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Dynamics of a double mathematical pendulum with variable mass in dimensionless coordinates

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Abstract

The article presents an analysis of the dynamics of a double mathematical pendulum with variable mass. In the analyzed system, with time, under the influence of gravity, the mass of the upper member of pendulum decreases and the mass of the lower member of pendulum increases. The total mass of the system doesn't change. For the analysis introduced dimensionless time and dimensionless parameters, which allows the presentation of the equations of motion in dimensionless form. It has been shown that the change of mass in the system has a significant impact on the dynamics. The increase in mass of the lower member reduces the amplitude of vibration of the pendulum. The numerical calculations were performer in Mathematica package.

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Keywords: variable mass systems; double mathematical pendulum; chaos;

1. Introduction

The purpose of this paper is an attempt to describe the mathematical phenomenon of mass exchange between the members of the double pendulum, in dimensionless coordinates and its impact on the dynamics of the whole system.

The work has included within the scope motion study of a double mathematical pendulum (Fig. 1) for two sets of initial conditions. These sets are different from each other only different, predetermined angle of the lower member of the pendulum β. Other initial conditions of the system, such as: L - lenght of the upper, l - lenght of the lower member of the pendulum, M - mass of the upper, m - mass of the lower member of the pendulum, α - angle of the upper member of the pendulum, α' - generalized velocity of the upper, β' - generalized velocity of the lower member, λ - coefficient of mass transfer, for both simulations are identical. It is assumed that the total mass of the double pendulum is constant, however the mass flow occurs between members.

The article is a continuation of the considerations discussed in the work of [1-6].

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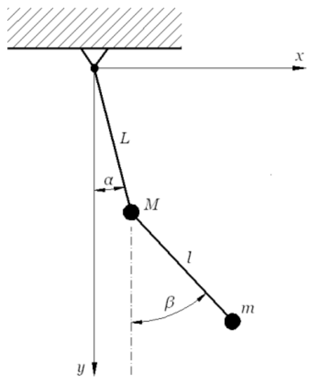

Fig. 1 Diagram of a double mathematical pendulum.

2. The dimensionless equations of motion

By introducing, on the basis of [7], to a system of nonlinear equations for the pendulum with variable mass, which is discussed in detail in the work [1-6], dimensionless parameters:

$$\varpi^2 = \frac{g}{L}; \quad \Omega^2 = \frac{g}{l}; \quad \chi = \frac{\omega^2}{\Omega^2} = \frac{l}{L}; \tag{1}$$

dimensionless time:

$$\tau = \varpi \cdot t \qquad \tau = \sqrt{\frac{g}{L}} \cdot t \tag{2}$$

$$t = \sqrt{\frac{L}{g}} \cdot \tau \qquad dt = \sqrt{\frac{L}{g}} \cdot d\tau$$

and introducing, additionally, time derivatives:

$$\frac{d}{dt}(\cdot) = \sqrt{\frac{L}{g}} \cdot \frac{d}{d\tau}(\cdot) \qquad \frac{d^2}{dt^2}(\cdot) = \frac{L}{g} \cdot \frac{d^2}{d\tau^2}(\cdot) \tag{3}$$

and dimensionless mass:

$$\mu = \frac{m}{M+m} = \frac{m}{M_0 + m_0} \tag{4}$$

$$m = \mu(M_0 + m_0) \tag{5}$$

$$\frac{dm}{dt} = (M_0 + m_0)\frac{d}{dt}\mu \tag{6}$$

$$\dot{m} = (M_0 + m_0)\sqrt{\frac{L}{g}}\frac{d}{d\tau}\mu \tag{7}$$

$$\frac{\dot{m}}{(M_0 + m_0)} = \frac{(M_0 + m_0)\sqrt{\frac{L}{g}}\frac{d}{d\tau}\mu}{(M_0 + m_0)} = \sqrt{\frac{L}{g}}\dot{\mu} \tag{8}$$

$$\frac{\dot{m}}{m} = \frac{(M_0 + m_0)\sqrt{\frac{L}{g}}\dot{\mu}}{(M_0 + m_0)\mu} = \sqrt{\frac{L}{g}}\frac{\dot{\mu}}{\mu} \tag{9}$$

we obtain finally, after several transformations, a **dimensionless form of nonlinear equations with variable mass**:

$$\begin{cases} \ddot{\alpha} + \chi\mu\cos(\alpha-\beta)\ddot{\beta} + \chi\mu\sin(\alpha-\beta)\dot{\alpha}\dot{\beta} + \chi\dot{\mu}\cos(\alpha-\beta)\dot{\beta} - \\ -\chi\mu\sin(\alpha-\beta)\left(\dot{\alpha}-\dot{\beta}\right)\dot{\beta} + \sin\alpha = 0 \\ \ddot{\alpha}\cos(\alpha-\beta) + \ddot{\beta}\chi + \sin(\alpha-\beta)\dot{\alpha}(-\dot{\alpha}+\dot{\beta}) + \frac{\dot{\mu}}{\mu}\cos(\alpha-\beta)\dot{\alpha} + \frac{\dot{\mu}}{\mu}\chi\dot{\beta} - \\ -\sin(\alpha-\beta)\dot{\alpha}\dot{\beta} + \sin\beta = 0 \end{cases} \tag{10}$$

where the dots above the symbols denote the derivative terms of dimensionless time *τ*.

3. Results

The study aims to analyze the impact of changes in the initial tilt of the lower member of the double pendulum (β) on the dynamics of the system. Initial angle β_0 for the first simulation is 0,1 and for the second simulation 0,2. Other initial conditions of the system are shown in table 1.

Table. 1. The initial conditions.

L [m]	*l [m]*	*M [kg]*	*m [kg]*	χ	μ_0
1	1	10	0,10	1	0,01

α_0 *[rad]*	β_0 *[rad]*	$\dot{\alpha}_0$ *[rad/s]*	$\dot{\beta}_0$ *[rad/s]*	λ *[1/s]*
0	0,10 / 0,20	0	0	0,015

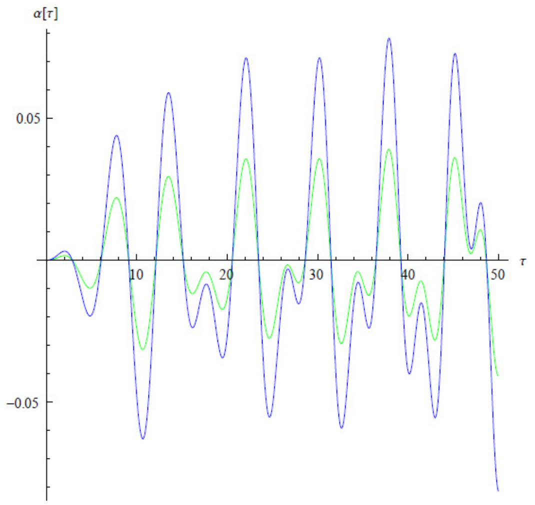

Fig.2. Graphs of generalized coordinates of the upper member *α*.

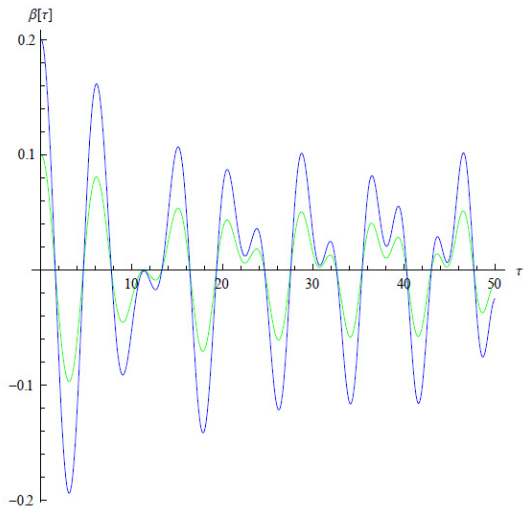

Fig.3. Graphs of generalized coordinates of the lower member *β*.

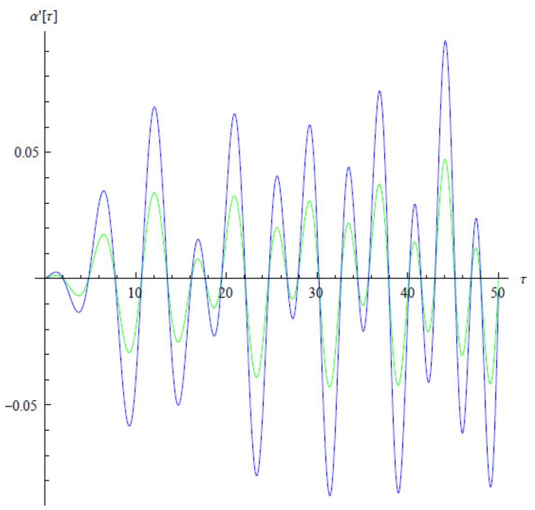

Fig.4. Graphs of generalized velocities of the upper member *α*.

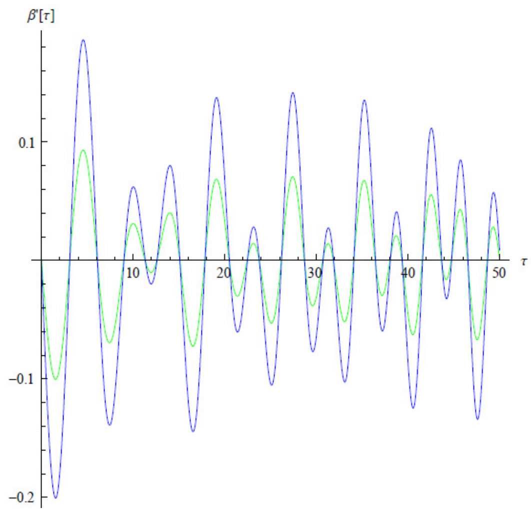

Fig.5. Graphs of generalized velocities of the upper member *β*.

4. Conclusion

With graphs of generalized coordinates α and generalized velocities α' of the upper member of the double mathematical pendulum, can be read that in the during a study, the amplitude of the vibration is chaotic. The values of the inclination of the upper member of the pendulum are changing within the range from 0 to 0.04 for the first and from 0 to 0.8 for the second simulation. Graphs of generalized velocities of the upper member of the pendulum for the first simulation are formed in the range from 0 to 0.045, and for the second from 0 to 0.09. Graphs of generalized coordinates β and generalized velocities β' of the lower member, indicate that a decrease in the mass of the upper member, and while increasing the mass of the lower member of the pendulum, occurs the damping of the latter. The size of the amplitude of the vibration decreases constantly during the studied period of time, from the initial set value of 0.1 to 0.05 for the first and of the initial set point size of 0.2 to 0.1 at the end of the course for the second simulation. The generalized velocity of the lower member of the pendulum, for both simulations reaches a maximum at the start of the test. For the first simulation 0.1 and for the second 0.2. Then the generalized velocity amplitude value for both simulations is gradually decreasing to 0.03 for the first and 0.06 for the second simulation. The main and most important conclusion of the study is to confirm that the phenomenon of mass flow can be used for vibration damping.

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