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A Traffic-Grooming Algorithm for Wavelength-Routed Optical Networks

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We consider the problem of grooming in all-optical networks to maximize traffic. We present an integer-programming formulation while constraining the number of optical transceivers at each node, the link load, and the capacity of each lightpath. Based on the structural properties of the problem, we develop a heuristic based on a column-generation technique. The algorithm is easy to implement, requires a modest amount of CPU time, and provides high-quality solutions. To ascertain the quality of solutions obtained by our algorithm, we present an alternative formulation that allows us to develop an upper bound using a Lagrangian-relaxation technique. An extensive computational study is presented.

Key words: optical networks; traffic grooming; wavelength division multiplexing; column generation; Lagrangian relaxation
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1. Introduction

Optical-fiber networks will play a major role in telecommunications (Cinkler 2003), prompting development of sophisticated techniques to allow increasing traffic on the same physical network. In the early phases of this evolution, wavelength division multiplexing (WDM) allowed multiple wavelengths of light to be transmitted on the same optical fiber. WDM networks use optical add drop multiplexes (OADMs) at the nodes to facilitate adding and dropping signals of specific wavelengths. Due to technological constraints, the OADMs performed these add/drop operations in the electrical domain (i.e., optical signals were converted to electrical signals before adding/dropping traffic). For such networks, each node requires expensive optical transmitters and receivers (also referred to as transponders/transceivers), motivating network designers to choose designs that minimize the number of transponders.

Current WDM systems use optical cross-connects (OXCs) in addition to OADMs at each node, and increasingly use topologies based on meshes, not rings (Zhu and Mukherjee 2002). OXCs switches direct specific wavelengths of light from input ports to output ports entirely in the optical domain (i.e., without converting light signals to electrical signals). These are called wavelength-routed switches or wavelength routers (Dutta and Rouskas 2002); the networks are called all-optical networks. The essential mechanism of transport in all-optical networks is a *lightpath*, which is a communication channel established between two nodes in the network of OXCs that uses the same wavelength of light for its entire path (Dutta and Rouskas 2002). Each such wavelength of light is capable of carrying up to 10 Gbps, while 40 Gbps systems are coming (Dutta and Rouskas 2002). To use such a high data-carrying capacity efficiently, a number of lower-rate traffic streams must be multiplexed onto a single lightpath. This multiplexing process gives rise to traffic-grooming techniques that combine lower-speed traffic requests into available wavelengths to meet network-design goals such as cost minimization or throughput maximization (Modiano and Lin 2001, Zhu and Mukherjee 2002). We examine the objective of throughput maximization for such all-optical networks.

Traffic demands (or connection requests) for such networks can be characterized by a source (origin), a destination (termination point), and a granularity (bandwidth requested). Each demand needs to be routed on a physical path in the network. All nodes other than the source and destination of each path are called intermediate nodes. The bandwidth of each demand is typically one of OC-48, OC-12, OC-3, or OC-1. An OC-1 is the lowest capacity optical communication channel, which has a capacity of 51.84 Mbps. An OC-n can carry 51.84n Mbps. Each physical path corresponds to one single lightpath (for *single-hop* networks) or could be a series of lightpaths (for *multi-hop* networks). A major advantage of single-hop networks is that the signal travels in an all-optical medium and a single wavelength needs to be allocated to the connection. In multi-hop networks, a single connection between two nodes can sequentially use multiple lightpaths, necessitating the ability to change/convert the wavelength of the transmission at network nodes. Wavelength conversion can be accomplished in the electrical domain but then the advantages of all-optical transmissions are lost. All-optical wavelength converters are available but are prohibitively expensive (Chu et al. 2003). We consider only single-hop networks in which wavelength conversion is not allowed.

We consider the problem of traffic grooming and routing in optical mesh networks. The objective is throughput maximization subject to constraints on the maximum link load and the number of transceivers available at each node. We develop a heuristic based on column generation and verify the quality of the heuristic solution through an upper bound computed using Lagrangian relaxation. We begin by reviewing the relevant literature in Section 2. Section 3 formalizes the notation, defines the problem, and illustrates it using a simple example. Section 4 presents an integer-programming formulation used as a basis for developing a heuristic. Section 5 describes our heuristic based on column generation. In Section 6, we develop a Lagrangian-relaxation-based approach for attaining a good upper bound on the objective. Section 7 reports the results of an extensive computational study and discusses the performance of our solution. Section 8 provides future research directions.

2. Literature Review

The literature has focused on many objectives (throughput, blocking probability, number of wavelengths, route distance, cost, etc., Chiu and Modiano 2000, Gerstel et al. 2000, Yoon 2001, Zhu and Mukherjee 2002). We seek to minimize traffic so restrict our literature review accordingly.

Zhu and Mukherjee (2002) provide models and solutions for grooming in all-optical networks to maximize traffic, considering both single and multi-hop all-optical networks. They provide a greedy heuristic without evaluating its quality, restricted to small networks of five nodes. While we focus only on single-hop networks, our heuristic is different in that it provides verifiably high-quality solutions for large networks.

Zhu et al. (2003) propose a generic network model for traffic grooming in heterogeneous mesh networks, which can be adapted for various objectives by using different grooming policies such as the number of transceivers, the number of wavelengths, and the wavelength-conversion capability. They also address partial wavelength conversion capabilities and different grooming capabilities at each node. They propose an integrated grooming procedure and traffic-selection scheme where connection requests are routed one request at a time. For static grooming (i.e., when all the traffic demands are known in advance), such a technique will be influenced by the order in which requests are routed. They propose two traffic-request-selection schemes. The least-cost-first scheme chooses the most cost-effective traffic request under the current network state and routes it, defining the cost of a traffic request as the weight of the shortest path for routing it on the corresponding auxiliary network, divided by the amount of the traffic (the granularity multiplied by the units of the traffic). The maximum-utilization-first scheme selects the connection with the highest utilization (total amount of the request divided by the number of hops from the source to the destination).

Hu and Leida (2002) consider traffic grooming with traffic routing and wavelength assignment (GRWA) to minimize the total number of transponders required via an integer program (IP). They decompose the problem into two smaller problems—traffic grooming (GR) and wavelength assignment (WA). A relaxed IP of GR is solved assuming that the lightpaths and their routes in the physical layer are given. For a given solution to GR, WA is solved using sequential node-coloring.

Lee et al. (2002) consider the routing and wavelength assignment to minimize the number of wavelengths used, via column generation. However, wavelength minimization is not critical for two reasons. First, using additional wavelengths has been found to increase the overall network costs only marginally (Zang et al. 2000). Second, wavelength capacity of optical networks has dramatically increased.

Mukherjee (1997) presents a multi-commodity flow formulation to minimize the link load in GRWA, via randomized rounding to route the lightpaths, relying on the assumption that there can be at most one lightpath for any source-destination pair. Limits on the number of transceivers at each node are not considered. Ozdalgar and Bertsekas (2000) provide an algorithm for routing and wavelength assignment (RWA) in optical networks with full wavelength conversion, addressing the routing and wavelength-assignment problem jointly. Their IP, based on multi-commodity network flows, minimizes the link load and thus the blocking probability. By adding a penalty function to the objective, they also address the problem of infeasibility of the formulation when some nodes have only a sparse wavelength-conversion capability.

3. Problem Definition

Consider a network $G = (N, A)$, where N is the set of nodes that represents wavelength-routed switches, and A is the set of arcs that represents directed fiber links between the nodes. We make the following assumptions:

1. Each arc in G represents a single fiber link between the corresponding pair of nodes in a mesh topology.
2. If arc $(i, j) \in A$, then arc $(j, i) \in A$.
3. Each fiber link can carry up to $\mathscr{C}$ wavelengths (lightpaths), the *maximum link load* allowed in the network.
4. There is no wavelength-conversion capability in the wavelength-routed switches at the nodes of the network.
5. Each node contains a pre-specified number of transmitters and receivers that are tunable to any wavelength on the fiber.
6. We are given a set of connection requests where each such request is represented by a tuple, $[(i, j), q]$. Here $i, j \in N$ are, respectively, the source and destination of the connection request, and $q \in Q = \{1, 3, 12, 48\}$ is the type of connection request. If the connection type is q, the bandwidth requirement for the connection is OC-q. Let $o(q)$ be the bandwidth requirement of connection type $q \in Q$ in multiples of OC-1 (i.e., 51.84 Mbps). We assume that a connection request between two nodes is indivisible, i.e., it must be routed on a single lightpath and cannot be divided into several slower connection requests that are routed separately.
7. Each wavelength-routed switch has unlimited capacity to multiplex/demultiplex channels. This implies that any number of slower connection requests of type $q \in Q$ can be multiplexed (respectively, demultiplexed) to (respectively, from) a lightpath originating (respectively, terminating) at that node. Each lightpath, however, has a finite capacity $\mathscr{M}$; we assume a lightpath capacity of OC-48, i.e., 48×51.84 Mbps or 2.5 Gbps.
8. Each communicating pair of nodes (i, j) is a specific commodity. Let R denote the set of distinct commodities (or, equivalently, the set of distinct communicating pairs of nodes). For each such commodity, there can be up to $|Q|$ different kinds of connection requests. We let $n_q^r \in \mathscr{Z}^+$ be the number of connection requests of bandwidth OC-q, $q \in Q$, for commodity $r \in R$. A lightpath is, therefore, a path for commodity $r \in R$ that uses a particular wavelength for transmission.

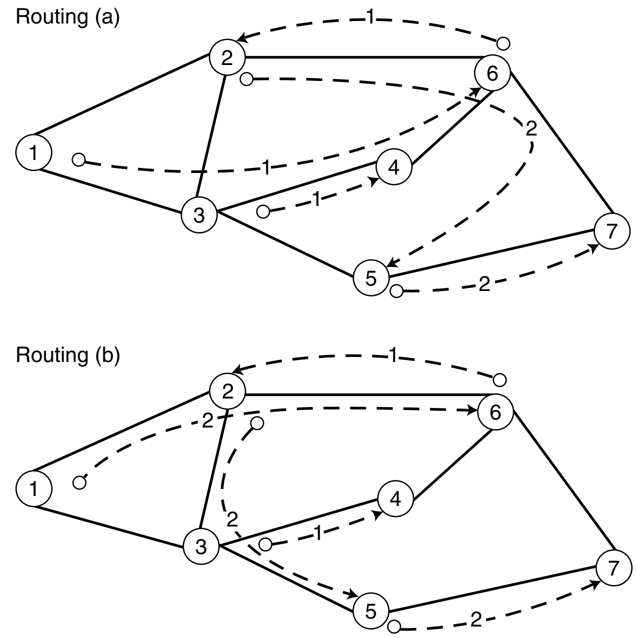

Figure 1 A Seven-Node Network with Two Transmitters and Two Receivers at Each Node

We seek to assign a set of lightpaths to each commodity r such that the bandwidth of communication requests assigned to these lightpaths is maximized. For example, in Figure 1 the traffic-routing problem is on a mesh network with 7 nodes and a link capacity of two lightpaths in each direction. Each node consists of two transmitters and receivers. The requested traffic connections are in Table 1.

Figures 1(a) and 1(b) indicate (by dashed lines) two feasible routings of lightpaths. The number corresponding to each dashed line is the number of lightpaths established along that line. The routing in Figure 1(a) allocates a total of 7 connection requests. Since the capacity of link $(3, 4)$ is two, this routing is able to satisfy only one connection request between nodes 1 and 6. The routing shown in Figure 1(b) accommodates all connection requests.

In the next section, we show that the problem is NP-hard and present an IP as a basis for a heuristic.

Table 1 Traffic Connections Required Between Pairs of Nodes for the Network in Figure 1

Origin	Destination	No. of connections requested in number of lightpaths
1	6	2
2	5	2
3	4	1
5	7	2
6	2	1

4. A Path-Based Formulation

In this section, we provide a path-based formulation for the traffic-grooming problem. Let d_r denote the aggregate demand for commodity $r \in R$. Thus, $d_r = \sum_{q \in Q} o(q) n_q^r$. d_r represents the total bandwidth requirement for commodity $r \in R$ in multiples of OC-1. For example, suppose a commodity $r \in R$ has the following demand requests: 15 OC-1 connections, 2 OC-3 connections, 1 OC-12 connection, and 1 OC-48 connection. Then, the aggregate demand for commodity r is $d_r = (1 \times 15) + (3 \times 2) + (12 \times 1) + (48 \times 1) = 81$. The number of lightpaths required to satisfy demand $d_r = 81$ fully is $\lceil d_r / \mathcal{M} \rceil = \lceil 81/48 \rceil = 2$, where $\mathcal{M} = 48$ is the capacity of a lightpath. We need the following additional notation:

$P_r =$ the index set of all lightpaths for commodity $r \in R$. Two lightpaths could have identical physical paths, in which case they require distinct wavelengths.

$$\delta_{mn}^{rp} = \begin{cases} 1 & \text{if link } (m,n) \text{ is on path } p \\ & \text{of commodity } r,\ (m,n) \in A; \\ 0 & \text{otherwise.} \end{cases}$$

$TT_u =$ the number of transmitters at node $u \in N$.

$RR_v =$ the number of receivers at node $v \in N$.

$R_u^o =$ the set of commodities originating at node $u \in N$.

$R_v^d =$ the set of commodities terminating at node $v \in N$.

$f_{rp} =$ flow on path $p \in P_r$ for commodity $r \in R$.

$$X_{rp} = \begin{cases} 1 & \text{if path } p \in P_r \text{ for commodity} \\ & r \in R \text{ is chosen;} \\ 0 & \text{otherwise.} \end{cases}$$

Problem Groom_Path:

$$\max_{f_{rp}, X_{rp}} \sum_{r \in R} \sum_{p=1}^{|P_r|} f_{rp}$$

$$\text{subject to: } \sum_{r \in R} \sum_{p=1}^{|P_r|} \delta_{mn}^{rp} X_{rp} \le \mathscr{C}, \quad \forall (m,n) \in A \tag{1}$$

$$f_{rp} \le \mathcal{M} X_{rp}, \quad \forall r \in R, \forall p \in P_r \tag{2}$$

$$\sum_{p=1}^{|P_r|} f_{rp} \le d_r, \quad \forall r \in R \tag{3}$$

$$\sum_{r \in R_u^o} \sum_{p=1}^{|P_r|} X_{rp} \le TT_u, \quad \forall u \in N \tag{4}$$

$$\sum_{r \in R_v^d} \sum_{p=1}^{|P_r|} X_{rp} \le RR_v, \quad \forall v \in N \tag{5}$$

$$X_{rp} \in \{0,1\}, \quad \forall i \in R, \forall p \in P_r \tag{6}$$

$$f_{rp} \in \mathcal{Z}^+, \quad \forall r \in R, \forall p \in P_r. \tag{7}$$

The objective function maximizes the total traffic flow over all commodities. Constraints (1) ensure that the number of lightpaths supported by each link is limited to $\mathscr{C}$. Constraints (2) limit the flow supported by each lightpath to the capacity of that lightpath. Constraints (3) restrict the total flow of a commodity to be at most the total aggregated demand for that commodity. Constraints (4) (respectively, (5)) limit the total number of lightpaths generated (respectively, received) by a node to the total number of transmitters (respectively, receivers) at that node.

Next, we observe two properties of Groom_Path that enable us to solve the problem more efficiently. Proofs are in the Online Supplement to this paper on the journal's website.

Property 1. *Given a solution to Groom_Path, where the total demand routed for commodity r is $\bar{d}_r = \sum_p f_{rp}$, there exists a feasible allocation of the original connection requests $[(i,j), q]$ (where the pair (i,j) corresponds to commodity r) such that the total allocation equals $\bar{d}_r$.*

The next property allows us to simplify the formulation and solve it more efficiently.

Property 2. *If the link capacities and the demands are integer valued, there always exists an optimal solution in which the flow variables f_{rp}, $p \in P_r$, $r \in R$ are integral. Consequently, we can relax the integrality constraints $f_{rp} \in \mathcal{Z}^+$ to $f_{rp} \ge 0$.*

A proof of the following result is in the Online Supplement.

Theorem 1. *The recognition version of Groom_Path is NP-complete in the strong sense.*

5. A Price-and-Branch Algorithm

Performance of IP solvers on the formulation of Section 4 is unsatisfactory. Table 2 summarizes the results of nine small instances we attempted to solve within one hour using CPLEX 8.1 on a Pentium IV (2.4 GHz, 512 MB RAM). We used a simple depth-first search to find all the paths for each commodity.

Typically, WDM-based all-optical networks are sparse when they operate over a wide geographical area. Given this, methods have been proposed to emulate their topology. The general principle is to distribute vertices at random locations in a plane and then add edges between pairs of vertices based on a probability distribution. For Table 2, the networks were generated as in Zegura et al. (1996) so that the probability of an edge decreases exponentially with the distance between the two vertices.

Columns 7 and 8 in Table 2 give the optimal solution of Groom_Path and its LP relaxation, respectively. Within the time limit, CPLEX 8.1 could solve to optimality, or obtain an integer feasible solution

Table 2 Results for One-Hour Runs Using CPLEX 8.1

Problem instance	Network size		Problem size		Results	Objective value			
	Nodes	Links	Constraints	Variables		Groom_Path	LP	Groom_Arc	FSA
1	6	7	1,890	6,210	Optimal	806	913	806	792
2	7	13	1,921	7,140	Optimal	1,031	1,206	1,031	968
3	8	11	2,043	8,903	Optimal	1,586	1,781	1,586	1,507
4	10	16	14,520	60,001	Feasible IP solution	2,392	2,914	—	2,266
5	12	22	18,323	65,731	No IP solution	—	5,312	—	4,126
6	14	29	18,920	66,391	No IP solution	—	6,881	—	5,062
7	16	33	19,034	78,181	No IP or LP solution	—	—	—	9,143
8	18	41	20,193	93,858	No IP or LP solution	—	—	—	15,214
9	20	56	21,284	249,231	No IP or LP solution	—	—	—	18,006

for, the instances with at most 10 nodes. For bigger networks (more than 14 nodes), even the LP relaxation of the problem could not be solved within an hour. With an increase in network size, the number of paths—and, hence, the number of binary variables—generated becomes prohibitively large. For example, the network with 16 nodes has a total of 37,821 directed paths. The last two columns in Table 2 indicate, respectively, the solution from an alternative formulation and a proposed feasible solution algorithm (FSA) introduced below.

Given the results in Table 2, our approach is to develop an efficient heuristic that can be used to get near-optimal solutions in reasonable time. Since the primary difficulty in solving Groom_Path is that of choosing from among an exponential (in the number of nodes) number of possible paths to route the demands, we use a well-known technique, *column generation*, to generate an optimal subset of paths efficiently. Section 5.1 describes how to price out and choose profitable paths as possible candidates to support the lightpaths. Following this, we describe the column generation and a heuristic algorithm to obtain solutions to Groom_Path in Sections 5.2 and 5.3.

5.1. The Pricing Procedure

Let Groom_PathR refer to the LP relaxation of Groom_Path. To formulate the dual of Groom_PathR, we associate dual multipliers λ_{mn}, μ_p^r, ν_r, ξ_u^o, and ϕ_v^d, respectively, with the link-capacity constraints (1), the lightpath-capacity constraints (2), the commodity-demand constraints (3), and the two transceiver constraints (4) and (5). Let Groom_PathRD refer to the dual problem of Groom_PathR. Thus, Groom_PathRD has the following constraints:

$$\mu_p^r+\nu_r\ge 1 \quad \text{and} \quad \sum_{\forall (m,n)\in A}\lambda_{mn}\delta_{mn}^{rp}-\mathcal{M}\mu_p^r+\xi_{s_r}^o+\phi_{t_r}^d\ge 0,$$
$$\forall p\in P_r,\ \forall r\in R$$
$$\lambda_{mn},\ \mu_p^r,\ \nu_r,\ \xi_u^o,\ \phi_v^d\ge 0,$$

where s_r and t_r are the origin and destination, respectively, of path $p\in P_r$.

We use the following iterative procedure to solve Groom_PathR. First, a restricted version of Groom_PathR, referred to as Groom_PathRR, is solved by choosing a subset of paths $\Psi=\bigcup_r \Psi_r$, where Ψ_r is a subset of paths chosen for each commodity $r\in R$. To initiate the iterative process, let Ψ be the set of shortest paths between each pair of nodes (i.e., for each commodity). The set Ψ is updated after each iteration.

Given an optimal solution to Groom_PathRR(Ψ), we price out the nonbasic columns as follows. A path $p\in P_r$ for commodity $r\in R$ is potentially profitable if it satisfies

$$1-\nu_r>\mu_p^r,\quad p\in P_r,\ r\in R \tag{8}$$

$$\mathcal{M}\mu_p^r-\xi_{s_r}^o-\phi_{t_r}^d>\sum_{\forall (m,n)\in A}\lambda_{mn}\delta_{mn}^{rp},$$
$$p\in P_r,\ r\in R,\ s_r=orig(r),\ t_r=dest(r), \tag{9}$$

where s_r and t_r are the origin and destination nodes, respectively, for commodity $r\in R$. The set of paths satisfying (10) below is a superset of that satisfying (8) and (9).

$$\mathcal{M}(1-\nu_r)-\xi_{s_r}^o-\phi_{t_r}^d>\sum_{\forall (m,n)\in A}\lambda_{mn}\delta_{mn}^{rp},$$
$$p\in P_r,\ r\in R,\ s_r=orig(r),\ t_r=dest(r) \tag{10}$$

In other words, a potentially profitable path $p\in P_r$ for commodity $r\in R$ satisfies

$$\zeta(r)>\mathcal{L}(r,p),\quad p\in P_r,\ r\in R, \tag{11}$$

where $\zeta(r)=\mathcal{M}(1-\nu_r)-\xi_{s_r}^o-\phi_{t_r}^d$, and $\mathcal{L}(r,p)=\sum_{\forall (m,n)\in A}\lambda_{mn}\delta_{mn}^{rp}$ is the length of path $p\in P_r$ with arc weights λ_{mn}.

It follows that if none of the shortest paths (with link weights λ_{mn}) satisfy (11), there are no profitable paths left to be added to Ψ. An optimal solution to Groom_PathRR(Ψ) is, therefore, an optimal solution to Groom_PathR. We formally state the column-generation procedure below.

5.2. A Column-Generation Procedure

Step 1. Find the aggregate demand d_r for each commodity $r \in R$.

Step 2. Solve the restricted version Groom_PathRR(Ψ), and let β_Ψ be an optimal basis.

Step 3. Check optimality of β_Ψ for Groom_PathR using (11). If β_Ψ is an optimal basis for Groom_PathR, go to Step 6.

Step 4. Identify profitable paths for each commodity using (8)–(9), and add them to the set of paths Ψ.

Step 5. Solve the updated Groom_PathRR(Ψ) to obtain an improved basis β_Ψ. Go to Step 3.

Step 6. Output an optimal solution to Groom_PathRR(Ψ) and the corresponding set of columns Ψ. Stop.

If the optimal solution (to Groom_PathR) returned by the procedure is integral, then it is also an optimal solution to Groom_Path. In general, however, this is not the case. We next describe a heuristic to obtain near-optimal integer feasible solutions to Groom_Path.

5.3. Feasible Solution Algorithm

The solution to the LP relaxation Groom_PathR (obtained from the above column generation) typically makes fractional use of the capacity of the paths in Ψ and is, therefore, infeasible for Groom_Path. In other words, some or all of the variables X_{rp}, $p \in P_r$, $r \in R$ are fractional in an optimal solution to Groom_PathR. Nevertheless, this solution provides a "good" set of paths Ψ. Let Groom_Path(Ψ) denote the restricted version of Groom_Path where the the set of available paths is restricted to Ψ. Since the cardinality of Ψ is a small fraction of the number of possible paths, Groom_Path(Ψ) should be solved quickly. To generate an integer feasible solution, we execute the following two steps after the column generation.

Step 7. Use the set of paths Ψ obtained in Step 6 to formulate Groom_Path(Ψ).

Step 8. Solve Groom_Path(Ψ) to optimality.

Note that this heuristic allocates a set of lightpaths to each commodity $r \in R$ based on its aggregate demand d_r; from our discussion in Section 4 (Property 1), it follows that a feasible disaggregated solution for the different types of connection requests is easy to obtain.

In general, one way to gauge the quality of the integer feasible solution obtained by solving Groom_Path(Ψ) is to measure its gap with the optimal solution to Groom_PathR. However, this gap may not necessarily be a good indicator due to the presence of an integrality gap between Groom_Path and its LP relaxation Groom_PathR. In other words, the optimal solution of the LP relaxation Groom_PathR may not necessarily be a good upper bound for Groom_Path. Motivated by this issue, we next develop an alternative formulation to Groom_Path using a Lagrangian relaxation to obtain a tighter upper bound.

6. A Lagrangian-Based Bounding Procedure

We adapt an alternative formulation for Groom_Path, referred to as Problem Groom_Arc, from Zhu and Mukheerjee (2002). Theorem 2 below proves the equivalence of the two formulations.

Decision Variables:

$$S_{ij}^{qt} = \begin{cases} 1 & \text{if the } t\text{th request for bandwidth } q \in Q \\ & \text{between } (i,j) \text{ is successfully routed;} \\ 0 & \text{otherwise.} \end{cases}$$

$t = 1, 2 \ldots, n_q^r$, $i = origin(r)$, $j = destination(r)$, $r \in R$.

$V_{ij} =$ the number of lightpaths from node i to j; $i = origin(r)$, $j = destination(r)$, $r \in R$.

$P_{mk}^{ij} =$ the number of lightpaths between nodes (i,j) routed through fiber link $(m,k) \in A$; $i = origin(r)$, $j = destination(r)$, $r \in R$.

Problem Groom_Arc:

$$\max_{V_{ij}, P_{mk}^{ij}, S_{ij}^{qt}} \sum_{q \in Q, i, j, t} o(q) S_{ij}^{qt}$$

subject to:

$$\sum_{j \in N} V_{ij} \le TT_i, \quad \forall i \tag{12}$$

$$\sum_{i \in N} V_{ij} \le RR_j, \quad \forall j \tag{13}$$

$$\sum_{m \in N} P_{mk}^{ij} = \sum_{n \in N} P_{kn}^{ij}, \quad \text{if } k \ne i, j \; \forall i, j, k \; (m,k), (k,n) \in A \tag{14}$$

$$\sum_{m \in N} P_{mi}^{ij} = 0, \quad \forall i, j \; (m,i) \in A \tag{15}$$

$$\sum_{n \in N} P_{jn}^{ij} = 0, \quad \forall i, j \; (j,n) \in A \tag{16}$$

$$\sum_{n \in N} P_{in}^{ij} = V_{ij}, \quad \forall i, j \; (i,n) \in A \tag{17}$$

$$\sum_{m \in N} P_{mj}^{ij} = V_{ij}, \quad \forall i, j \; (m,j) \in A \tag{18}$$

$$\sum_{i, j \in N} P_{mn}^{ij} \le \mathscr{C}, \quad \forall m, n \; (m,n) \in A \tag{19}$$

$$\sum_{q \in Q, t} o(q) S_{ij}^{qt} \le V_{ij} \times \mathscr{M}, \quad \forall i, j \tag{20}$$

$$V_{ij} \in \mathscr{Z}^+ \tag{21}$$

$$P_{mn}^{ij} \in \mathscr{Z}^+ \tag{22}$$

$$S_{ij}^{qt} \in \{0, 1\}. \tag{23}$$

The objective function maximizes the total bandwidth (traffic) routed. Constraints (12) limit the number of lightpaths beginning at a node to the number of transmitters at that node. Similarly, constraints (13) limit the number of lightpaths ending at a node to the number of receivers at that node. Constraints (14) are the lightpath-continuity constraints: for an intermediate node k, the number of lightpaths between nodes i and j entering and leaving node k are equal. Constraints (17) (respectively, (18)) limit the number of lightpaths originating (respectively, ending) at node i (respectively, j) to V_{ij}. Constraints (19) limit the number of lightpaths passing through link $(m, n) \in A$ to its maximum link load $\mathscr{C}$. Constraints (20) limit the bandwidth allocated to a node pair (i, j) to the capacity of the total lightpaths generated between nodes i and j.

Theorem 2. *IP formulations Groom_Path and Groom_Arc are equivalent.*

Proof. We show that an optimal solution to Groom_Arc can be converted into a feasible solution to Groom_Path of the same objective-function value and vice versa. Let the values of the variables S_{ij}^{qt} (respectively, V_{ij} and P_{mn}^{ij}) in an optimal solution to Groom_Arc be denoted by $\bar{S}_{ij}^{qt}$ (respectively, $\bar{V}_{ij}$ and $\bar{P}_{mn}^{ij}$). Note that $\bar{V}_{ij}$ is the number of lightpaths allocated to node pair (i, j). The variables $\bar{P}_{mn}^{ij}$ identify the physical paths used for routing the lightpaths. We can obtain a feasible solution to Groom_Path by allocating the same lightpath routing: for a commodity $r \in R$ corresponding to node pair (i, j), we set $X_{rp} = 1$ in Groom_Path for only those paths p (defined by the variables $\bar{P}_{mn}^{ij}$) used in the optimal solution to Groom_Arc. This solution is clearly feasible for Groom_Path because the values $\bar{V}_{ij}$ are feasible under the transceiver constraints, and the values $\bar{P}_{mn}^{ij}$ are feasible under the maximum-link-load constraints. In the resulting solution for Groom_Path, the value of the objective function component corresponding to commodity r is thus $\sum_p \bar{f}_{rp} = \sum_{q,t} o(q)\bar{S}_{ij}^{qt}$. As discussed in Property 1, $\sum_p \bar{f}_{rp}$ can be easily decomposed into the original demand requests between node pair (i, j).

Similarly, an optimal solution to Groom_Path ($\tilde{f}_{ip}$) can be converted to a feasible solution of Groom_Arc ($\tilde{S}_{ij}^{qt}$) with $\sum_{q,t} o(q)\tilde{S}_{ij}^{qt} = \sum_p \tilde{f}_{rp}$, $\forall\, i, j \in N$. The result follows. □

6.1. Lagrangian Relaxation

We relax constraints (13) and (19); after multiplying them, respectively, by nonnegative Lagrangian multipliers γ_j and α_{mn}, we include them in the objective function to obtain a Lagrangian relaxation Groom_ArcR.

Problem Groom_ArcR:

$$\max_{V_{ij},\, P_{mk}^{ij},\, S_{ij}^{qt}} \sum_{q,i,j,t} o(q)S_{ij}^{qt} + \sum_{m,n} \alpha_{mn}\left\{\mathscr{C} - \sum_{i,j} P_{mn}^{ij}\right\} + \sum_j \gamma_j\left\{RR_j - \sum_i V_{ij}\right\}$$

subject to: (12), (14)–(18), (20)–(23).

A significant advantage of Groom_ArcR is that it can be decomposed into separate problems; an individual problem in the decomposition corresponds to all commodities $r \in R$ starting at node $i \in N$.

Problem $(i, *)$:

$$\max_{V_{ij},\, P_{mk}^{ij},\, S_{ij}^{qt}} \sum_{q,j,t} o(q)S_{ij}^{qt} + \sum_{m,n,j} \alpha_{mn}\{-P_{mn}^{ij}\} + \sum_j \gamma_j\{-V_{ij}\}$$

subject to: $\sum_j V_{ij} \le TT_i$

$$\sum_m P_{mk}^{ij} = \sum_n P_{kn}^{ij}, \quad \text{if } k \ne i, j \ \forall\, j, k$$

$$\sum_m P_{mi}^{ij} = 0, \quad \sum_n P_{jn}^{ij} = 0, \quad \sum_n P_{in}^{ij} = V_{ij}, \quad \text{and}$$

$$\sum_m P_{mj}^{ij} = V_{ij} \quad \forall\, j$$

$$\sum_{q \in Q,\, t} o(q) \times S_{ij}^{qt} \le \mathscr{M} V_{ij}, \quad \forall\, j \tag{24}$$

$$S_{ij}^{qt} \in \{0, 1\}, \quad V_{ij} \in \mathscr{Z}^+, \quad P_{mn}^{ij} \in \mathscr{Z}^+.$$

The optimum solution of Groom_ArcR is the sum of $\sum_{m,n} \alpha_{mn}\mathscr{C} + \sum_j \gamma_j RR_j$ and the optimum solutions of Problems $(i, *)$, $i \in N$. We make two important observations for Problem $(i, *)$.

1. The LP relaxation is not integral, i.e., removing the integrality restrictions on S_{ij}^{qt}, V_{ij}, and P_{mn}^{ij} can lead to a fractional solution. This is easy to see: For any given realization of the variables S_{ij}^{qt}, the total demand routed between nodes i and j, $\sum_{q \in Q,\, t} o(q)S_{ij}^{qt}$, is typically not an exact multiple of the lightpath capacity $\mathscr{M}$. Consequently, the variables V_{ij} will be fractional due to constraints (24). An important consequence of the nonintegrality of the LP relaxation of Problem $(i, *)$, $i \in N$ is that, for some problem instance, the upper bound from solving the Lagrangian dual is strictly smaller than that obtained from solving the LP relaxation of Groom_Arc (Geoffrion 1974, Fisher 1985).

2. The integrality restrictions on P_{mn}^{ij} and S_{ij}^{qt} can be removed, i.e., we need only impose explicit integrality restrictions on V_{ij}. Integrality of V_{ij} implies integrality of P_{mn}^{ij}. Furthermore, Property 1 implies that integrality of V_{ij} guarantees at least one optimal solution in which all variables $S_{ij}^{qt} \in \{0, 1\}$.

The decomposition into separate subproblems, and the removal of explicit integrality restrictions on P_{mn}^{ij} and S_{ij}^{qt} makes computation of an optimal solution to Groom_ArcR straightforward. CPLEX 8.1 was able to solve Groom_ArcR in reasonable time.

For given values of the nonnegative Lagrangian multipliers γ_j and α_{mn}, the solution to Groom_ArcR is an upper bound to the solution of Groom_Arc. We employ a subgradient optimization algorithm to solve the Lagrangian dual. There are several approaches to

Table 3 Comparison of Heuristic, LP, and Lagrangian Solutions

Problem	Nodes	Transceivers	Link capacity	Heuristic	LP	Lagrangian
1	20	12	2	438,595	496,020	496,020
2	20	12	4	442,646	523,954	523,954
3	20	12	6	457,300	552,003	552,003
4	20	16	2	638,568	991,361	712,582
5	20	16	4	642,444	1,035,160	729,167
6	20	16	6	655,120	1,037,370	726,202
7	20	20	2	697,046	1,256,821	742,272
8	20	20	4	702,453	1,269,884	763,489
9	20	20	6	764,218	1,143,285	801,733

subgradient optimization; we used the one described in Fisher (1985). For a recent account of subgradient (and other nondifferentiable optimization) methods, see Crainic et al. (2001). Table 3 compares the heuristic solution, the solution of the LP relaxation of Groom_Arc, and the Lagrangian solution for nine sample instances of a network with 20 nodes.

7. Computational Experience

For all our experiments, we generated networks such that a path exists between each pair of nodes. Following Zhu and Mukherjee (2002), the bandwidth requirement for the connection requests was allowed to be any one of OC-1, OC-3, or OC-12. Thus, the type of each connection request is $q \in Q = \{1, 3, 12\}$. As suggested by Zhu and Mukherjee (2002), each OC-q demand was generated as a uniformly distributed random number between 0 and U_q where $U_1 = 16$, $U_3 = 8$, and $U_{12} = 2$. A typical demand matrix for OC-12 requests for a network with 10 nodes is in Table 4.

WDM-based all-optical networks operating over a wide geographical area are sparse, with in/out-degree values ranging from 2–5 (Ajmone et al. 2002, Banerjee and Mukherjee 1996, Jia 1998, Waxman 1988). Methods have been proposed to emulate the topology of such wide-area, sparse networks (Baroni and Bayvel 1997, Jia 1998, Jia et al. 2001, Waxman 1988). We use the

Table 4 A Typical Demand Matrix of OC-12 Requests for a Network with 10 Nodes

Node	0	1	2	3	4	5	6	7	8	9
0	0	0	0	2	0	1	2	1	0	0
1	1	0	2	0	2	0	1	2	1	2
2	1	1	0	1	0	1	2	1	1	1
3	1	2	2	0	1	1	1	1	0	0
4	2	2	0	1	0	1	0	2	2	0
5	1	1	2	1	0	0	0	1	1	2
6	2	1	0	2	0	0	0	0	0	0
7	1	0	1	0	2	0	2	0	0	1
8	1	0	0	2	0	2	0	0	0	1
9	1	1	1	0	0	0	2	2	0	0

procedure first developed in Waxman (1988) to generate a realistic network $G(N, A)$:

1. **Number of Nodes:** We considered $|N| = 20$, 40, 80, 100, and 150.
2. **Generation of Arcs:** We assumed that if two nodes u and v are adjacent, there exists a unidirectional fiber link from u to v and a separate unidirectional fiber link from v to u (Jia et al. 2002). Therefore, throughout the arc-generation process, whenever we add an arc (u, v) to the network, we also add the reverse arc (v, u).
   (i) To ensure that the network is connected, we first find a minimum spanning tree (using Euclidean distances as arc weights) and add the arcs corresponding to the tree.
   (ii) Arcs between the nodes are added until the average in-degree (or, equivalently, average out-degree since we always add bi-directional arcs) in the network reaches a specified maximum value. We considered average in-degree values of 2, 4, and 6, calling the associated networks sparse, medium, and dense, respectively.
3. **Transceivers:** We let the number of transceivers at each node be $0.6|N|$, $0.8|N|$, and $|N|$.
4. **Link Capacity:** We consider three values for the link capacity $\mathscr{C}$ of each fiber: 2, 4, and 6 lightpaths.

Thus, there are 135 problem settings and for each setting we generated 10 instances. The heuristic was coded in C++ (using the CPLEX 8.1 programming library) and run on a Pentium IV (2.4 GHz, 512 MB RAM) with Windows XP.

One cost component of an optical network is the number of unique wavelengths needed to accommodate demand (Ramaswami and Sivarajan 1995). Our procedure provides a set of routes that are feasible with respect to the maximum link load. To verify that the number of wavelengths needed is not inordinately large, we use a sequential network-coloring approach in Banerjee and Mukherjee (1996). Assigning wavelengths to a set of routes to minimize the number of distinct wavelengths needed is itself NP-hard (Garey and Johnson 1979). Hence, we use the network-coloring approach as a post-processing step once a good set of routes is obtained, providing us a measure $\mathscr{W}$ that is an upper bound on the maximum number of wavelengths needed for each solution provided by our heuristic.

7.1. Discussion

Tables S1–S5 in the Online Supplement summarize the computational results for our heuristic of Section 5.3 for our 1,350 instances. Each row in these tables refers to a summary of the 10 instances for the corresponding problem setting. The heuristic obtains good integer feasible solutions for large networks (up to 150 nodes). The average gap between our feasible solution and the upper bound (via the Lagrangian

Table 5 Summary of Tables S1–S5 in the Online Supplement

Number of nodes	Range of average gap	Average gap over all instances (%)
20	2.4%–6.4%	4.6
40	1.2%–8.8%	4.9
80	3.9%–9.2%	6.1
100	2.6%–10.1%	5.5
150	3.1%–8.7%	5.6
Overall average	—	5.3

relaxation of Section 6.1) over all instances is 5.3%; the ranges of the average gaps for different network sizes are in Table 5. The maximum gap is also modest, between 2.3% and 14.6%. The solution quality remains consistent with an increase in the number of transceivers at the nodes. For networks with 150 nodes, Figure 2 shows the change in the gap with an increase in the number of transceivers in each node. The average gap does not change appreciably with an increase in the number of transceivers. In some cases, we observed a modest increase in the gap primarily due to the inferior quality of the upper bound; for these instances, the optimal values of the LP relaxation and the Lagrangian relaxation were very close. Figure 3 illustrates the change in the percentage of total traffic routed with the number of transceivers at the nodes. The increase in traffic is more pronounced when the number of transceivers goes from 90 to 120, than from 120 to 150. This is because most of the traffic requests are fully routed, and most links in the network are saturated by the time the number of transceivers equals 120. Denser networks route more traffic than do less dense networks, as expected since denser networks imply more paths available to support the lightpaths.

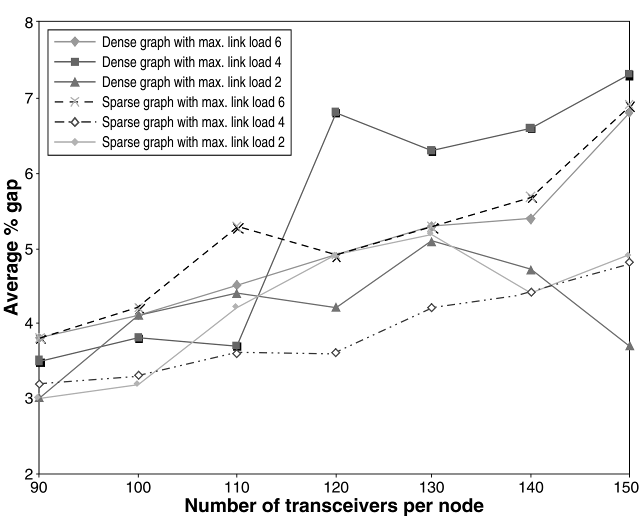

Figure 2 **Average Gap Between the Heuristic Solution and the Lagrangian Upper Bound, for $|N| = 150$**

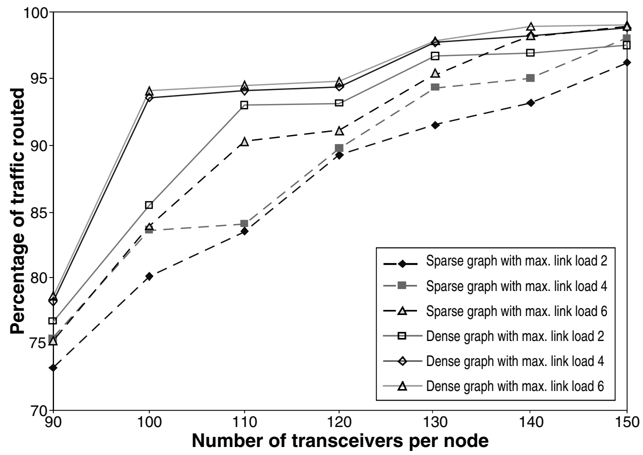

Figure 3 **Average Percentage of Demand Routed with Number of Transceivers, for $|N| = 150$**

The sequential wavelength-assignment technique indicates that the number of unique wavelengths ($\mathscr{W}$) needed for the routes is not inordinately large, typically less than 1.5 times the maximum link load. Figure 4 plots the average number of wavelengths required against the maximum link load. As expected, the number of distinct wavelengths required increases with the maximum link load. Dense networks require more wavelengths than do sparse networks for the same maximum link load because dense networks typically generate more lightpaths, thus increasing the possibility of more lightpaths sharing the same link; this requires more wavelengths.

Computationally, the heuristic is not very expensive. Computation times average 5 to 6 minutes for networks with 150 nodes, and much less (2 to 3 minutes) for networks with 100 nodes. Thus, the heuristic can also be used to benchmark efficiently the performance of greedy procedures such as Zhu et al. (2003).

We also applied our heuristic to two typical real-world network topologies (25 and 79 nodes) provided

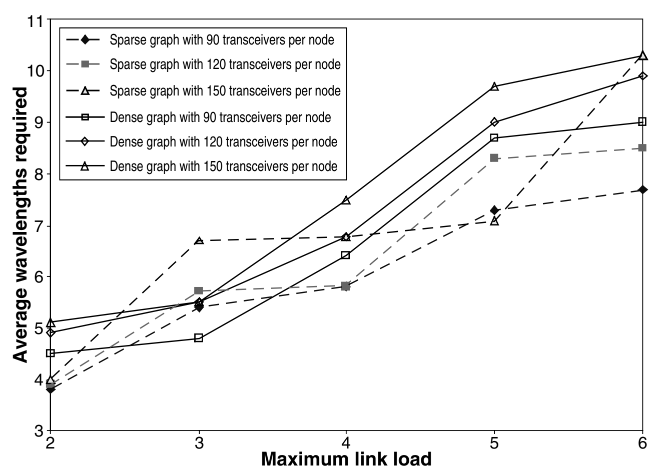

Figure 4 **Average Wavelength Requirements, for $|N| = 150$**

by a large telecommunication equipment manufacturer. We generated traffic data as before by varying the maximum degree, the number of transceivers, and the link capacity. For each parameter setting, we generated 10 traffic matrices; results are in Table S6 in the Online Supplement. The average gap between the feasible solution and the Lagrangian upper bound is between 2.6% and 8.0%; the average gap over all instances is 5.5%. The computation times and results for the number of distinct wavelengths required are also similar to those for the randomly generated instances.

8. Extensions

Our heuristic for approximating a traffic maximizing set of routes is based on a price-and-bound algorithm to solve the LP relaxation, followed by a branch-and-bound algorithm using only the columns generated at the root node. It might be possible to improve our heuristic with a branch-and-price algorithm that generates columns at every node of the tree. However, the improved solutions may be obtained at the expense of increased computing time. Another interesting extension could be routing and wavelength assignment together in a single formulation. Since we consider only single-hop networks, traffic routing using multi-hops is another extension.

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References

Ajmone, M., E. Leonardi, M. Mellia, A. Nucci. 2002. Design of local topologies in wavelength routed IP networks. *J. Photonic Network Comm.* **4** 423–442.

Banerjee, D., B. Mukherjee. 1996. A practical approach for routing and wavelength assignment in large wavelength-routed optical networks. *IEEE J. Selected Areas Comm.* **14** 903–908.

Baroni, S., P. Bayvel. 1997. Wavelength requirements in arbitrarily connected wavelength-routed optical networks. *J. Lightwave Tech.* **15** 242–251.

Chiu, A. L., E. H. Modiano. 2000. Traffic grooming algorithms for reducing electronic multiplexing costs in WDM ring networks. *J. Lightwave Tech.* **16** 2–12.

Chu, X., L. Bo, I. Chlamtac. 2003. Wavelength converter placement under different RWA algorithms in wavelength-routed all-optical networks. *IEEE Trans. Comm.* **51** 607–617.

Cinkler, T. 2003. Traffic and λ grooming. *IEEE Network* **17** 16–21.

Crainic, T. G., A. Frangioni, B. Gendron. 2001. Bundle-based relaxation methods for multicommodity capacitated fixed charge network design. *Discrete Appl. Math.* **112** 73–99.

Dutta, R., G. N. Rouskas. 2002. Traffic grooming in WDM networks: Past and future. *IEEE Network* **16** 46–56.

Fisher, M. L. 1985. An application oriented guide to Lagrangian relaxation. *Interfaces* **15** 1–18.

Garey, M. R., D. S. Johnson. 1979. *Computers and Intractability: A Guide to the Theory of NP-Completeness*. W. H. Freeman, San Francisco, CA.

Geoffrion, A. 1974. Lagrangian relaxation for integer programming. *Math. Programming Stud.* **2** 82–114.

Gerstel, O., R. Ramaswami, G. H. Sasaki. 2000. Cost-effective traffic grooming in WDM rings. *IEEE/ACM Trans. Networking* **8** 618–630.

Hu, J. Q., B. Leida. 2002. Traffic grooming, routing, and wavelength assignment in optical WDM mesh networks. *Sixth INFORMS Telecomm. Conf., Boca Raton, FL*, 156–164.

Jia, X. 1998. A distributed algorithm of delay-bounded multicast routing for multimedia applications in wide-area networks. *IEEE/ACM Trans. Networking* **6** 828–837.

Jia, X., X. Hu, L. Ruan, J. Sun. 2002. Multicast routing, load-balancing and wavelength assignment on a tree of rings. *IEEE Comm. Lett.* **6** 78–81.

Jia, X., D. Du, X. Hu, M. Lee, J. Gu. 2001. Optimization of wavelength assignment for QoS multicast in WDM networks. *IEEE Trans. Comm.* **49** 341–350.

Lee, K. L., K. C. Kang, T. Lee, S. Park. 2002. An optimization approach to routing and wavelength assignment in WDM all-optical mesh networks without wavelength conversion. *ETRI J.* **24** 131–141.

Modiano, E., P. J. Lin. 2001. Traffic grooming in WDM networks. *IEEE Comm. Magazine* **39** 124–129.

Mukherjee, B. 1997. *Optical Communication Networks*. McGraw-Hill, New York.

Ozdalgar, A. E., D. P. Bertsekas. 2000. Routing and wavelength assignment in optical networks. *IEEE/ACM Trans. Networking* **11** 633–644.

Ramaswami, R., K. N. Sivarajan. 1995. Routing and wavelength assignment in all-optical networks. *IEEE/ACM Trans. Networking* **3** 489–500.

Waxman, B. M. 1988. Routing of multipoint connections. *IEEE J. Selected Areas Comm.* **6** 1617–1622.

Yoon, M.-G. 2001. Traffic grooming and lightpath routing in WDM ring networks with hop-count constraints. *ICC 2001, IEEE Internat. Conf. Comm.* **3** 731–737.

Zang, H., J. P. Jue, B. Mukherjee. 2000. A review of routing and wavelength assignment approaches for wavelength-routed networks. *Optical Networks Magazine* **1** 47–59.

Zegura, E. W., K. L. Calvert, S. Bhattacharjee. 1996. How to model an internetwork. *Proc. 15th IEEE INFOCOM Annual Joint Conf.*, IEEE Comput. Comm. Societies, San Francisco, CA, 594–602.

Zhu, H., H. Zang, K. Zhu, B. Mukherjee. 2003. Novel generic graph model for traffic grooming in heterogeneous WDM mesh network. *IEEE/ACM Trans. Networking* **11** 432–447.

Zhu, K., B. Mukherjee. 2002. Traffic grooming in an optical WDM mesh network. *IEEE J. Selected Areas Comm.* **20** 122–133.