

The geodetic Radon number of grids

Mitre Costa Dourado

Dieter Rautenbach

Vinícius Gusmão Pereira de Sá

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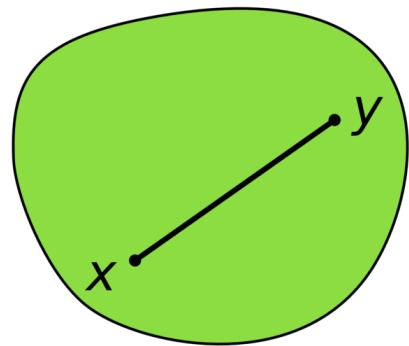
Convexity in Euclidean space

Ground set of the convexity space: the d -dimensional space R^d

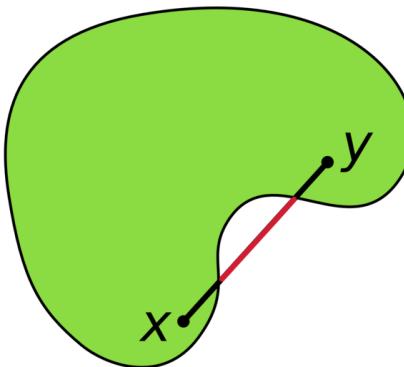
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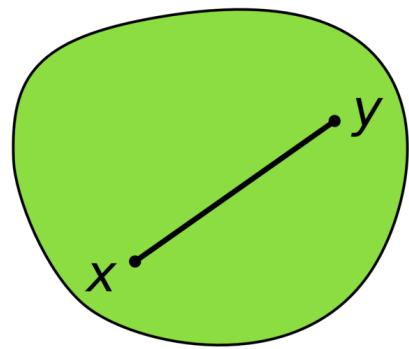
Non-convex set



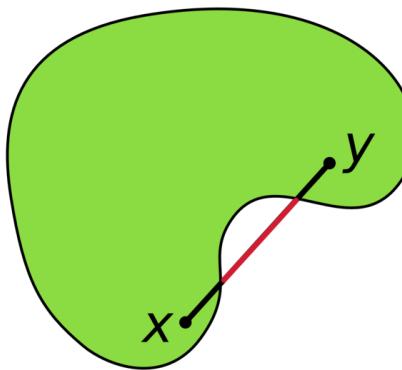
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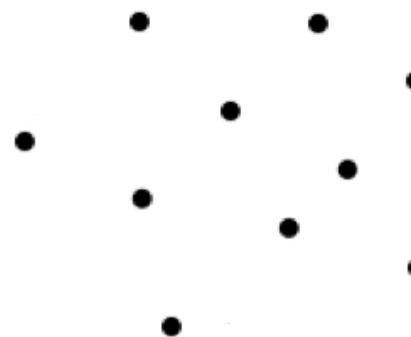
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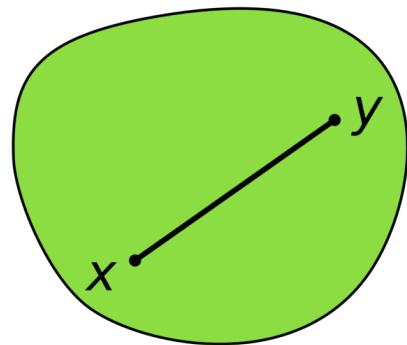
Convex hull



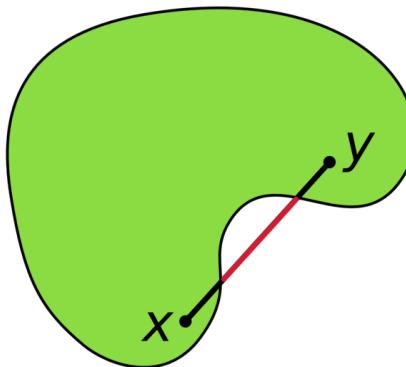
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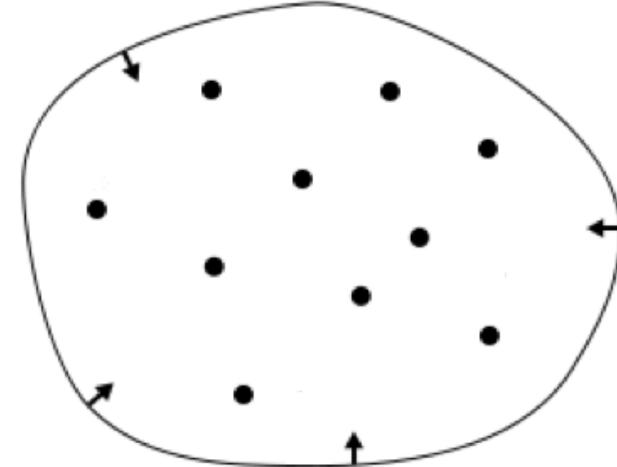
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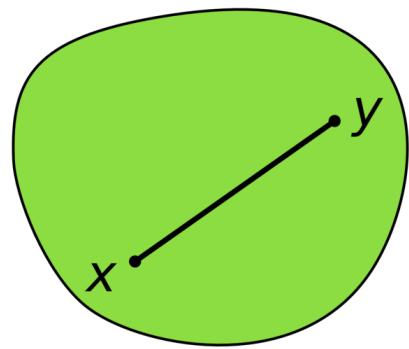
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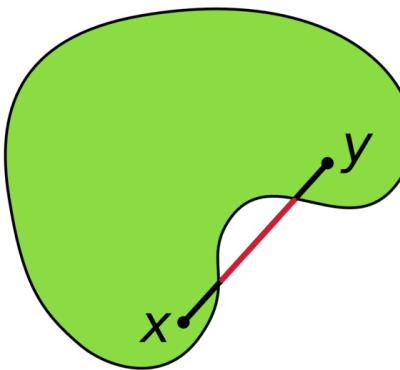
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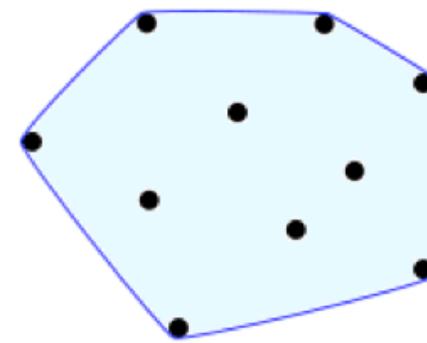
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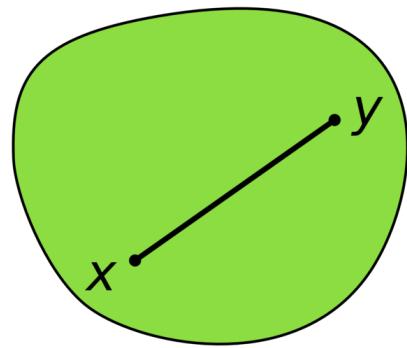
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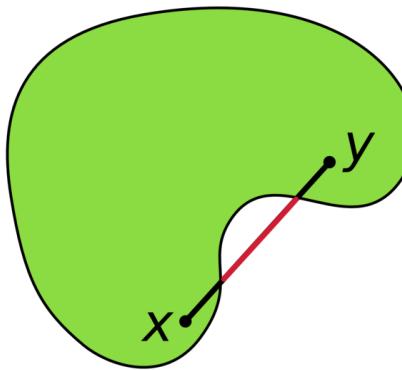
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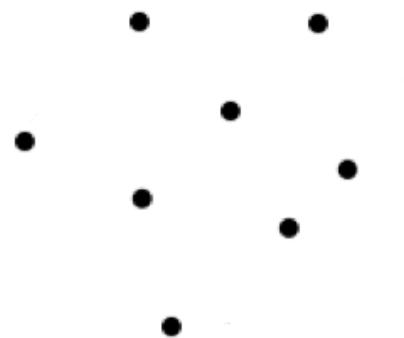
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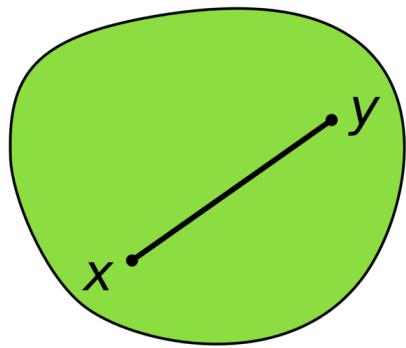


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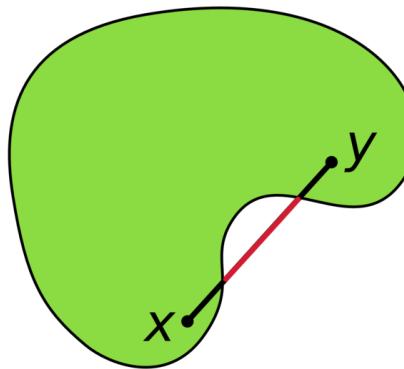
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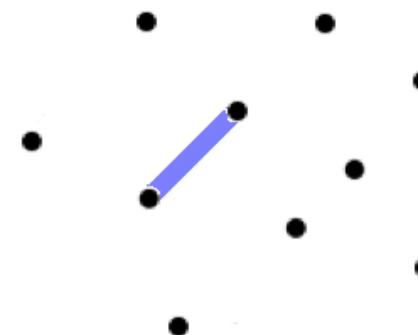
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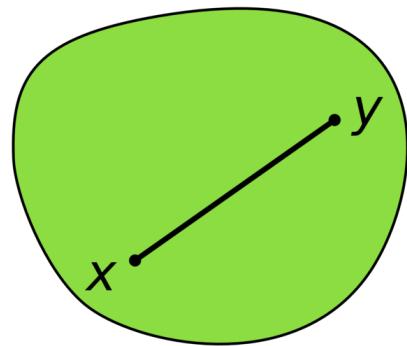


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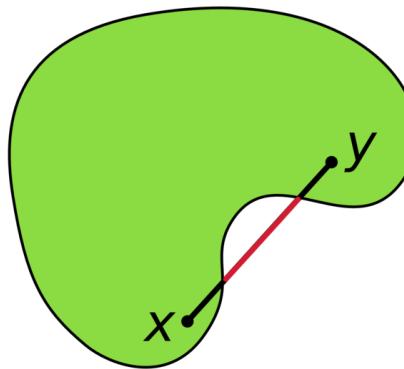
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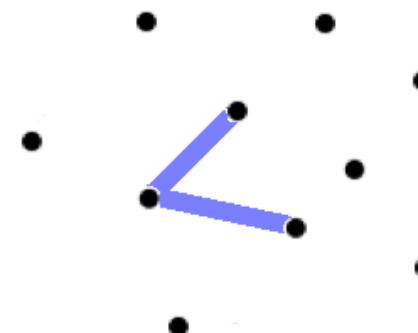
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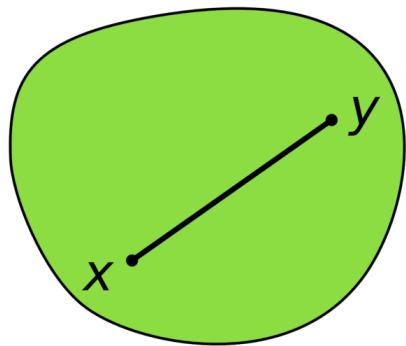


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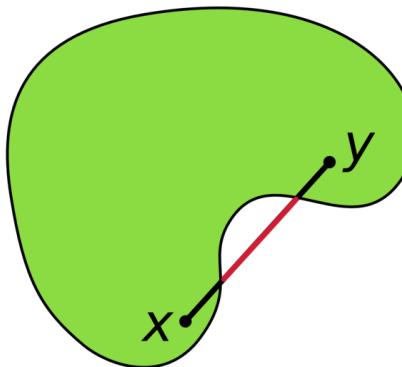
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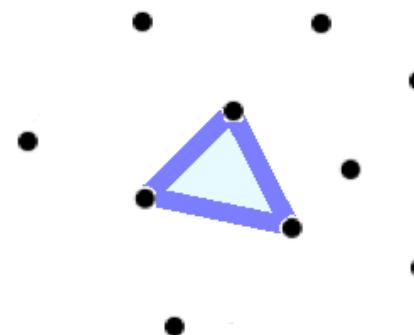
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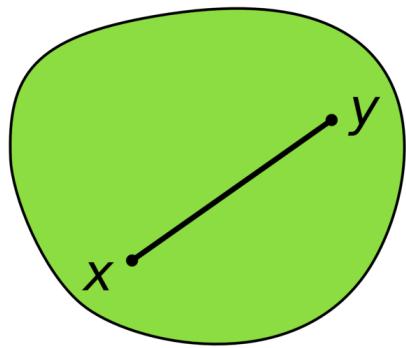


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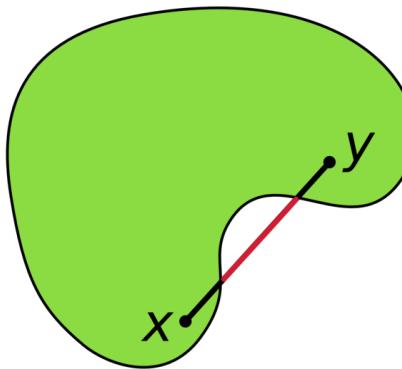
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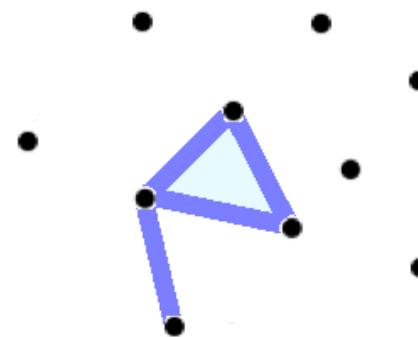
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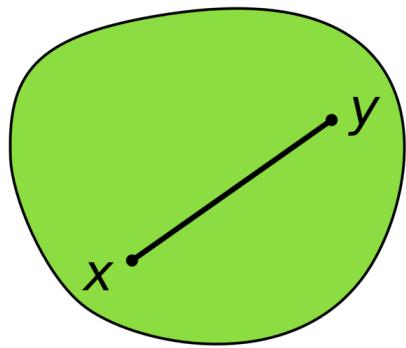


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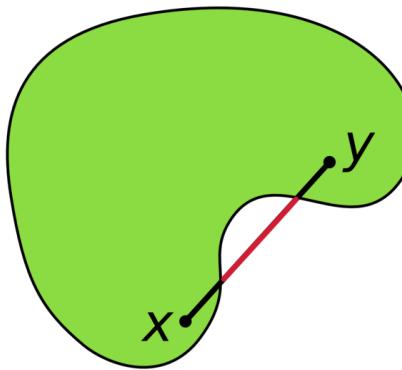
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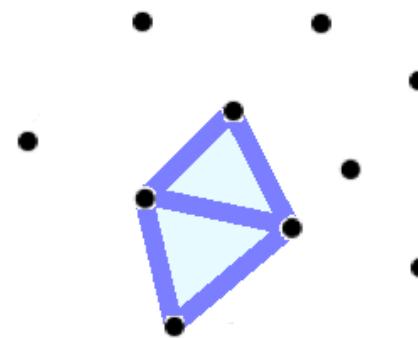
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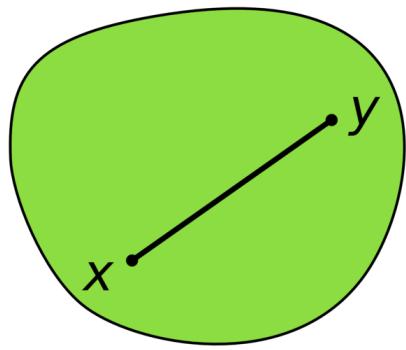


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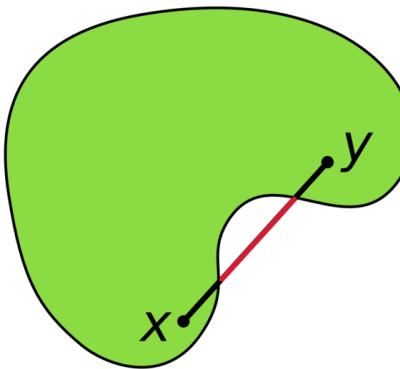
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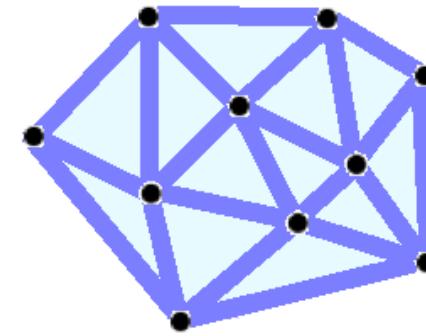
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Non-convex set



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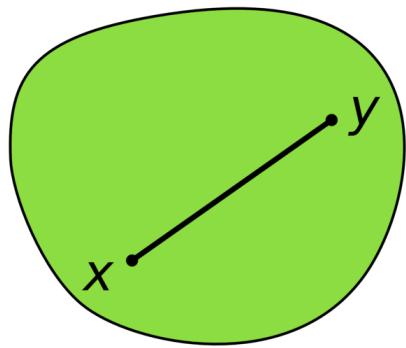


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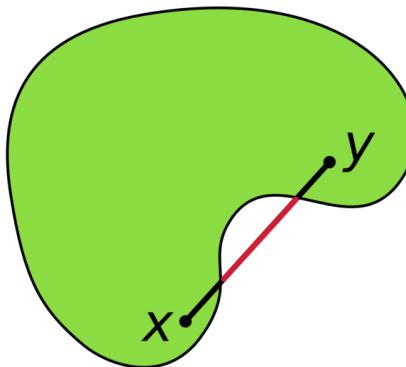
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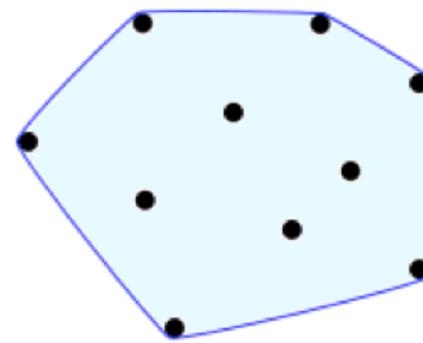
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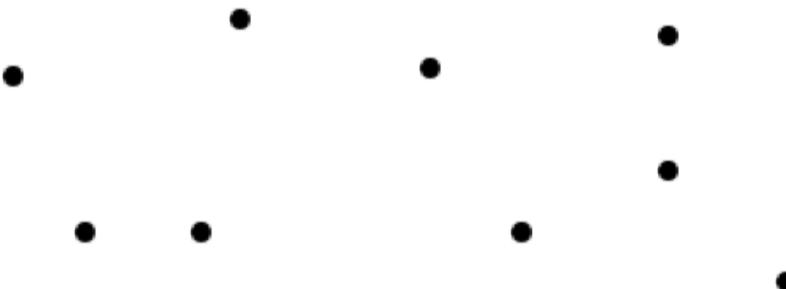
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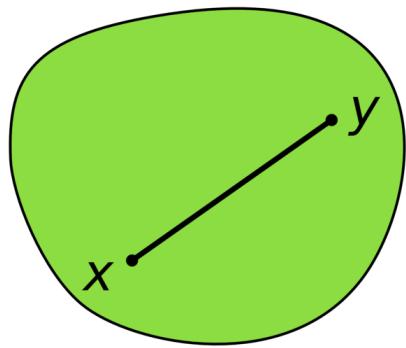


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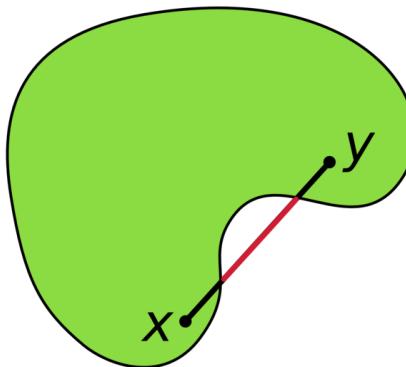
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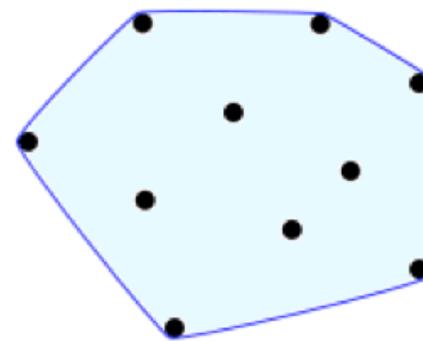
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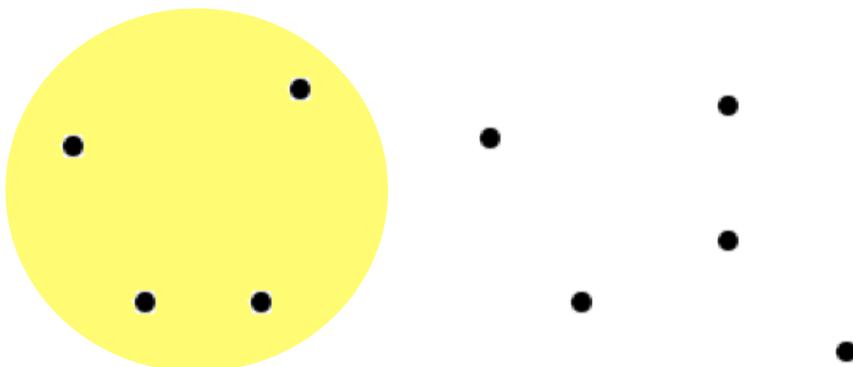
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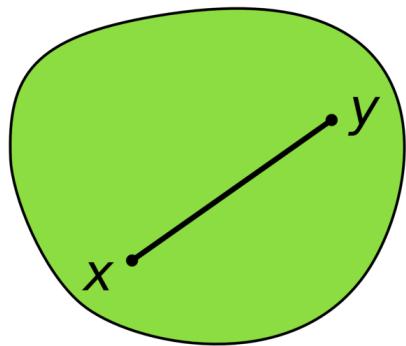


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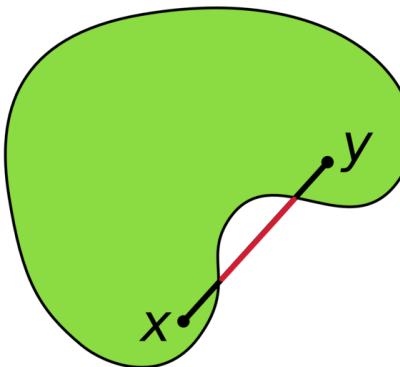
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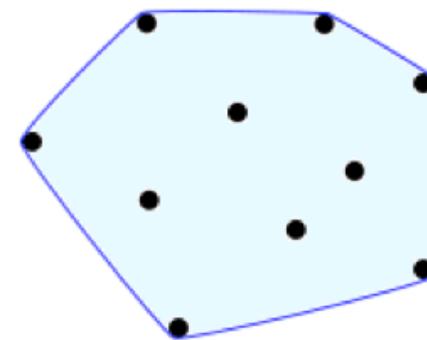
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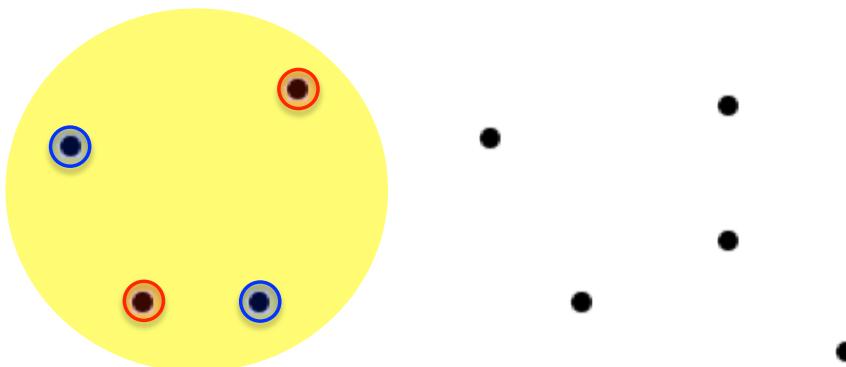
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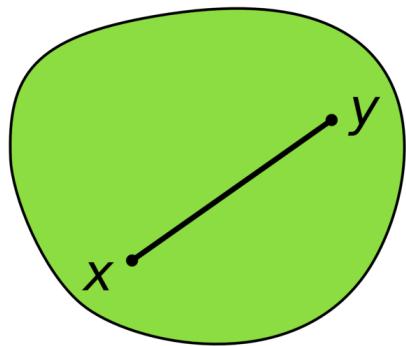


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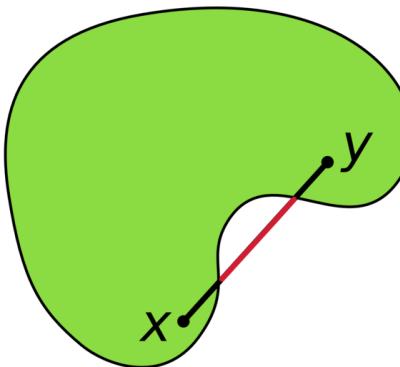
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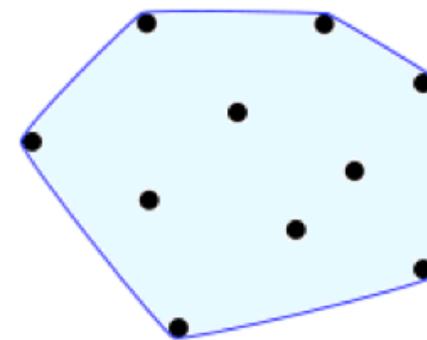
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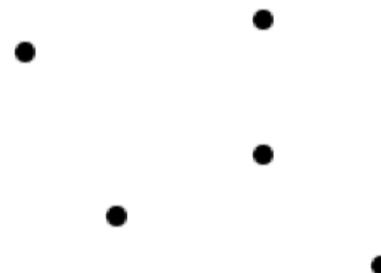
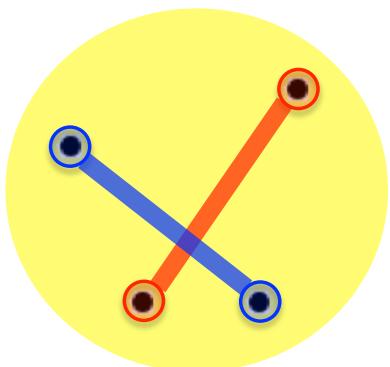
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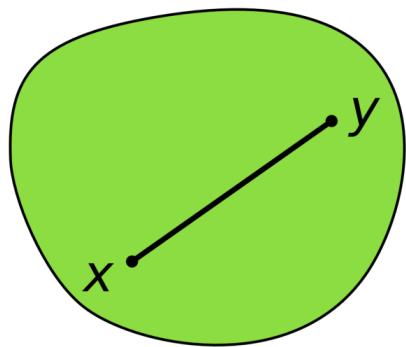


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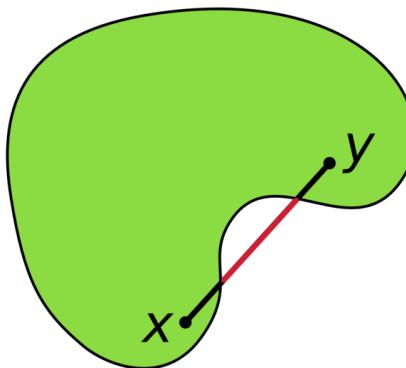
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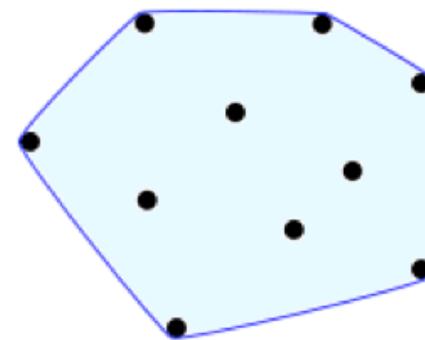
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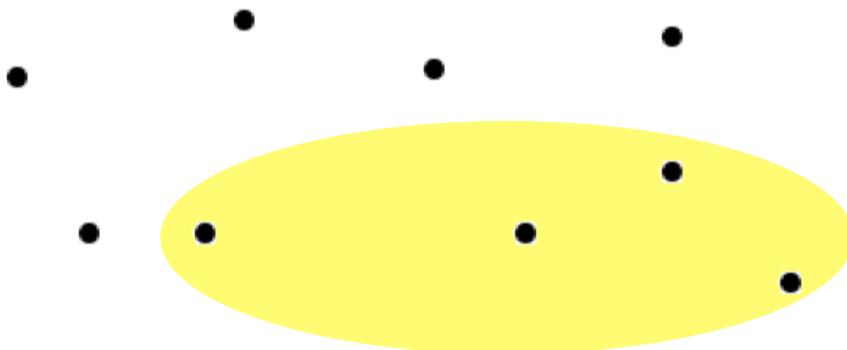
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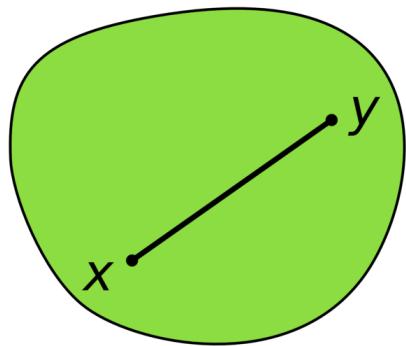


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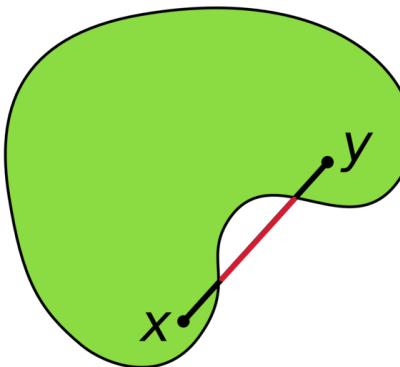
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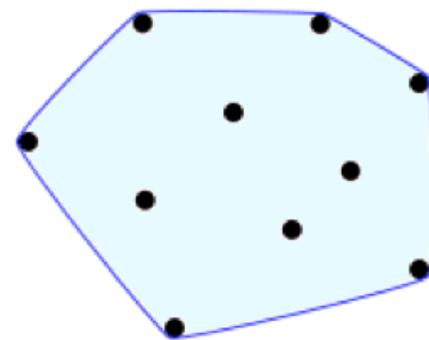
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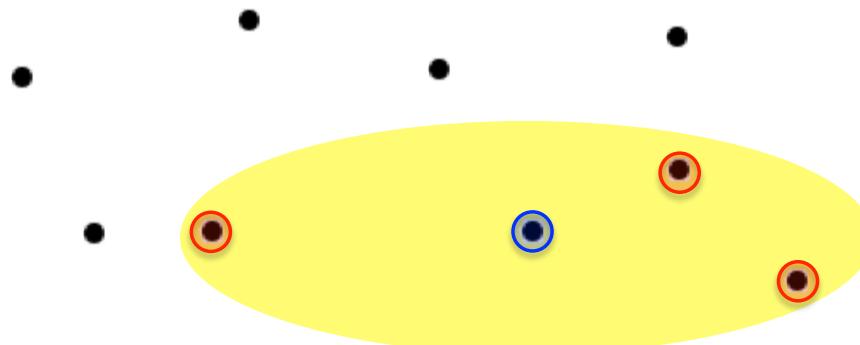
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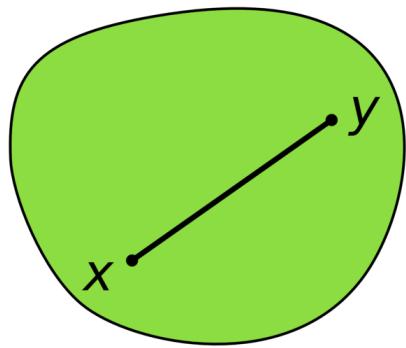


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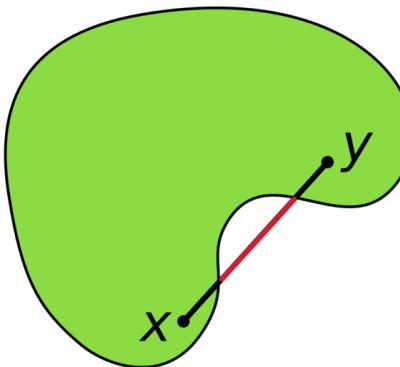
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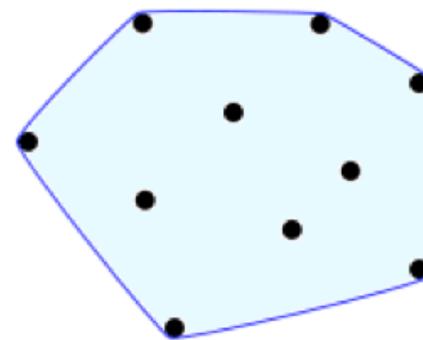
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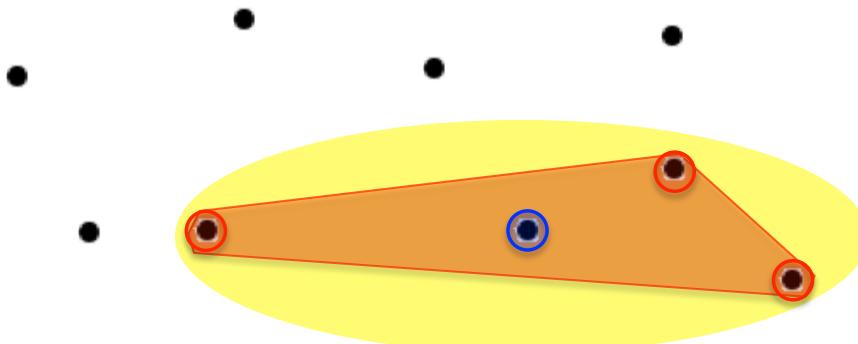
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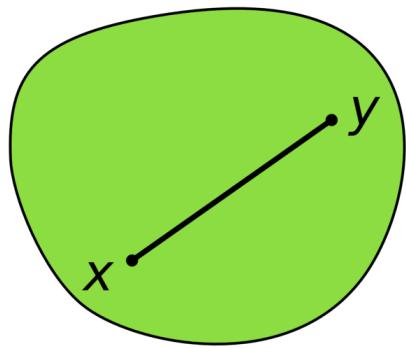


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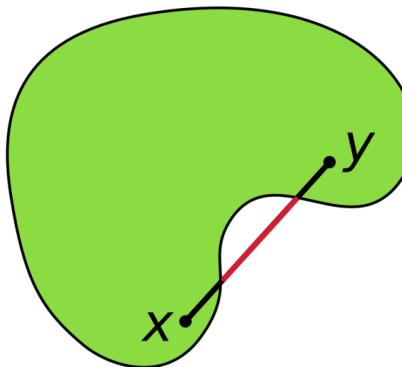
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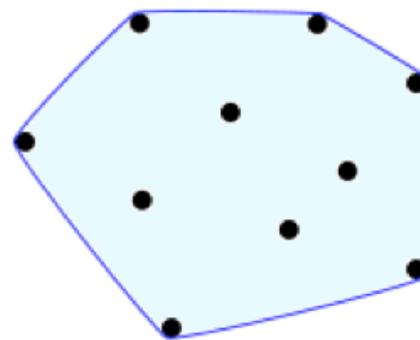
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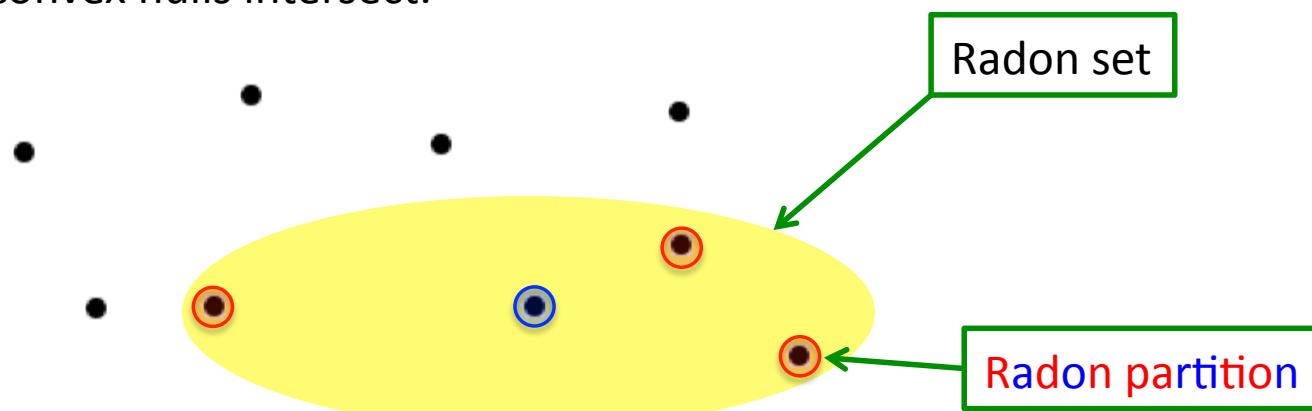
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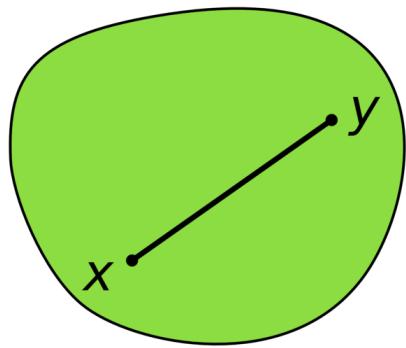


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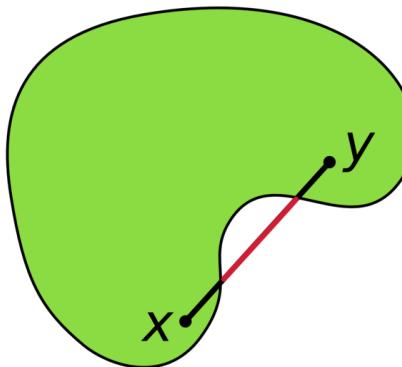
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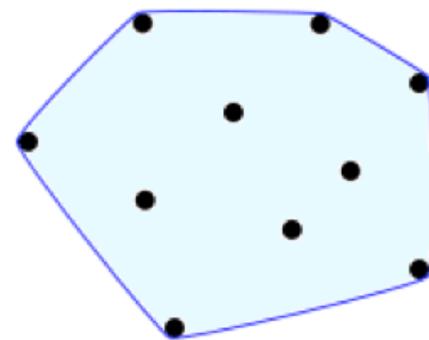
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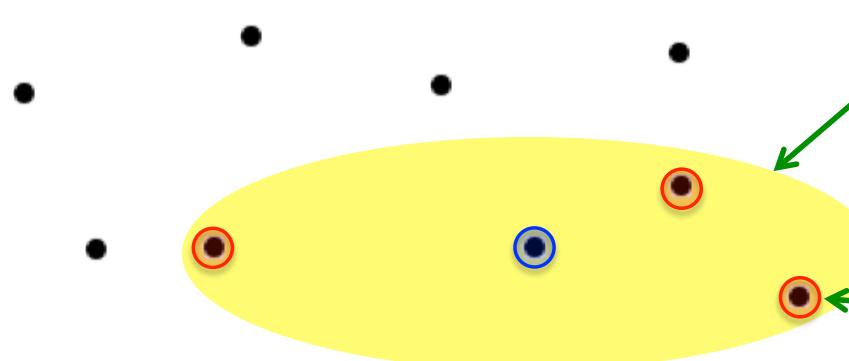


Radon's theorem (1921): every set of at least $d+2$ points in R^d can be partitioned into two sets whose convex hulls intersect.

Radon set

Radon number:
the smallest value r such that every set with at least r vertices is a Radon set

Radon partition

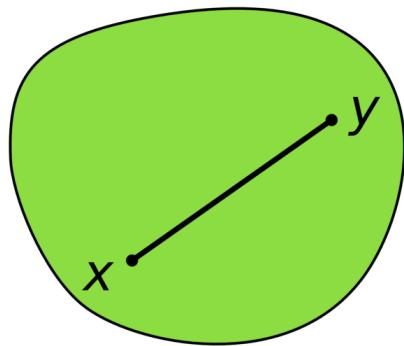


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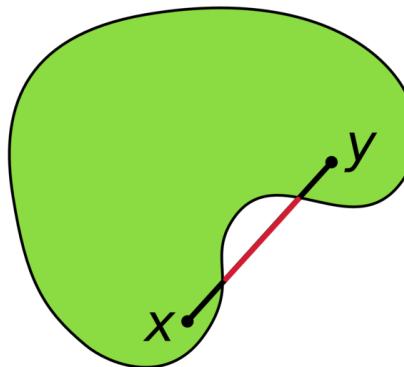
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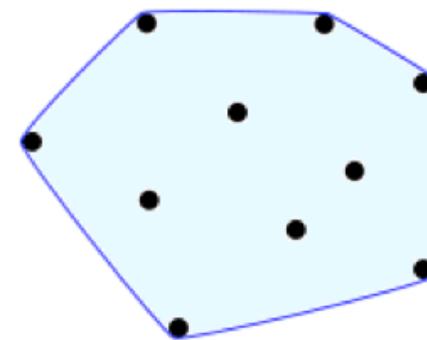
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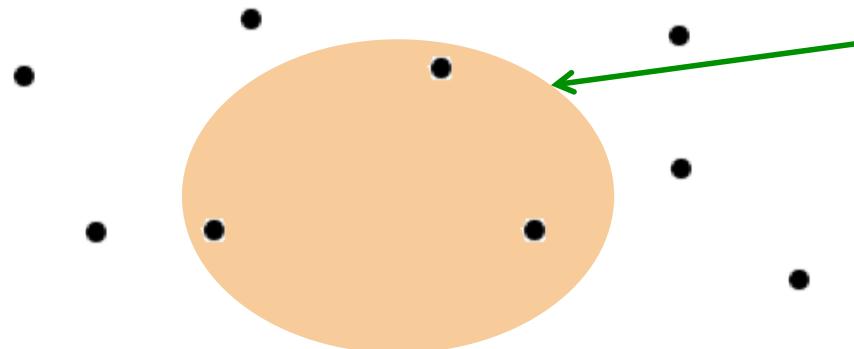
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Anti-Radon set

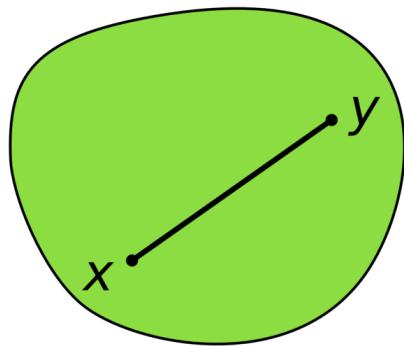
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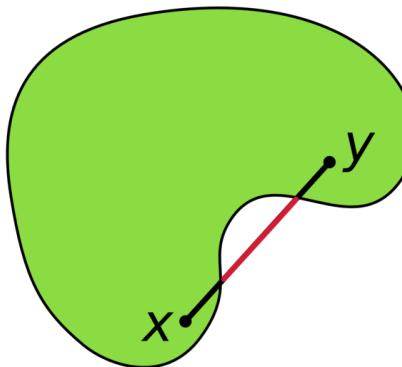
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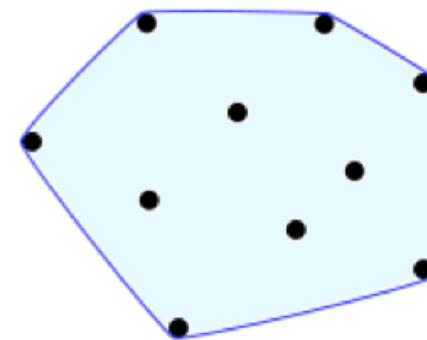
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Non-convex set

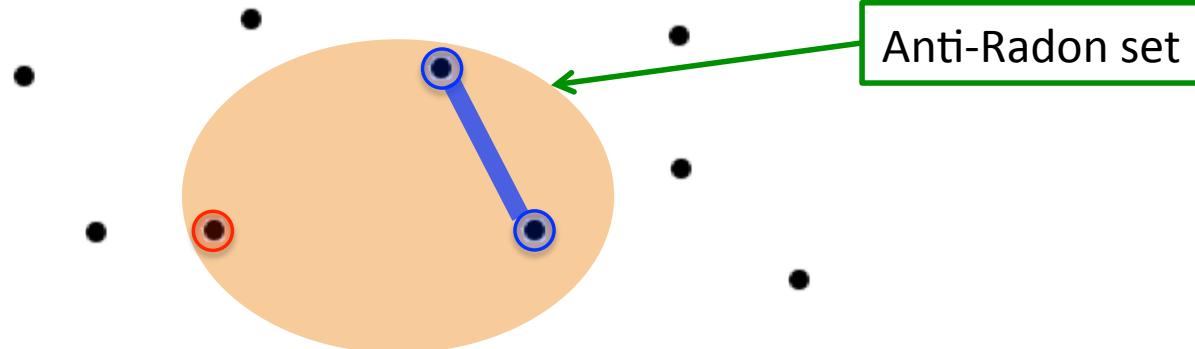


Convex hull



Radon's theorem (1921): every set of at least $d+2$ points in R^d can be partitioned into two sets whose convex hulls intersect.

Radon number:
the smallest value r such that every set with at least r vertices is a Radon set



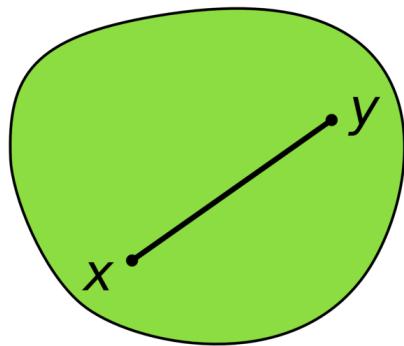
Anti-Radon set

Convexity in Euclidean space

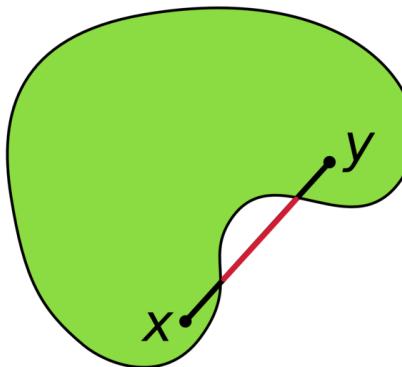
Ground set of the convexity space: the d -dimensional space R^d

Interval (x,y) = straight line segment between x and y

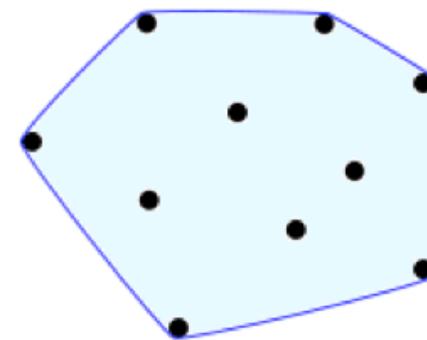
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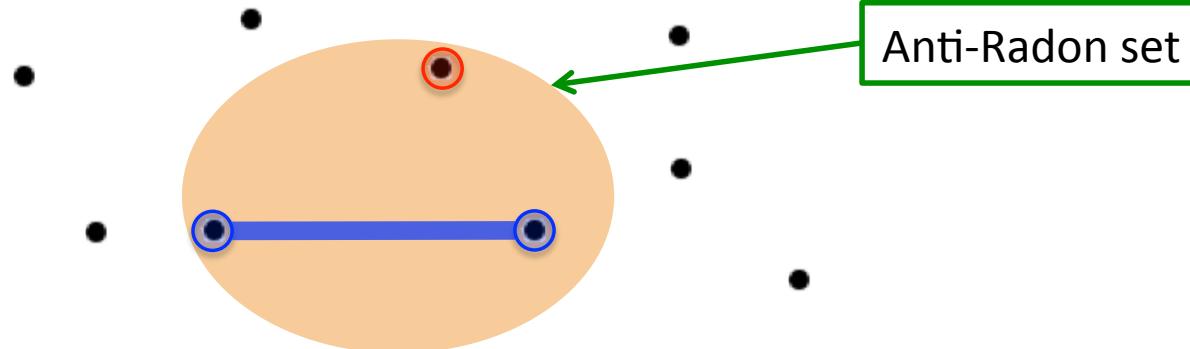
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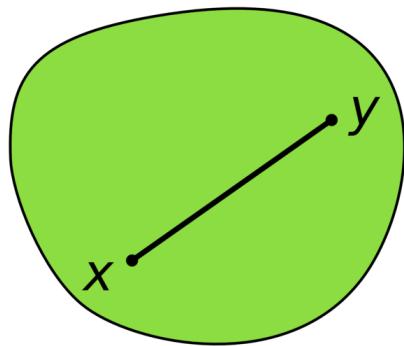
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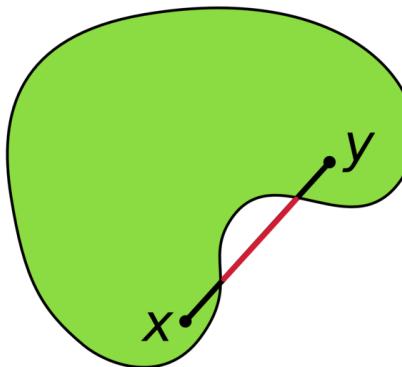
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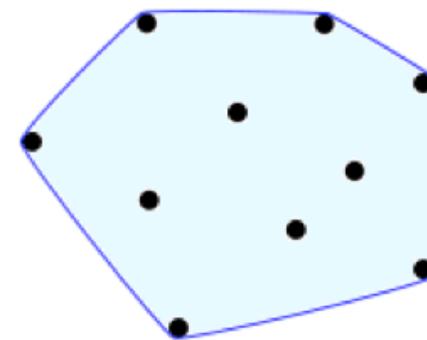
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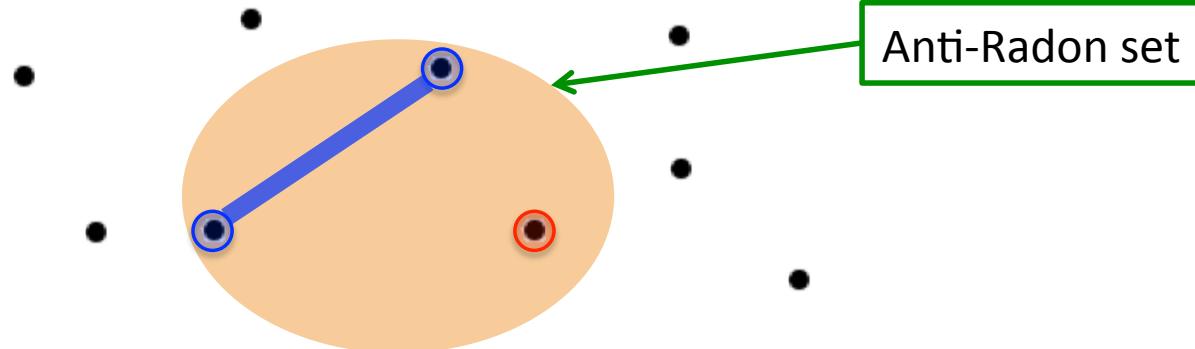


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Anti-Radon set

Geodetic convexity in graphs

Ground set of the convexity space: vertices V of some connected graph $G(V, E)$

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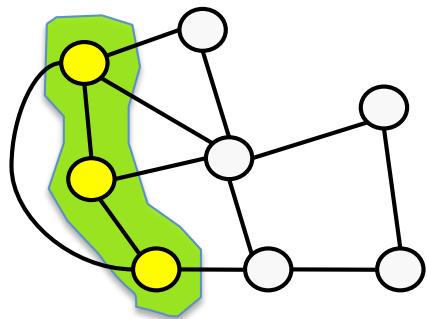
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Geodetic convexity in graphs

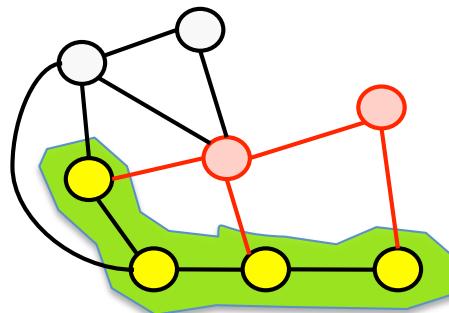
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Convex subset of V



Non-convex subset of V

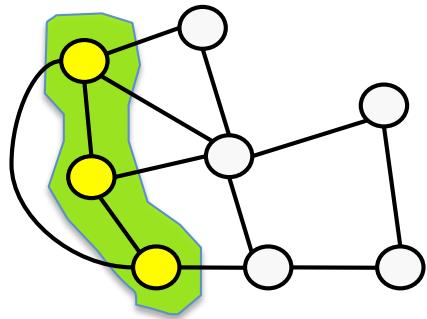


Geodetic convexity in graphs

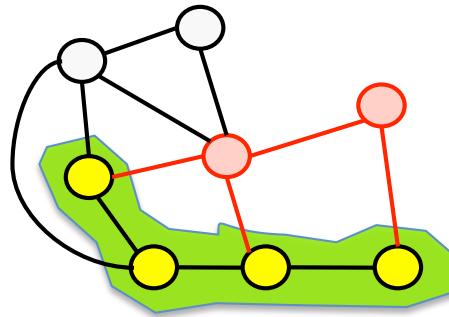
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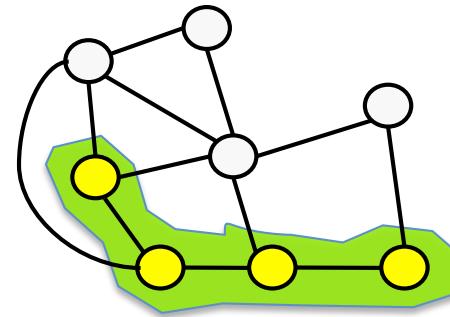
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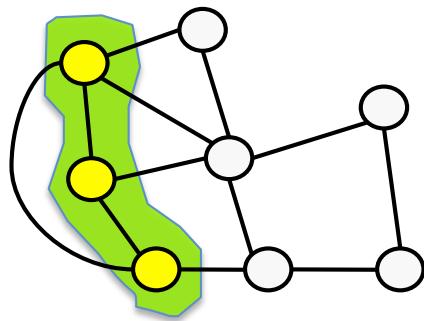


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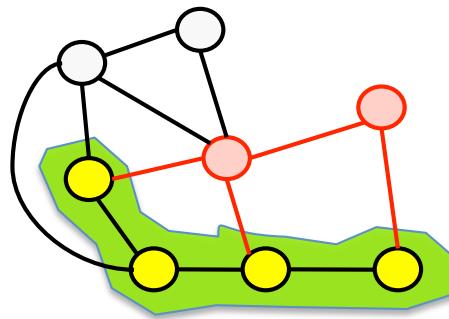
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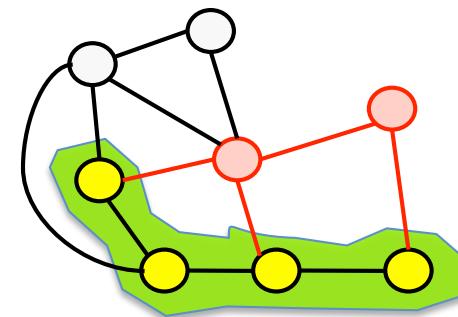
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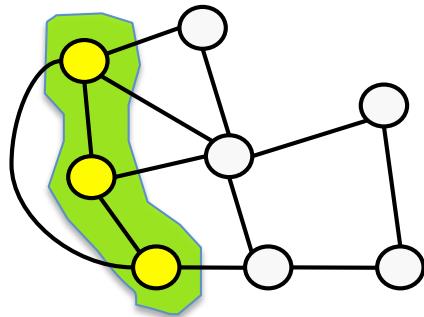


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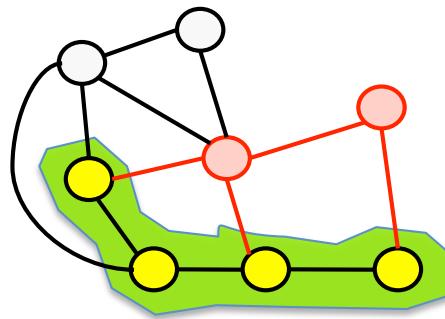
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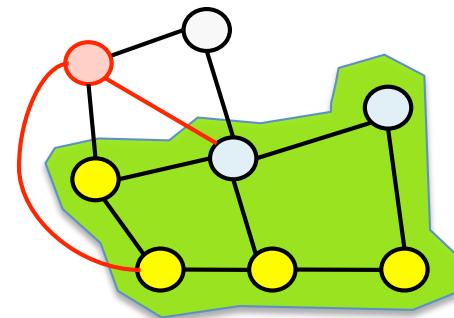
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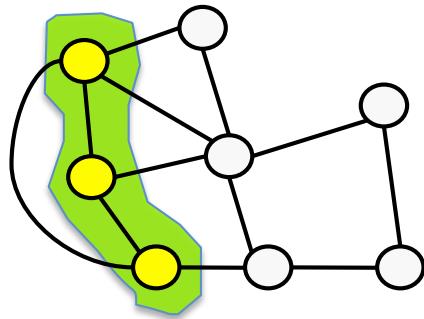


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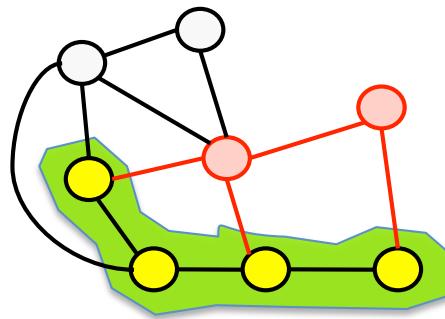
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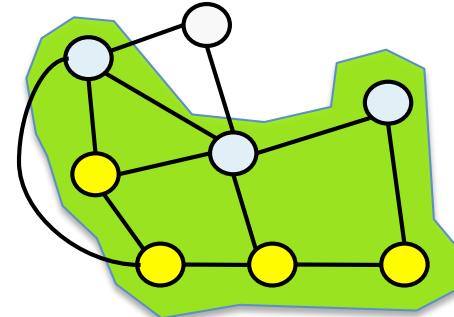
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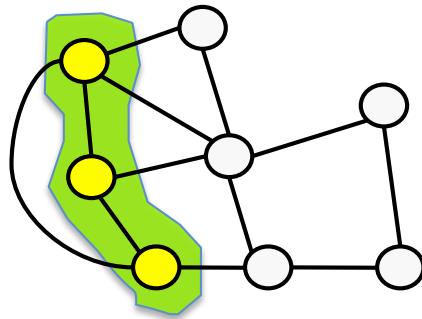


Geodetic convexity in graphs

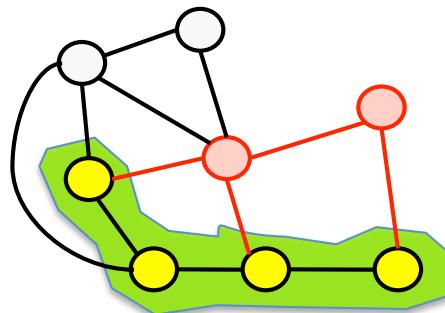
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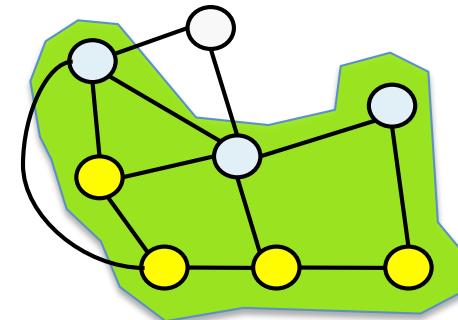
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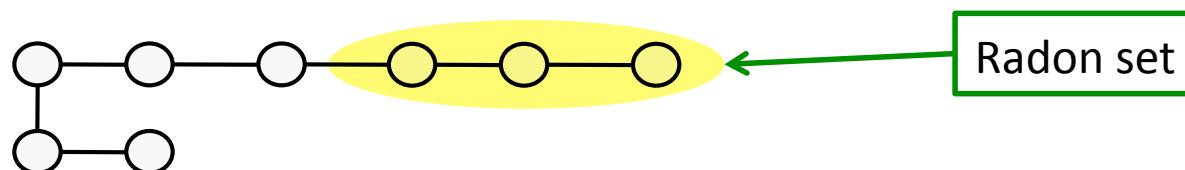


Convex hull



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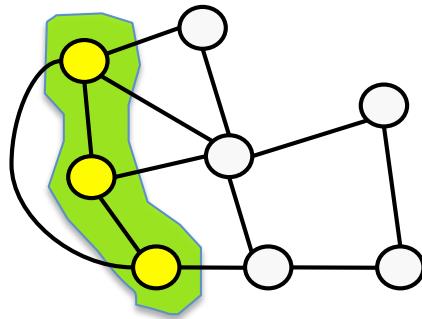


Ex.: a path

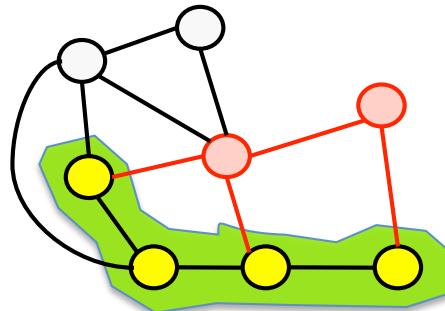
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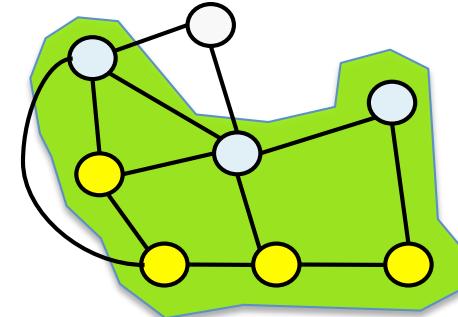
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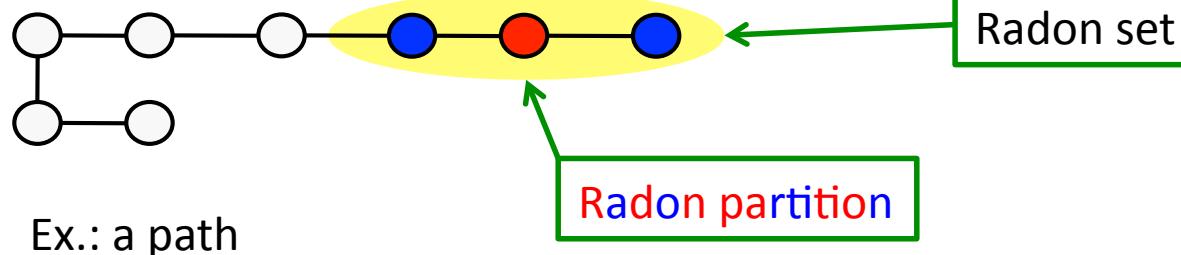


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Geodetic convexity in graphs

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Input: Graph $G (V,E)$

Output: the smallest r such that every subset S of V , $|S| \geq r$, is a Radon set.

or equivalently...

ANTI-RADON SET:

Input: Graph $G (V,E)$

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Geodetic convexity in graphs

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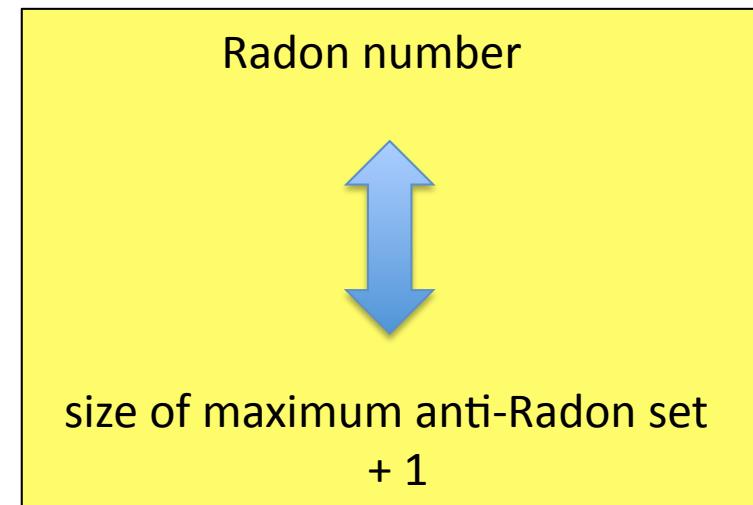
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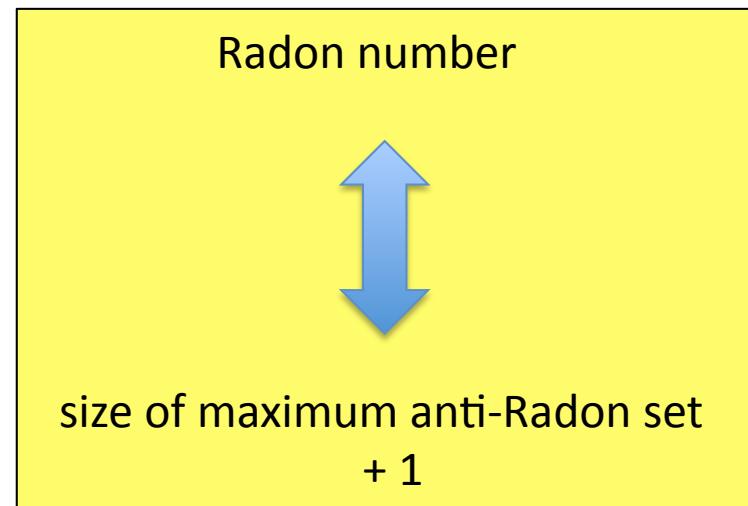
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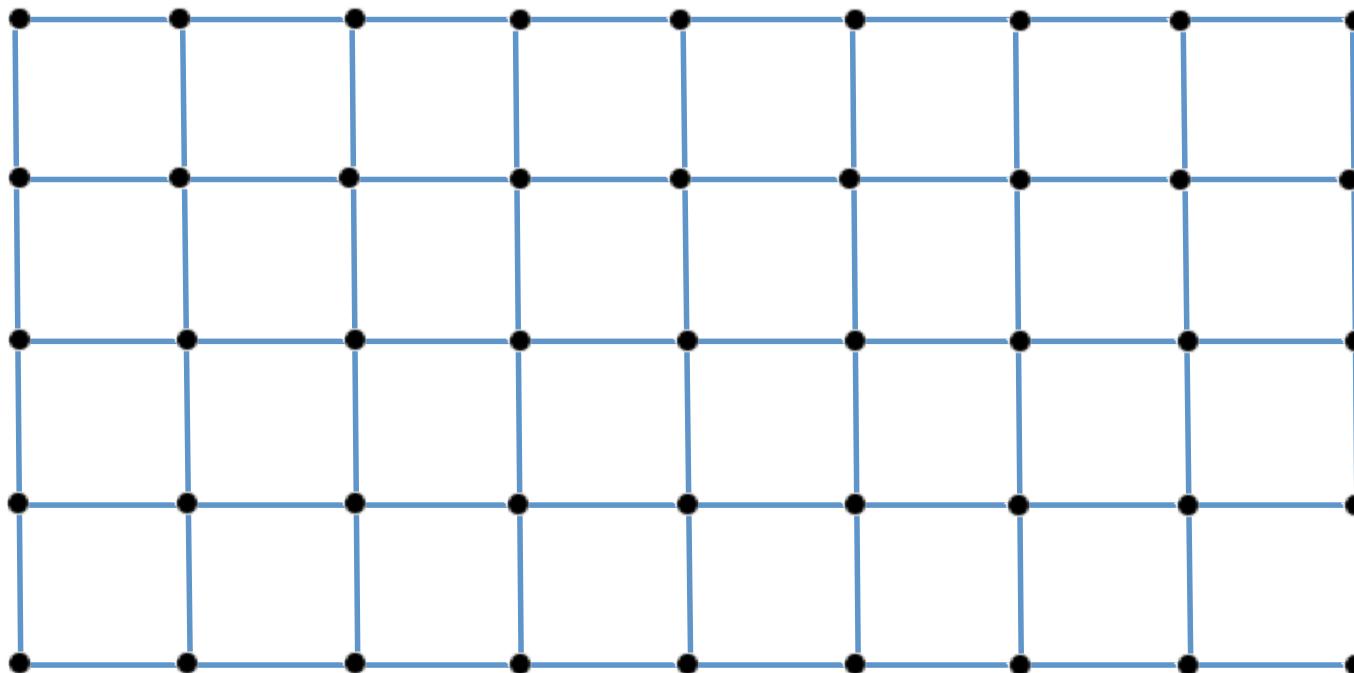
NP-hard, even for bipartite graphs.



Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

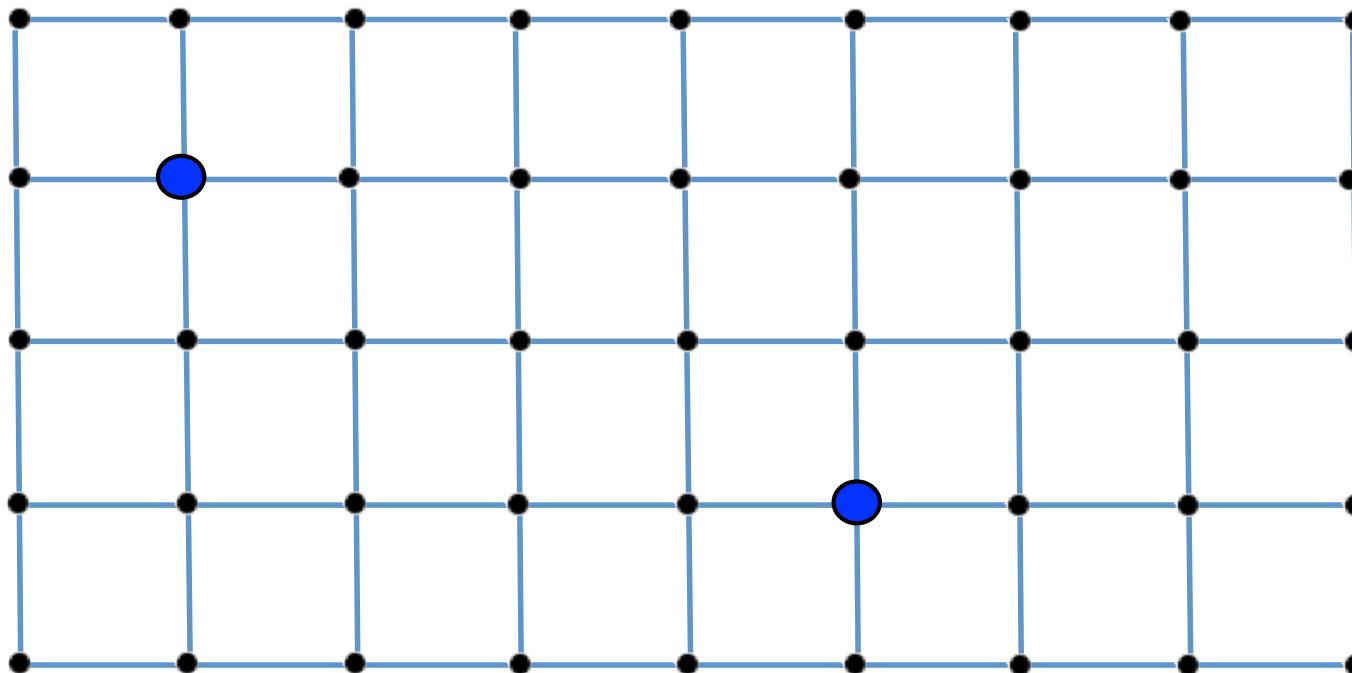
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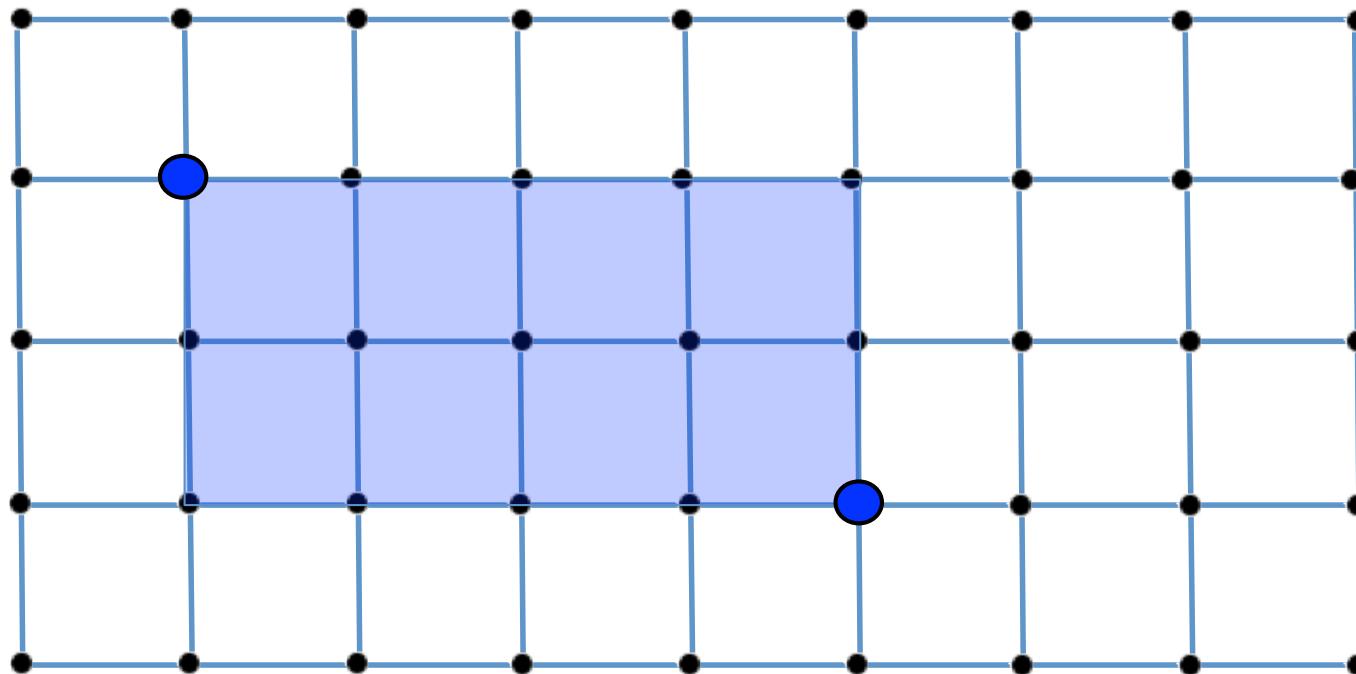
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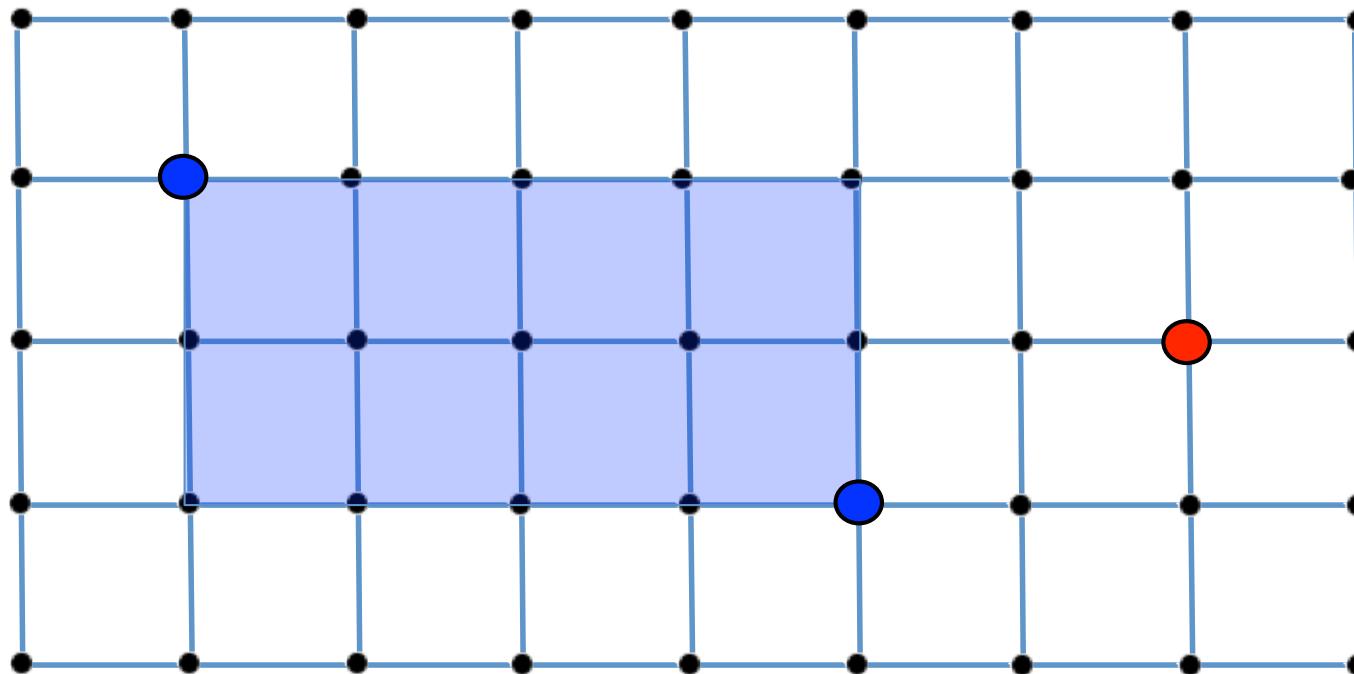
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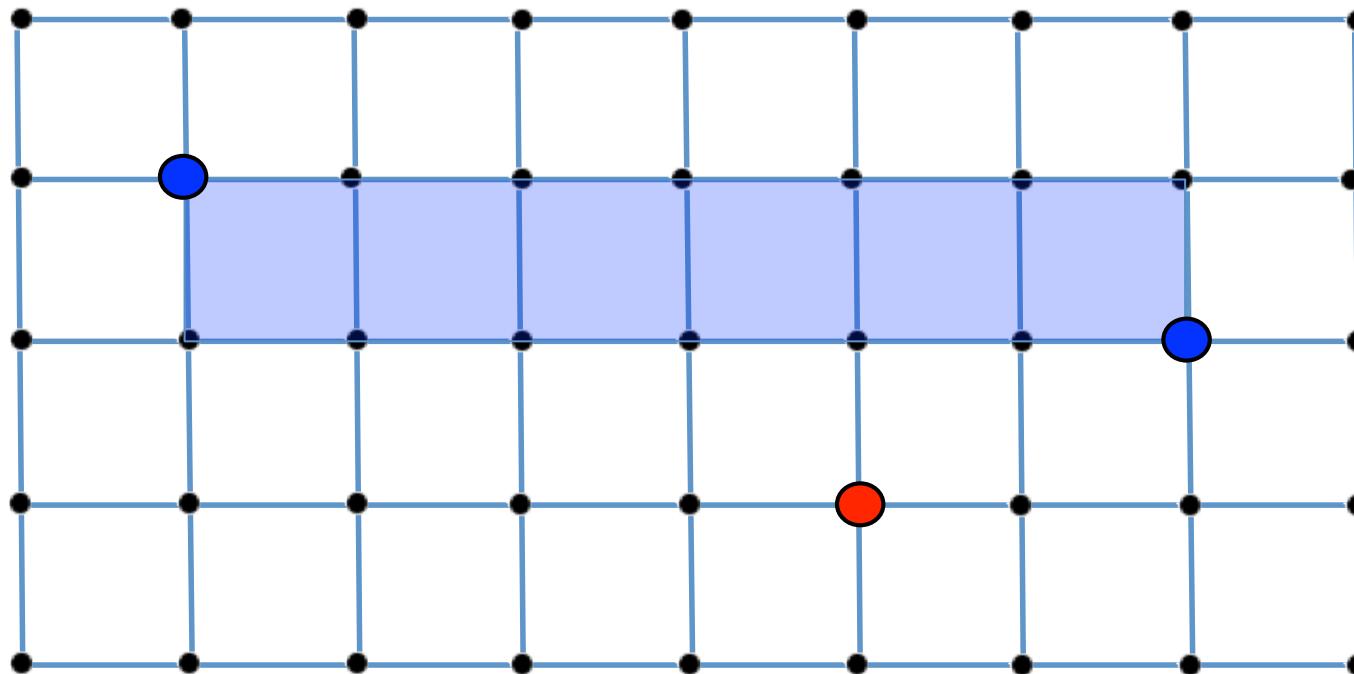
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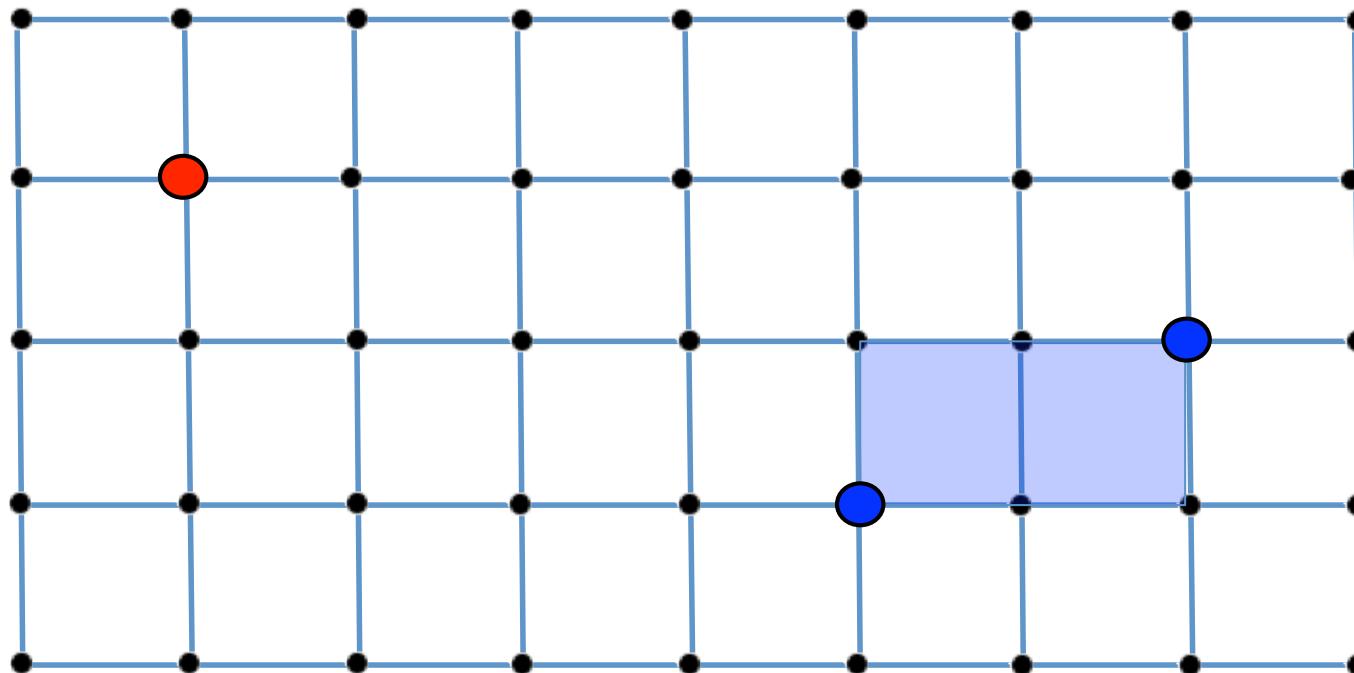
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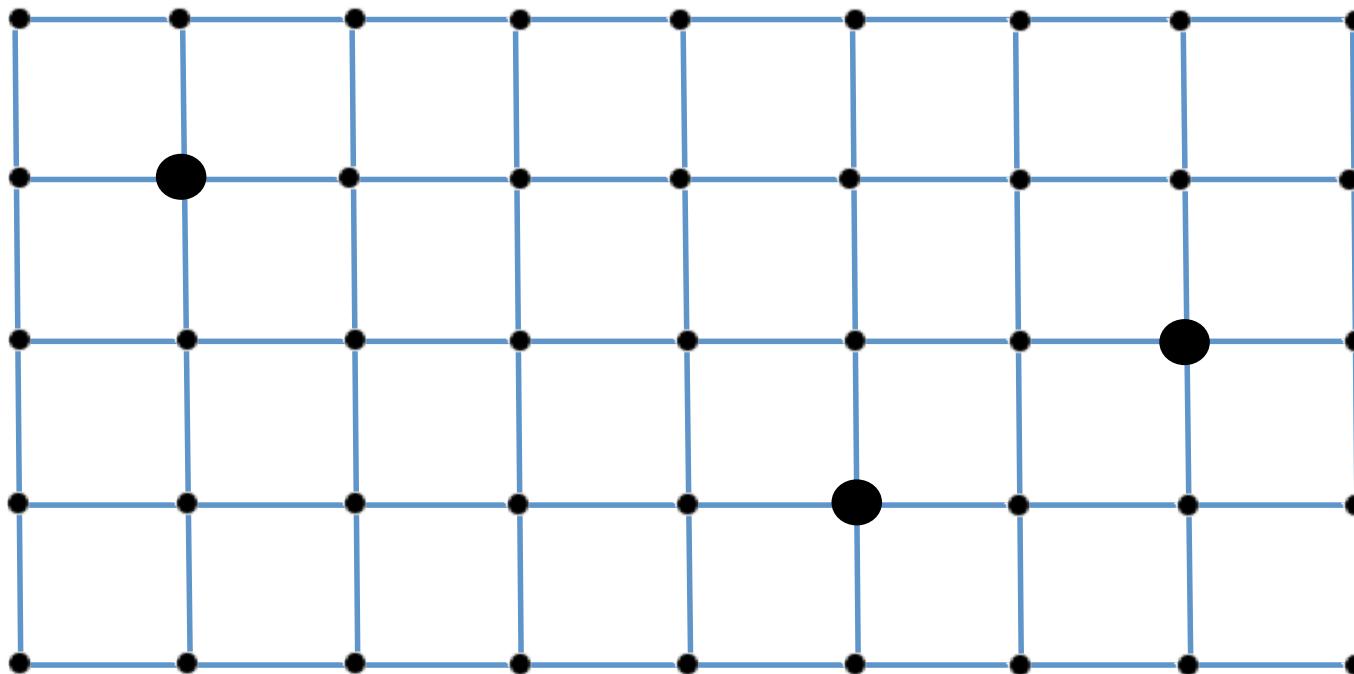
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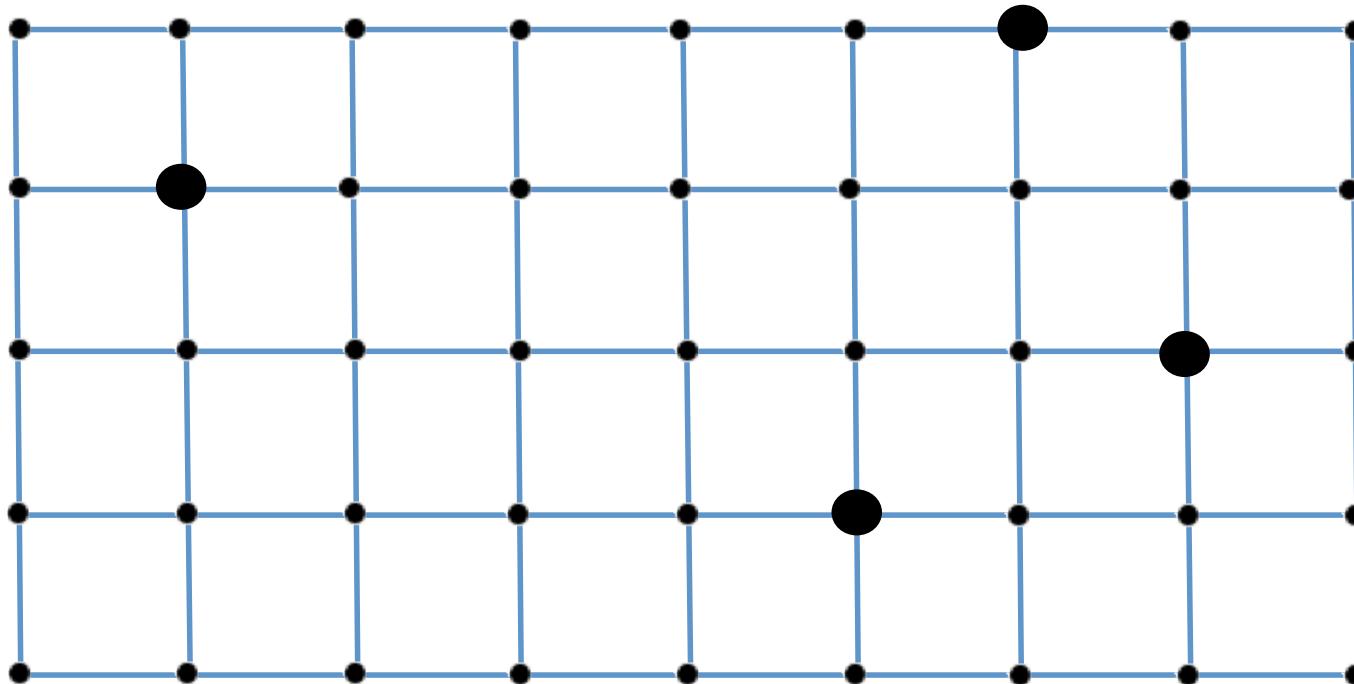


Anti-Radon of size 3 \Rightarrow Radon number ≥ 4

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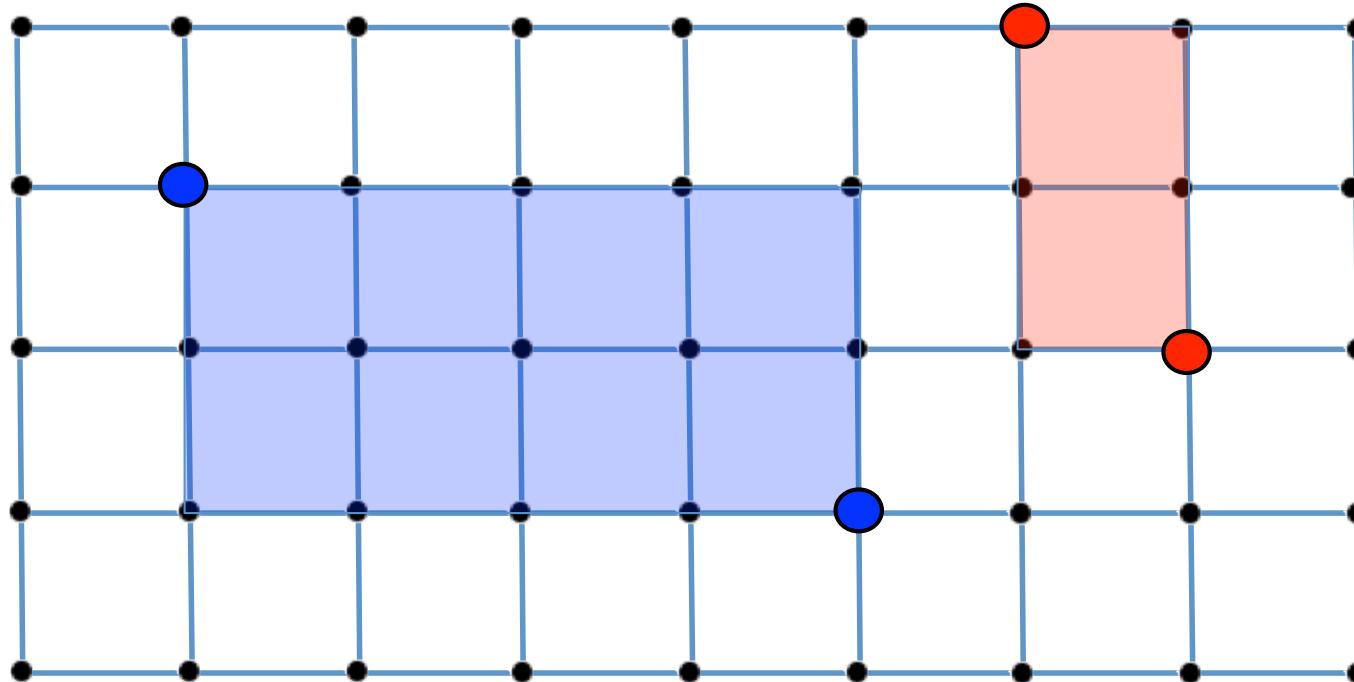
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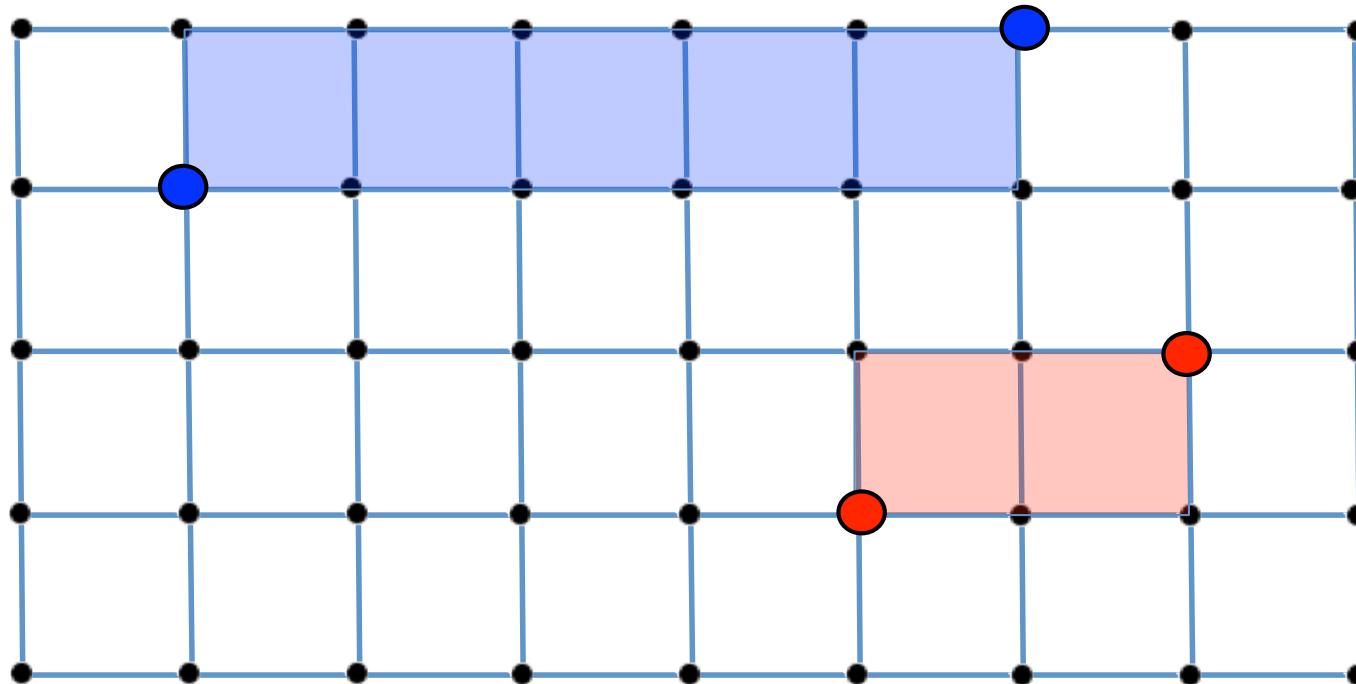
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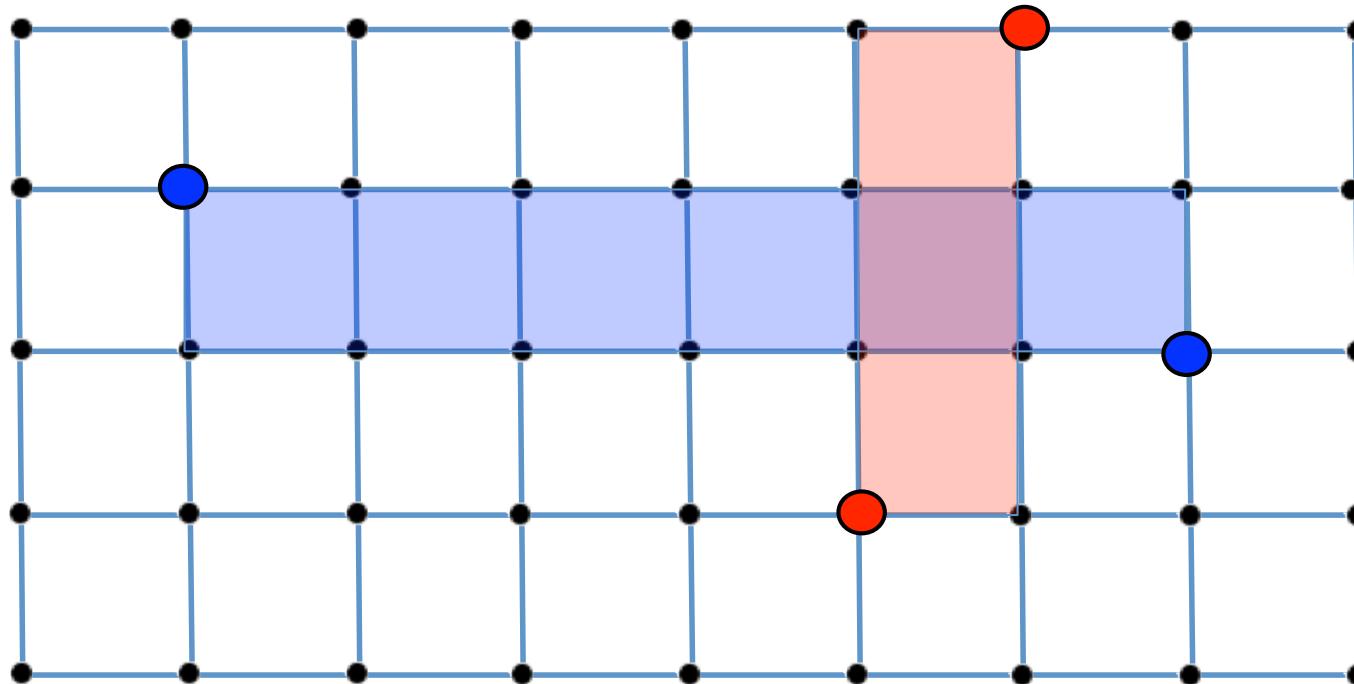
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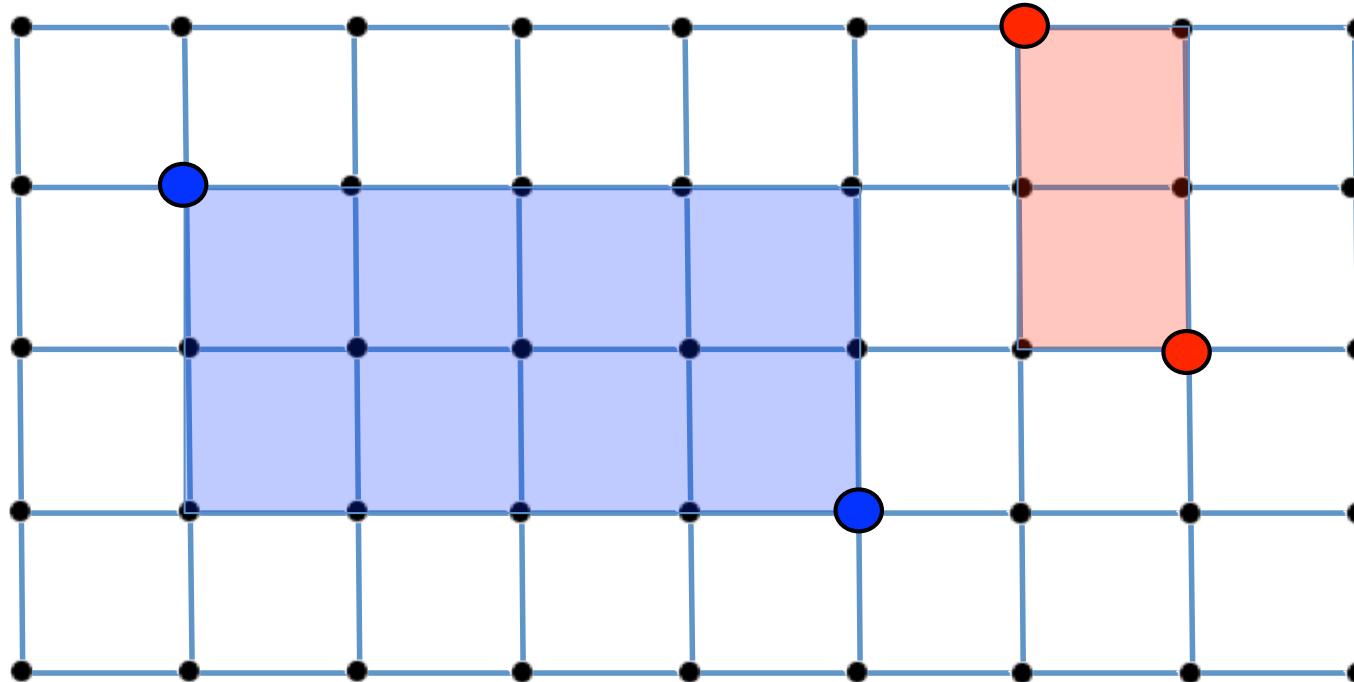
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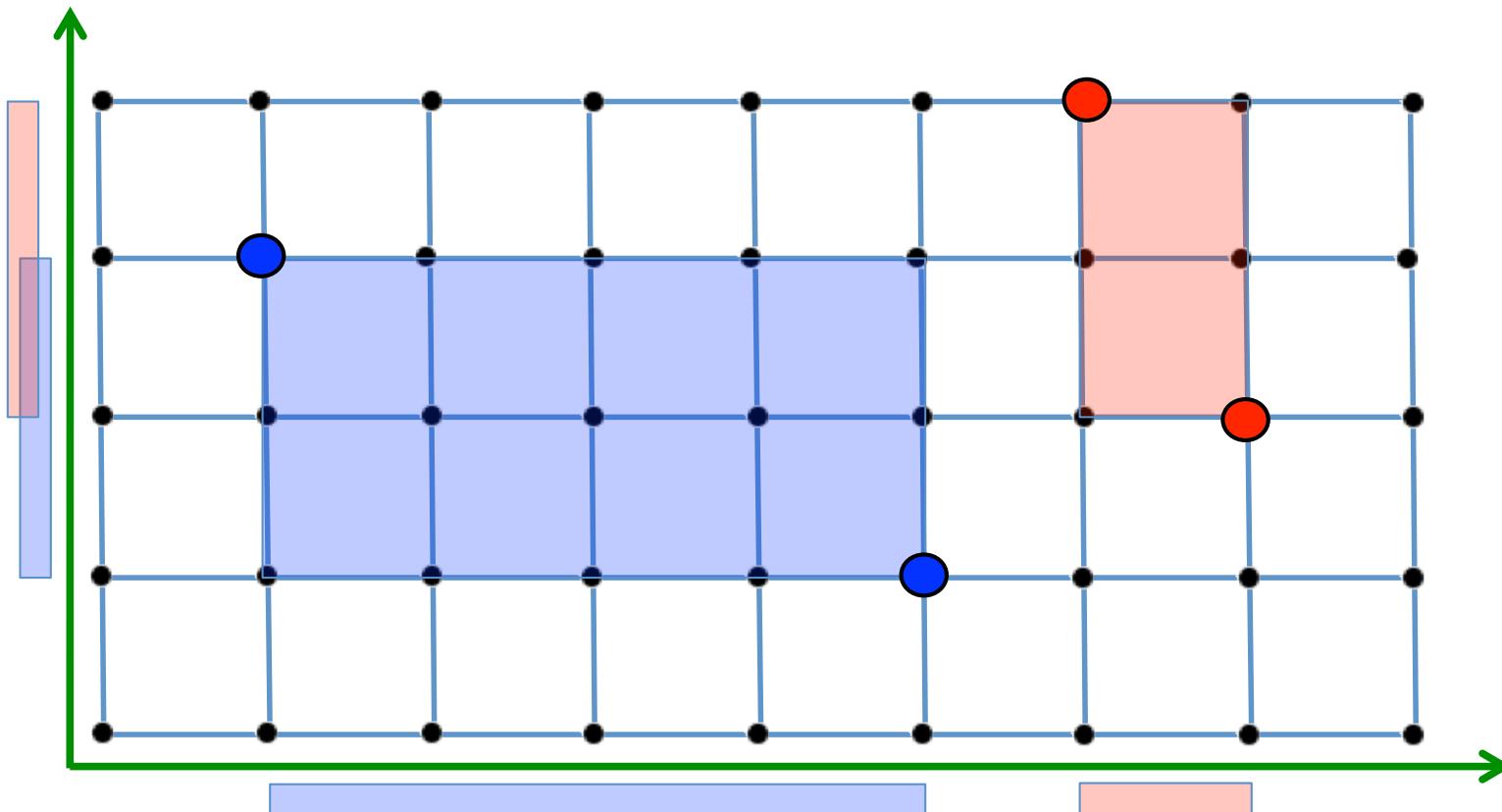
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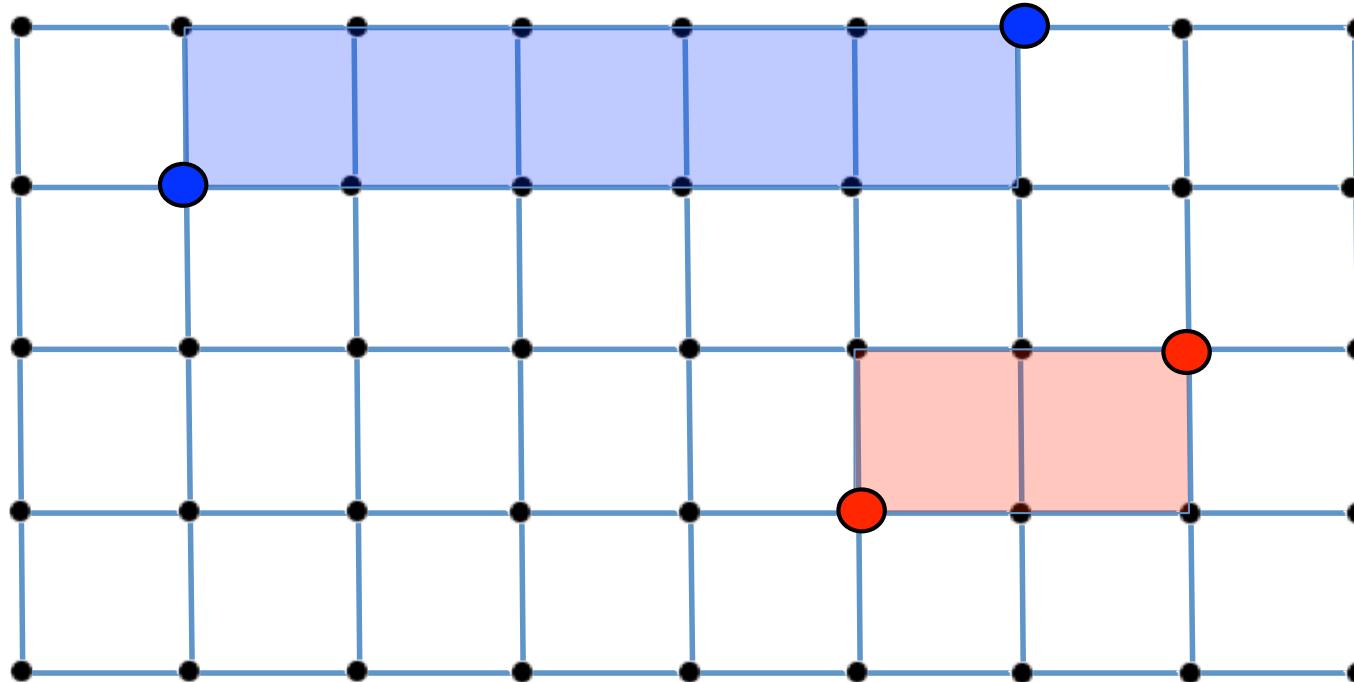
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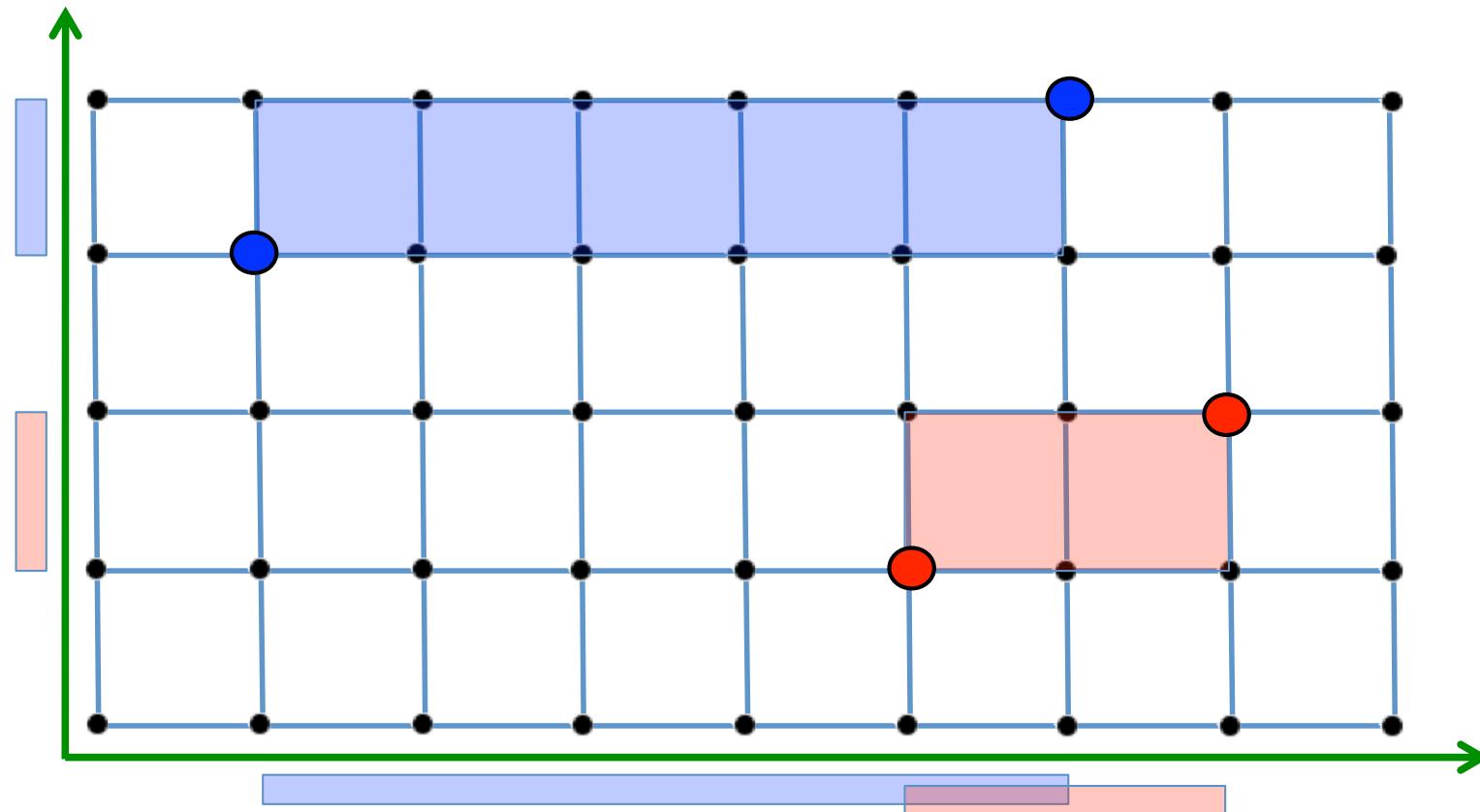
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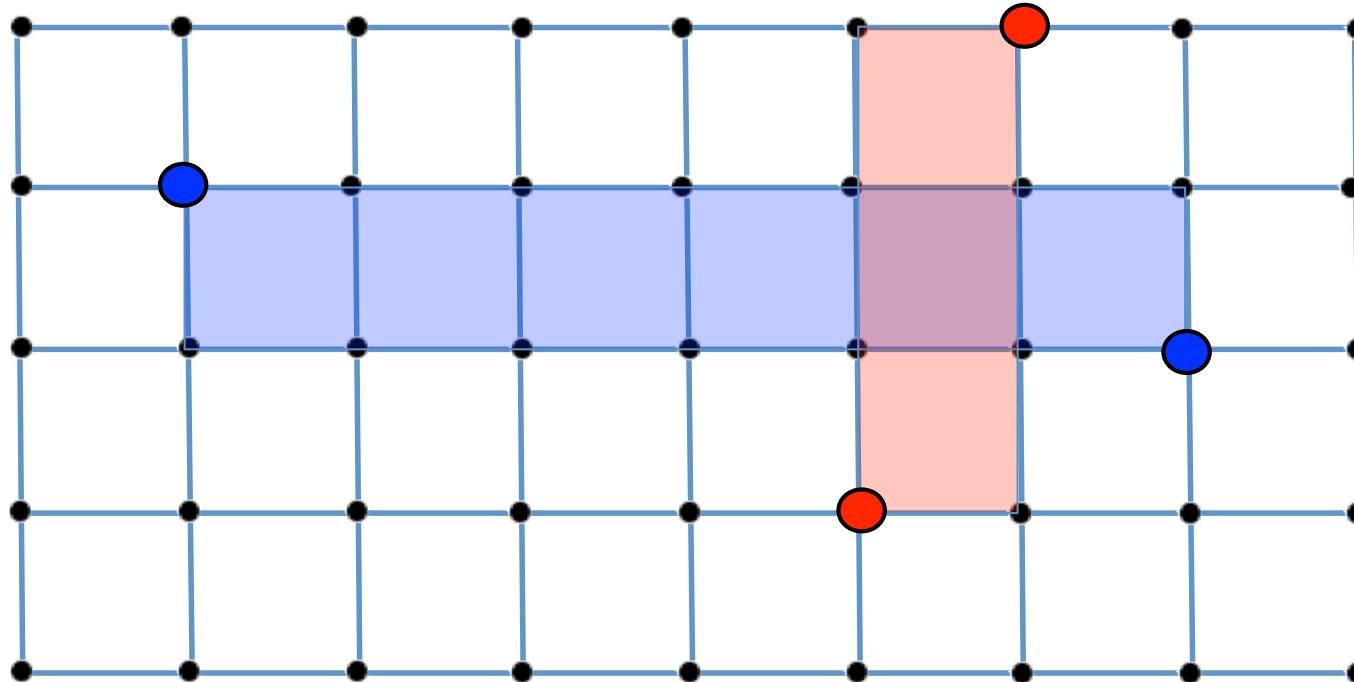
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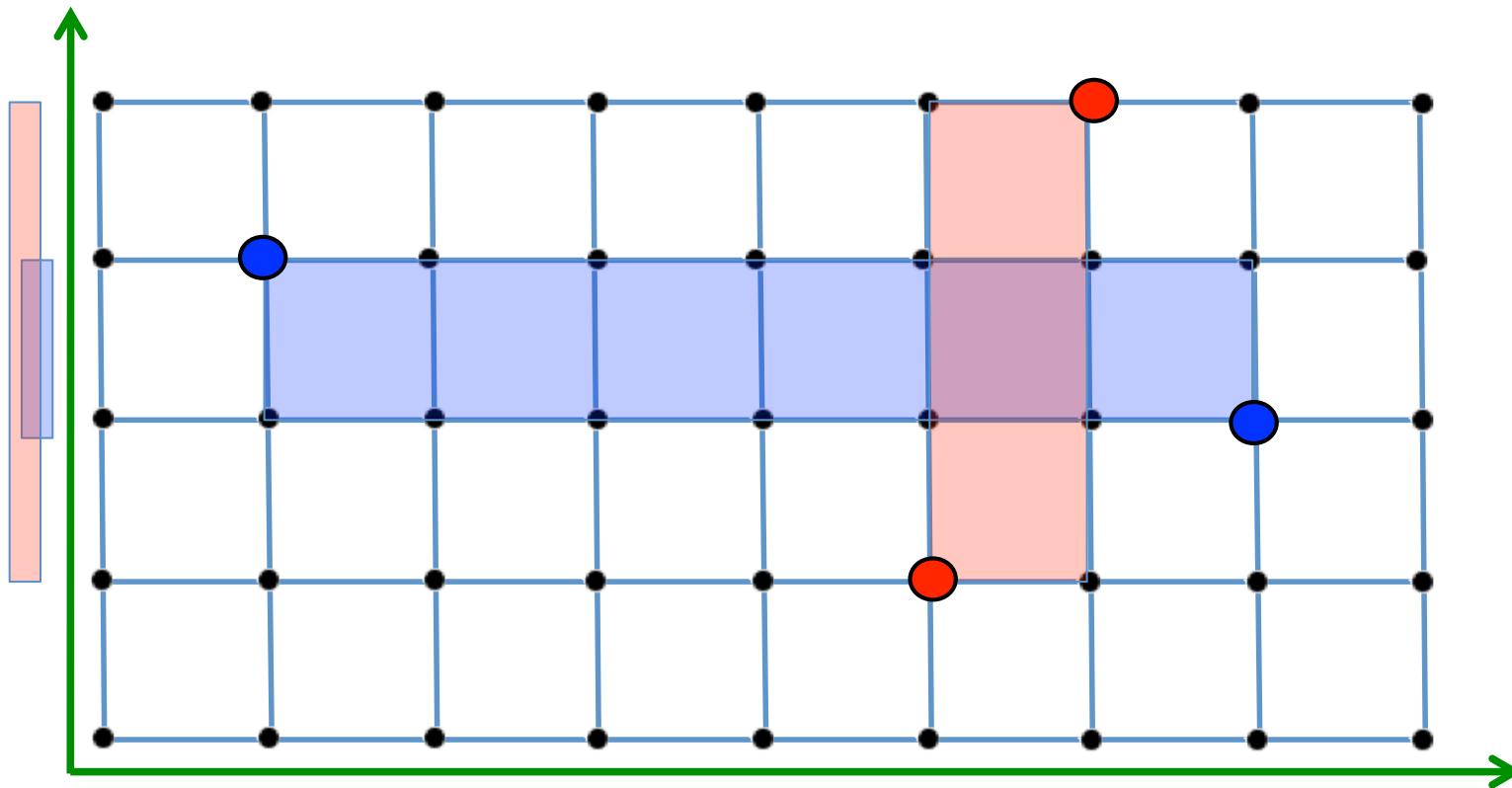
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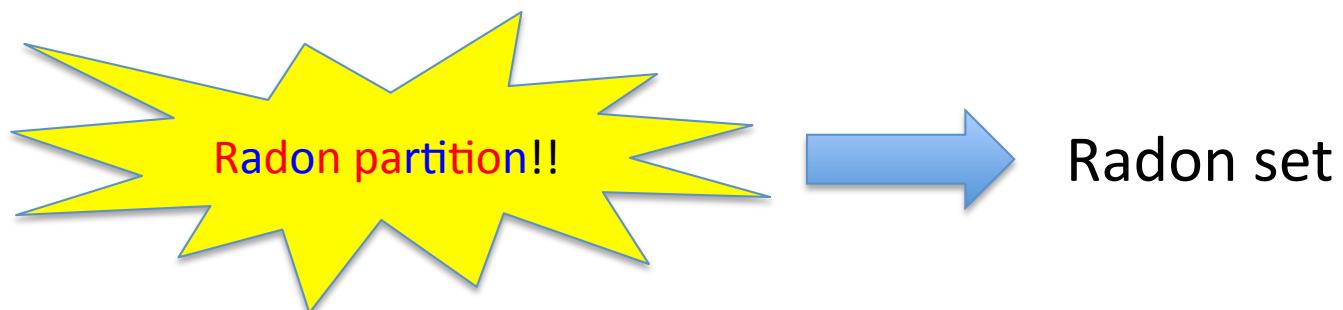
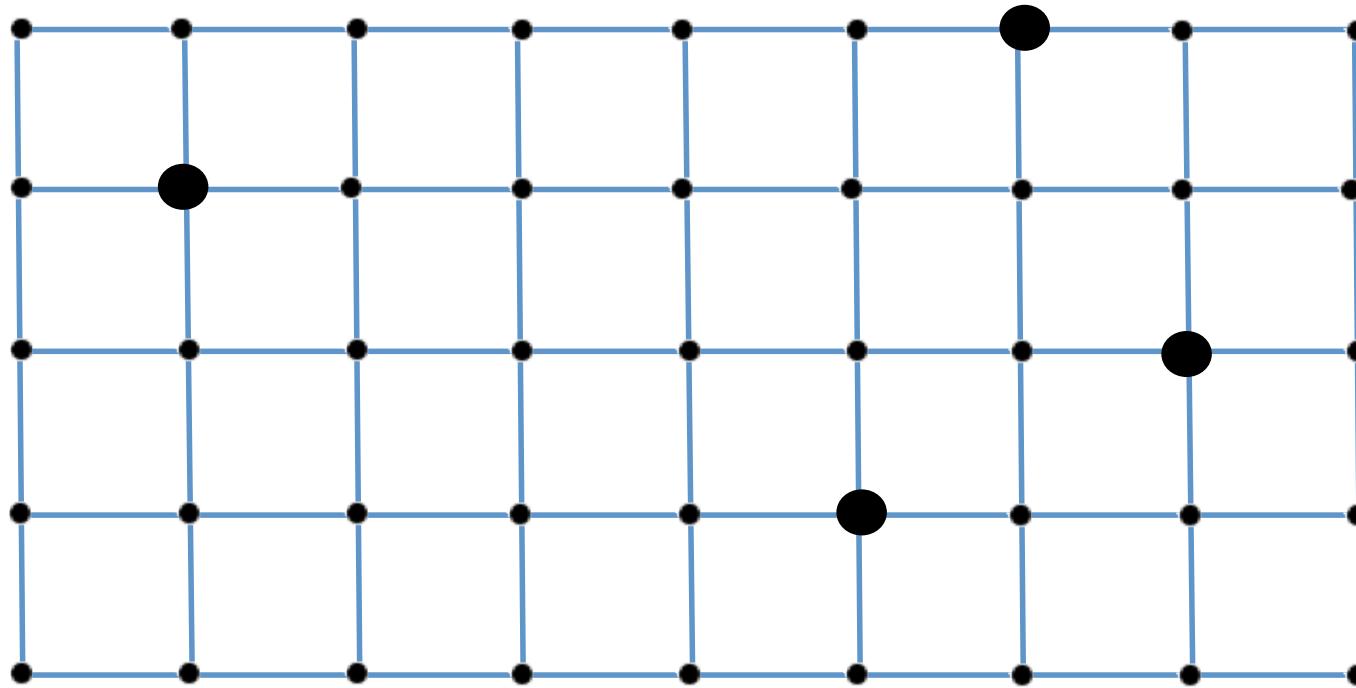
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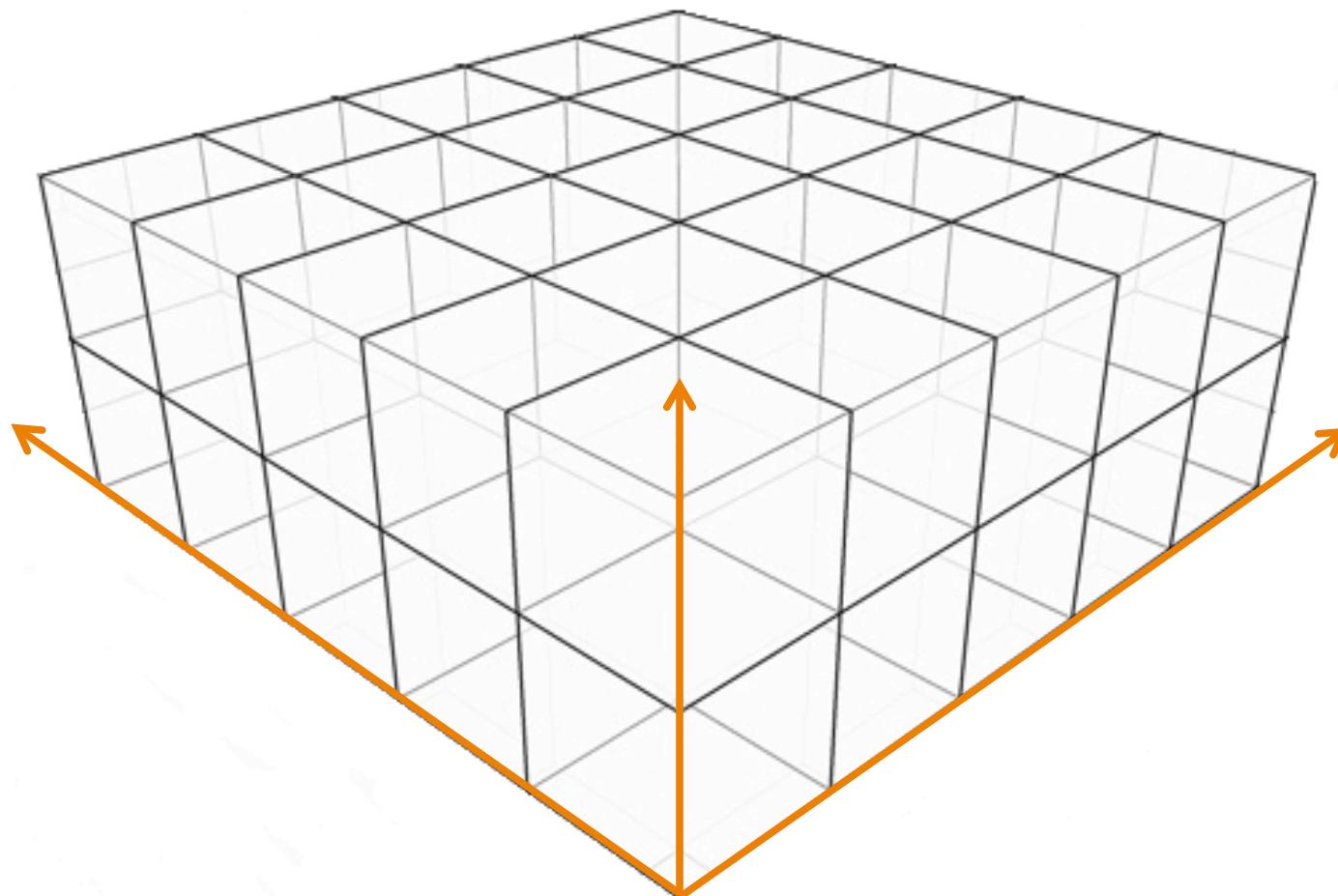
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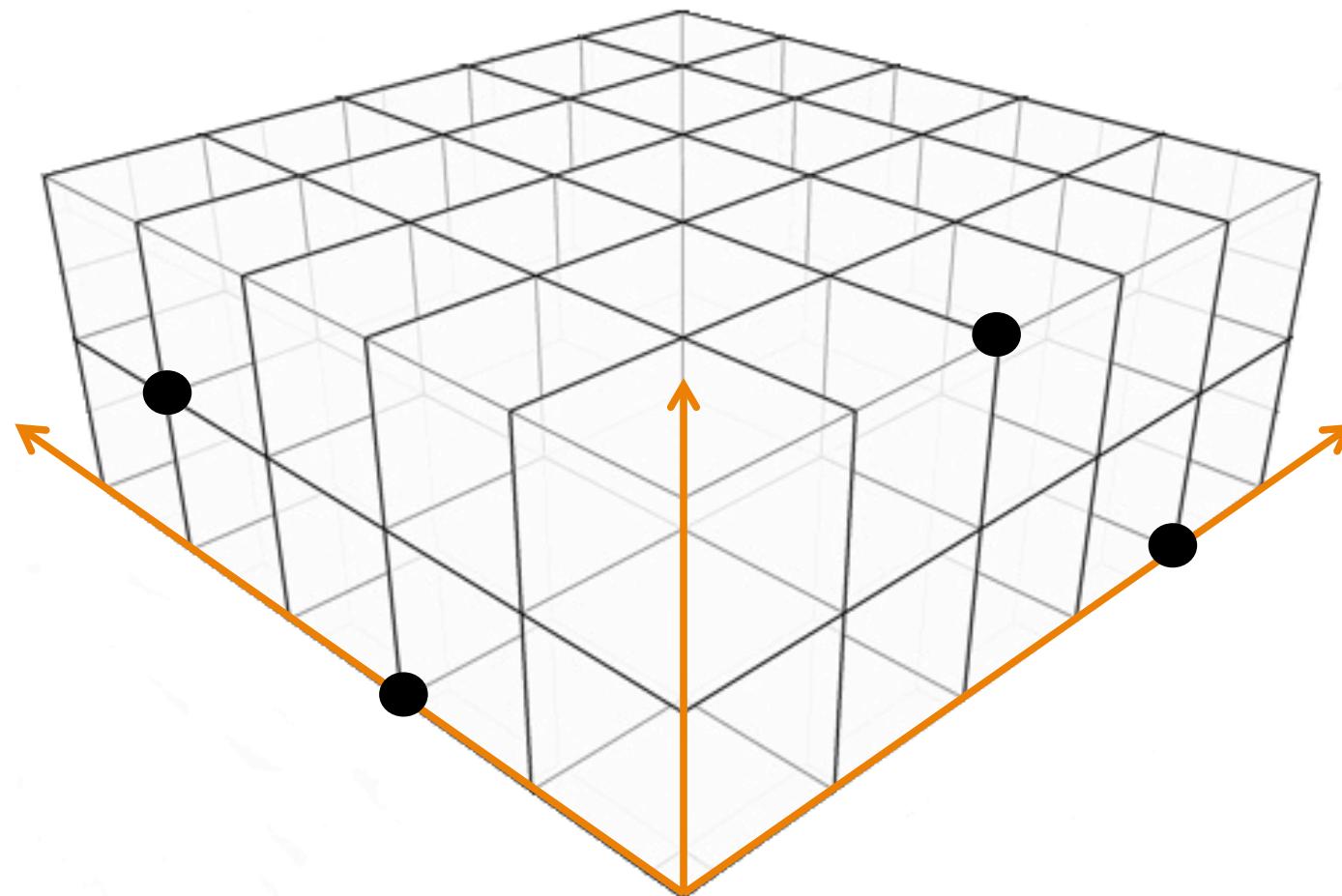
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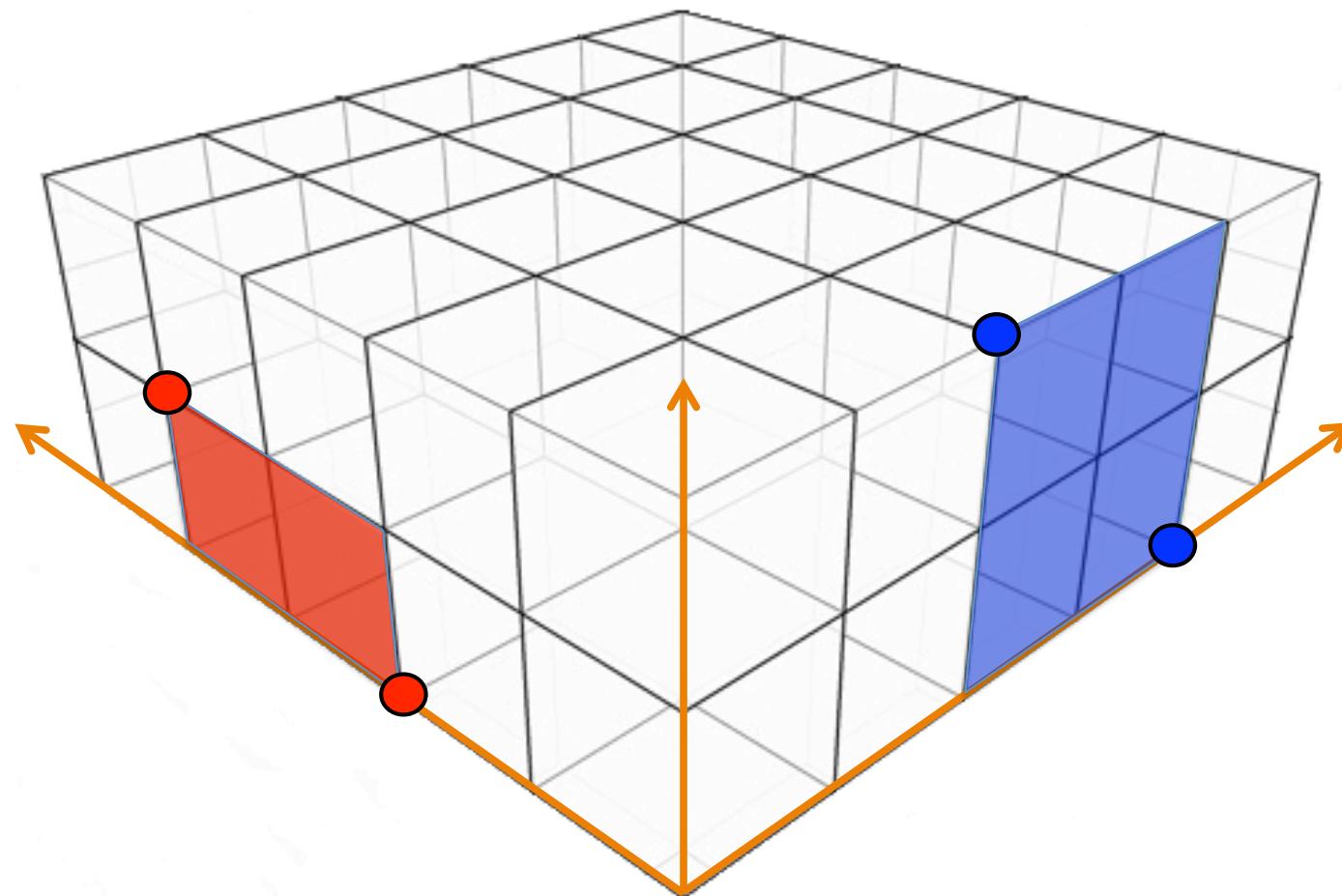
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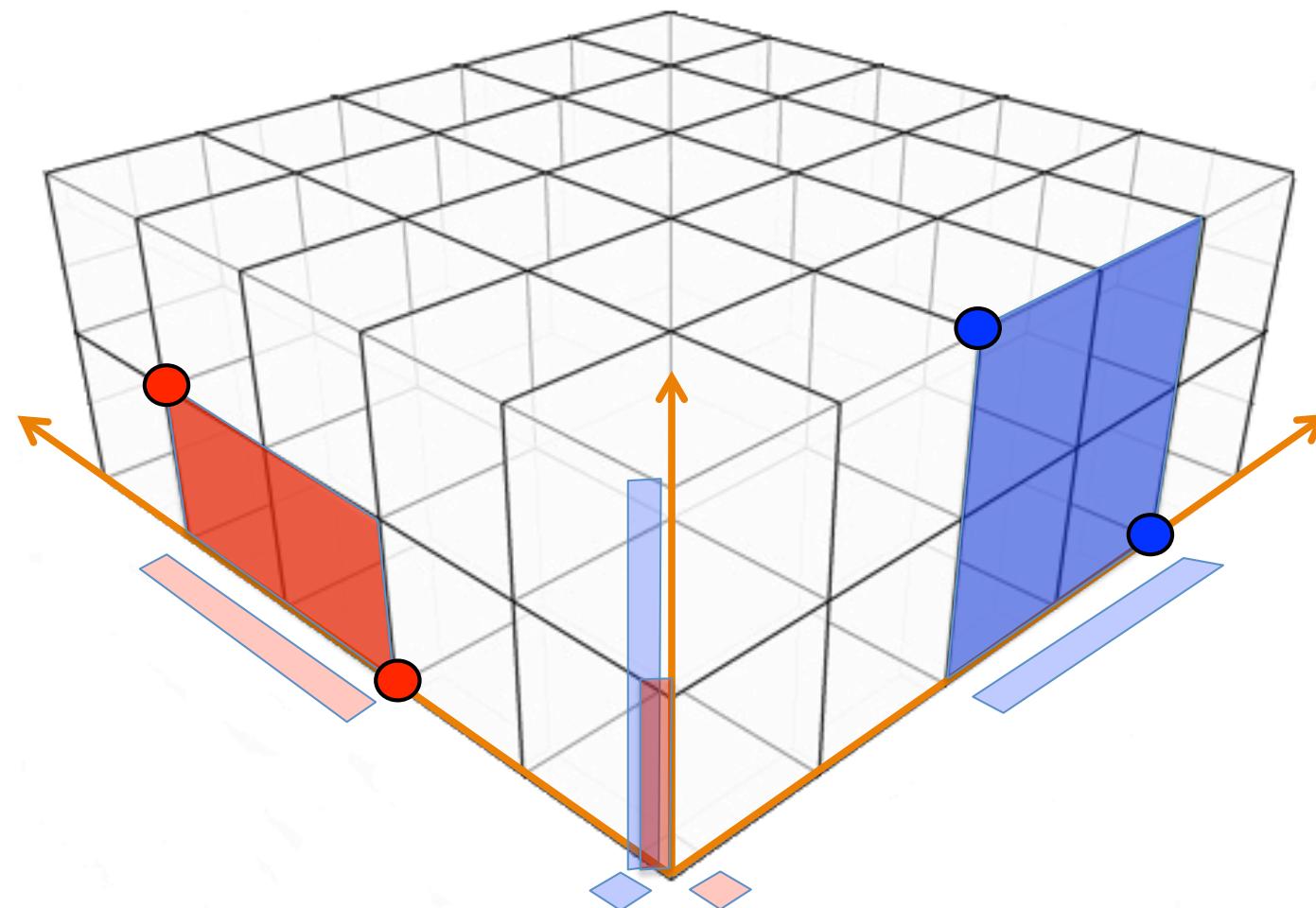
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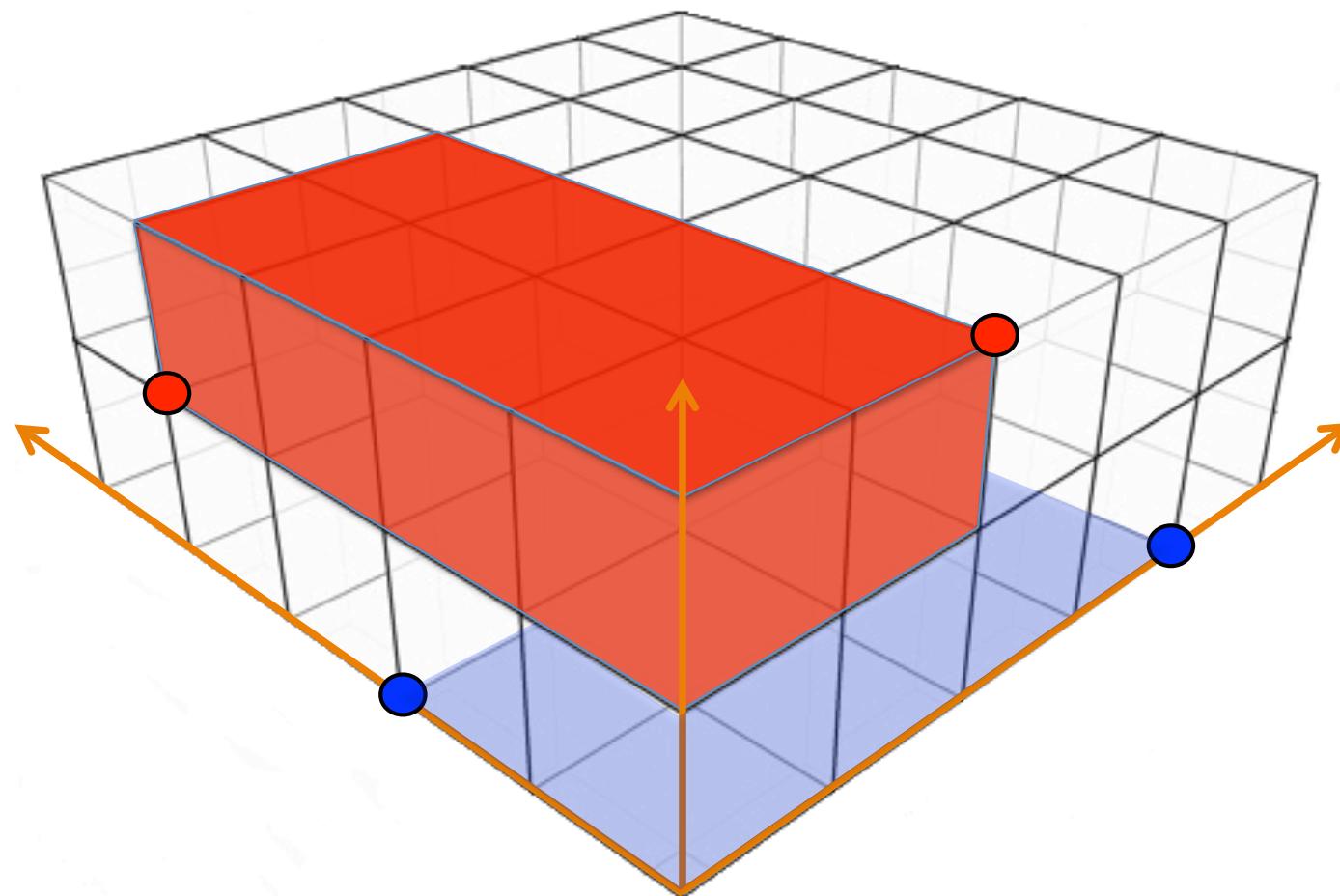
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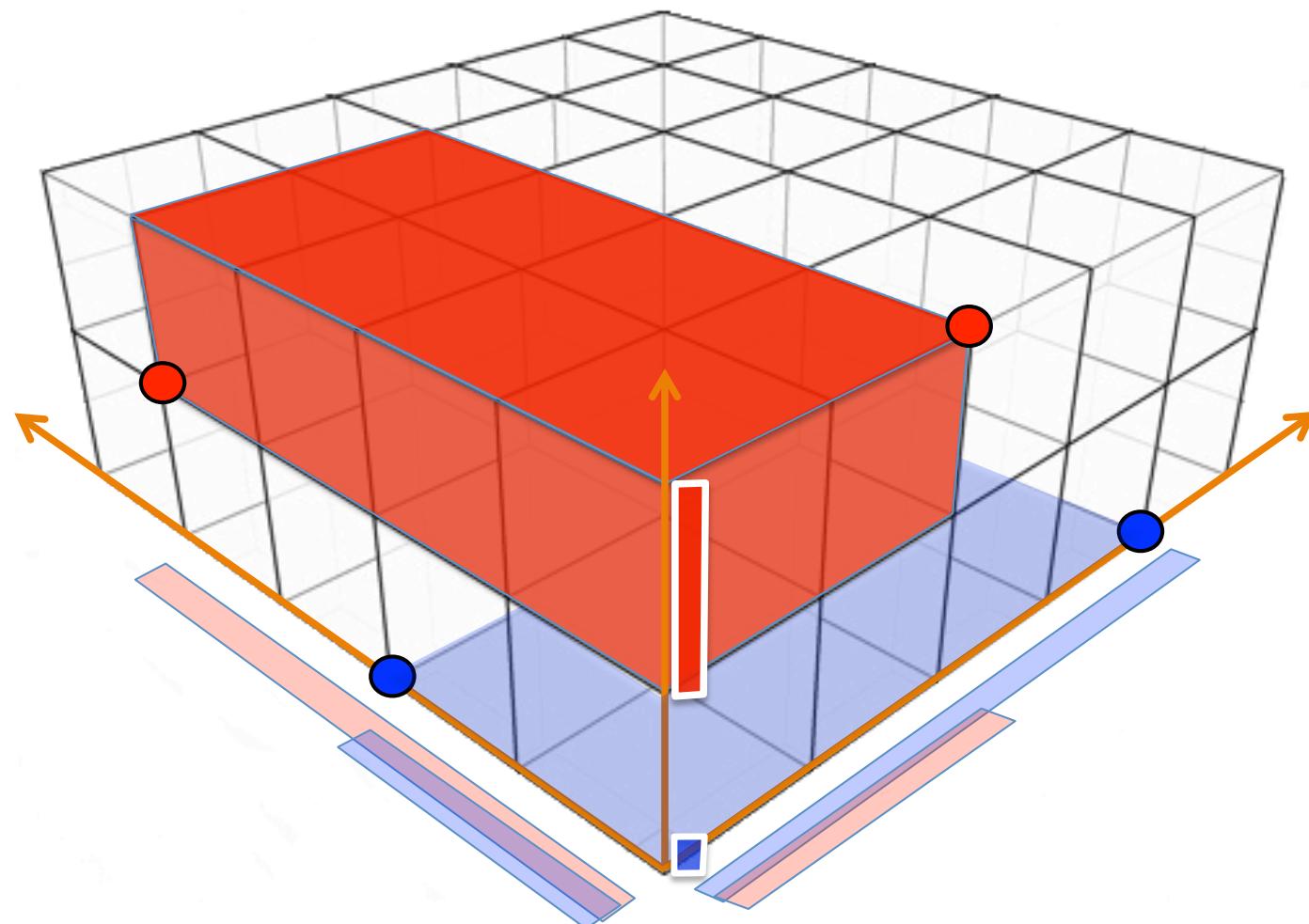
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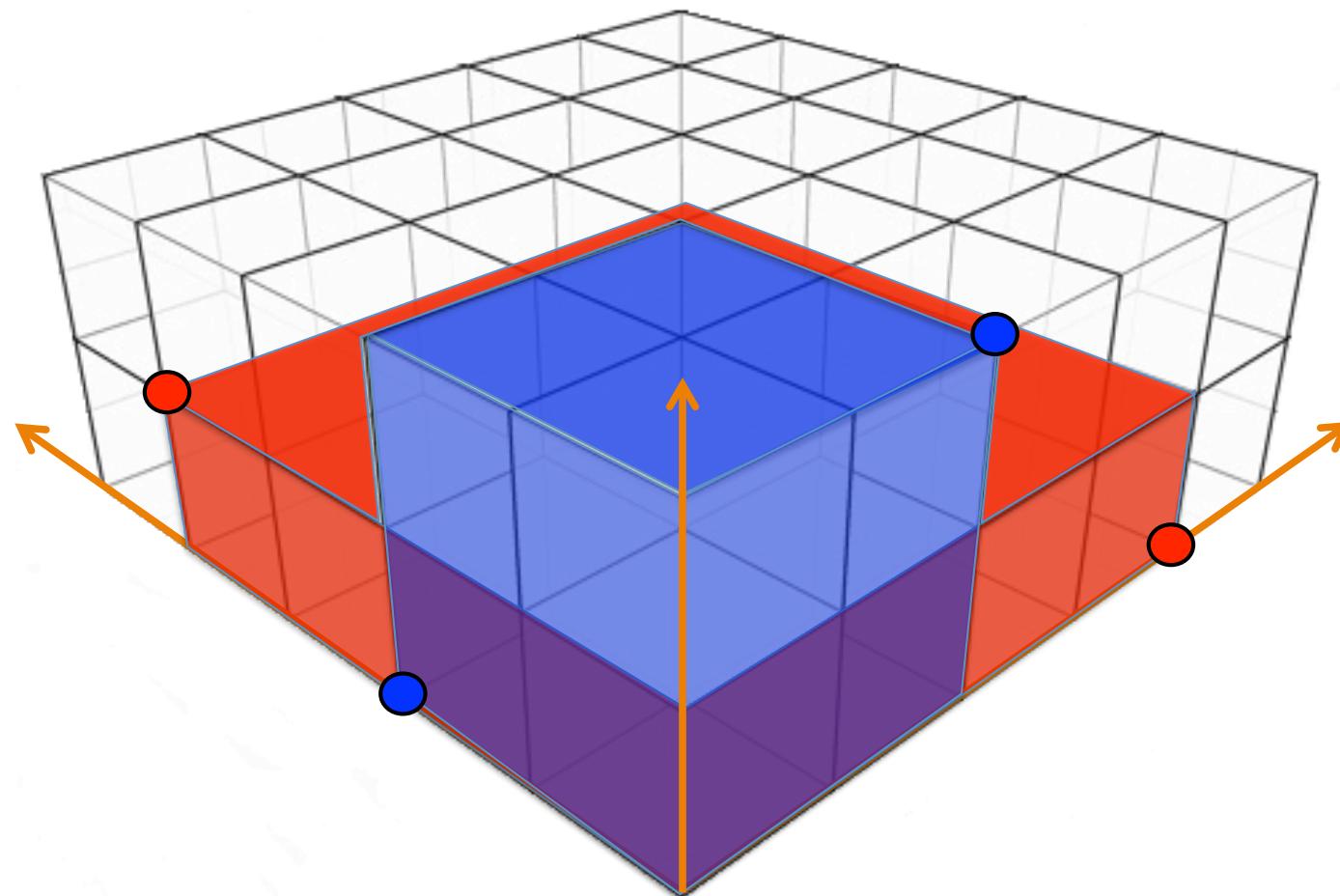
Ex.: Grid (6, 6, 3)



Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

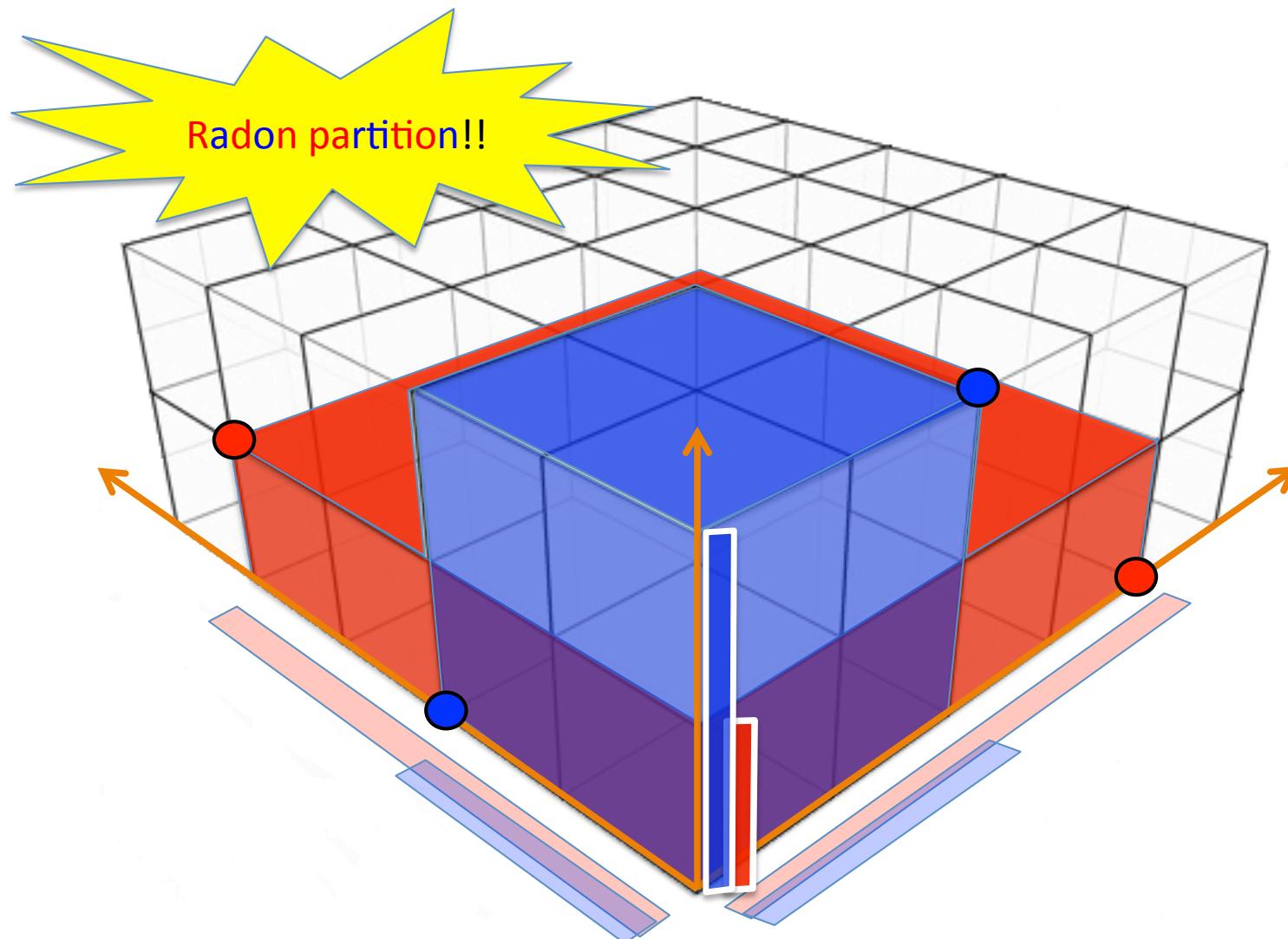
Ex.: Grid (6, 6, 3)



Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

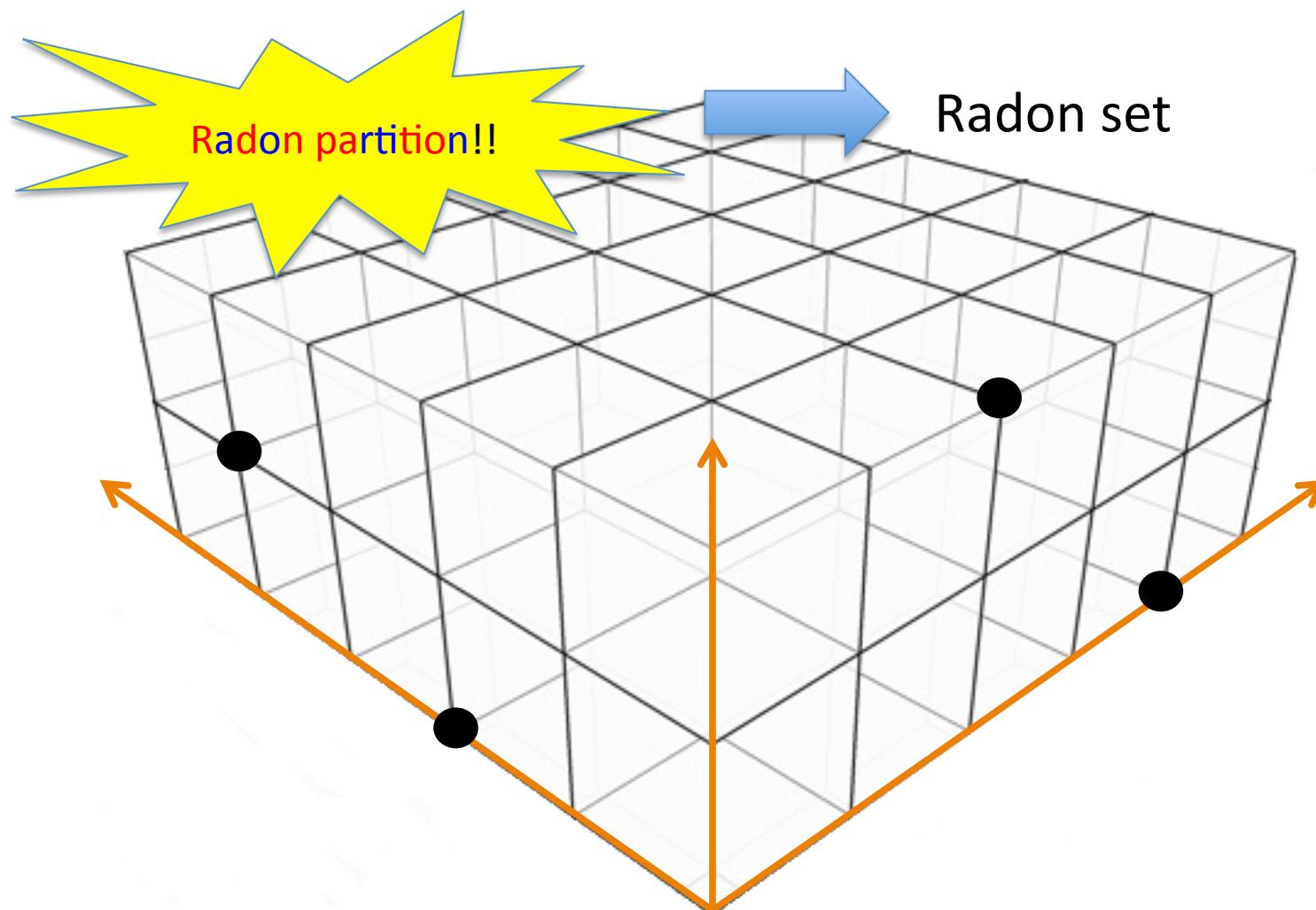
Ex.: Grid (6, 6, 3)



Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

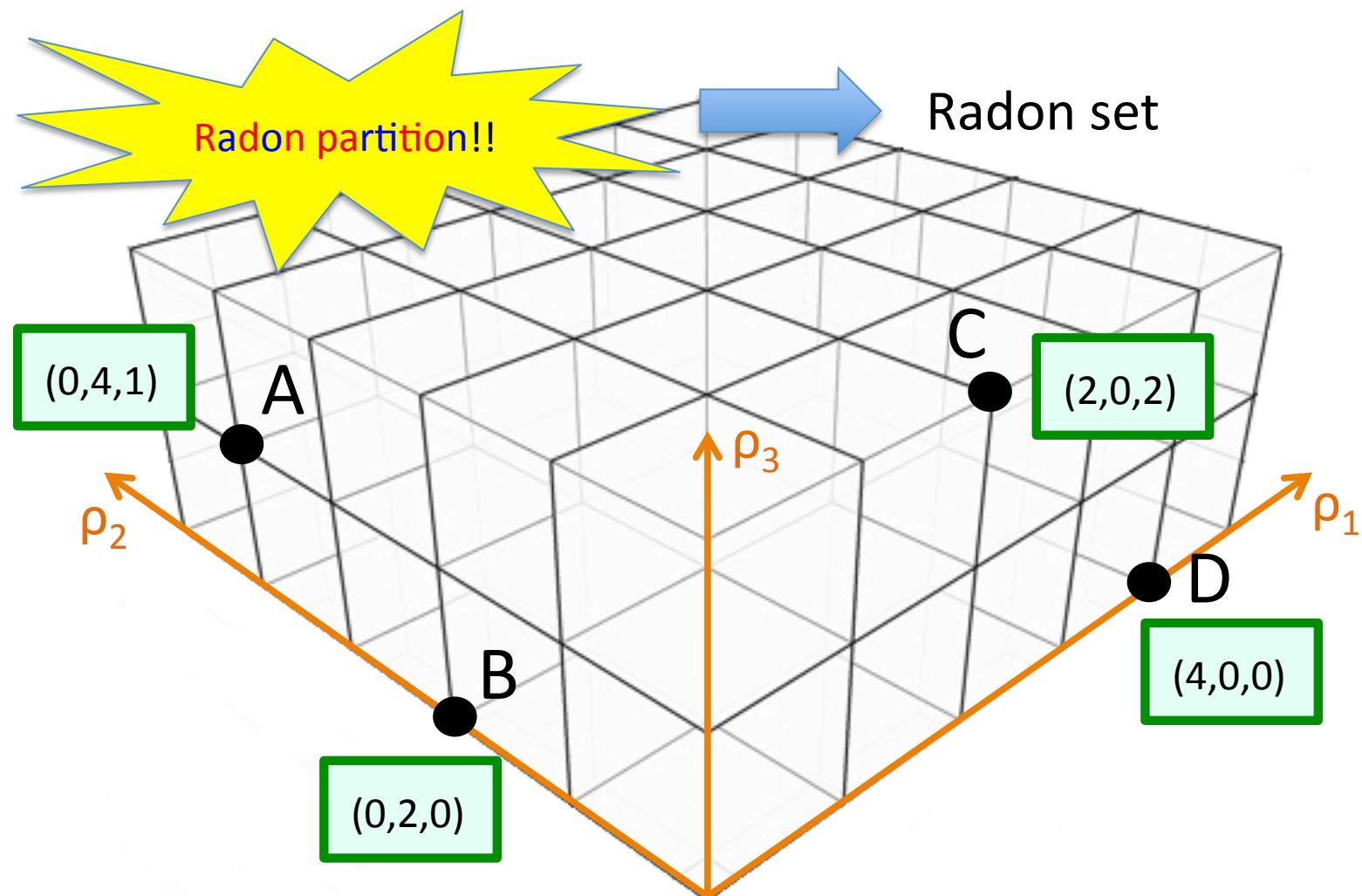
Ex.: Grid (6, 6, 3)



Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Ex.: Grid (6, 6, 3)



Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Ex.: Grid (6, 6, 3)

A (0,4,1)

B (0,2,0)

C (2,0,2)

D (4,0,0)

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Ex.: Grid (6, 6, 3)

Radon partition candidates:

A (0,4,1)

{A}, {B,C,D}

B (0,2,0)

{B}, {A,C,D}

C (2,0,2)

{C}, {A,B,D}

D (4,0,0)

{D}, {A,B,C}

{A,B}, {C,D}

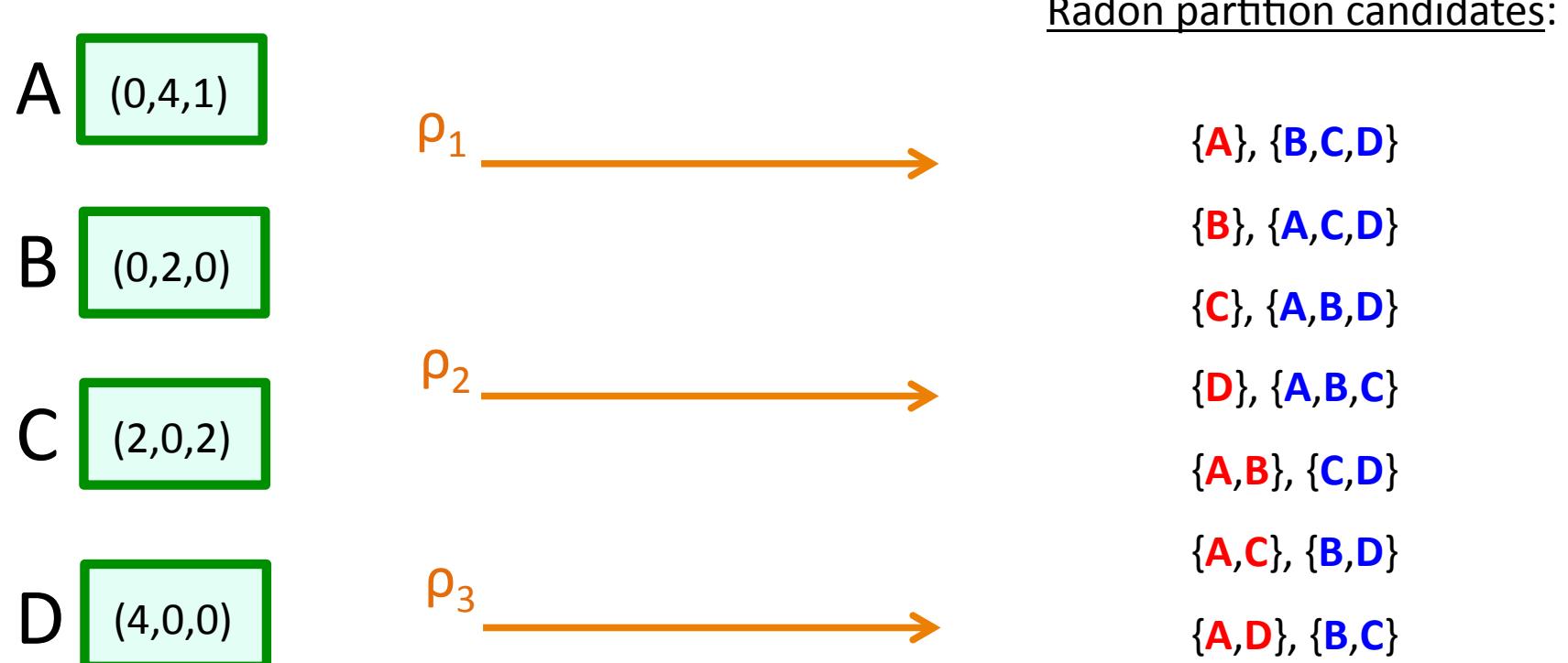
{A,C}, {B,D}

{A,D}, {B,C}

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Ex.: Grid (6, 6, 3)

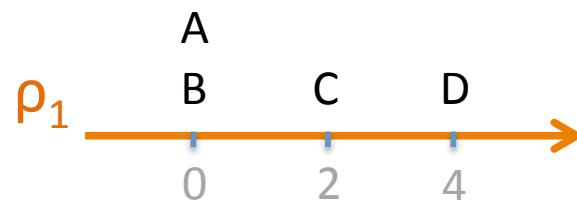


Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Ex.: Grid (6, 6, 3)

A (0,4,1)



B (0,2,0)



C (2,0,2)



D (4,0,0)

Radon partition candidates:

{A}, {B,C,D}

{B}, {A,C,D}

{C}, {A,B,D}

{D}, {A,B,C}

{A,B}, {C,D}

{A,C}, {B,D}

{A,D}, {B,C}

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

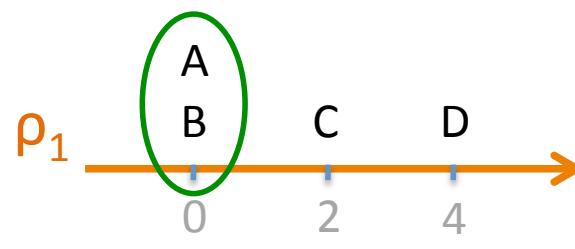
Ex.: Grid (6, 6, 3)

A (0,4,1)

B (0,2,0)

C (2,0,2)

D (4,0,0)



Radon partition candidates:

{A}, {B,C,D}

{B}, {A,C,D}

{C}, {A,B,D}

{D}, {A,B,C}

{A,B}, {C,D}

{A,C}, {B,D}

{A,D}, {B,C}

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

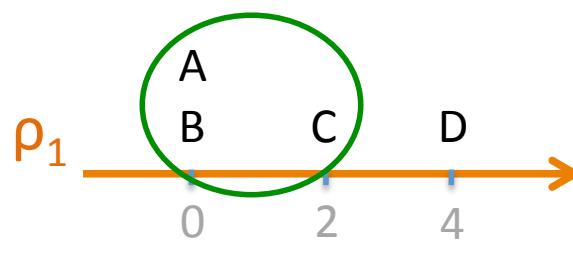
Ex.: Grid (6, 6, 3)

A (0,4,1)

B (0,2,0)

C (2,0,2)

D (4,0,0)



Radon partition candidates:

{A}, {B,C,D}

{B}, {A,C,D}

{C}, {A,B,D}

{D}, {A,B,C}

~~{A,B}, {C,D}~~

{A,C}, {B,D}

~~{A,D}, {B,C}~~

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

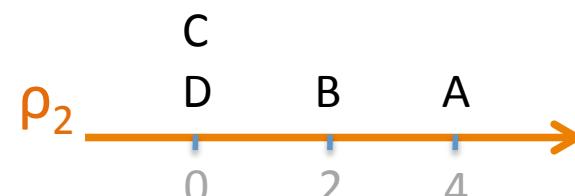
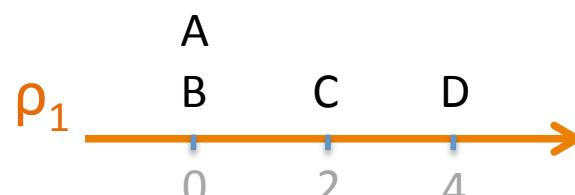
Ex.: Grid (6, 6, 3)

A (0, 4, 1)

B (0, 2, 0)

C (2, 0, 2)

D (4, 0, 0)



Radon partition candidates:

{A}, {B,C,D}

{B}, {A,C,D}

{C}, {A,B,D}

~~{D}, {A,B,C}~~

~~{A,B}, {C,D}~~

{A,C}, {B,D}

{A,D}, {B,C}

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

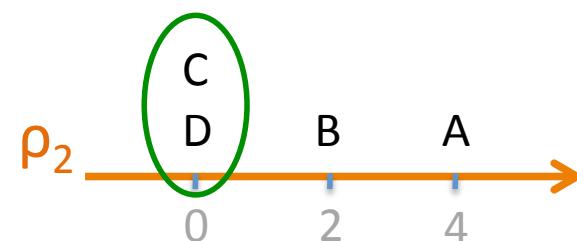
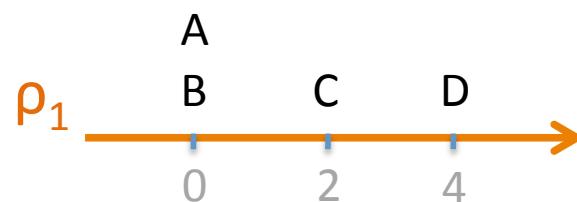
Ex.: Grid (6, 6, 3)

A (0, 4, 1)

B (0, 2, 0)

C (2, 0, 2)

D (4, 0, 0)



Radon partition candidates:

{A}, {B,C,D}

{B}, {A,C,D}

{C}, {A,B,D}

~~{D}, {A,B,C}~~

~~{A,B}, {C,D}~~

{A,C}, {B,D}

{A,D}, {B,C}

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

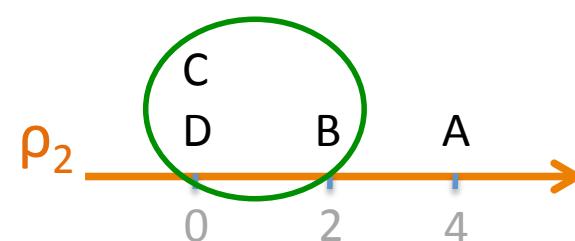
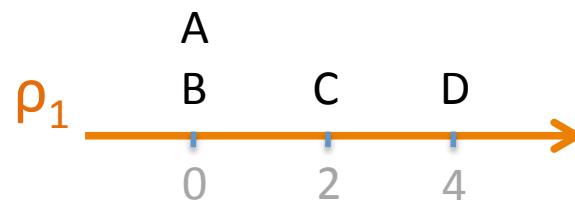
Ex.: Grid (6, 6, 3)

A (0, 4, 1)

B (0, 2, 0)

C (2, 0, 2)

D (4, 0, 0)



Radon partition candidates:

~~{A}, {B,C,D}~~

{B}, {A,C,D}

{C}, {A,B,D}

~~{D}, {A,B,C}~~

~~{A,B}, {C,D}~~

{A,C}, {B,D}

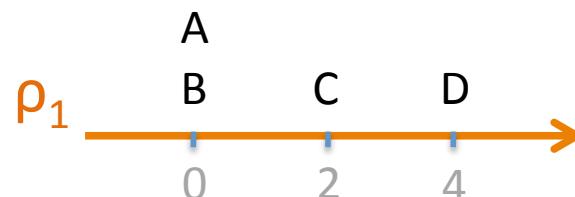
{A,D}, {B,C}

Geodetic Radon number of grids

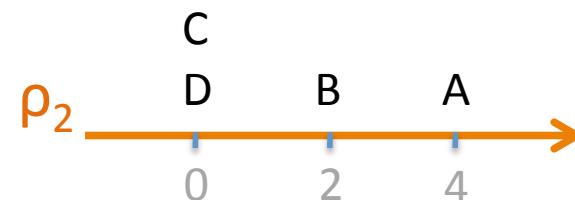
Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Ex.: Grid (6, 6, 3)

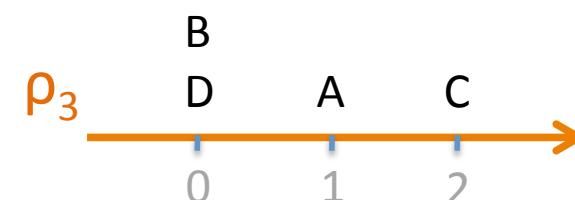
A (0,4,1)



B (0,2,0)



C (2,0,2)



D (4,0,0)

Radon partition candidates:

~~{A}, {B,C,D}~~

{B}, {A,C,D}

{C}, {A,B,D}

~~{D}, {A,B,C}~~

~~{A,B}, {C,D}~~

{A,C}, {B,D}

{A,D}, {B,C}

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

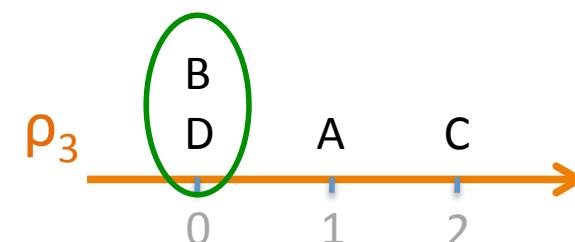
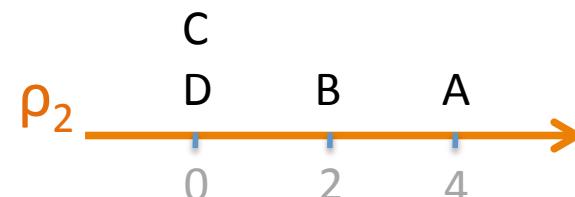
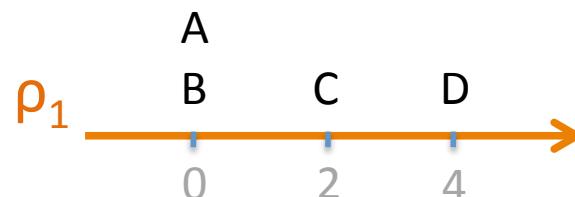
Ex.: Grid (6, 6, 3)

A (0,4,1)

B (0,2,0)

C (2,0,2)

D (4,0,0)



Radon partition candidates:

~~{A}~~, {B,C,D}

{B}, {A,C,D}

{C}, {A,B,D}

~~{D}~~, {A,B,C}

~~{A,B}~~, {C,D}

{A,C}, {B,D}

{A,D}, {B,C}

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

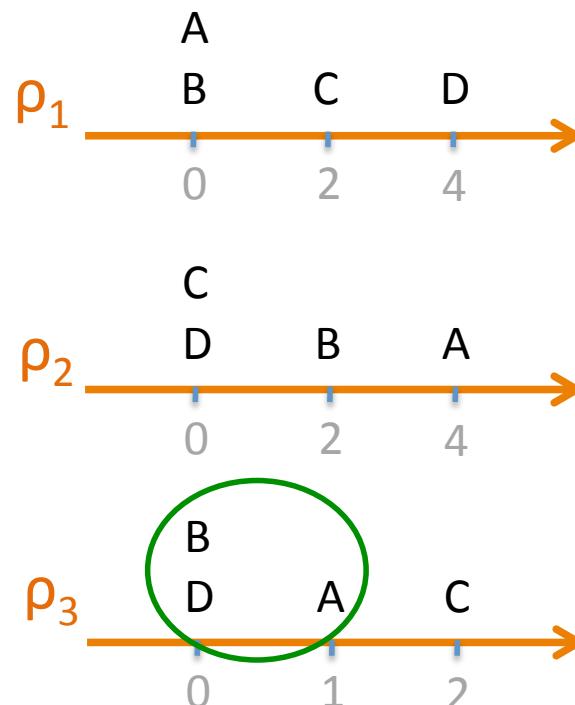
Ex.: Grid (6, 6, 3)

A (0,4,1)

B (0,2,0)

C (2,0,2)

D (4,0,0)



Radon partition candidates:

~~{A}, {B,C,D}~~

~~{B}, {A,C,D}~~

~~{C}, {A,B,D}~~

~~{D}, {A,B,C}~~

~~{A,B}, {C,D}~~

~~{A,C}, {B,D}~~

~~{A,D}, {B,C}~~

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

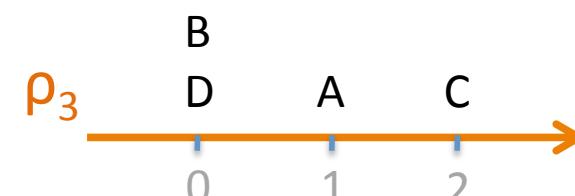
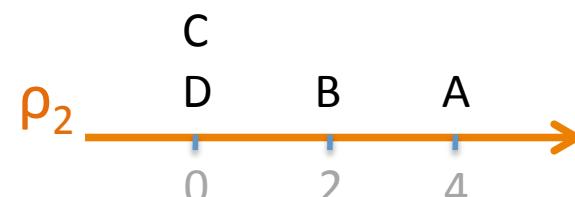
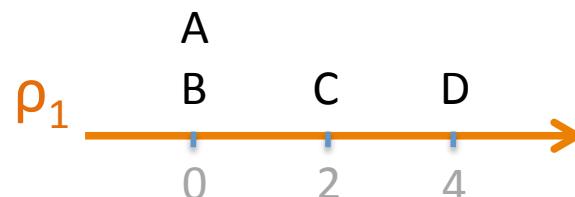
Ex.: Grid (6, 6, 3)

A (0,4,1)

B (0,2,0)

C (2,0,2)

D (4,0,0)



Radon partition candidates:

~~{A}, {B,C,D}~~

{B}, {A,C,D}

~~{C}, {A,B,D}~~

~~{D}, {A,B,C}~~

~~{A,B}, {C,D}~~

~~{A,C}, {B,D}~~

{A,D}, {B,C}

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

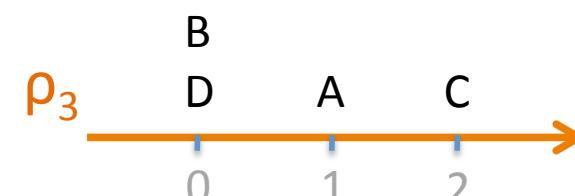
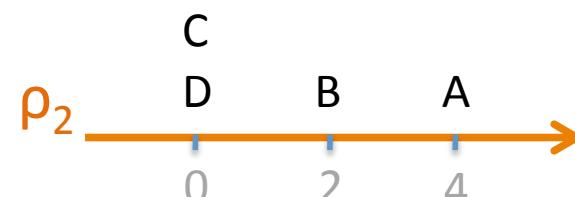
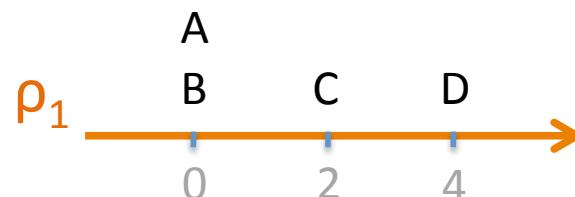
Ex.: Grid (6, 6, 3)

A (0,4,1)

B (0,2,0)

C (2,0,2)

D (4,0,0)



Radon partition candidates:

~~{A}, {B,C,D}~~

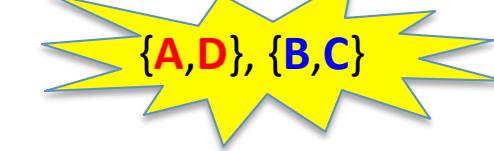
~~{B}, {A,C,D}~~

~~{C}, {A,B,D}~~

~~{D}, {A,B,C}~~

~~{A,B}, {C,D}~~

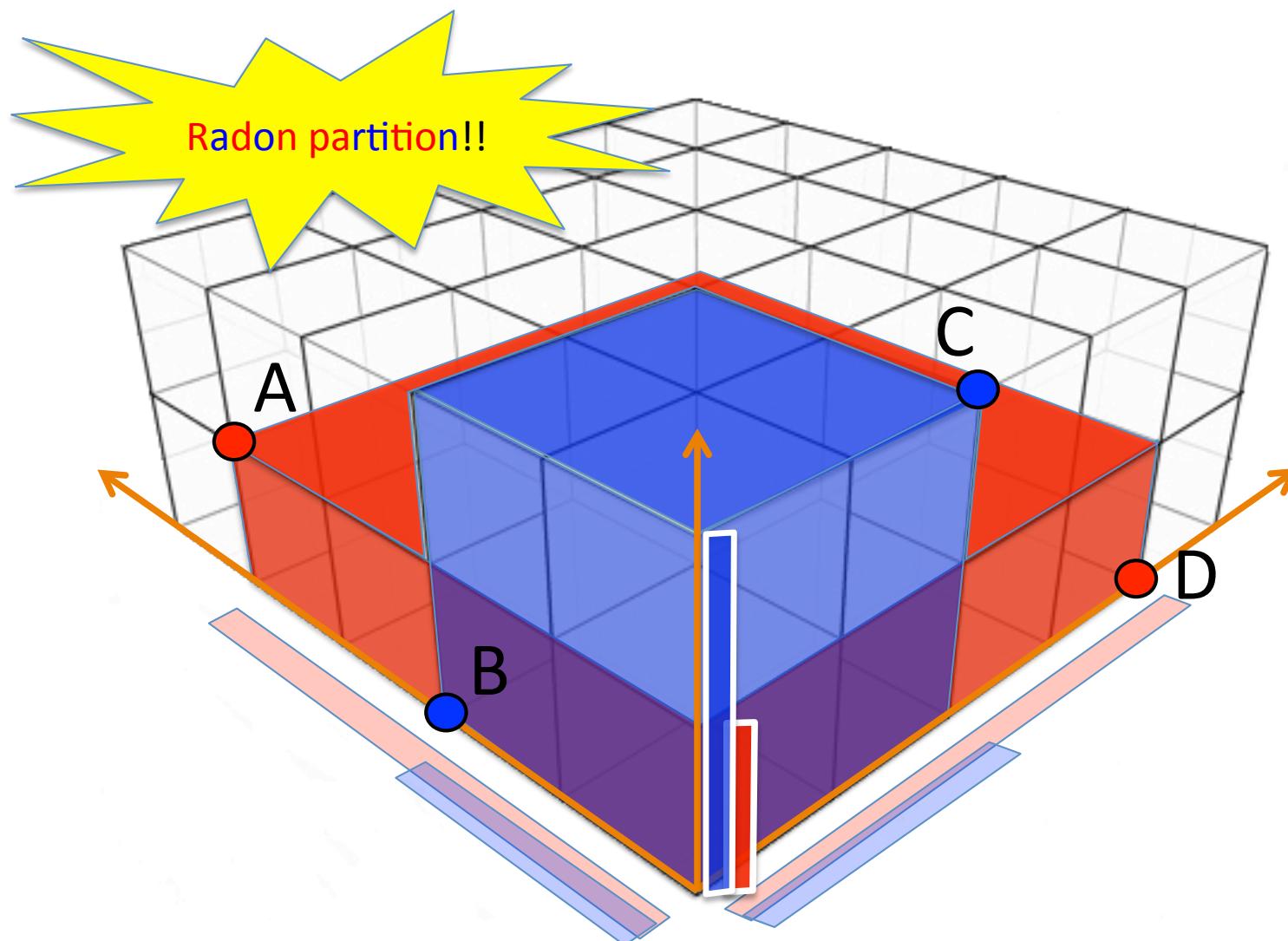
~~{A,C}, {B,D}~~



Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Ex.: Grid (6, 6, 3)

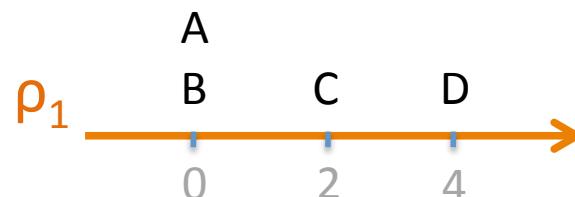


Geodetic Radon number of grids

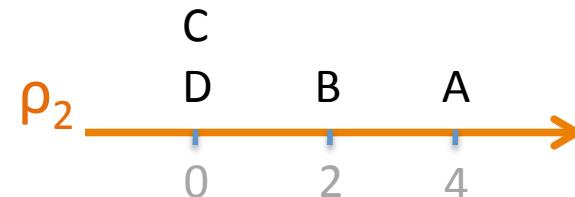
Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Ex.: Grid (6, 6, 3)

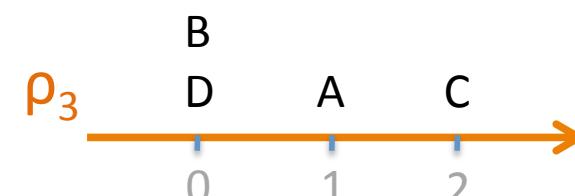
A (0,4,1)



B (0,2,0)



C (2,0,2)



D (4,0,0)

Radon partition candidates:

{A}, {B,C,D}

{B}, {A,C,D}

{C}, {A,B,D}

{D}, {A,B,C}

{A,B}, {C,D}

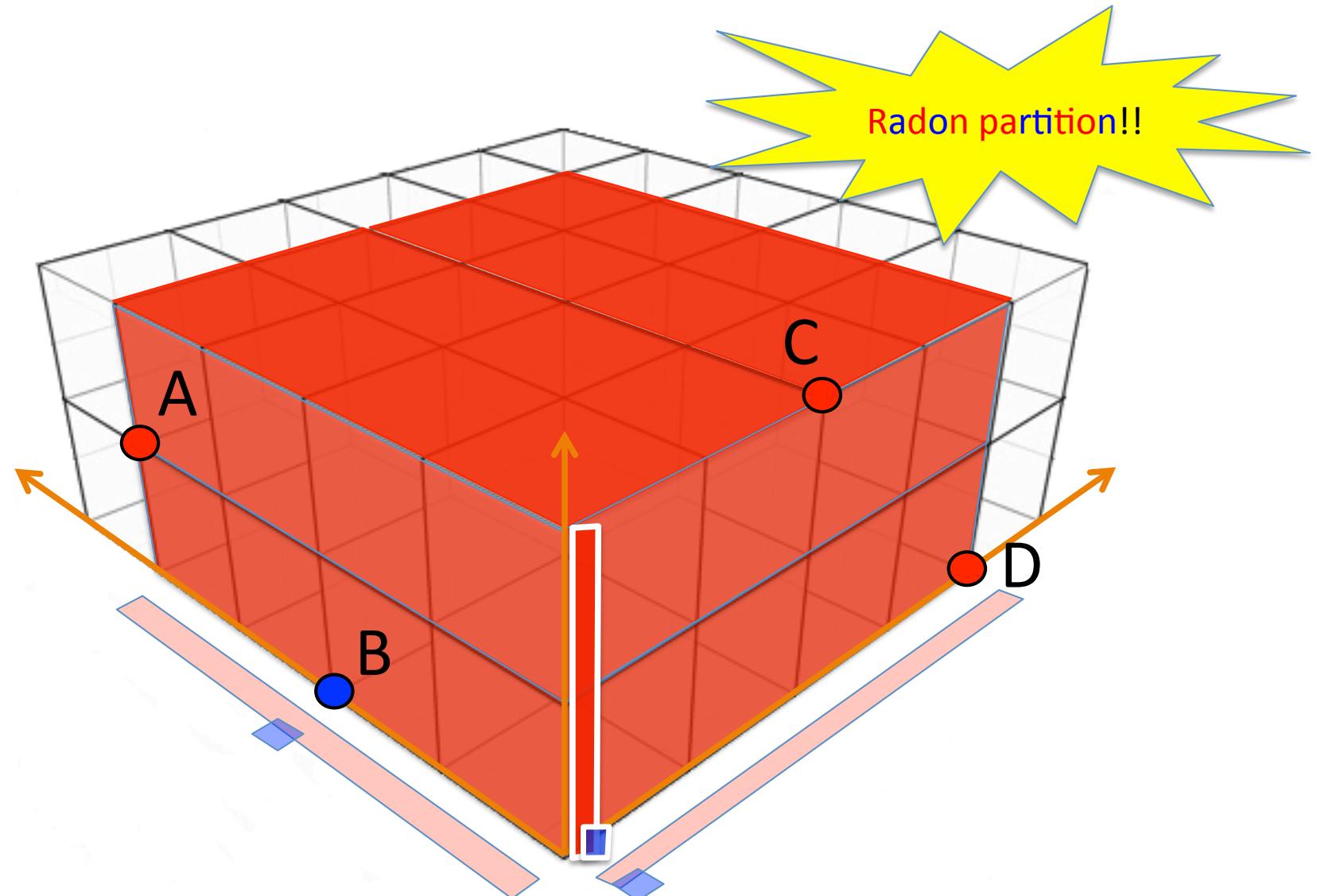
{A,C}, {B,D}

{A,D}, {B,C}

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Ex.: Grid (6, 6, 3)



Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Ex.: Grid (9, 9, 9, 9)

A (1,4,3,5)

B (2,2,0,8)

C (2,7,6,1)

D (0,0,0,0)

E (7,7,0,4)

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Ex.: Grid $(9, 9, 9, 9)$

Radon partition candidates:

A (1,4,3,5)

{A}, {B,C,D,E}

{B}, {A,C,D,E}

{C}, {A,B,D,E}

{D}, {A,B,C,E}

{E}, {A,B,C,D}

{A,B}, {C,D,E}

{A,C}, {B,D,E}

{A,D}, {B,C,E}

{A,E}, {B,C,E}

{B,C}, {A,D,E}

{B,D}, {A,C,E}

{B,E}, {A,C,D}

{C,D}, {A,B,E}

{C,E}, {A,B,D}

{D,E}, {A,B,C}

B (2,2,0,8)

C (2,7,6,1)

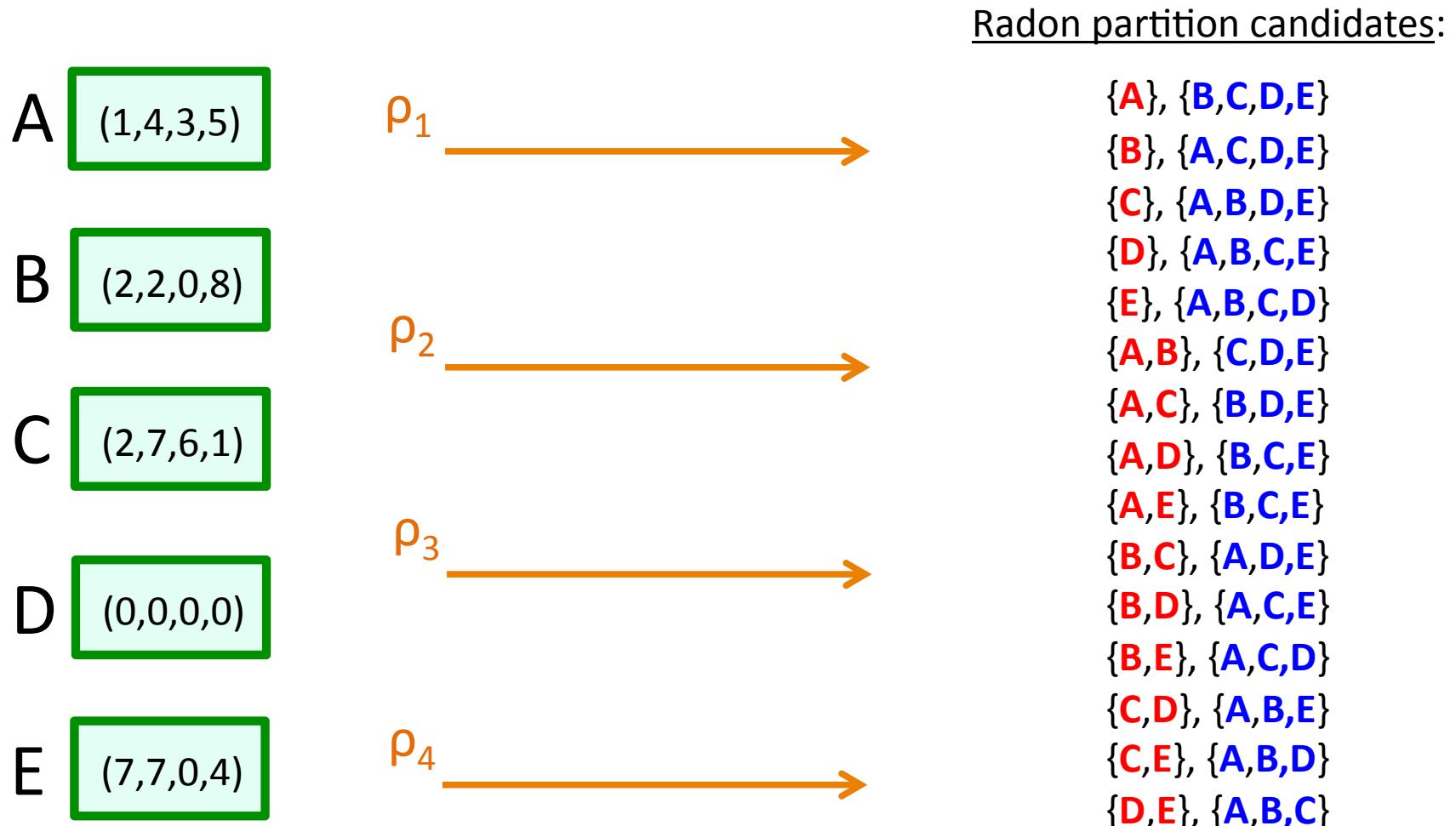
D (0,0,0,0)

E (7,7,0,4)

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

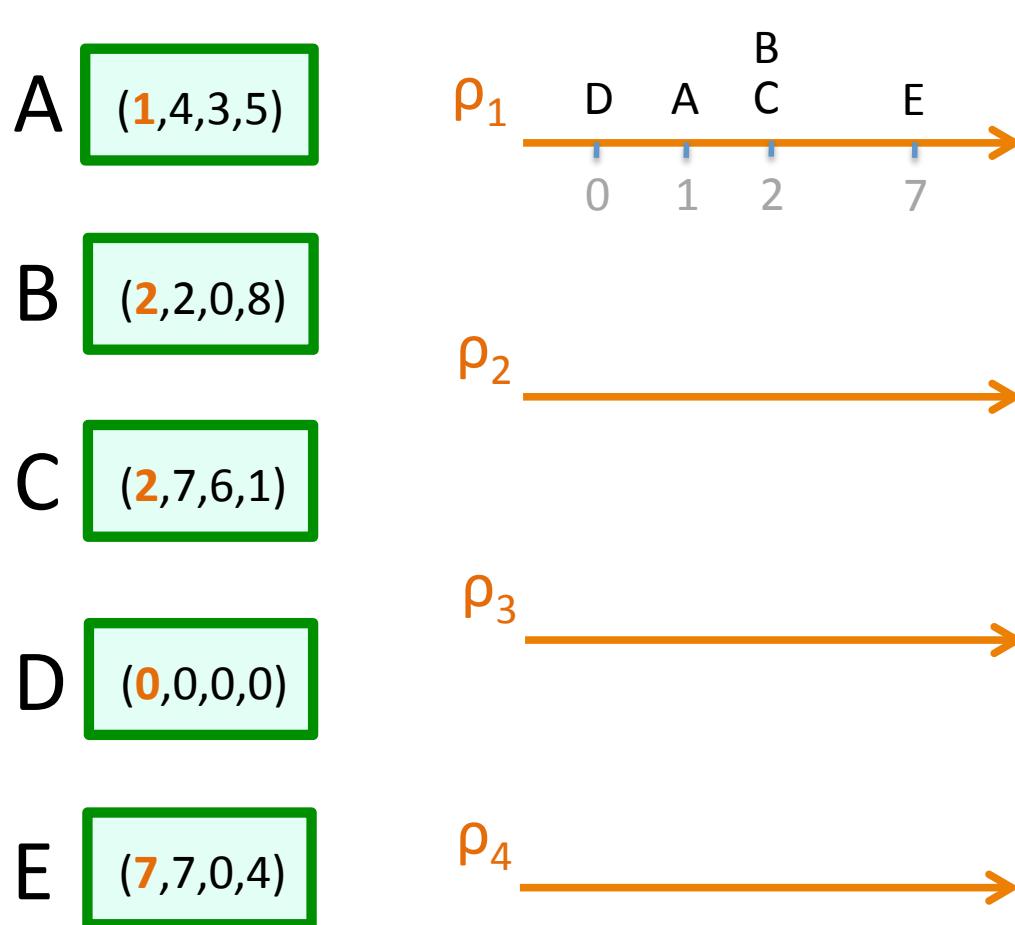
Ex.: Grid $(9, 9, 9, 9)$



Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Ex.: Grid $(9, 9, 9, 9)$



Radon partition candidates:

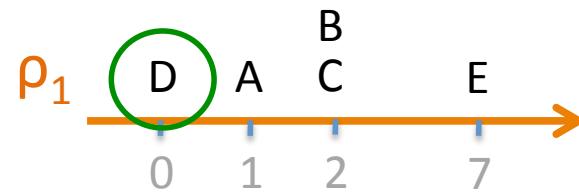
- {A}, {B,C,D,E}
- {B}, {A,C,D,E}
- {C}, {A,B,D,E}
- {D}, {A,B,C,E}
- {E}, {A,B,C,D}
- {A,B}, {C,D,E}
- {A,C}, {B,D,E}
- {A,D}, {B,C,E}
- {A,E}, {B,C,E}
- {B,C}, {A,D,E}
- {B,D}, {A,C,E}
- {B,E}, {A,C,D}
- {C,D}, {A,B,E}
- {C,E}, {A,B,D}
- {D,E}, {A,B,C}

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Ex.: Grid $(9, 9, 9, 9)$

A (1,4,3,5)



B (2,2,0,8)



C (2,7,6,1)



D (0,0,0,0)



E (7,7,0,4)

Radon partition candidates:

{A}, {B,C,D,E}

{B}, {A,C,D,E}

{C}, {A,B,D,E}

~~{D}, {A,B,C,E}~~

{E}, {A,B,C,D}

{A,B}, {C,D,E}

{A,C}, {B,D,E}

{A,D}, {B,C,E}

{A,E}, {B,C,E}

{B,C}, {A,D,E}

{B,D}, {A,C,E}

{B,E}, {A,C,D}

{C,D}, {A,B,E}

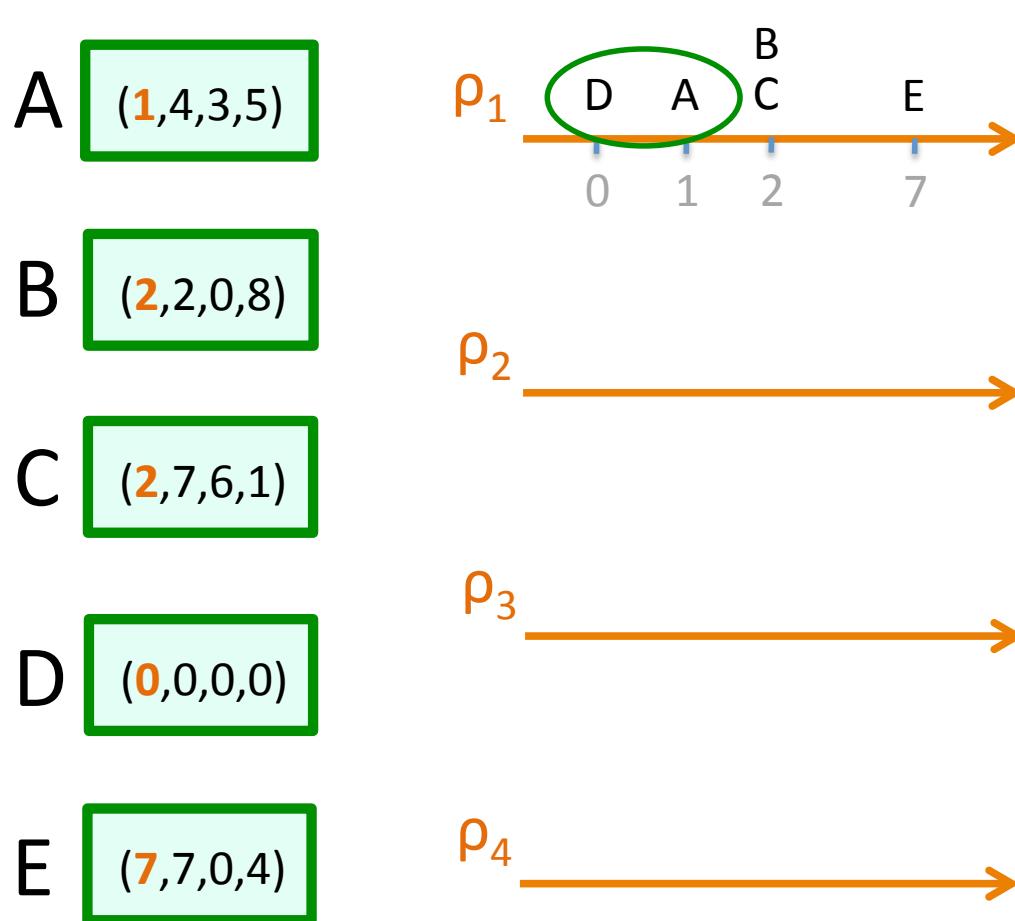
{C,E}, {A,B,D}

{D,E}, {A,B,C}

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Ex.: Grid $(9, 9, 9, 9)$



Radon partition candidates:

- {A}, {B,C,D,E}
- {B}, {A,C,D,E}
- {C}, {A,B,D,E}
- ~~{D}, {A,B,C,E}~~
- ~~{E}, {A,B,C,D}~~
- ~~{A,B}, {C,D,E}~~
- ~~{A,C}, {B,D,E}~~
- ~~{A,D}, {B,C,E}~~
- ~~{A,E}, {B,C,E}~~
- {B,C}, {A,D,E}
- ~~{B,D}, {A,C,E}~~
- ~~{B,E}, {A,C,D}~~
- ~~{C,D}, {A,B,E}~~
- ~~{C,E}, {A,B,D}~~
- ~~{D,E}, {A,B,C}~~

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Ex.: Grid $(9, 9, 9, 9)$

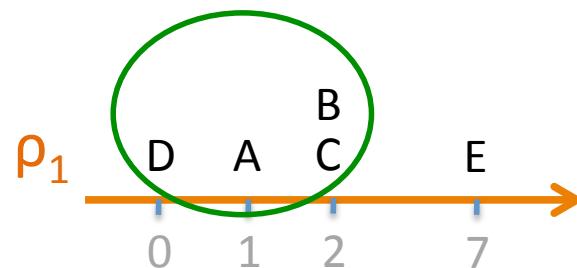
A (1,4,3,5)

B (2,2,0,8)

C (2,7,6,1)

D (0,0,0,0)

E (7,7,0,4)



Radon partition candidates:

{A}, {B,C,D,E}

{B}, {A,C,D,E}

{C}, {A,B,D,E}

~~{D}, {A,B,C,E}~~

~~{E}, {A,B,C,D}~~

{A,B}, {C,D,E}

{A,C}, {B,D,E}

~~{A,D}, {B,C,E}~~

{A,E}, {B,C,E}

{B,C}, {A,D,E}

{B,D}, {A,C,E}

{B,E}, {A,C,D}

{C,D}, {A,B,E}

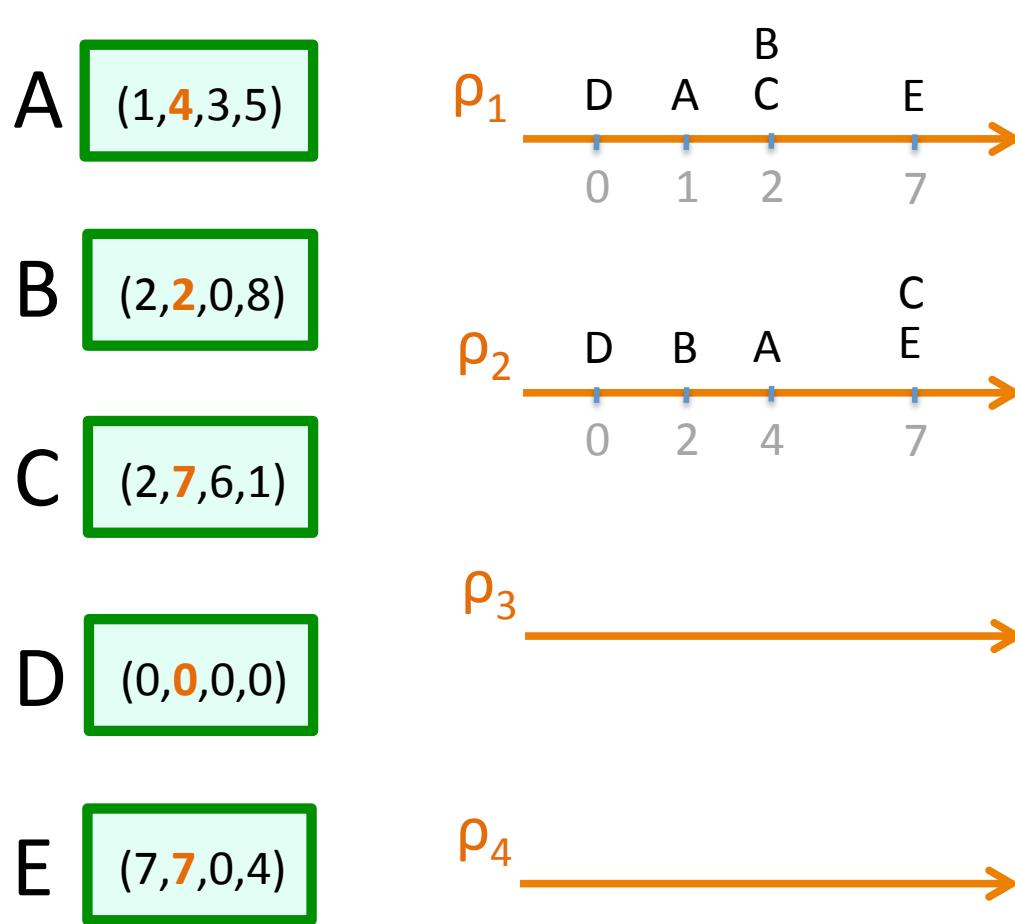
{C,E}, {A,B,D}

~~{D,E}, {A,B,C}~~

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Ex.: Grid $(9, 9, 9, 9)$



Radon partition candidates:

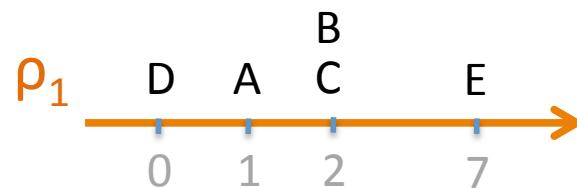
- {A}, {B,C,D,E}
- {B}, {A,C,D,E}
- {C}, {A,B,D,E}
- ~~{D}, {A,B,C,E}~~
- ~~{E}, {A,B,C,D}~~
- {A,B}, {C,D,E}
- {A,C}, {B,D,E}
- ~~{A,D}, {B,C,E}~~
- {A,E}, {B,C,E}
- {B,C}, {A,D,E}
- {B,D}, {A,C,E}
- {B,E}, {A,C,D}
- {C,D}, {A,B,E}
- {C,E}, {A,B,D}
- ~~{D,E}, {A,B,C}~~

Geodetic Radon number of grids

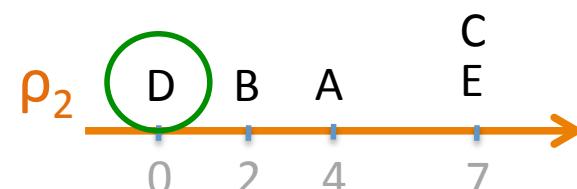
Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Ex.: Grid $(9, 9, 9, 9)$

A (1,4,3,5)



B (2,2,0,8)



C (2,7,6,1)

D (0,0,0,0)

E (7,7,0,4)



Radon partition candidates:

{A}, {B,C,D,E}

{B}, {A,C,D,E}

{C}, {A,B,D,E}

~~{D}, {A,B,C,E}~~

~~{E}, {A,B,C,D}~~

{A,B}, {C,D,E}

{A,C}, {B,D,E}

~~{A,D}, {B,C,E}~~

{A,E}, {B,C,E}

{B,C}, {A,D,E}

~~{B,D}, {A,C,E}~~

{B,E}, {A,C,D}

{C,D}, {A,B,E}

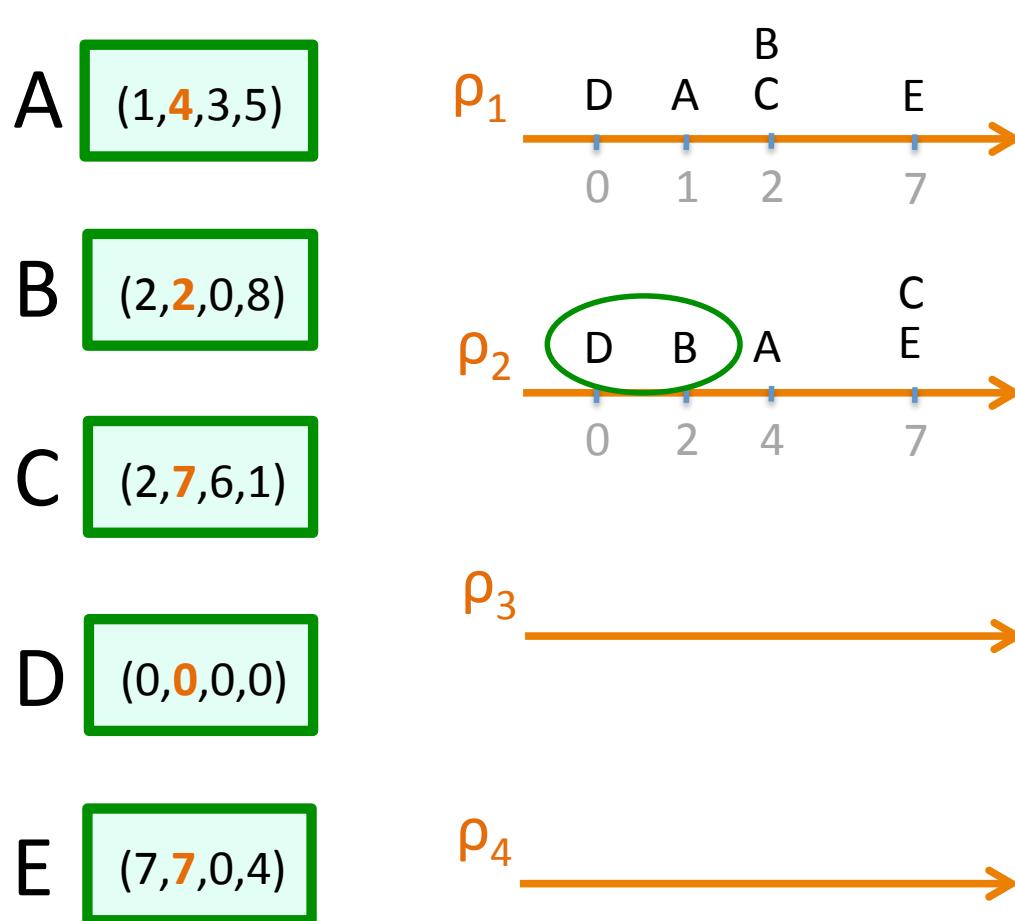
{C,E}, {A,B,D}

~~{D,E}, {A,B,C}~~

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Ex.: Grid $(9, 9, 9, 9)$



Radon partition candidates:

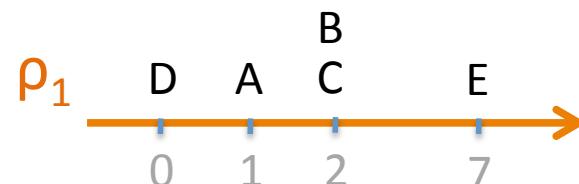
- {A}, {B,C,D,E}
- {B}, {A,C,D,E}
- {C}, {A,B,D,E}
- ~~{D}, {A,B,C,E}~~
- ~~{E}, {A,B,C,D}~~
- {A,B}, {C,D,E}
- {A,C}, {B,D,E}
- ~~{A,D}, {B,C,E}~~
- ~~{A,E}, {B,C,D}~~
- {B,C}, {A,D,E}
- ~~{B,D}, {A,C,E}~~
- ~~{B,E}, {A,C,D}~~
- {C,D}, {A,B,E}
- {C,E}, {A,B,D}
- {D,E}, {A,B,C}

Geodetic Radon number of grids

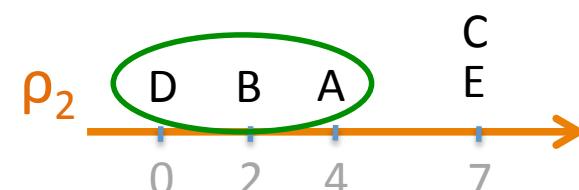
Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Ex.: Grid $(9, 9, 9, 9)$

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B (2,2,0,8)



C (2,7,6,1)



D (0,0,0,0)



E (7,7,0,4)

Radon partition candidates:

{A}, {B,C,D,E}

{B}, {A,C,D,E}

{C}, {A,B,D,E}

~~{D}, {A,B,C,E}~~

~~{E}, {A,B,C,D}~~

{A,B}, {C,D,E}

{A,C}, {B,D,E}

~~{A,D}, {B,C,E}~~

{A,E}, {B,C,E}

{B,C}, {A,D,E}

~~{B,D}, {A,C,E}~~

{B,E}, {A,C,D}

{C,D}, {A,B,E}

~~{C,E}, {A,B,D}~~

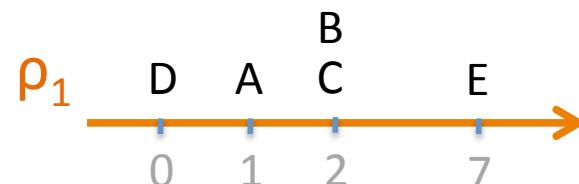
{D,E}, {A,B,C}

Geodetic Radon number of grids

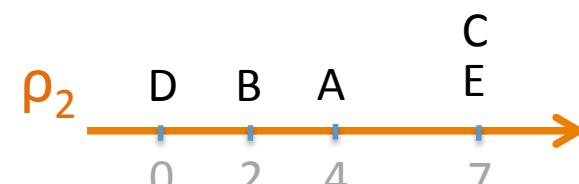
Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Ex.: Grid $(9, 9, 9, 9)$

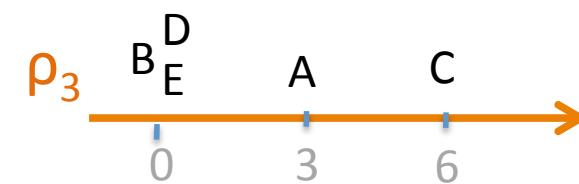
A (1,4,3,5)



B (2,2,0,8)



C (2,7,6,1)



D (0,0,0,0)



E (7,7,0,4)

Radon partition candidates:

{A}, {B,C,D,E}

{B}, {A,C,D,E}

{C}, {A,B,D,E}

~~{D}, {A,B,C,E}~~

~~{E}, {A,B,C,D}~~

{A,B}, {C,D,E}

{A,C}, {B,D,E}

~~{A,D}, {B,C,E}~~

{A,E}, {B,C,E}

{B,C}, {A,D,E}

~~{B,D}, {A,C,E}~~

{B,E}, {A,C,D}

{C,D}, {A,B,E}

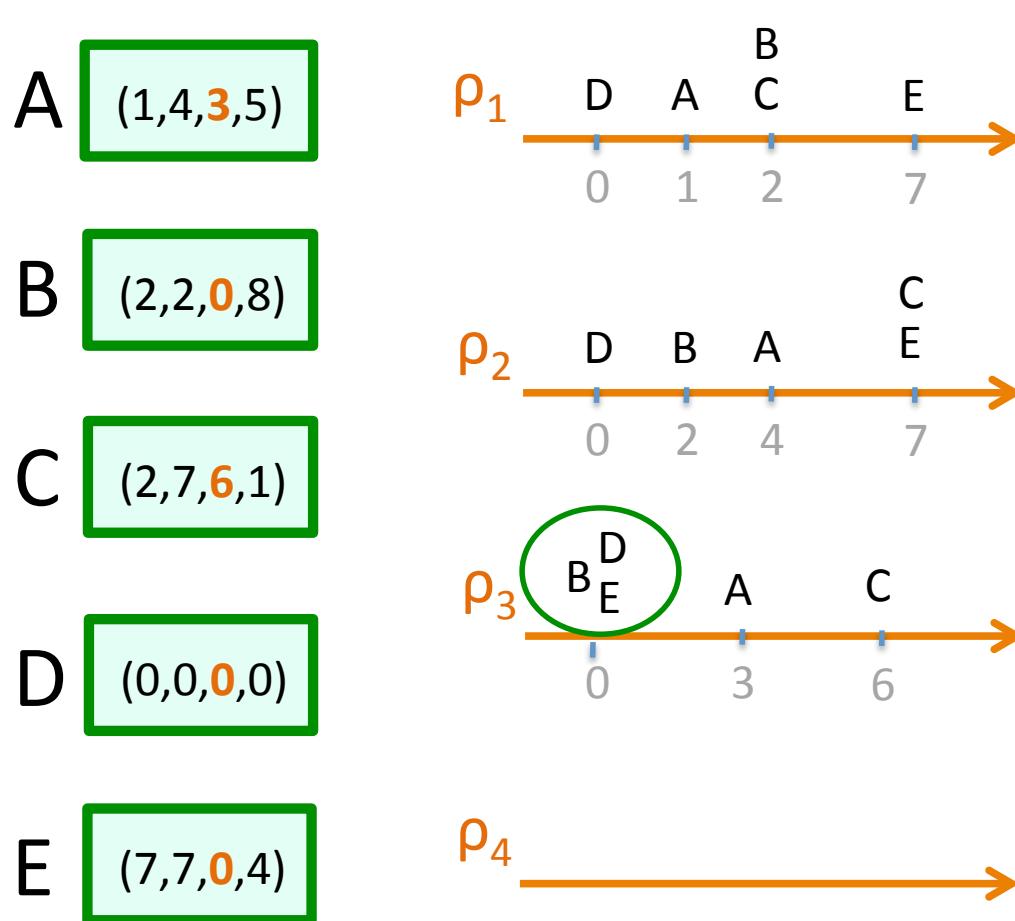
~~{C,E}, {A,B,D}~~

{D,E}, {A,B,C}

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Ex.: Grid $(9, 9, 9, 9)$



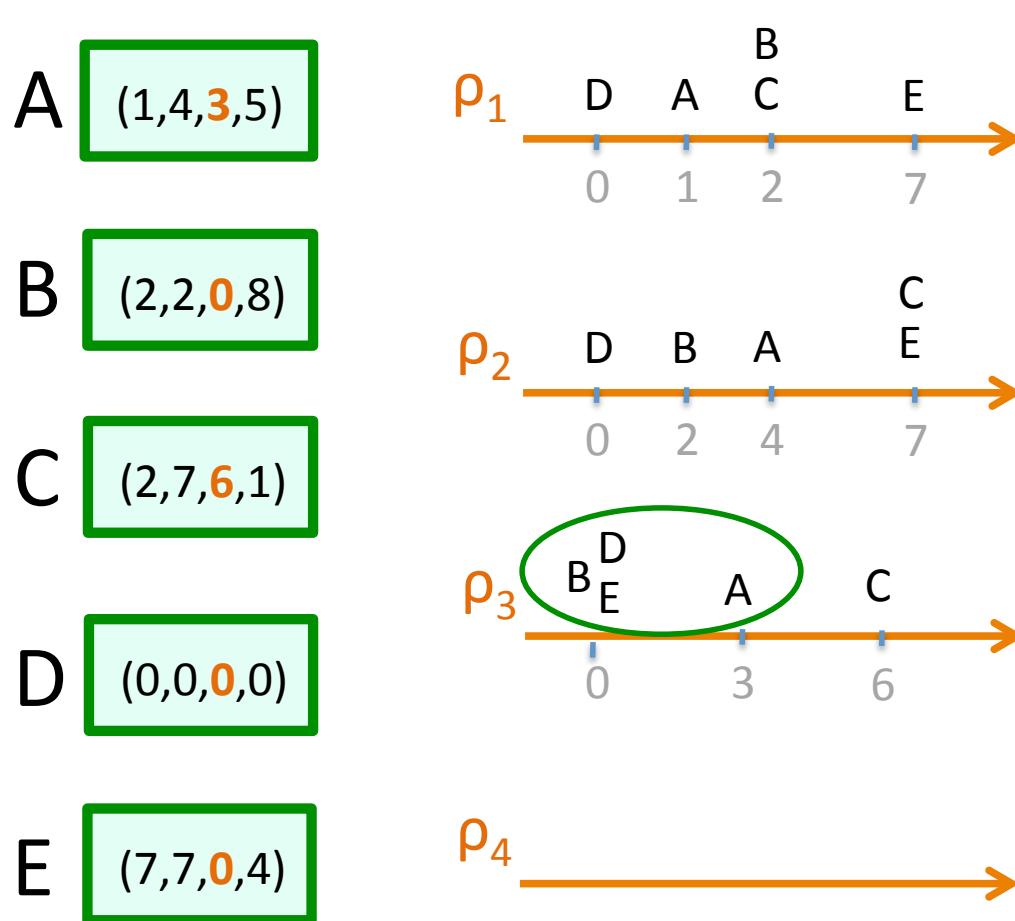
Radon partition candidates:

- $\{\textcolor{red}{A}\}, \{\textcolor{blue}{B, C, D, E}\}$
- $\{\textcolor{red}{B}\}, \{\textcolor{blue}{A, C, D, E}\}$
- $\{\textcolor{red}{C}\}, \{\textcolor{blue}{A, B, D, E}\}$
- $\{\textcolor{red}{D}\}, \{\textcolor{blue}{A, B, C, E}\}$
- $\{\textcolor{red}{E}\}, \{\textcolor{blue}{A, B, C, D}\}$
- $\{\textcolor{red}{A, B}\}, \{\textcolor{blue}{C, D, E}\}$
- $\{\textcolor{red}{A, C}\}, \{\textcolor{blue}{B, D, E}\}$
- $\{\textcolor{red}{A, D}\}, \{\textcolor{blue}{B, C, E}\}$
- $\{\textcolor{red}{A, E}\}, \{\textcolor{blue}{B, C, E}\}$
- $\{\textcolor{red}{B, C}\}, \{\textcolor{blue}{A, D, E}\}$
- $\{\textcolor{red}{B, D}\}, \{\textcolor{blue}{A, C, E}\}$
- $\{\textcolor{red}{B, E}\}, \{\textcolor{blue}{A, C, D}\}$
- $\{\textcolor{red}{C, D}\}, \{\textcolor{blue}{A, B, E}\}$
- $\{\textcolor{red}{C, E}\}, \{\textcolor{blue}{A, B, D}\}$
- $\{\textcolor{red}{D, E}\}, \{\textcolor{blue}{A, B, C}\}$

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Ex.: Grid $(9, 9, 9, 9)$



Radon partition candidates:

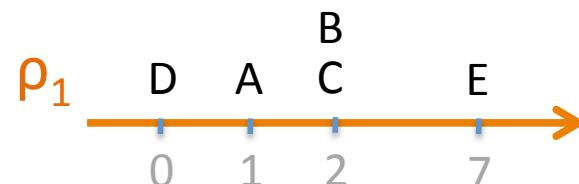
- $\{\textcolor{red}{A}\}, \{\textcolor{blue}{B, C, D, E}\}$
- $\{\textcolor{red}{B}\}, \{\textcolor{blue}{A, C, D, E}\}$
- $\{\textcolor{red}{C}\}, \{\textcolor{blue}{A, B, D, E}\}$
- $\{\textcolor{red}{D}\}, \{\textcolor{blue}{A, B, C, E}\}$
- $\{\textcolor{red}{E}\}, \{\textcolor{blue}{A, B, C, D}\}$
- $\{\textcolor{red}{A, B}\}, \{\textcolor{blue}{C, D, E}\}$
- $\{\textcolor{red}{A, C}\}, \{\textcolor{blue}{B, D, E}\}$
- $\{\textcolor{red}{A, D}\}, \{\textcolor{blue}{B, C, E}\}$
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- $\{\textcolor{red}{B, D}\}, \{\textcolor{blue}{A, C, E}\}$
- $\{\textcolor{red}{B, E}\}, \{\textcolor{blue}{A, C, D}\}$
- $\{\textcolor{red}{C, D}\}, \{\textcolor{blue}{A, B, E}\}$
- $\{\textcolor{red}{C, E}\}, \{\textcolor{blue}{A, B, D}\}$
- $\{\textcolor{red}{D, E}\}, \{\textcolor{blue}{A, B, C}\}$

Geodetic Radon number of grids

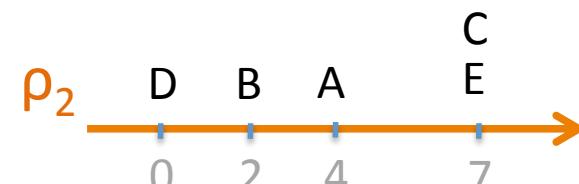
Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Ex.: Grid $(9, 9, 9, 9)$

A (1,4,3,5)



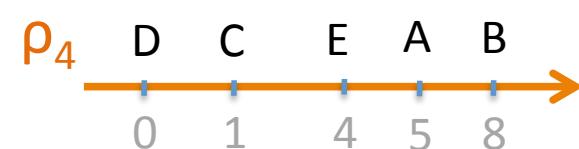
B (2,2,0,8)



C (2,7,6,1)



D (0,0,0,0)



E (7,7,0,4)

Radon partition candidates:

{A}, {B,C,D,E}

{B}, {A,C,D,E}

~~{C}, {A,B,D,E}~~

~~{D}, {A,B,C,E}~~

~~{E}, {A,B,C,D}~~

{A,B}, {C,D,E}

~~{A,C}, {B,D,E}~~

~~{A,D}, {B,C,E}~~

{A,E}, {B,C,E}

{B,C}, {A,D,E}

~~{B,D}, {A,C,E}~~

{B,E}, {A,C,D}

{C,D}, {A,B,E}

~~{C,E}, {A,B,D}~~

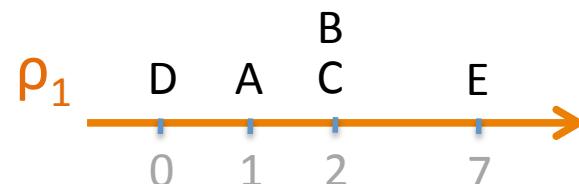
{D,E}, {A,B,C}

Geodetic Radon number of grids

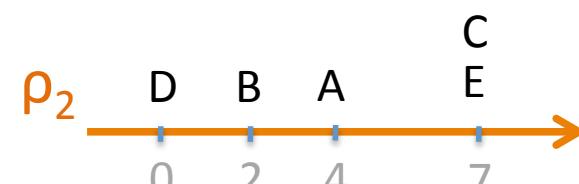
Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Ex.: Grid $(9, 9, 9, 9)$

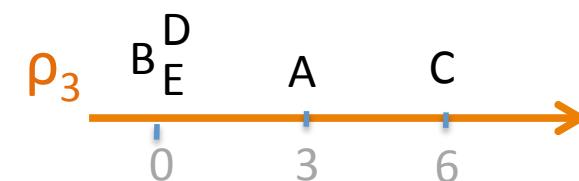
A (1,4,3,5)



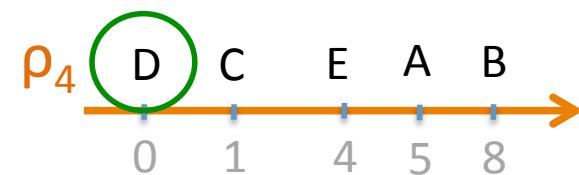
B (2,2,0,8)



C (2,7,6,1)



D (0,0,0,0)



Radon partition candidates:

{A}, {B,C,D,E}

{B}, {A,C,D,E}

~~{C}, {A,B,D,E}~~

~~{D}, {A,B,C,E}~~

~~{E}, {A,B,C,D}~~

{A,B}, {C,D,E}

~~{A,C}, {B,D,E}~~

~~{A,D}, {B,C,E}~~

{A,E}, {B,C,E}

{B,C}, {A,D,E}

~~{B,D}, {A,C,E}~~

{B,E}, {A,C,D}

{C,D}, {A,B,E}

~~{C,E}, {A,B,D}~~

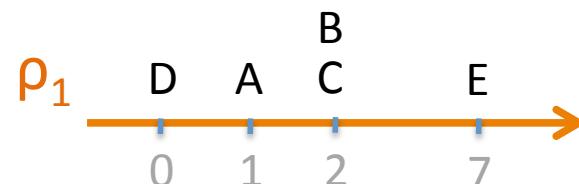
{D,E}, {A,B,C}

Geodetic Radon number of grids

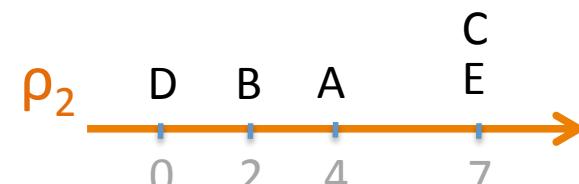
Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

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Radon partition candidates:

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{A,B}, {C,D,E}

~~{A,C}, {B,D,E}~~

~~{A,D}, {B,C,E}~~

{A,E}, {B,C,E}

{B,C}, {A,D,E}

~~{B,D}, {A,C,E}~~

{B,E}, {A,C,D}

~~{C,D}, {A,B,E}~~

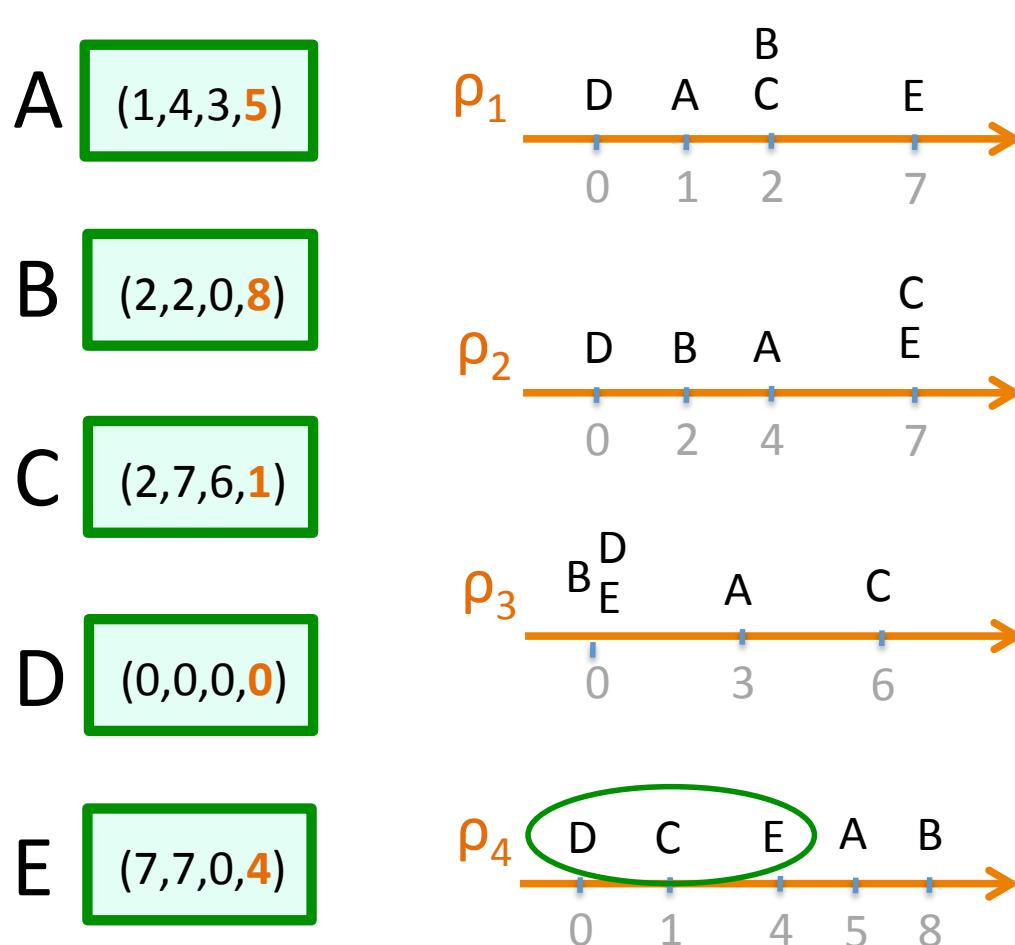
~~{C,E}, {A,B,D}~~

{D,E}, {A,B,C}

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Ex.: Grid $(9, 9, 9, 9)$



Radon partition candidates:

{A}, {B,C,D,E}

{B}, {A,C,D,E}

~~{C}, {A,B,D,E}~~

~~{D}, {A,B,C,E}~~

~~{E}, {A,B,C,D}~~

~~{A,B}, {C,D,E}~~

~~{A,C}, {B,D,E}~~

~~{A,D}, {B,C,E}~~

{A,E}, {B,C,E}

{B,C}, {A,D,E}

~~{B,D}, {A,C,E}~~

{B,E}, {A,C,D}

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~~{C,E}, {A,B,D}~~

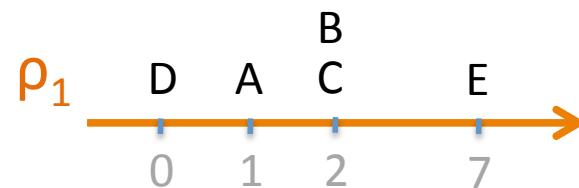
{D,E}, {A,B,C}

Geodetic Radon number of grids

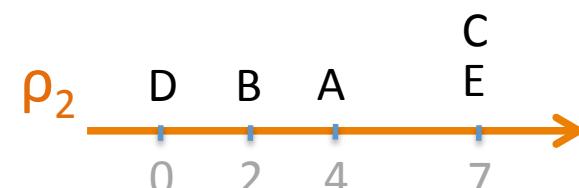
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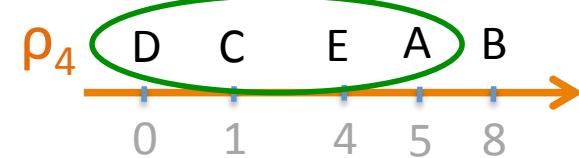
B (2,2,0,8)



C (2,7,6,1)



D (0,0,0,0)



E (7,7,0,4)

Radon partition candidates:

{A}, {B,C,D,E}

{B}, {A,C,D,E}

{C}, {A,B,D,E}

{D}, {A,B,C,E}

{E}, {A,B,C,D}

{A,B}, {C,D,E}

{A,C}, {B,D,E}

{A,D}, {B,C,E}

{A,E}, {B,C,E}

{B,C}, {A,D,E}

{B,D}, {A,C,E}

{B,E}, {A,C,D}

{C,D}, {A,B,E}

{C,E}, {A,B,D}

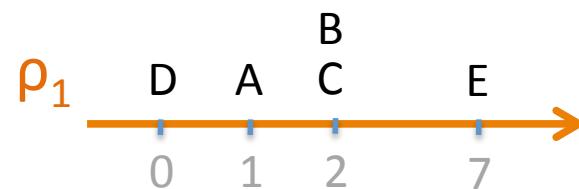
{D,E}, {A,B,C}

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Ex.: Grid $(9, 9, 9, 9)$

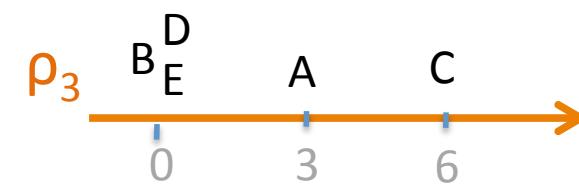
A (1,4,3,5)



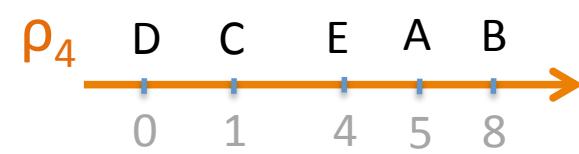
B (2,2,0,8)



C (2,7,6,1)

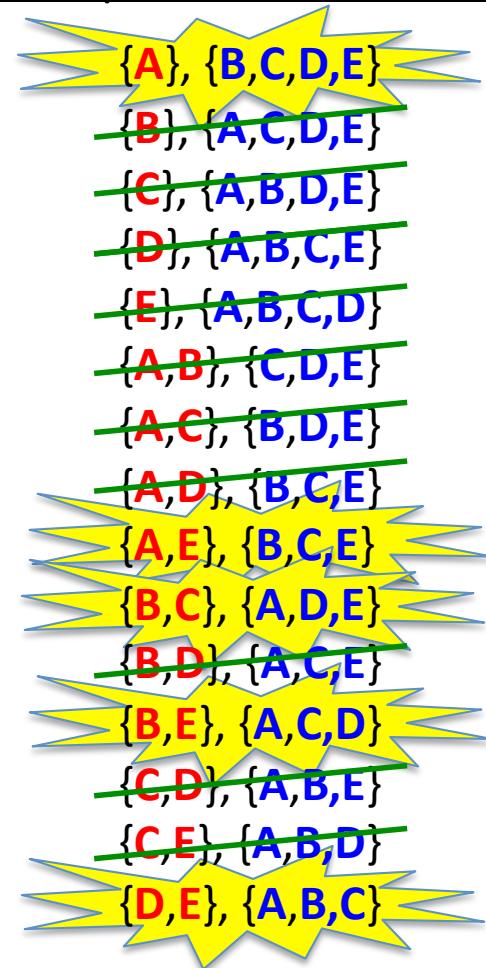


D (0,0,0,0)



E (7,7,0,4)

Radon partition candidates:



Geodetic Radon number of grids

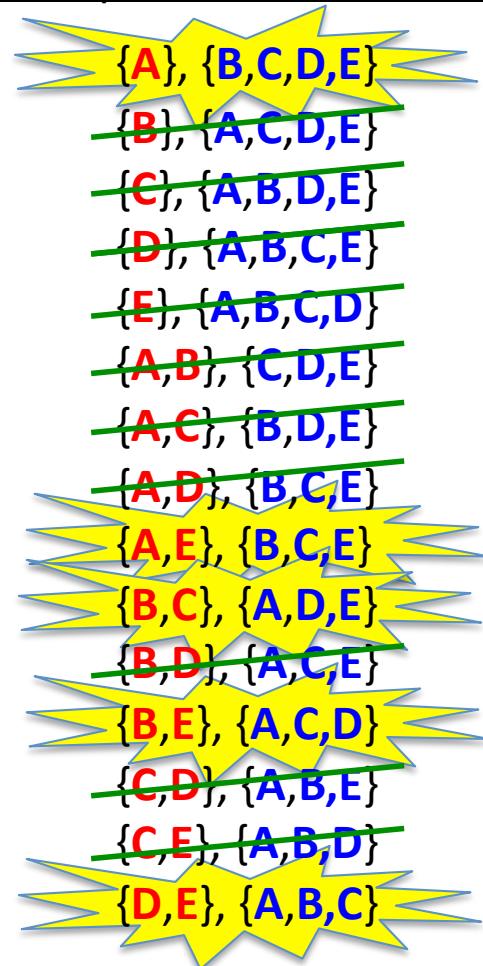
Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Ex.: Grid $(9, 9, 9, 9)$

- A (1,4,3,5)
- B (2,2,0,8)
- C (2,7,6,1)
- D (0,0,0,0)
- E (7,7,0,4)

Radon set

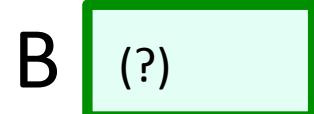
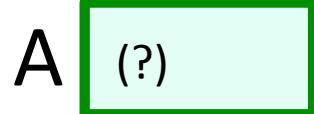
Radon partition candidates:



Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Anti-Radon set of size 2



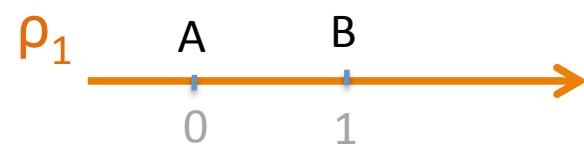
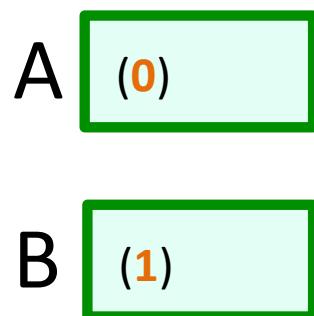
Radon partition candidate:

{A}, {B}

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Anti-Radon set of size 2



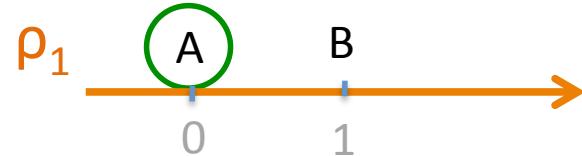
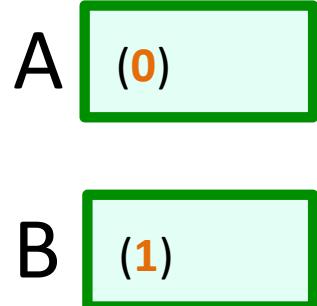
Radon partition candidate:

{A}, {B}

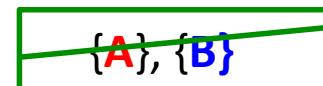
Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Anti-Radon set of size 2



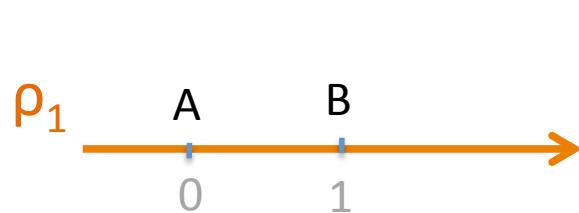
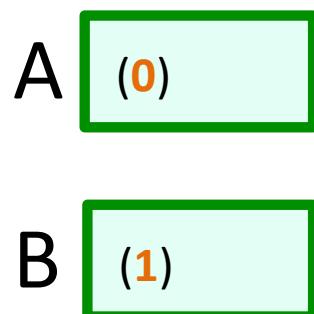
Radon partition candidate:



Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Anti-Radon set of size 2



Radon partition candidate:

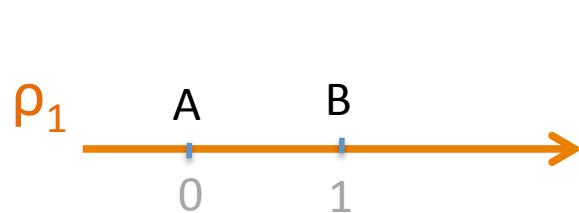
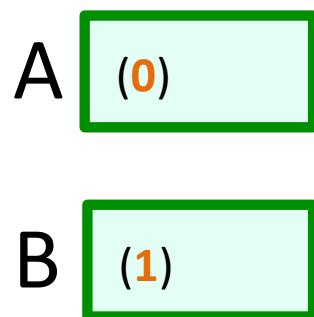
~~{A}, {B}~~

All grids have an anti-Radon set of size 2.

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Anti-Radon set of size 2



Radon partition candidate:

~~{A}, {B}~~

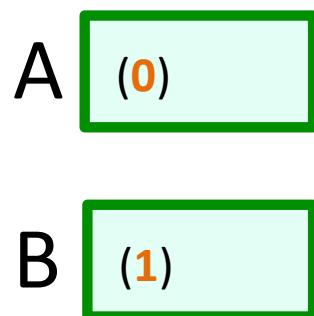
All grids have an anti-Radon set of size 2.

...is that true?

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Anti-Radon set of size 2



Radon partition candidate:

~~{A}, {B}~~

All grids have an anti-Radon set of size 2.

...is that true?



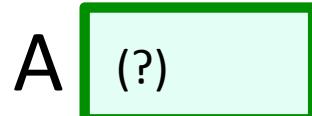
P_1 doesn't.

Geodetic Radon number of grids

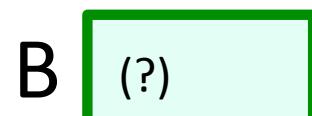
Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Anti-Radon set of size 3

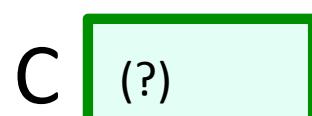
Radon partition candidates:



{A}, {B,C}



{B}, {A,C}

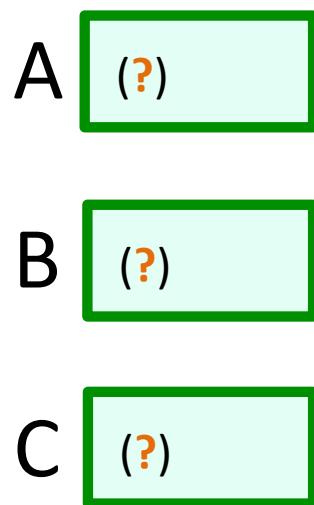


{C}, {A,B}

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Anti-Radon set of size 3



ρ_1



Radon partition candidates:

{**A**}, {**B,C**}

{**B**}, {**A,C**}

{**C**}, {**A,B**}

Geodetic Radon number of grids

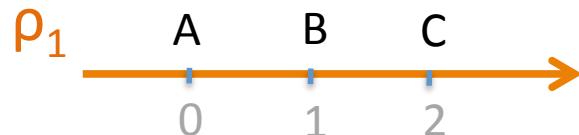
Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Anti-Radon set of size 3

A (0)

B (1)

C (2)



Radon partition candidates:

{A}, {B,C}

{B}, {A,C}

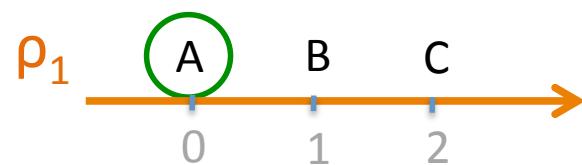
{C}, {A,B}

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Anti-Radon set of size 3

- A (0)
- B (1)
- C (2)



Radon partition candidates:

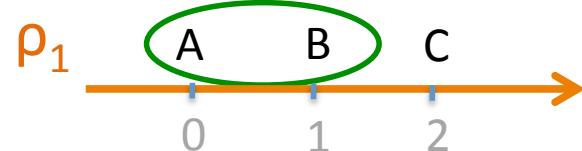
- {A}, {B,C}
- {B}, {A,C}
- {C}, {A,B}

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Anti-Radon set of size 3

- A (0)
- B (1)
- C (2)



Radon partition candidates:

- ~~{A}, {B,C}~~
- ~~{B}, {A,C}~~
- ~~{C}, {A,B}~~

Geodetic Radon number of grids

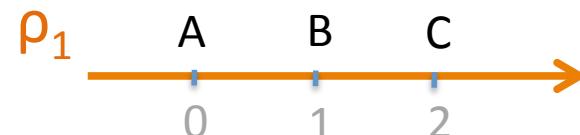
$$\text{Grid } (n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$$

Anti-Radon set of size 3

A (0)

B (1)

C (2)



Radon partition candidates:

~~{A}, {B,C}~~

~~{B}, {A,C}~~

~~{C}, {A,B}~~

1-dimensional grids (i.e. paths) have no anti-Radon set of size 3. Since, except for the trivial grid P_1 , they do have an anti-Radon set of size 2, their Radon number is 3.

Geodetic Radon number of grids

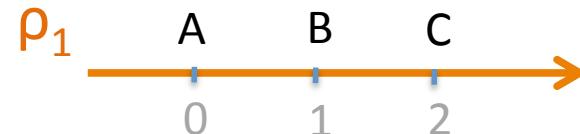
$$\text{Grid } (n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$$

Anti-Radon set of size 3

A (0)

B (1)

C (2)



Radon partition candidates:

~~{A}, {B,C}~~

~~{B}, {A,C}~~

~~{C}, {A,B}~~

1-dimensional grids (i.e. paths) have no anti-Radon set of size 3. Since, except for the trivial grid P_1 , they do have an anti-Radon set of size 2, their Radon number is 3.

...is that true?

Geodetic Radon number of grids

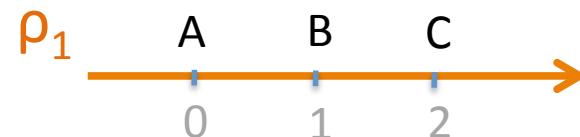
$$\text{Grid } (n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$$

Anti-Radon set of size 3

A (0)

B (1)

C (2)



Radon partition candidates:

~~{A}, {B,C}~~

~~{B}, {A,C}~~

~~{C}, {A,B}~~

1-dimensional grids (i.e. paths) have no anti-Radon set of size 3. Since, except for the trivial grid P_1 , they do have an anti-Radon set of size 2, their Radon number is 3.

...is that true?

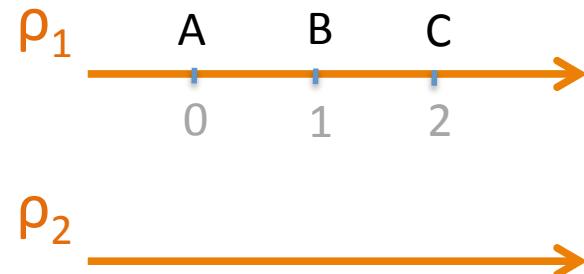


Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Anti-Radon set of size 3

- A (0,?)
- B (1,?)
- C (2,?)



Radon partition candidates:

- ~~{A}, {B,C}~~
- ~~{B}, {A,C}~~
- ~~{C}, {A,B}~~

Geodetic Radon number of grids

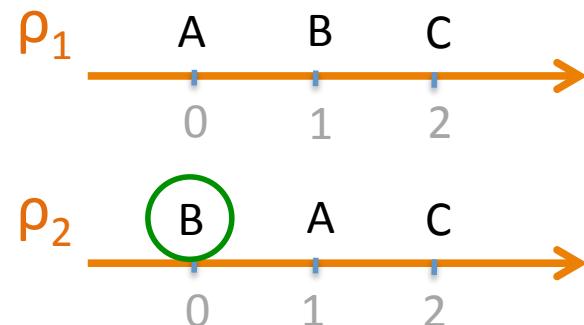
Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Anti-Radon set of size 3

A (0, 1)

B (1, 0)

C (2, 0)



Radon partition candidates:

~~{A}, {B, C}~~

~~{B}, {A, C}~~

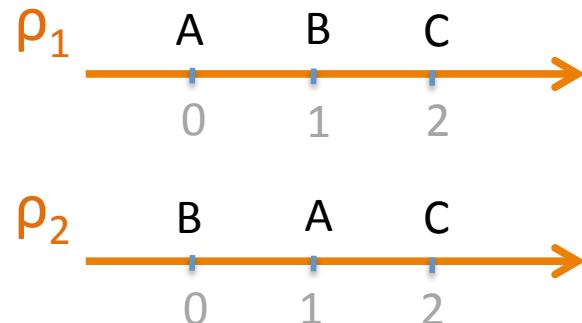
~~{C}, {A, B}~~

Geodetic Radon number of grids

$$\text{Grid } (n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$$

Anti-Radon set of size 3

- A (0,1)
- B (1,0)
- C (2,2)



Radon partition candidates:

- ~~{A}, {B,C}~~
- ~~{B}, {A,C}~~
- ~~{C}, {A,B}~~

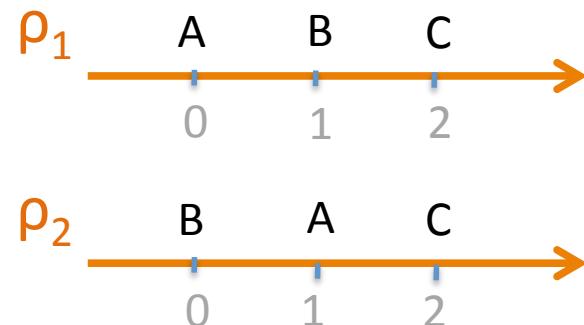
All d -dimensional grids ($d \geq 2$) have an anti-Radon set of size 3.

Geodetic Radon number of grids

$$\text{Grid } (n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$$

Anti-Radon set of size 3

- A (0,1)
- B (1,0)
- C (2,2)



Radon partition candidates:

- ~~{A}, {B,C}~~
- ~~{B}, {A,C}~~
- ~~{C}, {A,B}~~

All d -dimensional grids ($d \geq 2$) have an anti-Radon set of size 3.

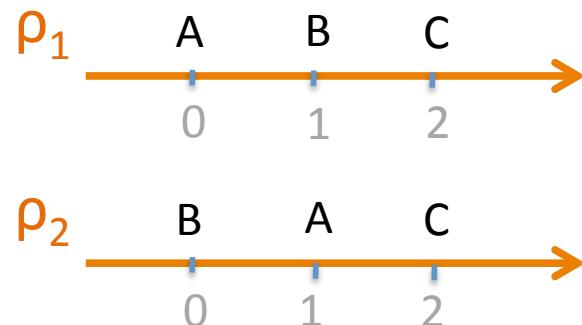
...is that true?

Geodetic Radon number of grids

$$\text{Grid } (n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$$

Anti-Radon set of size 3

- A (0,1)
- B (1,0)
- C (2,2)

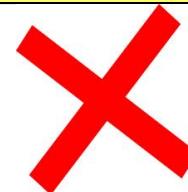


Radon partition candidates:

- ~~{A}, {B,C}~~
- ~~{B}, {A,C}~~
- ~~{C}, {A,B}~~

All d -dimensional grids ($d \geq 2$) have an anti-Radon set of size 3.

...is that true?



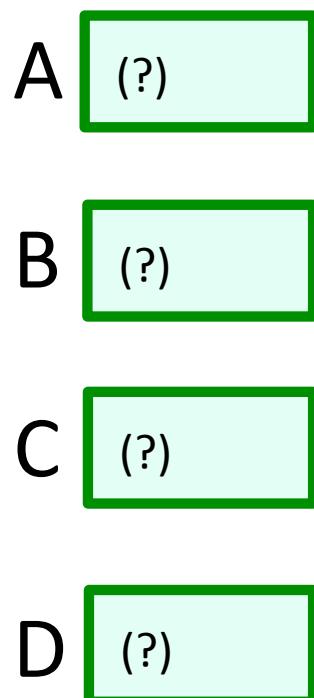
Grid(2,2) doesn't.

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Anti-Radon set of size 4

Radon partition candidates:



{A}, {B,C,D}

{B}, {A,C,D}

{C}, {A,B,D}

{D}, {A,B,C}

{A,B}, {C,D}

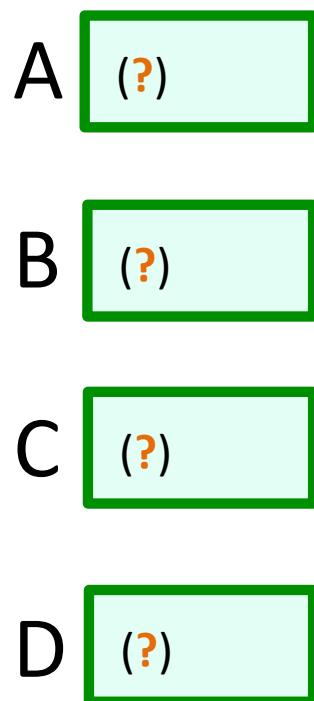
{A,C}, {B,D}

{A,D}, {B,C}

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Anti-Radon set of size 4



ρ_1



Radon partition candidates:

{A}, {B,C,D}

{B}, {A,C,D}

{C}, {A,B,D}

{D}, {A,B,C}

{A,B}, {C,D}

{A,C}, {B,D}

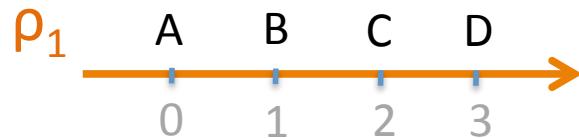
{A,D}, {B,C}

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Anti-Radon set of size 4

- A (0)
- B (1)
- C (2)
- D (3)



Radon partition candidates:

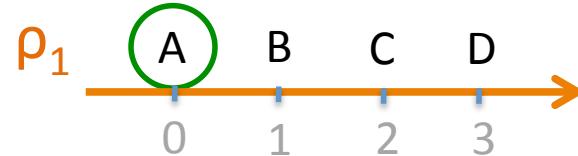
- {A}, {B,C,D}
- {B}, {A,C,D}
- {C}, {A,B,D}
- {D}, {A,B,C}
- {A,B}, {C,D}
- {A,C}, {B,D}
- {A,D}, {B,C}

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Anti-Radon set of size 4

- A (0)
- B (1)
- C (2)
- D (3)



Radon partition candidates:

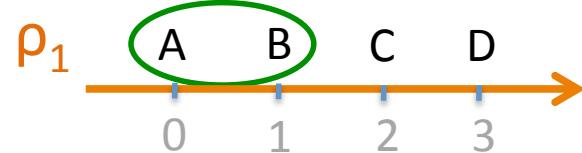
- {A}, {B,C,D}
- {B}, {A,C,D}
- {C}, {A,B,D}
- {D}, {A,B,C}
- {A,B}, {C,D}
- {A,C}, {B,D}
- {A,D}, {B,C}

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Anti-Radon set of size 4

- A (0)
- B (1)
- C (2)
- D (3)



Radon partition candidates:

- ~~{A}, {B,C,D}~~
- {B}, {A,C,D}
- {C}, {A,B,D}
- {D}, {A,B,C}

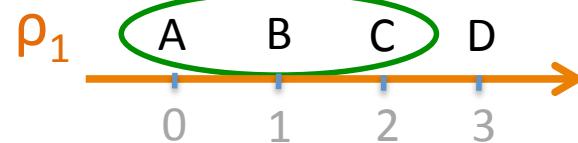
~~{A,B}, {C,D}~~- {A,C}, {B,D}
- ~~{A,D}, {B,C}~~

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Anti-Radon set of size 4

- A (0)
- B (1)
- C (2)
- D (3)



Radon partition candidates:

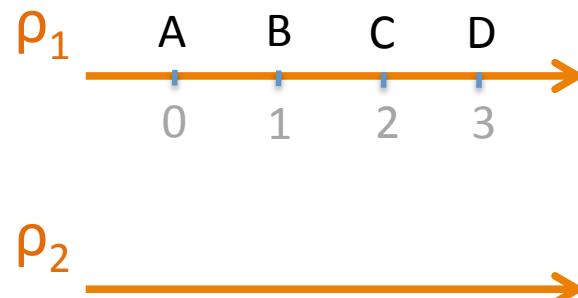
- ~~{A}, {B,C,D}~~
- ~~{B}, {A,C,D}~~
- ~~{C}, {A,B,D}~~
- ~~{D}, {A,B,C}~~
- ~~{A,B}, {C,D}~~
- ~~{A,C}, {B,D}~~
- ~~{A,D}, {B,C}~~

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Anti-Radon set of size 4

- A (0,?)
- B (1,?)
- C (2,?)
- D (3,?)



Radon partition candidates:

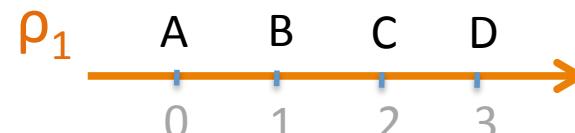
- ~~{A}, {B,C,D}~~
- {B}, {A,C,D}
- {C}, {A,B,D}
- ~~{D}, {A,B,C}~~
- ~~{A,B}, {C,D}~~
- {A,C}, {B,D}
- {A,D}, {B,C}

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Anti-Radon set of size 4

- A (0,2)
- B (1,0)
- C (2,3)
- D (3,1)



Radon partition candidates:

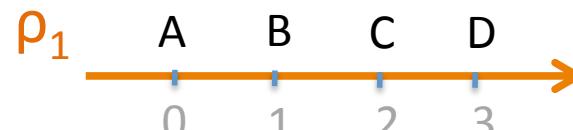
- ~~{A}, {B,C,D}~~
- {B}, {A,C,D}
- {C}, {A,B,D}
- ~~{D}, {A,B,C}~~
- ~~{A,B}, {C,D}~~
- {A,C}, {B,D}
- {A,D}, {B,C}

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Anti-Radon set of size 4

- A (0,2)
- B (1,0)
- C (2,3)
- D (3,1)



Radon partition candidates:

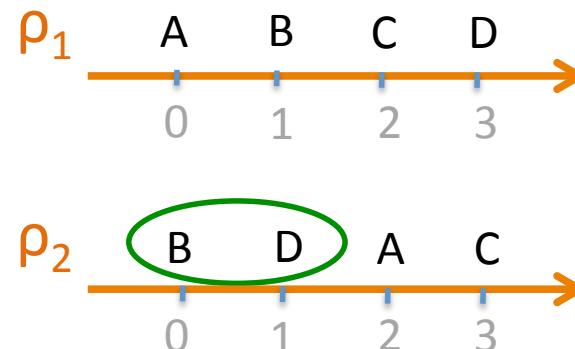
- ~~{A}, {B,C,D}~~
- ~~{B}, {A,C,D}~~
- {C}, {A,B,D}
- ~~{D}, {A,B,C}~~
- ~~{A,B}, {C,D}~~
- {A,C}, {B,D}
- {A,D}, {B,C}

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Anti-Radon set of size 4

- A (0,2)
- B (1,0)
- C (2,3)
- D (3,1)



Radon partition candidates:

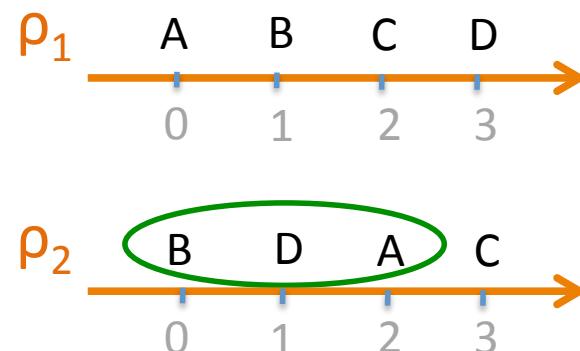
- ~~{A}, {B,C,D}~~
- ~~{B}, {A,C,D}~~
- ~~{C}, {A,B,D}~~
- ~~{D}, {A,B,C}~~
- ~~{A,B}, {C,D}~~
- ~~{A,C}, {B,D}~~
- {A,D}, {B,C}

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Anti-Radon set of size 4

- A (0,2)
- B (1,0)
- C (2,3)
- D (3,1)



Radon partition candidates:

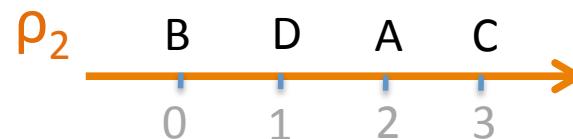
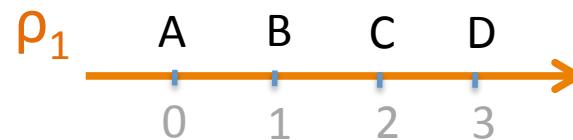
- ~~{A}, {B,C,D}~~
- ~~{B}, {A,C,D}~~
- ~~{C}, {A,B,D}~~
- ~~{D}, {A,B,C}~~
- ~~{A,B}, {C,D}~~
- ~~{A,C}, {B,D}~~
- {A,D}, {B,C}

Geodetic Radon number of grids

$$\text{Grid } (n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$$

Anti-Radon set of size 4

- A (0,2)
- B (1,0)
- C (2,3)
- D (3,1)



Radon partition candidates:

- ~~{A}, {B,C,D}~~
- ~~{B}, {A,C,D}~~
- ~~{C}, {A,B,D}~~
- ~~{D}, {A,B,C}~~
- ~~{A,B}, {C,D}~~
- ~~{A,C}, {B,D}~~
- {A,D}, {B,C}

2-dimensional grids have no anti-Radon set of size 4. Since, except for Grid(2,2), they *do* have an anti-Radon set of size 3, their Radon number is 4.

Geodetic Radon number of grids

$$\text{Grid } (n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$$

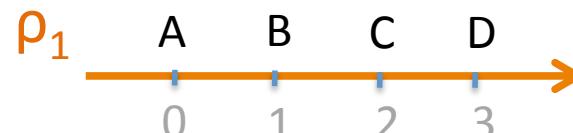
Anti-Radon set of size 4

A (0,2)

B (1,0)

C (2,3)

D (3,1)



Radon partition candidates:

~~{A}, {B,C,D}~~

~~{B}, {A,C,D}~~

~~{C}, {A,B,D}~~

~~{D}, {A,B,C}~~

~~{A,B}, {C,D}~~

~~{A,C}, {B,D}~~

{A,D}, {B,C}

2-dimensional grids have no anti-Radon set of size 4. Since, except for Grid(2,2), they *do* have an anti-Radon set of size 3, their Radon number is 4.

...true?

Geodetic Radon number of grids

$$\text{Grid } (n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$$

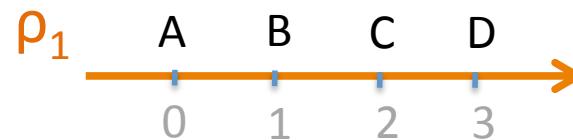
Anti-Radon set of size 4

A (0,2)

B (1,0)

C (2,3)

D (3,1)



2-dimensional grids have no anti-Radon set of size 4. Since, except for Grid(2,2), they *do* have an anti-Radon set of size 3, their Radon number is 4.

Radon partition candidates:

~~{A}, {B,C,D}~~

~~{B}, {A,C,D}~~

~~{C}, {A,B,D}~~

~~{D}, {A,B,C}~~

~~{A,B}, {C,D}~~

~~{A,C}, {B,D}~~

~~{A,D}, {B,C}~~

...true?



Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Anti-Radon set of size 4

A (0,2,?)

B (1,0,?)

C (2,3,?)

D (3,1,?)



Radon partition candidates:

~~{A}, {B,C,D}~~

~~{B}, {A,C,D}~~

~~{C}, {A,B,D}~~

~~{D}, {A,B,C}~~

~~{A,B}, {C,D}~~

~~{A,C}, {B,D}~~

{A,D}, {B,C}

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

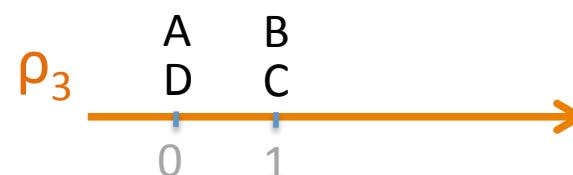
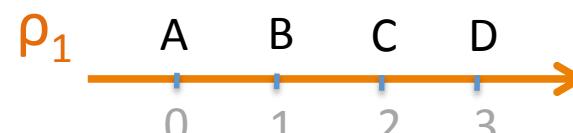
Anti-Radon set of size 4

A (0,2,0)

B (1,0,1)

C (2,3,1)

D (3,1,0)



Radon partition candidates:

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~~{B}, {A,C,D}~~

~~{C}, {A,B,D}~~

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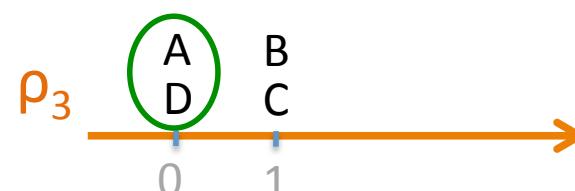
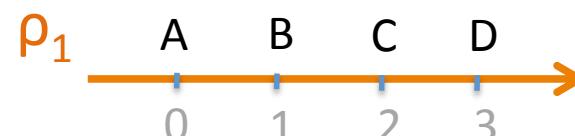
Anti-Radon set of size 4

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B (1,0,1)

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Radon partition candidates:

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Geodetic Radon number of grids

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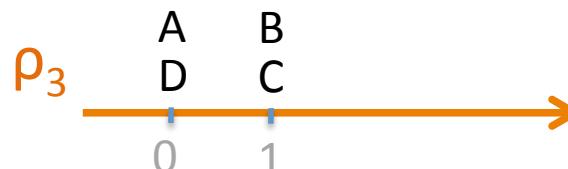
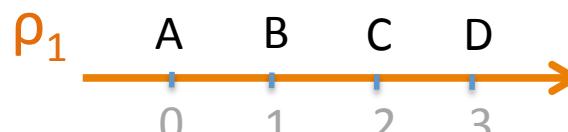
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~~{D}, {A,B,C}~~

~~{A,B}, {C,D}~~

~~{A,C}, {B,D}~~

~~{A,D}, {B,C}~~

All d -dimensional grids ($d \geq 3$) have an anti-Radon set of size 4.

Geodetic Radon number of grids

$$\text{Grid } (n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$$

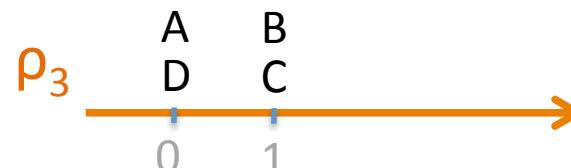
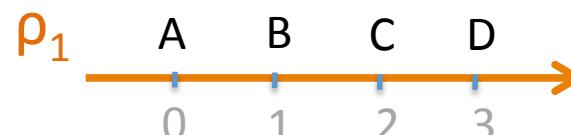
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C (2,3,1)

D (3,1,0)



Radon partition candidates:

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All d -dimensional grids ($d \geq 3$) have
an anti-Radon set of size 4.

...true?

Geodetic Radon number of grids

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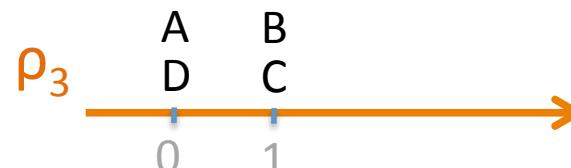
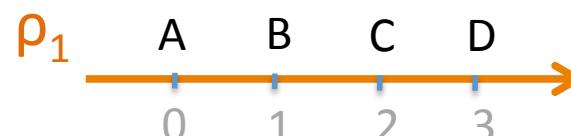
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~~{D}, {A,B,C}~~

~~{A,B}, {C,D}~~

~~{A,C}, {B,D}~~

~~{A,D}, {B,C}~~

...true? X
Several don't.

All d -dimensional grids ($d \geq 3$) have
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Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Anti-Radon set of size r

A_1 (?, ?, ..., ?)

A_2 (?, ?, ..., ?)

.

.

.

A_r (?, ?, ..., ?)

Radon partition candidates:

** HOW MANY ?? **

Geodetic Radon number of grids

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Anti-Radon set of size r

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A_2 (?, ?, ..., ?)

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A_r (?, ?, ..., ?)

Radon partition candidates:

** HOW MANY ?? **

$$(2^r - 2) / 2$$

Geodetic Radon number of grids

$$\text{Grid } (n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$$

Anti-Radon set of size r

A_1 (?, ?, ..., ?)

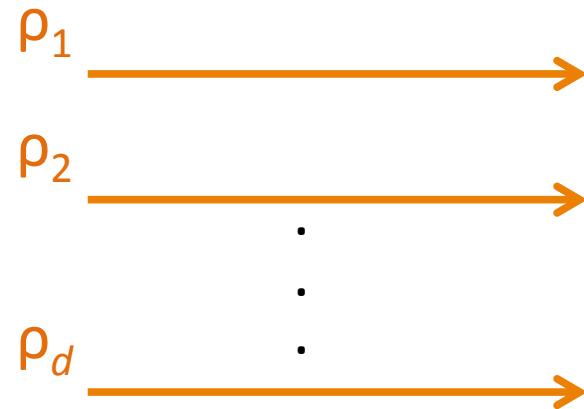
A_2 (?, ?, ..., ?)

.

.

.

A_r (?, ?, ..., ?)



Radon partition candidates:

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A_2 (?, ?, ..., ?)

⋮

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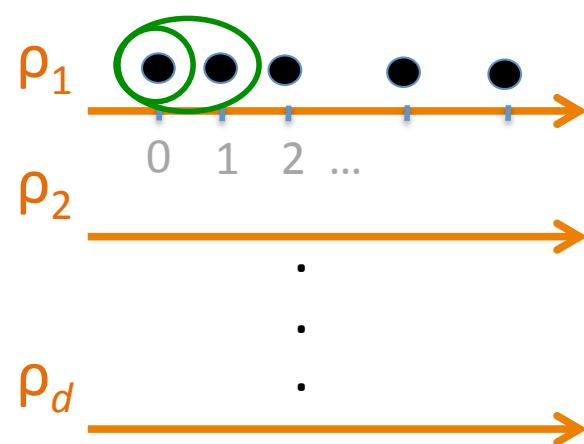
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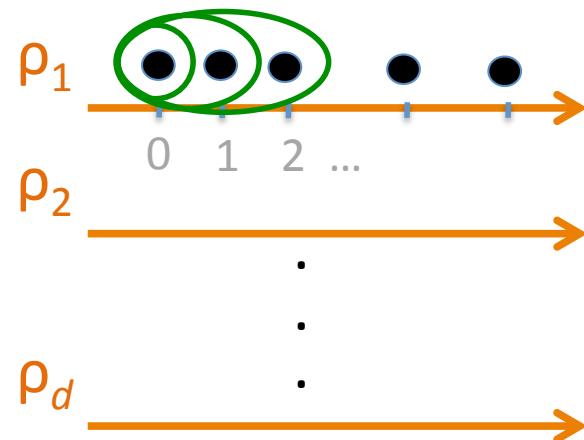
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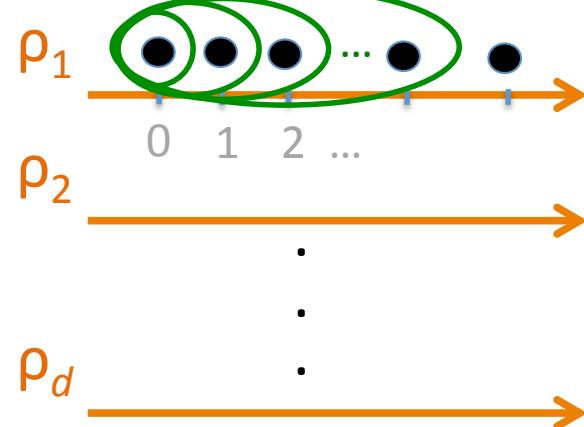
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Anti-Radon set of size r

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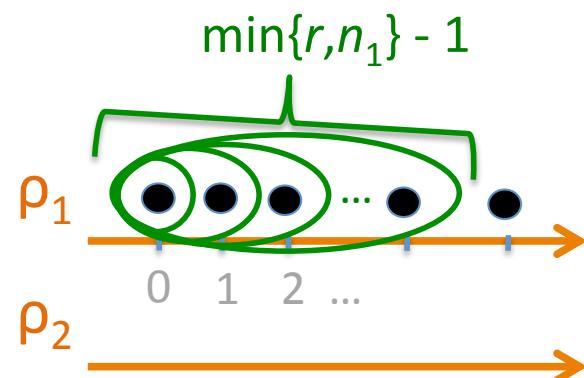
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Anti-Radon set of size r

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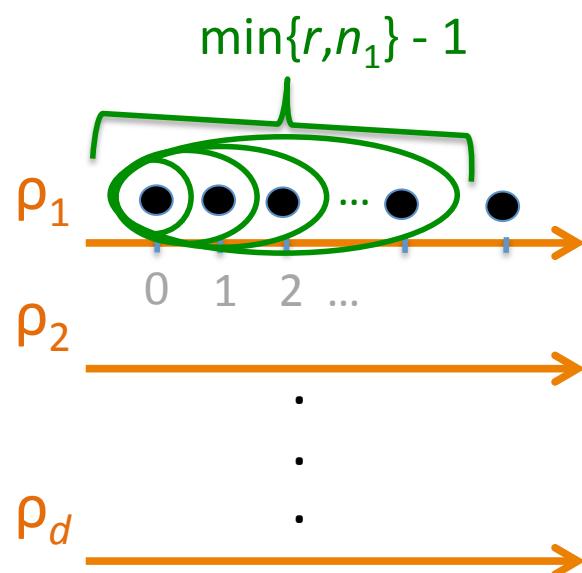
A_2 (?, ?, ..., ?)

.

.

.

A_r (?, ?, ..., ?)



Radon partition candidates:

** HOW MANY ?? **

$$(2^r - 2) / 2$$

Anti-Radon set
of size r



$$\sum_{i=1}^d [\min(r, n_i) - 1] \geq (2^r - 2) / 2$$

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Anti-Radon set
of size r



$$\sum_{i=1}^d [\min(r, n_i) - 1] \geq (2^r - 2) / 2$$

(1)

$r' :=$ the maximum integer that satisfies (1)

max anti-Radon set size $\leq r'$

Radon number $\leq r' + 1$

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

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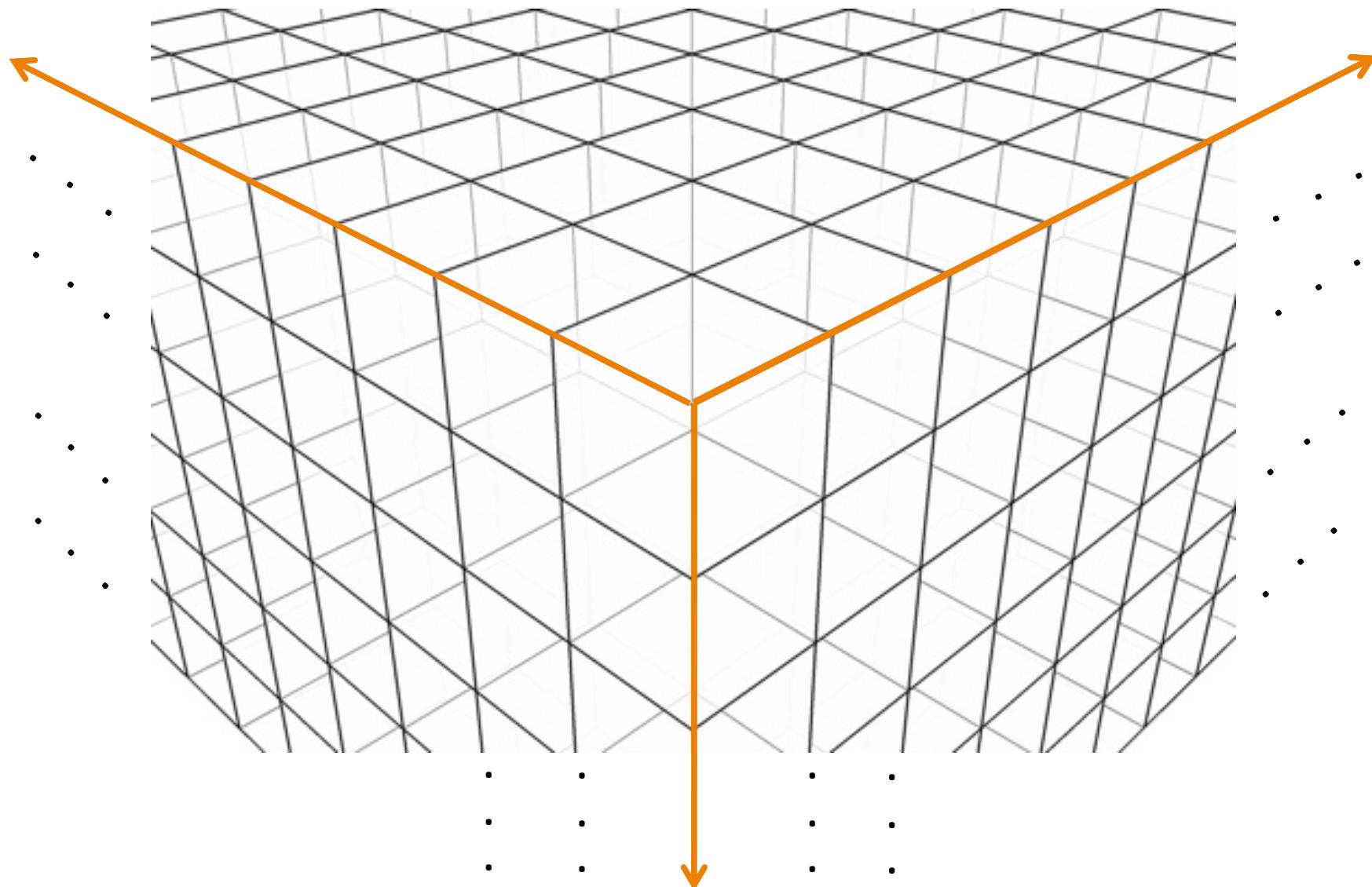
Equality does not always hold.

E.g. 5-dimensional Grid $(2,5,5,5,5) \rightarrow r' = 5$

max anti-Radon set size = 4

Geodetic Radon number of grids

What if all dimensions are “sufficiently large”?



Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

If all dimensions are “sufficiently large”...

Anti-Radon set
of size r

→

$$\sum_{i=1}^d [\min(r, n_i) - 1] \geq (2^r - 2) / 2 \quad (1)$$

$r' :=$ the maximum integer that satisfies (1)

max anti-Radon set size $\leq r'$

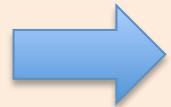
Radon number $\leq r' + 1$

Geodetic Radon number of grids

$$\text{Grid } (n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$$

If all dimensions are “sufficiently large”...

Anti-Radon set
of size r



$$d(r-1) \geq (2^r - 2) / 2$$

(2)

r'' := the maximum integer that satisfies (2)

$$\text{max anti-Radon set size} \leq r''$$

$$\text{Radon number} \leq r'' + 1$$

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

If all dimensions are “sufficiently large”...

Anti-Radon set
of size r



$$d(r-1) \geq (2^r - 2) / 2 \quad (2)$$

r'' := the maximum integer that satisfies (2)

max anti-Radon set size $\leq r''$

Radon number $\leq r'' + 1$

Equality does not always hold.

E.g. 7-dimensional Grid (9,9,9,9,9,9,9) $\rightarrow r'' = 6$
max anti-Radon set size = 5

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

A tighter necessary condition...

Anti-Radon set
of size r



$$2d \geq \binom{r}{\lfloor r/2 \rfloor} \quad (3)$$

Geodetic Radon number of grids

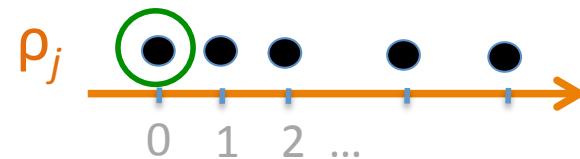
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Geodetic Radon number of grids

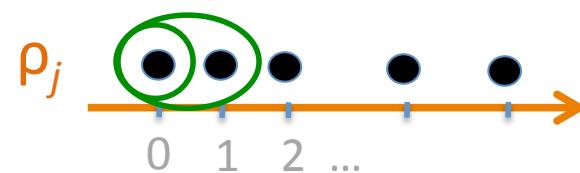
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Geodetic Radon number of grids

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Geodetic Radon number of grids

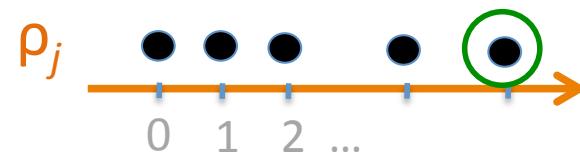
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Geodetic Radon number of grids

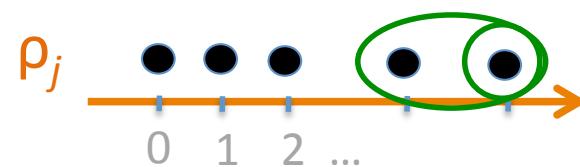
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Geodetic Radon number of grids

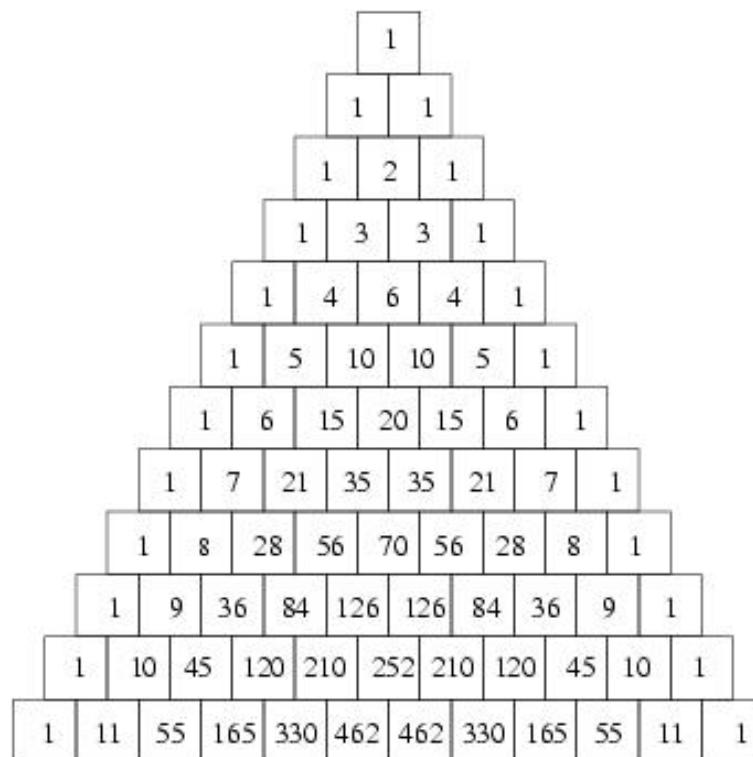
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Geodetic Radon number of grids

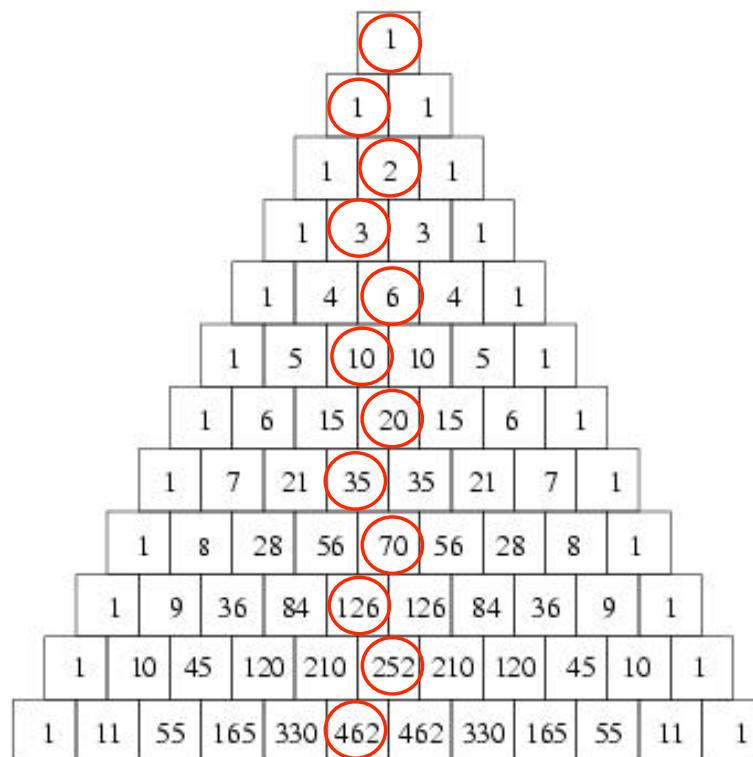
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Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

A tighter necessary condition...

Anti-Radon set
of size r



$$2d \geq \binom{r}{\lfloor r/2 \rfloor} \quad (3)$$

r''' := the maximum integer that satisfies (3)

max anti-Radon set size $\leq r'''$

Radon number $\leq r''' + 1$

Geodetic Radon number of grids

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

A tighter necessary condition...

Anti-Radon set
of size r



$$2d \geq \binom{r}{\lfloor r/2 \rfloor} \quad (3)$$

r''' := the maximum integer that satisfies (3)

max anti-Radon set size $\leq r'''$

Radon number $\leq r''' + 1$

...which is also sufficient!

(for “large enough” d -dimensional grids)

Theorem: Max anti-Radon set size $= r'''$
If $\min\{n_i\} \geq r''' \dots$ Radon number $= r''' + 1$

Geodetic Radon number of grids

Anti-Radon set
of size r



$$2d \geq \binom{r}{\lfloor r/2 \rfloor} \quad (3)$$

$r''' :=$ the maximum integer that satisfies (3)

Theorem: Max anti-Radon set size = r'''

If $\min\{n_i\} \geq r''' \dots$ Radon number = $r''' + 1$

Example: $d = 10$

$$\binom{5}{2} = 10 \leq 2d$$

$$\rightarrow \binom{6}{3} = 20 \leq 2d$$

$$\binom{7}{3} = 35 > 2d$$

$$r''' = 6$$

Anti-Radon set $R = \{A, B, C, D, E, F\}$

Geodetic Radon number of grids

Example: $d = 10$

$$\binom{5}{2} = 10 \leq 2d$$

→ $\binom{6}{3} = 20 \leq 2d$

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Anti-Radon set $R = \{A, B, C, D, E, F\}$

{A}	{A,B}	{A,B,C}
{B}	{A,C}	{A,B,D}
{C}	{A,D}	{A,B,E}
{D}	{A,E}	{A,B,F}
{E}	{A,F}	{A,C,D}
{F}	{B,C}	{A,C,E}
	{B,D}	{A,C,F}
	{B,E}	{A,D,E}
	{B,F}	{A,D,F}
	{C,D}	{A,E,F}
	{C,E}	{B,C,D}
	{C,F}	{B,C,E}
	{D,E}	{B,C,F}
	{D,F}	{B,D,E}
	{E,F}	{B,D,F}
		{B,E,F}
		{C,D,E}
		{C,D,F}
		{C,E,F}
		{D,E,F}

Subsets of R with size
at most $\lfloor 6/2 \rfloor = 3$

Geodetic Radon number of grids

Example: $d = 10$

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→ $\binom{6}{3} = 20 \leq 2d$

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Anti-Radon set $R = \{A, B, C, D, E, F\}$

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{E}	{A,F}	{A,C,D}
{F}	{B,C}	{A,C,E}
	{B,D}	{A,C,F}
	{B,E}	{A,D,E}
	{B,F}	{A,D,F}
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	{C,F}	{B,C,E}
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	{D,F}	{B,D,E}
	{E,F}	{B,D,F}
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Subsets of R with size
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Geodetic Radon number of grids

Example: $d = 10$

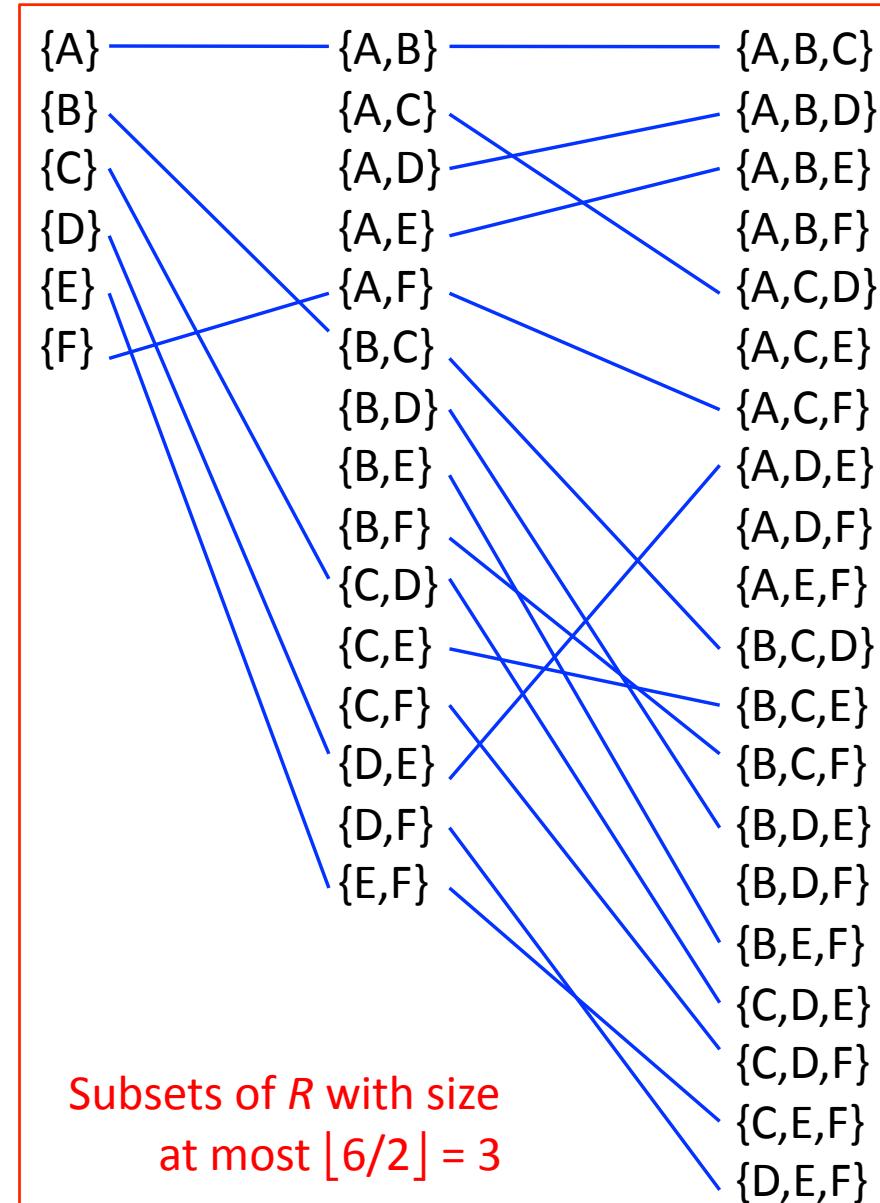
$$\binom{5}{2} = 10 \leq 2d$$

→ $\binom{6}{3} = 20 \leq 2d$

$$\binom{7}{3} = 35 > 2d$$

$$r''' = 6$$

Anti-Radon set $R = \{A, B, C, D, E, F\}$



Geodetic Radon number of grids

Example: $d = 10$

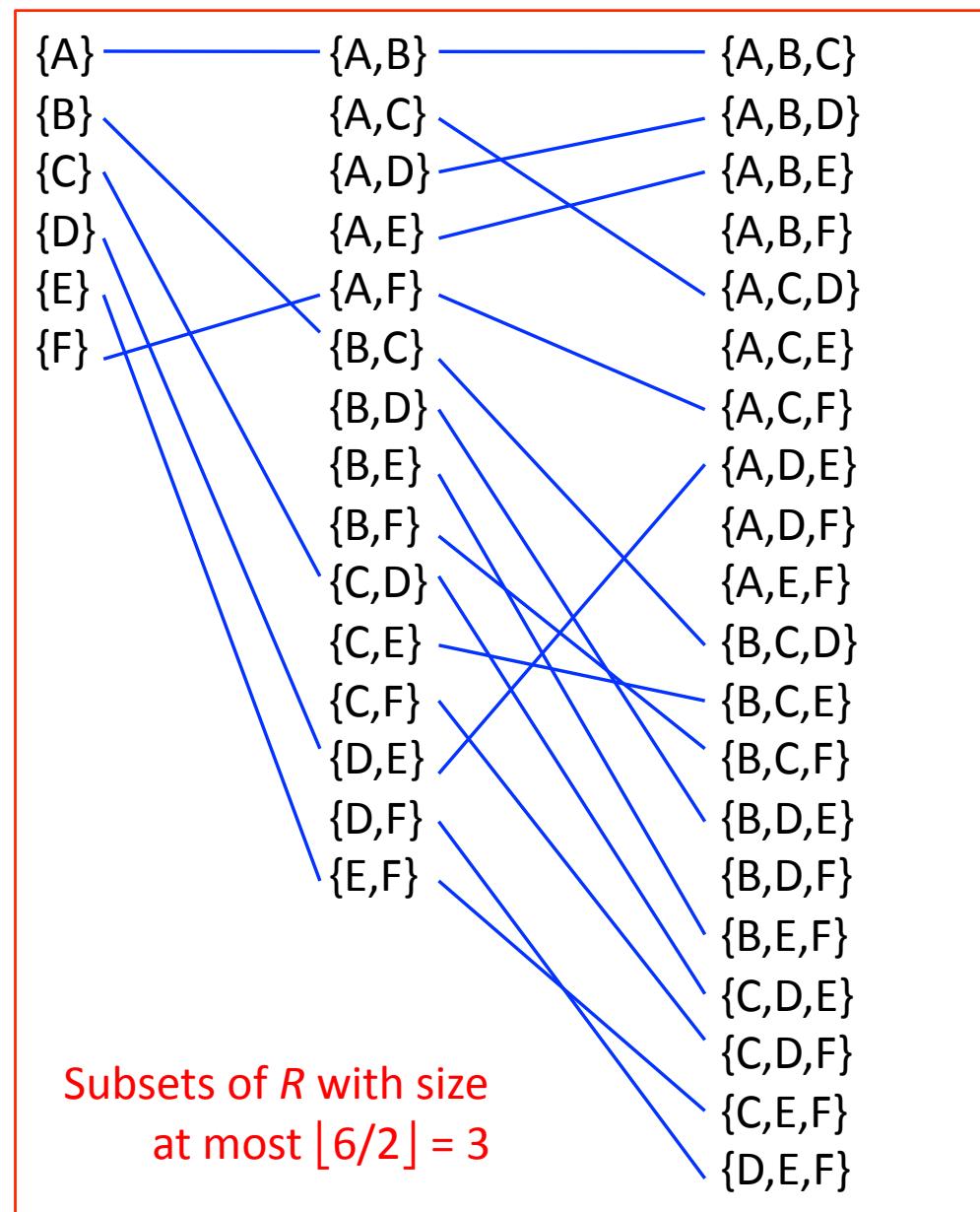
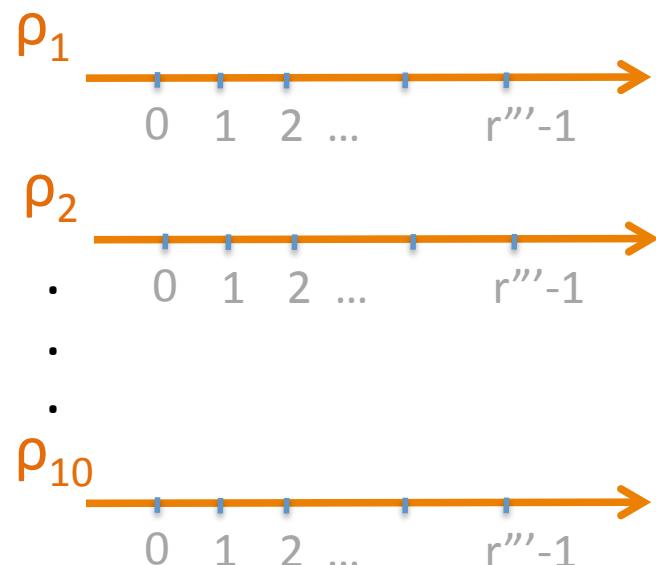
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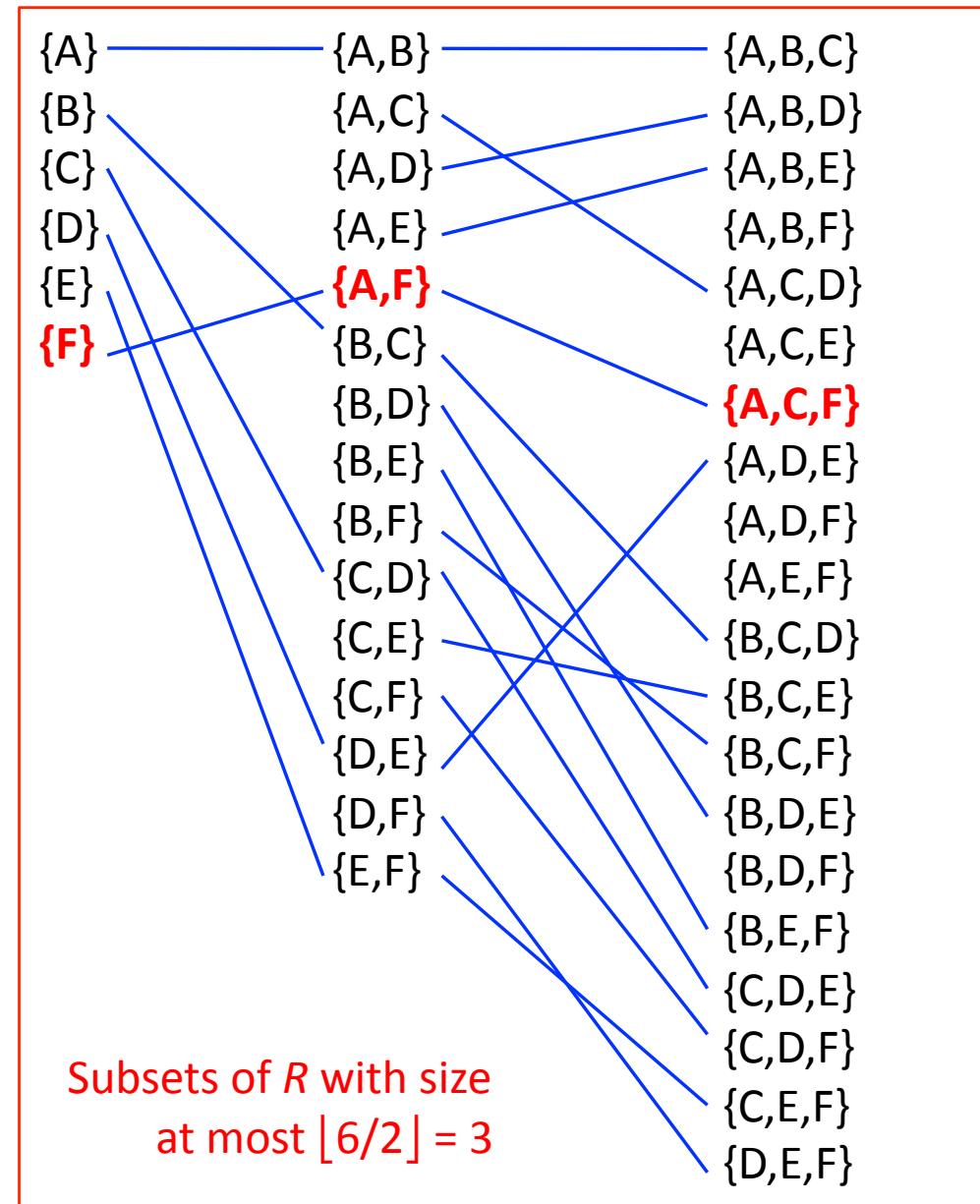
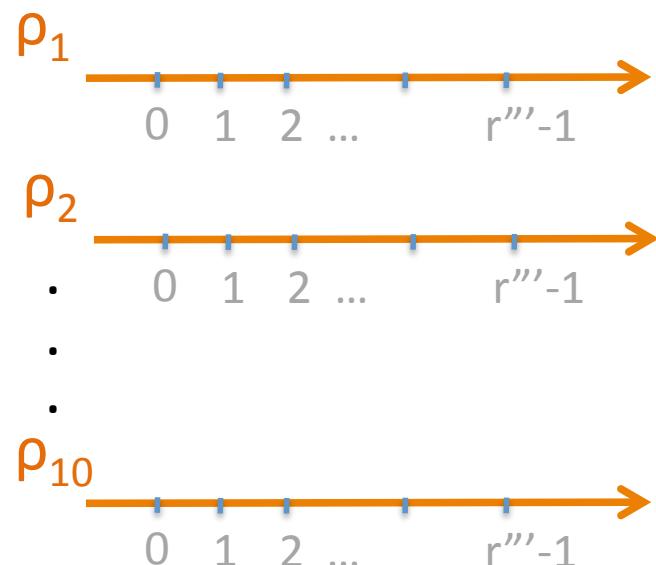
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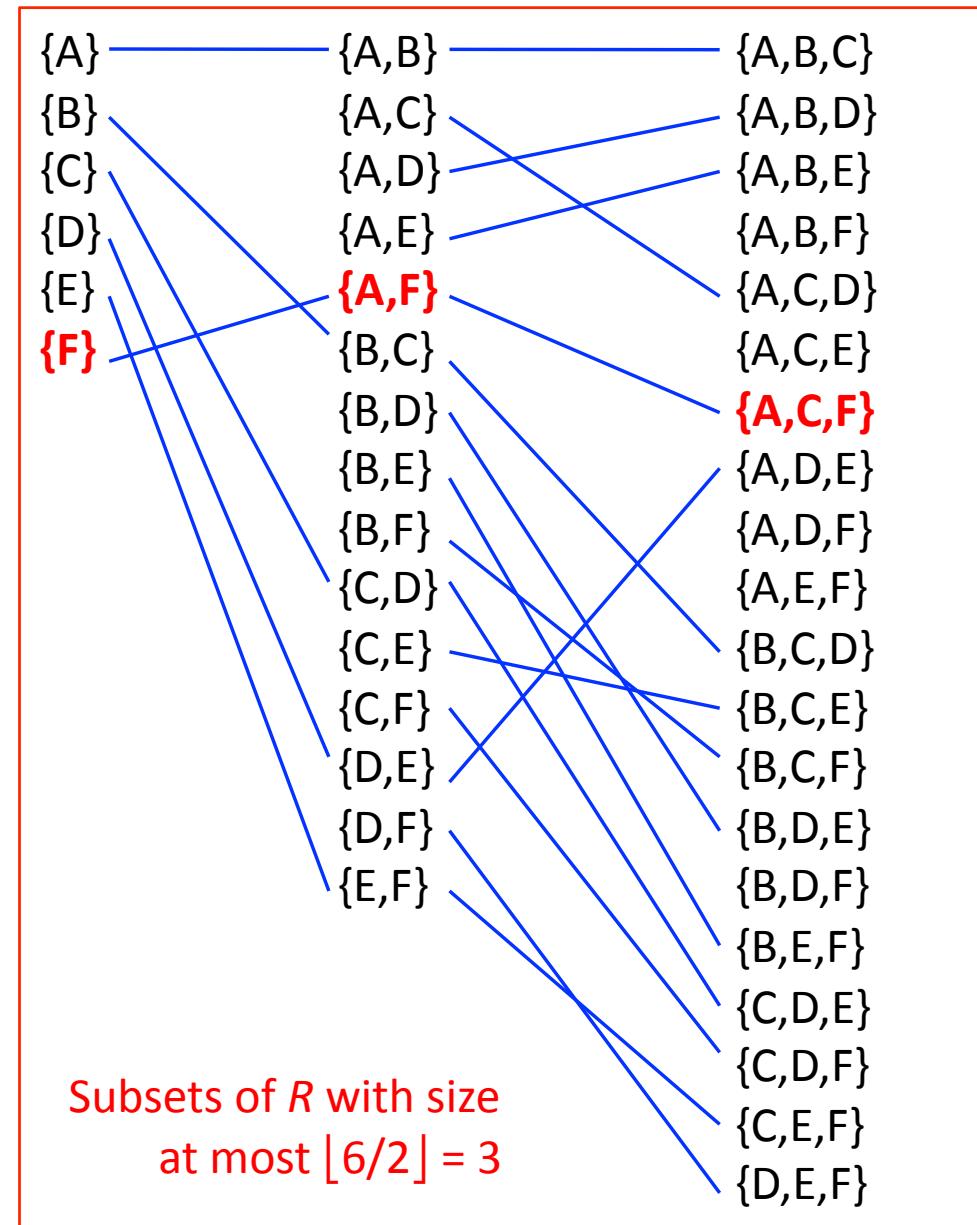
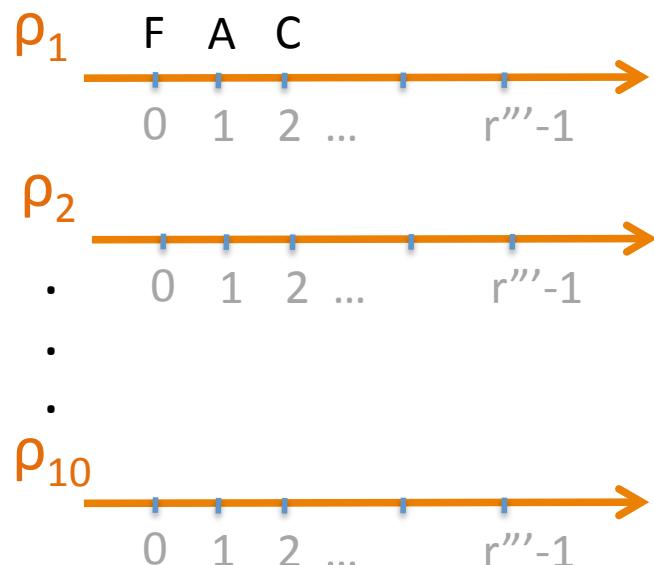
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Geodetic Radon number of grids

Example: $d = 10$

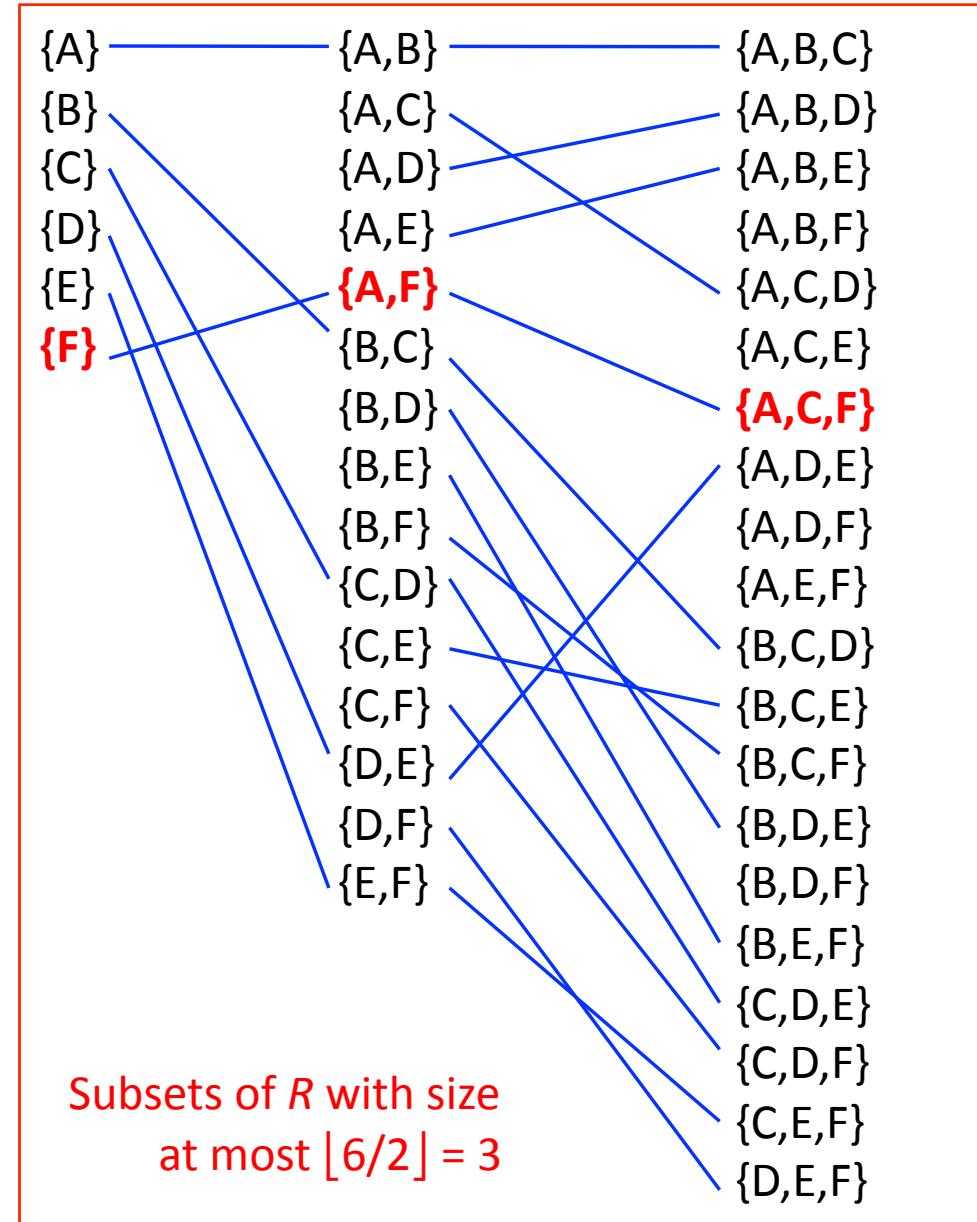
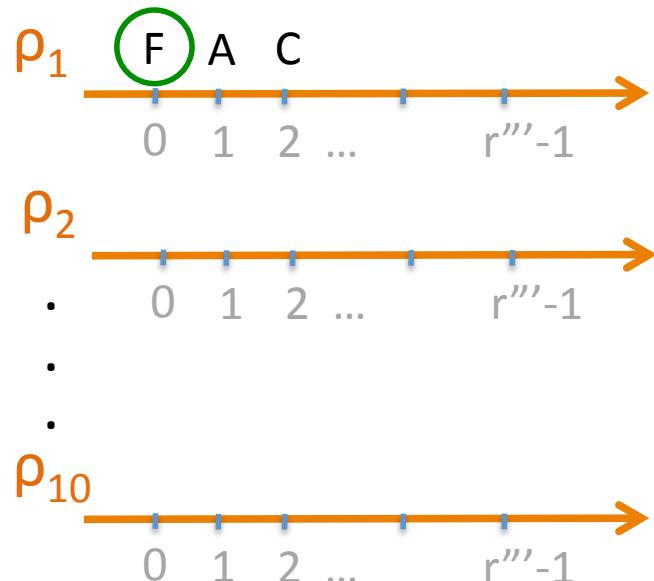
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Geodetic Radon number of grids

Example: $d = 10$

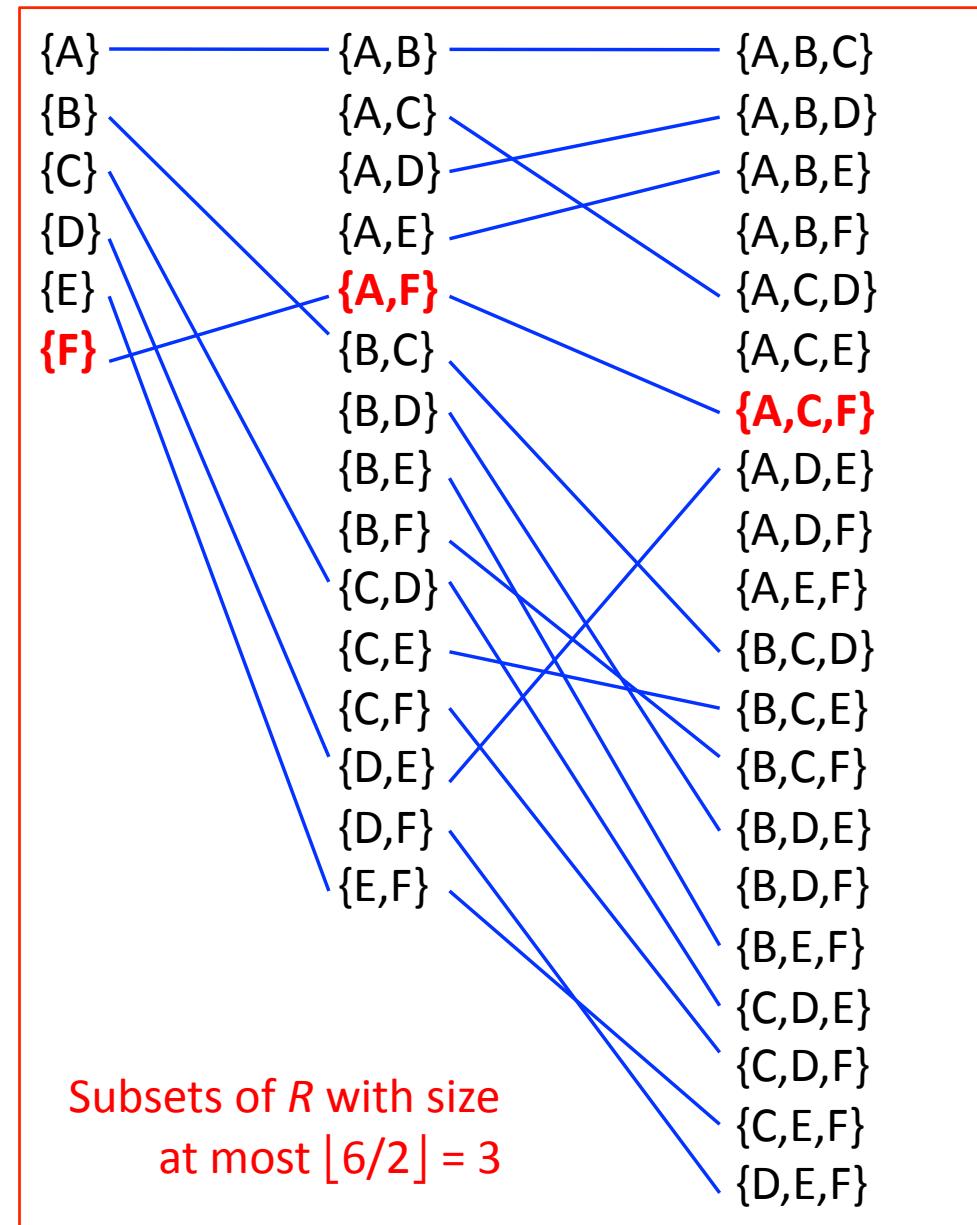
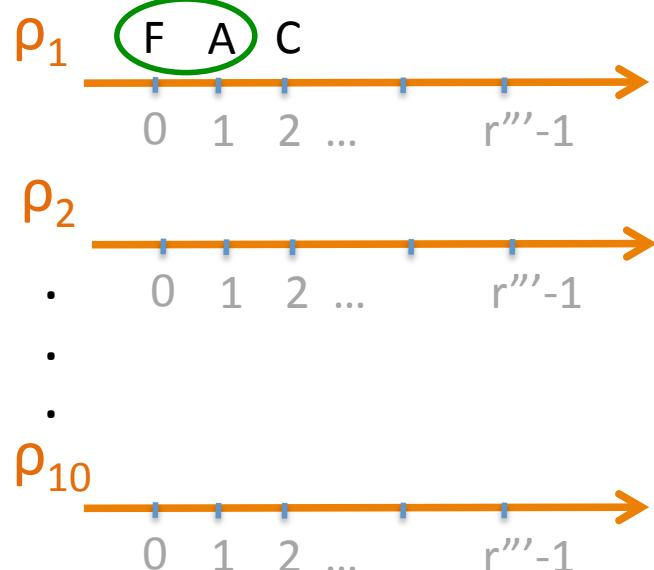
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Geodetic Radon number of grids

Example: $d = 10$

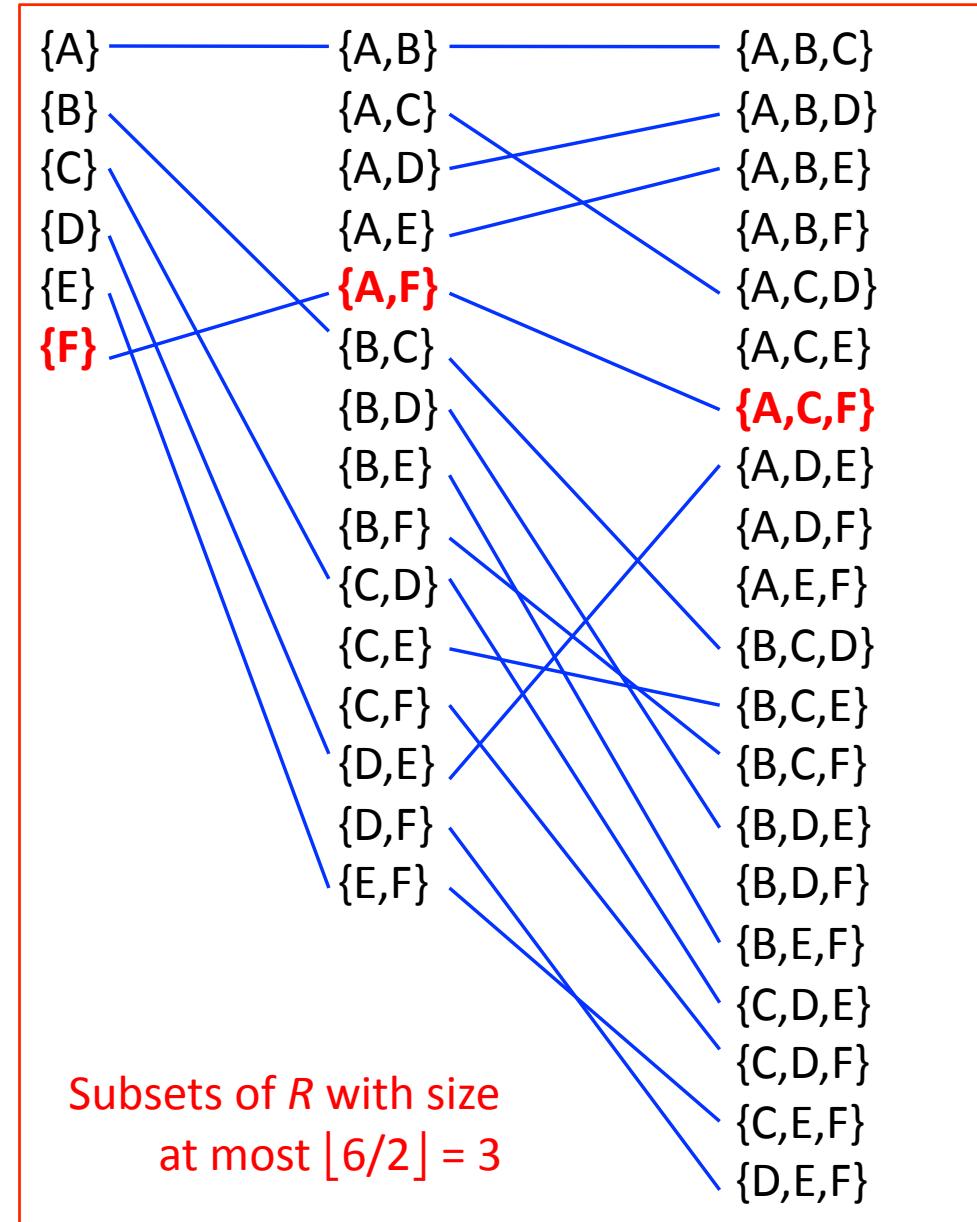
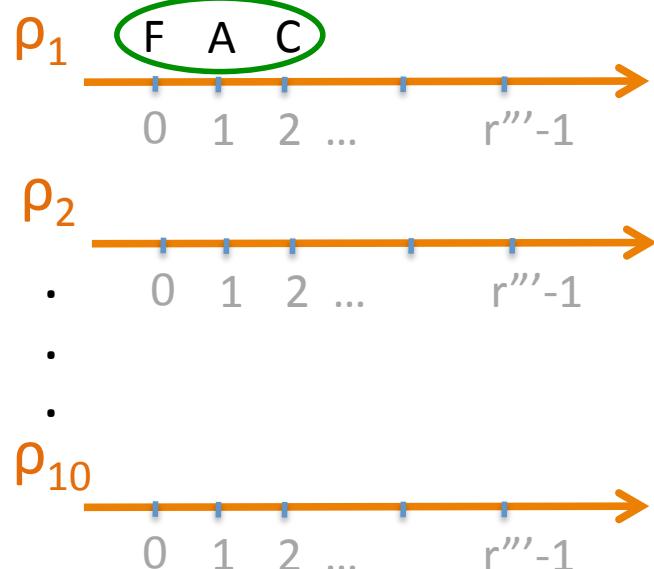
$$\binom{5}{2} = 10 \leq 2d$$

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Anti-Radon set $R = \{A, B, C, D, E, F\}$



Other results and particular cases

$$\text{Grid } (n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$$

Let $d, n_1, n_2, \dots, n_d \in \mathbb{N}$, and let $n^* = \min \{n_i\}$.

Grid (n_1, n_2, \dots, n_d) admits a *quasi-anti-Radon set* R of size r'''

s.t. R has no Radon partition $R_1 \cup R_2$ with

$$\lfloor r'''/2 \rfloor - \lfloor n^* / 2 \rfloor + 1 \leq |R_1| \leq |R_2|.$$

Other results and particular cases

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

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$$\lfloor r'''/2 \rfloor - \lfloor n^* / 2 \rfloor + 1 \leq |R_1| \leq |R_2|.$$

Let $d, n_1, n_2, \dots, n_d, r, p \in \mathbb{N}$ with $2p \leq r$ and $2p \leq n_i$ for $i = 1, \dots, d$.

If $\binom{r}{p} \leq 2d$, then there is a set R of vertices of Grid (n_1, n_2, \dots, n_d) of size r

s.t. R has no Radon partition $R_1 \cup R_2$ with

$$|R_1| \leq p.$$

Other results and particular cases

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

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If $\binom{r}{p} \leq 2d$, then there is a set R of vertices of Grid (n_1, n_2, \dots, n_d) of size r
s.t. R has no Radon partition $R_1 \cup R_2$ with

$$|R_1| \leq p.$$

Radon number of d -dimensional Grid $(2, 2, \dots, 2)$ = $\lfloor \log_2 (d+1) \rfloor + 2$.

Other results and particular cases

Grid $(n_1, n_2, \dots, n_d) := P_{n_1} \times P_{n_2} \times \dots \times P_{n_d}$

Let $d, n_1, n_2, \dots, n_d \in \mathbb{N}$, and let $n^* = \min \{n_i\}$.

Grid (n_1, n_2, \dots, n_d) admits a *quasi-anti-Radon set* R of size r'''
s.t. R has no Radon partition $R_1 \cup R_2$ with

$$\lfloor r'''/2 \rfloor - \lfloor n^* / 2 \rfloor + 1 \leq |R_1| \leq |R_2|.$$

Let $d, n_1, n_2, \dots, n_d, r, p \in \mathbb{N}$ with $2p \leq r$ and $2p \leq n_i$ for $i = 1, \dots, d$.

If $\binom{r}{p} \leq 2d$, then there is a set R of vertices of Grid (n_1, n_2, \dots, n_d) of size r
s.t. R has no Radon partition $R_1 \cup R_2$ with

$$|R_1| \leq p.$$

Radon number of d -dimensional Grid $(2, 2, \dots, 2) = \lfloor \log_2 (d+1) \rfloor + 2$.

Radon number of d -dimensional Grid $(3, 3, \dots, 3) = \lfloor \log_2 d \rfloor + 3$.

Complete results for $d \leq 9$

Complete results for $d \leq 9$

d	Dimension sizes	Cl 3	Cl 4	$r(G)$	Max anti-Radon set
1	1	3	2	2	(1)
	∞		3	3	(1), (2)
2	2, 2	4	3	3	(1, 1), (1, 2)
	2, ∞		4	4	(1, 3), (1, 1), (2, 2)
3	2, 2, 2	5	4	4	(1, 2, 2), (2, 2, 1), (1, 1, 1)
	2, 3, ∞		4	4	(1, 1, 1), (2, 1, 3), (1, 3, 3)
	2, ∞ , ∞		5	5	(2, 4, 3), (1, 3, 1), (1, 2, 4), (2, 1, 2)
	3, 3, 3		4	4	(1, 1, 3), (1, 1, 1), (3, 3, 2)
	3, 3, ∞		5	5	(2, 1, 4), (1, 1, 1), (3, 2, 2), (1, 3, 3)
4	2, 2, 2, 2	5	4	4	(1, 1, 2, 2), (2, 2, 2, 1), (1, 1, 1, 1)
	2, 2, 2, ∞		4	4	(1, 1, 1, 1), (2, 2, 2, 2), (1, 1, 1, 3)
	2, 2, 3, 3		4	4	(1, 1, 1, 1), (2, 2, 1, 3), (1, 1, 3, 3)
	2, 2, 3, ∞		5	5	(1, 2, 1, 3), (1, 1, 2, 1), (2, 2, 3, 2), (1, 1, 3, 4)
	2, 3, 3, 3		5	5	(1, 2, 1, 3), (1, 1, 2, 1), (2, 1, 3, 3), (1, 3, 3, 2)
5	2, 2, 2, 2, 2	6	4	4	(1, 2, 2, 1, 1), (1, 1, 1, 1, 1), (2, 2, 1, 2, 2)
	2, 2, 2, 2, 3		4	4	(1, 2, 2, 1, 1), (2, 2, 1, 2, 3), (1, 1, 1, 1, 2)
	2, 2, 2, 2, 4		5	5	(1, 2, 2, 1, 3), (2, 1, 2, 2, 2), (1, 1, 1, 1, 1), (1, 1, 1, 2, 4)
	2, 2, 2, 2, ∞		5	5	(1, 2, 2, 1, 3), (2, 1, 2, 2, 2), (1, 1, 1, 1, 1), (1, 1, 1, 2, 4)
	2, 2, 2, 3, 3		5	5	(1, 1, 1, 3, 3), (1, 1, 1, 1, 1), (2, 1, 2, 3, 1), (1, 2, 2, 2, 2)
	2, 2, 2, 3, ∞		5	5	(1, 2, 2, 2, 2), (2, 1, 2, 3, 4), (1, 1, 1, 3, 1), (1, 1, 1, 1, 3)
	2, 2, 3, 3, 3		5	5	(1, 1, 1, 3, 3), (1, 1, 1, 1, 1), (2, 1, 2, 3, 1), (1, 2, 2, 2, 2)
	2, 2, ∞ , ∞ , ∞		5	5	(2, 1, 4, 4, 3), (1, 2, 3, 1, 4), (2, 1, 2, 2, 1), (1, 1, 1, 3, 2)
	2, 3, 3, 3, 3		5	5	(1, 1, 1, 3, 3), (1, 1, 1, 1, 1), (2, 1, 2, 3, 1), (1, 2, 2, 2, 2)
	2, ∞ , ∞ , ∞ , ∞		6	5	(1, 4, 4, 2, 4), (2, 3, 2, 4, 2), (1, 2, 1, 1, 1), (1, 1, 3, 3, 3)
	3, 3, 3, 3, 3		5	5	(3, 3, 3, 2, 1), (3, 2, 2, 1, 3), (1, 1, 2, 1, 1), (2, 1, 1, 3, 2)
	3, 3, 4, 4, ∞		5	5	(2, 1, 1, 4, 3), (1, 1, 3, 2, 1), (3, 3, 4, 3, 4), (3, 2, 2, 1, 2)
	3, 3, 4, ∞ , ∞		6	6	(2, 1, 3, 4, 1), (3, 3, 1, 5, 3), (3, 2, 4, 2, 4), (1, 1, 1, 3, 5), (1, 3, 2, 1, 2)
	3, 4, 4, 4, 4		5	5	(2, 1, 4, 1, 3), (3, 4, 3, 4, 4), (3, 2, 1, 2, 2), (1, 3, 2, 3, 1)
	3, 4, 4, 4, ∞		6	6	(1, 1, 4, 2, 3), (3, 3, 2, 1, 2), (2, 4, 4, 4, 1), (1, 4, 1, 3, 4), (3, 2, 3, 4, 5)
	4, 4, 4, 4, 4		6	6	(4, 4, 3, 4, 1), (2, 4, 4, 1, 3), (3, 2, 2, 3, 4), (1, 3, 1, 2, 1), (1, 1, 4, 4, 2)

Complete results for $d \leq 9$

d	Dimension sizes	Cl 3	Cl 4	$r(G)$	Max anti-Radon set
6	2, 2, 2, 2, 2, 2	6	4	4	(1, 1, 2, 1, 1, 1), (2, 2, 2, 2, 2, 2), (1, 2, 1, 1, 1, 1)
	2, 2, 2, 2, 2, 3				(1, 1, 2, 1, 2, 3), (1, 2, 1, 2, 1, 3), (2, 1, 1, 2, 2, 2), (1, 1, 1, 1, 1, 1)
	2, 2, 2, ∞ , ∞ , ∞				(2, 1, 2, 2, 1, 1), (2, 2, 1, 3, 2, 2), (1, 1, 2, 4, 4, 4), (1, 1, 1, 1, 3, 3)
	2, 2, 3, 3, ∞ , ∞				(2, 1, 3, 1, 4, 4), (1, 1, 1, 1, 1, 3), (1, 2, 2, 2, 3, 1), (1, 1, 3, 3, 2, 2)
	2, 2, 3, 4, 4, ∞				(1, 1, 3, 4, 3, 4), (2, 1, 3, 2, 1, 2), (1, 1, 1, 1, 2, 3), (2, 2, 2, 3, 4, 1)
	2, 2, 3, 4, ∞ , ∞				(2, 2, 2, 3, 3, 1), (1, 1, 1, 1, 1, 2), (2, 1, 1, 2, 4, 5), (1, 2, 3, 1, 5, 3), (1, 1, 3, 4, 2, 4)
	2, 2, 4, 4, 4, 4				(2, 1, 2, 2, 1, 2), (1, 1, 1, 3, 2, 3), (2, 1, 4, 4, 3, 4), (1, 2, 3, 1, 4, 1)
	2, 2, 4, 4, 4, ∞				(2, 1, 1, 2, 4, 4), (2, 1, 3, 4, 2, 2), (1, 2, 4, 1, 3, 3), (1, 2, 2, 3, 1, 5), (1, 1, 1, 1, 1, 1)
	2, 3, 3, 3, 4, ∞				(1, 1, 2, 1, 1, 3), (1, 3, 2, 3, 2, 2), (2, 3, 3, 1, 4, 4), (1, 2, 1, 2, 3, 1)
	2, 3, 3, 4, 4, 4				(2, 2, 1, 3, 4, 1), (2, 3, 2, 2, 1, 2), (1, 3, 3, 4, 3, 4), (1, 1, 2, 1, 2, 3)
	2, 3, 3, 4, 4, ∞				(1, 1, 2, 1, 1, 1), (2, 2, 3, 2, 1, 5), (2, 3, 1, 3, 2, 2), (1, 1, 3, 4, 3, 3), (2, 1, 1, 1, 4, 4)
	2, 3, 4, 4, 4, 4				(1, 3, 1, 2, 1, 1), (2, 3, 4, 1, 2, 3), (1, 2, 2, 3, 3, 4), (2, 3, 3, 4, 4, 1), (1, 1, 4, 4, 1, 2)
	3, 3, 3, 3, 3, ∞				(3, 3, 3, 1, 3, 4), (2, 2, 1, 2, 2, 3), (1, 1, 2, 1, 1, 1), (3, 3, 2, 3, 3, 2)
	3, 3, 3, 3, 4, 4				(1, 1, 2, 1, 1, 3), (3, 3, 2, 3, 2, 2), (2, 2, 1, 2, 3, 1), (3, 3, 3, 1, 4, 4)
	3, 3, 3, 3, 4, ∞				(1, 2, 1, 2, 2, 5), (3, 1, 3, 3, 1, 4), (1, 1, 2, 1, 1, 1), (3, 1, 1, 1, 4, 3), (2, 3, 2, 3, 3, 2)
	3, 3, 3, 4, 4, 4				(3, 3, 2, 3, 4, 3), (1, 3, 1, 4, 2, 1), (2, 2, 3, 4, 1, 4), (1, 1, 3, 2, 3, 2), (3, 3, 3, 1, 1, 1)
7	2, 2, 2, 2, 2, 2, 2	6	5	5	(1, 1, 2, 1, 1, 1, 1), (1, 2, 1, 1, 1, 2, 2), (2, 2, 2, 1, 2, 2, 1), (1, 2, 2, 2, 2, 1, 2)
	2, 2, 2, 3, 4, 4, 4				(2, 1, 2, 3, 4, 3, 4), (1, 2, 2, 2, 1, 4, 1), (1, 1, 1, 1, 3, 2, 3), (1, 1, 2, 3, 2, 1, 2)
	2, 2, 2, 3, 4, 4, ∞				(2, 2, 1, 2, 4, 3, 3), (1, 2, 1, 3, 2, 1, 5), (1, 1, 1, 1, 1, 4, 4), (2, 1, 2, 3, 1, 2, 2), (1, 1, 1, 1, 3, 1, 1)
	2, 2, 2, 4, 4, 4, 4				(1, 1, 1, 1, 1, 1, 1), (1, 2, 2, 3, 2, 4, 1), (2, 1, 2, 4, 3, 1, 2), (2, 1, 2, 2, 1, 3, 4), (1, 2, 1, 2, 4, 2, 3)
	2, 2, 3, 3, 3, 3, 3				(1, 1, 2, 1, 1, 1, 1), (1, 2, 1, 1, 1, 2, 2), (2, 2, 2, 1, 2, 2, 1), (1, 2, 2, 2, 2, 1, 2)
	2, 2, 3, 3, ∞ , ∞ , ∞				(2, 2, 2, 2, 5, 5, 3), (1, 1, 1, 1, 1, 4, 4), (1, 1, 1, 3, 4, 2, 5), (2, 1, 1, 3, 2, 3, 1), (1, 1, 3, 1, 3, 1, 2)
	2, 2, 3, 4, 4, 4, 4				(2, 2, 2, 3, 4, 2, 1), (2, 2, 3, 1, 2, 4, 3), (1, 1, 3, 4, 1, 3, 1), (1, 2, 3, 4, 3, 1, 4), (1, 2, 1, 2, 1, 1, 2)
	2, 3, 3, 3, 3, 3, 4				(2, 3, 2, 3, 3, 2, 4), (1, 3, 1, 3, 3, 1, 2), (1, 1, 3, 1, 1, 1, 3), (1, 2, 3, 2, 2, 3, 1)
	2, 3, 3, 3, 3, 4, 4				(2, 3, 2, 3, 3, 2, 4), (1, 1, 3, 3, 1, 1, 3), (2, 3, 3, 1, 2, 1, 1), (1, 3, 1, 3, 1, 3, 1), (1, 2, 3, 2, 3, 4, 2)
	3, 3, 3, 3, 3, 3, 3				(3, 3, 1, 3, 3, 1, 1), (3, 3, 2, 3, 3, 2, 3), (2, 2, 3, 2, 2, 3, 1), (1, 1, 3, 1, 1, 1, 2)
	3, 3, 3, 3, 3, 3, 4				(3, 1, 3, 2, 1, 3, 1), (1, 3, 3, 3, 1, 1, 2), (3, 3, 1, 3, 3, 2, 1), (3, 2, 3, 2, 3, 1, 4), (2, 3, 2, 1, 2, 3, 3)

Complete results for $d \leq 9$

d	Dimension sizes	Cl 3	Cl 4	$r(G)$	Max anti-Radon set
8	2, 2, 2, 2, 2, 2, 2, 2	6	5	5	(1, 1, 2, 1, 1, 1, 1, 1), (1, 2, 1, 1, 1, 1, 2, 2), (2, 2, 2, 2, 1, 2, 2, 1), (1, 2, 2, 1, 2, 2, 1, 2)
	2, 2, 2, 2, 2, 4, 4, 4		5	5	(2, 1, 2, 1, 1, 4, 4, 4), (1, 2, 1, 2, 1, 3, 1, 1), (1, 1, 2, 2, 2, 2, 2, 2), (1, 1, 1, 1, 1, 1, 3, 3)
	2, 2, 2, 2, 2, 4, 4, ∞	6	6		(1, 1, 1, 1, 1, 2, 1), (2, 1, 1, 2, 2, 1, 4, 4), (1, 2, 1, 1, 2, 2, 1, 5), (1, 2, 1, 2, 1, 4, 3, 3), (2, 1, 2, 2, 2, 3, 1, 2)
	2, 2, 2, 2, 3, 3, 3, ∞	5	5		(1, 2, 1, 2, 2, 2, 2, 3), (2, 1, 2, 1, 1, 3, 3, 4), (1, 1, 2, 2, 3, 3, 3, 2), (1, 1, 1, 1, 1, 1, 1, 1)
	2, 2, 2, 2, 3, 3, 4, 4	5	5		(2, 1, 2, 1, 1, 3, 4, 4), (1, 1, 2, 2, 3, 3, 2, 2), (1, 1, 1, 1, 1, 1, 3), (1, 2, 1, 2, 2, 2, 3, 1)
	2, 2, 2, 2, 3, 3, 4, ∞	6	6		(2, 2, 1, 2, 3, 3, 1, 3), (1, 2, 1, 1, 2, 1, 2, 5), (1, 1, 1, 1, 1, 2, 1, 1), (1, 1, 1, 2, 1, 3, 4, 4), (2, 1, 2, 2, 2, 1, 3, 2)
	2, 2, 2, 2, 3, 4, 4, 4	6	6		(2, 2, 2, 1, 3, 2, 2, 4), (1, 1, 1, 2, 3, 2, 1, 1), (2, 2, 1, 2, 2, 3, 4, 2), (2, 1, 1, 2, 1, 4, 1, 4), (1, 1, 1, 1, 1, 1, 3, 3)
	2, 2, 2, 3, 3, 3, 3, 4	5	5		(1, 2, 1, 2, 2, 2, 2, 3), (1, 1, 2, 3, 3, 3, 3, 2), (2, 1, 2, 1, 3, 3, 3, 4), (1, 1, 1, 1, 1, 1, 1, 1)
	2, 2, 2, 3, 3, 3, 3, ∞	6	6		(1, 2, 1, 2, 1, 1, 2, 5), (2, 1, 1, 1, 3, 2, 3, 4), (1, 1, 1, 1, 2, 1, 1, 1), (1, 1, 2, 2, 1, 3, 3, 2), (2, 2, 1, 3, 2, 3, 1, 3)
	2, 2, 2, 3, 3, 3, 4, 4	6	6		(2, 1, 1, 1, 3, 3, 4, 1), (1, 2, 1, 1, 2, 2, 3, 4), (2, 2, 2, 2, 1, 3, 2, 2), (2, 1, 1, 3, 3, 1, 2, 3), (1, 1, 1, 1, 1, 1, 1, 1)
	2, 2, 3, 3, 3, 3, 3, 3	5	5		(2, 1, 3, 2, 2, 2, 2, 2), (1, 1, 2, 3, 3, 3, 3, 3), (1, 2, 3, 3, 1, 3, 3, 3), (1, 1, 1, 1, 1, 1, 1, 1)
	2, 2, 3, 3, 3, 3, 3, 4	6	6		(2, 1, 1, 1, 3, 2, 3, 4), (2, 1, 3, 2, 3, 1, 1, 2), (1, 1, 1, 1, 1, 2, 1, 1), (2, 2, 2, 2, 1, 3, 3, 2), (1, 2, 1, 3, 2, 1, 2, 3)
	2, 3, 3, 3, 3, 3, 3, 3	6	6		(1, 2, 1, 1, 3, 3, 2, 3), (2, 3, 2, 2, 2, 2, 1, 2), (1, 3, 1, 3, 1, 3, 3, 1), (1, 2, 3, 2, 1, 1, 3, 3), (1, 1, 1, 1, 1, 1, 1, 1)
9	2, 2, 2, 2, 2, 2, 2, 2, 2	6	5	5	(1, 2, 2, 1, 1, 2, 2, 1, 2), (2, 2, 2, 2, 2, 1, 2, 2, 1), (1, 1, 2, 1, 1, 1, 1, 1, 1, 1), (1, 2, 1, 1, 1, 1, 1, 2, 2)
	2, 2, 2, 2, 2, 2, 2, 4, ∞		5	5	(1, 1, 1, 1, 1, 1, 1, 3), (1, 1, 2, 2, 2, 1, 1, 2, 2), (1, 2, 1, 2, 1, 1, 1, 3, 1), (2, 1, 2, 1, 1, 2, 4, 4)
	2, 2, 2, 2, 2, 2, 3, 3, 3	5	5		(1, 1, 1, 2, 2, 2, 2, 3, 3), (2, 2, 1, 2, 1, 1, 3, 3, 3), (1, 1, 1, 1, 1, 1, 1, 1, 1), (1, 1, 2, 1, 2, 1, 3, 2, 2)
	2, 2, 2, 2, 2, 2, 3, 3, ∞	5	5		(2, 1, 2, 1, 1, 2, 3, 3, 4), (1, 1, 1, 1, 1, 1, 1, 1, 1), (1, 2, 1, 2, 1, 1, 2, 2, 3), (1, 1, 2, 2, 2, 1, 3, 3, 2)
	2, 2, 2, 2, 2, 2, 3, 4, 4	6	6		(1, 1, 1, 1, 1, 1, 1, 3, 1), (1, 2, 2, 1, 1, 3, 3, 2, 2), (2, 2, 1, 1, 2, 2, 2, 3, 4), (2, 1, 1, 2, 2, 3, 1, 2, 1), (1, 1, 1, 1, 1, 1, 1, 1)
	2, 2, 2, 2, 2, 3, 3, 3, 4	6	6		(1, 1, 1, 1, 1, 3, 3, 2, 4), (1, 1, 1, 1, 1, 1, 1, 1, 1), (1, 2, 2, 1, 2, 1, 3, 3, 2), (2, 2, 1, 1, 3, 2, 2, 1, 3, 2)
	2, 2, 2, 2, 3, 3, 3, 3	5	5		(2, 1, 1, 2, 2, 3, 3, 3, 3), (1, 1, 1, 1, 1, 1, 1, 1, 1), (1, 2, 1, 2, 3, 1, 3, 3, 3), (2, 1, 2, 1, 3, 2, 2, 2, 2)
	2, 2, 2, 2, 3, 3, 3, 4	6	6		(1, 1, 1, 1, 1, 3, 3, 2, 4), (1, 1, 1, 1, 1, 1, 1, 1, 1), (1, 2, 2, 1, 2, 1, 3, 3, 2), (2, 2, 1, 1, 3, 2, 2, 1, 3, 2)
	2, 2, 2, 3, 3, 3, 3, 3	6	6		(2, 1, 1, 2, 2, 3, 3, 2, 1, 1), (1, 2, 1, 3, 2, 2, 1, 2, 3), (2, 1, 2, 2, 2, 3, 1, 3, 2), (2, 1, 1, 2, 1, 3, 3, 3, 3), (1, 1, 1, 1, 1, 1, 1, 2, 1, 2)



On the geodetic Radon number of grids

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ABSTRACT

It is NP-hard to determine the Radon number of graphs in the geodetic convexity. However, for certain classes of graphs, this well-known convexity parameter can be determined efficiently. In this paper, we focus on geodetic convexity spaces built upon d -dimensional grids, which are the Cartesian products of d paths. After revisiting a result of Eckhoff concerning the Radon number of \mathbb{R}^d in the convexity defined by Manhattan distance, we present a series of theoretical findings that disclose some very nice combinatorial aspects of the problem for grids. We also give closed expressions for the Radon number of the product of P_2 's and the product of P_3 's, as well as computer-aided results covering the Radon number of all possible Cartesian products of d paths for $d \leq 9$.

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Thank you. ;-)



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The geodetic Radon number of grids

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