Application of Generative Models for Risk Measurement in Commodity Trading

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Abstract

Abstract. Sample references: Ref 1 [1], ref 2 Jebara [2].

1 Introduction

Introduction, related works, business case

2 Related works

Related works

3 Methodology

Methodology

3.1 Stochastic differential equations

Definition 3.1. A stochastic process is a random variable of the form

$$X_t(\omega) : T \times \Omega \to \mathbb{R}$$

Definition 3.2. A stochastic process $W = W_0, W_1, \ldots, W_T$ is called a Wiener process if the following properties hold:

1. $W_0 = 0$ with probability 1
2. $\mathbb{E}W_t = 0$
3. $\mathrm{Var}[W_b - W_a] = b - a$ for all $0 \leq a \leq b \leq T$
4. $W_b - W_a \sim \mathcal{N}(0, b - a)$
5. $W_b - W_a$ and $W_d - W_c$ are independent for all $a \leq b \leq c \leq d$

Definition 3.3. A stochastic differential equation is an equation of the form

$$\mathrm{d}S_t = \mu S_t \mathrm{d}t + \sigma S_t \mathrm{d}W_t$$

where μ and σ are some constants or some fucntions which represent the interest rate of nonrisky activities and is the market volatility respectively.

Definition 3.4. An Ito process is a stochastic process such that

$$X_t = X_0 + \int_0^t g(s)\mathrm{d}s + \int_0^t h(s)\mathrm{d}W_s$$

where $g(t,\omega)$ and $h(t,\omega)$ are random functions such that

$$P\left(\int_0^T |g(w,t)|\mathrm{d}t < \infty\right) = 1 \text{ and } P\left(\int_0^T h(w,t)^2\mathrm{d}t < \infty\right) = 1$$

Definition 3.5. A diffusion process is the solution of SDE of the form

$$\mathrm{d}X_t = b(t,X_t)\mathrm{d}t + \sigma(t,X_t)\mathrm{d}W_t$$

where some deterministic functions b and σ are called the *drift* and the *diffusion* coefficient of SDE such that

$$P\left\{\int_0^T \sup_{|x|\leq\mathbb{R}} (|b(t,x)| + \sigma^2(t,x))\mathrm{d}t < \infty\right\} = 1$$

A Black-Scholes-Merton or geometric Brownian motion

Definition 3.6. The Black-Scholes-Merton is described by the stochastic differential equation of the form

$$\mathrm{d}X_t = \theta_1 X_t \mathrm{d}t + \theta_2 X_t \mathrm{d}W_t$$

with $X_0 = x_0$ and $\theta_2 > 0$. The parameter θ_1 represents the constant interest rate and θ_2 — the volatility of risky activities.

The explicit solution is

$$S(t) = S(0)\exp\left[\left(r - \frac{\sigma^2}{2}\right)t + \sigma W(t)\right]$$

And the sumulation increment form is

$$S(t+\Delta t) = S(t)\exp\left[\left(r - \frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{\Delta t}Z\right]$$

Euler scheme

$$\Delta \log S = \left(r - \frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{\Delta t}Z$$

B Cox-Ingersoll-Ross

Cox-Ingersoll-Ross

3.2 Generative and Discriminative models

Generative and Discriminative models

A Bayesian Inference

Bayesian Inference

B Support Vector Machines

Support Vector Machines

C Generative adversarial networks

Generative adversarial networks

4 Case study

Case study

4.1 Model description

Model description

4.2 Model training

Model training

4.3 Dataset description

Dataset description

4.4 Data preprocessing

Data preprocessing

4.5 Quality metrics and results of testing

Quality metrics and results of testing

5 Conclusion

Conclusion

References

[1] Y. Chen, Y. Wang, D. Kirschen, and B. Zhang. Model-free renewable scenario generation using generative adversarial networks. *IEEE Transactions on Power Systems*, 33(3), 2018. doi: 10.1109/TPWRS.2018.2794541.

[2] T. Jebara. *MACHINE LEARNING: Discriminative and Generative.* Kluwer Academic Publishers, 2004.