Geodetic Number Versus Hull Number in P_3 Convexity

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Abstract

We study the graphs G for which the hull number $h(G)$ and the geodetic number $g(G)$ with respect to P_3-convexity coincide. These two parameters correspond to the minimum cardinality of a set U of vertices of G such that the simple expansion process which iteratively adds to U all vertices outside of U having two neighbors in U produces the whole vertex set of G either eventually or after one iteration, respectively. We establish numerous structural properties of the graphs G with $h(G) = g(G)$, allowing for the constructive characterization as well as the efficient recognition of all triangle-free such graphs. Furthermore, we characterize—in terms of forbidden induced subgraphs—the graphs G that satisfy $h(G') = g(G')$ for every induced subgraph G' of G.

Keywords. Hull number; geodetic number; P_3-convexity; irreversible 2-threshold processes