Music Generation with Wasserstein Autoencoders

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Introduction

Aim

Being able to generate music from scratch.

Methodology

Training deep Wasserstein Autoencoders (WAEs).

Data

Pianoroll samples derived from pop/rock songs.

Generative Models

- We have a set of *i.i.d.* data points $\{x^{(1)}, \dots, x^{(N)}\} \subseteq \mathcal{X}$.
- We do not know the true data distribution $P_X(X)$.
- We would like to find a distribution $P_G(X)$ over $\mathcal{X}$ that is *similar* to $P_X(X)$.

Latent Variable Model (LVM)

1. Introduce a latent space $\mathcal{Z}$.
2. Sample $P_Z(Z)$, a *prior* distribution on the latent space.
3. Map the sample to $\mathcal{X}$ through $P_G(X|Z)$, i.e. the *decoder*.

$$p_G(x) = \int_{\mathcal{Z}} p_G(x|z) p_z(z) dz \tag{1}$$

Variational Autoencoders (VAEs): Kingma & Welling, 2014 - [1]

Minimize the **KL-divergence** between P_X and P_G:

$$\inf_{P_G} \mathrm{KL}(P_X \| P_G) \iff \inf_{P_G} \mathbb{E}_{P_X}[-\log P_G] \tag{2}$$

Variational bound: for any conditional distribution $Q(Z|X)$ the following inequality holds

$$-\mathbb{E}_{P_X}[\log P_G(X)] \leq \mathbb{E}_{P_X}\left[\mathbb{E}_{Q(Z|X)}[-\log P_G(X|Z)] + \mathrm{KL}(Q(Z|X)\|P_Z(Z))\right] \tag{3}$$

- Decoders for which $\log P_G(X|Z)$ can be evaluated and differentiated.
- Often $P_Z(Z) = \mathcal{N}(Z; 0, I)$ and $Q(Z|X) = \mathcal{N}(Z; \mu(X), \Sigma(X))$.
- Everything is parametrized with Deep Neural Networks.
- The bound is minimized via Stochastic Gradient Descent.

Optimal Transport (OT)

Given a cost function $c : \mathbb{X} \times \mathbb{X} \to \mathbb{R}^+$ the OT problem is

$$W_c(P_X, P_G) = \inf_{\Gamma \in \mathcal{P}(X \sim P_X, Y \sim P_G)} \mathbb{E}_\Gamma[c(X, Y)] \tag{4}$$

For deterministic decoders, i.e. $P_G(X|Z) = \delta_{G(Z)}$, for any $G : \mathcal{Z} \to \mathcal{X}$ then

$$W_c(P_X, P_G) = \inf_{Q : Q_z = P_z} \mathbb{E}_{P_X}\left[\mathbb{E}_{Q(Z|X)}[c(X, G(Z))]\right] \tag{5}$$

where $Q_z = \mathbb{E}_{P_X}[Q(Z|X)]$ is the *aggregated posterior*.

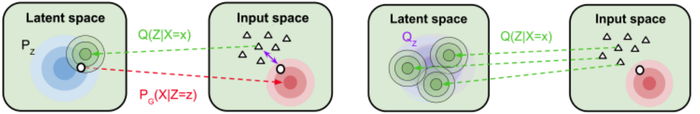


Figure 1: VAE approach (left) vs WAE approach (right)

Wasserstein Autoencoders (WAEs): Tolstikhin & al., 2018 - [2]

Minimizing the optimal transport:

$$\inf_{P_G} W_c(P_X, P_G) \iff \inf_{P_G} \inf_{Q: Q_z = P_z} \mathbb{E}_{P_X}\left[\mathbb{E}_{Q(Z|X)}[c(X, G(Z))]\right] \quad (6)$$

Relaxation:

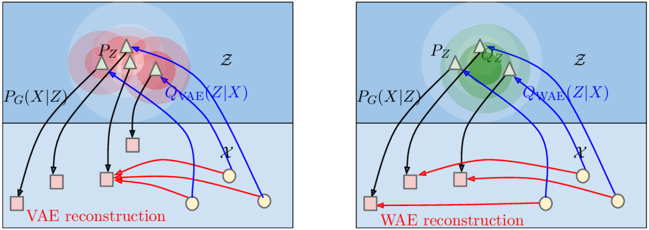
$$\inf_{P_G} \inf_{Q(Z|X) \in \mathcal{Q}} \mathbb{E}_{P_X}\left[\mathbb{E}_{Q(Z|X)}[c(X, G(Z))]\right] + \lambda \cdot \mathcal{D}_z(Q_z, P_z) \quad (7)$$

for any non-parametric set of encoders $\mathcal{Q}$ and divergence $\mathcal{D}_z$.

- Hyperparameter $\lambda \in \mathbb{R}^+$.
- Parametrize G (*decoder*) and Q (*encoder*) with DNNs.
- Run SGD on (7).

VAEs vs WAEs [2]

(a) VAE (b) WAE



VAE:

$$\inf_{P_G} \inf_{Q \in \mathcal{Q}} \mathbb{E}_{P_X} \left[\mathbb{E}_{Q(Z|X)} [- \log P_G(X|Z)] \right] + \mathbb{E}_{P_X} [\mathrm{KL}(Q(Z|X) \| P_Z(Z))]$$

WAE:

$$\inf_{P_G} \inf_{Q \in \mathcal{Q}} \mathbb{E}_{P_X} \left[\mathbb{E}_{Q(Z|X)} [c(X, G(Z))] \right] + \lambda \cdot \mathcal{D}_z(\mathbb{E}_{P_X}[Q(Z|X)], P_Z(Z))$$

WAE-GAN: adversarial penalty [2]

$$\mathcal{D}_z(Q_z, P_z) = D_{JS}(Q_z, P_z) = \frac{1}{2}\mathrm{KL}(P_Z \| M) + \frac{1}{2}\mathrm{KL}(Q_Z \| M)$$

Where $M = \frac{1}{2}(P_Z + Q_Z)$. A discriminator D is introduced in the latent space.

- Using the adversarial training:

$$D_{JS}(Q_Z, P_Z) \approx \sup_D \mathbb{E}_{Z \sim P_Z}[\log D(Z)] + \mathbb{E}_{Z^* \sim Q_Z}[\log(1 - D(Z^*))]$$

- **Con**: Leads to the $\min - \max$ problem.
- Pro: Should be easier than typical GANs (usually $\mathcal{Z}$ is low dimensional and P_Z uni-modal).

WAE-MMD: non-adversarial penalty [2] and [3]

$$\mathcal{D}_z(Q_Z, P_Z) = MMD_k(Q_Z, P_Z) = MMD_k(P_Z, Q_Z)$$

With

$$MMD_k(P_z, Q_z) = \left\| \int_{\mathcal{Z}} k(z, \cdot) dP_Z(z) - \int_{\mathcal{Z}} k(z, \cdot) dQ_Z(z) \right\|_{\mathcal{H}_k} \quad (8)$$

Where $k : \mathcal{Z} \times \mathcal{Z} \to \mathbb{R}$ is any p.d. reproducing kernel and $\mathcal{H}_k$ its RKHS. Given $\{z_i\}_{i=1}^N \sim P_Z$ and $\{z_i^*\}_{i=1}^N \sim Q_Z$, if k is characteristic then an unbiased estimate of the MMD is:

$$\frac{1}{N(N-1)} \left[\sum_{i=1}^{N} \sum_{j \neq i} k(z_i, z_j) + \sum_{i=1}^{N} \sum_{j \neq i} k(z_i^*, z_j^*) \right] - \frac{2}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} k(z_i, z_j^*) \quad (9)$$

Pianoroll

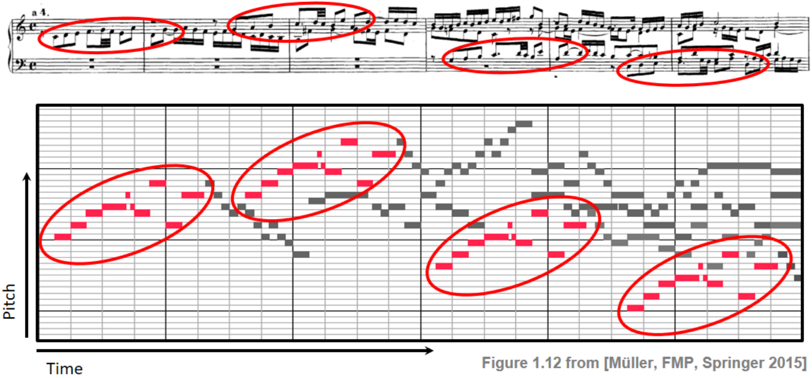


Figure 2: Six bars from Fugue BWV 846 in C major by J.S.Bach

Data Description: Structure - Dong & al., 2017 - [4]

- **Dataset**: Lakh Pianoroll Dataset (LPD)-5, *cleansed* version.
  - 21425 elements.
  - Every element is a phrase of 4 bars for 5 instruments.
  - Instruments are: *Drums*, *Guitar*, *Bass*, *Piano* and *Strings*.
  - Every element has a time signature of 4/4.
  - Quantization is set to 1/48th.
  - Spans from C1 to B7 (84 notes).
  - Majors to CMaj and minors to AMin.
  - Binary valued (*note-on* information).
- **Pro**: Simple, image-like, representation.
- **Con**: Partial information.

Data Description: Visualization

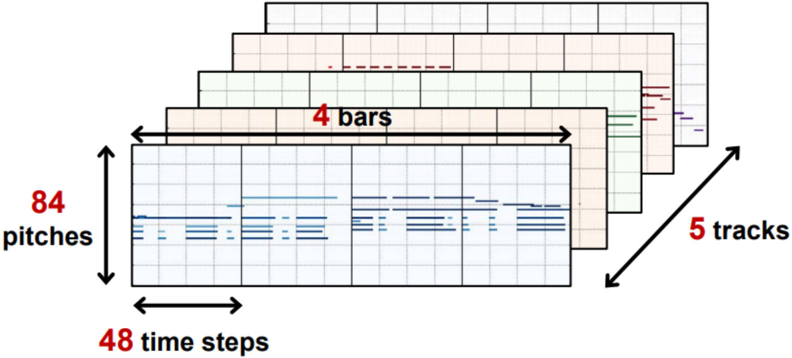


Figure 3: Visualization of a data point (image from dataset authors)

This results in a dataset whose elements live in $\mathbb{D} = \{0, 1\}^{84\times 192\times 5}$

Experiment details

Choices:

1. Euclidean latent space: $\mathcal{Z} = \mathbb{R}^{d_z}$.
2. Gaussian prior: $P_Z(Z) = \mathcal{N}(Z; 0, \sigma_z^2 \cdot I_{d_z})$.
3. Squared L2 norm as cost function: $c(x, y) = \|x - y\|_2^2$.

Implementation:

- Encoder Q_ϕ and Decoder G_θ as deep CNNs in a DCGAN-style (*Conv* / *Deconv* $\rightarrow$ *Batch Norm* $\rightarrow$ *ReLU*).
- Discriminator D_γ (only in the WAE-GAN algorithm) as a deep NN (*Dense* $\rightarrow$ *ReLU*).
- Adam optimizer.

Training Phase

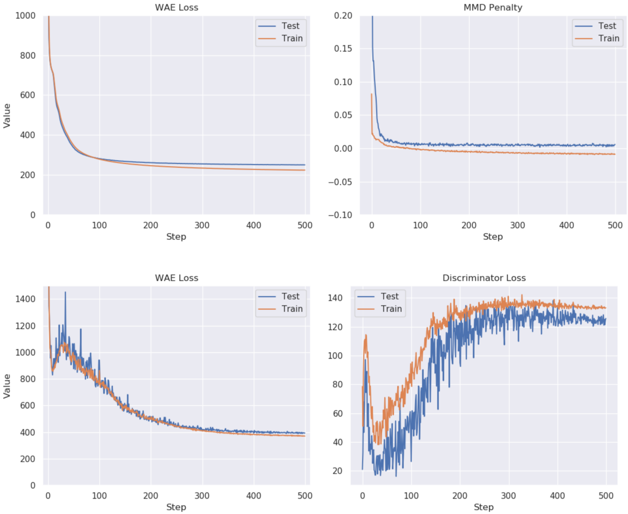


Figure 4: WAE-MMD (top) WAE-GAN (bottom)

- *Latent space dimension*: $d_z = 128$
- *Weight*: $\lambda = 100$
- *Convolutional filter size*: 6×6
- *Epochs*: 500
- *Number of filters*: $[8, 16, 32, 64]$
- *Learning rates*: $10^{-4} \rightarrow 10^{-5}$
- *Samples*: 8192 (train), 2048 (test)

Metrics

Lack of an objective measure of performance

- Reconstruction error: Mean Squared Error (**MSE**).

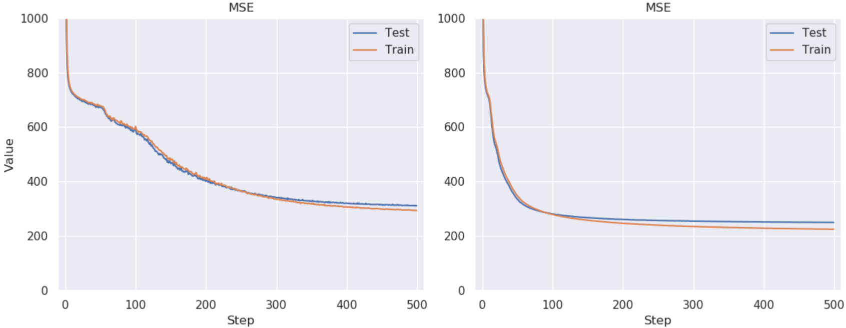
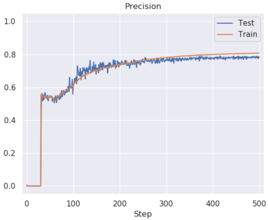
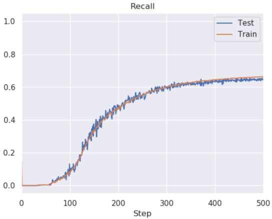


Figure 5: MSE: WAE-GAN (left) vs WAE-MMD (right)

Lack of an objective measure of performance

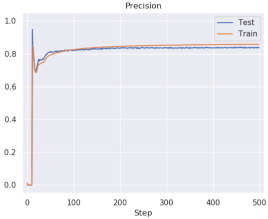
- Precision (**P**) and Recall (**R**) - Top: WAE-GAN, bottom: WAE-MMD.

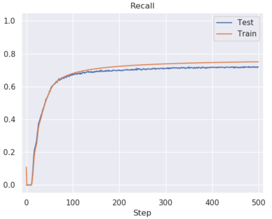




ALG	P	R
WAE-GAN	0.83	0.7
WAE-MMD	0.87	0.78

Table 1: Train Set





ALG	P	R
WAE-GAN	0.78	0.63
WAE-MMD	0.83	0.70

Table 2: Test Set

Metrics [4]

Data[1]	**EBR**	**QNR**	**POLY**
Test	0.05	0.88	0.51
Reconstruction - MMD	0.15	0.80	0.50
Reconstruction - GAN	0.14	0.76	0.49
Random Uniform on $\{0, 1\}$	0.0	0.24	1.0
Random Samples - MMD	0.13	0.65	0.29
Random Samples - GAN	0.16	0.64	0.38

Where these metrics are

$$\begin{cases} \text{Empty Bars Rate } (\textbf{EBR}) = \frac{\#\ [\text{empty bars}]}{\#\ [\text{bars}]} \\ \text{Qualified Notes Rate } (\textbf{QNR}) = \frac{\#\ [\text{notes no shorter than } 1/16th]}{\#\ [\text{notes}]} \\ \text{Polyphonicity } (\textbf{POLY}) = \frac{\#\ [\text{time steps where} \geq 2 \text{ pitches}]}{\#\ [\text{time steps}]} \end{cases}$$

[1] Every set in this column is composed of 2048 samples.

Conclusions

- WAEs are a promising learning framework.
  - Flexible and theoretically grounded.
  - Stable training.
- The lack of a “musical performance” metric is an obstacle
  - Qualitative analysis.
  - Hyperparameter optimization is difficult.

Listen to the results

`https://w00zie.github.io/post/wae`

Look at the code

`https://github.com/w00zie/wae_music`


Thank you for your
attention!

References I

[1] D. P. Kingma and M. Welling, *Auto-encoding variational bayes*, 2013. arXiv: 1312.6114 [stat.ML].

[2] I. Tolstikhin, O. Bousquet, S. Gelly, and B. Schölkopf, *Wasserstein auto-encoders*, 2017. arXiv: 1711.01558 [stat.ML].

[3] K. Muandet, K. Fukumizu, B. Sriperumbudur, and B. Schölkopf, "Kernel mean embedding of distributions: A review and beyond", *Foundations and Trends® in Machine Learning*, 2017. [Online]. Available: http://dx.doi.org/10.1561/2200000060.

References II

[4] H.-W. Dong, W.-Y. Hsiao, L.-C. Yang, and Y.-H. Yang, *Musegan: Multi-track sequential generative adversarial networks for symbolic music generation and accompaniment*, 2017. arXiv: 1709.06298 [eess.AS].

Deriving the ELBO in 1

$$\log p_\theta(x) = \int_{\mathbb{Z}} \log p_\theta(x) q_\phi(z|x) dz = \mathbb{E}_{q_\phi(z|x)}[\log p_\theta(x)] = \tag{10}$$

$$= \mathbb{E}_{q_\phi(z|x)}\left[\log \frac{p_\theta(x,z)}{p_\theta(z|x)}\right] = \tag{11}$$

$$= \mathbb{E}_{q_\phi(z|x)}\left[\log \left(\frac{p_\theta(x,z)}{q_\phi(z|x)} \frac{q_\phi(z|x)}{p_\theta(z|x)}\right)\right] = \tag{12}$$

$$= \mathbb{E}_{q_\phi(z|x)}\left[\log \frac{p_\theta(x,z)}{q_\phi(z|x)}\right] + \mathbb{E}_{q_\phi(z|x)}\left[\log \frac{q_\phi(z|x)}{p_\theta(z|x)}\right] = \tag{13}$$

$$= ELBO(\theta, \phi) + KL(q_\phi(z|x) \| p_\theta(z|x)) \tag{14}$$

Then

$$ELBO(\theta, \phi) = \mathbb{E}_{q_\phi(z|x)}\left[\log \frac{p_\theta(x,z)}{q_\phi(z|x)}\right] = \mathbb{E}_{q_\phi(z|x)}\left[\log \frac{p_\theta(x|z) p_\theta(z)}{q_\phi(z|x)}\right] = \tag{15}$$

$$= \mathbb{E}_{q_\phi(z|x)}[\log p_\theta(x|z)] - \mathbb{E}_{q_\phi(z|x)}\left[\log \frac{q_\phi(z|x)}{p_\theta(z)}\right] = \tag{16}$$

$$= \mathbb{E}_{q_\phi(z|x)}[\log p_\theta(x|z)] - KL(q_\phi(z|x) \| p_\theta(z)) \tag{17}$$

WAE-GAN

Init $G_\theta, Q_\phi, D_\gamma$
while (θ, ϕ, γ) not converged **do**
Sample $\{x^{(1)}, \dots, x^{(n)}\}$ from D.
Sample $\{z^{(1)}, \dots, z^{(n)}\}$ from $\mathcal{Z}$.
Sample $\tilde{z}^{(i)}$ from $Q(Z|X = x^{(i)})$ for $i = 1, \dots, n$.
Update D_γ by ascending:

$$\frac{\lambda}{n} \sum_{i=1}^{n} \log D(z^{(i)}) + \log(1 - D(\tilde{z}^{(i)})$$

Update (G_θ, Q_ϕ) by descending:

$$\frac{1}{n} \sum_{i=1}^{n} c(x^{(i)}, G_\theta(\tilde{z}^{(i)})) - \lambda \cdot \log D(\tilde{z}^{(i)})$$

end while

WAE-MMD

Init G_θ, Q_ϕ
while (θ, ϕ) not converged **do**
Sample $\{x^{(1)}, \dots, x^{(n)}\}$ from D.
Sample $\{z^{(1)}, \dots, z^{(n)}\}$ from $\mathcal{Z}$.
Sample $\tilde{z}^{(i)}$ from $Q(Z|X = x^{(i)})$ for $i = 1, \dots, n$.
Update (G_θ, Q_ϕ) by descending:

$$\frac{1}{n} \sum_{i=1}^{n} c(x^{(i)}, G_\theta(\tilde{z}^{(i)}))+$$

$$+ \frac{\lambda}{n(n-1)} \left[\sum_{i=1}^{n} \sum_{j \neq i} k(z^{(i)}, z^{(j)})+ \right.$$

$$\left. + \sum_{i=1}^{n} \sum_{j \neq i} k(\tilde{z}^{(i)}, \tilde{z}^{(j)}) \right] -$$

$$- \frac{2}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} k(z^{(i)}, \tilde{z}^{(j)})$$

end while

Kernels

Def (Positive definite kernel): Let $\mathcal{X}$ be a non-empty set. A function $k : \mathcal{X} \times \mathcal{X} \to \mathbb{R}$ is a positive definite kernel if it is symmetric and the Gram matrix is positive definite:

$$\sum_{i=1}^{n} \sum_{j=1}^{n} c_i c_j k(x_i, x_j) \geq 0$$

for any $n \in \mathbb{N}$, any $x_1, \ldots, x_n \in \mathcal{X}$ and $c_1, \ldots, c_n \in \mathbb{R}$. A p.d. kernel defines a space of functions from $\mathcal{X}$ to $\mathbb{R}$ called a Reproducing Kernel Hilbert Space (**RKHS**) $\mathcal{H}_k$, hence is also called a reproducing kernel. In particular the reproducing property says that for each $x \in \mathcal{X}$ there exists a function $k_x = k(x, \cdot) \in \mathcal{H}_k$ such that

$$\langle k_x, f \rangle_{\mathcal{H}_k} = \langle k(x, \cdot), f \rangle = f(x), \qquad \forall f \in \mathcal{H}_k$$

One may alternatively consider $k(x, \cdot)$ an implicit feature mapping $\varphi(x)$ from $\mathcal{X}$ to $\mathcal{H}_k$ (which is therefore also called the feature space), so that $k(x, x') = \langle \varphi(x), \varphi(x') \rangle_{\mathcal{H}_k}$ can be viewed as a measure of similarity between points $x, x' \in \mathcal{X}$.
Def (characteristic kernel): A kernel k is said to be characteristic if, named $\mathbb{P}$ the space of all distributions over $\mathcal{X}$, the mapping $\mu : P \to \mu_P$ is injective, for all $P \in \mathbb{P}$. The quantity μ_P is defined as

$$\mu_P = \mathbb{E}_P[k(X, \cdot)] = \mathbb{E}_P[\varphi(X)] = \int_{\mathcal{X}} \varphi(x) dP(x)$$

Kernel Mean Embeddings: MMD

Def (MMD): The maximum mean discrepancy is a distance-measure between distributions $P(X)$ and $Q(Y)$ (taking values on $\mathcal{X}$) which is defined as the squared distance between their embeddings in the RKHS $\mathcal{H}_k$:

$$
\begin{aligned}
MMD[\mathcal{H}_k, P, Q] &= \sup_{\|f\|_{\mathcal{H}_k} \leq 1} \left\{ \int_{\mathcal{X}} f(x) dP(x) - \int_{\mathcal{X}} f(y) dQ(y) \right\} = \\
&= \sup_{\|f\|_{\mathcal{H}_k} \leq 1} \left\{ \langle f, \int_{\mathcal{X}} k(x, \cdot) dP(x) \rangle - \langle f, \int_{\mathcal{X}} k(y, \cdot) dQ(y) \rangle \right\} = \\
&= \sup_{\|f\|_{\mathcal{H}_k} \leq 1} \{ \langle f, \mu_P - \mu_Q \rangle \} = \\
&= \|\mu_P - \mu_Q\|_{\mathcal{H}_k}
\end{aligned}
$$

Def (U-Statistic): Let $X_1, X_2, \ldots, X_m$ be be independent observations on a distribution $P(X)$. Let $\theta(P)$ be the functional defined by

$$
\theta(P) = \mathbb{E}_P[h(X_1, \ldots, X_m) = \int \cdots \int h(x_1, \ldots, x_m) dP(x_1) \ldots dP(x_m)
$$

for some real-valued measurable function h called a kernel. The corresponding U-statistic of order m for estimation of θ on the basis of a sample $x_1, \ldots, x_n$ of size $n \geq m$ is obtained by averaging the kernel h symmetrically over the observations:

$$
U_n = U(x_1, \ldots, x_m) = \frac{1}{\binom{n}{m}} \sum_c h(x_{i_1}, \ldots, x_{i_n})
$$

where $\sum_c$ denotes the sum over the $\binom{n}{m}$ combinations of m distinct elements $\{i_1, \ldots, i_m\}$ from $\{1, \ldots, n\}$.

Kernel used

The kernel used is the sum of inverse multiquadratics kernels at various scales:

$$k(x, y) = \sum_{s \in S} \frac{2sd_z\sigma_z^2}{2sd_z\sigma_z^2 + \|x - y\|_2^2}$$

where $S = \{0.1, 0.2, 0.5, 1, 2, 5, 10\}$.

#	Layer	Input	Ouput	Params
1	`Conv2D(32)`	$(B, 192, 84, 5)$	$(B, 96, 84, 32)$	5760
2	`BatchNorm`	$(B, 96, 84, 32)$	$(B, 96, 84, 32)$	128
3	`Conv2D(64)`	$(B, 96, 84, 32)$	$(B, 48, 42, 64)$	73728
4	`BatchNorm`	$(B, 48, 42, 64)$	$(B, 48, 42, 64)$	256
5	`Conv2D(128)`	$(B, 48, 42, 64)$	$(B, 24, 21, 128)$	294912
6	`BatchNorm`	$(B, 24, 21, 128)$	$(B, 24, 21, 128)$	512
7	`Conv2D(256)`	$(B, 24, 21, 128)$	$(B, 8, 7, 256)$	1179648
8	`BatchNorm`	$(B, 8, 7, 256)$	$(B, 8, 7, 256)$	1024
9	`Flatten`	$(B, 8, 7, 256)$	$(B, 14336)$	0
10	`Dropout(0.2)`	$(B, 14336)$	$(B, 14336)$	0
11	`Dense(128)`	$(B, 14336)$	$(B, 128)$	1835136

Table 3: Encoder

Divergence Plots

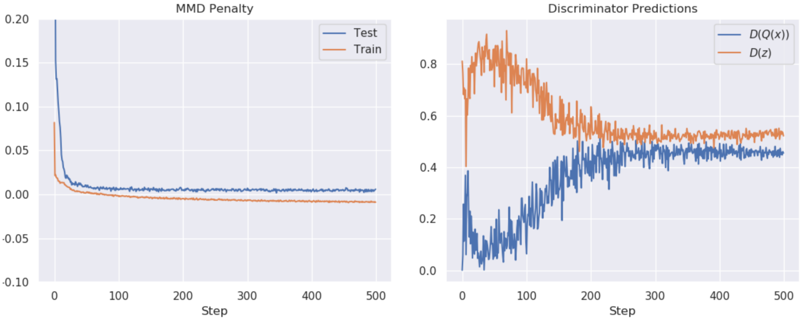


Figure 6: Plots for MMD (left) and discriminator values (right). Plot on the right is made on the test set only

Precision and Recall

Let

$$\hat{x}^{(i)} = G_\theta(Q_\phi(x^{(i)}))$$

where $x^{(i)} \in D$. Let the true positives (TP), true negatives (TN), false positives (FP) and false negatives (FN) defined as

$$\begin{cases} TP = \#[x_{jkl}^{(i)} = \hat{x}_{jkl}^{(i)} = 1] \\ TN = \#[x_{jkl}^{(i)} = \hat{x}_{jkl}^{(i)} = 0] \end{cases} \qquad \begin{cases} FP = \#[x_{jkl}^{(i)} = 0 \wedge \hat{x}_{jkl}^{(i)} = 1] \\ FN = \#[x_{jkl}^{(i)} = 1 \wedge \hat{x}_{jkl}^{(i)} = 0] \end{cases}$$

Then precision (**P**) and recall (**R**):

$$\begin{cases} \mathbf{P} & = \frac{TP}{TP+FP} \\ \mathbf{R} & = \frac{TP}{TP+FN} \end{cases}$$

Metrics from test set and relative reconstructions

Instrument	**EBR**	**QNR**	**POLY**	**ISR**
Drums	0.0471	-	-	-
Piano	0.0498	0.9076	0.6945	0.7144
Guitar	0.0426	0.8588	0.5822	0.7081
Bass	0.0173	0.8986	0.0229	0.4666
String	0.0753	0.8665	0.7453	0.7068

Table 4: Test set

Instrument	**EBR**	**QNR**	**POLY**	**ISR**
Drums	0.1831	-	-	-
Piano	0.1324	0.7305	0.6742	0.7395
Guitar	0.1870	0.6633	0.5653	0.7229
Bass	0.0610	0.7885	0.0138	0.4408
String	0.1172	0.8574	0.7195	0.7192

Table 5: Test set reconstructions with WAE-GAN

Instrument	**EBR**	**QNR**	**POLY**	**ISR**
Drums	0.3467	-	-	-
Piano	0.1031	0.7677	0.6747	0.7255
Guitar	0.1488	0.6966	0.5644	0.7055
Bass	0.0433	0.8269	0.0164	0.4565
String	0.1075	0.8951	0.7272	0.7117

Table 6: Test set reconstructions with WAE-MMD

Metrics from decoded samples

Instrument	**EBR**	**QNR**	**POLY**	**ISR**
Drums	0.1812	-	-	-
Piano	0.1187	0.6375	0.5139	0.7285
Guitar	0.2067	0.5759	0.4406	0.7047
Bass	0.1940	0.5949	0.0151	0.4507
String	0.0951	0.7384	0.5683	0.7177

Table 7: Decoded with WAE-GAN

Instrument	**EBR**	**QNR**	**POLY**	**ISR**
Drums	0.2841	-	-	-
Piano	0.0720	0.6316	0.3865	0.7413
Guitar	0.1128	0.5727	0.2920	0.7104
Bass	0.1000	0.6268	0.0234	0.4205
String	0.0643	0.7557	0.4601	0.7522

Table 8: Decoded with WAE-MMD