Applied Mathematical Modelling 40 (2016) 1554–1564

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Multicriteria group decision making with ELECTRE III method based on interval-valued intuitionistic fuzzy information

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ARTICLE INFO

Article history:
Received 25 March 2013
Revised 30 September 2014
Accepted 26 August 2015
Available online 26 September 2015

Keywords:
Multiple criteria decision making
ELECTRE III
Interval-valued intuitionistic fuzzy information
Investment project selection

ABSTRACT

A great number of real world problems can be associated with multicriteria decision making. These problems are often characterized by a high degree of uncertainty. Interval-valued intuitionistic fuzzy sets are a generalized form of an ordinal fuzzy set to deal with this natural uncertainty. In this paper, an enhanced version of the ELECTRE method, called ELECTRE III, is extended under the interval-valued intuitionistic fuzzy environment. The advantages and strengths of ELECTRE III as a decision aid technique and interval-valued intuitionistic fuzzy sets as an uncertain framework make the proposed method a suitable choice in solving practical problems. The application of the proposed method is illustrated by the solution of an investment project selection problem.

1. Introduction

The history of operations research science in its structured and exhaustive form shows that this field is a response to the needs of managers, decision makers, and resource owners for having the criteria to judge their decisions. In fact, decision makers always seek a criterion to determine the best decisions. Decision making plays an important role for managers at different organizational levels. They should make the best decisions to assign resources fairly and gain benefit for organizations and employees. Consequently, they should solve many problems and choose among the conflicting alternatives. Therefore, many effective methods of selecting the appropriate options have been developed for decision-making. The multiple-criteria decision making (MCDM) provides an effective framework for comparison, based on the evaluation of multiple conflict criteria. MCDM has been one of the fastest growing areas of operational research because it is often found that many concrete problems can be represented by several conflicting criteria. Since, usually, there are numerous and antithetic criteria in actual decision making problems, the MCDM methods have become an important branch of operations research in the last decades [1,2,3]. Decision makers should select the alternatives by comparing them based on several conflicting criteria and using multicriteria decision making methods. This comparison is not completely definitive because of unreliable and imprecise information available about

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http://dx.doi.org/10.1016/j.apm.2015.08.011

the considered alternatives. Therefore, it is very difficult to numerically express their estimates. Hence, uncertainty contexts are widely applied to MCDM problems.

A series of MCDM models use the so-called 'outranking relations' to rank a set of alternatives. The method of elimination and choice translating reality (ELECTRE) and its derivatives play a prominent role in this group. The ELECTRE approach was first introduced in 1966 [4]. The origin of ELECTRE methods goes back to 1965 to the European consultancy company SEMA. At that time, a research team from SEMA worked on a concrete, multiple criteria and real-world problem, regarding decisions on the development of new activities in the firms [1]. The main idea of this method is based on outranking relations and concordance and discordance concepts [5]. This method uses concordance and discordance indices to analyse the outranking relations [6]. Soon after the introduction of the first version known as ELECTRE I [7] this approach evolved into a number of other variants. Today, the most widely used versions are ELECTRE II [8], ELECTRE III [9], ELECTRE IV [10], ELECTRE IS [11] and ELECTRE TRI [12].

Under many conditions, exact data are inadequate to model the real-life situations. These situations are referred to as uncertainty, and researchers developed some structures, such as bounded data, ordinal data, fuzzy data, and grey numbers in response to these situations. Liu and Lin [13] believe that uncertainty frameworks can be classified into three distinct fields: probability and statistics, grey system theory and fuzzy set theory. The evolution of approaches to decision making problems is evident. As a generalization of the fuzzy set theory, Attanasov [14] developed intuitionistic fuzzy sets, where a non-membership degree is also associated with each element, in addition to ordinal membership degree in classic fuzzy sets. Recent years have witnessed a growing interest in the study of decision making problems with intuitionistic fuzzy sets/numbers.

Hung and Chen [15] applied intuitionistic fuzzy sets to a new fuzzy TOPSIS decision making model, using the entropy weight for dealing with multicriteria decision making problems under intuitionistic fuzzy environment. Ye [16] and Park et al. [17] developed different frameworks for the TOPSIS method under IVIF data. Li [18] applied the triangular intuitionistic fuzzy numbers to choose between three alternatives in a MADM problem. Dymova and Sevastjanov [19] presented an analysis of the basic definition of the theory of intuitionistic fuzzy sets and surveyed its application to MCDM problems. Daneshvar Rouyendegh [20] used Intuitionistic fuzzy TOPSIS in a model of group decision making for project selection. Yu et al. [21] proposed some IVIF aggregation operators and investigated their application to group decision making under IVIF environment. Chen et al. [22] also proposed a new multicriteria decision making method under IVIF environment.

In the context of the ELECTRE III method, as the method considered in this paper, Leyva Lopez [23] applied the ELECTRE III for student selecting problems. Papadopoulos and Karagiannidis [24] used this method for the optimization of decentralized energy systems. Li and Wang [25] used the ELECTRE III method based on fuzzy numbers for ranking the alternatives. Nosratian et al [26] proposed a new method of ranking alternatives based on interval grey data by ELECTRE method. Vahdani et al. [27] also extended another approach for applying ELECTRE method by interval weights and data. Atici and Ulucan [28] and Banias et al. [29] used this method for the optimization of energy systems. Radziszewska-Zielina [30] applied the ELECTREIII to selecting the best partner construction enterprise in terms of partnering relations. Zavadskas et al. [31] used the considered method for evaluating commercial construction projects for investment purposes. Marzouk [32], Thiel [33], Ulubeyli and Kazaz [34] used this method in value engineering applications, while Vahdani et al. [35] offered the extension of the ELECTRE method to fuzzy environment in flexible manufacturing.

The remaining part of the paper is organized as follows: in Section 2, the ELECTRE III method is briefly described. Then, in Section 3, a concept and basic calculation of triangular intuitionistic fuzzy is shortly reviewed (algebraic operations). In Section 4, following the introduction of MCDM problems with triangular intuitionistic fuzzy weights and data, an algorithm is presented to extend the ELECTRE III method to deal with triangular intuitionistic fuzzy weights and data. In Section 5, the proposed algorithmic method is illustrated by a case study. Section 6 presents the conclusions and outlines the aims of the future work.

2. The ELECTRE III method

The ELECTRE family in MCDM problems uses the concept of 'outranking relationship'. The outranking relationship of $A_k \rightarrow A_l$ means that, even though two alternatives k and l do not dominate each other mathematically, the DM accepts the risk of regarding A_k as almost surely better than A_l. This method includes pairwise comparison of alternatives, based on the degree, to which evaluations of the alternatives and the preference weights confirm or contradict the pairwise dominance relationship between the alternatives [36]. The versions, mentioned above, are based on the same foundation, but they differ slightly. Specifically, ELECTRE I is designed for selection problems, ELECTRE TRI for assignment problems and ELECTRE II, III and IV for ranking problems. ELECTRE III is used, when it is possible and desirable to quantify the relative importance of criteria, while ELECTRE IV is used in the cases, when this quantification is not possible. This paper is focused on the analysis and application of the ELECTRE III method.

ELECTRE III method includes different advantages in decision making problems. In comparison to ELEWCTRE II, the ELECTRE III is a type of method which uses a structured procedure to extract the relationship between alternatives. The main advantage of this method is the direct participation of decision maker in decision process. In fact, ELECTRE III is an interaction method. Another advantage of ELECTRE III is the indifference and preference threshold definition. When the data are equal and weighted similarly, the alternatives are accounted equally. Due to lower accuracy, the indifference and preference thresholds are defined to illustrate their preference or indifference compared to other alternatives. This means that when the performance of two alternatives is lower than the determined quantity in a certain criterion, these are considered as indifferent in the given criterion. Finally, the ELECTRE III provides the possibility for a decision maker to analyse both the qualitative and quantitative criteria at different levels of ambiguity. A difficulty in the application of this method is the determination of thresholds. The applications of this method emphasize the fact as evidences.

Let $A = (a, b, c, \ldots, n)$ be a set of alternatives and $(g_1, g_2, \ldots, g_m)$ a set of criteria for an MCDM problem; $g_j(a_j)$ represents the performance or the evaluation of the alternative $a \in A$ for the criterion g_j. Depending on whether the target is to maximize or to minimize the criterion $g_j(a_j)$, the higher or lower it is, the better the alternative meets the above criterion. Consequently, multi-criteria evaluation of the alternative $a \in A$ will be represented by the vector $g(a) = (g_1(a), g_2(a), \ldots, g_m(a))$.

The evaluation procedures of the ELECTRE III model encompass the establishment of a threshold function, disclosure of concordance and discordance indices, determination of credibility degree, and the ranking of the alternatives. Let $q(g)$ and $p(g)$ represent the indifference and preference thresholds, respectively [37].

If $g(a) \geq g(b)$, then,

$$g(a) \succ g(b) + p(g(b)) \Leftrightarrow aPb \tag{1}$$

$$g(b) + q(g(b)) \prec g(a) \prec g(b) + p(g(b)) \Leftrightarrow aQb \tag{2}$$

$$g(b) \prec g(a) \prec g(b) + q(g(b)) \Leftrightarrow aIb, \tag{3}$$

where P denotes a strong preference, Q denotes a weak preference, I denotes indifference, and $g(a)$ is the criterion value of the alternative a.

The steps of ELECTRE III and calculations are presented below.

Step 1. The concordance index $C(a, b)$ is computed for each pair of alternatives:

$$C(a, b) = \frac{\sum_{i=1}^{m} w_i C_i(a, b)}{\sum_{i=1}^{m} w_i}, \tag{4}$$

where $C_i(a, b)$ is the outranking degree of the alternative a and the alternative b under the criterion i, and

$$C_i(a, b) = \begin{cases} 0 \ \text{ if } \ g_i(b) - g_i(a) \succ p_i(g_i(a)) \\ 1 \ \text{ if } \ g_i(b) - g_i(a) \leq q_i(g_i(a)) \\ \dfrac{p_i + g_i(a) - g_i(b)}{p_i - q_i} \quad \text{otherwise} \end{cases} \tag{5}$$

Thus, $0 \leq c_i(a, b) \leq 1$.

The veto threshold $v_i(g_i(b))$ is defined for each criterion i as follows:

$$v_i(g_i(a)) = \alpha_v + \beta_v g_i(a) \tag{6}$$

The veto threshold, vi, allows for the possibility of a**S**b to be refused totally if, for any one criterion j, $g_i(b) \succ g_i(a) + v_i$.

Step 2. The discordance index $d(a, b)$ for each criterion is then defined as follows:

$$d_i(a, b) = \begin{cases} 0 \ \text{ if } g_i(b) - g_i(a) \leq p_i(g_i(a)) \\ 1 \ \text{ if } g_i(b) - g_i(a) \succ v_i(g_i(a)) \\ \dfrac{g_i(b) - g_i(a) - p_i}{v_i - p_i} \quad \text{otherwise} \end{cases} \tag{7}$$

Thus, $0 \leq d_i(a, b) \leq 1$

Step 3. Finally, the degree of outranking is defined by $S(a, b)$:

$$S(a, b) = \begin{cases} c(a, b) & \text{if } d_j(a, b) \leq c(a, b) \ \ \forall j \in J \\ c(a, b) \ \times \displaystyle\prod_{j \in J(a,b)} \dfrac{1 - d_j(a, b)}{1 - c(a, b)} & \text{otherwise} \end{cases}, \tag{8}$$

where $J(a, b)$ is the set of criteria for which $d_j(a, b) \succ c(a, b)$ [37].

Step 4. To obtain the complete ranking of the alternatives, the normal ranking method of ELECTRE III uses a structured algorithm via two intermediate ranking procedures: one is descending, where the alternatives are classified from the best to the worst (descending distillation), while the other is based on the ascending order from the worst to the best alternative (ascending distillation). However, according to Li and Wang [25], a new ranking method based on the introduction of three concepts, including the concordance credibility degree, the discordance credibility degree and the net credibility degree, is applied.

(i) The concordance credibility degree is defined by

$$\varphi^+(x_i) = \sum_{x_j \in X} S\left(x_i, x_j\right), \quad \forall x_i \in X \tag{9}$$

The concordance credibility degree is a measure of the outranking character of x_i (showing how x_i dominates all the other alternatives of X).

(ii) The discordance credibility degree is defined by

$$\varphi^-(x_i) = \sum_{x_j \in X} S\left(x_j, x_i\right), \quad \forall x_i \in X \tag{10}$$

The discordance credibility degree describes the outranked x_j (showing how x_j is dominated by all the other alternatives of X).

(iii) The net credibility degree is defined by

$$\varphi(x_i) = \varphi^+(x_i) - \varphi^-(x_i), \quad \forall x_i \in X \tag{11}$$

The net credibility degree represents the value function, where a higher value reflects higher attractiveness of the alternative x_i. Then, all the alternatives can be completely ranked by the net credibility degree.

In addition, to solve practical problems of fuzzy multiattribute decision making (FMADM), evaluators and decision makers must be provided for and should necessarily include various stakeholders and interest groups. The different backgrounds and positions of the members of these groups result in greatly varying subjective judgments. For example, the above thresholds (concordance, discordance, and veto) may be presented in fuzzy data. This shows that ELECTRE III and IV are more appropriate for the evaluation of real-world problems [37].

3. Interval-valued intuitionistic fuzzy sets

Intuitionistic fuzzy sets theory was introduced by Atanassov [14]. Let a set E be fixed. An intuitionistic fuzzy set A in E is defined as an object of the following form:

$$A = \{\langle x, \mu_A(x), \upsilon_A(x)\rangle | x \in E\} \tag{12}$$

where μ_A: $E \to [0, 1]$ indicates the degree of membership and υ_A: $E \to [0, 1]$ indicates the degree of non-membership of x in A.

$$0 \le \mu_A(x) + \upsilon_A(x) \le 1 \tag{13}$$

Atanassov and Gargov [38] generalized the IFS to interval-valued intuitionistic fuzzy sets (IVIFS) as follows:

Let $D[0, 1]$ be the set of all closed subintervals of the interval [0, 1]. Let $X(\neq \Phi)$ be a given set. The IVIFS in X is an expression given by $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), \upsilon_{\tilde{A}}(x)\rangle | x \in X\}$, where $\mu_{\tilde{A}} : X \to D[0, 1]$, $\upsilon_{\tilde{A}} : X \to D[0, 1]$ with the condition $0 \prec \sup_x \mu_{\tilde{A}}(x) + \sup_x \upsilon_{\tilde{A}}(x) \le 1$.

For each $x \in X$, $\mu_{\tilde{A}}(x)$ and $\upsilon_{\tilde{A}}(x)$ are closed intervals, whose lower and upper end points are denoted by $\mu_{AL}(x)$, $\mu_{AU}(x)$, $\upsilon_{AL}(x)$ and $\upsilon_{AU}(x)$. The IVIFS A is denoted by

$$A = \{\langle x, [\mu_{AL}(x), \mu_{AU}(x)], [\upsilon_{AL}(x), \upsilon_{AU}(x)]\rangle | x \in X\} \tag{14}$$

where $0 \prec \mu_{AU}(x) + \upsilon_{AU}(x) \le 1$, $\mu_{AL}(x), \upsilon_{AL}(x) \ge 0$. For convenience, the IVIFS value is denoted by $\tilde{A} = ([a, b], [c, d])$, referred to as an interval-valued intuitionistic fuzzy number (IVIFN).

Moreover, let $\tilde{A}_1 = ([a_1, b_1], [c_1, d_1])$ and $\tilde{A}_2 = ([a_2, b_2], [c_2, d_2])$ be IVIFNs. Their operational laws are defined as follows [39]:

$$\tilde{A}_1 + \tilde{A}_2 = ([a_1 + a_2 - a_1 a_2, b_1 + b_2 - b_1 b_2], [c_1 c_2, d_1 d_2]) \tag{15}$$

$$\tilde{A}_1 \cdot \tilde{A}_2 = ([a_1 a_2, b_1 b_2], [c_1 + c_2 - c_1 c_2, d_1 + d_2 - d_1 d_2]) \tag{16}$$

$$\lambda \tilde{A}_1 = \left([1 - (1 - a_1)^\lambda, 1 - (1 - b_1)^\lambda], \left[c_1^\lambda, d_1^\lambda\right]\right), \ \lambda \ge 0 \tag{17}$$

If $\tilde{A} = ([a, b], [c, d])$ is an IVIFN, then,

$$s\left(\tilde{A}\right) = \frac{1}{2}(a - c + b - d) \tag{18}$$

is the score function of $\tilde{A}$, where $s(\tilde{A}) \in [-1, 1]$, and

$$h\left(\tilde{A}\right) = \frac{1}{2}(a + c + b + d) \tag{19}$$

is the accuracy function of A, where $h(\tilde{A}) \in [0, 1]$ [39].

To compare two IVIFNs let us consider $\tilde{A}_a$ and $\tilde{A}_b$ as two IVIFNs. Therefore,

1. If $s(\tilde{A}_a) \prec s(\tilde{A}_b)$, then, $\tilde{A}_a$ is smaller than $\tilde{A}_b$, $\tilde{A}_a \prec \tilde{A}_b$.
2. If $s(\tilde{A}_a) = s(\tilde{A}_b)$, then,
   - 2.1. If $h(\tilde{A}_a) = h(\tilde{A}_b)$, then, $\tilde{A}_a = \tilde{A}_b$.
   - 2.2. If $h(\tilde{A}_a) \prec h(\tilde{A}_b)$, then, $\tilde{A}_a$ is smaller than $\tilde{A}_b$, $\tilde{A}_a \prec \tilde{A}_b$ [39].

Then, let $\tilde{A}_j = ([a_j, b_j], [c_j, d_j]),\ j = 1, 2, \ldots, n$ be a collection of IVIFNs. The generalized interval intuitionistic fuzzy weighted average $GIIFWA_w(\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n)$ is defined as follows:

$$GIIFWA_w(A_1, A_2, \ldots, A_n) = \left(w_1\tilde{A}_1^{\lambda} + w_2\tilde{A}_2^{\lambda} + \ldots + w_n\tilde{A}_n^{\lambda}\right)^{1/\lambda}, \tag{20}$$

where $\lambda \succ 0$, and $w = (w_1, w_2, \ldots, w_n)^T$ is the weight vector, with $w_j \geq 0,\ j = 1, 2, \ldots, n$, and $\sum_{j=1}^{n} w_j = 1$. It can be shown that GIIFWA is also an IVIFN and can be calculated as follows [40]:

$$GIIFWA_w\left(\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n\right) = \left(\left[\left(1 - \prod_{j=1}^{n}\left(1 - a_j^{\lambda}\right)^{w_j}\right)^{1/\lambda}, \left(1 - \prod_{j=1}^{n}\left(1 - b_j^{\lambda}\right)^{w_j}\right)^{1/\lambda}\right],\right.$$
$$\left.\left[1 - \left(1 - \prod_{j=1}^{n}\left(1 - \left(1 - c_j\right)^{\lambda}\right)^{w_j}\right)^{1/\lambda}, 1 - \left(1 - \prod_{j=1}^{n}\left(1 - \left(1 - d_j\right)^{\lambda}\right)^{w_j}\right)^{1/\lambda}\right]\right). \tag{21}$$

If $\lambda = 1$, then, GIIFWA is turned into the interval intuitionistic fuzzy weighted average (IIFWA).

4. The ELECTRE III method with the interval-valued intuitionistic fuzzy number

In this section, the ELECTRE III method is extended to include the IVIFNs data. First, let us consider a decision making problem consisting of m alternatives $A = (A_1, A_2, \ldots, A_m)$, which will be evaluated based on n criteria $(g_1, g_2, \ldots, g_m)$, while $\tilde{x}_{ij}$ is the value of ith alternative in the case of jth criterion, which is expressed as an IVIFN, and a group of K decision makers will assign their scores. In this case, the process of group decision making by ELECTRE III - IVIFN method is developed in the following steps:

Step 1. Determine the importance of each decision maker. First, the importance of each decision maker in the final decision should be determined. Suppose, that $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_K)$ is the decision makers' importance vector, where $\lambda_k \geq 0, k = 1, 2, \ldots, K$ is the importance of kth decision maker and $\sum_{k=1}^{K} \lambda_k = 1$. Note, that if decision makers are equally important, then, $\lambda_1 = \lambda_2 = \ldots = \lambda_K = 1/K$.

Step 2. Assign the scores. In this step, decision makers are asked to individually express their *opinions* and evaluate the alternatives based on criteria and their weights. Suppose, that $\tilde{x}_{ij}^k,\ i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n$ is the kth expert's evaluation rating of the alternative A_i based on criterion j, which is expressed by the IVIFN $\tilde{x}_{ij}^k = ([\mu_{Lij}^k, \mu_{Uij}^k], [\upsilon_{Lij}^k, \upsilon_{Uij}^k])$. Then, the kth expert's decision matrix is constructed as follows:

$$\tilde{X}^k = \begin{bmatrix} \tilde{x}_{11}^k & \tilde{x}_{12}^k & \ldots & \tilde{x}_{1n}^k \\ \tilde{x}_{21}^k & \tilde{x}_{22}^k & \ldots & \tilde{x}_{2n}^k \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{x}_{m1}^k & \tilde{x}_{m2}^k & \ldots & \tilde{x}_{mn}^k \end{bmatrix} \tag{22}$$

Then, aggregate the expert's matrices as follows:

$$\tilde{x}_{ij} = GIIFWA_{\lambda}\left(x_{ij}^1, x_{ij}^2, \ldots, x_{ij}^K\right) = \left(\left[\left(1 - \prod_{k=1}^{K}\left(1 - \mu_{Lij}^k\right)^{\lambda_k}\right), \left(1 - \prod_{k=1}^{K}\left(1 - \mu_{Uij}^k\right)^{\lambda_k}\right)\right],\right.$$
$$\left.\left[\prod_{k=1}^{K}\left(\upsilon_{Lij}^k\right)^{\lambda_k}, \prod_{k=1}^{K}\left(\upsilon_{Uij}^k\right)^{\lambda_k}\right]\right). \tag{23}$$

Step 3. Determine the weights of the criteria. Synchronously, expert k expresses his or her judgments with regard to the weights of criteria. Suppose, that $\tilde{w}_j^k = ([\mu_{Lj}^k, \mu_{Uj}^k], [\upsilon_{Lj}^k, \upsilon_{Uj}^k])$ is the kth expert's judgment about the importance of the jth criterion. In this step, the aggregated weights of criteria are determined by calculating the GIIFWA operator, Eq. (21) and the criteria weights, determined by decision makers. If $\tilde{w}_j^k, k = 1, 2, \ldots, K$ are the weights of the criterion j assigned by decision makers, then, the aggregated weight of the criterion j, $\tilde{w}_j = ([\mu_{Lj}, \mu_{Uj}], [\upsilon_{Lj}, \upsilon_{Uj}])$, will be calculated as described below:

$$\tilde{w}_j = GIIFWA_{\lambda}\left(\tilde{w}_j^1, \tilde{w}_j^2, \ldots, \tilde{w}_j^K\right) = \left(\left[\left(1 - \prod_{k=1}^{K}\left(1 - \mu_{Lj}^k\right)^{\lambda_k}\right), \left(1 - \prod_{k=1}^{K}\left(1 - \mu_{Uj}^k\right)^{\lambda_k}\right)\right],\right.$$
$$\left.\left[\prod_{k=1}^{K}\left(\upsilon_{Lj}^k\right)^{\lambda_k}, \prod_{k=1}^{K}\left(\upsilon_{Uj}^k\right)^{\lambda_k}\right]\right). \tag{24}$$

Step 4. Construct the concordance matrix. For this purpose, we should first determine the thresholds, where P denotes a strong preference, while Q denotes a weak preference, and I denotes indifference.

The alternatives' performance can usually be determined with 'certain accuracy', and the imperfect knowledge about the evaluations can be taken into account, when defining the thresholds for the model.

There are several techniques, which can be used in this case. Some of them are directly associated with the definition of a threshold, while others are based on the concept of a dispersion threshold. A dispersion threshold allows us to take into account the concept of probable value and the notion of optimistic and pessimistic values. It shows the plausible difference due to over- or under-estimation, which affects the evaluation of the consequence or performance level. It should be noted that there are no true values for thresholds. Therefore, the values chosen for thresholds are most convenient (best adapted) for expressing the imperfect character of the knowledge [1].

Then, a pair of alternatives should be compared and the concordance matrix should be calculated as follows:

If $s(\tilde{A}_a) \prec s(\tilde{A}_b)$, then, $\tilde{A}_a \prec \tilde{A}_b$, where $s(\tilde{A}_i)$ is calculated by Eq. (18).

Now, $C(a, b)$ is calculated by Eq. (25),

$$C(a,b) = \frac{\sum_{i=1}^{m} w_i C_i(a,b)}{\sum_{i=1}^{m} w_i}, \tag{25}$$

where

$$C_i(a,b) = \begin{cases} 0 \ \text{ if } s_i(b) - s_i(a) \succ s(\tilde{p}_i) \\ 1 \ \text{ if } s_i(b) - s_i(a) \le s(\tilde{q}_i) \\ \dfrac{s(\tilde{p}_i) + s(a) - s(b)}{s(\tilde{p}_i) - s(\tilde{q}_i)} \ \text{otherwise} \end{cases}. \tag{26}$$

Step 5. Determine the discordance index. The value of $d(a, b)$ for each criterion is defined as follows:

$$d_i(a,b) = \begin{cases} 0 \ \text{ if } s(b) - s(a) \le s(\tilde{p}_i) \\ 1 \ \text{ if } s\left(\tilde{A}_b\right) - s\left(\tilde{A}_a\right) \succ v_i\left(\tilde{A}_a\right) \\ \dfrac{s\left(\tilde{A}_b\right) - s\left(\tilde{A}_a\right) - s(\tilde{p}_i)}{s(\tilde{v}_i) - s(\tilde{p}_i)}, \ \text{otherwise} \end{cases}. \tag{27}$$

Thus, $0 \le d_i(a, b) \le 1$.

Step 6. Finally, the degree of outranking is defined by $S(a, b)$:

$$S(a,b) = \begin{cases} c(a,b) & \text{if } d_j(a,b) \le c(a,b) \ \ \forall j \in J \\ c(a,b) \times \prod\limits_{j \in J(a,b)} \dfrac{1 - d_j(a,b)}{1 - c(a,b)} & \text{otherwise} \end{cases}. \tag{28}$$

Step 7. According to Li and Wang [25], the alternatives could be ranked based on the concordance credibility degree, the discordance credibility degree and the net credibility degree by Eqs. (9), (10), and (11).

The algorithmic scheme of the proposed model is shown in Fig. 1.

5. The application of ELECTRE III with the interval-valued intuitionistic fuzzy number

Case 1. In this section, the application of ELECTRE III based on the interval-valued intuitionistic fuzzy numbers is presented. A management team is going to select the best among the investment projects. For this purpose, they asked managers of 5 departments to evaluate four projects based on different criteria. The economic and financial analysis of the project is based on the comparison of the cash flow of all costs and benefits, resulting from the project's activities. There are four common criteria for comparing the alternative investments [41,42]:

(1) Net present value (NPV),
(2) Rate of Return (ROR),
(3) Benefit-Cost analysis (CB),
(4) Pay Back Period (PBP).

First, a management team determined their importance in terms of the weight vector $\lambda = (0.25,\ 0.25,\ 0.25,\ 0.25)^T$.

Then, these managers were asked to express their opinion about the alternatives in the linguistic terms to show their preferences with respect to each criterion and to weigh them according to Table 1.

Table 2 shows Linguistic preference matrix of an expert.

The aggregation of the expert's preference matrices, based on Eq. (23), is shown in Table 3.

Table 4 demonstrates the proposed scale to show the significance of each criterion, while the experts are asked to define their opinions as shown in Table 5.

Now, the aggregated weight of the criteria determined by experts is calculated by Eq. (24):

$$\tilde{w}_1 = ([0.7648, 0.8712], [0, 0.1768]) \quad \tilde{w}_2 = ([0.7084, 0.8027], [0.1257, 0.2307])$$
$$\tilde{w}_3 = ([0.1549, 0.5838], [0, 0.3872]) \quad \tilde{w}_4 = ([0.7434, 0.8458], [0, 0.3872]).$$

Then, according to Eq. (26), the alternatives are compared, using the thresholds for this purpose, as shown in Table 6.

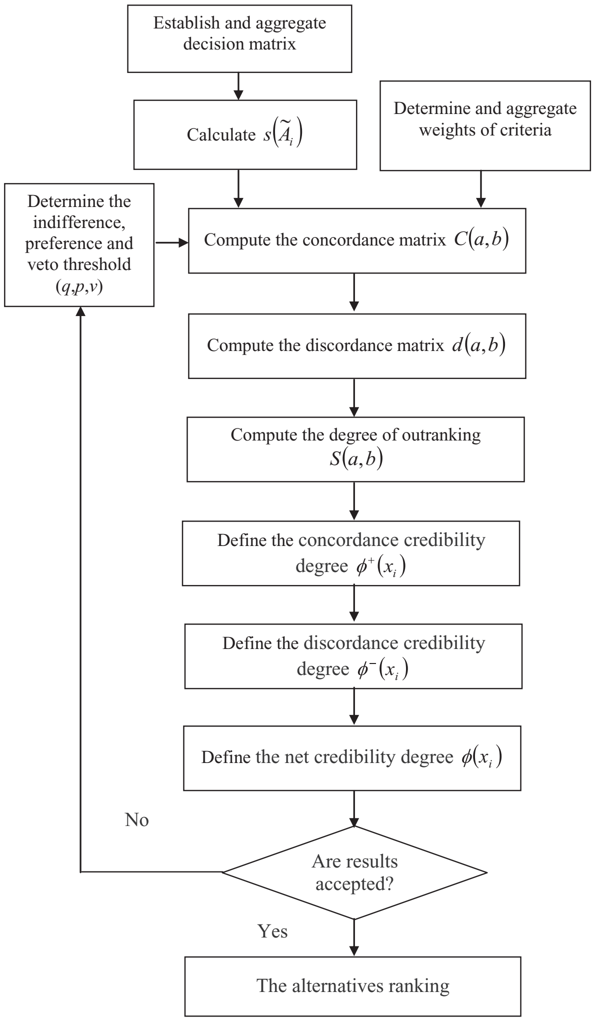

Fig. 1. The scheme of the ELECTRE III IVIF algorithm.

Table 1
The IVIFN scale for rating the alternatives with respect to various criteria.

Linguistic term	IVIFNs
Extremely good (EG)/ extremely high (EH)	([1, 1], [0, 0])
Very very good (VVG)/ very very high (VVH)	([0.9, 0.9], [0.1, 0.1])
Very good (VG)/ very high (VH)	([0.7333, 0.825], [0, 0.125])
Good (G)/ high (H)	([0.6333, 0.725], [0.1, 0.225])
Medium good (MG)/ medium high (MH)	([0.5333, 0.625], [0.2, 0.325])
Fair (F)/ medium (M)	([0.4333, 0.525], [0.3, 0.425])
Medium bad (MB)/ medium low (ML)	([0.3333, 0.425], [0.4, 0.525])
Bad (B)/ low (L)	([0.15, 0.2875], [0.45, 0.6375])
Very bad (VB)/ very low (VL)	([0, 0.1375], [0.6, 0.7875])
Very very bad (VVL)/ very very low (VVL)	([0.1, 0.1], [0.9, 0.9])

To construct the concordance matrix, it is necessary to calculate $s(\tilde{A}_j)$ and $s(\tilde{w}_i)$ first. The results of $s(\tilde{A}_j)$ are shown in Table 7.

$$s(\tilde{w}_1) = 0.729590194 \quad s(\tilde{w}_2) = 0.577357189$$
$$s(\tilde{w}_3) = 0.175780762 \quad s(\tilde{w}_4) = 0.601035037$$

Table 2
Linguistic preference matrix of an expert.

EX1	C_1	C_2	C_3	C_4	EX2	C_1	C_2	C_3	C_4	EX3	C_1	C_2	C_3	C_4
A_1	H	M	VH	H	A_1	M	M	M	ML	A_1	H	H	M	VH
A_2	M	ML	VL	M	A_2	M	H	L	H	A_2	M	VH	M	H
A_3	M	VVL	M	VH	A_3	L	M	VVL	H	A_3	VL	M	L	M
A_4	M	VH	H	M	A_4	H	MH	M	VVH	A_4	VH	H	VH	M

EX4	C_1	C_2	C_3	C_4	EX5	C_1	C_2	C_3	C_4
A_1	H	VH	H	ML	A_1	M	M	H	VH
A_2	M	H	VL	M	A_2	H	VH	ML	VH
A_3	VL	L	M	M	A_3	L	M	VL	M
A_4	VH	M	MH	H	A_4	M	H	H	VH

Table 3
The aggregated matrix of experts' preferences.

EXs	C_1	C_2	C_3	C_4
A_1	$\begin{pmatrix}[0.6452, 0.7382],\\ [0.0974, 0.2129]\end{pmatrix}$	$\begin{pmatrix}[0.6347, 0.7320],\\ [0, 0.2155]\end{pmatrix}$	$\begin{pmatrix}[0.6724, 0.7662],\\ [0, 0.1838]\end{pmatrix}$	$\begin{pmatrix}[0.6718, 0.7703],\\ [0, 0.1764]\end{pmatrix}$
A_2	$\begin{pmatrix}[0.5590, 0.6560],\\ [0.1687, 0.2927]\end{pmatrix}$	$\begin{pmatrix}[0.6997, 0.7984],\\ [0, 0.1498]\end{pmatrix}$	$\begin{pmatrix}[0.5148, 0.6388],\\ [0, 0.2934]\end{pmatrix}$	$\begin{pmatrix}[0.6724, 0.7662],\\ [0, 0.1838]\end{pmatrix}$
A_3	$\begin{pmatrix}[0.2000, 0.3492],\\ [0.3845, 0.5721]\end{pmatrix}$	$\begin{pmatrix}[0.3891, 0.4880],\\ [0.3233, 0.4581]\end{pmatrix}$	$\begin{pmatrix}[0.2959, 0.4056],\\ [0.3845, 0.5345]\end{pmatrix}$	$\begin{pmatrix}[0.6347, 0.7320],\\ [0, 0.2155]\end{pmatrix}$
A_4	$\begin{matrix}[0.6974, 0.7912],\\ [0, 0.1587]\end{matrix}$	$\begin{pmatrix}[0.6880, 0.7796],\\ [0, 0.1719]\end{pmatrix}$	$\begin{pmatrix}[0.6880, 0.7796],\\ [0, 0.1719]\end{pmatrix}$	$\begin{pmatrix}[0.7632, 0.8184],\\ [0, 0.1501]\end{pmatrix}$

Table 4
Linguistic terms used to define the significance of each criterion.

Linguistic term	IVIFNs
Very important (VI)	([0.9, 0.9], [0.1, 0.1])
Important (I)	([0.4, 0.7625], [0, 0.2115])
Medium (M)	([0.15, 0.5125], [0.25, 0.4625])
Unimportant (U)	([0, 0.3625], [0.4, 0.6125])
Very unimportant (VU)	([0.1, 0.1], [0.9, 0.9])

Table 5
The weighting matrix of experts.

	C_1	C_2	C_3	C_4
EX1	VI	VI	M	VI
EX2	I	VI	I	M
EX3	I	M	U	VI
EX4	M	U	U	M
EX5	VI	M	U	I

Table 6
The matrix of the alternative thresholds.

	C_1	C_2	C_3	C_4
Q	M	L	M	L
P	H	MH	MH	ML
V	VH	H	H	M

Table 7
Deterministic format of the alternatives and thresholds.

EXs	C_1	C_2	C_3	C_4
A_1	0.5365	0.5756	0.6274	0.6328
A_2	0.3768	0.6742	0.4301	0.6274
A_3	−0.2037	0.0478	−0.1087	0.5756
A_4	0.6650	0.6478	0.6478	0.7157
q	0.1166	−0.325	0.1166	−0.325
P	0.5166	0.3166	0.3166	−0.0833
v	0.7166	0.5166	0.5166	0.1166

Table 8
The concordance matrix.

	A_1	A_1	A_1	A_1
A_1	1	0.5286	0.7115	0.5297
A_2	0.5420	1	0.6272	0.3898
A_3	0	0	1	0.0843
A_4	0.6024	0.5658	0.7793	1

Table 9
The discordance matrix.

D1				D2				D3				D4			
1	0	0	0	1	0	0	0	1	0	0	0	1	0.38	0.13	0.83
0	1	0	0	0	1	0	0	0	1	0	0	0.44	1	0.15	0.85
1	0.32	1	1	1	1	1	1	1	1	1	1	0.70	0.67	1	1.11
0	0	0	1	0	0	0	1	0	0	0	1	0.002	0	0	1

Table 10
The comparison matrix.

$c_{11} = d_{11}$	$c_{12} \succ d_{12}$	$c_{13} \succ d_{13}$	$c_{14} \prec d_{14}$
$c_{21} \prec d_{21}$	$c_{22} = d_{22}$	$c_{23} \succ d_{23}$	$c_{24} \prec d_{24}$
$c_{31} \prec d_{31}$	$c_{32} \succ d_{32}$	$c_{33} = d_{33}$	$c_{34} \prec d_{34}$
$c_{41} \succ d_{41}$	$c_{42} \succ d_{42}$	$c_{43} \succ d_{43}$	$c_{44} = d_{44}$

Table 11
The credibility matrix.

1	0.5286	0.7115	3.449402
0.542	1	0.6272	1.020628
0	0	1	0
0.6024	0.5658	0.7793	1

Table 12
The ranking matrix.

Alternatives	Scores	Ranks
A_1	3.545102	1
A_2	1.095428	2
A_3	−2.118	3
A_4	−2.52253	4

Table 13
Comparison of IVIF-ELECTREE III results with the data yielded by different methods.

	A_1	A_2	A_3	A_4
Proposed method's results	1	2	3	4
TOPSIS [16]	1	2	4	3
TOPSIS [17]	1	2	3	4
COPRAS [43]	1	2	4	3

Now, the concordance matrix is constructed based on the comparison of the alternatives according to Eqs. (25) and (26). Table 8.

Then, the discordance matrix is calculated and the results are shown in Table 9.

In this step, the comparison between the concordance and discordance matrices is made, and $S(a, b)$ is determined by Eq. (28). The comparison results and $S(a, b)$ are presented in Tables 10 and 11, respectively.

In the final step, according to Li and Wang [25], the ranking is made and the results are shown in Table 12.

This case is solved by the methods proposed by Ye [16], Park et al. [17], and Razavi Hajiagha et al. [43].

Kendall's coefficient of concordance for the results of Table 13 is about 0.9. It is clear that there is an acceptable level of consistency among different methods in this example.

Case 2. Here, to demonstrate the application of the suggested method to other fields and to validate the results more thoroughly, the problem of Park et al. [17] is solved by this method. This problem is associated with determining the type of air-conditioning systems to be installed in a library. The contractor offered four feasible alternatives, A_1, A_2, A_3, A_4, which could be

Table 14
The aggregated matrix of experts' preferences.

	C_1	C_2	C_3	C_4	C_5
A_1	$\begin{pmatrix}[0.4385, 0.6199],\\ [0.1549, 0.2848]\end{pmatrix}$	$\begin{pmatrix}[0.3000, 0.4573],\\ [0.3404, 0.4710]\end{pmatrix}$	$\begin{pmatrix}[0.6116, 0.7117],\\ [0.1089, 0.2083]\end{pmatrix}$	$\begin{pmatrix}[0.5000, 0.6395],\\ [0.0980, 0.2567]\end{pmatrix}$	$\begin{pmatrix}[0.1323, 0.3623],\\ [0.3747, 0.5482]\end{pmatrix}$
A_2	$\begin{pmatrix}[0.3520, 0.4797],\\ [0.3114, 0.4681]\end{pmatrix}$	$\begin{pmatrix}[0.1138, 0.3010],\\ [0.2511, 0.4773]\end{pmatrix}$	$\begin{pmatrix}[0.3379, 0.4387],\\ [0.3872, 0.4887]\end{pmatrix}$	$\begin{pmatrix}[0.1758, 0.3134],\\ [0.5305, 0.6496]\end{pmatrix}$	$\begin{pmatrix}[0.6395, 0.7521],\\ [0.1089, 0.2830]\end{pmatrix}$
A_3	$\begin{pmatrix}[0.3516, 0.4906],\\ [0.2940, 0.4214]\end{pmatrix}$	$\begin{pmatrix}[0.6395, 0.7711],\\ [0.0980, 0.2263]\end{pmatrix}$	$\begin{pmatrix}[0.5213, 0.7804],\\ [0.0980, 0.2083]\end{pmatrix}$	$\begin{pmatrix}[0.4387, 0.6252],\\ [0.2263, 0.3262]\end{pmatrix}$	$\begin{pmatrix}[0.5452, 0.6502],\\ [0.1770, 0.3205]\end{pmatrix}$
A_4	$\begin{pmatrix}[0.3000, 0.4170],\\ [0.3114, 0.4887]\end{pmatrix}$	$\begin{pmatrix}[0.1000, 0.2103],\\ [0.6012, 0.7678]\end{pmatrix}$	$\begin{pmatrix}[0.1000, 0.2366],\\ [0.5577, 0.7569]\end{pmatrix}$	$\begin{pmatrix}[0.2103, 0.3109],\\ [0.4050, 0.5613]\end{pmatrix}$	$\begin{pmatrix}[0.1849, 0.3121],\\ [0.5031, 0.6118]\end{pmatrix}$

Table 15
The comparison of the results of the original research and the proposed method.

	A_1	A_2	A_3	A_4
Proposed method's results	2	3	1	4
TOPSIS [16]	2	3	1	4
TOPSIS [17]	3	2	1	4
COPRAS [43]	3	2	1	4

ranked, based on the following five attributes: (1) performance (C_1), (2) maintainability (C_2), (3) flexibility (C_3), (4) cost (C_4) and (5) safety (C_5). Table 14 shows the aggregated preference matrix of decision makers, while Table 15 illustrates the results yielded by the proposed method and the method described in the considered paper.

This case is also solved by the methods proposed by Ye [16], Park et al. [17], and Razavi Hajiagha et al. [43].

Kendall's coefficient of concordance for the results presented in Table 13 is about 0.9, which shows a high degree of concordance between these results.

It should be noted that the partial differences between the proposed method and other considered methods can be caused by the impact of the threshold values on the ELECTRE III results. Therefore, the selection of suitable values for these thresholds may hide these differences.

6. Conclusion

In the present paper, the application of the ELECTRE III with interval-valued intuitionistic fuzzy information is proposed to solving the MCDM problems. These types of decision making problems require managers to consider various aspects of the problem and the conflicting evaluation criteria. In this paper, the ELECTRE III is used as an appropriate method. The distinctive characteristic of ELECTRE III is its ability to handle a data set with a high degree of uncertainty. Another advantage of the ELECTRE III is associated with the fact that it is completely compatible with the environmental applications. Moreover, the ELECTRE III uses pairwise comparison of the alternatives. It means that one can select the best alternative according to predefined preference criteria. Each criterion is weighted to represent its relative importance according to the preference structure of the decision maker. Therefore, this ELECTRE version is preferable to its previous versions. In order to treat the ambiguity and ill-defined data, the interval-valued intuitionistic fuzzy numbers are used, which consider a membership and non-membership interval for each element and give a more tangible picture of the inaccuracy of the real world. The application of the interval-valued intuitionistic fuzzy numbers is extended so that the ELECTRE III method could help managers to make decisions based on nondeterministic data and to decrease the ambiguity of information. Hence, this combination makes the proposed method more widely applicable and reliable. Finally, the proposed method was applied to solving two problems of the investment project and air-conditioning system selection. In both cases, the results yielded by the proposed method were compared with the results yielded by some other methods, and only partial differences could be observed between them. Also, the comparison of the obtained results had shown an acceptable degree of consistency. In general, the MADM methods do not yield optimal results, but, in fact, they only lead to satisfactory conclusions. Therefore, the ELECTRE III, requiring decision makers' participation in threshold definition, yields a more desirable result. Also, the combination of this method with IVIF numbers can compensate for the lack of certainty, and allow decision makers to find an acceptable solution. This method can be considered to be a decision making system, which can be used in various fields of project management, construction decisions, supply chain decisions, etc.

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