Computational Photography

* Study the basics of computation and its impact on the entire workflow of photography, from capturing, manipulating and collaborating on, and sharing photographs.

Georgia Tech College of Computing

Image Processing and Filtering, via Convolution and Cross-Correlation

* Towards Edge Detection: Computing Image Gradients

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1. Detect Features in an image.
2. Edge of an image from the perspective of Information Theory.
3. Use of an Image Gradient to compute Edges.
4. Image Gradient in continuous form for a function; and in a discrete form for an image.

Recall: Convolution and Cross-Correlation

Cross-Correlation:

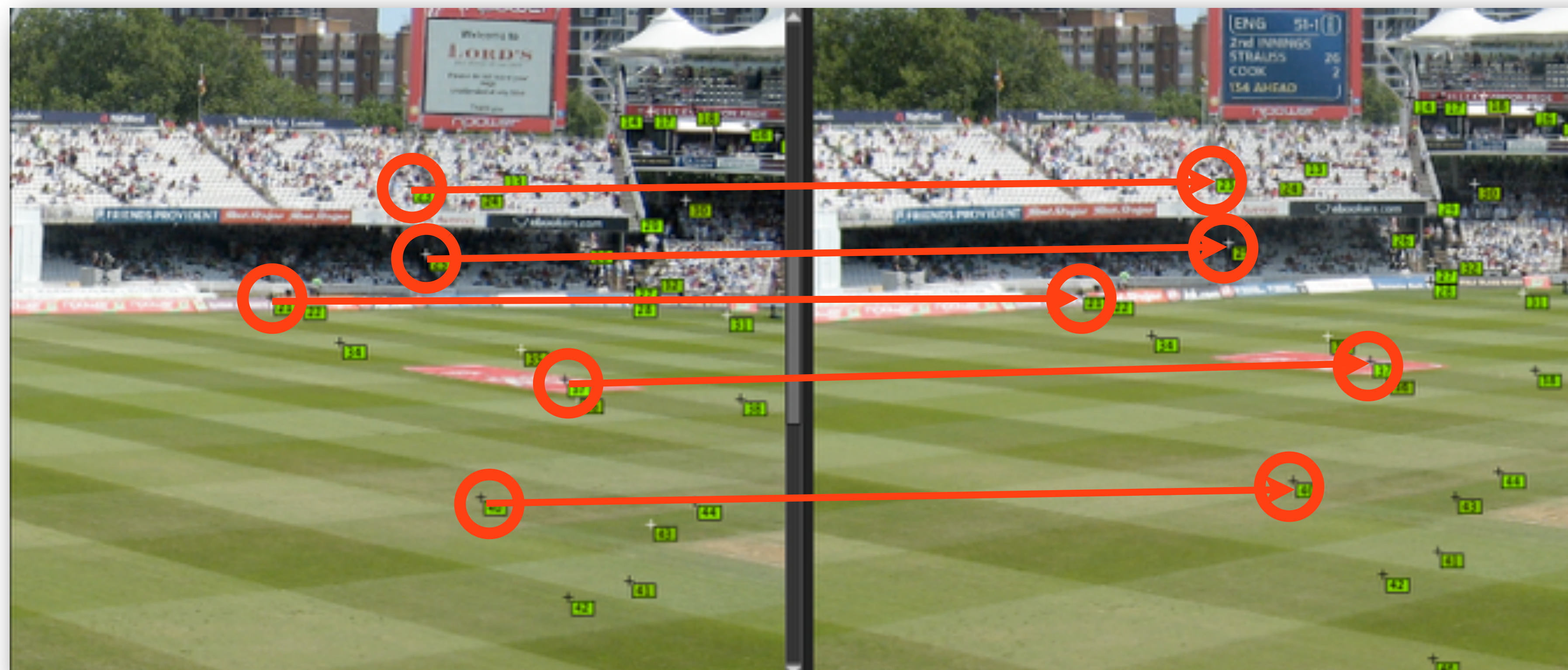
$$G = h \otimes F \qquad G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} h[u,v]F[i+u, j+v]$$

Convolution:

$$G = h * F \qquad G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} h[u,v]F[i-u, j-v]$$

* Filters can be applied to process images
* When **h** is symmetric, which one to use?

Using Filters to Find Features



Extract higher-level "features"

* map raw pixels to an intermediate representation
* Reduce amount of data, preserve useful information

Good Features to Match between Images

Features

* Parts of an image that encode it in a compact form

Edges

* Edges in an image??!!
* Information theory view: Edges encode change, therefore edges efficiently encode an image

What kind of discontinuities are in a scene?

surface normal

depth

surface color

illumination

Slide adapted from Aaron Bobick

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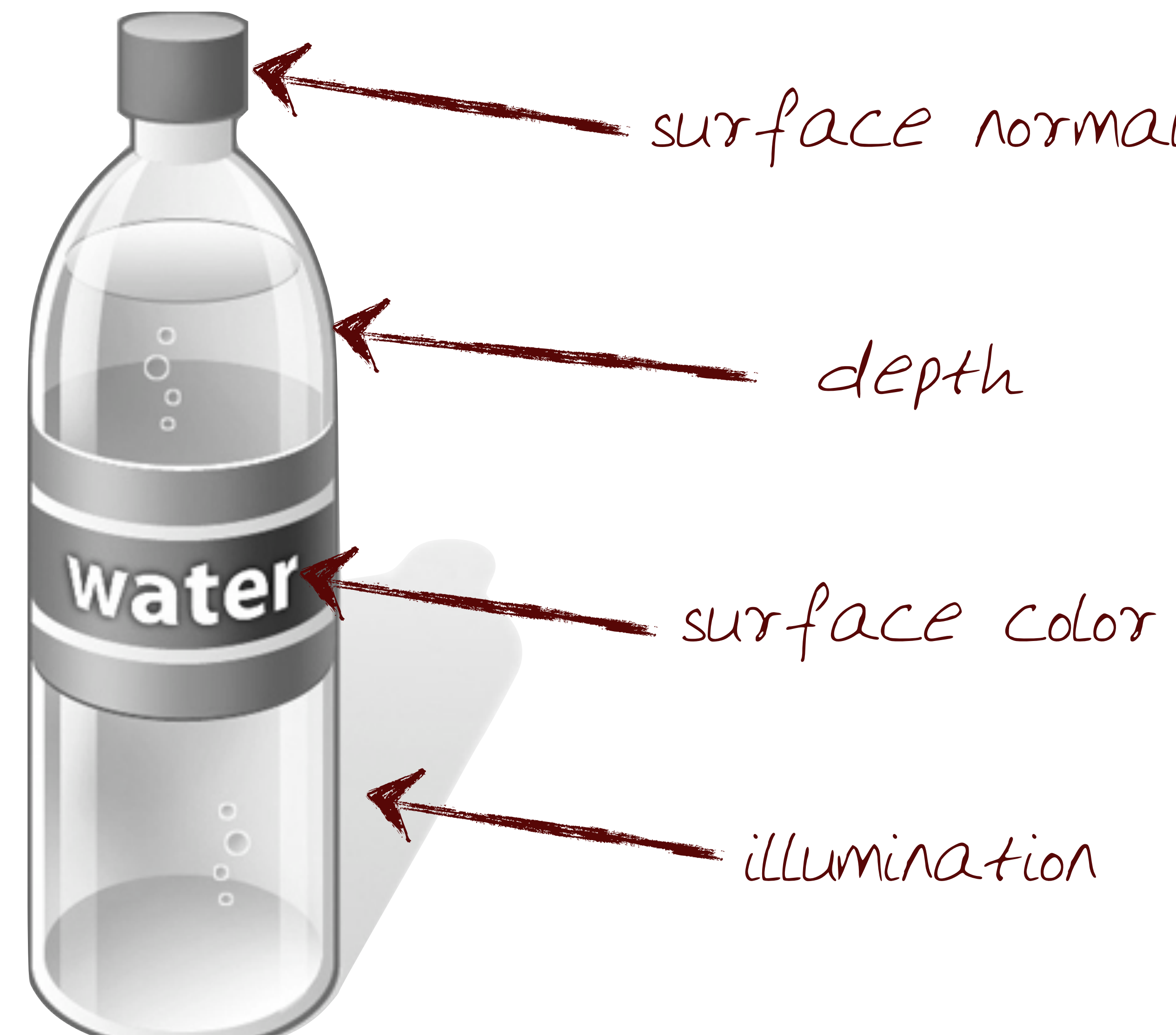

Slide adapted from Aaron Bobick

Edges in Real Images

depth discontinuity

reflectance change

surface normal discontinuity

object boundary

surface color/shape discontinuity

illumination discontinuity (shadows)

Image Courtesy Professor Henrik Christensen

Edges in Real Images

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reflectance change

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illumination discontinuity (shadows)

Image Courtesy Professor Henrik Christensen

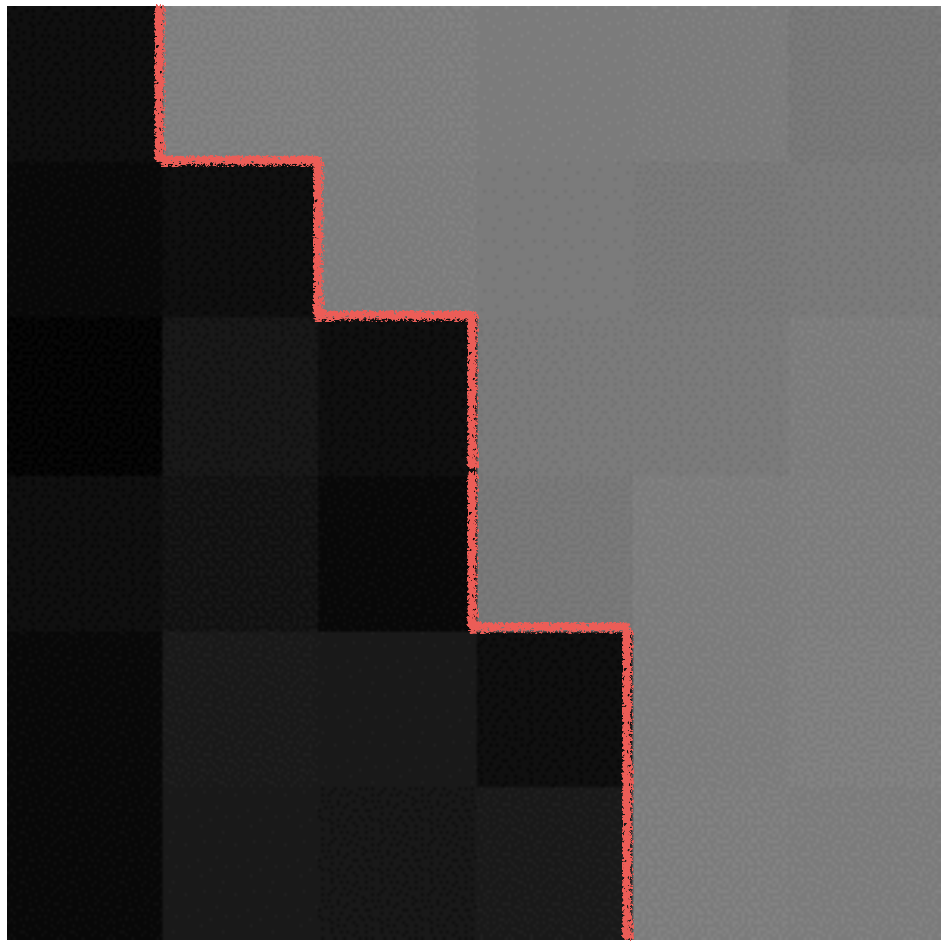
Recall: Images as Functions: $F(x,y)$

Edges appear as ridges in the 3D height map of an image

Edge Detection

Basic Idea:

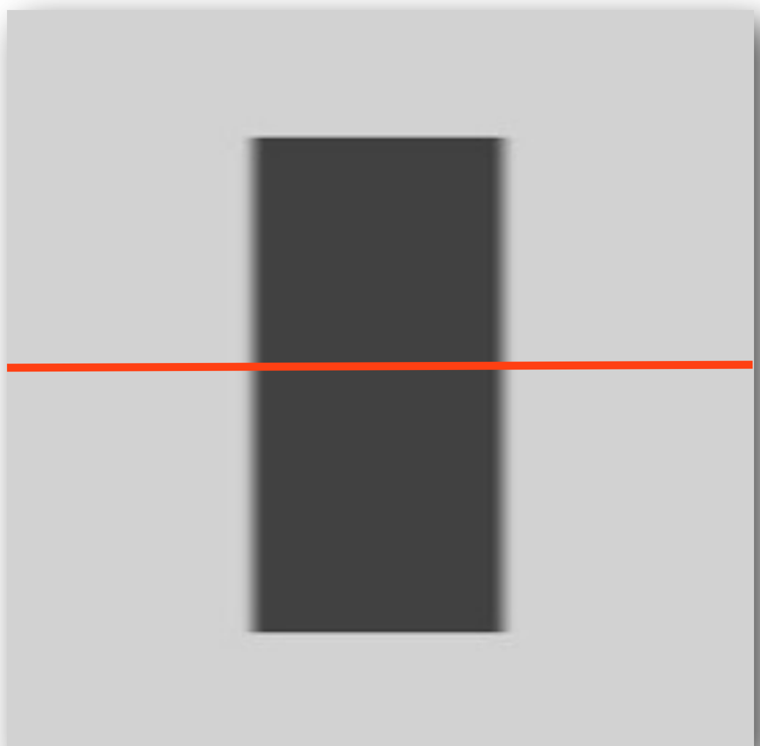
* Look for a neighborhood with strong signs of change



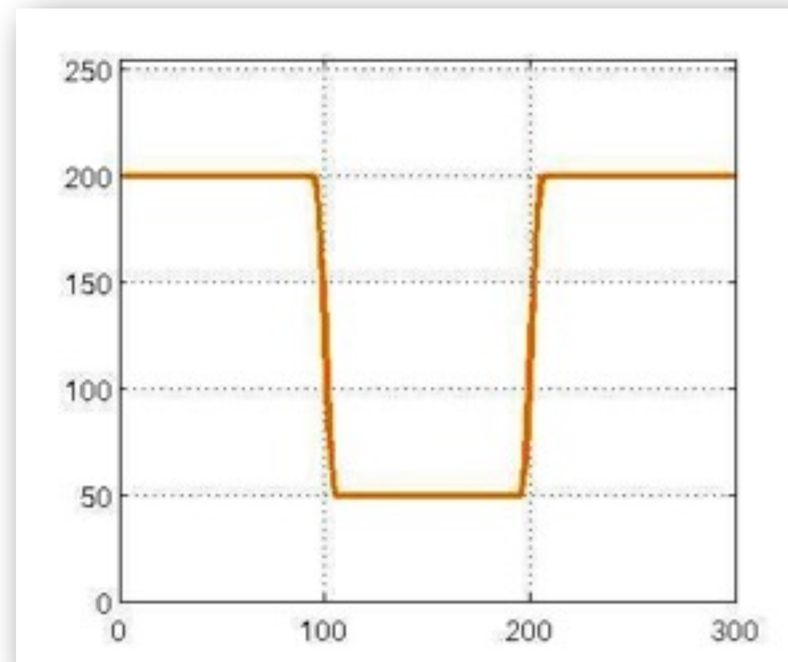
Issues to consider:

* Size of the neighborhood?
* What metrics represent a "change"?

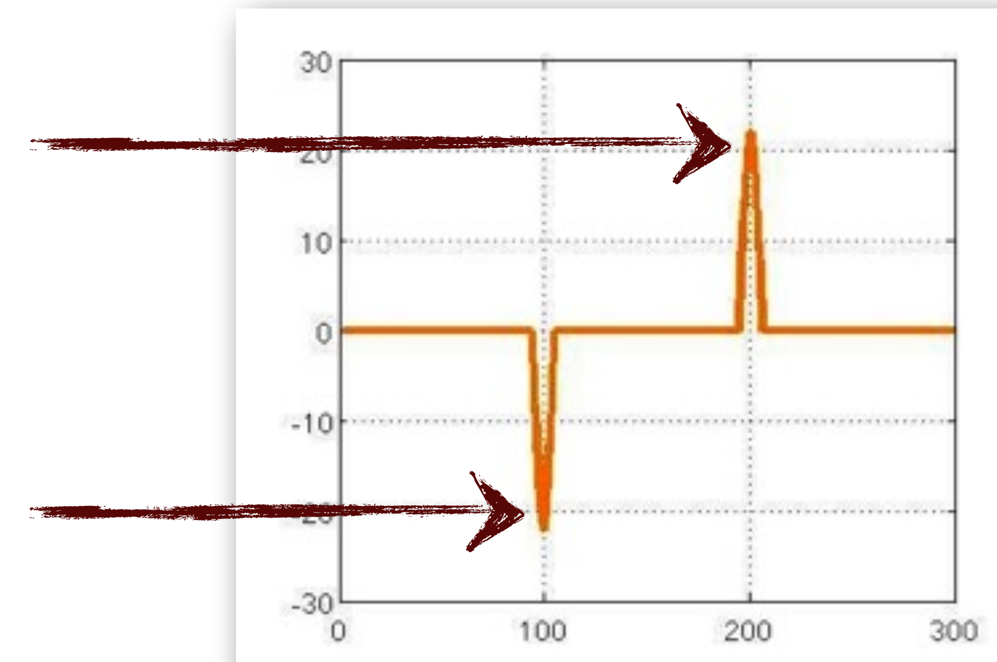
Derivatives of F(x,y) to get Edges



Test Image



Intensity along horizontal scan line (in red)

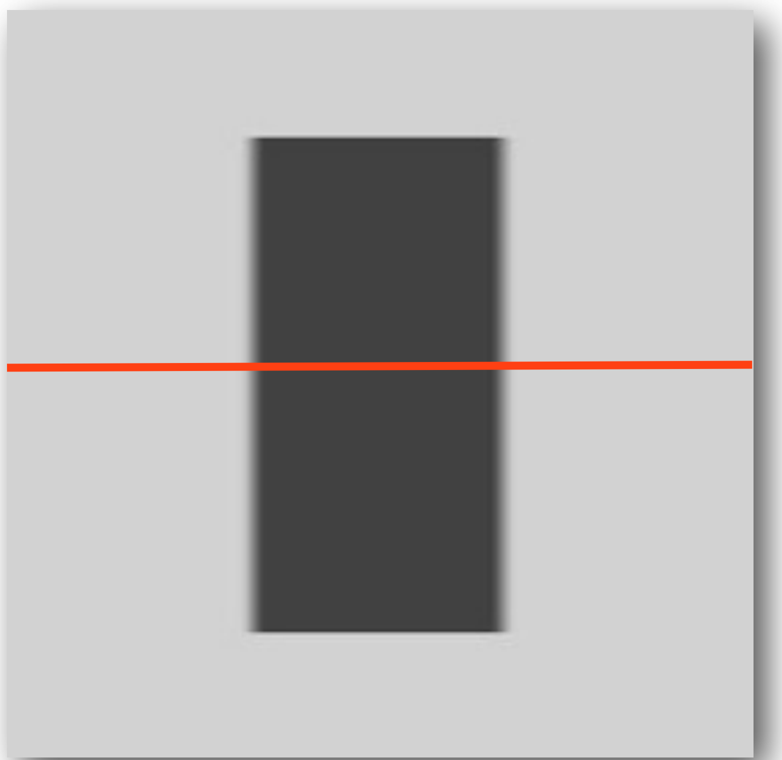


First Derivative

- An edge is where there is rapid change in the image intensity function
- Extrema indicate vertical edges (here we just looked for derivative in x)

Slide adapted from Aaron Bobick and Svetlana Lazebnik

Derivatives of $F(x,y)$ to get Edges



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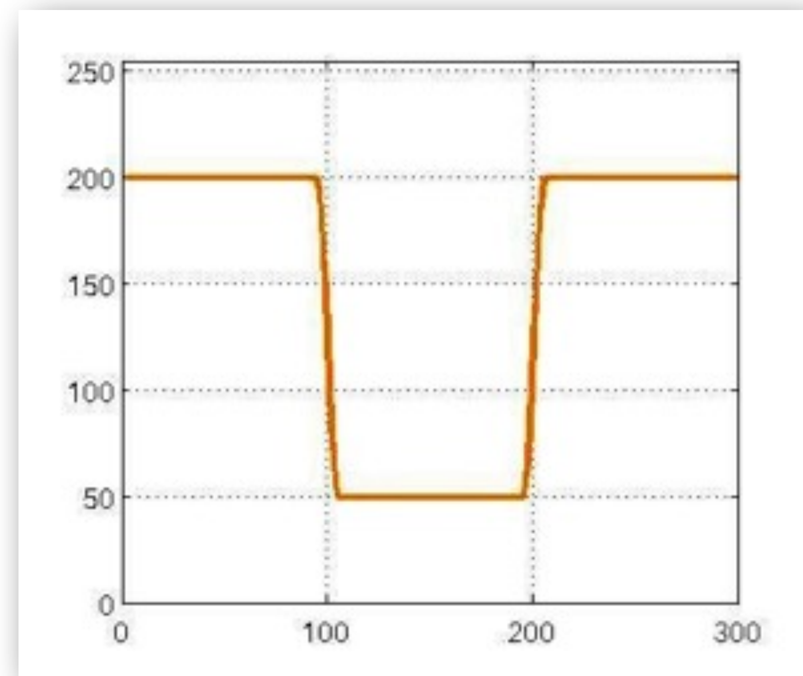

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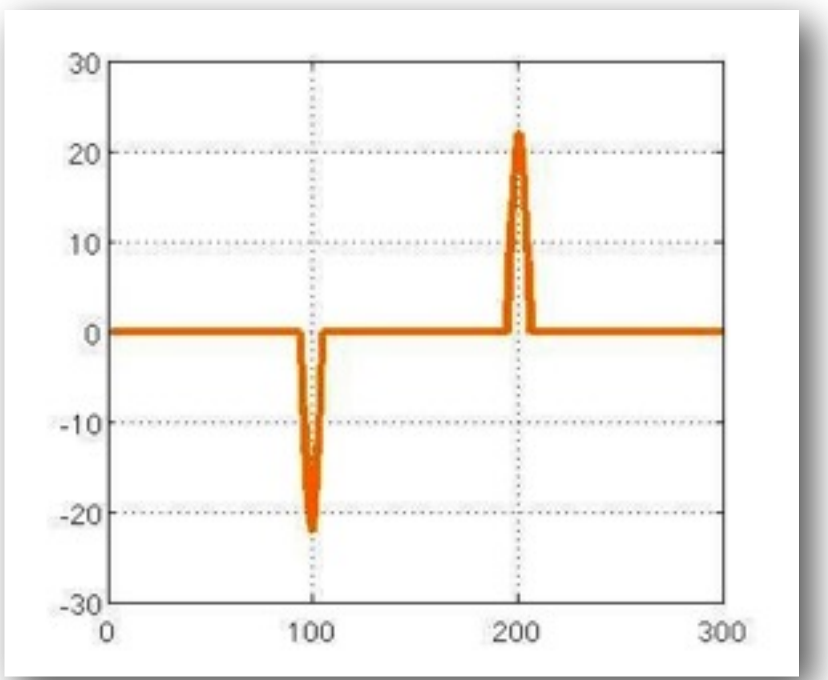

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Differential Operators for Images

Need an operation that when applied to an image returns its derivatives.

* Model these "operators" as masks/kernels.
  When applied, yields a new function that is the image gradient.
* Then "threshold" this gradient function to select edge pixels.

a	b	c
d	e	f
g	h	i

20	20	10	20	10	20	10	10	13
30	0	0	0	0	0	0	0	30
20	0	A	B	C	90	90	0	20
20	0	D	E	F	90	90	0	20
10	0	G	H	I	90	90	0	10
10	0	90	90	90	90	90	0	10
10	0	90	90	90	90	90	0	10
20	0	0	0	0	0	0	0	20
20	20	10	20	10	20	10	10	13

Image Gradient

Need to define "gradient" !

Gradient of an image =

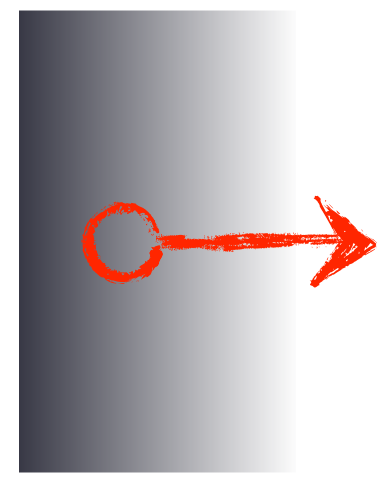
Measure of change in Image function, F(x,y) in x (across / columns) and y (down / rows)

20	20	10	20	10	20	10	10	13
30	0	0	0	0	0	0	0	30
20	0	A	B	C	90	90	0	20
20	0	D	E	F	90	90	0	20
10	0	G	H	I	90	90	0	10
10	0	90	90	90	90	90	0	10
10	0	90	90	90	90	90	0	10
20	0	0	0	0	0	0	0	20
20	20	10	20	10	20	10	10	13

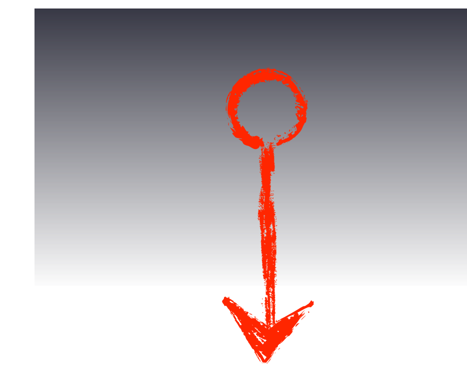
Definition: Image Gradient (Mathematically)

$$\nabla F = \left[\frac{\delta F}{\delta x}, \frac{\delta F}{\delta y}\right]$$

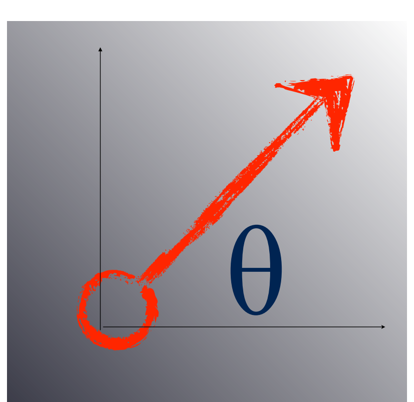
measure of change in Image function (**F**), in **x** and **y**



$$\nabla F = \left[\frac{\delta F}{\delta x}, 0\right]$$



$$\nabla F = \left[0, \frac{\delta F}{\delta y}\right]$$



$$\nabla F = \left[\frac{\delta F}{\delta x}, \frac{\delta F}{\delta y}\right]$$

Note: Gradient points in the direction of most rapid increase in intensity ($\boldsymbol{\theta}$)

Slide adapted from Aaron Bobick

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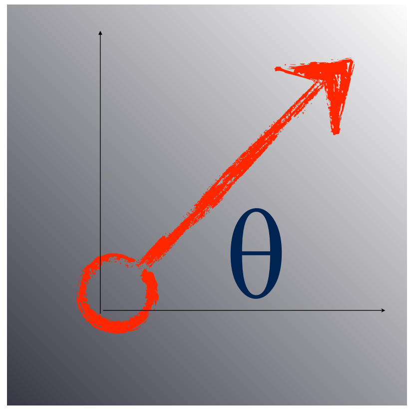
measure of change in Image function (**F**), in **x** and **y**



Gradient Direction is:

$$\theta = \tan^{-1} \left[\frac{\delta F}{\delta y} / \frac{\delta F}{\delta x} \right]$$

Gradient magnitude provides edge strength:

$$\| \nabla F \| = \sqrt{\left(\frac{\delta F}{\delta x} \right)^2 + \left(\frac{\delta F}{\delta y} \right)^2}$$

Q: How does this relate to edge direction?

Slide adapted from Aaron Bobick

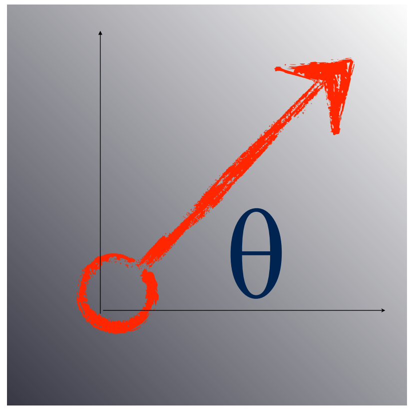
Definition: Image Gradient (Discrete)

For 2D function, F(x,y), the partial derivative is:

$$\frac{\delta F(x,y)}{\delta x} = \lim_{\epsilon \to 0} \frac{F(x+\epsilon,y) - F(x,y)}{\epsilon}$$

For discrete data, we can approximate using finite differences:

X : $$\frac{\delta F(x,y)}{\delta x} \approx \frac{F(x+1,y) - F(x,y)}{1}$$



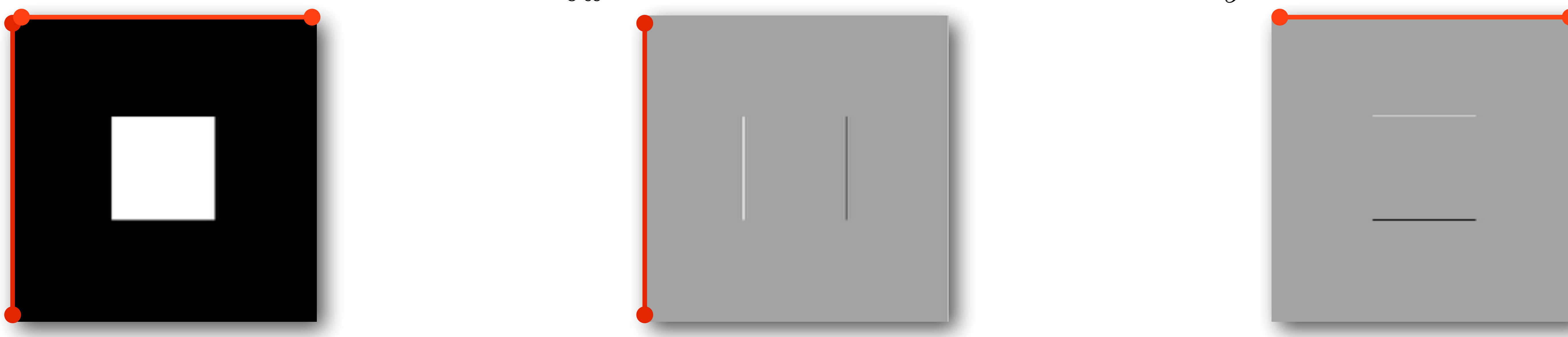
Y : $$\frac{\delta F(x,y)}{\delta y} \approx F(x,y+1) - F(x,y)$$

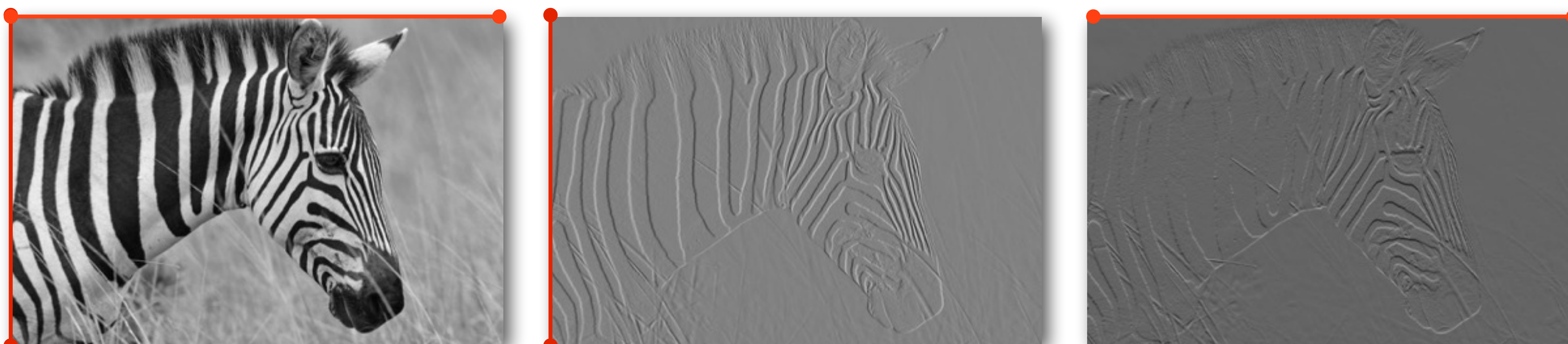
Slide adapted from Aaron Bobick

Differentiating an Image in X and Y

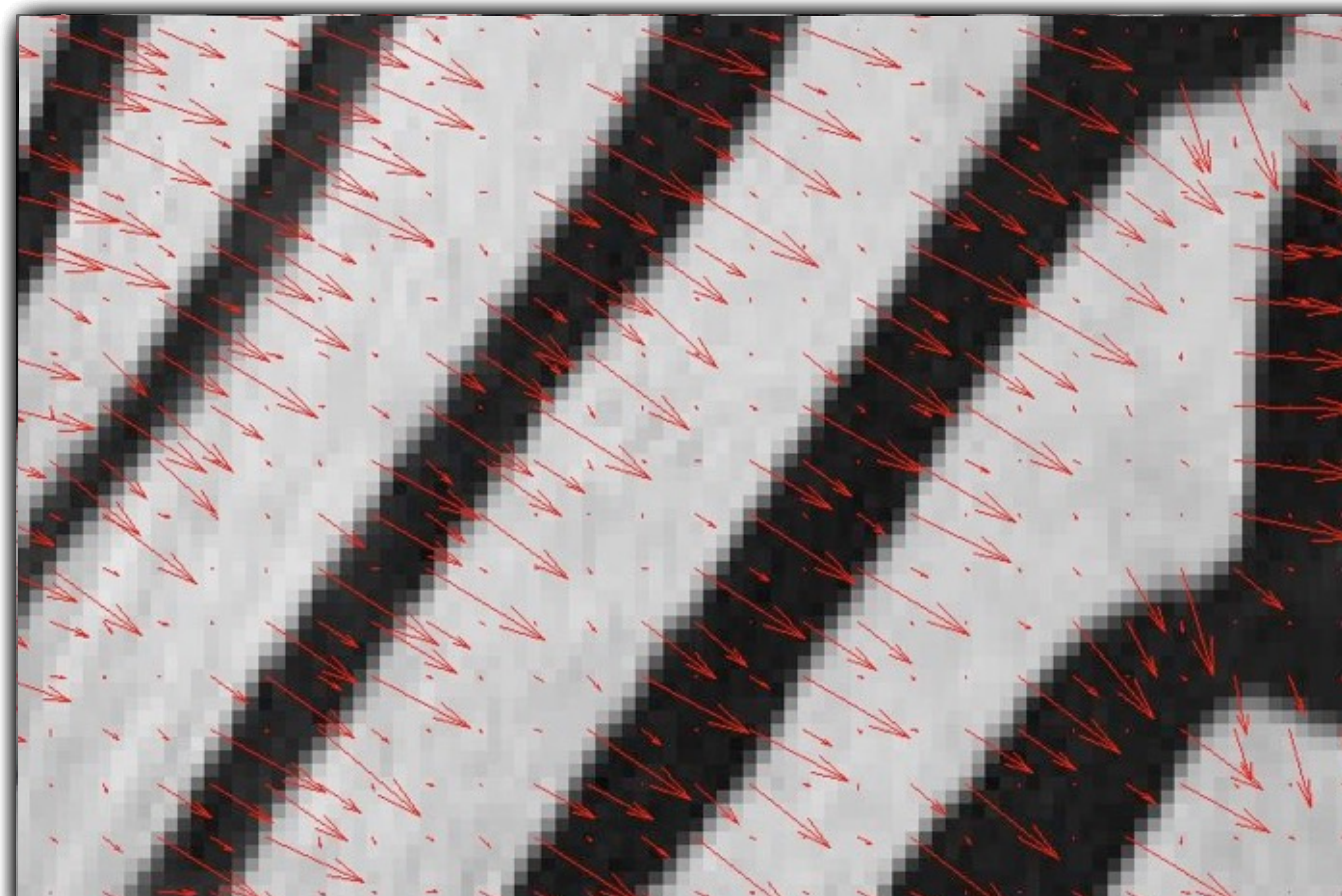
$$\frac{\delta F(x,y)}{\delta x} \approx F(x+1,y) - F(x,y)$$

$$\frac{\delta F(x,y)}{\delta y} \approx F(x,y+1) - F(x,y)$$





Gradient Images



$$\| \nabla F \| = \sqrt{\left(\frac{\delta F}{\delta x}\right)^2 + \left(\frac{\delta F}{\delta y}\right)^2}$$

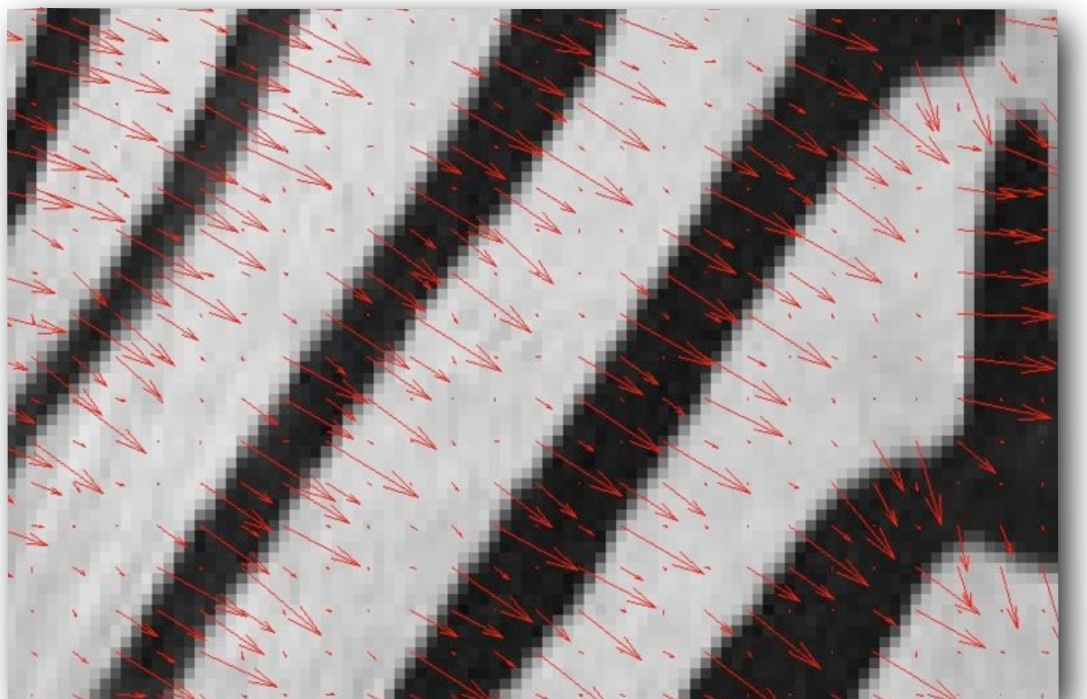
$$\theta = \tan^{-1}\left[\frac{\delta F}{\delta y} / \frac{\delta F}{\delta x}\right]$$

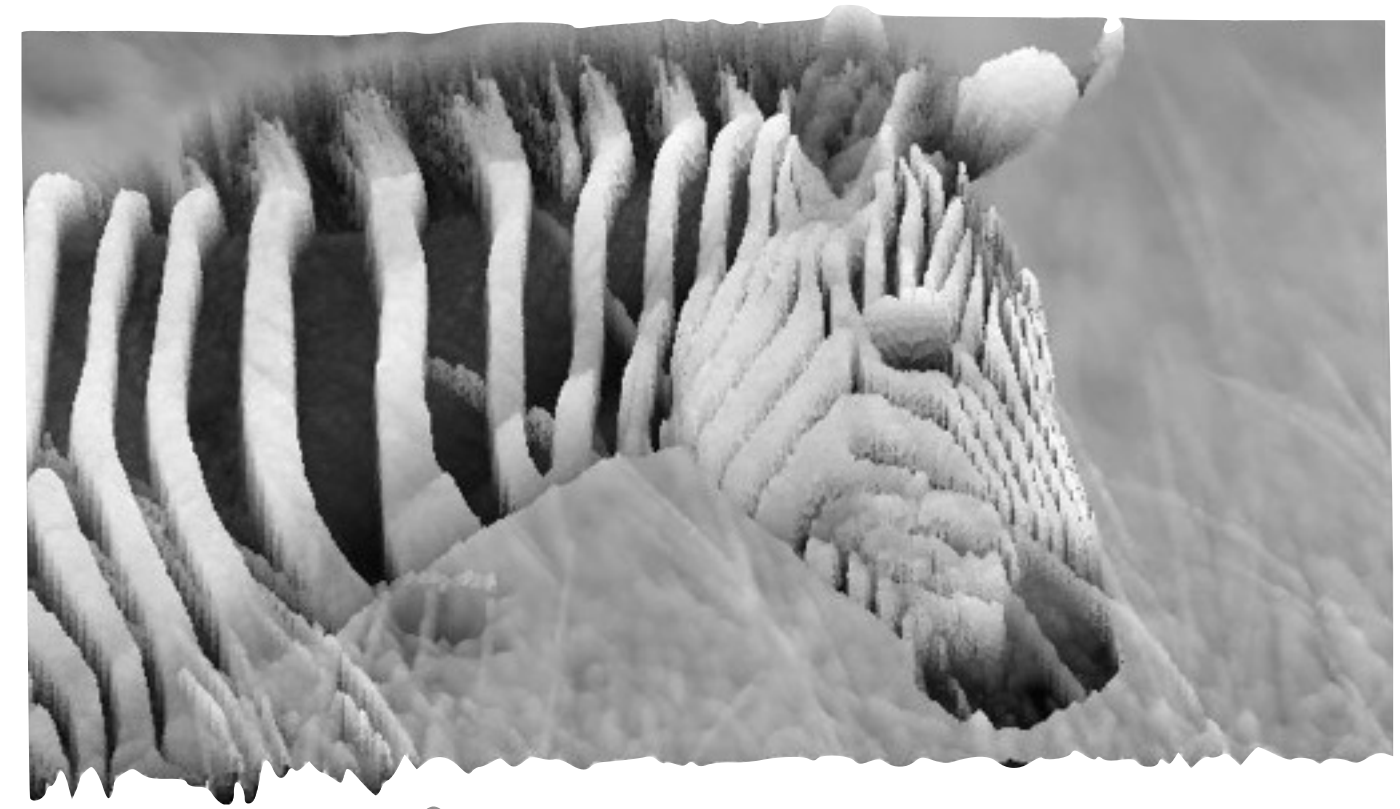
Visualizing Gradients

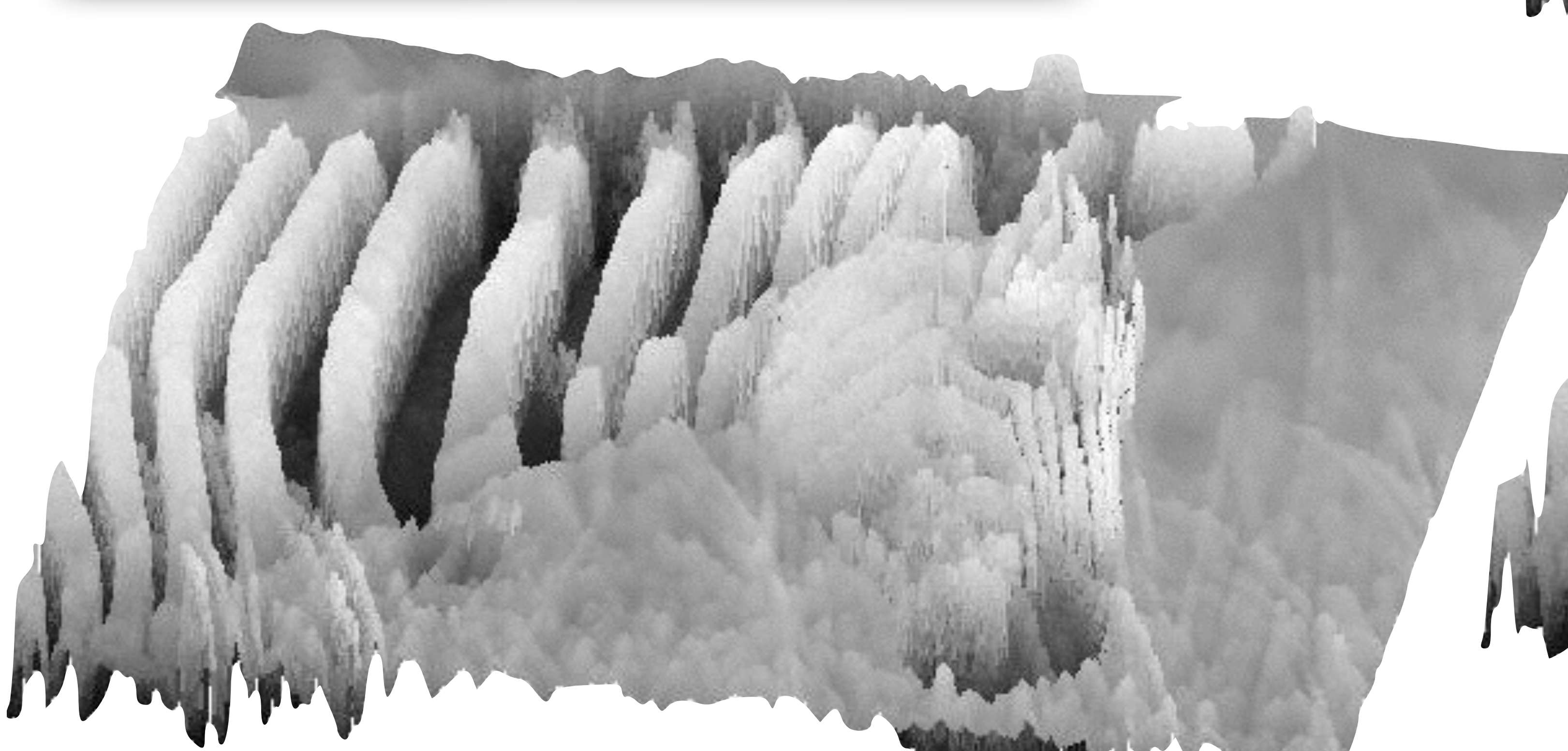
$$\| \nabla F \| = \sqrt{\left(\frac{\delta F}{\delta x}\right)^2 + \left(\frac{\delta F}{\delta y}\right)^2}$$

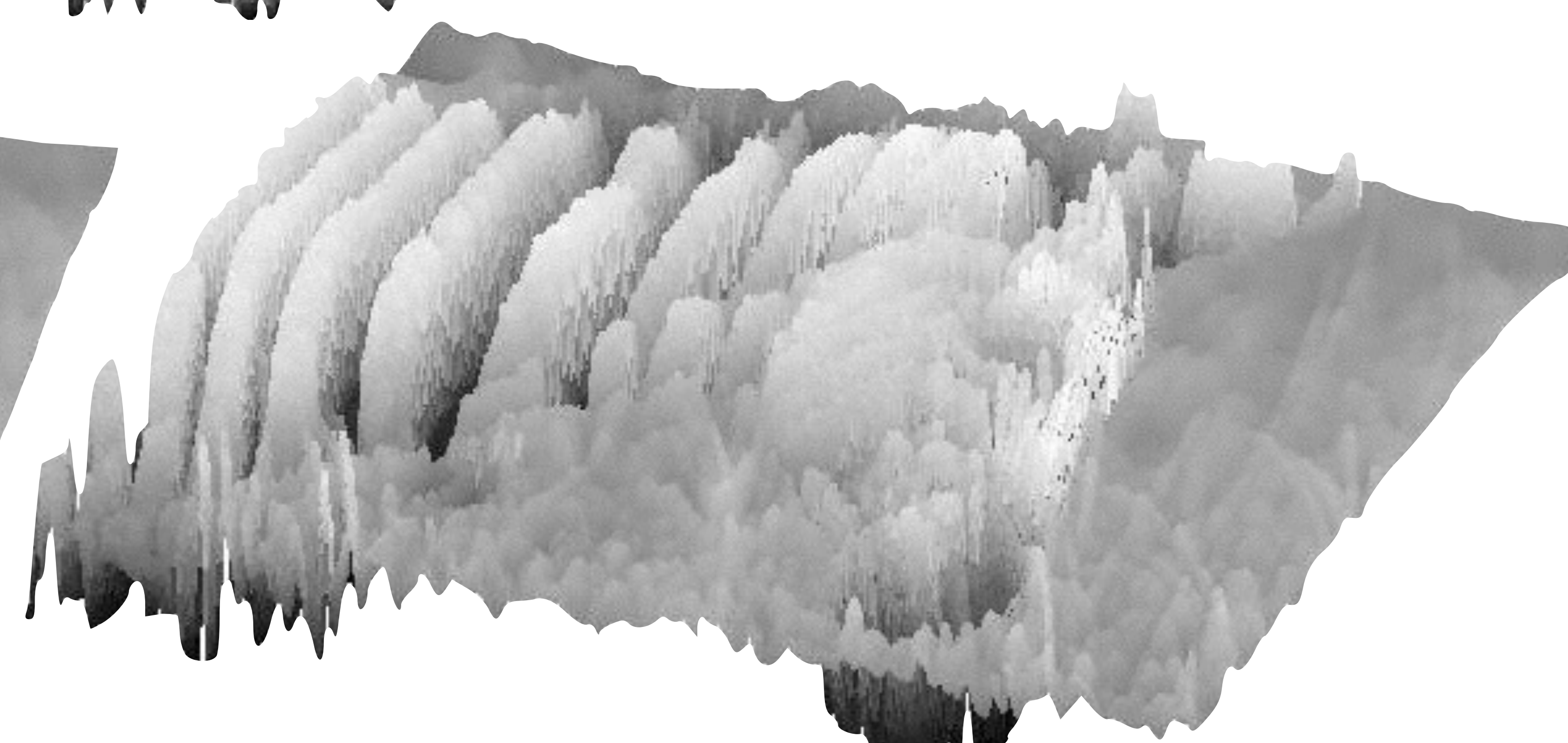
$$\theta = \tan^{-1}\left[\frac{\delta F}{\delta y} / \frac{\delta F}{\delta x}\right]$$

Visualizing Gradients









Summary

* Detecting features for matching
* Compute Edges by measuring changes across an image
* Use of derivatives to compute the gradient over an image

Next Class

Image Analysis: Edge Detection using Gradients

Credits

* Matlab software by Mathworks Inc.
* Some Slides adapted from Aaron Bobick
* For more information, see Szeliski OR Forsyth & Ponce Text Book.
* Images
    * Image Courtesy Professor Henrik Christensen
    * Images used from USC's Signal and Image Processing Institute's Image Database
    * Zebra Image by http://www.flickr.com/photos/lipkee/2904603582/