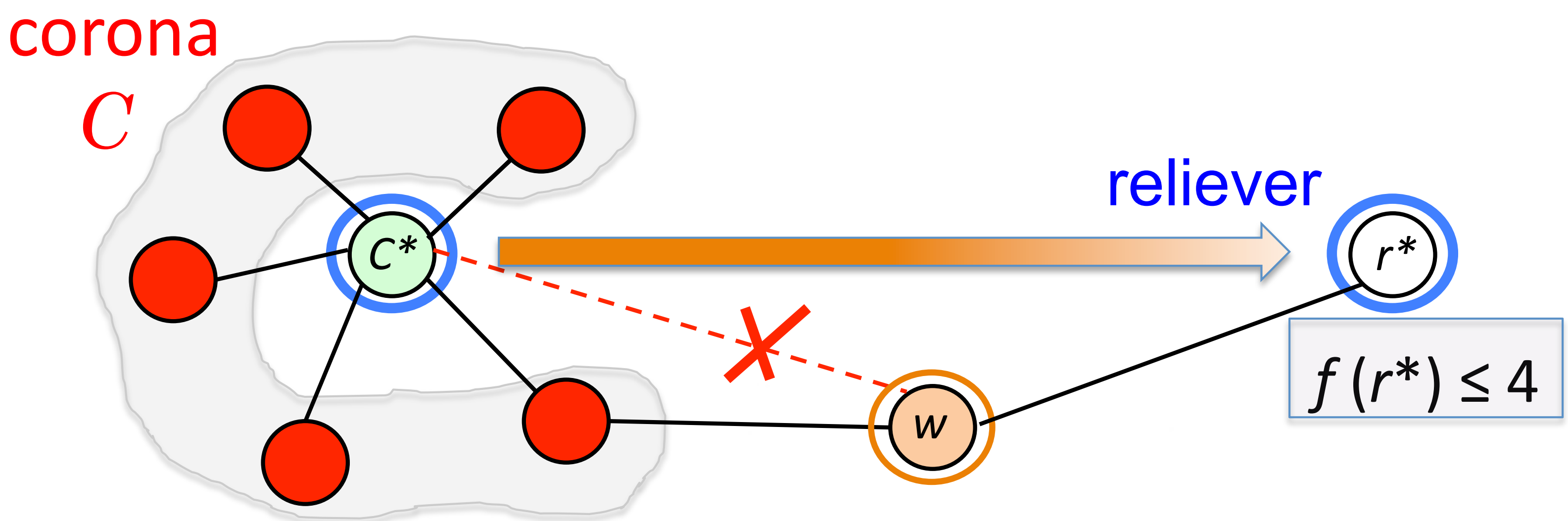

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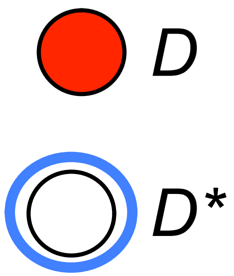

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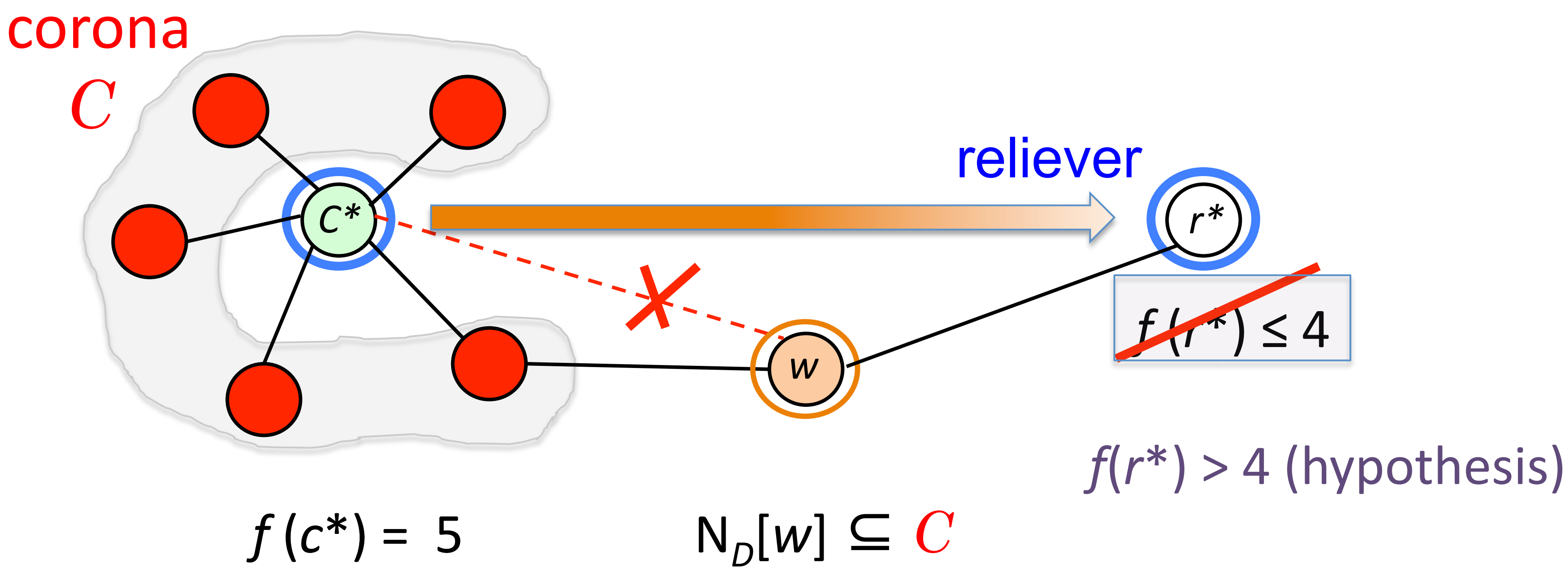

4.888...-approximation

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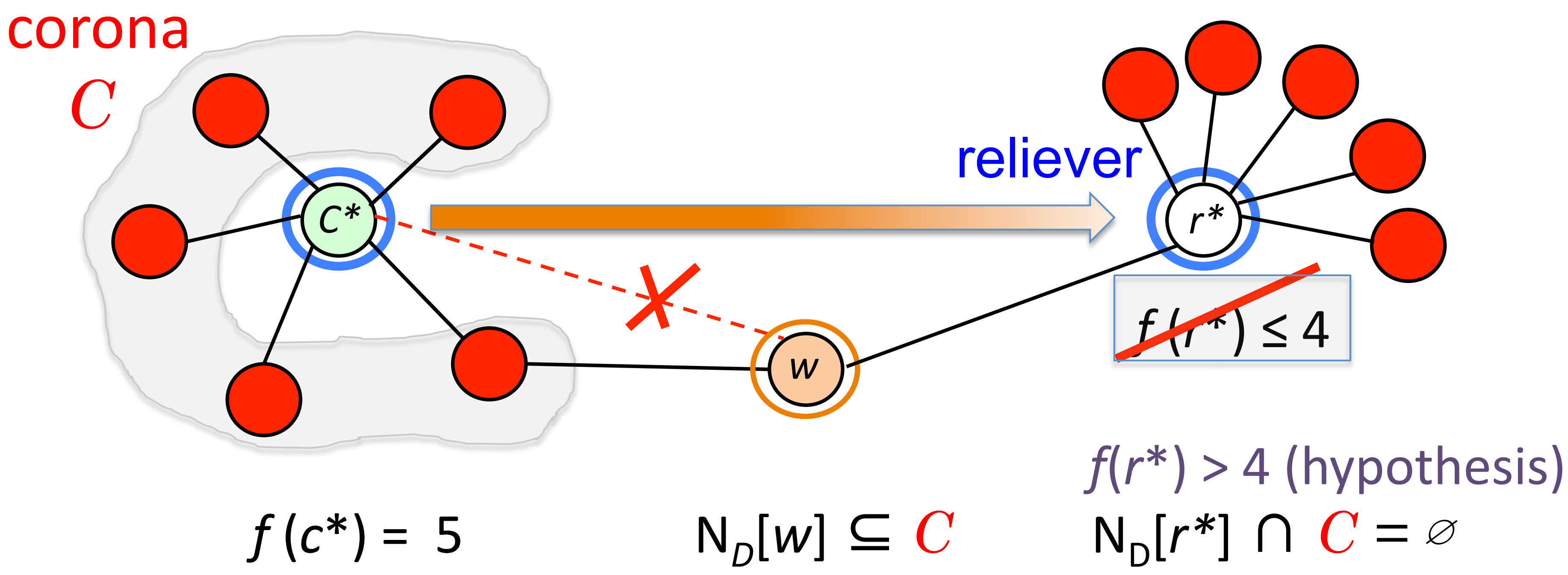

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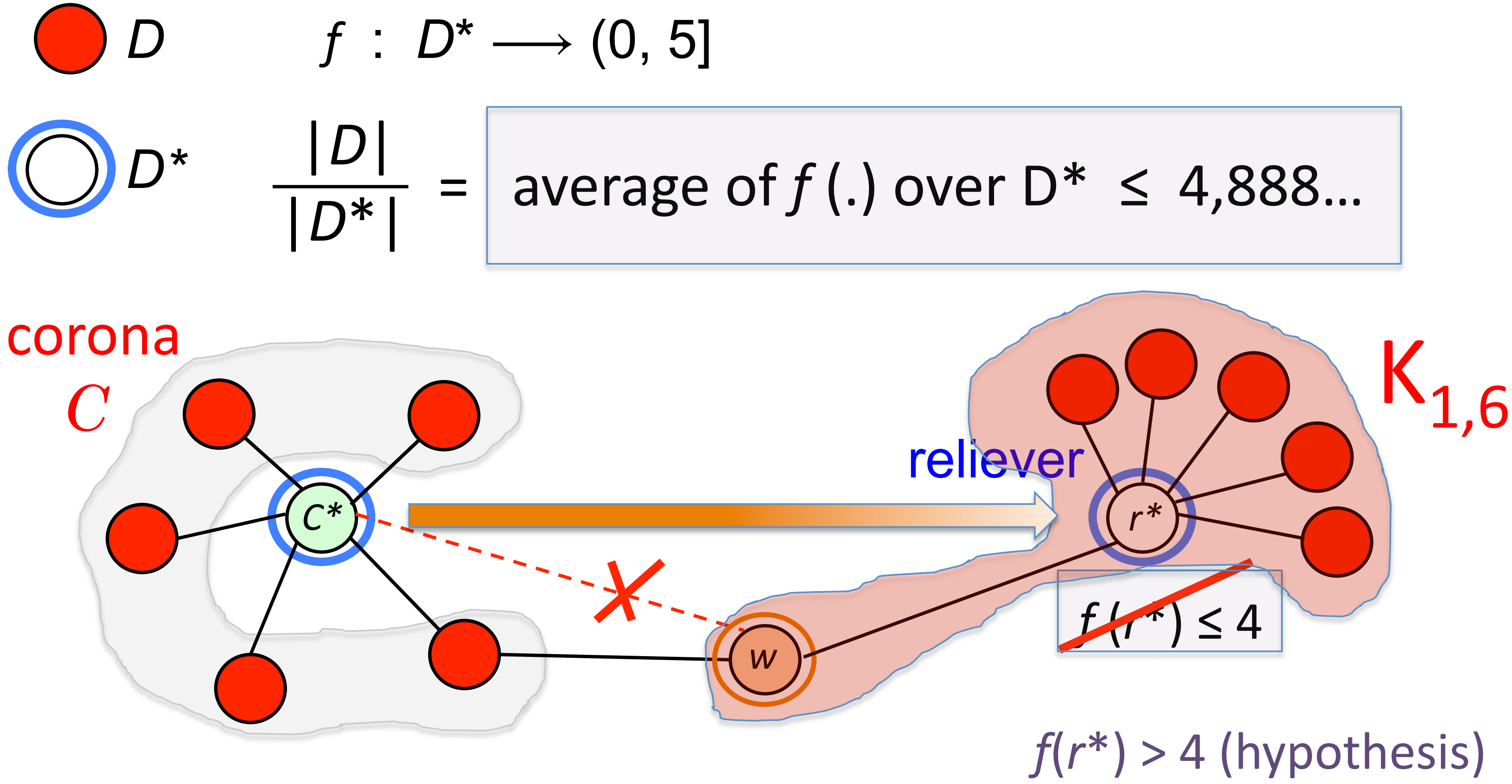

$f\,(c^*) = 5$ $\quad N_D[w] \subseteq C$ $\quad N_D[r^*] \cap C = \varnothing$

4.888...-approximation

D

D^*

$$f : D^* \longrightarrow (0, 5]$$

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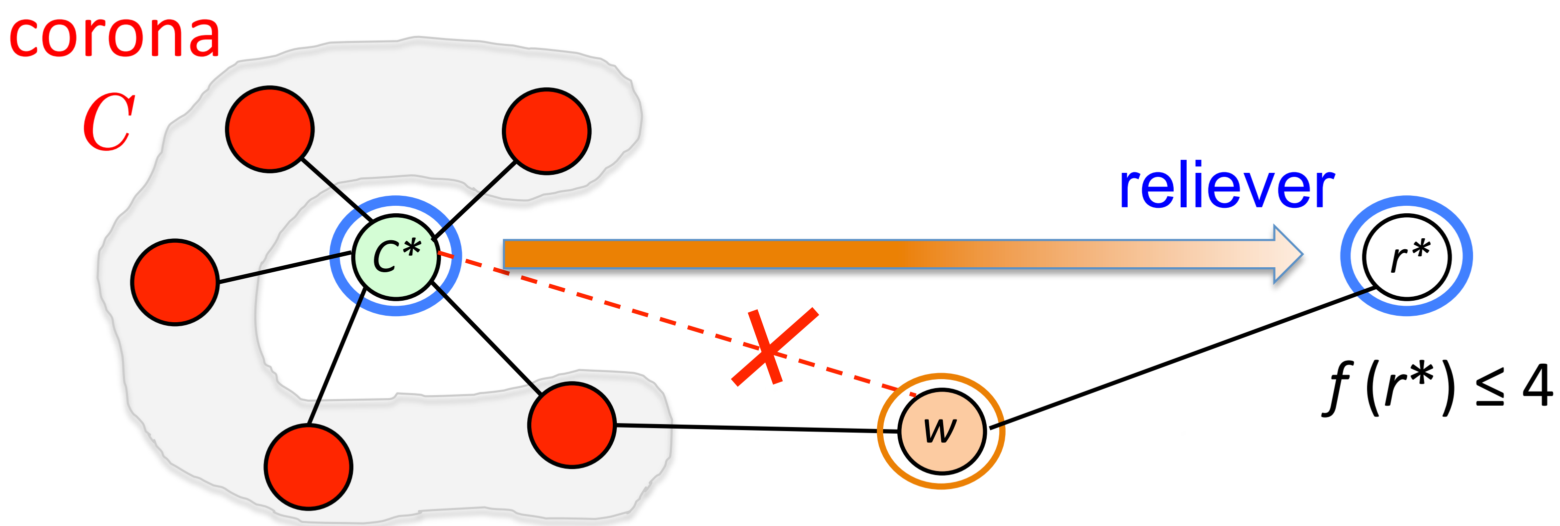


$f(c^*) = 5$ $\qquad$ $N_D[w] \subseteq C$

4.888...-approximation

D

$f : D^* \longrightarrow (0, 5]$

D^*

$$\frac{|D|}{|D^*|} = \boxed{\text{average of } f(.) \text{ over } D^* \leq 4{,}888\ldots}$$

$f(c^*) = 5$

$f(r^*) \leq 4$

4.888...-approximation

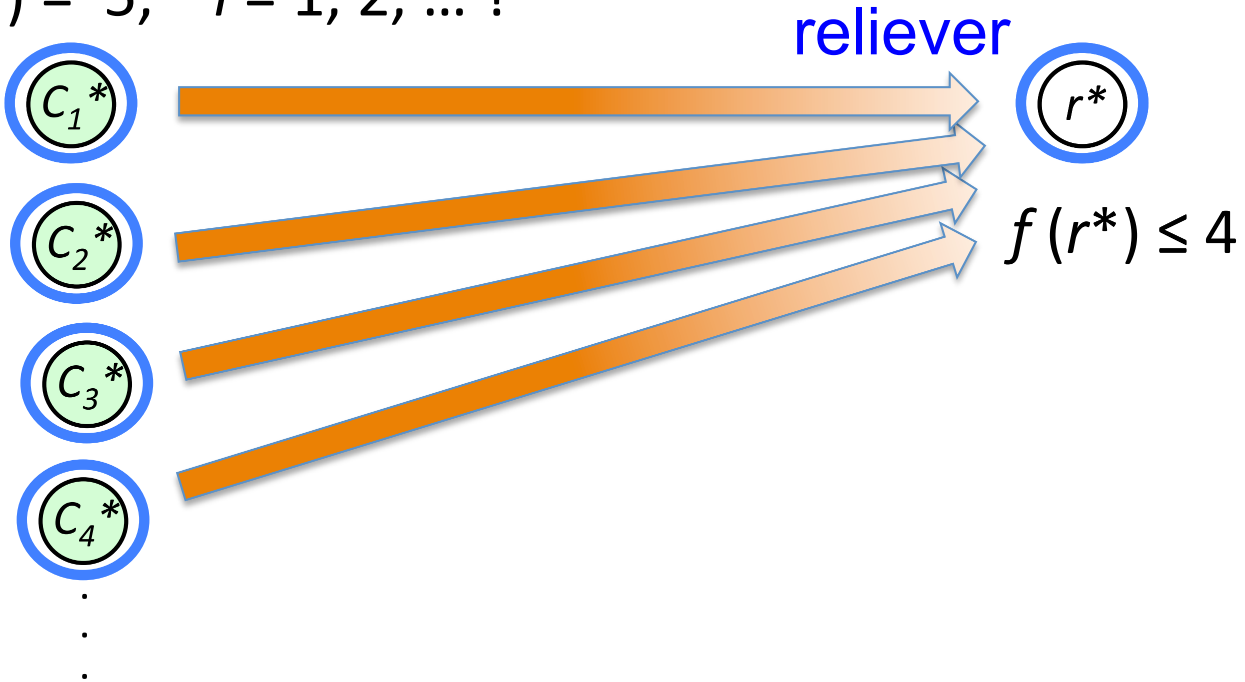
D

$f : D^* \longrightarrow (0, 5]$

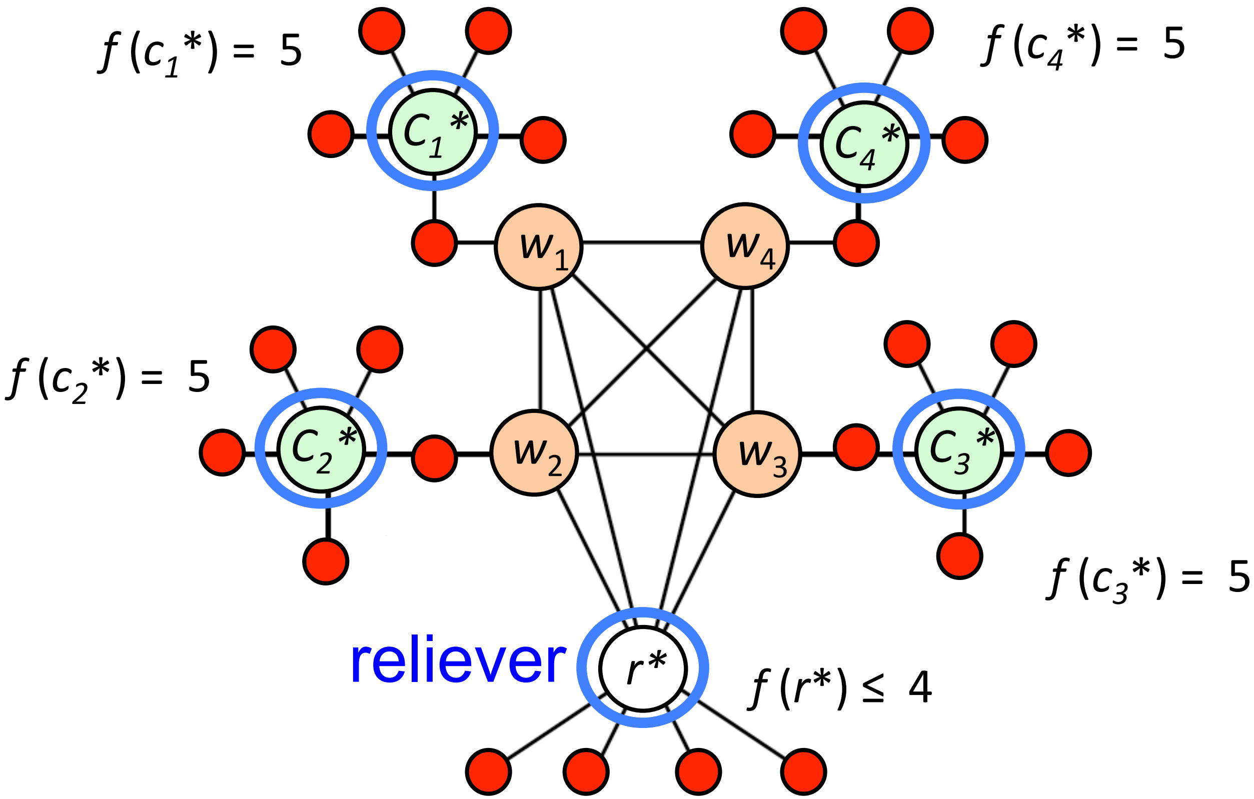
D^*

$$\frac{|D|}{|D^*|} = \boxed{\text{average of } f(.) \text{ over } D^* \le 4{,}888\ldots}$$

$f(c_i^*) = 5, \quad i = 1, 2, \ldots$?



4.888...-approximation



Some geometric lemmas

Lemma 1 (Pál 1921): If a set of points P has diameter 1, then P can be enclosed by a circle of radius $1/\sqrt{3}$.

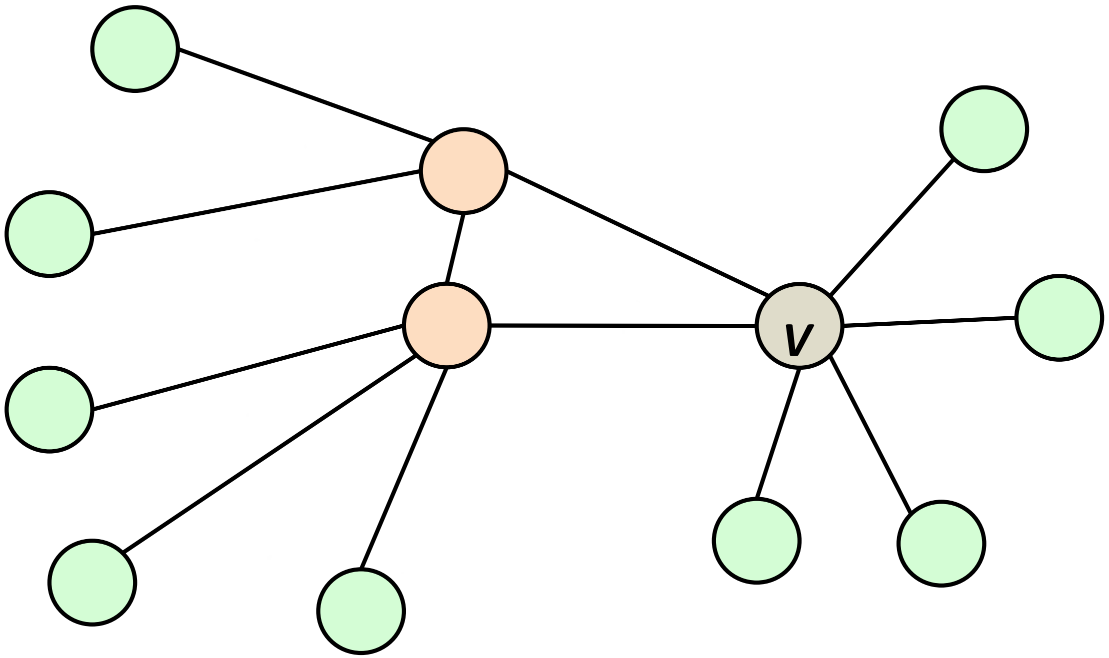
Lemma 2 (Fodor 2007): The radius of the smallest circle enclosing 13 points with mutual distance ≥ 1 is $(1+\sqrt{5})/2$.

Lemma 3 (Fejes Tóth 1953): Every packing of two or more congruent disks in a convex region has density at most $\pi/\sqrt{12}$.

(k,l)-pendant graphs

A **(k,l)-pendant** graph is a graph containing a vertex v with k pendant vertices in its open neighborhood and l pendant vertices in its open 2-neighborhood.

a (4,5)-pendant graph



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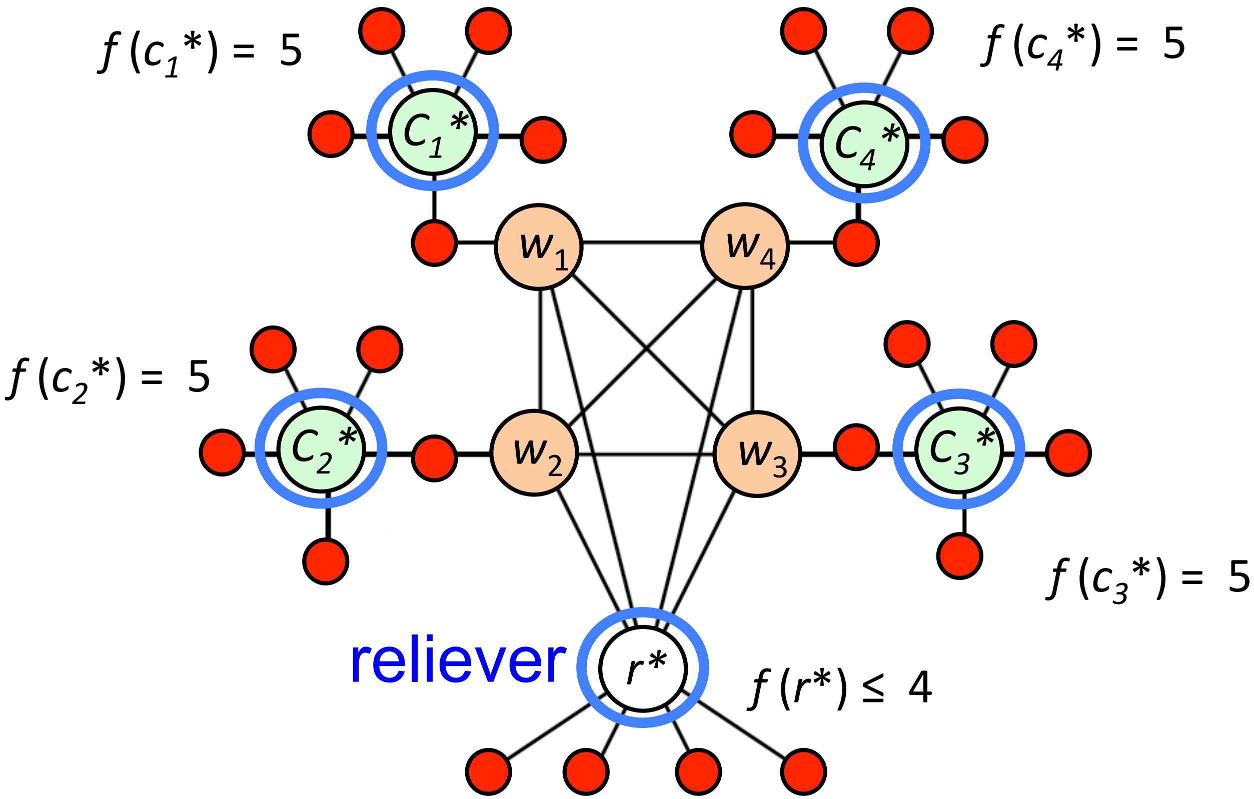
Lemma 6 (FFMS 2012): If G is a $(4,L)$-pendant unit disk graph, then $L \leq 8$.

Establishing the approximation factor



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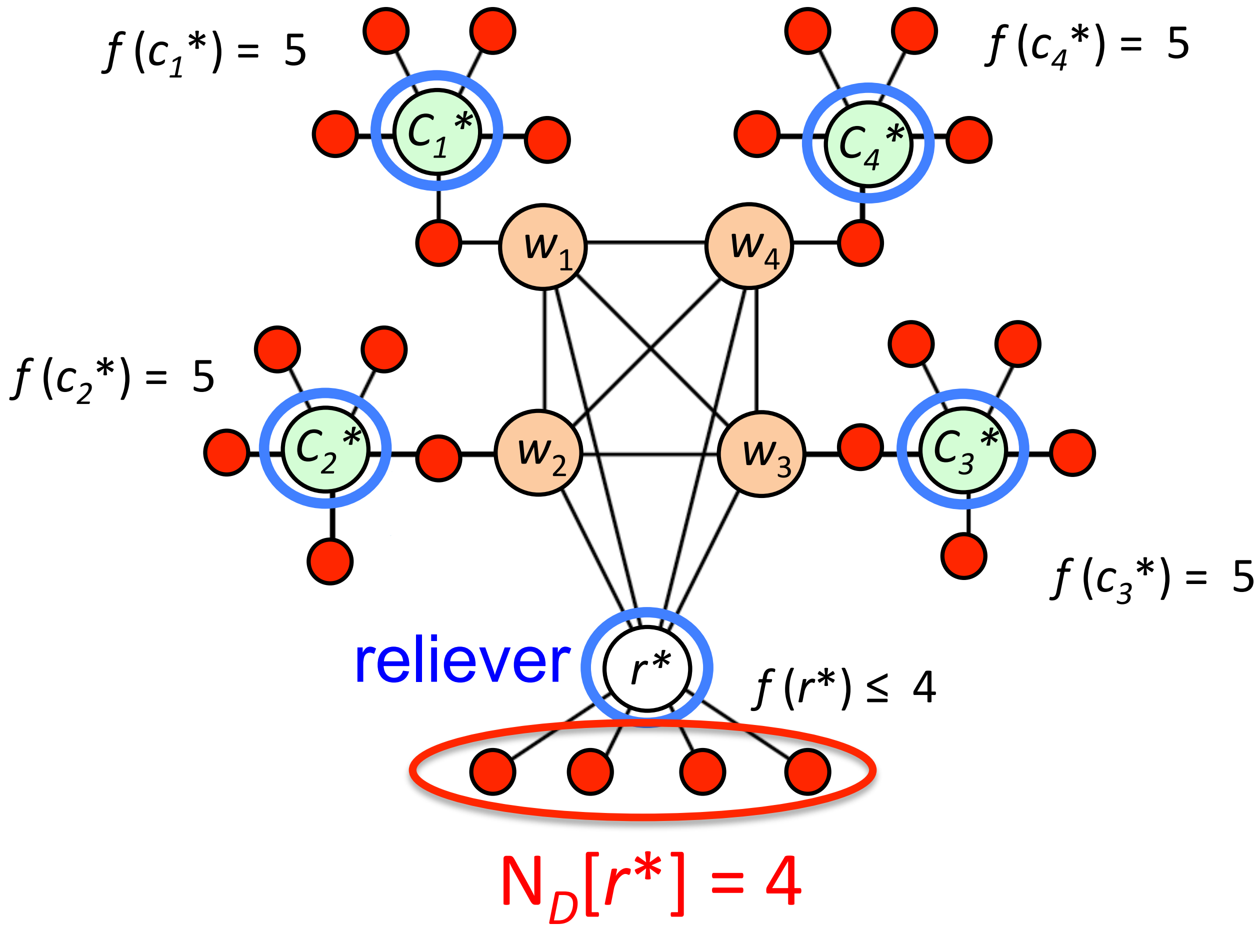

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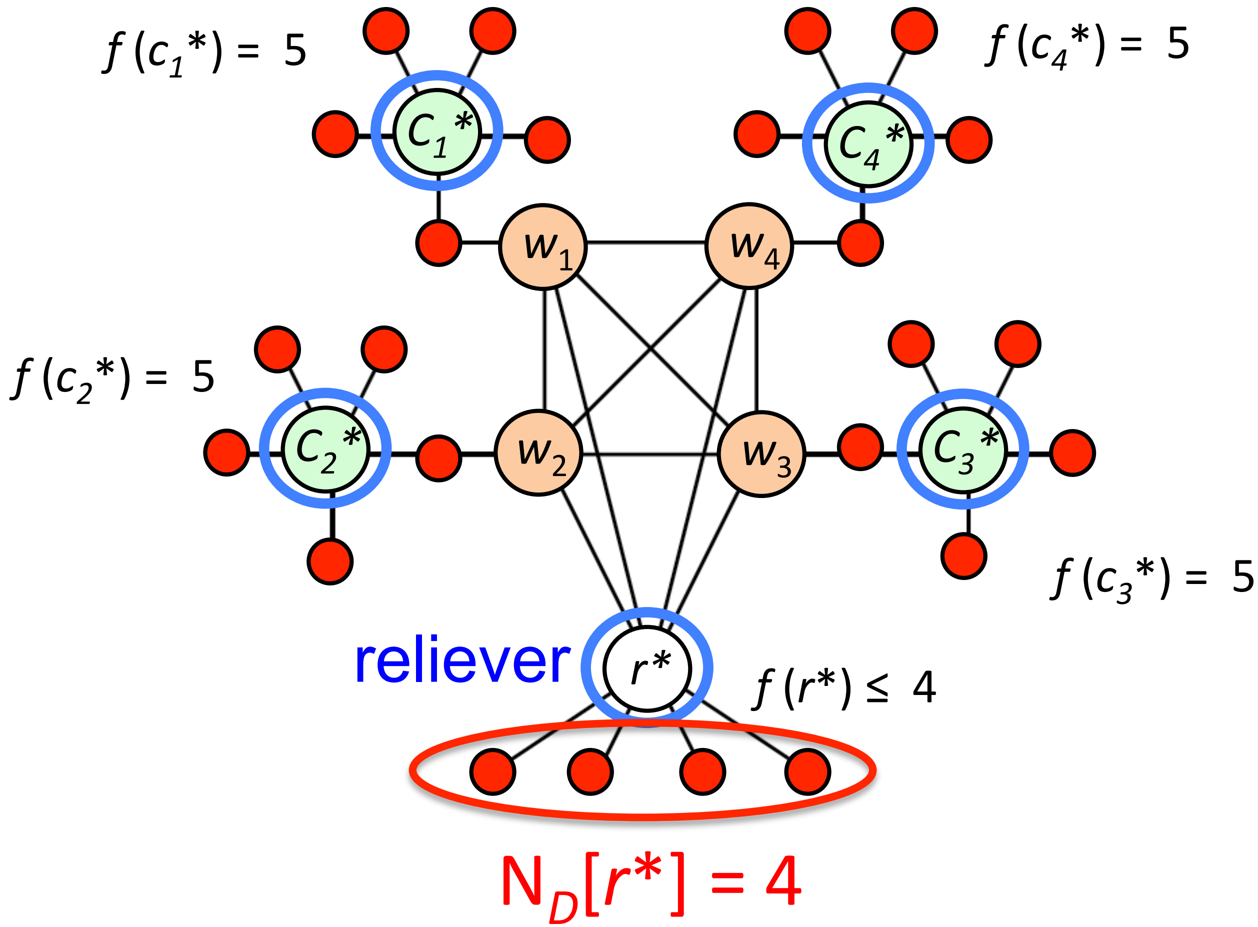

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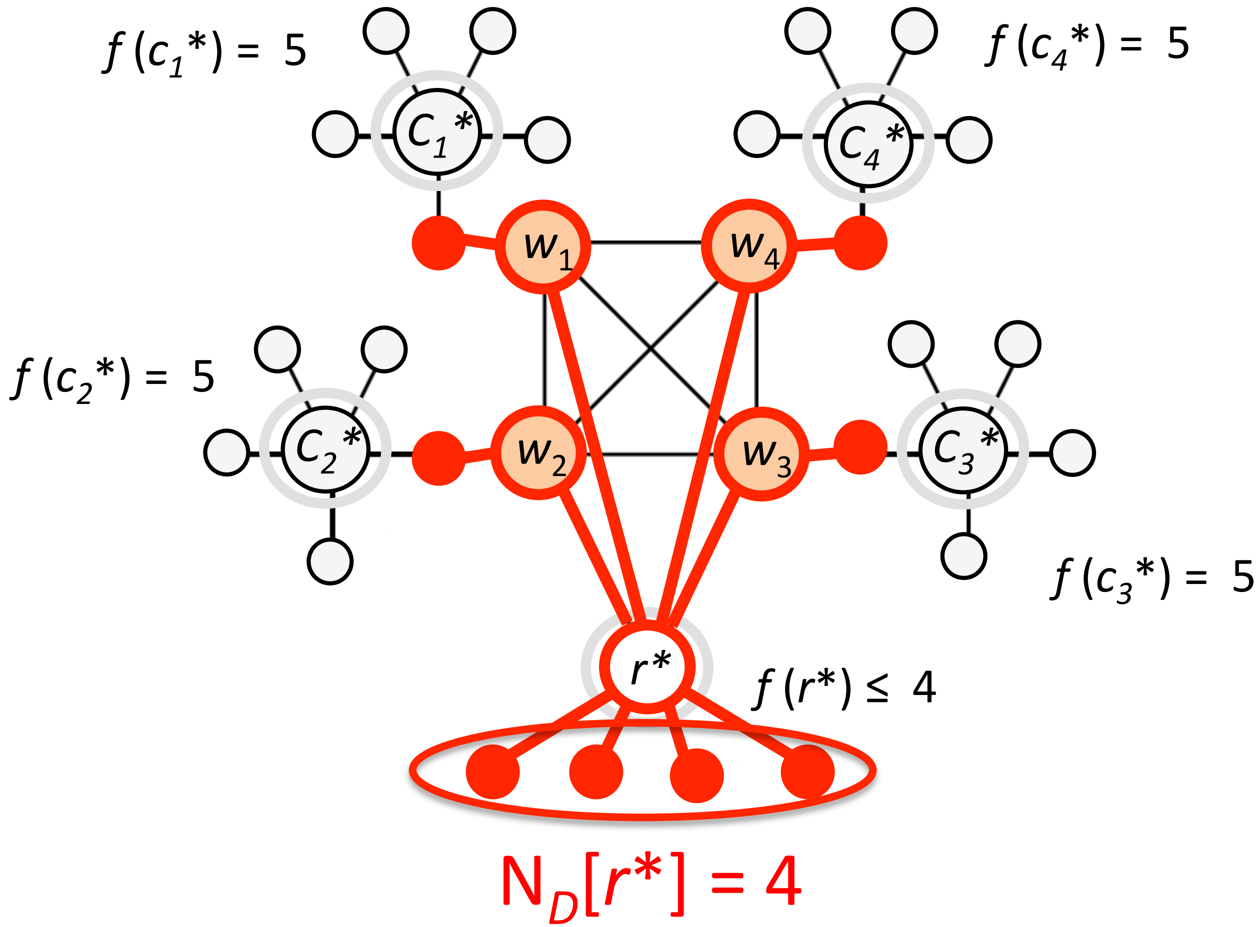

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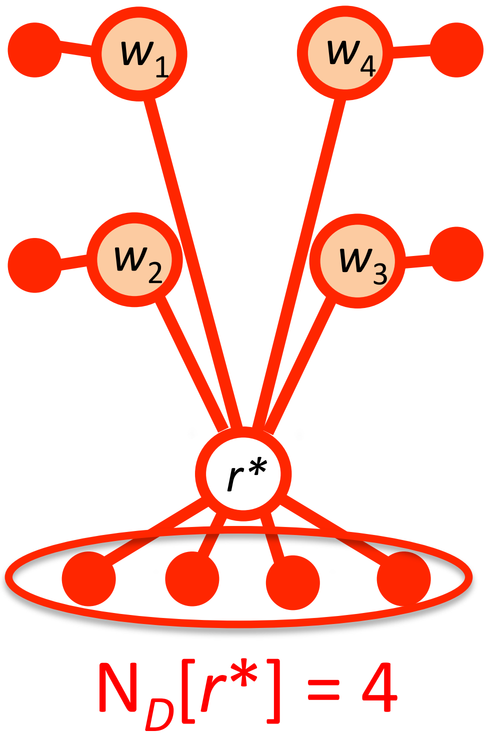

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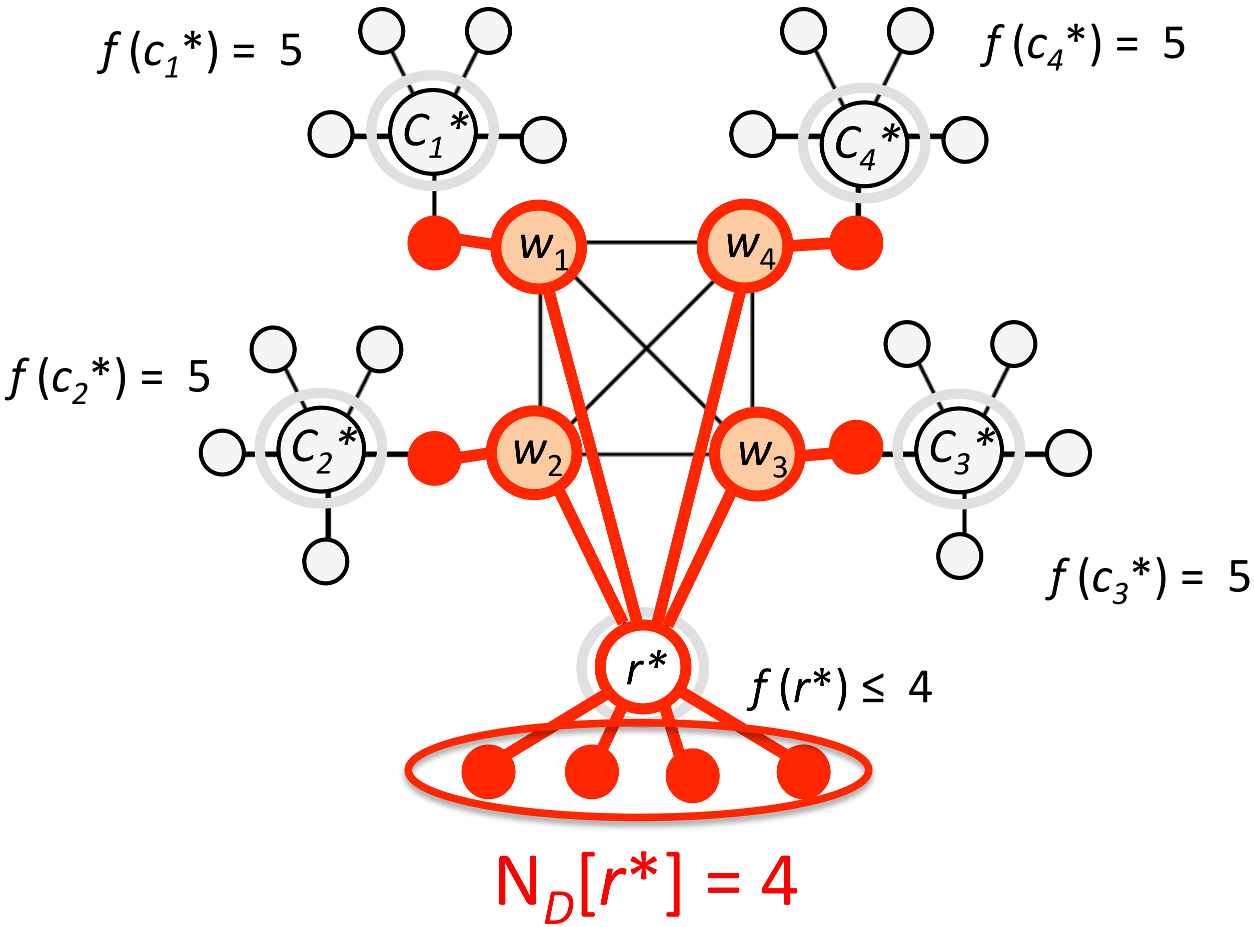

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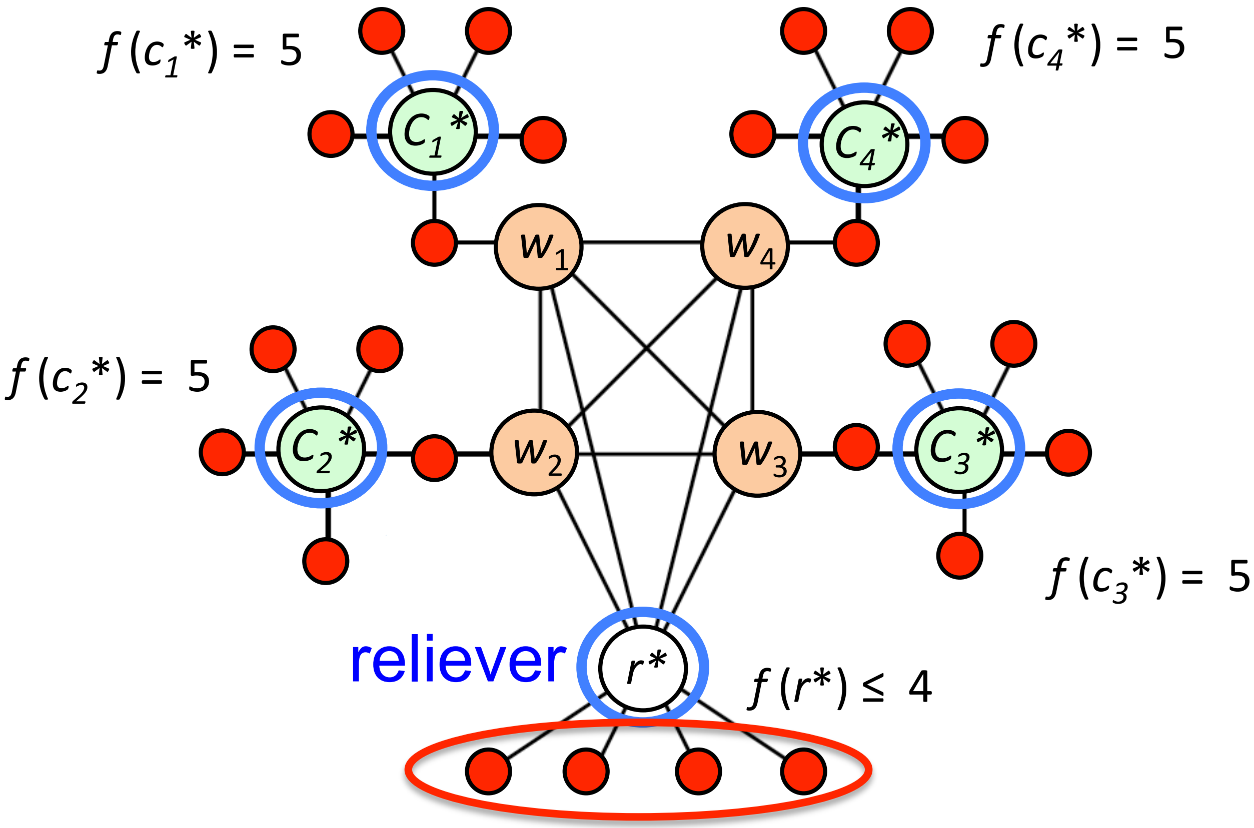
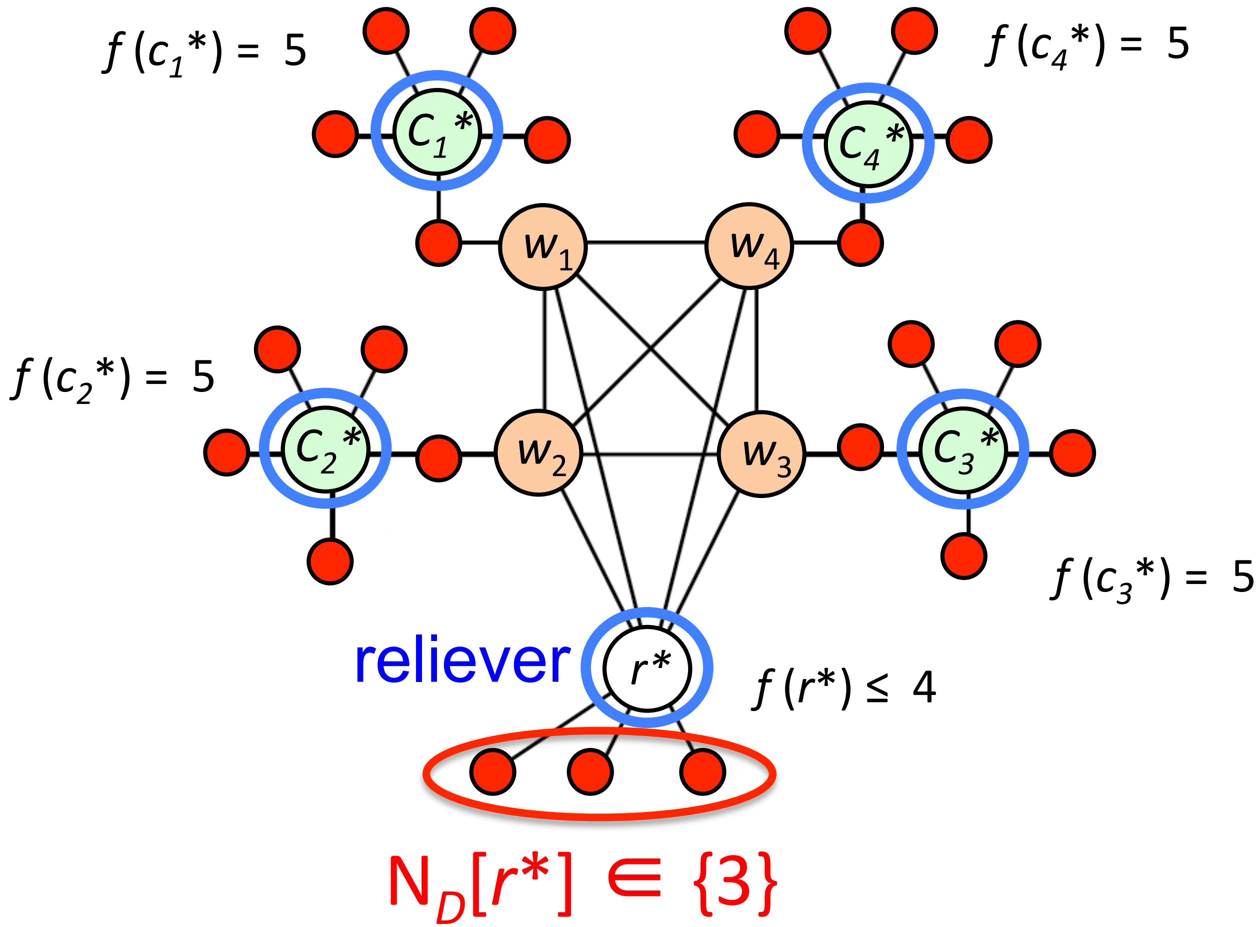


$N_D[r^*] = 4$ $\Rightarrow$ at most **8** cores per reliever

Establishing the approximation factor



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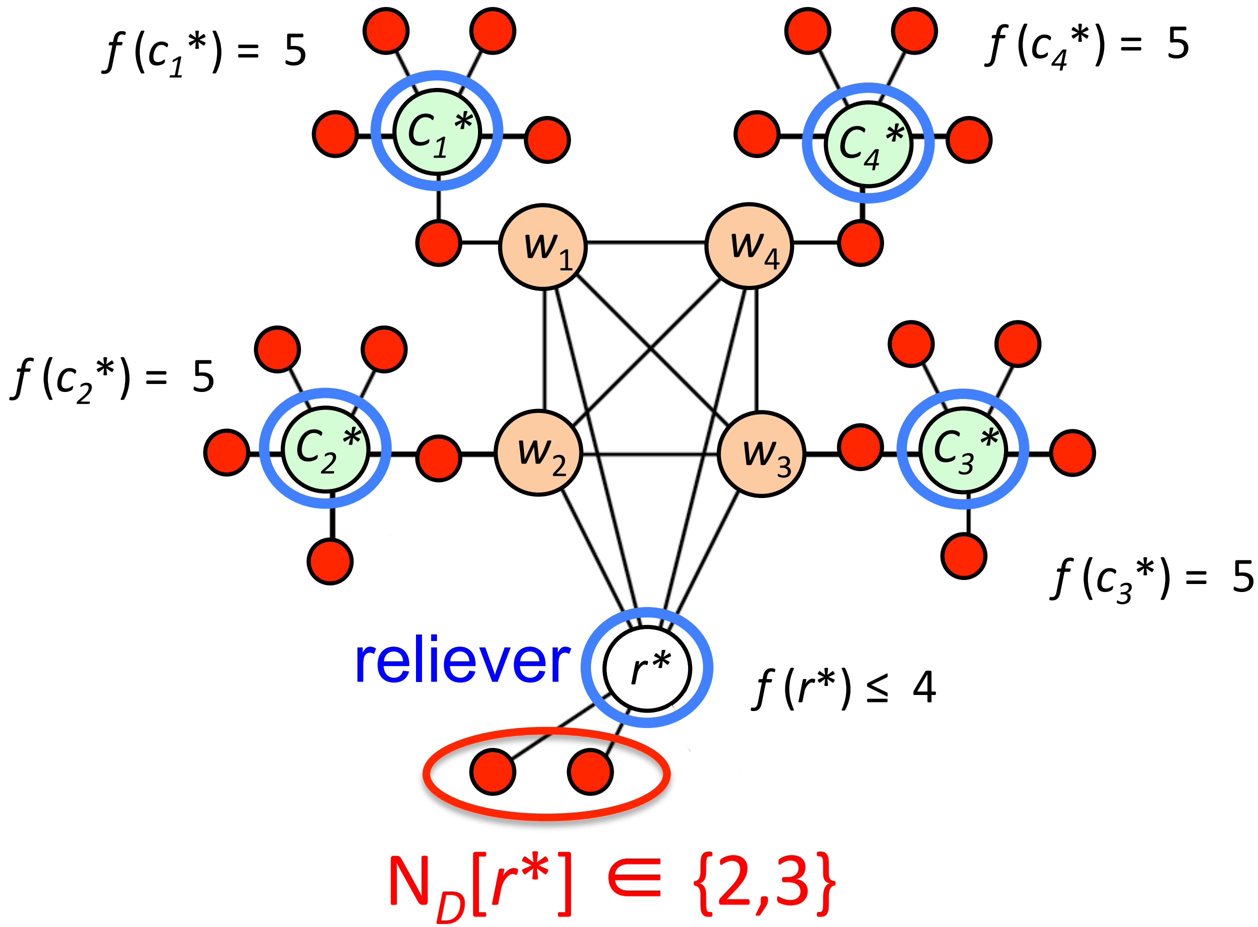

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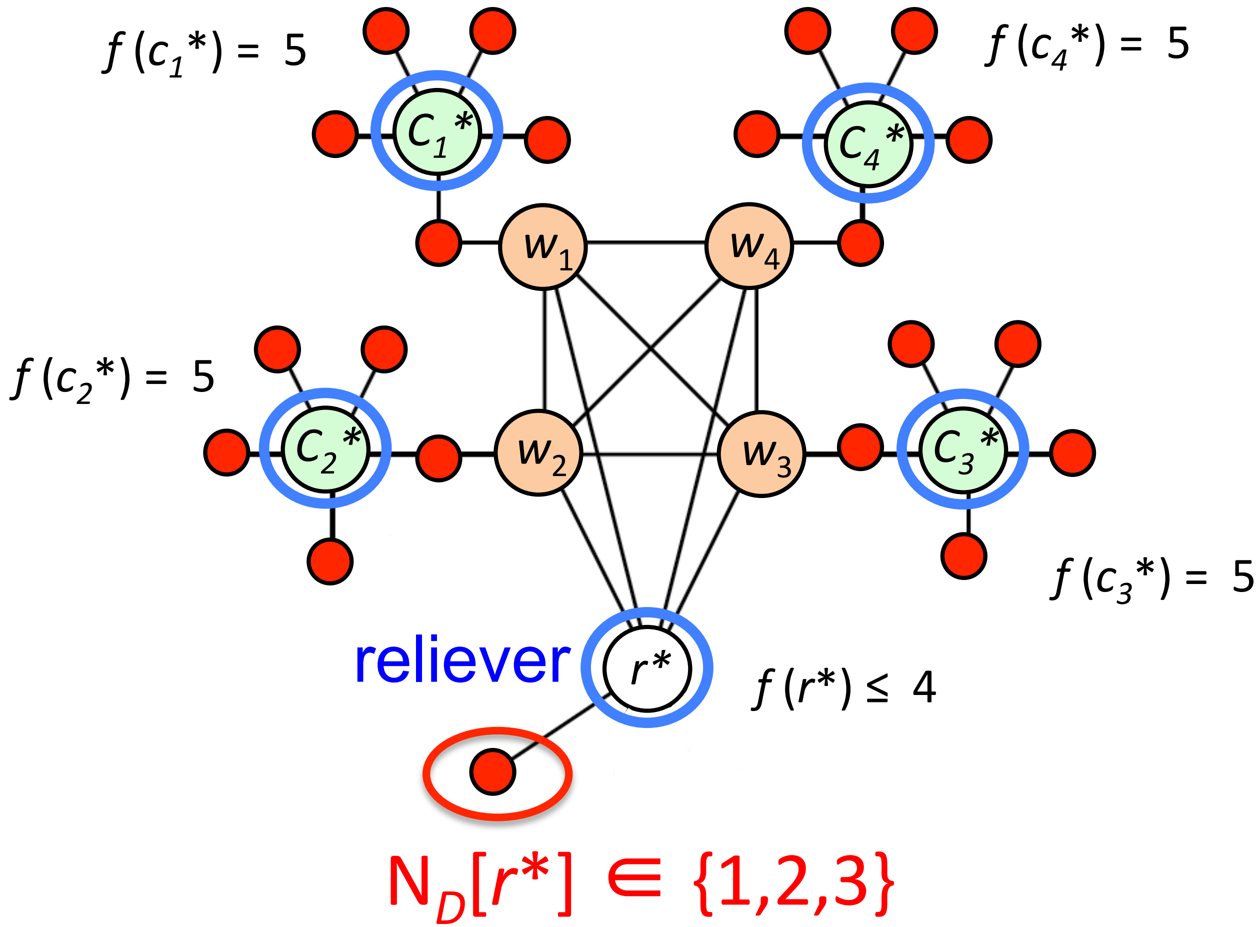

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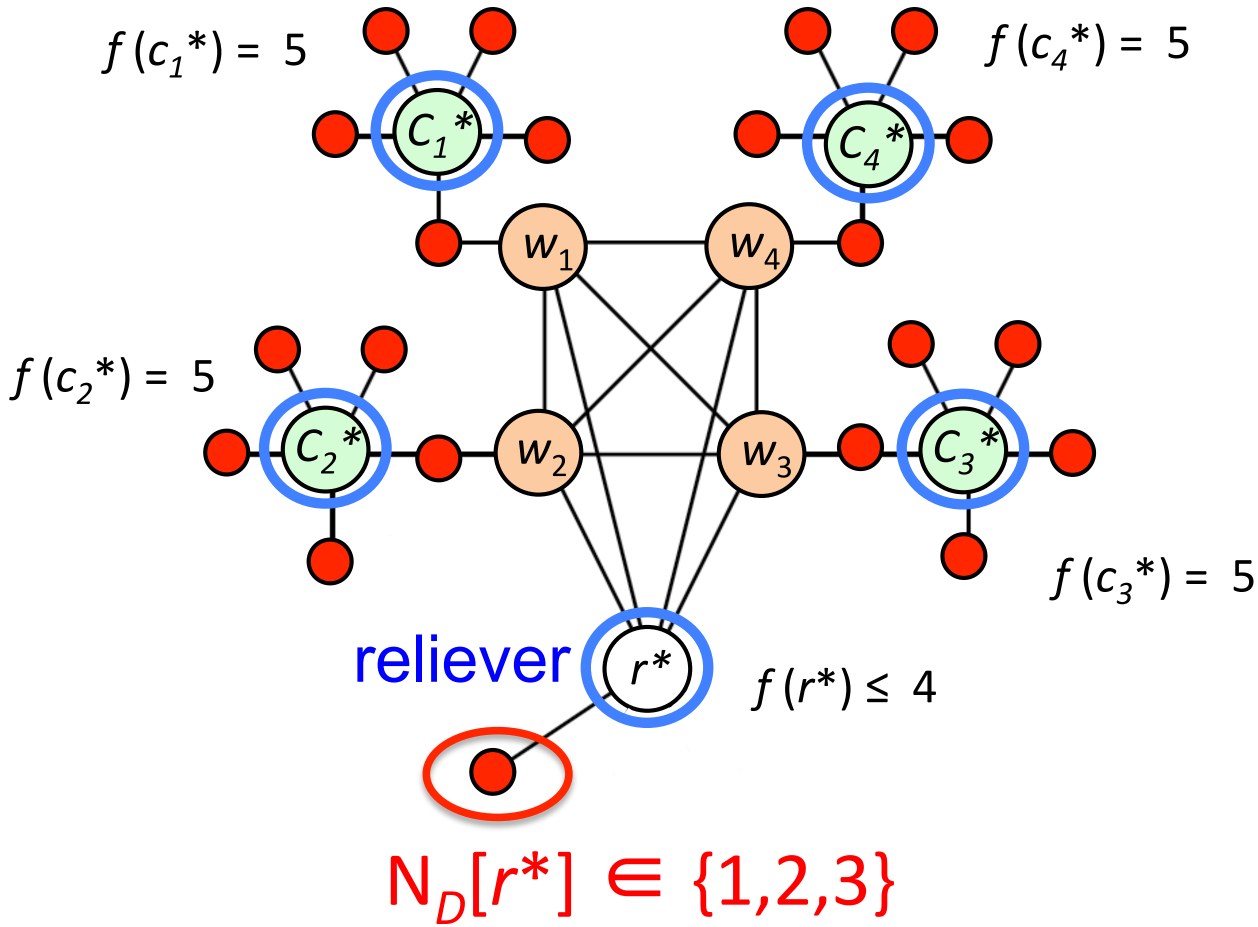

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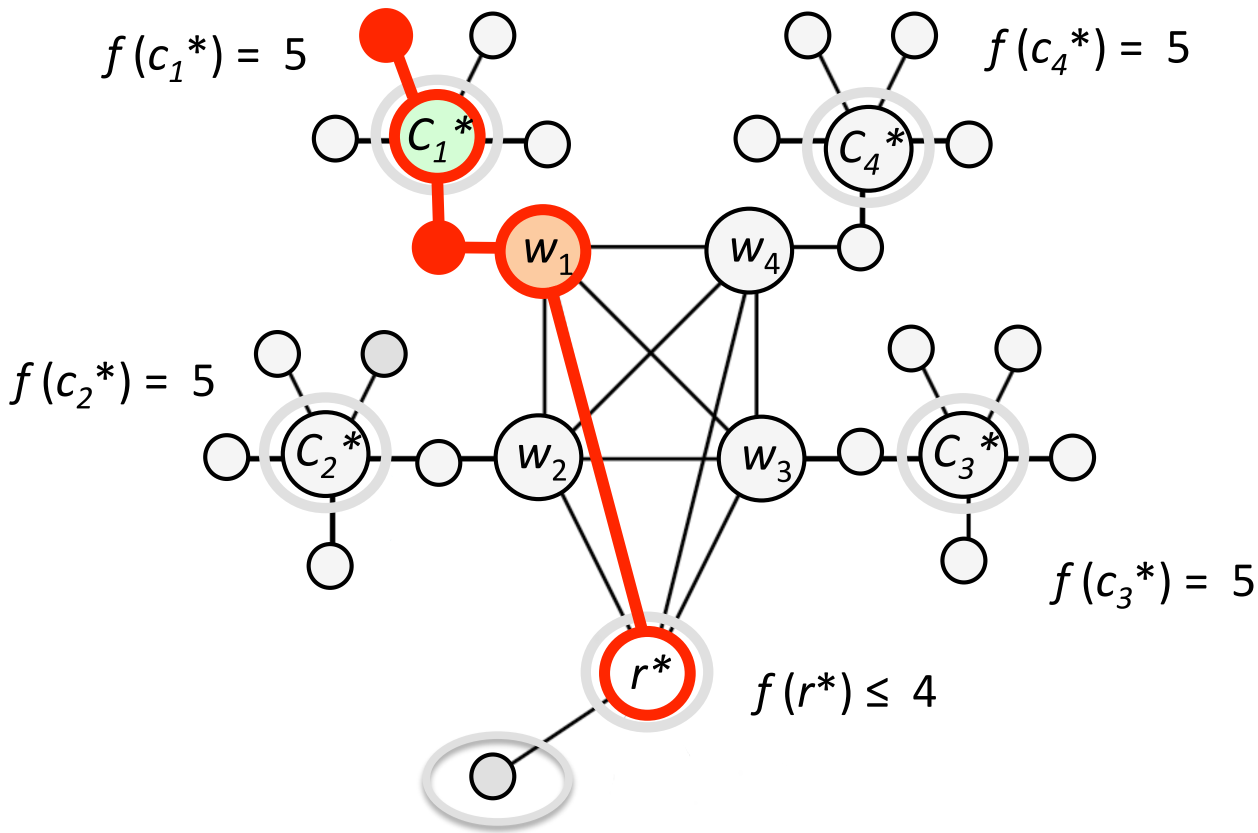
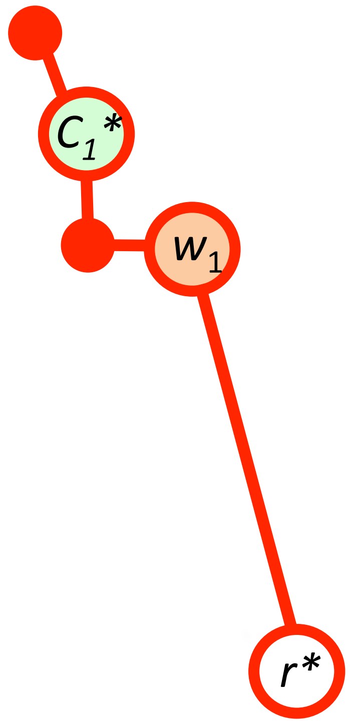


$N_D[r^*] \in \{1,2,3\}$

Establishing the approximation factor



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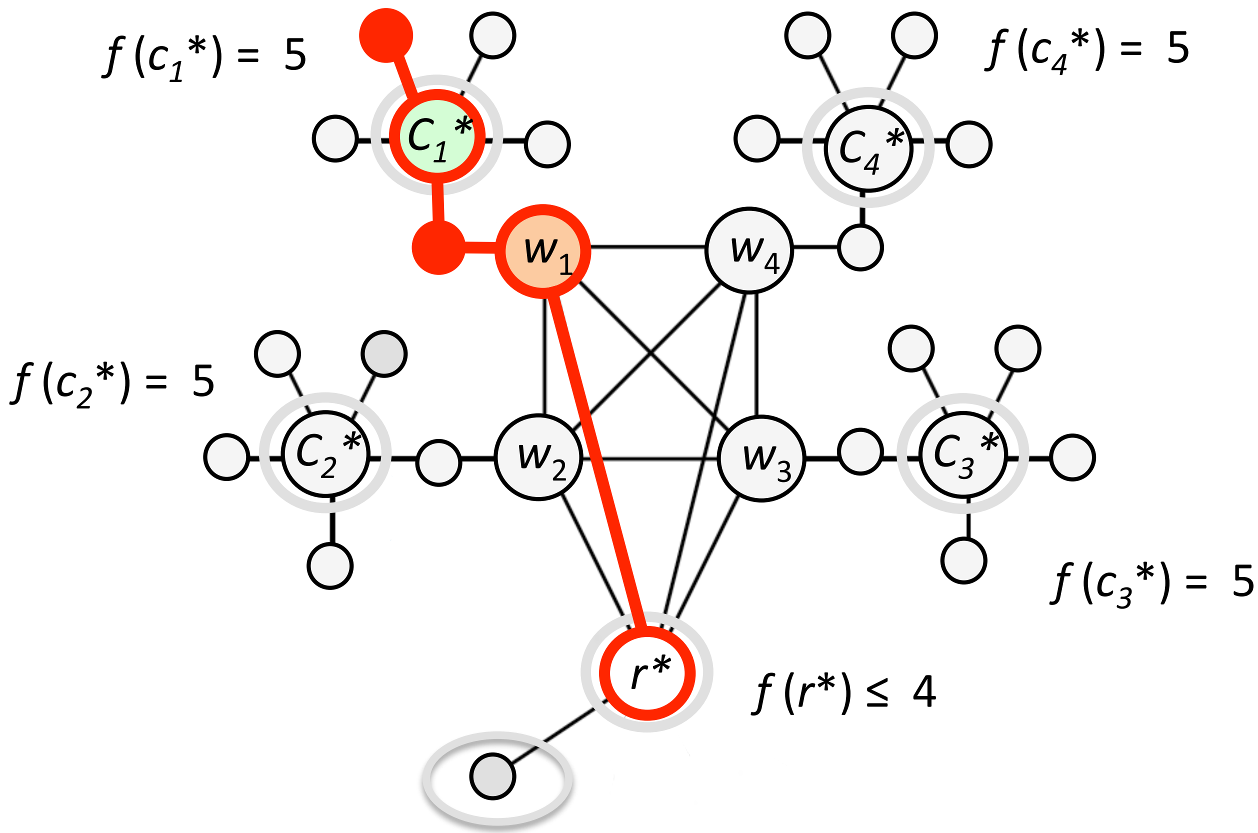

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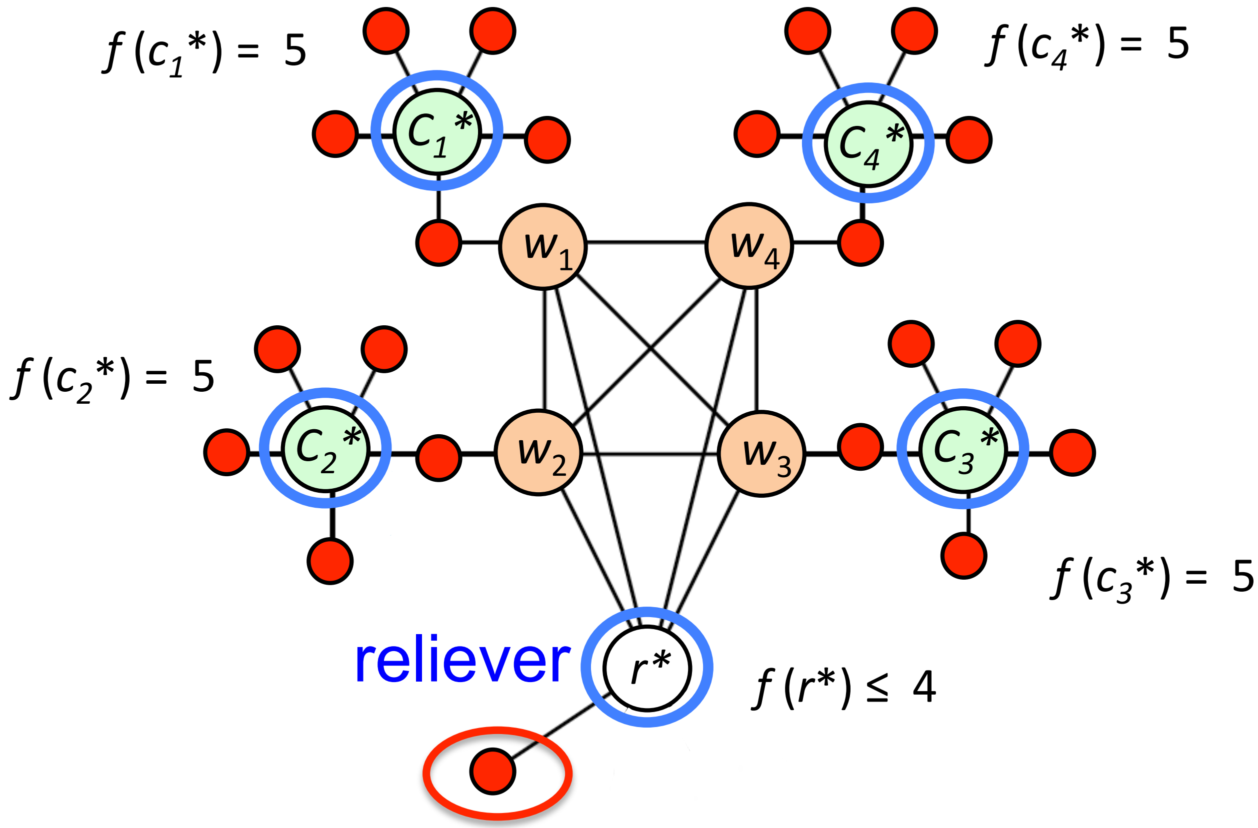


$N_D[r^*] \in \{1,2,3\} \Rightarrow$ at most **14** cores per relievers

Establishing the approximation factor

$\lvert N_D[r^*] \rvert$	Maximum number of cores c_i^* per reliever	Upper bound for $\lvert D \rvert / \lvert D^* \rvert$
1	14	$(1 \times 1 + 14 \times 5) / 15 =$ **4,7333…**
2	14	$(1 \times 2 + 14 \times 5) / 15 =$ **4,8**
3	14	$(1 \times 3 + 14 \times 5) / 15 =$ **4,8666…**
4	8	$(1 \times 4 + 8 \times 5) / 9 =$ **4,888…**

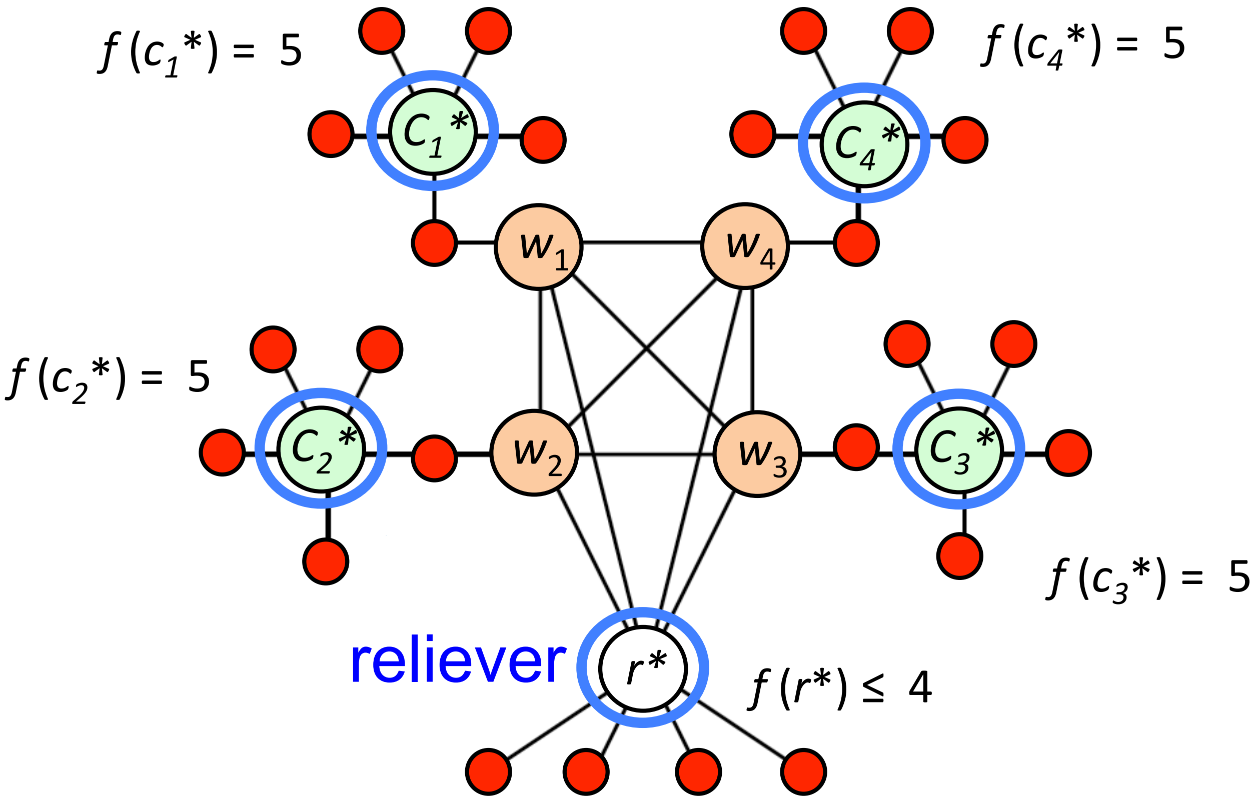
1 reliever

f(reliever)

max # of cores

f(core)

Lower bound

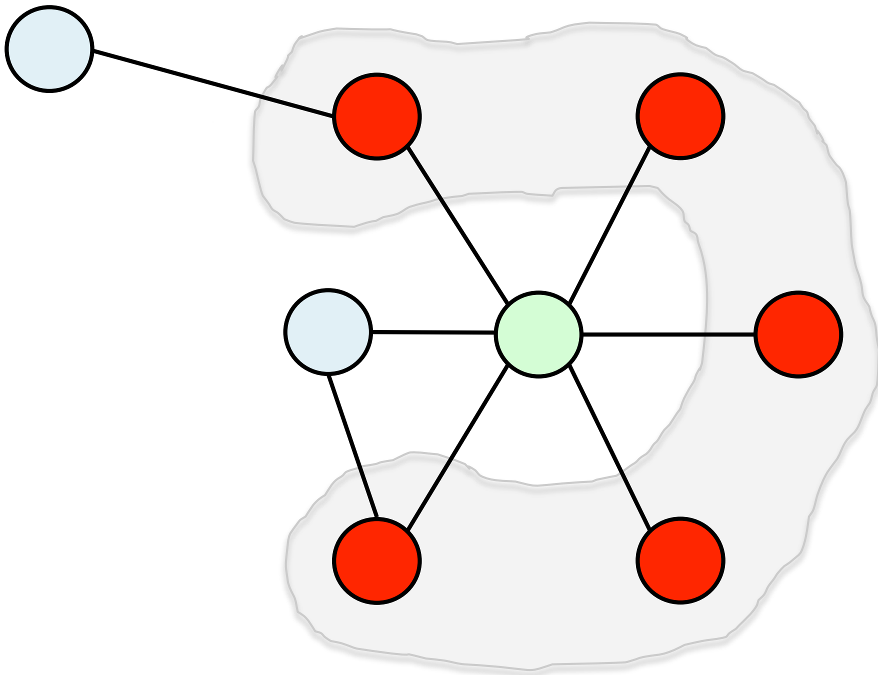


$$(1 \times 4 + 4 \times 5) \;/\; 5 = \mathbf{4.8}$$

Afterword: latest improvements

- Partial reductions

Irreducible corona



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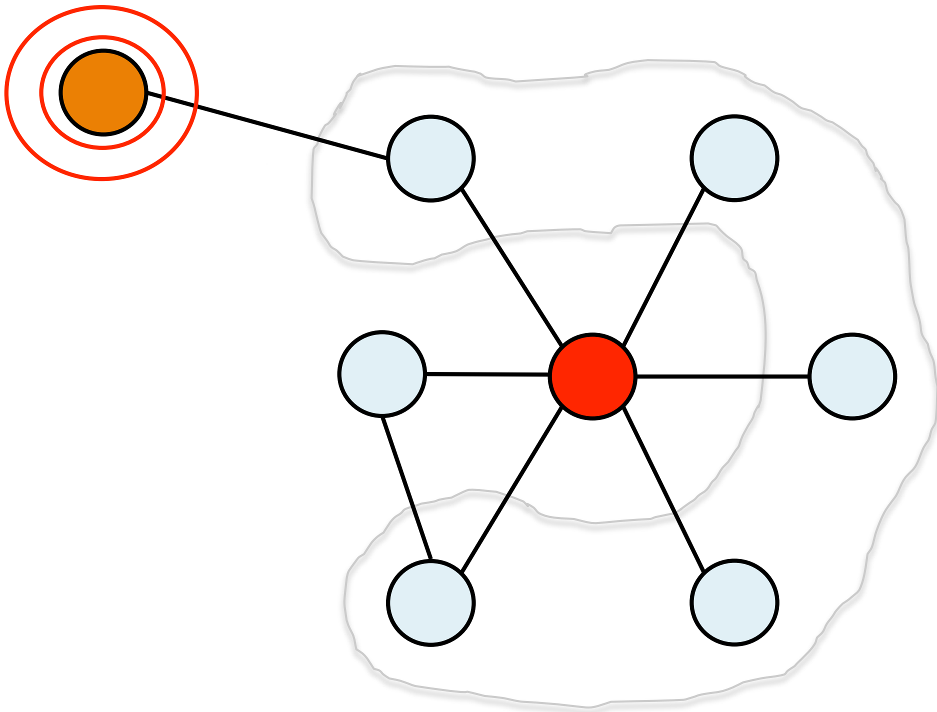
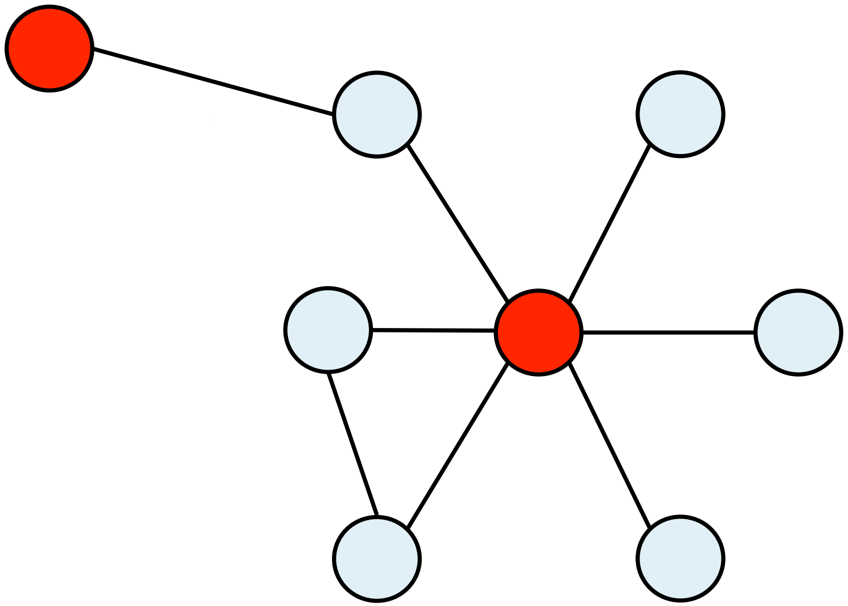
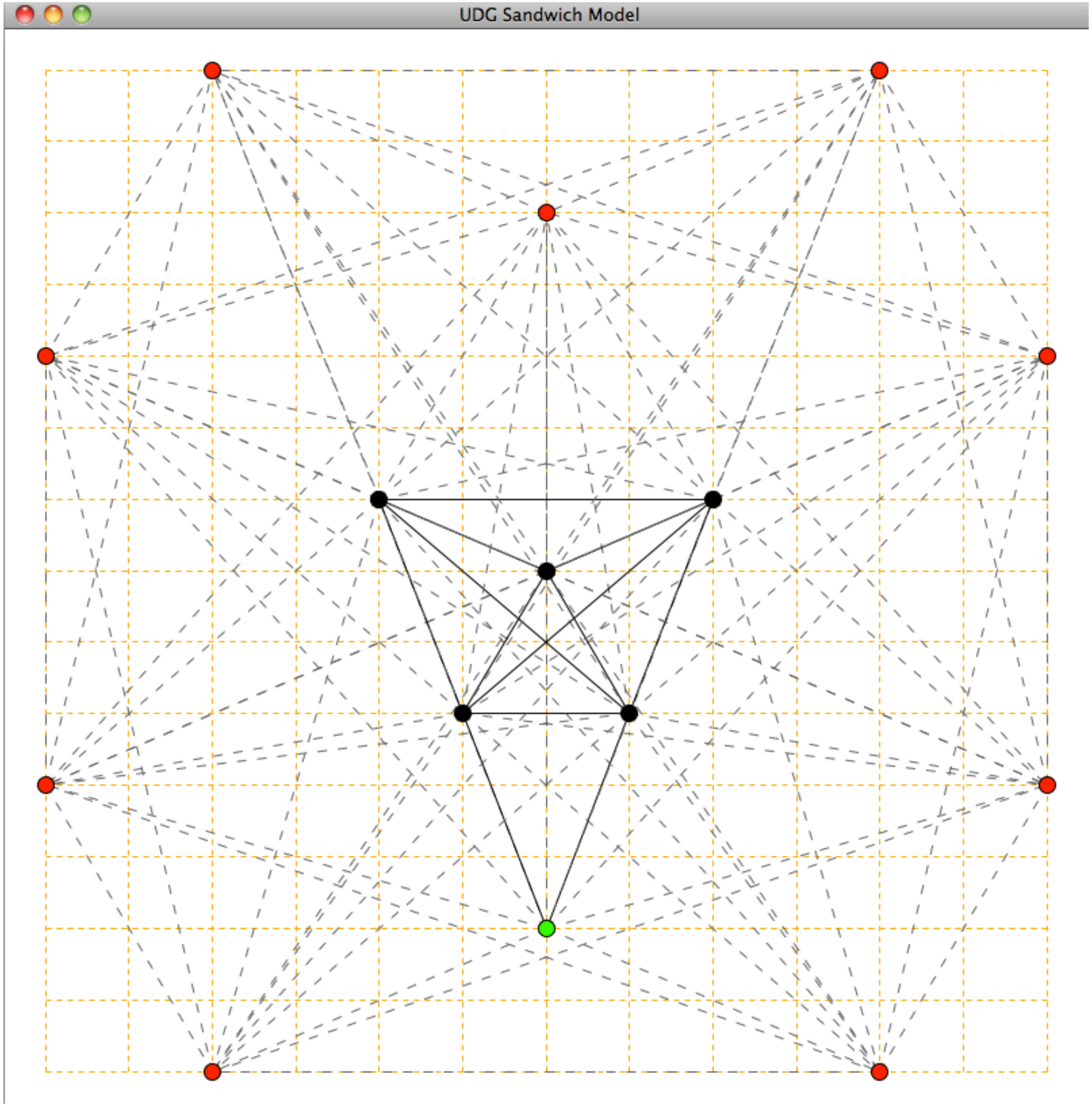


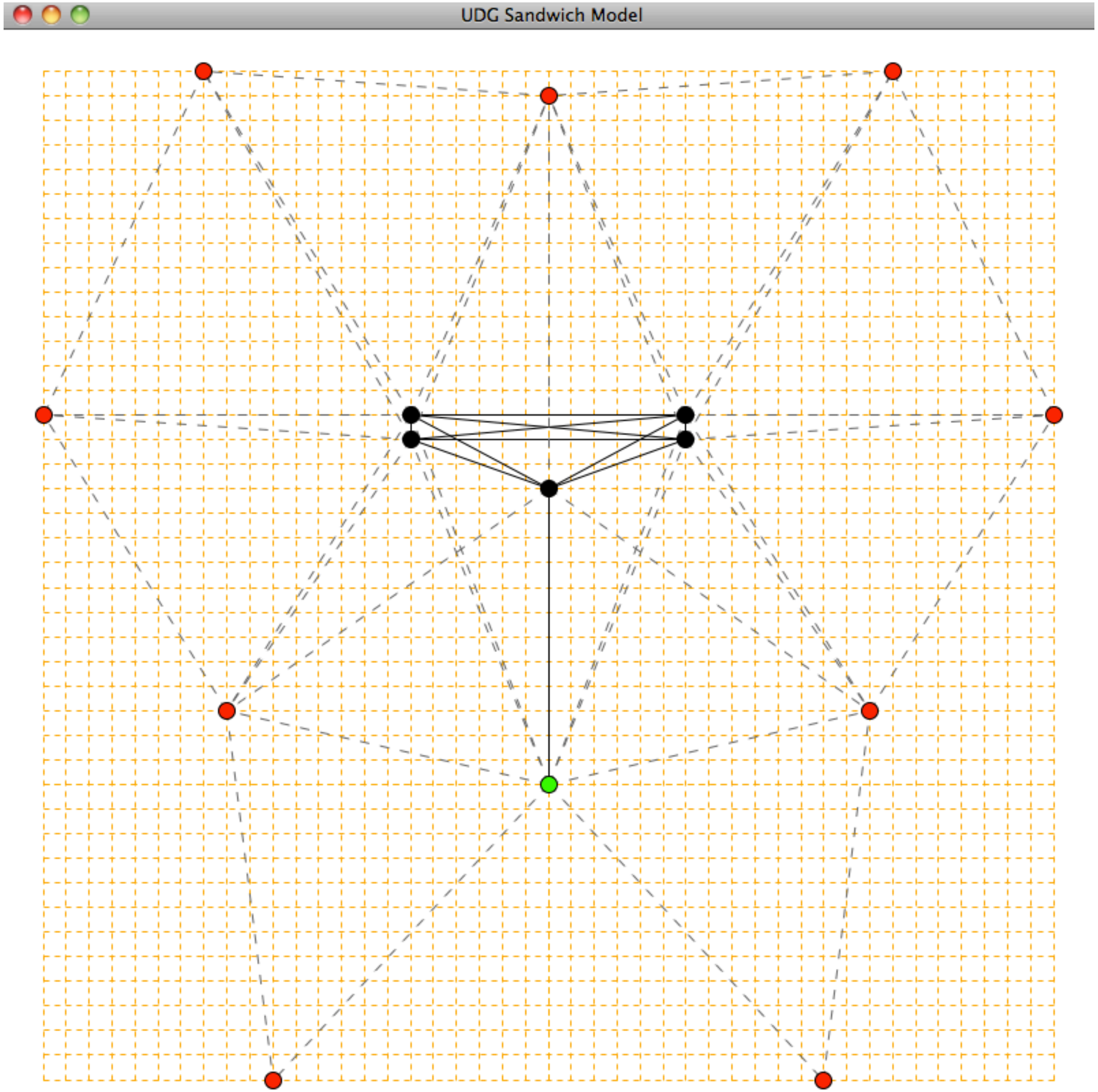
Afterword: latest improvements

- Partial reductions → 4,777...-approximation!

~~Irreducible~~ corona




UDG Sandwich Model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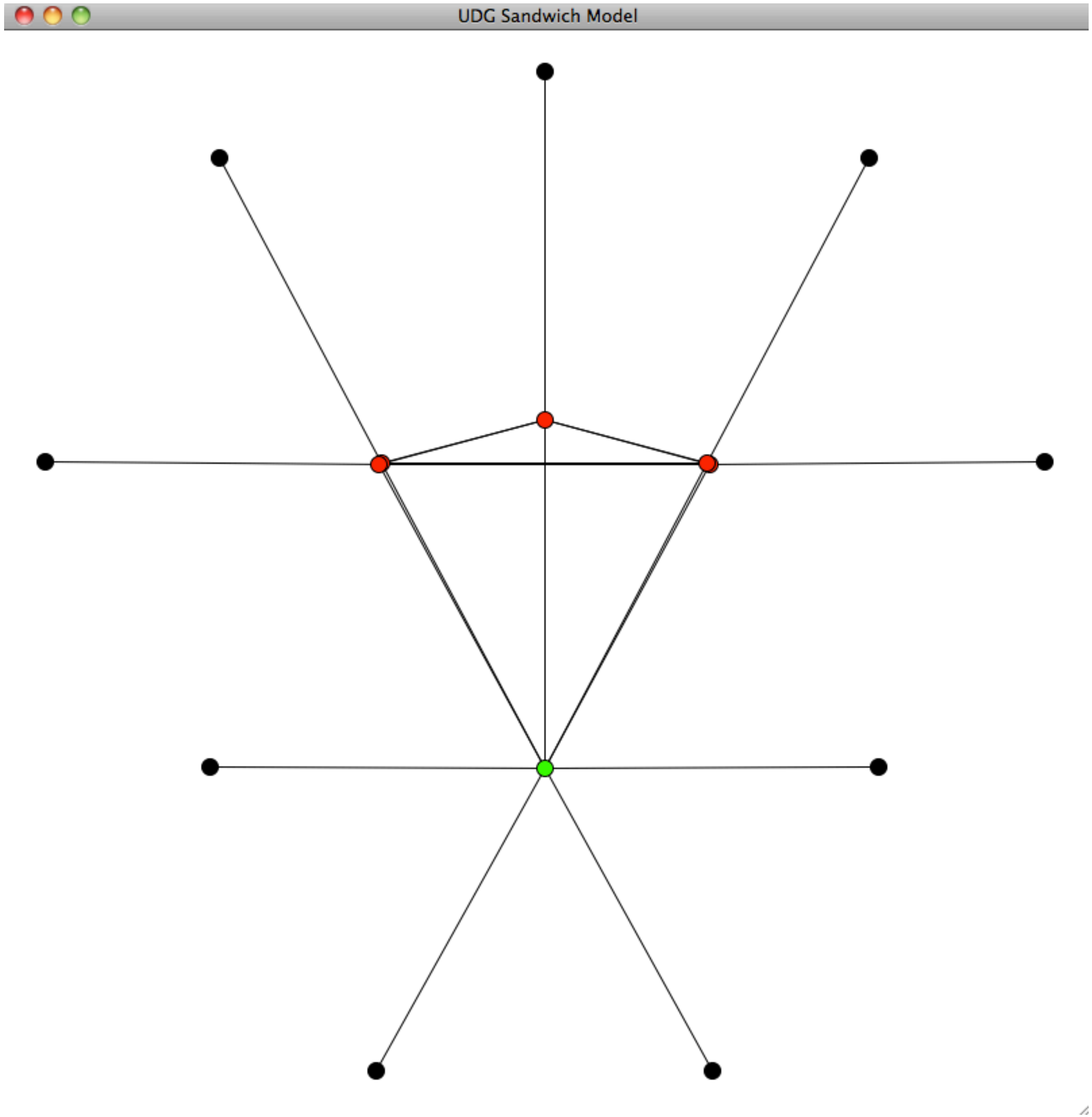
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