9 Лр

Вальнець В.
3курс 2група

4) $Ш(x_i, t_j) = \{(x_{i-1}, t_j), (x_i, t_j), (x_i, t_{j-1})\}$

$$Lu(x,t) = \frac{\partial u(x,t)}{\partial t} - a \cdot \frac{\partial u(x,t)}{\partial x} = \varphi(x,t)$$

$$L_h u^{(h)}(x_i, t_j) = A(x_i, t_j)u_{i-1}^j + B(x_i, t_j)u_i^j + C(x_i, t_j)u_i^{j-1}$$

$$R_h(u(x_i, t_j)) = \frac{\partial u(x_i, t_j)}{\partial t} - a\frac{\partial u(x_i, t_j)}{\partial x} -$$

$$-\left[A(x_i, t_j)u_{i-1}^j + B(x_i, t_j)u_i^j + C(x_i, t_j)u_i^{j-1}\right]$$

$\cancel{u_{i-1}^j} =$

$$u(x_{i-1}, t_j) = u(x_i, t_j) - \frac{h}{1!}\frac{\partial u(x_i, t_j)}{\partial x} + \frac{h^2}{2!}\frac{\partial^2 u(x_i, t_j)}{\partial x^2} - \frac{h^3}{3!}\frac{\partial^3 u(x_i, t_j)}{\partial x^3} + \ldots$$

$$u(x_i, t_{j-1}) = u(x_i, t_j) - \frac{\tau}{1!}\frac{\partial u(x_i, t_j)}{\partial t} + \frac{\tau^2}{2!}\frac{\partial^2 u(x_i, t_j)}{\partial t^2} - \frac{\tau^3}{3!}\frac{\partial^3 u(x_i, t_j)}{\partial t^3}$$

$$R_h(u(x_i, t_j)) = u(x_i, t_j)\underline{(-(A+B+C))} + \frac{\partial u(x_i, t_j)}{\partial x}\underline{(-a + hA)} - \frac{h^2}{2}A\frac{\partial^2 u(x_i, t_j)}{\partial x^2} +$$

$$+ \frac{h^3}{6}A\frac{\partial^3 u(x_i, t_j)}{\partial x^3} + \underline{(1 + \tau C)}\frac{\partial u(x_i, t_j)}{\partial t} - \frac{\tau^2}{2}C\frac{\partial^2 u(x_i, t_j)}{\partial t^2} + \frac{\tau^3}{6}C\frac{\partial^3 u(x_i, t_j)}{\partial t^3} + \ldots$$

$$\begin{cases} A + B + C = 0 \\ Ah = a \\ C\tau = -1 \end{cases} \qquad A = \frac{a}{h}, \quad C = -\frac{1}{\tau}$$

$$\frac{a}{h} + B - \frac{1}{\tau} = 0$$

$$B = \frac{1}{\tau} - \frac{a}{h}$$

$$R_h(u(x_i, t_j)) = -\frac{ah}{2}\frac{\partial^2 u(x_i, t_j)}{\partial x^2} + \frac{ah^2}{6}\frac{\partial^3 u(x_i, t_j)}{\partial x^3} + \frac{\tau}{2}\frac{\partial^2 u(x_i, t_j)}{\partial t^2} - \frac{\tau^2}{6}\frac{\partial^3 u(x_i, t_j)}{\partial t^3} + O(h^2) + O(\tau^2)$$

$$L_h u^{(h)}(x_i, t_j) = \frac{a}{h}u_{i-1}^j + \left(\frac{1}{\tau} - \frac{a}{h}\right)u_i^j + \frac{1}{\tau}u_i^{j-1} =$$

$$= -\frac{a}{h}(u_i^j - u_{i-1}^j) + \frac{1}{\tau}(u_i^j - u_i^{j-1}) = \varphi(x_i, t_j)$$