

Instructions: In this session, we will learn some more mathematical modeling. We will consider some scenarios where linear programming may be applied to solve a practical problem. Use these exercises to become more comfortable with writing optimisation models and solving them using AMPL.

Answer all the questions in this assignment. Only the questions marked **[R]** need to be answered in a lab-report. Do not include AMPL commands in your report. Write the report in your lab notebook after answering all such questions. Upload your models and code files on moodle. Also save all your files in the server in a new folder lab01. Show all your work to TA/instructor before leaving.

Answer all the questions in this assignment. Only the questions marked **[R]** need to be answered in a lab-report. All other questions must also be solved.

Exercise 1: A Simple Linear Program

Consider the optimization problem.

$$\begin{aligned} \min & 10x_1 + 5x_2 - 3x_3 - 6x_4 + 9x_5 - 7x_6 - 1.6x_7 \\ \text{s.t.} & \sum_{i=1}^7 x_i = 100, \\ & x_1 + x_2 \geq 10, \\ & 4x_3 + 2x_6 + 3x_7 \leq 150, \\ & 10x_1 - x_6 + 3x_5 \geq 20, \\ & x_3 - x_5 + 3x_7 \geq 10, \\ & x_1 \geq -1, \\ & x_2 \geq 3, \\ & x_3 \geq -10, \\ & x_4 \geq 8, \\ & x_5 \geq 9, \\ & x_6 \geq 0, \\ & x_7 \geq 0. \end{aligned}$$

1. Create a file 'lab01ex1.mod', with variables 'x1, x2, x3, x4, x5, x6, x7'.
2. Solve this optimisation problem.
3. Verify whether the solution obtained satisfies all the constraints and has the correct objective function value.
4. **[R]** Report the objective function value, the values of all variables at the optimal point, and the constraint activities (the value of each function in a constraint) for first four constraints. Use 'display' command.

Exercise 2: LP Reformulation

Consider the following optimisation problem:

$$\begin{aligned} \max & 0.043x_1 + 0.027x_2 + 0.025x_3 + 0.022x_4 + 0.045x_5 - 0.0275y \\ \text{s.t. } & x_1 + x_2 + x_3 + x_4 + x_5 - y \leq 100 \\ & x_2 + x_3 + x_4 \geq 40 \\ & \frac{2x_1 + 2x_2 + x_3 + x_4 + 5x_5}{x_1 + x_2 + x_3 + x_4 + x_5} \leq 1.4 \\ & \frac{9x_1 + 15x_2 + 4x_3 + 3x_4 + 2x_5}{x_1 + x_2 + x_3 + x_4 + x_5} \leq 5 \\ & x_1, x_2, x_3, x_4, x_5 \geq 0 \\ & y \in [0, 10] \end{aligned}$$

1. **[R]** Rewrite this optimisation problem as a linear program, and include it in your report.
2. Create a file 'lab01ex2.mod', containing the linear program in AMPL format.
3. Solve this model using gurobi solver.
4. **[R]** Report the optimal solution value, the values of variables at the optimal solution and the activities of all constraints of the LP model.

Exercise 3: Absolute Values, MinMax and MaxMin

Consider the following min-max optimisation problem

$$\begin{aligned} \min \quad & \max \{3x_1 + 5x_2 - 4x_3, 2x_1 - 9x_2 + 13x_3, 3x_2 + 3x_3 - 20\} \\ \text{s.t.} \quad & |x_1 + x_2 + x_3| \leq 100 \\ & x_1, x_2, x_3 \in [-150, 150] \end{aligned}$$

1. **[R]** Write the above problem as a linear optimisation problem.
2. Solve it using gurobi solver after writing a suitable AMPL model.
3. **[R]** Report the objective function value and the values of variables at the optimal solution.