MM3110 - Computational Materials Lab

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1 1D heat transfer Equation

1.1 General expression

Definition 1. The general heat transfer equations is derived by conserving energy and the equation takes the form

$$\frac{\partial T}{\partial t} = \nabla(\alpha \nabla T) + \frac{g}{\rho C_p} \ , \tag{1}$$

This equation can be written in one direction. Assuming that the material properties are independent of temperature and spacial coordinates the above equation can be reduced to

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} + \frac{g}{\rho C_p} \ , \tag{2}$$

Since, we are dealing with steady state, the transient term can be set to zero. The derivation for the heat equation for the situation under consideration is given in the next subsection

1.2 Generation Term

The generation term in equation 2 is per unit volume. In this situation the heat is flowing through the rod by conduction and some heat is lost to the surroundings by convective heat transfer. Since, the basic assumption in this case is that the temperature is constant along Y and Z direction, the convective heat loss through the surface can be converted to a volumetric heat loss.

Let Δx, Δy,Δz be the dimensions of the control volume. The temperature of the rod is changing along the X direction and the heat loss to the surrounding is along Y direction.
Thus, the amount of heat lost to the surrounding is given by

$$h(T - T_a) * \Delta x * \Delta z \ , \tag{3}$$

Dividing equation 3 by the volume of the control volume gives the volumetric heat loss. Thus,

$$\frac{h(T - T_a)}{\Delta y} \ , \tag{4}$$

1.3 1D heat transfer equation

Combing the conductive term and generation term the heat transfer equation governing the system is given by

$$\alpha \frac{d^2 T}{dx^2} - \frac{h(T - T_a)}{\Delta y} = 0 \ , \tag{5}$$

1.4 Results and Discussion

The equation 5 is solved and plotted using MATLAB by assuming all constants other than h to be equal to 1 in its respective SI unit and h is taken to be 0.02.

It is clearly visible from figure 1 that the temperature profile matches for N = 4,9. Where N is the number of points. This clearly suggests that the equation does not depend upon the length of the control volume.

Further, the temperature profile corresponding to h = 0.02 is below the curve corresponding to h = 0, which is as expected.

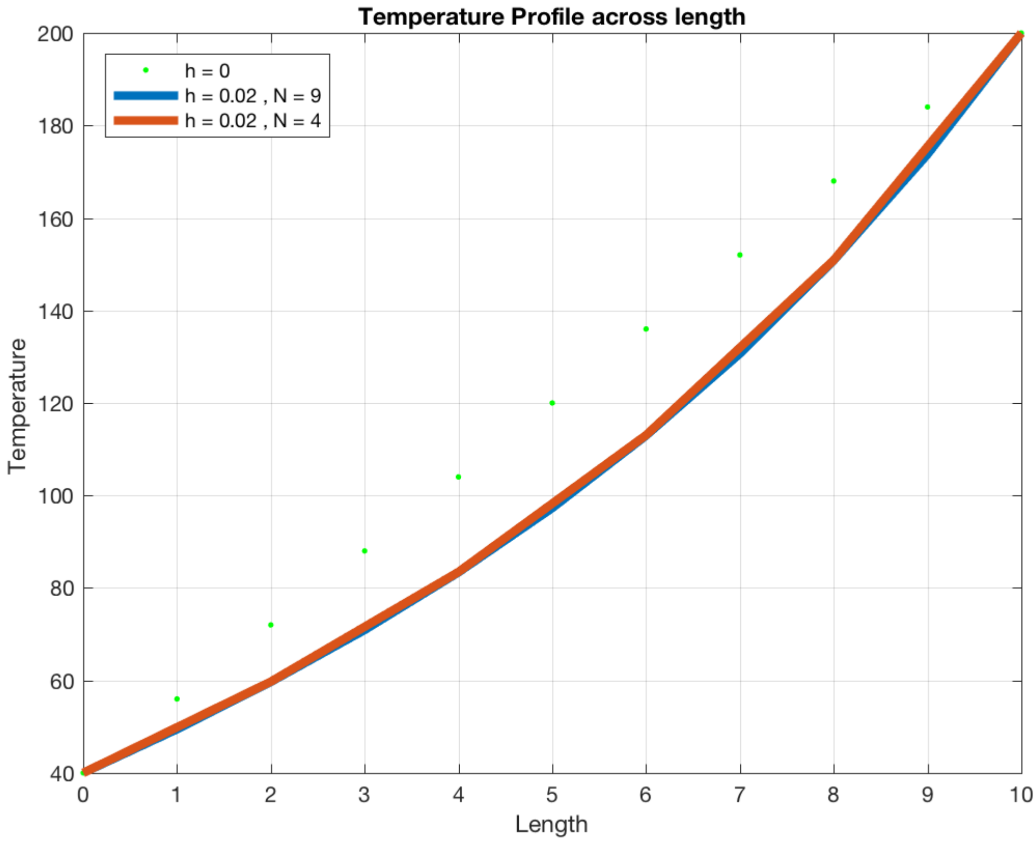

Figure 1: Temperature Profile across the rod.