S.102 (d)

s $\rightarrow$ supply by y, d $\rightarrow$defective

Probability that the computer was supplied by y, if the product

is defective $P(s/d) = \dfrac{P(s \cap d)}{P(d)}$

$P(s \cap d) = 0.3 \times 0.02 = 0.006$

$P(d) = 0.6 \times 0.01 + 0.3 \times 0.02 + 0.1 \times 0.03 = 0.015$

$P(s/d) = \dfrac{0.006}{0.015} = \mathbf{0.4}$

S.103 (c)

$P(A) = 0.2$ A$\rightarrow$ failing in paper 1

$P(B) = 0.3$ B$\rightarrow$ failing in paper 2

$P(A/B) = 0.6$

Prop. of failing in both $P(A \cap B)$

$= P(A/B) \times P(B)$

$= 0.6 \times 0.2 = \mathbf{0.12}$

S.104 (d)

Number of permutations with '2' in the first position = 19!

Number of permutations with '2' in the second position = $10 \times 18!$

(fill the first space with any of the 100 odd numbers and the 18 spaces after the 2 with 8 of the remaining number in 18! ways)

Number of permutations with '2' in 3^{rd} position $= 10 \times 9 \times 17!$

(fill the first 2 places with 2 of the 10 odd numbers and then the remaining 17 places with remaining 17 numbers)

and so on until '2' is in 11^{th} place. After that it is not possible to satisfy the given condition, since there are only 10 odd numbers available to fill before the '2'. So the desired number of permutations which satisfies the given condition is

$19! + 10 \times 18! + 10 \times 9 \times 17! + 10 \times 9 \times 8 \times 16! + ... + 10! \times 9!$

Now the probability of this happening is given by

$$\frac{19! + 10 \times 18! + 10 \times 9 \times 17! + 9 \times 8 \times 16! + ... + 10! \times 9!}{20!}$$

Which is clearly not choices (a), (b) or (c)

$\therefore$ Answer is (d) none of these.

S.105 (a)

$$\text{Sin } x = x - \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} - \frac{x^7}{\lfloor 7} + ...$$

$$\text{Cos } x = 1 - \frac{x^2}{\lfloor 2} + \frac{x^4}{\lfloor 4} - \frac{x^6}{\lfloor 6} + ...$$

S.106 (c)

Let C denote computes science study and M denote maths study.

P(C on Monday and C on Wednesday)

=P(C on monday, M on Tuesday and C on Wednesday)

+P(C on monday, C on Tuesday and C on Wednesday)

$= 1 \times 0.6 \times 0.4 + 1 \times 0.4 \times 0.4$

$= 0.24 + 0.16$

$= 0.40$

Alternate:

Aishwarya studies CS on Monday.

$\therefore$ Probability that she will study Maths on Tuesday = 0.6

and Probability that she will study Computer Science on Wednesday = 0.4

$\therefore$ Required probability = **0.4**

S.107 (a)

Given

$\mu_X = 1$, $\sigma_X^2 = 4 \Rightarrow \sigma_X = 2$ and $\mu_Y = -1$. σ_Y is unknown

given $P(X \le -1) = P(Y \ge 2)$

Converting into standard normal variates,

$$P\left(z \le \frac{-1-\mu_x}{\sigma_x}\right) = P\left(z \ge \frac{2-\mu_y}{\sigma_y}\right)$$

$$P\left(z \le \frac{-1-1}{2}\right) = p\left(z \ge \frac{2-(-1)}{\sigma_y}\right)$$

$$P(z \le -1) = P\left(z \ge \frac{3}{\sigma_V}\right) \quad \text{.......(i)}$$

Now since us know that in standard normal distribution,

$P(z \le -1) = p(z \ge 1)$ ......(ii)

Comparing (i) and (ii) we can say that

$$\frac{3}{\sigma_y} = 1 \Rightarrow \ = \sigma_y = 3$$

S.109 (b)

It is given that

$P(\text{odd}) = 0.9\ P(\text{even})$

Now since $\Sigma P(x) = 1$

$\therefore\ P(\text{odd}) + P(\text{even}) = 1$

$\Rightarrow 0.9\ P(\text{even}) + P(\text{even}) = 1$

$$\Rightarrow P(\text{even}) = \frac{1}{1.9} = 0.5263$$

Now, it is given that P (any even face) is same i.e., P (2) = P(4) = P(6)

Now since,

$P(\text{even}) = P(2) \text{or } P(4) \text{ or } P(6)$

$= P(2) + P(4) + P(6)$

$$\therefore\ P(2) = P(4) = P(6) = \frac{1}{6}P(\text{even})$$

$$= \frac{1}{3}(0.5263)$$

$= 0.1754$

It is given that

$P(\text{even/face}>3) = 0.75$

$$\Rightarrow \frac{P(\text{even} \cap \text{face} > 3)}{P(\text{face} > 3)} = 0.75$$

$$\Rightarrow \frac{P(\text{face} = 4, 6)}{P(\text{face} > 3)} = 0.75$$

$$\Rightarrow P(\text{face} > 3) = \frac{P(\text{face} = 4, 6)}{0.75} = \frac{P(4)+P(6)}{0.75}$$

$$= \frac{0.1754 + 0.1754}{0.75}$$

$= 0.4677 \approx$ **0.468**

12.3

Set Theory and Algebra

Level–1

Q.1 Let P, Q and R be sets. Let Δ denote the symmetric difference operator defined as $P \Delta Q = (P \cup Q) - (P \cap Q)$. Using Venn diagrams, determine which of the following is/are TRUE.

I. $P \Delta (Q \cap R) = (P \Delta Q) \cap (P \Delta R)$

II. $P \cap (Q \Delta R) = (P \cap Q) \Delta (P \cap R)$

(a) I only

(b) II only

(c) Neither I and II

(d) Both I and II

Q.2 If a finite set A has n elements, then P(A), the power set A has

(a) 2^{n+1} elements

(b) 2^3 elements

(c) 2^{n-1} elements

(d) 2^n elements

Q.3 Let A and B be finite sets with $|A| = n$ and $|B| = m$. How many functions are possible from A to B with A as the domain?

(a) m^n

(b) m^{m+1}

(c) n

(d) m

Q.4 If G is a group, then for every $a \in G$

(a) $(a^{-1})^{-1} = a$

(b) $(a^{-1})^{-1} = (a)^{-1}$

(c) $(a^{-1})^{-1} = a^{-1}$

(d) none of the above

Q.5 S is a set of powers p^n of a prime p, and $p^m \leq p^n$ means:

(a) $m \leq n$

(b) $m = 0$

(c) $m = n$

(d) $n = 0$

Q.6 S is a set of integers a, b, ... and $a \leq b$ means that $a - b$ is a negative integer or zero; in particular, $a < 0$ means that a is a

(a) complex number

(b) positive number

(c) negative number

(d) non-negative number

Q.7 An infinite chain isomorphic with the rationals of the form $\frac{n}{n+1}$ lie between

(a) 0 and 1

(b) 0 and 10

(c) 0 and ∞

(d) $-\infty$ and $+\infty$

Q.8 If $ab = a$, then $a + b$ will be equal to

(a) 0

(b) ab

(c) b

(d) a

Q.9 A lattice is modular if and only if does not contain a sub–lattice isomorphic with the

(a) triclinic lattice

(b) pentagonal lattice

(c) orthogonal lattice

(d) hexagonal lattice

Q.10 In the set Z of all integers ρ defined by $a \rho b$ iff ab is even is .....

(a) Symmetric & transitive but not reflexive

(b) Symmetric but neither reflexive nor transitive

(c) Reflexive & symmetric but not transitive

(d) None of above

Q.11 If H is a subgroup of G & N a normal subgroup of G which of the following statements is Not correct.

(a) $H \cap N$ is a normal subgroup of G

(b) NH is normal in G if H is a normal subgroup of G

(c) $H \cap N$ is a normal subgroup of H

(d) NH is a subgroup of G

Q.12 A subgroup of H in a group G is a normal subgroup of G if index of H in G is

(a) 3

(b) 2

(c) 1

(d) 0

Common Data For Questions 13 to 15:

In a language survey of students it is found that 80 students know English, 60 know French, 50 know German, 30 know English and French, 20 know French and German, 15 know English and German and 10 student know all the three languages.

Q.13 How many students know at least one language?

(a) 135

(b) 45

(c) 30

(d) 10

Q.14 How many students know at least two languages?

(a) 135

(b) 45

(c) 30

(d) 10

Q.15 How many students known French but not both out of English and German?

(a) 135

(b) 45

(c) 30

(d) 10

Q.16 'Subset' relation on a set of sets is

(a) a partial ordering

(b) transitive and symmetric only

(c) transitive and anti–symmetric only

(d) an equivalence relation

Q.17 If $n \geq 2$, then the number of surjections that can be defined from $\{1, 2, 3, ..., n\}$ onto $\{1, 2\}$ is

(a) $2^n - 2$

(b) 2^n

(c) ${}^nP_2$

(d) $2n$

Q.18 A relation on the integers 0 through 4 is defined by:

$$R = \{(x, y) : x + y \leq 2x\}.$$

Which of the properties listed below applies to this relation?

I. Transitivity

II. Symmetry

III. Reflexivity

(a) II and III

(b) I and III

(c) III only

(d) I only

Q.19 If Q be the set of non-zero rational number and the relation R be defined over the set Q by xRy if $x = 1/y$, x, $y \in Q$, which of the following are TRUE?

I. R is an equivalence relation

II. R is reflexive

III. R is symmetric

IV. R is transitive

(a) III only

(b) III and IV only

(c) II and III only

(d) I only

Q.20 In the set Z of all integers, the relation ρ defined by $a \rho b$ if $a + b$ is odd is

(a) Symmetric and transitive but not reflexive

(b) Reflexive and symmetric but not transitive.

(c) Only symmetric, neither reflexive nor transitive

(d) None of these

Q.21 In the set N of all natural numbers, $a \rho b$ iff a divides b, then ρ is:

(a) Reflexive and symmetric but not transitive.

(b) Symmetric and transitive but not reflexive

(c) transitive but neither reflexive nor symmetric.

(d) None of these

Q.22 In the set {1,2,3} a relation R = {(1,2),(2,3)}. How many more members must be included in R so that R will be an equivalence relation?

(a) 2

(b) 4

(c) 5

(d) 7

Q.23 The number of proper subset {1,2,3,4} is

(a) 13

(b) 15

(c) 9

(d) 14

Q.24 Let A = {1,2,3,4} and B {1,2}. Then the number of onto functions from A to B is:

(a) 16

(b) 8

(c) 14

(d) 12

Q.25 Which statement is incorrect?

(a) $A\cup(B\cap C) = (A\cup B)\cap(A\cap C)$

(b) $A\cap(B\cup C) = (A\cup B)\cup(A\cap C)$

(c) $A\cap(B\Delta C) = (A\cap B)\Delta(A\cap C)$

(d) $A\Delta(B\cap C) = (A\Delta B)\cup(A\Delta C)$

Q.26 In the set Z of all integer ρ is defined by $a \rho b$ iff 5 divides $a + 2b$ the ρ is ?

(a) Reflexive but neither symmetric nor transitive.

(b) Symmetric but neither reflexive not transitive.

(c) Transitive but neither reflexive nor symmetric.

(d) neither reflexive nor symmetric nor transitive.

Q.27 A set S contains precisely n distinct elements. How many subsets are there in S?

(a) 2

(b) n

(c) 2^n

(d) None of these

Q.28 In the set Z of all integers $a \rho b$ iff 3 divides $a-b$, then

(a) ρ is reflexive and symmetric but not transitive

(b) ρ is reflexive, symmetric and also transitive

(c) ρ is symmetric and transitive but not reflexive

(d) r is reflexive and transitive but not symmetric

Q.29 In the set R of all real number a r b iff $a \geq b$ the r is ?

(a) Reflexive, symmetric and transitive

(b) Reflexive and symmetric but not transitive

(c) Reflexive and transitive but not symmetric

(d) None of these

Q.30 In the set N of natural numbers if r is defined by apb iff $a^2-4ab+3ab^2=0$ then ρ is?

(a) Symmetric but neither reflexive nor transitive

(b) Transitive but neither reflexive nor transitive

(c) Reflexive but neither symmetric nor transitive

(d) None of these

Linked Questions 31 & 32:

Q.31 Set A has 3 elements set B has 6 elements, then the minimum no. of elements in $A \cup B$ is?

(a) 1

(b) 6

(c) 9

(d) None of these

Q.32 In above questions (31), the maximum no. of elements in $A \cup B$ is?

(a) 1

(b) 9

(c) 20

(d) None of these

Q.33 If X and Y are two sets, then $X \cap (Y \cap X)^c$ equals

(a) X

(b) Y

(c) ϕ

(d) None of these

Q.34 $A - (B \cap C)$ is

(a) $(A - B) \cup (A - C)$

(b) $A - B - C$

(c) $(A - B) \cap (A - C)$

(d) $A - \overline{(B \cap C)}$

Q.35 $A \cup B$ is NOT equivalent to

(a) $\left(\overline{\overline{A} \cap \overline{B}}\right)$

(b) $\left(\overline{\overline{A} \cap B}\right)$

(c) $\left(\overline{\overline{B} \cap A}\right)$

(d) $\left(\overline{A \cap B}\right)$

Q.36 Set A has n elements. The number of functions that can be defined from A into A is

(a) n^2

(b) $n!$

(c) n^n

(d) n

Q.37 Which of the following statements is true?

(a) Every equivalence relation is a partial ordering relation.

(b) The number of relations form A = {x,y,z} to B = {1,2}, is 6.

(c) The empty relation ϕ is reflexive.

(d) The properties of a relation being symmetric and being antisymmetric are negative of each other.

Q.38 Hasse diagrams are drawn for

(a) Partially ordered sets

(b) lattices

(c) boolean Algebra

(d) none of these

Q.39 A self complemented distributive lattice is called

(a) Boolean algebra

(b) Self dual lattice

(c) Complete lattice

(d) Modular lattice

Q.40 Principle of duality is defined as

(a) LUB becomes by $\geq$

(b) $\leq$ is replaced $\geq$

(c) all properties are unaltered when $\leq$ is replaced by $\geq$

(d) all properties are unaltered when is replaced by other than 0 and 1 element.

Q.41 Let F be the set of one to one functions from the set {1,2,3,......n} to the set {1,2........m} where $m \geq n \geq 1$.

How many functions are members of F?

(a) ${}^n p_m$

(b) m

(c) mc_n

(d) None of these

Q.42 A boolean algebra is a complemented and distributive

(a) lattice

(b) set

(c) sub-group

(d) group

Q.43 Let B be a Boolean algebra with 2^n elements. The number of sub-boolean algebras of B is equal to the number of partitions of a set with how many elements.

(a) n

(b) n − 1

(c) 1

(d) 2

Q.44 The statement which can take only two values i.e either true or false are called

(a) logical statements

(b) logic

(c) logical data

(d) none of the above

Q.45 Let R_e be the relation in A = {(2, p), (2, q), (3, r), (5, s), (6, s)} Then the domain and range of R will be:

(a) dom(R) = {2, 3, 5, 6} range(R) = [p, q, r, s}

(b) dom(R) = {p, q, r, s} range(R) = {2, 3, 5, 6}

(c) dom(R) = {2, 3, 6} range(R) = {p, q, r, s}

(d) dom(R) = {2, 5, 6} range(R) = {p, q, r, s}

Q.46 Let R and S be the relations from P = {2,5,7} to Q = {x,y,z} defined by

R = {2,x} (5,x), (7,y), (7,z)

S = {(2,z), (5,z), (7,z)}

Then $R \cap S$ is

(a) {7,2)}

(b) {(2,z)}

(c) {(7,z)}

(d) {2,s,7,z}

LEVEL–2

Q.47 A computer company requires 30 programmers to handle system programming jobs and 40 programmers for applications programming. If the company appoints 55 programmers to carry out these jobs, how many of these perform jobs of both types.

(a) 18

(b) 40

(c) 35

(d) 15

Q.48 For any strings u and v the value of |uv| is

(a) $| v |^2 - | u |^2$

(b) $| u^v | - 1$

(c) $| u | | v |$

(d) $| u | + | v |$

Q.49 For any two sets A and B the value of $A - (A \cap B)$ is

(a) B + A

(b) A − B

(c) B − A

(d) A

Q.50 A survey shows that 58% of Indians like coffee whereas 75% like tea. The percentage of Indian who like both coffee and tea is

(a) 55

(b) 44

(c) 33

(d) 22

Q.51 It is known that at a university 60% of the professors play tennis, 50% of them play bridge, 70% jog, 20% play tennis and bridge, 30% play tennis and jog and 40% play bridge and jog. The percentage of the professors that play three games is

(a) 70

(b) 30

(c) 50

(d) 10

Q.52 If A, B, C, D are non-empty sets, such that $A \subseteq C$ and $B \subseteq D$, then which of the following is valid?

(a) $A \times B \subset B \times C$

(b) $A \times B = B \times C$

(c) $A \times B \neq B \times C$

(d) $A \times B \subseteq C \times D$

Q.53 Let R be a relation from a set A to a set B, and A_1 and A_2 be subsets of A. If $A_1 \subseteq A_2$ then which of the following is true?

(a) $R(A_1) \geq R(A_2)$

(b) $R(A_1) \subseteq R(A_2)$

(c) $R(A_1) < R(A_2)$

(d) $R(A_1) > R(A_2)$

Q.54 Let A = {1, 2, 3, 4, 6} and B = {1, 3, 4, 5} and a relation R from A to B be defined by aRb iff $a < b$. Then the value of R(6) is

(a) {1, 2}

(b) {ϕ}

(c) {0}

(d) ϕ

Q.55 Let R be the relation "$\leq$" defined on the set of all integers Z. Let A_1 = {0, 1, 2} and A_2 = {3, 5} then which of the following relation is correct?

(a) $R(A_1 \cap A_2) = R(A_2) \cap R(A_3)$

(b) $R(A_1 \cap A_3) = R(A_2) \cap R(A_3)$

(c) $R(A_2 \cap A_3) = R(A_2) \cap R(A_3)$

(d) both (a) and (b)

Q.56 Let R be a relation from a set A to a set B. For all subset A_1, A_2 of A, $R(A_1 \cap A_2) = R(A_1) \cap R(A_2)$ for any distinct a, b$\in$A, then which of the following is satisfied?

(a) $R(a) \cap R(b) = \{0\}$

(b) $R(a) \cap R(b) = \phi$

(c) $R(a) \cap R(b) = R(a) \cup R(b)$

(d) $R(a) \cap R(b)$ not defined

Q.57 Let A = {0, ±1, ±2, 3}. Consider the function $f: A \to R$ where R is the set of all real numbers, defined by

$$f(x) = x^3 - 2x^2 + 3x + 1$$

for $x \in A$. The range of f is

(a) {2, 1, 3, −2}

(b) {1, 3, −5, 7, −21, 19}

(c) {1, 2, 3, −1, −2}

(d) {ϕ}

Q.58 If $X \to Y$ be any everywhere defined function and A and B be arbitrary non-empty subsets of X. Then which of the following is true?

(a) $f(A \cap B) \subseteq f(A) \cup f(B)$ and the equality holds if f is one-to-one

(b) $f(A \cup B) = f(A) \cap f(B)$

(c) If $A \subseteq B$, then $f(A) \subseteq (B)$

(d) none of these

Q.59 Let A = B = C = R, the set of all real numbers, and $f: A \to B$ and $g: B \to C$ be defined by

$f(a) = 2a + 1$, $g(b) = \frac{1}{3}b \ \forall a \in A, \forall b \in B$.

The value of $(gof)^{-1}$ is

(a) $\frac{1}{3}(2C-1)$

(b) $\frac{1}{2}(1-3C)$

(c) $\frac{1}{3}(3C-1)^2$

(d) $\frac{1}{2}(3C-1)$

Q.60 Let $f: A \to B$ and $g: B \to C$, then which of the following is true?

(a) gof is onto if f and g are both into.

(b) gof is onto if f and g are both onto.

(c) gof is onto if f and g are both one-one.

(d) gof is into if f and g are both onto.

Q.61 If G is a group then the inverse of every element of G is

(a) finite

(b) infinite

(c) non-unique

(d) unique

Q.62 If every element of the group G is its own inverse then G is

(a) zero

(b) abelian

(c) non-abelian

(d) unity element

Q.63 Let L be a lattice with 0, locally of finite length. If a, b are comparable elements, $a \leq b$, we have

$$h(b) - h(a) = l[a, b],$$

if b covers a, then $h(b) - h(a)$ will be

(a) 0

(b) 2

(c) 1

(d) 3

Q.64 $f:R \to R$ is given by $f(x) = 4x^3 - 7$ then f is:

(a) surjective but not injective.

(b) Injective but not surjective.

(c) Bijective

(d) Neither injective nor surjective

Q.65 The domain of the function

$$f(x) = \frac{1}{\sqrt{10x - (x^2 + 24)}} \text{ is}$$

(a) $\{x \in R: x < 4 \text{ or } x > 6\}$

(b) $4 < x \leq 6$

(c) $4 \leq x < 6$

(d) None of these

Q.66 Which of the following function is an Even function: f(X) =

(a) $\dfrac{a^x + a^{-x}}{a^x - a^{-x}}$

(b) $\dfrac{a^x + 1}{a^x - 1}$

(c) $x\dfrac{a^x - 1}{a^x + 1}$

(d) $\log_2\left(x + \sqrt{x^2 1}\right)$

Q.67 The domain of the function

$$f(x) = q\sin^{-1}\left(\log_2 \frac{x^2}{2}\right) \quad \text{is}$$

(a) $[-2,2]-(-1,1)$

(b) $[-2,2]-[-1,1]$

(c) $[-2,2]$

(d) None of these

Q.68 The domain of $y = \cos^{-1}\dfrac{2}{2+\sin x}$ contained in $[0, 2\pi]$ is

(a) $\left[0, \dfrac{\pi}{2}\right]$

(b) $[0, \pi]$

(c) $\left(0, \dfrac{\pi}{2}\right)$

(d) $(0, x)$

Q.69 The function $f(x) = \dfrac{\log(1+ax) - \log(1+bx)}{x}$, $x \neq 0$ what value should be assigned to f(0) so that the function is continuous at $x = 0$.

(a) $a + b$

(b) $a - b$

(c) $\log_a + \log_b$

(d) None of these

Q.70 If a lattice L is distributive in the sense $x \cup (y \cap z) = (x \cup y) \cap (x \cup z)$, then which of the following may not be true.

(a) $x \cup z \Rightarrow x \cap (y \cap z) = (x \cup y) \cap z$

(b) $x \cup (y \cap z) = (x \cap y) \cap (x \cap z)$

(c) $x \cup y = x^1 \cup y, x \cap y = x^1 \cap y \Rightarrow x = x^1$

(d) $(x \cup y) \cap (y \cup z) \cap (z \cup x) = (x \cap y) \cap (y \cap z) \cap (z \cap x)$

Q.71 If $f: R \to R$, $f(x) = 2x+3$ then $f^{-1}(x)$ is

(a) $\frac{x-3}{2}$

(b) $3x-2x$

(c) $2x-3$

(d) Does not exist

Q.72 Let A be the set of students in a college. Determine which of the following assignment defines a function on A

(i) To each student assign his or her age

(ii) To each student assign his or her spouse

(a) Only (ii)

(b) Only (i)

(c) Both (i)and (ii)

(d) None of these

Q.73 If A,B and C are subsets of a set P, then $C \times (A^c \cup B^c)^c$ equals

(a) $(C \times A) \cap (C \times B)$

(b) $(C \times B) \cap (C \times A)$

(c) $(C \times A) \cup (C \times B)$

(d) None of these

Q.74 Consider the set A = [{1,3,5}, {7,9,11},{13,15}] then determine which of the following is/ are true.

(i) $1 \in A$

(ii) $\phi \notin A$

(iii) $\{\{1,3,5\}\} \subseteq A$

(iv) $\phi \subseteq A$

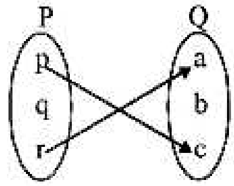
(a) (ii) is true

(b) (iii) and (iv) are true

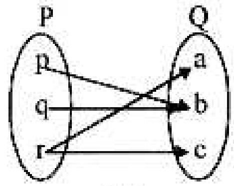
(c) (iii) and (ii) are true

(d) (iv) is true

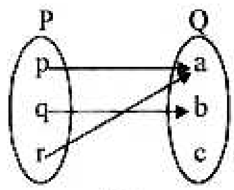
Q.75 Which of the following figure defines a function from P = {p,q,r} into Q = {a,b,c}



(i)



(ii)



(iii)

(a) both (ii) and (iii)

(b) (i)

(c) (ii)

(d) only (iii)

Q.76 Let R [mango, orange, pineapple, melon, pear]. Determine whether on each of the following is a partition of S

P_1 = [{mango},{pear, melon}]

P_2 = [{mango, orange, pineapple, melon, pear}]

(a) Both P_1 and P_2 are partition of R

(b) P_2 is partition of R

(c) P_1 is a partition of R

(d) None of these

Q.77 Let $A = \{(a,b): b=e^a, a \in R\}$

$B = \{(a,b): b=e^{-a}, a \in R\}$ Then,

(a) $A \cup B = R$

(b) $A \cap B = \phi$

(c) $A \cap B \neq \phi$

(d) None of these

Q.78 If A = {1,2,3,4} then which of the following are functions from A to itself.

(a) $f_1 = \{(x,y): y = x + 1\}$

(b) $f_2 = \{x,y\}: x + y.x + y>4\}$

(c) $f_3 = \{(x,y): y<x\}$

(d) $f_4 = \{(x,y): x+y = 5\}$

Q.79 If A and B are two non empty sets with non empty intersection, then the no. of elements in A Δ B is given | A Δ B| =

(a) $|A|+|B|-3|A\cap B|$

(b) $|A|+|B|-2|A\cap B|$

(c) $|A|+|B|-|A\cap B|$

(d) None of these

Q.80 If H is a subgroup of G. let

N(H) = g{g ∈ G | gHg $^{-1}$H} then which of the following statements in NOT correct:

(a) N(H) is subgroup of G

(b) H is normal is N (H)

(c) If HCK, K a subgroup of G, then H is normal in K.

(d) N(H) is the largest subgroup of G in which is normal

Q.81 If $f(x) = x^3 - x$ and $g(x) = \sin 2x$ then

(a) $f(f(1)) = 2$

(b) $g(f(1)) = 1$

(c) $f\left(g\left(\frac{\pi}{12}\right)\right) = -\frac{3}{8}$

(d) $g(f(2) = \sin 2$

Q.82 If A and B are sets. Which of the following is FALSE?

(a) $A-(A-B) = A\cap B$

(b) $A\subset B \Rightarrow B'\cap A'$

(c) $A-B' = A\cap B$

(d) none of these

Q.83 The number of functions

f: (1,2,3........,n), → {1,2 ......,m},

which are one to one is

(a) n (n-1) (n-2)............ (n-m+1)

(b) m (m-1) (m-2)........(m-n+1)

(c) n^m

(d) m^n

Q.84 The function f: R − {2} →R defined by $f(x) = (x^2+2x)(x-2)$ is

(a) one one and onto

(b) one one but not onto

(c) neither one one nor onto

(d) not one one but onto

Q.85 Let P (A) be the collection of all subsets of A = (a,b,c). Let R be a relation defined as "x is disjoint from y" over P (A). Then the number of elements in R is

(a) 17

(b) 9

(c) 8

(d) 11

Q.86 With respect to the relation 'x divides y' which of the following sets are totally ordered?

I. {36,3,9}

II. {7,77,11}

III {3,6,24,12}

IV. {1,2,3.....}

(a) II and III only

(b) I,II and III only

(c) I and III only

(d) All of these

Q.87 Let s(*w*) denote the set of all the letters in *w* where w is an English word. Let us denote set equality, subset and union relations by =, ⊂ and ∩ respectively.

(a) s (stored) = s (sorted)

(b) s(ten) ⊂ s(twenty)

(c) s (sixty) ⊂ (s (six) ∪ s (twenty)

(d) None of these

Q.88 Let f: R →R be defined by

$$f(x)=\begin{cases} x+2 & (x \le -1) \\ x^2 & (-1 \le x \le 1) \\ 2-x & (x \ge 1) \end{cases}$$

Then the value of f (–1.75)+f(0.5) + f(1.5) is

(a) 2

(b) –1

(c) 1

(d) 0

Q.89 If * is defined on R* as $a*b = \frac{ab}{2}$, then identity element in the group (R*,*) is

(a) $\frac{1}{3}$

(b) 2

(c) $\frac{1}{2}$

(d) 5

Q.90 If the binary operation * is defined on a set of ordered pairs of real numbers as

(a,b) * (c,d) = (ad+bc,bd)

and is associative, then

(1,2) * (3,5)* (3,4) equals

(a) (74, 40)

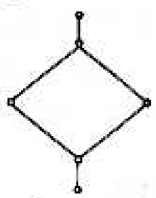
(b) (7, 5)

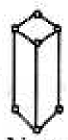
(c) (12, 24)

(d) (30, 45)

Q.91 Which one of the following lattices are Boolean?

(a) 

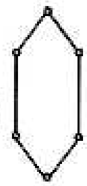
(b) 

(c) 

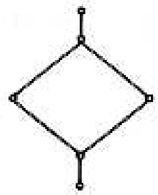
(d) None of these

Q.92 Which one of the following is not a "lattice" ?

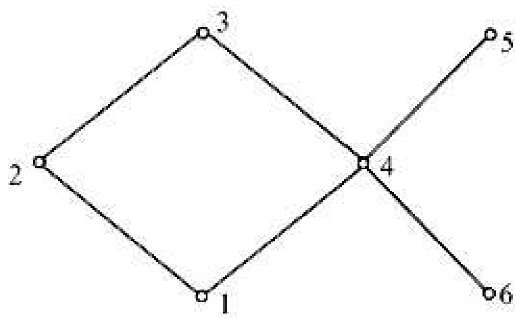
(a) 

(b) 

(c) 

(d) 

Q.93 Maximal and minimal elements of the Poset are



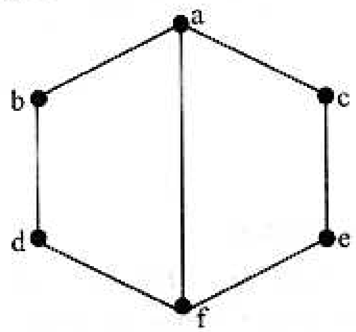
(a) Maximal 5,6; minimal 1

(b) Maximal 5,6; minimal 2

(c) Maximal 3,5; minimal 1,6

(d) None of these

Q.94 In the lattice defined by the Hasse diagram given below how many complements does the element 'e' have?



(a) 5
(b) 3
(c) 0
(d) 2

Q.95 Let $A = \{-2,-1,0,1,2\}$, $B = \{0,1,4\}$, and $f: A \to B$ is define as $f(x) = x^2$ is a function. f is

(a) one one
(b) Both (a) and (b)
(c) onto
(d) None of the above

Q.96 Inverse function of the following of $f(x) = 4x+7$

(a) $\frac{x}{2}$

(b) $\frac{x-7}{4}$

(c) $\frac{x}{4}$

(d) $\frac{x-3}{2}$

Q.97 Inverse function of $f(x) = \frac{2x+1}{3}$

(a) $\frac{x-3}{2}$

(b) $\frac{x}{2}$

(c) $\frac{3x-1}{2}$

(d) $\frac{3x}{2}$

Common Data For Questions 98 to 103 :

In a class of 100 students, 39 play Tennis, 58 play Cricket, 32 play Hockey, 10 Play Cricket and Hockey, 11 play Hockey and Tennis and 13 play Tennis and Cricket.

Q.98 The number of students who just play one game

(a) 77
(b) 65
(c) 67
(d) 73

Q.99 The number of students who play Tennis and Cricket but not Hockey

(a) 22
(b) 14
(c) 10
(d) 8

Q.100 The number of students who play only Tennis is

(a) 40
(b) 50
(c) 30
(d) 20

Q.101 The number of student who play only Hockey is

(a) 11
(b) 15
(c) 13
(d) 17

Q.102 The number of students who play all three games is

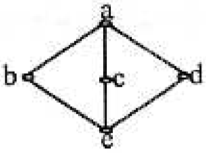
(a) 7
(b) 3
(c) 5
(d) 1

Q.103 The number of students who play only cricket is

(a) 40
(b) 38
(c) 33
(d) 35

LEVEL–3

Q.104 The following is the hasse diagram of the poset $[\{a,b,c,d,e,\},\leq]$



The poset is

(a) not a lattice

(b) a lattice but not a distributive lattice

(c) a Boolean algebra

(d) a distributive lattice but not a boolean algebra

Q.105 Let X be the set of all three digit integers; i.e.,

$X = \{ x \text{ is integer} : 100 \leq x \leq 999\}$.

If A_i is the set of number in X whose i^{th} digit is i, then the cardinality of the set $A_1 \cup A_2 \cup A_3$ is

(a) 270

(b) 220

(c) 252

(d) 292

Q.106 Using characteristic functions for any sets A, B, C contained in a universal set U the value of $(A \oplus B) \oplus C$ is

(a) $A \cup (B \oplus C)$

(b) $A \cup (B \cap C)$

(c) $A \oplus (B \oplus C)$

(d) $A \oplus (B \cap C)$

Q.107 Let R_1 and R_2 be two relations from a set A to a set B. If $R_1(x) = R_2(x)\ \forall\ x \leftarrow A$, then which relation is true?

(a) $R_2 \leq R_1$

(b) $R_2 \geq R_1$

(c) $R_2 > R_1$

(d) $R_1 = R_2$

Q.108 If (g(x)) is polynomial satisfying

$$g(x)g(y) = g(x) + g(y) + g(xy) - 2$$

for all real x and y and $g(2) = 5$, then $g(3)$ is equal to

(a) 20

(b) 30

(c) 10

(d) none of these

Q.109 If G is a finite group for all $a \in G$, there exists a positive integer N then, the value of a^N will be

(a) e

(b) 1

(c) ϕ

(d) 0

Q.110 A function f from the set of natural numbers to integers defined by

$$f(n) = \begin{cases} \dfrac{n-1}{2}, & \text{where n is odd} \\ n/2, & \text{where n is even} \end{cases}$$

is

(a) one-one but not onto

(b) one-one and onto both

(c) onto but not one-one

(d) neither one-one nor onto

Q.111 Let $f(x) = \sin x$ and $g(x) = \ln|x|$. If the ranges of the composite function fog and gof are R_1 and R_2 respectively, then

(a) $R_1 = [-1, 1]$, $R_2 = (-\infty, 0)$

(b) $R_1 = (-\infty, 0)$, $R_2 = [-1, 1]$

(c) $R_1 = [-1, 1]$, $R_2 = [-\infty, 0]$

(d) $R_1 = (-1, 1)$, $R_2 = (-\infty, 0)$

Q.112 Let $f(x) = (x-1)^2 - 1;\ x \geq 1$. Then the set $R = \{x : f(x) = f^{-1}(x)\}$ is

(a) $\{0, -1\}$

(b) $\{0, 1, -1\}$

(c) ϕ

(d) $\left\{0, -1, \dfrac{-3+i\sqrt{3}}{2}, \dfrac{-3-i\sqrt{3}}{2}\right\}$

Q.113 Let R be the relation on the set X = {1,2,3,4......} of integers defined by the equation $x^2+y^2 = 36$ then the sets of ordered pairs of R will be

(a) { }

(b) {(0,5),(3,4),(4,3),(5,0),}

(c) {(6,1),(2,5),(4,3),}

(d) {(1,6),(2,5),(3,4),}

Q.114 Let $U\{xyx^{-1}y^{-1} \mid x,y \in G\}$. Commutator subgroup G′ of G is the smallest subgroup of G which contains U. Which of the following statements is NOT correct.

(a) G/G is abelian

(b) G′ is normal in G

(c) If G/N is abelian, then $G \supset N$.

(d) If H is a sub group of G containing G then normal is G.

Q.115 A foreign language school has 200 students, of which 120 students study french (i.e.0(F) = 120). Denoting German by G and Russian by R if 0(G) = 90, 0(R) = 70. Further if $0(R \cap G) = 30$, $0(R \cap F) = 50$ & $0(G \cap F) = 40$ and $0(F \cap G \cap R) = 20$ then $0(R \cap G' \cap F')$____

(a) 58

(b) 10

(c) 25

(d) 20

Linked Questions 116 & 117:

Q.116 243 students took a test in Mathematics and Physics. 179 students passed in Mathematics and 197 students passed in physics. If the no. of students who failed in both the subjects is 28, the number of students who passed in both the subjects is?

(a) 170

(b) 161

(c) 150

(d) None of these

Q.117 In above question (116) the no. of students who failed in at least one of the two subjects and students who passed in at least one of the two subjects is

(a) 35,315

(b) 72,210

(c) 82,215

(d) None of these

Q.118 Let R denote the set of real numbers.

Let f: $R \times R \to R \times R$ be a bijective function defined by f(x,y) = (x+y,x−y). The inverse function of f is given by

(a) $f^{-1}(x,y) = 2\,[2(x-y), 2(x+y)]$

(b) $f^{-1}(x,y) = (x-y, x+y)$

(c) $f^{-1}(x,y) = \left[\frac{(x+y)}{2}, \frac{(x-y)}{2}\right]$

(d) $f^{-1}(x,y) = \left[\frac{1}{(x+y)}, \frac{1}{(x-y)}\right]$

Q.119 Consider the sets

I. $A = \{x \mid x = 2^n, n \in N\}$

II. $B = \{x \mid x = 3n, n \in N\}$

III. $C = \{x \mid x = 3n, n \in Z\}$

where N and Z above are sets of all natural numbers and integer

which of the above set (s) is (are) closed under additions subtractions and multiplications.

(a) I, III

(b) II

(c) III

(d) I

Q.120 The identity element in the group G $\left\{\begin{bmatrix} x & x \\ x & x \end{bmatrix}; x \in R, x \neq 0\right\}$ with respect to matrix multiplication is

(a) $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

(c) $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Q.121 Let * be a boolean operation defined as $A*B = AB + \overline{A}\overline{B}$. If C = A * B then A * A is?

(a) I

(b) 0

(c) A

(d) none of above

Q.122 The following function

(i) f: R→R is defined as $f(x) = \begin{cases} x, & x > 2 \\ 5x-2, & x \leq 2 \end{cases}$ is

(a) one-one

(b) onto

(c) both (a) and (b)

(d) none of these

Q.123 Function f: $[0,\infty) \rightarrow R$ defined as $f(x) = x^2$ is?

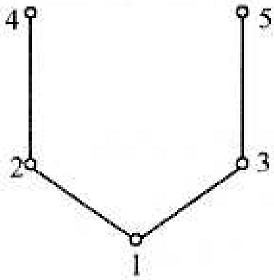
(a) one-one

(b) onto and one-one

(c) onto

(d) neither one-one nor onto

Q.124 Determine the matrix of the partial order whose Hasse diagram is given below:



(a) $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

(d) None of these

GATE QUESTIONS

Q.125 Let S be an infinite set and $S_1, \ldots, S_n$ be sets such that $S_1 \cup S_2 \cup \ldots \cup S_n = S$. Then, **[GATE 1993]**

(a) at least one of the set S_i is a finite set

(b) not more than one of the sets S_i can be finite

(c) at least one of the set S_i is an infinite set

(d) not more than one of the sets S_i can be infinite

(e) None of the above

Q.126 Let A be a finite set of size n. The number of elements in the power set A × A is **[GATE 1993]**

(a) 2^{2^n}

(b) 2^{n^2}

(c) $(2^n)^2$

(d) $(2^2)^n$

Q.127 The less-than relation, <, on reals is : **[GATE 1993]**

(a) a partial ordering since it is asymmetric and reflexive

(b) a partial ordering since it is antisymmetric and reflexive

(c) not a partial ordering because it is not asymmetric and not reflexive

(d) not a partial ordering because it is not antisymmetric and reflexive

(e) none of the above

Q.128 Let A and B be sets with cardinalities m and n respectively. The number of one-one mappings (injections) from A to B, when m < n, is : **[GATE 1993]**

(a) m^n

(b) ${}^nP_m$

(c) ${}^mC_n$

(d) ${}^nC_m$

(e) ${}^mP_n$

Q.129 Some group (G, o) is known to be abelian. Then, which one of the following is true for G? **[GATE 1994]**

(a) $g = g^{-1}$ for every $g \in G$

(b) $g = g^2$ for every $g \in G$

(c) $(goh)^2 = g^2oh^2$ for every $g, h \in G$

(d) G is of finite order

Q.130 Let R be a symmetric and transitive relation on a Set A. Then **[GATE 1995]** **[1-Mark]**

(a) R is reflexive and hence an equivalence relation

(b) R is reflexive and hence a partial order

(c) R is reflexive and hence not an equivalence relation

(d) None of the above

Q.131 The number of elements in the power set P(S) of the set $S=\{(\phi),1,(2,3)\}$ **[GATE 1995]** **[1-Mark]**

(a) 2

(b) 4

(c) 8

(d) None of the above

Q.132 Let R be a non-empty relation on a collection of sets defined by A R B if and only if $A \cap B = \phi$. Then **[GATE 1996]**

(a) R is reflexive and transitive

(b) R is symmetric and not transitive

(c) R is an equivalent relation

(d) R is not reflexive and not symmetric

Q.133 Which one of the following is false? **[GATE 1996]**

(a) The set of all bijective functions on a finite set forms a group under function composition.

(b) The set {1, 2, ..., p − 1} forms a group under multiplication mod p where p is a prime number.

(c) The set of all strings over a finite alphabet forms a group under concatenation.

(d) A subset $s \neq \phi$ of G is a subgroup of the group <G, *> if and only if any pair of element $a, b \in s$, $a * b^{-1} \in s$.

Q.134 Let A and B be sets and let A^c and B^c denote the complements of the sets A and B, the set $(A - B) \cup (B - A) \cup (A \cap B)$ is equal to. **[GATE 1996]**

(a) $A \cup B$

(b) $A^c \cup B^c$

(c) $A \cap B$

(d) $A^c \cap B^c$

Q.135 Let X = {2, 3, 6, 12, 24}, let ≤ be the partial order defined by X ≤ Y iff x/y. The number of edge as in the Hasse diagram of (X, ≤) is **[GATE 1996]**

(a) 3

(b) 4

(c) 9

(d) None of the above

Q.136 Let (Z, *) be an algebraic structure, where Z is the set of integers and the operation * is defined by n * m = maximum (n, m). Which of the following statement is true for (Z, *)? **[GATE 1997]** **[1-Mark]**

(a) (Z, *) is a monoid

(b) (Z, *) is an Abelian group

(c) (Z, *) is a group

(d) None of the above

Q.137 The number of equivalence relations of the set {1,2,3,4} is: **[GATE 1997]** **[2-Marks]**

(a) 15

(b) 16

(c) 24

(d) 4

Q.138 Let R_1 and R_2 be two equivalence relations on a set. Consider the following assertions:

I. $R_1 \cup R_2$ is an equivalence relation

II. $R_1 \cap R_2$ is an equivalence relation

Which of the following is correct? **[GATE 1998]** **[1-Mark]**

(a) Both assertions are true

(b) Assertion I is true but assertion II is not true

(c) Assertion II is true but assertion I is not true

(d) Neither I nor II is true

Q.139 Suppose A is a finite set with n elements. The number of elements in the largest equivalence relation of A is **[GATE 1998]** **[1-Mark]**

(a) n

(b) n^2

(c) 1

(d) n + 1

Q.140 The number of functions from an m elements set to an n element set is **[GATE 1998]** **[1-Mark]**

(a) m + n

(b) m^n

(c) n^m

(d) m * n

Q.141 Given two union compatible relations R_1(A, B) and R_2 (C, D), what is the result of the operation R_1 A = C A B = D R_2? **[GATE 1998]** **[1-Mark]**

(a) $R_1 \cup R_2$

(b) $R_1 \times R_2$

(c) $R_1 - R_2$

(d) $R_1 \cap R_2$

Q.142 The binary relation R = {(1, 1), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4)} on the set A = {1, 2, 3, 4} is **[GATE 1998]** **[2-Marks]**

(a) reflexive, symmetric and transitive

(b) neither reflexive, nor irreflexive but transitive

(c) irreflexive, symmetric and transitive

(d) irreflexive and antisymmetric

Q.143 In a room containing 28 people, there are 18 people who speak English, 15 people who speak Hindi and 22 people who speak Kannada. 9 persons speak both English and Hindi, 11 persons speak both Hindi and Kannada whereas 13 persons speak both Kannada and English. How many people speak all three languages? **[GATE 1998]** **[2-Marks]**

(a) 9

(b) 8

(c) 7

(d) 6

Q.144 The number of binary relations on a set with n elements is: **[GATE 1999]** **[1-Mark]**

(a) n^2
(b) 2^n
(c) 2^{n^2}
(d) None of the above

Q.145 Let L be a set with a relation R which is transitive, anti-symmetric and reflexive and for any two elements a, b ∈ L. Let the least upper bound lub (a,b) and the greatest lower bound glb (a,b) exist. Which of the following is/are true? **[GATE 1999]** **[2-Marks]**

(a) L is a poset
(b) L is Boolean algebra
(c) L is a lattice
(d) None of the above

Q.146 A relation R is defined on the set of integers as xRy iff (x + y) is even. Which of the following statement is true? **[GATE 2000]** **[2-Marks]**

(a) R is not an equivalence relation
(b) R is an equivalence relation having 1 equivalence class
(c) R is an equivalence relation having 2 equivalence class
(d) R is an equivalence relation having 3 equivalence classes

Q.147 Consider the following relations:

R1 (a,b) iff (a+b) is even over the set of integers

R2 (a,b) iff (a+b) is odd over the set of integers

R3 (a,b) iff a.b>0 over the set of non-zero rational numbers

R4 (a,b) iff $|a-b| \leq 2$ over the set of natural numbers

Which of the following statements is correct? **[GATE 2001]** **[1-Mark]**

(a) R1 and R2 are equivalent relations, R3 and R4 are not
(b) R1 and R3 are equivalent relations, R2 and R4 are not
(c) R1 and R4 are equivalent relations, R2 and R3 are not
(d) R1,R2,R3 and R4 are all equivalence relations

Q.148 The binary relation S = ϕ (empty set) on set A = {1,2,3} is: **[GATE 2002]** **[2-Marks]**

(a) Negative reflexive nor symmetric
(b) Symmetric and reflexive
(c) Transitive and reflexive
(d) Transitive and symmetric

Q.149 Let R_1 be a relation from A = {1, 3, 5, 7} to B = {2, 4, 6, 8} and R_2 be another relation from B to C = {1, 2, 3, 4} as defined below:

I. An element x in A is related to an element y in B (under R_1) if x + y is divisible by 3.

II. An element x in B is related to an element y in C (under R_2) if x + y is even but not divisible by 3.

Which is the composite relation R_1R_2 from A to C? **[IT-GATE 2004]** **[1-Mark]**

(a) $R_1R_2 = \{(1, 2), (1, 4), (3, 3), (5, 4), (7, 3)\}$
(b) $R_1R_2 = \{(1, 2), (1, 3), (3, 2), (5, 2), (7, 3)\}$
(c) $R_1R_2 = \{(1, 2), (3, 2), (3, 4), (5, 4), (7, 2)\}$
(d) $R_1R_2 = \{(3, 2), (3, 4), (5, 1), (5, 3), (7, 1)\}$

Q.150 Consider the binary relation

$S = \{(x, y) \mid y = x + 1 \text{ and } x, y \in \{0, 1, 2, \ldots\}\}$

The reflexive transitive closure of S is **[GATE 2004]** **[1-Mark]**

(a) $\{(x, y) \mid y > x \text{ and } x, y \in \{0, 1, 2, \ldots\}\}$
(b) $\{(x, y) \mid y \geq x \text{ and } x, y \in \{0, 1, 2, \ldots\}\}$
(c) $\{(x, y) \mid y < x \text{ and } x, y \in \{0, 1, 2, \ldots\}\}$
(d) $\{(x, y) \mid y \leq x \text{ and } x, y \in \{0, 1, 2, \ldots\}\}$

Q.151 In a class of 200 students, 125 have taken Programming Language course, 85 students have taken Data Structures Course, 65 student have taken Computer Organization course; 50 students have taken both Programming Language and Data Structures, 35 students have taken both Data Structures and Computer Organization; 30 students have taken both programming language and Computer Organization; 15 students have taken all the three course. How many students have not taken any of the three courses? **[IT-GATE 2004]** **[1-Mark]**

(a) 15
(b) 20
(c) 25
(d) 35

Q.152 Let f be a function from a set A to a set B, g a function from B to C, and h a function from A to C, such that h(a) = g(f(a)) for all a ∈ A. Which of the following statements is always true for all such functions f and g? **[IT-GATE 2005]**

[2-Marks]

(a) g is onto ⇒ h is onto

(b) h is onto ⇒ f is onto

(c) h is onto ⇒ g is onto

(d) h is onto ⇒ f and g are onto

Q.153 Let A be a set with n elements. Let C be a collection of distinct subsets of A such that for any two subsets S_1 and S_2 in C, either $S_1 \subset S_2$ or $S_2 \subset S_1$. What is the maximum cardinality of C? **[IT-GATE 2005]**

[2-Marks]

(a) n

(b) n + 1

(c) $2^{n-1} + 1$

(d) n!

Q.154 Let R and S be any two equivalence relations on a non-empty set A. Which one of the following statements is TRUE? **[GATE 2005]**

[2-Marks]

(a) R ∪ S, R ∩ S are both equivalence relations

(b) R ∪ S is an equivalence relation

(c) R ∩ S is an equivalence relation

(d) Neither R ∪ S nor R ∩ S is an equivalence relation

Q.155 Let f: B→C and g: A→B be two function let h = fog. Given that h is an onto function. Which one of the following is TRUE? **[GATE 2005]**

[2-Marks]

(a) f and g should both be onto functions

(b) f should be onto but g need not be onto

(c) g should be onto but f need not be onto

(d) both g and f need not be onto

Q.156 Let A, b and C are non empty sets and let

X = (A–B)–C and Y = (A–C)–(B–C)

Which one is true? **[GATE 2005]**

[1-Mark]

(a) X = Y

(b) Y ⊂ X

(c) X ⊂ Y

(d) None of these

Q.157 Let X,Y, Z be sets of sizes x,y and z respectively. Let W = X × Y and E be the set of all subsets of W. The number of function from Z to E is **[GATE 2006]**

[1-Mark]

(a) $Z^{2^{xy}}$

(b) $Z \times 2^{xy}$

(c) Z^{2x+y}

(d) 2^{xyz}

Q.158 A relation R is defined on ordered pairs of integers follows (x,y) R (u,v) if x < u and y > v. Then R is **[GATE 2006]**

[1-Mark]

(a) Neither a partial order nor an equivalence Relation

(b) A total order

(c) A partial order but not a total order

(d) An Equivalence relation

Q.159 The set {1,2,3,5,7,8,9} under multiplication modulo 10 is not a group. Given below are four plausible reasons. Which one of them is false? **[GATE 2006]**

[1-Mark]

(a) 8 does not have an inverse

(b) 2 does not have an inverse

(c) 3 does not have an inverse

(d) It is not closed

Q.160 Let E,F and G be finite sets. Let $X = (E \cap F) - (F \cap G)$ and $Y = (E - (E \cap G)) - (E - F)$. Which one of the following is true? **[GATE 2006]**

[2-Marks]

(a) $X - Y \neq \phi$ and $Y - X \neq \phi$

(b) $X \supset Y$

(c) $X = Y$

(d) $X \subset Y$

Q.161 How many different non-isomorphic Abelian groups of order 4 are there? **[GATE 2007]**

[2-Marks]

(a) 2

(b) 3

(c) 4

(d) 5

Q.162 Which of the following graphs has an eulerian circuit? **[GATE 2007]**

[2-Marks]

(a) Any k-regular graph where k is an even number

(b) A complete graph on 90 vertices

(c) The complement of a cycle on 25 vertices

(d) None of the above

Q.163 Let S be a set of n elements. The number of ordered pairs in the largest and the smallest equivalence relations on S are: **[GATE 2007]**

[1-Mark]

(a) n and 1

(b) n^2 and n

(c) n^2 and 0

(d) n and n

Q.164 Consider the following two statements about the function $f(x) = |x|$: **[GATE 2007]**

[1-Mark]

P: f (x) is continuous for all real values of x

Q: f (x) is differentiable for all real values of x

Which of the following is TRUE?

(a) P is true and Q is false.

(b) Both P and Q are true.

(c) P is false and Q is true.

(d) Both P and Q are false

Q.165 If P,Q,R are subsets of the universal set U, then $(P \cap Q \cap R) \cup (P \cap Q \cap R) \cup Q^c \cup R^c$ **[GATE 2008]**

[1-Mark]

(a) $P^c \cup Q^c \cup R^c$

(b) $P \cup Q^c \cup R^c$

(c) $Q^c \cup R^c$

(d) U

Q.166 Consider the binary relation

$R = \{(x,y),(x,z),(x,y),(z,y)\}$

on the set $\{x,y,z\}$. Which one of the following is TRUE? **[GATE 2009]**

[1-Mark]

(a) R is neither symmetric nor antisymmetric

(b) R is NOT symmetric but antisymmetric

(c) R is both symmetric and antisymmetric

(d) R is symmetric but NOT antisymmetric

Q.167 For the composition table of a cyclic group shown below.

*	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

which one of the following choices is correct? **[GATE 2009]**

[2-Marks]

(a) d,a are generators

(b) b,c are generators

(c) c,d are generators

(d) a,b are generators

ANSWER KEY

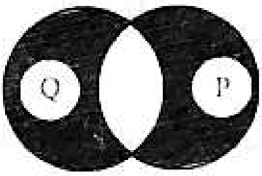
1	d	2	d	3	a	4	a	5	a
6	c	7	a	8	c	9	b	10	a
11	a	12	b	13	a	14	b	15	c
16	a	17	a	18	c	19	a	20	a
21	d	22	d	23	b	24	c	25	b
26	d	27	c	28	b	29	c	30	c
31	b	32	b	33	c	34	c	35	a
36	c	37	b	38	a, b	39	a	40	d
41	a	42	a	43	a	44	a	45	a
46	c	47	d	48	d	49	b	50	c
51	d	52	d	53	b	54	d	55	c
56	b	57	b	58	c	59	d	60	b
61	d	62	b	63	c	64	c	65	d
66	c	67	b	68	b	69	a	70	c
71	a	72	b	73	a	74	b	75	d
76	b	77	c	78	d	79	b	80	c
81	c	82	d	83	b	84	c	85	d
86	c	87	d	88	c	89	b	90	a
91	d	92	b	93	c	94	b	95	b
96	b	97	c	98	d	99	d	100	d
101	c	102	c	103	a	104	b	105	c
106	c	107	d	108	c	109	a	110	a
111	a	112	c	113	a	114	c	115	b
116	b	117	c	118	c	119	c	120	c
121	a	122	b	123	a	124	c	125	c
126	b	127	e	128	b	129	c	130	d
131	c	132	b	133	c	134	a	135	b
136	d	137	a	138	c	139	b	140	c
141	a	142	b	143	d	144	c	145	c
146	c	147	b	148	d	149	c	150	b
151	c	152	c	153	c	154	c	155	b
156	a	157	d	158	a	159	d	160	c
161	c	162	a	163	b	164	a	165	d
166	a	167	c						

SOLUTIONS

S.1 (d)

Both I and II.

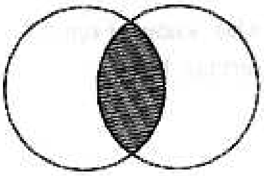
Given that $P\Delta Q = (P\cup Q) - (P\cap Q)$



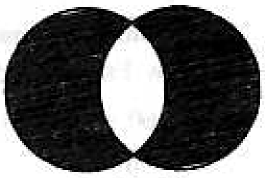
$(P\Delta Q)$

(i) $P\Delta(Q\cap R) = (P\Delta Q) \cap (P\Delta R)$

$P\Delta(Q\cap R) \Rightarrow$



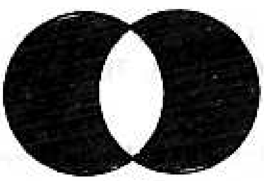
$(Q\cap R)$



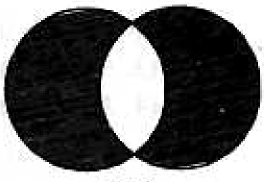
$P\Delta(Q\cap R)$

.....(i)

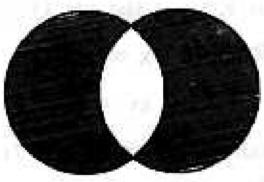
$(P\Delta Q) \cap (P\Delta R)$



$P\Delta Q$



$P\Delta R$



$(P\Delta Q)\cap(P\Delta R)$

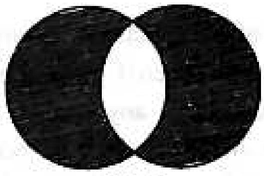
.....(ii)

(ii) is same as (i)

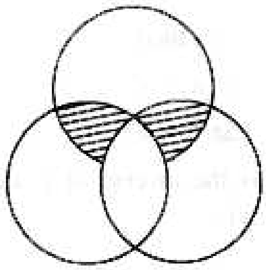
$\therefore$ **(I) is true.**

(iii) $P\cap(Q\Delta R) = (P\cap Q)\Delta(P\cap R)$

$P\cap(Q\Delta R) \Rightarrow$



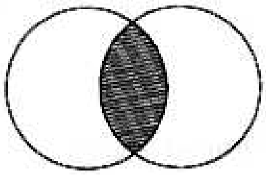
$Q\Delta R$



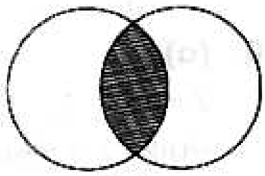
$P\cap(Q\Delta R)$

.....(iii)

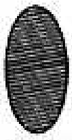
$(P\cap Q)\Delta(P\cap R)$



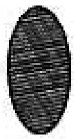
$P\cap Q$



$P\cap R$

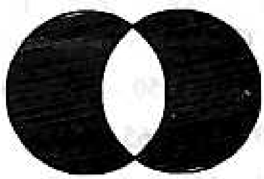


$P\cap Q$



$P\cap R$

$(P\cap Q)\Delta(P\cap R) \Rightarrow$



.....(iv)

(iv) is same as (iii)

$\therefore$ **(II) is also true.**

S.2 (d)

The power set A is given by 2^n elements.

S.3 (a)

In each of the functions having A as the domain. There are n ordered pairs. Further, any element $a \in A$ can have any one of the m elements of B as its image. Therefore, there are $\mathbf{m^n}$ possible functions from A to B with A as the domain.

S.4 (a)

By property of group,

Let $a^{-1} = e$, then

$$ea = a^{-1}a = e$$

$$ae = aa^{-1} = e$$

Thus a is the inverse of e, i.e., $a = e^{-1}$. Thus

$\mathbf{a = (a^{-1})^{-1}}$

S.5 (a)

$p^m \le p^n$ satisfies only when $m \le n$.

S.8 (c)

$$a + b = ab + b$$
$$= b(1 + a)$$
$$= b \qquad [\because \ 1 + a = 1]$$

S.10 (a)

$Z = \{0, 1, 2, ...\}$

relation $a \rho b$ such that ab = even

So, possible set element of the relation are

$\{\{1, 2\}, \{2, 3\}, \{2, 2\}, \{3, 2\}, \{2, 1\}, ...\}$

By the example it is clear that given relation is not reflexive because every element is not related to itself. But relation is symmetric because $(a, b) \in R$ and $(b, a) \in R$ and transitive because $(a, b) \in R$, $(b, c) \in R$, then $(a, c) \in R$.

S.13 (a)

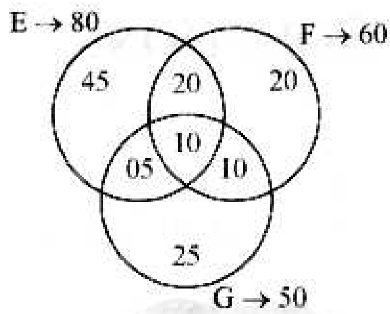
$|E \cup F \cup G| = |E| + |F| + |G| - |E \cap F| - |F \cap G| - |G \cap E| + |E \cap F \cap G|$

$= 80 + 60 + 50 - 30 - 20 - 15 + 10$

$= 190 + 10 - 65$

$= 200 - 65$

$= \mathbf{135}$



E → English
F → French
G → German

S.14 (b)

From the Venn diagram

$$20 + 05 + 10 + 10 = \mathbf{45}$$

students those know at least two languages.

S.15 (c)

Student those know French but not both out of English and German.

= Student those who know French – those know English and German both

$= 60 - (20 + 10)$

$= \mathbf{30}$

S.18 (c)

Relation R on integer $\{0, 1, 2, 3, 4\}$ is defined as

$R = \{(x, y) \ \ x + y \le 2x\}$

So, possible set of elements are

$R = \{(0, 0), (1, 0), (1, 1), (2, 0), (2, 1), (2, 2), (3, 0), (3, 1), (3, 2), (3, 3), (4, 0), (4, 1), (4, 3), (4, 4)\}$

From the set it is clear that **relation is reflexive but it is not symmetric and transitive.**

S.19 (a)

Relation R over the set Q is

$$x R y \text{ if } x = 1/y$$
$$\Rightarrow xy = 1$$

So, possible sets are $\{(0.5, 2), (2, 0.5), (0.25, 4), (4, 0.25), ...\}$

from the set we can say that relation is **symmetric but not reflexive and transitive.**

S.20 (a)

$Z = \{0, 1, 2, ...\}$

Relation ρ is $a \rho b$ such that $a + b$ = odd

So, possible sets are

$\rho = \{(0, 1), (1, 0), (1, 2), (2, 1), (0, 3), (3, 0), (2, 3), (3, 2), ...\}$

So, we can say that relation is **symmetric and transitive but not reflexive.**

S.21 (d)

Relation ρ on set N is a ρ b such that

b/a = integer

So, possible set of relation are

$\rho = \{(1, 1), (1, 2), (1, 3), ..., (2, 2), (2, 4), ..., (3, 3), (3, 6), ...\}$

From the set it is clear that relation is **reflexive and transitive but not symmetric.**

S.22 (d)

Any relation is said to be equivalence if the relation is reflexive, symmetric and transitive.

For reflexive, the relation is a R a.

For symmetric, the relation is a R b, then b R a.

For transitive, the relation is a R b and b R c, then a R c.

$\therefore$ 7 more numbers must be inclued in R. i.e., (1, 1), (2, 2), (3, 3), (3, 1), (2, 1), (3, 2), (1, 3)

S.23 (b)

Total number of subsets $= 2^4 = 16$

These include the subset {1,2,3,4} which is not the proper subset. Hence number of proper subsets = 16–1=15

S.24 (c)

The number of onto functions from A to B $= 2^n - 2 = 2^4 - 2 = 16 - 2 = \mathbf{14}$

S.28 (b)

Relation ρ on Z such that,

a ρ b $\forall$ a, b, 3 divides (a – b)

So, possible set of relation is

$\rho = \{(0, 0), (3, 0), (0, 3), (3, 3), (6, 0), (0, 6), (3, 6), (6, 3), (6, 6) ...\}$

It is clear that relation is **symmetric, reflexive and transitive.**

S.29 (c)

The relation r in all real number is

a r b $\forall$ a $\geq$ b

Possible set of r is

$r = \{(0, 0), (0, -1), (0, -2), ...,$
$(1, 1), (1, 0), (1, -1), ...,$
$(2, 2), (2, 1), (2, 0), ...,$
$.........................\}$

From the set it is clear that relation **r is reflexive and transitive but not symmetric.**

S.30 (c)

A relation r is a r b such that

$a^2 - 4ab + 3ab^2 = 0$

Possible set of the relation is

$r = \{(1, 1), ...\}$

So, we can say that this **relation is reflexive but neither symmetric nor transitive.**

S.31 (b)

$n(A) = 3, n(B) = 6$

$n(A \cap B) = 3$ [For min^m elements]

$\therefore$ $n(A \cup B) = 9 - 3 = \mathbf{6}$

S.32 (b)

For max^m $n(A \cap B) = 0$

$\therefore$ $n(A \cup B) = 9$

S.33 (c)

$x \cap (y \cap x)^c = x \cap (y^c \cup x^c)$

$= (x \cap y^c) \cup (x \cap x^c)$

$= x \cap y^c \cup 0$

$= x \cap y^c$

$= \phi$

S.38 (a, b)

Hasse diagrams are drawn for partially ordered sets and lattices for visualizing.

S.41 (a)

$${}^n p_m = \frac{n}{(n-m)!}$$

S.45 (a)

Domain is the set of first co-ordinates of the ordered pairs of a relation R

$\therefore$ domain(R) = **{2, 3, 5, 6}**

range(R) = **{p, q, r, s}**

S.46 (c)

R and S must be treated as sets

$\therefore$ $R \cap S$ = **{(7, z)}**

S.47 (d)

$|A| = 30$, $|B| = 40$, $|A \cup B| = 55$ from Additional rule,

$|A \cup B| = |A| + |B| - |A \cap B|$ gives,

$|A \cap B| = |A| + |B| - |A \cup B|$

$= 30 + 40 - 55 = \mathbf{15}$

S.48 (d)

Let $|u| = r$, $|v| = s$, then uv will consist of the r letters of u followed by the s letters of v Therefore,

$|uv| = |u| + |v| = s + r$

Similarly, $|vu| = s + r = |v| + |u|$

Thus, $\mathbf{|uv| = |u| + |v|}$

S.49 (b)

For any x

$x \in A - (A \cap B)$

$\Leftrightarrow x \in \{x : x \in A \cap x \notin (A \cap B)\}$

$\Leftrightarrow x \in A \cap \rceil (x \in A \cap x \in B)$

$\Leftrightarrow x \in A \cap (x \notin A \cup x \notin B)$

$\Leftrightarrow (x \in A \cap x \notin A) \cup (x \in A \cap x \notin B)$

$\Leftrightarrow x \in A \cap x \notin B$

$\Leftrightarrow x \in \{x : x \in A \cap x \notin B\} = \mathbf{A - B}$

S.50 (c)

Let A = % of Indian who drink coffee

B = % of Indian who drink tea

$\therefore$ $A \cap B \rightarrow$ % of Indian who drink both coffee and tea

$|A| = 58$, $|B| = 75$ and $|A \cup B| = 100$

to find $|A \cap B|$

We know that,

$|A \cup B| = |A| + |B| - |A \cap B|$

$100 = 58 + 75 - |A \cap B|$

$(A \cap B) = 133 - 100 = \mathbf{33}$

S.51 (d)

$|A| = 60$, $|B| = 50$, $|C| = 70$

$|A \cap B| = 20$, $|A \cap C| = 30$, $|B \cap C| = 40$

$|A \cup B \cup C| = 100$

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

$\Rightarrow 100 = 60 + 50 + 70 - 20 - 30 - 40 + |A \cap B \cap C|$

i.e., $|A \cap B \cap C| = \mathbf{10}$

S.52 (d)

Take any $(x, y) \in A \times B$. Then $x \in A$ and $y \in B$. Since $A \subseteq C$ and $B \subseteq D$, it follows that $x \in C$ and $y \in D$.

Therefore, $(x, y) \in C \times D$

Thus, $(x, y) \in A \times B \Rightarrow (x, y) \in C \times D$.

Hence, $\mathbf{(A \times B) \subseteq (C \times D)}$.

S.53 (b)

Take any $y \in R(A_1)$, then x R y for some $x \in A_1$. Since $A_1 \subseteq A_2$, it follows that $x \in A_2$. Thus x R y for some $x \in R_2$, so that $y \in R(A_2)$.

Hence $\mathbf{R(A_1) \subseteq R(A_2)}$.

S.54 (d)

We first note that,

R = {(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5), (3, 4), (3, 5), (4, 5)}

from this, we find that,

Dom (R) = {1, 2, 3, 4}, Ran(R) = {3, 4, 5},

R(3) = {4, 5}, R(6) = ϕ, R({2, 3, 4}) = {3, 4, 5}.

S.55 (c)

$R(A_3) = \{2, 3, 4, \ldots\}$

$R(A_2 \cap A_3) = R(\{3\}) = \{3, 4, 5, \ldots\} = R(A_2)$

$R(A_2) \cap R(A_3) = \{3, 4, 5, \ldots\} = R(A_2)$

S.56 (b)

$a, b \in A$

$R(a) \cap R(b) = R(\{a\}) \cap R(\{b\})$

$= R(\{a\} \cap \{b\})$

$= R(\phi) = \phi$

S.57 (b)

We may find that,

f(0) = 1, f(1) = 3, f(–1) = 5, f(2) = 7, f(–2) = –

21, f(3) = 19.

Hence, the range of f is

Ran(f) = {1, 3, –5, 7, –21, 19}

S.58 (c)

Take any $y \in Y$ then,

$y \in f(A) \Rightarrow y = f(x)$ for some $x \in A$.

$\Rightarrow y = f(x)$ for some $x \in B$, because $A \subseteq B$

$\Rightarrow y \in f(B)$

$\Rightarrow f(A) \subseteq f(B)$

S.59 (d)

$$(gof)^{-1}(c) = (f^{-1}og^{-1})(c) = f^{-1}\{g^{-1}(c)\} = f^{-1}(3c) = \frac{1}{2}(3C-1)$$

S.60 (b)

Let c be any arbitrary element of C.

Since $g : B \to C$ is onto, there exists some $b \in B$ such that

$g(b) = c$ .....(1)

Since $f : A \to B$ is onto, there exists some element $a \in A$ such that,

$f(a) = b$ .....(2)

From (1) and (2),

$g(f(a)) = c$

or $(gof)(a) = c$

where $a \in A$.

Hence **gof is onto.**

S.61 (d)

Suppose $a \in G$ has two inverse elements b and c in G. Then,

$a \,.\, b = b \,.\, a = e$ .....(i)

$a \,.\, c = c \,.\, a = e$ .....(ii)

Consider $b = b \,.\, e$, since e is the identity in G

$= b \,.\, (a \,.\, c)$, using (ii)

$= (b \,.\, a) \,.\, c$, by Associative law in G

$= e \,.\, c$, using (i)

$\therefore$ b = c, since e is the identity in G.

Hence inverse of $a \in G$ is **unique.**

S.62 (b)

A group is said to Abelian if every element of the group a is its own inverse.

We have $a = a^{-1}\ \forall\ a \in G$. .....(1)

Let $a, b \in G$ so that $a \,.\, b \in G$.

From(1), we have

$a = a^{-1}$, $b = b^{-1}$ and $(a \cdot b)^{-1} = a \cdot b$ .....(2)

Now, $(a \cdot b)^{-1} = a \,.\, b \Rightarrow b^{-1} \cdot a^{-1} = a \cdot b$

$\Rightarrow b \cdot a = a \cdot b$, using (2)

Hence G is **abelian.**

S.63 (c)

Since $0 \le a \le b$, whereas

$h(b) - h(a) = l[0, b] - l[0, a]$

$= l[0, a] + l[a, b] - l[0, a] = l[a, b]$

If b covers a, then there is no element x in L such that $a < x < b$, so that [a, b] is a two elements chain and $l[a, b] = 1$

S.64 (c)

$f : R \to R$, $f(x) = 4x^3 - 7$

f is bijective if it is both injective and surjective.

$\because$ $f : A \to B$ is said to be ijective if for each pair of distinct elements of A, their f-images are distinct.

It is clear that $f(x) = 4x^3 - 7$, $f : R \to R$ has all distinct f-images, so it is injective.

$\because$ $f : A \to B$ is said to be surjective if $f(A) = B$.

$\because$ By the relation $f : R \to R$ for $f(x) = 4x^3 - 7$, we can get elements such that $f(R) = R$.

So, it is a surjective relation.

So, the given relation is bijective.

S.65 (d)

$10x - (x^2 + 24) > 0$ should be

$x^2 + 24 < 10x$

$\Rightarrow x^2 - 10x + 24 < 0$

$\Rightarrow (x - 6)(x - 4) < 0$

So, x must be **4 < x < 6.**

S.66 (c)

A given function is even if $f(x) = f(-x)$

Take, $f(x) = x\frac{a^x - 1}{a^x + 1}$

$$f(-x) = (-x)\frac{a^{-x} - 1}{a^{-x} + 1}$$

$$= (-x)\frac{1 - a^x}{1 + a^x}$$

$$= x\frac{a^x - 1}{a^x + 1}$$

$$= f(x)$$

So, this is an even function.

S.67 (b)

Domain will be

$$-1 \le \log_2 \frac{x^2}{2} \le 1$$

$$\Rightarrow 2^{-1} \le \frac{x^2}{2} \le 2$$

$$\Rightarrow 1 \le x^2 \le 4$$

$\because$ $x^2 \le 4 \Rightarrow x \in [-2, 2]$

$\because$ $1 \le x^2 \Rightarrow x \le -1, x \ge 1$

So, domain is **[−2, 2] − [−1, 1]**

S.68 (b)

Domain

$$-1 \le \frac{2}{2 + \sin x} \le 1$$

$-(2 + \sin x) \le 2 \le 2 + \sin x$

$\because$ $-(2 + \sin x) \le 2$

$\Rightarrow \sin x \ge -4 \Rightarrow x \ge \pi$

$\because$ $2 + \sin x \ge 2$

$\Rightarrow \sin x \ge 0 \Rightarrow x \ge 0$

So, domain is $\mathbf{x \in [0, \pi]}$.

S.71 (a)

$y = f(x) \Rightarrow x = f^{-1}(y)$

$\Rightarrow y = 2x + 3$

$$\Rightarrow x = \frac{y - 3}{2}$$

$$\Rightarrow f^{-1}(y) = \frac{y - 3}{2}$$

$$\Rightarrow f^{-1}(x) = \frac{x - 3}{2}$$

S.72 (b)

When each element of A is assigned a unique element of B; the collection of such assignments is called a function from A into B.

$f : A \to B$

In (i) Each student has one and only one age.

$\therefore$ This assignment defines a function on A.

(ii) To each student assign his or her spouse.

This case is not possible to define a function because it is possible that some student may not be married.

S.73 (a)

$C \times (A^c \cup B^c)^c$, by De Morgan's low

$C \times ((A^c)^c \cap (B^c)^c)$

$C \times (A \cap B) = (C \times A) \cap (C \times B)$

S.74 (b)

A is a class of set. Its elements are the sets {1,3,5}, {7,9,11} and {13,15}

(i) $1 \in A$ is false since 1 is not one of the elements of A.

(ii) $\phi \notin A$ is false the empty set ϕ is not an element of A i.e. it is not one of the three sets listed in the problem statement.

(iii) $\{\{1,3,5\}\} \subseteq A$ is true because the set consisting of the element {1,3,5} is a subset of A.

(iv) $\phi \subseteq A$ is true because the empty set is a subset of every set; even a class of sets.

S.75 (d)

(i) There is no element of Q assigned to the element $q \in p$. So it does not defined a function.

(ii) Two elements, a and c, are assigned to $r \in p$. So it does not defined a function.

(iii) Each element of P assigned a unique element of B.

S.76 (b)

Partition of S is a collection p = $\{A_i\}$ of nonempty subsets of S such that

(i) each a in S belongs to one of the A_i

P_1 = [{mango}], {pear, melon}] is not partition of R

Since orange, pineapple does not belong to any cell.

P_2 = [{mango, orange, pineapple, melon, pear}] is a partition of R whose only element is S itself

(ii) The sets of P are mutually disjoints ie $A_i \neq A_j$ the $A_i \wedge A_j \neq \phi$ the $A_i \cap A_j \neq \phi$ subsets on a partition are called cells.

S.77 (c)

$y = e^x$ and

$y = e^{-x}$

$e^x = e^{-x}$

$e^{2x} = 1$

$\therefore x = 0$

$\therefore y = 1$

$\therefore y = 1$

$\therefore$ A and B meet on (0,1)

S.78 (d)

$f_1 = \{(1,2), (2,3), (3,4)\}$

$f_2 = \{(1,4), (2,3), (2,4), (3,3), (3,4), (4,4), (4,1), (4,2), (4,3)\}$

$f_3 = \{(2,1), (3,2), (3,1), (3,2), (4,1), (4,2), (4,3)\}$

$f_4 = \{(1,4), (2,3), (3,2), (4,1)\}$

$\therefore f_4$ is a function from a to itself and f_1, f_2, f_3 are not function from A to itself.

S.79 (b)

$|A \Delta B| = |A \cup B| - |A \cap B|$

$= |A| + |B| - |A \cap B| - |A \cap B|$

$= |A| + |B| - 2|A \cap B|$

S.81 (c)

$f(x) = x^3 - x$

$g(x) = \sin 2x$

$f(g(x)) = (\sin 2x)^3 - \sin 2x$

$= \sin^3 2x - \sin 2x$

$$f\left(g\left(\frac{\pi}{12}\right)\right) = \sin^3\frac{\pi}{6} - \sin\frac{\pi}{6}$$

$$= \left(\frac{1}{2}\right)^3 - \frac{1}{2}$$

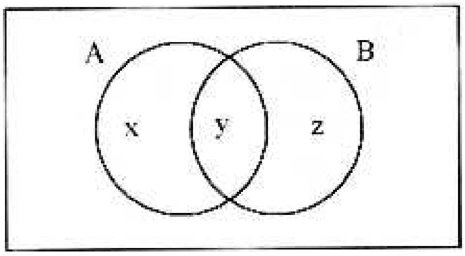
$$= \frac{1}{8} - \frac{1}{2}$$

$$= \frac{1-4}{8}$$

$$= -\frac{3}{8}$$

S.82 (d)

Let set A and B are defined as given in venn-diagram.



$\because$ (a) $A - (A - B) = A \cap B$

L.H.S. $\rightarrow A - (A - B) = (x + y) - x$

R.H.S. $\rightarrow A \cap B = y$

L.H.S. = R.H.S.

So, it is true

(b) $A \subset B \Rightarrow B' \cap A'$

R.H.S. $\rightarrow (x + 0) \cap (z + 0) \Rightarrow 0$

That is possible only if $A \subset B$ = L.H.S.

So, it is ture.

(c) $A - B' = A \cap B$

L.H.S. $\rightarrow (x + y) - (x) = y$

R.H.S. $\rightarrow (x + y) \cap (y + z) = y$

L.H.S. = R.H.S.

So, it is true.

S.84 (c)

$f: R - \{2\} \to R$

$f(x) = (x^2 + 2x)(x - 2)$

Function values of $x = 1, y = -3$

$x = 0, y = 0$

$x = -1, y = 3$

$x = -2, y = 0$

$\because$ Function is one-one if for $x_1 \neq x_2$

$\Rightarrow f(x_1) \neq f(x_2)$

So, given function is not one-one.

$\because f: A \to B$ is said to be onto if $f(A) = B$

But $f(R - \{2\}) \neq R$. So, it is not onto.

S.86 (c)

A totally ordered set is a set plus a relation on the set (called a total order) that satisfies condition for a partial order plus an additional known as the comparability condition.

S.88 (c)

Using function definition,

$f(-1.75) + f(0.5) + f(1.5)$

$= (-1.75 + 2) + (0.5)^2 + (2 - 1.5)$

$= 1$

S.89 (b)

As only option (b) satisfies the condition.

S.90 (a)

$\because (a, b) * (c, d) = (ad + bc, bd)$

Then $(1, 2) * (3, 5) * (3, 4)$

$= (1 \times 5 + 2 \times 3, 2 \times 5) * (3, 4)$

$= (11, 10) * (3, 4)$

$= (11 \times 4 + 10 \times 3, 10 \times 4)$

$= (74, 40)$

S.91 (d)

As none of the figure satisfies property of boolean lattice.

S.93 (c)

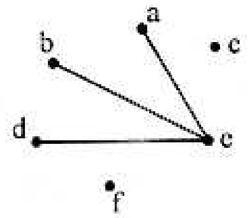
From the diagram it is clear that

Maximal elements are **3, 5** and

Minimal elements are **1, 6.**

S.94 (b)

Complements of element e are **3**.



S.95 (b)

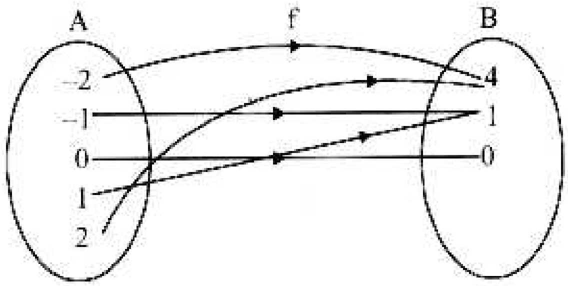
$f(x) = x^2$

$f(-2) = 4, f(-1) = 1$

$f(0) = 0, f(1) = 1, f(2) = 4.$

F: A→B is a function, and f: A→B is onto (since f (A) = (B).

$f(-1) = f(1) = 1$



hence f: A→B is not one one, since f: A→B is not bijection.

S.96 (b)

$y = f(x) = 4x+7$

$\Rightarrow x = \frac{y-7}{4}$

$\therefore$ Inverse function $f^{-1}(y) = \frac{y-7}{4}$

and inverse function $f^{-1}(x) = \frac{x-7}{4}$

S.97 (c)

$y = f(x) = \frac{2x+1}{3}$

$\therefore f^{-1}(y) = x = \frac{3y-1}{2}$

and $f^{-1}(x) = \frac{3x-1}{2}$

S.98 (d)

No. of students who just play one game

$= 40 + 20 + 13 = \mathbf{73}$

S.99 (d)

No. of students who play Tennis and Cricket but not Hockey

$= |T \cap C| - |T \cap C \cap H|$

$= 13 - 5 = \mathbf{8}$

S.100 (d)

No. of students play only Tennis

$= |T| - |T \cap C| - |T \cap H| + |T \cap C \cap H|$

$= 39 - 13 - 11 + 5 = \mathbf{20}$

S.101 (c)

No. of students play only Hockey

$= |H| - |T \cap H| - |T \cap C| + |T \cap C \cap H|$

$= 32 - 11 - 13 + 5 = \mathbf{13}$

S.102 (c)

$|T \cup C \cup H| = |T| + |C| + |H| - |T \cap C| - |T \cap H| - |C \cap H| + |T \cap C \cap H|$

$\Rightarrow 100 = 39 + 58 + 32 - 13 - 11 - 10 + |T \cap C \cap H|$

or $|T \cap C \cap H| = 5$

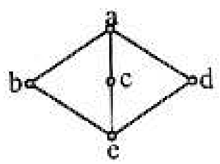
S.103 (a)

No. of students play only cricket

$= |C| - |T \cap C| - |C \cap H| + |C \cap H \cap T|$

$= 58 - 13 - 10 + 5 = \mathbf{40}$

S.104 (b)



$[\{a,b,c,d,e\},\leq]$

The poset $[\{a,b,c,d,e\},\leq]$ is a lattice but is not a distributive lattice. Because distributive lattice satisfy the following conditions

$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$

$x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$

where $\wedge$ and $\vee$ are meet and join operation but for given poset

$[\{a,b,c,d,e\},\leq]$

$b \wedge (c \vee d) = b \wedge a = b$

$(b \wedge c) \vee (b \wedge d) = e \vee e = e$

So it is not distributive. (Also, element b has 2 complements c and d, which is not possible in a distributive lattice, since, in a distributive lattice, complement if it exists, is always unique.)

So it is not distributive.

S.105 (c)

We first observe the following:

$A_1 = \{100, 101, 102, \ldots\ldots 199\}$

i.e. $|A_1| = 100$

$A_2 = \{120, 121, 122, \ldots\ldots 129, 220, 221, \ldots\ldots 229, \ldots\ldots 320, \ldots\ldots 929\}$

i.e. $|A_2| = 90$

and $A_3 = \{103, 113, 123, \ldots\ldots 923, \ldots\ldots 993\}$

so that $|A_3| = 90$.

Further we may find that,

$A_1 \cap A_2 = \{120, 121, 122, \ldots\ldots 229\}$ so that

$|A_1 \cap A_2| = 10$

$A_1 \cap A_3 = \{103, 113, \ldots\ldots 193\}$ so that

$|A_1 \cap A_3| = 10$

$A_2 \cap A_3 = \{123, 223, 323, \ldots\ldots 923\}$ so that

$|A_2 \cap A_3| = 9$

$A_1 \cap A_2 \cap A_3 = \{123\}$ so that

$|A_1 \cap A_2 \cap A_3| = 1.$

$\therefore\ |A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3| - |A_1 \cap A_2| - |A_2 \cap A_3| - |A_3 \cap A_1| + |A_1 \cap A_2 \cap A_3|$

$= 100 + 90 + 90 - 10 - 10 - 9 + 1 = \mathbf{252}$

S.106 (c)

Let $D = A \oplus B$ and $E = B \oplus C$. Then for all $x \in U$ we have

$f(A \oplus B) \oplus C(x) = f(D \oplus C)(x)$

$= F_D(x) + f_C(x) - 2F_D(x)f_C(x)$

$= \{1 - 2f_C(x)\}f_D(x) + f_C(x)$

$= \{1 - 2f_C(x)\}\{f_A(x) + f_B(x) - 2f_A(x)f_B(x)\} + f_C(x)$

$= f_A(x) + f_B(x) + f_C(x) - f_C(x)f_A(x) - 2f_C(x)f_B(x) - 2f_A(x)f_B(x) + 4f_A(x)f_B(x)f_C(x)$

$= f_A(x) + \{1 - 2f_A(x)\}\{f_B(x) + f_C(x) - 2f_B(x)f_C(x)\}$

$= f_A(x) + \{1 - 2f_A(x)\}F_E(x)$

$= f_A(x) + f_E(x) - 2f_A(x)f_E(x)$

$= f_{A \oplus E}(x) = f_{A \oplus (B \oplus C)}(x)$

$(A \oplus B) \oplus C = \mathbf{A \oplus B \oplus C.}$

S.107 (d)

$(a, b) \in R_1$. Then $a \in A$, $b \in R_1(a) \subseteq B$. Since $R_1(x) = R_2(x)\ \forall\ x \in A$, we have $R_1(a) = R_2(a)$. So $b \in R_2(a)$. Hence $(a, b) \in R_2$. Thus $R_1 \subseteq R_2$.

A similar argument shows that $R_2 \subseteq R_1$.

Accordingly, $\mathbf{R_1 = R_2}$

S.108 (c)

$g(x)g(y) = g(x) + g(y) + g(xy) - 2$

Put $x = 1$ and $y = 2$

$g(1) = 2$

Put $y = 1/x$

$$g(x)g\left(\frac{1}{x}\right) = g(x) + g\left(\frac{1}{x}\right) + g(1) - 2$$

$g(x) = x^n + 1$

$g(2) = 2^n + 1 = 5$

$2^n = 4$

$n = 2$

$\Rightarrow g(x) = x^2 + 1$

$g(3) = 3^2 + 1 = \mathbf{10}$

S.109 (a)

Let $a \in G$ be arbitrary.

Since G is a group, by the closure property,

$a^2 = a \cdot a \in G,\ a^3 = a^2 \cdot a \in G$ and so on

Thus $a^2, a^3, a^4, \ldots\ldots$ are all elements of G. Since G is finite, three must be repetitions in the above collection of elements i.e.,

$a^i = a^j$ for some integers i and j. $i > j$.

$\Rightarrow a^i \cdot a^{-j} = a^j \cdot a^{-j}$

$\Rightarrow a^{i-j} = a^0 = e \Rightarrow a^N = e$.

where $N = i - j > 0$.

Hence there exists a positive integer N such that,

$\mathbf{a^N = e}$ for all $a \in G$.

S.110 (a)

$f(n) = \dfrac{n-1}{2}$, when n is odd

$= \dfrac{n}{2}$, when n is even

For every $n \in N$ whether odd or even, we get a unique value of f(n) which belongs to set of integers. Thus f(n) is one-one.

Let $x = \dfrac{n-1}{2}$, when n is odd and $x \in I$.

$\therefore n = 2x + 1$

For negative values of x, $n \notin N$

Thus f(n) is not onto

Thus, the given function is **one-one but not onto.**

S.111 (a)

$fog = (\ln|x|) = \sin(\ln x)$

$\ln(x)$ is defined for $x > 0$, $\sin(\ln x) \in \mathbf{[-1, 1]}$

$\therefore$ gof $= g(\sin x) = \ln(\sin x)$ is defined

$\therefore\ 0 < \sin \le 1$

$\ln(\sin x) \in (-\infty, 0)$

S.112 (c)

f(x) is a objective function.

For $f^{-1}(x)$

let $f(x) = (x-1)^2 - 1,\ x \ge 1$

$y = (x-1)^2 - 1$

$\Rightarrow x = \sqrt{y+1} + 1$

$\Rightarrow f^{-}(y) = \sqrt{y+1} + 1$

$\Rightarrow f^{-}(x) = \sqrt{x+1} + 1$

or

Given that

$f(x) = f^{-1}(x)$

This equalify is not satisfied for each value of $x \ge 1$.

So, $\mathbf{R = \phi}$.

S.113 (a)

When $x = 1$

$y^2 = 36 - 1$ $[\because x^2 + y^2 = 36]$

$\therefore y = \sqrt{35}$

when $x = 2$

$\therefore y^2 = 36 - 4 = 32$

$\therefore y = \sqrt{32}$

when $x = 3$

$\therefore y^2 = 36 - 9$

$y = \sqrt{27}$

when $x = 4$

$\therefore y = 36 - 16 = 20$

$y = \sqrt{11}$

when $x = 6$

$\therefore y^2 = 36 - 36$

$y = \sqrt{0}$

when $x = 7$

$y^2 = 36 - 49$

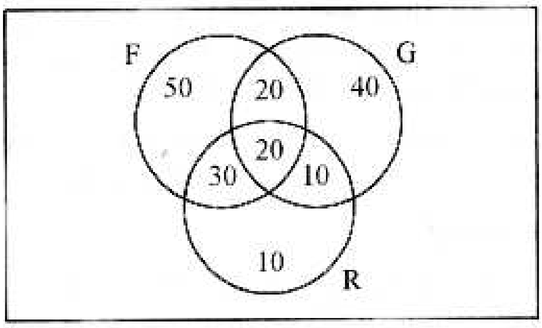
$= -13$

$\therefore$ There is no positive integer solutions of the given equation

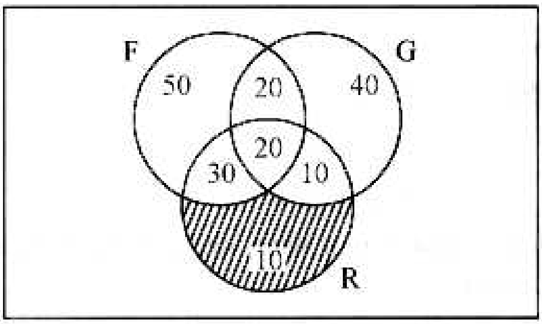
$\therefore$ **R = ϕ (null set) = { }**

S.115 (b)

By the given Data, distribution of students is represented in Venn-diagram.



Now $R \cap G' \cap F'$ is represented by shaded area.



So, **$O(R \cap G' \cap F') = 10$**

S.116 (b)

Total students = 243

28 fail, so remaining students = 243 – 28 = 215

Students passed in Maths (M) = 179

Students passed in Physics (P) = 197

$\therefore n(P \cap M) = n(P) + n(M) - n(P \cap M)$

$\Rightarrow 215 = 179 + 197 - n(P \cap M)$

$\Rightarrow n(P \cap M) = \mathbf{161}$

S.117 (c)

Students who failed in at least one of the two subjects is = fail in Maths + fail in Physics + fail in Both

$= (243 - 179 - 28) + (243 - 197 - 28) + 28$

$= \mathbf{82}$

and

Students who passed in at least one of the two subjects is = Total students – fail in both subjects

$= 243 - 28$

$= \mathbf{215}$

S.119 (c)

Set is closed under addition, substraction and multiplication, if it can find all elements of given set applying these operations.

I. $A = \{x = 2^n, n \in N\}$ $N = \{1, 2, ...\}$

So, we can't find all values applying given operations.

II. $B = \{x = 3n, n \in N\}$

Similarly, It does not satisfy.

III. $C = \{x = 3n, n \in Z\}$ $Z = \{0, 1, 2, ...\}$

This set satisfy the properties, So it is closed under given operations.

S.121 (a)

$A^* A = AA + \overline{A}\overline{A} = A + \overline{A} = 1$

S.122 (b)

(i) $3 > 2$, then $f(3) = 3$

$1 < 2$, then $f(10)= 5\ (1)-2=3$

Thus 1 and 3 have same image. Hence it is not one-one

Let $y \in R$

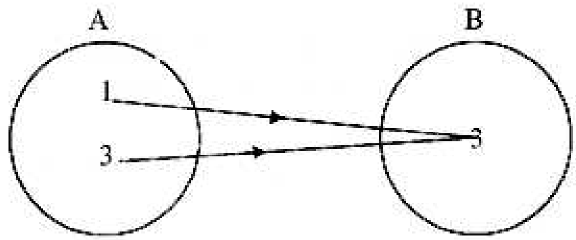
If $y > 2$, take $f(x) = y$

then $y = x$

if $y \leq 2$, take $x = \frac{y+2}{5}$

Then $x \leq \frac{4}{5}$

Hence $f(x) = 5x - 2 = 5\left(\frac{y+2}{5}\right) - 2 = y$



Hence it is onto.

S.123 (a)

$f(x) = x^2$

$f\ [0,\infty) \to R$ is a function.

Let $a_1, a_2 \in [0, \infty)$ and $f(a_1) = f(a_2)$

Now $f(a_1) = f(a_2)$

$\Rightarrow \quad a_1^2 = a_2^2$

$\Rightarrow \quad a_1 = a_2$

$\therefore\ [0, \infty) \to R$ is **one-one.**

There is no pre image in $[0,\infty]$ to (-1)

$\therefore$ f is not onto

$\therefore\ f\ [0,\infty) \to R$ is not a bijection

S.124 (c)

Assign value 1 if there is a path between any two nodes, otherwise assign '0'.

S.125 (c)

Note that if one or more of the sets S_i be infinite, then

$S_i \cup S_i = S$ will be infinite.

$S_i \cup S_i = S$ will be finite iff all S_i be finite.

Hence it follows that at least **one of the sets S_i be infinite.**

S.126 (b)

The power set $A \times A$ has cardinality n^2.

$\Rightarrow$ The cardinality of $P(A \times A)$ is $\mathbf{2^{n^2}}$.

(Note that if cardinality of A be n, then the cardinality of P(A) is 2^n. P(A) = set of all subsets of A including the empty set ϕ and the set A itself.)

S.127 (e)

A relation is partial order, if it is reflexive, antisymmetric and transitive.

Given relation is $X\ R\ Y\ \forall\ X < Y$

and $X, Y \in$ Real.

$\because$ X is always less than Y, so it can not be a reflexive relation.

$\because$ A relation is symmetric if $(a, b) \in R$ then $(b, a) \in R$ must be in the relation. But in this relation it is not possible. So it is an asymmetric relation.

$\because$ This is antisymmetric relation because $(a, b) \in R$ but $(b, a) \notin R$ and $(a, a) \notin R$.

S.128 (b)

Any element of A may be associated with any element of B in n ways. Hence, by multiplication principle, all m elements of A may be associated in $n \times n \times n \ldots$ m times $= n^m$ ways. Hence n^m functions are possible from A to B.

Out of these, exactly $^n\mathbf{P_m}$ function are injective.

There are no surjective possible from A to B.

For example,

$A = \{a, b\}$, $B = \{1, 2, 3\}$ $m = 2$, $n = 3$

There are $3^2 = 9$ function from A to B. They are

$\begin{pmatrix} a & b \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} a & b \\ 2 & 2 \end{pmatrix}, \begin{pmatrix} a & b \\ 3 & 3 \end{pmatrix}, \begin{pmatrix} a & b \\ 1 & 2 \end{pmatrix}, \begin{pmatrix} a & b \\ 2 & 1 \end{pmatrix},$

$\begin{pmatrix} a & b \\ 2 & 3 \end{pmatrix}, \begin{pmatrix} a & b \\ 3 & 2 \end{pmatrix}, \begin{pmatrix} a & b \\ 1 & 3 \end{pmatrix}, \begin{pmatrix} a & b \\ 3 & 1 \end{pmatrix}$

out of these 6 are injective, i.e. ${}^3P_2$ and no surjective.

S.129 (c)

(G, o) is Abelian

$\Rightarrow$ goh $=$ hog $\forall$ g, h $\in$ G

$(goh)^2 = (goh)o(goh)$

$= go(hog)oh$ ($\because$ o is associative)

$= go(goh)oh$ ($\because$ hog = goh)

$= (gog)o(hoh)$

$= g^2oh^2$

(Note that $g = g^{-1}$ implies that evey element in G is self inverse. In that case G is Abelian. But converse is not true.

$g = g^2$ doesn't follow when G is Abelian.

G may or may not be finite order, since there are infinite Abelian groups).

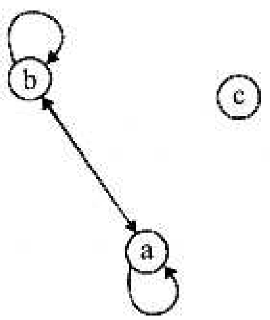
S.130 (d)

A relation that is symmetric and transitive need not be reflexive.

For example,

A = {a, b, c} and consider

R = {(a, a), (b, b), (a, b), (b, a)}



Certainly R is symmetric and transitive but not reflexive.

Since (c, c) $\notin$ R.

S.131 (c)

S has 3 elements.

$\Rightarrow$ P(S) has 2^3 = **8 elements.**

Note that P(S) = $\{\phi\}$, {1}, {(2, 3)}, $\{\{\phi\}, 1\}$, $\{\{\phi\}, (2, 3)\}$, {1, (2, 3)}, $\{\{\phi\}, 1, (2, 3)\}\}$.

S.132 (b)

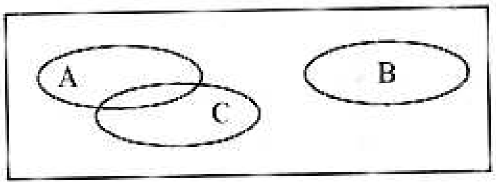
R is certainly not reflexive since $A \cap A = A$ for every A.

$\Rightarrow$ R cannot be equivalence.

$A \cap B = \phi \Rightarrow B \cap A = \phi$

Hence A R B $\Rightarrow$ B R A

$\Rightarrow$ R is symmetric.



$A \cap B = \phi$, $B \cap C = \phi \not\Rightarrow A \cap C = \phi$ (See the figure)

$\Rightarrow$ R is not transitive.

Hence R is symmetric but not transitive.

S.133 (c)

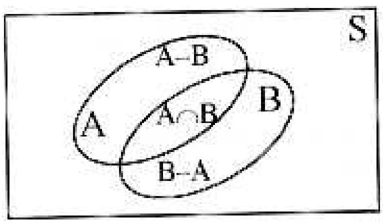
The statement (a), (b), (d) are all true.

The set of all strings over a finite alphabet forms a monoid under concatenation, but not a group, since inverses do not exist under concatenation.

S.134 (a)

It is **A $\cup$ B**

(from the figure)



(Note: $(A - B) \cup (B - A) = A \Delta B$ is called symmetric difference of A and B.

We have $A \cup B = (A \Delta B) \cup (A \wedge B)$.

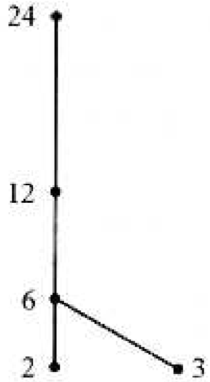
S.135 (b)

A relation is called Partial order if it is reflexive, antisymmetric and transitive.

Hasse diagarm is derived in the form of $X \leq Y$.

So, [(3, 6), (6, 24), (12, 24), (2, 12)]

Hasse diagram is



So, number of edges is **4.**

S.136 (d)

(i) $n * m$ = maximum (n, m)

= n or m

$\because$ n, m $\in$ Z

n * m $\in$ Z

$\therefore$ (Z, *) satisfies closure axiom.

(ii) a * (b * c) = maximum (a, b * c)

= maximum (a, maximum (b, c)) .....(1)

(a * b) * c = maximum (a * b, c)

= maximum (maximum (a, b), c) .....(2)

Both (1) and (2) will give us the maximum of a, b and c

$\therefore$ (Z, *) satisfies associatively

(iii) But here identify element does not exist.

If Z had is a been set of natural numbers, then e = 0. But for integers we have no identity element.

$\therefore$ **(Z, *) is a semi-group.**

S.137 (a)

The number of equivalence relations on a set is precisely equal to the number of partition of that set. A set with four elements has exactly 15 partitions. Therefore, the number of equivalence relations on {1, 2, 3, 4} is **15**

(**Note**: The partition are

1) {{1}, {2}, {3}, {4}

2) {{1, 2}, {3, 4}}

3) {{1, 3}, {2, 4}}

4) {{1, 4}, {2, 3}}

5) {{1}, {2, 3, 4}}

6) {{2}, {1, 3, 4}}

7) {{3}, {1, 2, 4}}

8) {{4}, {1, 2, 3}}

9) {{1}, {2}, {3, 4}}

10) {{1}, {3}, {2, 4}}

11) {{1}, {4}, {2, 3}}

12) {{2}, {3}, {1, 4}}

13) {{2}, {4}, {1, 3}}

14) {{3}, {4}, {1, 2}}

15) {{1, 2, 3, 4}}

S.138 (c)

Consider,

R_1 = {(a, a), (b, b), (a, b), (b, a)}

R_2 = {(b, b), (c, c), (c, b), (b, c)}

(a) $R_1 \cup R_2$ = {(a, a), (b, b), (a, b), (b, a), (c, c), (c, b), (b, c)}

In this $R_1 \cup R_2$ is not transitive.

Suppose (a, b) $\in$ R and (b, c) $\in$ R then (a, c) $\notin$ R.

Both it is not true for $R_1 \cup R_2$.

(b) $R_1 \cap R_2$ = {(b, b)}

which is reflexive, transitive and symmetric.

$\therefore$ **$R_1 \cap R_2$ is an equivalence relation.**

This can be proved for any two equivalence relations 'R_1' and 'R_2'.

S.139 (b)

Any relation on A is some subset of A $\times$ A and the largest equivalence relation on A is A $\times$ A itself. Note that A $\times$ A has **n^2 elements.**

S.140 (c)

Any one element of the m–element set may be associated with any one element of the n–element set in exactly n ways.

$\Rightarrow$ By MP, all m elements of the first set may be associated with those of the n–element set is $n \times n \times \ldots$ m times $=$ **n^m ways.** Hence n^m functions are possible.

S.141 (a)

Two relations $R_1(A, B)$ and $R_2(C, D)$

$R_1A = C\ A\ B = DR_2$

Result for this operations is $\mathbf{R_1 \cup R_2}$.

S.142 (b)

Let R be the relation on set A.

Reflexive Relation

If$(x, x) \in R\ \forall\ x \in A$

i.e., $xRx\ \forall\ x \in A$

Irreflexive Relation:

$x \not R x\ \forall\ x \in A$

Given $R = \{(1, 1), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4)\}$

$A = \{1, 2, 3, 4\}$

Now (4, 4) is not an element of R.

$\therefore$ R is not reflexive

$\therefore$ Option(a) eliminates.

Also, $\because$ (1, 1), (2, 2), (3, 3) are elements of R.

$\therefore$ R is also not irreflexive.

$\therefore$ Option (c) and (d) eliminates.

S.143 (d)

$E \to$ English, $H \to$ Hindi, $K \to$ Kannada

$E \cup H \cup K = 28$, $E = 18$, $H = 15$, $K = 22$

$E \cap H = 9$, $H \cap K = 11$, $K \cap E = 13$

$\therefore$ $E \cup H \cup K = E + H + K - E \cap H - H \cap K - E \cap K + E \cap H \cap K$

$\Rightarrow$ $28 = 18 + 15 + 22 - 9 - 11 - 13 + E \cap H \cap K$

$\Rightarrow$ $E \cap H \cap K = 28 - 22 = \mathbf{6}$

S.144 (c)

Any binray relation on a set A is some subset of $A \times A$.

$\Rightarrow$ The number of binary relation on A = cardinality of $P(A \times A)$

If cardinality of A = n, then cardinality of $A \times A = n^2$.

$\Rightarrow$ Cardinality of $P(A \times A) = 2^{n^2}$

Hence $\mathbf{2^{n^2}}$ binary relations are possible.

S.145 (c)

L is a lattice because as 'R' is a relation on set 'L' satisfying the following three properties:

1. Reflexive

   for any $a \in L$, we have aRa

2. Antisymmetric

   If aRb and bRa, then $a = b$.

3. Transitive

   If aRb and bRc, then aRc.

Thus 'R' is defined as partial ordering of 'L'.

Thus 'L' is a poset.

Moreover lattice is a poset $(L, \leq)$ in which every subset $\{a, b\}$ consisting two elements a, b has least upper bound and greatest lower bound.

Here,

lub of 'a' and 'b' $= a \vee b$

glb of 'a' and 'b' $= a \wedge b$

Hence **'L' is a lattice.**

S.146 (c)

R is an equivalence relation having 2 eqivalence classes.

Given,

Relation 'R' on the set of integers is defined as xRy iff $(x + y)$ is even.

This can be proved as follows.

'R' is an equivalence relation if it is Reflexive, Symmetric and transitive

(i) Reflexive:

Given $x + y$

$\therefore$ $x + x = 2x$ which is divisible by 2 hence it is even.

$\Rightarrow$ $(x, x) \in R$ for all $x \in I$

$\therefore$ R is reflexive.

(ii) Symmetric

Let $(x, y) \in R$

$\Rightarrow$ $x + y$ is divisible by 2.

$$\Rightarrow \frac{x+y}{2} = k$$

$$\Rightarrow \frac{y+x}{2} = k$$

$\Rightarrow$ $(y, x) \in R$

$\Rightarrow$ R is symmetric

(iii) Transitive:

Let $(x, y) \in R$ and $(y, z) \in R$

$\Rightarrow$ $(x + y)$ and $(y + z)$ are divisible by 2.

$$\Rightarrow \frac{y+x}{2} = m_1 \ \& \ \frac{y+z}{2} = m_2$$

$$\Rightarrow \frac{x+y+y+z}{2} = m_1 + m_2$$

$$\Rightarrow \frac{x+2y+z}{2} = k(\text{let})$$

$$\Rightarrow \frac{x+z}{2} + y = k$$

$$\Rightarrow \frac{x+z}{2} = k - y = k_1$$

$\Rightarrow$ $(x, z) \in R$

$\therefore$ 'R' is transitive.

(i), (ii) & (iii) $\Rightarrow$ 'R' is an equivalence relation

'R' is an equivalence relation on a set of integers then the equivalence class is defined.

$\bar{x} = \{a|aRx\}$

$aRx \Rightarrow x + a$ is divisible by 2

i.e $a \equiv x(\text{mod } 2)$

$\therefore$ $\bar{0} = \{\ldots\ldots,-4, -2, 0, 2, 4,\ldots\ldots\}$

$\bar{1} = \{\ldots\ldots,-5, -3, -1, 1, 3, 5,\ldots\ldots\}$

Thus it has two equivalence classes.

S.147 (b)

(i) Consider $R_1(a, b)$ iff

$(a + b)$ is even over the set of integers.

$(a + b)$ even

$\Rightarrow$ $(a + b)$ is divisible by 2.

Reflexive: $(a + a) = 2a$ which is divisible by 2.

$\Rightarrow$ $(a, a) \in R$ for all $a \in I$.

$\therefore$ 'R_1' is reflexive. .....(1)

Symmetric: Let $(a, b) \in R$

$\Rightarrow$ $(a + b)$ is divisible by 2.

$$\Rightarrow \frac{(a+b)}{2} = k$$

$\Rightarrow$ $(a, b) \in R$

$\Rightarrow$ 'R_1' is symmetric. .....(2)

Transitive: Let $(a, b) \in R_1$ and $(b, c) \in R_1$

$\Rightarrow$ $(a + b)$ is divisible by 2 and

$(b + c)$ is divisible by 2.

$$\Rightarrow \frac{a+b}{2} = m_1 \ \& \ \frac{b+c}{2} = m_2$$

$$\Rightarrow \frac{a+b+b+c}{2} = m_1 + m_2$$

$$\Rightarrow \frac{a+2b+c}{2} = k \ (\text{let})$$

$$\Rightarrow \frac{a+c}{2} + b = k$$

$$\Rightarrow \frac{a+c}{2} = k - b = k_1$$

$\Rightarrow$ $(a + c)$ is divisible by 2.

$\Rightarrow$ $(a + c)$ Î R_1

$\Rightarrow$ 'R_1' is transitive. .....(3)

{From (1), (2) and (3)}

$\therefore$ 'R_1' is an equivalence relation.

(ii) Consider, $R_3(a, b)$ iff $a.b > 0$ over the set of non-zero rational numbers.

Reflexive

$a.a = a^2 > 0$

$\Rightarrow$ (a, a) $\in$ 'R_3' for all a belongs to non-zero rational numbers.

$\Rightarrow$ 'R_3' is reflexive. .....(4)

Symmetric:

(a, b) $\in R_3$

$\Rightarrow$ a.b > 0

$\Rightarrow$ b.a > 0

given a, b belongs to set of non-zero rational numbers.

$\Rightarrow$ 'R_3' is symmetric .....(5)

Transitive:

Let (a, b) $\in R_3$ and (b, c) $\in R_3$

$\Rightarrow$ a.b > 0 and b.c > 0

$\Rightarrow$ $a.b^2.c > 0$

$\Rightarrow$ $\frac{a.b^2.c}{b^2} > \frac{0}{b^2}$

$\Rightarrow$ a.c > 0

$\Rightarrow$ (a, c) Î R_3

$\Rightarrow$ 'R_3' is transitive. .....(6)

From (4), (5) & (6)

'R_3' is an equivalence relation.

And R_2 & R_4 are not equivalent relation.

S.148 (d)

A relation R on A is reflexive, iff $\forall$ a $\in$ A, aRa

Here, S is not reflexive

For S to be reflexive, it should atleast contain (1, 1), (2, 2) and (3, 3)

A relation R on A is symmetric, if aRb, then bRa.

$\therefore$ S is symmetric.

A relation R on A is transitive, if aRb and bRa, then aRc.

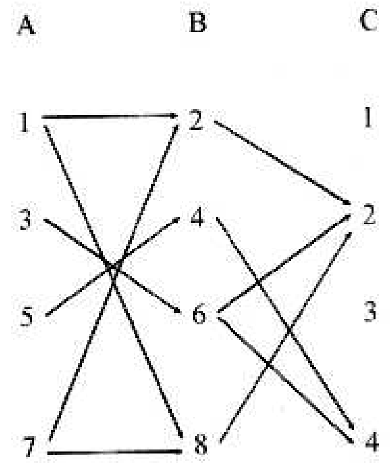
$\therefore$ S is transitive.

Hence S is **transitive and symmetric.**

S.149 (c)

R_1 = {(1, 2), (1, 8), (3, 6), (5, 4), (7, 2), (7, 8)}

R_2 = {(2, 2), (4, 4), (6, 2), (6, 4), (8, 2)}



From the above figure, A and C can be related

R_1R_2 = **{(1, 2), (3, 2), (3, 4), (5, 4), (7, 2)}**

S.151 (c)

Total number student who have taken one or more courses

= (125 + 85 + 65) − (50 + 35 + 30) + 15

= 275 − 115 + 15 = 175

Number of students who have not taken any courses

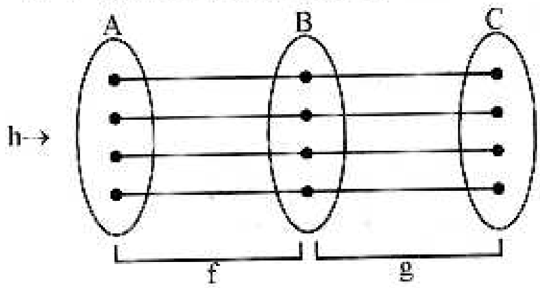
= 200 − 175 = **25**

S.152 (c)

f: A $\rightarrow$ B, g: B $\rightarrow$ C, h: A $\rightarrow$ C

h(a) = g(f(a)) $\forall$ a ε A

If 'h' have to be onto then 'g' must be onto as



S.153 (c)

Let A = {a, b, c}

C = collection of distinct subsets of A such that

S1 $\subset$ S2 or S2 $\subset$ S1 .....(i)

So distinct subsets of A are

$= \{\wedge, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{c,a\}, \{a,b,c\}\}$

if subset satisfy the condition (i) then

$C \quad = \{\wedge, \{a,b\}, \{b,c\}, \{c,a\}, \{a,b,c\}\}$

So **cardinality of C is $2^{n-1} + 1$**

$= 2^{3-1} + 1 = 5$

S.154 (c)

If R and S both be any two equivalent relation, then $R \cap S$ is reflexive relation (because they have n elements which are related to itself).

Similarly symmetric is also true ant transitive too.

Here noticeable point is that transitive property is always closed with intersection only.

Consider an example on set [1, 2, 3]

$R = \{<1, 1>, <2, 2>, <3, 3>, <1, 2>, <2, 1>\}$

It is an equivalence relation

$S = \{<1, 1>, <2, 2>, <3, 3>, <2, 3>, <3, 2>\}$

It is an equivalence relation

$R \cap S = \{<1, 1>, <2, 2>, <3, 3>\}$

It is an equivalence relation

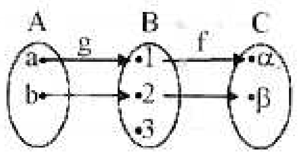
$R \cup S = \{<1, 1>, <2, 2>, <3, 3>, <1, 2>, <2, 1>, <2, 3>, <3, 2>\}$

It is not an equivalence relation

$<1, 3>$ is not true because transitive is not closed under union. Same with symmetric.

S.155 (b)

Consider the arrow diagram shown below.



$h(a) = f.g(a) = \alpha$

$h(b) = f.g(b) = \beta$

Here f is onto but g is not onto, yet h is onto

As can be seen from diagram if f is not onto, h cannot be onto.

$\therefore$ **f should be onto, but g need not be onto.**

S.156 (a)

$X = (A-B) - C$

$= (A \cap B') - C$

$= (A \cap B') \cap C'$

$= AB'C'$

$Y = (A-C) - (B-C)$

$= (A \cap C') - (B \cap C')$

$= (AC') \cap (BC')'$

$= (AC') \cap (B' + C)$

$= (AC') \cdot (B'+C)$

$= AC'B' + AC'C$

$= AC'B' = AB'C' \qquad$ (Since $C'C = 0$)

$X = Y \qquad$ (Commutative property)

S.157 (d)

Given $|X| = x$, $|Y| = y$ and $|Z| = z$

$W = x \times y$

so $|W| = x\,y$

$|E| = 2^{|W|} = 2^{xy}$

so the number of function from Z to E $= |E|^{|Z|} = (2^{xy})^z = \mathbf{2^{xyz}}$

S.158 (a)

$(x,y)\ R\ (u,v)$ iff $x < u$ and $y > v$

$(x,x)\ \not R\ (x,x)$ since $x \not< x$ and $x \not> x$

(x,x) is not related to (x,x) since x is not greater than x and x is not less than x.

So R is not reflexive.

So R is **neither partial order nor equivalence relation.**

S.159 (d)

Let $A = \{1,2,3,5,7,8,9\}$

Construct the table for any $x, y, \in A$ such that $x * y = (x, y) \bmod 10$

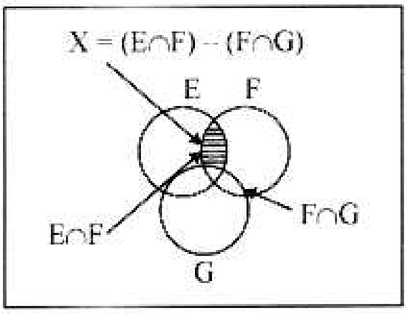
•	1	2	3	5	7	8	9
1	1	2	3	5	7	8	9
2	2	4	6	0	4	6	8
3	3	6	9	5	1	4	7
5	5	0	5	5	5	0	5
7	7	4	1	5	9	6	3
8	8	6	4	0	6	4	2
9	9	8	7	5	3	2	1

We know that $0 \notin A$. So, **it is not closed.**

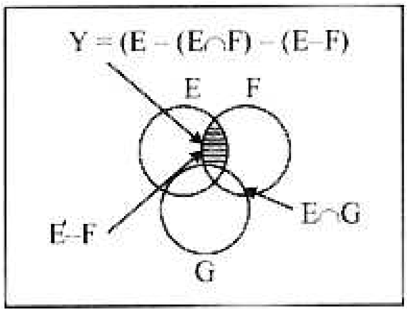
S.160 (c)

Using Venn diagram,

Venn diagram for X



Venn diagram for Y



So **X = Y**

Alternate:

X $= (E \cap F) - (F \cap G)$

$= EF - FG$

$= EF \cap (FG)'$

$= EF.\ (F'+G')$

$= EFF' + EFG' = EFG'$

Similarly

Y $= (E-(E \cap G)) - (E-F)$

$= (E-EG) - (E.F')$

$= E.\ (EG)' - EF'D$

$= E.\ (EG)' - EF'$

$= E.\ (E'+G') - EF'$

$= EG' - EF'$

$= EG' - (EF)'$

$= EG'\ (E' + F)$

$= EE\ G' + EF\ G'$

$= EFG'$

Therefore, **X = Y**

S.162 (a)

A **k-regular graph** is one in which vertex is of degree k. If k is even then, such a graph will have an Eulerian circuit.

S.163 (b)

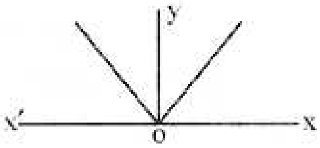
Let S be a set of n element say {1,2,3 .... n}. Now the smallest set equivalence relation on S must contain all reflexive elements {(1,1), (2,2),(3,3)......... (4,4)}. So largest equivalence relation on S is S×S has cardinality n^2. So the answer is n and n^2. The largest and smallest equivalence relations on S have cardinalities of **n^2 and n** respectively.

S.164 (a)

$f(x) = |x|$

$$\text{or } f(x) = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

The graph of f (x) is



f(x) is continuous for all real values of x

$$\lim_{x \to 0} (x) = \lim_{x \to 0} (x) = 0$$

as can be seen from graph of $|x|$

$$\lim_{x \to 0^-} f'(x) = -1$$

and $\lim_{x \to 0^+} f'(x) = +1$ by the graph.

left derivative $\neq$ right derivative.

So |x| **is continuous but not differential at x = 0.**

S.165 (d)

This statement can be converted in the following form

$(P \cap q \cap r) \cup (p^c \cap q \cap r) \cup q^c \cup r^c$

$= p.q.r + p'.q.r + q' + r'$

$= qr\ (p+p') + q' + r'$

$= qr + q' + r' = (q+q')\ (r+q') + r'$

$= r + q' + r'$

$= 1 + q' = 1 = U$

S.166 (a)

Given R = {(x,y),(x,z),(z,x),(z,y),} on set (x,y,z)

Since (x,y) $\in$ R and (y,x) $\notin$ R, R is not symmetric. (x,z) $\in$ R and (z,x) $\in$ R, R is not antisymmetric.

Since in antisymmetric digraph, all arrows are unidirectional. So **R is neither symmetric nor anti-symmetric.**

S.167 (c)

If an element is a generator then all element must be generated as power of element.

Try a,b,c,d one by one to see which are the generator.

$a = a,\ a^2 = a \cdot a = a \cdot a^3 = a^2 \cdot a = a$ and so on

$\therefore$ a is not the generator.

$b = b\ ,\ b^2 = b.b\ = a$

$b^3 = b.b^2 = b.a = b$

$b^4 = b.b^3 = b.b = a$ and so on

$\therefore$ b is not the generator

$c = c,\ c^2 = c.c = b$

$c^3 = c.c^2 = c.b = d$

$c^4 = c.c^3 = c{\cdot}d = a$. Since all a,b,c,d have been generated as power of c, so **c is generator.**

Similarly,

$d = d,\ d^2 = d.d = b,\ d^3 = d.d^2 = d.b = c$

$d^4 = d.d^3 = d.d^3 = d.c = a$

$\therefore$ **d is also generator.**

12.4

Combinatorics

Level-1

Common Data For Questions 1 & 2:

There are four bus lines between A and B; and three bus lines between B and C.

Q.1 The number of ways a person can travel by bus from A and C by way of B will be

(a) 24

(b) 14

(c) 12

(d) 10

Q.2 The number of way a person roundtrip by bus form A to C by way of B will be

(a) 264

(b) 144

(c) 48

(d) 12

Q.3 In how many ways can 5 red and 4 white balls be drawn from a bag containing 10 red and 8 white balls?

(a) $^{18}C_9$

(b) $^{8}C_5 \times {}^{10}C_4$

(c) $^{10}C_5 \times {}^{8}C_4$

(d) None of these

Q.4 In how many ways can 5 prizes be distributed among 4 boys when every boy can take one or more prizes?

(a) 600

(b) 120

(c) 625

(d) 1024

Q.5 Six teachers and six students have to sit round a circular table such that there is a teacher between any two students. The number of ways in which they can sit is

(a) $5! \times 5!$

(b) $5! \times 6!$

(c) $6! \times 6!$

(d) none of these

Q.6 If $^{n}C_4 = {}^{n}C_5$ then the value of $^{n}C_8$ is

(a) 90

(b) 109

(c) 19

(d) 9

Q.7 ${}^nC_r + 2\,{}^nC_{r+1} + {}^nC_{r+2}$ is equal to $(2 \le r \le n)$

(a) $2\,{}^nC_{r+2}$

(b) ${}^{n+2}C_{r+2}$

(c) ${}^{n+1}C_{r+1}$

(d) None of these

Q.8 The total number of ways of selecting six coins out of 20 one rupee coin, 10 fifty paise coin and 7 twenty paise coins is ______.

(a) 28

(b) 38

(c) 18

(d) 80

Q.9 If ${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$, then r is equal to

(a) 1

(b) 2

(c) 3

(d) 5

Q.10 If ${}^np_r = {}^np_r + 1$ and ${}^nC_r = {}^nC_{r-1}$, the (n, r) are

(a) 2, 3

(b) 3, 2

(c) 4, 2

(d) 2, 4

Q.11 The given word is ALGEBRA which is to be arranged.

Total words with the given word are

(a) $\dfrac{\lfloor 7}{\lfloor 2}$

(b) $\dfrac{\lfloor 7}{\lfloor 3}$

(c) $\dfrac{\lfloor 7}{\lfloor 4}$

(d) $\dfrac{\lfloor 7}{3}$

Q.12 For the correct answer of Q.12, how many words are there with two A's are together.

(a) $\lfloor 6$

(b) $\lfloor 5$

(c) $\lfloor 4$

(d) $\lfloor 7$

LEVEL–2

Q.13 How many 10 digits numbers can be written by using the digits 1 and 2?

(a) 10 !

(b) ${}^{10}C_2$

(c) 2^{10}

(d) ${}^{10}C_1 \times {}^9C_2$

Q.14 In an examination there are three multiple choice questions and each question has 4 choices. Number of ways in which a student can fail to get all answers correct is

(a) 63

(b) 27

(c) 12

(d) 11

Q.15 The number of words that can be formed out of the letters of the word 'COMMITTEE' is

(a) 9 !

(b) $\dfrac{9!}{2!}$

(c) $\dfrac{9!}{(2!)^2}$

(d) $\dfrac{9!}{(2!)^3}$

Q.16 How many different committees of 5 can be formed from 6 men and 4 women on which exact 3 men and 2 women serve?

(a) 120

(b) 60

(c) 20

(d) 6

Q.17 A student can take one of four Mathematics sections and one of five English sections. The number n of ways he can register for the two courses, is

(a) 30

(b) 20

(c) 5

(d) 4

Q.18 There are 10 lamps in a hall. Each one of them can be switched on independently. The number of ways in which the hall can be illuminated is

(a) 10!

(b) 2^{10}

(c) 1023

(d) 10^2

Q.19 Number n of license plates that can be made where each plate contains two distinct letters followed by three different digits is

(a) 489000

(b) 498000

(c) 468000

(d) 486000

Q.20 In a school of 400 students, every student reads 5 magazines and every magazine is read by 50 students. The number of magazines is

(a) exactly 40

(b) at least 45

(c) at most 35

(d) none of these

Q.21 The value of the expression ${}^{47}C_4 + \sum_{j=1}^{5} {}^{52-j}C_3$ is equal to

(a) ${}^{52}C_3$

(b) ${}^{52}C_4$

(c) ${}^{52}C_5$

(d) ${}^{47}C_5$

Q.22 The number of words that can be formed from the letter of the word "MALAYALAM"

(a) $\frac{9!}{4!(2!)^2}$

(b) $\frac{9!}{4!}$

(c) $\frac{9!}{4!(2!)^3}$

(d) $\frac{9!}{(2!)^3}$

Q.23 The number of words that can be formed by considering all possible permutations of the letters of the word "FATHER". How many of these words begin with U and end with R.

(a) 6!4!

(b) 4!6!

(c) 5!6!

(d) none of these

Q.24 All letters of the word "AGAIN" are permuted in all possible ways and the words so formed (with or without meaning) are written as in dictionary, then the 50^{th} word is

(a) INAGA

(b) IAANG

(c) NAAIG

(d) NAAGI

Q.25 If repetition of digits are not allowed, then how many numbers can be formed, each lying between 100 and 1000, using the following digits : 0, 3, 4, 5, 8, 9.

(a) $^5P_1 + 5!$

(b) $^5P_1 + {}^5P_2$

(c) 6!

(d) none of these

Q.26 If $^{28}C_{2r} : {}^{24}C_{2r-4} = 225 : 11$, then

(a) r = 24

(b) r = 14

(c) r = 7

(d) none of the these

Q.27 If $^{2n+1}P_{n-1} : {}^{2n-1}P_n = 3 : 5$, the value of n is

(a) 4

(b) 6

(c) 3

(d) 8

Q.28 The number of different words ending and beginning with a consonant which can be made out of the letters of the words EQUATION is

(a) 5200

(b) 4320

(c) 1295

(d) 1240

Q.29 All the letters of the word E AM LET are arranged in all possible ways. The number of such arrangements in which no two vowels are adjacent to each other is

(a) 360

(b) 144

(c) 72

(d) 54

Q.30 The given word is MATHEMATICS which can be arranged in different ways, by taking some or all the letters used in the word.

The number of words that can be formed by taking 4 letters at a time out of the letters of the given word is

(a) 2454

(b) 2250

(c) 2000

(d) 2500

Linked Questions 31 & 32:

Q.31 There are 10 things which are equally divided. Given things are to be divided equally between two persons, then total number of such ways

(a) 252

(b) 152

(c) 126

(d) 150

Q.32 For the correct answer of Q.31, how many ways are there in which things are divided equally in two heaps.

(a) 252

(b) 152

(c) 126

(d) 150

Q.33 Let A be a sequence of 8 distinct integers sorted in ascending order. How many distinct pairs of sequences, B and C are there such that (i) each is sorted in ascending order, (ii) B has 5 element and C has 3 element, and (iii) the result of merging B and C gives A?

(a) 2

(b) 30

(c) 56

(d) 256

Q.34 What is the minimum number of ordered pairs of non-negative numbers that should be chosen to ensure that there are two pairs (a, b) and (c, d) in the chosen set such that $a \equiv c \mod 3$ and $b \equiv d \mod 5$

(a) 4

(b) 6

(c) 16

(d) 24

Q.35 If ${}^{K+5}C_{K+1} = \frac{11(K-1)(K+3)}{2} P_K$ then the values of 'K' are:

(a) 6 and 7

(b) 7 and 11

(c) 2 and 11

(d) 2 and 6

Q.36 In an examination of 9 papers a candidate has to pass in more papers than the number of papers in which he fails in order to be successful. The number of ways in which he can be unsuccessful is

(a) $\frac{9!}{2}+1$

(b) $\frac{9!}{2}-1$

(c) 256

(d) 255

LEVEL–3

Q.37 m men and n women are to be seated in a row so that no two women sit together. If m > n, then the number of ways in which they can be seated is

(a) $\frac{m!n!}{(m+n)!}$

(b) $\frac{m!n!}{(m-n+1)!}$

(c) $\frac{m!(m+1)!}{(m-n+1)!}$

(d) none of these

Q.38 Suppose a licence plate contains two letters followed by three digits with the first digit not zero. How many different licence plates can be printed?

(a) 608600

(b) 608400

(c) 608200

(d) 608000

Q.39 12 persons are to be arranged on a round table. If two particular persons among them are not to be side by side, the total number of arrangements is

(a) 10!

(b) 45(8!)

(c) 2(10!)

(d) 9(10!)

Q.40 If 'x' and 'y' are positive integers, then ${}^xC_r + {}^xC_{r-1}{}^yC_1 + {}^xC_{r-2}{}^yC_2 + \ldots\ldots\ldots + {}^yC_1 =$

(a) ${}^{xy}C_r$

(b) ${}^{x+y}C_r$

(c) $\frac{(x+y)!}{r!}$

(d) $\frac{x!y!}{r!}$

Q.41 The number of ways of arranging the letter AAAAABBBCCCDEEF in a row when no two C's are together is

(a) $\frac{12!}{5!3!2!} \times {}^{13}P_3$

(b) $\frac{12!}{5!3!2!} \times \frac{{}^{13}P_3}{3!}$

(c) $\frac{15!}{5!3!2!} - \frac{13!}{5!3!2!}$

(d) $\frac{15!}{5!3!3!2!} - 3!$

Q.42 There are three coplanar parallel ends. If any 'p' points are taken on each of the lines, the maximum number of triangles with vertices at these points is

(a) $p^2(4p-3)$

(b) $3p^2(p-1)$

(c) $3p^2(p-1)+1$

(d) none of these

Q.43 $\sum_{r=0}^{m} {}^{n+r}C_n$ is equal to

(a) ${}^{n+m+1}C_{n-1}$

(b) ${}^{m+n+2}C_n$

(c) ${}^{m+n+3}C_{n-1}$

(d) none of these

Q.44 The number of straight lines obtained by joining the given points in fair is

(a) 57

(b) 60

(c) 75

(d) 98

Q.45 The number of triangles that can be formed with vertices at these points is

(a) 200

(b) 210

(c) 220

(d) 180

Q.46 There are 2 green balls 2 blue balls and 1 red ball.

In how many ways can these balls be arranged such that the red ball is always in the middle.

(a) 5!

(b) 4!

(c) 6

(d) 18

Q.47 How many arrangements are possible if all same color balls are to be together

(a) 12

(b) 6

(c) 5

(d) None of these

GATE QUESTIONS

Q.48 The recurrence relation that arises in relation with the complexity of binary search is **[GATE 1994]**

(a) T(n) = T(n/2) + k, k a constant

(b) T(n) = 2T(n/2) + k, k is a constant.

(c) T(n) = T(n/2) + log n

(d) T(n) = T(n/2) + n

Q.49 Which of the following permutations can be obtained in the output (in the same order) using a stack assuming that the input is the sequence 1, 2, 3, 4, 5 in that order? **[GATE 1994]**

(a) 3, 4, 5, 1, 2

(b) 3, 4, 5, 2, 1

(c) 1, 5, 2, 3, 4

(d) 5, 4, 3, 1, 2

Q.50 The recurrence relation

$$T(1) = 2$$

$$T(n) = 3T\left(\frac{n}{4}\right)+n$$

has the solution T(n) equal to **[GATE 1996]**

(a) O(n)

(b) O(log n)

(c) $O(n^{3/4})$

(d) None of these

Q.51 Two girls have picked 10 roses, 15 sunflowers and 15 daffodils. What is the number of ways they can divide the flowers amongst themselves? **[GATE 1999]**

[2-Marks]

(a) 1638

(b) 2100

(c) 2640

(d) None of the above

Q.52 The minimum number of cards to be dealt from an arbitrarily shuffled deck of 53 cards to guarantee that three cards are from some same suit is **[GATE 2000]**

[1-Mark]

(a) 3

(b) 8

(c) 9

(d) 12

Q.53 The solution to the recurrence equation $T(2^k) = 3T(2^{k-1}) + 1$, $T(1) = 1$ is:

[GATE 2002]

[1-Mark]

(a) 2^k

(b) $(3^{k+1} - 1)/2$

(c) $3^{\log_2 k}$

(d) $2^{\log_3 k}$

Q.54 Consider the following recurrence relation

$T(1) = 1$

$T(n + 1) = T(n) + \lfloor \sqrt{n+1} \rfloor$ for all $n \geq 1$

The value of $T(m^2)$ for $m \geq 1$ is

[GATE 2003]

[2-Marks]

(a) $\frac{m}{6}(21m - 39) + 4$

(b) $\frac{m}{6}(4m^2 - 3m + 5)$

(c) $\frac{m}{2}(3m^{2.5} - 11m + 20) - 5$

(d) $\frac{m}{6}(5m^3 - 34m^2 + 137m - 104) + \frac{5}{6}$

Common Data For Questions 55 & 56:

In a permutation $a_1, \ldots, a_n$ of n distinct integers, an inversion is a pain (a_i, a_j) such that $i < j$ and $a_i > a_j$.

Q.55 If all permutations are equally likely, what is the expected number of inversions in a randomly chosen permutation of 1...n? **[GATE 2003]**

[2-Marks]

(a) $n(n - 1)/2$

(b) $n(n - 1)/4$

(c) $n(n + 1)/4$

(d) $2n[\log_2 n]$

Q.56 What would be the worst case time complexity of the insertion sort algorithm, if the inputs are restricted to permutations of 1 ... n with at most n inversions? **[GATE 2003]**

[2-Marks]

(a) $O(n^2)$

(b) $O(n \log n)$

(c) $O(n^{1.5})$

(d) $O(n)$

Q.57 n couples are invited to a party with the condition that every husband should be accompanied by his wife. However, a wife need not be accompanied by her husband. The number of different gatherings possible at the party is

[GATE 2003]

[1-Mark]

(a) $({}^{2n}C_n)$

(b) $\frac{(2n)!}{2^n}$

(c) 3^n

(d) $({}^{2n}C_n)2^n$

Q.58 m identical balls are to be placed in n distinct bags. You are given that $m \geq kn$, where k is a natural number ≥ 1. In how many ways can the balls be placed in the bags if each bag must contain at least k balls? **[GATE 2003]**

[2-Marks]

(a) $\binom{m-kn+n+k-2}{n-k}$

(b) $\binom{m-1}{n-k}$

(c) $\binom{m-kn+n-1}{n-1}$

(d) $\binom{m-k}{n-1}$

Q.59 Mala has a colouring book in which each English letter is drawn two times. She wants to paint each of these 52 prints with one of k colours, such that the colour-pairs used to colour any two letters are different. Both prints of a letter can also be coloured with the same colour. What is the minimum value of k that satisfies this requirement? **[GATE 2004]**

[2-Marks]

(a) 6

(b) 7

(c) 8

(d) 9

Q.60 In how many ways can we distribute 5 distinct balls, $B_1, B_2, \ldots, B_5$ in distinct cells, $C_1 C_2 \ldots, C_5$ such that Bal! B_i is not in cell C_i, $\forall i = 1,2 \ldots, 5$ and each cell contains exactly one ball?

[IT-GATE 2004]

[2-Marks]

(a) 44

(b) 96

(c) 120

(d) 3125

Q.61 Consider the following recurrence

$T(n) = 2T(\lceil \sqrt{n} \rceil) + 1, T(1) = 1$ Which one of the following is true? **[GATE 2006]**

[2-Marks]

(a) $T(n) = \theta(\log \log n)$

(b) $T(n) = \theta(\log n)$

(c) $T(n) = \theta(\sqrt{n})$

(d) $T(n) = \theta(n)$

Q.62 Given a set of elements $N = \{1, 2, \ldots. n\}$ and two arbitrary subsets $A \subseteq N$ and $B \subseteq N$, how many of the n! permutations π from N to N satisfy min $[\pi(A)]$ = min $[\pi(B)]$, where min (S) is the smallest integer in the set of integers S, and $\pi(S)$ is the set of integers obtained by applying permutation π to each element of S? **[GATE 2006]**

[2-Marks]

(a) $(n - |A \cup B|)\, |A|\, |B|$

(b) $(|A|^2 + |B|^2)n^2$

(c) $n! \dfrac{|A \cap B|}{|A \cup B|}$

(d) $\dfrac{|A \cap B|^2}{\binom{n}{|A \cup B|}}$

Common Data For Questions 63 & 64:

Suppose that a robot is placed on the cartesian plane. At each step it is allowed to move either one unit up or one unit right, i.e, if it is at (i,j) then it can move to either (i+1,j) or (i,j+1).

Q.63 How many distinct paths are there for the robot to reach the point (10, 10) starting from the initial position (0, 0)? **[GATE 2007]**

[2-Marks]

(a) $({}^{20}C_{10})$

(b) 2^{20}

(c) 2^{10}

(d) None of these

Q.64 Suppose that the robot is not allowed to traverse the line segment from (4, 4) to (5, 4). With this constraint, how many distinct paths are there for the robot to reach (10, 10) starting from (0, 0)? **[GATE 2007]**

(a) 2^9 **[2-Marks]**

(b) 2^{19}

(c) $({}^{8}C_4) \times ({}^{11}C_5)$

(d) $({}^{20}C_{10}) - ({}^{8}C_4) \times ({}^{11}C_5)$

Common Data For Questions 65 & 66:

Let X_n denote the number of binary strings of length n that contain no consecutives 0s.

Q.65 Which of the following recurrences does x_n satisfy? **[GATE 2008]**

(a) $x_n = 2x_{n-1}$ **[2-Marks]**

(b) $x_n = x_{\lfloor n/2 \rfloor + 1}$

(c) $x_n = x_{\lfloor n/2 \rfloor} + n$

(d) $x_n = x_{n-1} + x_{n-2}$

Q.66 The value of x_5 is **[GATE 2008]**

(a) 5 **[2-Marks]**

(b) 7

(c) 8

(d) 16

ANSWER KEY

1	c	2	b	3	c	4	d	5	b
6	d	7	b	8	a	9	c	10	b
11	a	12	a	13	c	14	a	15	d
16	a	17	b	18	c	19	c	20	a
21	b	22	a	23	a	24	c	25	d
26	c	27	a	28	b	29	c	30	a
31	a	32	c	33	c	34	c	35	a
36	c	37	c	38	b	39	d	40	b
41	b	42	a	43	a	44	a	45	b
46	c	47	b	48	c	49	b	50	a
51	d	52	c	53	b	54	b	55	b
56	a	57	c	58	c	59	a	60	a
61	a	62	d	63	a	64	d	65	d
66	d								

SOLUTIONS

S.1 (c)

There are four ways to go from A to B and three ways to go from B to C; hence there are $4 \times 3 = 12$ ways to go from A to C by way of B.

S.2 (b)

There are twelve ways to go from A to C by way of B, and 12 ways to return. Hence there are $12 \times 12 = 144$ ways to travel round trip.

S.4 (d)

As there is no restriction regarding the number of prizes a boy can get, each prize can be given in 4 ways. The total no, of ways $= 4^5 = \mathbf{1024}$.

S.6 (d)

$^nC_4 = {}^nC_5$

$$\frac{n!}{4!(n-4)!} = \frac{n!}{5!(n-5)!}$$

$$\frac{1}{n-4} = \frac{1}{5}$$

$\therefore\ n - 4 = 5$

$\therefore\ n = 9$

$\therefore\ {}^9C_8 = {}^9C_{9-8} = {}^9C_1 = 9$

S.7 (b)

$^nC_r + 2.{}^nC_{r+1} + {}^nC_{r+2}$

$= ({}^nC_r + {}^nC_{r+1}) + ({}^nC_{r+1} + {}^nC_{r+2})$

$= {}^{n+1}C_{r+1} + {}^{n+1}C_{r+2} = \mathbf{{}^{n+2}C_{r+2}}$

S.8 (a)

If there are n_1 items of one kind, n items of another kind and so on, then the number of ways of choosing r items out of these items is = co-eff. of x^r in $(1 + x + x^2 + ... + x^m)(1 + x + x^2 + ... + x^n)$ ....

= co-eff. of x^6 in $(1 + x + x^2 + ... + x^{20})(1 + x + x^2 + ... + x^{10})(1 + x + ... + x^7)$

= co-eff. of x^6 in $(1 + 2x + 3x^2 + 4x^3 + 5x^4 + 6x^5 + 7x^6 + ...)(1 + x + ... + x^7)$

$= 1 + 2 + 3 + 4 + 5 + 6 + 7 = \mathbf{28}$

S.9 (c)

We have $\dfrac{n-r+1}{r} = \dfrac{84}{36} = \dfrac{7}{3}$

and $\dfrac{n-r}{r+1} = \dfrac{126}{84} = \dfrac{3}{2}$

$\therefore\ \dfrac{7}{3}r - 1 = n - r = \dfrac{3}{2}(r+1)$

or $14r - 6 = 9r + 9$ or $\mathbf{r = 3.}$

S.10 (b) $^nP_r = {}^nP_{r+1}$

$$\therefore\ \frac{n!}{(n-r)!} = \frac{n!}{(n-r-1)!}$$

$$\frac{1}{n-r} = 1 \text{ or } n - r = 1 \quad .....(i)$$

$$^nC_r = {}^nC_{r-1}$$

$$\therefore\ r + r - 1 = n \quad .....(ii)$$

or $2r - n = 1$

Solving (i) and (ii), we get

$\mathbf{n = 3,\ r = 2.}$

S.11 (a)

Total letters in the word ALGEBRA are 7, in which A appears twice.

$\therefore$ Required number of words $= \dfrac{\lfloor 7}{\lfloor 2}$.

S.12 (a)

When 2As will remain together then total words

$= \dfrac{\lfloor 5+1}{\lfloor 2} \times \lfloor 2 = \lfloor 6$.

S.16 (a)

The no. of committees formed is:

$^6C_3 \times {}^4C_2 = \mathbf{120}$

S.17 (b)

There are four choices for Mathematics and five choices for English.

Hence, $n = 4 \times 5 = \mathbf{20}$

S.18 (c)

The no. of ways in which the hall can be illuminated is:

$^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4 + {}^{10}C_5 + {}^{10}C_6 + {}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10} = \mathbf{1023}$

S.19 (c)

Each plate contains 2 distinct letters, so the no. of permutations formed is $= {}^{26}P_2$

Each plate contains 3 different digits, so the no. of permutations formed is $= {}^{10}P_3$

Total no. of permutations $= {}^{26}P_2 \times {}^{10}P_3$

$= \mathbf{468000}$

S.20 (a)

Let the number of magazines be m If each student reads only one magazine, then

the no. of students = 50 m

Given each students reads 5 magazine

$\therefore$ No. of students $= 400 = \dfrac{50m}{5}$

$\therefore\ m = \dfrac{400 \times 5}{50} = \mathbf{40}$

S.21 (b)

$^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3$

$= {}^{47}C_4 + {}^{47}C_3 + {}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3$

$= {}^{48}C_4 + {}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3$

$= {}^{49}C_4 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3$

$= {}^{50}C_4 + {}^{50}C_3 + {}^{51}C_3$

$= {}^{51}C_4 + {}^{51}C_3$

$= \mathbf{{}^{52}C_4}$

S.22 (a)

Total no. of letters = 9

No. of M's = 2

No. of A's = 4

No. of L's = 2

$\therefore$ The number of words that can be formed

$= \dfrac{9!}{2!4!2!} = \dfrac{9!}{4!(2!)^2}$

S.23 (a)

FATHER contain six alphabets. These all can be permuted in $^6P_6$.

i.e. 6! ways.

$\therefore$ The no. of words that can be formed is 6! words. If words start with U and end with R there are four words which can occupy position between U and R. They can be permuted in $^4P_4$ i.e. 24 ways. Thus, there are 24 words beginning with U and ending with R.

S.24 (c)

$4! = 24$ word being with A, $\dfrac{4!}{2!} = 12$ words begin with G and $\dfrac{4!}{2!} = 12$ words begin with I. So 49^{th} and 50^{th} words begin with N and in dictionary order 49^{th} word is NAAGI and 50^{th} is **NAAIG**.

S.25 (d)

Numbers should start from 101 and maximum number is 999. Therefore all numbers are of 3 digits.

Restricted	Non Restricted
3 or 4 or 5 or 8 or 9	
5 ways	$^5P_2$ ways

Here hundredth place cannot use the digit 0 (zero), but can be filled by any of the other five digits. So, permutation of restricted place $^5P_1 = 5$. Now tenth place and unit place can be filled by remaining 5 digits taking two at a time (repetition of digits not allowed). So, permutation of non-restricted place $= {}^5P_2 = 20$.

$\therefore$ By multiplication theorem,

Total number of ways $= 5 \times 20 = \mathbf{100.}$

S.26 (c)

$$^{28}C_{2r} \div {}^{24}C_{2r-4} = \frac{225}{11}$$

$$\therefore\ \frac{28!}{(28-2r)!2r!} \times \frac{(28-2r)!(2r-4)!}{24!} = \frac{225}{11}$$

$$\Rightarrow \frac{28 \times 27 \times 26 \times 25}{2r(2r-1)(2r-2)(2r-3)} = \frac{225}{11}$$

$\therefore\ 2r(2r-1)(2r-2)(2r-3)$

$$= \frac{11\times28\times27\times26\times25}{225} = 11\times28\times3\times26$$

$= 11 \times 12 \times 13 \times 14 = 14(14-1)(14-2)(14-3)$

$\Rightarrow 2r = 14$ or $r = 7$

S.27 (a)

$$\frac{(2n+1)!(n-1)!}{(n+2)!(2n-1)!} = \frac{3}{5}$$

$$\frac{(2n+1)(2n)}{(n+2)(n+1)n} = \frac{3}{5}$$

or $10(2n+1) = 3(n^2+3n+2)$

or $3n^2 - 11n - 4 = 0$

or $(n-4)(3n+1) = 0$

$\therefore$ $n = 4$

S.28 (b)

8 letter i.e. 3 consonants and 5 vowels. The consonants are to occupy 1st and last place and it can be done in ${}^3P_2$ ways. We will now be let with 5 vowels and 1 consonants i.e., 6 letter which can be arranged in 6! ways. Hence the number of words under given condition is ${}^3P_2 \times 6! = 6 \times 720 =$ **4320.**

S.29 (c)

Consonants M, L, T, in 3! = 6 ways and 4 gaps and 3 vowels (2 alike) in ${}^4P_3 \cdot \frac{1}{2!} = 12$ ways.

By fundamental theorem the number is $12 \times 6 =$ **72.**

S.30 (a)

Total letters = 11 out of these M appeared twice, A appeared twice, T appeared twice.

(i) when all the four selected letter are different then such word

$$= {}^8C_4 \times \underline{|4} = 1680 \quad .....(1)$$

(ii) When two different and two alike

Then such words ${}^7C_2 \times 3\frac{\underline{|4}}{\underline{|2}} = 756 \quad .....(2)$

(iii) When two alike of one kind and two alike of other kind then such words.

$$\frac{\underline{|4}}{\underline{|2}\cdot\underline{|2}} \times 3 = 18 \quad .....(3)$$

Required ways = 1680 + 756 + 18 = **2454.**

S.31 (a)

The groups are distinct

$\therefore$ Required number of ways

$$= \frac{\underline{|10}}{(\underline{|5})^2} = \mathbf{252}$$

S.32 (c)

Now 10 things are to be equally divided in two heaps,

$\therefore$ Required number of ways

$$= \frac{\underline{|10}}{(\underline{|5})^2} \times \frac{1}{2} = \frac{252}{2} = \mathbf{126}$$

S.33 (c)

This corresponds to an ordered partition of 8 elements into two groups, the first with 5 elements and second with 3 elements. The number of ways of doing this is

$$P(8;5,3) = \frac{8!}{5!3!} = \mathbf{56}$$

S.34 (c)

The number of combinations of pairs (a mod 3, b mod 5) is

$$3 \times 5 = 15$$

(since a mod 3 can be 0, 1, or 2) and b mod 5 can be 0, 1, 2, 3 or 4)

$\therefore$ If 16 different ordered pairs are chosen at least 2 of them must have a (a mod 3, b mod 5) as same (basic pigeon hole principle).

Let such two pairs be (a, b) and (c, d) then

a mod 3 = c mod 3 $\Rightarrow$ a $\equiv$ c mod 3

and b mod 5 = d mod 5 $\Rightarrow$ b $\equiv$ d mod 5

S.35 (a)

Since $${}^{K+5}P_{K+1} = \frac{11(K-1)(K+3)}{2}\,{}^{P}_{K}$$

$\therefore \quad \frac{K+5!}{4!} = \frac{11(K-1)}{2}\frac{K+3!}{3!}$

or $\quad \frac{(K+5)(K+4)}{4} = \frac{11(K-1)}{2}$

or $\quad K^2 + 9K + 20 = 22K - 22$

or $\quad K^2 - 13K + 42 = 0$

$\therefore \quad$ **K = 6, 7**

S.36 (c)

The candidate is unsuccessful if he fails in 9 or 8 or 6 or 5 papers.

$\therefore$ the number of ways to be unsuccessful

$= {}^9C_9 + {}^9C_8 + {}^9C_7 + {}^9C_6 + {}^9C_5$

$= {}^9C_0 + {}^9C_1 + {}^9C_2 + {}^9C_3 + {}^9C_4$

$= \frac{1}{2}({}^9C_0 + {}^9C_1 + {}^9C_2 + {}^9C_3 + ... + {}^9C_9)$

$= \frac{1}{2}(2^9) = 2^8 =$ **256**

S.37 (c)

There are m men and n women; where m > n.

Let the men be denoted by M and Women by W.

The condition given in the question is that no two women can sit together. Therefore, a women will sit either in the left of a men or in the right. There are total m men so women can sit on m + 1 places.

The ways in which women can sit = ${}^{m+1}P_n$

The ways in which men can sit = m!

Total no. of ways = m! ${}^{m+1}P_n$

$= \frac{m!(m+1)!}{(m+1-n)!}$

$= \mathbf{\frac{m!(m+1)!}{(m-n+1)!}}$

S.38 (b)

Each letter can be printed in twenty-six different ways, the first digit in nine ways and each of the other two digits in ten ways.

Hence 26 × 26 × 9 × 10 × 10 = **608400** different plates can be printed.

S.39 (d)

There are in total 12 persons to be arranged around a round table.

The condition given is that 2 particular out-person of them cannot sit together.

Therefore, the remaining 10 person can be arranged in 10! ways.

Now the remaining two person are to be arranged in the middle of already seated 10 persons.

The no. of ways of arranging these persons is 10 – 1 = 9

Total no. of arrangements = 10! × 9

= **9(10!)**

S.40 (b)

Consider the product

$(1 + t)^x(1 + t)^y = (1 + t)^{x+y}$

i.e. $({}^xC_0 + {}^xC_1t + {}^xC_2t^2 + ... + {}^xC_xt^n)({}^yC_0 + {}^yC_1t + {}^yC_2t^2 + ... + {}^yC_3t^y) = (1 + t)^{x+y}$

Equating coefficient of t^r on both sides, we get

${}^xC_0{}^yC_r + {}^xC_1{}^yC_{r-1} + {}^xC_r{}^yC_0 = {}^{x+y}C_r$

$\Rightarrow {}^xC_r + {}^xC_{r-1}{}^yC_1 + {}^xC_{r-2}{}^yC_2 + ... + {}^xC_1{}^yC_{r-1} + {}^yC_r = \mathbf{{}^{x+y}C_r}$ $[\because {}^yC_0 = {}^xC_0 = 1]$

S.41 (b)

Total no. of letters = 15

no. of C's = 3.

First place 12 letters other than C's at dot places.

X.X.X.X.X.X.X.X.X.X.X.X

The no. of ways $= \frac{12!}{5!3!2!}$

Since no two C's are together.

Thus Place C's at cross places whose number = 13

Their arrangements are $\frac{{}^{13}P_3}{3!}$

Total no. of ways $= \mathbf{\frac{12!}{5!3!2!} \times \frac{{}^{13}P_3}{3!}}$

S.42 (a)

The number of triangles with vertices on different lines

$^pC_1 \times {^pC_1} \times {^pC_1} = p^3$

The number of triangles with 2 vertices on one line and the third vertex on any one of the other two lines.

$= \ {^3c_1}\ ({^pc_2} \times {^pc_1})$

$= \ 6p.\dfrac{p(p-1)}{2} = 3p^2(p-1)$

$\therefore$ the required number of triangles

$= \ p^3 + 3p^2\ (p-1) = 4p^3 - 3p^2 = \mathbf{p^2(4p-3)}$

(The word "maximum" shows that no selection of points out of the three lines are collinear).

S.43 (a)

$${^nC_r} = {^nC_{r-1}}$$

and $\quad {^nC_{r-1}} + {^nC_r} = {^{n+1}C_r}$, we have

$$\sum_{r=0}^{m} {^{n+r}C_n} = \sum_{r=0}^{m} {^{n+r}C_r}$$

$= {^nC_0} + {^{n+1}C_1} + {^{n+2}C_2} + \ldots + {^{n+m}C_m}$

$= [1 + (n+1)] + {^{n+2}C_2} + {^{n+3}C_3} + \ldots + {^{n+m}C_m}$

$= ({^{n+2}C_1} + {^{n+2}C_2}) + {^{n+3}C_3} + \ldots + {^{n+m}C_m}$

$\because \quad n + 2 = {^{n+2}C_1}$ or ${^nC_1} = n$

$= ({^{n+3}C_2} + {^{n+3}C_3}) + \ldots + {^{n+m}C_m}$

$= ({^{n+4}C_3} + {^{n+4}C_4}) + \ldots + {^{n+m}C_m}$

....

....

$= {^{n+m}C_{m-1}} + {^{n+m}C_m} = {^{n+m+1}C_m}$

$= {^{n+m+1}C_{n+1}} \qquad [{^nC_r} = {^nC_{n-r}}]$

S.44 (a)

Total points $= 12$

Collinear points $= 5$

$\therefore$ Required straight lines $= {^{12}C_2} - {^5C_2} + 1$

$= \dfrac{12\times11}{2} - \dfrac{5\times4}{2} + 1$

$= 66 - 10 + 1 = \mathbf{57}$

S.45 (b)

Required triangles $= {^{12}C_3} - {^5C_3}$

$= \dfrac{12\times11\times10}{3\times2\times1} - \dfrac{5\times4\times3}{3\times2\times1}$

$= 220 - 10 = \mathbf{210}$

S.46 (c)

Here 2 green and 2 blue ball are identical. So here we will use the concept of permutations of objects when some of them are identical. Also position of red ball is fixed, we will permute only remaining 4 balls.

$\therefore$ No. of ways $= \dfrac{4!}{2!2!} \times 1 = \mathbf{6}$

S.47 (b)

Since all the same colour balls are to be together, 2 green balls will act as one single ball. Similarly 2 blue balls will act as one single blue ball. Also there is 1 red ball. So in effect, we have to arrange 3 balls.

$\therefore$ No. of arrangements $= 3! = 6$.

S.48 (c)

For recurrence relation, worst case number of comparisons is given by

$$\begin{aligned} w(n) &= 1 + w(n/2) \\ &= 1 + 1 + w(n/2^2) \\ &= 1 + 1 + 1 + w(n/2^3) \end{aligned}$$

i.e. $\quad w(n) = \log_2 n + 1$

Since, w(n) depends upon $\log_2 n$ thus T(n) should also be dependent on $\log_2(n)$. Only one option has $\log_2(n)$.

S.49 (b)

Input sequence is 1, 2, 3, 4, 5

PUSH 1, 2, 3 initially $\begin{array}{|c|}\hline 3\\\hline 2\\\hline 1\\\hline\end{array}$

POP 3 $\begin{array}{|c|}\hline 2\\\hline 1\\\hline\end{array}$ Output seq. : 3
Input seq. : 4 5
(remaining part)

PUSH 4 and POP it $\begin{array}{|c|}\hline 2\\\hline 1\\\hline\end{array}$ Output seq. : 3 4
Input seq. : 5

PUSH 5 and POP it out $\begin{array}{|c|}\hline 2\\\hline 1\\\hline\end{array}$ Output seq. : 3 4 5
Input seq. : empty

Now, pop the remaining elements

$\therefore$ Output seq. = 3, 4, 5, 2, 1

S.51 (d)

The roses can be divided amongst the two girls in 11 ways corresponding to one girl getting 0, 1, 2, ..., 10 roses and the other girl getting the remaining. Similarly the sunflowers can be divided in 16 ways and the daffodils in 16 ways.

So, the total number of ways in 11 * 16 * 16 = **2816**.

S.52 (c)

Pigeon Hole Principle states if 'n' pigeons are assigned to 'm' pigeon holes (m < n) then one pigeon hole must have atleast two pigeons.

Extended Pigeon Hole Principle:

If 'n' pigeon are assigned to 'm' pigeonholes and m << n, then one pigeonhole must have atleast

$\left\lfloor \frac{n-1}{m} \right\rfloor + 1$ pigeons.

Here m = 4. Since we have 4 suits in a pack of card. Given no. of pigeons = 3.

$$\therefore \quad \left\lfloor \frac{n-1}{4} \right\rfloor + 1 = 3$$

$$\therefore \quad \left\lfloor \frac{n-1}{4} \right\rfloor = 2$$

$$\therefore \quad n-1 = 8$$

$$\therefore \quad n = 9$$

$\therefore$ Minimum number of cards to guarantee that three cards are from the same suit is 9.

S.53 (b)

$T(2^k) = 3T(2^{k-1}) + 1,\ T(1) = 1$

Put k = 1, we have

$$T(2) = 3T(2^0) + 1$$

$$= 3T(1) + 1 = 4$$

For k = 2, $T(4) = 3T(2) + 1 = 13$

For k = 3, $T(8) = 3T(4) + 1 = 40$

$$T(k) = \frac{(3^{k+1} - 1)}{2},$$

For $T(1) = 4$ for k = 1

$T(2) = 13$ for k = 2

$T(3) = 40$ for k = 3

S.54 (b)

$$T(1) = 1$$

$$T(2) = T(1) + \lfloor \sqrt{2} \rfloor$$

$$= 1 + 1 = 2$$

$$T(3) = T(2) + \lfloor \sqrt{3} \rfloor$$

$$= 2 + 1 = 3$$

$$T(4) = T(3) + \lfloor \sqrt{4} \rfloor$$

$$= 3 + 2 = 5$$

$$T(5) = T(4) + \lfloor \sqrt{5} \rfloor$$

$$= 5 + 2 = 7$$

$$T(6) = T(5) + \lfloor \sqrt{6} \rfloor$$

$$= 7 + 2 = 9$$

$$T(7) = 9 + 2 = 11$$

$$T(8) = 11 + 2 = 13$$

$$T(9) = 13 + \sqrt{9} = 16 \text{ and so on till,}$$

$$T(16) = 16 + 6 \times 3 + 4 = 38$$

$\therefore$ $T(1) = 1$, $T(4) = 5$, $T(9) = 16$ and $T(16) = 38$

Choice (a) does not satisfy T(16).

Choice (c) does not satisfy T(4).

Choice (d) does not satisfy T(1).

$\therefore$ answer is choice (b) which satisfies T(1) upto T(16).

S.55 (b)

$$\text{Expected number of inversion} = \frac{n(n-1)}{2}$$

$$\text{For randomly} = \frac{n(n-1)}{2} \times \frac{1}{2} = \frac{n(n-1)}{4}$$

S.57 (c)

For each of the n couples invited to the party one of three thing is possible

1. both husband and wife attend the party.
2. wife only attends the party.
3. neither husband nor wife attends the party.

Since there are n such couples, total number of possibilities = 3^n

S.58 (c)

Total number of bags = n

Total number of balls = m

$$m \geq kn$$

$$m - kn \geq 0$$

So the total number of ways such that each bag contains at least k balls is the solution to $x_1 + x_2 + x_3 + \ldots\ldots + x_n = m - kn$, which is

$$\binom{n-1+m-kn}{m-kn} = \binom{n-1+m-kn}{n-1}$$

$$= \binom{\mathbf{m-kn+n-1}}{\mathbf{n-1}}$$

S.59 (a)

The problem reduces to finding how many distinct ordered colour pair (C_1, C_2) are possible with k colors.

Since the first color C_1 can be any one of the k colours and the second color C_2 also can be any one of the k colors (both prints of a letter can be colored with same color), the total no. of such order color pairs is equal to $k \times k = k^2$.

Since each pair of letters must be colored with different color pairs, at least 26 color pairs are required to do this.

Therefore the requirement is $K^2 \geq 26$.

The minimum value of k that satisfies this equation is **k = 6.**

S.60 (a)

No. of ways in which no ball goes into the

Correct cell = Total no. of ways – No. of ways in which atleast 1 ball goes correctly + No. of ways in which atleast 2 balls go correctly – Total no. of ways in which atleast 3 balls go correctly + Total no. of ways in which atleast 4 balls go correctly – Total no. of ways in which all balls go correctly

$= 5! - {}^5C_1 \times 4! + {}^5C_2 \times 3! - {}^5C_3 \times 2! + {}^5C_4 \times 1! - {}^5C_5$

$= 120 - 120 + 60 - 20 + 5 - 1$

$=$ **44 ways.**

S.62 (d)

Given $N = \{1, 2, ..., n\}$

$A \subseteq N$

and $B \subseteq N$

$\pi : N \to N$

$\min(\pi(A)) = \min(\pi(B))$

Total number of permutation to find x such that

$$x \in A \cup B = \binom{n}{|A \cup B|}$$

$\min(\pi(A)) = \min(\pi(B))$

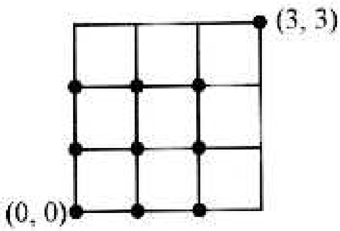
if $x \in |A \cup B|$

Total number of n! permutation

$$= \frac{|A \cap B||A \cap B|}{\binom{n}{|A \cup B|}} = \frac{|A \cap B|^2}{\binom{n}{|A \cup B|}}$$

S.63 (a)

Consider the following diagram.



The robot can move only right or up as defined in problem.

Let us denote right move by 'R' and up move by 'U'. Now to reach (3, 3) from (0,0), the robot has to make exactly 3 'R' moves and 3 'U' moves in any order.

Similarly to reach (10, 10) from (0, 0), the robot has to make 10 'R' moves and 10 'U' moves in any order. The number of ways this can be done is same as number of permutations of a word consisting of 10 'R's and 10 'U's.

Applying formula of permutation with limited repetitions we get the

answer as $\dfrac{20!}{10!10!} = {}^{20}C_{10}$

S.64 (d)

The robot can reach (4, 4) from (0, 0) in ${}^8C_4$ ways as argued in previous problem. Now after reaching (4, 4) robot is not allowed to go to (5, 4).

Let us count how many parts are there from (0, 0) to (10, 10) if robot goes from (4, 4) to (5, 4) and then we can subtract this from total number of ways to get the answer.

Now there are ${}^8C_4$ ways for robot to reach (4, 4) from (0, 0) and then robot takes the 'U' move from (4, 4) to (5, 4). Now from (5, 4) to (10, 10) the robot has to make 5 'U' moves and 6 'R' moves in any order which can be done in $\dfrac{11!}{5!6!}$ ways $= {}^{11}C_5$ ways.

$\therefore$ The number of ways robot can move from (0, 0) (10, (10) via (4, 4) – (5, 4) move is.

${}^8C_4 \times {}^{11}C_5 = ({}^8C_4)({}^{11}C_5)$

$\therefore$ No of ways robot can move from (0, 0) to (10, 10) without using (4, 4) to (5, 4) move is $({}^{20}C_{10}) - ({}^8C_4) \times ({}^{11}C_5)$ ways.

S.65 (d)

If x_n denote the number of binary strings of length n. There are 2^n possible sequences of outcomes. Suppose we want to know the binary string that contain no consecutive 0'S. In each string of n–1 0'S and 1'S in which there are no consecutive 0'S, we can append a 1 to obtain a string of length n. For each string of n – 2 0'S and 1's in which there are no consecutive 0'S we can append a 1. So the recurrence equation is

$$x_1 = 2$$

$$x_2 = 3$$

$$x_3 = 5$$

$$x_n = x_{n-1} + x_{n-2}$$

S.66 (d)

$$x_1 = 2,\ x_2 = 3,\ x_3 = 2 + 3 = 5$$

$$x_4 = x_2 + x_3 = 3 + 5 = 8$$

$$x_5 = x_3 + x_4 = 5 + 8 = 13$$

Since 13 is not one of the answers, so nearest answer is 16.

12.5

Graph Theory

Level–1

Q.1 The minimum number of edges in a connected cyclic graph on n vertices is

(a) n–1

(b) n

(c) n+1

(d) none of these

Q.2 The number of distinct simple graphs with up to three nodes is

(a) 15

(b) 10

(c) 7

(d) 9

Q.3 The minimum number of colours required to colour the vertices of a cycle with n nodes in such a ways that no two adjacent nodes have the same colour is

(a) 2

(b) 3

(c) 4

(d) $n-2\left[\frac{n}{2}\right]+2$

Q.4 Consider a simple connected graph G with n Vertices and n edges (n>2). Then which of the following statements are TRUE?

(a) G has no cycles.

(b) G has atleast one cycle.

(c) The graph obtained by removing any edge from G is not connected.

(d) None

Q.5 In any undirected graph, the sum of degrees of all the nodes

(a) need not be EVEN

(b) must be ODD

(c) is twice the number of edges

(d) must be even

Q.6 The number of vertices of ODD degree in a graph is

(a) always ZERO

(b) either EVEN or ODD

(c) always ODD

(d) always EVEN

Q.7 A graph in which all nodes are of equal degree is known as

(a) Complete graph

(b) Regular graph

(c) Non-regular graph

(d) Multi graph

Q.8 The maximum degree of any node in a simple graph with n vertices is

(a) $n - 2$

(b) n

(c) $\frac{n}{2}$

(d) $n - 1$

Q.9 Two isomorphic graphs must have

(a) equal number of vertices

(b) same number of edges

(c) same number of vertices

(d) all of these

Q.10 A given connected graph G is a Eular graph if and only if all vertices of G are of

(a) Different degree

(b) Odd degree

(c) Even degree

(d) Same degree

Q.11 A tree with "n" nodes has

(a) $n + 1$ edges

(b) $n - 1$ edges

(c) n edges

(d) $\frac{n}{2}$ edges

Q.12 The number of circuits in a tree with 'n' nodes is

(a) $\frac{n}{2}$

(b) $n - 1$

(c) one

(d) zero

Q.13 A graph is a tree if and only if it

(a) is planar

(b) contains a circuit

(c) is minimally connected

(d) is completely connected

Q.14 A graph with "n" vertices and $n - 1$ edges that is not a tree, is

(a) A circuit

(b) Eular

(c) Disconnected

(d) Connected

Q.15 T is a graph with n vertices. T is connected and has exactly $n - 1$ edges, then

(a) addition of a new edge will create a cycle.

(b) every pair of vertices in T is connected by exactly one path

(c) T contains no cycles

(d) T is a tree

Q.16 A simple graph in which there exists an edge between every pair of vertices is called

(a) Regular graph

(b) Planner graph

(c) Eular graph

(d) Complete graph

Q.17 The minimum number of spanning trees in a connected graph with "n" nodes is

(a) $\frac{n}{2}$

(b) $n - 1$

(c) 2

(d) 1

Q.18 For any graph G, exactly ONE of the following statements is FALSE.

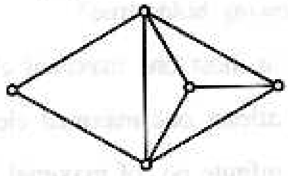
(a) If the minimum degree of G equals the maximum degree of G, then all vertices of G have the same degree

(b) The number of vertices of odd degree is even

(c) The number of vertices of even degree is even

(d) the sum of the degrees of the vertices of G is twice the number of edges of G

Q.19 The given graph is k-colorable, where



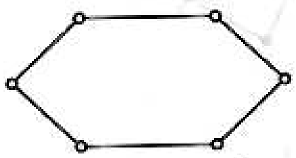
(a) K = 5
(b) K = 4
(c) K = 3
(d) K = 2

Q.20 Which of the following statement is FALSE?
(a) K_q is q – regular
(b) K_q is always q – partite
(c) K_q is p – partite for same p < q
(d) Any complete graph is never bipartite

Q.21 If G is a connected graph, then
(a) G is unicursal if it has no vertex of odd degree
(b) G is never unicursal
(c) G is unicursal if it has exactly one vertex of even degree
(d) G is unicursal if it has exactly one vertex of even degree

Q.22 Which of the following statements is FALSE?
(a) The radius of a tree is not necessarily half its diameter.
(b) A circuit having 2n edges has radius n and has exactly 2n centres.
(c) A simple chain having (2n + 1) edges has radius (n + 1) and has exactly one centre.
(d) A simple chain having 2n edges has diameter 2n and has exactly one centre.

Q.23 The given graph has



(a) radius 3 and diameter 3
(b) radius 2 and diameter 2
(c) radius 2 and diameter 3
(d) none of these

Q.24 The number of distinct graphs with p vertices and q edges ($p \neq q$) is always equal to
(a) min (p, q)
(b) q
(c) p
(d) none of these

Q.25 Every cutset of a connected Euler graph has
(a) no such characterization
(b) at least three edges
(c) an even number of edges
(d) an odd number of edges

Q.26 In connection with "Hamiltonian" property,
(a) A tree can possess a Hamiltonian chain under certain conditions
(b) Any bipartite graph having an odd number of vertices possesses a Hamiltonian circuit.
(c) If a graph possesses a Hamiltonian circuit, then it always possesses Hamiltonian chain
(d) Every complete graph has several Hamiltonian circuits

Q.27 Which of the following statements is FALSE?
(a) If G having p vertices is not complete, then its chromatic number is less than p.
(b) Every tree is bipartite
(c) The chromatic number of G is the largest K such that G is K – partite.
(d) Every tree has chromatic number 2

Q.28 Every (nontrivial) tree has
(a) chromatic number 3 and radius R = diameter T
(b) chromatic number 3 and radius R < diameter T
(c) chromatic number 2 and radius R = diameter T
(d) chromatic number 2 and radius R < diameter T

Q.29 The minimal spanning tree is determined by using
(a) cyclic generation algorithm
(b) decomposition algorithm
(c) dantzig's algorithm
(d) prim's algorithm

LEVEL–2

Q.30 Maximum number of edges in a n node undirected graph without self loop is

(a) n^2

(b) $\frac{n(n-1)}{2}$

(c) n–1

(d) $\frac{n(n+1)}{2}$

Q.31 Let G be a graph with 100 vertices numbered 1 to 100. Two vertices i and j are adjacent if $|i-j| = 8$ or $|i-j| = 12$. The number of connected components in G is

(a) 8

(b) 4

(c) 12

(d) 25

Q.32 The number of different rooted labeled trees with n vertices is

(a) n^n

(b) n^{n-1}

(c) 2^n

(d) $2^n - 1$

Q.33 If a graph requires k different colours for its proper coloring, then the chromatic number of the graph is

(a) $\frac{k}{2}$

(b) k – 1

(c) k

(d) 1

Q.34 Let G = G(V, E) has 6 vertices. Then the maximum number n of edges in E if G is a graph and if G is a multigraph will be

(a) undetermined, undetermined

(b) 30, undetermined

(c) 3!, 6!

(d) 30, 6!

Q.35 Suppose S is a linearly ordered set. Then which of the following holds true?

I. S has at most one maximal element.

II. S has atleast one maximal element

III. S has infinite no. of maximal element.

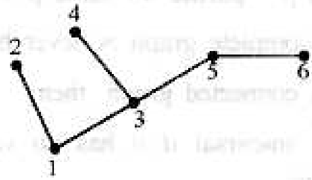
(a) III

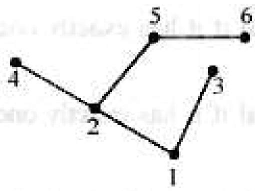
(b) II

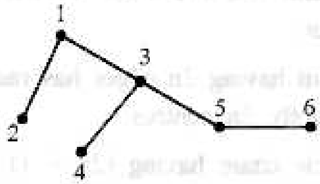
(c) I

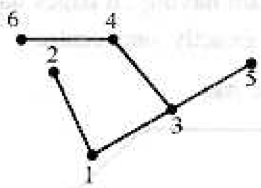
(d) none of these

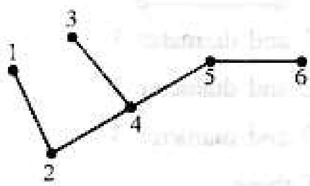
Q.36 The Hasse diagram of dual poset of the poset whose Hasse diagram is given by



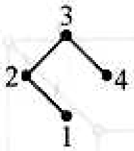
(a) 

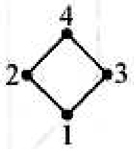
(b) 

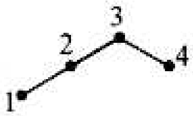
(c) 

(d) 

Q.37 The hasse diagram on A = {1, 2, 3, 4} of R = {(1, 1), (1, 2), (2, 2), (2, 4), (1, 3), (3, 3), (3, 4), (1, 4), (4, 4)}. Diagraph will be

(a) 

(b) 

(c) 

(d) 

Q.38 Determine which of the following graph G(V, E) is a simple graph where V = {a, b, c, d} and

(i) E = [{a, b}, {a, c}, {b, a}, {d, d}, {b, c}, {c, d}, {d, a}]

(ii) E = [{a, b}, {c, d}, {a, b}, {d, b}]

(iii) E = [{a, a}, {b, c}, {d, a}, {d, c}]

(iv) E = [{a, b}, {a, c}, {a, d}, {b, c }, {c, d}]

(a) only (iv)

(b) (iii) and (iv)

(c) (i) and (ii)

(d) (ii) and (iv)

Q.39 Let S = {5, 10, 25, 27, 35} be ordered by divisibility. Then the maximal and minimal elements of S.

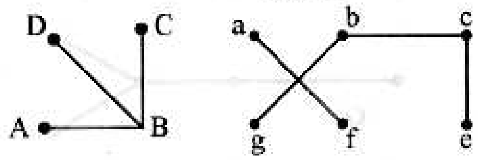
(a) (25, 35), (5)

(b) (25, 35), (5, 10)

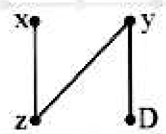
(c) (35), (5)

(d) (10, 25, 35), (5)

Q.40 Which of the following graphs is connected



(i) (ii)



(iii)

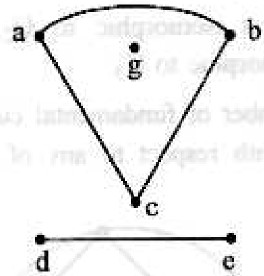
(a) all are disconnected

(b) both (i) and (iii)

(c) only (ii)

(d) only (iii)

Q.41 Consider the adjoining multigraph:



Then list the connected components of the graph.

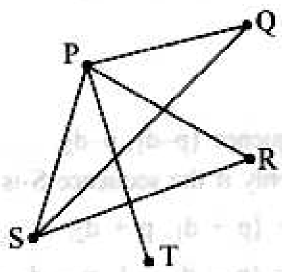
(a) {a, b, c}, {d, e}

(b) {g}, {d, e}

(c) {a, b, c}, {d, e}, {g}

(d) none of these

Q.42 Consider the adjoining figure:



What will be the bridge/s of the graph?

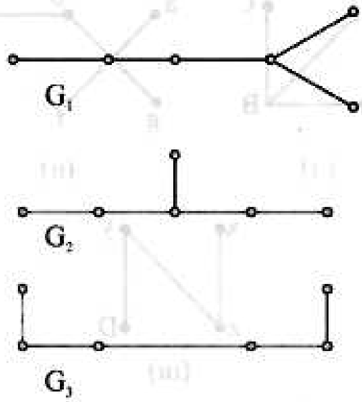
(a) {P, T}

(b) {Q, S}

(c) {S, R}

(d) {P, R}

Q.43 For given graphs G_1, G_2 and G_3.



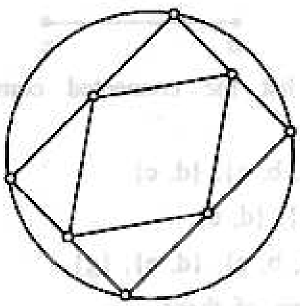
(a) G_1 is not isomorphic to G_2, but G_2 is isomorphic to G_3.

(b) G_1 is isomorphic to G_2 and G_2 is isomorphic to G_3.

(c) G_1 is not isomorphic to G_2 and G_2 is not isomorphic to G_3.

(d) G_1 is isomorphic to G_2, but G_2 is not isomorphic to G_3.

Q.44 The number of fundamental cutsets of the given graph with respect to any of its spanning tree equals



(a) 8

(b) 7

(c) 6

(d) 5

Q.45 The sequence $\{p-d_1, p-d_2, \ldots\ldots, d_p\}$ is graphical if and only if the sequence S is graphical, where

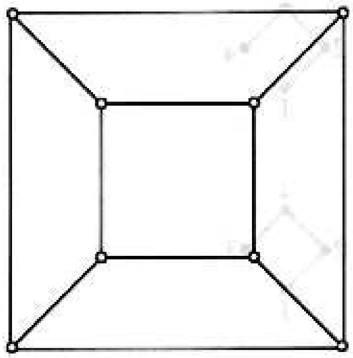
(a) $S = \{p + d_1, p + d_2, \ldots\ldots, p + d_p\}$

(b) $S = \{p - d_1 - 1, p - d_2 - 1, \ldots\ldots, p - d_p - 1\}$

(c) $S = \{p - d_1 + 1, p - d_2 + 1, \ldots\ldots, p - d_p + 1\}$

(d) $S = \{p - d_1, p - d_2, \ldots\ldots, p - d_p\}$

Q.46 If K and λ respectively denote the vertex connectivity and the edge connectivity of the given graph then



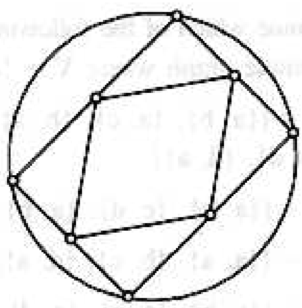
(a) $K = \lambda = 3$

(b) $K = \lambda = 2$

(c) $K = 3,\ \lambda = 2$

(d) $K = 2,\ \lambda = 3$

Q.47 If K and λ respectively denote the vertex connectivity and the edge connectivity of the given graph, then



(a) $K = 4,\ \lambda = 3$

(b) $K = 3,\ \lambda = 4$

(c) $K = \lambda = 4$

(d) $K = \lambda = 3$

Q.48 Let δ and Δ respectively denote the minimum and the maximum degree of a graph whose chromatic number is X. Then

(a) $X \leq 1 + \delta$

(b) $X > 1 + \delta$

(c) $X \leq 1 + \Delta$

(d) $X > 1 + \Delta$

Q.49 The graph whose chromatic polynomial is $\lambda(\lambda-1)^5$, is

(a) K_6

(b) K_5

(c) a tree having 6 vertices

(d) a tree having 5 vertices

Q.50 G(p, q) denotes the graph on p vertices and q edges. Given G_1 (p_1, q_1) and $G_2(p_2, q_2)$, then $G_1 \cup G_2$ has

(a) p_1p_2 vertices and q_1q_2 edges

(b) $p_1 + p_2$ vertices and $q_1 + q_2$ edges

(c) p_1p_2 vertices and $q_1 + q_2$ edges

(d) $p_1 + p_2$ vertices and q_1q_2 edges

Q.51 If $p_1 + p_2$ are the vertices and $q_1 + q_2 + p_1p_2$ are the edges for a graph G, then graph G can be defined by,

(a) $G_1 \cap G_2$

(b) $G_1 \cup G_2$

(c) $G_1 \times G_2$

(d) None of these

Q.52 If the complete graph K_q is p – connected then

(a) $\left\{\frac{q}{2}\right\}$

(b) $p = \left[\frac{q}{2}\right]$

(b) $p > q - 1$

(d) $p \le q - 1$

LEVEL–3

Q.53 Let A = {x | x is a real no and $-10 \le x \le 100$}. Then usual relation < will ______ on A.

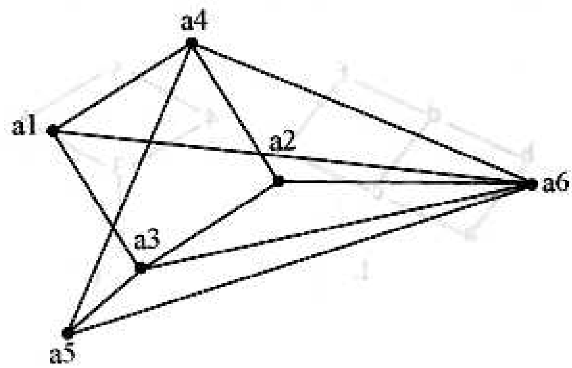
(a) Poset Isomorphism

(b) Quasi order

(c) Dual poset

(d) Partial order

Q.54 Consider the graph given below.



Then the no. of even and odd vertices will be

(a) 5, 1

(b) 4, 2

(c) 2, 4

(d) 3, 2

Q.55 Consider the graph as a polygon with its n edges. Then the sum of the degree of all the vertices of this graph will be ______

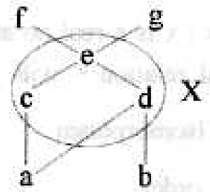
(a) n^2

(b) 2n

(c) $\left(\frac{n+1}{2}\right)$

(d) 2(n + 1)

Q.56 Let V = {a, b, c, d, e, f, g} be ordered as shown below and let X = {c, d, e}, the upper and lower bound of X will be



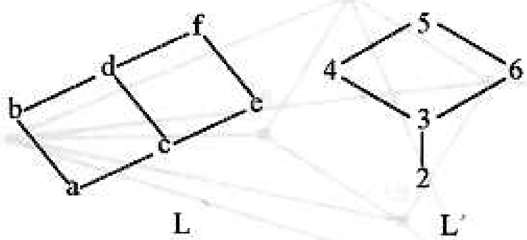
(a) {e, f, g}, {a, b}

(b) {f, g}, {b}

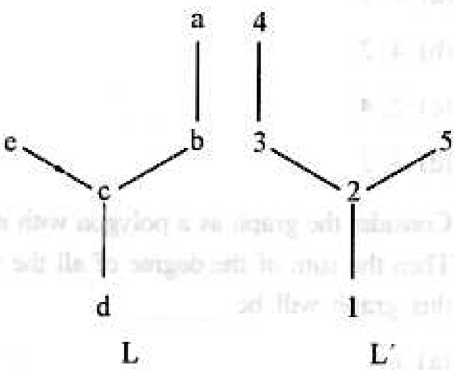
(c) {e, f, g} and {a}

(d) {f, g}, {a, b}

Q.57 Are the lattices L and L' isomorphic?



(i)



(ii)

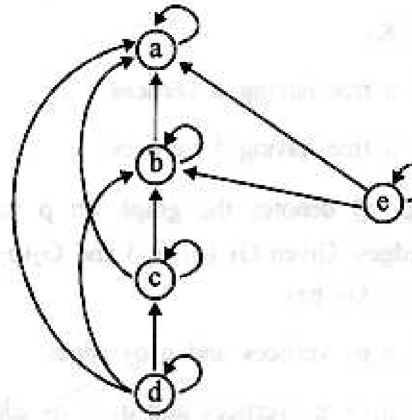
(a) L and L' in (i) and (ii) are not isomorphic.

(b) L and L' in (i) and (ii) are isomorphic.

(c) L and L' in (ii) are isomorphic.

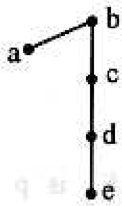
(d) L and L' in (i) are isomorphic.

Q.58 The Hasse diagram of the partial order having the given diagraph will be

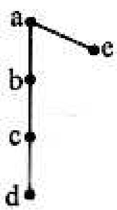


(a)

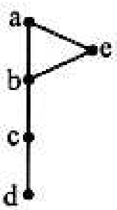
(b)



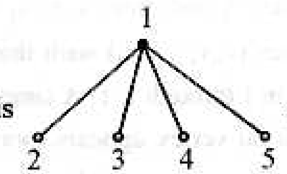
(c)



(d)



Q.59 The matrix of partial order whose Hasse diagram is



will be

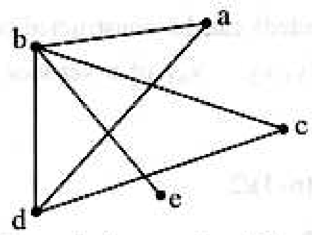
(a) $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

Q.60 Consider the following graph



Determine whether each of the following is a closed path, trail, simple path or cycle:

(i) (b, a, d, c)

(ii) (d, a, b, e)

(iii) (b, d, e, b)

(a) (i) is closed, (ii) cycle, (iii) trail

(b) (i) is cycle, (ii) is simple path, (iii) not a path

(c) (i) is cycle, (ii) is trail, (iii) closed

(d) none of these

GATE QUESTIONS

Q.61 A graph is planar if and only if, **[GATE1990]**

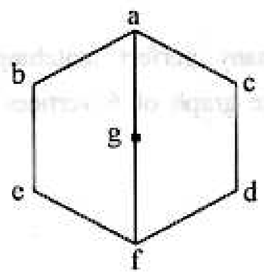
(a) it does not contains subgraphs homeomorphic to K_5 and $K_{3,3}$.

(b) it does not contains subgraphs isomorphic to K_5 or $k_{3,3}$

(c) it does not contains subgraphs isomorphic to K_5 or $K_{3,3}$.

(d) it does not contains subgraph homeomorphic to K_5 or $K_{3,3}$.

Q.62 In the lattice defined by the Hasse diagram given in following figure, how many complements does the element 'e' have? **[GATE 1997]** **[1-Mark]**



(a) 2

(b) 3

(c) 0

(d) 1

Q.63 How many undirected graphs (not necessarily connected) can be constructed out of a given set $V = \{v_1, v_2, \ldots, v_n\}$ of n vertices ? **[GATE 2001]**

[2-Marks]

(a) $n(n-1)/2$

(b) 2^n

(c) $n!$

(d) $2^{n(n-1)/2}$

Q.64 Let G be an arbitrary graph with n nodes and k components. If a vertex is removed from G, the number of components in the resultant graph must necessarily lie between. **[GATE 2003]**

[1-Mark]

(a) k + 1 and n – k

(b) k – 1 and n – 1

(c) k – 1 and k + 1

(d) k and n

Q.65 A Graph G = (V, E) satisfies $|E| \le 3|V| - 6$. The min-degree of G is defined as $\min_{v \in V}\{\text{degree}(v)\}$. Therefore, min-degree of G cannot be

[GATE 2003]

(a) 6

(b) 5

(c) 4

(d) 3

Q.66 How many perfect matchings are there in a complete graph of 6 vertices? **[GATE 2003]**

[2-Marks]

(a) 60

(b) 30

(c) 24

(d) 15

Q.67 Let G = (V,E) be a directed graph with n vertices. A path from v_i to v_j in G is sequence of vertices $(v_i, v_{i+1}, \ldots v_j)$ such that $(v_k, v_{k+1}) \in E$ for all k in i through j–1. A simple path is a path in which no vertex appears more than once. Let A be an n×n array initialized as follow.

$$A[i, j] = \begin{cases} 1 & \text{if } (j,k) \in E \\ 0 & \text{otherwise} \end{cases}$$

Consider the following algorithm.

for i =1 to n

for j = 1 to n

for k = 1 to n

A [j,k] = max (A[j,k](A[j,i]+A[i,k]);

Which of the following statements is necessarily true for all j and k after termination of the above algorithm? **[GATE 2003]**

[2-Marks]

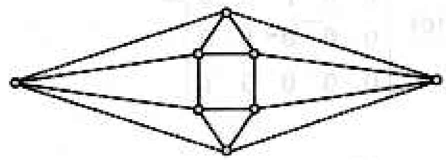
(a) A [j,k]≤n

(b) If A [j,j]≥n–1, then G has a Hamiltonian cycle

(c) If there exists a path from j to k, A [j,k] contains the longest path lens from j to k.

(d) If there exists a pth from j to k, every simple path from j to k contains most A [j,k] edges.

Q.68 The minimum number of colours required to colour the following graph, such that no two adjacent vertices are assigned the same colour, is



[GATE 2004]
[2-Marks]

(a) 5

(b) 4

(c) 3

(d) 2

Q.69 Let $G_1 = (V, E_1)$ and $G_2 = (V, E_2)$ be connected graphs on the same vertex set V with more than two vertices. If $G_1 \cap G_2 = (V, E_1 \cap E_2)$ is not a connected graph, then the graph $G_1 \cup G_2 = (V, E_1 \cup E_2)$ **[GATE 2004]**

[2-Marks]

(a) has chromatic number strictly greater than those of G_1 and G_2
(b) must have a cut-edge (bridge)
(c) must have a cycle
(d) cannot have a cut vertex

Q.70 What is the maximum number of edges in an acyclic undirected graph with n vertices **[IT-GATE 2004]**

[2-Marks]

(a) n–1
(b) n
(c) n+1
(d) 2n–1

Q.71 What is the number of vertices in an undirected connected graph with 27 edges, 6 vertices of degree 2,3 vertices of degree 4 and remaining of degree 3? **[IT-GATE 2004]**

[2-Marks]

(a) 10
(b) 11
(c) 18
(d) 19

Q.72 How many graph on n labeled vertices exist which have at least $(n^2-3n)/2$ edges? **[GATE 2004]**

[2-Marks]

(a) $\sum_{k=0}^{n} {}^{\frac{n^2-n}{2}}C_k$

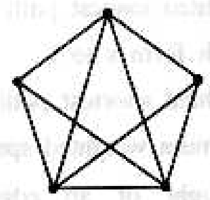
(b) ${}^{(n^2-n)/2}C_n$

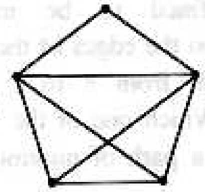
(c) $\sum_{k=0}^{(n^2-3n)/2} {}^{(n^2-n)}C_k$

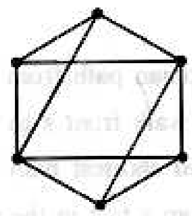
(d) ${}^{(n^2-n)/2}C_{(n^2-3n)/2}$

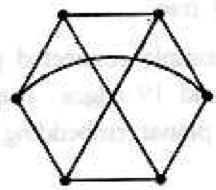
Q.73 Which one of the following graphs is NOT planar? **[GATE 2005]**

[2-Marks]

(a) G_4:


(b) G_3:


(c) G_2:


(d) G_1:


Q.74 Let G be a simple graph will 20 vertices and 100 edges. The size of the minimum vertex cover of G is 8. Then, the size of the maximum independent set of G is. **[GATE 2005]**

[1-Mark]

(a) more than 12
(b) less than 8
(c) 8
(d) 12

Common Data For Questions 75 & 76:

Let s and t be two vertices in an undirected graph G = (V,E) having distinct positive edge weights. Let[X,Y] be a partition of V such that $s \in X$ and $t \in Y$. Consider the edge e having the minimum weight amongst all those edges that have one vertex in X and one vertex in Y.

Q.75 The edge e must definitely belong to:

[GATE 2005]

[2-Marks]

(a) the weighted longest path from s to t

(b) each path form s to t

(c) the weighted shortest path from s to t

(d) the minimum weighted spanning tree of G

Q.76 Let the weight of an edge e denote the congestion on that edge. The congestion on a path is defined to be maximum of the congestions on the edges of the path. We wish to find the path from s to t having minimum congestion. Which one of the following paths is always such a path of minimum congestion?

[GATE 2005]

[2-Marks]

(a) a Hamiltonian path from s to t

(b) an Euler walk from s to t

(c) a weighted shortest path from s to t

(d) a path from s to t in the minimum weighted spanning tree

Q.77 Let G be a simple connected planar graph with 13 vertices and 19 edges. Then, the number of faces in the planar embedding of the graph is

[GATE 2005]

[1-Mark]

(a) 13

(b) 9

(c) 8

(d) 6

Common Data For Questions 78 to 80:

The 2^n vertices of graph G corresponds to all subsets of a set of size n, for $n \geq 6$. Two vertices of G are adjacent if and only if the corresponding sets intersect in exactly two elements.

Q.78 The number of vertices of degree zero in G is

[GATE 2006]

[2-Marks]

(a) 2^n

(b) $n + 1$

(c) n

(d) 1

Q.79 The maximum degree of a vertex in G is

[GATE 2006]

[2-Marks]

(a) 2^{n-1}

(b) $2^{n-3} \times 3$

(c) 2^{n-2}

(d) $({}^{n/2}C_2)2^{n/2}$

Q.80 The number of connected components in G is

[GATE 2006]

[2-Marks]

(a) $\frac{2^n}{n}$

(b) $2^{n/2}$

(c) $n + 2$

(d) n

Q.81 Which of the following graphs has an Eulerian circuit?

[GATE 2007]

[2-Marks]

(a) The complement of a cycle on 25 vertices

(b) A complete graph on 90 vertices

(c) Any k-regular graph where k is an even number

(d) None of these

Q.82 Let G be the non-planar graph with the minimum possible number of edges. Then G has

[GATE 2007]

[1-Mark]

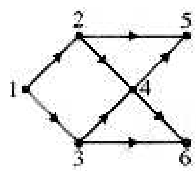
(a) 10 edges and 6 vertices

(b) 10 edges and 5 vertices

(c) 9 edges and 6 vertices

(d) 9 edges and 5 vertices

Q.83 Consider the DAG with V = {1, 2, 3, 4, 5, 6}, shown below



Which of the following is NOT a topological ordering?

[GATE 2007]

[1-Mark]

(a) 3 2 4 1 6 5

(b) 1 3 2 4 5 6

(c) 1 3 2 4 6 5

(d) 1 2 3 4 5 6

Q.84 Which of the following statements is true for every planar graph on n vertices. **[GATE 2008]** **[2-Marks]**

(a) The graph has an independent set of size at least n/3

(b) The graph has a vertex-cover of size at most 3n/4

(c) The graph is Eulerian

(d) The graph is connected

Q.85 What is the chromatic number of an n-vertex simple connected graph which does not contain any odd length cycle? Assume $n \geq 2$. **[GATE 2009]** **[1-Mark]**

(a) n

(b) n – 1

(c) 3

(d) 2

Q.86 Which one of the following is TRUE for any simple connected undirected graph with more than 2 vertices? **[GATE 2009]** **[1-Mark]**

(a) All vertices have the same degree

(b) At least three vertices have the same degree

(c) At least two vertices have the same degree

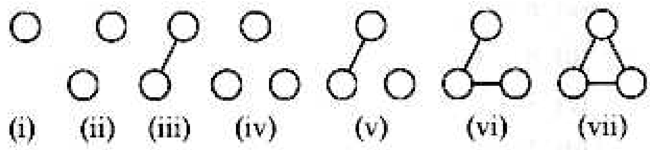
(d) No two vertices have the same degree

ANSWER KEY

1	b	2	c	3	b	4	b, c	5	c
6	d	7	b	8	d	9	d	10	c
11	b	12	d	13	c	14	c	15	d
16	d	17	c	18	c	19	b	20	b
21	a	22	c	23	a	24	d	25	c
26	b	27	c	28	d	29	d	30	b
31	b	32	b	33	a	34	b	35	c
36	b	37	b	38	a	39	d	40	b
41	c	42	a	43	c	44	b	45	b
46	a	47	c	48	c	49	c	50	b
51	c	52	d	53	b	54	c	55	b
56	c	57	c	58	d	59	d	60	b
61	d	62	b	63	d	64	b	65	a
66	d	67	a	68	b	69	b	70	a
71	a	72	a	73	d	74	a	75	d
76	d	77	c	78	b	79	b	80	c
81	c	82	c	83	a	84	b	85	d
86	c								

SOLUTIONS

S.2 (c)



S.34 (b)

30, undetermined.

If G is a graph, then there are ${}^6C_2 = 6 \times 5 = 30$ ways of choosing two vertices from V, hence n = 30

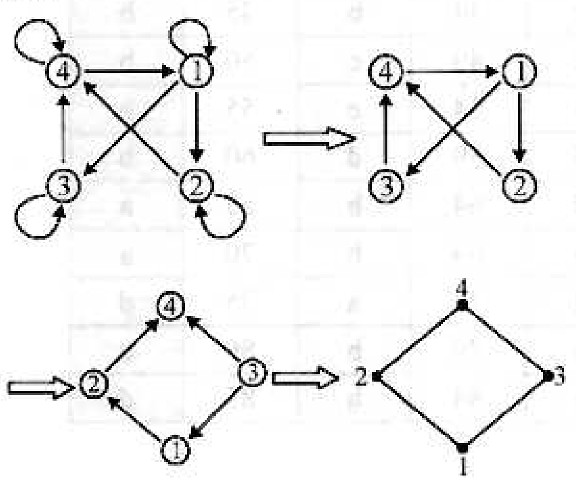
If G is a multigraph, then multiple edges are allowed, so G can have any number of edges and loops, finite or infinite hence there is no way to determine it.

S.35 (c)

A linearly ordered subset is one in which every pair of elements is comparable. Any set with one element is linearly ordered and any pair of comparable elements is linearly ordered

Assume that a and b are distinct maximal elements of S. Since all elements of S are comparable, either a < b or b< a. But this implies that a=b by the definition of a maximal elements, hence we contradict the assumption that a and b are distinct.

S.37 (b)



S.38 (a)

A multigraph G(V, E) is a graph if it has neither multiple edges nor loops.

(i) No, since {a, b}, {b, a} are multiple edges and {d, d} is loop.

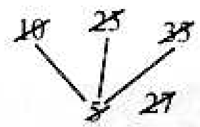
(ii) No, since {a, b} and {a, b} are multiple edges

(iii) No, since {a, a} is a loop

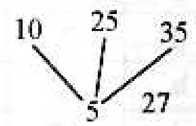
(iv) Yes, since there is no multiple edge or loop.

S.39 (d)

An element a in S is said to be maximal if no other element succeeds a, i.e. if $a \le x$ implies $a = x$, similarly an element b in a poset S is said to be minimal if no other element proceeds b i.e., if $y \le b$ implies $y = b$.



There can be more than one maximal or more than one minimal element



10, 25, 35 is divisible by 5, 27 is not divisible by any one of these number.

$\therefore$ **5 is minimal.**

10, 25, 35 are maximal.

S.40 (b)

(i) Yes, it is a connected graph since there is a path between any two vertices of the graph.

(ii) No, it is not connected graph since g, b, c, e are connected, and a and f are connected, but there is no path from g, b, c, e to either a and f.

(iii) Yes, there is a path between any two vertices of the graph.

S.41 (c)

A component of G is a subgraph of G which is not connected subgraph of G. A connected component is the full subgraph spanned by its vertices so connected component is designated by listing its vertices. G can be partitioned into its connected components.

S.42 (a)

G – e is the graph obtained by simply deleting e from the edge set of G

Thus V(G, e) = V(G) and E(G, e) = E(G)/{e}.

In the given graph only **{P, T}** disconnects G and hence **{P, T}** is the only bridge of G.

S.53 (b)

A relation 'R' on 'A' is called quasi order if it is transitive and irreflexive.

Let A = {x | x is a real no. and $-10 \le x \le 100$}

Irreflexive : x is a real no.

$\therefore$ x not less than x $\forall$ x $\in$ A

eg: 10 not less than 10

$\therefore$ < is irreflexive.

Transitive : Let $x < y$, $y < z$

$\therefore$ $x < z$

$\therefore$ x '<' is transitive

$\therefore$ **< is Quasi order.**

For Partial order relation set must be reflexive, antisymmetric and Transitive.

For poset isomorphism we need two posets.

Dual poset (A, R) is poset, then (A, R^{-1}) is also a poset which is dual poset.

S.54 (c)

The degree of a vertex V in a graph G is equal to the number of edges which are incident on V i.e. the number of edges which contain V as an end point. The vertex V is even or odd according as deg(V) is even or odd.

Consider a_1 has 3 edges

$\therefore$ deg(V) = 3

$\therefore$ a_1 is odd vertex

a_2 has 3 edges

$\therefore$ deg(V) = 3

$\therefore$ a_2 is odd vertex

a_3 has 4 edges

$\therefore$ deg(V) = 4

$\therefore$ a_3 is even vertex

a_4 has 4 edges

$\therefore$ deg(V) = 4

$\therefore$ a_4 is even vertex

a_5 has 3 edges

$\therefore$ deg(V) = 3

$\therefore$ a_5 is odd vertex

a_6 has 5 edges

$\therefore$ deg(V) = 5

$\therefore$ a_6 is even vertex

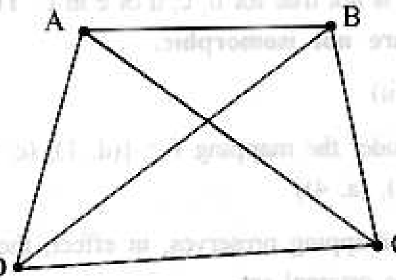
$\therefore$ **No. of odd vertices = 4**

$\therefore$ **No. of even vertices = 2**

S.55 (b)

Theorem:

The sum of the degree of the vertices of a graph is equal to twice the number of edges. Consider the example of a quadrilateral with its two diagonals.



No. of edges = 6

$\therefore$ sum of the degrees of the vertices = 2 × no. of edges

= 2 × 6 = 12

$\therefore$ **For polygon with n edges the sum of the**

degrees of the vertices = 2 × n.

Note : The above theorem holds for a multigraph also, in multigraph a loop must be counted twice towards the degree of its end point.

S.56 (c)

An element M in S is called an upper bound of A if M succeeds every element of A. i.e. M is an upper bound of A, if, for every x in A, we have $x \le M$.

The element e, f and g succeed every element of X. **Hence e, f, g are the upper bounds of X.**

An element m in S is called a lower bound of A if m proceeds every element of A i.e. m is a lower bound of A if, for every x in A, $m \le x$

The element a proceeds every element of X.

Hence **a is the lower bound of X.**

Note: b is not a lower bound since b does not proceeds c, b and c are not comparable.

S.57 (c)

Two lattices L and L' are said to be isomorphic if there is a bijective function $f : L \to L'$ such that $f(a \wedge b) = f(a) \wedge f(b)$ and $f(a \vee b) = f(a) \vee f(b)$. Consider the fig. (i) (for any elements a, b in L.)

a must map to 1

f must map to 5

Then in $f(2, x) = 2$ for $x \ne 1$ in L'.

This is not true for b, c, d or e in L. **Thus L and L' are not isomorphic.**

fig. (ii)

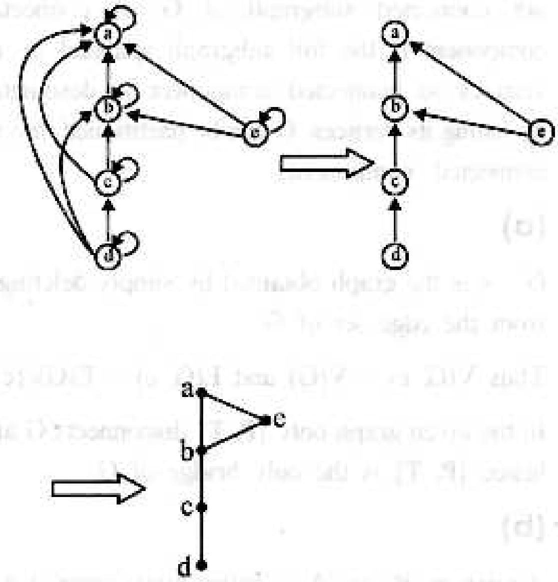
Consider the mapping f = {(d, 1), (c, 2), (e, 5), (b, 3), (a, 4)}

This mapping preserves, in effect, the structure of the original set.

For example $f(c \wedge b) = f(c) = 2$ and $f(c) \wedge f(b) = 3 \wedge 5 = 2$

So, L and L' are isomorphic.

S.58 (d)



S.59 (d)

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

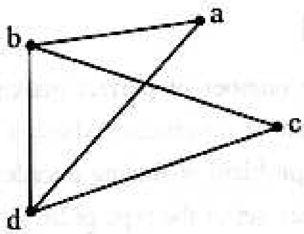
Poset: Let A be a set and let R be a relation defined on A. If R is reflexive, antisymmetric and transitive then R is called as partial order relation or a partial order and the set A with the partial order R i.e. (A, R) is called a partially ordered set or poset.

When the relation is a partial order, its diagraph can be simplified and such simplified graph of a partial order is called Hasse diagram. When the partial order is a total order its Hasse diagram is a straight line and the corresponding poset is called a chain. For Representation of matrix of partial order. Represent each relation by 1 if there is a connection between them, otherwise it will be zero.

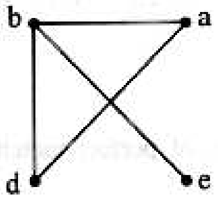
S.60 (b)

Trail : A path $\alpha = (V_0, V_1, \ldots, V_n)$ is simple if all the vertices are distinct. The path is trail if all the edges are distinct.

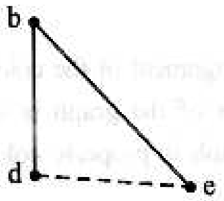
(i) This path is **cycle**. Since it is closed and has distinct vertices



(ii) This path is **simple** since its vertices are distinct. It is not a cycle since it is not closed.



(iii) This is **not even a path** since (d, e) is not an edge.



S.61 (d)

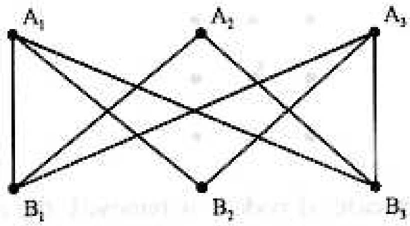
Let 'G' be a corrected planar graph with 'p' vertices & 'q' edges, where $p \geq 3$.

Then $q \geq 3p - 6$.

Suppose we consider Kuratowski's Theorem.

Nonplanar Graphs, Kuratowski's Theorem:

We give two examples of nonplanar graphs. Consider first the utility graph; that is, three houses A_1, A_2, A_3 are to be connected to outlets for water, gas and electricity. B_1, B_2, B_3 as in the Figure(a). Observe that this is the graph $K_{3,3}$ and it has $p = 6$ vertices and $q = 9$ edges. Suppose the graph is planar. By Euler's formula a planar representation has $r = 5$ regions. Observe that no three vertices are connected to each other; hence the degree of each region must be 4 or more and so the sum of the degrees of the regions must be 20 or more. The graph must have 10 or more edges. This contradicts the fact that the graph has $q = 9$ edges. Thus the utility graph $K_{3,3}$ is nonplanar.



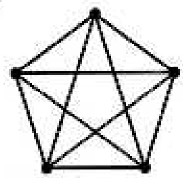


Fig. (a) : $K_{3,3}$ Fig. (b) : K_5

Consider next the star graph in Figure(b). This is the complete graph K_5 on $p = 5$ vertices and has $q = 10$ edges. If the graph is planar, then

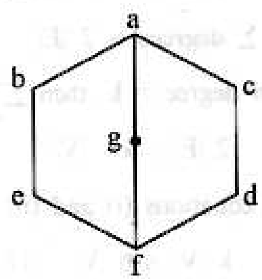
$$10 = q \leq 3p - 6 = 15 - 6 = 9$$

which is impossible. Thus K_5 is nonplanar.

A graph is planar if and only if it does not contains subgraph homeomorphic to $K_{3,3}$ or K_5.

S.62 (b)

It follows that:



$e \vee d = a,\ e \wedge d = f \Rightarrow e' = d$

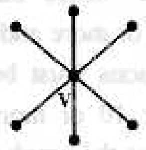
$e \vee c = a,\ e \wedge c = f \Rightarrow e' = c$

$e \vee g = a,\ e \wedge g = f \Rightarrow e' = g$

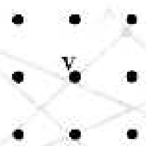
Hence 'e' has 3 components.

S.64 (b)

Maximum components will result after removal of a node, if graph G is a star graph as shown below



or a null graph of n vertices as shown below:



In either case, if node v is removed, the number of components will be n – 1, where n is the total number of nodes in the star graph.

∴ n – 1 is the maximum number of components possible. Minimum components will result if the node being removed is a lone vertex in which case, the number of components will be k – 1.

∴ **The number of components must necessarily lie between k – 1 and n – 1.**

S.65 (a)

Given $|E| \le 3\ |V| - 6$

$\therefore \quad 2\ |E| \le 6\ |V| - 12$ .....(i)

Now Σ degrees = 2 |E|

If min degree = k, then Σ degrees $\ge$ k |V|

i.e. $\quad 2\ |E| \ge k\ .\ |V|$ .....(ii)

From equations (i) and (ii) we can say that

$k\ |V| \le 6\ |V| - 12$

Now substitute k = 3, 4, 5, 6 in order

$k = 3 \Rightarrow \quad 3\ |V| \le 6\ |V| - 12$

$\Rightarrow \quad |V| \ge 4$ which is possible

$k = 4 \Rightarrow \quad 4\ |V| \le 6\ |V| - 12$

$\Rightarrow \quad |V| \ge 6$ which is possible

$k = 5 \Rightarrow \quad 5\ |V| \le 6\ |V| - 12$

$\Rightarrow \quad |V| \ge 12$ which is possible

$k = 6 \Rightarrow \quad 6\ |V| \le 6\ |V| - 12$

which is not possible

∴ **minimum degree cannot be 6.**

S.66 (d)

The number of perfect matchings in a complete graph of n vertices, where n is even, reduces to the problem of finding unordered partitions of the vertex set of the type p(2n; 2, 2, 2, ..., n times)

$$= \frac{(2n)!}{(2!)^n n!}$$

For n = 3, 2n = 6, i.e. complete graph K_6, we have

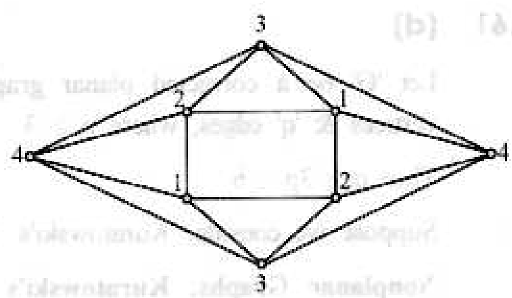
$$\text{Number of perfect matchings} = \frac{6!}{(2!)^3 3!}$$

$$= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 2 \times 2 \times 6}$$

$$= 15$$

S.68 (b)

An assignment of the colors 1, 2, 3 and 4 to the vertices of the graph is shown below such that the graph is properly colored.



So **4 colours are required.**

S.69 (b)

$G_1 = (V, E_1)$

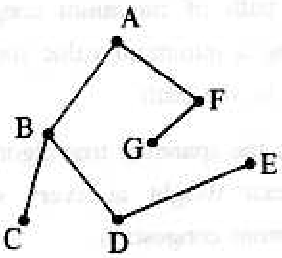
$G_2 = (V, E_2)$

$G_1 \cap G_2 = (V, E_1 \cap E_2)$ is not connected

then $G_1 \cup G_2 = (V, E_1 \cup E_2)$ **must have cut-edge** because there exists a partition of E into subset E_1 and E_2 for any edge $a_1 \in E_1$ and $a_2 \in E_2$.

S.70 (a)

Consider the following acyclic graph:



Here total number of edges = 6, and

total number of vertices = 7.

$\therefore$ Vertices = edges + 1

$\therefore$ **The maximum number of edges in an acyclic graph with n vertices is n – 1.**

S.71 (a)

Total no. of edges = 27

We have 6 vertices of degree 2 = 12 edges

3 vertices of degree 4 = 12 edges

$\therefore$ 12 + 12 = 24 edges

$\therefore$ 27(total) – 24 = 3

It is given that the remaining vertices have 3 edges and only 3 edges are left, therefore only 1 vertex is possible.

$\therefore$ **Total no. of vertices = 6 + 3 + 1 = 10**

S.72 (a)

Take number of edges available in n labelled vertices is

$${}^nC_2 = \frac{n(n-1)}{2} = \frac{n^2-n}{2} \text{ edges}$$

Now from this we need to choose $\frac{n^2-3n}{2}$ edges or more upto a maximum of $\frac{n^2-n}{2}$ edges.

Each such choice of edges represents a distinct graph on n labelled vertices.

Total number of such graphs

$$= {}^{\frac{n^2-n}{2}}C_{\frac{n^2-3n}{2}} + {}^{\frac{n^2-n}{2}}C_{\frac{n^2-3n}{2}+1} + {}^{\frac{n^2-n}{2}}C_{\frac{n^2-3n}{2}+2} + \dots + {}^{\frac{n^2-n}{2}}C_{\frac{n^2-n}{2}}$$

Now since ${}^nC_r = {}^nC_{n-r}$

$${}^{\frac{n^2-n}{2}}C_{\frac{n^2-3n}{2}} = {}^{\frac{n^2-n}{2}}C_{\left[\frac{n^2-n}{2}\right]-\left[\frac{n^2-3n}{2}\right]}$$

$$= {}^{\frac{n^2-n}{2}}C_n$$

Similarly,

$${}^{\frac{n^2-n}{2}}C_{\left[\frac{n^2-3n}{2}+1\right]} = {}^{\frac{n^2-n}{2}}C_{n-1}$$

and so on until,

$${}^{\frac{n^2-n}{2}}C_{\frac{n^2-n}{2}} = {}^{\frac{n^2-n}{2}}C_0$$

So the required summation reduces to

$${}^{\frac{n^2-n}{2}}C_n + {}^{\frac{n^2-n}{2}}C_{n-1} + {}^{\frac{n^2-n}{2}}C_{n-2} + \ldots + {}^{\frac{n^2-n}{2}}C_0$$

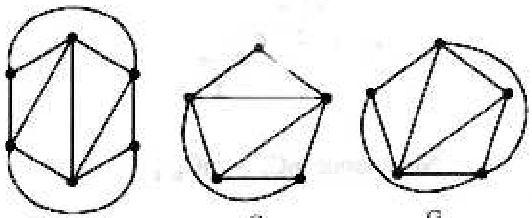
$$= {}^{\frac{n^2-n}{2}}C_0 + {}^{\frac{n^2-n}{2}}C_1 + \ldots + {}^{\frac{n^2-n}{2}}C_n$$

$$= \sum_{k=0}^{n} {}^{\frac{n^2-n}{2}}C_k$$

S.73 (d)

G_1 is same $K_{3,3}$ which is known to be non planar

G_2, G_3 and G_4 can be redrawn as follows so that they are planar.



G_2 G_3 G_4

S.74 (a)

Given $V = 20$, $E = 100$

maximum vertex cover $= \propto_0 = 8$

maximum edge cover $= \propto_1$

Let β_0 denotes the maximum vertex independent set and β_1 denotes the maximum edge independent set then for any graph G.

$\propto_0 \leq V - \beta_0$

$\beta_0 \geq V - \propto_0$

$\beta_0 \geq 20 - 8$

$\beta_0 \geq \mathbf{12}$

S.75 (d)

Given $G = (V, E)$ be undirected positive edge weighted graph. $[X, Y]$ be the partition of vertices V such that $s \in X$ and $t \in Y$. In the given problem we partition the vertex set of the graph let $X = \{V_1, V_2, \ldots\ldots, V_i\}$ and $Y = \{V_{i+1}, V_{i+2}, \ldots\ldots, V_n\}$

If e does not belong to minimum wt spanning tree, then we can remove one of the edges other than e which must span X and Y and replace it with e, therefore resulting in a more minimum weight spanning tree, which is a contradiction. Therefore, **e must belong to a minimum weight spanning tree of G.**

S.76 (d)

The path of minimum congestion is the path having a minimum value for the biggest weight edge in the path.

Since the spanning tree algorithm includes edges of least weight at every stage, it will have minimum congestion.

S.77 (c)

Given $V = 13$, $E = 19$

Let R be the number of regions.

Apply Euler's Formula, (Here faces and regions mean one and the same.)

$$R = E - V + 2$$

or $$\mathbf{R = 19 - 13 + 2 = 8}$$

S.78 (b)

Let S contain n elements then S have 2^n subsets

Graph G contain 2^n vertices.

Let $S = \{v_1, v_2, \ldots\ldots\ldots, v_n\}$

Two vertices of G are adjacent if and only if the corresponding sets intersect in exactly two elements.

So $|\{V_i\} \cap \{V_j\}| = 2$

For this to happen, the subset must have at least 2 elements.

There are n sets which contains a single element for V_1 to V_n who doesn't intersect another set such that it contains two elements.

Therefore the degree of all these n vertices is zero. G also contains a vertex ϕ whose degree is zero. **So number of vertices whose degree is zero is n + 1.**

S.79 (b)

Let the set be S = {1, 2, 3, 4, ... n}

Consider a subset containing 2 elements of the form {1, 2}

Now {1, 2} will be adjacent to any subset with which it has exactly 2 elements in common. These sets can be formed by adding zero or more elements from remaining n – 2 elements, to the set {1, 2}. Since each of these elements may be either added or not added, number of ways of making such sets containing 1 and 2 is 2^{n-2}.

$\therefore$ Vertices with 2 elements will have 2^{n-2} degrees.

Now consider subsets of 3 elements say {1, 2, 3}

Since we want exactly 2 elements in common, we choose these in ${}^3C_2$ ways and then we can add or not add remaining n–3 elements. This can be done in 2^{n-3} ways.

$\therefore$ Total number of subsets with at least 2 elements common with {1, 2, 3} is given by ${}^3C_2 \times 2^{n-3}$.

Similarly, we can argue that the number of degrees of 4 element subsets is ${}^4C_2 \times 2^{n-4}$ and for 5 element subsets is ${}^5C_2 \times 2^{n-5}$ and so on.

Out of these

$2^{n-2} = 2.2^{n-3}$ is less than ${}^3C_2 \times 2^{n-3} = 3 \times 2^{n-3}$

Then ${}^3C_2 \times 2^{n-3} = 3 \times 2^{n-3}$ is same as

$$4C_2 \times 2^{n-4} = 6 \times 2^{n-4} = 3 \times 2^{n-3}$$

and ${}^4C_2 \times 2^{n-4} = 3 \times 2^{n-3}$ is greater than

$${}^5C_2 \times 2^{n-5} = 10 \times 2^{n-5} = 2.5 \times 2^{n-3}$$

$\therefore$ **maximum degrees** in this graph is occurring for 3 elements and 4 element subsets both of which have **$3 \times 2^{n-3}$ degrees which is the correct answer.**

S.80 (c)

The number of connected component of G is determined by the degree and edges of vertices. There are n + 1 vertices whose degree is zero, so they can form n + 1 connected component. The remaining vertices of the graph G are all connected as a single component. **So total number of connected component is n + 2.**

S.81 (c)

Whenever in a graph all vertices have even degrees, it will surely have an Euler circuit. Since in **a k-regular graph**, every vertex has exactly k degrees and if k is even, every vertex in the graph has **even degrees,** such a graph must have an Euler circuit.

S.82 (c)

K_5 and $K_{3,3}$ are the smallest non planar graphs.

Out of this K_5 has 5 vertices and ${}^5C_2 = 10$ edges and $K_{3,3}$ has 6 vertices and $3 \times 3 = 9$ edges. So, the non planar graph with **minimum number of edges is $K_{3,3}$ with 9 edges and 6 vertices.**

Note : K_5 is the non planar graph with minimum number of vertices.

S.83 (a)

In topological sorting the partial ordering of the DAG must be preserved i.e. if a ≤ b in the DAG then in the topological order, b must come after a, not before. Consider the ordering 3 2 4 1 6 5. 1 ≤ 4 in the given DAG but 4 is coming before 1 in 3 2 4 1 6 5 order which means that **3 2 4 1 6 5 is not a topological order of the given DAG.**

S.84 (b)

Every planar graph with n vertices **has a vertex cover of size at most 3n/4.**

S.85 (d)

If a n-vertex simple connected graph contains no cycles of odd length, then its chromatic number is two, since the vertices can be alternately colored with the first color, then the second color, then the first color & then the second colour and so on.

Alternatively, since a simple connected graph with no cycles of odd length must be bipartite, and since the **chromatic number of a bipartite graph is always 2** (in a bipartite graph each partition requires one color, there are no edges within a partition of a bipartite graph) and there are only two partitions.

S.86 (c)

In a simple connected undirected graph (with more than two vertices), **at least 2 vertices must have same degree,** since if this is not true, then all vertices would have different degrees, A graph with all vertices having different degrees is not possible to construct. Notice that it is possible to construct graphs satisfying choices a, c & d.

○ ○ ○

12.6

LINEAR ALGEBRA

LEVEL–1

Q.1 $\nabla \times \nabla \times P$, where P is a vector is equal to
(a) $\nabla(\nabla \cdot P) - \nabla^2 P$
(b) $\nabla^2 P + \nabla \times P$
(c) $\nabla^2 P + \nabla(\nabla \times P)$
(d) $P \times \nabla \times P - \nabla^2 P$

Q.2 The rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ is
(a) 3
(b) 2
(c) 1
(d) 0

Q.3 If A, B, C are square matrices of the same order, then $(ABC)^{-1}$ is equal to
(a) $C^{-1}A^{-1}B^{-1}$
(b) $A^{-1}C^{-1}B^{-1}$
(c) $A^{-1}B^{-1}C^{-1}$
(d) $C^{-1}B^{-1}A^{-1}$

Q.4 If A is a 3-rowed square matrix, such that $|A| = 3$ then $|adj(adj\ A)|$ is equal to
(a) $|A|^2$
(b) $|A|^4$
(c) $|A|^3$
(d) $|A|^6$

Q.5 A square matrix each of whose diagonal elements are '1' and non–diagonal matrix are '0' is called
(a) Null matrix
(b) row matrix
(c) Skey symmetric matrix
(d) Identity matrix

Q.6 Matrix multiplication is associative if ______ condition satisfied
(a) Matrices should be non–negative
(b) Matrices should be identical matrices
(c) Matrices should be conformable
(d) All of these

Q.7 If A and B are two matrices such that AB and A + B are both defined then A, B are
(a) cannot predict
(b) matrices of different order
(c) square matrices of same order
(d) scalar matrices of same order

Q.8 If A is non–zero column matrix and B is a non–zero row matrix then rank of matrix AB is
(a) does not depends on rows & columns
(b) always = No. of elements in column
(c) always = No. of elements in row
(d) always 1

Q.9 Rank of matrix which is in echelon form is equal to

(a) Can't say

(b) It is independent of No. of rows and columns

(c) No. of columns in the matrix

(d) No. of rows in the matrix

Q.10 Cayley Hamilton theorem is

(a) Every square matrix satisfies it's own characteristic equation

(b) A matrix can be expressed as sum of symmetric and skew-symmetric matrices

(c) Inverse of a matrix exists if it is singular

(d) None of these

Q.11 Inverse of a matrix

(a) is unique

(b) is not unique

(c) exist of matrix is singular

(d) $\frac{|A|}{\text{adj}A}$

Q.12 If $P=\begin{bmatrix} p & q \\ -q & p \end{bmatrix}$, $Q=\begin{bmatrix} r & s \\ -s & r \end{bmatrix}$, then PQ is equal to

(a) $\begin{bmatrix} pq-rs & pr-qs \\ qr+ps & -qs+pr \end{bmatrix}$

(b) $\begin{bmatrix} pr-qs & ps+qr \\ -qr-ps & -qs+pr \end{bmatrix}$

(c) $\begin{bmatrix} pq+rs & -pr+qs \\ -qp-rs & qs-pr \end{bmatrix}$

(d) $\begin{bmatrix} pr+qs & ps-qr \\ qr+ps & qs-pr \end{bmatrix}$

Q.13 If for a matrix A, we have $AA^2 = I$, then which is wrong?

(a) $|A| = 0$

(b) A is orthogonal

(c) A^{-1} exists

(d) A is a square matrix

Q.14 Eigen value of inverse of matrix is

(a) Inverse of the matrix values

(b) Negative of matrix values

(c) No any relation between then

(d) Same as the matrix

Q.15 Rank 'n' of non-zero matrix

(a) may be $n \geq 1$

(b) may be $n > 1$

(c) may be $n = 1$

(d) may be $n = 0$

Q.16 For what value of λ do the equations $x + 2y = 1$, $3x + \lambda y = 3$ have unique solution

(a) $\lambda = 5$

(b) $\lambda \neq 5$

(c) $\lambda = 6$

(d) $\lambda \neq 6$

Q.17 Matrix A and B are square matrices of order n. Then B is said to be similar to A if there exists a non-singular matrix P such that:

(a) $B = PAP^{-1}$

(b) $B = P^{-1}AP$

(c) $P'AP$

(d) $B = PAP'$

Q.18 A square matirx U is called unitary if

(a) $U = U^{-1}$

(b) $U = -U^{-1}$

(c) $U' = U^{-1}$

(d) $U' = -U^{-1}$

Q.19 For $A=\begin{bmatrix} 0 & 2 & 3 \\ -1 & 6 & 5 \\ -3 & -5 & 0 \end{bmatrix}$ matrix which of following statement is false

(a) A is square matrix

(b) A is symmetric matrix

(c) A is skew symmetric matrix

(d) $|A| \neq 0$

Q.20 If the matrix AB = 0, then we must have

(a) Both A & B should zero

(b) At least one of A & B should be zero

(c) A + B = 0

(d) Neither A nor B may be zero

Q.21 If A & B are arbitrary square matrices of the same order, then

(a) $(AB)' = B' A'$

(b) $(A') (B')= B' A'$

(c) $(A + B)' = A' - B'$

(d) $(AB)' = A'B'$

Q.22 Transpose of the product of two matrices is

(a) product of matrices in matrices in same order

(b) product of transpose of matrices in reverse order

(c) product of transposes of matrices in same order

(d) product of matrices in same order

Q.23 Rank of a matrix A is said to be 'r' if

(a) atleast one square matrix of A or order 'r' whose determinant is not equal to zero.

(b) the matrix A contains any square sub-matrix of order r + 1, then the determinant of every square sub matrix of order r + 1 should be zero.

(c) both (a) and (b) true

(d) only (b) true

Q.24 The equations are said to be inconsistent if

(a) the solution is zero vector

(b) the equations have no solution

(c) there are infinite number of solutions

(d) solution is possible is in the form of complex number

Q.25 The matrix A is said to be skew hermitial if

(a) $A' = \overline{A}$

(b) $A' = A^{-1}$

(c) $A' = -\overline{A}$

(d) $A' = \text{Adj } A$

Q.26 If I_n is the identity matrix of order n, then $(I_n)^{-1}$

(a) does not exist

(b) $= nI_n$

(c) $= 0$

(d) $= I_n$

Q.27 If A′ is the transpose of a square matrix A, then

(a) $|A'| = |A|$

(b) $|A'| \neq |A|$

(c) $|A| + |A'| = 0$

(d) $|A'| = |A|$ only when A is symmetric

Q.28 Eigen values of the matrix $A = \begin{bmatrix} a & h & g \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ are

(a) 0, a, b

(b) a, b, c

(c) b, a, g

(d) h, b, g

Q.29 If A is a non-singular matrix & A′ is the transpose of A then

(a) $|A| = |A'|$

(b) $|A' . A| \neq |A'|^2$

(c) $|A| + |A'| \neq 0$

(d) $|A . A'| \neq |A^2|$

Q.30 For what values of x, the matrix $\begin{bmatrix} 3-x & 2 & 2 \\ 2 & 4-x & 1 \\ -2 & -4 & -1-x \end{bmatrix}$ is singular

(a) 0, 3

(b) 0, –3

(c) –1,4

(d) 3, 4

Q.31 Evaluate

$$\Delta = \begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$$ where ω is one of the imaginary cube root of unity

(a) 0
(b) 1
(c) 2
(d) None of these

Q.32 If $A = [x\ y\ z]$, $B = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$, $C = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Then
(a) ABC is not possible
(b) $[ax^2 + by^2 + cz^2 + 2hxy + 2gzx + 2fyz]$
(c) $[a^2x + b^2y + c^2z + 2abc + 2xyz + 2fgh]$
(d) $\begin{bmatrix} ax^2 + byz + fcz^2 \\ 2hxy + 2gzx + 2fyz \end{bmatrix}$

Q.33 The inverse of a matrix $\begin{bmatrix} 8 & 7 & 9 \\ 5 & 10 & 15 \\ 1 & 2 & 3 \end{bmatrix}$ is

(a) $\dfrac{1}{2}\begin{bmatrix} 15 & 7 & 8 \\ 10 & 5 & 3 \\ 5 & 2 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} -15 & 7 & 8 \\ -10 & -5 & 3 \\ 5 & 2 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} -15 & 7 & 8 \\ 10 & -5 & 3 \\ 5 & 2 & 1 \end{bmatrix}$

(d) none of these

Q.34 If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$ & $A^2 = kA + 14I$, then k = ...

(a) 3
(b) 5
(c) 1
(d) –5

Q.35 Characteristic root of matrix $A = \begin{bmatrix} 1 & 2 & 5 \\ 0 & 3 & 5 \\ 0 & 0 & 2 \end{bmatrix}$ are

(a) –1, –3, –2
(b) 1, 3, 2
(c) –1, –2, 3
(d) –3, 1, 2

Q.36 The eigen vectors of the matrix

$$\begin{bmatrix} 0 & 0 & \alpha \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \alpha \neq 0 \text{ is (are)}$$

(i) $(a, 0, \alpha)$ (ii) $(\alpha, 0, 0)$
(iii) $(0, 1, 1)$ (iv) $(0, \alpha, 0)$

(a) (i), (ii)
(b) (iii), (iv)
(c) (ii), (iv)
(d) (i), (iii)

Q.37 The rank of the following $(n + 1) \times (n + 1)$ matrix, where a is a real number is

$$\begin{bmatrix} 1 & a & a^2 & \cdots & a^n \\ 1 & a & a^2 & \cdots & a^n \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & a & a^2 & \cdots & a^n \end{bmatrix}$$

(a) 1
(b) 2
(c) n
(d) depends on the value of a

Q.38 The quardratic from of the symmetrical matrix diagram $[\lambda_1, \lambda_2, ..., \lambda_n]$ is

(a) $\lambda_1 + \lambda_2 + ... + \lambda_n$

(b) $\lambda_1^2 x_1 + \lambda_2^2 x_2 + ... + \lambda_n^2 x_n$

(c) $\lambda_1 x_1^2 + \lambda_2 x_2^2 + ... + \lambda_n x_n^2$

(d) $\lambda_1 x_1 + \lambda_2 x_2 + ... + \lambda_n x_n$

Q.39 A and B are idempotent then AB is also idempotent if

(a) $A = B^T$

(b) A and B are non-singular

(c) AB is symmetric

(d) AB = BA

Q.40 The value of the determinant $\begin{vmatrix} a & -a & -a \\ b & b & -b \\ c & -c & -c \end{vmatrix}$ is

(a) ab + bc + ca

(b) a + b + c

(c) –abc

(d) 0

Q.41 The determinant $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$ is divisible by

(a) b – c

(b) bc

(c) a + b

(d) a/c

LEVEL–2

Q.42 All the four entries of the 2 × 2 matrix $P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$ are nonzero, and one of its eigenvalues is zero. Which of the following statements is true?

(a) $P_{11}P_{22} + P_{12}P_{21} = 0$

(b) $P_{11}P_{22} - P_{12}P_{21} = 0$

(c) $P_{11}P_{22} - P_{12}P_{21} = -1$

(d) $P_{11}P_{22} - P_{12}P_{21} = 1$

Q.43 For matrix $\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$ the eigen value corresponding to the eigen vector $\begin{bmatrix} 101 \\ 101 \end{bmatrix}$ is

(a) 8

(b) 6

(c) 4

(d) 2

Q.44 Let, $A = \begin{bmatrix} 2 & -0.1 \\ 0 & 3 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} \frac{1}{2} & a \\ 0 & b \end{bmatrix}$.

Then (a + b) = ?

(a) 11/20

(b) 19/60

(c) 3/20

(d) 7/20

Q.45 Eigen values of the matrix $\begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix}$ are

(a) 1, 2, 4

(b) 1, 4, 4

(c) 1, 1, 1

(d) 1, 1, 2

Q.46 If the product of matrix

$$A = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$$

is a null matrix, then θ and ϕ differ by

(a) an odd multiple of $\pi/2$

(b) an even multiple of π

(c) an odd multiple of π

(d) an even multiple of $\pi/2$

Q.47 Inverse of the matrix $\begin{bmatrix} ab & 0 \\ 0 & 1 \end{bmatrix}$ is

(a) $\begin{bmatrix} a & 0 \\ 0 & 1/b \end{bmatrix}$

(b) $\begin{bmatrix} b & 0 \\ a & b \end{bmatrix}$

(c) $\begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$

(d) $\begin{bmatrix} 1/a & 0 \\ 0 & 1/b \end{bmatrix}$

Q.48 The sum of the eigen value of the matrix $\begin{bmatrix} 3 & 4 \\ x & 1 \end{bmatrix}$ for real and negative value of x is

(a) zero

(b) greater than zero

(c) less than zero

(d) dependent on the value of x

Q.49 Let A be an invertible matrix and suppose that the inverse of 7A is $\begin{bmatrix} -1 & 2 \\ 4 & -7 \end{bmatrix}$. The matrix A is

(a) $\begin{bmatrix} 7 & 4 \\ 2 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 7 & 2 \\ 4 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -4/7 \\ -2/7 & 1/7 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2/7 \\ 4/7 & 1/7 \end{bmatrix}$

Q.50 $A = \begin{bmatrix} 5 & 0 & 2 \\ 0 & 3 & 0 \\ 2 & 0 & 1 \end{bmatrix}$. The inverse of A is

(a) $\begin{bmatrix} 5 & 0 & 2 \\ 0 & -\frac{1}{3} & 0 \\ 2 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} \frac{1}{5} & 0 & -\frac{1}{2} \\ 0 & \frac{1}{3} & 0 \\ -\frac{1}{2} & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & -2 \\ 0 & \frac{1}{3} & 0 \\ -2 & 0 & 5 \end{bmatrix}$

(d) $\begin{bmatrix} \frac{1}{5} & 0 & \frac{1}{2} \\ 0 & \frac{1}{3} & 0 \\ \frac{1}{2} & 0 & 1 \end{bmatrix}$

Q.51 Matrix D is an orthogonal matrix $D = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}$. The value of | B | is

(a) 0

(b) 1

(c) $\frac{1}{2}$

(d) $\frac{1}{\sqrt{2}}$

Q.52 If $A = \begin{bmatrix} t^2 & \cos t \\ e^t & \sin t \end{bmatrix}$, then $\frac{dA}{dt}$ will be

(a) Undefined

(b) $\begin{bmatrix} 2t & \cos t \\ e^t & \sin t \end{bmatrix}$

(c) $\begin{bmatrix} t^2 & \sin t \\ e^t & \sin t \end{bmatrix}$

(d) $\begin{bmatrix} 2t & -\sin t \\ e^t & \cos t \end{bmatrix}$

Q.53 A matrix A has x rows and x + 5 columns, matrix B has y rows and 11 – y columns. Both AB and BA exists. Then values of x and y respectively are

(a) 3, –8

(b) 5, –11

(c) 3, 8

(d) –5, 11

Q.54 If a given matrix [A] m × n has r linearly independent vectors (rows or columns) and the remaining vectors are combination of these r vectors. Then rank of matrix is

(a) r

(b) m – n

(c) n

(d) m

Q.55 Rank of matrix $\begin{bmatrix} 0 & 2 & 2 \\ 7 & 4 & 8 \\ -7 & 0 & -4 \end{bmatrix}$ is

(a) 2

(b) 3

(c) 1

(d) 4

Q.56 Eigen values of matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 1 \\ 0 & 2 & 4 \end{bmatrix}$

(a) 1, 3, 4

(b) 1, 2, 5

(c) –1, 3, 4

(d) 1, –2, 4

Q.57 Whether the following equations are consistent

$x + y + z = -3$

$3x + y - 2z = -2$

$2x + 4y + 7z = 7$

(a) can't be determined

(b) can't say

(c) no

(d) yes

Q.58 If $A = \begin{bmatrix} 5 & 4 \\ 3 & 2 \\ -2 & 3 \end{bmatrix}$ & $B = \begin{bmatrix} -1 & 6 & 3 \\ -2 & 5 & 1 \end{bmatrix}$ & $A + B' - X = 0$ then $X =$

(a) $\begin{bmatrix} 4 & 2 \\ 9 & 7 \\ 1 & -4 \end{bmatrix}$

(b) $\begin{bmatrix} 4 & 2 \\ 9 & 7 \\ 1 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & 2 \\ 9 & -7 \\ 1 & 4 \end{bmatrix}$

(d) $\begin{bmatrix} 4 & -2 \\ 9 & -7 \\ 1 & 4 \end{bmatrix}$

Q.59 If $A = \begin{bmatrix} 5 & -1 \\ 3 & 2 \end{bmatrix}$ & $B = \begin{bmatrix} -4 & 3 \\ 1 & -2 \end{bmatrix}$ then the matrix X which satisfies the equation 3A + X = 2B is given by X =

(a) $\begin{bmatrix} -23 & 9 \\ -7 & -10 \end{bmatrix}$

(b) $\begin{bmatrix} -23 & 9 \\ -7 & 10 \end{bmatrix}$

(c) $\begin{bmatrix} 23 & 9 \\ 7 & 10 \end{bmatrix}$

(d) None of these

Q.60 If $A=\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}$, then the value of k for which $A^2 = 8A + kI$ is ......

(a) –7
(b) –5
(c) 5
(d) 7

Q.61 If $A=\begin{bmatrix}2x & 0\\ x & x\end{bmatrix}$ and $A^{-1}=\begin{bmatrix}1 & 0\\ -1 & 2\end{bmatrix}$, then value of x is

(a) 1/2
(b) 2
(c) 1
(d) none of these

Q.62 For what value of λ, the system of equations

$3x - y + z = 0$
$15x - 6y + 5z = 0$
$\lambda x - 2y + 2z = 0$

has non–zero solution.

(a) 6
(b) 5
(c) 4
(d) 1

Q.63 Given $A=\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$, indicate the statement which is not correct for A,

(a) It is singular
(b) A^{-1} exists
(c) It is non singular
(d) It is orthogonal

Q.64 The following set of equations

$3x + 2y + z = 4$
$x - y + z = 2$
$-2x + 2z = 5$ have

(a) No solution
(b) Multiple solution
(c) Unique solution
(d) An inconsistency

Q.65 If $\begin{bmatrix}x & 1\\ -1 & -y\end{bmatrix}+\begin{bmatrix}y & 1\\ 3 & x\end{bmatrix}=\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix}$, then (x, y) = ...

(a) (1, 1)
(b) (1, 0)
(c) (0, 1)
(d) (2, 1)

Q.66 If $\Delta=\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix}$, then....|Δ| is

(a) $\Delta = 2\,abc$
(b) $\Delta = a^2 + b^2 + c^2$
(c) $\Delta = -abc$
(d) $\Delta = 0$

Q.67 $\Delta=\begin{bmatrix}1 & bc & a(b+c)\\ 1 & ca & b(c+a)\\ 1 & ab & c(a+b)\end{bmatrix}$ is equal to

(a) 1
(b) 0
(c) a + b + c
(d) ab + bc + cd

Q.68 The eigen value of a matrix $\begin{bmatrix}5 & 8 & 5\\ 0 & 7 & 12\\ 0 & 0 & 13\end{bmatrix}$ are

(a) 5, 7, 13
(b) –6, –7, –13
(c) 5, 8, 5
(d) 5, 12, 13

Q.69 Determine the eigen values of the matrix

$$A=\begin{bmatrix}2 & 1-2i\\ 1+2i & -2\end{bmatrix}$$

(a) –2, 2,
(b) 3, –3
(c) 1, –1
(d) 1, 3

Q.70 The value of the determinant $\begin{bmatrix} 1 & 2 & 3 \\ 0 & \sec x & \tan x \\ 0 & \tan x & \sec x \end{bmatrix}$ is ...

(a) $\tan^2 x$

(b) 0

(c) 1

(d) $\sec^2 x$

Q.71 Write quadratic equation in matrix form

$2x_1^2 + x_2^2 - 3x_3^2 - 8x_2x_3 - 4x_3x_1 + 12x_1x_2$

(a) $\begin{bmatrix} 2 & 12 & -4 \\ 12 & 1 & -8 \\ -4 & -8 & -3 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 6 & -2 \\ 6 & 1 & -4 \\ -2 & -4 & -3 \end{bmatrix}$

(c) $\begin{bmatrix} -2 & -6 & 2 \\ -6 & -1 & 4 \\ 2 & 4 & 3 \end{bmatrix}$

(d) $\begin{bmatrix} -2 & -12 & 4 \\ -12 & -1 & 8 \\ 4 & 8 & 3 \end{bmatrix}$

Q.72 The eigen values of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ are

(a) 0, 0, 0

(b) 0, 0, 1

(c) 1, 1, 1

(d) 0, 0, 3

Q.73 The rank of matrix $\begin{bmatrix} 8 & 1 & 2 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}$ is

(a) 3

(b) 2

(c) 1

(d) 4

Q.74 Consider the following determinant $\Delta = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$ which of the following is a factor of Δ?

(a) a – b

(d) a + b + c

(c) a + b

(d) abc

Q.75 Consider the following four possible properties of a matrix A.

(i) It is square matrix

(ii) $a_{ij} = +a_{ji}$

(iii) $a_{ij} = -a_{ji}$

(iv) All leading diagonal elements are zero.

If A is skew-symmetric, which of the following is True?

(a) (i), (iii), (iv)

(b) (iii) & (iv)

(c) (i), (ii), (iv)

(d) None of the above

Q.76 Eigen value of matrix $\begin{bmatrix} 5 & 3 \\ 3 & -3 \end{bmatrix}$ are

(a) 5, 3

(b) 6, –4

(c) –3, 5

(d) –4, –6

Q.77 Adjoint of matrix A = $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 0 \\ 2 & 4 & 3 \end{bmatrix}$ is

(a) $\begin{bmatrix} 15 & 0 & -10 \\ 6 & -3 & 0 \\ -15 & 6 & 5 \end{bmatrix}$

(b) $\begin{bmatrix} 15 & -3 & 5 \\ -10 & -6 & 0 \\ 0 & 15 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 15 & 6 & -15 \\ 0 & -3 & 0 \\ -10 & 0 & 5 \end{bmatrix}$

(d) $\begin{bmatrix} 15 & -10 & 0 \\ -3 & 6 & 15 \\ 5 & 0 & 0 \end{bmatrix}$

Q.78 If A + B + C = π, then the value of

$$\begin{vmatrix} \sin(A+B+C) & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A+B) & -\tan A & 0 \end{vmatrix}$$ is equal to

(a) 1

(b) 0

(c) 2 sin B tan A cos C

(d) None of these

Q.79 If $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $Y - 2X = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$, then X = ...

(a) $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$

Q.80 If $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the latent roots of matrix A then A^3 has latent roots

(a) $\lambda_1 \ldots \lambda_n$

(b) $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \ldots, \frac{1}{\lambda_n}$

(c) $\lambda_1^3, \lambda_2^3, \ldots, \lambda_n^3$

(d) $\frac{1}{\lambda_1^3}, \frac{1}{\lambda_2^3}, \ldots, \frac{1}{\lambda_n^3}$

Q.81 If the rank of the matrix $A = \begin{bmatrix} -1 & \lambda & 1 \\ 1 & 1 & \lambda \end{bmatrix}$ is 1, then the value of λ is

(a) 1

(b) –1

(c) ±1

(d) none of the above

Q.82 Let A, B and C be three n × n matrices, AB = AC implies B = C only if

(a) A^{-1} exist

(b) rank $(A)^{5n}$

(c) B ≠ 0 and C ≠ 0

(d) both (a) and (b) above

Q.83 If A is orthogonal ($AA^T = I = A^TA$) then | A | is

(a) ≠ 0

(b) 1

(c) 1 or –1

(d) can be any value.

Q.84 Find the eigen values of the matrix and state that whether it is diagonal or not

$$A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -3 \\ 3 & -4 & 1 \end{bmatrix}$$

(a) Eigen value 1, 2, 3 & diagonal

(b) Eigen value 1, 2, 3 and non diagonal

(c) Eigen value 1, 1, 3 and non diagonal

(d) Eigen value 1, 1, 3 and diagonal

Q.85 The matrix form of quadratic equations $6x_1^2 + 3x_2^2 + 14x_3^2 + 4x_2x_3 + 18x_3x_1 + 4x_1x_2$.

(a) $\begin{bmatrix} 6 & 2 & 9 \\ 2 & 3 & 2 \\ 9 & 2 & 14 \end{bmatrix}$

(b) $\begin{bmatrix} 6 & 4 & 18 \\ 4 & 3 & 4 \\ 18 & 4 & 14 \end{bmatrix}$

(c) $\begin{bmatrix} -6 & -2 & -9 \\ -2 & -3 & -2 \\ 9 & 2 & -14 \end{bmatrix}$

(d) $\begin{bmatrix} -6 & 4 & 18 \\ 4 & -3 & 4 \\ 18 & 4 & -14 \end{bmatrix}$

Q.86 If $\begin{bmatrix} 4 & 1 & 2 \\ 0 & 5 & 3 \end{bmatrix} \begin{bmatrix} 3 & 4 & 5 \\ -1 & 0 & -2 \\ 3 & 4 & 7 \end{bmatrix}$

$= \begin{bmatrix} 8x+3y & 6z & 32 \\ 4 & 12 & 26x-5y \end{bmatrix}$, the values of x, y, z are

(a) $x = 3, y = 4, z = 1$

(b) $x = 0, y = 1, z = 4$

(c) $x = 1, y = 3, z = 4$

(d) $x = 1, y = -3, z = -4$

Q.87 The quadratic expression in matrix form are $x^2 + 4y^2 + 9z^2 + t^2 - 12yz + 6zx - 4xy - 2xt - 6zt$.

(a) $\begin{bmatrix} 1 & -2 & 3 & -1 \\ -2 & 4 & -6 & 0 \\ 3 & -6 & 9 & -3 \\ -1 & 0 & -3 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 4 & -6 & 2 \\ 4 & -4 & 12 & 0 \\ -6 & 12 & -9 & 6 \\ 2 & 0 & 6 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 2 & -3 & 1 \\ 2 & -4 & 6 & 0 \\ -3 & 6 & -9 & 3 \\ 1 & 0 & 3 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} -1 & -4 & 6 & -2 \\ -4 & 4 & -12 & 0 \\ 6 & -12 & 9 & -6 \\ -2 & 0 & -6 & 1 \end{bmatrix}$

Q.88 Which of the following statement is wrong?

(a) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

(b) $(ABC)^T = C^TA^TB^T$

(c) $A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$ provided A is square matrix

(d) A = LU, where A is square matrix, L – Lower triangular matrix, U – Upper triangular matrix

Q.89 T_p, T_q, T_r are the p^{th}, q^{th} & r^{th} terms of an A.P. then $\begin{vmatrix} T_p & T_q & T_r \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$ equals

(a) 1

(b) –1

(c) 0

(d) p + q + r

Q.90 The value of $\begin{vmatrix} p & 0 & 0 & 0 \\ a & q & 0 & 0 \\ b & c & r & 0 \\ d & e & f & s \end{vmatrix}$ is ....

(a) p + q + r + s

(b) 1

(c) ab + cd + ef

(d) pqrs

Q.91 If a_1, a_2, ... form a G.P. & $a_i > 0$ for all $i \geq 1$, then $\begin{vmatrix} \log a_m & \log a_{m+1} & \log a_{m+2} \\ \log a_{m+3} & \log a_{m+4} & \log a_{m+5} \\ \log a_{m+6} & \log a_{m+7} & \log a_{m+8} \end{vmatrix}$ is equal to

(a) $\log a_{m+8} - \log a_m$

(b) $\log a_{m+8} + \log a_m$

(c) zero

(d) $\log a^2_{m+4}$

Q.92 The relationship between following vectors $x_1 = (1, 3, 4, 2)$, $x_2 = (3, -5, 2, 2)$, $x_3 = (2, -1, 3, 2)$ is

(a) $2x_1 + x_2 - 2x_3 = 0$

(b) $x_4 + x_2 + x_3 = 0$

(c) $x_1 + x_2 - 2x_3 = 0$

(d) $x_1 - 2x_2 + x_3 = 0$

Q.93 The matrices $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ commute under multiplication

(a) if $a = b\theta^2$, $\theta = n\pi$, is integer

(b) always

(c) if a = b

(d) never

Q.94 If A is 3 × 4 matrix and B is a matrix such that both A'B and BA' are defined, then B is a ....

(a) 4 × 3 matrix
(b) 3 × 4 matrix
(c) 3 × 3 matrix
(d) 4 × 4 matrix

Q.95 If $a\mu^3 + b\mu^2 + c\mu + d = \begin{vmatrix} 3\mu & \mu+1 & \mu-1 \\ \mu-3 & -2\mu & \mu+2 \\ \mu+3 & \mu-4 & 5\mu \end{vmatrix}$ be an identity in μ, where a, b, c, d are constants, then the value of d is

(a) 5
(b) −6
(c) 9
(d) 0

Q.96 $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} =$

(a) $1+\Sigma\frac{1}{a}$
(b) $abc\left(1+\Sigma\frac{1}{a}\right)$
(c) $1 + \Sigma a$
(d) none of these

Q.97 The value of the determinant, $\Delta = \begin{vmatrix} 1! & 2! & 3! \\ 2! & 3! & 4! \\ 3! & 4! & 5! \end{vmatrix}$ is

(a) 5!
(b) 4!
(c) 3!
(d) 2!

LEVEL–3

Q.98 The eigen value of the following matrix are

$$\begin{bmatrix} -1 & 3 & 5 \\ -3 & -1 & 6 \\ 0 & 0 & 3 \end{bmatrix}$$

(a) −6 + 5j, 3 + j, 3 − j
(b) 3, 3 + 5j, 6 − j
(c) 3, −1 + 3j, −1−3j
(d) 3 + j, 3 − j, 5 + j

Q.99 Consider the matrix $P = \begin{bmatrix} 0 & 1 \\ 3 & -3 \end{bmatrix}$. The value of e^P is

(a) $\begin{bmatrix} 2e^{-1}-e^{-2} & e^{-1}-e^{-2} \\ -2e^{-1}+2e^{-2} & -e^{-1}+2e^{-2} \end{bmatrix}$

(b) $\begin{bmatrix} 5e^{-2}-e^{-1} & 3e^{-1}-e^{-2} \\ 2e^{-2}-6e^{-1} & 4e^{-2}+e^{-1} \end{bmatrix}$

(c) $\begin{bmatrix} e^{-1}+e^{-2} & 2e^{-2}-e^{-1} \\ 2e^{-1}-4e^{2} & 3e^{-1}+2e^{-2} \end{bmatrix}$

(d) $\begin{bmatrix} 2e^{-2}-3e^{-1} & e^{-1}-e^{-2} \\ 2e^{-2}-2e^{-1} & 5e^{-2}-e^{-1} \end{bmatrix}$

Q.100 In the given matrix $A = \begin{bmatrix} 3 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 6 & 1 \end{bmatrix}$, then eigen value are

(a) (2, 3, 5)
(b) (3, 4, 2)
(c) (3, 4, 5)
(d) (3, 4, 1)

Q.101 If $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$ & $B=\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$

then $\cos\theta(A)+\sin\theta(B)=$

(a) $\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$

(b) $\begin{bmatrix}0 & -1\\ -1 & 0\end{bmatrix}$

(c) $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$

(d) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

Q.102 The value of determinant of matrix

$$\begin{bmatrix}1+a & 1 & 1 & 1\\ 1 & 1+b & 1 & 1\\ 1 & 1 & 1+c & 1\\ 1 & 1 & 1 & 1+d\end{bmatrix}$$

(a) $dbc\left[1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right]$

(b) $abcd\left[1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right]$

(c) $abcd\left[1+\frac{1}{a}\frac{1}{b}\frac{1}{c}\frac{1}{d}\right]$

(d) $abd\left[1+\frac{1}{a}+\frac{1}{b}+\frac{1}{d}\right]$

Q.103 If $A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$, then value of A^n is

(a) $\begin{bmatrix}\cos n\alpha & \sin n\alpha\\ -\sin n\alpha & \cos n\alpha\end{bmatrix}$

(b) $\begin{bmatrix}-\cos n\alpha & -\sin n\alpha\\ \sin n\alpha & -\cos n\alpha\end{bmatrix}$

(c) $\begin{bmatrix}n\cos\alpha & n\sin\alpha\\ -n\sin\alpha & n\cos\alpha\end{bmatrix}$

(d) $\begin{bmatrix}\cos n\alpha & -\sin n\alpha\\ \sin n\alpha & \cos n\alpha\end{bmatrix}$

Q.104 The matrix $\begin{bmatrix}0 & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}}\\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{3}}\\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}}\end{bmatrix}$ is

(a) singular

(b) not orthogonal

(c) orthogonal

(d) none of these

Q.105 If α, β, γ, are real numbers, then

$\begin{bmatrix}1 & \cos(\beta-\alpha) & \cos(\gamma-\alpha)\\ \cos(\alpha-\beta) & 1 & \cos(\gamma-\beta)\\ \cos(\alpha-\gamma) & \cos(\beta-\gamma) & 1\end{bmatrix}$ is equal to

(a) -1

(b) $\cos\alpha\cos\beta\cos\gamma$

(c) $\cos\alpha+\cos\beta+\cos\gamma$

(d) None of these

Q.106 If $\Delta_1=\begin{bmatrix}2 & 2^2 & 2^3\\ 3 & 3^2 & 3^3\\ 4 & 4^2 & 4^3\end{bmatrix}$ & $\Delta=\begin{bmatrix}1 & 2 & 4\\ 1 & 3 & 9\\ 1 & 4 & 16\end{bmatrix}$, then

(a) $\Delta_1 = 2\Delta$

(b) $\Delta_1 = 3\Delta$

(c) $\Delta_1 = 4\Delta$

(d) $\Delta_1 = 24\Delta$

Q.107 If $A=\begin{bmatrix}0 & -\tan\alpha/2\\ \tan\alpha/2 & 0\end{bmatrix}$

then $\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}=....$

(a) $\frac{|I+A|}{|I-A|}$

(b) $(I-A)^1$

(c) $1-A^2$

(d) $(I+A)^2$

Q.108 The points (x_1, y_1), (x_2, y_2), (x_3, y_3) are collinear if and only if the rank of matrix $\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$ is

(a) equal to 3
(b) equal to or less than 3
(c) less than 3
(d) can't say

Q.109 If $A = \begin{bmatrix} 1 & 3 & 5 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix}$, the eigen values of the matrix $I + A + A^2$ are

(a) 1, 2, 3
(b) 1, 1, 1
(c) 1, 7, 13
(d) 3, 7, 13

Q.110 What are values of λ and μ so that the equations have no solution

$$2x + 3y + 5z = 9$$
$$7x + 3y - 2z = 8$$
$$2x + 3y + \lambda z = \mu$$

(a) $\mu \neq 9, \lambda = 5$
(b) $\lambda = 5, \mu = 9$
(c) $\mu = 9, \lambda = 5$
(d) $\mu \neq 9, \lambda \neq 5$

Q.111 $A = (a_{ij})$ is a 3×2 matrix, whose elements are given by

$$a_{ij} = 2i - j \quad \text{if } i > j$$
$$= 2j - i \quad \text{if } i \leq j$$

Then the matrix A is

(a) $\begin{bmatrix} 1 & 0 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 3 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 3 \\ 3 & 2 \\ 5 & 4 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}$

Q.112 The value of determinant, $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix}$ is

(a) $a^3\left(1+\frac{2}{a}\right)$

(b) $a^3\left(1-\frac{3}{a}\right)$

(c) $a^3\left(1+\frac{3}{a}\right)$

(d) $a^3\left(1-\frac{2}{a}\right)$

Q.113 The system of linear equations

$$4x + 2y = 7$$
$$2x + y = 6$$

has

(a) exactly two distinct solutions
(b) an infinite number of solutions
(c) a unique solution
(d) no solution

Q.114 How many solutions does the following system of linear equations have?

$$-x + 5y = -1$$
$$x - y = 2$$
$$x + 3y = 3$$

(a) infinitely many
(b) two distinct solutions
(c) unique
(d) none

GATE QUESTIONS

Q.115 A square matrix is singular whenever **[GATE 1987]**

(a) the rows are linearly independent
(b) the columns are linearly independent
(c) the rows are linearly dependent
(d) None of the above

Q.116 The rank of matrix $\begin{bmatrix} 0 & 0 & -3 \\ 9 & 3 & 5 \\ 3 & 1 & 1 \end{bmatrix}$ is **[GATE 1994]**

(a) 0

(b) 1

(c) 2

(d) 3

Q.117 Let A be the set of all non-singular matrices over real number and let * be the matrix multiplication operation. Then, **[GATE 1995]**

(a) A is closed under * but <A, *> is not a semi-group

(b) <A, *> is a semi-group but to a monoid

(c) <A, *> is a monoid but not a group

(d) <A, *> is a group but not an Abelian group

Q.118 The determinant of the matrix

$$\begin{bmatrix} 6 & -8 & 1 & 1 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & 4 & 8 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$ is **[GATE 1997]**

[1Mark]

(a) 11

(b) –48

(c) 0

(d) –24

Q.119 Consider the following set of equations:

$$x + 2y = 5$$

$$4x + 8y = 12$$

$$3x + 6y + 3z = 15$$

This set **[GATE 1998]**
[1Mark]

(a) has unique solution

(b) has no solution

(c) has finite number of solutions

(d) has infinite number of solutions

Q.120 The rank of the matrix given below is:

$$\begin{bmatrix} 1 & 4 & 8 & 7 \\ 0 & 0 & 3 & 0 \\ 4 & 2 & 3 & 1 \\ 3 & 12 & 24 & 2 \end{bmatrix}$$ **[GATE 1998]**

[2Marks]

(a) 3

(b) 1

(c) 2

(d) 4

Q.121 The determinant of the matrix $\begin{bmatrix} 2 & 0 & 0 & 0 \\ 8 & 1 & 7 & 2 \\ 2 & 0 & 2 & 0 \\ 9 & 0 & 6 & 1 \end{bmatrix}$ is **[GATE 2000]**

[1Mark]

(a) 4

(b) 0

(c) 15

(d) 20

Q.122 The rank of the matrix $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ is **[GATE 2002]**

[1Mark]

(a) 4

(b) 2

(c) 1

(d) 0

Q.123 Consider the following system of linear equations

$$\begin{bmatrix} 2 & 1 & -4 \\ 4 & 3 & -12 \\ 1 & 2 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \alpha \\ 5 \\ 7 \end{bmatrix}$$

Notice that the second and the third column of the coefficient matrix are linearly dependent. For how many values of α, does this system of equations have infinitely many solutions?

[GATE 2003]

(a) 0

(b) 1

(c) 2

(d) infinitely many

Q.124 In an M × N matrix such that all non-zero entries are covered in 'a' rows and 'b' columns. Then the maximum number of non-zero entries, such that no two are on the same row or column, is **[GATE 2004]** **[2Marks]**

(a) $\leq a + b$

(b) $\leq \max(a, b)$

(c) $\leq \min[M - a, N - b]$

(d) $\leq \min\{a, b\}$

Q.125 What values of x, y and z satisfy the following system of linear equations?

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 2 & 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ 12 \end{bmatrix}$$

[IT-GATE 2004] **[1 Mark]**

(a) $x = 6, y = 3, z = 2$

(b) $x = 12, y = 3, z = -4$

(c) $x = 6, y = 6, z = -4$

(d) $x = 12, y = -3, z = 0$

Q.126 Let A be an n × n matrix of the following form.

$$A = \begin{bmatrix} 3 & 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 1 & 3 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 3 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 & \cdots & 0 & 0 & 0 \\ \cdots \\ \cdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 3 \end{bmatrix}_{n \times n}$$

What is the value of the determinant of A?

[IT-GATE 2004] **[2-Marks]**

(a) $\left(\frac{5+\sqrt{3}}{2}\right)^{n-1}\left(\frac{5\sqrt{3}+7}{2\sqrt{3}}\right)+\left(\frac{5-\sqrt{3}}{2}\right)^{n-1}\left(\frac{5\sqrt{3}-7}{2\sqrt{3}}\right)$

(b) $\left(\frac{7+\sqrt{5}}{2}\right)^{n-1}\left(\frac{7\sqrt{5}+3}{2\sqrt{5}}\right)+\left(\frac{7-\sqrt{5}}{2}\right)^{n-1}\left(\frac{7\sqrt{5}-3}{2\sqrt{5}}\right)$

(c) $\left(\frac{3+\sqrt{7}}{2}\right)^{n-1}\left(\frac{7\sqrt{5}+5}{2\sqrt{7}}\right)+\left(\frac{3-\sqrt{7}}{2}\right)^{n-1}\left(\frac{3\sqrt{7}-5}{2\sqrt{7}}\right)$

(d) $\left(\frac{3+\sqrt{5}}{2}\right)^{n-1}\left(\frac{3\sqrt{5}+7}{2\sqrt{5}}\right)+\left(\frac{3-\sqrt{5}}{2}\right)^{n-1}\left(\frac{3\sqrt{5}-7}{2\sqrt{5}}\right)$

Q.127 If matrix $X = \begin{bmatrix} a & 1 \\ -a^2+a-1 & 1-a \end{bmatrix}$ and $X^2 - X + I = 0$ (I is the identity matrix and O is the zero matrix), then the inverse of X is:

[IT-GATE 2004] **[2-Marks]**

(a) $\begin{bmatrix} 1-a & -1 \\ a^2 & a \end{bmatrix}$

(b) $\begin{bmatrix} 1-a & -1 \\ a^2-a+1 & a \end{bmatrix}$

(c) $\begin{bmatrix} -a & 1 \\ -a^2+a-1 & a-1 \end{bmatrix}$

(d) $\begin{bmatrix} a^2-a+1 & a \\ 1 & 1-a \end{bmatrix}$

Q.128 Consider the following system of equations in three real variables x_1, x_2 and x_3

$$2x_1 - x_2 + 3x_3 = 1$$
$$3x_1 - 2x_2 + 5x_3 = 2$$
$$-x_1 - 4x_2 + x_3 = 3$$

This system of equations has **[GATE 2005]** **[2 Marks]**

(a) no solution

(b) a unique solution

(c) more than one but a finite number of solutions

(d) an infinite number of solutions

Q.129 What are the eigenvalues of the following 2 × 2 matrix: $\begin{bmatrix} 2 & -1 \\ -4 & 5 \end{bmatrix}$ **[GATE 2005]** **[2Marks]**

(a) −1 and 1

(b) 1 and 6

(c) 2 and 5

(d) 4 and −1

Q.130 Consider the set H of all 3 × 3 matrices of the type

$$\begin{bmatrix} a & f & e \\ 0 & b & d \\ 0 & 0 & c \end{bmatrix}$$

where a, b, c, d, e and f are real numbers and abc ≠ 0. Under the matrix multiplication operation, the set H is **[GATE 2005]**

(a) a group **[2Marks]**

(b) a monoid but not group

(c) a semigroup but not monoid

(d) neither a group nor a semigroup

Q.131 The determinant of the matrix given below is

$$\begin{bmatrix} 0 & 1 & 0 & 2 \\ -1 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 \\ 1 & -2 & 0 & 1 \end{bmatrix}$$

[IT-GATE 2005]

(a) –1 **[1-Mark]**

(b) 0

(c) 1

(d) 2

Q.132 F is an n × n real matrix. b is an n × 1 real vector. Suppose these are two n × 1 vectors, u and v such that u ≠ v, and Fu = b, Fv = b. Which one of the following statements is false?

[GATE 2006]

(a) Determinant of F is zero **[2Marks]**

(b) There are an infinite number of solutions to Fx = b

(c) There is an x ≠ 0 such that Fx = 0

(d) F must have two identical rows

Q.133 Let A be a 4 × 4 matrix with eigenvalues –5, –2, 1, 4. Which of the following is an eigenvalue of

$$\begin{bmatrix} A & I \\ I & A \end{bmatrix}$$

where I is the 4 × 4 identity matrix?

[GATE 2007]

(a) –5 **[2Marks]**

(b) –7

(c) 2

(d) 1

Q.134 The following system of equations

$x_1 + x_2 + 2x_3 = 1$

$x_1 + 2x_2 + 3x_3 = 2$

$x_1 + 4x_2 + ax_3 = 4$

has a unique solution. The only possible value(s) for a is/are **[GATE 2008]**

(a) 0 **[1Mark]**

(b) either 0 or 1

(c) one of 0, 1 or –1

(d) any real number other than 5

Q.135 How many of the following matrices have an eigenvalue 1?

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \text{and} \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix}$$

[GATE 2008]

(a) one **[2Marks]**

(b) two

(c) three

(d) four

ANSWER KEY

1	a	2	b	3	d	4	b	5	d
6	c	7	c	8	d	9	d	10	a
11	a	12	b	13	a	14	a	15	a
16	d	17	b	18	c	19	b	20	d
21	a	22	b	23	c	24	b	25	c
26	d	27	a	28	b	29	a	30	a
31	a	32	b	33	d	34	b	35	b
36	c	37	a	38	c	39	d	40	d
41	a	42	b	43	b	44	d	45	b
46	a	47	d	48	b	49	d	50	c
51	b	52	d	53	c	54	a	55	a
56	b	57	c	58	b	59	a	60	a
61	a	62	a	63	a	64	c	65	b
66	d	67	b	68	a	69	b	70	c
71	b	72	d	73	a	74	a	75	a
76	b	77	c	78	b	79	d	80	c
81	b	82	c	83	c	84	a	85	a
86	c	87	a	88	b	89	c	90	d
91	c	92	c	93	c	94	b	95	b
96	b	97	b	98	c	99	a	100	d
101	d	102	b	103	a	104	c	105	d
106	d	107	a	108	c	109	d	110	a
111	c	112	c	113	d	114	c	115	c
116	c	117	d	118	b	119	b	120	d
121	a	122	c	123	b	124	d	125	c
126	d	127	b	128	b	129	b	130	a
131	a	132	d	133	d	134	d	135	a

SOLUTIONS

S.1 (a)

From vector triple product.

$A \times (B \times C) = B(A \cdot C) - C(A \cdot B)$

$A = \nabla$, $B = \nabla$, $C = P$

$\nabla \times \nabla \times P = \nabla(\nabla \cdot P) - P(\nabla \cdot \nabla)$

$= \nabla(\nabla \cdot P) - \nabla^2 P$

S.2 (b)

$R_3 \to R_1 - R_3$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ Rank = 2

S.4 (b)

We have $|\text{adj (adj } A)| = |A|^{n-2} \cdot A$.

Here $n = 3$ and $|A| = 3$.

So $\text{adj(adj } A) = 3^{(3-2)} \cdot A = 3A$

& $|\text{adj(adj}A)| = |A|^4$.

S.6 (c)

Conformable for multiplication means "The product of AB of two matrices A and B exists if and only if the number of column in A is equal to the number of rows in B."

S.13 (a)

If $AA^2 = I$ $|A|$ cannot be equal to zero, i.e. matrix A should be non-singular.

i.e. $|A| = 0$ **is wrong.**

S.14 (a)

As per theorem for inverse matrix, that eigen values of inverse matrix are inverse of eigen values of the matrix

i.e. if λ is eigen value of A, then $1/\lambda$ is eigen value of A^{-1}.

S.15 (a)

For non zero matrix rank must be greater than or equal to 1, but not equal to zero.

S.16 (d)

For given equations

$$\begin{bmatrix} 1 & 2 \\ 3 & \lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$

$$\begin{bmatrix} 1 & 2 \\ 0 & \lambda-6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

i.e. $(\lambda - 6)y = 0$

i.e. $\lambda - 6 = 0$

$\lambda = 6$ will give no solution to have unique solution **$\lambda \neq 6$.**

S.17 (b)

Here P diagonals matrix B by prepost multiplication such that

$B = P^{-1}$ AP.

S.18 (c)

A per the definition of unitary square matrix.

S.19 (b)

As A is not symmetric matrix.

S.23 (c)

As per definition of rank of matrix.

S.25 (c)

As per the property of the skew Hermitian matrix.

S.28 (b)

For triangular matrix eigen values are same as major principal diagonal elements.

S.30 (a)

A is singular matrix means $|A| = 0$

$$\therefore \quad \begin{bmatrix} 3-x & 2 & 2 \\ 2 & 4-x & 1 \\ -2 & -4 & -1-x \end{bmatrix} = 0$$

$R_2 \to R_2 + R_3$

$$\begin{vmatrix} 3-x & 2 & 2 \\ 0 & -x & -x \\ -2 & -4 & -1-x \end{vmatrix} = 0 \Rightarrow x(3-x)(x-3) = 0$$

$\therefore$ **x = 0, 3**

S.31 (a)

$$\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$$

$c_1 \to c_1 + c_2 + c_3$, we get

$$\Delta = \begin{vmatrix} 1+\omega+\omega^2 & \omega & \omega^2 \\ 1+\omega+\omega^2 & \omega^2 & 1 \\ 1+\omega+\omega^2 & 1 & \omega \end{vmatrix}$$

$$= \begin{vmatrix} 0 & \omega & \omega^2 \\ 0 & \omega^2 & 1 \\ 0 & 1 & \omega \end{vmatrix} = \mathbf{0} \qquad \text{as } 1 + \omega + \omega^2 = 0$$

S.32 (b)

By performing

$$ABC = [x\ y\ z] \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= [x\ y\ z] \begin{bmatrix} ax+hy+gz \\ hx+by+fz \\ gx+fy+cz \end{bmatrix}$$

$$\mathbf{= ax^2 + by^2 + cz^2 + 2hxy + 2gzx + 2fyz}$$

S.33 (d)

As $|A| = 0$

So inverse does not exist.

S.34 (b)

$A^2 = KA + 14I$

$$A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9+20 & -15-10 \\ -12-8 & 20+4 \end{bmatrix}$$

$$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix}$$

$$\begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} = \begin{bmatrix} 3K & -5K \\ -4K & 2K \end{bmatrix} + \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

Equating both sides, we have

$$3K + 14 = 29 \quad \text{OR} \quad 2K + 14 = 24$$

$$3K = 15 \qquad 2K = 24 - 14$$

$$2K = 10$$

$$K = 5 \qquad K = 5$$

S.35 (b)

For triangular matrix eigen values are same as diagonal elements

$$\mathbf{\lambda_1 = 1, \quad \lambda_2 = 3, \quad \lambda_3 = 2.}$$

S.36 (c)

$|A - \lambda I| = 0$

$$\begin{vmatrix} -\lambda & 0 & \alpha \\ 0 & -\lambda & 0 \\ 0 & 0 & -\lambda \end{vmatrix} = 0 \quad \Rightarrow \quad -\lambda^3 = 0$$

$\therefore\ \lambda = 0$

Hence if $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is eigen vector then,

$AX = \lambda X$

$$\therefore \begin{bmatrix} 0 & 0 & \alpha \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore\ \alpha z = 0,\ \alpha \neq 0$, so $z = 0$

$\therefore$ The vectors must be $\begin{bmatrix} \alpha \\ 0 \\ 0 \end{bmatrix}$ or $\begin{bmatrix} 0 \\ \alpha \\ 0 \end{bmatrix}$

As the vectors of the form $\begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$.

S.37 (a)

Hint : After taking the common part from each column, we get matrix having all values = 1 and we get after reducing all rows and columns to zero, the rank of matrix is 1 only.

S.38 (c)

The diagonal matrix is

$$= [x_1 \ldots\ldots x_n]\begin{bmatrix}\lambda_1 & 0 & 0\\ 0 & \lambda_2 & 0\\ 0 & 0 & \lambda_n\end{bmatrix}\begin{bmatrix}x_1\\ \vdots\\ x_n\end{bmatrix}$$

$$= \lambda_1 x_1^2 + \lambda_2 x_2^2 + \ldots + \lambda_n x_n^2$$

S.39 (d)

As per property of idempotent matrix if A & B are idempotent implies **AB = BA.**

S.40 (d)

$$\begin{vmatrix}a & -a & -a\\ b & b & -b\\ c & -c & -c\end{vmatrix} = a\begin{vmatrix}1 & -1 & -1\\ b & b & -b\\ c & -c & -c\end{vmatrix} = abc\begin{vmatrix}1 & -1 & -1\\ 1 & 1 & -1\\ 1 & -1 & -1\end{vmatrix}$$

$$R_1 - R_2 \quad abc\begin{vmatrix}0 & 0 & 0\\ 1 & 1 & -1\\ 1 & -1 & -1\end{vmatrix} = 0$$

S.41 (a)

$$\begin{vmatrix}1 & 1 & 1\\ a & b & c\\ a^2 & b^2 & c^2\end{vmatrix}$$

$C_1 - C_2$ and $C_2 - C_3$

$$\begin{vmatrix}0 & 0 & 1\\ a-b & b-c & c\\ a^2-b^2 & b^2-c^2 & c^2\end{vmatrix}$$

$$= (a-b)(b-c)\begin{vmatrix}0 & 0 & 1\\ 1 & 1 & c\\ a+b & b+c & c^2\end{vmatrix}$$

$$= (a - b)(b - c)(0 + 0 + 1((b + c) - (a + b)))$$

$$= (a - b)(b - c)(c - a)$$

So divisible by **(b–c)**

S.42 (b)

Eigen values are the roots of the determinant formed by matrix $[P - I_S]$

$$[P - I_S] = \begin{bmatrix}P_{11} - S & P_{12}\\ P_{21} & P_{22} - S\end{bmatrix}$$

$$[P - I_S] = 0 \Rightarrow (s - P_{11})(s - P_{22}) - P_{12}P_{21} = 0$$

$$\Rightarrow s^2 - (P_{11} + P_{22})s + P_{11}P_{22} - P_{12}P_{21} = 0$$

Since, one of the its eigen values is zero, therefore, putting s = 0

$$P_{11}P_{22} - P_{12}P_{21} = 0$$

Which is the desired condition.

S.43 (b)

$$M = \begin{bmatrix}4 & 2\\ 2 & 4\end{bmatrix} \qquad [M - \lambda I] = \begin{bmatrix}4-\lambda & 2\\ 2 & 4-\lambda\end{bmatrix}$$

Given eigen vector

$$\therefore \begin{bmatrix}4-\lambda & 2\\ 2 & 4-\lambda\end{bmatrix}\begin{bmatrix}101\\ 101\end{bmatrix} = 0$$

$\Rightarrow (4 - \lambda)(101) + 2(101) = 0$

$\Rightarrow 4 - \lambda + 2 = 0$

$\lambda = 6$

S.44 (d)

$[AA^{-1}] = I$

$$\Rightarrow \begin{bmatrix}2 & -0.1\\ 0 & 3\end{bmatrix}\begin{bmatrix}\frac{1}{2} & a\\ 0 & b\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}1 & 2a - 0.1b\\ 0 & 3b\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$\Rightarrow 2a - 0.1b = 0, \qquad 3b = 1 \Rightarrow b = 1/3$

$2a = 0.1/3 \Rightarrow a = 0.1/6 = 1/60$

$$a + b = \frac{1}{60} + \frac{1}{3} = \frac{1+20}{60} = \frac{21}{60} = \mathbf{\frac{7}{20}}$$

S.45 (b)

The characteristic equation is

$$[A - \lambda I] = 0 = \begin{bmatrix}3-\lambda & -1 & -1\\ -1 & 3-\lambda & -1\\ -1 & -1 & 3-\lambda\end{bmatrix}$$

or $(3 - \lambda)\{(3 - \lambda)^2 - 1\} + (-3 - \lambda) - 1\}$
$- \{1 + (3 - \lambda)\} = 0$

or $(3 - \lambda)\{(3 - \lambda) - 1\}(-3 - \lambda) + 1\}$
$-2\{(3 - \lambda) + 1\} = 0$

or $\{(3 - \lambda) + 1\}[(3 - \lambda)(3 - \lambda - 1) - 2] = 0$

or $(4 - \lambda)(6 - 3\lambda - 2\lambda + \lambda^2 - 2) = 0$

or $(4-\lambda)(4-4\lambda-\lambda+\lambda^2)=0$

or $(4-\lambda)(\lambda-1)(\lambda-4)=0$

$\therefore$ **$\lambda = 1, 4, 4$ are the eigen values.**

S.46 (a)

$$AB=\begin{bmatrix}\cos\theta\cdot\cos\phi\cdot\cos(\theta-\phi) & \cos\theta\cdot\sin\phi\cdot\cos(\theta-\phi)\\ \cos\phi\cdot\sin\theta\cdot\cos(\theta-\phi) & \sin\theta\cdot\sin\phi\cdot\cos(\theta-\phi)\end{bmatrix}$$

= A null matrix, when $\cos(\theta-\phi)=0$

i.e. if $(\theta-\phi)$ is an odd multiple of $\left(\frac{\pi}{2}\right)$.

S.48 (b)

Eigen values are given by the solution of equation,

$$\begin{vmatrix}3-\lambda & 4\\ x & 1-\lambda\end{vmatrix}=0$$

Since x is real and negative, put $x=-k$, where k is positive constant

$\therefore\ (3-\lambda)(1-\lambda)+4k=0$

$\Rightarrow\ \lambda^2-4\lambda+3+4k=0$

If λ_1 and λ_2 be the solutions of the above equation, then λ_1 and λ_2 are eigen values.

Now,

Sum of eigen values = Sum of roots of the above equation

i.e. $\quad \lambda_1+\lambda_2=\frac{-(-4)}{1}=4(>0)$

S.49 (d)

Given

$$(7A)^{-1}=\begin{bmatrix}-1 & 2\\ 4 & -7\end{bmatrix}$$

Let $(7A)^{-1}=B \quad \therefore (7A)=B^{-1}$

$$\therefore\ B^{-1}=\frac{1}{|B|}\text{adj}B$$

$$|B|=\begin{bmatrix}-1 & 2\\ 4 & -7\end{bmatrix}=7-8=-1$$

$$\text{adj}B=\begin{bmatrix}-7 & -2\\ -4 & -1\end{bmatrix}$$

$$\therefore\ B^{-1}=\frac{1}{-1}\begin{bmatrix}-7 & -2\\ -4 & -1\end{bmatrix}$$

$$B^{-1}=\begin{bmatrix}7 & 2\\ 4 & 1\end{bmatrix}$$

$$7A=\begin{bmatrix}7 & 2\\ 4 & 1\end{bmatrix}$$

$$A=\begin{bmatrix}1 & 2/7\\ 4/7 & 1/7\end{bmatrix}$$

S.50 (c)

$\det A=|A|=5[3-0]-0[0-2]+2[0-6]$

$=15-12=3$

$$\text{Adj } A=\begin{bmatrix}\begin{vmatrix}3&0\\0&1\end{vmatrix} & -\begin{vmatrix}0&2\\0&1\end{vmatrix} & \begin{vmatrix}0&2\\3&0\end{vmatrix}\\ -\begin{vmatrix}0&0\\2&1\end{vmatrix} & \begin{vmatrix}5&2\\2&1\end{vmatrix} & -\begin{vmatrix}5&2\\0&0\end{vmatrix}\\ \begin{vmatrix}0&3\\2&0\end{vmatrix} & -\begin{vmatrix}5&0\\2&0\end{vmatrix} & \begin{vmatrix}5&0\\0&3\end{vmatrix}\end{bmatrix}$$

$$\text{Adj } A=\begin{bmatrix}3 & 0 & -6\\ 0 & 1 & 0\\ -6 & 0 & 15\end{bmatrix}$$

$$A^{-1}=\frac{\text{Adj } A}{|A|}$$

$$=\frac{1}{3}\begin{bmatrix}3 & 0 & -6\\ 0 & 1 & 0\\ -6 & 0 & 15\end{bmatrix}$$

$$A^{-1}=\begin{bmatrix}1 & 0 & -2\\ 0 & 1/3 & 0\\ -2 & 0 & 5\end{bmatrix}$$

S.51 (b)

For orthogonal matrix

$\det M=1$ and $M^{-1}=M^T$, therefore Hence $D^{-1}=D^T$

$$D^T=\begin{bmatrix}A & C\\ B & 0\end{bmatrix}=D^{-1}=\frac{1}{-BC}\begin{bmatrix}0 & -B\\ -C & A\end{bmatrix}$$

This implies

$$B=\frac{-C}{-BC} \quad \Rightarrow B=\frac{1}{B} \quad \Rightarrow B=\pm 1$$

Hence **B = 1.**

S.52 (d)

$$\frac{dA}{dt}=\begin{bmatrix}\frac{d(t^2)}{dt} & \frac{d(\cos t)}{dt}\\ \frac{d(e^t)}{dt} & \frac{d(\sin t)}{dt}\end{bmatrix}=\begin{bmatrix}2t & -\sin t\\ e^t & \cos t\end{bmatrix}$$

S.53 (c)

$[A]_{(x)\times(x+5)}$ $[B]_{(y)\times(11-y)}$

AB exists $\Rightarrow$ $x + 5 = y$ .....(1)

BA exists $\Rightarrow$ $x = 11 - y$ .....(2)

Solving (1) and (2) gives **$x = 3, y = 8$.**

S.54 (a)

For a matrix rank is equal to no. of independent vectors. So here rank is r.

S.55 (a)

Let $A=\begin{bmatrix}0 & 2 & 2\\ 7 & 4 & 8\\ -7 & 0 & -4\end{bmatrix}$

$R_2 \to R_2 + R_3$

$$A=\begin{bmatrix}0 & 2 & 2\\ 0 & 4 & 4\\ -7 & 0 & -4\end{bmatrix}$$

$\therefore$ $|A| = 0$ ..... rank < 3

Now minor $\begin{bmatrix}0 & 2\\ 7 & 4\end{bmatrix}\Rightarrow\begin{vmatrix}0 & 2\\ 7 & 4\end{vmatrix}=-14\neq 0$

$\therefore$ **rank = 2**

S.56 (b)

Eigen values are obtained by

$$|A - \lambda I| = 0$$

$$\begin{vmatrix}1-\lambda & 0 & 0\\ 2 & 3-\lambda & 1\\ 0 & 2 & 4-\lambda\end{vmatrix}=0$$

which gives **$\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 5$**

S.57 (c)

Given equations are AX = B

It will be consistant if Rank A = Rank [A B]

We have

$$AX=\begin{bmatrix}1 & 1 & 1\\ 3 & 1 & -2\\ 2 & 4 & 7\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}-3\\ -2\\ 7\end{bmatrix}=B$$

Augmented matrix

$$[A\,B]\sim\begin{bmatrix}1 & 1 & 1 & : & -3\\ 3 & 1 & -2 & : & -2\\ 2 & 4 & 7 & : & 7\end{bmatrix}$$

We reduce [A B] to Echelon form by applying successively.

$R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - 2R_1$ transformations

$$\therefore\quad [A\,B]\sim\begin{bmatrix}1 & 1 & 1 & : & -3\\ 0 & -2 & -5 & : & 7\\ 0 & 2 & 5 & : & 13\end{bmatrix}$$

Apply $R_3 \to R_3 + R_2$

$$[A\,B]\sim\begin{bmatrix}1 & 1 & 1 & : & -3\\ 0 & -2 & -5 & : & 7\\ 0 & 0 & 0 & : & 20\end{bmatrix}\Rightarrow \text{Rank}[A\,B]=3$$

= No. of non zero rows in Echelon form.

$$\therefore\quad A\sim\begin{bmatrix}1 & 1 & 1\\ 0 & -2 & -5\\ 0 & 0 & 0\end{bmatrix}$$

Rank A = 2

Since Rank A $\neq$ Rank [A B]

$\therefore$ **Inconsistent equations.**

S.58 (b)

$A + B' - X = 0$

$$=\begin{bmatrix}5 & 4\\ 3 & 2\\ -2 & 3\end{bmatrix}+\begin{bmatrix}-1 & -2\\ 6 & 5\\ 3 & 1\end{bmatrix}-X=0$$

$$\Rightarrow\quad X=\begin{bmatrix}4 & 2\\ 9 & 7\\ 1 & 4\end{bmatrix}$$

S.59 (a)

$3A + X = 2B$

$\Rightarrow X = 2B - 3A$

$$\Rightarrow X = \begin{bmatrix} -8 & 6 \\ 2 & -4 \end{bmatrix} - \begin{bmatrix} 15 & -3 \\ 9 & 6 \end{bmatrix}$$

$$X = \begin{bmatrix} -23 & 9 \\ -7 & -10 \end{bmatrix}$$

S.60 (a)

$A^2 = 8A + KI$

$\Rightarrow KI = A^2 - 8A$

$$K = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} = 8\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$$

$$\begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix}$$

$$\begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix} = \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix}$$

By comparing $K = -7$

S.61 (a)

As $AA^{-1} = I$

$$\therefore \quad \begin{bmatrix} 2x & 0 \\ 0 & 2x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\therefore \quad 2x = 1$

$x = 1/2$

S.62 (a)

$$A = \begin{bmatrix} 3 & -1 & 1 \\ 15 & -6 & 5 \\ \lambda & -2 & 2 \end{bmatrix}$$

For non-trivial solution $\rho(A) < 3$

$\therefore \ |A| = 0 \Rightarrow -6 + \lambda(0 + 1) = 0$

$\lambda = 6$

S.63 (a)

As determinant of A i.e. $|A| \neq 0$.

S.64 (c)

By solving

$$\begin{bmatrix} 3 & 2 & 1 \\ 1 & -1 & 1 \\ -2 & 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 5 \end{bmatrix}$$

We get the values only one for each parameter, so it has **unique solution.**

S.65 (b)

$$\begin{bmatrix} x+y & 2 \\ 2 & x-y \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

comparing

$x + y = 1$ .....(1)

$x - y = 1$ .....(2)

solving $x = 1$, $y = 0$.

S.66 (d)

$$|\Delta| = \begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix}$$

$= -a(0 + bc) + b(ac - 0)$

$= -abc + abc$

$= 0$

S.67 (b)

$$\Delta = \begin{bmatrix} 1 & bc+ca+ab & a(b+c) \\ 1 & bc+ca+ab & b(c+a) \\ 1 & bc+ca+ab & c(a+b) \end{bmatrix}$$

$C_2 \to C_2 + C_3$

$$= (ab+bc+ca)\begin{bmatrix} 1 & 1 & a(b+c) \\ 1 & 1 & b(c+a) \\ 1 & 1 & c(a+b) \end{bmatrix}$$

$= (ab + bc + ca)\ 0$

$= 0$

S.68 (a)

Hint: If the matrix is in the form of upper/lower triangular matirx, then the diagonal elements are the eigen values of the element.

S.69 (b)

$$\begin{bmatrix} 2 & 1-2i \\ 1+2i & -2 \end{bmatrix}$$ have eigen values as

$$\begin{bmatrix} 2-\lambda & 1-2i \\ 1+2i & -2-\lambda \end{bmatrix} = 0$$

$\therefore \ (\lambda - 2)(\lambda + 2) - (1 + 2i)(1 - 2i) = 0$

$$\lambda^2 - 4 - (1 + 4) = 0$$

$$\lambda^2 - 9 = 0$$

$\therefore$ The given values are $\lambda = +3, \lambda = -3$

S.70 (c)

$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & \sec x & \tan x \\ 0 & \tan x & \sec x \end{vmatrix}$$

$$= 1(\sec^2 x - \tan^2 x) - 2(0) + 3(0)$$

$$= \sec^2 x - \tan^2 x$$

$$= \mathbf{1}$$

S.71 (b)

Quadratic form is written as

$$2x_1x_1 + x_2x_2 - 3x_3x_3 - 4x_2x_3 - 4x_3x_2 - 2x_3x_1 - 2x_1x_3 + 6x_1x_2 + 6x_2x_1$$

which can be written in matrix as

$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} 2 & 6 & -2 \\ 6 & 1 & -4 \\ -2 & -4 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

S.72 (d)

We have $(A - \lambda I) = 0$

$$\begin{bmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{bmatrix} = 0$$

$\therefore$ $(1 - \lambda)[(1 - \lambda)^2 - 1] - 1[(1 - \lambda) - 1] + 1[1 - (1 - \lambda)] = 0$

$\therefore$ $(1 - \lambda)(\lambda^2 - 2\lambda) + 2\lambda = 0$

$\therefore$ $\lambda^2(3 - \lambda) = 0$

$\therefore$ $\mathbf{\lambda = 0, 0, 3}$

S.73 (a)

We have $C_1 \to \frac{1}{8} C_1$

$$\begin{bmatrix} 1 & 1 & 2 & 6 \\ 0 & 3 & 2 & 2 \\ -1 & -1 & -3 & 4 \end{bmatrix}$$

Now $R_3 \to R_3 + R_1$

$$\begin{bmatrix} 1 & 1 & 2 & 6 \\ 0 & 3 & 2 & 2 \\ 0 & 0 & -1 & 10 \end{bmatrix}$$

which is in Echelon form rank is equal to no. of nonzero rows **Rank = 3**

S.74 (a)

$$\Delta = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

Performing $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$,

$$\begin{bmatrix} 1 & a & bc \\ 0 & b-a & ca-bc \\ 0 & c-a & ab-bc \end{bmatrix} = 0$$

$\therefore$ $(b - a)(ab - bc) - (c - a)(ca - bc) = 0$

$(b - a).\ b(a - c) - (a - c)\ c(a - b) = 0$

$(a - c)(b - a)(b - c) = 0$

$\therefore$ **$(a - b)$ is a factor of Δ**

S.75 (a)

Let a skew symmetric matrix is

$$A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

This matrix satisfy the (i), (iii) and (v) property.

S.76 (b)

Characteristic equation are

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 5-\lambda & 3 \\ 3 & -3-\lambda \end{bmatrix} = 0$$

$$(5 - \lambda)(-3 - \lambda) - 9 = 0$$

$$-15 + \lambda^2 + 2\lambda - 9 = 0$$

$$\lambda^2 - 2\lambda - 24 = 0$$

$\therefore$ **$\lambda = 6, -4$**

S.77 (c)

Adjoint A = [Cofactor matrix A]^T

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 0 \\ 2 & 4 & 3 \end{bmatrix}$$

$$\text{adj}(A) = \begin{bmatrix} -\begin{vmatrix} 5 & 0 \\ 4 & 3 \end{vmatrix} & -\begin{vmatrix} 8 & 0 \\ 2 & 3 \end{vmatrix} & \begin{vmatrix} 0 & 5 \\ 2 & 4 \end{vmatrix} \\ -\begin{vmatrix} 2 & 3 \\ 4 & 3 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 2 & 3 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} \\ -\begin{vmatrix} 2 & 3 \\ 5 & 0 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ 0 & 0 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 0 & 5 \end{vmatrix} \end{bmatrix}^T$$

$$=\begin{bmatrix}(15-0) & -(0-0) & (0-10)\\ -(6-12) & (3-6) & -(4-4)\\ (0-15) & -(0-0) & (5-0)\end{bmatrix}^T$$

$$=\begin{bmatrix}15 & 0 & -10\\ 6 & -3 & 0\\ -15 & 0 & 5\end{bmatrix}^T$$

$$\text{adj}(A)=\begin{bmatrix}15 & 6 & -15\\ 0 & -3 & 0\\ -10 & 0 & 5\end{bmatrix}$$

S.78 (b)

$A + B + C = \pi$

$\Rightarrow\ A + B = \pi - C$

putting values.

$$\begin{vmatrix}\sin\pi & \sin B & \sin C\\ -\sin B & 0 & \tan A\\ \cos(\pi-C) & -\tan A & 0\end{vmatrix}$$

$$=\begin{vmatrix}0 & \sin B & \cos C\\ -\sin B & 0 & \tan A\\ -\cos C & -\tan A & 0\end{vmatrix}$$

$= 0 - \sin B(0 + \tan A\cos C)$
$\quad + \cos C(\tan A\sin B - 0)$

$= \tan A\sin B(-\cos C + \cos C)$

$= \mathbf{0}$

S.79 (d)

$$Y-2X=\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}$$

$$\Rightarrow\ 2X=\begin{bmatrix}3 & 2\\ 1 & 4\end{bmatrix}-\begin{bmatrix}1 & 0\\ -3 & 2\end{bmatrix}$$

$$\Rightarrow\ 2X=\begin{bmatrix}2 & 2\\ 4 & 2\end{bmatrix}$$

$$\Rightarrow\ X=\begin{bmatrix}1 & 1\\ 2 & 1\end{bmatrix}$$

S.80 (c)

$AX = \lambda X$

$A^2(AX) = A^2(\lambda X)$

$A^3X = A\lambda(AX)$

$\quad = A\lambda(\lambda X)$

$\quad = \lambda^2(AX)$

$A^3X = \lambda^3X$

S.81 (b)

For given matrix of rank = 1

$\lambda = -1$

S.82 (c)

To satisfy that condition B & C should not be equal to zero.

S.83 (c)

$|A^TA| = |I| = |A^T|\cdot|A| = 1$

$\Rightarrow\ |A|\cdot|A| = 1$

$\Rightarrow\ (|A|)^2 = 1$

$\Rightarrow\ |A| = \pm 1$

S.84 (a)

$$A=\begin{vmatrix}8-\lambda & -8 & -2\\ 4 & -3-\lambda & -2\\ 3 & -4 & 1-\lambda\end{vmatrix}=0$$

$$\begin{vmatrix}1-\lambda & -1+\lambda & -1+\lambda\\ 4 & -3-\lambda & -2\\ 3 & -4 & 1-\lambda\end{vmatrix}=0$$

.....By $R_1-(R_2+R_3)$

or $$(1-\lambda)\begin{vmatrix}1 & -1 & -1\\ 4 & 1-\lambda & -2\\ 3 & -4 & 1-\lambda\end{vmatrix}=0$$

$(1 - \lambda)[(1 - \lambda)(4 - \lambda) + 2] = 0$

$(1 - \lambda)(\lambda^2 - 5\lambda + 6) = 0$

$\Rightarrow\ (1 - \lambda)(\lambda - 2)(\lambda - 3) = 0$

Roots of equation are **1, 2 and 3.**

As the eigen values of the matrix A are all distinct hence **A is similar to diagonal matrix.**

S.85 (a)

Given quadratic equation is obtained from

$$[x_1\ x_2\ x_3]\begin{bmatrix}6 & 2 & 9\\ 2 & 3 & 2\\ 9 & 2 & 14\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

$$= 6x_1^2 + 3x_2^2 + 14x_3^2 + 4x_2x_3 + 18x_3x_1 + 4x_1x_2$$

S.86 (c)

$$\begin{bmatrix}4 & 1 & 2\\ 0 & 5 & 3\end{bmatrix}\begin{bmatrix}8 & 4 & 5\\ -1 & 0 & -2\\ 3 & 4 & 7\end{bmatrix} \Rightarrow \begin{bmatrix}17 & 29 & 32\\ 4 & 12 & 11\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}17 & 29 & 32\\ 4 & 12 & 11\end{bmatrix} = \begin{bmatrix}8x+3y & 6z & 3z\\ 4 & 12 & 26x-5y\end{bmatrix}$$

$\Rightarrow$ **$x = 1, y = 3, z = 4$**

S.87 (a)

Quadratic expression is written as

$$[x\ y\ z\ t]\begin{bmatrix}1 & -2 & 3 & -1\\ -2 & 4 & -6 & 0\\ 3 & -6 & 9 & -3\\ -1 & 0 & -3 & 1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\\ t\end{bmatrix}$$

$= x^2 + 4y^2 + 9z^2 + t^2 - 12yz + 6zx - 4xy + 2xt - 6zt$

S.88 (b)

As $(ABC)^T = C^TB^TA^T$

$\therefore$ (b) is wrong.

S.89 (c)

Let first term and difference of terms in AB are a and d respectively then

$T_p = a + (p - 1)d$

$T_q = a + (q - 1)d$

$T_r = a + (r - 1)d$

Putting values in matrix

$$\begin{bmatrix}a+(p-1)d & a+(q-1)d & a+(r-1)d\\ p & q & r\\ 1 & 1 & 1\end{bmatrix}$$

$= 0$

S.90 (d)

$$\begin{vmatrix}p & 0 & 0 & 0\\ a & q & 0 & 0\\ b & c & r & 0\\ d & e & f & s\end{vmatrix}$$

$$= p\begin{vmatrix}q & 0 & 0\\ c & r & 0\\ e & f & s\end{vmatrix}$$

$$= pq\begin{vmatrix}r & 0\\ f & s\end{vmatrix}$$

$= pr(rs - 0) =$ **pqrs**

S.91 (c)

Hint: $a_1, a_2, \ldots$ are in G.P.

So $\log a_1, \log a_2, \ldots$ are also in G.P.

S.92 (c)

Relationship obtained by using

$$\lambda_1x_1 + \lambda_2x_2 + \lambda_3x_3 = 0$$

$\therefore\ \lambda_1(1, 3, 4, 2) + \lambda_2(3, -5, 2, 2) + \lambda_3(2, -1, 3, 2) = 0$

gives 4 equations

i.e. $\lambda_1 + 3\lambda_2 + 2\lambda_3 = 0$

$3\lambda_1 - 5\lambda_2 - \lambda_3 = 0$

$4\lambda_1 + 2\lambda_2 + 3\lambda_3 = 0$

$2\lambda_1 + 2\lambda_2 + 2\lambda_3 = 0$

By solving these equations we get

$\lambda_1 = 1, \lambda_2 = 1, \lambda_3 = -2$

$\therefore$ Equation is **$x_1 + x_2 - 2x_3 = 0$**

S.93 (c)

Let $A = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$ $B = \begin{bmatrix}a & 0\\ 0 & b\end{bmatrix}$

$$AB = \begin{bmatrix}a\cos\theta & -b\sin\theta\\ a\sin\theta & b\cos\theta\end{bmatrix} \quad .....(1)$$

$$BA = \begin{bmatrix}a\cos\theta & -a\sin\theta\\ b\sin\theta & b\cos\theta\end{bmatrix} \quad .....(2)$$

from (1) and (2), we have

AB = BA only if a = b.

S.94 (b)

Hint: $A_{3\times4}$

$\because\ A'_{4\times3}$

so for A'B and BA' M must of **3 × 4 order.**

S.95 (b)

$$a\mu^3+b\mu^2+c\mu+d=\begin{vmatrix}3\mu & \mu+1 & \mu-1\\ \mu-3 & -2\mu & \mu+2\\ \mu+3 & \mu-4 & 5\mu\end{vmatrix}$$

$$=3\mu[-10\mu^2-(\mu^2-2\mu-8)]$$
$$-(\mu+1)[(5\mu^2-15\mu)-(\mu^2+5\mu+6)]$$
$$+(\mu-1)[(\mu^2-7\mu+12)-(-2\mu^2-6\mu)]$$

Solving we get **d = –6**

S.96 (b)

On solving we get

abc + bc

$$\boxed{abc\left(1+\frac{1}{a}\right)}$$

S.97 (b)

$$\Delta=\begin{vmatrix}1! & 2! & 3!\\ 2! & 3! & 4!\\ 3! & 4! & 5!\end{vmatrix}$$

$$=\begin{vmatrix}1 & 2 & 6\\ 2 & 6 & 24\\ 6 & 24 & 120\end{vmatrix}$$

$$=2\times6\begin{vmatrix}1 & 1 & 1\\ 2 & 3 & 4\\ 6 & 12 & 20\end{vmatrix}$$

$C_3 \to C_3 - C_2$ and $C_2 \to C_2 - C_1$

$$=2\times6\begin{vmatrix}1 & 0 & 0\\ 2 & 1 & 1\\ 6 & 6 & 8\end{vmatrix}$$

$= 2 \times 6\ (8 - 2)$

$= 24$

$= 4!$

S.98 (c)

Sum of the eigen values are the sun of the principle diagonal element of the matrix

Sum of the diagonal current

$= 3 - 1 - 1$

$= 1$

Sum of the eigen value

$= 3 - 1 + 3j - 1 - 3j$

$= 3 - 1 - 1$

$= 1$

S.99 (a)

$$P=\begin{vmatrix}0 & 1\\ 3 & -3\end{vmatrix}$$

$= -3$

So $e^P = e^{-3}$

and $\begin{vmatrix}2e^{-1}-e^{-2} & e^{-1}-e^{-2}\\ -2e^{-1}+2e^{-2} & -e^{-1}+2e^{-2}\end{vmatrix}$

$= (2e^{-1} - 2e^{-2})(-e^{-1} + 2e^{-2}) - (e^{-1} - e^{-2})(-2e^{-1} + 2e^{-2})$

$= e^{-3}$

So option (a) is correct

S.100 (d)

Subtracting λ from the each diagonal element.

$$A-\lambda I=\begin{bmatrix}3-\lambda & 0 & 0\\ 5 & 4-\lambda & 0\\ 3 & 6 & 1-\lambda\end{bmatrix}$$

Characteristic equation of A is

$[A - \lambda I] = 0$

i.e. $$\begin{bmatrix}3-\lambda & 0 & 0\\ 5 & 4-\lambda & 0\\ 3 & 6 & 1-\lambda\end{bmatrix}=0$$

Expanding from first row, we get $(3-\lambda)(4-\lambda)(1-\lambda)=0$

Hence eigen values of given matrix are **3, 4 and 1.**

S.101 (d)

$\cos\theta(A) + \sin\theta(B)$

$$=\cos\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ -\cos\theta\sin\theta & \cos^2\theta \end{bmatrix}$$

$$+ \begin{bmatrix} \sin^2\theta & -\cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & 0 \\ 0 & \cos^2\theta+\sin^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

S.102 (b)

Value of determinant is obtained by taking a, b, c, d common from 1st, 2nd, 3rd, 4th column.

$$= abcd \begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d} & \frac{1}{b} & \frac{1}{c} & \frac{1}{d} \\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d} & \frac{1}{b}+1 & \frac{1}{c} & \frac{1}{d} \\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d} & \frac{1}{b} & \frac{1}{c}+1 & \frac{1}{d} \\ 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d} & \frac{1}{b} & \frac{1}{c} & \frac{1}{d}+1 \end{vmatrix}$$

Applying $a \to c_1 + c_2 + c_3 + c_4$

$$= (abcd)\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right) \begin{vmatrix} 1 & \frac{1}{b} & \frac{1}{c} & \frac{1}{d} \\ 1 & \frac{1}{b}+1 & \frac{1}{c} & \frac{1}{d} \\ 1 & \frac{1}{b} & \frac{1}{c}+1 & \frac{1}{d} \\ 1 & \frac{1}{b} & \frac{1}{c} & \frac{1}{d}+1 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$, $R_4 \to R_4 - R_1$

$$= abcd\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right).$$

S.103 (a)

$$A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$\begin{bmatrix} \cos^2\alpha - \sin^2\alpha & \cos\alpha\sin\alpha + \sin\alpha\cos\alpha \\ -\sin\alpha\cos\alpha - \sin\alpha\cos\alpha & \cos^2\alpha - \sin^2\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}$$

As $\cos^2\alpha - \sin^2\alpha = \cos 2\alpha$

& $2\sin\alpha\cos\alpha = \sin 2\alpha$

Similarly, we get

$$A^n = \begin{bmatrix} \cos n\alpha & \sin n\alpha \\ -\sin n\alpha & \cos n\alpha \end{bmatrix}$$

S.104 (c)

A matrix is said to be orthogonal if $A^T A = I$

$$\begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} 0 & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$\therefore$ **A is orthogonal.**

S.106 (d)

$$\Delta = \begin{vmatrix} 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \end{vmatrix}$$

$$\Delta_1 = \begin{vmatrix} 2 & 2^2 & 2^3 \\ 3 & 3^2 & 3^3 \\ 4 & 4^2 & 4^3 \end{vmatrix}$$

$$= 2\times3\times4 \begin{vmatrix} 1 & 2 & 2^2 \\ 1 & 3 & 3^2 \\ 1 & 4 & 4^2 \end{vmatrix}$$

$\Delta_1 = 24\Delta$

S.107 (a)

$$\therefore \frac{I+A}{I-A}$$

$$= \frac{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan\alpha/2 \\ \tan\alpha/2 & 0 \end{bmatrix}}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -\tan\alpha/2 \\ \tan\alpha/2 & 0 \end{bmatrix}}$$

$$= \frac{\begin{bmatrix} 1 & -\tan\alpha/2 \\ \tan\alpha/2 & 1 \end{bmatrix}}{\begin{bmatrix} 1 & \tan\alpha/2 \\ -\tan\alpha/2 & 1 \end{bmatrix}} = \frac{(1+\tan^2\alpha/2)}{(1+\tan^2\alpha/2)} = 1 \qquad .....(1)$$

and $\begin{vmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{vmatrix} = \cos^2\alpha + \sin^2\alpha = 1$ .....(2)

So option (a) is correct from (1) and (2)

$$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = \frac{I+A}{I-A}$$

S.108 (c)

Suppose these points are collinear on line

$ax + by + c = 0$

Then $ax_1 + by_1 + c = 0$ .....(i)

$ax_2 + by_2 + c = 0$ .....(ii)

$ax_3 + by_3 + c = 0$ .....(iii)

Eliminate a, b, c, between (i), (ii) and (iii)

$$\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix} = 0$$

Thus rank of matrix $\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$ is less than 3

Conversely if rank of matrix A is less than 3, then

$$\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix} = 0$$

$\therefore$ Area of triangle with vertices (x_1, y_1) (x_2, y_2) (x_3, y_3) is equal to 0

$\therefore$ **They are collinear points.**

S.109 (d)

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix}$$

$\therefore$ $I + A + A^2$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 3 & 5 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 3 & 5 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\times \begin{bmatrix} 1 & 3 & 5 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & 5 \\ 0 & 3 & -1 \\ 0 & 0 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 9 & 17 \\ 0 & 4 & -5 \\ 0 & 0 & 9 \end{bmatrix} = \begin{bmatrix} 3 & 12 & 22 \\ 0 & 7 & -6 \\ 0 & 0 & 13 \end{bmatrix}$$

Now for triangular matrix eigen values are same as principal diagonal element.

$\therefore$ $\lambda_1 = 3$

$\lambda_2 = 7$

$\lambda_3 = 13$ are eigen values.

S.110 (a)

$$\left|\begin{matrix} 2 & 3 & 5 & \vdots & 9 \\ 7 & 3 & -2 & \vdots & 8 \\ 2 & 3 & \lambda & \vdots & \mu \end{matrix}\right|$$

The system admits of unique solution if and only if the coefficient matrix is of rank 3.

This needs

$$\begin{vmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 2 & 3 & \lambda \end{vmatrix} = 15(5-\lambda) \neq 0$$

$\therefore$ $\lambda \neq 5$

S.112 (c)

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+a & 1 \\ 1 & 1 & 1+a \end{vmatrix}$$

$= (1 + a)[(1 + a)(1 + a) - 1] - 1\cdot[1(1 + a) - 1)]$
$\quad + 1\cdot[1 - (1 + a)]$

$= (1 + a)[a^2 + 2a]a - a$

$= a^3 + 2a^2 + a^2 + 2a - 2a$

$= a^3 + 3a^2$

$= a^3\left(1+\frac{3}{a}\right)$

S.113 (d)

The system can be written in matrix form as

$$\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

The Augmented matrix [A|B] is given by

$$\left[\begin{array}{cc|c} 4 & 2 & 7 \\ 2 & 1 & 6 \end{array}\right]$$

Performing Gauss elimination on this [A|B] as follows:

$$\left[\begin{array}{cc|c} 4 & 2 & 7 \\ 2 & 1 & 6 \end{array}\right] \xrightarrow[=R_2-\frac{1}{2}R_1]{R_2-\frac{R_1}{2}} \left[\begin{array}{cc|c} 4 & 2 & 7 \\ 0 & 0 & 5/2 \end{array}\right]$$

Now Rank [A|B] = 2

(The number of non-zero rows in [A|B])

Rank [A] = 1

(The number of non-zero rows in [A])

Since, **Rank [A|B] ≠ Rank [A],**

The system has no solution.

S.114 (c)

$-x + 5y = -1$

$x - y = 2$

$x + 3y = 3$

The augmented matrix is

$$\left[\begin{array}{cc|c} -1 & 5 & -1 \\ 1 & -1 & 2 \\ 1 & 3 & 3 \end{array}\right]$$

Using gauss-elimination on above matrix we get,

$$\left[\begin{array}{cc|c} -1 & 5 & -1 \\ 1 & -1 & 2 \\ 1 & 3 & 3 \end{array}\right] \xrightarrow[R_3+R_1]{R_2+R_1} \left[\begin{array}{cc|c} -1 & 5 & -1 \\ 0 & 4 & 1 \\ 0 & 8 & 2 \end{array}\right]$$

$$\xrightarrow{R_3-2R_2} \left[\begin{array}{cc|c} -1 & 5 & -1 \\ 0 & 4 & 1 \\ 0 & 0 & 0 \end{array}\right]$$

Rank [A|B] = 2 (number of non zero rows in [A|B])

Rank [A] = 2 (number of non zero rows in [A])

Rank [A|B] = Rank [A] = 2 = number of variables

∴ **Unique solution exists.**

S.115 (c)

A square matrix $n \times n$ is non-singular if its rank $r = n$.

⇒ It is singular if its rank $r \le n$.

⇒ A row or (rows) is a linear combination of remaining rows.

⇒ **Rows are linearly dependent.**

S.116 (c)

$$X = \begin{bmatrix} 0 & 0 & -3 \\ 9 & 3 & 5 \\ 3 & 1 & 1 \end{bmatrix}$$

$\to R_2 \to R_2 - 3R_3$

$$= \begin{bmatrix} 0 & 0 & -3 \\ 0 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$$

$\to -\frac{1}{3}R_1$ and $\frac{1}{2}R_2$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

$\to R_1 \to R_1 - R_2$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

⇒ Rank of X is **2**.

S.117 (d)

Whenever A and B are non–singular, AB is also non–singular.

($\because$ $|AB| = |A||B|$)

$\Rightarrow$ closure follows

Matrix multiplication is always associative.

The unit matrix is the identity.

Since A is non singular A^{-1} exists.

Matrix multiplication is not commutative.

$\Rightarrow$ $\langle A, * \rangle$ is a group but not an Abelian group.

(Note: A semi–group is an algebraic system satisfying closure and associatively. A monoid is a semi–group with identity.)

S.118 (b)

Consider

$$\begin{bmatrix} 6 & -8 & 1 & 1 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & 4 & 8 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$= 6\begin{bmatrix} 2 & 4 & 6 \\ 0 & 4 & 8 \\ 0 & 0 & -1 \end{bmatrix} + 8\begin{bmatrix} 0 & 4 & 6 \\ 0 & 4 & 8 \\ 0 & 0 & -1 \end{bmatrix}$$

$$+ 1\begin{bmatrix} 0 & 2 & 6 \\ 0 & 0 & 8 \\ 0 & 0 & -1 \end{bmatrix} - 1\begin{bmatrix} 0 & 2 & 4 \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

$= 6[2(-4-0) - 4(0-0) + 6(0-0)] + [0(-4-0) - 4(0-0) + 6(0-0)] + 1[0(0-0) - 2(0-0) + 6(0-0)] - 1[0(0-0) - 2(0-0) + 4(0-0)]$

$= 6[(-8) + 8(0) + 1(0) - 1(0)]$

$= -48$

S.119 (b)

The set is inconsistent. The first two equations are

$$x + 2y = 5$$

and $\quad x + 2y = 3$

This implies inconsistency.

Hence no solution.

S.120 (d)

Given matrix

$$\Rightarrow \begin{bmatrix} 1 & 4 & 8 & 7 \\ 0 & 0 & 3 & 0 \\ 4 & 2 & 3 & 1 \\ 3 & 12 & 24 & 2 \end{bmatrix}$$

Step 1. $R_4 \to R_{4/3} - R_1$

$$= \begin{bmatrix} 1 & 4 & 8 & 7 \\ 0 & 0 & 3 & 0 \\ 4 & 2 & 3 & 1 \\ 0 & 0 & 0 & -19/3 \end{bmatrix}$$

Step 2. $R_3 \to R_3 - 4R_1$

$$= \begin{bmatrix} 1 & 4 & 8 & 7 \\ 0 & 0 & 3 & 0 \\ 0 & -14 & -29 & -27 \\ 0 & 0 & 0 & -19/3 \end{bmatrix}$$

determinant of 2×2 matrix $= 0$

determinant of 3×3 matrix $\neq 0$

determinant of 4×4 matrix $\neq 0$

Hence **Rank** $= 4$

S.121 (a)

$$\begin{vmatrix} 2 & 0 & 0 & 0 \\ 8 & 1 & 7 & 2 \\ 2 & 0 & 2 & 0 \\ 9 & 0 & 6 & 1 \end{vmatrix} = 2\begin{vmatrix} 1 & 7 & 2 \\ 0 & 2 & 0 \\ 0 & 6 & 1 \end{vmatrix} - 0 + 0 - 0$$

$= 2(1(2-0) - 7(0-0) + 2(0-0)$

$= 2[2 + 0 + 0]$

$= 4$

S.122 (c)

The given matrix is $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

$C_2 \to C_2 - C_1 \quad A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

which is of the form $A = \begin{bmatrix} I_1 & 0 \\ 0 & 0 \end{bmatrix}$

$\because$ order of matrix 1 is one, Rank is 1.

S.123 (b)

The augmented matrix for the given system is

$$\left[\begin{array}{ccc|c} 2 & 1 & -4 & \alpha \\ 4 & 3 & -12 & 5 \\ 1 & 2 & -8 & 7 \end{array}\right]$$

Performing Gauss-Elimination on the above matrix

$$\left[\begin{array}{ccc|c} 2 & 1 & -4 & \alpha \\ 4 & 3 & -12 & 5 \\ 1 & 2 & -8 & 7 \end{array}\right] \xrightarrow[R_3-1/2R_1]{R_2-2R_1} \left[\begin{array}{ccc|c} 2 & 1 & -4 & \alpha \\ 0 & 1 & -4 & 5-2\alpha \\ 0 & 3/2 & -6 & 7-\alpha/2 \end{array}\right] \xrightarrow{R_2-3/2R_2}$$

$$\left[\begin{array}{ccc|c} 2 & 1 & -4 & \alpha \\ 0 & 1 & -4 & 5-2\alpha \\ 0 & 0 & 0 & \frac{5\alpha-1}{2} \end{array}\right]$$

Now for infinite solution it is necessary that at least one row must be completely zero.

$$\therefore \quad \frac{5\alpha-1}{2}=0$$

α = **1/5 is the solution**

$\therefore$ There is only one value of α for which infinite solution exists.

S.124 (d)

Every entry will remove one row and one column from further consideration of availability, since no two entries should be in same row or column. Proceeding in this way we can add a maximum of either 'a' entries or 'b' entries depending on which is lesser, since if a < b we will run out of rows first and if b < a we will run out of columns and if a = b then we run out of both rows and columns. Therefore maximum entries that can be added $\leq$ min {a, b}.

S.125 (c)

By using Cramer's rule, we have

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 2 & 2 & 3 \end{vmatrix}$$

$= 1(9-8) - 2(3-8) + 3(2-6)$

$= 1 - 2(-5) + 3(-4)$

$= 1 + 10 - 12$

$= -1$

$$|A_1| = \begin{vmatrix} 6 & 2 & 3 \\ 8 & 3 & 4 \\ 12 & 2 & 3 \end{vmatrix}$$

$= 6(9-8) - 2(24-48) + 3(16-36)$

$= 6 - 2(-24) + 3(-20)$

$= 6 + 48 - 60$

$= -6$

$$|A_2| = \begin{vmatrix} 1 & 6 & 3 \\ 1 & 8 & 4 \\ 2 & 12 & 3 \end{vmatrix}$$

$= 1(24-48) - 6(3-8) + 3(12-16)$

$= -24 - 6(-5) + 3(-4)$

$= -24 + 30 - 12$

$= -6$

$$|A_3| = \begin{vmatrix} 1 & 2 & 6 \\ 1 & 3 & 8 \\ 2 & 2 & 12 \end{vmatrix} = 4$$

$$\therefore \quad x = \frac{|A_1|}{|A|} = \frac{-6}{-1} = 6$$

$$y = \frac{|A_2|}{|A|} = \frac{-6}{-1} = 6$$

$$z = \frac{|A_3|}{|A|} = \frac{4}{-1} = -4$$

$\therefore$ **x = 6, y = 6, z = −4**

S.126 (d)

Taking subparts of the matrix $[A]_{n\times n}$

$$A_1 = \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix}_{2\times 2} = 9 - 1 = 8$$

$$A_2 = \begin{vmatrix} 3 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{vmatrix}_{3\times 3} = 3|9-1|-1|3-1| = 21$$

$$A_3 = \begin{vmatrix} 3 & 1 & 0 & 0 \\ 1 & 3 & 1 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 3 \end{vmatrix}_{4\times 4} = 3\begin{vmatrix} 3 & 1 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 & 0 \\ 0 & 3 & 1 \\ 0 & 1 & 3 \end{vmatrix}$$

$= 63 - 2$

$= 61$

Similarly we can find determinants of other subparts.

Now check the option that satisfy the values of determinants of subparts of the matrix.

take option (d).

$$\left(\frac{3+\sqrt{5}}{2}\right)^{n-1}\left(\frac{3\sqrt{5}+7}{2\sqrt{5}}\right)+\left(\frac{3-\sqrt{5}}{2}\right)^{n-1}\left(\frac{3\sqrt{5}-7}{2\sqrt{5}}\right)$$

for A_1 put n = 2

$$\left(\frac{3+\sqrt{5}}{2}\right)\left(\frac{3\sqrt{5}+7}{2\sqrt{5}}\right)+\left(\frac{3-\sqrt{5}}{2}\right)\left(\frac{3\sqrt{5}-7}{2\sqrt{5}}\right)$$

$$= \frac{\left(9\sqrt{5}+21+15+7\sqrt{5}\right)+\left(9\sqrt{5}-21-15+7\sqrt{5}\right)}{4\sqrt{5}}$$

$$= \frac{(9\sqrt{5}+7\sqrt{5})\times 2}{4\sqrt{5}} = 8 \qquad \text{(Some value as } A_1)$$

for A_2 put n = 3

$$\left(\frac{3+\sqrt{5}}{2}\right)^2\left(\frac{3\sqrt{5}+7}{2\sqrt{5}}\right)+\left(\frac{3-\sqrt{5}}{2}\right)^2\left(\frac{3\sqrt{5}-7}{2\sqrt{5}}\right)$$

$$= \frac{(42+\sqrt{5}+42\sqrt{5})\times 2}{8\times\sqrt{5}} = \frac{84}{4} = 21$$

(Same vlaue as A_2)

Similarly we can check for $A_{3\times 3}$ and $A_{n\times n}$.

So Current option is (D)

S.127 (b)

Given: $X^2 - X + 1 = 0$

Multiply both side by X^{-1}, we have

$X - I + X^{-1} = 0$

$X^{-1} = I - X$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} a & 1 \\ -a^2+a-1 & 1-a \end{bmatrix}$$

$$\text{Hence, } X^{-1} = \begin{bmatrix} 1-a & -1 \\ a^2-a+1 & a \end{bmatrix}$$

S.128 (b)

The augmented matrix for the given system is

$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 1 \\ 3 & -2 & 5 & 2 \\ -1 & -4 & 1 & 3 \end{array}\right]$$

Using Gauss-elimination method on above matrix we get,

$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 1 \\ 3 & -2 & 5 & 2 \\ -1 & -4 & 1 & 3 \end{array}\right] \xrightarrow[R_3+\frac{1}{2}R_1]{R_2-\frac{3}{2}R_1}$$

$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 1 \\ 0 & -1/2 & 1/2 & 1/2 \\ 0 & -9/2 & 5/2 & 7/2 \end{array}\right] \xrightarrow{R_3-9R_2}$$

$$\left[\begin{array}{ccc|c} 2 & -1 & 3 & 1 \\ 0 & -1/2 & 1/2 & 1/2 \\ 0 & 0 & -2 & -1 \end{array}\right]$$

Rank ([A | B]) = 3

Rank ([A]) = 3

Since **Rank ([A | B]) = Rank ([A])** = number of variables, the system has unique solution.

S.129 (b)

$$A = \begin{bmatrix} 2 & -1 \\ -4 & 5 \end{bmatrix}$$

The characterisitc equation of this matrix is given by

$|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & -1 \\ -4 & 5-\lambda \end{vmatrix} = 0$$

$(2 - \lambda)(5 - \lambda) - 4 = 0$

$\lambda^2 - 7\lambda + 6 = 0$

$\lambda = 1, 6$

$\therefore$ The eigen values of A are 1 and 6.

S.130 (a)

(i) The set H is closed, since multiplication of upper triangular matrices will result only in upper triangular matrix.

(ii) Matrix multiplication is associative, i.e. A*(B*C) = (A*B)*C

(iii) Identity element is

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and this belongs to H as I is an upper triangular as well as lower triangular matrix.

(iv) If $A \in H$, then $|A| = abc$. Since it is given that $abc \neq 0$, this means that $|A| \neq 0$ i.e. every matrix belonging to H is non-singular and has a unique inverse.

$\therefore$ the set H along with matrix multiplication is a group.

S.131 (a)

$$\begin{vmatrix} 0 & 1 & 0 & 2 \\ -1 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 \\ 1 & -2 & 0 & 1 \end{vmatrix} \; R_1 \leftrightarrow R_3 \; - \begin{vmatrix} 0 & 0 & 0 & 1 \\ -1 & 1 & 1 & 3 \\ 0 & 1 & 0 & 2 \\ 1 & -2 & 0 & 1 \end{vmatrix}$$

$$= -\left| 0+0+0-1 \begin{vmatrix} -1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & -2 & 0 \end{vmatrix} \right|$$

$$= \begin{vmatrix} -1 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & -2 & 0 \end{vmatrix} \; R_1 \leftrightarrow R_2 \; - \begin{vmatrix} 0 & 1 & 0 \\ -1 & 1 & 1 \\ 1 & -2 & 0 \end{vmatrix}$$

$= -[0 - 1.(0 - 1) + 0]$

$= -1$

OR

$$\text{Let } M = \begin{vmatrix} 0 & 1 & 0 & 2 \\ -1 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 \\ 1 & -2 & 0 & 1 \end{vmatrix}$$

$$\text{so, } |M| = -1 \times \begin{vmatrix} -1 & 1 & 3 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{vmatrix} - 2 \times \begin{vmatrix} -1 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & -2 & 0 \end{vmatrix}$$

$= -1[-1(0) -1(-1) +3(0)] -2[-1(0) -1(0) +1(0)]$

$= -1(1)$

$|M| = -1$

S.132 (d)

Given that Fu = b and Fv = b

If F is non singular, then it has a unique inverse.

Now, $u = F^{-1}b$ and $v = F^{-1}b$

Since F^{-1} is unique u = v but it is given that $u \neq v$. This is a contradiction. So F must be singular. This means that

(a) Determinate of F is zero is true. Also

(b) There are infinite number of solution to Fx = B is true since | F | = 0.

(c) There is an $X \neq 0$ such that FX = 0 is also true, since X has infinite number of solutions, including the X = 0 solution.

(d) F must have 2 identical rows is false, since a determinant may becomes zero, even if two identical columns are present. It is not necessary that 2 identical rows must be present for | F | to becomes zero.

S.133 (d)

Eigenvalues of A is –5, –2, 1, 4.

$$\begin{bmatrix} A & I \\ I & A \end{bmatrix}$$

$$[A - \lambda I] = 0$$

So $\begin{vmatrix} A & I \\ I & A \end{vmatrix} - \begin{vmatrix} \lambda & 0 \\ 0 & \lambda \end{vmatrix} = 0$

after solving we get $\lambda = 1$.

S.134 (d)

The augmented matrix for above system is

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 1 & 4 & a & 4 \end{array}\right] \xrightarrow[R_3-R_1]{R_2-R_1} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 3 & a-2 & 3 \end{array}\right]$$

$$\xrightarrow{R_3-3R_2} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & a-5 & 0 \end{array}\right]$$

Now as long as $a - 5 \neq 0$, rank (A) = rank (A | B) = 3

∴ **a can take any real value except 5.**

S.135 (a)

Eigen values of $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$$\begin{vmatrix} 1-\lambda & 0 \\ 0 & 0-\lambda \end{vmatrix} = 0$$

$$(1 - \lambda)(-\lambda) = 0$$

$$\lambda = 0 \text{ or } \lambda = 1$$

Eigen value of $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$$\begin{bmatrix} -\lambda & 1 \\ 0 & -\lambda \end{bmatrix} = 0$$

$$\lambda^2 = 0$$

$$\boldsymbol{\lambda = 0, 0}$$

Eigen value of $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1-\lambda & -1 \\ 1 & 1-\lambda \end{bmatrix} = 0$$

$$(1 - \lambda)^2 + 1 = 0$$

$$(1 - \lambda)^2 = -1$$

$$1 - \lambda = i \text{ or } -i$$

$$\lambda = 1 - i \text{ or } 1 + i$$

Eigen values of $\begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix}$

$$\begin{bmatrix} -1-\lambda & 0 \\ 1 & -1-\lambda \end{bmatrix} = 0$$

$$(-1 - \lambda)(-1 - \lambda) = 0$$

$$(1 + \lambda)^2 = 0$$

$$\lambda = -1, -1$$

Only one matrix has an eigen value of 1 which is $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$.

12.7

NUMERICAL METHODS

LEVEL–1

Q.1 Match the following and choose the correct combination:

Group 1	Group 2
E. Newton-Raphson Method	1. Solving nonlinear equations
F. Rung-Kutta method	2. Solving linear simultaneous equations
G. Simpson's Rule	3. Solving ordinary differential equations
H. Gauss elimination	4. Numerical integration
	5. Interpolation
	6. Calculation of Eigen values

(a) E – 5, F – 3, G – 4, H – 1
(b) E – 1, F – 3, G – 4, H – 2
(c) E – 1, F – 6, G – 4, H – 3
(d) E – 6, F – 1, G – 5, H – 3

Q.2 If the trapezoidal method is used to evaluate the integral $\int_0^1 x^2 dx$, then the value obtained
(a) is always > 1/3
(b) is always < 1/3
(c) is always = 1/3
(d) may be greater or lesser than 1/3

Q.3 The Newton's-Raphson iterative formula for finding $f(x) = x^2 - 1$, is

(a) $x_{i+1} = \dfrac{2x_i}{2x_i^2 + 1}$

(b) $x_{i+1} = \dfrac{2x_i^2 + 1}{2x_i}$

(c) $x_{i+1} = \dfrac{x_i^2 + 1}{2x_i}$

(d) $x_{i+1} = \dfrac{x_i^2 - 1}{2x_i}$

Q.4 The convergence of which of the following method is sensitive to starting value?
(a) Newton-Raphson method
(b) Gauss seidal method
(c) False position
(d) All of these

Q.5 Newton-Raphson method is applicable to the solution of
(a) transcedental equation only
(b) algebraic equations only
(c) both algebraic and transcendental and also used when the roots are complex
(d) both algebraic and transcendental equations

Q.6 For Trapezoidal rule, the interpolating polynomial is a

(a) hyperbola

(b) parabola

(c) straight line

(d) none of these

Q.7 For Simpson's 1/3rd rule, the interpolating polynomial is of degree

(a) fourth

(b) third

(c) second

(d) first

Q.8 For Simpson's 3/8 rule, the interpolating polynomial is a

(a) cubic curve

(b) parabola

(c) straight line

(d) none of these

Q.9 The equation $x^3 - x - 1 = 0$ has a root in the interval [1, 2], using the bisection method, at the end of third iteration, the approximate value of the root will be

(a) 1.375

(b) 1.75

(c) 1.25

(d) None of these

Q.10 The convergence of the bisection method is

(a) Cubic

(b) Quadratic

(c) Linear

(d) None of these

Q.11 In the Gauss elimination method for solving a system of linear algebraic equations, triangularzation leads to

(a) singular matrix

(b) upper triangular matrix

(c) lower triangular matrix

(d) diagonal matrix

LEVEL–2

Q.12 The Taylor series expansion of $\frac{\sin x}{x-\pi}$ at $x = \pi$ is given by

(a) $-1+\frac{(x-\pi)^2}{3!}+\ldots\ldots\ldots$

(b) $1-\frac{(x-\pi)^2}{3!}+\ldots\ldots\ldots$

(c) $-1-\frac{(x-\pi)^2}{3!}+\ldots\ldots\ldots$

(d) $1+\frac{(x-\pi)^2}{3!}+\ldots\ldots\ldots$

Q.13 The recursion relation to solve $x = e^{-x}$ using Newton-Raphson method is

(a) $x_{n+1} = \frac{x_n^2 - e^{-x_n}(1+x_n)-1}{x_n - e^{-x_n}}$

(b) $x_{n+1} = (1+x_n)\frac{e^{-x_n}}{1+e^{-x_n}}$

(c) $x_{n+1} = x_n - e^{-x_n}$

(d) $x_{n+1} = e^{-x_n}$

Q.14 In the Taylor series expansion of exp(x) + sin (x) about the point $x = \pi$, the coefficient of $(x-\pi)^2$ is

(a) exp (π) – 1

(b) exp (π) + 1

(c) 0.5 exp (π)

(d) exp (π)

Q.15 The equation $x^3 - x^2 + 4x - 4 = 0$ is to be solved using the Newton-Raphson method. If x = 2 is taken as the initial approximation of the solution, then the next approximation using this method will be

(a) 3/2

(b) 1

(c) 4/3

(d) 2/3

Q.16 Consider the ordinary differential equation $\frac{dy}{dx} = x^2 + y^2$, with $y(0) = 0$. Take $h = 0.2$, using Runge-Kutta fourth order method, $y(0.2)$ will be approximately equal to

(a) 0.002667

(b) 0.26670

(c) 0.02667

(d) None of these

Q.17 Let A be the matrix:

$$\begin{bmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 6 & 9 & 8 \end{bmatrix}$$

Let A = LU, where L is the lower triangular matrix and U is the upper triangular matrix. Using Doolittle LU decomposition (i.e. with L having unit diagonal values) the matrix L will be

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 3 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$

Q.18 A real root of the equation $x^3 - x - 4 = 0$ in the interval [1, 2] obtained by the method of bisection at the end of fourth iteration is

(a) 1.96875

(b) 1.875

(c) 1.435

(d) None of these

Q.19 Let A be the matrix:

$$\begin{bmatrix} 2 & 2 & -1 \\ 4 & 5 & 2 \\ -2 & 1 & 2 \end{bmatrix}$$

Let A = LU, where L is the lower triangular matrix and U is the upper triangular matrix. using Dolittle LU decomposition (i.e. with L having unit diagonal values) the matrix U will be

(a) $\begin{bmatrix} 2 & 2 & -1 \\ 0 & 1 & 4 \\ 0 & 0 & 11 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 2 & 1 \\ 0 & 1 & 4 \\ 0 & 0 & -11 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & 2 & -1 \\ 0 & 1 & 4 \\ 0 & 0 & -11 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & 2 & -1 \\ 0 & 1 & 4 \\ 0 & 0 & -11 \end{bmatrix}$

Q.20 Consider the integral $I = \int_1^2 (x^3 + 1)dx$. Using Simpson's 3/8th rule, and taking 3 intervals, I will be approximately equal to

(a) 5.75

(b) 6.75

(c) 4.75

(d) None of these

Q.21 Consider the initial value problem:

$$\frac{dy}{dx} = 1 - xy \; ; \; y(0) = 1. \text{ Take } h = 0.2$$

Using Runge-Kutta fourth order method, y(0.2) will be approximately equal to

(a) 1.10755

(b) 0.82245

(c) 1.17755

(d) None of these

Q.22 Consider the differential equation $\frac{dy}{dx} = -2t + y$, with $y(0) = 3$. Take the step size $h = 0.1$. Using Euler's method $y(0.2)$ will be approximately equal to

(a) 2.999

(b) 3.610

(c) 3.100

(d) None of these

Q.23 Find from the following table, the area bounded by the curve and the x – axis from x = 7.47 to x = 7.52

x	7.47	7.48	7.49	7.50	7.51	7.52
f(x)	1.93	1.95	1.98	2.01	2.03	2.06

(a) 0.996

(b) 0.0916

(c) 0.0996

(d) 0.0699

Q.24 A solid of revolution is formed by rotating about the x – axis the area between the x – axis, the lines x = 0 and x = 11 and to a curve through the points with the following coordinates.

x	0.00	0.25	0.50	0.75	1.00
y	1.000	0.9896	0.9589	0.9089	0.8415

(a) 2.8192

(b) 2.6992

(c) 2.9812

(d) 2.7918

Q.25 Solve following equations by Gauss-elimination method.

$$x + y + z = 1,\ 3x + y - 3z = 5,$$
$$x - 2y - 5z = 10$$

(a) 2, –7, 6

(b) 6, –7, 2

(c) –7, 6, 2

(d) –7, 2, 6

Q.26 A curve is drawn to pass through the points given by the following table:

x	1	1.5	2	2.5	3	3.5	4
y	2	2.4	2.7	2.8	3	2.6	2.1

Estimate the area bounded by the curve, the x–axis and the lines x = 1 and x = 4.

(a) 8.77

(b) 7.87

(c) 7.78

(d) 7.078

Q.27 Evaluate $\int_{-1.6}^{-1} e^x\,dx$ by Simpson's rule with six intervals.

(a) 0.1660

(b) 0.1760

(c) 0.1560

(d) 0.1860

Q.28 The smallest positive root of the equation $f(x) = x^3 - 5x + 1 = 0$ by using bisection method is

(a) 0.203125

(b) 0.203159

(c) 0.219532

(d) 0.251732

Q.29 When the factorization of system of linear equations is effected, the inverse of A can be computed from the formula

(a) $A^{-1} = (LU)^{-1} = U^{-1}L^{-1}$

(b) $A^{-1} = I^{-1}A$

(c) $A^{-1} = A^{-1}A$

(d) $A^{-1} = \frac{AA}{A^{-1}}$

Common Data for Question 30 to 32

Equation $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots a_n$ can be solved by graphical method, direct method and iterative method.

Q.30 Which one of the following is correct?

(a) Bisection method is used for iteration

(b) Regula falsi method is direct method

(c) Secant method is direct method

(d) Newton Raphson method is not iterative method

Q.31 If $n = 3$, $a_0 = 1$, $a_1 = 0$, $a_2 = -1$, $a_3 = -11$ then the root of the equation between 2 and 3 by bisection method is

(a) 2.37

(b) 2.1

(c) 2.9

(d) 2.7

Q.32 If $n = 3$, $a_0 = 1$, $a_1 = 0$, $a_2 = -2$, $a_3 = -5$ then the root of the equation between 1.75 and 2.5 by regula falsi method is

(a) 2.0

(b) 2.09

(c) 2.9

(d) 2.2

LEVEL-3

Q.33 A solid of revolution, is formed by rotating about x axis, the area between x axis, $x = 0$ & $x = 1$ and a curve through points with following coordinates

x	0	0.25	0.5	0.75	1
y	1	0.98	0.95	0.91	0.84

The volume of solid is

(a) 2.65

(b) 2.81

(c) 2.4

(d) 3

Q.34 Let $I = \int_0^{\pi/2} \sqrt{\sin x}\, dx$ Taking 8 intervals and using Simpson's 1/3 rule, I will be approximately equal to (the values of sqrtf (sin x)) are given in the table below:

0.261799	0.508743
0.523599	0.707107
0.785398	0.840896
1.047198	0.930605
1.308997	0.982815
1.570796	1

(a) 1.18728

(b) 1.28728

(c) 1.38728

(d) None of these

Q.35 Let $I = \int_{-1}^{1} \exp(x)\,dx$

Using Trapezoidal rule, with $h = 0.25$, I will be approximately equal to

(a) 2.59917

(b) 2.49917

(c) 2.36263

(d) None of these

Q.36 The depth d of a river at a distance x from one of the banks is given in the table below:

X	D
0	0
10	4.1
20	6.9
30	9
40	12.1
50	14.9
60	13.9
70	8.1
80	3

Using Trapezoidal rule, the area of the cross-section of the river is approximately

(a) 664

(b) 662

(c) 667

(d) 666

Q.37 Compute the value of $\int_0^6 \frac{dx}{1+x^2}$ by using Trapezoidal rule.

(a) 1.0410

(b) 1.4108

(c) 2.0410

(d) 2.4108

Q.38 Compute the value of $\int_0^6 \frac{dx}{1+x^2}$ by the Simpson's 1/3 rule.

(a) 1.78424

(b) 0.07842

(c) 1.3662

(d) 1.0784

Q.39 Use Gauss elimination method to solve:

$x + y + z = 6$

$3x + 3y + 4z = 20$

$2x + y + 3z = 13$

(a) $x = 1, y = 2, z = 3$

(b) $x = 2, y = 1, z = 3$

(c) $x = 3, y = 1, z = 2$

(d) $x = 1, y = 3, z = 2$

Q.40 Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using trapezoidal rule with $h = 0.2$ and hence determine the values of π.

(a) 3.1418

(b) 3.1513

(c) 3.1348

(d) 3.1601

Q.41 Apply Picard's method to obtain solution of the following differential equation correct to four places of decimals.

$\frac{dy}{dx} = x^2 + y^2$ with $y = 0$ when $x = 0$.

for $x = 0.4$.

(a) 0.03125

(b) 0.02135

(c) 0.02590

(d) 0.02199

Q.42 Use Runge-Kutta method of 4th order to find $y(0.1)$ given that

$\frac{dy}{dx} = \frac{1}{x+y}$, $y(0) = 1$

(a) 1.9104

(b) 1.9410

(c) 1.0914

(d) 1.0419

Q.43 If $a = 0$, $b = 6$, $f(x) = \frac{1}{1+x^2}$ then the value of $\int_a^b f(x)dx$ by Simpson three-8th rule is

(a) 1.9

(b) 1.99

(c) 1.35

(d) 1.1

GATE QUESTIONS

Q.44 Which of the following statements is true in respect of the convergence of the Newton-Raphson procedure? **[GATE 1987]**

(a) It converges always under all circumstances.

(b) It does not converge to a root where the second differential coefficient changes sign.

(c) It does not converge to a root where the second differential coefficient vanishes.

(d) None of the above

Q.45 Simpson's rule for integration gives exact result when f(x) is a polynomial of degree **[GATE 1993]**

(a) 1 **[1 Mark]**

(b) 2

(c) 3

(d) 4

Q.46 The iteration formula to find the square root of a positive real number b using the Newton Raphson method is **[GATE 1995]**

(a) $x_{k+1} = 3(x_k + b)/2x_k$ **[2 Marks]**

(b) $x_{k+1} = (x_k^2 + b)/2x_k$

(c) $x_{k+1} = x_k - 2k_k/(x_k^2 + b)$

(d) None of the above

Q.47 Newton-Raphson iteration formula for finding $\sqrt[3]{c}$, where $c > 0$ is, **[GATE 1996]**

(a) $x_{n+1} = \frac{2x_n^3 + \sqrt[3]{c}}{2x_n^2}$

(b) $x_{n+1} = \frac{2x_n^3 - \sqrt[3]{c}}{2x_n^2}$

(c) $x_{n+1} = \frac{2x_n^3 + c}{3x_n^2}$

(d) $x_{n+1} = \frac{2x_n^3 - c}{2x_n^2}$

Q.48 The trapezoidal method to numerically obtain $\int_a^b f(x)dx$ has an error E bounded by

$$\frac{b-a}{12}h^2 \max f''(x)\ x \in [a, b]$$ where h is the width of the trapezoids. The minimum number of trapezoids guaranteed to ensure $E \leq 10^{-4}$ in computing ln 7 using $f = \frac{1}{x}$ is **[GATE 1997]**

(a) 60

(b) 100

(c) 600

(d) 10,000

Q.49 The Newton -Raphson method is used to find the root of the equation $x^2 - 2 = 0$. If the iterations are started from −1, the iterations will

[GATE 1997]
[1 Mark]

(a) converge to −1

(b) converge to $\sqrt{2}$

(c) converge to $-\sqrt{2}$

(d) not converge

Q.50 Which of the following statements applies to the bisection method used for finding roots of functions **[GATE 1998]**

(a) converges within a few iterations

(b) guaranteed to work for all continuous functions

(c) is faster than the Newton-Raphson method

(d) requires that there be no error in determining the sign of the function

Q.51 The Newton-Raphson method is to be used to find the root of the equation f(x) = 0 where x_0 is the initial approximation and f' is the derivative of f. The method converges **[GATE 1999]**
[1 Mark]

(a) always

(b) only if f is a polynomial

(c) only if $f(x_0) < 0$

(d) None of the above

Q.52 The trapezoidal rule for integration gives exact result when the integrand is a polynomial of degree

[GATE 2002]
[1 Mark]

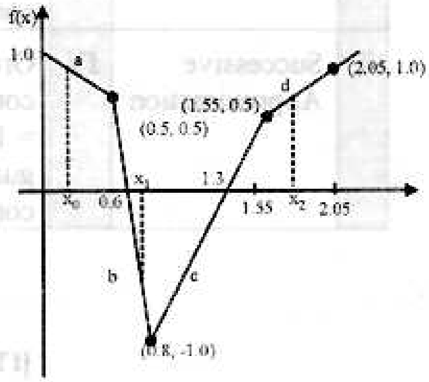
(a) 0 but not 1

(b) 1 but not 0

(c) 0 or 1

(d) 2

Q.53 A piecewise linear function f(x) is plotted using thick solid lines in the figure below (the plot is drawn to scale).



If we use the Newton-Raphson method to find the roots of f(x) = 0 using x_0, x_1 and x_2 respectively as initial guesses, the roots obtained would be

[GATE 2003]
[2 Marks]

(a) 1.3, 0.6 and 0.6 respectively

(b) 0.6, 0.6 and 1.3 respectively

(c) 1.3, 1.3 and 0.6 respectively

(d) 1.3, 0.6 and 1.3 respectively

Q.54 Consider the following iterative root finding methods and convergence properties:

Iterative root finding methods		Convergence properties	
Q	False Position	I	Order of convergence = 1.62
R	Newton Raphson	II	Order of convergence = 2
S	Secant	III	Order of convergence = 1 with guarantee of convergence
T	Successive Approximation	IV	Order of convergence = 1 with no guarantee of convergence

[IT-GATE 2004]
[2-Marks]

(a) Q – II, R – IV, S – III, T – I

(b) Q – III, R – II, S – I, T – IV

(c) Q – II, R – I, S – IV, T – III

(d) Q – I, R – IV, S – II, T – III

Q.55 If f(1) = 2, f(2) = 4 and f(4) = 16, what is the value of f(3) using Lagrange's interpolation formula? **[IT-GATE 2004]**
[2-Marks]

(a) 8

(b) $8\frac{1}{3}$

(c) $8\frac{2}{3}$

(d) 9

Q.56 If the trapezoidal method is used to evaluate the integral $\int_0^1 X^2\,dx$, then the value obtained

[IT-GATE 2005]
[1 Mark]

(a) is always $> \frac{1}{3}$

(b) is always $< \frac{1}{3}$

(c) is always $= \frac{1}{3}$

(d) may be greater or less than $\frac{1}{3}$

Q.57 Consider the series $x_{n+1} = \frac{x_n}{2} + \frac{9}{8x_n}, x_0 = 0.5$ obtained from the Newton-Raphson method. The series converges to **[GATE 2007]**
[2 Marks]

(a) 1.5

(b) $\sqrt{2}$

(c) 1.6

(d) 1.4

Q.58 The Newton-Raphson iteration $x_{n+1} = \frac{1}{2}\left(x_n + \frac{R}{x_n}\right)$ can be used to compute the **[GATE 2008]**
[1 Mark]

(a) square of R

(b) reciprocal of R

(c) square root of R

(d) logarithm of R

Q.59 The minimum Number of equal length subintervals needed to approximate $\int_1^2 xe^x\,dx$ to an accuracy of at least $\frac{1}{3} \times 10^{-6}$ using the trapezoidal rule is **[GATE 2008]**
[2 Marks]

(a) 1000e

(b) 1000

(c) 100e

(d) 100

ANSWER KEY

1	b	2	a	3	c	4	a	5	d
6	c	7	c	8	a	9	a	10	c
11	b	12	a	13	b	14	c	15	c
16	a	17	c	18	a	19	d	20	c
21	c	22	b	23	c	24	a	25	b
26	c	27	a	28	a	29	a	30	a
31	a	32	b	33	b	34	a	35	c
36	a	37	b	38	c	39	c	40	c
41	b	42	c	43	c	44	c	45	b, d
46	b	47	c	48	c	49	c	50	b
51	d	52	c	53	d	54	b	55	c
56	a	57	a	58	c	59	a		

SOLUTIONS

S.2 (b)

$$\int_0^1 x^2 dx$$

As per Trapezoidal rule,

$$\int_a^b f(x)dx \approx (b-a)\frac{f(a)+f(b)}{2}\sqrt{b^2-4ac}$$

Hence

$$\int_0^1 (x^2 \cdot dx) = (1-0)\left(\frac{0+1}{2}\right)$$

$$\approx (1)\left(\frac{1}{2}\right) = \frac{1}{2}$$

$$\frac{1}{2} > \frac{1}{3}$$

S.9 (a)

The bisection method produces the following sequence

{1.5, 1.25, 1.375, 1.315, ...}

Hence, at the end of third iteration, the approximate value of the root will be 1.315.

S.10 (c)

Initially, let the root lie in $\Delta x = [b - a]$.

In n iterations, the interval will be $= \frac{\Delta x}{2^{n+1}}$

In (n + 1) iteration, the interval reduces by a factor of 1/2 (i.e. a constant factor). Hence the convergence of the bisection method is linear.

S.12 (a)

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} \ldots$$

$$\therefore \quad \sin(x-\pi) = (x-\pi) - \frac{(x-\pi)^3}{3!} + \frac{(x-\pi)^5}{5!} \ldots$$

$$-\sin(x) = (x-\pi)\left[1 - \frac{(x-\pi)^2}{3!} + \frac{(x-\pi)^4}{5!} \ldots\right]$$

$$-\frac{\sin x}{(x-\pi)} = \left[1 - \frac{(x-\pi)^2}{3!} + \frac{(x-\pi)^4}{5!} \ldots\right]$$

$$\frac{\sin x}{(x-\pi)} = -1 + \frac{(x-\pi)^2}{3!} - \frac{(x-\pi)^4}{5!} \ldots$$

S.13 (b)

The given equation to be solved is $x = e^{-x}$

Which can be written as

$$f(x) = x - e^{-x} = 0$$

$$f'(x) = 1 + e^{-x}$$

The Newton-raphson iterative formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Here $f(x_n) = x_n - e^{-x_n}$

$$f'(x_n) = 1 + e^{-x_n}$$

∴ The Newton-raphson iterative formula is

$$x_{n+1} = x_n - \frac{x_n - e^{-x_n}}{1+e^{-x_n}}$$

$$= \frac{e^{-x_n}x_n + e^{-x_n}}{1+e^{-x_n}}$$

$$= (1+x_n)\frac{e^{-x_n}}{1+e^{-x_n}}$$

S.14 (c)

$f(x) = e^x + \sin x$

We wish to expand about $x = \pi$

Taylor's series expansion about $x = a$ is

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a)........$$

Now about $x = \pi$

$$f(x) = f(\pi) + (x-\pi)f'(\pi) + \frac{(x-\pi)^2}{2!}f''(\pi) + ...$$

The coefficient of $(x - \pi)^2$ is $\frac{f''(\pi)}{2!}$

Here $f(x) = e^x + \sin x$

$$f'(x) = e^x + \cos x$$

$$f''(x) = e^x - \sin x$$

$$f''(\pi) = e^\pi - \sin \pi = e^\pi - 0 = e^\pi$$

The coefficient of $(x - \pi)^2$ is therefore

$$\frac{e^\pi}{2!} = 0.5\exp(\pi)$$

S.15 (c)

$$f(x) = x^3 - x^2 + 4x - 4$$

$$f'(x) = 3x^2 - 2x + 4$$

$$f(2) = 8$$

$$f'(2) = 12 \Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 2 - \frac{8}{12} = \frac{4}{3}$$

S.16 (a)

Here

$$m_1 = f(x_0, y_0) = 0,$$

$$m = f\left(x_0 + \frac{h}{2}, y_0 + \frac{m_1 h}{2}\right) = f(0.1, 0)$$

$$= 0.01,$$

$$m_3 = f\left(x_0 + \frac{h}{2}, y_0 + \frac{m_2 h}{2}\right) = f\left(0.1, \frac{0.01 \times 0.2}{2}\right)$$

$$= 0.01$$

$$m_4 = f(x_0 + h, y_0 + m_3 h) = f(0.2, 0.01 \times 0.2)$$

$$= 0.04$$

$$\therefore \quad y(0.2) = 0 + \left(\frac{0 + 2\times0.01 + 2\times0.01 + 0.04}{6}\right)0.2$$

$$= \mathbf{0.002667.}$$

S.17 (c)

$$\begin{bmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 6 & 9 & 8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} . \begin{bmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 0 & 0 & -1 \end{bmatrix}$$

S.18 (a)

The bisection method produces the following sequence

{1.5, 1.75, 1.875, 1.9375, 1.96875 ....}

S.19 (d)

$$\begin{bmatrix} 2 & 2 & -1 \\ 4 & 5 & 2 \\ -2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 3 & 1 \end{bmatrix} . \begin{bmatrix} 2 & 2 & -1 \\ 0 & 1 & 4 \\ 0 & 0 & -11 \end{bmatrix}$$

S.20 (c)

$$\int_{x_0}^{x_3} f(x)dx \approx \frac{3h}{8}(f(x_0)+3f(x_1)+3f(x_2)+f(x_3))$$

here $x_0 = 1$

and $h = \frac{2-1}{3} = 0.333$

$\therefore\ x_0 = 1 \Rightarrow f(x_0) = 2$

$x_1 = x_0 + h \Rightarrow f(x_1) = 3.70193$

$x_2 = x_0 + 2h \Rightarrow f(x_2) = 5.629907$

$x_3 = 2 \Rightarrow f(x_3) = 9$

$\therefore\ \int_1^2 (1+x^3)dx = \mathbf{4.750038}$

S.21 (c)

$f(x, y) = 1 - xy,$

Hence using Runge-kutta formula,

$m_1 = 1$

$m_2 = 0.89$

$m_3 = 0.8911$

$m_4 = 0.7644$

We get $\mathbf{y(0.2) = 1.17755}$ approximately.

S.22 (b)

$\frac{dy}{dx} = -2t + y;\ x_0 = 0;\ y_0 = 3;\ h = 0.1$

Euler's method gives

$y_{n+1} = y_n + hf(x_n, y_n)$

$y_1 = y_0 + hf(x_0, y_0) = 3 + (0.1)[-2(0) + 3]$

$= 3 + 0.3 = 3.3$

$y_2 = y_1 + hf(x_1, y_1) = 3.3 + 0.1[-2(0.1)+3.3]$

$= 3.3 + 0.31 = 3.61$

$\therefore\ y_2 = y(0.2) = \mathbf{3.61}$

S.23 (c)

We know that area $= \int_{7.47}^{7.52} f(x)dx$

with h = 0.01 the trapezoidal rule gives area

$$= \frac{0.01}{2}[1.93 + 2(1.95 + 1.98 + 2.01 + 2.03) + 2.06] = \mathbf{0.0996}$$

S.24 (a)

Estimate the volume of the solid formed.

If V is the volume of the solid formed.

$$V = \pi\int_0^1 y^2\,dx$$

x	0.00	0.25	0.50	0.75	1.00
y	1.000	0.9793	0.9195	0.8261	0.7081

with h = 0.25, Simpson's rule gives

$$V = \frac{\pi(0.25)}{3}[1.0000 + 4(0.9793 + 0.8261) + 2(0.9195) + 0.7081]$$

$$= \mathbf{2.8192}$$

S.25 (b)

In first stage, the multipliers are $-\frac{3}{1}$ and $-\frac{1}{1}$.

In second stage, the multiplier is $-\frac{(-3)}{(-2)} = -\frac{3}{2}$

Augmented matrix

$$= \begin{vmatrix} 1 & 1 & 1 & 1 \\ 3 & 1 & -3 & 5 \\ 1 & -2 & -5 & 10 \end{vmatrix} \begin{matrix} \\ = -3 \\ -1 \end{matrix} \begin{vmatrix} 1 & 1 & 1 & 1 \\ 3 & 1 & -3 & 5 \\ 1 & -2 & -5 & 10 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & -2 & -6 & 2 \\ 0 & -3 & -6 & -9 \end{vmatrix} = \frac{-3}{2} \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & -2 & -6 & 2 \\ 0 & -3 & -6 & 9 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & -2 & -6 & 2 \\ 0 & 0 & 3 & 6 \end{vmatrix}$$

Hence, the system becomes

$x + y + z = 1$

$-2y - 6z = 2$

and $3z = 6 \Rightarrow z = 2$

$\therefore\ -2y - 6(2) = 1 \Rightarrow y = -7$

and $x + y + z = 1$

$\Rightarrow x - 7 + 2 = 1$

$\therefore\ \mathbf{x = 6,\ y = -7,\ z = 2}$

S.26 (c)

Here $h = 0.5$, $y_0 = 2$, $y_1 = 2.4$, $y_2 = 2.7$, $y_3 = 2.8$, $y_4 = 3$, $y_5 = 2.6$, $y_6 = 2.1$

by Simpson's 1/3 rule, the required area is

$$A = \int_1^4 y\,dx$$

$$= \frac{h}{3}\left[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$$

$$= \frac{0.5}{3}\left[(2 + 2.1) + 4(2.4 + 2.8 + 2.6) + 2(2.7 + 3)\right]$$

$$= \frac{0.5}{3}[4.1 + 31.2 + 11.4] = \frac{0.5}{3}(46.7)$$

$\Rightarrow$ **Area = 7.78**

S.27 (a)

X	f(x)
–1.6	$e^{-1.6} = 0.2019$
–1.5	$e^{-1.5} = 0.2231$
–1.4	$e^{-1.4} = 0.2466$
–1.3	$e^{-1.3} = 0.2725$
–1.2	$e^{-1.2} = 0.3012$
–1.1	$e^{-1.1} = 0.3329$
–1	$e^{-1.0} = 0.3679$

$$\int_{-1.6}^{-1} e^x\,dx = \frac{h}{3}[y_0 + y_6 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)]$$

$$= \frac{1}{30}[0.2019 + 0.3679 + 4(0.2231 + 0.2725 + 0.3329) + 2(0.2466 + 0.3012)]$$

$$= \frac{1}{30}[0.5698 + 3.314 + 2(0.5478)] = \mathbf{0.1660}$$

S.28 (a)

$f(x) > 0$, $f(1) < 0$, the smallest positive root lies in the interval (0, 1). Take $a_0 = 0$, $b_0 = 1$ we get,

$$m_1 = \frac{1}{2}(a_o + b_o) = \frac{1}{2}(0 + 1) = 0.5$$

and $f(m_1) = -1.375$ and $f(a_0)f(m_1) < 0$.

Thus the root lies in (0, 0.5). Then the root lies (0.1875, 0.21875). Then the approx. root is taken as **0.203125.**

S.30 (a)

Bisection method, regula falsi method, secant method, Newton Raphson method are iterative methods

S.31 (a)

If $n = 3$, $a_0 = 1$, $a_2 = -2$, $a_3 = -5$ then equation becomes $x^3 - x - 11 = 0$

$\therefore$ using $m_1 = \dfrac{x_0 + x_1}{2}$ rule.

We get root **2.37** between 2 and 3.

S.32 (b)

If $x = 3$, $a_0 = 1$, $a_1 = 0$, $a_2 = -2$, $a_3 = -5$ then the root of the equation between 1.75 and 2.5 by regula falsi method is obtained from the equation $x^3-2x-5 = 0$.

Using approximation

$$x_1 = \frac{bf(a) - af(b)}{f(a) - f(b)}$$

where $a = 1.75$, $b = 2.5$

$\therefore \quad x_1 = 2.0187$

and $\quad f(x_1) = f(2.0187) = -0.8901 < 0$

Repeating this process, we get, the required approximate value = **2.094.**

S.33 (b)

Here $h = 0.25$, $y_0 = 1$, $y_1 = 0.98$, $y_2 = 0.95$ etc.

Required volume $= \int_0^1 \pi y^2\,dx$

$$= \pi\frac{h}{3}\left[\left(y_0^2 + y_4^2\right) + 4\left(y_1^2 + y_3^2\right) + 2y_2^2\right]$$

$$= \frac{0.25\pi}{3}[1 + (0.84)^2 + 4(0.98)^2 + (0.91)^2 + 2(0.95)^2]$$

= **2.8192**

S.34 (a)

Simpson's Approximation:

$A = dx/3\{f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \ldots + 4f(x_{n-1}) + f(x_n)\}$

(where $dx = x_{i+1}-x_i$)

0.261799	0.258819	0.508743
0.523599	0.5	0.707107
0.785398	0.707107	0.840896
1.047198	0.866025	0.930605
1.308997	0.965926	0.982815
1.570796	1	1

Ans.: **1.187281**

S.35 (c)

Trapazoid Approximation

$A = dx/2\{f(x_0) + f(x_n)\} + dx\{f(x_1) + f(x_2) + \ldots\ldots + f(x_{n-1})\}$

(where $dx = x_{i+1} - x_i$)

-1	0.367879
-0.75	0.472367
-0.5	0.606531
-0.25	0.778801
0	1
0.25	1.284025
0.5	2.117
1	2.718282

Ans.: **2.362631**

S.36 (a)

Trapazoid Approximation

$A = dx/2\{f(x_0) + f(x_n)\} + dx\{f(x_1) + f(x_2) + f(x_{n-1})\}$

(where $dx = x_{i+1} - x_i$)

X	D
0	0
10	4.1
20	6.9
30	9
40	12.1
50	14.9
60	13.9
70	8.1
80	3

Cross section area = **664.**

S.37 (b)

By trapezidal rule, we have : Here h = 1

$$\int_0^6 f(x)dx = \frac{h}{2}[(y_0+y_6)+2(y_1+y_2+y_3+y_4+y_5)]$$

$$= \frac{1}{2}[(1+0.0270)+2(0.5+0.2+0.1+0.05882 +0.03846)]$$

$$= \frac{1}{2}[(1.02702+1.79456)]$$

$= \mathbf{1.4108}$

S.38 (c)

By Simpson's 1/3 rule

$$\int_0^6 f(x)dx = \frac{h}{2}[(y_0+y_6)+4(y_1+y_3+y_5) +2(y_2+y_4)]$$

$$= \frac{1}{3}[(1+0.0270)+4(0.5+0.1+0.03846) +2(0.2+0.05882)]$$

$$= \frac{1}{3}[1.0270)+2.55384+0.517764]$$

$= \mathbf{1.36620}$

S.39 (c)

Here the segmented matrix is

$$C = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 3 & 3 & 4 & 20 \\ 2 & 1 & 3 & 13 \end{bmatrix}$$

operate $R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - 2R_1$, we get

$$C = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 0 & 1 & 2 \\ 0 & -1 & 1 & 1 \end{bmatrix}$$

Since the pivot $a_{22} = 0$ so interchange R_2 and R_3 to get

$$C = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

This gives

$x + y + z = 6$

$-y + z = 1$

$z = 2$

$\therefore$ by back substitution, we get -

$z = 2, y = 1, x = 3$

or $\mathbf{x = 3, y = 1, z = 2}$

S.40 (c)

Given $f(x) = \frac{1}{1+x^2}$, h = 0.2. The values of x and y = f(x) are tabulated below:

x	Y = f(x)
$X_0 = 0$	$Y_0 = 1$
$X_1 = 0.2$	$Y_1 = 0.9615$
$X_2 = 0.4$	$Y_2 = 0.8621$
$X_3 = 0.6$	$Y_3 = 0.7353$
$X_4 = 0.8$	$Y_4 = 0.6098$
$X_5 = 1.0$	$Y_5 = 0.5$

By trapezoidal rule

$$\int_{x_0}^{x_n} f(x)dx = \frac{h}{2}[(y_0 + y_5) + 2(y_1 + y_2 + y_3 + y_4)]$$

$$\therefore \quad \int_0^1 \frac{dx}{1+x^2} = \frac{0.2}{2}[(1+0.5) + 2(0.9615$$

$$+0.8621 + 0.7353 + 0.6098)]$$

$$= \mathbf{0.7837} \quad .....(i)$$

To find the value of π:

by actual immigration

$$\int_0^1 \frac{dx}{1+x^2} = \left(\tan^{-1} x\right)_0^1 = \tan^{-1}(1) - \tan^{-1}(0) = \frac{\pi}{4} \quad .....(ii)$$

$\therefore$ from (i) and (ii), we get

$\pi/4 = 0.7837$

$\pi = \mathbf{3.1348.}$

S.41 (b)

Here $f(x, y) = x^2 + y^2$, $x_0 = y_0 = 0$

The Picard's formula is

$$y = y_0 + \int_{x_0}^{x} f(x,y)dx \quad ....(i)$$

The first approximation to y is given by

$$y^{(1)} = y_0 + \int_{x_0}^{x} f(x, y^0)dx \quad \text{where } y^0 = y_0$$

$$= 0 + \int_0^x f(x,0)dx$$

$$= \int_0^x x^2 dx$$

$$= \left[\frac{x^3}{3}\right]_0^x = \frac{x^3}{3}$$

$$y^{(1)} = \frac{x^3}{3} \quad .....(ii)$$

This second approximation to y is given by

$$y^{(2)} = y_0 + \int_{x_0}^{x} f\left(x, y^{(1)}\right)dx$$

$$= 0 + \int_{x_0}^{x} f\left(x, \frac{x^3}{3}\right)dx$$

$$= 0 + \int_{x_0}^{x} \left(x^2 + \frac{x^6}{9}\right)dx$$

$$= \left[\frac{x^3}{3} + \frac{x^7}{63}\right]_0^x$$

$$\therefore \quad y^{(2)} = \frac{x^3}{3} + \frac{x^7}{63}$$

Now $y(0.4) = \frac{(0.4)^3}{3} + \frac{(0.4)^7}{63}$

$= 0.02133 + 0.000026$

$= \mathbf{0.02135}$

S.42 (c)

Here $f(x,y) = \frac{1}{x+y}$, $x_0 = 0, y_0 = 1$ and $h = 0.1$.

Now $K_1 = hf(x_0, y_0) = (0.1)\left(\frac{1}{0+1}\right) = 0.1$

$$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$$

$$= (0.1)\left(\frac{1}{0.05+1.05}\right) = 0.0909$$

$$K_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right)$$

$$= (0.1)\left[\frac{1}{0.05+1.045}\right] = 0.0913$$

$$K_4 = hf(x_0 + h, y_0 + K_3)$$

$$= (0.1)\frac{1}{(0.1+1.0913)} = 0.0839$$

Now $K = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4) = 0.0914$

$\therefore$ $y_1 = y(0.1) = y_0 + K = 1 + 0.0914$

$= 1.0914$

$\therefore$ **$y(0.1) = 1.0914$**

S.43 (c)

If $a = 0$, $b = 6$, $f(x) = \frac{1}{1+x^2}$ then

By Simpson's three 8th rule

$$\int_a^b f(x)dx = \frac{3h}{8}[(y_0 + y_n) + 2(y_3 + y_6 + y_9 + \ldots\ldots + y_{n-3}) + 3(y_1 + y_2 + y_4 + \ldots\ldots)]$$

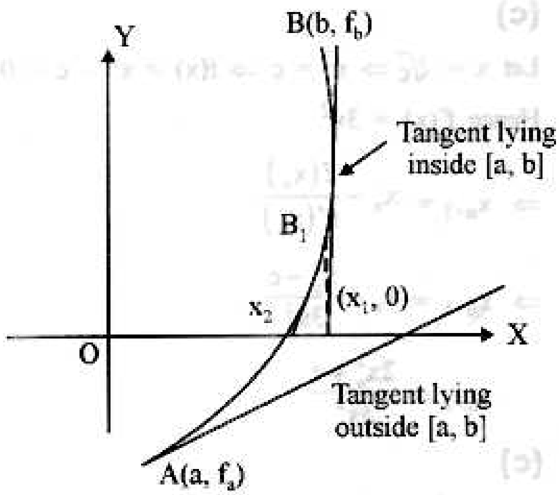
gives the value 1.35.

S.44 (c)

For a curve $y = f(x)$ through points $A(a, f_a)$ and $B(b, f_b)$ choosing $b = x_0$ as first approximation. Tangent to curve at $B(x_0, f_0)$ meets x axis at $x = x_1$ which is first approximation. Through the point $B(x_1, f_1)$ we draw a tangent which meet X-axis at $x = x_2$ which is second approximation. Therefore we go on getting values of x closer to the root.

If we choose $a = x_0$ as first approximation, the tangent to the curve at $A(x_0, f_0)$ meets X-axis at a point x', which is outside the interval (a, b). In other words the method becomes impractical for such a choice of initial approximation.

If we assume $f''(x) > 0$ for $a \le x \le b$ and $f(b) > 0$ then, $f''(x)\,f(b) > 0$ for $x_0 = b$ is $f''(x)\,f(x_0) > 0$. In this case tangent to curve at $B(b, f_b)$ meets X-axis at a point within (a, b).



However, if we choose $x_0 = a$ and $f(a) < 0$ then $f''(x)\,f(a) < 0 \Rightarrow$ Tangent to curve meets X-axis outside (a, b).

This gives a general rule for a choice of initial approximation.

If $f(a)\,f(b) < 0$ for $x = a$ and $x = b$ then root lies between [a, b] then choose end of the interval as initial approximation x_0 such that $f''(x)$ and $f(x_0)$ have same sign.

S.45 (b) and (d).

Simpson's rule gives exact result when f(x) is a polynomial of even degree.

S.46 (b)

We have the equation $x = \sqrt{b}$

$\Rightarrow \quad x^2 = b$

$\Rightarrow \quad f(x) = x^2 - b = 0$

$\Rightarrow \quad f'(x) = 2x$

The Newton-Raphson iteration formulas is

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$x_{k+1} = x_k - \frac{x_k^2 - b}{2x_k}$$

$$2x_k x_{k+1} = 2x_k^2 - x_k^2 + b$$

$$2x_k x_{k+1} = x_k^2 + b$$

$$x_{k+1} = \frac{x_k^2 + b}{2x_k}$$

S.47 (c)

Let $x = \sqrt[3]{c} \Rightarrow x^3 = c \Rightarrow f(x) = x^3 - c = 0$

Hence $f'(x) = 3x^2$

$$\Rightarrow x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$\Rightarrow x_{n+1} = x_n - \frac{x_n^3 - c}{3x_n^2}$$

$$x_{n+1} = \frac{2x_n^3 + c}{3x_n^2}$$

S.49 (c)

$f(x) = x^2 - 2 = 0$

$\Rightarrow f(x) = 2x$

First iteration $x_1 = -1 - \frac{f(-1)}{f'(-1)} = -1 - \frac{(-1)}{(-2)}$

$$= -1 - \frac{1}{2} = -\frac{3}{2} = -1.5$$

Second iteration $x_2 = -1.5 - \frac{f(-1.5)}{f'(-1.5)}$

$$= -1.5 - \frac{(0.25)}{(-3)}$$

$$= -1.5 + 0.083 = -1.417$$

and so on.

It follow that the iterations converge to $-\sqrt{2} = -1.4142$

(**Note**: $x^2 - 2 = 0 \Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2}$)

S.50 (b)

Bisection Method:

This method can locate the root of the equation $f(x) = 0$ between a and b. If $f(x)$ is continuous between a and b, the $f(a)$ and $f(b)$ are of opposite signs, then there is a root between a and b.

If $f(a)$ is negative and $f(b)$ is positive, then the first approximation to the root is

$$x_1 = \frac{1}{2}(a + b)$$

If $f(x_1)$ is positive, then a lies between a and x_1, then 2nd approximation to the roots is

$$x_2 = \frac{1}{2}(a + x_1)$$

Third approximation is, $x_3 = \frac{1}{2}(x_1 + x_2)$.

S.51 (d)

The method converges only if f' is continuous.

S.52 (c)

The error term in Trapezoidal Rule is given by $\frac{-h^3}{12} f''(\xi)$

For any polynomial of degree 1 and 0 will have $f''(\xi)$ always zero.

$\therefore$ We will get the exact result.

S.53 (d)

Starting from x_o,

$$\text{slope of line a} = \frac{1 - 0.5}{0 - 0.5} = -1$$

y-intercept = 1

Equation of a is $y = mx + c = -1x + 1$

This line will cut x axis (i.e., $y = 0$), at $x = 1$

Since $x = 1$ is > than $x = 0.8$, a perpendicular at $x = 1$ will cut the line c and not line b.

$\therefore$ root will be 1.3

Starting from x_1,

the perpendicular at x_1 is cutting line b and root will be 0.6.

Starting from x_2,

$$\text{Slope of line d} = \frac{1 - 0.5}{2.05 - 1.55} = 1$$

Equation of d is $y - 0.5 = 1(x - 1.55)$

i.e. $y = x - 1.05$

This line will cut x axis at $x = 1.05$

Since $x = 1.05$ is > than $x = 0.8$, the perpendicular at $x = 1.05$ will cut the line c and not line b. The root will be therefore equal to 1.3. So starting from x_0, x_1 and x_2 the roots will be respectively **1.3, 0.6 and 1.3.**

S.54 (b)

Iterative root finding methods		Convergence properties	
Q	False Position	III	Order of convergence = 1 with guarantee of convergence
R	Newton Raphson	II	Order of convergence = 2
S	Secant	I	Order of convergence = 1.62
T	Successive Approximation	IV	Order of convergence = 1 with no guarantee of convergence

S.55 (c)

$f(1) = 2$

$f(2) = 4$

$f(4) = 16$

$f(3) = ?$

Through given data,

$$x_0 = 1;\ x_1 = 2;\ x_2 = 3;\ x_3 = 4$$

also $y_0 = 2;\ y_1 = 4;\ y_2 = ?;\ y_3 = 16$

Now, by Language Interpretation

$$f(x) = y_0 * \frac{(x-x_1)(x-x_3)}{(x_0-x_1)(x_0-x_3)} + y_1 * \frac{(x-x_0)(x-x_3)}{(x_1-x_0)(x_1-x_3)} + y_3 * \frac{(x-x_0)(x-x_1)}{(x_3-x_0)(x_3-x_1)}$$

$$= 2*\frac{(3-2)(3-4)}{(1-2)(1-4)} + 4*\frac{(3-1)(3-4)}{(2-1)(2-4)} + 16*\frac{(3-1)(3-2)}{(4-1)(4-2)}$$

$$= -\frac{2}{3} + 4 + \frac{16}{3}$$

$$= \frac{26}{3} = 8\frac{2}{3}$$

S.56 (a)

By the trapezoidal method integral value is given

as $= \frac{1}{2}n[(a_0 + a_n) + 2(a_1 + a_2 + a_3 + \ldots + a_{n-1})]$

where $n = 0.50 - 0.25 = 0.25$

∴ x	x^2	
0.25	0.0625	$\to a_0$
0.50	0.25	$\to a_1$
0.75	0.5625	$\to a_2$
1.00	1	$\to a_n$

$$\therefore \int_0^1 x^2 dx = \frac{1}{2} \times 0.25[1.0625 + 2(0.25 + 0.5625)]$$

$$= \frac{0.25}{2}[1.0625 + 2(0.8125)]$$

$$= \frac{0.25}{2}[2.6875]$$

$$= 0.3359$$

S.57

Given

$$x_{n+1} = \frac{x_n}{2} + \frac{9}{8x_n}, \quad x_0 = 0.5$$

When the series converges $x_{n+1} = x_n = \alpha$

$$\alpha = \frac{\alpha}{2} + \frac{9}{8\alpha}$$

$$\alpha = \frac{4\alpha^2 + 9}{8\alpha}$$

$$\Rightarrow \quad 8\alpha^2 = 4\alpha^2 + 9$$

$$\Rightarrow \quad \alpha^2 = \frac{9}{4}$$

$$\alpha = \frac{3}{2} = \mathbf{1.5}$$

S.58 (c)

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{R}{x_n}\right)$$

at convergence

$$x_{n+1} = x_n = \alpha$$

$$\alpha = \frac{1}{2}\left(\alpha + \frac{R}{\alpha}\right)$$

$$2\alpha = \alpha + \frac{R}{\alpha}$$

$$2\alpha^2 = \alpha^2 + R$$

$$\Rightarrow \quad \alpha^2 = R$$

$$\boldsymbol{\alpha = \sqrt{R}}$$

So this iteration will compute the square roof of R.

Correct choice is (c).

S.59 (a)

Here, the function being integrated is

$$f(x) = xe^x$$

$$f'(x) = xe^x + e^x = e^x(x + 1)$$

$$f''(x) = xe^x + e^x + e^x = e^x(x + 2)$$

Truncation Error for trapezoidal rule

= TE (bound)

$$= \frac{h^3}{12}\max|f''(\zeta)|(N_i)$$

where N_i is number of subintervals

$$N_i = \frac{b-a}{h}$$

$$\therefore \quad T_E = \frac{h^3}{12}\max|f''(\zeta)|\left(\frac{b-a}{h}\right)$$

$$= \frac{h^2}{12}(b-a)\max|f''(\zeta)| \quad 1 \le \zeta \le 2$$

$$= \frac{h^2}{12}(2-1)[e^2(2+2)]$$

$$= \frac{h^2}{3}e^2 = \frac{1}{3}\times 10^{-6}$$

$$\Rightarrow \quad h^2 = \frac{10^{-6}}{e^2}$$

$$\Rightarrow \quad h = \frac{10^{-3}}{e}$$

$$N_i = \frac{b-a}{h}$$

$$= \frac{2-1}{(10^{-3}/e)} = \mathbf{1000e}$$

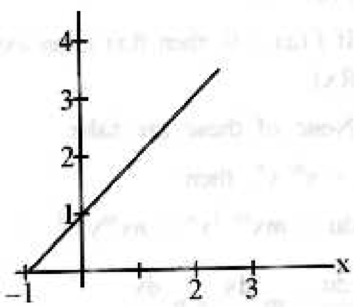
12.8

CALCULUS

LEVEL–1

Q.1 The following plot shows a function y which varies linearly with x. The value of the integral

$$I = \int_1^2 y\,dx \text{ is}$$



(a) 5.0

(b) 4.0

(c) 2.5

(d) 1.0

Q.2 $\lim\limits_{\theta \to 0} \dfrac{\sin(\theta/2)}{\theta}$ is

(a) 0.5

(b) 1

(c) 2

(d) not defined

Q.3 The integral $\int_0^{\pi} \sin^3\theta\, d\theta$ is given by

(a) 1/2

(b) 2/3

(c) 4/3

(d) 8/3

Q.4 The discontinuity of the function

$$f(x) = \frac{|x|}{x},\ x \neq 0$$

$$= 0, \quad x = 0$$

at the origin is known as

(a) Discontinuity of 2nd kind

(b) Discontinuity of 1st kind

(c) Mixed discontinuity

(d) None of these

Q.5 $f(x) = x + \dfrac{1}{x}$ is

(a) maximum at $x = -1$

(b) maximum at $x = 2$

(c) maximum at $x = 1$

(d) maximum at $x = -2$

Q.6 $\lim_{x\to 0}\left(\frac{1}{x}\right)^{\tan x}$ equal to

(a) 0
(b) 1
(c) −1
(d) None of these

Q.7 $I = \int_0^{\pi/2} x^2 \sin x\,dx$

(a) $\frac{x+2}{2}$
(b) $2\pi - 2$
(c) $\pi - 2$
(d) $\pi + 2$

Q.8 The maximum value of $f(x) = \frac{\sin x - \cos x}{\sqrt{2}}$ is

(a) 3
(b) $\sqrt{2}$
(c) $1/\sqrt{2}$
(d) 1

Q.9 $\lim_{x\to 8}\frac{\sqrt{x}-2\sqrt{2}}{x-8}$

(a) $\frac{1}{4\sqrt{2}}$
(b) $\frac{1}{2}$
(c) $\frac{0}{0}$
(d) $\frac{1}{\sqrt{2}}$

Q.10 Value of the integral $I \int_0^{\pi/4} \cos^2 x\,dx$, is

(a) $\frac{\pi}{8}+\frac{1}{4}$
(b) $-\frac{\pi}{8}+\frac{1}{4}$
(c) $-\frac{\pi}{8}-\frac{1}{4}$
(d) $\frac{\pi}{8}-\frac{1}{4}$

Q.11 The value of $\int_0^{5\pi}(2-\sin x)dx$ is

(a) 2
(b) >0
(c) 0
(d) unfind

Q.12 The integral $\lim_{a\to\infty}\int_1^a x^{-4}dx$

(a) converges to $-\frac{1}{a^3}$
(b) converges to $\frac{1}{3}$
(c) diverges
(d) converges to 0

Q.13 Which of the following is false?

(a) f(a) is an extreme value of f(x) if f'(a) = 0
(b) If f(a) is an extreme value of f(x), then f'(a) = 0
(c) If f'(a) = 0, then f(a) is an extreme value of f(x)
(d) None of these are false

Q.14 If $u = x^m y^n$, then

(a) $du = mx^{m-1}y^n + nx^m y^{n-1}$
(b) $\frac{du}{u} = m\frac{dx}{x} + n\frac{dy}{y}$
(c) $udu = mxdx + nydy$
(d) $du = mdx + ndy$

Q.15 $f(x) = x^9 + 3x^7 + 6$ is increasing for

(a) all positive real values of x
(b) all negative real values of x
(c) all non-zero real values of x
(d) None of these

Q.16 $f(x) = x^2e^{-x}$ is increasing in the interval

(a)] 0, 2 [
(b)] −2, 0 [
(c)] 2, ∞ [
(d)] −∞, ∞ [

Q.17 The value of $\int_0^1 |5x-3|\,dx$ is

(a) −1/2
(b) 13/10
(c) 1/2
(d) 23/10

Q.18 $\int_0^1 \frac{e^{\sqrt{1-x^2}} \cdot x}{\sqrt{1-x^2}}\,dx$ is equal to

(a) $\frac{1}{2}$
(b) $-\frac{1}{2}$
(c) e − 1
(d) $\frac{e+1}{2}$

Q.19 $\int_{-1}^{1} \frac{|x|}{x}\,dx$ is equal to

(a) 2
(b) 0
(c) 1
(d) $\frac{1}{2}$

Q.20 $\lim\limits_{x\to 0} \frac{x^2}{1-\cos x}$ equals to

(a) 2
(b) 3
(c) 1
(d) 0

Q.21 $\lim\limits_{x\to\infty} x^n e^{-x}$ is equal to

(a) ∞
(b) 0
(c) 4
(d) n−1!

Q.22 The value of $\lim\limits_{\theta\to\pi/2} \frac{\cot\theta}{(\pi/2)-\theta}$ is

(a) 1
(b) 2
(c) 0
(d) $\frac{1}{2}$

Q.23 $f(x) = x^3 - 12x^2 + 45x + 11$ is

(a) minimum at x = 5
(b) minimum at x = −5
(c) minimum at x = 4
(d) minimum at x = −4

Q.24 $\lim\limits_{x\to 2} \frac{|x-2|}{x-2}$ is equal to

(a) 1
(b) −1
(c) 0
(d) does not exist

Q.25 Evaluate $\lim\limits_{x\to 0} e^{-x/\log x}$

(a) 1
(b) 0
(c) ∞
(d) does not exist

Q.26 Evaluate $I = m\int_0^{\pi} \frac{\sin 2kx}{\sin x}\,dx$

(a) 0
(b) 1/2
(c) −1
(d) π/2

Q.27 $\lim\limits_{x\to 0} \frac{(1+x)^n - 1}{x}$ is equal to

(a) n
(b) n−1
(c) 0
(d) does not exist

Q.28 $\int_2^{\infty} \frac{dx}{e^x + e^{-x}}$ is equal to

(a) $\tan^{-1}e - \pi/2$
(b) $\tan^{-1}e - \pi/4$
(c) $\tan^{-1}e + \pi/4$
(d) $\tan^{-1}e + \pi/2$

Q.29 Which one of the following function is strictly bounded?

(a) $\frac{1}{x^2}$
(b) e^x
(c) x^2
(d) e^{-x^2}

Q.30 The value of $\int_0^{5\pi}(2-\sin\pi)\,dx$ is

(a) > 0
(b) 2
(c) 0
(d) undefined

Q.31 The function $f(x) = 3x(x-2)$ has

(a) minimum at x = 1
(b) maximum at x = 1
(c) minimum at x = 2
(d) maximum at x = 2

Q.32 What is the derivative of $f(x) = |x|$ at $x = 0$?

(a) 1
(b) –1
(c) 0
(d) does not exist

Q.33 The function $f(x) = x^3 - 6x^2 + 9x + 25$ has

(a) a maxima at x = 1, but no minima
(b) no maxima, but a minima at x = 1
(c) a maxima at x = 3 and a minima at x = 1
(d) a maxima at x = 1 and a minima at x = 3

Q.34 if $\phi(x) = \int_0^{x^2} \sqrt{t}\,dt$, then $\frac{d\phi}{dx}$ is

(a) 1
(b) 0
(c) $\sqrt{x}$
(d) $\frac{2x^3}{3}$

Q.35 Given: the function

$$f(x) = 2x^3 - 15x^2 + 36x + 10$$

Maximum and minimum will occur at

(a) Maximum at x = 2
Minimum at x = 4
(b) Maximum at x = 2
Minimum at x = 3
(c) Maximum at x = 3
Minimum at x = 1
(d) Maximum at x = 1
Minimum at x = 2

Q.36 A point on the curve $y = \sqrt{x-2}$ is given for interval [2, 3], where the tangent is parallel to the chord joining the end points of the curve is

(a) $\left(\frac{9}{2}, \frac{1}{4}\right)$
(b) $\left(\frac{7}{4}, \frac{1}{2}\right)$
(c) $\left(\frac{7}{2}, \frac{1}{4}\right)$
(d) $\left(\frac{9}{4}, \frac{1}{2}\right)$

LEVEL-2

Q.37 Consider the function $f(x) = x^2 - x - 2$. The maximum value of f(x) in the closed interval [–4, 4] is

(a) 18
(b) 10
(c) –2.25
(d) indeterminate

Q.38 The value of the integral

$$I = \frac{1}{\sqrt{2\pi}}\int_0^{\infty} \exp\left(-\frac{x^2}{8}\right) dx \text{ is}$$

(a) 1
(b) π
(c) 2
(d) 2π

Q.39 It is given that $f(x) = \frac{ax+b}{x+1}$, $\lim_{x \to 0} f(x) = 2$ and $\lim_{x \to \infty} f(x) = 1$, then value of f(–2) is

(a) 1
(b) 0
(c) e
(d) ∞

Q.40 The value of $\lim_{x \to (\pi/2)} \frac{1+\cos 2x}{(\pi - 2x)^2}$ is

(a) $\frac{1}{2}$
(b) $\frac{1}{3}$
(c) $\frac{1}{4}$
(d) 0

Q.41 The series

$\frac{1}{a+1} - \frac{1}{a+2} + \frac{1}{a+3} - \frac{1}{a+4} + \ldots$ is convergent if

(a) $a < -1$
(b) $a < 0$
(c) $a \geq 0$
(d) none of these

Q.42 If $f(x) = 1$, if $x < 3$
$= ax + b$, if $3 \leq x < 5$
$= 7$ if $5 \leq x$

Determine the value of 'a' and 'b' so that f(x) is continuous.

(a) a = 3, b = –8
(b) a = 4, b = –7
(c) a = 1, b = –9
(d) a = 2, b = 3

Q.43 Which of the following limit exist?

(a) $\lim_{x \to 0} e^{1/x}$
(b) $\lim_{x \to 0} \frac{1}{x^2}$
(c) $\lim_{x \to 0} \frac{1}{x}$
(d) $\lim_{x \to 0} \frac{1}{1-e^{1/x}}$

Q.44 The first three non-zero terms of Taylor's series for ln x around 2 is

(a) $\ln 2 + \frac{1}{2}(x-2) - \frac{1}{8}(x-2)^2$
(b) $\ln 2 + (x-2) + \frac{1}{8}(x-2)^2$
(c) $\ln 2 - \frac{1}{2}(x-2) + \frac{1}{4}(x-2)^2$
(d) $\ln 2 + (x-2) - \frac{1}{4}(x-2)^2$

Q.45 Evaluate $I = \int \frac{x - \sin x}{1 - \cos x} dx$

(a) $-x + \cot\frac{x}{2} + C$
(b) $x \cot\frac{x}{2} + C$
(c) $-x \cot\frac{x}{2} + C$
(d) $x + \tan\frac{x}{2} + C$

Q.46 $\int_1^5 \frac{dx}{\sqrt[3]{x-2}}$ is equal to

(a) $\frac{3}{2}\left(\sqrt[3]{9} - 1\right)$
(b) $\frac{\sqrt[3]{9}+1}{2}$
(c) $3\left(\sqrt[3]{9}+1\right)$
(d) None of these

Q.47 $\lim_{x \to 0} \frac{x - \sin x}{x^3}$

(a) 1/3
(b) 1/6
(c) –1/3
(d) none of these

Q.48 The function f defined is

$$f(x)=\frac{\left(\sin x^2\right)}{x} \text{ for } x \neq 0 \text{ and } f(0)=0 \text{ is}$$

(a) neither continuous nor derivable at x = 0
(b) continuous and derivable at x = 0
(c) continuous but not derivable at x = 0
(d) none of these

Q.49 If $f(x) = x\left[\sqrt{x}-\sqrt{x+1}\right]$ then
(a) f(x) is differentiable at x = 0
(b) f(x) is not differentiable at x = 0
(c) f(x) is not differentiable at x = 1
(d) None of these.

Q.50 The integration of $\int \frac{1}{x^2 \cos^2(1/x)} dx$

(a) $-\tan\left(\frac{1}{x}\right)+C$

(b) $\tan\left(\frac{1}{x}\right)+C$

(c) $\cot\left(\frac{1}{x}\right)+C$

(d) None of these

Q.51 The integration of $\int \frac{1}{4x^2+4x+5} dx =$

(a) $\frac{1}{4}\tan^{-1}\left(x^2+\frac{1}{2}\right)+C$

(b) $\frac{1}{4}\tan^{-1}\left(x-\frac{1}{2}\right)+C$

(c) $\frac{1}{4}\tan^{-1}\left(x+\frac{1}{2}\right)+C$

(d) $\frac{1}{4}\tan^{-1}\left(x^2-\frac{1}{2}\right)+C$

Q.52 $\int_0^{\pi/2}\left(\sqrt{\tan x}+\sqrt{\cot x}\right) dx =$

(a) $\frac{\pi}{2\sqrt{2}}$

(b) $\frac{\pi}{\sqrt{2}}$

(c) $\frac{1}{2\sqrt{2}}$

(d) None of these.

Q.53 At x = 0, the function $y = e^{-|x|}$ is
(a) differentiable with derivative = 1
(b) continuous and not differentiable
(c) continuous
(d) differentiable with derivative = –1

Q.54 Let f(a) > 0, and let f(x) be a non, decreasing continuous function in [a, b] then

$\frac{1}{b-a}\int_a^b f(x)\,dx$ has the
(a) minimum value f(a)
(b) maximum value f(b)
(c) maximum value of f(a)
(d) (a) and (b) both

Q.55 If f(0) = 2 and f(x) $\frac{1}{5-x^2}$, then lower and upper bounds of f(1) estimated by the mean value theorem are
(a) 1.9, 2.2
(b) 2.25, 2.5
(c) 2.2, 2.25
(d) None of these

Q.56 The third term in the Taylor series expansion of e^x about a would be

(a) $\frac{x^2}{2}$

(b) $\frac{e^a}{2}(x-a)$

(c) e^a (x a)

(d) $\frac{e^a}{6}(x-a)^3$

Q.57 Let $f(x) = x(x + 3)e^{\frac{x}{2}}$, $-3 \le x \le 0$. Let $c \leftrightarrow]-3, 0[$ such that $f'(c) = 0$. Then, the value of c is

(a) 3

(b) -3

(c) $-\frac{1}{2}$

(d) -2

Q.58 $\tan\left(\frac{\pi}{4}+x\right)$ when expanded in Taylor's series, gives

(a) $1+2x+2x^2+\frac{8}{3}x^3+...$

(b) $1+x+x^2+\frac{4}{3}x^3+...$

(c) $1+\frac{x^2}{2!}+\frac{x^4}{4!}+...$

(d) None of these

Q.59 If $z=xyf\left(\frac{y}{x}\right)$, then $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}$ is equal to

(a) z

(b) xz

(c) 2z

(d) yz

Q.60 If $y = a \log x + bx^2 + x$ has its extreme values at $x = -1$ and $x = 2$, then

(a) $a=2, b=-\frac{1}{2}$

(b) $a = 2, b = -1$

(c) $a=-\frac{1}{2}, b=2$

(d) None of these

Q.61 $\int \frac{2x+3}{\sqrt{x^2+x+1}}dx$ is equal to

(a) $2\sqrt{x^2+x+1}+2\sinh^{-1}\frac{2x+1}{\sqrt{3}}+C$

(b) $\sqrt{x^2+x+1}+2\sinh^{-1}\frac{2x+1}{\sqrt{3}}+C$

(c) $2\sqrt{x^2+x+1}+\sinh^{-1}\frac{2x+1}{\sqrt{3}}+C$

(d) $2\sqrt{x^2+x+1}+\sinh^{-1}\frac{2x+1}{\sqrt{3}}+C$

Q.62 $\int_0^{\pi} xF(\sin x)dx$ is equal to

(a) $\frac{1}{2}\int_0^{\pi} \pi F(\sin x)\,dx$

(b) $\pi\int_0^{\pi} F(\sin x)\,dx$

(c) $\pi\int_0^{\pi/2} F(\sin x)\,dx$

(d) $\frac{\pi}{4}\int_0^{\pi} F(\sin x)\,dx$

Q.63 The area of the region bounded by the curve $y(x^2 + 2) = 3x$ and $4y = x^2$ is given by

(a) $\int_0^1\int_{y=0}^{x^2/4} dxdy$

(b) $\int_0^1\int_{y=0}^{x^2/4} dydx$

(c) $\int_0^2\int_{y=x^2/4}^{3x/(x^2+2)} dydx$

(d) $\int_{y=0}^1\int_{y=x^2/4}^{3x/(x^2+2)} dxdy$

Q.64 Match the list I with List II

List I		List II	
(i)	$\lim_{x\to\infty} \sinh x$	(1)	0
(ii)	$\lim_{x\to\infty} \tanh x$	(2)	does not exist
(iii)	$\lim_{x\to\infty} \frac{x^n}{\lfloor n}, n \in N$	(3)	1
(iv)	$\lim_{x\to 0} \log x$	(4)	∞

(a) (i)-(4), (ii)-(3), (iii)-(1), (iv)-(2)
(b) (i)-(3), (ii)-(4), (iii)-(1), (iv)-(1)
(c) (i)-(1), (ii)-(2), (iii)-(4), (iv)-(2)
(d) (i)-(2),(ii)-(1),(iii)-(3),(iv)-(4)

Q.65 $\lim_{x\to\infty}\left(\sqrt{x^2+1}-\sqrt{x+1}\right)$ equals to
(a) 0
(b) ∞
(c) 1
(d) e

Q.66 First rule for maximum and minimum values of y = f(x) are as follows.

Get $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. Solve $\frac{dy}{dx} = 0$ and consider its roots. These are the value of x which make $\frac{dy}{dx} = 0$. For each of these values of x, calculate the corresponding value of y, and examine the sign of $\frac{d^2y}{dx^2}$. Now pick the correct statement from the following alternatives.
(a) It the sign is +, the corresponding value of y is maximum
(b) If the sign is –, the corresponding value of y is maximum
(c) Can not be determined
(d) None of these

Q.67 The function sin x (1+cos x) is maximum in the interval $(0,\pi)$, when
(a) $x = \frac{5\pi}{4}$
(b) $x = \frac{3\pi}{2}$
(c) $x = \frac{\pi}{3}$
(d) $x = \frac{2\pi}{3}$

Q.68 The absolute maximum of $f(x) = x^4 - 2x^3 - x^2 - 4x + 3$ on the interval [0,4] is
(a) 80
(b) 36
(c) –18
(d) None of these

Q.69 The absolute minimum of $f(x) = \frac{x^3}{(x+2)}$ on the interval [–1,1] is
(a) 0
(b) –1
(c) 1/3
(d) None of these

Q.70 The limiting value of

$\frac{1^2}{n^3}+\frac{2^2}{n^3}+\frac{3^2}{n^3}+........+\frac{n^2}{n^3}$ as $n\to\infty$ is
(a) 1
(b) $\frac{1}{2}$
(c) $\frac{1}{3}$
(d) $\frac{1}{4}$

Q.71 The value of the coefficients' such that the function f defined as

$$f(x) = ax^2-2 \quad x \le 1$$
$$= -1 \quad x \ge 1$$

is continuous at
(a) a = 1
(b) a = 2
(c) a = 0
(d) a = 3

Q.72 The maximum value of

$5\cos\theta + 3\cos\left(\theta+\frac{\pi}{3}\right)+3$ is

(a) 5

(b) 10

(c) 11

(d) –1

Q.73 Use Lagrange's mean value theorem for $f(x) = e^x$. Which of the following is true?

(a) $1 + x < e^x < 1 + xe^x$

(b) $1 + xe^x < e^x < 1 + x$

(c) $e^x < 1 + xe^x < 1 + x$

(d) None of these

Q.74 Evaluate $I = \int_0^{\pi/2} \sin 2x \log \tan x \, dx$

(a) 0

(b) $-\pi\log 2$

(c) $\pi\log 2$

(d) $\pi/4 \log 2$

Q.75 The function

$$f(x) = \begin{cases} ax^2+b & \text{for } x>2 \\ 2 & x=2 \\ 2ax-b & x<2 \end{cases}$$

is continuous at x = 2 if

(a) a = 0, b = 1/2

(b) a = 1/2, b = 0

(c) a = –1/2, b = 0

(d) b = –1/2, a = 0

Q.76 The value of $\lim\limits_{x\to 0}\left[\frac{n\sin x}{x}\right]$, where [,] is the greatest integer function

(a) n + 1

(b) n

(c) n – 1

(d) none of these

Q.77 The area between the curves $y = 1/x$ and $y = \frac{1}{x+1}$ to right of line x = 1 is

(a) ln 3

(b) ln 2

(c) 2 ln 3

(d) 2 ln 2

Q.78 If $f(x) = \begin{Bmatrix} x-1 & 1\le x\le 2 \\ 2x-3 & 2\le x\le 3 \end{Bmatrix}$

Tick the following alternative which is appropriate for the above function.

(a) continues at x = 1

(b) continuous at x = 2

(c) discontinuous at x = 1

(d) discontinuous at x = 2

Q.79 Consider the following statement

Assertion (A) : The function $f(x) = x-[x]$, $x \in z$ is discontinuous at x = 1

Reason(R): Left $\lim\limits_{x\to 1} f(x) \neq$ Right $\lim\limits_{x\to 1} f(x)$

Of these statements

(a) Both A and R are true and R is the correct explanation of A

(b) Both aand R are true and R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

Q.80 The minimum value of $|x^2 - 5x + 2|$ is

(a) 1

(b) 0

(c) $1/\sqrt{2}$

(d) 3

Q.81 $\lim\limits_{x\to 2} \frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}$ equal to

(a) 8

(b) –8

(c) 4

(d) –4

Q.82 The general solution of $\frac{\partial^2 f}{\partial x^2} = 0$ is

(a) f(x,y) = x g(y) + h(y)

(b) f(x,y) = y g(x) + h(x)

(c) f(x,y) = x g(y) + h (x)

(d) f(x,y) = g(x) + h(y)

Q.83 If $f(x) = e^x$ and $g(x) = e^{-x}$ then value of c in interval, [a,b] by cauchy's mean value theorem

(a) $\frac{a+b}{2}$

(b) $\sqrt{ab}$

(c) $\frac{2ab}{a+b}$

(d) none of these

Q.84 $I = \int_0^b \int_0^b y(2b+y)\,dx\,dy$

(a) $\frac{b^4}{3}$

(b) $\frac{2b^4}{3}$

(c) b^4

(d) $\frac{4b^4}{3}$

Q.85 If $I = \iint_R y^2 dA$ where R is region bounded by y = 2x, y = 5x and x = 1. Then I is equal to

(a) $\frac{39}{2}$

(b) $\frac{39}{4}$

(c) $\frac{39}{8}$

(d) $\frac{39}{16}$

Q.86 The value of integral $\int_{-2}^{2} \frac{dx}{x^2}$ is

(a) 0

(b) 0.25

(c) 1

(d) ∞

Q.87 The minimum point of the function $f(x) = (x^2/3) - x$ is at

(a) x = 1

(b) x = –1

(c) x = 0

(d) $x = \frac{1}{\sqrt{3}}$

Q.88 The function, $y = x^2 + \frac{250}{x}$ at x = 5 attains

(a) Maximum

(b) Minimum

(c) Neither

(d) 1

Q.89 If f(x) = |x|, then f(x) is

(a) discontinuous at x = 0

(b) continuous only at x = 0

(c) continuous at all values of x

(d) discontinuous at x = 1

Q.90 The interval in which the lagrange's theorem is applicable of the function $f(x) = \frac{1}{x}$ is

(a) [–3,3]

(b) [–2,2]

(c) [2, 3]

(d) [–1,1]

Q.91 If f(x) = |x|, then the interval [–1,1], f (x) is

(a) satisfied all the conditions of rolle's theorem

(b) satisfied all the conditions of Mean Value Theorem

(c) does not satisfied the conditions Mean value theorem

(d) none of these

Q.92 The maximum slope of the curve $-x^3 + 6x^2 + 2x + 1$ is

(a) 14

(b) 16

(c) 19

(d) −13

Q.93 The function $f(x) = x^5 - 5x^4 + 5x^3 - 1$ has

(a) one minima and two maxima

(b) two minima and one maxima

(c) two minima and two maxima

(d) one minima and one maxima

Q.94 $\int_0^{\pi/2}\int_0^{\pi/2} \sin(x+y)\,dx\,dy$ is

(a) 2

(b) $\pi/2$

(c) π

(d) 0

Q.95 The greatest and the least value of $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 1$ in [0, 2] are

(a) 1, −8

(b) 1, 8

(c) 0, −8

(d) 0, 8

Q.96 If $f(x) = x^3 - 6x^2 + 11x - 6$ is on {1, 3}, then the point $c \leftrightarrow]1, 3[$ such that $f'(c) = 0$ is given by

(a) $c = 2 \pm \frac{1}{2}$

(b) $c = 2 \pm \frac{1}{\sqrt{3}}$

(c) $c = 2 \pm \frac{1}{\sqrt{2}}$

(d) none of these

Q.97 If Rolle's theorem holds for $f(x) = x^3 - 6x^2 + kx + 5$ on [1, 3] with $c = 2 + \frac{1}{\sqrt{3}}$, the value of k is

(a) 11

(b) 7

(c) 3

(d) −3

Q.98 Let $f(x) = \sqrt{x^2 - 4}$ be defined in [2, 4]. Then, the value of c of the mean value theorem is

(a) $2\sqrt{3}$

(b) $\sqrt{3}$

(c) $\sqrt{6}$

(d) $-\sqrt{6}$

Q.99 If $u = \tan^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = ?$

(a) 2 tan 2u

(b) $\frac{1}{4}\tan u$

(c) $\frac{1}{4}\sin 2u$

(d) 2 cos 2u

Q.100 If $u = \phi\left(\frac{y}{x}\right) + x\Psi\left(\frac{y}{x}\right)$, then the value of

$$x^2\frac{\partial^2 u}{dx^2} + 2xy\frac{\partial^2 u}{dx\,dy} + y^2\frac{\partial^2 u}{\partial y^2}$$, is

(a) −u

(b) 2u

(c) u

(d) 0

Q.101 If $z = e^x \sin y$, $x = \log_e t$ and $y = t^2$, then $\frac{dz}{dt}$ is given by the expression

(a) $\frac{e^x}{t}(\cos y - 2t^2 \sin y)$

(b) $\frac{e^x}{t}(\cos y + 2t^2 \sin y)$

(c) $\frac{e^x}{t}(\sin y + 2t^2 \cos y)$

(d) $\frac{e^x}{t}(\sin y - 2t^2 \cos y)$

Q.102 For what value of $x\left(0 \le x \le \frac{\pi}{2}\right)$, the function $z = \frac{1}{x} + \tan x$ has a maxima?

(a) cos x

(b) cot x

(c) 0

(d) tan x

Q.103 $\int \frac{dx}{\sqrt{2x^2 + 3x + 4}}$ is equal to

(a) $\frac{1}{\sqrt{2}} \cosh^{-1} \frac{4x+3}{\sqrt{23}}$

(b) $\frac{1}{\sqrt{2}} \sinh^{-1} \frac{4x+3}{\sqrt{23}}$

(c) $\frac{1}{\sqrt{2}} \sin^{-1} \frac{4x+3}{\sqrt{23}}$

(d) None of these

Q.104 $\int \frac{1}{(x+1)\sqrt{1-2x-x^2}} dx$ is equal to

(a) $-\frac{1}{\sqrt{2}} \cosh^{-1}\left(\frac{\sqrt{2}}{1+x}\right)$

(b) $-\sqrt{2} \cosh^{-1}\left(\frac{\sqrt{2}}{1+x}\right)$

(c) $\frac{1}{\sqrt{2}} \cosh^{-1}\left(\frac{\sqrt{2}}{1+x}\right)$

(d) $\sqrt{2} \cosh^{-1}\left(\frac{\sqrt{2}}{1+x}\right)$

Q.105 The value of $\int_0^{2a} \frac{f(x)}{f(x) + f(2a-x)} dx$ is

(a) 2a

(b) a

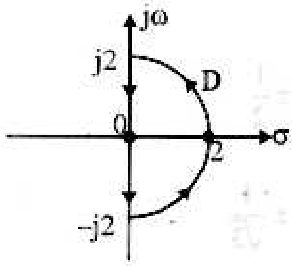
(c) 1

(d) 0

LEVEL–3

Q.106 If the semi circular contour D of radius 2 is as shown in the figure, then the value of the integral

$\oint_D \frac{1}{(s^2 - 1)} ds$ is



(a) jπ

(b) –jπ

(c) –π

(d) π

Q.107 The value of the contour integral

$$\oint_{|z-j|=2} \frac{1}{z^2+4} dz$$ in positive sense is

(a) $j\pi/2$

(b) $-\pi/2$

(c) $-j\pi/2$

(d) $\pi/2$

Q.108 A window formed by a rectangle surrounded by a semicircle have a fixed parameter p. Find the dimensions that will admit the most light.

(a) $\ell = \frac{p}{2+\pi}, b = \frac{p}{4+\pi}$

(b) $\ell = b = \frac{p}{e+\pi}$

(c) $\ell = b = \frac{p}{4+\pi}$

(d) none of these

Q.109 If $z = e^{y/x}$ then

(a) $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y} = 0$

(b) $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial z} = 2$

(c) $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y} = 2$

(d) $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y} = -2$

Q.110 Find the general solution of $\frac{\partial^2 f}{\partial x \partial y} = 1$ is

(a) $f(x, y) = yA(x) + B(y)$

(b) $f(x, y) = A(x) + B(y) + xy$

(c) $f(x, y) = A(x) + B(y)$

(d) none of these

Q.111 Let f(x) be defined as follows:

$f(x) = x^6, \; x^2 > 1$

$\quad\quad\;\; x^3, \; x^2 \le 1$

Then f(x) is

(a) discontinuous at $x = -1$

(b) not differentiable at $x = 1$

(c) (a) and (b) both

(d) neither (a) nor (b).

Q.112 If $u = e^{xyz}$, then $\frac{\partial^3 u}{\partial x\, \partial y\, \partial z}$ is equal to

(a) $e^{xyz}[1 + 3xyz + x^2y^2z^2]$

(b) $e^{xyz}[1 + xyz + x^3y^3z^3]$

(c) $e^{xyz}[1 + xyz + 3x^2y^2z^2]$

(d) $e^{xyz}[1 + 3xyz + x^3y^3z^3]$

Q.113 $\int \frac{dx}{\sin(x-a)\sin(x-b)}$ is equal to

(a) $\sin(x - a) \log \sin(x - b)$

(b) $\log\sin\left(\frac{x-a}{x-b}\right)$

(c) $\sin(a-b)\log\left\{\frac{\sin(x-a)}{\sin(x-b)}\right\}$

(d) $\frac{1}{\sin(a-b)}\log\left\{\frac{\sin(x-a)}{\sin(x-b)}\right\}$

Q.114 The height h and radius r of cylinder of greatest volume that can be cut from a sphere of radius b is

(a) $h = \frac{2b}{\sqrt{3}}, r = b\sqrt{\frac{2}{3}}$

(b) $h = \frac{b}{\sqrt{3}}, r = b\sqrt{\frac{2}{3}}$

(c) $h = \frac{b}{2\sqrt{3}}, r\frac{b}{\sqrt{6}}$

(d) $h = \frac{2b}{\sqrt{3}}, r = b\sqrt{\frac{1}{3}}$

Q.115 For $f(x) = x^3$ suppose $f(b) - f(a) = (b-a)f'(c)$ holds then c is

(a) $\frac{1}{2}\sqrt{a^2+b^2}$

(b) $\frac{a+b}{2}$

(c) $\sqrt{\frac{a^2+b^2+ab}{3}}$

(d) $\sqrt{\frac{a^2+b^2-ab}{3}}$

Q.116 If $f(x) = \frac{1}{x^2+x+1}$, then $f^{36}(0)$ is

(a) $-36!$

(b) $36!$

(c) 2^{36}

(d) none of these

Q.117 $I = \int_0^{\pi/2} \cos^4 x \sin^3 dx$

(a) $\frac{1}{35}$

(b) $\frac{2}{35}$

(c) $\frac{3}{35}$

(d) $\frac{4}{35}$

Q.118 Evaluate $I = \int \frac{d\theta}{\sin^4\theta + \cos^4\theta}$

(a) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan^2\theta - 1}{\sqrt{2}\tan\theta}\right)$

(b) $\tan^{-1}\left(\frac{\tan^2\theta - 1}{\tan\theta}\right)$

(c) $\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{2}\tan\theta}\right)$

(d) $\tan^{-1}\left(\frac{1}{\sqrt{2}\tan\theta}\right)$

Q.119 The function $f(x)\ \sqrt{1-\sqrt{1-x^2}}$ is

(a) not continuous at $x = 0$

(b) is continuous but not differentiable at $x = 0$

(c) differentiable at $x = 0$

(d) none of these

Q.120 The greatest and least value of

$\frac{x^3}{3} - \frac{3x^2}{2} + 2x$ in $[0,2]$ are

(a) $\frac{5}{6}$ and $\frac{2}{3}$

(b) $\frac{11}{6}$ and $\frac{1}{6}$

(c) $\frac{8}{3}$ and $\frac{2}{3}$

(d) $\frac{13}{3}$ and $\frac{1}{3}$

Q.121 A point P is moving along curve of intersection of parabola $\frac{x^2}{16} - \frac{y^2}{9} = z$ and cylinder $x^2 + y^2 = 5$.

If x is increasing at the rate of 5cm/s how fast is z changing when x = 2m and y =1 cm?

(a) $\frac{25}{36}$

(b) $\frac{85}{36}$

(c) $\frac{144}{25}$

(d) $\frac{18}{25}$

Q.122 The area included between curve $r = a\ (\sec\theta + \cos\theta)$ and its asymptote $r = a \sec\theta$ is

(a) πa^2

(b) $\frac{5\pi a^2}{4}$

(c) $\frac{2\pi a^2}{3}$

(d) $2\pi a^2$

Q.123 $I = \int_0^{\pi}\int_0^{a}(1+\cos\theta)r^4\cos\theta\, dr\, d\theta$

(a) $\frac{5}{16}\pi a^2$

(b) $\frac{a^5\pi}{10}$

(c) $\frac{3}{16}\pi a^5$

(d) $\frac{1}{4}\pi a^4$

Q.124 $I = \int_0^{\pi}\int_0^{a(1+\cos\theta)} 2\pi r^4 \sin^3\theta\, dr\, d\theta$

(a) $\frac{2}{35}\pi a^2$

(b) $\frac{2^4}{35}\pi a^5$

(c) $\frac{2^6\pi a^5}{35}$

(d) $\frac{2^8\pi a^5}{35}$

Q.125 The value of $I = \iint_R \frac{1}{\sqrt{2y-y^2}}\, dA$ where R is region in the first quadrant bounded by $x^2 = 4 - 2y$ is

(a) 0

(b) 2

(c) 4

(d) −1

Q.126 $I = \int_0^1\int_{x^2}^{\sqrt{x}}\int_0^{2-x-y} dx\, dy\, dz$

(a) $\frac{11}{30}$

(b) $\frac{13}{30}$

(c) $\frac{1}{8}$

(d) 16

Q.127 The area bounded by the parabola $2y = x^2$ and the line $x = y - 4$ is equal to

(a) 6

(b) 18

(c) ∞

(d) None of these

Q.128 The sum of the perimeters of a circle and a square is 1. If the sum of area is least, then

(a) side of the square is double the radius of the circle

(b) side of the square is 1/2 of the radius of the circle

(c) side of the square is equal to the radius of the circle

(d) none of these

Q.129 If $z = \sqrt{x^2+y^2}$ and $x^3 + y^3 + 3axy = 5a^2$, then at $x = a$, $y = a$, $\frac{dz}{dx}$ is equal to

(a) a^3

(b) $2a^2$

(c) 0

(d) 2a

Q.130 Match the List-I with List-II.

List I		List II	
(i)	if $u = \frac{x^2y}{x+y}$ then $x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial x \partial y}$	(1)	$-\frac{3}{16}u$
(ii)	if $u = \frac{x^{1/2} - y^{1/2}}{x^{1/4} - y^{1/4}}$ then $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$	(2)	$\frac{\partial u}{\partial x}$
(iii)	if $u = x^{1/2} + y^{1/2}$ then $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$	(3)	0
(iv)	if $u = f\left(\frac{y}{x}\right)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$	(4)	$-\frac{1}{4}u$

Correct match is –

	(i)	(ii)	(iii)	(iv)
(a)	1	2	4	3
(b)	2	1	3	4
(c)	2	1	4	3
(d)	1	2	3	4

Q.131 The value of $\int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx$ is

(a) $e^x \cot\frac{x}{2} + c$

(b) $e^x + \cot\frac{x}{2}$

(c) $e^x f(x)$

(d) $e^x \tan\frac{x}{2} + c$

Q.132 $\int_0^{\pi/2} \frac{e^x}{2}\left(\sec^2\frac{x}{2} + 2\tan\frac{x}{2}\right) dx$ is equal to

(a) $e^{\frac{\pi}{4}}$

(b) e

(c) $e^{\frac{\pi}{2}}$

(d) e^x

Q.133 The value of $\int_0^1 \int_0^{\sqrt{1+x^2}} dy\,dx$ is

(a) $\frac{\pi}{2}\log\left(\sqrt{2}+1\right)$

(b) $\frac{\pi}{4}\log\left(\sqrt{2}-1\right)$

(c) $\frac{\pi}{4}\log\left(\sqrt{2}+1\right)$

(d) None of these

Q.134 The area of the cardioid $r = a(1 + \cos\theta)$ is given by

(a) $\int_0^{\pi}\int_{r=0}^{a(1+\cos\theta)} r\,dr\,d\theta$

(b) $\int_0^{\pi/2}\int_{r=0}^{a(1+\cos\theta)} r\,dr\,d\theta$

(c) $2\int_0^{\pi}\int_{r=a}^{a(1+\cos\theta)} r\,dr\,d\theta$

(d) $2\int_{\theta=0}^{\pi/2}\int_{r=a}^{a(1+\cos\theta)} r\,dr\,d\theta$

GATE QUESTIONS

Q.135 Which of the following predicate calculus statements (s) is/are valid? **[Gate 1992]**

(a) $(\forall x) P(x) \vee (\forall x) Q(x) \to (\forall x) \{P(x) \vee Q(x)\}$

(b) $(\exists x) P(x) \vee (\exists x) Q(x) \to (\exists x) \{P(x) \wedge Q(x)\}$

(c) $(\forall x) \{P(x) \vee Q(x)\} \to (\forall x) P(x) \vee (\forall x) Q(x)$

(d) $(\exists x) \{P(x) \vee Q(x)\} \to \sim (\forall x) P(x) \vee (\exists x) Q(x)$

Q.136 The differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} + \sin y = 0$ is **[Gate 1993]**

(a) linear

(b) non-linear

(c) homogeneous

(d) of degree two

Q.137 Backward Euler method for solving the differential equation $\frac{dy}{dx} = f(x, y)$ is specified by, **[Gate 1994]**

(a) $y_{n+1} = y_n + hf(x_n, y_n)$

(b) $y_{n+1} = y_n + hf(x_{n+1}, y_{n+1})$

(c) $y_{n+1} = y_{n-1} + 2hf(x_n, y_n)$

(d) $y_{n+1} = (1 + h) f(x_{n+1}, y_{n+1})$

Q.138 In the interval $[0, \pi]$ the equation $X = \cos x$ has **[Gate 1995]** **[1-Mark]**

(a) No solution

(b) Exactly one solution

(c) Exactly two solution

(d) An infinite number of solutions

Q.139 The solution of differential equation $y'' + 3y' + 2y = 0$ is the form. **[Gate 1995]**

(a) $C_1 e^{x} + C_2 e^{2x}$

(b) $C_1 e^{-x} + C_2 e^{3x}$

(c) $C_1 e^{-x} + C_2 e^{-2x}$

(d) $C_1 e^{-2x} + C_2 e^{-x}$

Q.140 Let R denotes the set of real numbers. Let $f : R \times R \to R \times R$ be a bijective function defined by $f(x, y) = (x + y, x - y)$. The inverse function of f is given by **[Gate 1996]**

(a) $f^{-1}(x,y) = \left(\frac{1}{x+y}, \frac{1}{x-y}\right)$

(b) $f^{-1}(x, y) = (x - y, x + y)$

(c) $f^{-1}(x,y) = \left(\frac{x+y}{2}, \frac{x-y}{2}\right)$

(d) $f^{-1}(x, y) = [2(x - y), 2(x + y)]$

Q.141 The formula used to compute an approximation for the second derivative of a function f at a point X_0 is **[Gate 1996]**

(a) $\frac{f(x_0+h)+f(x_0-h)}{2}$

(b) $\frac{f(x_0+h)-f(x_0-h)}{2h}$

(c) $\frac{f(x_0+h)+2f(x_0)+f(x_0-h)}{h^2}$

(d) $\frac{f(x_0+h)-2f(x_0)+f(x_0-h)}{h^2}$

Q.142 Consider the function $y = |x|$ in the interval $[-1, 1]$. In this interval, the function is **[Gate 1998]**

(a) Continuous and differentiable

(b) Continuous and not differentiable

(c) Differentiable but not continuous

(d) Neither continuous nor differentiable

Q.143 $\lim\limits_{x \to \infty} \dfrac{x - \sin x}{x + \cos x}$ equals **[Gate 2008]** **[1-Mark]**

(a) 1

(b) −1

(c) ∞

(d) −∞

Q.144 $\int_0^{\pi/4} (1 - \tan x)/(1 + \tan x)dx$ evaluates to **[Gate 2009]** **[2-Marks]**

(a) 0

(b) 1

(c) ln 2

(d) 1/2 ln 2

ANSWER KEY

1	c	2	a	3	c	4	b	5	a
6	b	7	c	8	d	9	a	10	a
11	b	12	b	13	d	14	b	15	c
16	a	17	b	18	c	19	b	20	a
21	b	22	a	23	a	24	d	25	d
26	a	27	a	28	b	29	d	30	a
31	a	32	d	33	d	34	d	35	b
36	d	37	a	38	a	39	b	40	a
41	c	42	a	43	d	44	a	45	c
46	a	47	b	48	b	49	a	50	a
51	c	52	a	53	b	54	d	55	c
56	a	57	d	58	a	59	c	60	a
61	a	62	a	63	c	64	a	65	b
66	b	67	c	68	d	69	b	70	c
71	d	72	b	73	a	74	a	75	b
76	c	77	b	78	b	79	a	80	b
81	b	82	a	83	a	84	d	85	b
86	c	87	a	88	b	89	c	90	c
91	c	92	a	93	d	94	a	95	a
96	b	97	a	98	c	99	c	100	d
101	c	102	a	103	b	104	a	105	b
106	a	107	d	108	c	109	a	110	b
111	c	112	a	113	d	114	a	115	c
116	b	117	b	118	a	119	b	120	a
121	b	122	b	123	b	124	c	125	c
126	a	127	b	128	a	129	c	130	c
131	d	132	c	133	d	134	d	135	a,d
136	b	137	a	138	b	139	d	140	c
141	d	142	b	143	a	144	d		

SOLUTIONS

S.1 (c)

$$y = x + 1$$

$$I = \int_1^2 y\,dx$$

$$= \int_1^2 (x+1)\,dx = \left.\frac{(x+1)^2}{2}\right|_1^2$$

$$= \frac{1}{2}(9-4) = \mathbf{2.5}$$

S.2 (a)

$$\lim_{\theta\to 0}\frac{\frac{1}{2}\times\sin\left(\frac{\theta}{2}\right)}{\theta\times\frac{1}{2}} = \frac{1}{2}\lim_{\theta\to 0}\frac{\sin\theta/2}{\theta/2} = \frac{1}{2} = \mathbf{0.5}$$

S.3 (c)

$$\int_0^{\pi}\sin^3\theta\,d\theta = \int_0^{\pi}\left(1-\cos^2\theta\right)\sin\theta\,d\theta$$

Let $\cos\theta = t \Rightarrow -\sin\theta\,d\theta = dt$,

$\theta = 0,\ \cos 0 = 1 = t$

$\theta = \pi,\ \cos\pi = -1 = t$

$$= \int_{+1}^{-1} -\left(1-t^2\right)dt$$

$$= \int_{-1}^{+1}\left(1-t^2\right)dt \quad \left[\because \int_a^b f(x)\,dx = -\int_b^a f(x)\,dx\right]$$

$$= \left|t-\frac{t^3}{3}\right|_{-1}^{+1} = \left(1-\frac{1}{3}\right)-\left(-1+\frac{1}{3}\right)$$

$$I = \frac{2}{3}+\frac{2}{3} = \mathbf{\frac{4}{3}}$$

S.4 (b)

Here $f(0) = 0$

Left Hand Limit

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\left(\frac{|x|}{x}\right)$$

$$= \lim_{x\to 0^+}\left(\frac{x}{x}\right) = 1 \quad \left[\because |x| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0\end{cases}\right]$$

Right Hand Limit

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}\left(\frac{|x|}{x}\right)$$

$$= \lim_{x\to 0^-}\left(\frac{-x}{x}\right)$$

$$= -1$$

$\therefore$ R.H.L. $\neq$ L.H.L.

The function f(x) is discontinuous at x = 0 and the discontinuity at x = 0 is of first kind because R.H.L. and L.H.L. both exist finitely but are unequal.

S.5 (a)

$$f(x) = x + \frac{1}{x}$$

$$f'(x) = 1 - \frac{1}{x^2}$$

$$f''(x) = \frac{2}{x^3}$$

for maximum and minimum $f'(x) = 0$

or $\quad 1 - \frac{1}{x^2} = 0$

so $\quad x = \pm 1$

$\quad f''(-1) = -2 < 0$

& $\quad f''(1) = 2 > 0$

$\therefore$ f(x) is maximum at **x = –1.**

S.6 (b)

$$A = \lim_{x\to 0}\left(\frac{1}{x}\right)^{\tan x}$$

$$\Rightarrow \log_e A = \lim_{x\to 0}\tan x\log\frac{1}{x}$$

$$= \lim_{x\to 0}\frac{\log 1/x}{\cot x} = \lim_{x\to 0}\frac{x(-1)}{x^2(-\operatorname{cosec}^2 x)}$$

[By L'Hospital rule]

$$= \lim_{x\to 0}\frac{\sin^2 x}{x} = 0$$

$$\log_e A = 0 \Rightarrow A = e^0 = \mathbf{1}$$

S.7 (c)

Integrating by parts

$$I = \int x^2 \sin x\,dx$$

$= -x^2\cos x + \int 2x\cos x\,dx$

$= -x^2\cos x + 2x\sin x - 2\int \sin x\,dx$

$= -x^2\cos x + 2x\sin x + 2\cos x\Big|_0^{\pi/2}$

$= \pi - 2$

S.8 (d)

Let $f(x) = \frac{(\sin x - \cos x)}{\sqrt{2}} = \sin(x - \pi/4)$

since maximum value of $\sin\theta$ is 1

$\therefore$ maximum is **1**.

S.9 (a)

$\frac{0}{0}$ form

So applying L' hospital rule

$\lim_{x\to 8} \frac{\frac{1}{2\sqrt{r}}}{1} = \frac{1}{2\sqrt{8}}$

$= \frac{1}{4\sqrt{2}}$

S.10 (a)

$I = \int_0^{\pi/4} \cos^2 x\,dx = \int_0^{\pi/4} \frac{1+\cos 2x}{2}dx$

$= \int_0^{\pi/4}\frac{dx}{2} + \int_0^{\pi/4}\frac{\cos 2x}{2}dx$

$= \frac{1}{2}[x]_0^{\pi/4} + \frac{1}{2}\left[\frac{\sin 2x}{2}\right]_0^{\pi/4}$

$= \frac{\pi}{8} + \frac{1}{4}\left[\sin\frac{\pi}{2} - \sin x\right] = \frac{\pi}{8} + \frac{1}{4}\times 1$

$= \frac{\pi}{8} + \frac{1}{4}$

S.11 (b) $\int_0^{5\pi}(2 - \sin x)dx = [2x + \cos x]_0^{5\pi}$

$= 10\pi - 1 - 1 = 10\pi - 2 > 0$

S.12 (b) $\lim_{a\to\infty}\int_1^a x^{-4}dx = \lim_{a\to\infty}\int_1^a\left[-\frac{1}{3}\left(x^{-3}\right)_1^a\right]$

$= \lim_{a\to\infty} -\frac{1}{3}\left[a^{-3} - 1\right] = -\frac{1}{3}[0 - 1] = \frac{1}{3}$

S.13 (d)

Since statement (a), (b), (c) are all correct hence the option (d) is answer.

S.14 (b)

Given that $u = x^m y^n$

Taking logarithm of both sides, we get

$\log u = m \log x + n \log y$

Differentiating with respect to x, we get

$\frac{1}{u}\frac{du}{dx} = m.\frac{1}{x} + n.\frac{1}{y}\frac{dy}{dx}$

$\Rightarrow \frac{du}{u} = m\frac{dx}{x} + n.\frac{dy}{y}$

S.15 (c)

$f'(x) = 9x^8 + 21x^6 > 0$ for all non-zero real values of x.

S.16 (a)

$f'(x) = -x^2e^{-x} + 2xe^{-x} = e^{-x}x(2 - x)$.

Clearly, $f'(x) > 0$ when $x > 0$ and $x < 2$.

S.17 (b)

$\int_0^1 |5x - 3|dx = -\int_0^{3/5}(5x - 3)dx + \int_{3/5}^1(5x - 3)dx$

$= \left(-\frac{5}{2}x^2 + 3x\right)_0^{3/5} + \left(\frac{5x^2}{2} - 3x\right)_{3/5}^1$

$= \left(-\frac{9}{10} + \frac{9}{5}\right) + \left[\left(\frac{5}{2} - 3\right) - \left(\frac{9}{10} - \frac{9}{5}\right)\right]$

$= \frac{9}{10} + \left(\frac{-1}{2} + \frac{9}{10}\right) = \frac{13}{10}$

S.18 (c)

Let $I = \int_0^1 \frac{e^{\sqrt{1-x^2}}}{\sqrt{1-x^2}}x\,dx$

Put $\sqrt{1-x^2} = t$

$\Rightarrow \frac{1}{2\sqrt{1-x^2}}(-2x)dx = dt$

when $x = 0$, $t = 1$, when $x = 1$, $t = 0$

$I = \int_1^0 -e^t dt = -[e^t]_1^0 = -[e^0 - e^1] = e - 1$

S.19 (b)

$$\int_1^0 \frac{-x}{x}dx + \int_0^1 \frac{1-x}{x}dx$$

$$= -\int_{-1}^0 1dx + \int_0^1 1.dx$$

$$= -(x)_{-1}^0 + (x)_0^1$$

$$= -(0+1) + (1-0) = \mathbf{0}$$

S.20 (a)

$$\lim_{x\to 0}\frac{x^2}{1-\cos x} = 2\lim_{x\to 0}\left(\frac{(x/2)}{\sin(x/2)}\right)^2 = 2$$

S.21 (b)

$$\lim_{x\to\infty} x^n e^{-x} = \lim_{x\to\infty}\frac{x^n}{e^x}\left[\text{from}\frac{\infty}{\infty}\right]$$

By L Hospital rule differentiating denominator and numerator 'n' times, we get

$$\lim_{x\to\infty}\frac{x^n}{e^x} = \lim_{x\to\infty}\frac{\lfloor n}{e^x} = 0$$

S.22 (a) Put $\frac{\pi}{2} - \theta = y$, so that as $\theta \to \frac{\pi}{2}$, $y \to 0$

$$= \lim_{y\to 0}\frac{\tan y}{y}$$

$$= 1$$

S.23 (a)

$f(x) = x^3 - 12x^2 + 45x + 11$

$f'(x) = 3x^2 - 24x + 45$

$f''(x) = 6x - 24$

$f'(x) = 0$ or $3x^2 - 24x + 45 = 0$

$x = 3, x = 5$

$f''(5) = (6 \times 5) - 24$

$= 6 > 0$

$\therefore$ f(x) is minimum at **x = 5.**

S.24 (d)

L.H.S. $\lim_{x\to 2^-}\frac{|x-2|}{x-2} = +1$

R.H.S $\lim_{x\to 2^+}\frac{|x-2|}{x-2} = -1$

$\because$ L.H.S. $\neq$ R.H.S

So does not exist

S.26 (a)

$$I = m\int_0^\pi \sin 2Kx \cdot \operatorname{cosec} x \cdot dx$$

It is an odd function, so **I = 0**.

S.27 (a)

Put $1 + x = y$, so that $x \to 0 \Rightarrow y \to 1$

$$\frac{(1+x)^n - 1}{x} = \frac{y^n - 1}{y-1}$$

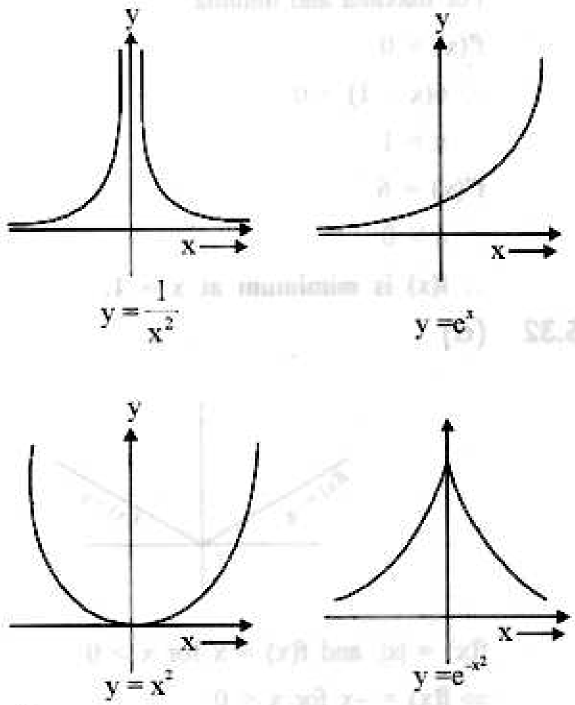
$$\therefore \lim_{x\to 0}\frac{(1+x)^n - 1}{x}$$

$$= \lim_{y\to 1}\frac{y^n - 1}{y-1}$$

$$= n(1)^{n-1}$$

$$= \mathbf{n}$$

S.29 (d)



Alternate:

We have, $\lim_{x\to 0}\frac{1}{x^2} = \infty$

$$\lim_{x\to\infty} x^2 = \infty$$

$$\lim_{x\to\infty} e^{-x^2} = 0$$

$$\lim_{x\to 0} e^{-x^2} = 1$$

Thus, e^{-x^2} is strictly bounded.

So, (d) is correct option.

S.30 (a) $\int_0^{5\pi} (2 - \sin \pi)\, dx = \int_0^{5\pi} 2.dx$

$= 2[x]_0^{5\pi}$

$= 2[5\pi]$

$= \mathbf{10\pi > 0}$

S.31 (a) $f(x) = 3x(x - 2)$

$= 3x^2 - 6x$

$f'(x) = 6x - 6$

$f''(x) = 6$

For maxima and minima

$f'(x) = 0$

$\therefore\ 6(x - 1) = 0$

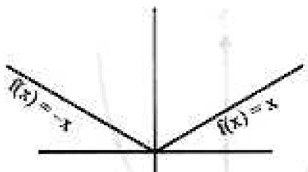
$x = 1$

$f''(x) = 6$

$6 > 0$

$\therefore$ **f(x) is mimimum at x = 1.**

S.32 (d)



$f(x) = |x|$, and $f(x) = x$ for $x > 0$

$\Rightarrow f(x) = -x$ for $x < 0$

Hence from graph it is clear that derivative does not exist for $x = 0$.

S.33 (d)

$f'(x) = 3(x^2 - 4x + 3)$, $f'(x) = 0 \Rightarrow x = 1, 3$

At $x = 1$, $f''(1) = -ve$; and

at $x = 3$, $f''(3) = +ve$

S.34 (d)

$\phi(x) = \int_0^{x^2} \sqrt{t}dt$

let $t = p^2 \Rightarrow dt = 2p.dp$

when $t = 0$, $p = 0$

$t = x^2$, $p = x$

So $\phi(x) = \int_0^x 2p.\sqrt{p^2}.dp$

$= \int_0^x 2p^2.dp$

$= 2\left[\frac{p^3}{3}\right]_0^x = \frac{\mathbf{2x^3}}{\mathbf{3}}.$

S.35 (b)

$f(x) = 2x^3 - 15x^2 + 36x + 10$

$f'(x) = 6x^2 - 30x + 36$ .....(i)

$f''(x) = 12x - 30$ .....(ii)

For maxima and minima, $f'(x) = 0$

$\therefore\ 6x^2 - 30x + 36 = 0$

$\Rightarrow x^2 - 5x + 6 = 0$

$\Rightarrow x = 3, 2$

Putting these values of x in equation (ii), we get

$f''(3) = 36 - 30 = +6$ positive

Hence **minimum value at 3**

$\therefore$ $f''(2) = 24 - 30 = -6$ negative, hence **maximum value at 2**

S.36 (d)

Let $f(x) = \sqrt{x-2}$. For each $x \leftrightarrow [2, 3]$, f(x) has a definite and unique value.

So, f(x) is continuous on [2, 3].

Also, $f'(x) = \frac{1}{2\sqrt{x-2}}$,

which exists for all $x \leftrightarrow]2, 3[$

f(x) is differentiable in]2, 3[.

So, there exists $c \leftrightarrow]2, 3[$ such that

$f'(c) = \frac{f(3) - f(2)}{(3-2)} = 1 \Rightarrow \frac{1}{2\sqrt{c-2}} = 1$

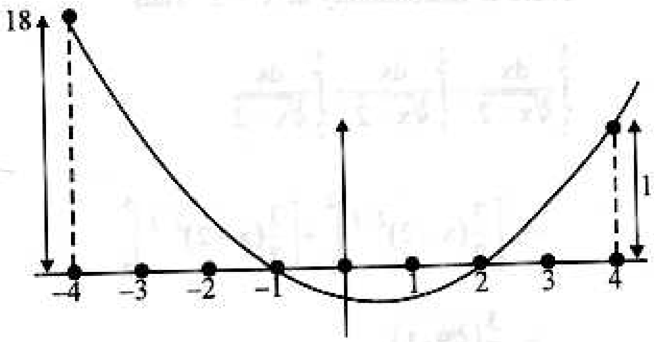
$c = \frac{9}{4} \leftrightarrow]2,3[$

$x = \frac{9}{4}$ and $y = \sqrt{x-2} \Rightarrow y = \frac{1}{2}$

Thus required point is $\left(\frac{\mathbf{9}}{\mathbf{4}}, \frac{\mathbf{1}}{\mathbf{2}}\right)$.

S.37 (a)

$$f(x) = x^2 - x - 2$$
$$= (x+1)(x-2)$$
$$f(-4) = 18$$
$$f(+4) = 10$$



f(x) is maximum in interval [–4,4] at x = –4

S.38 (a)

$$I = \frac{1}{\sqrt{2\pi}}\int_0^{\infty} e^{\left(-x^2/8\right)dx}$$

$$I = \frac{2\sqrt{2}}{\sqrt{2\pi}}\int_0^{\infty} e^{-z^2}dz \qquad \left[\text{Putting } \frac{x}{2\sqrt{2}} = z\right]$$

$$= \frac{2}{\sqrt{\pi}}\int_0^{\infty} e^{-z^2}dz$$

$$= \frac{2}{\sqrt{\pi}}\left[\frac{\sqrt{\pi}}{2}\right] \qquad \left[\because \int_0^{\infty} e^{-t^2}dt = \frac{\sqrt{\pi}}{2}\right]$$

$$= 1$$

S.39 (b)

$$\lim_{x\to 0}\frac{ax+b}{x+1} = \frac{\lim\limits_{x\to 0} ax+b}{\lim\limits_{x\to 0} x+1} = b \qquad .....(1)$$

But $\lim\limits_{x\to\infty} f(x) = 2$ (Given)

$\therefore$ b = 2 {from equation (1)]

Also, $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty}\frac{ax+b}{x+1}$

$$= \lim_{x\to\infty}\frac{a+(b/x)}{1+(1/x)} = a \qquad .....(2)$$

But $\lim\limits_{x\to\infty} f(x) = 1$ (Given)

$\therefore$ a = 1 (from equation (2))

$$\therefore f(x) = \frac{x+2}{x+1}$$

$$f(z) = \frac{-2+2}{-2+1} = 0$$

S.40 (a)

Put $x - \frac{\pi}{2} = t$, so that,

$$x = \frac{\pi}{2} + t \quad \text{or} \quad 2x = \pi + 2t$$

when $x \to \frac{\pi}{2}$ then $t \to 0$

$$\text{Now} \lim_{x\to(\pi/2)}\frac{1+\cos 2x}{(\pi-2x)^2} = \lim_{t\to 0}\frac{1+\cos(\pi+2t)}{(-2t)^2}$$

$$= \lim_{t\to 0}\frac{1-\cos 2t}{4t^2}$$

$$= \lim_{t\to 0}\frac{2\sin^2 t}{4t^2}$$

$$= \lim_{t\to 0}\left(\frac{\sin t}{t}\right)^2 \cdot \frac{1}{2}$$

$$= \frac{1}{2}$$

S.42 (a)

Here f(x) = 1

Also, Left Hand Limit

$$\lim_{f(x)\to 3^-} 1 = 1$$

Right Hand Limit

$$\lim_{x\to 3^+}(ax+b) = 3a+b$$

Also, f(3) = 1

Since f(x) is continuous, therefore

$3a + b = 1$ .....(1)

Also, $f(5) = 7$

and $\lim_{f(x)\to 5^-} f(x) = \lim_{x\to 5^-} (ax+b)$

$= 5a + b$

Since f (5) is continuous at x = 5, therefore

$5a + b = 7$ .....(2)

Solving equation (1) and equation (2), we get

$a = 3,\ b = -8$

S.43 (d)

$$\lim_{x\to 0} \frac{1}{1-e^{1/x}}$$

$$= \frac{1}{1-e^{\infty}} = 0$$

So limit exist

S.44 (a)

Let $f(x) = \ln x$. then $f'(x) = \frac{1}{x}$, $f''(x) = -\frac{1}{x^2}$

$f'''(x) = \frac{2}{x^3}$

$f''''(x) = \frac{-2(3)}{x^4}$ & in general

$f^n(x) = (-1)^{n+1}\frac{(n-1)!}{x^n}$

So $f(2) = \ln 2$,

$f^n(2) = (-1)^{n+1}\frac{(n-1)!}{2^n}$

Thus Taylor series is $\sum_{n=0}^{\infty}\frac{f^n(2)}{n!}(x-2)^n$

$$= \ln 2 + \left(\frac{x-2}{2}\right) - \frac{1}{8}(x-2)^2 + \ldots$$

S.45 (c)

$$I = \int \frac{x - \sin x}{1-\cos x}dx$$

$$= \int \frac{x}{1-\cos x}dx - \int \frac{\sin x}{1-\cos x}dx$$

$$= \int \frac{x}{2\sin^2 x/2}dx - \int \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2 x/2}$$

$$= \int \frac{x}{2}\operatorname{cosec}^2\frac{x}{2}dx - \int \cot\frac{x}{2}dx$$

$$= -x\cot\frac{x}{2} + C$$

S.46 (a)

There is discontinuity at x = 2. Thus

$$\int_1^5 \frac{dx}{\sqrt[3]{x-2}} = \int_1^2 \frac{dx}{\sqrt[3]{x-2}} + \int_2^5 \frac{dx}{\sqrt[3]{x-2}}$$

$$= \left[\frac{3}{2}(x-2)^{2/3}\right]_1^2 + \left[\frac{3}{2}(x-2)^{2/3}\right]_2^5$$

$$= \frac{3}{2}\left(\sqrt[3]{9}-1\right)$$

S.47 (b)

$$\lim_{x\to 0}\frac{x-\sin x}{x^3} \quad \left\{\begin{array}{l}\because \frac{0}{0}\text{form} \\ \text{so By L'Hospital rule}\end{array}\right\}$$

$$= \lim_{x\to 0}\frac{1-\cos x}{3x^2}$$

$$= \lim_{x\to 0}\frac{-\sin x}{6x} = \frac{1}{6}$$

S.48 (b)

$$f(0+0) = \lim_{h\to 0}\frac{\sin(0+h)^2}{0+h} = \lim_{h\to 0}\frac{\sin h^2}{h}$$

$$= \lim_{h\to 0}\frac{\sin h^2}{h^2}.h = 1\times 0 = 0$$

Similarly $f(0-0) = 0$

Hence f(x) is continuous at x = 0.

Now $Rf'(0) = \lim_{h\to 0}\frac{\sin(0+h)^2/(0+h) - 0}{h}$

$$= \lim_{h\to 0}\frac{\sin h^2}{h^2} = 1$$

and $Lf'(0) = \lim_{h\to 0}\frac{\sin(0-h)^2/(0-h) - 0}{-h}$

$$= \lim_{h\to 0}\frac{\sin h^2}{h^2} = 1$$

Since, $Rf'(0) = Lf'$ **the function f(x) is derivable at x = 0.**

S.49 (a)

It is easy to see that,

$f(0+0) = f(0-0) = f(0) = 0$

Hence f(x) is continuous at x = 0

Now

$$Lf'(0) = \lim_{h\to 0} (0-h)\frac{\left[\sqrt{0-h}-\sqrt{0-h+1}\right]-0}{-h}$$

$$= \lim_{h\to 0}\left[\sqrt{-h}-\sqrt{-h+1}\right]$$

$$= 0-\sqrt{1}=-1$$

and

$$Rf'(0) = \lim_{h\to 0}\frac{(0+h)\left[\sqrt{0+h}-\sqrt{0+h+1}\right]-0}{h}$$

$$= \lim_{h\to 0}\left[\sqrt{h}-\sqrt{h+1}\right]=0-\sqrt{1}=-1$$

Since, $Lf'(0) = Rf'(0) = -1$ the function **f(x) is differentiable at x = 0.**

S.50 (a)

$$I = \int \frac{1}{x^2}\sec^2\frac{1}{x}dx$$

Put, $\frac{1}{x}=t$

$$\therefore \quad -\frac{1}{x^2}dx = dt$$

$$\therefore \quad I=-\int \sec^2 t\,dt=-\tan t$$

$$= -\tan\left(\frac{1}{x}\right)+C$$

S.51 (c)

We have

$$4x^2+4x+5 = 4\left(x^2+x+\frac{5}{4}\right)$$

$$= 4\left[x^2+x+\frac{1}{4}+\frac{5}{4}-\frac{1}{4}\right]$$

$$= 4\left[\left(x+\frac{1}{2}\right)^2+1^1\right]$$

$$\therefore \; I = \frac{1}{4}\int\frac{dx}{\left(x+\frac{1}{2}\right)^2+1^2}$$

$$= \frac{1}{4}\tan^{-1}\left(x+\frac{1}{2}\right)+C$$

S.52 (a)

$$I = \int_0^{\pi/2}\frac{(\sin x+\cos x)}{\sqrt{\sin x\cos x}}dx$$

$$= \int_0^{\pi/2}\frac{\sqrt{2}(\sin x+\cos x)}{\sqrt{\sin 2x}}$$

$$= \int_0^{\pi/2}\frac{\sqrt{2}(\sin x+\cos x)}{\sqrt{1-(\sin x-\cos x)^2}}dx$$

let $\sin x - \cos = t$

$(\cos x + \sin x)\,dx = dt$

$$\int\frac{\sqrt{2}\,dt}{\sqrt{1-t^2}}$$

$$= \sqrt{2}\sin^{-1}t$$

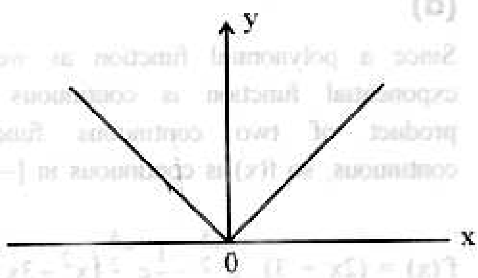
$$= \sqrt{2}\sin^{-1}(\sin x-\cos x)$$

$$\therefore \; I = \left[\sqrt{2}\sin^{-1}(\sin x-\cos x)\right]_0^{\pi/2} = \sqrt{2}.\frac{\pi}{4}$$

$$= \frac{\pi}{2\sqrt{2}}$$

S.53 (b)

$f(x) = e^{-|x|}$



from graph it is clear that f(x) is continuous but not differentiable at x = 0.

S.54 (d)

Here $f'(x) \geq 0$ in [a, b], so, f(x) is m.i. Hence, $f(a) \leq f(x) \leq f(b)$.

Therefore, $\int_a^b f(a)dx \leq \int_a^b f(x)dx \leq \int_a^b f(b)dx$

or, $f(a).(b-a) \leq \int_a^b f(x)dx \leq f(b).(b-a)$

$\therefore \; f(a) \leq \frac{1}{b-a}\int_a^b f(x)dx \leq f(b).$

S.55 (c)

From mean value theorem, $\frac{f(1)-f(0)}{1-0}$

$= f(x), x \in [0,1]$

$f(1) = 2 + \frac{1}{5-x^2}$ .....since f(0) = 2

Lower bound of $f(1) = 2 + \frac{1}{5-0} = \mathbf{2.2}$

Upper bound of $f(1) = 2 + \frac{1}{5-1} = \mathbf{2.25}$

S.56 (a)

Taylor series expansion of

$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$\therefore = 1 + x + \frac{n^2}{2} + ...$

Hence third

term = $\frac{\mathbf{x^2}}{\mathbf{2}}$

S.57 (d)

Since a polynomial function as well as an exponential function is continuous and the product of two continuous functions is continuous, so f(x) is continuous in [–3, 0].

$f'(x) = (2x+3).\, e^{-\frac{x}{2}} - \frac{1}{2}e^{-\frac{x}{2}}\left(x^2+3x\right)$

$= e^{\frac{x}{2}}\left[\frac{x+6-x^2}{2}\right]$

which clearly exists for all x ↔] –3, 0 [.

f(x) is differentiable in] 3, 0 [.

Also, f(–3) = f(0) = 0.

By Rolle's theorem c ↔] –3, 0 [such that f' (c) = 0.

Now, $f'(c) = 0 \Rightarrow e^{\frac{c}{2}}\left[\frac{c+6-c^2}{2}\right] = 0$

$c + 6 - c^2 = 0$ i.e., $c^2 - c - 6 = 0$

$\Rightarrow (c+2)(3-c) = 0 \;\Rightarrow c = -2,\; c = 3.$

Hence, **c = –2** ↔] –3, 0 [.

S.58 (a)

Let f(x) = tan x Then,

$f\left(\frac{\pi}{4}+x\right) = f\left(\frac{\pi}{4}\right) + xf'\left(\frac{\pi}{4}\right) + \frac{x^2}{2!}.f''\left(\frac{\pi}{4}\right) + ...$

$f'(x) = \sec^2$, $f''(x) = 2\sec^2 x \tan x$,

$f'''(x) = 2\sec^4 x + 4\sec^2 x \tan^2 x$ etc.

Now,

$f\left(\frac{\pi}{4}\right) = 1, f'\left(\frac{\pi}{4}\right) = 2,\; f''\left(\frac{\pi}{4}\right) = 4,\; f'''\left(\frac{\pi}{4}\right) = 16, ...$

Thus $\tan\left(\frac{\pi}{4}+x\right) = 1 + 2x + \frac{x^2}{2}.4 + \frac{x^3}{6}.16 + ...$

$= \mathbf{1 + 2x + 2x^2 + \frac{8}{3}x^3 + ...}$

S.59 (c)

The given function is homogeneous of degree 2.

Euler's theorem $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \mathbf{2z}$

S.60 (a)

$\frac{dy}{dx} = \frac{a}{x} + 2bx + 1$

$\left[\frac{dy}{dx}\right]_{(x=-1)} = 0 \Rightarrow -a - 2b + 1 = 0$

$\Rightarrow a + 2b = 1$ .....(i)

$\left[\frac{dy}{dx}\right]_{(x=2)} = 0 \Rightarrow \frac{a}{2} + 4b + 1 = 0$

$\Rightarrow a + 8b = -2$ .....(ii)

Solving (i) and (ii) we get $\mathbf{b = -\frac{1}{2}}$ and **a = 2.**

S.61 (a)

$$I = \int \frac{2x+3}{\sqrt{x^2+x+1}}.dx$$

$$I = \int \frac{2x+1}{\sqrt{x^2+x+1}}dx + \int \frac{2}{\sqrt{x^2+x+1}}.dx$$

let $x^2 + x + 1 = t^2$

$(2x + 1)\ dx = 2t.\ dt$

putting values and after solving we get

$$I = 2\sqrt{x^2+x+1} + 2\sinh^{-1}\frac{2x+1}{\sqrt{3}} + C$$

S.62 (a)

Let $I = \int_0^{\pi} xF(\sin x)\,dx$ .....(1)

$$= \int_0^{\pi}(\pi - x)F[\sin(\pi - x)]dx$$

$I = \int_0^{\pi}(\pi - x)F[\sin(\pi - x)]dx$ .....(2)

Adding (1) and (2), we get

$$2I = \int_0^{\pi}\pi F(\sin x)dx$$

$$\Rightarrow I = \frac{1}{2}\int_0^{\pi}\pi F(\sin x)dx$$

S.63 (c)

The equation of given curves are

$y(x^2 + 2) = 3x$ .....(i)

and $4y = x^2$ .....(ii)

The curve (i) and (ii) intersect at A (2, 1).

If a figures is drawn then from fig. the required area is $\int_0^2\int_{y=x^2/4}^{3x/(x^2+2)} dydx$

S.64 (a)

$$\lim_{x\to\infty}\sin hx = \lim_{x\to\infty}\frac{e^x - e^{-x}}{2}$$

$$= \frac{1}{2}\left(\lim_{x\to\infty}e^x - \lim_{x\to\infty}e^{-x}\right) = \frac{1}{2}(\infty - 0) = \infty$$

$$\lim_{x\to\infty}\tan hx = \lim_{x\to\infty}\frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$= \lim_{x\to\infty}\frac{1-e^{-2x}}{1+e^{-2x}} = \frac{1-0}{1+0}$$

$$= 1$$

$$\lim_{x\to\infty}\frac{x^n}{\lfloor n} = 0$$

$\lim_{x\to 0}\log x$ (does not exist).

S.65 (b) $\lim_{x\to\infty}\left(\sqrt{x^2+1} - \sqrt{x+1}\right)$

$$= \lim_{x\to\infty}\left(\sqrt{x^2+1} - \sqrt{x^2+1}\right)\left(\frac{\sqrt{x^2+1}+\sqrt{x+1}}{\sqrt{x^2+1}+\sqrt{x+1}}\right)$$

$$= \lim_{x\to\infty}\frac{(x^2+1)-(x+1)}{\left(\sqrt{x^2+1}+\sqrt{x+1}\right)}$$

$$= \lim_{x\to\infty}\frac{x(x-1)}{x\left[\sqrt{1+\frac{1}{x^2}} + \frac{1}{\sqrt{x}}\sqrt{1+\frac{1}{\sqrt{x}}}\right]}$$

$$= \frac{\lim_{x\to\infty}(x-1)}{\left[\sqrt{1+\frac{1}{x^2}} + \frac{1}{\sqrt{x}}\sqrt{1+\frac{1}{x}}\right]}$$

$$= \frac{\infty}{0+1} = \infty$$

S.67 (c)

$y = \sin x(1 + \cos x)$

$$\frac{dy}{dx} = \cos x - \sin^2 x + \cos^2 x$$

$$= \cos x + \cos 2x$$

$$\frac{d^2y}{dx^2} = -(\sin x + 2\sin 2x)$$

From maximum and minimum, $\frac{dy}{dx} = 0$

$\cos x + \cos 2x = 0$

or $2\cos^2 x + \cos x - 1 = 0$

or $\cos x = \frac{1}{2}, \cos = -1$

$\therefore x = \frac{\pi}{3}$ or $x = \pi$

But $\frac{\pi}{3} \in (0, x)$ but $\pi \in (0, x)$

$\frac{d^2y}{dx^2} < 0$ at $x = \frac{\pi}{3}$

S.68 (d)

$f'(x) = 4x^3 - 6x^2 - 2x - 4$

$= 2(x-2)(2x^2 + x + 1)$

Thus only critical number is $x = 2$

So $f(0) = 3$

$f(2) = -9$

$f(4) = 99$

S.69 (b)

$$f'(x) = \frac{(x+2)(3x^2) - x^3}{(x+2)^3} = \frac{2x^2(x+3)}{(x+2)^2} = 0.$$

$x = 0$ and $x = -3$ are critical. However $x = -3$ does not lie in interval so we list 0 and end points -1 and 1.

$f(0) = 0$, $f(-1) = -1$, $f(1) = 1/3$

S.70 (c)

Here $\lim\limits_{n\to\infty}\left[\frac{1^2}{n^3} + \frac{2^2}{n^3} + \frac{3^2}{n^3} + \ldots\ldots + \frac{n^2}{n^3}\right]$

$$= \lim_{n\to\infty}\left(\frac{1^2 + 2^2 + 3^2 + \ldots\ldots + n^2}{n^3}\right)$$

$$= \lim_{n\to\infty}\left[\frac{n(n+1)(2n+1)}{6n^3}\right]$$

$$\left[\therefore 1^2 + 2^2 + 3^3 + \ldots\ldots n^2 = \frac{n(n+1)(2n+1)}{6}\right]$$

$$= \lim_{n\to\infty}\left[\frac{2n^2 + 3n + 1}{6n^2}\right]$$

$$= \lim_{n\to\infty}\frac{1}{6}\left[2 + \frac{3}{n} + \frac{1}{n^2}\right]$$

$$= \frac{2}{6} = \frac{1}{3}$$

S.71 (d)

Here $f(1) = -1$

Also, Right hand limit

$\lim\limits_{x\to 1^+} f(x) = -1$

Left Hand Limit

$\lim\limits_{x\to 1^-} f(x) = a - 2$

Now, L.H.L = R.H.L

$\Rightarrow a - 2 = -1$

$\Rightarrow a = 3$

S.72 (b)

for maximum vlaue put $0 = 0$

$5\cos\theta + 3\cos(\theta + \pi/3) + 3$

$= 5 + 3\cos\pi/3 + 3$

$= 5 + 1.5 + 3$

≈ 10

S.73 (a)

Using lagrange's mean value theorem

$$f'(c) = \frac{f(x) - f(0)}{x - 0} \quad \text{or} \quad e^c = \frac{e^x - 1}{x} \qquad \ldots\ldots(i)$$

Now $0 <<< x \Rightarrow e^0 < e^c < e^x \qquad \ldots\ldots(ii)$

From equation (i) and (ii) $1 < \frac{e^x - 1}{x} < e^x$

$\Rightarrow x < e^x - 1 < xe^x$

$\Rightarrow 1 + x < e^x < 1 + xe^x$

S.74 (a)

$$I = \int_0^{\pi/2} \sin 2(\pi/2 - x)\log\tan(\pi/2 - x)dx$$

$$2I = \int_0^{\pi/2}\left[\sin 2x\log(\tan x) + \sin 2x\log\cot x\right]dx$$

$$= \int_0^{\pi/2}\left[\sin 2x\log(\tan x\cot x)\right]dx$$

$$= \int_0^{\pi/2}\sin 2x\log_1 dx = \mathbf{0}$$

S.75 (b)

L. $\lim_{x\to 2^-} f(x) = \lim_{h\to 0} 2a(2-h) - b = 4a - b$

R. $\lim_{x\to 2^+} f(x) = \lim_{h\to 0} a(2+h)^2 + b = 4a + b$

L. $\lim_{x\to 2} f(x) = $ R. $\lim_{x\to 2} f(2)$

$4a - b = 2$ and $4a + b = 2$

$\Rightarrow$ **$a = 1/2$, $b = 0$**

S.76 (c)

For $x \in (0,\pi/2)$, $0 < \sin x < x$

$$0 < \frac{\sin x}{x} < 1 \Rightarrow 0 < \frac{n\sin x}{x} < n \quad .....(i)$$

$$\therefore \left[\frac{n\sin x}{x}\right] = n - 1 \quad .....(ii)$$

$$\lim_{x\to 0}\left[\frac{n\sin x}{x}\right] = \mathbf{n - 1}$$

S.77 (b)

$$A = \int_1^{\infty}\left(\frac{1}{x} - \frac{1}{x+1}\right)dx = [\ln|x| - \ln|x+1|]_1^{\infty}$$

$= \mathbf{\ln 2}$

S.78 (b)

Left Hand Limit, $\lim_{h\to 0} f(2-h)$

$$= \lim_{h\to 0} (2 - h - 1)$$
$$= 1 \quad ......(i)$$

Right Hand Limit, $\lim_{h\to 0} f(2+h)$

$$= \lim_{h\to 0} [2(2+h) - 3)$$
$$= 1 \quad ......(ii)$$

From equation (i) and equation (ii)

L.H.L = R.H.L = $\lim_{x\to 2} f(x)$ exists

$\lim_{x\to 2} f(x) = f(2) \Rightarrow$ f(x) is continuous at $x = 2$

S.79 (a)

Left Hand Limit,

$$\lim_{x\to 1} f(x) = \lim_{h\to 0} (1-h) - [1-h]$$

$$= \lim_{h\to 0} (1+h) - 0 = 1$$

Right Hand Limit,

$$\lim_{x\to 1} f(x) = \lim_{h\to 0} (1+h) - [1+h]$$

$$= \lim_{h\to 0} (1+h) - 1$$

$$= \lim_{h\to 0} h = 0$$

L.H.L $\lim_{x\to 1} f(x) \neq$ R.H.L $\lim_{x\to 1} f(x)$

$f(x) = x - [x]$ is discontinuous at $x = 0$.

S.80 (b)

The value of mod function cannot be less then 0

$f(x) = |x^2 - 5x + 2| = \mathbf{0}$

S.81 (b)

$$\lim_{x\to 2}\frac{x^2 - 4}{\sqrt{x+2} - \sqrt{3x-2}}$$

$$= \lim_{x\to 2}\frac{x^2 - 4}{\sqrt{x+2} - \sqrt{3x-2}} \times \frac{\sqrt{x+2} + \sqrt{3x-2}}{\sqrt{x+2} + \sqrt{3x-2}}$$

$$= \lim_{x\to 2}\frac{(x+2)(x-2)\left(\sqrt{(x+2)} + \sqrt{3x-2}\right)}{(x+2) - (3x-2)}$$

$$= \lim_{x\to 2}\frac{(x+2)(x-2)\left(\sqrt{x+2} + \sqrt{3x-2}\right)}{-2(x-2)}$$

$$= \lim_{x\to 2}\frac{(x+2)\left(\sqrt{x+2} + \sqrt{3x-2}\right)}{-2} = \frac{4\left(\sqrt{4} + \sqrt{4}\right)}{-2}$$

$= \mathbf{-8}$

S.82 (a)

Let $k(x,y) = \frac{\partial f}{\partial x} = g(y)$ then $\frac{\partial k}{\partial x} = 0$ then

$k(x,y) = g(y)$.

Hence $\frac{\partial f}{\partial x} = g(y)$ then $f(x,y) = xg(y) + h(y)$ for a suitable function h

S.83 (a)

By cauchy's, $\dfrac{f(b)-f(a)}{g(b)-g(a)}=\dfrac{f'(c)}{g'(c)}$

Choosing $f(x) = e^x$, $g(x) = e^{-x}$

$$\therefore \frac{e^b-e^a}{e^{-b}-e^{-a}}=\frac{e^c}{-e^{-c}} \Rightarrow \mathbf{c=\frac{a+b}{2}}$$

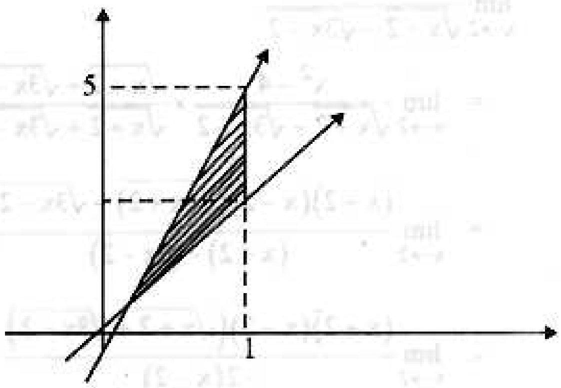
S.84 (d)

$$I = \int_0^b\int_0^b y(2b+y)\,dx\,dy$$

$$= \int_0^b\left[by^2+\frac{y^3}{3}\right]_0^b dx$$

$$= \int_0^b\left(b^3+\frac{b^3}{3}\right)dx = \mathbf{\frac{4b^4}{3}}$$

S.85 (b)



$$I = \int_0^1\int_{2x}^{5x} y^2\,dy\,dx$$

$$= \int_0^1 \frac{1}{3}y^3\Big]_{2x}^{5x} dx = \frac{1}{3}\int_0^1\left(125x^3-8x^3\right)dx$$

$$= \frac{39}{4}x^4\Big]_0^1 = \mathbf{\frac{39}{4}}$$

S.86 (c)

$$\int_{-2}^{2}\frac{dx}{x^1}=\int_{-2}^{2}\frac{1}{x^2}dx$$

$f(x) = \dfrac{1}{x^2}$ = even function

For even function,

$$\int_{-a}^{a}f(x)dx=\int_{-a}^{a}f(x)dx=2\int_0^2\frac{1}{x^2}dx=-\left[-1\frac{1}{x}\right]=1$$

$$\therefore \int_{-2}^{2}\frac{1}{x^2} = \mathbf{1}$$

S.87 (a)

$$f(x)=\frac{x^3}{3}-x \Rightarrow f'(x)=x^2-1$$

For maximum and minimum value, $f(x) = 0$

$\therefore \quad x^2-1 = 0$

or $\quad x = \pm 1$

Again $f''(x) = 2x$

At $x = 1, f''(x) = 2 > 0$

$f(x) = -2 < 0$

Hence **minimum at x =1**

S.88 (b)

Given, $y = x^2+\dfrac{250}{x}$

$$\therefore \frac{dy}{dx}=2x-\frac{250}{x^2}, \text{ and } \frac{d^2y}{dx^2}=2+\frac{250}{x^3}$$

When $x = 5$,

$$\frac{dy}{dx}=10-10=0 \text{ and } \frac{d^2y}{dx^2}=\text{is positive}$$

Therefore function is **minimum at x = 5.**

S.89 (c)

Let $a \in R$ (real number)

L. $\lim\limits_{x\to a} f(x)=\lim\limits_{h\to a}|a-h|=\lim\limits_{h\to 0}(a-h)=a$

R. $\lim\limits_{x\to a} f(x)=\lim\limits_{h\to a}|a+h|=\lim\limits_{h\to 0}(a+h)=a$

and $\quad f(a) = |a| = a$

$\therefore \lim\limits_{x\to a}|x|=a=f(x)$

Hence, the $\lim\limits_{x\to a}|x|$ continuous for all values of x.

S.90 (c)

Since $f(x) = \frac{1}{x}$ is not continuous in [–3, 3] [–4, 2] or [–1, 1], the point of discontinuity is '0'. Only in [2, 3] the function is continuous, and differentiable hence mean value theorem is applicable in [2, 3].

S.91 (c)

since f(x) = (x) is continuous in [–1, 1] but it is not differentiable at x= 0 ε (–1, 1)

S.92 (a)

Given equation

$\Rightarrow \quad x^3 + 6x^2 + 2x + 1$

Its slope $\Rightarrow \; -3x^2 + 12x + 2$

Let $F(x) = -3x^2 + 12x + 2$

$F'(x) = -6x + 12$

$F''(x) = -6$

For maxima and minima $F'(x) = 0$

$\therefore \quad -6x + 12 = 0$

$x = 2$

$\therefore$ at $x = 2$, $F''(x) = -6 < 0$

it is maxima

$\therefore$ for maximum vlaue.

$F(2) = -3(2)^2 + 12(2) + 2$

$= -3(4) + 24 + 2$

$= -12 + 24 + 2$

$F(2) = 14$

this is maximum value of slope

S.93 (d)

$f(x) = x^5\; 5x^4 + 5x^3 - 1$

$f'(x) = 5x^4 - 20x^3 + 15x^2$

$f''(x) = 20x^3 - 60x^2 + 30x$

For maximum and minimum, $f'(x) = 0$

or $\quad x^4 - 4x^3 + 3x^2 = 0$

or $\quad x^2(x-1)(x-3) = 0$

$\therefore \quad x = 0, 1, 3$

$f'(0) = 0$

$f''(0) = 0 = 0$

Hence f(x) is neither maximum nor minimum at x = 0.

$f''(1) = 20 - 60 + 30 = -10 < 10$

$\therefore$ f (x) is maximum = 1

$f''(3) = 90 > 0$

$\therefore$ f(x) is minimum at x = 3

Hence there is one maxima and one minima for f(x).

S.94 (a)

$$\int_0^{\pi/2}\int_0^{\pi/2} \sin(x+y)\,dx.dy$$

$$\Rightarrow \int_0^{\pi/2}\int_0^{\pi/2} \sin(x.\cos y + \cos n.\sin y)\,dx.dy$$

$$\Rightarrow \int_0^{\pi/2} (\sin x + \cos x)dx$$

$\Rightarrow 1 + 1$

$\Rightarrow 2$

S.95 (a)

$f(x) = x^4 - 8x^3 + 22x^2 - 24x + 1, \qquad f(0) = 1$

$f(2) = 2^4 - 8.(2)^3 + 22.(2)^2 - 24.(2) + 1$

$= -7$

Now $f'(x) = 4x^3 - 24x^2 + 44x - 24$

For maximum and minimum, $\qquad f'(x) = 0$

or $\quad 4x^3 - 24x^2 + 44x - 24 = 0$

or $\quad 4(x-1)(x-2)(x-3) = 0$

$\therefore \quad x = 1, 2, 3$

Since x = 3 does not lie in [0, 2]

Therefore, consider only x = 1 and x = 2.

We have,

$f(1) = 1^4 - 8.(1)^3 + 22.(1)^2 - 24.(1) + 1 = -8$

Greatest of f(x) = largest of {1, –7, –8} = 1

Least of f(x) = smallest of {1, –7, –8} = –8

S.96 (b)

A polynomial function is continuous as well as differentiable. So, the given function is continuous and differentiable.

$f(1) = 0$ and $f(3) = 0$. So $f(1) = f(3)$.

By Rolle's theorem c such that $f'(c) = 0$.

Now, $f'(x) = 3x^2 - 12x + 11$

$\Rightarrow f'(c) = 3c^2 - 12c + 11$

Now, $f'(c) = 0 \Rightarrow 3c^2 - 12c + 11 = 0$

$$\Rightarrow c = \left(2 \pm \frac{1}{\sqrt{3}}\right)$$

S.97 (a)

$f'(x) = 3x^2 - 12x + k$

$f'(c) = 0 \Rightarrow 3c^2 - 12c + k = 0$

$$\Rightarrow c = \frac{12 \pm \sqrt{144 - 12k}}{6} \Rightarrow \frac{144 - 12k}{36} = \frac{1}{3}$$

$\Rightarrow 144 - 12k = 12 \Rightarrow$ **k = 11.**

S.98 (c)

Let $f(x) = \sqrt{x^2 - 4}$.

Clearly, f(x) has a definite and unique value for each $x \leftrightarrow [2, 4]$. So it is continuous on [2, 4].

$f'(x) = \dfrac{x}{\sqrt{x^2 - 4}}$ which exists for each $x \leftrightarrow]2, 4[$.

$$f'(c) = \frac{f(4) - f(2)}{(4 - 2)} = \frac{\sqrt{12} - 0}{2} = \sqrt{3}$$

$$\text{Now, } \frac{c}{\sqrt{c^2 - 4}} = \sqrt{3} \Rightarrow c^2 = 3(c^2 - 4)$$

$$\Rightarrow \quad c = \pm\sqrt{6}$$

But, $\sqrt{6} \leftrightarrow]2, 4[$; So, $\mathbf{c = \sqrt{6}}$.

S.99 (c)

$$u = \tan^{-1}\left(\frac{x + y}{\sqrt{x} + \sqrt{y}}\right)$$

$$\Rightarrow \quad \tan u = \frac{x + y}{\sqrt{x} + \sqrt{y}} = f(\text{say})$$

which is a homogeneous equation of degree 1/2.

By Euler's theorem. $x\dfrac{\partial f}{\partial x} + y\dfrac{\partial f}{\partial y} = \dfrac{1}{2}f$

$$\Rightarrow \quad x\frac{\partial(\tan u)}{\partial x} + y\frac{\partial(\tan u)}{\partial y} = \frac{1}{2}\tan u$$

$$x\sec^2 u\frac{\partial u}{\partial x} + y\sec^2 u\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$$

$$\Rightarrow \quad x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\sin u\cos u = \frac{1}{4}\sin 2u$$

S.100 (d)

Let $v = \phi\left(\dfrac{y}{x}\right)$ and $w = x\Psi\left(\dfrac{y}{x}\right)$

Then $u = v + w$

Now v is homogeneous of degree zero and w is homogeneous of degree one

$$\Rightarrow x^2\frac{\partial^2 v}{\partial x^2} + 2xy\frac{\partial^2 v}{\partial x \partial y} + y^2\frac{\partial^2 v}{\partial y^2} = 0 \qquad \text{.....(i)}$$

$$\text{and } x^2\frac{\partial^2 w}{\partial x^2} + 2xy\frac{\partial^2 w}{\partial x \partial y} + y^2\frac{\partial^2 w}{\partial y^2} = 0 \qquad \text{.....(ii)}$$

Adding (i) and (ii), we get

$$x^2\frac{\partial^2}{\partial x^2}(v + w) + 2xy\frac{\partial^2}{\partial x \partial y}(v + w)$$

$$+ y^2\frac{\partial^2}{\partial y^2}(v + w) = 0$$

$$\Rightarrow \quad x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0$$

S.101 (c)

$z = e^x \sin y \Rightarrow \dfrac{\partial z}{\partial x} = e^x \sin y$

And $\dfrac{\partial z}{\partial y} = e^x \cos y$, $x = \log_e t \Rightarrow \dfrac{dx}{dt} = \dfrac{1}{t}$

And $y = t^2 \Rightarrow \dfrac{dy}{dt} = 2t$

$$\frac{dz}{dt}=\frac{\partial z}{\partial x}.\frac{dx}{dt}+\frac{\partial z}{\partial y}.\frac{dy}{dt}$$

$$= e^x \sin y.\frac{1}{t}+e^x\cos y.2t$$

$$= \frac{e^x}{t}(\sin y+2t^2\cos y)$$

S.102 (a)

Let $z=\frac{1}{x}+\tan x$

$$\frac{dz}{dx} = \frac{-1}{x^2}+\sec^2 x$$

$$\frac{dz^2}{dx^2}=\frac{2}{x^3}+2\sec^2 x\tan x$$

$$\frac{dz}{dx}=0\Rightarrow -\frac{1}{x^2}+\sec^2 x=0$$

$\Rightarrow$ x = cos x

$$\left[\frac{d^2z}{dx^2}\right]_{x=\cos x} = 2\cos^3x + 2\sec^2x\tan x > 0$$

Thus z has a minima and therefore y has maxima at x = cos x.

S.103 (b)

$$\int\frac{dx}{\sqrt{2x^2+3x+4}}=\frac{1}{\sqrt{2}}\int\frac{dx}{\sqrt{\left(x+\frac{3}{4}\right)^2+\left(\frac{\sqrt{23}}{4}\right)^2}}$$

$$=\frac{1}{\sqrt{2}}\sinh^{-1}\frac{x+\frac{3}{4}}{\left(\frac{\sqrt{23}}{4}\right)}$$

$$=\frac{1}{\sqrt{2}}\sinh^{-1}\frac{4x+3}{\sqrt{23}}$$

S.104 (a)

Let $I=\int\frac{1}{(x+1)\sqrt{1-2x-x^2}}dx$

Put $x+1=\frac{1}{t}$ $\Rightarrow$ $dx=-\frac{1}{t^2}dt$

$$I=\int\frac{-\frac{1}{t^2}dt}{\frac{1}{t}\sqrt{1-2\left(\frac{1}{t}-1\right)-\left(\frac{1}{t}-1\right)^2}}=-\int\frac{dt}{\sqrt{2t^2-1}}$$

$$=-\frac{1}{\sqrt{2}}\int\frac{dt}{\sqrt{t^2-\left(\frac{1}{\sqrt{2}}\right)^2}}=-\frac{1}{\sqrt{2}}\cosh^{-1}\frac{t}{1/\sqrt{2}}$$

$$=-\frac{1}{\sqrt{2}}\cosh^{-1}\left(\frac{\sqrt{2}}{x+1}\right)$$

S.105 (b)

$$\text{Let } I=\int_0^{2a}\frac{f(x)}{f(x)+f(2a-x)}dx \quad .....(i)$$

$$I=\int_0^{2a}\frac{f(2a-x)}{f(2a-x)+f(x)}dx \quad .....(ii)$$

Adding (i) and (ii), we get

$$2I=\int_0^{2a}\frac{f(x)+f(2a-x)}{f(x)+f(2a-x)}dx=\int_0^{2a}1.dx=[x]_0^{2a}=2a$$

$\Rightarrow$ **I = a**

S.106 (a)

$$\oint_D\frac{1}{(s^2-1)}ds=\oint_D\frac{1}{(s+1)(s-1)}ds$$

= $2\pi j$ × (Sum of residue)

residue at pole s = –1 is 0

[∵ It lies outside the given contour]

and residue at pole s = 1is

$$= \lim_{x\to 1}\frac{(s-1)}{(s-1)(s+1)} = \frac{1}{2}$$

$$\Rightarrow \oint_D \frac{1}{(s^2-1)}ds = 2\pi\times\frac{1}{2} = j\pi$$

S.107 (d)

$$\frac{1}{z^2+4} = \frac{1}{(z+2j)(z-2j)}$$

Pole (0,2) lies inside the circle $|z-j| = 2$

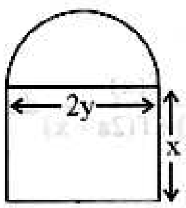
$\therefore$ By Cauchy's integral formula,

$$I = \oint \frac{1}{Z^2+4}dz$$

$|z - j| = 2$

$$= \frac{2\pi j}{2j+2j} = \frac{\pi}{2}$$

S.108 (c)



Let 2y be length of side on which semicircle rests

let x be length of other side

$p = 2x + 2y + \pi y$

$$0 = 2\frac{dx}{dy}+2+\pi \Rightarrow \frac{dx}{dy} = -(2+\pi)/2$$

To admit most light area must be maximum

$A = 2xy + \pi y^2/2$

$$\frac{dA}{dy} = 2\left(x+\frac{dx}{dy}.y\right)+\pi y = 2x-2y = 0 \Rightarrow x = y$$

$$\therefore x = y = \frac{p}{4+\pi}$$

$$\left.\frac{d^2A}{dy^2}\right]_{p/4+\pi} \text{ is } -\text{ ve.}$$

So area will be maximum at

$$l = b = \frac{P}{4+\pi}$$

S.109 (a) $Z = e^{y/x}$

$$\frac{\partial z}{\partial x} = e^{y/x}\left(\frac{-y}{x^2}\right) \quad .....(1)$$

$$\text{and } \frac{\partial z}{\partial x} = e^{y/x}\left(\frac{1}{x}\right) \quad .....(2)$$

$$x.\frac{\partial z}{\partial x}+y.\frac{\partial z}{\partial y}$$

$$= x.e^{y/x}\left(\frac{-y}{x^2}\right)+y.e^{y/x}\left(\frac{1}{x}\right) \quad \text{[from equ. (1) and (2)]}$$

$$= -\frac{y}{x}e^{y/x}+\frac{y}{x}e^{y/x}$$

$= \mathbf{0}$

S.110 (b)

Let $c(x, y) = f(x, y) - xy$, then $\dfrac{\partial^2 c}{\partial x \partial y}$

$= 1 - 1 = 0$ then

$c(x, y) = A(x) + B(y) \therefore f(x, y)$

$= \mathbf{A(x) + B(y) + xy}$

S.111 (c)

$$fx = \begin{cases} x^6 & x < -1 \text{ and } x > 1 \\ x^3 & -1 \le x \le 1 \end{cases}$$

at $x = -1$

$$\lim_{h\to -1} f(x) = \lim_{h\to -1} x^3 = -1$$

$$\text{L.H.S} \lim_{h\to -1^-} f(x) = \lim_{-1-1} x^6 = 1$$

R.H.S $\lim\limits_{x\to -1^+} f(x) = \lim\limits_{-1^+} x^3 = -1$

L.H.S ≠ R.H.S. So discontinuous at x = −1

Now at x = 1. $\lim\limits_{x\to 1} f(x) = \lim\limits_{x\to 1} x^3 = 1$

L.H.S $\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-} x^6 = 1$

R.H.S $\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} x^6 = 1$

L.H.S = R.H.S So continuous at x = 1

Differentiatbility

L.H.S $F(1^-) = \lim\limits_{h\to 0} \frac{f(1+h)-f(1)}{-h} = -1$

R.H.S $F(1^+) = \lim\limits_{h\to 0} \frac{f(1+h)-f(1)}{h} = 1$

L.H.S ≠ R.H.S So not differentiable at x = 1.

S.112 (a)

Here $u = e^{xyz} \Rightarrow \frac{\partial u}{\partial x} = e^{xyz}.yz$

$$\frac{\partial^2 u}{\partial x \partial y} = ze^{xyz} + yze^{xyz}.xz = e^{xyz}\left(z + xyz^2\right)$$

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = e^{xyz}.(1+2xyz) + \left(z+xyz^2\right)e^{xyz}.xy$$

$$= \mathbf{e^{xyz}(1 + 3xyz + x^2y^2z^2)}$$

S.113 (d)

$$\int \frac{dx}{\sin(x-a)\sin(x-b)}$$

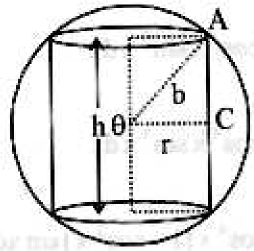
$$= \frac{1}{\sin(a-b)} \int \frac{\sin(a-b)dx}{\sin(x-a)\sin(x-b)}$$

$$= \frac{1}{\sin(a-b)} \int \frac{\sin[(x-b)-(x-a)]}{\sin(x-a)\sin(x-b)}$$

$$= \frac{1}{\sin(a-b)} \times \int \frac{\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)}{\sin(x-a)\sin(x-b)}dx$$

$$= \frac{1}{\sin(a-b)} \log\left\{\frac{\sin(x-a)}{\sin(x-b)}\right\}$$

S.114 (a)



using pythagoras theorem

in Δ AOC

we get $AO^2 = OC^2 + AC^2$

$b^2 = r^2 + (h/2)^2$

$b^2 = r^2 + (h/2)^2$

$V = \pi r^2 h = \pi (b^2 - h^2/4)h$

$$\frac{d^2V}{dh^2} = -\frac{3\pi h}{2} \Rightarrow \left.\frac{d^2V}{dh^2}\right]_{2b/\sqrt{3}} = \frac{-3\pi}{2} \times \frac{2b}{\sqrt{3}} - ve$$

Hence there is relative maxima at $\mathbf{h = 2b/\sqrt{3}}$

$\therefore$ $\mathbf{r = b\sqrt{2/3}}$

S.115 (c)

$f(x) = x^3 \Rightarrow f'(x) = 3x^2$

$f(b) - f(a) = (b-a)\ f'(c)$

$b^3 - a^3 = (b-a) \times 3c^2$

$$\Rightarrow 3c^2 = \frac{(b-a)\left(a^2+ab+b^2\right)}{b-a}$$

$$\Rightarrow \mathbf{c} = \sqrt{\frac{\mathbf{a^2+b^2+ab}}{\mathbf{3}}}$$

S.116 (b)

$$f(x) = \frac{1-x}{1-x^3} \text{ from } \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

$$\text{and so } f(x) = \frac{1}{1-x^3} - x\frac{1}{1-x^3}$$

$$= \sum_{n=0}^{\infty} x^{3n} - \sum_{n=0}^{\infty} x^{3n+1}$$

From Maclaurin expansion, $\frac{f^{36}(0)}{36!} = a_{36} = +1$

Hence $\mathbf{f^{36}(0) = 36!}$

S.117 (b)

$$I = \int_0^{\pi/2} \cos^4 x \sin^3 x\, dx$$

$$= \int_0^{\pi/2} \cos^4 x \sin^3 x\, dx$$

$$= \int_0^{\pi/2} \cos^4 x\left(1-\cos^2 x\right)\sin x dx$$

let cosx = t

$$I = -\int_1^0 t^4\left(1-t^2\right)dt = \left[\frac{t^5}{5}-\frac{t^7}{7}\right]_0^1 = \frac{2}{35}$$

S.118 (a)

$$I = \int \frac{\sec^4\theta}{\tan^4+1}d\theta = \int\frac{\left(1+\tan^2\theta\right)\sec^2\theta}{1+\tan^4\theta}d\theta$$

Put tan θ = x; $\sec^2\theta\, d\theta = dx$

$$I = \int\frac{\left(1+x^2\right)dx}{1+x^4} = \int\frac{\left(1+1/x^2\right)dx}{\left(x-\frac{1}{x}\right)^2+2}$$

Put $x-\frac{1}{x} = t \Rightarrow \left(x+\frac{1}{x^2}\right)dx = dt$

$$I = \int\frac{dt}{t^2+\left(\sqrt{2}\right)^2} = \frac{1}{\sqrt{2}}\tan^{-1}\frac{t}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}\tan\left(\frac{x-1/x}{\sqrt{2}}\right)+C$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan^2\theta-1}{\sqrt{2}\tan\theta}\right)+C$$

S.119 (b)

f(0) = 0

$$\lim_{x\to 0} f(h) = 0$$

$\therefore$ f (x) is continuous at x = 0

Next

$$f(0^+) = \lim_{x\to 0^+}\frac{\sqrt{1-\sqrt{1-x^2}}-0}{x}$$

$$= \lim_{x\to 0^+}\frac{\sqrt{1\sqrt{1-x^2}}}{x\sqrt{1+\sqrt{1-x^2}}}$$

$$= \lim_{x\to 0^+}\frac{\sqrt{x^2}}{x\sqrt{1+\sqrt{1-x^2}}}$$

$$= \lim_{x\to 0^+}\frac{x}{x\sqrt{1+\sqrt{1-x^2}}}$$

because $\sqrt{x^2} = x \quad$ if $x \ge 0$

$\qquad\qquad\qquad x \quad$ if $x < 0$

$$= \frac{1}{\sqrt{2}}$$

But $f(0^-) = -\frac{1}{\sqrt{2}}$

$\therefore$ F is not differentiable of x =0.

S.120 (a)

$f(x) = x^3/3 - 3x^2/2 + 2x$

$f'(x) = x^2 - 3x + 2$

$f''(x) = 2x - 3$

for maxima minima $f'(x) = 0$; $x^2-3x+2 = 0$

$\Rightarrow (x-1)(x-2) = 0$

$\Rightarrow x = 1, 2$

$f''(1) = 2 - 3 = -1 < 0$, maxima

$f''(2) = 2 \times 3 - 2 = 1 > 0$, minima

f(1) 1/3 − 3/2 + 2 = **5/6 (greatest)**

f(2) = 8/3 −6+4 = 4/6 = **2/3 (least)**

S.121 (b)

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{dt}+\frac{\partial z}{\partial y}\frac{dy}{dt} = \frac{x}{8}\frac{dy}{dt}\cdot\frac{2y}{9}\frac{dy}{dt}$$

$$\text{since}\frac{dx}{dt} = 5 \text{ and } 5x+y\frac{dy}{dt} = 0$$

so when x = 2 and y = 1, $\frac{dy}{dt} = -10$

$$\therefore \frac{dz}{dt} = \frac{2}{8}(5)-\frac{1}{9}(-10) = \frac{85}{36}\text{cm/s}$$

S.122 (b)

$$A = 2\int_0^{\pi/2}\int_{a\sec\theta}^{a(\sec\theta+\cos\theta)} r\,dr\,d\theta$$

$$= \int_0^{\pi/2}\left[r^2\right]_{a\sec\theta}^{a(\sec\theta+\cos\theta)} .d\theta$$

$$= a^2\int_0^{\pi/2}\left\{(\sec\theta+\cos\theta)^2-\sec^2\theta\right\}d\theta$$

$$= a^2\int_0^{\pi/2}\left[2+\cos^2\theta\right]d\theta$$

$$= \frac{5\pi a^2}{4}$$

S.123 (b)

$$\int_0^{\pi}\int_0^{a}(1+\cos\theta).\cos\theta.\, r^4.\, dr.d\theta$$

$$= \frac{a^5}{5}\int_0^{\pi}(\cos\theta+\cos^2\theta)d\theta$$

$$= \frac{a^5}{5}\int_0^{\pi}\left(\cos\theta+\frac{1}{2}+\frac{\cos 2\theta}{2}\right)d\theta$$

$$= \frac{a^5}{5}\left\{[\sin\theta]_0^{\pi}+\frac{1}{2}[\theta]_0^{\pi}+\frac{1}{4}[\sin 2\theta]_0^{\pi}\right\}$$

$$= \frac{a^5}{5}\left[\frac{\pi}{2}\right]$$

$$= \frac{a^5\pi}{10}$$

S.124 (c)

$$I = 2\pi\int_0^{\pi}\left[\frac{r^5}{5}\right]_0^{a(1+\cos\theta)}\sin^3\theta\, d\theta$$

$$= \frac{2\pi a^5}{5}\int_0^{\pi}(1+\cos\theta)^5\sin\theta.\sin^2\theta d\theta$$

$$= \frac{2\pi a^5}{5}\int_0^{\pi}(1+\cos\theta)^5.\sin\theta.(1+\cos^2\theta)d\theta$$

$$= \frac{2\pi a^5}{5}\int_0^{\pi}(1+\cos\theta)^5(1-\cos\theta)(1+\cos\theta)\sin\theta.d\theta$$

$$= \frac{2\pi a^5}{5}\int_0^{\pi}(1+\cos\theta)^6(1-\cos\theta)\sin\theta.d\theta$$

put $(1+\cos\theta) = t$; $-\sin\theta\, d\theta = dt$

$\theta \to 0,\ t = 2$ and $\theta \to \pi,\ t = 0$

$\cos\theta = t - 1 \quad \therefore \quad 1 - \cos\theta = 2 - t$

$$\therefore I = -\frac{2\pi a^5}{5}\int_{\alpha}^{0}(t)^6(2-t)dt$$

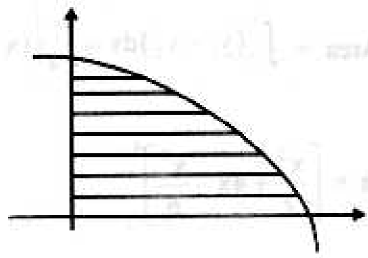
$$= \frac{2\pi a^5}{5}\int_0^{2}(2t^6-t^7)dt$$

$$= \frac{2\pi a^5}{5}\left[\frac{2t^7}{7}-\frac{t^{\theta}}{\theta}\right]_0^2$$

$$= \frac{2^9\pi a^5}{5\times 56}$$

$$= \frac{2^6\pi a^5}{35}$$

S.125 (c)



$$I = \int_0^{2}\int_0^{\sqrt{4-2y}}\frac{1}{\sqrt{2y-y^2}}dxdy$$

$$= \int_0^{2}\left[\frac{x}{\sqrt{2y-y^2}}\right]_0^{\sqrt{4-2y}}dy = \int_0^{2}\frac{\sqrt{2}}{\sqrt{y}}\frac{\sqrt{2-y}}{\sqrt{2-y}}dy$$

$$= \sqrt{2}.2y^{1/2}\Big]_0^2 = 4$$

S.126 (a)

$$I = \int_0^1 dx \int_{x^2}^{\sqrt{x}} dy \int_0^{2-x-y} dz$$

$$= \int_0^1 dx \int_{x^2}^{\sqrt{x}} dy [z]_0^{2-x-y}$$

$$= \int_0^1 dx \int_{x^2}^{\sqrt{x}} (2-x-y) dy$$

$$= \int_0^1 \left[(2-x)\sqrt{x} - \frac{x}{2} - (2-x)x^2 + \frac{x^2}{2}\right] dx = \frac{11}{30}$$

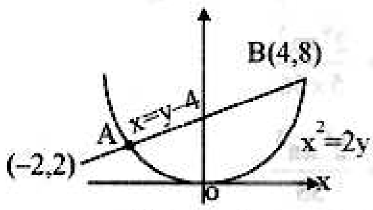
S.127 (b)

Given $2y = x^2$

$$y = \frac{x^2}{2}$$

and $y = x + 4$

$\therefore \quad x^2 - 2x - 8 = 0$



Given curve and the line intersect at A(–2,2) and B (4,8)

$$\therefore \text{ Area } = \int_{-2}^{4} (y_1 - y_2) dx = \int_{-2}^{4} \left\{(x+4) - \frac{x^2}{2}\right\}$$

$$\text{Area} = \left[\frac{x^2}{2} + 4x - \frac{x^3}{6}\right]_{-2}^{4}$$

$$= \left(8 + 16 - \frac{32}{3}\right) - \left(2 - 8 + \frac{4}{3}\right) = 18$$

S.128 (a)

Let radius of circle is r and side of a square is x.

Perimeter = $2\pi r + 4\pi = 1$ (given) .....(i)

$$A = \pi r^2 + x^2 = \pi r^2 + \left(\frac{1-2\pi r}{4}\right)^2$$

$$\frac{dA}{dr} = 2\pi r + \frac{2}{16}(1-2\pi r).(-2\pi)$$

and $$\frac{d^2A}{dr^2} = 2\pi + \frac{\pi^2}{2}$$

For maximum and minimum, $\frac{dA}{dr} = 0$

or $$2\pi r - \frac{\pi}{4}(1-2\pi r) = 0$$

$$2r - \frac{1}{4}(1-2\pi r) = 0$$

$$2r = \frac{1}{4}(1-2\pi r)$$

$8r = 1 - 2\pi r$

$8r + 2\pi r = 1$ – equation (ii)

using equation (1) and (ii)

$8r + 2\pi r = 4x + 2\pi r$

$8r = 4x$

$x = 2r$

Hence side of square is twice of radius of circle

S.129 (c)

Given that $z = \sqrt{x^2 + y^2}$ .....(i)

and $x^3 + y^3 + 3axy = 5a^2$ .....(ii)

$$\frac{dz}{dx} = \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}\frac{dy}{dx}$$

from (i),

$$\frac{\partial z}{\partial x} = \frac{1}{2\sqrt{x^2+y^2}}.2x,$$

$$\frac{\partial z}{\partial y} = \frac{1}{2\sqrt{x^2+y^2}}.2y$$

and $$3x^2 + 3y^2\frac{dy}{dx} + 3ax\frac{dy}{dx} + 3ay.1 = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\left(\frac{x^2+ay}{y^2+ax}\right)$$

Substituting these value in (ii), we get

$$\frac{dz}{dx} = \frac{x}{\sqrt{x^2+y^2}} + \frac{y}{\sqrt{x^2+y^2}}\left(-\frac{x^2+ay}{y^2+ax}\right)$$

$$\left(\frac{dz}{dx}\right)_{(a,a)} = \frac{a}{\sqrt{a^2+a^2}} + \frac{a}{\sqrt{a^2+a^2}}\left(-\frac{a^2+a.a}{a^2+a.a}\right)$$

$$= 0$$

S.130 (c)

In (i) $u = \frac{x^2 y}{x+y}$ It is a homogeneous function of degree 2.

$$x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial x \partial y} = (n-1)\frac{\partial u}{\partial x} = \frac{\partial u}{\partial x}$$

In (ii), $u = \frac{x^{\frac{1}{2}} - y^{\frac{1}{2}}}{x^{\frac{1}{4}} - y^{\frac{1}{4}}}$ It is a homogeneous function of degree $\left(\frac{1}{2}-\frac{1}{4}\right) = \frac{1}{4}$

$$x\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + x\frac{\partial^2 u}{\partial y^2} = n(n-1)u$$

$$= \frac{1}{4}\left(\frac{1}{4}-1\right)u = -\frac{3}{16}u$$

In (iii) $u = x^{\frac{1}{2}} + y^{\frac{1}{2}}$ It is a homogeneous function of degree $\frac{1}{2}$.

$$x\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + x\frac{\partial^2 u}{\partial y^2} = n(n-1)u$$

$$= \frac{1}{2}\left(\frac{1}{2}-1\right)u = -\frac{1}{4}u$$

In (iv) $u = f\left(\frac{y}{x}\right)$ It is a homogeneous function of degree zero.

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0.u = 0$$

Hence correct match is

(i)	(ii)	(iii)	(iv)
2	1	3	4

S.131 (d)

Let $I = \int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx$

$$= \int e^x\left(\frac{1+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)dx$$

$$= \frac{1}{2}\int e^x \sec^2\frac{x}{2}dx + \int e^x \tan\frac{x}{2}dx$$

$$= \frac{1}{2}\left\{e^x.2\tan\frac{x}{2} - \int e^x.2\tan\frac{x}{2}dx\right\} + \int e^x \tan\frac{x}{2}dx$$

$$= \quad e^x \tan\frac{x}{2} + c$$

S.132 (c)

Let $I = \int_0^{\pi/2} \frac{e^x}{2}\left(\sec^2\frac{x}{2} + 2\tan\frac{x}{2}\right)dx$

$$= \int_0^{\pi/2} \frac{1}{2}e^x \sec^2\frac{x}{2}dx + \int_0^{\pi/2} e^x \tan\frac{x}{2}dx = I_1 + I_2$$

Here, $I_1 = \int_0^{\pi/2} \frac{1}{2}e^x \sec^2\frac{x}{2}dx$

$$= \left[\frac{1}{2}e^x.2\tan\frac{x}{2}\right]_0^{\pi/2} - \frac{1}{2}\int e^x.2\tan\frac{x}{2}dx$$

$$= \left(e^{\pi/2}\tan\frac{\pi}{4} - 0\right) - \int_0^{\pi/2} e^x \tan\frac{x}{2}dx$$

$= e^{\pi/2} - I_2$, $I_1 + I_2 = e^{\pi/2}$

$I = I_1 + I_2 = e^{\pi/2}$

S.133 (d)

$$\int_0^1 \int_0^{\sqrt{1+x^2}} dy\,dx = \int_0^1 [y]_0^{\sqrt{1+x^2}}\,dx$$

$$= \int_0^1 \sqrt{1+x^2}\,dx$$

$$= \frac{1}{2}\left[x\sqrt{1+x^2} + \log\left(x+\sqrt{1+x^2}\right)\right]_0^1$$

$$= \frac{1}{2}\left[\sqrt{2} + \log\left(1+\sqrt{2}\right)\right]$$

S.134 (d)

The equation of the cardioid is

$r = a(1+\cos\theta)$ .....(i)

If a figure is drawn then from fig. the required area is Required area $A = 2\int_{\theta=0}^{\pi/2}\int_{r=0}^{a(1+\cos\theta)} r\,dr\,d\theta$

S.135 (a,d)

(a) and (d) are valid and (b) and (c) are invalid

If we want to prove $P \to q$; it is enough to prove if P = T then q = T.

for statement (a)

$(\forall x)\ P(x)\ V\ (\forall x)\ Q(x) \to (\forall x)(P(x) V Q(x))$

L.H.S is true. That is either all P's are true or all Q's are true. then $\forall$ (P(x) V Q (x)) are true. Hence RHS is true.

that is (a) is valid.

for statement (b)

$(\exists x)\ P(x) \wedge (\exists x)\ Q(x) \to (\exists x)\ [P(x) \wedge Q(x)]$

Let we have only two elements in are universal set and here LHS is true but R.H.S is false Hence (b) is invalid.

for statement (c)

$(\forall x)\ \{P(x)\ VQ(x)] \to (\forall x)\ P(x)V(\forall x)\ Q(x)$

$(\forall(x))\ [\neg (p(x)\ V\ Q(x)V(\forall x)(P(x)V(\forall(x)\ Q(x)]$

for statement (d)

Let R.H.S be false then

$\sim (\forall x)\ P(x)\ V(\exists x)\ Q(x) = F$

that is $\sim (\forall x)\ P(x) = F$

and $(\exists x)\ Q(x) = F$

$(\forall x)\ P(x) = T$ and $(\exists x)\ Q(x) = F$

$P(y) \to Q(y) = F$ as $(\forall x)$ {Let Q(y) = F}

this implies that $(\exists x)\ \{P(x) \to Q(x)\} = F$

that is L.H.S is false

Hence (d) is a valid statement.

S.136 (b)

This is a differential equation of first degree and second order.

A linear D.E. is of the form $\frac{dy}{dx} + Py = Q$ where P, Q are functions of x. A homogenous D.E. is of the form

$$a_0x^n\frac{d^ny}{dx^n} + a_1x^{n-1}\frac{d^{n-1}y}{dx^{n-1}} + a_2x^{n-2}\frac{d^{n-2}y}{dx^{n-2}} + ...$$

$$+ a_ny = P$$

where P is a function of x.

Hence the D.E. is non-linear

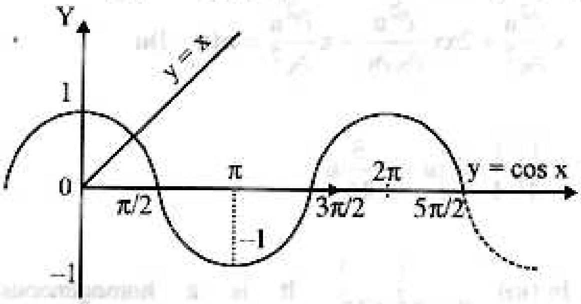
S.138 (b)

Given equation

X = cos x

X − cos x = 0

By ploting graph of x and cos x, we have



From the graph it follows that the line y = x meets the curve exactly once in the domain [0, π]

⇒ the equation X = cos x has exactly one solution.

(Note that the root of the equation X = cos x lies in the neighbourhood of π/4).

S.139 (d)

D.E. is $y'' + 3y' + 2y = 0$

$\Rightarrow (D^2 + 3D + 2)y = 0$

$\Rightarrow$ Auxiliary equation is

$D^2 + 3D + 2 = 0$

$\Rightarrow (D + 2)(D + 1) = 0$

$\Rightarrow D = -2, -1$

$\Rightarrow$ The solution is $y = C_1e^{-2x} + C_2e^{-x}$

or $= C_1e^{-x} + C_2e^{-2x}$ Since, C_1C_2 are arbitrary.

S.140 (c)

$f(x, y) = (x + y, x - y) = (u, v)$ say.

$\rightarrow x + y = u$

$x - y = v$

$$\Rightarrow x = \frac{u+v}{2}, \; y = \frac{u-v}{2}$$

$\because f(x,y) = (u,v)$

$$\text{Now, } f^{-1}(u, v) = (x, y) = \left(\frac{u+v}{2}, \frac{u-v}{2}\right)$$

$$\textbf{Hence } f^{-1}(x, y) = \left(\frac{x+y}{2}, \frac{x-y}{2}\right)$$

S.141 (d)

By Taylor's theorem,

$$f(x_0 + h) = f(x_0) + hf'(x_0) + \frac{h^2}{2!}f''(x_0)$$

$$\text{and } \quad f(x_0 - h) = f(x_0) - hf'(x_0) + \frac{h^2}{2!}f''(x_0)$$

$$\Rightarrow f(x_0 + h) + f(x_0 - h) = 2f(x_0) + h^2f''(x_0)$$

$$\Rightarrow \frac{f(x_0+h) - 2f(x_0) + f(x_0-h)}{h^2} = f''(x_0)$$

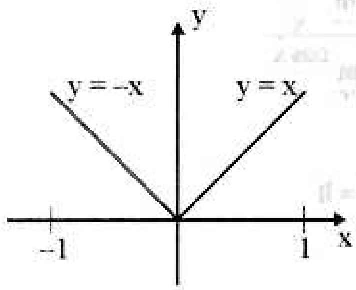
S.142 (b)

The function $y = |x|$ is certainly continuous in $[-1, 1]$.

$y = |x| = x, \qquad 0 \le x \le 1$

$\quad = -x, \qquad -1 \le x \le 0$

$|x|$ is not differentiable at $x = 0$



$$f'(0^+) = \lim_{h\to 0^+} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h\to 0^+} \frac{f(h) - f(0)}{h}$$

$$= \lim_{h\to 0^+} \frac{|h|}{h} \qquad [\because f(0) = 0]$$

$$= \lim_{h\to 0^+} \frac{h}{h} = 1 \quad [\because |h| = h \text{ when } h \to 0^+]$$

$$f'(0^-) = \lim_{h\to 0^-} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h\to 0^-} \frac{f(h) - f(0)}{h}$$

$$= \lim_{h\to 0^-} \frac{|h|}{h}$$

$$= \lim_{h\to 0^-} \frac{-h}{h} = -1$$

$[\because |h| = -h \text{ when } h \to 0^-]$

$\Rightarrow f'(0^+) \neq f'(0^-)$

$\Rightarrow f'(0)$ doesn't exist.

$\Rightarrow f$ is not differentiable in $[-1, 1]$.

S.143 (a)

$$\lim_{x\to\infty} \frac{x - \sin x}{x + \cos x} = \lim_{x\to\infty} \frac{1 - \sin x/x}{1 + \cos x/x}$$

$$= \frac{\lim_{x\to\infty}(1 - \sin x/x)}{\lim_{x\to\infty}(1 + \cos x/x)}$$

$$= \frac{1 - \lim_{x\to\infty} \frac{\sin x}{x}}{1 + \lim_{x\to\infty} \frac{\cos x}{x}}$$

$$= \frac{1-0}{1+0} = 1$$

S.144 (d)

Since

$$\int_0^a f(x)dx = \int_0^a f(a-x)dx$$

$$I = \int_0^{\pi/4} \frac{1-\tan x}{1+\tan x} dx$$

$$= \int_0^{\pi/4} \frac{1-\tan\left(\frac{\pi}{4}-x\right)}{1+\tan\left(\frac{\pi}{4}-x\right)}$$

Since $\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$

$$= \frac{1-\left[\dfrac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right]}{1+\left[\dfrac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right]} dx$$

$$= \int_0^{\pi/4} \frac{1-\left[\frac{1-\tan x}{1+\tan x}\right]}{1+\left[\frac{1-\tan x}{1+\tan x}\right]} dx$$

$$= \int_0^{\pi/4} \frac{(1+\tan x)-(1-\tan x)}{(1+\tan x)+(1-\tan x)} dx$$

$$= \int_0^{\pi/4} \frac{2\tan x}{2} dx$$

$$= \int_0^{\pi/4} \tan x dx$$

$$= \left[\log(\sec x)\right]_0^{\pi/4}$$

$$= \ln\left(\sec\frac{\pi}{4}\right) - \ln(\sec 0)$$

$$= \ln\left(\sqrt{2}\right) - \ln(1)$$

$$= \ln\left(2^{1/2}\right) - 0 = \frac{1}{2}\ln 2$$

UNIT–13

GENERAL APTITUDE

13.1

VERBAL ABILITY

ANALOGY

Directions :

In each of the following questions, a related pair of words or phrases is followed by five lettered pairs of words or phrases. Select the lettered pair that best expresses a relationship similar to that expressed in the original pair.

Q.1 DUNGEON : CONFINEMENT ::
(a) church : chapel
(b) school : truancy
(c) asylum : refuge
(d) hospital : mercy

Q.2 HERMIT : GREGARIOUS ::
(a) miser : penurious
(b) ascetic : hedonistic
(c) coward : pusillanimous
(d) scholar : literate

Q.3 MENDACITY : HONESTY ::
(a) courage : cravenness
(b) truth : beauty
(c) courage : fortitude
(d) unsophistication : ingenuousness

Q.4 MARATHON : STAMINA ::
(a) relay : independence
(b) hurdle : perseverance
(c) sprint : celerity
(d) jog : weariness

Q.5 HACKNEYED : ORIGINAL ::
(a) mature : juvenile
(b) trite : morbid
(c) withdrawn : reserved
(d) evasive : elusive

Q.6 MEANDERING : DIRECTNESS ::
(a) menacing : ambition
(b) affable : permissiveness
(c) digressive : conciseness
(d) circuitous rotation

Q.7 SIGNATURE : ILLUSTRATION ::
(a) byline : column
(b) alias : charge
(c) credit : purchase
(d) note : scale

Q.8 EULOGY : BLAME :

(a) elegy : loss

(b) satire : mockery

(c) tirade : abuse

(d) benediction : curse

Q.9 SNICKER : DISRESPECT ::

(a) whimper : impatience

(b) chortle : glee

(c) frown : indifference

(d) sneer : detachment

Q.10 IMPROMPTU : REHEARSAL ::

(a) practiced : technique

(b) makeshift : whim

(c) offhand : premeditation

(d) glib : fluency

Q.11 SCULPTOR : STONE ::

(a) essayist : words

(b) painter : turpentine

(c) composer : symphony

(d) logger : timber

Q.12 STORY : BUILDING ::

(a) plot : outline

(b) rung : ladder

(c) cable : elevator

(d) foundation : skyscraper

Q.13 STARE : GLANCE ::

(a) participate : observe

(b) scorn : admire

(c) hunt : stalk

(d) gulp : sip

Q.14 RANCID : TASTE ::

(a) tepid : temperature

(b) glossy : look

(c) rank : smell

(d) dulcet : sound

Q.15 WHISPER : SPEAK ::

(a) brush : touch

(b) skip : walk

(c) listen : hear

(d) request : ask

Q.16 TRAILER : MOTION PICTURE ::

(a) truck : cargo

(b) theater : play

(c) edition novel

(d) commercial : product

Q.17 SIGN : ZODIAC ::

(a) poster : billboard

(b) letter : alphabet

(c) prediction : prophecy

(d) signal : beacon

Q.18 BUFFOON : DIGNITY ::

(a) braggart : modesty

(b) blackguard : strength

(c) laughingstock : ridicule

(d) imposter : identification

Q.19 METAPHOR : FIGURATIVE ::

(a) fable : contemporary

(b) adage : paradoxical

(c) precept : instructive

(d) irony : dramatic

Q.20 JUGGERNAUT : INEXORABLE ::

(a) cosmonaut : worldly

(b) colossus : gigantic

(c) demagogue : liberal

(d) philistine : cultivated

SENTENCE COMPLITION

Directions :

Each sentence below has one or two blanks, each blank indicating that something has been omitted. Beneath the sentence are five lettered words or sets of words. Choose the word or set of words for each blank that best fits the meaning of the sentence as a whole.

Q.21 We lost confidence in him because he never —— the grandiose promises he had made.

(a) forgot about

(b) reneged on

(c) tired on

(d) delivered on

Q.22 We were amazed that a man who had been heretofore the most —— of public speakers could, in a single speech, electrify an audience and bring them cheering to their feet.

(a) enthralling

(b) accomplished

(c) pedestrian

(d) auspicious

Q.23 Despite the mixture's —— nature, we found that by lowering its temperature in the laboratory we could dramatically reduce its tendency to vaporize.

(a) resilient

(b) volatile

(c) homogeneous

(d) insipid

Q.24 In one shocking instance of —— research, one of the nation's most influential researchers in the field of genetics reported on experiments that were never carried out and published deliberately —— scientific papers on his nonexistent work.

(a) comprehensive...abstract

(b) theoretical...challenging

(c) fraudulent...deceptive

(d) derivative...authoritative

Q.25 To alleviate the problem of contaminated chicken, the study panel recommends that the federal government shift its inspection emphasis from cursory bird-by-bird visual checks to a more —— random sampling for bacterial and chemical contamination.

(a) rigorous

(b) perfunctory

(c) symbolic

(d) discreet

Q.26 To the dismay of the student body, the class president was —— berated by the principal at a school assembly.

(a) ignominiously

(b) privately

(c) magnanimously

(d) fortuitously

Q.27 The authority of voice in Frazer's writing strikes many readers today as —— colonialism; his prose seems as invulnerable and expansive as something on which the sun was presumed never to set.

(a) consonant with

(b) independent of

(c) ambivalent toward

(d) cognizant of

Q.28 Few other plants can grow beneath the canopy of the sycamore tree, whose leaves and pods produce a natural herbicide that leaches into the surrounding soil, —— other plants that might compete for water and nutrients.

(a) inhibiting

(b) distinguishing

(c) nourishing

(d) encouraging

Q.29 Despite an affected —— which convinced casual observers that he was indifferent about his painting and enjoyed only frivolity, Warhol cared deeply about his art and labored at it ——.

(a) nonchalance...diligently

(b) empathy...methodically

(c) fevor...secretly

(d) gloom...intermittently

Q.30 Just as disloyalty is the mark of the renegade, —— is the mark of the ——.

(a) timorousness...hero

(b) temerity...coward

(c) avarice...philanthropist

(d) cowardice...craven

Q.31 One of the most —— educators in New York, Dr. Shalala ignited a controversy in 1984 by calling the city public schools a "rotten barrel" in need of —— reform.

(a) disputations...little

(b) outspoken...systemic

(c) caustic...partial

(d) indifferent...pretentious

Q.32 The reasoning in this editorial is so —— that we cannot see how anyone can be deceived by it.

(a) coherent

(b) astute

(c) cogent

(d) specious

Q.33 Chatwin has devoted his life to a kind of Grail quest, hoping to prove–by study and direct experience with primitive people–that human nature is gentle and defensive rather than ——, and that man is ——, not a predator.

(a) belligerent...an apostate

(b) martial...a crusader.

(c) aggressive...a pilgrim

(d) truculent...a gladiator

Q.34 A —— statement is an —— comparison: it does not compare things explicitly, but suggests a likeness between them.

(a) sarcastic...unfair

(b) blatant...overt

(c) sanguine...inherent

(d) metaphorical...implied

Q.35 I have no —— motive in offering this advice; I seek no personal advantage or honor.

(a) nominal

(b) altruistic

(c) incongruous

(d) ulterior

Q.36 If you carry this —— attitude to the conference, you will —— any supporters you may have at this moment.

(a) belligerent...delight

(b) truculent...alienate

(c) conciliatory...defer

(d) supercilious...attract

Q.37 For Miro, art became a —— ritual; paper and pencils were holy objects to him and he worked as though he were performing a religious rite.

(a) superficial

(b) sacred

(c) banal

(d) cryptic

Q.38 Shy and hypochondriacal, Madison was uncomfortable at public gatherings; his character made him a most —— lawmaker and practicing politician.

(a) conscientious

(b) unlikely

(c) fervent

(d) gregarious

Q.39 We must try to understand his momentary —— for he has —— more strain and anxiety than any among us,

(a) outcry...described

(b) senility...understood

(c) vision...forgotten

(d) aberration...undergone

Q.40 Truculent in defending their individual rights of sovereignty under the Articles of Confederation, the newly formed states —— constantly.

(a) apologized

(b) digressed

(c) conferred

(d) squabbled

Q.41 I am not attracted by the —— life of the ——, always wandering through the countryside, begging for charity.

(a) proud...almsgiver

(b) noble...philanthropist

(c) affluent...mendicant

(d) peripatetic...vagabond

Q.42 The sugar dissolved in water ——; finally all that remained was an almost —— residue on the bottom of the glass.

(a) quickly...lumpy

(b) immediately...fragrant

(c) gradually...imperceptible

(d) subsequently...glassy

Q.43 Although the economy suffers downturns, it also has strong —— and self-correcting tendencies.

(a) unstable

(b) recidivist

(c) inauspicious

(d) recuperative

Q.44 After the Japanese attack on Pearl Harbor on December 7, 1941, Japanese-Americans were —— of being spies for Japan, although there was no —— to back up this accusation.

(a) acquitted...buttress

(b) tired...witness

(c) reminded...reason

(d) suspected...evidence

Q.45 The mind of a bigot is like the pupil of the eye: the more light you pour upon it, the more it will——.

(a) blink

(b) veer

(c) stare

(d) contract

Q.46 After having worked in the soup kitchen feeding the hungry, the volunteer began to see her own good fortune as —— and her difference from the —— as chance rather than destiny.

(a) an omen...homeless

(b) a fluke...impoverished

(c) a threat...destitute

(d) a reward...indigent

Q.47 Japan's industrial success is —— in part to its tradition of group effort and ——, as opposed to the emphasis on personal achievement that is a prominent aspect of other industrial nations.

(a) responsive...independence

(b) related...introspection

(c) equivalent...solidarity

(d) attributed...cooperation

Q.48 His listeners enjoyed his —— wit but his victims often —— at its satire.

(a) lugubrious...suffered

(b) caustic...laughed

(c) kindly...smarted

(d) trenchant...winced

Q.49 We are —— the intellects of the past; or, rather, like children we take it for granted that somebody must supply us with our supper and our ——.

(a) ungrateful to...ideas

(b) dependent on...repose

(c) unfaithful to...needs

(d) fortunate in...allowance

Q.50 Faced with these massive changes, the government keeps its own counsel; although generally benevolent, it has always been —— regime.

(a) an altruistic

(b) an unpredictable

(c) a reticent

(d) a sanguine

SYNONYMS

Directions :

Each of the questions below consists of a word in capital letters, followed by five lettered words or phrases. Choose the lettered word or phrase that is most nearly similar in meaning to the word in capital letters and write the letter of your choice on your answer paper.

Q.51 ABLUTION

(a) censure

(b) forgiveness

(c) survival

(d) washing

Q.52 ABSTRUSE

(a) profound

(b) irrespective

(c) suspended

(d) protesting

Q.53 ACCEDE

(a) fail

(b) compromise

(c) correct

(d) consent

Q.54 AVARICE

(a) easiness

(b) greed

(c) statement

(d) power

Q.55 BANAL

(a) philosophical

(b) trite

(c) dramatic

(d) heedless

Q.56 BOISTEROUS

(a) conflicting

(b) noisy

(c) testimonial

(d) grateful

Q.57 BRINDLED

(a) equine

(b) pathetic

(c) hasty

(d) spotted

Q.58 CALLOW

(a) youthful

(b) holy

(c) mild

(d) colored

Q.59 CIRCUITOUS
(a) indirect
(b) complete
(c) obvious
(d) aware

Q.60 CLANDESTINE
(a) abortive
(b) secret
(c) tangible
(d) doomed

Q.61 COGNIZANCE
(a) policy
(b) knowledge
(c) advance
(d) examination

Q.62 CONSENSUS
(a) general agreement
(b) project
(c) insignificance
(d) sheaf

Q.63 CONTRITE
(a) smart
(b) penitent
(c) restful
(d) recognized

Q.64 COPIOUS
(a) plentiful
(b) cheating
(c) dishonorable
(d) adventurous

Q.65 DECIMATE
(a) kill
(b) disgrace
(c) search
(d) collide

Q.66 DECREPITUDE
(a) feebleness
(b) disease
(c) coolness
(d) melee

Q.67 DENIGRATE
(a) refuse
(b) blacken
(c) terrify
(d) admit

Q.68 DEPRAVITY
(a) wickedness
(b) sadness
(c) heaviness
(d) tidiness

Q.69 EDIFY
(a) mystify
(b) suffice
(c) improve
(d) erect

Q.70 EGREGIOUS
(a) pious
(b) shocking
(c) anxious
(d) sociable

Q.71 ELUSIVE
(a) deadly
(b) eloping
(c) evasive
(d) simple

Q.72 EMBROIL
(a) cherish
(b) overheat
(c) entangle
(d) assure

Q.73 FLORID
(a) ruddy
(b) rusty
(c) ruined
(d) patient

Q.74 FORMIDABLE
(a) dangerous
(b) outlandish
(c) grandiloquent
(d) impenetrable

Q.75 FURTIVE
(a) underhanded
(b) coy
(c) brilliant
(d) quick

Q.76 GARRULITY
(a) credulity
(b) senility
(c) loquaciousness
(d) speciousness

Q.77 GAUCHE
(a) rigid
(b) swift
(c) awkward
(d) taciturn

Q.78 GAUNT
(a) victorious
(b) tiny
(c) stylish
(d) haggard

Q.79 INCLEMENT
(a) unfavorable
(b) abandoned
(c) kindly
(d) selfish

Q.80 INCORRIGIBLE
(a) narrow
(b) straight
(c) inconceivable
(d) unreformable

Q.81 INEBRIETY
(a) revelation
(b) drunkenness
(c) felony
(d) starvation

Q.82 INFIRMITY
(a) disability
(b) age
(c) inoculation
(d) hospital

Q.83 INNOCUOUS
(a) not capable
(b) not dangerous
(c) not eager
(d) not frank

Q.84 INSINUATE
(a) resist
(b) suggest
(c) report
(d) rectify

Q.85 INTREPID
(a) cold
(b) hot
(c) understood
(d) courageous

Q.86 MENIAL
(a) intellectual
(b) clairvoyant
(c) servile
(d) arrogant

Q.87 MINION
(a) quorum
(b) majority
(d) host
(d) dependent

Q.88 MISANTHROPE
(a) benefactor
(b) philanderer
(c) man-hater
(d) epicure

Q.89 MOLLIFY
(a) avenge
(b) attenuate
(c) attribute
(d) appease

Q.90 MORIBUD
(a) dying
(b) appropriate
(c) leather bound
(d) answering

Q.91 MUNIFICENT
(a) grandiose
(b) puny
(c) philanthropic
(d) poor

Q.92 PHLEGMATIC
(a) calm
(b) cryptic
(c) practical
(d) salivary

Q.93 PLAGIARISM
(a) theft of funds
(b) theft of ideas
(c) belief in God
(d) arson

Q.94 PLATITUDE
(a) fatness
(b) bravery
(c) dimension
(d) trite remark

Q.95 REGAL
(a) oppressive
(b) royal
(c) major
(d) basic

Q.96 REITERATE
(a) gainsay
(b) revive
(c) revenge
(d) repeat

Q.97 REPERCUSSION
(a) reaction
(b) restitution
(c) resistance
(d) magnificence

Q.98 REPRISAL
(a) reevaluation
(b) assessment
(c) loss
(d) retaliation

Q.99 REPUGNANCE
(a) tenacity
(b) renewal
(c) pity
(d) loathing

Q.100 TAWDRY
(a) orderly
(b) meretricious
(c) reclaimed
(d) filtered
(e) proper

Q.101 TEMPORAL
(a) priestly
(b) scholarly
(c) secular
(d) sleepy

Q.102 TEPID
(a) boiling
(b) lukewarm
(c) freezing
(d) gaseous

Q.103 THESPIAN
(a) foreigner
(b) skeptic
(c) daydreamer
(d) actor

Q.104 VOLUBLE
(a) worthwhile
(b) serious
(c) terminal
(d) loquacious

Q.105 VORACIOUS
(a) ravenous
(b) spacious
(c) truthful
(d) pacific

Q.106 WANTON
(a) needy
(b) passive
(c) rumored
(d) unchaste

Q.107 ZEALOT
(a) beginner
(b) patron
(c) fanatic
(d) murderer

ANTONYMS

Directions :

Each of the questions below consists of a word in capital letters, followed by five lettered words or phrases. Choose the lettered word or phrase that is most nearly similar in meaning to the word in capital letters and write the letter of your choice on your answer paper.

Q.108 ALACRITY
(a) slowness
(b) plenty
(c) filth
(d) courtesy

Q.109 ALLEVIATE
(a) endure
(b) worsen
(c) enlighten
(d) maneuver

Q.110 AMELIORATE
(a) make slow
(b) make sure
(c) make young
(d) make worse

Q.111 ANTITHESIS
(a) velocity
(b) maxim
(c) similarity
(d) acceleration

Q.112 APHASIA
(a) volubility
(b) necessity
(c) pain
(d) crack

Q.113 BENIGN
(a) tenfold
(b) peaceful
(c) wavering
(d) **malignant**

Q.114 BERATE
(a) grant
(b) **praise**
(c) refer
(d) purchase

Q.115 BIGOTRY
(a) arrogance
(b) approval
(c) promptness
(d) **tolerance**

Q.116 CAPRICIOUS
(a) satisfied
(b) photographic
(c) scattered
(d) **steadfast**

Q.117 CARNAL
(a) impresssive
(b) minute
(c) **spiritual**
(d) actual

Q.118 CENSURE
(a) process
(b) enclose
(c) interest
(d) **praise**

Q.119 CUPIDITY
(a) anxiety
(b) tragedy
(c) **generosity**
(d) entertainment

Q.120 DANK
(a) **dry**
(b) guiltless
(c) warm
(d) babbling

Q.121 DEBONAIR
(a) **awkward**
(b) windy
(c) balmy
(d) strong

Q.122 DIABOLICAL
(a) mischievous
(b) lavish
(c) **seraphic**
(d) azure

Q.123 DIN
(a) lightness
(b) safety
(c) **silence**
(d) hunger

Q.124 ENERVATE
(a) **strengthen**
(b) sputter
(c) arrange
(d) scrutinize

Q.125 ENUNCIATE
(a) pray
(b) request
(c) deliver
(d) **mumble**

Q.126 ERUDITE
(a) professorial
(b) stately
(c) short
(d) **ignorant**

Q.127 EULOGISTIC
(a) pretty
(b) critical
(c) brief
(d) stern

Q.128 EXASPERATE
(a) confide
(b) formalize
(c) placate
(d) betray

Q.129 EXHUME
(a) decipher
(b) sadden
(c) integrate
(d) inter

Q.130 EXORBITANT
(a) moderate
(b) partisan
(c) military
(d) barbaric

Q.131 GRANDIOSE
(a) false
(b) ideal
(c) proud
(d) simple

Q.132 GREGARIOUS
(a) antisocial
(b) anticipatory
(c) glorious
(d) horrendous

Q.133 HACKNEYED
(a) carried
(b) original
(c) banned
(d) timely

Q.134 HALCYON
(a) wasteful
(b) prior
(c) subsequent
(d) martial

Q.135 HIBERNAL
(a) musical
(b) summerlike
(c) local
(d) seasonal

Q.136 ILLUSIVE
(a) not deceptive
(b) not certain
(c) not obvious
(d) not coherent

COMPREHENSION

Directions :

Each passage in this group is followed by questions based on its content. After reading a passage, choose the best answer to each question. Answer all questions following a passage on the basis of what is stated or implied in that passage.

Passage 1 :

Both plants and animals of many sorts show remarkable changes in form, structure, growth habits, and even mode of reproduction in becoming adapted to different climatic environment, types of food supply, or mode of living. This divergence in response to evolution is commonly expressed by altering the form and function of some part or parts of the organism, the original identity of which is clearly discernible. For example, the creeping foot of the snail is seen in related marine pteropods to be modified into a flapping organ useful for swimming, and is changed into prehensile arms that bear suctorial disks in the squids and other cephalopods. The limbs of various mammals are modified according to several different modes of life–for swift running (cursorial) as in the horse and monkeys, for digging (fossorial) as in the moles and gophers, for flying (volant) as in the bats, for swimming (aquatic) as in the seals, whales and dolphins and for other adaptations. The structures or organs that show main change in connection with this adaptive divergence are commonly identified readily as homologous, in spite of great alterations.

Thus, the finger and wristbones of a bat and whale, for instance, have virtually nothing in common except that they are definitely equivalent elements of the mammalian limb.

Q.137 Which of the following is the most appropriate title for the passage, based on its content?

(a) Adaptive Divergence

(b) Evolution

(c) Unusual Structures

(d) Changes in Organs

Q.138 The author provides information that would answer which of the following questions?

I. What factors cause change in organisms?

II. What is the theory of evolution?

III. How are horses 'legs related to seals' flippers?

(a) I only

(b) II only

(c) I and II only

(d) I and III only

Q.139 Which of the following words could best be substituted for "homologous" (line 24) without substantially changing the author's meaning?

(a) altered

(b) mammalian

(c) corresponding

(d) divergent

Q.140 The author's style can best be described as

(a) humorous

(b) objective

(c) patronizing

(d) esoteric

Passage 2 :

An essay which appeals chiefly to the intellect is Francis Bacon's "Of Studies". His careful tripartite division of studies expressed succinctly in aphoristic prose demands the complete attention of the mind of the reader. He considers studies as they should be: for pleasure, for self-improvement, for business. He considers the evils of excess study: laziness, affectation, and preciosity. Bacon divides books into three categories: those to be read in part, those to be read cursorily, and those to be read with care. Studies should include reading, which gives depth, . speaking, which adds readiness of thought; and writing, which trains in preciseness. Some what mistakenly, the author ascribes certain virtues to individual fields of study: wisdom to history, wit to poetry, subtlety to mathematics, and depth to natural philosophy. Bacon's four–hundred–word essay, studded with Latin phrases and highly compressed in thought, has intellectual appeal indeed.

Q.141 Which of the following is the most appropriate title for the passage, based on its content?

(a) Francis Bacon and the Appeal of the Essay

(b) "Of Studies" A Tripartite Division

(c) An Intellectual Exercise : Francis Bacon's "Of Studies"

(d) The Categorization of Books According to Bacon

Q.142 Which of the following words could best be substituted for "aphoristic" (lines 3–4) without substantially changing the author's meaning?

(a) abstruse

(b) pithy

(c) tripartite

(d) realistic

Q.143 The passage suggests that the author would be most likely to agree with which of the following statements?

(a) "Of Studies" belongs in the category of works that demand to be read with care.

(b) Scholar's personalities are shaped by the academic discipline in which they are engaged.

(c) It is an affectation to use foreign words in one's writing.

(d) An author can be more persuasive in a long work than in a shorter one.

Passage 3 :

Although vocal cords are lacking in cetaceans, phonation is undoubtedly centered in the larynx.

The toothed whales or odontocetes (sperm whale and porpoises) are much more vociferous than the whalebone whales, or mysticetes. In this country observers have recorded only occasional sounds from two species of mysticetes (the humpback and right whale). A Russian cetologist reports hearing sounds from at least five species of whalebone whales but gives no details of the circumstances or descriptions of the sounds themselves. Although comparison of the sound–producing apparatus in the two whale groups cannot yet be made, it is interesting to note that the auditory centers of the brain are much more highly developed in the odontocetes than in the mysticetes, in fact, to a degree unsurpassed by any other mammalian group.

Q.144 The passage contains information that would answer which of the following questions?

I. What are odontocetes and mysticetes?

II. In which part of the body do whales produce sounds?

III. In which animals is the auditory center of the brain most developed?

(a) I only

(b) II only

(c) I and II only

(d) II and III only

Q.145 The author's attitude toward the observations reported by the Russian cetologist mentioned in lines 8–11 is best described as one of

(a) admiration

(b) indignation

(c) surprise

(d) skepticism

Q.146 It can be inferred from the passage that

(a) animals with more highly developed auditory apparatuses tend to produce more sounds

(b) animals without vocal cords tend to produce as much sound as those with vocal cords

(c) highly intelligent animals tend to produce more sound than less intelligent species

(d) the absence of vocal cords has hindered the adaptation of cetaceans

Passage 4 :

"The emancipation of women," James Joyce told one of his friends, "has caused the greatest revolution in our time in the most important relationship there is– that between men and women." Other modernists agreed: Virginia Woolf, claiming that in about 1910 "human character changed," and, illustrating the new balance between the sexes, urged, "Read the 'Agamemnon,' and see whether... your sympathies are not almost entirely with Clytemnestra." D.H. Lawrence wrote, "perhaps the deepest fight for 2000 years and more, has been the light for women's independence."

But if modernist writers considered women's revolt against men's domination one of their "greatest" and "deepest" themes, only recently–in perhaps the past 15 years–has literary criticism begun to catch up with it. Not that the images of sexual antagonism that abound in modern literature have gone unremarked; far from it. But what we are able to see in literary works depends on the perspectives we bring to them, and now that women–enough to make a difference–are reforming canons and interpreting literature, the landscapes of literary history and the features of individual books have begun to change.

Q.147 According to the passage, women are changing literary criticism by

(a) noting instances of hostility between men and women

(b) seeing literature from fresh points of view

(c) studying the works of early twentieth-century writers

(d) reviewing books written by feminists

Q.148 The author quotes James Joyce, Virginia Woolf, and D.H. Lawrence primarily in order to show that

(a) these were feminist writers

(b) although well–meaning, they were ineffectual

(c) modern literature is dependent on the women's movement

(d) the interest in feminist issues is not new

Q.149 The author's attitude toward women's reformation of literary canons can best be described as one of

(a) ambivalence

(b) antagonism

(c) indifference

(d) endorsement

Q.150 Which of the following titles best describes the content of the passage?

(a) Modernist Writers and the Search for Equality

(b) The Meaning of Literary Works

(c) Toward a New Criticism

(d) Women in Literature, from 1910 On

Passage 5 :

Observe the dilemma of the fungus: it is a plant, but it possesses no chlorophyll. While all other plants put the sun's energy to work for them combining the nutrients of ground and air into the body structure, the chlorophylless fungus must look else-where for an energy supply. It finds it in those other plants which, having received their energy free from the sun, relinquish it at some point in their cycle either to animals (like us humans) or to fungi.

In this search for energy the fungus become the earth's major source of rot and decay. Wherever you see mold forming on a piece of bread, or a pile of leaves turning to compost, or a blown–down tree becoming pulp on the ground, you are watching a fungus eating. Without fungus action the earth would be piled high with the dead plant life of past centuries. In fact, certain plants which contain resins that are toxic to fungi will last indefinitely; specimens of the redwood, for instance, can still be found resting on the forest floor centuries after having been blown down.

Q.151 Which of the following words best describes the fungus as depicted in the passage?

(a) Unevolved

(b) Sporadic

(c) Enigmatic

(d) Parasitic

Q.152 The passage states all the following about fungi EXCEPT:

(a) They are responsible for the decomposition of much plant life.

(b) They cannot live completely apart from other plants.

(c) They are vastly different from other plants.

(d) They are poisonous to resin–producing plants.

Q.153 The author's statement that "you are watching a fungus eating" (lines 15–16) is best described as

(a) figurative

(b) ironical

(c) parenthetical

(d) erroneous

Q.154 The author's is primarily concerned with

(a) warning people of the dangers of fungi

(b) writing a humorous essay on fungi

(c) relating how most plants use solar energy

(d) describing the actions of fungi

ANSWER KEY

1	c	2	b	3	a	4	c	5	a
6	c	7	a	8	d	9	b	10	c
11	a	12	b	13	d	14	c	15	a
16	d	17	b	18	a	19	c	20	b
21	d.	22	c	23	b	24	c	25	a
26	a	27	a	28	a	29	a	30	d
31	b	32	d	33	c	34	d	35	d
36	b	37	b	38	b	39	d	40	d
41	d	42	c	43	d	44	d	45	d
46	b	47	d	48	d	49	a	50	c
51	d	52	a	53	d	54	b	55	b
56	b	57	d	58	a	59	a	60	b
61	b	62	a	63	b	64	a	65	a
66	a	67	b	68	a	69	c	70	b
71	c	72	c	73	a	74	a	75	a
76	c	77	c	78	d	79	a	80	d
81	b	82	a	83	b	84	b	85	d
86	c	87	d	88	c	89	d	90	a
91	c	92	a	93	b	94	d	95	b
96	d	97	a	98	d	99	d	100	d
101	c	102	b	103	d	104	d	105	a
106	d	107	c	108	a	109	b	110	d
111	c	112	a	113	d	114	b	115	d
116	d	117	c	118	d	119	c	120	a
121	a	122	c	123	c	124	a	125	d
126	d	127	b	128	c	129	d	130	a
131	d	132	a	133	b	134	d	135	b
136	a	137	a	138	d	139	c	140	b
141	c	142	b	143	a	144	c	145	d
146	a	147	b	148	d	149	d	150	c
151	d	152	d	153	a	154	d		

13.2

NUMERICAL ABILITY

Q.1 The unit's digit in the product $(7^{71} \times 6^{59} \times 3^{65})$ is :

(a) 1
(b) 2
(c) 4
(d) 6

Q.2 If x is a whole number, then $x^2(x^2 - 1)$ is always divisible by:

(a) 12
(b) 24
(c) 12 – x
(d) multiple of 12

Q.3 Three different containers contain 496 liters, 403 liters and 713 liters of mixtures of milk and water respectively. What biggest measure can measure all the different quantities exactly?

(a) 1 litre
(b) 7 litres
(c) 31 litres
(d) 41 litres

Q.4 The H.C.F. and L.C.M. of two numbers are 11 and 385 respectively. If one number lies between 75 and 125, then that number is:

(a) 77
(b) 88
(c) 99
(d) 110

Q.5 The number of students in each section of a school is 24. After admitting new students, three new sections were started. Now, the total number of section is 16 and there are 21 students in each section. The number of new students admitted is:

(a) 14
(b) 24
(c) 48
(d) 114

Q.6 A light was seen at intervals of 13 seconds. It was seen for the first time at 1 hr. 54 min. 50 secs. a.m. and the last time at 3 hrs. 17 min. 49 secs. a.m. How many times was the light seen?

(a) 360
(b) 375
(c) 378
(d) 384

Q.7 $\frac{1}{10}$ of a pole is coloured red, $\frac{1}{20}$ white, $\frac{1}{30}$ blue, $\frac{1}{40}$ black, $\frac{1}{50}$ violet, $\frac{1}{60}$ yellow and the rest is green. If the length of the green portion of the pole is 12.08 meters, then the length of the pole is :

(a) 16 m
(b) 18 m
(c) 20 m
(d) 30 m

Q.8 At an International Dinner, $\frac{1}{5}$ of the people attending were French men. If the number of French women at the dinner was $\frac{2}{3}$ greater than the number of French men, and there were no other French people at the dinner, then what fraction of the people at the dinner were not French?

(a) $\frac{1}{5}$

(b) $\frac{2}{5}$

(c) $\frac{2}{3}$

(d) $\frac{7}{15}$

Q.9 There are two examination rooms A and B. If 10 students are sent from A to B, then the number of students in each room is the same. If 20 candidates are sent from B to A, then the number of students in A is double the number of students in B. The number of students in room A is :

(a) 20

(b) 80

(c) 100

(d) 200

Q.10 The average temperature of the town in the first four days of a month was 58 degrees. The average for the second, third, fourth and fifth days was 60 degrees. If the temperature of the first and fifth days were in the ratio 7 : 8, then what is the temperature on the fifth day?

(a) 64 degrees

(b) 62 degrees

(c) 56 degrees

(d) None of these

Q.11 The average age of 3 children in a family is 20% of the average age of the father and the eldest child. The total age of the mother and the youngest child is 39 years. If the father's age is 26 years, what is the age of second child?

(a) 15 years

(b) 18 years

(c) 20 years

(d) Cannot be determined

Q.12 A number consists of two digits such that the digit in the ten's place is less by 2 than the digit in the unit's place. Three times the number added to $\frac{6}{7}$ times the number obtained by reversing the digits equals 108. The sum of the digits in the number is :

(a) 6

(b) 7

(c) 8

(d) 9

Q.13 In a two–digit number, the digit in the unit's place is more than twice the digit in ten's place by 1. If the digits in the unit's place and the ten's place are interchanged, difference between the newly formed number and the original number is less than the original number by 1. What is the original number?

(a) 25

(b) 37

(c) 49

(d) 52

Q.14 Tanya's grandfather was 8 times older to her 16 years ago. He would be 3 times of her age 8 years from now. Eight years ago, what was the ratio of Tanya's age to that to her grandfather?

(a) 1 : 2

(b) 1 : 5

(c) 3 : 8

(d) None of these

Q.15 Q is as much younger than R as he is older than T. If the sum of the ages of R and T is 50 years, what is definitely the difference between R and Q's age?

(a) 1 year

(b) 2 years

(c) 25 years

(d) Data inadequate

Q.16 Father is aged three times more than his son Ronit. After 8 years, he would be two and a half times of Ronit's age. After further 8 years, how many times would he be of Ronit's age?

(a) 2 times

(b) $2\frac{1}{2}$ times

(c) $2\frac{3}{4}$ times

(d) 3 times

Q.17 My brother is 3 years elder to me. My father was 28 years of age when my sister was born while my mother was 26 years of age when I was born. If my sister was 4 years of age when my brother was born, then, what was the age of my father and mother respectively when my brother was born?

(a) 32 yrs, 23 yrs

(b) 32 yrs, 29 yrs

(c) 35 yrs, 29 yrs

(d) 35 yrs, 33 yrs

Q.18 In a competitive examination is State A, 6% candidates got selected from the total appeared candidates. State B had an equal number of candidates appeared and 7% candidates got selected with 80 more candidates got selected than A. What was the number of candidates appeared from each State?

(a) 7600

(b) 8000

(c) 8400

(d) Data inadequate

Q.19 8% of the people eligible to vote are between 18 and 21 years of age. In an election, 85% of those eligible to vote, who were between 18 and 21, actually voted. In that election, the number of persons between 18 and 21, who actually voted, was what percent of those eligible to vote?

(a) 4.2

(b) 6.4

(c) 6.8

(d) 8

Q.20 In an election, 30% of the votes voted for candidate A whereas 60% of the remaining voted for candidate B. The remaining votes did not vote. If the difference between those who voted for candidate A and those who did not vote was 1200, how many individuals were eligible for casting vote in that election?

(a) 10,000

(b) 45,000

(c) 60,000

(d) 72,000

Q.21 $\frac{5}{9}$ part of the population in a village are males. If 30% of the males are married, the percentage of unmarried females in the total population is:

(a) 20%

(b) $27\frac{7}{9}\%$

(c) 40%

(d) 70%

Q.22 The income of a broker remains unchanged though the rate of commission is increased from 4% to 5%. The percentage of slump in business is:

(a) 1%

(b) 8%

(c) 20%

(d) 80%

Q.23 In an examination, 34% of the students failed in Mathematics and 42% failed in English. If 20% of the students failed in both the subjects, then the percentage of students who passed in both the subjects was:

(a) 44

(b) 50

(c) 54

(d) 56

Q.24 In a hotel, 60% had vegetarian lunch while 30% had non-vegetarian lunch and 15% had both types of lunch. If 96 people were present, how many did not eat either type of lunch?

(a) 20

(b) 24

(c) 26

(d) 28

Q.25 In an examination, 65% students passed in Civics and 60% in History, 40% passed in both of these subjects. If 90 students failed in History and Civics both, then what is the total number of students?

(a) 600

(b) 650

(c) 700

(d) 750

Q.26 Pure ghee costs Rs.100 per kg. After adulterating it with vegetable oil costing Rs.50 per kg, a shopkeeper sells the mixture at the rate of Rs.96 per kg, thereby making a profit of 20%. In what ratio does he mix the two?

(a) 3 : 2

(b) 3 : 1

(c) 2 : 3

(d) 1 : 3

Q.27 A dairyman pays Rs. 6.40 per litre of milk. He adds water and sells the mixture at Rs. 8 per litre, thereby making 37.5% profit. The proportion of water to milk received by the customer is:

(a) 1 : 10

(b) 1 : 12

(c) 1 : 15

(d) 1 : 20

Q.28 On selling a chair at 7% loss and a table at 17% gain, a man gain Rs. 296. If he sells the chair at 7% gain and the table at 12% gain, then he gains Rs. 400. The actual price of the table is:

(a) Rs. 1600

(b) Rs. 1800

(c) Rs. 2200

(d) Rs. 2400

Q.29 An industrial loom weaves 0.128 meters of cloth every second. Approximately, how many seconds will it take for the loom to weave 25 metres of cloth?

(a) 178

(b) 195

(c) 204

(d) 488

Q.30 A wheel that has 6 cogs is meshed with a larger wheel of 14 cogs. When the smaller wheel has made 21 revolutions, then the number of revolutions made by the larger wheel is:

(a) 4

(b) 9

(c) 12

(d) 49

Q.31 If 80 lamps can be lighted, 5 hours per day for 10 days for Rs. 21.25, then the number of lamps, which can be lighted 4 hours daily for 30 days, for Rs. 76.50, is :

(a) 100

(b) 120

(c) 150

(d) 160

Q.32 400 persons, working 9 hours per day complete $\frac{1}{4}$th of the work in 10 days. The number of additional persons, working 8 hours per day, required to complete the remaining work in 20 days, is :

(a) 675

(b) 275

(c) 250

(d) 225

Q.33 A contractor employed 30 men to do a piece of work in 38 days. After 25 days, he employed 5 men more and the work was finished one day earlier. How many days he would have been behind, if he had not employed additional men?

(a) 1

(b) $1\frac{1}{4}$

(c) $1\frac{3}{4}$

(d) $1\frac{1}{2}$

Q.34 2 men and 7 boys can do a piece of work in 14 days; 3 men and 8 boys can do the same in 11 days. Then, 8 men and 6 boys can do three times the amount of this work in :

(a) 18 days

(b) 21 days

(c) 24 days

(d) 30 days

Q.35 P, Q and R are three typists who working simultaneously can type 216 pages in 4 hours. In one hour, R can type as many pages more than Q as Q can type more than P. During a period of five hours, R can type as many pages as P can during seven hours. How many pages does each of them type per hour?

(a) 14, 17, 20

(b) 15, 17, 22

(c) 15, 18, 21

(d) 16, 18, 22

Q.36 A and B can do a piece of work in 5 days; B and C can do it in 7 days; A and C can do it in 4 days. Who among these will take least time if put to do it alone?

(a) A

(b) B

(c) C

(d) Data inadequate

Q.37 A can do a piece of work in 4 hours; B and C together can do it in 3 hours, while A and C together can do it in 2 hours. How long will B alone take to do it?

(a) 8 hours

(b) 10 hours

(c) 12 hours

(d) 24 hours

Q.38 A does half as much work as B in three-fourth of the time. If together they take 18 days to complete the work, how much time shall B take to do it?

(a) 30 days

(b) 35 days

(c) 40 days

(d) None of these

Q.39 A can finish a work in 24 days, B in 9 days and C in 12 days. B and C start the work but are forced to leave after 3 days. The remaining work was done by A in :

(a) 5 days

(b) 6 days

(c) 10 days

(d) $10\frac{1}{2}$ days

Q.40 X and Y can do a piece of work in 20 days and 12 days respectively. X started the work alone and then after 4 days Y joined him till the completion of the work. How long did the work last?

(a) 6 days

(b) 10 days

(c) 15 days

(d) 20 days

Q.41 A man, a woman and a boy can complete a job in 3, 4 and 12 days respectively. How many boys must assist 1 man and 1 woman to complete the job in $\frac{1}{4}$ of a day?

(a) 1

(b) 4

(c) 19

(d) 41

Q.42 One man, 3 women and 4 boys can do a piece of work in 96 hours, 2 men and 8 boys can do it in 80 hours, 2 men and 3 women can do it in 120 hours. 5 men and 12 boys can do it in :

(a) $39\frac{1}{11}$ hours

(b) $42\frac{7}{11}$ hours

(c) $43\frac{7}{11}$ hours

(d) 44 hours

Q.43 A tank is filled by three pipes with uniform flow. The first two pipes operating simultaneously fill the tank in the same time during which the tank is filled by the third pipe alone. The second pipe fills the tank 5 hours faster than the first pipe and 4 hours slower than the third pipe. The time required by the first pipe is:

(a) 6 hrs.

(b) 10 hrs.

(c) 15 hrs.

(d) 30 hrs.

Q.44 Two pipes A and B can fill a tank in 12 minutes and 15 minutes respectively. If both the taps are opened simultaneously, and the tap A is closed after 3 minutes, then how much more time will it take to fill the tank by tap B?

(a) 7 min 15 sec

(b) 7 min 45 sec

(c) 8 min 5 sec

(d) 8 min 15 sec

Q.45 Three taps A, B and C can fill a tank in 12, 15 and 20 hours respectively. If A is open all the time and B and C are open for one hour each alternately, the tank will be full in :

(a) 6 hrs

(b) $6\frac{2}{3}$ hrs

(c) 7 hrs

(d) $7\frac{1}{2}$ hrs

Q.46 An express train travelled at an average speed of 100 km/hr, stopping for 3 minutes after every 75 km. How long did it take to reach its destination 600 km from the starting point?

(a) 6 hrs 21 min

(b) 6 hrs 24 min

(c) 6 hrs 27 min

(d) 6 hrs 30 min

Q.47 A motor car starts with the speed of 70 km/hr with its speed increasing every two hours by 10 kmph. In how hours will it cover 345 kms?

(a) $2\frac{1}{4}$ hrs

(b) 4 hrs 5 min

(c) $4\frac{1}{2}$ hrs

(d) Cannot be determined

Q.48 A is faster than B. A and B each walk 24 km. The sum of their speeds is 7 km/hr and the sum of times taken by them is 14 hours. Then, A's speed is equal to :

(a) 3 km/hr

(b) 4 km/hr

(c) 5 km/hr

(d) 7 km/hr

Q.49 The average speed of a train in the onward journey is 25% more than that is return journey. The train halts for one hour on reaching the destination. The total time taken for the complete to and fro journey is 17 hours, covering a distance of 800 km. The speed of the train in the onward journey is :

(a) 45 km/hr

(b) 47.5 km/hr

(c) 52 km/hr

(d) 56.25 km/hr

Q.50 A motorist covers a distance of 39 km in 45 minutes by moving at a speed of x kmph for the first 15 minutes, then moving at double the speed for the next 20 minutes and then again moving at his original speed for the rest of the journey. Then, x is equal to :

(a) 31.2

(b) 36

(c) 40

(d) 52

Q.51 Walking $\frac{6}{7}$ th of his usual speed, a man is 12 minutes too late. The usual time taken by him to cover that distance is:

(a) 1 hour

(b) 1 hr 12 min.

(c) 1 hr 15 min.

(d) 1 hr 20 min.

Q.52 Excluding stoppages, the speed of a bus is 54 kmph and including stoppages, it is 45 kmph. For how many minutes does the bus stop per hour?

(a) 9

(b) 10

(c) 12

(d) 20

Q.53 It takes eight hours for a 600 km journey, if 120 km is done by train and the rest by car. It takes 20 minutes more, if 200 km is done by train and the rest by car. The ratio of the speed of the train to that of the car is:

(a) 2 : 3

(b) 3 : 2

(c) 3 : 4

(d) 4 : 3

Q.54 A walks around a circular field at the rate of one round per hour while B runs around it at the rate of six rounds per hour. They start in the same direction from the same point at 7.30 a.m. They shall first cross each other at :

(a) 7.42 a.m.

(b) 7.48 a.m.

(c) 8.10 a.m.

(d) 8.30 a.m.

Q.55 A train M leaves Meerut at 5 a.m. and reaches Delhi at 9 a.m. Another train leaves Delhi at 7 a.m. and reaches Meerut at 10.30 a.m. At what time do the two train cross each other?

(a) 7:36 a.m.

(b) 7:56 a.m.

(c) 8:00 a.m.

(d) 8:26 a.m.

Q.56 A man takes 5 hours 45 min. in walking to a certain place and riding back. He would have gained 2 hours by riding both ways. The time he would take to walk both ways, is :

(a) 3 hrs 45 min.

(b) 7 hrs 30 min.

(c) 7 hrs 45 min.

(d) 11 hrs 45 min.

Q.57 A train passes a station platform in 36 seconds and a man standing on the platform in 20 seconds. If the speed of the train is 54 km/hr, what is the length of the platform?

(a) 120 m

(b) 240 m

(c) 300 m

(d) None of these

Q.58 How many seconds will a 500 metre long train take to cross a man walking with a speed of 3 km/hr in the direction of the moving train if the speed of the train is 63 km/hr?

(a) 25

(b) 30

(c) 40

(d) 45

Q.59 Two trains are moving in opposite direction @ 60 km/hr and 90 km/hr. Their lengths are 1.10 km and 0.9 km respectively. The time taken by the slower train to cross the faster train in seconds is :

(a) 36

(b) 45

(c) 48

(d) 49

Q.60 A train overtakes two persons who are walking in the same direction in which the train is going, at the rate of 2 kmph and 4 kmph and passes them completely in 9 and 10 seconds respectively. The length of the train is :

(a) 45 m

(b) 50 m

(c) 54 m

(d) 72 m

Q.61 A train travelling at 48 kmph completely crosses another train having half its length and travelling in opposite direction at 42 kmph, in 12 seconds. It also passes a railway platform in 45 seconds. The length of the platform is:

(a) 400 m

(b) 450 m

(c) 560 m

(d) 600 m

Q.62 A train X starts from Meerut at 4 p.m. and reaches Ghaziabad at 5 p.m. while another train Y starts from Ghaziabad at 4 p.m. and reaches Meerut at 5.30 p.m. The two trains will cross each other at :

(a) 4:36 p.m.

(b) 4:42 p.m.

(c) 4:48 p.m.

(d) 4:50 p.m.

Q.63 A boat running upstream takes 8 hours 48 minutes to cover a certain distance, while it takes 4 hours to cover the same distance running downstream. What is the ratio between the speed of the boat and speed of the water current respectively?

(a) 2 : 1

(b) 3 : 2

(c) 8 : 3

(d) None of these

Q.64 A man can row $9\frac{1}{3}$ kmph in still water and finds that it takes him thrice as much time to row up than as to row down the same distance in the river. The speed of the current is :

(a) $3\frac{1}{3}$ km / hr

(b) $3\frac{1}{9}$ km / hr

(c) $4\frac{2}{3}$ km / hr

(d) $4\frac{1}{2}$ km / hr

Q.65 A boat covers 24 km upstream and 36 km downstream in 6 hours while it covers 36 km upstream and 24 downstream in $6\frac{1}{2}$ hours. The velocity of the current is :

(a) 1 km/hr

(b) 1.5 km/hr

(c) 2 km/hr

(d) 2.5 km/hr

Q.66 In what ratio must a grocer mix two varieties of tea worth Rs. 60 a kg and Rs. 65 a kg so that by selling the mixture at Rs. 68.20 a kg he may gain 10%?

(a) 3 : 2

(b) 3 : 4

(c) 3 : 5

(d) 4 : 5

Q.67 At his usual rowing rate, Rahul can travel 12 miles downstream in a certain river in 6 hours less than it takes him to travel the same distance upstream. But if he could double his usual rowing rate for his 24-mile round trip, the downstream 12 miles would then take only one hour less than the upstream 12 miles. What is the speed of the current in miles per hour?

(a) $1\frac{1}{3}$

(b) $1\frac{2}{3}$

(c) $2\frac{1}{3}$

(d) $2\frac{2}{3}$

Q.68 Two vessels A and B contain spirit and water mixed in the ratio 5 : 2 and 7 : 6 respectively. Find the ratio in which these mixture be mixed to obtain a new mixture in vessel C containing spirit and water in the ratio 8 : 5?

(a) 4 : 3

(b) 3 : 4

(c) 5 : 6

(d) 7 : 9

Q.69 A milk vendor has 2 cans of milk. The first contains 25% water and the rest milk. The second contain 50% water. How much milk should he mix from each of the containers so as to get 12 litres of milk such that the ratio of water to milk is 3 : 5?

(a) 4 litres, 8 litres

(b) 6 litres, 6 litres

(c) 5 litres, 7 litres

(d) 7 litres, 5 litres

Q.70 A vessel is filled with liquid, 3 parts of which are water and 5 parts syrup. How much of the mixture must be drawn off and replaced with water so that the mixture may be half water and half syrup?

(a) $\frac{1}{3}$

(b) $\frac{1}{4}$

(c) $\frac{1}{5}$

(d) $\frac{1}{7}$

Q.71 The sides of a triangle are 3 cm, 4 cm and 5 cm. The area (in cm^2) of the triangle formed by joining the mid-points of the sides of this triangle is :

(a) $\frac{3}{4}$

(b) $\frac{3}{2}$

(c) 3

(d) 6

Q.72 The perimeter of a triangle is 30 cm and its area is 30 cm^2. If the largest side measures 13 cm, then what is the length of the smallest side of the triangle?

(a) 3 cm

(b) 4 cm

(c) 5 cm

(d) 6 cm

Q.73 If the height of a triangle is decreased by 40% and its base is increased by 40%, what will be the effect on its area?

(a) No change

(b) 8% decrease

(c) 16% decrease

(d) None of these

Q.74 A circular swimming pool is surrounded by a concrete wall 4 ft. wide. If the area of the concrete wall surrounding the pool is $\frac{11}{25}$ that of the pool, then the radius of the pool is :

(a) 8 ft

(b) 16 ft

(c) 20 ft

(d) 30 ft

Q.75 Three circles of radius 3.5 can are placed in such a way that each circle touches the other two. The area of the portion enclosed by the circles is :

(a) 1.967 cm^2

(b) 1.975 cm^2

(c) 19.67 cm^2

(d) 21.21 cm^2

Q.76 The volume of a rectangular block of stone is 10368 dm^3. Its dimensions are in the ratio of 3 : 2 : 1. If its entire surface is polished at 2 paise per dm^2, then the total cost will be :

(a) Rs. 31.50

(b) Rs. 31.68

(c) Rs. 63

(d) Rs. 63.36

Q.77 If each edge of a cube is increased by 25%, then the percentage increase in its surface area is :
(a) 25%
(b) 48.75%
(c) 50%
(d) 56.25%

Q.78 The radius of the cylinder is half its height and area of the inner part is 616 sq. cms. Approximately how many litres of milk can it contain?
(a) 1.4
(b) 1.5
(c) 1.7
(d) 1.9

Q.79 If both the radius and height of a right circular cone are increased by 20%, its volume will be increased by :
(a) 20%
(b) 40%
(c) 60%
(d) 72.8%

Q.80 In a 500 m race, the ratio of the speeds of two contestants A and B is 3 : 4. A has a start of 140 m. Then, A wins by :
(a) 60 m
(b) 40 m
(c) 20 m
(d) 10 m

Q.81 In a race of 200 m, A can beat B by 31 m and C by 18 m. In a race of 350 m, C will beat B by
(a) 22.75 m
(b) 25 m
(c) 19.5 m
(d) $7\frac{4}{7}$ m

Q.82 What was the day of the week on 17th June, 1998?
(a) Monday
(b) Tuesday
(c) Wednesday
(d) Thursday

Q.83 The last day of a century cannot be:
(a) Monday
(b) Wednesday
(c) Friday
(d) Tuesday

Q.84 The first Republic Day of India was celebrated on 26th January, 1950. It was:
(a) Tuesday
(b) Wednesday
(c) Thursday
(d) Friday

Q.85 At what angle the hands of a clock are inclined at 15 minutes past 5?
(a) $58\frac{1}{2}^\circ$
(b) 64°
(c) $67\frac{1}{2}^\circ$
(d) $72\frac{1}{2}^\circ$

Q.86 How much does a watch lose per day, if its hands coincide every 64 minutes?
(a) $32\frac{8}{11}$ min.
(b) $36\frac{5}{11}$ min.
(c) 90 min.
(d) 96 min.

Q.87 At what time between 7 and 8 o'clock will the hands of a clock be in the same straight line but, not together?
(a) 5 min. past 7
(b) $5\frac{2}{11}$ min. past 7
(c) $5\frac{3}{11}$ min. past 7
(d) $5\frac{5}{11}$ min. past 7

Q.88 At what time between 5.30 and 6 will the hands of a clock be at right angles?

(a) $43\frac{5}{11}$ min. past 5

(b) $43\frac{7}{11}$ min. past 5

(c) 40 min. past 5

(d) 45 min. past 5

Q.89 In how many different ways can the letters of the word 'LEADING' be arranged in such a way that the vowels always come together?

(a) 360

(b) 480

(c) 720

(d) 5040

Q.90 In how many different ways can the letters of the word 'CORPORATION' be arranged so that the vowels always come together?

(a) 810

(b) 1440

(c) 2880

(d) 50400

Q.91 In how many different ways can the letters of the word 'DETAIL' be arranged in such a way that the vowels occupy only the odd positions?

(a) 32

(b) 48

(c) 36

(d) 60

Q.92 In a group of 6 boys and 4 girls, four children are to be selected. In how many different ways can they be selected such that at least one boy should be there?

(a) 159

(b) 194

(c) 205

(d) 209

Q.93 Two dice are thrown simultaneously. What is the probability of getting two numbers whose product is even?

(a) $\frac{1}{2}$

(b) $\frac{3}{4}$

(c) $\frac{3}{8}$

(d) $\frac{5}{16}$

Q.94 Two cards are drawn from a pack of 52 cards. The probability that either both are red or both are kings, is:

(a) $\frac{7}{13}$

(b) $\frac{3}{26}$

(c) $\frac{63}{221}$

(d) $\frac{55}{221}$

Q.95 A bag contains 2 red, 3 green and 2 blue balls. Two balls are drawn at random. What is the probability that none of the balls drawn is blue?

(a) $\frac{10}{21}$

(b) $\frac{11}{21}$

(c) $\frac{2}{7}$

(d) $\frac{5}{7}$

Q.96 Four persons are chosen at random from a group of 3 men, 2 women and 4 children. The chance that exactly 2 of them are children, is:

(a) $\frac{1}{9}$

(b) $\frac{1}{5}$

(c) $\frac{1}{12}$

(d) $\frac{10}{21}$

Q.97 A speaks truth in 75% cases and B in 80% of the cases. In what percentage of cases are they likely to contradict each other, narrating the same incident?

(a) 5%

(b) 15%

(c) 35%

(d) 45%

Q.98 Two ships are sailing in the sea on the two sides of a lighthouse. The angles of elevation of the top of the lighthouse as observed from the two ships are 30° and 45° respectively. If the lighthouse is 100 m high, the distance between the two ships is:

(a) 173 m

(b) 200 m

(c) 273 m

(d) 300 m

Q.99 A man is watching from the top of a tower a boat speeding away from the tower. The boat makes an angle of depression of 45° with the man's eye when at a distance of 60 metres from the tower. After 5 seconds, the angle of depression becomes 30°. What is the approximate speed of the boat, assuming that it is running in still water?

(a) 32 kmph

(b) 36 kmph

(c) 38 kmph

(d) 40 kmph

Q.100 A man on the top of a vertical observation tower observes a car moving at a uniform speed coming directly towards it. If it takes 12 minutes for the angle of depression to change from 30° to 45°, how soon after this will the car reach the observation tower?

(a) 14 min. 35 sec.

(b) 15 min. 49 sec.

(c) 16 min. 23 sec.

(d) 18 min. 5 sec.

Q.101 The top of a 15 metre high tower makes an angle of elevation of 60° with the bottom of an electric pole and angle of elevation of 30° with the top of the pole. What is the height of the electric pole?

(a) 5 metres

(b) 8 metres

(c) 10 metres

(d) 12 metres

ANSWER KEY

1	c	2	a	3	c	4	a	5	b
6	d	7	a	8	d	9	c	10	a
11	d	12	a	13	b	14	d	15	d
16	a	17	a	18	b	19	c	20	c
21	b	22	c	23	a	24	b	25	a
26	*	27	a	28	d	29	b	30	b
31	b	32	b	33	a	34	b	35	c
36	a	37	c	38	a	39	c	40	b
41	d	42	c	43	c	44	d	45	c
46	a	47	c	48	b	49	d	50	b
51	b	52	b	53	c	54	a	55	b
56	c	57	b	58	b	59	c	60	b
61	a	62	a	63	c	64	c	65	c
66	a	67	d	68	d	69	b	70	c
71	b	72	c	73	c	74	c	75	a
76	d	77	d	78	b	79	d	80	c
81	b	82	c	83	d	84	c	85	c
86	a	87	d	88	b	89	c	90	d
91	c	92	d	93	b	94	d	95	a
96	d	97	c	98	c	99	a	100	c
101	c								

SOLUTIONS

S.1 (c)

Unit digit in 7^4 is 1.

$\therefore$ Unit digit in 7^{68} is 1.

Unit digit in 7^{71} is 3.

[$1 \times 7 \times 7 \times 7$ gives unit digit 3]

Again, every power of 6 will give unit digit 6.

$\therefore$ Unit digit in 6^{59} is 6.

Unit digit in 3^4 is 1.

$\therefore$ Unit digit in 3^{64} is 1. Unit digit in 3^{65} is 3.

$\therefore$ Unit digit in $(7^{71} \times 6^{59} \times 3^{65})$ = Unit digit in $(3 \times 6 \times 3) = 4$.

S.2 (a)

Putting $x = 2$, we get $2^2(2^2 - 1) = 12$.

So, $x^2(x^2 - 1)$ is always divisible by **12**.

S.3 (c)

Required measurement = (H.C.F. of 496, 403 713) litres = **31 litres**.

S.4 (a)

Product of numbers = $11 \times 385 = 4235$.

Let the numbers be 11a and 11b. Then, $11a \times 11b = 4235 \Rightarrow ab = 35$.

Now, co-primes with product 35 are (1, 35) and (5, 7).

So, the numbers are (11 × 1, 11 × 35) and (11 ×5, 11 × 7).

Since one number lies between 75 and 125, the suitable pair is (55, 77).

Hence, required number = **77**.

S.5 (b)

Original number of sections = (16 – 3) = 13.

Original number of students = (24 × 13) = 312.

Present number of students = (21 × 16) = 336.

Number of new students admitted = (336 – 312) = **24**.

S.6 (d)

Hrs.	Min.	Sec.
3	17	49
(-) 1	54	50
1	22	59

Total time $= (1 \times 60 + 22)$ min. + 59 sec.

$= (82 \times 60 + 59)$ sec.

$= 4979$ sec.

∴ Number of times the light is seen

$$= \left(\frac{4979}{13} + 1\right) = \mathbf{384}.$$

S.7 (a)

Green portion

$$= \left[1 - \left(\frac{1}{10} + \frac{1}{20} + \frac{1}{30} + \frac{1}{40} + \frac{1}{50} + \frac{1}{60}\right)\right]$$

$$= \left[1 - \frac{1}{10}\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}\right)\right]$$

$$= 1 - \frac{1}{10} \times \frac{147}{60} = 1 - \frac{147}{600} = \frac{453}{600}$$

Let the length of the pole be x meters.

Then, $\frac{453}{600}x = 12.08 \Leftrightarrow x = \left(\frac{12.08 \times 600}{453}\right) = \mathbf{16}$

S.8 (d)

French men $= \frac{1}{5}$;

French women= $\left(\frac{1}{5} + \frac{2}{3} \times \frac{1}{5}\right) = \frac{5}{15} = \frac{1}{3}$

French people= $\left(\frac{1}{5} + \frac{1}{3}\right) = \frac{8}{15}$

∴ Not–French= $\left(1 - \frac{8}{15}\right) = \mathbf{\frac{7}{15}}$

S.9 (c)

Let the number of students in rooms A and B be x and y respectively.

Then, $x - 10 = y + 10 \Rightarrow x - y = 20$ .....(i)

and $x + 20 = 2(y - 20) \Rightarrow x - 2y = -60$ .....(ii)

Solving (i) and (ii), we get : **x = 100, y = 80**.

S.10 (a)

Sum of temperatures on 1st, 2nd, 3rd and 4th days = (58 × 4) = 232 degrees .....(i)

Sum of temperatures on 2nd, 3rd 4th and 5th days = (60 × 4) = 240 degrees .....(ii)

Subtracting (i) from (ii), we get :

Temp. on 5th day – Temp. on 1st day = 8 degrees.

Let the temperatures on 1st and 5th days be 7x and 8x degrees respectively.

Then, 8x – 7x = 8 or x = 8.

∴ Temperature on the 5th day = 8x = **64 degrees**.

S.11 (d)

Since the total or average age of all the family members is not given, the given data is inadequate. So, the age of second child **cannot be determined**.

S.12 (a)

Let the unit's digit be x.

Then, ten's digit = (x – 2).

$$\therefore\ 3[10(x-2)+x] + \frac{6}{7}[10x + (x-2)] = 108$$

$\Leftrightarrow 231x - 420 + 66x - 12 = 756 \Leftrightarrow 297x = 1188$

$\Leftrightarrow x = 4.$

Hence, sum of the digits = x + (x – 2) = 2x – 2 = **6**.

S.13 (b)

Let the ten's digit be x, then, unit's digit = 2x+1.

$[10x+(2x+1)]-[\{10(2x+1)+x\}-\{10x+(2x+1)\}]=1$

$\Rightarrow (12x+1)-(9x+9)=1 \Leftrightarrow 3x=9$

$\Rightarrow x=3.$

So, ten's digit = 3 and unit's digit = 7.

Hence, original number = **37**.

S.14 (d)

16 years ago, let T = x years and G = 8x years.

After 8 years from now, T = (x + 16 + 8) years and G = (8x + 16 + 8) years.

$\therefore\ 8x+24=3(x+24) \Leftrightarrow 5x=48.$

8 years ago,

$$\frac{T}{g}=\frac{x+8}{8x+8}=\frac{\frac{48}{5}+8}{8\times\frac{48}{5}+8}=\frac{88}{424}$$

$$=\frac{\mathbf{11}}{\mathbf{53}}$$

S.15 (d)

$R-Q=R-T \Rightarrow Q=T.$

Also, $R+T=50 \Rightarrow R+Q=50$

So, (R – Q) cannot be determined.

S.16 (a)

Let Ronit's present age be x years.

Then, father's present age = (x + 3x) years = 4x years.

$\therefore\ (4x+8)=\frac{5}{2}(x+8) \Leftrightarrow 8x+16$

$=5x+40 \Leftrightarrow 3x=24 \Leftrightarrow x=8$

Hence, required ratio $=\frac{(4x+16)}{(x+16)}=\frac{48}{24}=2$

S.17 (a)

Clearly, my brother was born 3 years before I was born and 4 years after my sister was born.

So, father's age when brother was born

= (28 + 4) years = **32 years**

mother's age when brother was born

= (26 – 3) years = **23 years**

S.18 (b)

Let the number of candidates appeared from each state be x.

Then, 7% of x – 6% of x = 80 ⇔ 1% of x = 80

⇔ x = 80 × 100 = **8000**.

S.19 (c)

Let the number of persons eligible to vote be x. Then,

Number of eligible persons between 18 and 21 = 8% of x.

Number of persons between 18 and 21, who voted = 85% of (8% of x)

$$=\left(\frac{85}{100}\times\frac{8}{100}\times x\right)=\frac{68}{1000}x$$

∴ Required percentage

$$=\left(\frac{68x}{1000}\times\frac{1}{x}\times 100\right)=\mathbf{6.8\%}$$

S.20 (c)

Let the number of person eligible to vote be x.

Then, voters who voted for A = 30% of x.

Voters who voted for B = 60% of (70% of x).

$$=\left(\frac{60}{100}\times\frac{70}{100}\times 100\right)\%\text{of } x$$

$$=42\%\text{ of } x$$

Voters who did not vote = [100 – (30 + 42)]% of x = 28% of x.

∴ 30% of x – 28% of x = 1200 ⇔ 2% of x =

$1200 \Leftrightarrow x=\left(\frac{1200\times 100}{2}\right)=60{,}000$

S.21 (b)

Let total population = x.

Then, number of males $=\frac{5}{9}x$

Married males = 30% of $\frac{5}{9}x=\left(\frac{30}{100}\times\frac{5}{9}x\right)=\frac{x}{6}$

Married females $=\frac{x}{6}$;

Number of females $= \left(x - \frac{5}{9}x\right) = \frac{4x}{9}$

Unmarried females $= \left(\frac{4x}{9} - \frac{x}{6}\right) = \frac{5x}{18}$

$\therefore$ Required percentage $= \left(\frac{5x}{18} \times \frac{1}{x} \times 100\right)\%$

$= 27\frac{7}{9}\%$

S.22 (c)

Suppose the business value changes from x to y.

4% of x = 5% of y

$\Rightarrow \frac{4}{100}x = \frac{5}{100}y \Rightarrow y = \frac{4}{5}x$

$\therefore$ Changes in business $= \left(x - \frac{4}{5}x\right) = \frac{x}{5}$

Percentage slump $= \left(\frac{x}{5} \times \frac{1}{x} \times \frac{1}{100}\right)$ **= 20%**

S.23 (a)

$n(A) = 34,$

$n(B) = 42,$

$n(A \cap B) = 20$

So, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 34 + 42 - 20 = 56$

$\therefore$ Percentage failed in either or both the subjects = 56.

Hence, percentage passed = (100 − 56)% **= 44%**.

S.24 (b)

$n(A) = \left(\frac{60}{100} \times 96\right) = \frac{288}{5}$

$n(B) = \left(\frac{30}{100} \times 96\right) = \frac{144}{5}$

$n(A \cap B) = \left(\frac{15}{100} \times 96\right) = \frac{72}{5}$

$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= \frac{288}{5} + \frac{144}{5} - \frac{72}{5} = \frac{360}{5} = 72$

So, people who had either or both types of lunch = 72

Hence, people who had neither type of lunch = (96 − 72) = **24**

S.25 (a)

Let the total number of students be x.

Number passed in one or both is given by:

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

= 65% of x + 60% of x − 40 % of x

$= \left(\frac{65}{100}x + \frac{60}{100}x - \frac{40}{100}x\right)$

$= \frac{85}{100}x = \frac{17}{20}x$

Failed in both $= \left(x - \frac{17}{20}x\right) = \frac{3x}{20}$

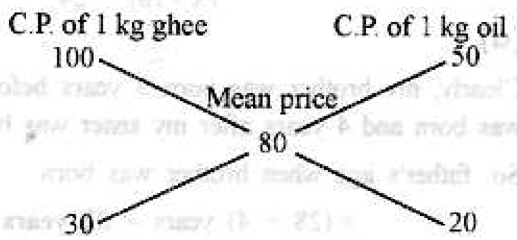
$\therefore \frac{3x}{20} = 90 \Leftrightarrow x = \left(\frac{90 \times 20}{3}\right)$ **= 600**

S.26 (a)

Mean cost price = Rs. $\left(\frac{100}{120} \times 96\right)$

= Rs. 80 per kg.

By the rule of alligation :



$\therefore$ Required ratio = 30 : 20 = **3 : 2**

S.27 (a)

Mean cost price $= \text{Rs.}\left(\frac{100}{137.5}\times 8\right) = \text{Rs.}\frac{64}{11}$

By the rule of alligation :

C.P. of 1 litre water — C.P. of 1 litre milk

0 — 6.40

Mean price $\frac{64}{11}$

$\frac{64}{110}$ — $\frac{64}{11}$

$\therefore$ Required ratio $= \frac{64}{110} : \frac{64}{11} = \mathbf{1 : 10}$

S.28 (d)

Let C.P. of the chair be Rs. x and that of the table be Rs. y.

Then, 17% of y – 7% of x = 296

$\Rightarrow 17y - 7x = 29600$ .....(i)

And, 12% of y + 7% of x = 400

$\Rightarrow 12y + 7x = 40000$ .....(ii)

Solving (i) and (ii), we get :

$y = 2400$ and $x = 1600$

$\therefore$ C.P. of table = **Rs. 2400**

S.29 (b)

Let the required time be x seconds. Then, **More metres, more time (Direct Proportion)**

$\therefore\ 0.128 : 25 :: 1 : x$

$\Leftrightarrow 0.128 \times x = 25 \times 1$

$\Leftrightarrow x = \frac{25}{0.128} = \frac{25\times 1000}{128}$

$\Leftrightarrow x = 195.31$

$\therefore$ Required time = **195 sec** (approximately).

S.30 (b)

Let the required number of revolutions made by larger wheel be x.

Then, **More cogs, Less revolutions (Indirect Proportion)**

$\therefore\ 14 : 6 :: 21 : x$

$\Leftrightarrow 14 \times x = 6 \times 21$

$\Leftrightarrow x = \left(\frac{6\times 21}{14}\right) = \mathbf{9}$

S.31 (b)

Let the required number of lamps be x.

Less hours per day, More lamps **(Indirect Proportion)**
More money, More lamps **(Direct Proportion)**
More days, Less lamps **(Indirect Proportion)**

$$\left.\begin{array}{ll}\text{Hours per day} & 4:5\\ \text{Money} & 21.25:76.50\\ \text{Number of days} & 30:10\end{array}\right\} :: 80:x$$

$\therefore\ 4 \times 21.25 \times 30 \times x = 5 \times 76.50 \times 10 \times 80$

$\Leftrightarrow x = \frac{5\times 76.50\times 10\times 80}{4\times 21.25\times 30} \Leftrightarrow x = \mathbf{120}$

S.32 (b)

Let the number of persons completing the work in 20 days be x.

Work done $= \frac{1}{4}$,

Remaining work $= \left(1-\frac{1}{4}\right) = \frac{3}{4}$.

Less hours per day, More men required **(Indirect Proportion)**
More work, More men required **(Direct Proportion)**
More days, Less men required **(Indirect Proportion)**

$$\left.\begin{array}{ll}\text{Hours per day} & 8:9\\ \text{Work} & \frac{1}{4}:\frac{3}{4}\\ \text{Days} & 20:10\end{array}\right\} :: 400:x$$

$\therefore\ 8\times\frac{1}{4}\times 20\times x = 9\times\frac{3}{4}\times 10\times 400$

$\Leftrightarrow 40x = 27000 \Leftrightarrow x = 675$

$\therefore$ Additional men = (675 – 400) = **275**

S.33 (a)

After 25 days, 35 men complete the work in 12 days.

Thus, 35 men can finish the remaining work in 12 days.

$\therefore$ 30 men can do it in $\dfrac{(12\times35)}{30} = 14$ days, **which is 1 day behind.**

S.34 (b)

(2×14) men + (7×14) boys $\equiv (3 \times 11)$ men + (8×11) boys.

$\Leftrightarrow$ 5 men $\equiv$ 10 boys $\Leftrightarrow$ 1 man $\equiv$ 2 boys.

$\therefore$ (2 men + 7 boys) $\equiv (2 \times 2 + 7)$ boys $= 11$ boys.

(8 men + 6 boys) $\equiv (8 \times 2 + 6)$ boys $= 22$ boys.

Let the required number of days be x.

Now,

More boys, Less days (Indirect Proportion)

More work, More days (Direct Proportion)

$$\left.\begin{array}{ll}\text{Boys} & 22:11\\ \text{Work} & 1:3\end{array}\right\} :: 14:x$$

$\therefore (22 \times 1 \times x) = (11 \times 3 \times 14)$

$\therefore x = \dfrac{462}{22} = 21$

Hence, the required number of days = **21.**

S.35 (c)

Let the number of pages typed in one hour by P, Q and R be x, y and z respectively.

Then,

$$x + y + z = \frac{216}{4} \Rightarrow x + y + z = 54 \quad .....(i)$$

$$z - y = y - x \quad \Rightarrow 2y = x + z \quad .....(ii)$$

$$5z = 7x \quad \Rightarrow x = \frac{5}{7}z \quad .....(iii)$$

Solving (i), (ii) and (iii),

we get **x = 15, y = 18, z = 21**

S.36 (a)

(A + B)'s 1 day's work $= \dfrac{1}{5}$;

(B + C)'s 1 day's work $= \dfrac{1}{7}$;

(A + C)'s 1 day's work $= \dfrac{1}{4}$.

Adding, we get :

2(A + B + C)'s 1 day work $= \left(\dfrac{1}{5}+\dfrac{1}{7}+\dfrac{1}{4}\right) = \dfrac{83}{140}$

(A + B + C)'s 1 day work $= \dfrac{83}{280}$

A's 1 day work $= \left(\dfrac{83}{280}-\dfrac{1}{7}\right) = \dfrac{43}{280}$

B's 1 day work $= \left(\dfrac{83}{280}-\dfrac{1}{4}\right) = \dfrac{13}{280}$

C's 1 day work $= \left(\dfrac{83}{280}-\dfrac{1}{5}\right) = \dfrac{27}{280}$

Thus time taken by A, B, C is $\dfrac{280}{43}$ days, $\dfrac{280}{13}$ days, $\dfrac{280}{27}$ days respectively.

Clearly, the time taken by A is least.

S.37 (c)

A's 1 hour's work $= \dfrac{1}{4}$;

(B + C)'s 1 hour's work $= \dfrac{1}{3}$;

(A + C)'s 1 hour's work $= \dfrac{1}{2}$.

(A + B + C)'s 1 hour's work $= \left(\dfrac{1}{4}+\dfrac{1}{3}\right) = \dfrac{7}{12}$

B's 1 hour's work $= \left(\dfrac{7}{12}-\dfrac{1}{2}\right) = \dfrac{1}{12}$

$\therefore$ B alone will take **12 hours** to do the work.

S.38 (a)

Suppose B takes x days to do the work.

$\therefore$ A takes $\left(2\times\frac{3}{4}x\right) = \frac{3x}{2}$ days to do it.

(A + B)'s 1 day's work = $\frac{1}{18}$

$\therefore \frac{1}{x}+\frac{2}{3x} = \frac{1}{18}$ or x = **30**

S.39 (c)

(B + C)'s 1 day's work = $\left(\frac{1}{9}+\frac{1}{12}\right)=\frac{7}{36}$

Work done by B and C in 3 days

$= \left(\frac{7}{36}\times 3\right)=\frac{7}{12}$

Remaining work = $\left(1-\frac{7}{12}\right)=\frac{5}{12}$

Now, $\frac{1}{24}$ work is done by A in 1 day.

So, $\frac{5}{12}$ work is done by A in

$= \left(24\times\frac{5}{12}\right)$ = **10 days**

S.40 (b)

Work done by X in 4 days = $\left(\frac{1}{20}\times 4\right)=\frac{1}{5}$

Remaining work = $\left(1-\frac{1}{5}\right)=\frac{4}{5}$

(X + Y)'s 1 day work = $\left(\frac{1}{20}+\frac{1}{12}\right)=\frac{8}{60}=\frac{2}{15}$

Now, $\frac{2}{15}$ work done by X and Y in 1 day.

So, $\frac{4}{5}$ work will be done by X and Y in

$\left(\frac{15}{2}\times\frac{4}{5}\right)=6\text{days}$

Hence, total time taken = (6 + 4) days = **10 days.**

S.41 (d)

(1 man + 1 woman)'s 1 day's work

$= \left(\frac{1}{3}+\frac{1}{4}\right)=\frac{7}{12}$

Work done by 1 man and 1 woman in $\frac{1}{4}$ day

$= \left(\frac{7}{12}\times\frac{1}{4}\right)=\frac{7}{48}$

Remaining work = $\left(1-\frac{7}{48}\right)=\frac{41}{48}$

Work done by 1 boy in $\frac{1}{4}$ day = $\left(\frac{1}{12}\times\frac{1}{4}\right)=\frac{1}{48}$

$\therefore$ Number of boys required = $\left(\frac{41}{48}\times 48\right)$ = **41**

S.42 (c)

Let 1 man's 1 hour's work = x; 1 woman's 1 hour's work = y and 1 boy's 1 hour's work = z.

Then,

$$x + 3y + 4z = \frac{1}{96} \quad .....(i)$$

$$2x + 8z = \frac{1}{80} \quad .....(ii)$$

$$2x + 3y = \frac{1}{120} \quad .....(iii)$$

Adding (ii) and (iii) and subtracting (i) from it, we get : $3x + 4z = \frac{1}{96}$ .....(iv)

From (ii) and (iv), we get

$$x = \frac{1}{480}.$$

Substituting, we get

$$y = \frac{1}{720}, z = \frac{1}{960}$$

(5 men + 12 boys)'s 1 hour's work

$$= \left(\frac{5}{480} + \frac{12}{960}\right) = \left(\frac{1}{96} + \frac{1}{80}\right) = \frac{11}{480}$$

$\therefore$ 5 men and 12 boys can do the work in $\frac{480}{11}$

i.e., **$43\frac{7}{11}$ hours**.

S.43 (c)

Suppose first pipe alone takes x hours to fill the tank. Then, second and third pipes will take (x – 5) and (x – 9) hours respectively to fill the tank.

$$\therefore \frac{1}{x} + \frac{1}{(x-5)} = \frac{1}{(x-9)} \Leftrightarrow \frac{x-5+x}{x(x-5)} = \frac{1}{(x-9)}$$

$\Leftrightarrow (2x - 5)(x - 9) = x(x - 5) \Leftrightarrow x^2 - 18x + 45 = 0$

$\Leftrightarrow (x - 15)(x - 3) = 0 \Leftrightarrow x = 15$ [neglecting x = 3]

S.44 (d)

Part filled in 3 min. $= 3\left(\frac{1}{12} + \frac{1}{15}\right) = \left(3 \times \frac{9}{60}\right)$

$$= \frac{9}{20}$$

Remaining part $= \left(1 - \frac{9}{20}\right) = \frac{11}{20}$

Part filled by B in 1 min. $= \frac{1}{15}$

$\frac{1}{15} : \frac{11}{20} :: 1 : x$ or $x = \left(\frac{11}{20} \times 1 \times 15\right) = 8\frac{1}{4}$ min.

= 8 min. 15 sec.

$\therefore$ Remaining part is filled by B in **8 min. 15 sec.**

S.45 (c)

(A + B)'s 1 hour's work $= \left(\frac{1}{12} + \frac{1}{15}\right) = \frac{9}{60} = \frac{3}{20}$

(A + C)'s 1 hour's work $= \left(\frac{1}{12} + \frac{1}{20}\right) = \frac{8}{60} = \frac{2}{15}$

Part filled in 2 hrs $= \left(\frac{3}{20} + \frac{2}{15}\right) = \frac{17}{60}$;

Part filled in 6 hrs $= \left(3 \times \frac{17}{60}\right) = \frac{17}{20}$

Remaining part $= \left(1 - \frac{17}{20}\right) = \frac{3}{20}$

Now, it is the turn of A and B and $\frac{3}{20}$ part is filled by A and B in 1 hour.

$\therefore$ Total time taken to fill the tank = (6 + 1)hrs = **7 hrs**.

S.46 (a)

Time taken to cover 600 km $= \left(\frac{600}{100}\right)$hrs = 6 hrs

Number of stoppages $= \frac{600}{75} - 1 = 7$

Total time of stoppage = (3 × 7)min. = 21 min.

Hence, total time taken = **6 hrs 21 min**.

S.47 (c)

Distance covered in first 2 hours = (70 × 2)km = 140 km

Distance covered in next 2 hours = (80 × 2)km = 160 km

Remaining distance = 345 – (140 + 160) = 45 km

Speed in the fifth hour = 90 km/hr.

Time taken to cover 45 km $= \left(\frac{45}{90}\right)$hr $= \frac{1}{2}$ hr

$\therefore$ Total time taken $= \left(2 + 2 + \frac{1}{2}\right) =$ **$4\frac{1}{2}$ hrs**

S.48 (b)

Let A's speed = x km/hr

Then, B's speed = (7 – x)km/hr

So, $\frac{24}{x}+\frac{24}{(7-x)}$ = 14 ⇔ 24(7 – x) + 24x = 14x(7 – x)

⇔ $14x^2 - 98x + 168 = 0 \Leftrightarrow x^2 - 7x + 12 = 0$

⇔ (x – 3) (x – 4) = 0 ⇔ x = 3 or x = 4.

Since. A is faster than B.

So, A's speed = **4 km/hr**

and B's speed = 3 km/hr.

S.49 (d)

Let the speed in return journey be x km/hr.

Then, speed in onward journey = $\frac{125}{100}x=\left(\frac{5}{4}x\right)$ km / hr.

Average speed

$$=\left(\frac{2\times\frac{5}{4}x\times x}{\frac{5}{4}x+x}\right)\text{km / hr}=\frac{10x}{9}\text{ km / hr.}$$

$$\therefore\ \left(800\times\frac{9}{10x}\right)=16\ \Leftrightarrow\ x=\left(\frac{800\times9}{16\times10}\right)=45$$

So, speed in onward journey = $\left(\frac{5}{4}\times45\right)$km / hr.

= **56.25 km/hr.**

S.50 (b)

$$x\times\frac{15}{60}+2x\times\frac{20}{60}+x\times\frac{10}{60}=39$$

$$\Rightarrow\quad \frac{x}{4}+\frac{2x}{3}+\frac{x}{6}=39$$

⇒ 3x + 8x + 2x = 468 ⇒ x = **36**

S.51 (b)

New speed = $\frac{6}{7}$ of usual speed.

New time = $\frac{7}{6}$ of usual time.

$\therefore\ \left(\frac{7}{6}\text{ of usual time}\right)-(\text{usual time})=\frac{1}{5}\text{hr.}$

$\Rightarrow \frac{1}{6}$ of usual time = $\frac{1}{5}$ hr

⇒ usual time = $\frac{6}{5}$ hr = **1 hr 12 min.**

S.52 (b)

Due to stoppages, it covers 9 km less.

Time taken to cover 9 km

$$=\left(\frac{9}{54}\times60\right)\text{min}=\mathbf{10\ min.}$$

S.53 (c)

Let the speed of the train be x km/hr and that of the car be y km/hr.

Then, $\frac{120}{x}+\frac{480}{y}=8$ or $\frac{1}{x}+\frac{4}{y}=\frac{1}{15}$

And, $\frac{200}{x}+\frac{400}{y}=\frac{25}{3}$ or $\frac{1}{x}+\frac{2}{y}=\frac{1}{24}$

Solving (i) and (ii), we get x = 60 and y = 80.

∴ Ratio of speeds = 60 : 80 = **3 : 4**

S.54 (a)

Since A and B move in the same direction along the circle, so they will first meet each other when there is a difference of one round between the two.

Relative speed of A and B = (6 – 1) = 5 rounds per hour.

Time taken to complete one round at this speed

= $\frac{1}{5}$ hr = 12 min.

They will cross each other at = 7:30 + 12 min = **7:42 am**. (for the first time)

S.55 (b)

Let the distance between Meerut and Delhi be x km and let the trains meet y hours after 7 a.m.

Clearly, M covers x km in 4 hrs and N covers x km in (7/2) hrs.

$\therefore$ Speed of M= $\frac{x}{4}$ kmph,

Speed of N= $\frac{2x}{7}$ kmph

Distance covered by M in (y + 2) hrs + Distance covered in y hrs = x

$$\therefore \frac{x}{4}(y+2)+\frac{2x}{7}\times y = x \Leftrightarrow \frac{(y+2)}{4}+\frac{2y}{7}=1$$

$$\Leftrightarrow y = \frac{14}{15}\text{ hrs} = \left(\frac{14}{15}\times 60\right)\text{ min.} = 56\text{ min.}$$

Hence, the train meet at **7:56 a.m.**

S.56 (c)

Let the distance be x km. Then,

(Time taken to walk x km) + (Time taken to ride x km) = $\frac{23}{4}$ hrs.

$\Rightarrow$ (Time taken to walk 2x km) + (Time taken to ride 2x km) = $\frac{23}{2}$ hrs.

But, time taken to ride 2x km = $\frac{15}{4}$ hrs.

$\therefore$ Time taken to walk 2x km = $\left(\frac{23}{2}-\frac{15}{4}\right)$ hrs

$= \frac{31}{4}$ hrs = **7 hrs 45 min**

S.57 (b)

$$\text{Speed} = \left(54\times\frac{5}{18}\right)\text{m/sec} = 15\text{m/sec}$$

Length of the train = (15 × 20)m = 300 m.

Let the length of the platform be x meters.

Then, $\frac{x+300}{36} = \frac{50}{3} \Leftrightarrow 3(x + 300)$

$= 1950 \Leftrightarrow$ **x = 240 m.**

S.58 (b)

Speed of train relative to man= (63 − 3) km/hr

= 60 km/hr

$$= \left(60\times\frac{5}{18}\right)\text{m/sec} = \frac{50}{3}\text{ m/sec}$$

$\therefore$ Time taken to pass the man

$$= \left(500\times\frac{3}{50}\right)\text{sec} = \mathbf{30\ sec}$$

S.59 (c)

Relative speed= (60 + 90) km/hr

$$= \left(150\times\frac{5}{18}\right)\text{m/sec} = \left(\frac{125}{3}\right)\text{m/sec}$$

Distance covered = (1.10 + 0.9) km = 2 km = 2000 m.

$$\text{Required time} = \left(2000\times\frac{3}{125}\right)\text{sec} = \mathbf{48\ sec}$$

S.60 (b)

$$2\text{ kmph} = \left(2\times\frac{5}{18}\right)\text{m/sec} = \frac{5}{9}\text{ m/sec and}$$

$$4\text{ kmph} = \frac{10}{9}\text{ m/sec}$$

Let the length of the train be x meters and its speed be y m/sec.

$$\text{Then, } \frac{x}{\left(y-\frac{5}{9}\right)} = 9 \text{ and } \frac{x}{\left(y-\frac{10}{9}\right)} = 10$$

$\therefore$ 9y − 5 = x and 10 (9y − 10) = 9x

$\Rightarrow$ 9y − x = 5 and 90y − 9x = 100.

On solving, we get : x = 50

$\therefore$ Length of the train is **50 m.**

S.61 (a)

Let the length of the first train be x meters.

Then, the length of second train is $\left(\frac{x}{2}\right)$ meters.

Relative speed= (48 + 42) kmph

$$= \left(90\times\frac{5}{18}\right)\text{m/sec} = 25\text{ m/sec.}$$

$\therefore \dfrac{\left(x+\dfrac{x}{2}\right)}{25} = 12$ or $\dfrac{3x}{2} = 300$ or $x = 200$

$\therefore$ Length of first train $= 200$ m.

Let the length of platform be y metres.

Speed of the first train

$= \left(48 \times \dfrac{5}{18}\right) \text{m/sec} = \dfrac{40}{3}$ m/sec

$\therefore (200+y) \times \dfrac{3}{40} = 45 \Leftrightarrow 600 + 3y = 1800$

$\Leftrightarrow$ **y = 400 m**.

S.62 (a)

Suppose, the distance between Meerut and Ghaziabad is x km.

Time taken by X to cover x km = 1 hour.

Time taken by Y to cover x km = $\dfrac{3}{2}$ hours

$\therefore$ Speed of X = x kmph,

Speed of Y = $\left(\dfrac{2x}{3}\right)$ kmph.

Let them meet y hours after 4 p.m. Then,

$xy + \dfrac{2xy}{3} = x \Leftrightarrow y\left(1+\dfrac{2}{3}\right) = 1$

$\Leftrightarrow y = \dfrac{3}{5}\text{ hours} = \left(\dfrac{3}{5} \times 60\right)\text{min} = 36\text{ min}.$

So, the two trains meet at **4:36 p.m.**

S.63 (c)

Let the man's rate upstream be x kmph and that downstream be y kmph. Then, Distance covered upstream in 8 hrs 48 min. = Distance covered downstream in 4 hrs.

$\Rightarrow \left(x \times 8\dfrac{4}{5}\right) = (y \times 4)$

$\Rightarrow \dfrac{44}{5}x = 4y \Rightarrow y = \dfrac{11}{5}x$

$\therefore$ Required ratio =

$\left(\dfrac{y+x}{2}\right) : \left(\dfrac{y-x}{2}\right) = \left(\dfrac{16x}{5} \times \dfrac{1}{2}\right) : \left(\dfrac{6x}{5} \times \dfrac{1}{2}\right)$

$= \dfrac{8}{5} : \dfrac{3}{5} = \mathbf{8 : 3}$

S.64 (c)

Let speed upstream be x kmph. Then, speed downstream = 3x kmph.

Speed in still water $= \dfrac{1}{2}(3x+x)$ kmph

= 2x kmph

$\therefore 2x = \dfrac{28}{3} \Rightarrow x = \dfrac{14}{3}$

So, Speed upstream $= \dfrac{14}{3}$ km/hr

Speed downstream = 14 km/hr.

Hence, speed of the current

$= \dfrac{1}{2}\left(14 - \dfrac{14}{3}\right)$ km/hr

$= \dfrac{14}{3}$ km/hr $= \mathbf{4\dfrac{2}{3}}$ **km/hr.**

S.65 (c)

Let rate upstream = x kmph and rate downstream = y kmph.

Then, $\dfrac{24}{x} + \dfrac{36}{y} = 36$ .....(i)

and $\dfrac{36}{x} + \dfrac{24}{y} = \dfrac{13}{2}$ .....(ii)

Adding (i) and (ii), we get:

$60\left(\dfrac{1}{x} + \dfrac{1}{y}\right) = \dfrac{25}{2}$ or $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{5}{24}$ .....(iii)

Subtracting (i) from (ii), we get:

$12\left(\dfrac{1}{x} - \dfrac{1}{y}\right) = \dfrac{1}{2}$ or $\dfrac{1}{x} - \dfrac{1}{y} = \dfrac{1}{24}$ .....(iv)

Adding (iii) and (iv), we get: $\frac{2}{x} = \frac{6}{24}$ or $x = 8$

So, $\frac{1}{8} + \frac{1}{y} = \frac{5}{24} \Leftrightarrow \frac{1}{y} = \left(\frac{5}{24} - \frac{1}{8}\right) = \frac{1}{12} \Leftrightarrow y = 12$

$\therefore$ Speed upstream = 8 kmph

Speed downstream = 12 kmph.

Hence, rate of current = $\frac{1}{2}(12 - 8)$ kmph

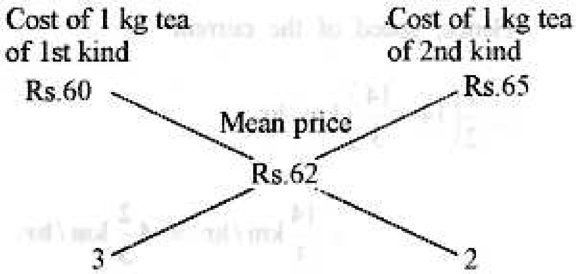
= **2 kmph**

S.66 (a)

S.P. of 1 kg of the mixture = Rs. 68.20, Gain = 10%

C.P. of 1 kg of the mixture = Rs. $\left(\frac{100}{110} \times 68.20\right)$ = Rs. 62

By the rule of alligation, we have:



$\therefore$ Required ratio = **3 : 2**

S.67 (d)

Let the speed in still water be x mph and the speed of the current be y mph. Then,

Speed upstream = (x − y)

Speed downstream = (x + y)

$\therefore \frac{12}{(x-y)} - \frac{12}{(x+y)} = 6$

$\Leftrightarrow 6(x^2 - y^2) = 24y \Leftrightarrow x^2 - y^2 = 4y$

$\Leftrightarrow x^2 = (4y + y^2)$ .....(i)

And, $\frac{12}{(2x-y)} - \frac{12}{(2x+y)} = 1$

$\Leftrightarrow 4x^2 - y^2 = 24y \Leftrightarrow x^2 = \frac{24y + y^2}{4}$ .....(ii)

From (i) and (ii), we have

$4y + y^2 = \frac{24y + y^2}{4}$

$\Leftrightarrow 16y + 4y^2 = 24y + y^2$

$\Leftrightarrow 3y^2 = 8y$

$\Leftrightarrow y = \frac{8}{3}$

$\therefore$ Speed of the current = $\frac{8}{3}$ mph = $\mathbf{2\frac{2}{3}}$ **mph**

S.68 (d)

Let the C.P. of spirit be Rs. 1 per litre.

Spirit in 1 litre mix. of A = $\frac{5}{7}$ litre ;

C.P. of 1 litre mix. in A = Re. $\frac{5}{7}$;

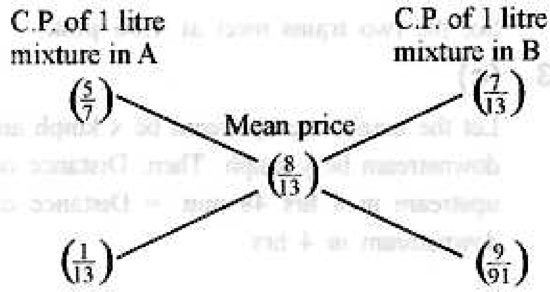
Spirit in 1 litre mix. of B = $\frac{7}{13}$ litre ;

C.P. of 1 litre mix. in B = Re. $\frac{7}{13}$.

Spirit in 1 litre mix. of C = $\frac{8}{13}$ litre ;

Mean price = Re. $\frac{8}{13}$

By the rule of alligation, we have:



$\therefore$ Required ratio = $\frac{1}{13} : \frac{9}{91}$ = **7: 9**

S.69 (b)

Let cost of 1 litre milk be Rs. 1.

Milk in 1 litre mix. in 1st can = $\frac{3}{4}$ litre ;

C.P. of 1 litre mix. in 1st can = Re. $\frac{3}{4}$;

Milk in 1 litre mix. in 2nd can = $\frac{1}{2}$ litre ;

C.P. of 1 litre mix. in 2nd can = Re. $\frac{1}{2}$.

Milk in 1 litre of final mix. = $\frac{5}{8}$ litre ;

Mean price = Re. $\frac{5}{8}$

By the rule of alligation, we have:

C.P. of 1 litre mixture in 1st can | C.P. of 1 litre mixture in 2nd can

$\frac{3}{4}$ | $\frac{1}{2}$

Mean price $\frac{5}{8}$

$\frac{1}{8}$ | $\frac{1}{8}$

$\therefore$ Ratio of two mixture = $\frac{1}{8}:\frac{1}{8}=1:1$

So, quantity of mixture taken from each can

$= \left(\frac{1}{2}\times 12\right)$ = **6 litres**

S.70 (c)

Suppose the vessel initially contains 8 litres of liquid.

Let x litres of this liquid be replaced with water.

Quantity of water in new mixture

$= \left(3-\frac{3x}{8}+x\right)$ litres

Quantity of syrup in new mixture

$= \left(5-\frac{5x}{8}\right)$ litres

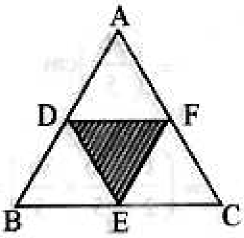
$\therefore \left(3-\frac{3x}{8}+x\right)=\left(5-\frac{5x}{8}\right)$

$\Rightarrow 5x + 24 = 40 - 5x$

$\Rightarrow 10x = 16 \Rightarrow x=\frac{8}{5}$

So, part of the mixture replaced = $\left(\frac{8}{5}\times\frac{1}{8}\right)$ = $\mathbf{\frac{1}{5}}$

S.71 (b)



a = 3 cm, b = 4 cm and c = 5 cm.

It is a right-angled triangle with base = 3 cm and height = 4 cm.

$\therefore$ Its area = $\left(\frac{1}{2}\times 3\times 4\right)cm^2 = 6\ cm^2$

Area of required triangle = $\left(\frac{1}{4}\times 6\right)cm^2$

$= \mathbf{\frac{3}{2}\ cm^2}$

S.72 (c)

Let the smallest side be x cm.

Then, other sides are 13 cm and (17 – x) cm.

Let a = 13, b = x, c = (17 – x). So, s = 15.

$\therefore$ Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{15\times 2\times (15-x)(x-2)}$

$\Leftrightarrow 30 \times (15 - x)(x - 2) = (30)^2$

$\Leftrightarrow (15 - x)(x - 2) = 30 \Leftrightarrow x^2 - 17x + 60 = 0$

$\Leftrightarrow (x - 12)(x - 5) = 0 \Leftrightarrow x = 12$ or $x = 5$

$\therefore$ Smallest side = **5 cm**.

S.73 (c)

Let initial base = b cm and initial height = h cm.

Then, initial area = $\left(\frac{1}{2}bh\right)$ cm^2

New base $= (140\% \text{ of } b) \text{ cm} = \left(\frac{140b}{100}\right)\text{cm}$

$= \left(\frac{7b}{5}\right)\text{cm}$

New height $= (60\% \text{ of } h) \text{ cm} = \left(\frac{60h}{100}\right)\text{cm}$

$= \left(\frac{3h}{5}\right)\text{cm}$

New area $= \left(\frac{1}{2}\times\frac{7b}{5}\times\frac{3h}{5}\right) = \left(\frac{21}{50}bh\right)\text{cm}^2$

Area decreased $= \left(\frac{1}{2}bh - \frac{21}{50}bh\right) = \left(\frac{4}{50}bh\right)\text{cm}^2$

Percentage decrease $= \left(\frac{4bh}{50}\times\frac{2}{bh}\times 100\right)\%$

$= \mathbf{16\%}$

S.74 (c)

Let the radius of the pool be R ft. Radius of the pool including the wall = (R + 4) ft.

Area of the concrete wall

$= \pi[(R+4)^2 - R^2]$ sq. ft

$= [\pi(R+4+R)(R+4-R)]$ sq. ft

$= 8\pi(R+2)$ sq. ft

$8\pi(R+2) = \frac{11}{25}\pi R^2$

$\Leftrightarrow 11R^2 = 200(R+2) \Leftrightarrow 11R^2 - 200R - 400 = 0$

$\Leftrightarrow 11R^2 - 220R + 20R - 400 = 0$

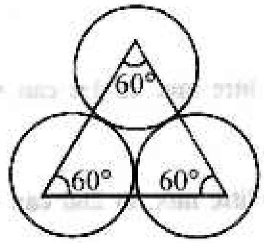
$\Leftrightarrow 11R(R-20) + 20(R-20) = 0$

$\Leftrightarrow (R-20)(11R+20) = \Leftrightarrow R = 20$

$\therefore$ Radius of the pool = **20 ft.**

S.75 (a)

Required area = (Area of an equilateral Δ of side 7 cm) – (3 × area of sector with $\theta = 60°$ & r = 3.5 cm)



$= \left[\left(\frac{\sqrt{3}}{4}\times 7\times 7\right) - \left(3 - \frac{22}{7}\times 3.5\times 3.5\times\frac{60}{360}\right)\right]\text{cm}^2$

$= \left(\frac{49\sqrt{3}}{4} - 11\times 0.5\times 3.5\right)\text{cm}^2$

$= (21.217 - 19.25)\ \text{cm}^2 = \mathbf{1.967\ cm^2}.$

S.76 (d)

Let the dimensions be 3x, 2x and x respectively. Then,

$3x \times 2x \times x = 10368 \Leftrightarrow x^3 = \left(\frac{10368}{6}\right) = 1728$

$\Leftrightarrow x = 12$

So, the dimensions of the block are 36 dm, 24 dm and 12 dm.

Surface area= $[2(36\times 24 + 24\times 12 + 36\times 12)]$ dm^2

$= [2\times 144\ (6+2+3)]\ \text{dm}^2 = 3168\ \text{dm}^2.$

$\therefore$ Cost of polishing = Rs. $\left(\frac{2\times 3168}{100}\right)$

= **Rs. 63.36**

S.77 (d)

Let original edge = a. Then, surface area = $6a^2$.

New edge $= \frac{125}{100}a = \frac{5a}{4}$

New surface area $= 6\times\left(\frac{5a}{4}\right)^2 = \frac{75a^2}{8}$

Increase in surface area $= \left(\dfrac{75a^2}{8} - 6a^2\right)$

$= \dfrac{27a^2}{8}$

$\therefore$ Increase% $= \left(\dfrac{27a^2}{8} \times \dfrac{1}{6a^2} \times 100\right) = \mathbf{56.25\%}$

S.78 (b)

It is given that $r = \dfrac{1}{2}h$ and $2\pi rh + \pi r^2 = 616\,m^2$

$\therefore\ 2\pi \times \dfrac{1}{2}h \times h + \pi \times \dfrac{1}{4}h^2 = 616$

$\Rightarrow \dfrac{5}{4} \times \dfrac{22}{7} \times h^2 = 616$

$\Rightarrow h^2 = \left(616 \times \dfrac{28}{110}\right) = \dfrac{28 \times 28}{5}$

$\therefore$ Volume $= \pi r^2 h = \dfrac{22}{7} \times \dfrac{1}{4}h^2 \times h$

$= \dfrac{22}{7} \times \dfrac{1}{4} \times \dfrac{28 \times 28}{5} \times \dfrac{28}{\sqrt{5}}\ cm^3$

$= \left(\dfrac{22 \times 28 \times 28}{25} \times \sqrt{5}\right) cm^3$

$= \left(\dfrac{22 \times 28 \times 28 \times 2.23}{25 \times 1000}\right)$ litre $= \mathbf{1.53\ litre}$

S.79 (d)

Let the original radius and height of the cone be r and h respectively.

Then, Original volume $= \dfrac{1}{3}\pi r^2 h$

New radius $= \dfrac{120}{100}r = \dfrac{6}{5}r$, New height $= \dfrac{6}{5}h$

New volume $= \dfrac{1}{3}\pi \times \left(\dfrac{6}{5}r\right)^2 \times \left(\dfrac{6}{5}h\right)$

$= \dfrac{216}{125} \times \dfrac{1}{3}\pi r^2 h$

Increase in volume $= \dfrac{91}{125} \times \dfrac{1}{3}\pi r^2 h$

$\therefore$ Increase% $= \left(\dfrac{\dfrac{91}{125} \times \dfrac{1}{3}\pi r^2 h}{\dfrac{1}{3}\pi r^2 h} \times 100\right)\%$

$= \mathbf{72.8\%}$

S.80 (c)

To reach the winning post A will have to cover a distance of (500 – 140) m, i.e., 360 m.

While A covers 3 m, B covers 4 m.

While A covers 360 m,

B covers $\left(\dfrac{4}{3} \times 360\right) m = 480 m$

Thus, when A reaches the winning post, B covers 480 m and therefore remain 20 m behind.

$\therefore$ **A wins by 20 m.**

S.81 (b)

A : B = 200 : 169 and

A : C = 200 : 182

$\dfrac{C}{B} = \left(\dfrac{C}{A} \times \dfrac{A}{B}\right) = \left(\dfrac{182}{200} \times \dfrac{200}{169}\right)$

= 182 : 169

When C covers 182 m, B covers 169 m.

When C covers 350 m,

B covers $\left(\dfrac{169}{182} \times 350\right) m = 325\ m$

So, C will beat B by = (350 – 325)m = **25 m**

S.82 (c)

1600 years have 0 odd day. 300 years have 1 odd day.

97 years = 24 leap years + 73 ordinary years

= [(24 × 2) + (73 × 1)] odd days = 121 odd days

= (17 weeks + 2 days) odd days = 2 odd days

Jan. Feb. March April May June

31 + 28 + 31 + 30 + 31 + 17 = 168 days = 0 odd day

Total number of odd days = (0 + 1 + 2 + 0) = 3 odd days

Hence, the required day was **"Wednesday"**.

S.83 (d)

100 years contain 5 odd days. So, last day of 1st century is 'Friday'.

200 years contain (5 × 2) = 10 odd days = 3 odd days.

So, last day of 2nd century is 'Wednesday'.

300 years contain (5 × 3) = 15 odd days = 1 odd day.

∴ Last day of 3rd century is 'Monday'.

400 years contain 0 odd day.

∴ Last day of 4th century is 'Sunday'.

Since the order is continually kept in successive cycles, we see that **the last day of a century cannot be Tuesday, Thursday or Saturday.**

S.84 (c)

26th Jan.,1950 = (1949 years + Period from 1st Jan., 1950 to 26th Jan., 1950) 1600 years have 0 odd day. 300 years have 1 odd day.

49 years = (12 leap years + 37 ordinary years)

= [(12 × 2) + (37 × 1)] odd days

= 61 odd days = 5 odd days.

Number of days from 1st Jan. to 26th Jan. = 26 = 5 odd days.

Total number of odd days = (0 + 1 + 5 + 5) = 11 = 4 odd days.

∴ The required day was **"Thursday"**.

S.85 (c)

Angle traced by hour hand in $\frac{21}{4}$ hrs

$$=\left(\frac{360}{12}\times\frac{21}{4}\right)^{\circ}=157\frac{1}{2}^{\circ}$$

Angle traced by min. hand in 15 min.

$$=\left(\frac{360}{12}\times 15\right)^{\circ}=90^{\circ}$$

∴ Required angle = $\left(157\frac{1}{2}\right)^{\circ}-90^{\circ}$ = $\mathbf{67\frac{1}{2}^{\circ}}$

S.86 (a)

55 min. spaces are covered in 60 min.

60 min. spaces are covered in $\left(\frac{60}{55}\times 60\right)$ min.

$$= 65\frac{5}{11}\text{ min.}$$

Loss in 64 min.= $\left(65\frac{5}{11}-64\right)=\frac{16}{11}$ min.

Loss in 24 hrs= $\left(\frac{16}{11}\times\frac{1}{64}\times 24\times 60\right)$min.

$$= \mathbf{32\frac{8}{11}\ min.}$$

S.87 (d)

When the hands of the clock are in the same straight line but not together, they are 30 minute spaces apart.

At 7 o'clock, they are 25 min. spaces apart.

∴ Minute hand will have to gain only 5 min. spaces.

55 min. spaces are gained in 60 min.

5 min. spaces are gained in $\left(\frac{60}{55}\times 5\right)$ min.

$$= 5\frac{5}{11}\text{ min.}$$

∴ Required time = $\mathbf{5\frac{5}{11}}$ **min. past 7**

S.88 (b)

At 5 o'clock, the hands are 25 min. spaces apart.

To be at right angles and that too between 5.30 and 6, the minute hand has to gain (25 + 15) = 40 min. spaces.

55 min. spaces are gained in 60 min.

40 min. spaces are gained in $\left(\frac{60}{55}\times 40\right)$ min.

$= 43\frac{7}{11}$ min.

$\therefore$ Required time = $\mathbf{43\frac{7}{11}}$ **min. past 5**

S.89 (c)

The word 'LEADING' has 7 different letters.

When the vowels EAI are always together, they can be supposed to form one letter. Then, we have to arrange the letters LDNG (EAI).

Now, 5 letters can be arranged in 5! = 120 ways.

The vowels (EAI) can be arranged among themselves in 3! = 6 ways.

$\therefore$ Required number of ways = (120×6) = **720**

S.90 (d)

In the word 'CORPORATION', we treat the vowels OOAIO as one letter. Thus, we have CRPRTN (OOAIO).

This has 7 letters of which R occurs 2 times and the rest are different.

Number of ways of arranging these letters = $\frac{7!}{2!} = 2520$

Now, 5 vowels in which O occurs 3 times and the rest are different, can be arranged in $\frac{5!}{3!} = 20$ ways.

$\therefore$ Required number of ways = (2520 × 20) = **50400**.

S.91 (c)

There are 6 letters in the given word, out of which there are 3 vowels and 3 consonants.

Let us mark these positions as under :

$(^1)(^2)(^3)(^4)(^5)(^6)$

Now, 3 vowels can be placed at any of the three places out of 4, marked 1, 3, 5.

Number of ways of arranging the vowels

$= {}^3P_3 = 3! = 6.$

Also, the 3 consonants can be arranged at the remaining 3 positions.

Number of ways of these arrangements

$= {}^3P_3 = 3! = 6.$

Total number of ways = (6 × 6) = **36**.

S.92 (d)

We may have (1 boy and 3 girls) or (2 boys and 2 girls) or (3 boys and 1 girl) or (4 boys).

$\therefore$ Required number of ways

$= ({}^6C_1\times{}^4C_3) + ({}^6C_2\times{}^4C_2) + ({}^6C_3\times{}^4C_1) + ({}^6C_4)$

$= ({}^6C_1\times{}^4C_1) + ({}^6C_2\times{}^4C_2) + ({}^6C_3\times{}^4C_1) + ({}^6C_2)$

$= (6\times 4) + \left(\frac{6\times 5}{2\times 1}\times\frac{4\times 3}{2\times 1}\right) + \left(\frac{6\times 5\times 4}{3\times 2\times 1}\times 4\right) + \left(\frac{6\times 5}{2\times 1}\right)$

= (24 + 90 + 80 + 15) = **209**.

S.93 (b)

In a simultaneous throw of two dice, we have n(S) = (6 × 6) = 36.

Let E = event of getting two numbers whose product is even.

Then,

E = {(1, 2), (1, 4), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 2), (3, 4), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4,), (4, 5), (4, 6), (5, 2), (5, 4), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}.

$\therefore$ n(E) = 27

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{27}{36} = \frac{3}{4}$

S.94 (d)

Clearly, $n(S) = {}^{52}C_2 = \frac{(52 \times 51)}{2} = 1326$

Let E_1 = event of getting both red cards.

E_2 = event of getting both kings.

Then, $E_1 \cap E_2$ = event of getting 2 kings of red cards.

$\therefore \quad n(E_1) = {}^{26}C_2 = \frac{(26 \times 25)}{(2 \times 1)} = 325;$

$n(E_2) = {}^4C_2 = \frac{(4 \times 3)}{(2 \times 1)} = 6;$

$n(E_1 \cap E_2) = {}^2C_2 = 1$

$\therefore \quad P(E_1) = \frac{n(E_1)}{n(S)} = \frac{325}{1326}$

$P(E_1) = \frac{n(E_2)}{n(S)} = \frac{6}{1326}$

$P(E_1 \cap E_2) = \frac{1}{1326}$

$\therefore$ P(both red or both kings) $= P(E_1 \cup E_2)$

$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$= \left(\frac{325}{1326} + \frac{6}{1326} - \frac{1}{1326}\right) = \frac{330}{1326} = \mathbf{\frac{55}{221}}$

S.95 (a)

Total number of balls = (2 + 3 + 2) = 7

Let S be the sample space. Then,

n(S) = Number of ways of drawing 2 balls out of 7

$= {}^7C_2 = \frac{(7 \times 6)}{(2 \times 1)} = 21$

Let E = Event of drawing 2 balls, none of which is blue.

$\therefore$ n(E) = Number of ways of drawing 2 balls out of (2 + 3) balls

$= {}^5C_2 = \frac{(5 \times 4)}{(2 \times 1)} = 10$

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \mathbf{\frac{10}{21}}$

S.96 (d)

Let S be the sample space and E be the event of choosing four persons such that 2 of them are children. Then,

n(S) = Number of ways of choosing 4 persons out of 9

$= {}^9C_4 = \frac{(9 \times 8 \times 7 \times 6)}{(4 \times 3 \times 2 \times 1)} = 126$

n(E) =Number of ways of choosing 2 children out of 4 and 3 persons out of (3 + 2) persons

$= \left({}^4C_2 \times {}^5C_2\right) = \frac{(4 \times 3)}{(2 \times 1)} \times \frac{(5 \times 4)}{(2 \times 1)} = 60$

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{60}{126} = \mathbf{\frac{10}{21}}$

S.97 (c)

Let A = Event that A speaks the truth and B = Event that B speaks the truth.

Then, $P(A) = \frac{75}{100} = \frac{3}{4}$

$P(B) = \frac{80}{100} = \frac{4}{5}$

$\therefore \quad P(\overline{A}) = \left(1 - \frac{3}{4}\right) = \frac{1}{4}$

and $\quad P(\overline{B}) = \left(1 - \frac{4}{5}\right) = \frac{1}{5}$

P(A and B contradict each other)

= P[(A speaks the truth and B tells a lie) or (A tells a lie and B speaks the truth)]

$= P[(A \text{ and } \overline{B}) \text{ or } (\overline{A} \text{ and } B)]$

$= P(A \text{ and } \overline{B}) + P(\overline{A} \text{ and } B)$

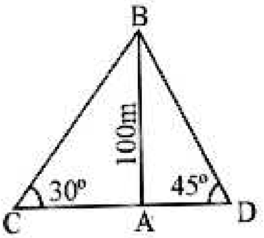
$= P(A).P(\overline{B}) + P(\overline{A}).P(B)$

$= \left(\frac{3}{4} \times \frac{1}{5}\right) + \left(\frac{1}{4} \times \frac{4}{5}\right) = \left(\frac{3}{20} + \frac{1}{5}\right) = \frac{7}{20}$

$= \left(\frac{7}{20} \times 100\right)\% = 35\%$

$\therefore$ **A and B contradict each other in 35% of the cases.**

S.98 (c)



Let AB be the lighthouse and C and D be the positions of the ships. Then,

AB = 100 m, ∠ACB = 30° and ∠ADB = 45°

$$\frac{AB}{AC} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad AC = AB \times \sqrt{3} = 100\sqrt{3} \text{ m}$$

$$\frac{AB}{AD} = \tan 45° = 1$$

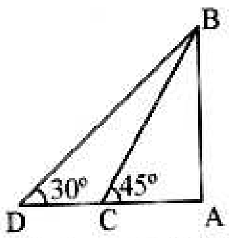
$$\Rightarrow \quad AD = AB = 100 \text{ m}$$

$$\therefore \quad CD = (AC + AD) = (100\sqrt{3} + 100) \text{ m}$$

$$= 100(\sqrt{3} + 1) \text{ m} = (100 \times 2.73) \text{ m}$$

$$= \mathbf{273 \ m}$$

S.99 (a)



Let AB be the tower and C and D be the two positions of the boats. Then,

∠ACB = 45°, ∠ADB = 30° and AC = 60 m

Let AB = h.

$$\text{Then,} \quad \frac{AB}{AC} = \tan 45° = 1$$

$$\Rightarrow \quad AB = AC \Rightarrow h = 60 \text{ m}.$$

$$\text{And,} \quad \frac{AB}{AD} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad AD = (AB \times \sqrt{3}) = 60\sqrt{3} \text{ m}$$

$$\therefore \quad CD = (AD - AC) = 60(\sqrt{3} - 1) \text{ m}$$

Hence,

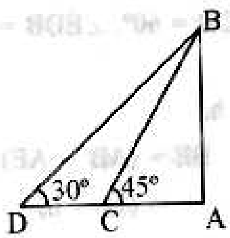
$$\text{required speed} = \left[\frac{60(\sqrt{3}-1)}{5}\right] \text{m/s}$$

$$= (12 \times 0.73) \text{ m/s}$$

$$= \left(12 \times 0.73 \times \frac{18}{5}\right) \text{km/hr}$$

$$= 31.5 \text{ km/hr} = \mathbf{32 \ km/hr.}$$

S.100 (c)



Let AB be the tower and C and D be the two positions of the car.

Then, ∠ACB = 45°, ∠ADB = 30°

Let AB = h, CD = x and AC = y.

$$\frac{AB}{AC} = \tan 45° = 1$$

$$\Rightarrow \quad \frac{h}{y} = 1 \Rightarrow y = h.$$

$$\frac{AB}{AD} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{h}{x+y} = \frac{1}{\sqrt{3}} \Rightarrow x + y = \sqrt{3}h$$

$$\therefore \quad x = (x + y) - y$$

$$= \sqrt{3}h - h = h(\sqrt{3} - 1)$$

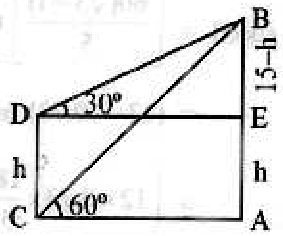
Now, $h(\sqrt{3}-1)$ is covered in 12 min.

So, h will be covered in $\left[\frac{12}{h(\sqrt{3}-1)} \times h\right]$

$$= \frac{12}{(\sqrt{3}-1)} \text{ min.}$$

$$= \left(\frac{1200}{73}\right) \text{min} = \mathbf{16 \ min. \ 23 \ sec.}$$

S.101 (c)



Let AB be the tower and CD be the electric pole.

Then, $\angle ACB = 60°$, $\angle EDB = 30°$ and AB = 15 m.

Let CD = h.

Then, $BE = (AB - AE) = (AB - CD)$
$= (15 - h)$

$$\frac{AB}{AC} = \tan 60° = \sqrt{3}$$

$\Rightarrow$ $$AC = \frac{AB}{\sqrt{3}} = \frac{15}{\sqrt{3}}$$

And, $$\frac{BE}{DE} = \tan 30° = \frac{1}{\sqrt{3}}$$

$\Rightarrow$ $$DE = (BE \times \sqrt{3}) = \sqrt{3}(15 - h)$$

$$AC = DE$$

$\Rightarrow$ $$\frac{15}{\sqrt{3}} = \sqrt{3}(15 - h)$$

$\Rightarrow$ $3h = (45 - 15) \Rightarrow$ **h = 10 m.**

PAPER

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GP

Q.1 to Q.25 Carry One Mark Each.

Q.1 Let G = (V, E) be a graph. Define $\xi(G) = \sum_d i_d \times d$, where i_d is the number of vertices of degree d in G. If S and T are two different trees with $\xi(S) = \xi(T)$, then

(a) $|S| = 2|T|$ (b) $|S| = |T| - 1$

(c) $|S| = |T|$ (d) $|S| = |T| + 1$

Q.2 Newton-Raphson method is used to compute a root of the equation $x^2 - 13 = 0$ with 3.5 as the initial value. The approximation after one iteration is

(a) 3.575 (b) 3.676

(c) 3.667 (d) 3.607

Q.3 What is the possible number of reflexive relations on a set of 5 elements?

(a) 2^{10} (b) 2^{15}

(c) 2^{20} (d) 2^{25}

Q.4 Consider the set $S = \{1, \omega, \omega^2\}$, where ω and ω^2 are cube roots of unity. If * denotes the multiplication operation, the structure (S, *) forms

(a) A group (b) A ring

(c) An integral domain (d) A field

Q.5 What is the value of $\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{2n}$?

(a) 0 (b) e^{-2}

(c) $e^{-1/2}$ (d) 1

Q.6 The minterm expansion of $f(P,Q,R) = PQ + Q\bar{R} + P\bar{R}$ is

(a) $m_2 + m_4 + m_6 + m_7$

(b) $m_0 + m_1 + m_3 + m_5$

(c) $m_0 + m_1 + m_6 + m_7$

(d) $m_2 + m_3 + m_4 + m_5$

Q.7 A main memory unit with a capacity of 4 megabytes is built using 1M × 1-bit DRAM chips. Each DRAM chip has 1K rows of cells with 1K cells in each row. The time taken for a single refresh operation is 100 nanoseconds. The time required to perform one refresh operation on all the cells in the memory unit is

(a) 100 nanoseconds

(b) $100*2^{10}$ nanoseconds

(c) $100*2^{20}$ nanoseconds

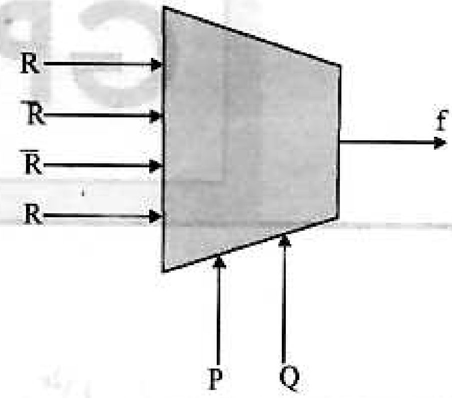
(d) $3200*2^{20}$ nanoseconds

Q.8 P is a 16-bit signed integer. The 2's complement representation of P is $(F87B)_{16}$. The 2's complement representation of 8*P is

(a) $(C3D8)_{16}$ (b) $(187B)_{16}$

(c) $(F878)_{16}$ (d) $(987B)_{16}$

Q.9 The Boolean expression for the output f of the multiplexer shown below is



(a) $\overline{P \oplus Q \oplus R}$ (b) $P \oplus Q \oplus R$

(c) $P + Q + R$ (d) $\overline{P + Q + R}$

Q.10 In a binary tree with n nodes, every node has an odd number of descendants. Every node is considered to be its own descendant. What is the number of nodes in the tree that have exactly one child?

(a) 0 (b) 1

(c) $(n - 1) / 2$ (d) $n - 1$

Q.11 What does the following program print?

```
#include < stdio.h >
void f (int * p, int * q){
        p = q;
        * p = 2;
}
int i=0, j = 1;
int main ( ){
        f(&i, & j);
        print f("%d %d\n", i, j);
        return 0;
}
```

(a) 2 2 (b) 2 1

(c) 0 1 (d) 0 2

Q.12 Two alternative packages A and B are available for processing a database having 10^k records. Package A requires $0.0001n^2$ time units and package B requires $10n\log_{10}n$ time units to process n records. What is the smallest value of k for which package B will be preferred over A?

(a) 12 (b) 10

(c) 6 (d) 5

Q.13 Which data structure in a compiler is used for managing information about variables and their attributes?

(a) Abstract syntax tree

(b) Symbol table

(c) Semantic stack

(d) Parse table

Q.14 Which languages necessarily need heap allocation in the runtime environment?

(a) Those that support recursion

(b) Those that use dynamic scoping

(c) Those that allow dynamic data structures

(d) Those that use global variables

Q.15 One of the header fields in an IP datagram is the Time to Live (TTL) field. Which of the following statements best explains the need for this field?

(a) It can be used to prioritize packets

(b) It can be used to reduce delays

(c) It can be used to optimize throughput

(d) It can be used to prevent packet looping

Q.16 Which one of the following is not a client server application?

(a) Internet chat

(b) Web browsing

(c) E-mail

(d) Ping

Q.17 Let L1 be a recursive language. Let L2 and L3 be languages that are recursively enumerable but not recursive. Which of the following statements is not necessarily true?

(a) L2 – L1 is recursively enumerable

(b) L1 – L3 is recursively enumerable

(c) L2 $\cap$ L1 is recursively enumerable

(d) L2 $\cup$ L1 is recursively enumerable

Q.18 Consider a B^+-tree in which the maximum number of keys in a node is 5. What is the minimum number of keys in any non-root node?

(a) 1 (b) 2

(c) 3 (d) 4

Q.19 A relational schema for a train reservation database is given below:

Passenger(pid, pname, age)

Reservation(pid, cass, tid)

Table : Passenger

pid	pname	Age
0	'Sachin'	65
1	'Rahul'	66
2	'Sourav'	67
3	'Anil'	69

Table : Reservation

pid	class	tid
0	'AC'	8200
1	'AC'	8201
2	'SC'	8201
5	'AC'	8203
1	'SC'	8204
3	'AC'	8202

What pids are returned by the following SQL query for the above instance of the tables?

```
SELECT      pid
FROM        Reservation
WHERE       class ='AC' AND
      EXISTS (SELECT *
      FROM Passenger
      WHERE age > 65 AND
      Passenger.pid = Reservation.pid)
```

(a) 1, 0 (b) 1, 2

(c) 1, 3 (d) 1, 5

Q.20 Which of the following concurrency control protocols ensure both conflict serializability and freedom from deadlock?

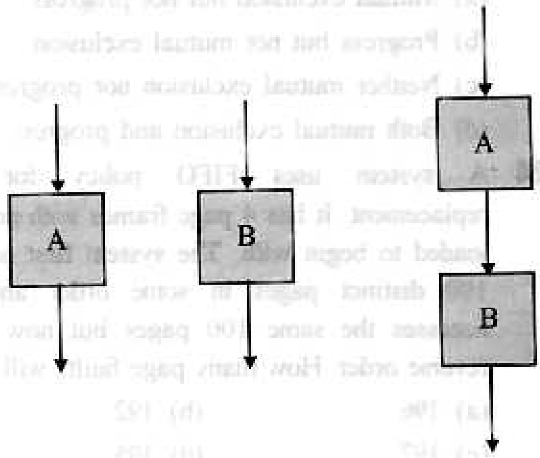
I. 2-phase locking

II. Time-stamp ordering

(a) I only (b) II only

(c) Both I and II (d) Neither I nor II

Q.21 The cyclomatic complexity of each of the modules A and B shown below is 10. What is the cyclomatic complexity of the sequential integration shown on the right hand side?



(a) 19 (b) 21

(c) 20 (d) 10

Q.22 What is the appropriate pairing of items in the two columns listing various activities encountered in a software life cycle?

Group I		Group II	
P.	Requirement Capture	1.	Module Development and Integration
Q.	Design	2.	Domain Analysis
R.	Implementation	3.	Structural and Behavioral Modeling
S.	Maintenance	4.	Performance Tuning

(a) P–3, Q–2, R–4, S–1

(b) P–2, Q–3, R–1, S–4

(c) P–3, Q–2, R–1, S–4

(d) P–2, Q–3, R–4, S–1

Q.23 Consider the methods used by processes P1 and P2 for accessing their critical sections whenever needed, as given below. The initial values of shared boolean variables S1 and S2 are randomly assigned.

Method used by P1	Method used by P2
while (S1 = = S2) ; Critical Section S1 = S2;	while (S1 != S2) Critical Section S2 = not S1

Which one of the following statements describes the properties achieved?

(a) Mutual exclusion but not progress

(b) Progress but not mutual exclusion

(c) Neither mutual exclusion nor progress

(d) Both mutual exclusion and progress

Q.24 A system uses FIFO policy for page replacement. It has 4 page frames with no pages loaded to begin with. The system first accesses 100 distinct pages in some order and then accesses the same 100 pages but now in the reverse order. How many page faults will occur?

(a) 196 (b) 192

(c) 197 (d) 195

Q.25 Which of the following statements are true?

I. Shortest remaining time first scheduling may cause starvation

II. Preemptive scheduling may cause starvation

III. Round robin is better than FCFS in terms of response time

(a) I only (b) I and III only

(c) II and III only (d) I, II and III

Q.26 to Q.55 Carry Two Mark Each.

Q.26 Consider a company that assembles computers. The probability of a faulty assembly of any computer is p. The company therefore subjects each computer to a testing process. This testing process gives the correct result for any computer with a probability of q. What is the probability of a computer being declared faulty?

(a) pq + (1 – p) (1– q)

(b) (1– q)p

(c) (1– p)q

(d) pq

Q.27 What is the probability that divisor of 10^{99} is a multiple of 10^{96}?

(a) 1/625 (b) 4/625

(c) 12/625 (d) 16/625

Q.28 The degree sequence of a simple graph is the sequence of the degrees of the nodes in the graph in decreasing order. Which of the following sequences can not be the degree sequence of any graph?

I. 7, 6, 5, 4, 4, 3, 2, 1

II. 6, 6, 6, 6, 3, 3, 2, 2

III. 7, 6, 6, 4, 4, 3, 2, 2

IV. 8, 7, 7, 6, 4, 2, 1, 1

(a) I and II

(b) III and IV

(c) IV only

(d) II and IV

Q.29 Consider the following matrix

$$A = \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$$

If the eigenvalues of A are 4 and 8, then

(a) x = 4, y = 10

(b) x = 5, y = 8

(c) x = –3, y = 9

(d) x = –4, y = 10

Q.30 Suppose the predicate F(x, y, t) is used to represent the statement that person x can fool person y at time t. which one of the statements below expresses best the meaning of the formula $\forall x \exists y \exists t(\neg F(x, y, t))$?

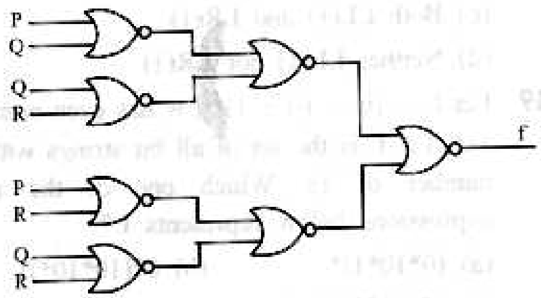
(a) Everyone can fool some person at some time

(b) No one can fool everyone all the time

(c) Everyone cannot fool some person all the time

(d) No one can fool some person at some time

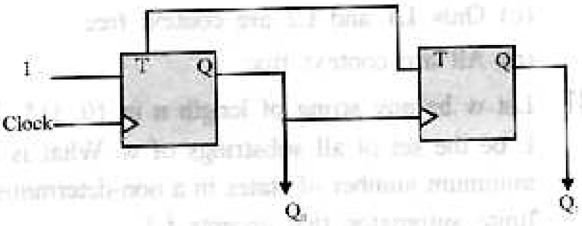
Q.31 What is the Boolean expression for the output f of the combinational logic circuit of NOR gates given below?



(a) $\overline{Q+R}$ (b) $\overline{P+Q}$

(c) $\overline{P+R}$ (d) $\overline{P+Q+R}$

Q.32 In the sequential circuit shown below, if the initial value of the output Q_1Q_0 is 00, what are the next four values of Q_1Q_0?



(a) 11,10,01,00 (b) 10,11,01,00

(c) 10,00,01,11 (d) 11,10,00,01

Q.33 A 5-stage pipelined processor has Instruction Fetch (IF), Instruction Decode (ID), Operand Fetch (OF), Perform Operation (PO) and Write Operand (WO) stages. The IF, ID, OF and WO stages take 1 clock cycle each for any instruction. The PO stage takes 1 clock cycle for ADD and SUB instructions, 3 clock cycles for MUL instruction, and 6 clock cycles for DIV instruction respectively. Operand forwarding is used in the pipeline. What is the number of clock cycles needed to execute the following sequence of instructions?

Instruction	Meaning of instruction
I_0 : MUL R_2, R_0, R_1	$R_2 \leftarrow R_0 * R_1$
I_1 : DIV R_5, R_3, R_4	$R_5 \leftarrow R_3 / R_4$
I_2 : ADD R_2, R_5, R_2	$R_2 \leftarrow R_5 + R_2$
I_3 : SUB R_5, R_2, R_6	$R_5 \leftarrow R_2 - R_6$

(a) 13 (b) 15

(c) 17 (d) 19

Q.34 The weight of a sequence $a_0, a_1, \ldots, a_{n-1}$ of real numbers is defined as $a_0 + a_1/2 + \ldots + a_{n-1}/2^{n-1}$. A subsequence of a sequence is obtained by deleting some elements from the sequence, keeping the order of the remaining elements the same. Let X denote the maximum possible weight of a subsequence of $a_0, a_1, \ldots, a_{n-1}$. Then X is equal to

(a) max(Y, a_0 + Y)

(b) max(Y, a_0 + Y/2)

(c) max(Y, a_0 + 2Y)

(d) a_0 + Y / 2

Q.35 What is the value printed by the following C program?

```
#include < stdio.h >

int f(int * a, int n)

{

    if (n <= O)return 0;

    else if(*a % 2 = = 0) return * a + f(a +1,
    n -1);

    else return * a - f(a + 1, n - 1);

}

int main (          )

{

    int a[ ] = {12, 7, 13, 4, 11, 6};

    printf ("%d", f(a, 6));

    return 0;

}
```

(a) –9

(b) 5

(c) 15

(d) 19

Q.36 The following C function takes a simply-linked list as input argument. It modifies the list by moving the last element to the front of the list and returns the modified list. Some part of the code is left blank.

```
typedef struct node {
    int value;
    struct node *next;
}   Node;
Node *move_to_front(Node *head) {
    Node *p, *q;
    if ((head = = NULL: || (head->next = =
    NULL)) return head;
    q = NULL; p = head;
while (p-> next !=NULL) {
    q = P;
    p = p->next;
}
____________________________________
return head;
}
```

Choose the correct alternative to replace the blank line.

(a) q = NULL; p->next = head; head = p;

(b) q->next = NULL; head = p; p->next = head;

(c) head = p; p->next = q; q->next = NULL;

(d) q->next = NULL; p->next = head; head = p;

Q.37 The program below uses six temporary variables a, b, c, d, e, f.

```
a = 1
b = 10
c = 20
d = a + b
e = c + d
f = c + e
b = c + e
e = b + f
d = 5 + e
return d + f
```

Assuming that all operations take their operands from registers, what is the minimum number of registers needed to execute this program without spilling?

(a) 2 (b) 3

(c) 4 (d) 6

Q.38 The grammar $S \rightarrow aSa|bS|c$ is

(a) LL(1) but not LR(1)

(b) LR(1) but not LL(1)

(c) Both LL(1) and LR(1)

(d) Neither LL(1) nor LR(1)

Q.39 Let $L = \{w \in (0 + 1)^* \mid w$ has even number of 1s$\}$, i.e. L is the set of all bit strings with even number of 1s. Which one of the regular expressions below represents L?

(a) $(0^*10^*1)^*$ (b) $0^*(10^*10^*)^*$

(c) $0^*(10^*1^*)^*0^*$ (d) $0^*1(10^*1)^*10^*$

Q.40 Consider the languages $L1 = \{0^i1^j \mid i \neq j\}$. $L2 = \{0^i1^j \mid i = j\}$. $L3 = \{0^i1^j \mid i = 2j + 1\}$. $L4 = \{0^i1^j \mid i \neq 2j\}$. Which one of the following statements is true?

(a) Only L2 is context free

(b) Only L2 and L3 are context free

(c) Only L1 and L2 are context free

(d) All are context free

Q.41 Let w be any string of length n in $\{0, 1\}^*$. Let L be the set of all substrings of w. What is the minimum number of states in a non-deterministic finite automaton that accepts L?

(a) n–1 (b) n

(c) n + 1 (d) 2^{n-1}

Q.42 Consider the following schedule for transactions T1, T2 and T3:

T1	T2	T3
Read(X)		
	Read(Y)	
		Read(Y)
	Write(Y)	
Write(X)		
	Write(X)	
Read(X)		
Writ(X)		

Which one of the schedules below is the correct serialization of the above?

(a) $T_1 \rightarrow T3 \rightarrow T2$

(b) $T2 \rightarrow T1 \rightarrow T3$

(c) $T2 \rightarrow T3 \rightarrow T1$

(d) $T3 \rightarrow T1 \rightarrow T2$

Q.43 The following functional dependencies hold for relations R(A, B, C) and S(B, D, E)

$B \rightarrow A$,

$A \rightarrow C$

The relation R contains 200 tuples and the relation S contains 100 tuples. What is the maximum number of tuples possible in the natural join $R \bowtie S$?

(a) 100

(b) 200

(c) 300

(d) 2000

Q.44 The following program is to be tested for statement coverage:

```
begin
if (a = = b)          {S1; exit;}
else if (c = = d)     {S2;}
else {S3; exit;}
S4;
end
```

The test cases T1, T2, T3 and T4 given below are expressed in terms of the properties satisfied by the values of variables a, b, c and d. The exact values are not given.

T1 : a, b, c and d are all equal

T2 : a, b, c and d are all distinct

T3 : a=b and c !=d

T4 : a !=b and c=d

Which of the test suites given below ensures coverage of statements S1, S2, S3 and S4?

(a) T1, T2, T3

(b) T2, T4

(c) T3, T4

(d) T1, T2, T4

Q.45 The following program consists of 3 concurrent processes and 3 binary semaphores. The semaphores are initialized as S0 = 1, S1 = 0, S2 = 0.

Process P0	Process P1	Process P2

while (true) {
wait (S0);
print '0'
 	
release (S1);
release (S2);
}
	wait (S1);
	Release (S0);
		wait (S2);
		release (S0);

How many times will process P0 print '0'?

(a) At least twice (b) Exactly twice

(c) Exactly thrice (d) Exactly once

Q.46 A system has n resources $R_0,\ldots,R_{n-1}$, and k processes $P_0,\ldots,P_{k-1}$. The implementation of the resource request logic of each process P_i, is as follows:

```
if (i% 2== 0) {
if (i<n) request R_i;
if (i+2<n)request R_{i+2};
}
else {
if (i<n) request R_{n-i};
if (i+2<n)request R_{n-i-2};
}
```

In which one of the following situations is a deadlock possible?

(a) n = 40, k = 26 (b) n = 21, k = 12

(c) n = 20, k = 10 (d) n = 41, k = 19

Q.47 Suppose computers A and B have IP addresses 10.105.1.113 and 10.105.1.91 respectively and they both use the same net mask N. Which of the values of N given below should not be used if A and B should belong to the same network?

(a) 255.255.255.0

(b) 255.255.255.128

(c) 255.255.255.192

(d) 255.255.255.224

Common Data For Questions 48 & 49:

A computer system has an L1 cache, an L2 cache, and a main memory unit connected as shown below. The block size in L1 cache is 4 words. The block size in L2 cache is 16 words. The memory access times are 2 nanoseconds, 20 nanoseconds and 200 nanoseconds for L1 cache, L2 cache and main memory unit respectively.

Q.48 When there is a miss in L1 cache and a hit in L2 cache, a block is transferred from L2 cache to L1 cache. What is the time taken for this transfer?

(a) 2 nanoseconds (b) 20 nanoseconds

(c) 22 nanoseconds (d) 88 nanoseconds

Q.49 When there is a miss in both L1 cache and L2 cache, first a block is transferred from main memory to L2 cache, and then a block is transferred from L2 cache to L1 cache. What is the total time taken for these transfers?

(a) 222 nanoseconds (b) 888 nanoseconds

(c) 902 nanoseconds (d) 968 nanoseconds

Common Data For Questions 50 & 51:

Consider a complete undirected graph with vertex set {0, 1, 2, 3, 4}. Entry W_{ij} in the matrix W below is the weight of the edge {i, j}.

$$W = \begin{pmatrix} 0 & 1 & 8 & 1 & 4 \\ 1 & 0 & 12 & 4 & 9 \\ 8 & 12 & 0 & 7 & 3 \\ 1 & 4 & 7 & 0 & 2 \\ 4 & 9 & 3 & 2 & 0 \end{pmatrix}$$

Q.50 What is the minimum possible weight of a spanning tree T in this graph such that vertex 0 is a leaf node in the tree T?

(a) 7 (b) 8

(c) 9 (d) 10

Q.51 What is the minimum possible weight of a path P from vertex 1 to vertex 2 in this graph such that P contains at most 3 edges?

(a) 7 (b) 8

(c) 9 (d) 10

Common Data For Questions 52 & 53:

A hash table of length 10 uses open addressing with hash function h(k) = k mod 10, and linear probing. After inserting 6 values into an empty hash table, the table is as shown below

0	
1	
2	42
3	23
4	34
5	52
6	46
7	33
8	
9	

Q.52 Which one of the following choices gives a possible order in which the key values could have been inserted in the table?

(a) 46, 42, 34, 52, 23, 33

(b) 34, 42, 23, 52, 33, 46

(c) 46, 34, 42, 23, 52, 33

(d) 42, 46, 33, 23, 34, 52

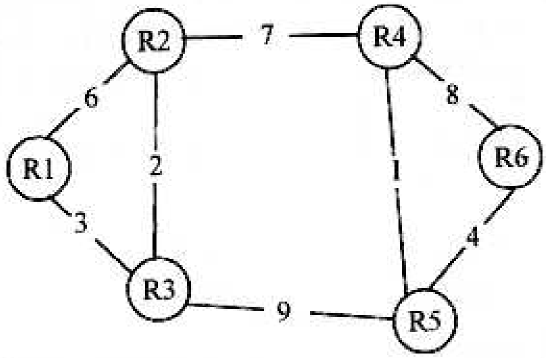
Q.53 How many different insertion sequences of the key values using the same hash function and linear probing will result in the hash table shown above?

(a) 10 (b) 20

(c) 30 (d) 40

Common Data For Questions 54 & 55:

Consider a network with 6 routers R1 to R6 connected with links having weights as shown in the following diagram



Q.54 All the routers use the distance vector based routing algorithm to update their routing tables. Each router starts with its routing table initialized to contain an entry for each neighbour with the weight of the respective connecting link. After all the routing tables stabilize, how many links in the network will never be used for carrying any data?

(a) 4 (b) 3

(c) 2 (d) 1

Q.55 Suppose the weights of all unused links in the previous question are changed to 2 and the distance vector algorithm is used again until all routing tables stabilize. How many links will now remain unused?

(a) 0 (b) 1

(c) 2 (d) 3

Q.56 to Q.60 Carry One Mark Each.

Q.56 Choose the most appropriate word from the options given below to the complete the following sentence:

His rather casual remarks on politics____his lack of seriousness about the subject.

(a) masked (b) belied

(c) betrayed (d) suppressed

Q.57 Which of the following options is closest in meaning to the word Circuitous.

(a) cyclic (b) indirect

(c) confusing (d) crooked

Q.58 Choose the most appropriate word from the options given below to complete the following sentence:

If we manage to _____ our natural resources, we would leave a better planet for our children.

(a) uphold (b) restrain

(c) cherish (d) conserve

Q.59 25 persons are in a room. 15 of them play hockey, 17 of them play football and 10 of them play both hockey and football. Then the number of persons playing neither hockey nor football is:

(a) 2 (b) 17

(c) 13 (d) 3

Q.60 The question below consists of a pair of related words followed by four pairs of words. Select the pair that best expresses the relation in the original pair.

Unemployed: Worker

(a) fallow: land (b) unaware: sleeper

(c) wit: jester (d) renovated: house

Q.61 to Q.65 Carry Two Mark Each.

Q.61 If 137 + 276 = 435 how much is 731 + 672?

(a) 534 (b) 1403

(c) 1623 (d) 1513

Q.62 Hari (H), Gita (G), Irfan (I) and Saira (S) are siblings (i.e. brothers and sisters). All were born on 1st January. The age difference between any two successive siblings (that is born one after another) is less than 3 years. Given the following facts:

i. Hari's age + Gita's age > Irfan's age + Saira's age

ii. The age difference between Gita and Saira is 1 year. However Gita is not the oldest and Saira is not the youngest.

iii. There are no twins.

In what order were they born (oldest first)?

(a) HSIG (b) SGHI

(c) IGSH (d) IHSG

Q.63 Modern warfare has changed from large scale clashes of armies to suppression of civilian populations. Chemical agents that do their work silently appear to be suited to such warfare; and regretfully, there exist people in military establishments who think that chemical agents are useful tools for their cause.

Which of the following statements best sums up the meaning of the above passage:

(a) Modern warfare has resulted in civil strife.

(b) Chemical agents are useful in modern warfare.

(c) Use of chemical agents in warfare would be undesirable.

(d) People in military establishments like to use chemical agents in war.

Q.64 5 skilled workers can build a wall in 20 days; 8 semi-skilled workers can build a wall in 25 days; 10 unskilled workers can build a wall in 30 days. If a team has 2 skilled, 6 semi-skilled and 5 unskilled workers, how long will it take to build the wall?

(a) 20 (b) 18

(c) 16 (d) 15

Q.65 Given digits 2,2,3,3,4,4,4,4 how many distinct 4 digit numbers greater than 3000 can be formed?

(a) 50 (b) 51

(c) 52 (d) 54

ANSWER KEY

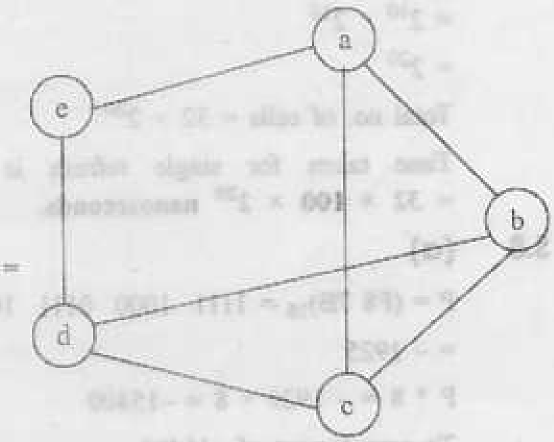
1	c	2	d	3	c	4	a	5	b
6	a	7	d	8	a	9	b	10	a
11	d	12	c	13	b	14	c	15	d
16	d	17	b	18	b	19	c	20	b
21	a	22	b	23	a	24	a	25	d
26	a	27	a	28	d	29	d	30	b
31	a	32	a	33	b	34	a	35	c
36	d	37	c	38	c	39	b	40	f
41	c	42	a	43	a	44	a	45	a
46	b	47	d	48	c	49	a	50	d
51	b	52	c	53	b	54	d	55	a
56	c	57	b	58	d	59	d	60	a
61	c	62	c	63	c	64	d	65	b

SOLUTIONS

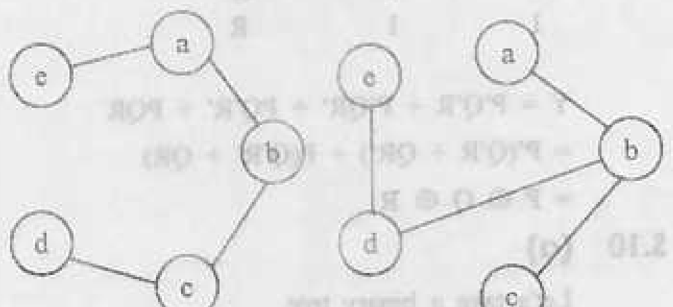
S.1 (c)

Let's assume a graph

a =



Given $\xi(G) = \sum_{d} i_d \times d$ where i_d is number of vertices of degree d. Let two graph S and T are as following



S T

$\xi(S) = i_1 * 1 + i_2 * 2 + \ldots\ldots$

$= 2 \times 1 + 3 \times 2$

$= 8$

$\xi(T) = i_1 * 1 + i_2 * 2 + i_3 * 3 + \ldots\ldots$

$= 3 \times 1 + 1 \times 2 + 1 \times 3$

$= 8$

So $|S| = |T|$

S.2 (d)

Given equation :

$x^2 - 13 = 0$

$x_0 = 3.5$ (given)

By Newton-Raphsom method, we have

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

After first iteration n = 0

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 3.5 - \frac{(3.5)^2 - 13}{2 \times x_0}$$

$$= 3.5 - \frac{12.25 - 13}{2 \times 3.5}$$

$$= 3.5 - \frac{(-0.75)}{7}$$

$$= \mathbf{3.607}$$

S.3 (c)

Total number of reflexive relations given by

formula $= 2^{n^2 - n} = 2^{25-5} = 2^{20}$

S.4 (a)

A group satisfies four properties

(a) closure (b) Associative

(c) Identify (d) Inverse

$S = \{(1, \omega, \omega^2), *\}$

(a) Closure

$1 * \omega = \omega \in S$

$\omega * \omega^2 = \omega^3 = 1 \in S$

$\omega^2 * 1 = \omega^2 \in S$

Closed under set S

(b) Associate

$a * (b * c) = (a * b) * c$

$1 * (\omega * \omega^2) = (1 * \omega) * \omega^2$

$\omega^3 = \omega^3$

Follows associative property

(c) Identity

$a * e = a \in G$, there exit "$\exists e$" $\in G$

$1 * \omega = \omega \in G$

$1 * \omega^2 = \omega^2 \in G$

$1 * 1 = 1 \in G$

Holds identity property.

(d) Inverse

$x * a = e$, $\{x, a, e \in G\}$

$1 * 1 = 1$

$\omega * \omega^2 = 1$ Thus given set is **group.**

$\omega^2 * \omega = 1$

S.5 (b)

$$\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{2n}$$

$$= \left\{\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{-n}\right\}^{-2}$$

$$= e^{-2}$$

S.6 (a)

$$f(P, Q, R) = PQ + Q\bar{R} + P\bar{R}$$

P \ QR	00	01	11	10
0	m_0	m_1	m_3	m_2 1
1	m_4 1	m_3	m_7 1	m_6 1

Minterms are (m_2 + m_4 + m_6 + m_7)

Alternate:

$$= PQ(R+\bar{R}) + (P+\bar{P})Q\bar{R} + P\bar{R}(\bar{Q}+Q)$$

$$= PQR + PQ\bar{R} + PQ\bar{R} + \bar{P}Q\bar{R} + P\bar{Q}\bar{R} + PQ\bar{R}$$

$$= PQR + PQ\bar{R} + \bar{P}Q\bar{R} + P\bar{Q}\bar{R} + PQ\bar{R}$$

$$= PQR + PQ\bar{R} + \bar{P}Q\bar{R} + P\bar{Q}\bar{R} \quad (\because x + x = x)$$

$$= m_7 + m_6 + m_2 + m_4 = \mathbf{m_2 + m_4 + m_6 + m_7}$$

S.7 (d)

Total memory size is $4M \times 1B = 4 \times 10^6 \times 2^3$ bits

Size of one chip is $1M \times 1\text{-bits} = 10^6$ bits

So total no of chips = $(4 \times 10^6 \times 2^3)/10^6 = 32$

Each DRAM chip has number of cell

$= 1k \times 1k$

$= 2^{10} \times 2^{10}$

$= 2^{20}$

Total no. of cells = 32×2^{20}

Time taken for single refresh is 100 ns = **$32 \times 100 \times 2^{20}$ nanoseconds.**

S.8 (a)

$P = (F8\ 7B)_{16} = 1111\ \ 1000\ \ 0111\ \ 1011$

$= -1925$

$P * 8 = -1925 \times 8 = -15400$

2's complement of -15400

$= 1100\ \ 0011\ 1101\ \ \ 1000 = \mathbf{(C3D8)_{16}}$

S.9 (b)

P	Q	f
0	0	R
0	1	R′
1	0	R′
1	1	R

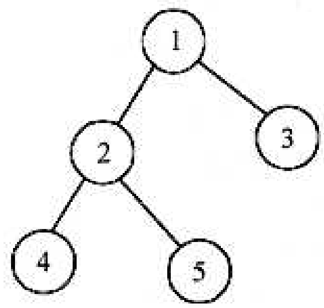
$Y = P'Q'R + P'QR' + PQ'R' + PQR$

$= P'(Q'R + QR') + P(Q'R' + QR)$

$= \mathbf{P \oplus Q \oplus R}$

S.10 (a)

Let's take a binary tree



There are 5 nodes in the binary tree

According to question there is odd number of descendents of every node considering itself.

Consider the tree, at every node satisfy the given properties, only if in the tree every node must have two as zero child nodes.

So number of nodes in the tree have exactly one child is ZERO.

S.11 (d)

Assume that variable i & j has following addresses 1000 & 2000 respectively.

	i	j
value	0	1
address	1000	2000

After passing address in pointer such that

p	q
1000	2000

After assignment

p	q
2000	2000

After assigning value *p = 2. Value of j will be equal to 2. No effect on i because p is not pointing it.

So, **i = 0 and j = 2.**

S.12 (c)

Given $n = 10^k$

Package A requires $0.0001 \times n^2$ time units

$t_A = 0.0001 \times (10^k)^2$

$= 10^{2k-4}$

Package B requires $10\ n \log_{10} n$ time units

$t_B = 10 \times 10^k \times \log_{10} 10^k$

$= k \times 10^{k+1}$

Package B will be preferred area A if $t_B < t_A$ this is true for smallest value of **k = 6.**

S.13 (b)

Symbol table in compiler is used for managing information about variables and their attributes.

S.15 (d)

In an IP datagram the Time to Live (T T L) field is **used to prevent packet looping.**

TTL is measured using addition of maximum of a datagram sent time and acknowledgment receive time of that datagram.

S.16 (d)

Ping is a command that is used to know connection between two computer.

S.17 (b)

Union of two recursive enumerable is not recursive enumerable and also the intersection, $L_2 - L_1$ is recursive enumerable is true but $L_1 - L_3$ need not to be recursive enumerable.

S.18 (b)

Minimum number of keys in any non-root node

$= \left\lceil \frac{n}{2} \right\rceil$

$= \left\lceil \frac{5}{2} \right\rceil$ where $\lceil\ \rceil \rightarrow$ roof

$= 2$

S.19 (c)

By internal query we get the passenger are

pid	pname	Age
1	'Rahul'	66
2	'Sourva'	67
3	'Anil'	69

By outside query pid from AC class are

pid
1
3

S.20 (b)

A transaction is conflict serializability if all serial transactions execute in cascaded order such that result will not change.

In 2-phase looking first resource are allocated then de-allocate in same order. So it is deadlock free but not conflict serializability.

S.21 (a)

Cyclomatic complexity is given by

$Cc = e - n + 2$

where e = number of edges in the control flow graph

n = number of decision nodes in the control flow graph.

$CC_A = 10\ CC_B = 10$

Due to sequential integration one edge is reduced. So cyclomatic complexity of the final result is

$CC_f = CC_A + CC_B - 1$

$CC_f = 10 + 10 - 1$

$CC_f = 19$

S.22 (b)

The corresponding activities in a software life cycle are

(1) Requirements capture → Domain Analysis

(2) Design → Structure and behavioral modeling

(3) Implementation → Module Development and integration

(4) Maintenance → Performance Tuning.

S.23 (a)

Both the processes cannot be simultaneously in critical section, so **it will satisfy mutual exclusion,** but after a process entering in its critical it is not able to signal the other process, so there is no progress.

S.24 (a)

Since we have 4 page frames, 1 to 100 pages will get 100 faults, but in reverse, these will not be any fault for page No's 100, 99, 98, 97 and so remaining 96 faults.

Total faults = 100 + 96 = 196

S.25 (d)

Any preemptive scheduling strategy may cause starvation, SRTF is also preemptive. So causes starvation.

In the FCFS late arriving short Job wait for long time, but in RR, the time quantum expiry will preempt the present job and give the response to the next job in queue so **all are true.**

S.26 (a)

The probability of a faulty assembly of the computers = p

The probability of non faulty assembly of the computers = 1 – p

The probability of correct result for any computer in testing process = q

The probability of correct result for any computer in testing process = 1 – q

The probability of a computer being declared faulty = probability of being faulty and probability of declaring incorrectly = **pq + (1 – p) (1 – q)**

S.27 (a)

The divisor of 10^{99} which are multiples of 10^{96} are 10^{96}, 2×10^{96}, 5×10^{96}, 8×10^{96}, 20×10^{96}, 25×10^{96}, 40×10^{96}, 50×10^{96}, 100×10^{96}, 125×10^{96}, 200×10^{96}, 250×96, 500×10^{96}, 1000×10^{96}

that is a total of 16.

The total number of divisor of $10^{99} = 2^{99} \times 5^{99}$

$= (99 + 1)(99 + 1)$

$= 10000$

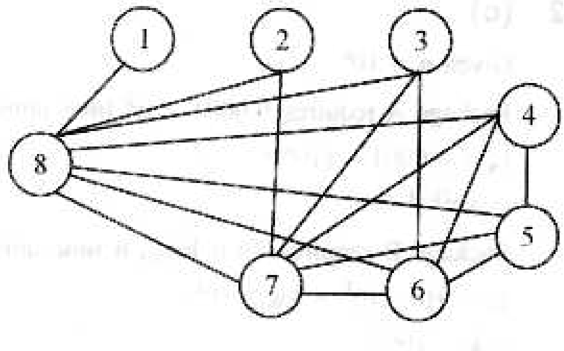
The required probability = 16/10000

= 1/625

S.28 (d)

Degree sequence I ⇒ 7, 6, 5, 4, 4, 3, 2, 1 there is minimum 8 vertex required to form a graph.

Graph can be represented as



In the above graph vertex

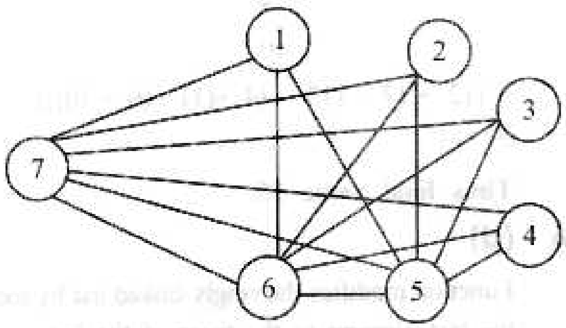
"	7	"	6	"
"	8	"	5	"
"	5	"	4	"
"	4	"	4	"
"	3	"	3	"
"	2	"	2	"
"	1	"	1	"

Thus for the given degree sequence graph is possible.

Degree sequence II $\Rightarrow$ 6, 6, 6, 6, 3, 3, 2, 2

In this case we need 7 vertex required to form a graph.

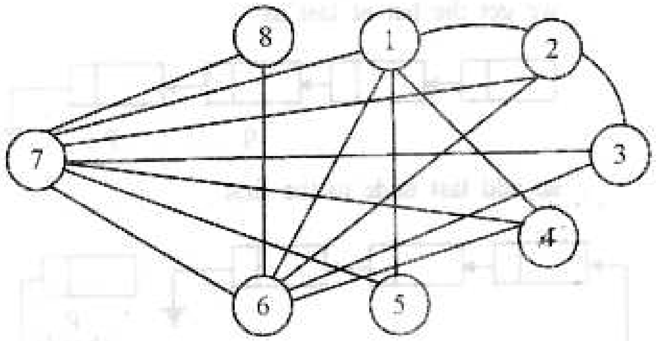
Graph can be required as



We observe that in above graph of we make degree 6 at vertex 7, 6, and 5, then degree 2 is not possible for any other vertex in the graph. So this degree sequence is not possible.

Degree sequence III $\Rightarrow$ 7, 6, 6, 4, 4, 3, 2, 2

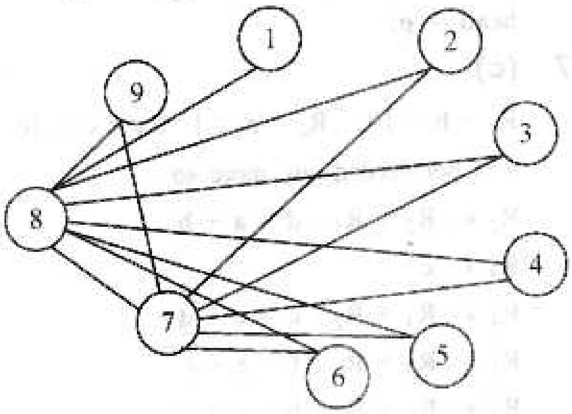
In this case minimum 8 vertex required as highest degree is 7.



For the given sequence graph is possible.

Degree sequence IV $\Rightarrow$ 8, 7, 7, 6, 4, 2, 1, 1

In this case minimum 9 vertex needed as the highest degree is 8.



We observe that we make degree 8 and 7 at vertex 8 and 7 respectively, degree 1 is not possible for two vertices. So this graph sequence is not possible.

Thus degree sequence II and IV are not possible.

S.29 (d)

$$A = \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & 3 \\ x & y-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)(y-\lambda) - 3x = 0$$

Given eigen values 4, 8

If we use 4, then equation becomes,

$$(2-4)(y-4) - 3x = 0 \qquad \text{.....(i)}$$

If we use 8, the equation becomes,

$$(2-8)(y-8) - 3x = 0 \qquad \text{.....(ii)}$$

Solving (1) & (ii), we get

$\mathbf{x = -4, \quad y = 10}$

S.30 (b)

$\because \forall x \exists y \exists t (\neg F(x, y, t))$

$\Rightarrow \neg \exists x \forall y \forall t (F(x, y, t))$

Meaning **there does not exist a person who can fool everyone everytime/no one can fool everyone everytime.**

S.31 (a)

$$\overline{\overline{\overline{(P+Q)}+\overline{(Q+R)}}+\overline{\overline{(P+R)}+\overline{(Q+R)}}}$$

So, it can be written as

$$\overline{\overline{\overline{(P+Q)}+\overline{(Q+R)}}}\;\overline{\overline{\overline{(P+R)}+\overline{(Q+R)}}}$$

$$= \left(\overline{(P+Q)}+\overline{(Q+R)}\right).\left(\overline{(P+R)}+\overline{(Q+R)}\right)$$

$$= (\bar{P}\bar{Q}+\bar{Q}\bar{R}).(\bar{P}\bar{R}+\bar{Q}\bar{R})$$

$$= \bar{P}\bar{Q}\bar{P}\bar{R}+\bar{P}\bar{Q}\bar{Q}\bar{R}+\bar{Q}\bar{R}\bar{P}\bar{R}+\bar{Q}\bar{R}\bar{Q}\bar{R}$$

$= \overline{P}\overline{Q}\overline{R} + \overline{P}\overline{Q}\overline{R} + \overline{Q}P\overline{R} + \overline{Q}\overline{R} \quad [\because X.X = X]$

$= \overline{P}\overline{Q}\overline{R} + \overline{Q}P\overline{R} + \overline{Q}\overline{R} \quad [X + X = X]$

$= \overline{Q}\overline{R}\left[\overline{P} + 1\right]$

$= Q'R'$

$= \overline{(Q+R)}$

S.32 (a)

Initially Q_0 Q_1

0 0

Here T is T –flip-flop

If T = 0 the next state is equal to present state when T = 1, the next state is compliment of present state.

For both flip flops the T input is '1' and the output 'Q_0' is given as a clock to the next T-flip flop.

Initially both will be complemented it will become 11, next second Q_1 remains 1 since the Q_0 become '0' when T input is 1, so when clock is not present it remain in same state.

So it will go for sequence 11, 10, 01, 00.

S.33 (b)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
10	IF	ID	OF	PO	PO	PO	WO									
11		IF	ID	OF	Stall	Stall	PO	PO	PO	PO	PO	PO	WO			
12			IF	ID	OF	-	-	-	-	-	-	-	PO	WO		
13				IF	ID	OF	-	-	-	-	-	-	-	PO	WO	

So total 15 cycles are required.

S.35 (c)

Given a [ ] = | 12 | 7 | 13 | 4 | 11 | 6 |

Iterations for given function are

f(a, 6) → 12 + f(a + 1, n – 1)

↓

7 – f(a + 1, n – 1)

↓

13 – f(a + 1, n – 1)

↓

4 + f(a + 1, n – 1)

↓

11 – f(a + 1, n –1)

↓

6 + f(a + 1, 0)

↓

0

= (12 + (7 – (13 – (4 +(11 –(6 + 0))))

= 15

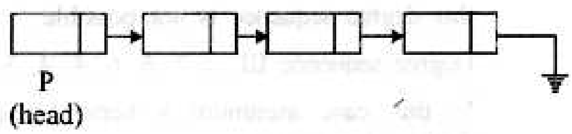
Thus final value 15.

S.36 (d)

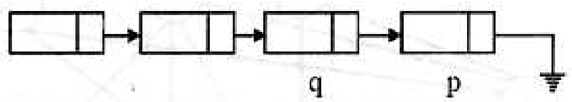
Function modifies the singly-linked list by moving the last element to the front of the list.

Initially q = NULL , p = head

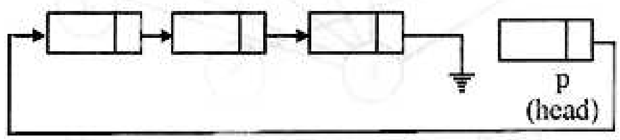
Lets assume a linked list



after applying given operation on the linked list we get the list at last as



so add last node to the first



So do this

operation we must code as

q → next = NULL; p → next = head;

head = p;

S.37 (c)

$R_1 = b = 10$, $R_2 = a = 1$ $R_3 = c = 20$

a is not needed any more so,

$R_2 \leftarrow R_2 + R_3$; d = a + b

$R_3 \leftarrow c$

$R_4 \leftarrow R_3 + R_2$; e = c + d

$R_1 \leftarrow R_3 + R_4$; f = c + e

$R_3 \leftarrow R_3 + R_4$; b = c + e

$R_3 \leftarrow R_3 + R_1$; $e = b + f$

$R_2 \leftarrow R_3 + 5$; $d = e + 5$

return R2 + R1 ;

So we need only 4 registers.

S.38 (c)

grammar S = aSa|bS|c

First (S) = {a, b, c}

Follow (S) = {a, S}

LL(1)

	a	b	c	$
S	$S \rightarrow aSa$	$S \rightarrow bS$	$S \rightarrow c$	

No multiple entries, so this is **LL(1) grammar.**

LR(1) : $S \rightarrow aSa|bS|c$

1. $S \rightarrow aSa$
2. $S \rightarrow bS$
3. $S \rightarrow c$

I_0 : $S' \rightarrow .S, \$$

$S' \rightarrow .aSa, \$$

$S \rightarrow .bS, \$$

$S \rightarrow .c, \$$

I_1 : $S' \rightarrow S., \$$

I_2 : $S \rightarrow a.Sa, \$$

I_{10} : $S \rightarrow b.S, \$$

: $S \rightarrow .aSa, \$$

: $S \rightarrow .bS, \$$

: $S \rightarrow .c, \$$

I_{11} : $S \rightarrow .aSa, \$$

I_3 : $S \rightarrow b.S, \$$

$S \rightarrow .aSa, \$$

$S \rightarrow .bS, \$$

$S \rightarrow .c,\$$

I_4 : $S \rightarrow c., \$$

I_5 : $S \rightarrow aS.a, \$$

I_6 : $S \rightarrow a.Sa, a$

$S \rightarrow .aSa, a$

$S \rightarrow .bS, a$

$S \rightarrow .c, a$

I_9 : $S \rightarrow bS., \$$

I_{10} : $S \rightarrow b.S, \$$

I_{12} : $S \rightarrow aS.a, a$

I_{13} : $S \rightarrow bS., a$

I_{14} : $S \rightarrow bS., \$$

I_{15} : $S \rightarrow aSa.,a$

I_7 : $S \rightarrow b.S, a$

$S \rightarrow .aSa, a$

$S \rightarrow .bS, a$

$S \rightarrow .c, a$

I_8 : $S \rightarrow c.,a$

	a	b	c	$	S
0	S_2	S_3	S_4		1
1					
2	S_6	S_7	S_8		5
3	S_2	S_{10}	S_4		9
4	- - -	- - -	- - -	- - - r_3	
5	S_{11}				
6	S_6	S_7	S_8		12
7	S_6	S_7	S_8		13
8	R_3				
9	- - -	- - -	- - -	- - - r_2	
10	S_2	S_3	S_4		14
11	- - -	- - -	- - -	- - - r_1	
12	S_{15}				
13	r_2				
14	- - -	- - -	- - -	- - - r_2	
15	- - - r_1				

No multiple entries so **given grammar is LR(1)** also.

S.39 (b)

Every string should contain even number of '1's. min string are $\wedge$, 0, 11, 101, 1010.... We get that string from the expression

0* (10* 10*)*

In option C and D Min. string is not $\in$. In option A we don't get, 0, 00, 000......

S.40 (d)

All the given languages can be recognized by the push-Down Automaton, so all are context free languages.

L1 = number of zero is not equal to number of 1's. so no need of count, so it is context free.

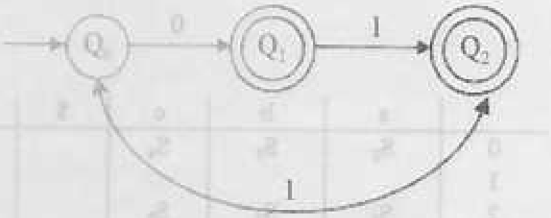
L2 = number of zero equal to number of 1's that can recognize by a deterministic push down automata similarly L3 & L4.

S.41 (c)

Let w = 01

substring are 0, 1, 01

so for this string NFA is



It requires Min 3 states.

So, If string length is 'n', It required 'n + 1' states.

S.42 (a)

Conflicts between two transactions is, if only they are performing write-write or read-write or write-read operation on same items.

There is conflict between T1 to T3 for read-write operation on X.

similarly conflict between T3 to T2 for write-write operation on X.

So correct serialization for T1, T2 and T3 is

T1 → T3 → T2

S.43 (a)

There is no effect of dependencies on the natural join because it is in same relation.

R contains 200 tuples

S contains 100 tuples

natural join of R and S

R ⋈ S = min tuples (R, S)

because corresponding matching of minimum one attribute must be present

So maximum number of tuples in natural join is 100.

S.44 (a)

Using test cases T1, T2, T3 and T4 we have to cover all statements

that can be done using only T1, T2, T4 test cases

By T1, statement S1 will cover

By T2, statement S3 will cover

By T4, statement S2 and S4 will cover

S.45 (a)

Initially neither P1 nor P2 can go, only P0 can go since it uses the semaphore whose value is '1'. It prints '0' then,

after releasing s1, s2, again either P1 or P2 releases S0 so that it will go to print '0' atleast twice.

S.47 (d)

Net mask is meant for giving the network address of a given IP Address. When Both IP-Addresses are in the same Network, After performing

the Mask operation, same Id must be there for both, It will not come when we use 255.255.255.224

IP of A is 10.105.1.113

its last eight bit are $(113)_{10} = (01101111)_2$

IP of B is 10.105.1.91

Its last eight bit are $(91)_{10} = (01011011)_2$

for mask 225.225.225.224

its last eight bits are $(224)_{10} = (11100000)_2$

Adding last eight bits of mask with last eight bits of A and Adding last eight bits of mask with last eight bits of B, don't give the same result, so this mask can not be used for same netmask of A and B.

S.48 (c)

A block to access in L2 cache requires 20 nanoseconds, and 2 nanoseconds to place in L1 cache, so it require total 22 nanoseconds.

There is the term block transfer only for to create confusion.

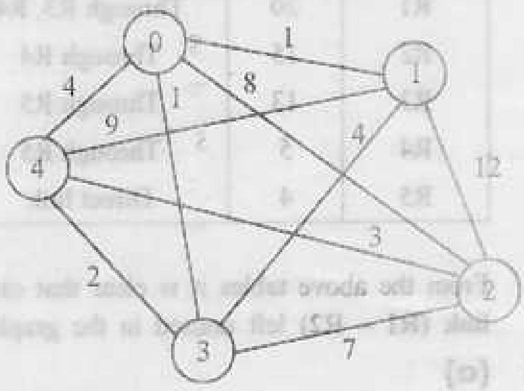
S.49 (a)

When there is a miss in both L1, L2, it has to be access in Main memory which takes 200 nanoseconds, and 20 nanoseconds to transfer to L2 cache and 2 for L1 cache, so

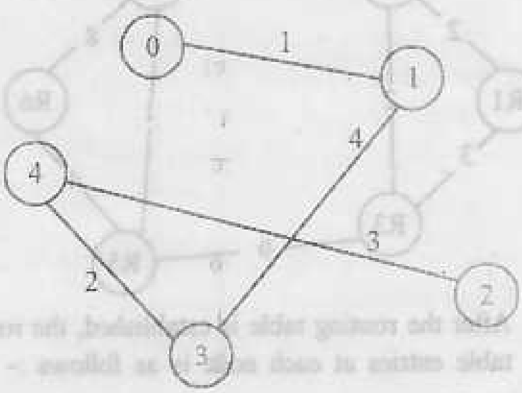
Total = 200 + 20 + 2 = 222 nanoseconds.

S.50 (d)

Graph from the given matrix is



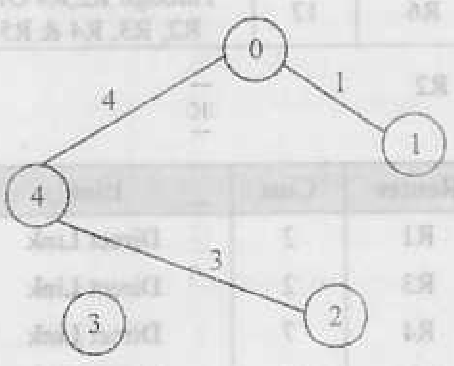
The possible minimum weight spanning tree T in this graph taking vertex 0 as a leaf node is



So minimum weight is = 10.

S.51 (b)

Using the graph in Q.50 a path P from vertex 1 to vertex 2 such that P contains at most 3 edges is



So minimum possible weight of path P is 8.

S.52 (c)

Values in hash table are 46, 34, 42, 23, 52, 33.

hash function h(k) = kmod10

So, for 46, h(46) = 46 mod 10 = 6. (index)

So, 46 will be stored on index 6

Similarly, 34

h (34) = 34 mod 10 = 4

42 → 2

23 → 3

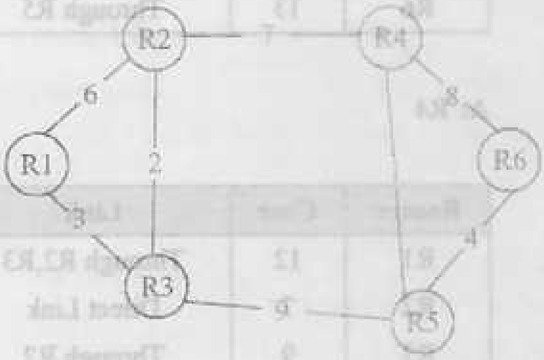
52 → 2(location is occupied)

So it will be stored next at next empty location

33 → 3 (location is occupied)

So it will be stored next at next empty location, hence this sequence satisfies the table as shown.

S.54 (d)



After the routing tables stabilized, the routing Table entries at each node is as follows

At R1

Router	Cost	Link
R2	5	Through R3
R3	3	Direct Link
R4	12	Through R2, R3
R5	12	Through R3
R6	16	Through R3, R5

At R2

Router	Cost	Link
R1	5	Through R3
R3	2	Direct Link
R4	7	Direct Link
R5	8	Through R4
R6	12	Through R4, R5

At R3

Router	Cost	Link
R1	3	Direct Link
R2	2	Direct Link
R4	9	Through R2
R5	9	Direct Link
R6	13	Through R5

At R4

Router	Cost	Link
R1	12	Through R2,R3
R2	7	Direct Link
R3	9	Through R2
R5	1	Direct Link
R6	5	Through R5

At R5

Router	Cost	Link
R1	12	Through R3
R2	8	Through R4
R3	9	Direct Link
R4	1	Direct Link
R6	4	Through R5

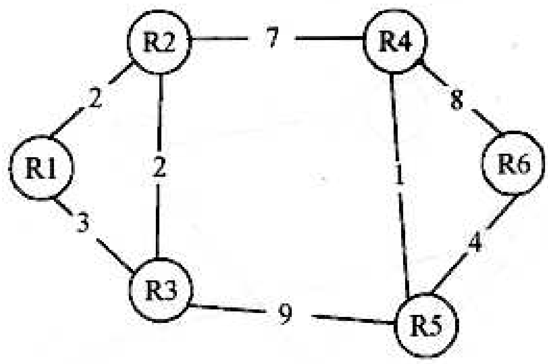
At R6

Router	Cost	Link
R1	20	Through R3, R4
R2	15	Through R4
R3	13	Through R5
R4	5	Through R5
R5	4	Direct link

From the above tables it is clear that only **one link (R1 – R2)** left unused in the graph.

S.55 (a)

After changing the cost of line R1 – R2, graph is



After the routing table is established, the routing table entries at each node is as follows :-

At R1

Router	Cost	Link
R2	2	Direct Link
R3	3	Direct Link
R4	9	Through R2
R5	10	Through R2,R4
R6	17	Through R2,R4 OR R2, R3, R4 & R5

At R2

Router	Cost	Link
R1	2	Direct Link
R3	2	Direct Link
R4	7	Direct Link
R5	8	Through R4
R6	12	Through R4, R6

At R3

Router	Cost	Link
R1	3	Direct Link
R2	2	Direct Link
R4	9	Through R2
R5	9	Direct Link
R6	10	Through R2,R4

At R4

Router	Cost	Link
R1	9	Through R2
R2	7	Direct Link
R3	9	Through R2
R5	1	Direct Link
R6	5	Through R5

At R5

Router	Cost	Link
R1	10	Through R2,R4
R2	8	Through R4
R3	9	Through R2, R4
R4	1	Direct Link
R6	4	Direct Link

At R6

Router	Cost	Link
R1	14	Through R2, R4, R5
R2	12	Through R4, R5
R3	13	Through R5
R4	5	Through R5
R5	4	Direct link

From the above tables it is clear that no link will remain unused if at R1 it uses route for R6, through R2 & R4. Otherwise if it uses route R2, R3, R4 & R6 then one link will be unused.

So there is ambiguity in question but most appropriate.

S.56 (c)

The key words in the statement are 'casual remarks' and 'lack of seriousness'. The blank should be filled with a word meaning 'showed' or 'revealed'. Hence, **'betrayed'** is the correct answer.

S.57 (b)

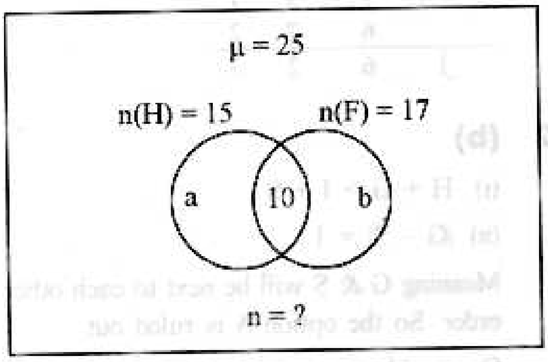
Circuitous means round about or not direct. Therefore the closest in meaning will be **indirect.**

S.58 (d)

The clue in this sentence is 'If we manage to __ our natural resources' and 'better planet'. This implies that the blank should be filled by a word which means 'preserve' or 'keep for long time'. Therefore the word **'conserve'** is the right answer.

S.59 (d)

Representing the given information in the Venn Diagram, we have



Let, the number of people who play only hockey = a

The number of people who play only football = b

Now, $a = n(H) - 10 = 15 - 10 = 5$

$b = n(F) - 10 = 17 - 10 = 7$

Clearly, $a + b + 10 + n = 25$

$\Rightarrow n = 25 - 7 - 5 - 10 \Rightarrow n = 3$

The number of people who play neither Hockey nor Football is 3

S.60 (a)

A worker who is inactive or not working is termed as unemployed, similarly **land** which is inactive or not in use is called **Fallow**.

S.61 (c)

Given,

137 + 276 = 435

It is addition in octal system as all digit is less than 8

i.e. 0 + 7 = 7

1 + 7 = 0 (carry 1)

2 + 7 = 2 (carry 1)

Therefore

$$\begin{array}{rrr} +1 & +1 & \\ 1 & 3 & 7 \\ 2 & 7 & 6 \\ \hline 4 & 3 & 5 \end{array}$$

Using same logic we have

$$\begin{array}{rrrr} +1 & +1 & & \\ & 7 & 3 & 1 \\ & 6 & 7 & 2 \\ \hline 1 & 6 & 2 & 3 \end{array}$$

S.62 (b)

(i) H + G > I + S

(ii) |G − S| = 1

Meaning G & S will be next to each other in the order. So the option A is ruled out.

G not oldest

S not youngest

(iii) No twins.

Going by the options, we will try to solve the equation.

Taking an example with youngest aged 1, we can try to solve the equation and correct the age (started with ages 4,3,2,1) to suit condition (i) and (ii) which gives 5,4,3,1

S	I + 4	Generalizing, we can take their ages in terms of I's age.
G	I + 3	In this case, H + G > I + S
H	I + 2	Since 2I + 5 > 2I + 4
I	I	

I	In this order, G is always less than I and H is always less than S.
G	So G < I and H < S
S	Implies G + H < I + S, all values are positive
H	Defies condition (i) Hence incorrect.

I	In this order H < I, G < S
H	Hence H + G < I + S
S	Defies Condition (i)
G	Hence incorrect.

S.63 (c)

Among the answer choices, the three options B, C and D can be inferred from the passage. But the main conclusion of the passage is that chemical agents are being used my military establishments in warfare which is not desirable. Therefore option C.

S.64 (d)

Given,

5 skilled workers can build a wall in 20 days i.e., 1 skilled worker can build the same wall in 100 days.

The capacity of each skilled worker is 1/100

8 semi-skilled workers can build a wall in 25 days i.e., 1 semi-skilled worker can build the same wall in 200 days.

The capacity of each semi-skilled worker is 1/200

Similarly, the capacity of 1 unskilled worker is 1/300

Now the capacity of 2 skilled + 6 semi-skilled + 5 unskilled workers = 2/100 + 6/200 + 5/300 = 1/15

Therefore the required number of days is 15.

S.65 (b)

Given digits : 2, 2, 3, 3, 3, 4, 4, 4, 4

Number that are greater than 3000 can be formed in the following way.

1st Digit	2nd Digit	3rd Digit	4th Digit	
3	2	2	3 or 4	4 Ways
4	2	2	3 or 4	
3	2	3	2 or 3 or 4	6 Ways
4	2	3	2 or 3 or 4	
3	3	2	2 or 3 or 4	6 Ways
4	3	2	2 or 3 or 4	
3	3	3	2 or 4	5 Ways
4	3	3	2 or 3 or 4	
3	2	4	2 or 3 or 4	6 Ways
4	2	4	2 or 3 or 4	
3	4	2	2 or 3 or 4	6 Ways
4	4	2	2 or 3 or 4	
3	3	4	2 or 3 or 4	6 Ways
4	3	4	2 or 3 or 4	
3	4	3	2 or 3 or 4	6 Ways
4	4	3	2 or 3 or 4	
3	4	4	2 or 3 or 4	6 Ways
4	4	4	2 or 3 or 4	

∴ Total no. of ways ⇒ 4 + 6 + 6 + 5 + 6 + 6 + 6 + 6 + 6 = 51 ways.

Hence there can be 51 numbers greater than 3000.

Alternet:

The given digits are 2, 2, 3, 3, 3, 4, 4, 4, 4 we have to find the numbers that are greater than 3000.

∴ The first digit can be 3 or 4 but not 2.

Now, let us fix the first, second and third digits as 3, 2, 2 and then the fourth place can be filled in 3 ways.

i.e.,

3	2	2	2 or 3 or 4	3 ways

∴ the number of ways is 3.

Similarly, we fix first, third and fourth places as 3, 2 and 2 respectively, so the second place can be filled in 3 ways again.

i.e.,

3	2 or 3 or 4	2	2

The number of ways is 3.

Now, we fix first, second and fourth places just as previous cases and we obtain the same result.

∴ The number of ways is 3 so; the total number of ways is 9.

Similarly this can do by fixing the numbers as 3 and 4 (instead of 2) and thereby we obtain the 9 ways in each case.

∴ The number of numbers greater than 3000 starting with 3 is 27.

Similarly by taking 4 as the first digit and applying the same process, we get 27 numbers.

∴ The total number of numbers that are greater than 3000 is 27 + 27 = 54

But, 3222 and 4222 is not possible as there are only two 2's (given), 3333 is also not possible as there are only three 3's (given)

∴ The total number of numbers that are greater than 3000 is $54 - 3 = 51$.