Constrained Model Predictive Control for Learned Dynamics
CS 294-112

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1 Abstract

Deep reinforcement learning has recently made progress in traditionally unsolvable problems for continuous control. Such continuous control problems are characterized by a MDP setting, where an agent interacts with the world, characterized by a set of states $\mathcal{S}$, by taking actions chosen from a set of actions $\mathcal{A}$ to maximize the expected net present value of a reward given over time. Episodes of a single problem share the same MDP setting, but differ in the initial state. We are interested in spaces where the dimension of $\mathcal{A}$ is moderately large, continuous, or both; adversarial and stochastic worst-case or high-probability methods are not applicable.

We consider a model-based method, Model Predictive Control (MPC), with unknown dynamics. In other words, both the transition model and policy for an unknown MDP must be learned, only by interacting with the environment. We find that even for simple continuous control problems, dynamics learning can be difficult. The usual MPC algorithm, which seeks to optimize over any actions and states over a small horizon, exhibits pathological behavior even for short horizons with poor dynamics due to a blunt optimization approach.

We show that the root of the pathological behavior with unconstrained MPC derives from over-optimism during the planning phase. As an MPC agent improves its planning using a trained dynamics model, the agent actually tends to perform worse when using the real dynamics. In particular, the objective value of the reward function at the end of the planning phase can be viewed as the planner's estimator for future rewards. Treating it this way we find that this value over-inflates as the MPC planning optimization quality improves.

We handicap MPC planning by only allowing optimization along the first action, so while planning may account for future steps in the simulated horizon, no new actions can be taken. This handicapped optimization performs uniformly better on a variety of small continuous control tasks.

By taking into account, even crudely, for some uncertainty in the dynamics model, we can improve MPC performance. This indicates that a more careful approach which carefully analyzes dynamics uncertainty may be a promising direction for future work.

2 Introduction

Model-free reinforcement learning algorithms directly learn a policy that maps a state to the action that, when taken, maximizes the expected reward returned by the environment. Model-free methods perform all planning *implicitly* and only with state information. Model-based reinforcement learning algorithms, on the other hand, construct models of the environment's dynamics and use the models to *explicitly* plan. These model-based methods tend to have improved sample complexities relative to their model-free counterparts.

However, constructing a model of the world—even in its restricted form as a simple dynamics prediction problem in continuous control—complicates the existing deep RL architecture. Purely model-based methods assume they have a completely accurate dynamics model and optimize expected reward under this assumption. Understandably, such methods are inherently limited by how accurate the dynamics model is.

For instance, one such pure model-based algorithm, iLQG, was outperformed by an algorithm which used a learned model, Guided Policy Search [4]. Moreover, generic global models, such as neural networks, have not historically performed as well as simple, local linear models [2]. Altogether, pure model-based methods are fundamentally limited by *model bias* and pure model-free methods are fundamentally limited by *policy variance*.

As is common in machine learning, we need to find the right trade-off between bias and variance. One way to bridge this gap is to explicitly include model-predicted rewards in the value estimates. In the style of SVG [3], we might accelerate value learning by mixing in a model-based expansion of our value estimates, where the value $V^\pi : \mathcal{S} \to \mathbb{R}$ (the true net present value for the reward at a given state) is approximated with the help of dynamics $f : \mathcal{S} \times \mathcal{A} \to \mathcal{S}$ describing transitions deterministically for simplicity and the reward $r : \mathcal{S} \times \mathcal{A} \times \mathcal{S} \to \mathbb{R}$ for a policy $\pi : \mathcal{S} \to \mathcal{A}$:

$$V^\pi(s_1) \approx \left(\sum_{t=1}^{H} r(s_t, a_t, s_{t+1}) \right) + \hat{V}(s_{H+1}); \quad s_{t+1} \triangleq f(s_t, a_t), a_t \triangleq \pi(s_t)$$

While a viable approach, an implementation must take care to account for bias in f, which may skew estimates in V, no matter how good $\hat{V}$ is. Second, this introduces a nuanced coupling of the training of $f, \hat{V}, \pi$ at the same time.

3 Background

We investigate a use of dynamics models that is presumably less-coupled with policy learning. We will revisit the potential for fused model-free and model-based methods at the end of the report. MPC chooses actions using an open-loop planner P that accepts a dynamics model f and a horizon H and returns the next H actions to play. MPC iteratively queries the planner at each time step, playing only the first action in the plan (Algorithm 1).

Algorithm 1 The MPC algorithm acts as a policy, returning an action for a given state, using an internal planner and a dynamics model.

```
1: procedure MPC(state s, horizon H, reward r, dynamics f, planner P)
2:     {a_t}_{t=1}^H = P(s, H, r, f)
3:     return a_1
```

Most planners treat f as the true dynamics and attempt to solve the following maximization, which is frequently intractable:

$$\begin{aligned} \max_{\{a_t, s_t\}_{t=1}^H} \quad & \sum_{t=1}^{H} r(s_t, a_t) \\ \text{s.t.} \quad & s_{t+1} = f(s_t, a_t) \end{aligned} \tag{1}$$

A typical way of estimating a solution to Equation 1 is with the random shooter method, which samples the t-th action from the distribution $A_t(s_t)$ (Algorithm 2).

We believe RandomShooter, or more generally any solution to Equation 1, works well when the corresponding reward R_i is a decent estimator for the true H-step reward of the agent. It is important to note that the agent takes $a_1^{(i_*)}$ now and continues with MPC later, so there will necessarily be some mismatch. In fact, one might expect positive bias, since the true reward comes from planning H steps ahead for each of the agent's next H steps, looking ahead $2H$ timesteps, as opposed to their initial estimate of its reward through Algorithm 2, which only looks H steps ahead.

Assuming a good reward estimator R_i, choosing the first action $a_1^{(i_*)}$ that results in the best R_i would be the smartest move one could make. Usually, one specifies $A_t(s) = \text{Uniform}\,\mathcal{A}$ for a rectangle $\mathcal{A}$, independent

Algorithm 2 The RandomShooter algorithm is a planner that approximately solves Equation 1, where the quality of the approximation improves with the number of trials it attempts. Altogether, RandomShooter is a stochastic policy available only in this generative form, returning an action a for a provided state s. The RandomShooter's properties highly depend on the state-conditional time-dependent action sampling distributions A_t.

1: **procedure** RandomShooter(state s, horizon H, reward r, dynamics f, samplers $\{A_t\}_{t=1}^H$)
2: $\left\{s_1^{(i)} \leftarrow s\right\}_{i=1}^K$.
3: **for** $i \leftarrow 1, \ldots, K$ **do**
4: **for** $t \leftarrow 1, \ldots, H$ **do**
5: sample $a_t^{(i)} \sim A_t(s_t)$
6: $s_{t+1}^{(i)} \leftarrow f\left(s_t^{(i)}, a_t^{(i)}\right)$
7: $R_i \leftarrow \sum_{t=1}^H r\left(s_t^{(i)}, a_t^{(i)}, s_{t+1}^{(i)}\right)$
8: $i_* = \mathrm{argmax}_i\, R_i$
9: **return** $\left\{s_t^{(i*)}\right\}_{t=1}^H$

of t, s (Uniform RS MPC). But a uniform distribution is a poor representation of future MPC steps: it stands to reason that a more intelligent selection of A_t might improve performance.

4 Contributions

We find that one needs to be concerned with both underoptimizing and overoptimizing Equation 1. In particular, because f is learned and partially inaccurate, improvements in Equation 1 do not correspond to improvements in actual reward. For instance, letting $\tilde{R}$ be the true H-step reward, the reward predicted by Algorithm 2 with Uniform RS MPC, $\tilde{R} - R_{i_*}$, becomes increasingly negative as optimization quality is improved. As a function of optimization quality, the reward curve is actually U-shaped: too little optimization, and poor actions are chosen. Too much, and bias prevents or hurts improvement. Given our findings, we present a new MPC objective as an alternative to Equation 1 and propose several methods to investigate in future work.

5 Method

We propose learning a stationary action distribution conditional on the state, $\pi_\theta(s)$. This policy is used to regularize the MPC optimization. In particular, if the planner is RandomShooter, then we select $A_t^{(\pi_\theta)}$ to be a distribution that is some function of π_θ. Overall, we consider a joint training algorithm, Algorithm 3.

Since the dynamics are learned from historical data, we propose a planner that optimizes Equation 2, where $\mathcal{F}_t$ is a region where we consider our dynamics model to be accurate.

$$\begin{aligned} \max_{\{a_t, s_t\}_{t=1}^H} \quad & \sum_{t=1}^H r(s_t, a_t) \\ \text{s.t.} \quad & s_{t+1} = f(s_t, a_t) \\ & (a_t, s_t) \in \mathcal{F}_t \end{aligned} \tag{2}$$

For instance, we might claim that our dynamics model is accurate for actions and states within some ϵ-ball of historical action and state distributions. In other words, let J be the cost for fitting a state-conditional

Algorithm 3 This algorithm assumes that we have the testing environment with which we may perform real rollouts to learn from. We consider some fixed class of deterministic functions F to model dynamics, and a parameterization π_θ of generative action distributions conditioned on states, coupled with an associated loss J.

procedure TRAINMPC(iterations N, starter transition dataset $\mathcal{D}$, reward r)
  for $i \leftarrow 1, \dots, N$ **do**
    $f \leftarrow \operatorname{argmin}_{f \in F} \mathbb{E}\left[\|f(s,a) - s'\|^2\right]$, where $(s, a, s') \sim \text{Uniform}\,\mathcal{D}$
    $\theta \leftarrow \operatorname{argmin}_\theta J(\theta, \mathcal{D})$
    sample E episodes $\mathcal{D}'$ of $\text{MPC}(\cdot, H, r, f, P_\theta)$, where P_θ is some planner parameterized by θ.
    $\mathcal{D} \leftarrow \mathcal{D}' \cup \mathcal{D}$
  return $\text{MPC}(\cdot, H, r, f, P_\theta)$

action distribution π_θ on our historical data $\mathcal{D}$. Then we might define $\mathcal{F}_t \triangleq B_{\epsilon_t}(\pi_\theta(s_t)) \times B_{\epsilon'_t}(\bar{s}_t)$, where $\bar{s}_{t+1} = f(\bar{s}_t, \pi_\theta(\bar{s}_t))$: actions must be near a historical mean of actions taken in the corresponding state and states must be near the states that would come from a historical trajectory. $B_r(z)$ indicates a ball of radius r around z.

An approximate way of optimizing the above objective for $\epsilon_1 = \infty$ and $\epsilon_t, \epsilon'_t = 0$ otherwise would be to only allow RANDOMSHOOTER to sample randomly on the first action, and otherwise stick to a policy representative of past actions (Equation 3). In general, setting the support of the sampling distributions for RANDOMSHOOTER results in Monte Carlo optimization of the constrained problem.

$$
\begin{gathered}
A_t(s) = \begin{cases} \text{Uniform}\,(\mathcal{A}) & t = 1 \\ \pi_\theta(s) \triangleq \delta_{g_\theta(s)} & \text{otherwise} \end{cases} \\
J(\theta, \mathcal{D}) = \mathbb{E}_{(s,a)\sim\mathcal{D}} \|a - A\|^2 = \mathbb{E}_{(s,a)\sim\mathcal{D}} \|a - g_\theta(s)\|^2\,;\ A \sim \delta_{g_\theta(s)}
\end{gathered}
\tag{3}
$$

Above, g is some function parameterized by θ, such as a neural network.

We refer to TRAINMPC with RANDOMSHOOTER using the distributions of Equation 3 as δ RS MPC. We also consider a simple, parameter-less regularization $g_\theta(s) = \mathbf{0}$, called 0 RS MPC.

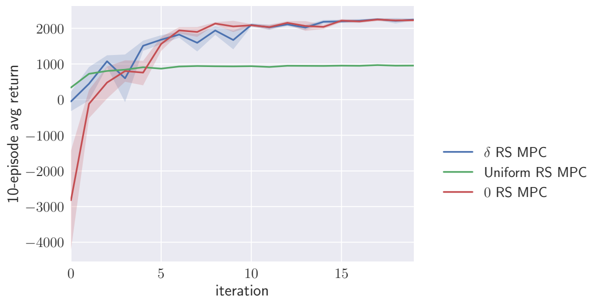
6 Results

6.1 Easy Cheetah

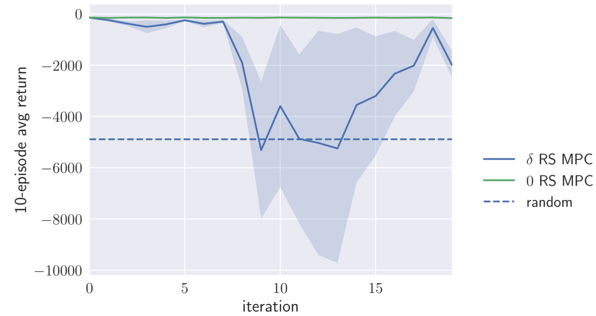
For preliminary validation of our ideas, we consider the `HalfCheetah-v1` environment with the reward function provided in Homework 4: an environment we call easy cheetah. In particular, the reward function includes additional penalties for undesirable behavior, making this an easy baseline to test on.[1]

The performance of the various MPC policies is shown in Figure 1a. The underlying π_θ policy (henceforth the "learner") is poor (Figure 1b), so the improvement it induces in MPC is likely coming from its regularizing properties rather than any smarter selection of actions that may be induced by the learner. Notice that both regularized versions reduce the bias of the predicted reward from the MPC optimization, $\tilde{R} - R_i$ (Figure 1c). This is not due to catastrophic cancellation either, as the MSE $(\tilde{R} - R_i)^2$ figures demonstrate (Figure 1d).

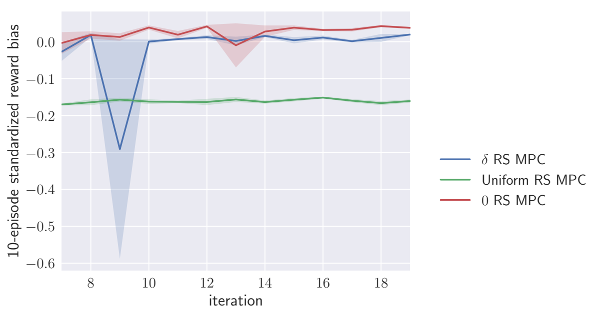
[1]Here our dynamics model is represented by a neural network of depth 2 and width 500. With an MPC horizon $H = 15$ and $K = 1000$ simulation paths, we evaluate the performance of the proposed methods over several iterations of the training procedure (Algorithm 3). Every iteration takes 10 sample episodes, and trains the dynamics for 60 epochs with a 10^{-3} learning rate. The policy optimization step, if present, does 100 epochs with 10^{-3} learning rate on a neural network g_θ of depth 5 and width 32. The universal batch size was 512. Reported results are averages over 4 seeds.



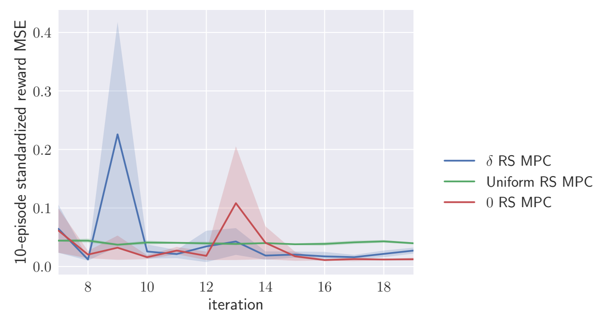
(a) Performance during the course of optimization in the easy cheetah setting. Note both regularized versions significantly outperform Uniform MPC.



(b) Performance of the underlying learner π_θ in the easy cheetah setting, during the course of optimization. We compare to the performance of an agent taking random actions uniformly on the action space.



(c) Easy cheetah MPC predicted reward bias is calculated as the true H-step reward minus the MPC's planner objective value H steps ago. A very negative value indicates overoptimism in the expected reward during MPC planning. We will refer to this as reward bias from this point.



(d) Easy cheetah MPC predicted reward MSE. This is the average bias of Figure 1c, squared.

Figure 1: Easy cheetah experiments

7 Reward Bias

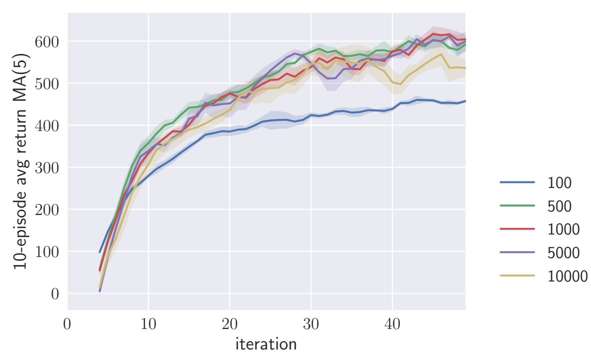
We now investigate more carefully the over-optimization that occurs when MPC optimizes the unconstrained Equation 1. We consider the usual `HalfCheetah-v1` environment.[2] In particular, we improve the quality of the RandomShooter optimization of Equation 1 in the Uniform RS MPC procedure by increasing K. We note that as K increases, so does the over-optimism in the expected reward from the MPC objective (Figure 2a, Figure 2b). Note that the predicted reward is a function of the states projected by the MPC planner $\{s_t\}_{t=1}^{H}$. Then, a mismatch in the reward function implies a mismatch in predicted states, because the state space topology is necessary finer than the pushforward topology on the state space induced by the (continuous) reward function. This may explain why increases in K do not result in improvements in episode reward, and start to hamper performance for large K (Figure 2c).[3]

[2] We use a minor modification to the observations in this environment to include additional position information in the state so that the usual Half Cheetah reward function is a function of the transitions. Otherwise MPC optimization would be impossible because usually the Half Cheetah enviroment determines the reward based on a latent coordinate position.

[3] Hyperparameters are as in Section 6.1, except for K which is modified as indicated.

(a) Half Cheetah predicted reward bias for Uniform RS MPC on a variety of K, smoothed over the 5 previous iterations.

(b) Half Cheetah predicted reward MSE for Uniform RS MPC on a variety of K, smoothed.



(c) Half Cheetah reward for Uniform RS MPC on a variety of K, smoothed. Note that both underoptimizing and overoptimizing reduces performance.

Figure 2: `HalfCheetah-v1` experiments

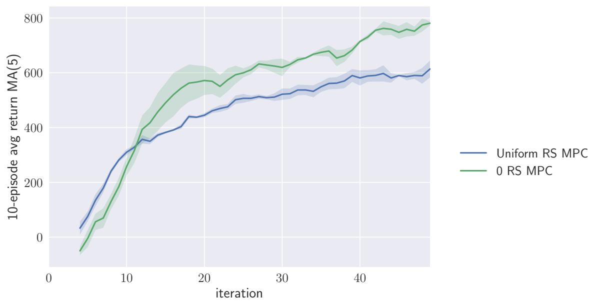
8 Validation on Other Environments

We validate that regularizing MPC to solve Equation 2 with a Monte Carlo RandomShooter method (but supports of sampling distributions appropriately constrained) improves MPC performance across a variety of continuous control environments. We test on the `HalfCheetah-v1`, `Ant-v1`, `Walker2d-v1` environments, with observations slightly expanded to make the reward a function of the transition.[4]

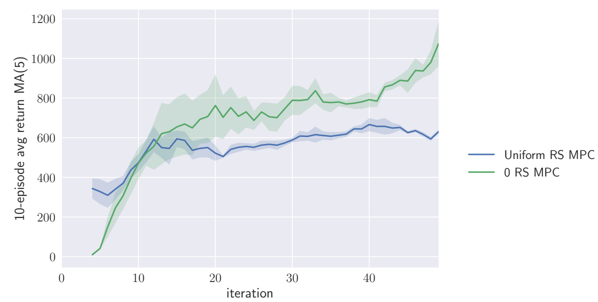
9 Conclusion

We have provided evidence for the hypothesis that one needs to use constrained MPC per Equation 2 when using learned dynamics. We have shown the explicit relationship between over-optimization and over-optimism for one environment, and a generic but crude regularization strategy has shown promise in improving MPC performance.

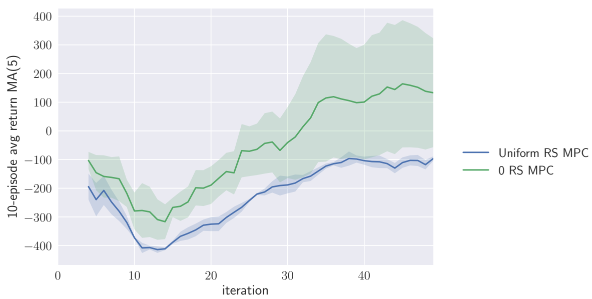
[4]Hyperparameters are as in Section 6.1, except we only run for three random seeds and modify the simulation horizon and number of simulation paths as indicated.



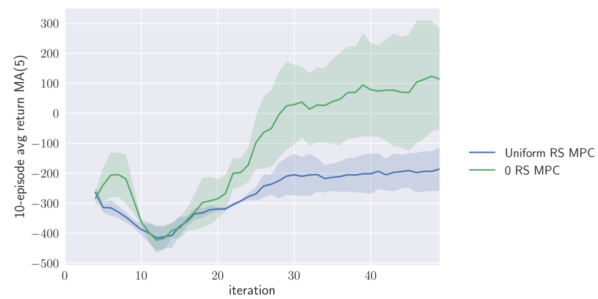
(a) Half Cheetah reward for Uniform RS MPC vs 0 RS MPC with $K = 1000$ and $H = 15$.



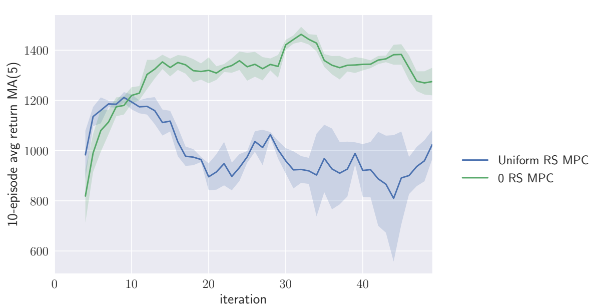
(b) Half Cheetah reward for Uniform RS MPC vs 0 RS MPC with $K = 3000$ and $H = 30$.



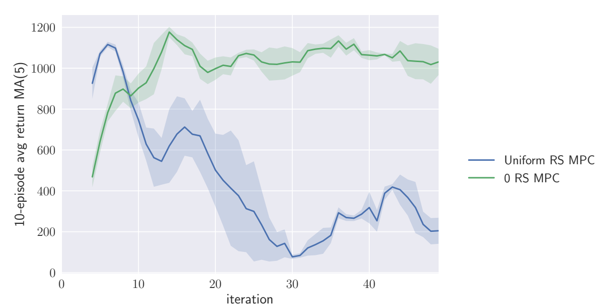
(c) Ant reward for Uniform RS MPC vs 0 RS MPC with $K = 1000$ and $H = 15$.



(d) Ant reward for Uniform RS MPC vs 0 RS MPC with $K = 3000$ and $H = 30$.



(e) Walker2d reward for Uniform RS MPC vs 0 RS MPC with $K = 1000$ and $H = 15$.



(f) Walker2d reward for Uniform RS MPC vs 0 RS MPC with $K = 3000$ and $H = 30$.

Figure 3: `HalfCheetah-v1`, `Ant-v1`, and `Walker2d-v1` experiments.

9.1 Future Work

We believe that our work provides sufficient evidence to explore several promising directions. First, we propose fusing the model-based approach with the model-free approach, by virtue of using an off-policy model-free method to train π_θ with an appropriate choice of loss J in Algorithm 3. Centering MPC around π_θ may lead to performance improvements over the model-free method. However, a more valuable contribution would be to accelerate learning. In particular, by generating more performant episodes thanks to MPC, the off-policy algorithm may exhibit more stable or accelerated convergence properties. Early experiments show that this may be viable, but ϵ_t, ϵ_t' from Equation 2 may require additional nuance over the course of training. Note that using a model-free approach here requires that the resulting trajectories are not too distant from those of the off-policy method: if the model-free method's value functions are not accurate on the support of the MPC agent's trajectories, then it will not learn from the off-policy data.

Second, we plan to move from using the RandomShooter for optimization to using an explicit colocation method directly solving Equation 2 with dual gradient ascent. This should reduce variance in the MPC runs by consistently finding a good estimator. Note that the MPC solution from a previous timestep can be re-used to warm-start the next MPC solution, making the method tractable.

Third, we plan to explore a more nuanced set of constraints than proximity to the mean. The phenomenon we are interesting in capturing is dynamics accuracy, so one approach might be to directly retrieve dynamics variance. Then we can solve the an MPC objective while constraining ourselves to likely trajectories (Equation 4).

$$\begin{aligned} \max_{\{a_t,s_t\}_{t=1}^H} \quad & \sum_{t=1}^{H} r(s_t, a_t) \\ \text{s.t.} \quad & s_{t+1} = \mathbb{E}\, f(s_t, a_t) \\ & \sum_t \operatorname{var} f(s_t, a_t) \le \epsilon \end{aligned} \tag{4}$$

We can approximately solve a soft version of the above by fixing Lagrange multipliers and treating the dynamics neural network model f as an approximate variational Gaussian Process, so that approximate variance can be computed using dropout samples [1].

References

[1] Yarin Gal and Zoubin Ghahramani. Dropout as a bayesian approximation: Representing model uncertainty in deep learning. In *international conference on machine learning*, pages 1050–1059, 2016.

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