Computational Photography

* Study the basics of computation and its impact on the entire workflow of photography, from capturing, manipulating and collaborating on, and sharing photographs.

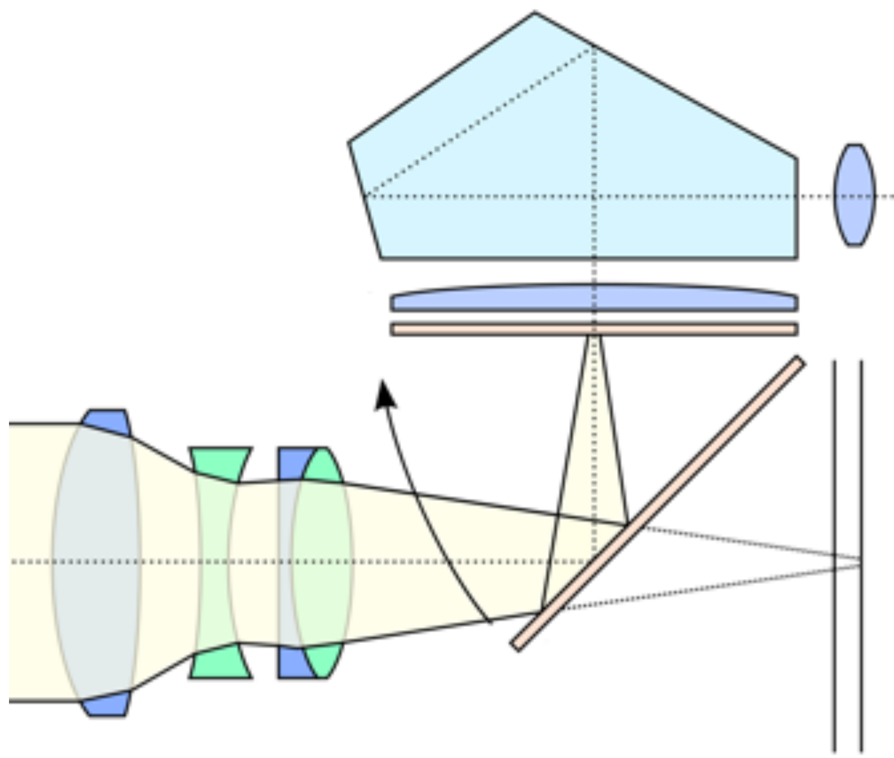
Georgia Tech College of Computing

Digital Images: Into the Frequency Domain

* Images are samples of light information stored in array of pixels.

1. Using sines and cosines to reconstruct a signal
2. The Fourier Transform
3. Frequency Domains for a Signal
4. Three properties of Convolution relating to Fourier Transform

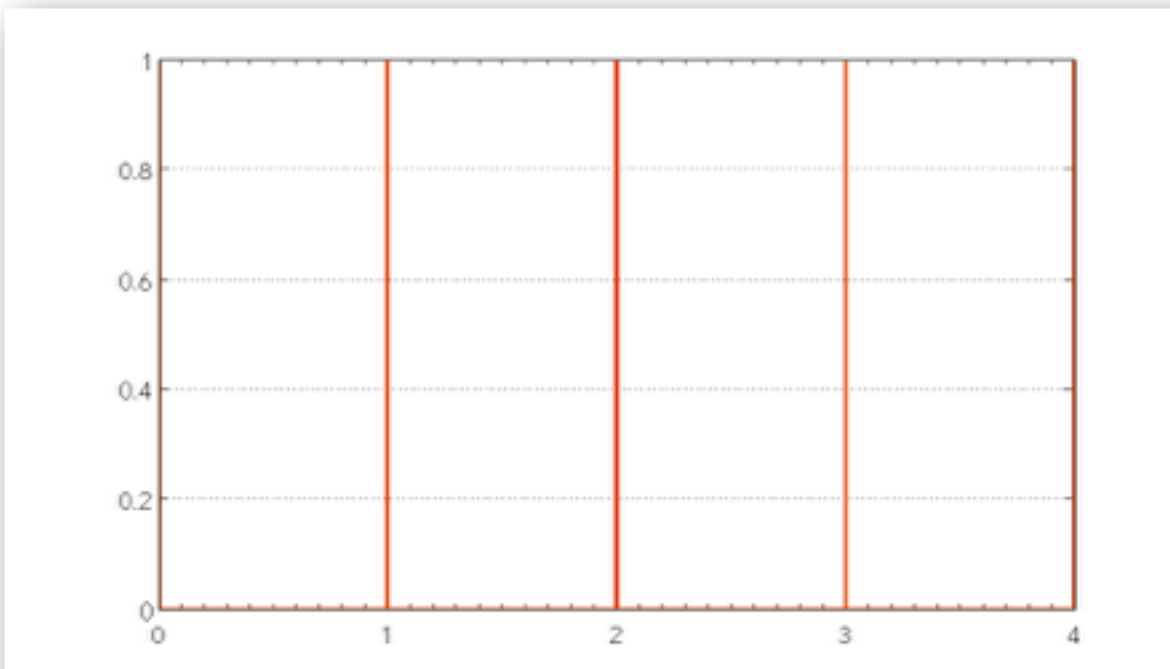
Recall: Images and Camera



- Rays of Light go through a Camera / Optics / Lens
- Aperture, Shutter, and Film Sensitivity
- Sensor to Convert Light to Digital Information
- Process the Image

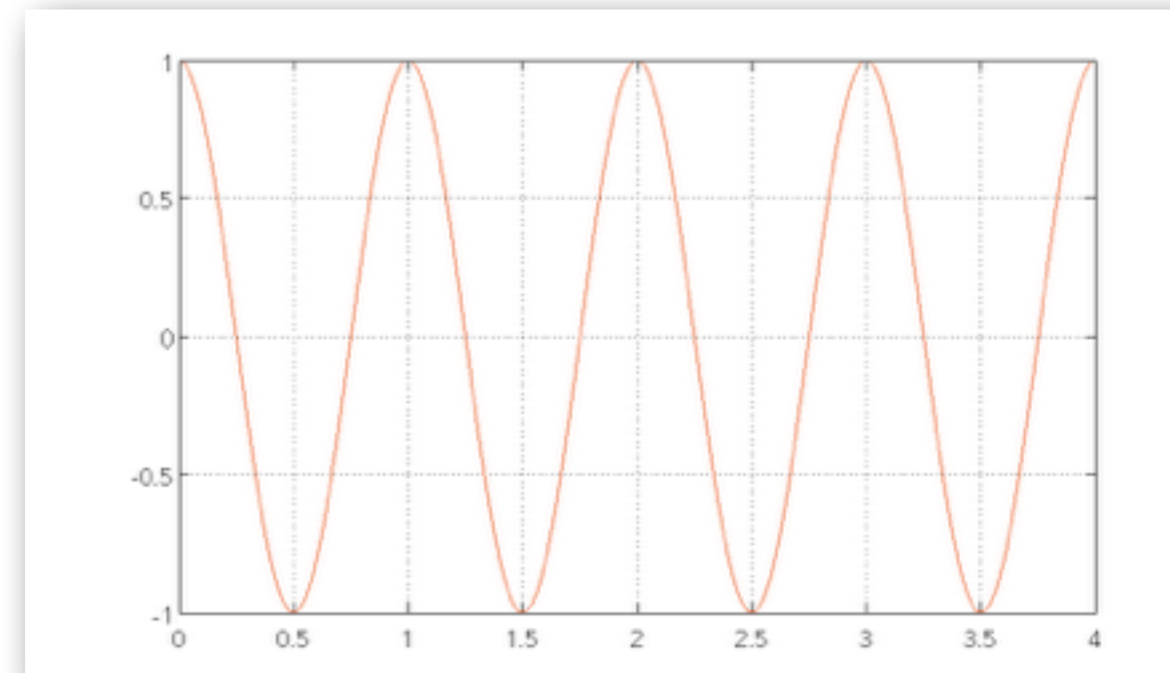
Reconstructing a Signal

Target Signal: $f^T(t)$



Repeating Impulse Function

f_1

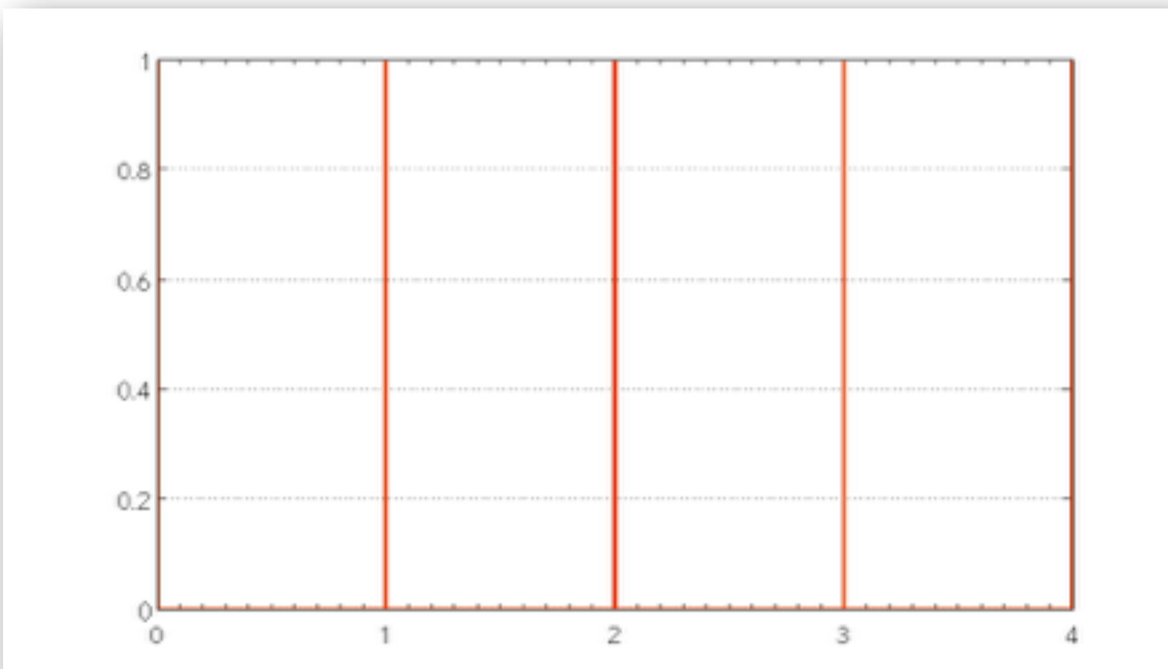


* Basic building block
  * A: Amplitude, w: frequency

$$f(t) = A\cos(n\omega t)$$

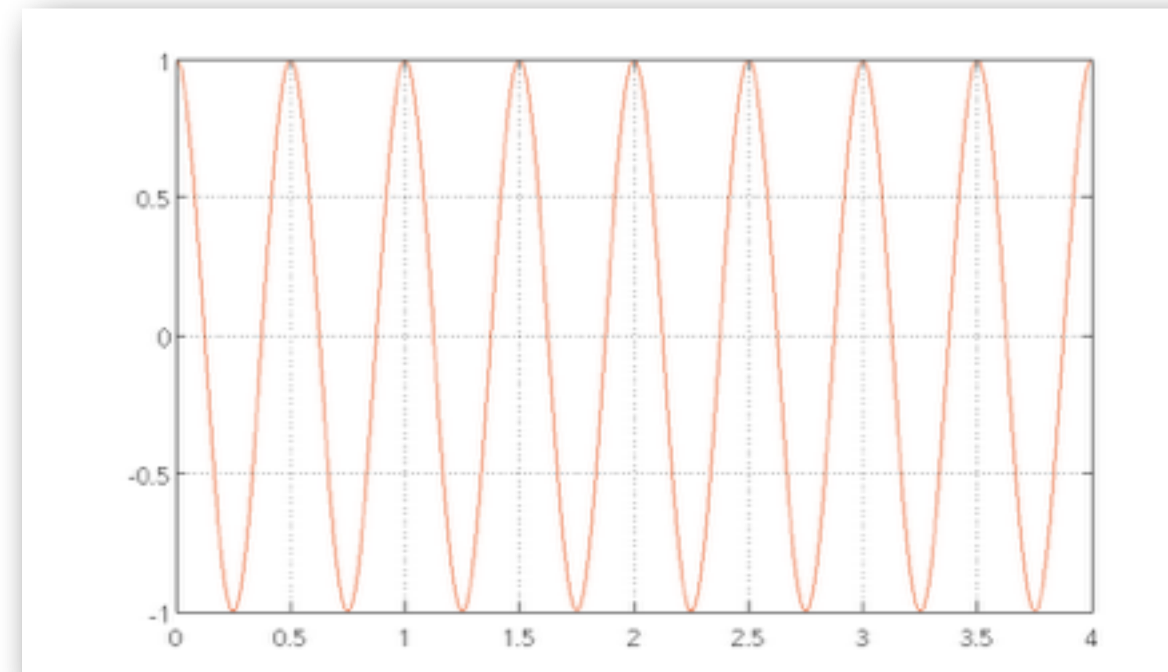
Reconstructing a Signal

Target Signal: $f^T(t)$



Repeating Impulse Function

f_2

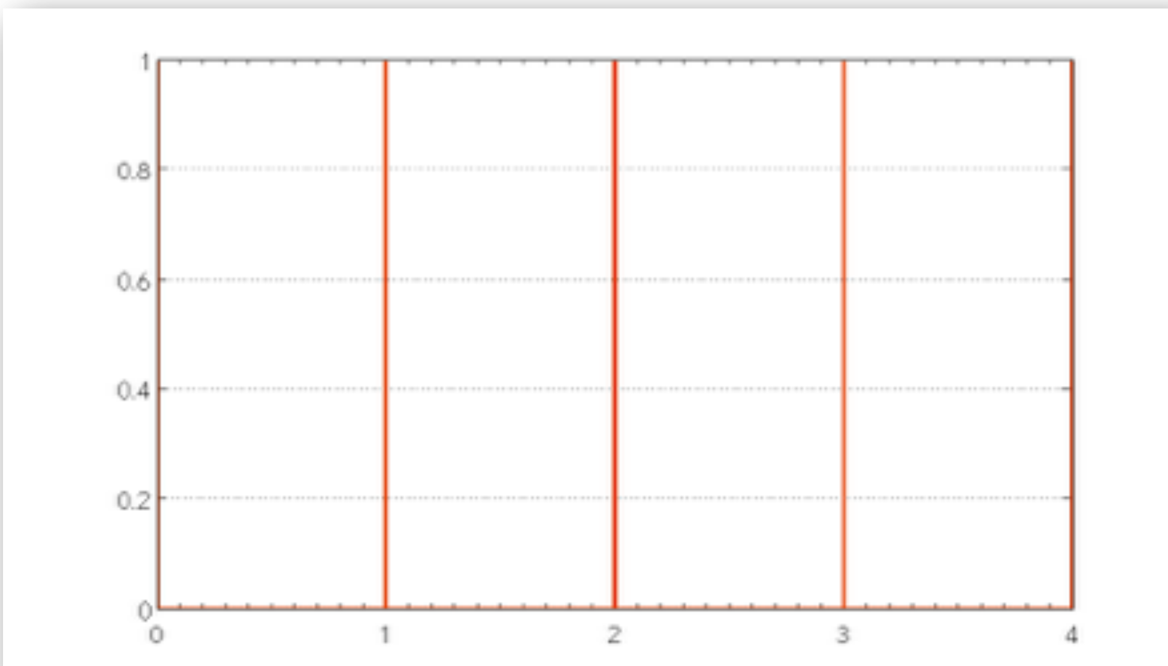


* Basic building block
  * A: Amplitude, w: frequency

$$f(t) = A\cos(n\omega t)$$

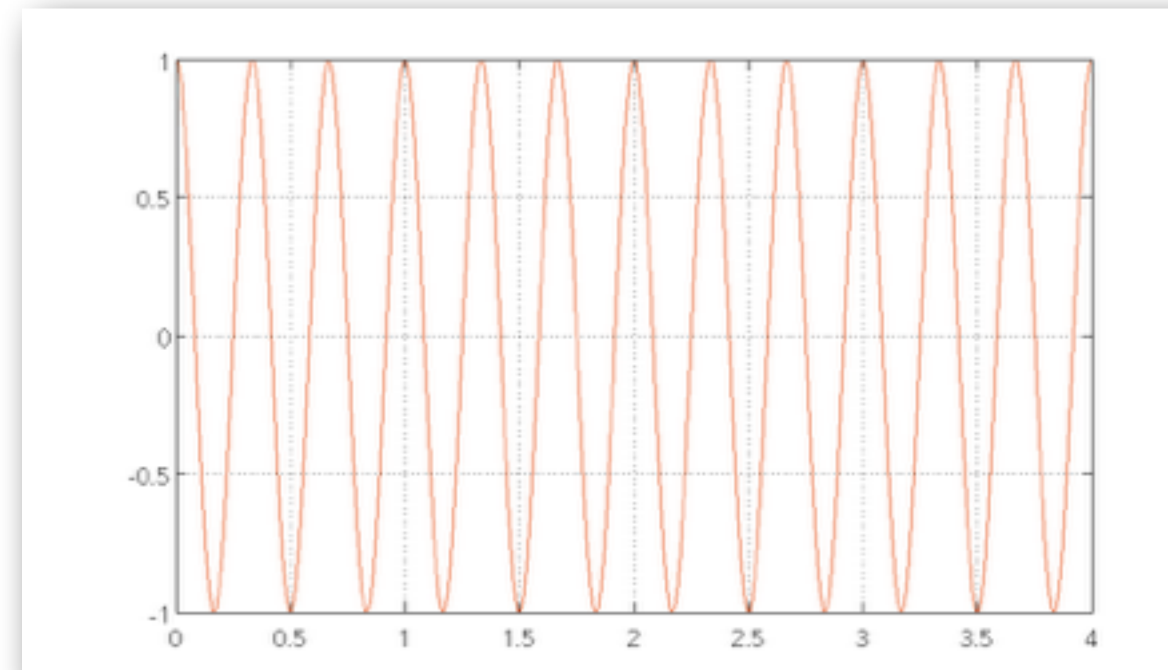
Reconstructing a Signal

Target Signal: $f^T(t)$



Repeating Impulse Function

f_3



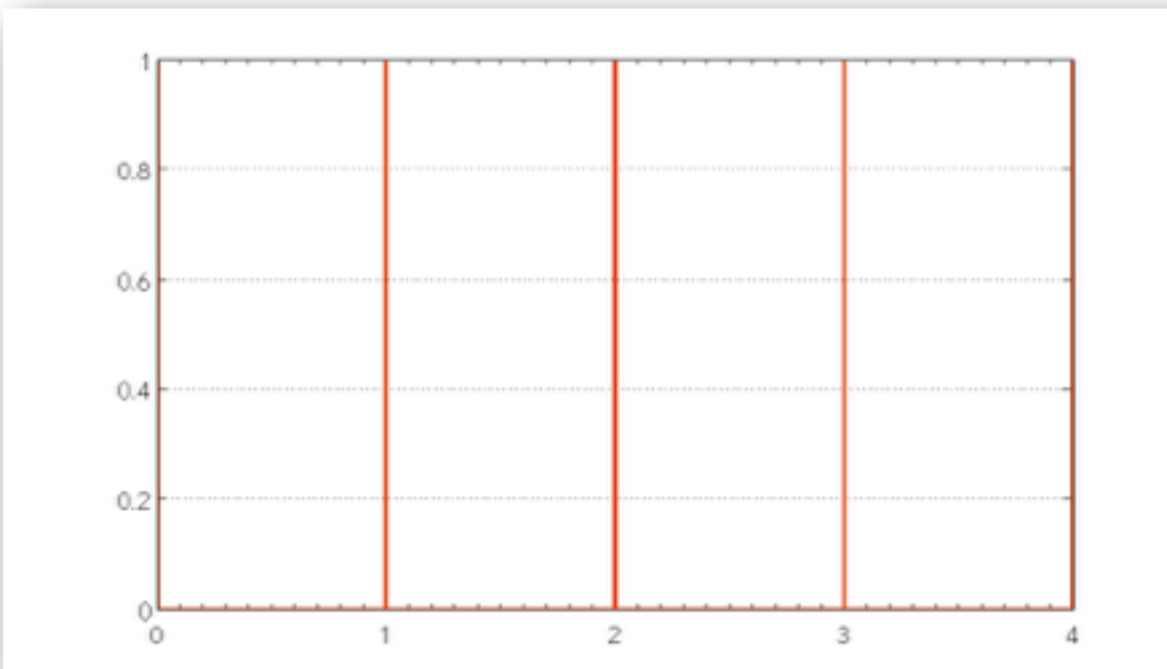
* Basic building block

$$f(t) = A\cos(n\omega t)$$

  * A: Amplitude, w: frequency

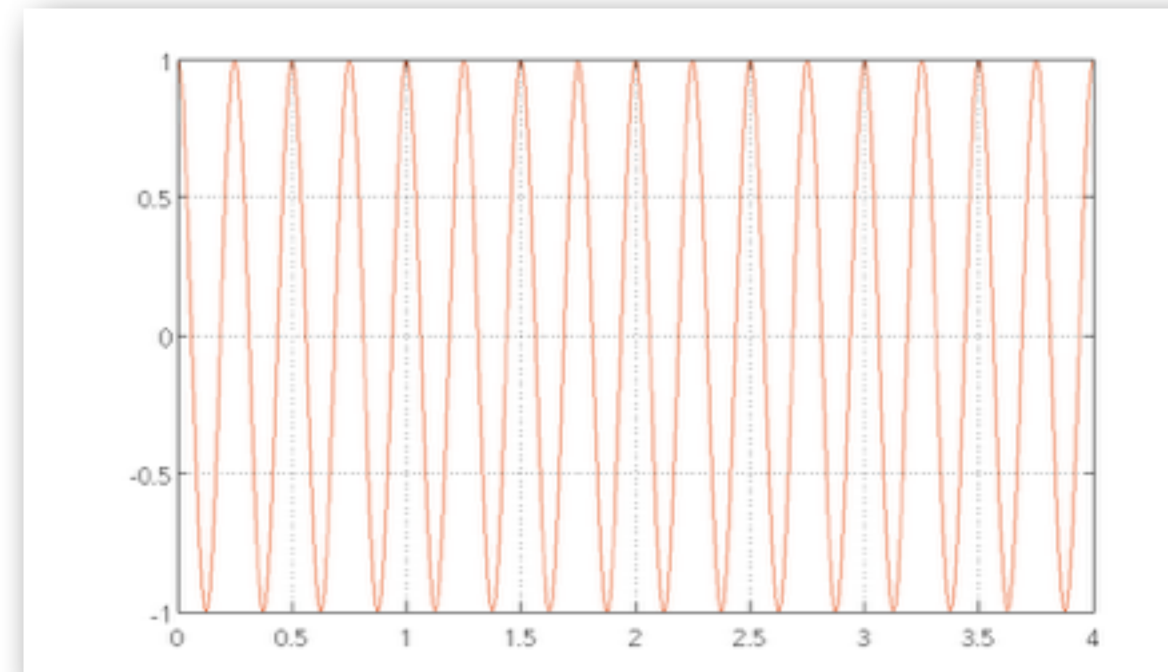
Reconstructing a Signal

Target Signal: $f^T(t)$



Repeating Impulse Function

f_4

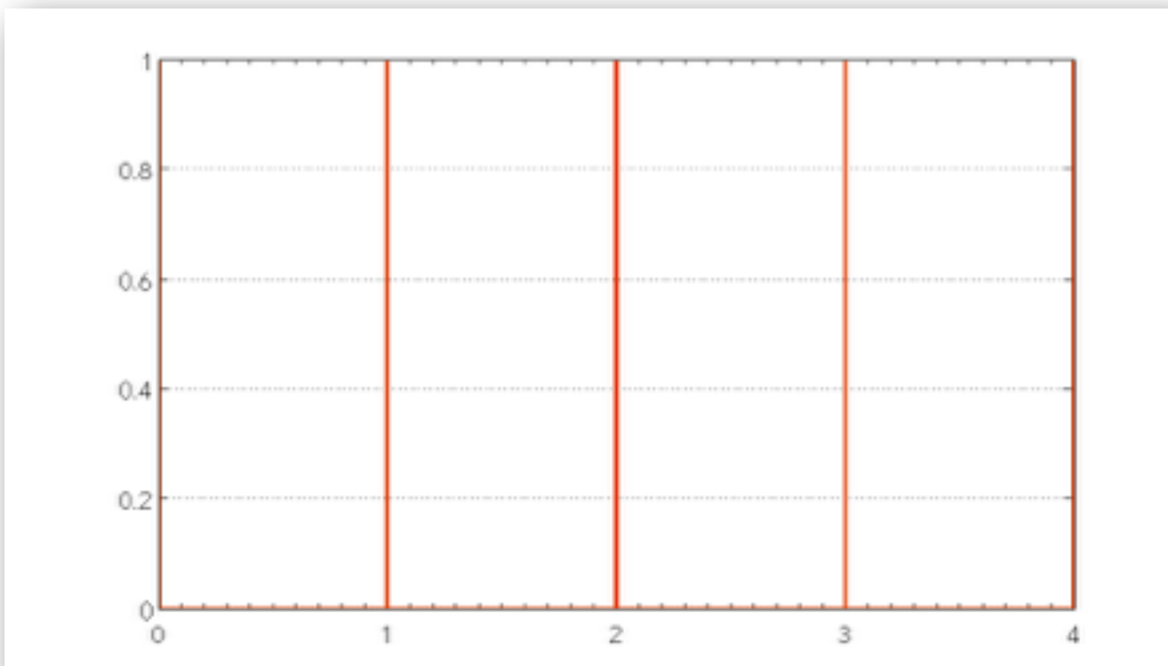


* Basic building block
  * A: Amplitude, w: frequency

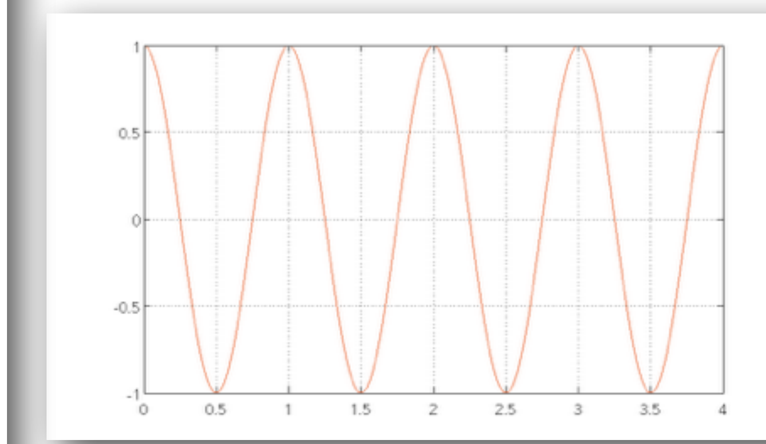
$$f(t) = A\cos(n\omega t)$$

Reconstructing a Signal

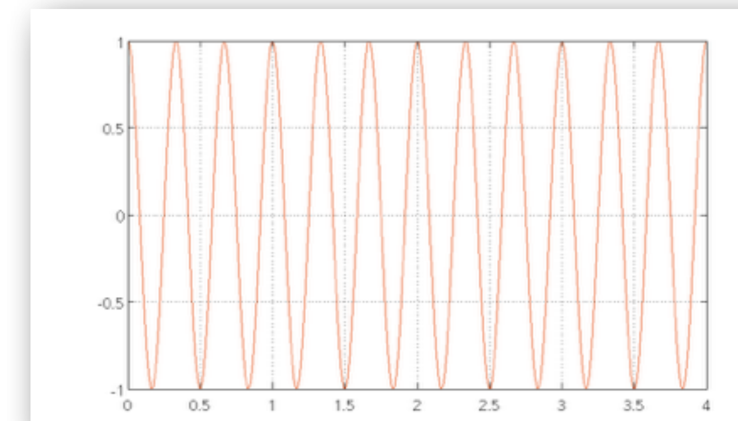
Target Signal: $f^T(t)$



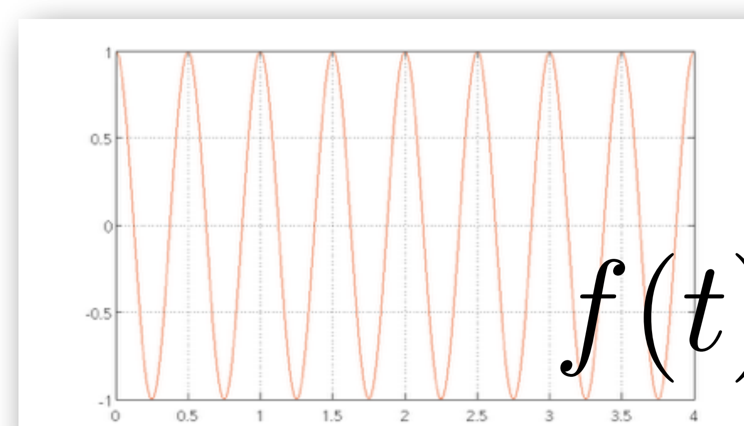
Repeating Impulse Function



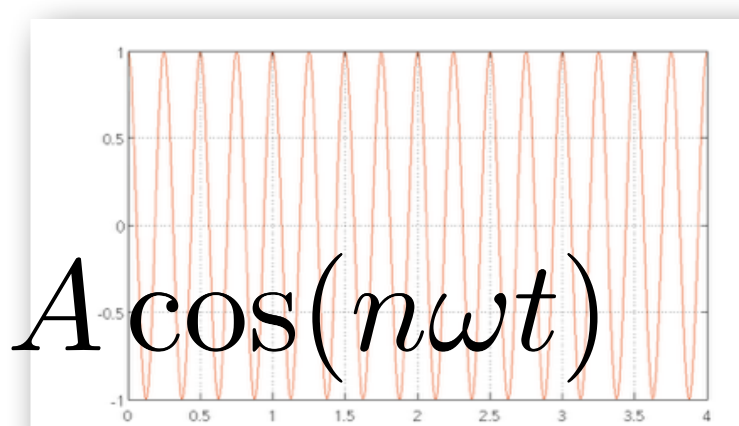
f_1



f_3



f_2

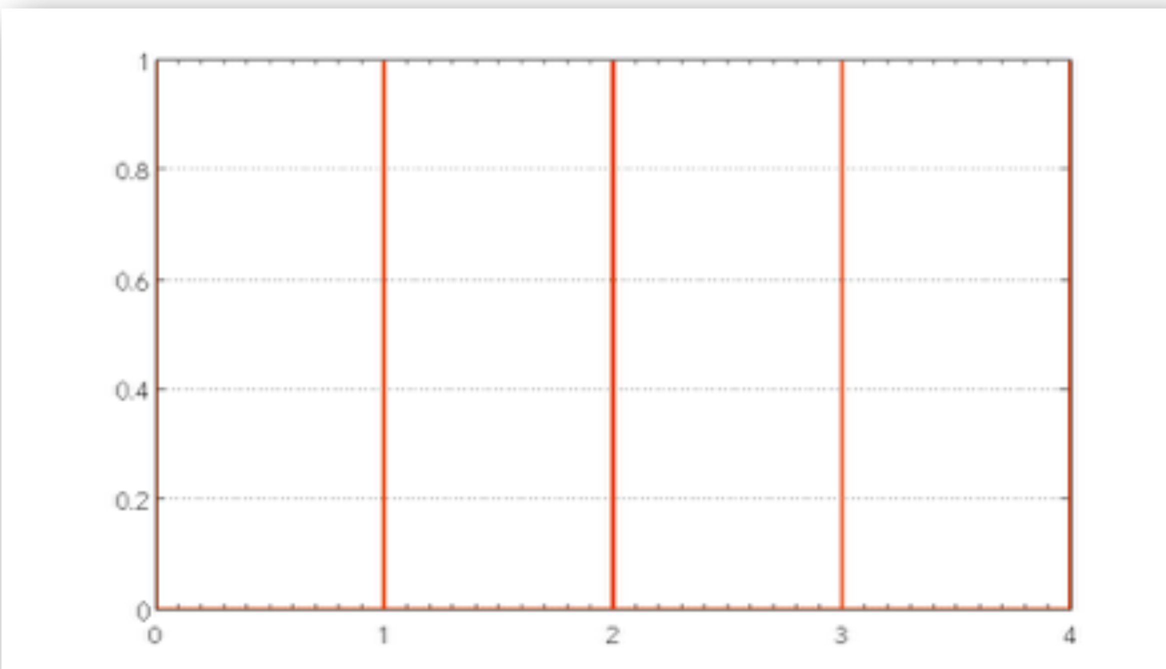


f_4

$$f(t) = A\cos(n\omega t)$$

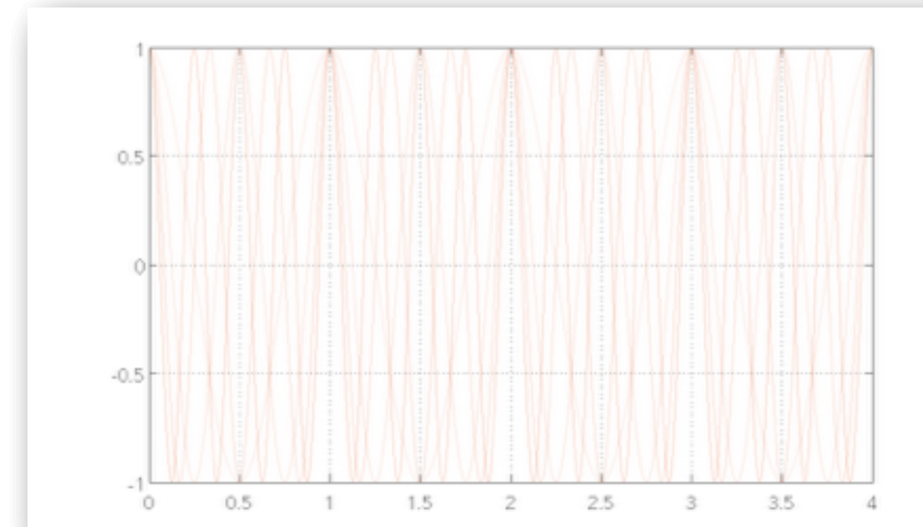
Sum of Cosines

Target Signal: $f^T(t)$



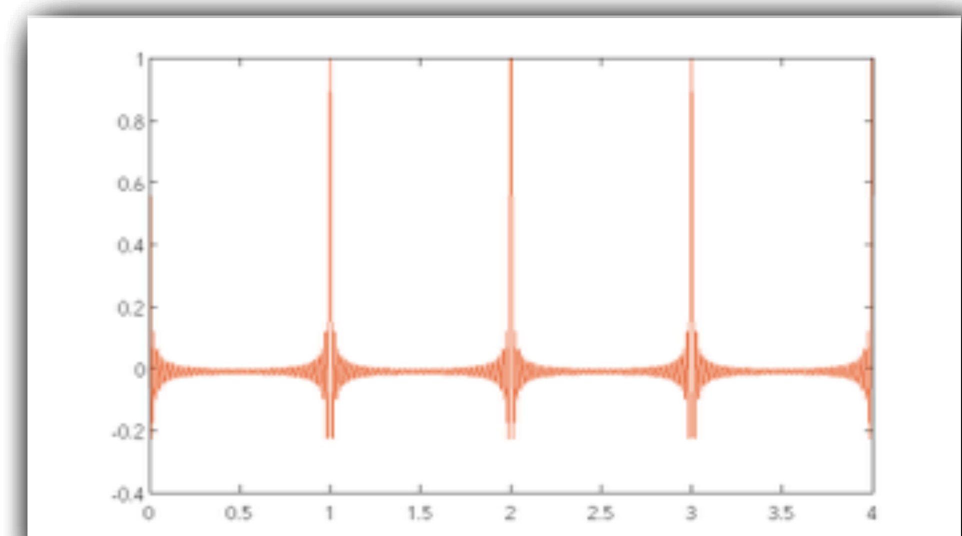
Repeating Impulse Function

$f_1 \quad f_2 \quad f_3 \quad f_4$



N=10

N=50



$f_1 + f_2 + f_3 + f_4$

* Adding Cosines (N of them)

$$f(t) = A\cos(n\omega t)$$

$$f^T(t) = \sum_{n=1}^{N} A\cos(n\omega t)$$

A Fourier Transform

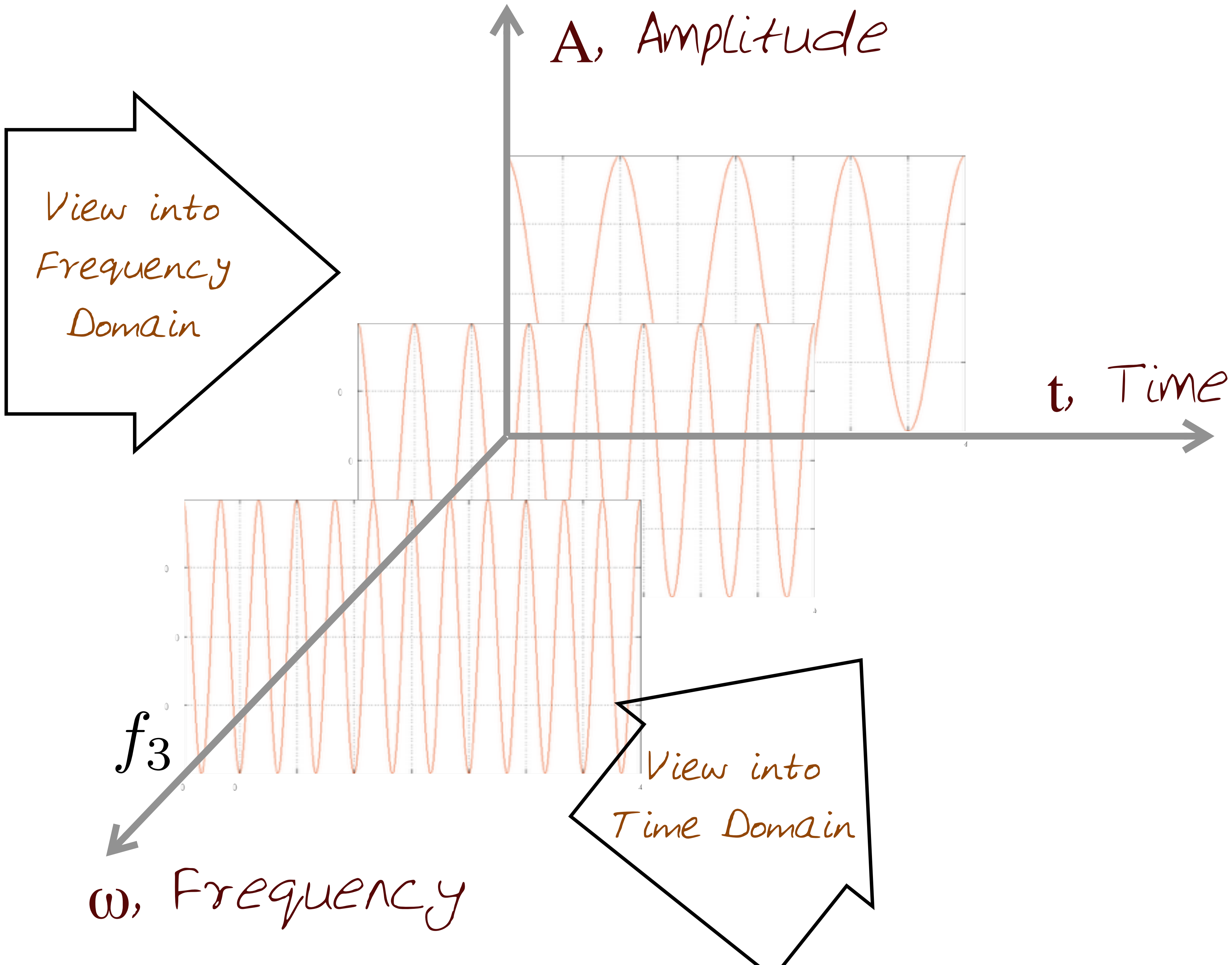
Jean Baptiste Joseph Fourier (1768–1830)

* Periodic function ⇒ a weighted sum of sines and cosines of different frequencies
* Transforms **f(t)**, into a **F(ω)**
* Frequency spectrum of the function **f**
* A reversible operation
* For every **ω** from **0** to ∞ (infinity) **F(ω)** holds the Amplitude **A** and phase **ϕ** of a sine function

$$F(\omega) = A\cos(\omega t + \phi)$$

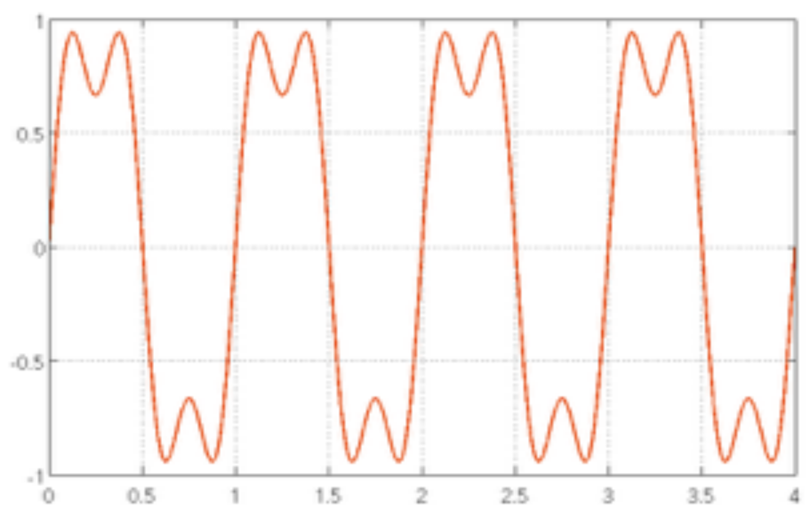
http://en.wikipedia.org/wiki/File:Fourier2.jpg

Frequency Domain of a Signal



- How many N?
- What does each control?
- Coarse vs. fine structure

Time, Frequency, and Frequency Spectra



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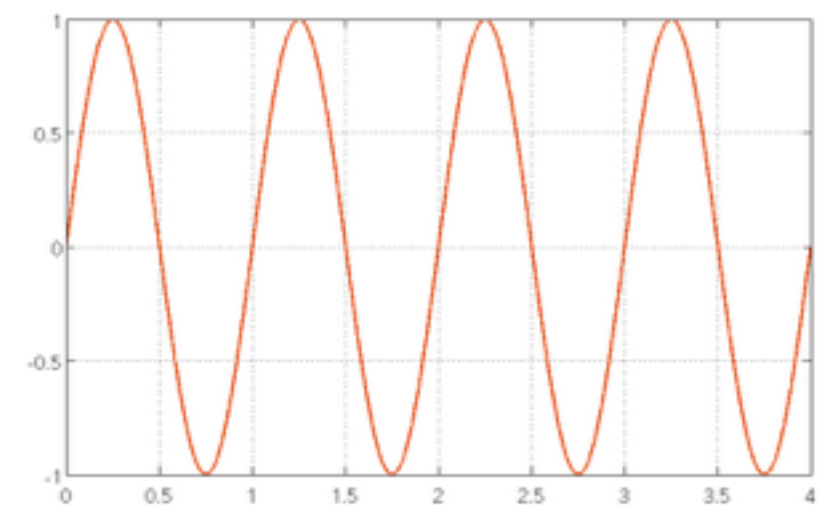
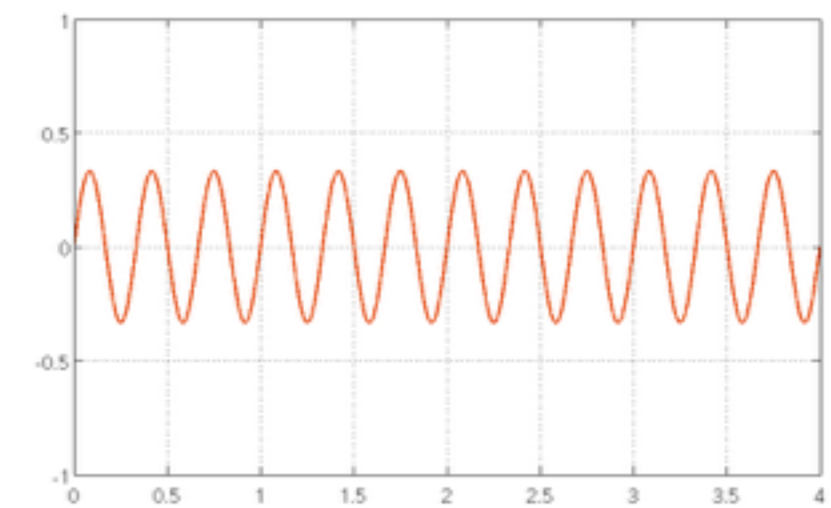
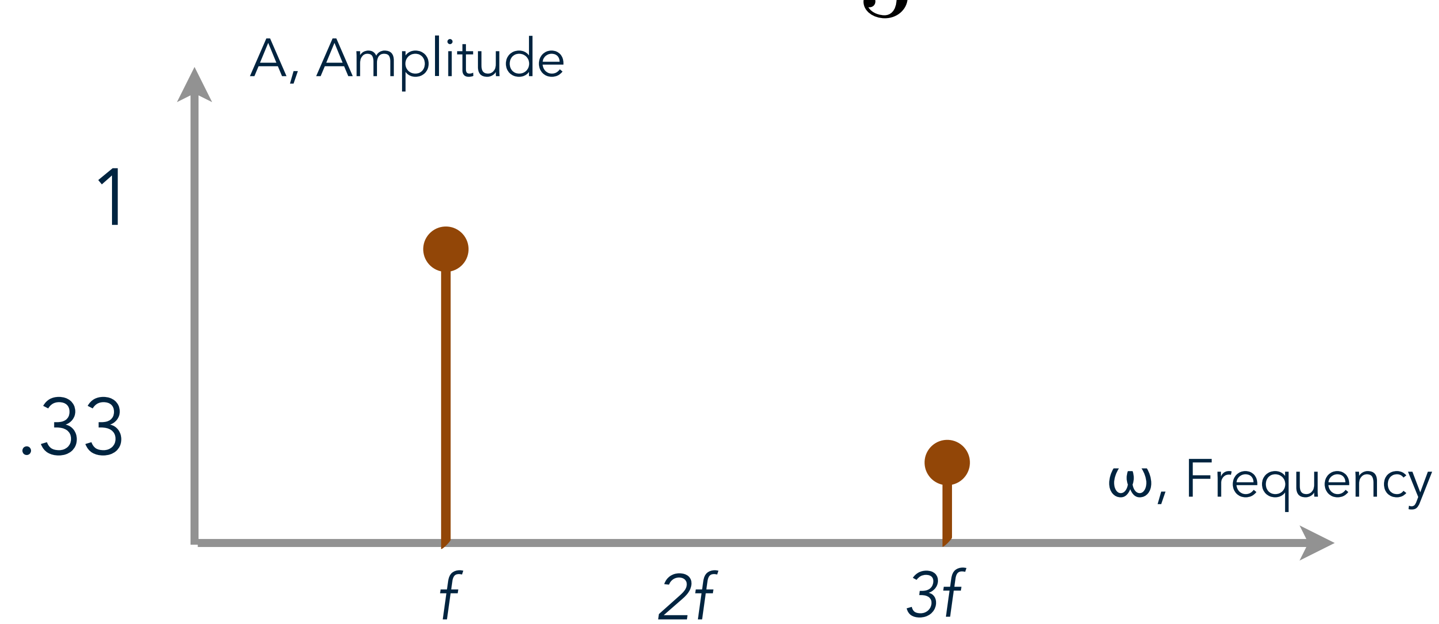


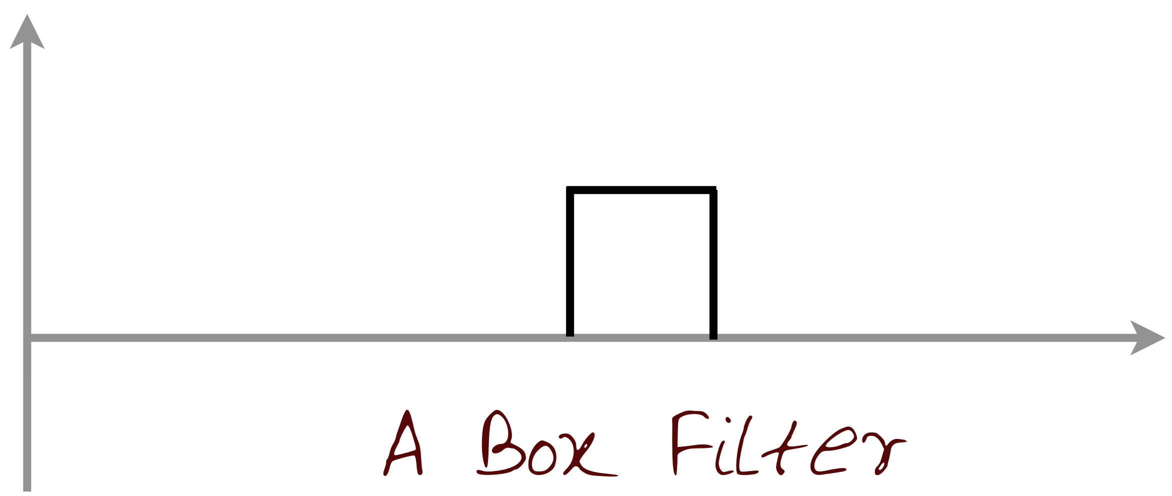
+



$$g(t) = \sin(2p\omega t) + \frac{1}{3}\sin(2p(3\omega)t)$$

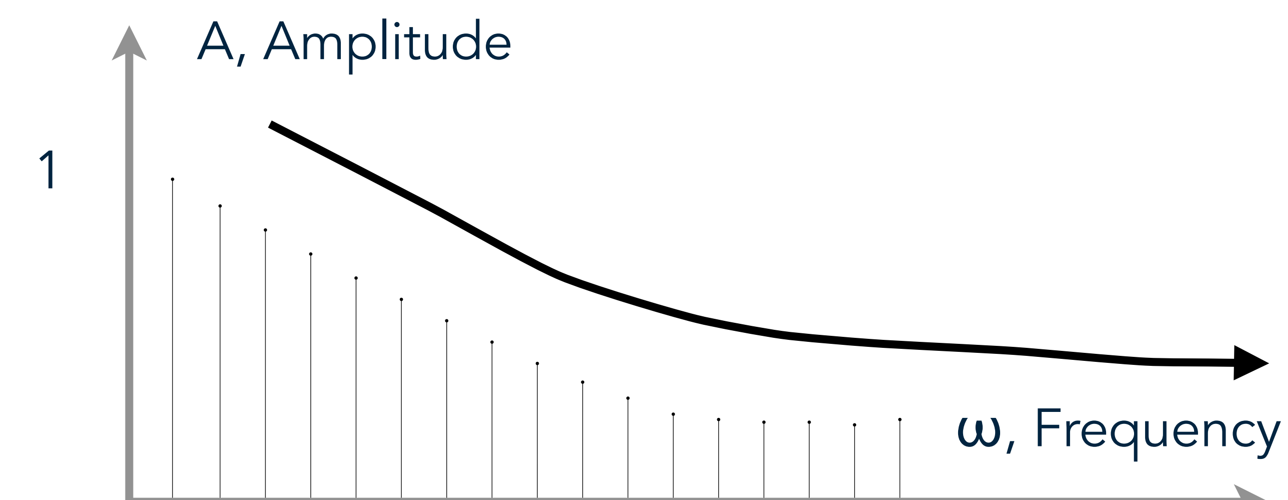


Frequency Spectra



A Box Filter

$$A \sum_{k=1}^{\infty} \frac{1}{k} \sin(2\pi k t)$$



Convolution Theorem and the Fourier Transform

* **Fourier transform** of a convolution of two functions = product of their Fourier transforms

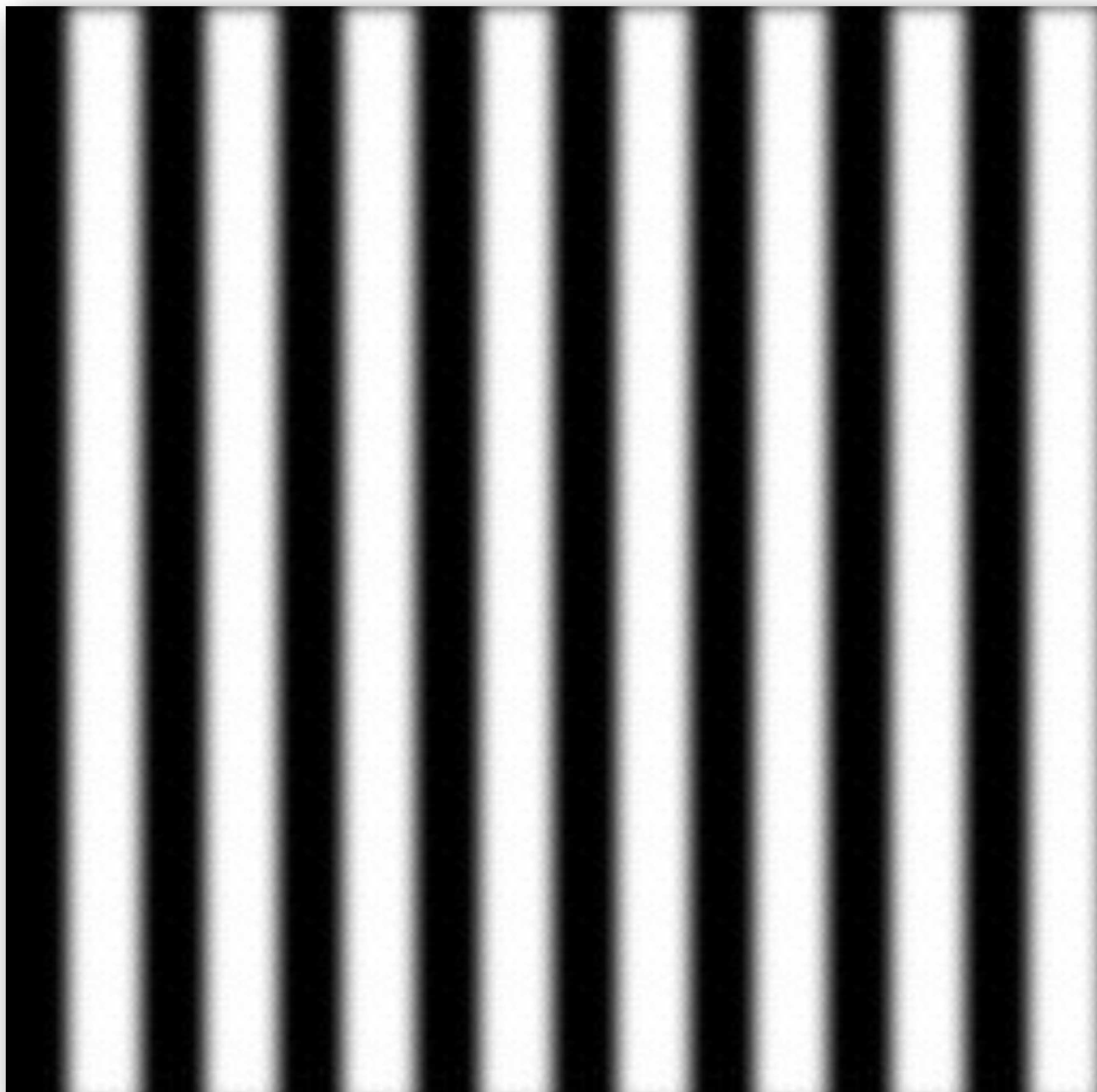
$$F[g * h] = F[g]F[h]$$

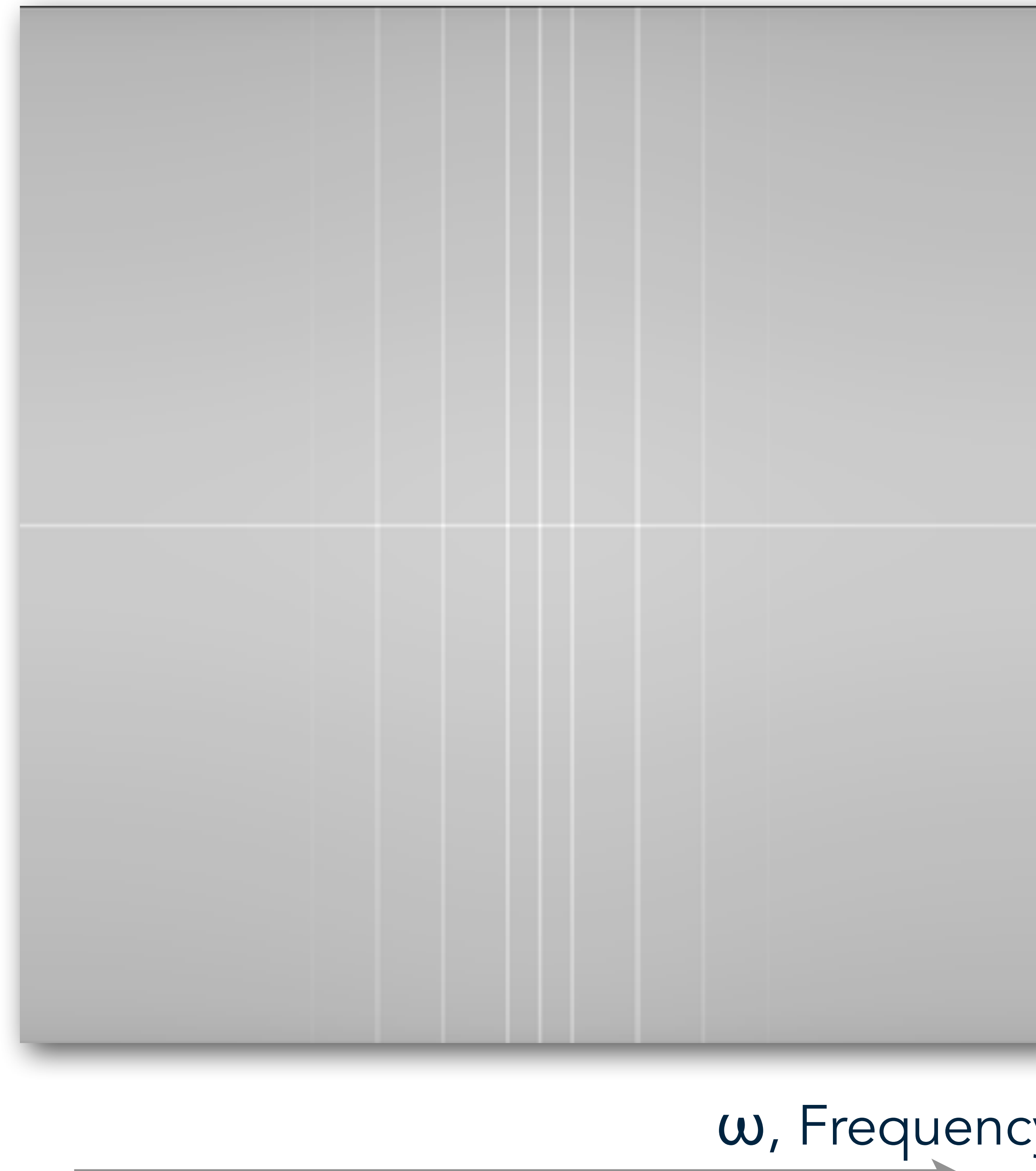
* **Inverse Fourier transform** of the product of two Fourier transforms = convolution of the two inverse Fourier transforms

$$F^{-1}[gh] = F^{-1}[g] * F^{-1}[h]$$

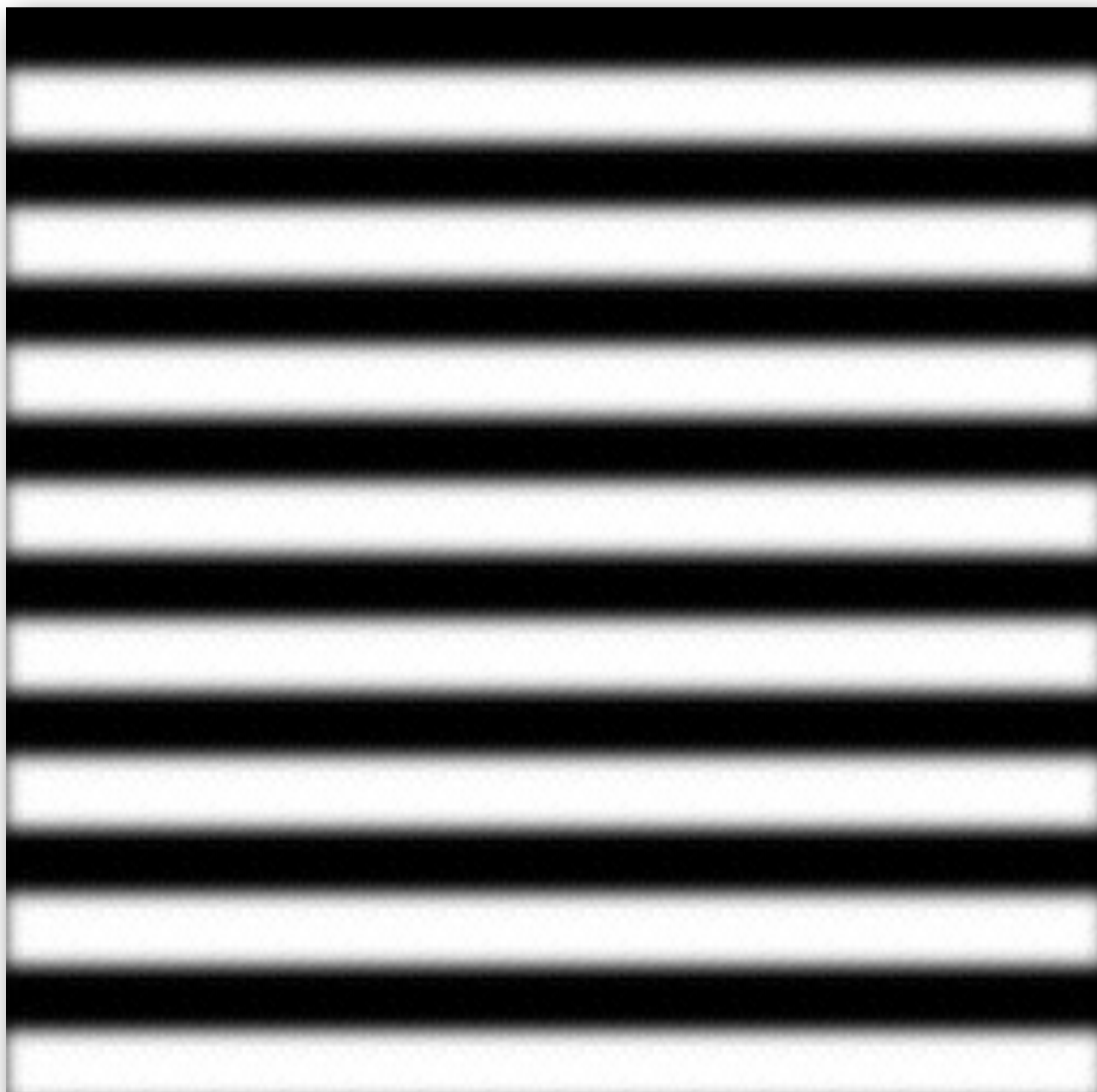
* Convolution in spatial domain is equivalent to multiplication in frequency domain!

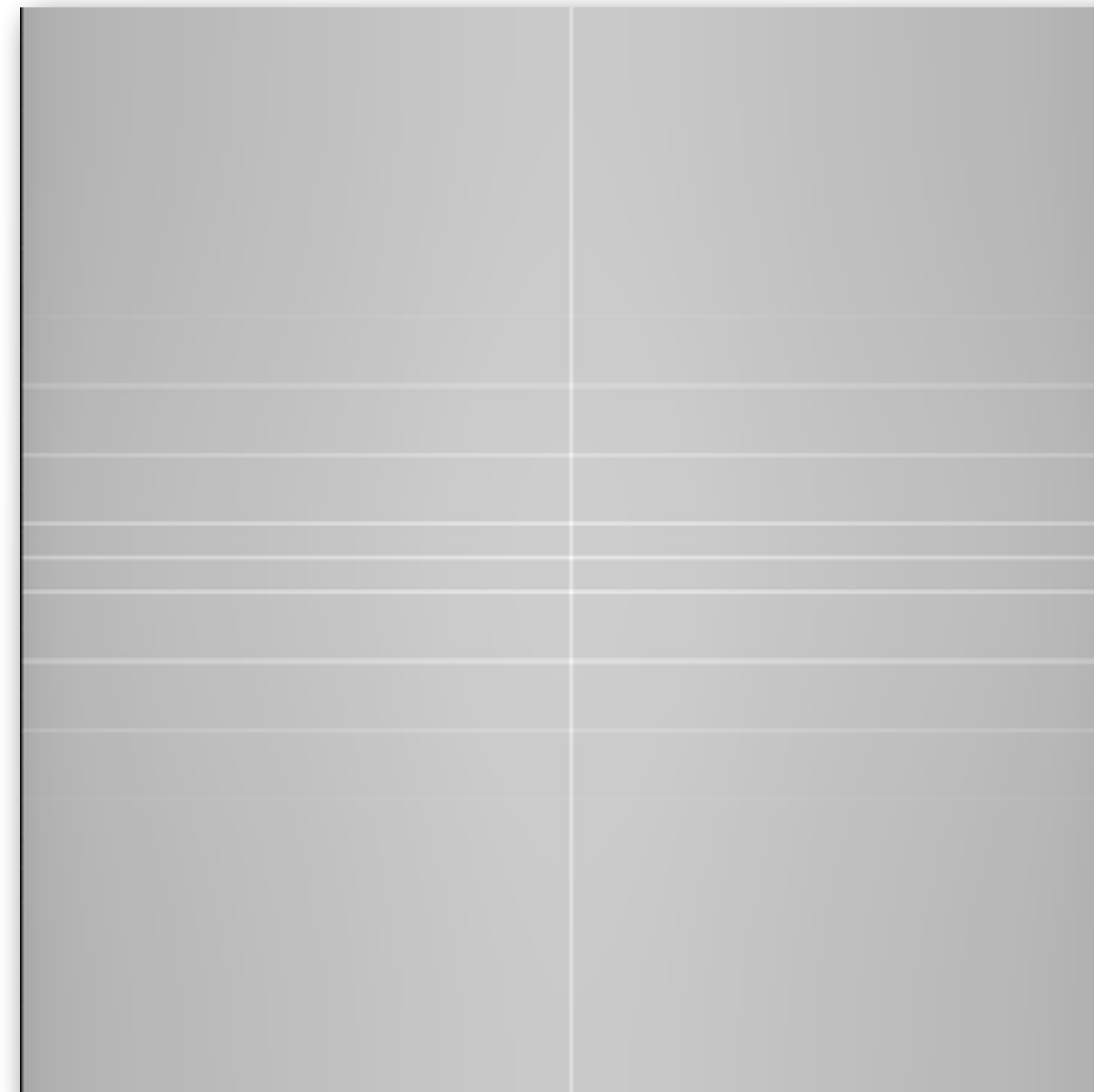
Now, Frequency Spectra for Images



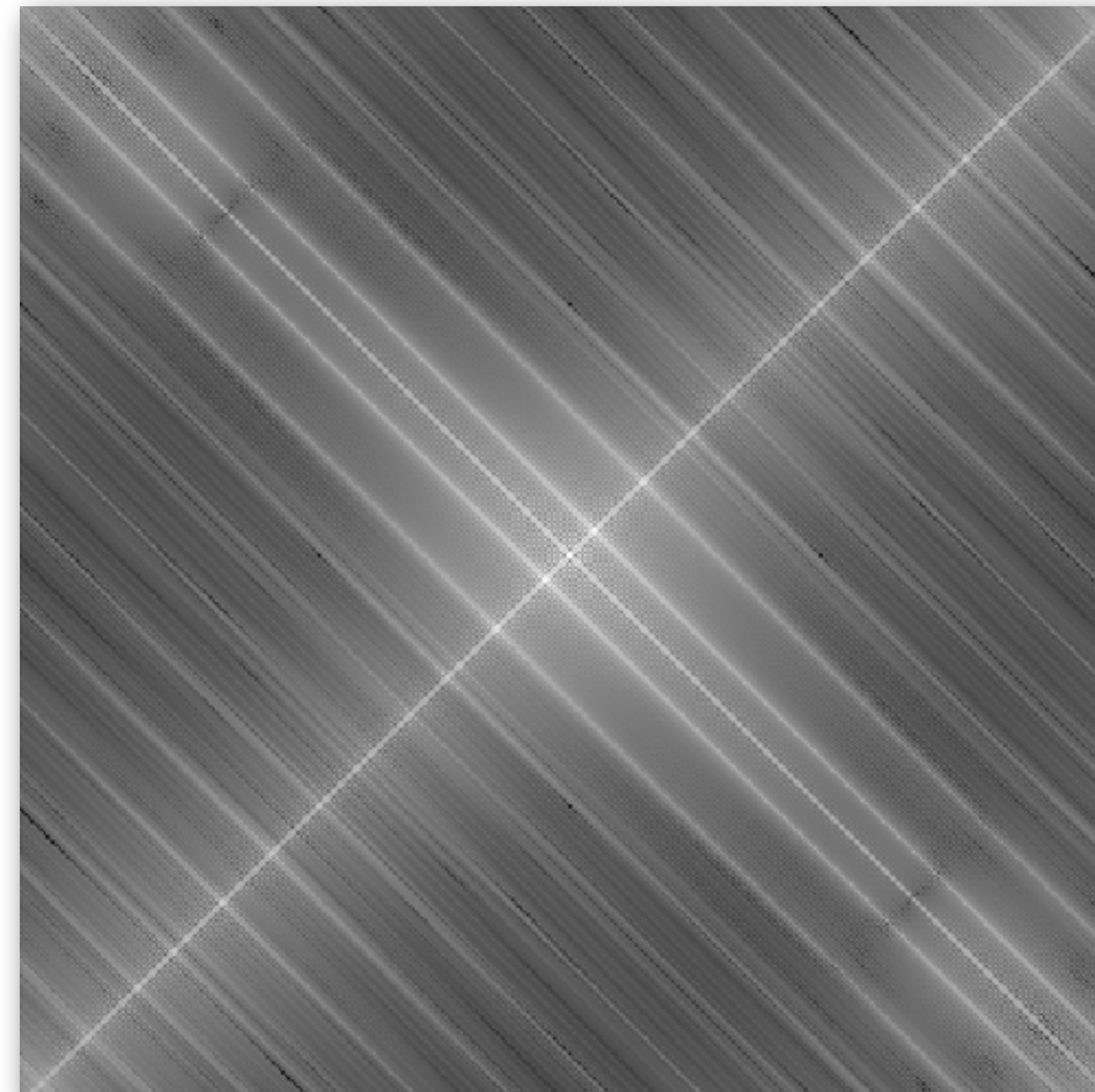


Now, Frequency Spectra for Images

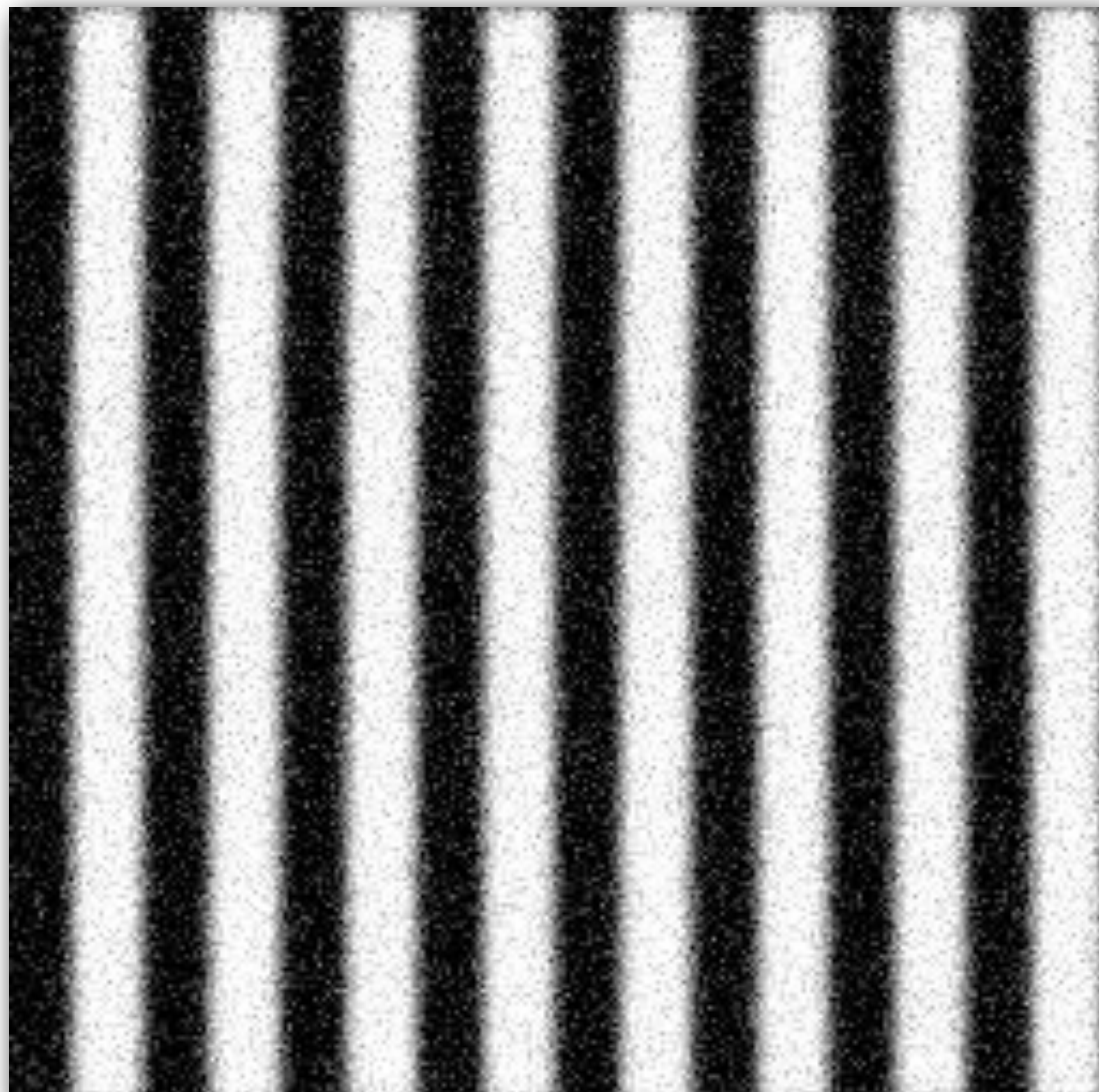




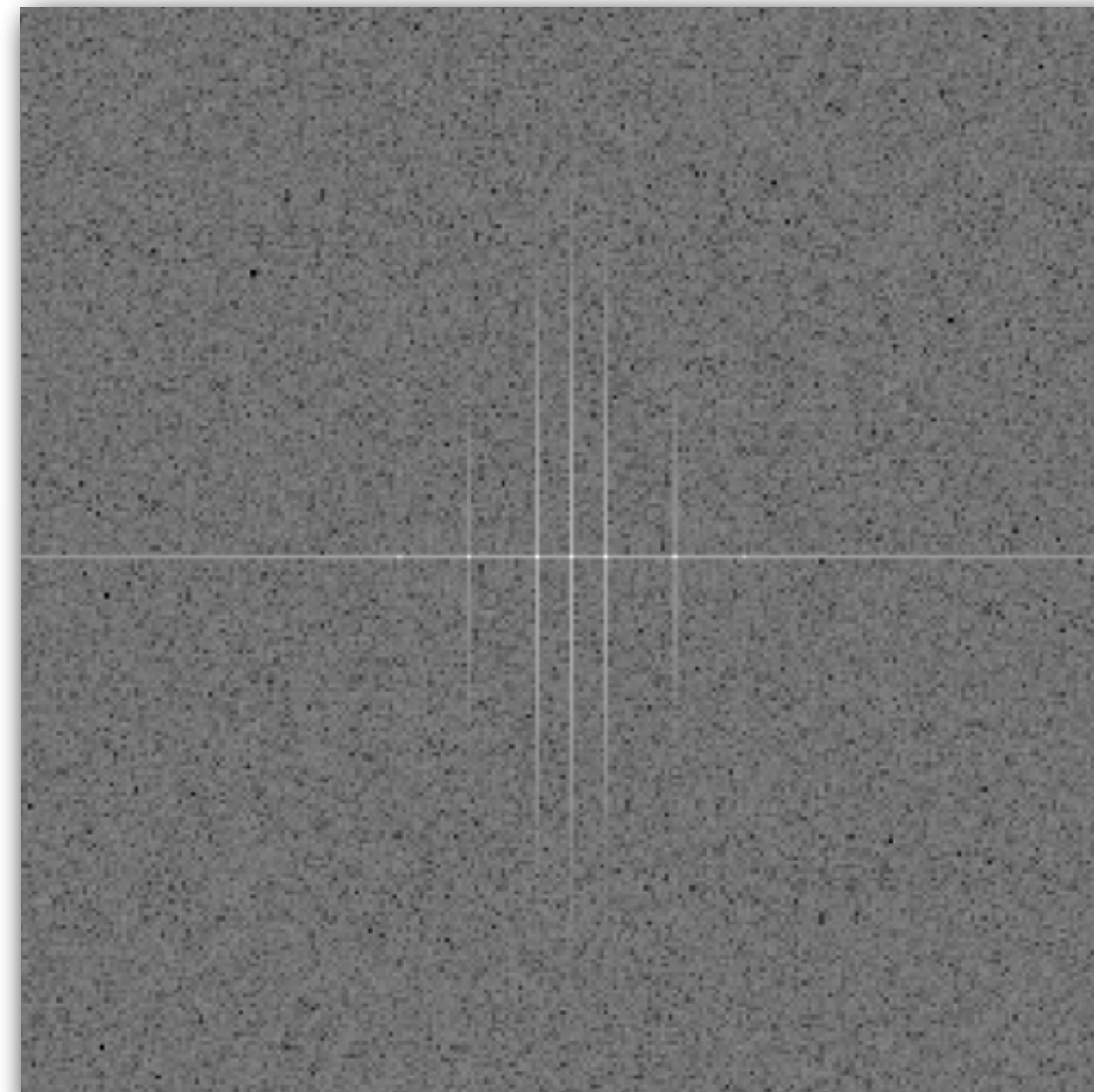
Now, Frequency Spectra for Images



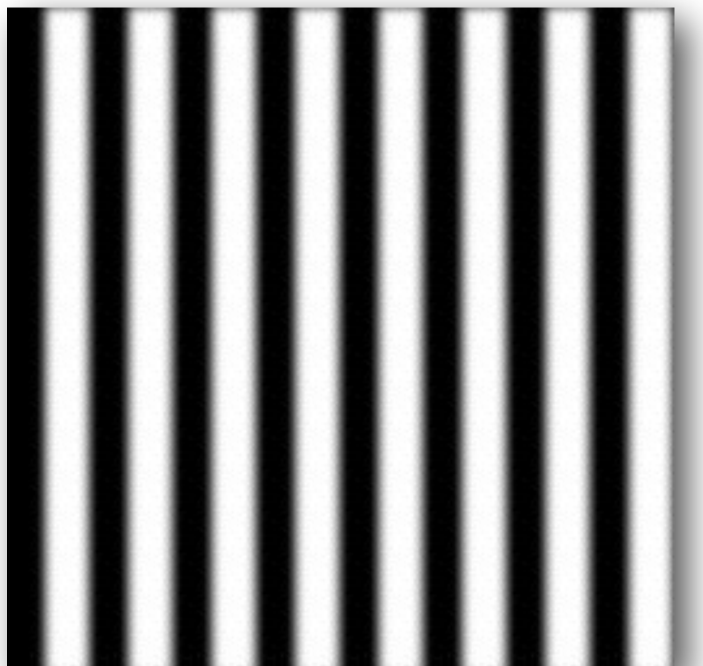
Now, Frequency Spectra for Images

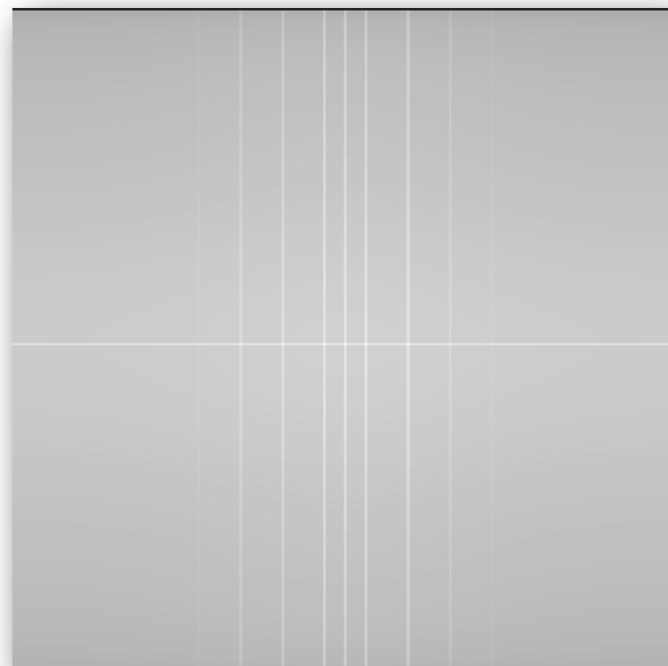


Noise Added

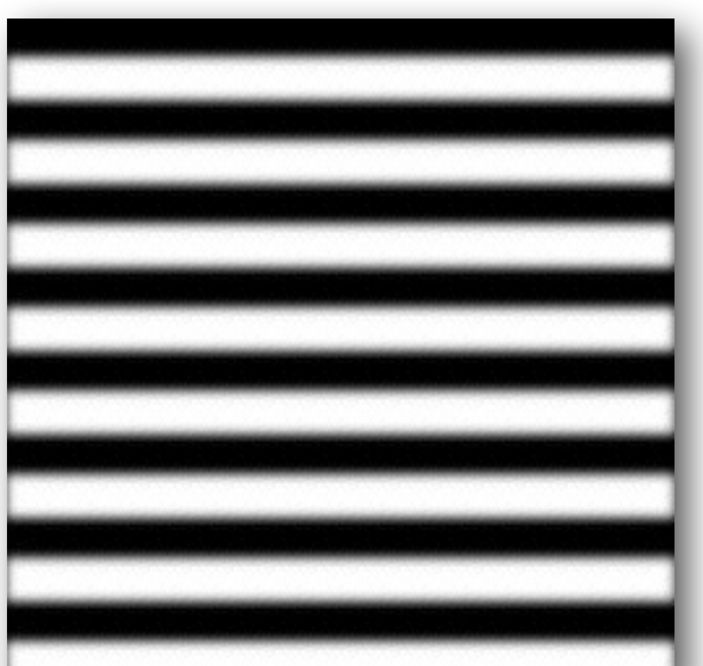


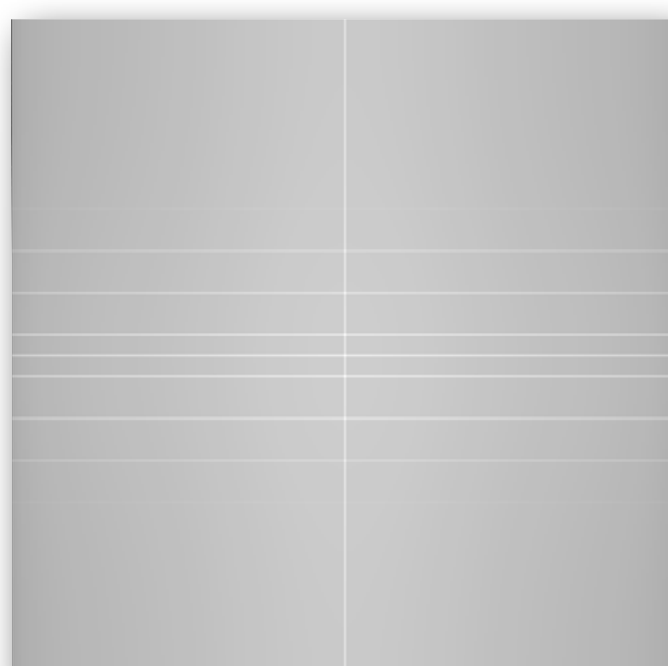
Now, Frequency Spectra for Images

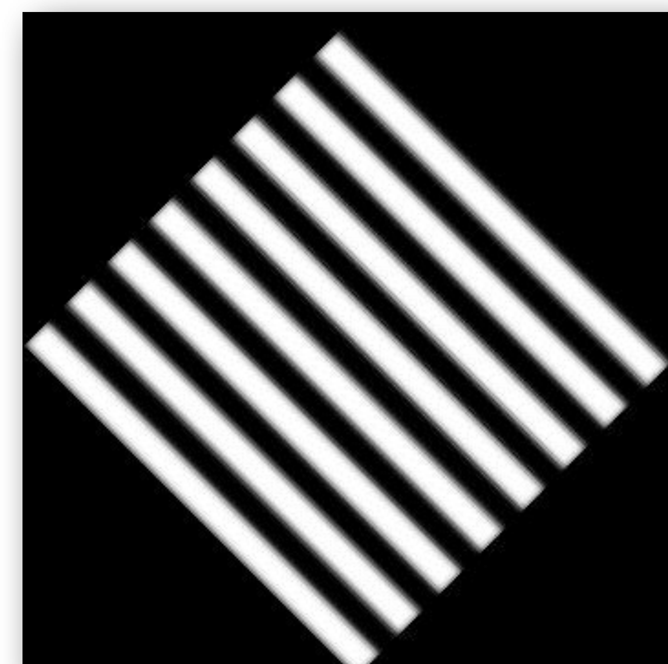


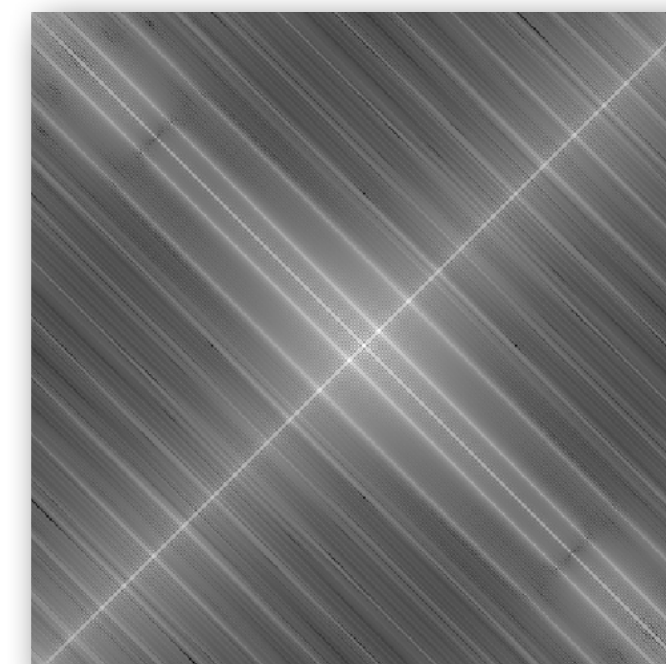


Noise Added

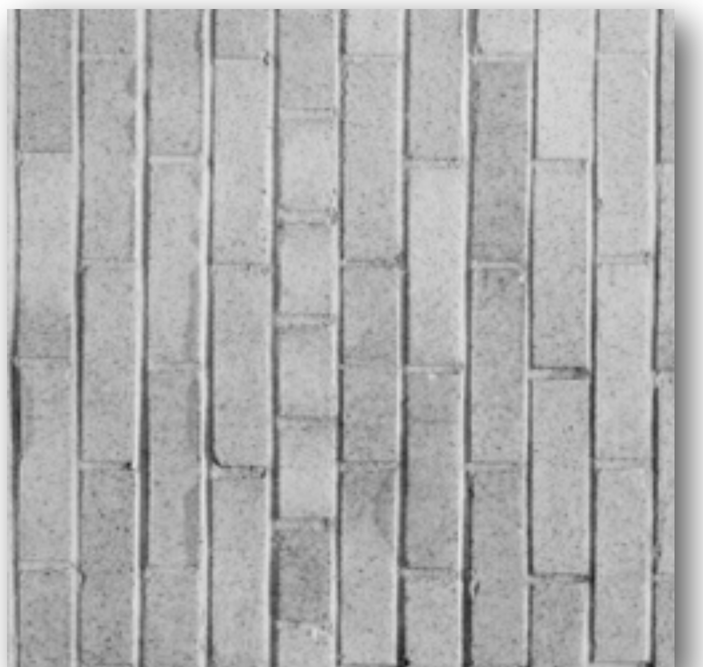


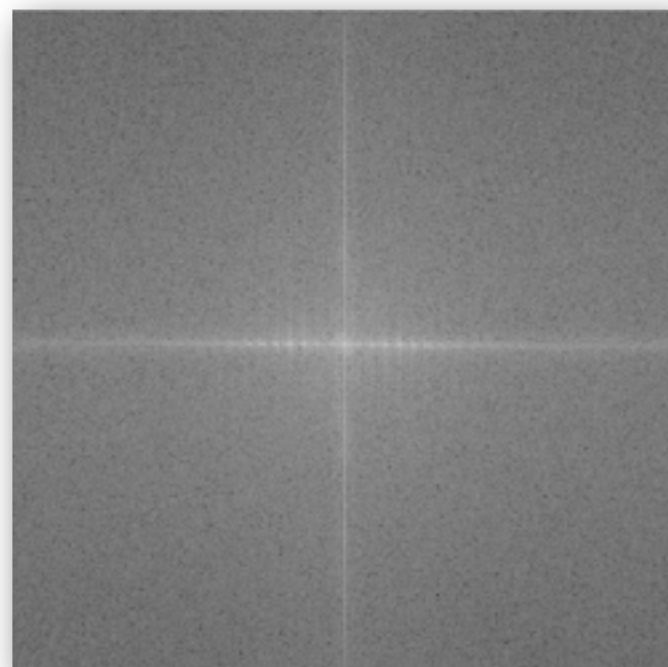


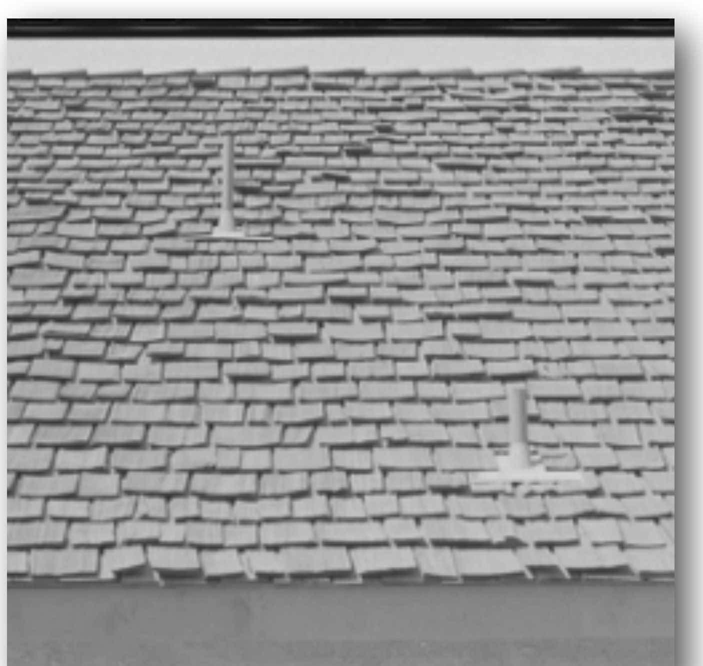


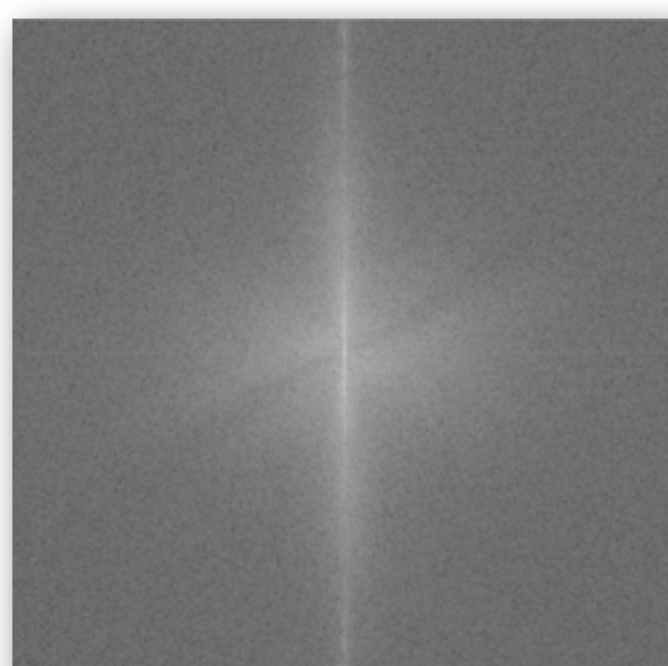


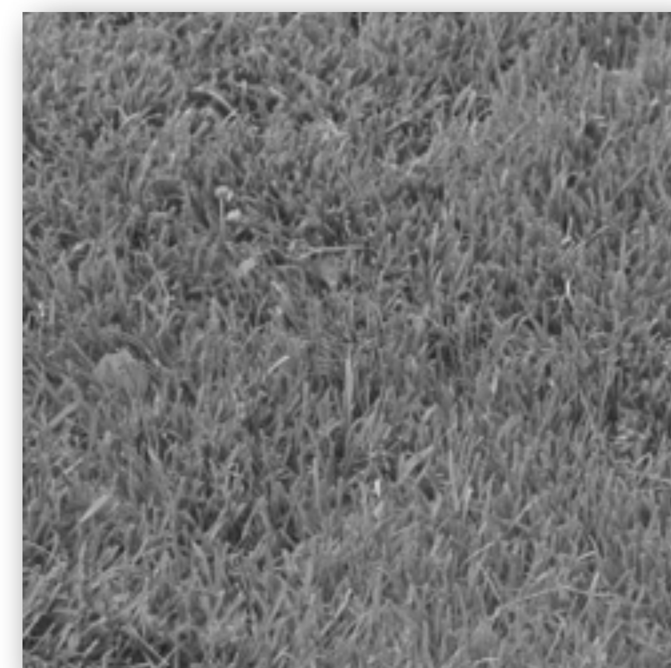
Frequency Spectra for Real Images

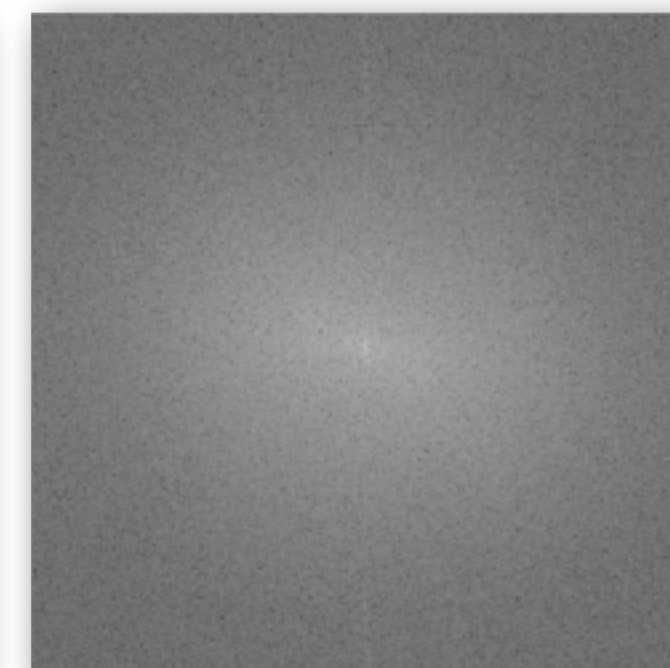












Fourier Transform: Some Observations

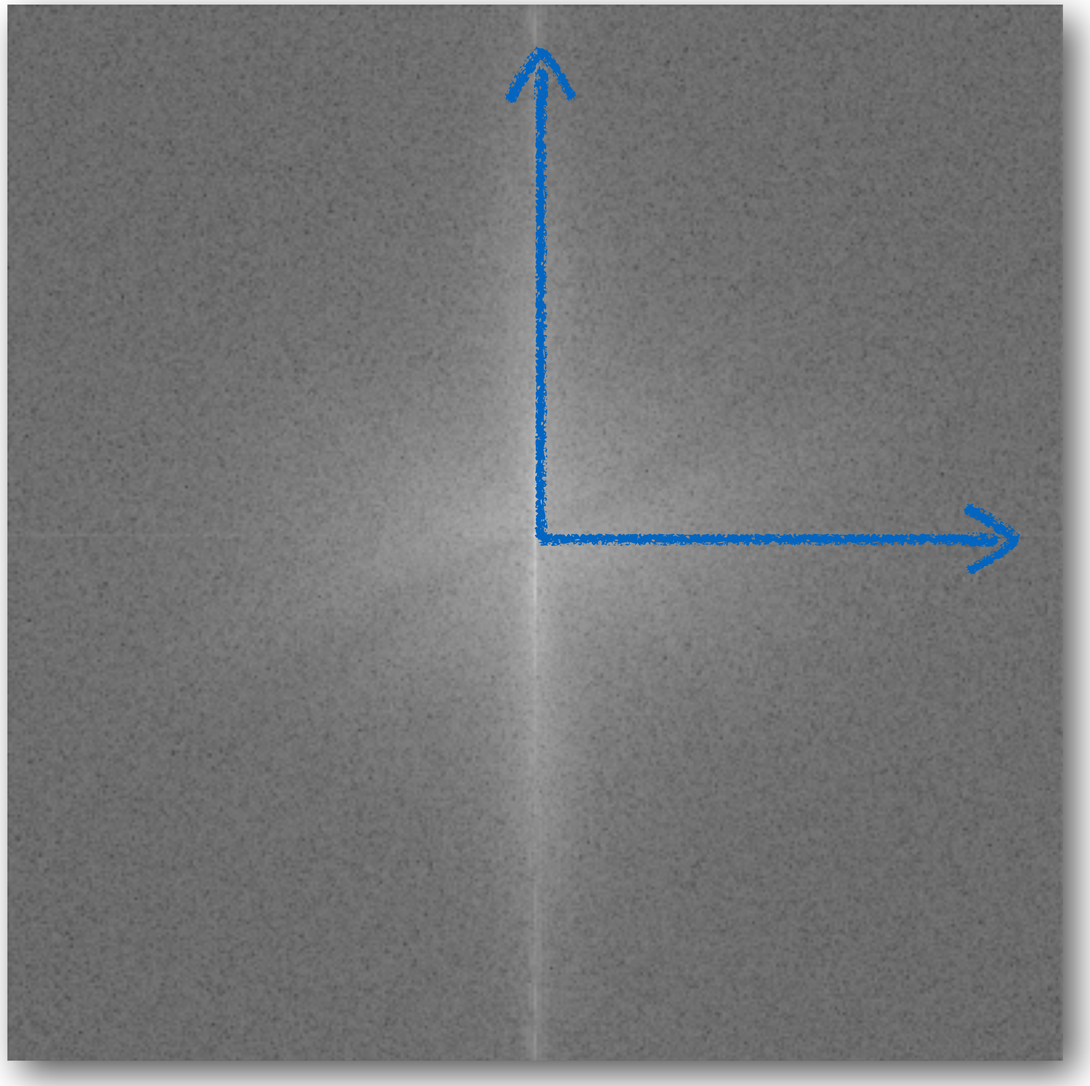
* For every ω from 0 to ∞ (infinity) F(ω) holds the Amplitude A and phase ϕ of a sine function.
* It uses Real and Complex Numbers to achieve this

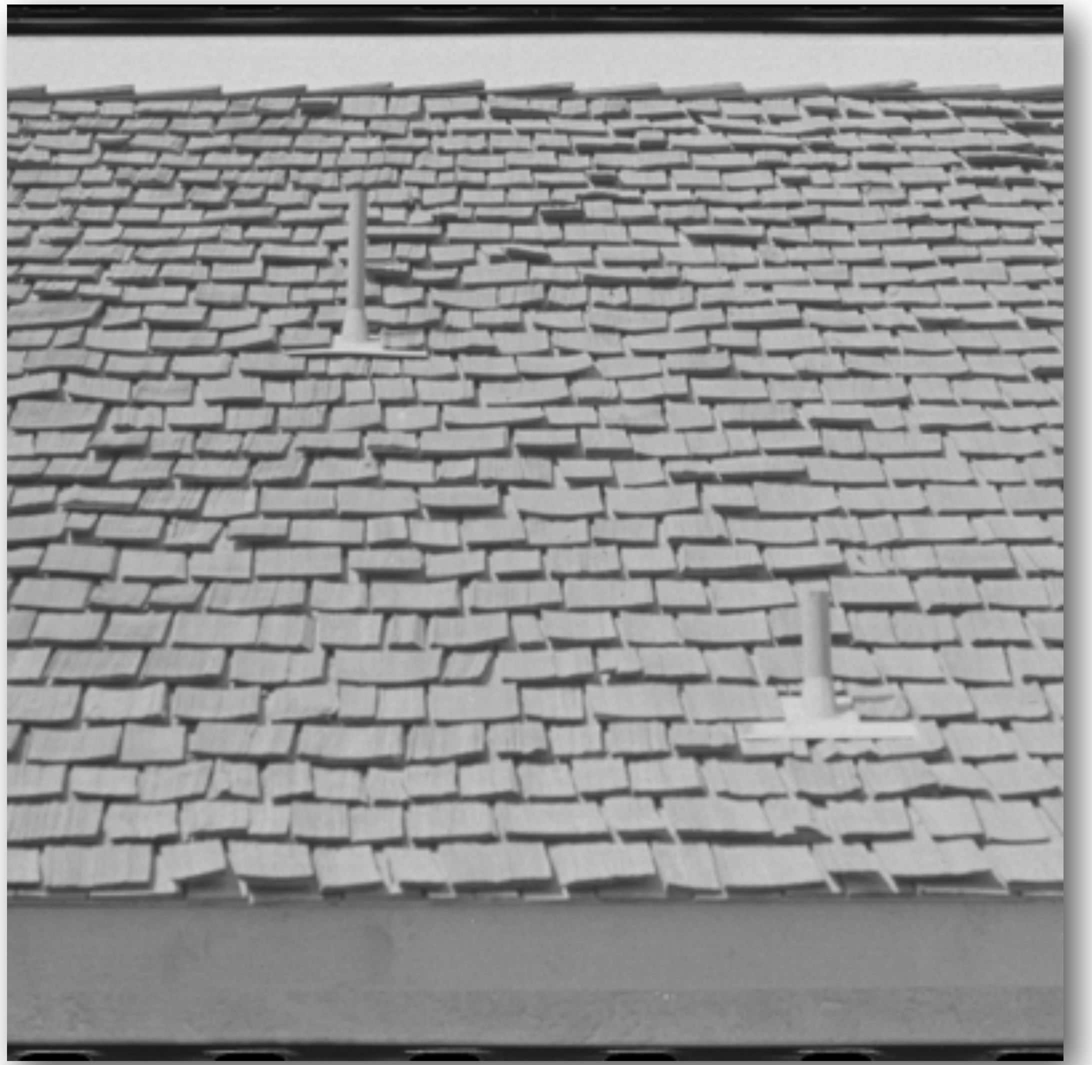
$$A \sin(\omega t + \phi)$$

$$F(\omega) = R(\omega) + jI(\omega)$$

$$A = \pm\sqrt{R(\omega)^2 + I(\omega)^2}$$

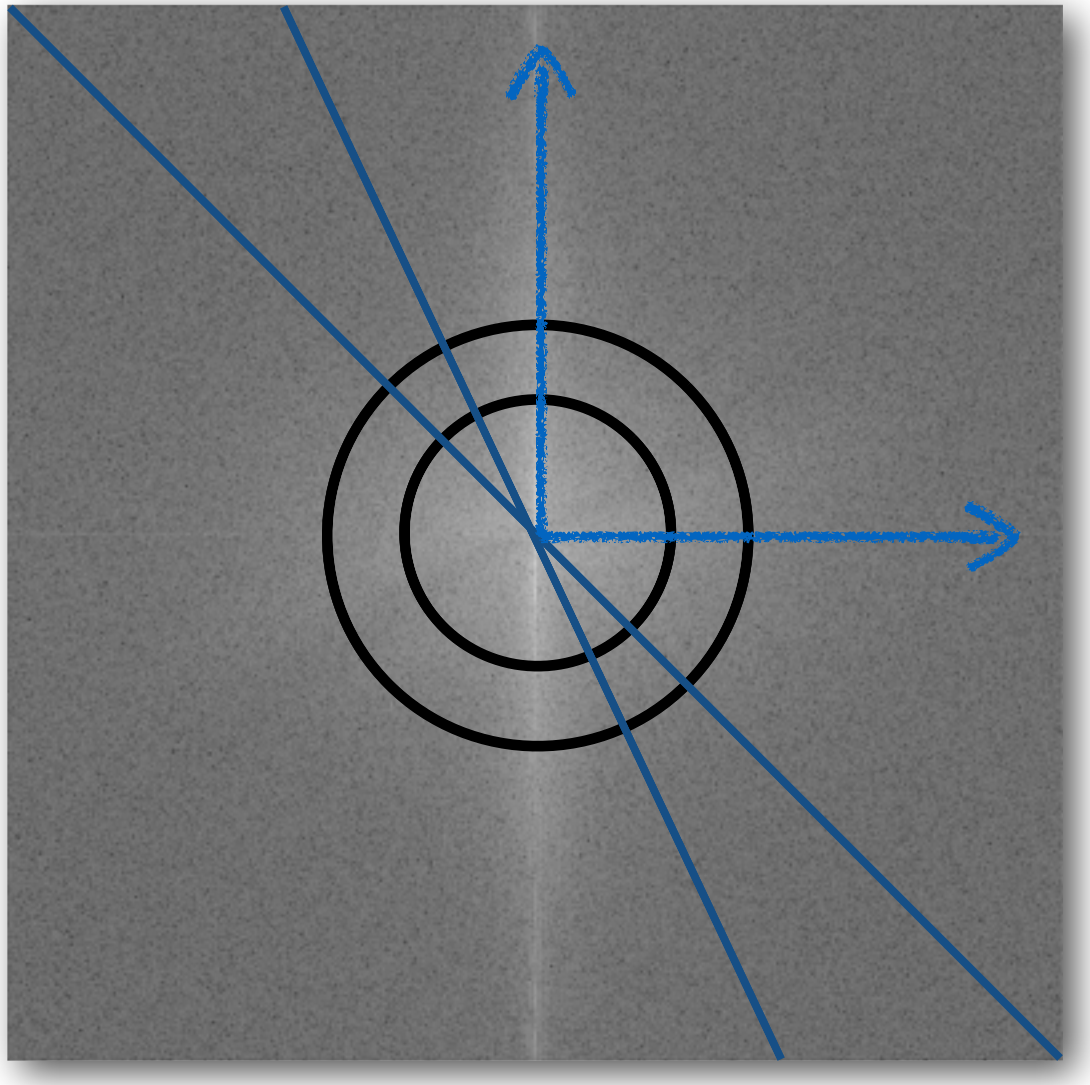
$$\phi = \tan^{-1}\frac{I(\omega)}{R(\omega)}$$

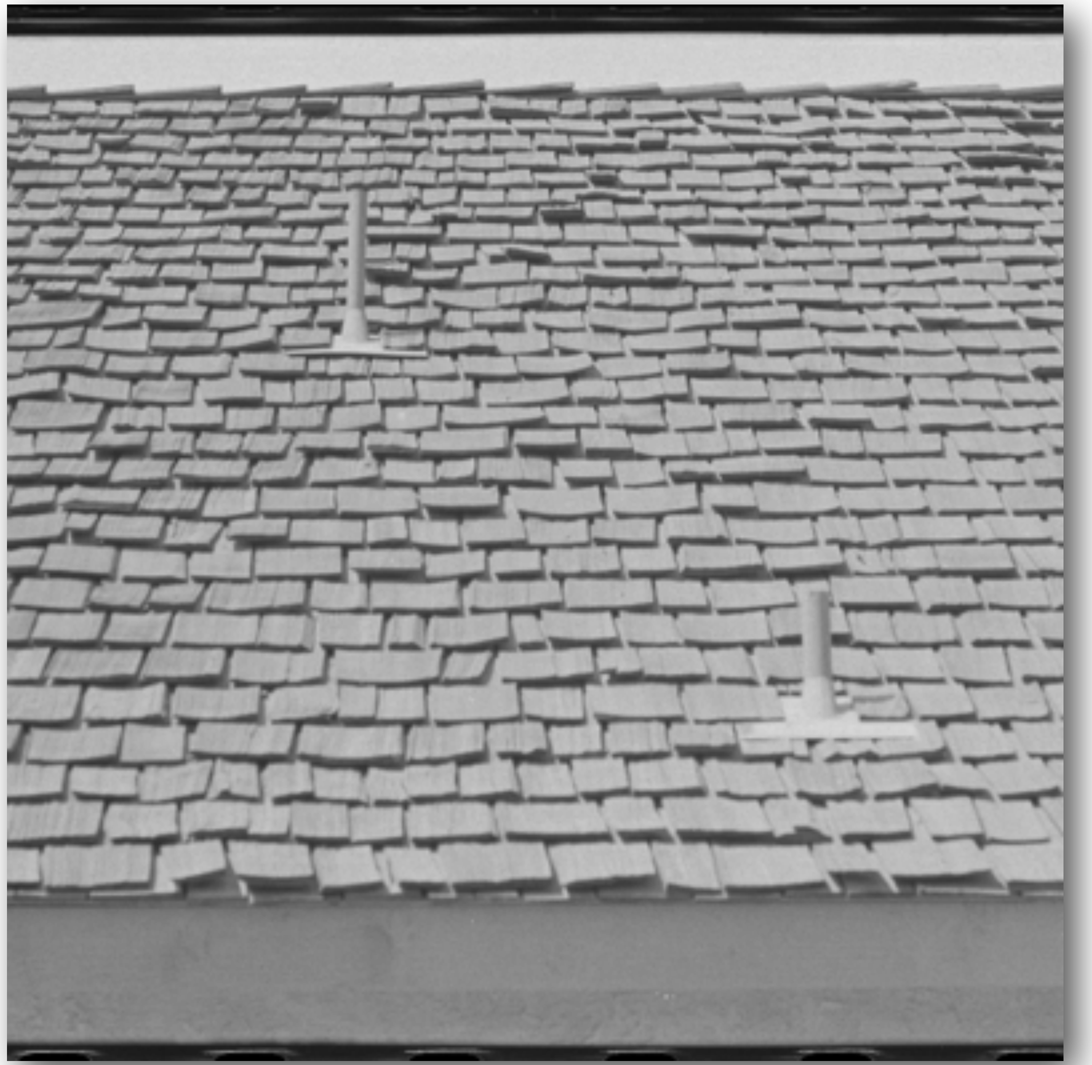




Using the Frequency Spectra

* Low-pass,
* High-Pass,
* Band-pass Filtering
* Change the spectrum and reconstruct





Blurring and Frequencies

Original Image

Gaussian 5x5 Blur

Smooth - Original

Summary

- Introduced how sines and cosines can be used to reconstruct a signal
- Characterized the Fourier Transform using Frequency, Amplitude and Phase
- Introduced the Frequency Domains for a Signal
- Identified the three properties of Convolution as it is associated with the Fourier Transform

Next Class

* Merging and Blending of Images

Credits

* For more information, see
  * Richard Szeliski (2010) Computer Vision: Algorithms and Applications, Springer
  * Forsyth & Ponce (2012), Computer Vision: A Modern Approach, Pearson
* Some concepts in slides motivated by similar slides by A. Efros and J. Hays
* Some images retrieved from
  * http://commons.wikimedia.org/
  * List will be available on website

Computational Photography

* Study the basics of computation and its impact on the entire workflow of photography, from capturing, manipulating and collaborating on, and sharing photographs.

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