#Course: Differential Equations (Lec 1)

23/3/21

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$$\frac{d}{dx} = D$$

$$\frac{dy}{dx} + y = 0$$

$$Dy + y = 0$$

1. Ordinary Differential Equation
2. Partial " "

$$\frac{\partial x}{\partial y}$$

power)

সরল সরল

(degree)

power

$$\# \left(\frac{d^2y}{dx^2}\right)^{1} + 3\left(\frac{d^1y}{dx}\right)^{2} + y = 5x$$

degree

$$\# \left(\frac{d^2y}{dx^2}\right) + 2\left(\frac{dy}{dx}\right)^2 + 6y = 3x$$

Non-linear

cause $\frac{dy}{dx}$ গুণ উচ্চ power

আছে।

$$\# \frac{d^2y}{dx^2} + 2y\frac{dy}{dx} + 6y = 3x$$

y আগের গুণ হওয়ায় কারণে

Non-linear [$y \cdot \frac{dy}{dx}$, 2টো y]

$$\# \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 6y^2 = 3x$$

Linear

Ex:3 :: Page-2

$$c(y+c)^2 = x^3 \quad \text{----} \quad (i)$$

D. w. r. to x

$$2c(y+c)\frac{dy}{dx} = 3x^2 \quad \text{----} \quad (ii)$$

Dividing (i) by (ii)

$$\frac{c(y+c)^2}{2c(y+c)\frac{dy}{dx}} = \frac{x^3}{3x^2}$$

$$\frac{y+c}{2\frac{dy}{dx}} = \frac{x}{3}$$

$$y+c = \frac{x}{3} \cdot 2\frac{dy}{dx}$$

$$3y+3c = 2x\frac{dy}{dx}$$

$$3c = 2x\frac{dy}{dx} - 3y$$

$$\therefore c = \frac{1}{3}\left[2x\frac{dy}{dx} - 3y\right]$$

Putting the value of c in 2

$$\Rightarrow \frac{2}{3}\left(2x\frac{dy}{dx}-3y\right)\left\{y+\frac{1}{3}\left(2x\frac{dy}{dx}-3y\right)\right\}\Big/\frac{dy}{dx} = 3x^2$$

$$\Rightarrow 8x\left(\frac{dy}{dx}\right)^3-12y\left(\frac{dy}{dx}\right)^2 = 27x$$

order 1
degree 3

Non-linear

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Pg: 8 - 10

$$dx = \frac{f_1(x,y)}{f_2(x,y)}$$ x, y এর degree same

$$y = vx$$

$$\frac{dy}{dx} = v.1 + x\frac{dv}{dx}$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

~~Ex 02 : Pg~~

Ex: 02 :: Pg: 10

$$x^2 y\,dx - (x^3 + y^3)\,dy = 0$$

$$\frac{dy}{dx} = \frac{x^2 y}{x^3 + y^3} \quad \text{--- (i)}$$

উপরে
নিচে
দুটোতেই power 3

হোমোজেনাস ফাংশন

Let $y = vx$

$$\therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$(i) \Rightarrow v + x\frac{dv}{dx} = \frac{x^2 vx}{x^3 + v^3 x^3}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v}{1+v^3} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v - v - v^4}{1+v^3}$$

$$\Rightarrow x\frac{dv}{dx} = -\frac{v^4}{1+v^3}$$

$$\Rightarrow \frac{1+v^4}{v^4}\,dv = -\frac{dx}{x}$$

$$\Rightarrow \left(\frac{1}{v^4}+\frac{1}{v}\right)dv = -\frac{dx}{x}$$

$$\Rightarrow -\frac{1}{3v^3} + \ln v = -\ln x + c$$

$$\Rightarrow \ln v + \ln x = \frac{1}{3v^3} + c$$

$$\Rightarrow \ln vx = \frac{x^3}{3y^3} + c$$

$$\Rightarrow \ln \frac{y}{x}\cdot x = \frac{x^3}{3y^3} + c$$

$$\Rightarrow \ln y = \frac{x^3}{3y^3} + c \quad \text{(Ans)}$$

Ex:4 Pg: 11

$$y^2 + x^2\frac{dy}{dx} = xy\frac{dy}{dx}$$

$$\Rightarrow (xy - x^2)\frac{dy}{dx} = y^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^2}{xy - x^2} \quad \text{------ (i)}$$

Let $y = vx$

$$\therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$$

(i) $\Rightarrow v + x\frac{dv}{dx} = \frac{v^2x^2}{xvx - x^2}$

$\Rightarrow x\frac{dv}{dx} = \frac{v^2}{v-1} - v$

$\Rightarrow x\frac{dv}{dx} = \frac{v^2 - v^2 + v}{v-1}$

$\Rightarrow x\frac{dv}{dx} = \frac{v}{v-1}$

$\Rightarrow \frac{v-1}{v} dv = \frac{dx}{x}$

$\Rightarrow (1 - \frac{1}{v}) dv = \frac{dx}{x}$

Integrating

$v - \ln v = \ln x + c$

$\Rightarrow v = \ln x + \ln v + c$

$\Rightarrow v = \ln vx + c$

$\Rightarrow \frac{y}{x} = \ln \frac{y}{x} \cdot x + c$

$\Rightarrow y = x \ln y + cx$ (Ans)

$\Rightarrow \ln vx = v - c$

$\Rightarrow \ln vx = v + \ln c_1$

$\Rightarrow \ln vx = \ln e^v + \ln c_1$

$\Rightarrow \ln vx = \ln e^v \cdot c_1$

$\Rightarrow vx = e^v c_1$

$\Rightarrow \frac{y}{x} \cdot x = c_1 e^{y/x}$

$\Rightarrow y = c_1 e^{y/x}$ (Ans)

Pg: 28

Linear Differential Equation:

$\frac{dy}{dx} + Py = Q$

Integrating Factor $= e^{\int P dx}$

It's ~~sotion~~ solution is

$y \cdot I.F = \int Q \cdot I.F \, dx + c$

$$\Rightarrow y\sqrt{1-x^2} = \int \frac{dx}{\sqrt{1-x^2}}$$

$$\Rightarrow y\sqrt{1-x^2} = \sin^{-1}x + c \quad \text{(Ans)}$$

P.18 :: Ex-1

$$(1-x^2)\frac{dy}{dx} - xy = 1$$

$$\frac{dy}{dx} - \frac{xy}{1-x^2} = \frac{1}{1-x^2}$$

which is the form of

$$\frac{dy}{dx} + Py = Q$$

where, $P = \frac{-x}{1-x^2}$, $Q = \frac{1}{1-x^2}$

So, the eq^n is L.D.E,

$$\therefore I.F = e^{\int P dx}$$

$$= e^{\int \frac{-x}{1-x^2}dx}$$

$$= e^{\int \frac{d(1-x^2)}{2(1-x^2)}}$$

$$= e^{1/2 \ln(1-x^2)}$$

$$= e^{\ln\sqrt{1-x^2}}$$

$$= \sqrt{1-x^2}$$

$$\boxed{y.I.F = \int Q.I.F\,dx}$$

It's solution is,

$$y = \sqrt{1-x^2} = \int \frac{1}{1-x^2}\sqrt{1-x^2}\,dx$$

Pg : 21 :: Ex-10

$$\frac{dy}{dx} - 2y\cos x = -2\sin 2x \quad \text{---(i)}$$

which is the form of

$$\frac{dy}{dx} + Py = Q$$

$$P = -2\cos x,\ Q = -2\sin 2x$$

$$\therefore I.F = e^{\int P dx}$$

$$= e^{-\int 2\cos x\,dx}$$

$$= e^{-2\sin x}$$

It's solution is

$$y \cdot e^{-2\sin x} = \int -2\sin 2x \cdot e^{-2\sin x}\,dx + c$$

$$= c - 4\int \sin x \cos x\, e^{-2\sin x}\,dx$$

Let $-2\sin x = t$

$-2\cos x \, dx = dt$

$\therefore \; ye^{-2\sin x} = c + \int te^{t}dt$

$= c + te^{t} - \int e^{t}dt$

$= c + te^{t} - e^{t}$

$= c + (t-1)\,e^{t}$

$\Rightarrow ye^{-2\sin x} = c + (-2\sin x - 1)e^{-2\sin x}$

$\Rightarrow y = ce^{2\sin x} - (2\sin x + 1)$

(Ans)

Math

Differential Equations

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Pg: 22

Bernoulli Equation

$$\frac{dy}{dx} + Py = Qy^n \qquad \Big| \quad \frac{dy}{dx} + Py = Q$$

$$\Rightarrow \frac{1}{y^n} \cdot \frac{dy}{dx} + \frac{Py}{y^n} = Q$$

$$\Rightarrow y^{-n} \cdot \frac{dy}{dx} + Py^{-n+1} = Q$$

Let, $y^{n+1} = v$

$$(1-n) y^{-n} \frac{dy}{dx} = \frac{dv}{dx}$$

$$\Rightarrow \frac{1}{1-n} \frac{dv}{dx} + PV = Q$$

$$\Rightarrow \frac{dv}{dx} + (1-n) Pv = (1-n)Q \qquad \Bigg| \quad y^{-n}\frac{dy}{dx} = \frac{1}{1-n}\frac{dv}{dx}$$

$\longrightarrow$ Linear Equation

Pg: 23 :: Ex - 01

$$\frac{dy}{dx} = x^3y^3 - xy$$

$$\Rightarrow \frac{dy}{dx} + xy = x^3y^3$$

$$\Rightarrow y^{-3}\frac{dy}{dx} + xy^{-2} = x^3$$

Let,

$$y^{-2} = v$$

$$\Rightarrow -2y^{-3}\frac{dy}{dx} = \frac{dv}{dx}$$

$$\Rightarrow y^{-2}\frac{dy}{dx} = -\frac{1}{2}\frac{dv}{dx}$$

$$\therefore -\frac{1}{2}\frac{dv}{dx} + xV = x^3$$

$$\Rightarrow \frac{dv}{dx} - 2xV = -2x^3$$

which is linear equation,

$$\therefore I.F = e^{\int -2x\,dx} = e^{-x^2}$$

It's solution is, $v.e^{-x^2} = \int -2x^3 \cdot x\, e^{-x^2}\,dx + c$

$$\Rightarrow y^{-2} e^{-x^2} = \int x^2 e^{-x^2} \cdot (-2x) dx + c$$

Let.
$-x^2 = t$
$\Rightarrow -2x dx = dt$

$$= \int -t e^t dt + c$$

$$= \int -t e^t + \int e^t dt + c$$

$$= -t e^t + e^t + c$$

$$\therefore y^{-2} e^{-x^2} = x^2 e^{-x^2} + e^{-x^2} + c$$

$$\Rightarrow y^{-2} = x^2 + 1 + c e^{x^2}$$

$$\Rightarrow \frac{1}{y^2} = x^2 + 1 + c e^{x^2} \quad \text{(Ans)}.$$

Pg: 25 :: Ex- 05

$$\frac{dy}{dx} = 1 - x(y-x) - x^3 (y-x)^3 \quad \text{----(i)}$$

Let, $y - x = v$

$$\frac{dy}{dx} - 1 = \frac{dv}{dx} \quad \therefore \frac{dy}{dx} = \frac{dv}{dx} + 1$$

(i) $\Rightarrow \dfrac{dv}{dx} + 1 = 1 - xv - x^3v^3$

$\Rightarrow \dfrac{dv}{dx} + xv = -x^3 v^3$

Let,

$v^{-2} = u$

$\Rightarrow v^{-3} \dfrac{dv}{dx} + x v^{-2} = -x^3$

$-2v^{-3}\dfrac{dv}{dx} = \dfrac{du}{dx}$

$v^{-3}\dfrac{dv}{dx} = -\dfrac{1}{2}\cdot\dfrac{du}{dx}$

$\Rightarrow -\dfrac{1}{2}\cdot\dfrac{du}{dx} + xu = -x^3$

$\Rightarrow \dfrac{du}{dx} - 2xu = 2x^3$

which is linear equation.

$\therefore$ I. F $= e^{\int -2x\,dx} = e^{-x^2}$

It's solution is.

$u\cdot e^{-x^2} = \int 2x^3 e^{-x^2} dx + c$

$\Rightarrow ue^{-x^2} = \int x^2 e^{-x^2}(2x)\,dx + c$

Let,

$-x^2 = t$

$\Rightarrow 2x\,dx = -dt$

$\Rightarrow x^2 = -t$

$\Rightarrow ue^{-x^2} = \int -te^{t}(-dt) + c$

$= te^t - e^t + c$

$$\Rightarrow v^{-2}e^{-x^2} = -x^2e^{-x^2} - e^{-x^2} + c$$

$$\Rightarrow (y-x)^2 e^{-x^2} = -(x^2+1)e^{-x^2} + c$$

$$\Rightarrow (y-x)^{-2} = -(x^2+1) + ce^{x^2} \quad \text{(Ans)}.$$

Differential Equations:

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Pg: 26 :: Ex: 7

Linear Equation

$$x\frac{dy}{dx} + y = y^2 \log x$$

$$\Rightarrow y^{-2}\frac{dy}{dx} + \frac{1}{x}y^{-1} = \frac{1}{x}\log x$$

$$\Rightarrow -\frac{dv}{dx} + \frac{v}{x} = \frac{1}{x}\log x$$

Let, $y^{-1} = v$

$$\Rightarrow -1 \cdot y^{-2}\frac{dy}{dx} = \frac{dv}{dx}$$

$$\Rightarrow y^{-2}\frac{dy}{dx} = -\frac{dv}{dx}$$

$$\Rightarrow \frac{dv}{dx} - \frac{1}{x} v = -\frac{1}{x} \log x$$

which is linear equation.

$$\therefore \text{I.F} = e^{-\int \frac{dx}{x}} = e^{-\log x} = e^{\log \frac{1}{x}} = \frac{1}{x}$$

It's solution is

$$v \cdot \frac{1}{x} = -\int \frac{1}{x} \log x \cdot \frac{1}{x} \, dx$$

$$= -\int \frac{1}{x^2} \log x \, dx + c$$

$$\Rightarrow \frac{1}{y} \cdot \frac{1}{x} = -\log x \int \frac{dx}{x^2} + \int \left\{ \frac{d}{dx} \log x \int \frac{dx}{x^2} \right\} dx + c$$

$$\Rightarrow \frac{1}{xy} = -\log x \left(-\frac{1}{x}\right) + \int \frac{1}{x} \cdot \frac{-1}{x} \, dx + c$$

$$\Rightarrow \frac{1}{xy} = \frac{1}{x} \log x - \left(-\frac{1}{x}\right) + c$$

$$\Rightarrow \frac{1}{xy} = \frac{1}{x} \log x + \frac{1}{x} + c$$

$$\Rightarrow \frac{1}{y} = \log x + 1 + cx \quad \text{(Ans)}$$

Pg: 27 :: Ex: 10

$$\frac{dy}{dx} - 2y\tan x = y^2 \tan^2 x$$

$$\Rightarrow y^{-2}\frac{dy}{dx} - 2y^{-1}\tan x = \tan^2 x$$

$$\Rightarrow -\frac{dv}{dx} - 2v\tan x = \tan^2 x$$

$$\Rightarrow \frac{dv}{dx} + 2\tan x \cdot v = -\tan^2 x$$

Let, $y^{-1} = v$

$$\Rightarrow -y^{-2}\frac{dy}{dx} = \frac{dv}{dx}$$

$$\Rightarrow +y^{-2}\frac{dy}{dx} = -\frac{dv}{dx}$$

which is linear equation.

$$\therefore \text{I.F} = e^{\int 2\tan x} = e^{-2\log\cos x}$$

$$= e^{-\log\cos^2 x} = e^{\log\frac{1}{\cos^2 x}}$$

$$= \frac{1}{\cos^2 x} = \sec^2 x$$

It's solution is

$$\sec^2 x \cdot v = \int \tan^2 x \cdot \sec^2 x \, dx + c$$

$\tan x = z$

$\sec^2 x \, dx = dz$

$\int (\tan x)^2 dx (\tan x)$

$= \frac{\tan^3 x}{3}$

$$\Rightarrow \frac{\sec^2 x}{y} = \frac{1}{3}\tan^3 x + c \quad \text{(Ans)}$$

Pg: 29 :: Ex: 16

$$\frac{dy}{dx} + y\cos x = y^n \sin 2x$$

$$\Rightarrow y^{-n}\frac{dy}{dx} + \cos x \cdot y^{1-n} = \sin 2x$$

Let $y^{1-n} = v$

$$\therefore (1-n)y^{-n}\frac{dy}{dx} = \frac{dv}{dx}$$

$$\therefore y^{-n}\frac{dy}{dx} = \frac{1}{1-n}\frac{dv}{dx}$$

$$\text{i)} \Rightarrow \frac{1}{1-n}\frac{dv}{dx} + \cos x \cdot v = \sin 2x$$

$$\Rightarrow \frac{dv}{dx} + (1-n)\cos x \cdot v = (1-n)\sin 2x$$

which is linear equation.

$$\therefore I.F = e^{\int (1-n)\cos x\, dx} = e^{(1-n)\sin x}$$

It's solution is

$$v \cdot e^{(1-n)\sin x} = \int (1-n)\sin 2x \cdot e^{(1-n)\sin x} dx$$

$$\Rightarrow y^{1-n} \cdot e^{(1-n)\sin x} = \int (1-n)e^{(1-n)\sin x} \cdot 2\sin x\cos x\, dx + c \quad \text{---- (ii)}$$

Let $(1-n)\sin x = t$

$(1-n)\cos x\, dx = dt$

$\sin x = \frac{1}{1-n}$

$$(ii) \Rightarrow y^{1-n} \cdot e^{(1-n)\sin x} = 2\int \frac{t}{1-n} \cdot e^{t}dt + c$$

$$\Rightarrow y^{1-n} \cdot e^{(1-n)\sin x} = \frac{2}{1-n}\left[te^{t} - \int e^{t}dt\right] + c$$

$$\Rightarrow y^{1-n} \cdot e^{(1-n)\sin x} = \frac{2}{1-n}\left[te^{t} - e^{t}\right] + c$$

$$\Rightarrow y^{1-n} \cdot e^{(1-n)\sin x} = \frac{2}{1-n}\left\{(1-n)\sin x - 1\right\}e^{(1-n)\sin x} + c$$

$$\Rightarrow y^{1-n} = \frac{2}{1-n}\left\{(1-n)\sin x - 1\right\} + ce^{-(1-n)\sin x}$$

(Ans)

Math

P: 36

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3.2

To find the necessary and sufficient condition for a differential equation of first degree being exact.

Soln: Let the differential equation be,

$$M + N\frac{dy}{dx} = 0 \quad \cdots\cdots (i)$$

Let $u = c$ be its primitive. (একাধিক উপায় আছে solution এর)

If ~~di~~ D.E. (i) is exact.

D. ② ; we get

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\cdot\frac{dy}{dx} = 0$$

$$\frac{\partial u}{\partial x}\cdot dx + \frac{\partial u}{\partial y}\cdot dy = 0 \quad \cdots\cdots (iii)$$

$u = c$

$u(x, y),\ y(x)$

partial w.r. to x

$$\frac{\partial u}{\partial x}\left(\frac{dx}{dx}\right) + \frac{\partial y}{\partial y}\cdot\frac{dy}{dx} = 0$$

From (i) and (iii) we get

$$M = \frac{\partial u}{\partial x}, \quad N = \frac{\partial u}{\partial y}$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial^2 u}{\partial x \, dy}, \quad \frac{\partial N}{\partial x} = \frac{\partial^2 u}{\partial x \, dy}$$

From above relation. we can write

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Necessary condition is complete.

Let, $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$, we have to show that

$M + N\dfrac{dy}{dx} = 0$, $M\,dx + N\,dy = 0$ is an exact D.E.

Let $\int M\,dx = U \Rightarrow M = \dfrac{\partial U}{\partial x}$

$$\text{ie} \quad \frac{\partial U}{\partial x} = M \qquad \therefore \frac{\partial^2 U}{\partial x \partial y} = \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\therefore \frac{\partial N}{\partial x} = \frac{\partial^2 U}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial U}{\partial y}\right)$$

$$\Rightarrow \frac{\partial N}{\partial x} = \frac{\partial}{\partial x}\left(\frac{\partial U}{\partial y}\right)$$

Integrating, we get x এর সাপেক্ষে integration

~~term~~

$$\therefore N = \frac{\partial U}{\partial y} + f(y)$$ Integrating constent

$$\therefore M + N\frac{dy}{dx} = \frac{\partial U}{\partial x} + \left[\frac{\partial U}{\partial y} + f(y)\right]\frac{dy}{dx}$$

$$= \frac{d}{dx}\left[U + \int f(y)\frac{dy}{dx}\,dx\right]$$

$$= \frac{d}{dx}\underbrace{\left[U + F(y)\right]}$$

x এর function বা, constant

$\Rightarrow N \frac{dy}{dx} = 0$ is an exact D.E..

Pg: 38 **Ex: 3**

$$(x^2 - 2xy + 3y^2)dx + (4y^3 + 6xy - x^2)dy = 0$$

$$Mdx + Ndy = 0$$

Here, $M = x^2 - 2xy + 3y^2$, $N = 4y^3 + 6xy - x^2$

$$\therefore \frac{\partial M}{\partial y} = -2x + 6y, \quad \frac{\partial N}{\partial y} = 6y - 2x$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x},$$

Hence the given equation is exact D.E.

$\therefore$ It's sol^n is

$$\cancel{\int Mdx + \int Ndy \text{ (exact } x \text{ term) } dy = c}$$

x मुक्त पद

$$\int Mdx + \int N \text{ (free term of } x) \, dy = c$$

$$\Rightarrow \int (x^2 - 2xy + 3y^2)\,dx + \int 4y^3\,dy = c$$

$$\Rightarrow \frac{x^3}{3} - x^2 + 3xy^2 + y^4 = c$$

$$\Rightarrow x^3 - 3x^2y + 9xy^2 + 3y^4 = 3c = c_1 \quad \text{(Ans)}$$

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Math

$$Mdx + Ndy = 0$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \text{ ; Exact D.E}$$

It's sol^n is

$$\int Mdx + \int N(\text{excluding } x \text{ function})dy = c$$

Pg: 39: Ex - 6

$$(1+e^{x/y})\,dx + \left(1-\frac{x}{y}\right)e^{x/y}\,dy = 0$$

$$M\,dx + N\,dy = 0$$

$$M = 1+e^{x/y} \qquad N = \left(1-\frac{x}{y}\right)e^{x/y}$$

$$\frac{\partial M}{\partial y} = e^{x/y}\cdot\frac{-x}{y^2}$$

$$= -\frac{x}{y^2}e^{x/y}$$

$$\frac{\partial N}{\partial x} = \left(1-\frac{x}{y}\right)e^{x/y}\cdot\frac{1}{y} + \left(0-\frac{1}{y}\right)e^{x/y}$$

$$= \frac{1}{y}e^{x/y} - \frac{x}{y^2}e^{x/y} - \frac{1}{y}e^{x/y}$$

$$= -\frac{x}{y^2}e^{x/y}$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Hence the given equation is an exact D.E.

It's solution is

$$\int (1+e^{x/y})\,dx + \int 0\,dy = c$$

$$\Rightarrow x + e^{x/y}\cdot y + c$$

$$\Rightarrow x + y e^{x/y} = c \qquad \text{(Ans).}$$

Pg:41,42 (Formula)

$$x\,dy - y\,dx = 0 \longrightarrow \text{I.F} = \frac{1}{x^2}$$

Pg: 42 :: Ex: 03

$$x\,dy - y\,dx - x(x^2-y^2)^{1/2}\,dx = 0$$

$$\Rightarrow \frac{x\,dy - y\,dx}{(x^2-y^2)^{1/2}} - x\,dx = 0$$

$$\Rightarrow \frac{x\,dy - y\,dx}{x\sqrt{1-\frac{y^2}{x^2}}} - x\,dx = 0.$$

$$\Rightarrow \frac{\frac{x\,dy - y\,dx}{x^2}}{\sqrt{1-\left(\frac{y}{x}\right)^2}} - dx = 0$$

$$\Rightarrow \frac{d\left(\frac{y}{x}\right)}{\sqrt{1-\left(\frac{y}{x}\right)^2}} - dx = 0$$

$$d\left(\frac{u}{v}\right)$$

$$= d\left(\frac{y}{x}\right)$$

$$= \frac{x\,dy - y\,dx}{x^2}$$

Integrating, we get

$$\sin^{-1}\frac{y}{x} - x = c \quad \text{(Ans).}$$

Pg. 43 : Ex : 8

$$y(2xy + e^x)\,dx - e^x\,dy = 0$$

$$\Rightarrow e^x \frac{dy}{dx} = y\,(2xy + e^x) = 2xy^2 + ye^x.$$

$$\Rightarrow \frac{dy}{dx} - y = 2xy^2 e^{-x}$$

$$\Rightarrow y^{-2}\frac{dy}{dx} - \frac{1}{y} = 2xe^{-x}$$

$$\Rightarrow -y^{-2}\frac{dy}{dx} + \frac{1}{y} = -2xe^{-x}$$

$$\Rightarrow \frac{dt}{dx} + t = -2xe^{-x}$$

Let $\frac{1}{y} = t$

$$-\frac{1}{y^2} \cdot \frac{dy}{dx} = \frac{dt}{dx}$$

which is L.D.E.

$$\therefore \text{I.F} = e^{\int dx} = e^{x}$$

It's sol^n is

$$t \cdot e^{x} = -\int 2xe^{-x}dx + c$$

$$\Rightarrow \frac{1}{y} e^{x} = -\int 2x\,dx + c$$

$$\Rightarrow e^{x} = -yx^2 + cy \quad \text{(Ans)}$$

Math

Differential Equations

15/4/21

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \quad ; \quad \text{Not exact}$$

(R-I) Rules

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = f(x) \; ; \; I.F = e^{\int f(x)dx}$$

(R-II) ;

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M} = g(y) \; ; \; I.F = e^{\int -g(y)dy}$$

$$\frac{\partial N/\partial x - \frac{\partial M}{\partial y}}{M} = g(y); \; I.F = e^{\int g(y)dy}$$

Pg:95: Ex:9

$$(x^2+y^2+2x)dx + 2ydy = 0$$

$$Mdx + Ndy = 0$$

$$M = x^2 + y^2 + 2x;$$

$$\therefore \frac{\partial M}{\partial y} = 2y \qquad \frac{\partial N}{\partial x} = 0$$

$$\therefore \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x};$$ So, given eqn is not exact

$$\therefore \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{2y - 0}{2y} = 1x^0 = f(x)$$

$$\therefore I.F = e^{\int 1 \cdot dx} = e^x$$

Multiply e^x of (i)

$$\therefore e^x(x^2 + y^2 + 2x)dx + 2ye^x dy = 0$$

$$\Rightarrow (x^2 + 2x)e^x dx + y^2 e^x dx + 2ye^x dy = 0$$

$$\Rightarrow x^2 e^x dx + 2xe^x dx + y^2 e^x dx + 2ye^x dy = 0$$

$$\Rightarrow d(x^2 e^x) + d(y^2 e^x) = 0$$

Now integrating,

$$\therefore x^2 e^x + y^2 e^x = c$$

$$\Rightarrow (x^2+y^2)\, e^x = c \qquad \text{(Ans).}$$

Pg: 46 :: Ex-8

$$(y^4+2y)\,dx + (2y^3 + 2y^4 - 4x)\,dy = 0$$

$$M\,dx + N\,dy = 0$$

$M = y^4 + 2y;$ $\qquad$ $N = xy^3 + 2y^4 \cancel{+} - 4x$

$$\frac{\partial M}{\partial y} = 4y^3 + 2; \qquad \frac{\partial N}{\partial x} = y^3 - 4;$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$; , so eq^n (i) is not exact

$$\Rightarrow \frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \frac{y^3 - 4 - 4y^3 - 2}{y^4 + 2y}$$

$$= \frac{-3y^3 - 6}{y(y^3+2)}$$

$$= \frac{-3(y^3+2)}{y(y^3+2)}$$

$$= -\frac{3}{y}$$

$$= g(y)$$

$$\therefore \text{I.F} = e^{\int \frac{3}{y} dy}$$

$$= e^{-3\log y}$$

$$= e^{\log y^{-3}}$$

$$= y^{-3}$$

$$= \frac{1}{y^3}$$

Multiply (i) by $\frac{1}{y^3}$

$$\Rightarrow \frac{y^4+2y}{y^3}\, dx + \left(\frac{xy^3+2y^4-4x}{y^3}\right) dy = 0$$

$$\Rightarrow \left(y + \frac{2}{y^2}\right)dx + \left(x + 2y - \frac{4x}{y^3}\right)dy = 0$$

which is exact D.E.

$$M_1 = y + \frac{2}{y^2}, \quad N_1 = x + 2y - \frac{4x}{y^3}$$

$$\Rightarrow \frac{\partial M_1}{\partial y} = 1 - \frac{4}{y^3}, \quad \frac{\partial N_1}{\partial x} = 1 - \frac{4}{y^2}$$

It's solution is

$$\int \left(y + \frac{2}{y^2}\right)dx + \int 2y\,dy = c$$

$$\Rightarrow xy + \frac{2x}{y^2} + y^2 = c \quad \text{(Ans)}.$$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = g(y); \quad I.F = e^{\int -g(x)\,dy}$$

$$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = g(y); \quad I.F = e^{\int g(y)\,dy}$$

(Rules $\overset{*}{-}$III) $Mdx + Ndy = 0$. [Homogenious]

$$Mx + Ny \neq 0; \; I.F = \frac{1}{Mx + Ny}$$

(Rules - IV) $yf(xy)dx + xg(xy)dy = 0$

$$f(xy) \neq g(xy)$$

$$I.F = \frac{1}{xy[f(xy) - g(xy)]}$$

Pg: 48 :: Ex:2

$$(x^4 + y^4)\,dx - xy^3 dy = 0$$

$$Mdx + Ndy = 0$$

$$Mx + Ny = (x^4 + y^4)x - xy^3 \cdot y$$

$$= x^5 + xy^4 - xy^4$$

$$= x^5 \neq 0$$

Multiplying (i) by I.F.

$$\left(\frac{x^4}{x^5} + \frac{y^4}{x^5}\right) dx - \frac{xy^3}{x^5}\, dy = 0$$

$$\Rightarrow \left(\frac{1}{x} + \frac{y^4}{x^5}\right) dx - \left(\frac{y^3}{x^4}\right) dy = 0$$

which is exact D.E.

$$\therefore \int \left(\frac{1}{x} + \frac{y^4}{x^5}\right) dx + \int 0\, dy = c$$

$$\Rightarrow \quad - \frac{y^4}{4x^4} = c \quad \text{(Ans)}$$

Pg: 49:: Ex: 4

$$y\,(xy + 2x^2y^2)\, dx + x\,(xy - x^2y^2)\, dy = 0$$

$$y\, f(xy)\, dx + x\, g(xy)\, dy = 0$$

$$Mx - Ny = xy(xy + 2x^2y^2) - yx(xy - x^2y^2)$$

$$= x^2y^2 + 2x^3y^3 - x^2y^2 + x^3y^3$$

$$= 3x^3y^3 \neq 0$$

$$I.F = \frac{1}{Mx - Ny} = \frac{1}{3x^3y^3}$$

Multiplying (i) by I.F

$$\therefore \quad \frac{y(xy + 2x^2y^2)}{3x^3y^3}\,dx + \frac{x(xy - x^2y^2)}{3x^3y^3}\,dy = 0$$

$$\Rightarrow \left(\frac{1}{3x^2y} + \frac{2y}{3x}\right)dx + \left(\frac{1}{3xy^2} - \frac{1}{3y}\right)dy = 0$$

which is exact D.E.

It's solution.

$$\int \left(\frac{1}{3x^2y} + \frac{2}{3x}\right) dx + \int -\frac{1}{3y}\, dy = c$$

$$\Rightarrow \frac{-1}{3xy} + \frac{2}{3}\log x - \frac{1}{3}\log y = c$$

$$\Rightarrow \frac{1}{xy} - 2\log x + \log y = 3c = c_1 \quad \text{(Ans)}.$$

- - - - -

Math

D.E

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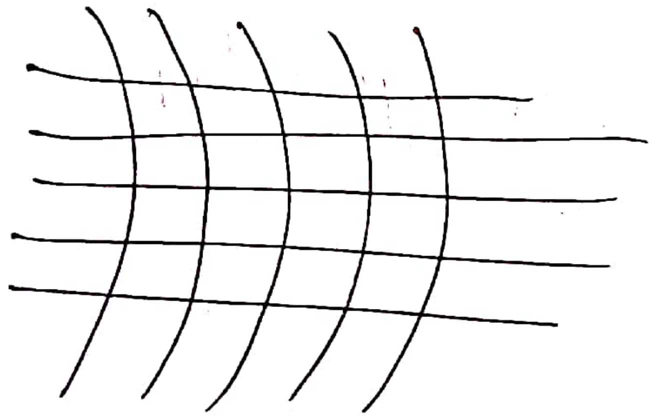
Chapter-4

Orthogonal Trajectories



$\alpha = 90^\circ$

Slops

$m = \dfrac{dy}{dx}$ $\quad m_1 m_2 = -1$

$$\frac{dy}{dx} \cdot \frac{dy_1}{dx_1} = -1$$

$$\frac{dy}{dx} = -\frac{dx_1}{dy_1}$$

Orthogonal Trajectories

$$\boxed{\frac{dy}{dx} = -\frac{dx}{dy}}$$

Polar equation

$$\boxed{\frac{dr}{d\theta} = -r^2\frac{d\theta}{dr}}$$

Pg: 54 :: Ex: 01

$xy = c^2$ ——(i) [rectengular hyperbola]

D.w.r to x, we get

$$y + x\frac{dy}{dx} = 0 \quad \text{———} (ii)$$

Replacing $\frac{dy}{dx}$ by $-\frac{dx}{dy}$,

ii) $y - x\frac{dx}{dy} = 0$

$\Rightarrow y\,dy - x\,dx = 0$

$\Rightarrow x\,dx - y\,dy = 0$

Integrating we get,

$$\frac{x^2}{2} - \frac{y^2}{2} = c_1$$

$\Rightarrow x^2 - y^2 = 2c_1 = c$

Pg: 55 :: Ex: 4

$$y^2 = 4a(x+a)$$

$\Rightarrow y^2 = 4ax + 4a^2$ ——— (i)

$\therefore 2y\frac{dy}{dx} = 4a$

$2y \cdot P = \cancel{2}\,4a$ $\qquad$ [assume, $\frac{dy}{dx} = P$].

i) $\Rightarrow y^2 = 4x \cdot \frac{yP}{2} + 4 \cdot \frac{y^2p^2}{4}$

$$\Rightarrow y^2 = 2Pyx + y^2p^2 \quad \text{——— (ii)}$$

Replacing p by $-\frac{1}{p}$

$$\therefore y^2 = -\frac{2yx}{p} + \frac{y^2}{p^2}$$

$$\Rightarrow p^2y^2 + 2pxy = y^2$$

which is the same as (ii)

<u>Pg:56 :: Ex:6</u>

$$x^2+y^2=c^2 \quad \text{——— (i)}$$

D. w. r to x

$$2x+2y\frac{dy}{dx} = 0$$

$$\Rightarrow x+Py = 0 \quad \text{- - - - (ii)}$$

$\boxed{P = \frac{dy}{dx}}$

we shall replace p by

$$\frac{P-\tan 45^\circ}{1+P\tan 45^\circ} = \frac{P-1}{1+P}$$

(ii) $\Rightarrow x + y \cdot \frac{p-1}{p+1} = 0$

$\Rightarrow (p+1)x + (p-1)y = 0$

$\Rightarrow \left(\frac{dy}{dx} + 1\right)x + \left(\frac{dy}{dx} - 1\right)y = 0$ $\quad [\, p = \frac{dy}{dx} \,]$

$\Rightarrow (dy + dx)x + (dy - dx)y = 0$

$\Rightarrow x\,dy + x\,dx + y\,dy - y\,dx = 0$

$\Rightarrow (x+y)dy + (x-y)dx = 0$

$\Rightarrow \frac{dy}{dx} + \frac{x-y}{x+y} = 0$ ———— (iii)

which is homogenious D.E

Let $y = vx$ $\quad \therefore \frac{dy}{dx} = v + \frac{x\,dv}{dx}$

(iii) $\Rightarrow v + x\frac{dv}{dx} + \frac{x - vx}{x + vx} = 0$

$\Rightarrow x\frac{dv}{dx} + v + \frac{1-v}{1+v} = 0$

$$\Rightarrow x\frac{dv}{dx} + \frac{v + v^2 + 1 - v}{1+v} = 0$$

$$\Rightarrow \frac{1+v}{v^2+1}\,dv + \frac{dx}{x} = 0$$

$$\Rightarrow \left(\frac{1}{v^2+1} + \frac{v}{v^2+1}\right)dv + \frac{dx}{x} = 0$$

Integrating,

$$\tan^{-1} v + \frac{1}{2}\log(v^2+1) + \log x = \log c_1$$

$$\Rightarrow 2\tan^{-1} v + \log(v^2+1) + 2\log x = 2\log c_1$$

$$\Rightarrow \log x^2 + \log(v^2+1) = \log c - 2\tan^{-1} v$$

$$\Rightarrow \log x^2(v^2+1) = \log c + \log e^{-2\tan^{-1} v} = \log c\, e^{-2\tan^{-1} v}$$

$$\Rightarrow x^2(v^2+1) = ce^{-2\tan^{-1} v}$$

$$\Rightarrow x^2\left(\frac{y^2}{x^2} + 1\right) = ce^{-2\tan^{-1}\frac{y}{x}}$$

$$\Rightarrow x^2(x^2+y^2) = ce^{-2\tan^{-1}\frac{y}{x}}$$

Pg: 57 :: Ex-8

$$r = a(1-\cos\theta) \quad \text{---} \quad (i)$$

D.w.r to θ

$$\therefore \frac{dr}{d\theta} = a(0 + \sin\theta)$$

$$= a\sin\theta$$

$$\therefore a = \frac{1}{\sin\theta} \cdot \frac{dr}{d\theta}$$

$$\therefore \text{ i)} \Rightarrow r = \frac{1}{\sin\theta} \cdot \frac{dr}{d\theta}(1-\cos\theta)$$

$$\Rightarrow \frac{1}{r}\frac{dr}{d\theta} = \frac{\sin\theta}{1-\cos\theta} \quad \text{———} \quad (ii)$$

Replacing $\frac{dr}{d\theta}$ by $-x^2\frac{d\theta}{dx}$

$$(ii) \Rightarrow -\frac{1}{r} \cdot r^2\frac{d\theta}{dr} = \frac{\sin\theta}{1-\cos\theta}$$

$$\Rightarrow -r\frac{d\theta}{dr} = \frac{\sin\theta}{1-\cos\theta}$$

$$\Rightarrow \frac{1-\cancel{\sin\theta}\cos\theta}{\sin\theta}\,d\theta+\frac{dr}{r}=0$$

$$\Rightarrow \frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\,d\theta+\frac{dr}{r}=0$$

$$\Rightarrow \tan\frac{\theta}{2}\,d\theta+\frac{dr}{r}=0$$

Integrating we get,

$$-2\log\cos\frac{\theta}{2}+\log r=\log c_1$$

$$\Rightarrow \log r=\log\cos^2\frac{\theta}{2}+\log c_1$$

$$\Rightarrow \log r=\log\cos^2\frac{\theta}{2}+\log 2c$$

$$\Rightarrow \log r=\log 2c\cos^2\frac{\theta}{2}$$

$$\Rightarrow r=c(1+\cos\theta)\quad\text{(Ans).}$$

Pg: 58 :: Ex: 12

$$r^n \cos n\theta = c^n \qquad \text{———} (i)$$

$$\Rightarrow n \log r + \log \cos n\theta = n \log c$$

D. w. r to θ.

$$n \cdot \frac{1}{r} \cdot \frac{dr}{d\theta} + \frac{1}{\cos n\theta} \cdot -(\sin n\theta) n = 0$$

$$\Rightarrow \frac{1}{r} \cdot \frac{dr}{d\theta} - \tan n\theta = 0 \qquad \text{———} (ii)$$

Replacing $\frac{dr}{d\theta}$ by $-r^2 \frac{d\theta}{dr}$.

$$\therefore (ii) \to -\frac{1}{r} r^2 \frac{d\theta}{dr} - \tan n\theta = 0$$

$$\Rightarrow r \frac{d\theta}{dr} + \tan n\theta = 0$$

$$\Rightarrow \frac{1}{\tan n\theta} d\theta + \frac{dr}{r} = 0$$

$$\Rightarrow \cot n\theta \, d\theta + \frac{dr}{r} = 0$$

$$\Rightarrow \frac{1}{n} \log(\sin n\theta) + \log r = \log c_1$$

$$\Rightarrow \log(\sin n\theta) + n\log r = n\log c_1$$

$$\Rightarrow \log(\sin n\theta) + \log r^n = \log c_1^n$$

$$\Rightarrow \log(r^n \sin n\theta) = \log c_1^n$$

$$\Rightarrow \therefore r^n \sin\theta = c_1^n \quad \text{(Ans)}$$

D.E

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Chapter 5 :: Pg: 60

$$P_0 \frac{d^n y}{dx^n} + P_1 \frac{d^{n-1} y}{dx^{n-1}} + P_2 \frac{d^{n-2} y}{dx^{n-2}} + \dots$$

$$\dots + P_n y = x$$

$$D = \frac{d}{dx}, \quad D^2 = \frac{d^2}{dx^2}$$

A. E, $y = e^{m}$, $\frac{dy}{dx} = me^{mx}$, $e^{mx} \neq 0$

$m = m_1, m_2, m_3$

$y = c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x}$

$m = m_1, m_2, m_3$

$y = (c_1 + c_2 x) e^{m_1 x} + c_3 e^{m_2 x}$

Pg: 61 :: Ex: 1

$(D^3 - 13D - 12)y = 0$

$\frac{d^3y}{dx^3} - 13\frac{dy}{dx} - 12y = 0$ ---- (i)

Let,

$y = e^{mx}$

be the trial solution

$\frac{dy}{dx} = me^{mx}$

$\frac{d^2y}{dx^2} = m^2 e^{mx}$

$\frac{d^3y}{dx^3} = m^3 e^{mx}$

(i) $\Rightarrow (m^3 - 13m - 12)e^{mx} = 0$

Auxiliary equation is

$$m^3 - 13m - 12 = 0, \quad e^{mx} \neq 0$$

$$\Rightarrow m^2(m+1) - m(m+1) - 12(m+1) = 0$$

$$\Rightarrow (m+1)(m^2 - m - 12) = 0$$

$$\Rightarrow (m+1)\{m(m+3) - 4(m+3)\} = 0$$

$$\Rightarrow (m+1)(m+3)(m-4) = 0$$

$$\therefore m = -1, -3, 4$$

$\therefore$ The complete solution is

$$y = c_1 e^{-x} + c_2 e^{-3x} + c_3 e^{4x}$$

Equal roots ; $y = (c_1 + c_2 x) e^{m_1 x} + c_3 e^{m_2 x}$

Pg: 62 :: Ex: 1

$$\frac{d^4y}{dx^4} - \frac{d^3y}{dx^3} - 9\frac{d^2y}{dx^2} - 11\frac{dy}{dx} - 4y = 0 \quad \text{--- (i)}$$

Let $y = e^{mx}$ be the trial solution of (i)

$$\frac{dy}{dx} = me^{mx}, \quad \frac{d^2y}{dx^2} = m^2e^{mx}, \quad \frac{d^3y}{dx^3} = m^3e^{mx},$$

$$\frac{d^4y}{dx^4} = m^4e^{mx}$$

$$\Rightarrow (m^4 - m^3 - 9m^2 - 11m - 4)\, e^{mx} = 0$$

A. E is

$$m^4 - m^3 - 9m^2 - 11m - 4 = 0 \qquad [e^{mx} \neq 0]$$

$$\Rightarrow m^3(m+1) - 2m^2(m+1) - 7m(m+1) - 4(m+1) = 0$$

$$\Rightarrow (m+1)(m^3 - 2m^2 - 7m - 4) = 0$$

$$\Rightarrow (m+1)\{m^2(m+1) - 3m(m+1) - 4(m+1)\} = 0$$

$\Rightarrow (m+1)^2 (m^2-3m-4) = 0$

$\Rightarrow (m+1)^2 \{m(m+1) - 4(m+1)\} = 0$

$\Rightarrow (m+1)^3 (m-4) = 0$

$\therefore m = -1, -1, -1, 4$

$\therefore$ Complete solution is,

$$y = (c_1 + c_2 x + c_3 x^2) e^{-x} + c_4 e^{4x}$$

$m = a \pm ib$ $\Rightarrow m_1 = a+ib, \quad m_2 = a-ib$

$y = c_1 e^{(a+ib)x} + c_2 e^{(a-ib)x}$

$= c_1 e^{ax} \cdot e^{ibx} + c_2 e^{ax} \cdot e^{-ibx}$

$= c_1 e^{ax} \cdot [\cos bx + i \sin bx] + c_2 e^{ax} [\cos bx - i \sin bx]$

$= e^{ax} [c_1 \cos bx + c_2 \cos bx + i c_1 \sin bx - i c_2 \sin bx$

$$= e^{ax}[(c_1+c_2)\cos bx + (c_1 i - c_2 i)\sin bx]$$

$$\boxed{\begin{gathered} y = e^{ax}[A\cos bx + B\sin bx] \\ m = a \pm ib \end{gathered}}$$

Imaginary root repeated

$$y = e^{ax}[(c_1+c_2x)\cos bx + (c_3+c_4x)\sin bx]$$

Pg-63 :: Ex-1

$$(D^4 + 5D^2 + 6)y = 0 \quad \text{- - - - (i)}$$

Let $y = e^{mx}$ be the trial solution of (i)

$\therefore Dy = m e^{mx}$, $D^2y = m^2 e^{mx}$, $D^3y = m^3 e^{mx}$,

$$D^4y = m^4 e^{mx}$$

$$\Rightarrow (m^4 + 5m^2 + 6)e^{mx} = 0$$

A.E is $m^4 + 5m^2 + 6 = 0$, $e^{mx} \neq 0$

$$\Rightarrow m^2(m^2+2) + 3(m^2+2) = 0$$

$$\Rightarrow (m^2+2)(m^2+3) = 0$$

$$\therefore m = \pm\sqrt{2}\,i, \quad m = \pm\sqrt{3}\,i$$

The complete solution is

$$y = c_1 \cos\sqrt{2}x + c_2 \sin\sqrt{2}x + c_3 \cos\sqrt{3}x$$
$$+ c_4 \sin\sqrt{3}x$$

Math

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$y = e^{mx}$ be the trial sol^n.

$$D = \frac{d}{dx}, \quad D^2 = \frac{d^2}{dx^2}$$

A. E .....

General Sol^n / Complete sol^n:

$$y = c_1 e^{m_1 x} + c_2 e^{m_2 x} + \dots \dots$$

$m_1 \neq m_2 \neq m_3$

$$y = c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x}$$

$m_1 = m_2, m_3$

$$y = (c_1 + c_2 x) e^{m_1 x} + c_3 e^{m_3 x}$$

$m = a \pm ib$

$$y = e^{ax} (A \cos bx + B \sin bx)$$

Pg: 63 :: Ex: 2

$$(D^4 - D^3 - D + 1)y = 0 \quad \ldots\ldots (i)$$

Let $y = e^{mx}$ be the trial sol^n of (i)

$\therefore Dy = me^{mx}$, $D^2y = m^2e^{mx}$, $D^3y = m^3e^{mx}$,

$D^4y = m^4e^{mx}$

(i) $\Rightarrow (m^4 - m^3 - m + 1)\, e^{mx} = 0$

$\therefore$ A. E is $m^4 - m^3 - m + 1 = 0$, $e^{mx} \neq 0$

$\Rightarrow m^3(m-1) - 1(m-1) = 0$

$\Rightarrow (m-1)(m^3 - 1) = 0$

$\Rightarrow (m-1)(m-1)(m^2 + m + 1) = 0$

$\therefore m = 1, 1,$ $\quad m = \dfrac{-1 \pm \sqrt{1^2 - 4.1.1}}{2.1}$

$= \frac{1}{2}(-1 \pm \sqrt{3}i)$

$m = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$

∴ The general sol^n is

$$y = (c_1 + c_2 x)e^{x} + e^{-1/2 x}\left(A\cos\frac{\sqrt{3}}{2}x + B\sin\frac{\sqrt{3}}{2}x\right)$$

Particular Integral: (P.I):

$$P.I = \frac{1}{f(D)} \cdot x$$

$$(1-x)^{-1} = 1 + x + x^2 + \dots$$

$$(1+x)^{-1} = 1 - x + x^2 - x^3 + \dots$$

Pg: 69 ∴ Ex: 1

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \sin 2x$$

Let $y = e^{mx}$ be the trial sol^n of.

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$$

$$\therefore \frac{dy}{dx} = me^{mx}, \quad \frac{d^2y}{dx^2} = m^2 e^{mx}$$

$\therefore$ A.E is $m^2 + m + 1 = 0$, $e^{mx} \neq 0$

$$\therefore m = \frac{-1 \pm \sqrt{1^2 - 4.1.1}}{2.1} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$$

$\therefore$ Complementary Function (C.F) is

$$y_c = e^{-1/2 x}\left(A \cos \frac{\sqrt{3}}{2} x + B \sin \frac{\sqrt{3}}{2} x\right)$$

Particular Integral be (P.I)

$$y_p = \frac{1}{D^2 + D + 1} \cdot \sin 2x$$

$$= \frac{1}{-2^2 + D + 1} \cdot \sin 2x \qquad \Big| \; D^2 = -2^2$$

$$= \frac{1}{D - 3} \cdot \sin 2x$$

$$= \frac{D+3}{D^2 - 9} \cdot \sin 2x$$

$$= \frac{D+3}{-2^2-9} \cdot \sin 2x$$

$$= -\frac{1}{13} (D+3) \cdot \sin 2x$$ differentiation

$$= -\frac{1}{13} (2\cos 2x + 3\sin 2x)$$

The complete sol^n is $y = y_c + y_p$

$$y = e^{-1/2x} \left(A\cos \frac{\sqrt{3}}{2} x + B\sin \frac{\sqrt{3}}{2} x\right)$$

$$- \frac{1}{13} (2\cos 2x + 3\sin 2x)$$

Ans.

* উপরে D থাকলে Differentiation

নিচে D থাকলে Integration

Math

D.E

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$x = e^{ax}$, $f(D) = f(a)$; $f(a) \neq 0$

$f(a) = 0$; $f(D - a)$

Pg: 71 :: Ex-1

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^x$$

Let $y = e^{mx}$ be the trial sol^n of

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^x$$

$\therefore$ A.E is $m^2 - 3m + 2 = 0$, $e^{mx} \neq 0$

$\Rightarrow (m-1)(m-2) = 0$ $\therefore m = 1, 2$

$\therefore$ C.F in

$$y_c = c_1 e^x + c_2 e^{2x}$$

P.S. be $y_p = \dfrac{1}{D^2 - 3D + 2} \cdot e^x$ | $x \cdot \dfrac{1}{f'(D)} \cdot e^{ax}$

$y_p = x \cdot \dfrac{1}{2D - 3} \cdot e^x$

$= x \cdot \dfrac{1}{2 \cdot 1 - 3} \, e^x$ → x গুণ হবে

$= -xe^x$

$\therefore$ The general soln is $y = y_c + y_p$

$\therefore y = c_1 e^x + c_2 e^{2x} - xe^x$

(Ans)

Or

P.I. $y_p = \dfrac{1}{D^2 - 3D + 2} \cdot e^x$

$= \dfrac{1}{(D-1)^2 - 3D - }$

$f(a) = 0$

$y_p = x \cdot \dfrac{1}{f'(D)} \cdot e^{ax}$

Pg: 72 :: Ex: 4

$$2\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + y = e^x + 1$$

Auxiliary Eq^n is.

$$2m^3 - 3m^2 + 1 = 0 \quad , e^{mx} \neq 0$$

$$\Rightarrow 2m^2(m-1) - m(m-1) - 1(m-1) = 0$$

$$\Rightarrow (m-1)(2m^2 - m - 1) = 0$$

$$\Rightarrow (m-1)\{2m(m-1) + 1(m-1)\} = 0$$

$$\Rightarrow (m-1)(m-1)(2m+1) = 0$$

$$\therefore m = 1, 1, -\frac{1}{2}$$

$\therefore$ Complementary Function is

$$y_c = (c_1 + c_2x)e^x + c_3e^{-1/2\,x}$$

P. I be $y_p = \frac{1}{2D^3 - 3D^2 + 1} \cdot (e^x + 1)$

$$y_p = \frac{1}{2D^3 - 3D^2 + 1} \cdot e^x + \frac{1}{2D^3 - 3D^2 + 1} \cdot e^{0x}$$

$$= x \cdot \frac{1}{6D^2 - 6D} \cdot e^x + \frac{1}{2.0 - 3.0 + 1} \cdot e^{0x}$$

$$= x \cdot x \cdot \frac{1}{12D - 6} \cdot e^x + 1$$

$$= x^2 \frac{1}{12.1 - 6} \cdot e^x + 1$$

$$= \frac{x^2 e^x}{6} + 1$$

$\therefore$ The complete sol^n is,

$$y = y_c + y_p$$

$$y = (c_1 + c_2 x) e^x + c_3 e^{-1/2 x} + \frac{1}{6} x^2 e^x + 1$$

(Ans)

D.E

25/4/21

Pg: 74 :: Ex-8

$$\frac{d^6 y}{dx^6} + y = \sin\frac{3}{2}x \sin\frac{1}{2}x \quad ----(i)$$

Let $y = e^{mx}$ be the trial sol^n of

$$\frac{d^6 y}{dx^6} + y = 0 \quad ----(ii)$$

A.E is $m^6 + 1 = 0$; $e^{mx} \neq 0$

$$\Rightarrow (m^2)^3 + 1^3 = 0$$

$$\Rightarrow (m^2+1)(m^4 - m^2 + 1) = 0$$

$$\Rightarrow (m^2+1)\{(m^2)^2 - m^2 + 1\} = 0$$

$$\Rightarrow (m^2+1)\{(m^2)^2 + 2m^2 + 1 - 3m^2\} = 0$$

$$\Rightarrow (m^2+1)\{(m^2+1)^2 - 3m^2\} = 0$$

$$\Rightarrow (m^2+1)\{(m^2+1)^2 - (\sqrt{3}m)^2\} = 0$$

$$\Rightarrow (m^2+1)(m^2+1+\sqrt{3}m)(m^2+1-\sqrt{3}m) = 0$$

$\therefore m^2 + 1 = 0$, $\quad m = \dfrac{-\sqrt{3} \pm \sqrt{3-4}}{2.1}$

$\therefore m = \pm i$ $\quad \therefore m = \dfrac{1}{2}(-\sqrt{3} \pm i)$

$$m = \frac{\sqrt{3} \pm \sqrt{3-4}}{2.1}$$

$$\therefore m = \frac{1}{2}(\sqrt{3} \pm i)$$

∴ The Complementary function is

~~$y_c = A\cos x + B\sin x +$~~

$$\boxed{\begin{aligned} m &= a \pm ib. \\ y &= e^{ax}(A\cos bx + B\sin bx) \end{aligned}}$$

$$y_c = c_1 \cos x + c_2 \sin x + e^{-\frac{\sqrt{3}}{2}x}\left(c_3 \cos \frac{1}{2}x + c_4 \cancel{\cos} \sin \frac{1}{2}x\right) + e^{\sqrt{3}/2\, x}\left(c_5 \cos \frac{1}{2}x + c_6 \sin \frac{1}{2}x\right)$$

Particular integral be

$$y_p = \frac{1}{D^6+1} \cdot \sin \frac{3}{2}x \sin \frac{1}{2}x$$

$$= \frac{1}{D^6+1} \cdot \frac{1}{2}(\cos x - \cos 2x)$$

$$= \frac{1}{2} \cdot \frac{1}{D^6+1} \cdot \cos x - \frac{1}{2} \cdot \frac{1}{D^6+1} \cos 2x$$

$$\left[\begin{aligned} &2\sin A \sin B \\ &\quad = \cos(A-B) - \cos(A+B) \end{aligned} \right.$$

$$= \frac{1}{2} x \frac{1}{6D^5} \cdot \cos x - \frac{1}{2} \cdot \frac{1}{-(2^2)^3+1} \cdot \cos 2x$$

$$= \frac{x}{12} \cdot \frac{1}{1^2 \cdot D} \cdot x \cancel{\alpha} - \frac{1}{2} \cdot \frac{1}{-63} \cdot \cos 2x$$

$$= \frac{x}{12} \cdot \frac{1}{D} \cos x + \frac{1}{126} \cos 2x$$

$$\therefore y_p = \frac{x}{12} \cdot \sin x + \frac{1}{126} \cos 2x$$

$\therefore$ The complete solution is

$$y = y_c + y_p$$

$$\boxed{\begin{aligned} D &= \frac{d}{dx} \\ \frac{1}{D} &= \int dx \end{aligned}}$$

Pg : 76 :: Ex

$$(D^3 + 2D^2 + D)y = e^{2x} + x^2 + x \quad \text{---(i)}$$

Let $y = e^{mx}$ be the trial soln of

$$(D^3 + 2D^2 + D)y = 0$$

$\therefore$ A.E is $m^3 + 2m^2 + m = 0$; $e^{mx} \neq 0$

$$\Rightarrow m(m^2 + 2m + 1) = 0$$

$$\Rightarrow m(m+1)^2 = 0$$

$$\Rightarrow m = 0, -1, -1$$

$\therefore$ C.F. is $y_c = c_1 + (c_2 + c_3 x)e^{-x}$

$\therefore$ P.S be $y_p = \dfrac{1}{D^3 + 2D^2 + D}(e^{2x} + x^2 + x)$

$$= \frac{1}{D(D+1)^2} \cdot e^{2x} + \frac{1}{D(D+1)^2}(x^2 + x)$$

$$y_p = \frac{1}{2(2+1)^2} \cdot e^{2x} + \frac{1}{D}(D+1)^{-2}(x^2+x)$$

$$= \frac{1}{18} e^{2x} + \frac{1}{D}(1 - 2D + 3D^2 - 4D^3 + \dots)(x^2+x)$$

$$= \frac{e^{2x}}{18} + \frac{1}{D}(x^2+x - 4x - 2 + 6)$$

$$= \frac{1}{18} e^{2x} + \frac{1}{D}(x^2 - 3x + 4)$$

$$= \frac{1}{18} e^{2x} + \frac{x^3}{3} - \frac{3x^2}{2} + 4x$$

$$\left[D^2(x^2+x) \right.$$
$$= D \cdot D(x^2+x)$$
$$= D(2x+1)$$
$$\left. = 2 + 0 = 2 \right]$$

$\therefore$ The complete solⁿ is

$$y = y_c + y_p$$

Math

DE

29/4/21

$$y_p = \frac{1}{f(D)}(e^{ax}.V)$$ $\longrightarrow$ x এর ফাংশন

$$= e^{ax}\frac{1}{f(D+a)}.V$$

Pg: 77 :: Ex-2

$$\frac{d^3y}{dx^3} - \frac{3d^2y}{dx^2} + 3\frac{dy}{dx} - y = xe^x + e^x$$

Let $y = e^{mx}$ be the trial soln of ....

Last এ

* A.E is $m^3 - 3m^2 + 3m - 1 = 0$, $e^{mx} \neq 0$

$$\Rightarrow (m-1)^3 = 0$$

$$\therefore m = 1, 1, 1.$$

The C.F is $y_c = (c_1 + c_2x + c_3x^2)e^x$

$$y_p = \frac{1}{D^3 - 3D^2 + 3D - 1} \cdot e^x (x+1)$$

$$= \frac{1}{(D-1)^3} \cdot e^x (1+x)$$

$$= e^x \cdot \frac{1}{(D+1-1)^3} \cdot (x+1)$$

$$= e^x \cdot \frac{1}{D^3} (x+1)$$

$$= e^x \cdot \frac{1}{D^2} \cdot \frac{(x+1)^2}{2}$$

$$= \frac{1}{2} \cdot e^x \cdot \frac{1}{D} \cdot \frac{(x+1)^3}{3}$$

$$= \frac{1}{6} \cdot e^x \cdot \frac{(x+1)^4}{4}$$

$$= \frac{1}{24} \cdot e^x (x+1)^4$$

Complete solution is $y = y_c + y_p$

Pg : 78 :: Ex : 4

$$\frac{d^3y}{dx^3} - 2\frac{dy}{dx} + 4y = e^x \cos x \quad \text{------ (i)}$$

Let $y = e^{mx}$ be the trial solution of

$$\frac{d^3y}{dx^3} - 2\frac{dy}{dx} + 4y = 0 \quad \text{------ (ii)}$$

$\therefore$ A.E is $m^3 - 2m + 4 = 0$, $e^{mx} \neq 0$

$\Rightarrow m^2(m+2) - 2m(m+2) + 2(m+2) = 0$

$\Rightarrow (m+2)(m^2 - 2m + 2) = 0$

$\therefore m = -2$, $m = \dfrac{2 \pm \sqrt{2^2 - 4.1.2}}{2.1} = 1 \pm i$

Last 9 (**) $\therefore y_c = c_1 e^{-2x} + e^x (A\cos x + B\sin x)$

$$y_p = \frac{1}{D^3 - 2D + 4} \cdot e^x \cos x$$

$$= e^x \cdot \frac{1}{(D+1)^3 - 2(D+1) + 4} \cos x$$

$$\Rightarrow y_P = e^x \cdot \frac{1}{D^3+3D^2+D+3} \cdot \cos x$$

$$= e^x \cdot x \cdot \frac{1}{3D^2+6D+1} \cdot \cos x$$

$$= x \cdot e^x \cdot \frac{1}{3(-1)^2+6D+1} \cos x$$

$$= x \cdot e^x \cdot \frac{1}{6D-2} \cdot \cos x$$

$$= xe^x \cdot \frac{3D+1}{2(3D-1)(3D+1)} \cos x$$

$$= \frac{1}{2} xe^x \frac{3D+1}{9D^2-1} \cos x$$

$$= \frac{1}{2} \cdot xe^x - \frac{3D+1}{9(-1^2)-1} \cdot \cos x$$

$$= -\frac{1}{20} x \cdot e^x (\cancel{3\sin x} - 3\sin x + \cos x)$$

$$= \frac{1}{20} x e^x (3\sin x - \cos x)$$

$\therefore\ y = y_c + y_p$

$= c_1 e^{-2x} + e^{x}(A\cos x + B\sin x)$

$+ \frac{1}{20} x e^{x}(3\sin x - \cos x)$

(Ans)

⊛ $y = e^{mx} \longrightarrow \frac{dy}{dx} = me^{mx}, \longrightarrow \frac{d^2y}{dx^2} = m^2 e^{mx}$

$\longrightarrow \frac{d^3y}{dx^3} = m^3 e^{mx}$

$$\frac{d^3y}{dx^3} - \frac{3d^2y}{dx^2} + 3\frac{dy}{dx} - y = 0$$

$\Rightarrow m^3 e^{mx} - 3m^2 e^{mx} + 3me^{mx} - e^{mx} = 0$

$\Rightarrow e^{mx}(m^3 - 3m^2 + 3m - 1) = 0 \quad [e^{mx} \neq 0]$

$\therefore$ A.E $\boxed{m^3 - 3m^2 + 3m - 1 = 0}$

** $m = -2,\ 1 \pm i \quad (1+i,\ 1-i)$

$$y_c = c_1 e^{-2x} + c_2 e^{(1+i)x} + c_3 e^{(1-i)x}$$

$$= c_1 e^{-2x} + c_2 e^{x} \cdot e^{ix} + c_3 e^{x} \cdot e^{-ix}$$

$$= c_1 e^{-2x} + c_2 e^{x} \cdot (\cos x + i \sin x)$$

$$+ c_3 e^{x} (\cos x - i \sin x)$$

$$= c_1 e^{-2x} + e^{x} \underline{(c_2 + c_3)} \cos x$$

$$+ e^{x} (\underline{c_2 i - c_3 i}) \sin x$$

$$= c_1 e^{-2x} + e^{x} [A \cos x + B \sin x]$$

DE (Math) 2 May 2021

Particular Integral $= \frac{1}{f(D)}(xV)$

$$= x\frac{1}{f(D)}.V - \frac{f'(D)}{[f(D)]^2}.V$$

Pg: 80:: Ex-1

$$\frac{d^4y}{dx^4} - y = x\sin x \quad ---- (i)$$

Let $y=e^{mx}$ be the trial solution of

$$\frac{d^4y}{dx^4} - y = 0 \quad ----- (ii)$$

$\therefore$ A.E is $m^4 - 1 = 0$, $e^{mx} \neq 0$

$$\Rightarrow (m^2-1)(m^2+1) = 0$$

$$\therefore m = \pm 1, \pm i$$

$$\boxed{e^{ix} = \cos x + i\sin x}$$

$$\boxed{\begin{aligned} \cos x &= \text{Real Part of } e^{ix} \\ \sin x &= \text{Imaginary part of } e^{ix} \end{aligned}}$$

$$y_p = \frac{1}{D^4-1}(x\sin x)$$

$$= \text{Imaginary part of } \frac{1}{D^4-1} \cdot xe^{ix}$$

$$= \text{I.P. } e^{ix} \frac{1}{(D+i)^4 - 1} \cdot x$$

$$= \text{I.P. } e^{ix} \frac{1}{D^4 + 4iD^3 - 6D^2 - 4iD + i^4 - 1} \cdot x$$

$$= \text{I.P. } e^{ix} \frac{1}{-4iD - 6D^2 + 4iD^3 + D^4} \cdot x$$

$$= I.P\ e^{ix} \frac{1}{-4iD\left(1+\frac{6D^2-4iD^3-D^4}{4iD}\right)} \cdot x$$

$$= I.P\ -\frac{e^{ix}}{4iD} \left(1+\frac{6D^2-4iD^3-D^4}{4iD}\right)^{-1} \cdot x$$

$$= I.P.\ -\frac{e^{ix}}{4iD} \left(1+\frac{6D-4iD^2-D^3}{4i}\right)^{-1} \cdot x$$

$$= I.P.\ -\frac{e^{ix}}{4iD} \left[1-\frac{6D-4iD^2-D^3}{4i}+\left(\frac{6D-4iD^2-D^3}{4i}\right)^2+\dots\right] \cdot x$$

$$= I.P\ \frac{-e^{ix}}{4iD} \left[x-\frac{6}{4i}\right]$$

$$= I.P\ -e^{ix} \frac{1}{4iD} \left(x+\frac{3i}{2}\right)$$

$$= I.P\ -e^{ix} \frac{1}{4i} \left(\frac{x^2}{2}+\frac{3i}{2}x\right)$$

$$= -I.P\ (\cos x + i\sin x)\left(\frac{x^2 i}{-8}+\frac{3}{8}x\right)$$

$$= -I.P\left[-\frac{x^2}{8}\cos i+\frac{3}{8}x\cos x+\frac{x^2}{8}\sin x+\frac{3}{8}x\sin x\, i\right.$$

$$\therefore y_p = \frac{x^2}{8}\cos x - \frac{3}{8}x\sin x$$

$\therefore$ The complete solution is

$$y = y_c + y_p$$

Pg: 83 :: Ex : 6(a)

$$\frac{d^2y}{dx^2} - y = x^2\cos x \quad \text{-----} \quad (i)$$

Let $y = e^{mx}$ be the trial solution of

$$\frac{d^2y}{dx^2} - y = 0 \quad \text{-----} \quad (ii)$$

$\therefore$ A.E is $m^2 - 1 = 0$, $e^{mx} \neq 0$

$$\therefore m = \pm 1$$

C.F is $y_c = C_1 e^{x} + C_2 e^{-x}$

P.I be $y_p = \frac{1}{D^2-1} \cdot x^2 \cos x$

$= \text{Real part of } \frac{1}{D^2-1} \cdot x^2 e^{ix}$

$= R.P \text{ of } e^{ix} \frac{1}{(D+i)^2 - 1} x^2$

$= R.P \text{ of } e^{ix} \frac{1}{D^2 - 2iD - 1 - 1} x^2$

$= R.P \text{ of } e^{ix} \frac{1}{-2 + 2iD + D^2} \cdot x^2$

$= R.P \text{ of } e^{ix} \cdot \frac{1}{-2\left(1 - \frac{2iD + D^2}{2}\right)} \cdot x^2$

$= -\frac{1}{2} \cdot R.P \text{ of } e^{ix} \left(1 - \frac{2iD + D^2}{2}\right)^{-1} \cdot x^2$

$= -\frac{1}{2} R.P \text{ of } e^{ix} \left[1 + \frac{2iD + D^2}{2} + \left(\frac{2iD + D^2}{2}\right)^2 + \dots\dots\right] x^2$

$$= -\frac{1}{2} \text{ R.P of } e^{ix} \left[1 + iD + \frac{D^2}{2} - D^2 + iD^3 + \frac{D^4}{4} + \cdots\right] x^2$$

$$\Rightarrow y_p = -\frac{1}{2} \text{ R.P of } e^{ix} [x^2 + 2ix + 1 - 2]$$

$$= -\frac{1}{2} \text{ R.P of } e^{ix} (x^2 - 1 + 2ix)$$

$$= -\frac{1}{2} \text{ R.P of } (\cos x + i\sin x)(x^2 - 1 + 2ix)$$

$$= -\frac{1}{2} \text{ R.P of } [(x^2-1)\cos x + 2ix\cos x + i\sin x(x^2-1) - 2x\sin x]$$

$$= -\frac{1}{2}[(x^2-1)\cos x - 2x\sin x]$$

$$= x\sin x - \frac{1}{2}(x^2-1)\cos x$$

$$\therefore y = y_c + y_p$$

$$= c_1 e^x + c_2 e^{-x} + x\sin x - \frac{1}{2}(x^2-1)\cos x$$

(Ans).

Math

4/5/21

Linear Homogenious Differential Equation

$$\boxed{\begin{aligned} &x^2 \frac{d^2y}{dx^2} \\ &x^2 \frac{d^ny}{dx^n} \end{aligned}}$$

$$x = e^z$$

$$\log x = \log e^z = z \log e = z$$

$$z = \log x \; ; \; \frac{dz}{dx} = \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx} = \frac{1}{x} \cdot \frac{dy}{dz}$$

$$x \frac{dy}{dx} = \frac{dy}{dz} \; ; \; x^2 \frac{d^2y}{dx^2} = \frac{d^2y}{dz^2} - \frac{dy}{dz}$$

$$\boxed{\frac{dy}{dx} = Dy \, , \quad \frac{dy}{dz} = D_1 y}$$

$$D = \frac{d}{dx}$$

$$D_1 = \frac{d}{dz}$$

$$x \frac{dy}{dx} = xDy = D_1 y$$

$$x^2 \frac{d^2y}{dx^2} = x^2 D^2 y = D_1 (D_1 - y) y$$

$$x^3 \frac{d^3y}{dx^3} = x^3 D^3 y = D_1 (D_1 - 1)(D_1 - 2) y$$

$$x^4 \frac{d^4y}{dx^4} = x^4 D^4 y = D_1 (D_1 - 1)(D_1 - 2)(D_1 - 3) y$$

$$\cdots \quad \cdots \quad \cdots$$

$$x^n \cdot \frac{d^ny}{dx^n} = x^n D^n y = D_1 (D_1 - 1)(D_1 - 2) \cdots \{D_1 - (n-1)\} y$$

Pg: 96 :: Ex-2

$$x^2 \frac{d^2y}{dx^2} - 2y = x^2 + \frac{1}{x} \quad \cdots \cdots (i)$$

Let, $x = e^z \quad \therefore z = \log x$

$$\therefore x^2 \frac{d^2y}{dx^2} = x^2 D^2 y = D_1 (D_1 - 1) y$$

$D = \frac{d}{dx}$

$D_1 = \frac{d}{dz}$

$$\therefore (i) \Rightarrow D_1 (D_1 - 1) y - 2y = e^{2z} + e^{-z}$$

$$\Rightarrow (D_1^2 - D_1 - 2)y = e^{2z} + e^{-z} \quad --- (ii)$$

Let $y = e^{mz}$ be the trial solution of

$$(D_1^2 - D_1 - 2)y = 0$$

A. E is

$$m^2 - m - 2 = 0, \quad e^{mz} \neq 0$$

$$\Rightarrow (m-2)(m+1) = 0$$

$$\therefore m = -1, 2$$

$\therefore$ The complementary function is

$$y_c = c_1 e^{-z} + c_2 e^{2z}$$

$$= c_1 x^{-1} + c_2 x^2$$

$x = e^z$

$\frac{1}{x} = e^{-z}$

$x^{-1} = e^{-z}$

The particular integral be $y_p = \frac{1}{D^2 - D_1 - 2}(e^{2z} + e^{-z})$

$$\cancel{= \frac{1}{(D_1-2)(D_1+1)}(e^{2z} + e^{-z})}$$

$$y_p = \frac{1}{D_1^2 - D_1 - 2} \cdot e^{2z} + \frac{1}{D_1^2 - D_1 - 2} e^{-z}$$

$$= z \cdot \frac{1}{2D_1 - 1} \cdot e^{2z} + z \cdot \frac{1}{2D_1 - 1} e^{-z}$$

$$= z \cdot \frac{1}{2 \cdot 2 - 1} \cdot e^{2z} + z \cdot \frac{1}{2(-1) - 1} e^{-z}$$

$$= \frac{ze^{2z}}{3} - \frac{ze^{-z}}{3}$$

$$= \frac{z}{3} \left(e^{2z} - e^{-z} \right)$$

$$= \frac{\log x}{3} \left(x^2 - x^{-1} \right)$$

The complete solution is

$$y = y_c + y_p$$

$$= c_1 x^{-1} + c_2 x^2 + \frac{\log x}{3} (x^2 - x^{-1})$$

(Ans)

Pg : 97 :: Ex - 4 (a)

$$x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 4y = 2x^2 \quad \text{---} \quad (i)$$

Let, $x = e^z$, $z = \log x$,

$$x \frac{dy}{dx} = x D y = D$$

$$x^2 \frac{d^2y^2}{dx^2} = x^2 D^2 y = D_1 (D_1 - 1) y$$

$D = \frac{d}{dx}$

$D_1 = \frac{d}{dz}$

$\therefore (i) \Rightarrow D_1 (D_1 - 1) y - 3D_1 y + 4y = 2e^{2z}$

$\Rightarrow (D_1^2 - D_1 - 3D_1 + 4) y = 2e^{2z}$

$\Rightarrow (D_1^2 - 4D_1 + 4) y = 2e^{2z} \quad \text{----} \quad (ii)$

Let $y = e^{mz}$ be the trial solution of

$(D_1 - 4D_1 + 4) y =$

$\therefore$ A.E is $m^2 - 4m + 4 = 0$

$\Rightarrow (m-2)^2 = 0 \Rightarrow m = 2, 2$

The C.F is $y_c = (c_1 + c_2 z)e^{2z}$

$$= (c_1 + c_2 \log x)x^2$$

$\therefore$ The P.I be $y_p = \dfrac{1}{(D_1-2)^2} \cdot 2e^{2z}$

$$= z \cdot \frac{1}{2(D_1-2)} \cdot 2e^{2z}$$

$$\cancel{= z \cdot \frac{1}{D_1+2-2} e^{2z}}$$

$$\cancel{= z \cdot \frac{1}{D_1} \cdot e^{2z}}$$

$$= z \cdot z \cdot \frac{1}{1-0} e^{2z}$$

$$= z^2 \cdot e^{2z}$$

$$= (\log x)^2 x^2$$

$\therefore$ The complete solution is, $y = y_c + y_p$

$$= (c_1 + c_2 \log x)x^2 + (\log x)^2 x^2$$

(Ans)

Differential Equations

Linear Algebra

Lec: 16

29/8/21

Pg: 100, Ex-9

$$x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right) \text{ --- (i)}$$

Let $x = e^z$, $x \frac{dx}{dy} = Dy$ $\quad D = \frac{d}{dz}$

$$\Rightarrow x^2 \frac{d^2y}{dx^2} = D(D-1)y$$

$$\Rightarrow x^3 \frac{d^3y}{dx^3} = D(D-1)(D-2)y$$

$$\text{(i)} \Rightarrow D(D-1)(D-2)y + 2D(D-1)y + 2y = 10(e^z + e^{-z})$$

$$\Rightarrow [D(D^2 - 3D + 2) + 2D^2 - 2D + 2]y = 10(e^z + e^{-z})$$

$$\Rightarrow (D^3 - 3D^2 + 2D + 2D^2 - 2D + 2)y = 10(e^z + e^{-z})$$

$$\Rightarrow (D^3 - D^2 + 2)y = 10(e^z + e^{-z})$$

Let $y = e^{mz}$ be the trial solⁿ of

$$(D^3 - D^2 + 2)y = 0 \quad \text{---} \; (ii)$$

$\therefore$ A.E is $m^3 - m^2 + 2 = 0$

$\Rightarrow m^2(m+1) - 2m(m+1) + 2(m+1) = 0$

$\Rightarrow (m+1)(m^2 - 2m + 2) = 0$

$\therefore m+1 = 0, \quad m^2 - 2m + 2 = 0$

$\Rightarrow m = -1, \quad m = \dfrac{2 \pm \sqrt{2^2 - 4.1.2}}{2.1}$

$= \dfrac{2 \pm 2i}{2}$

$= 1 \pm i$

$\therefore$ Complementary function is

$$y_c = c_1 e^{-z} + e^{z}(c_2 \cos z + c_3 \sin z)$$

$$= c_1 x^{-1} + x[c_2 \cos(\log x) + c_3 \sin(\log x)]$$

P.I be

$$y_p = \frac{1}{D^3 - D^2 + 2} \cdot 10 \cdot (e^z + e^{-z})$$

$$= \frac{10}{(D+1)\ (D^2 - 2D + 2)} e^z + \frac{10}{(D+1)(D^2 - 2D + 2)} e^{-z}$$

$$= \frac{10 e^z}{(1+1)\ (1^2 - 2.1 + 2)} + \frac{10}{D^3 - D^2 + 2} e^{-z}$$

$$= \frac{10 e^z}{2} + z \cdot \frac{10}{3D^2 - 2D} e^{-z}$$

$$= 5e^z + z \cdot \frac{10}{3 \cdot (-1)^2 - 2 \cdot (-1)} \cdot e^{-z}$$

$$= 5e^z + \frac{10 z e^{-z}}{5}$$

$$= 5e^z + 2z e^{-z}$$

$$= 5x + 2 \log x \cdot x^{-1}$$

The complete solution is

$$y = y_c + y_p$$
$$= c_1 x^{-1} + x\,[c_2 c_3 (\log x) + c_3 \sin(\log x)]$$
$$+ 5x + 2x^{-1} \log x \quad \text{(Ans)}.$$

~~101 pg : Ex~~

Differential Equations: (Lec 17) 31/8/21

Pg: 103 :: Ex: 14

$$\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} = \frac{12\log x}{x^2}$$

$$\Rightarrow x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} = 12 \log x$$

$$\Rightarrow x^2 D^2 y + x D y = 12 \log x \quad \text{---- (i)} \qquad \Big|\ D = \frac{d}{dx}$$

Let ~~Lim~~ $x = e^z$, $z = \log x$

$$xDy = D_1 y, \quad D_1 = \frac{d}{dz}$$

$x^2 D^2 y = D_1(D_1 - 1)y$

(i) $\Rightarrow D_1(D_1-1)y + D_1 y = 12z$

$\Rightarrow D_1^2 y - D_1 y + D_1 y = 12z$

$\Rightarrow D_1^2 y = 12z$ ---- (ii)

$y = e^{mz}$ be the trial soln. of

$D_1^2 y = 0$

A.E is,

$m^2 = 0$,

$\therefore m = 0, 0$

$\therefore$ The C.F is

$y_c = (c_1 + c_2 z) e^{0z}$

$= c_1 + c_2 \log x$

Let P.I be $y_p = \frac{1}{D_1^2} \cdot 12z$

$$= 12 \frac{1}{D_1} \left(\frac{z^2}{2}\right)$$

$$= 6 \cdot \frac{z^3}{3}$$

$$= 2(\log x)^3$$

$\therefore$ General sol^n is

$$y = y_c + y_p$$

$$= c_1 + c_2 \log x + 2(\log x)^3 \quad \text{(Ans)}.$$

Pg: 105 :: Ex: 18

$$x^3 \frac{d^3y}{dx^3} + 3x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = x \log x$$

$$\Rightarrow x^3 D^3 y + 3x^2 D^2 y + xDy + y = x \log x \quad \Big| \; D = \frac{d}{dx}$$

---- (i)

Let $x = e^z$, $\therefore z = \log x$.

$x Dy = D_1 y \qquad \Big| \; D_1 = \frac{d}{dz}$

$x^2 D^2 y = D_1 (D_1 - 1) y$

$x^3 D^3 y = D_1 (D_1 - 1)(D_1 - 2) y$

$\therefore$ (i) $\Rightarrow D_1 (D_1 - 1)(D_1 - 2) y + 3 D_1 (D_1 - 1) y + D_1 y + y = e^z . z$

$\Rightarrow D_1 (D_1^2 - 3D_1 + 2) y + 3 D_1^2 y - 3 D_1 y + D_1 y + y = z . e^z$

$\Rightarrow D_1^3 y - 3 D_1^2 y + 2 D_1 y + 3 D_1^2 y - 3 D_1 y + D_1 y + y = z . e^z$

$\Rightarrow (D_1^3 + 1) y = \cancel{2e^z} \; z . e^z$ ---------- (ii)

Let, $y = e^{mz}$ be the trial solⁿ of $(D_1^3 + 1) y = 0$

$\therefore$ A.E is $m^3 + 1 = 0$, $e^{mz} \neq 0$

$\Rightarrow (m+1)(2m^2 - m + 1) = 0$

$\therefore m = -1$, $m = \dfrac{1 \pm \sqrt{1^2 - 4.1.1}}{2.1}$

$= \dfrac{1 \pm \sqrt{3}i}{2}$

$= \dfrac{1}{2} \pm i\dfrac{\sqrt{3}}{2}$

$\therefore$ C.F is $y_c = c_1 e^{-z} + e^{z/2}\left(c_2 \cos\frac{\sqrt{3}}{2}z\right.$

$\left. + c_2 \sin \frac{\sqrt{3}}{2} z\right)$

$= c_1 x^{-1} + \sqrt{x}\left[c_2' \cos\left(\frac{\sqrt{3}}{2}\log x\right)\right.$

$\left. + c_3 \sin\left(\frac{\sqrt{3}}{2}\log x\right)\right]$

$y_p = \dfrac{1}{D_1^3 + 1} \cdot z.e^z$

$= e^z . \dfrac{1}{(D_1+1)^3 + 1} z$

$$= e^{z} \cdot \frac{1}{1+3D_1^2+3D_1+D_1^3+1} \cdot z$$

$$= e^{z} \cdot \frac{1}{2+3D_1+3D_1^2+D_3} \cdot z$$

$$\therefore y_p = e^{z} \frac{1}{2\left\{1+\frac{1}{2}(3D_1+3D_1^2+D_1^3)\right\}} \cdot z$$

$$= e^{z} \left\{1+\frac{1}{2}(3D_1+3D_1^2+D_1^3)\right\} \cdot z$$

$$= e^{z} \cdot \left[1+\frac{1}{2}(3D_1+3D_1^2+D_1^3)\right.$$

$$+ \left\{\frac{1}{2}(3D_1+3D_1^2+D_1^3)^2\right.$$

$$\left. + \ldots\ldots \right] . z$$

$$= e^{z} \left[z - \frac{1}{2} \cdot 3.1.\right)$$

$$= e^{z} \left(z-\frac{3}{2}\right)$$

$$= x\left(\log x - \frac{3}{2}\right)$$

$\therefore$ The general sol^n is,

$$y = y_c + y_p$$

$$= c_1 x^{-1} + \sqrt{x}\left[c_2 \cos\left(\frac{\sqrt{3}}{2}\log x\right) + c_3 \sin\left(\frac{\sqrt{3}}{2}\log x\right)\right] + x\left(\log x - \frac{3}{2}\right)$$

(Ans).

Differential Equations (Lec 18) 5/9/21

Apply the method of variation of parameters to solve the D.E

$$\frac{d^2y}{dx^2} + n^2y = \sec nx$$

Solⁿ:

We have,

$$\frac{d^2y}{dx^2} + n^2y = \sec nx \quad \ldots\ldots (i)$$

Let $y = e^{mx}$ be the trial solⁿ of i

$$\frac{d^2y}{dx^2} + n^2y = 0 \quad \ldots\ldots (ii)$$

$\therefore$ A.E is $m^2+n^2 = 0$

$\therefore m = \pm in$

$\therefore$ C.F is $y_c = c_1 \cos nx + c_2 \sin nx$

where c_1 and c_2 are constant.

Let us suppose that

$$y = A \cos nx + B \sin nx \quad \text{---- (iii)}$$ be the

soln ~~where~~ of (i), where A and B are

functions of x.

D.w.r to x,

$$\frac{dy}{dx} = -A \sin nx . n + A_1 \cos nx + B \cos nx . n + B_1 \sin nx$$

$$= -An \sin nx + Bn \cos nx + A_1 \cos nx + B_1 \sin nx \quad \text{---- (iv)}$$

choose A and B such that,

$$A_1 \cos nx + B_1 \sin nx = 0 \quad \text{---- (v)}$$

$$\text{(iv)} \Rightarrow \frac{dy}{dx} = -An \cos nx + Bn \cos nx \quad \text{---- (vi)}$$

D. (vi) w. r to x.

$$\therefore \frac{d^2y}{dx^2} = -An\cos nx.n - A_1 n\sin nx - Bn\sin nx.n + B_1 n\cos nx$$

$$= -n^2(A\cos nx + B\sin nx) - A_1 n\sin nx + B_1 n\cos nx$$

$$\frac{d^2y}{dx^2} = -n^2y - A_1 n\sin nx + B_1 n\cos nx$$

Putting the value of $\cancel{\frac{d^2x}{dy^2}}$ $\frac{d^2y}{dx^2}$ in (i) we get,

$$(i) \Rightarrow -n^2y - A_1 n\sin nx + B_1 n\cos nx + n^2y = \sec nx$$

$$\Rightarrow -A_1 n\sin nx + B_1 n\cos nx = \sec nx, \quad \text{---(vii)}$$

(v) $\times\, n\sin nx$ + (vii) $\times \cos nx$ we get,

$\therefore$ $A_1 n \sin nx \cos nx + B_1 n \sin^2 nx$

$- A_1 n \sin nx \cos nx + B_1 n \cos^2 nx = \sec nx \cdot \cos nx$

$\Rightarrow B_1 n (\sin^2 nx + \cos^2 nx) = 1$

$\Rightarrow B_1 n = 1.$

$\Rightarrow B_1 = \frac{1}{n}$

$\therefore B = \int \frac{1}{n} dx + c_2 = \frac{x}{n} + c_2$

Putting the value of B_1 in (5)

(v) $\Rightarrow A_1 \cos x + \frac{1}{n} \sin x = 0$

$$A_1 = -\frac{1}{n} \frac{\sin nx}{\cos nx} = \frac{-1}{n} \tan nx$$

Integrating we get,

$$A = -\frac{1}{n} \int \tan nx \, dx + c_1$$

$$= \frac{1}{n^2} \log (\cos nx) + c_1$$

~~D (vi) w.~~

Putting the value of A and B in (iii) we get.

The complete solution is,

$$y = \left\{ \frac{1}{n^2} \log(\cos nx) + c_1 \right\} \cos nx + \left\{ \frac{x}{n} + c_2 \right\} \sin nx$$

$$= c_1 \cos nx + c_2 \sin nx + \frac{\cos nx}{n^2} \log(\cos nx) + \frac{x}{n} \sin nx$$

(Ans)

Differential Equations (Lec 19) 9/9/21

* $(D^2+4)y = 4\tan 2x$ ---- (i)

Let $y = e^{mx}$ be the trial solution of

$(D^2+4)y = 0$

$\therefore$ Auxiliary eq^n $m^2+4=0$

$\Rightarrow m = \pm 2i$

$\therefore$ C.F is $y_c = Ay_1 + By_2$

$\Rightarrow y_c = A\cos 2x + B\sin 2x$

where $y_1 = \cos 2x$, $y_2 = \sin 2x$

Wronskian

C.F is $y_c = c_1 \cos 2x + c_2 \sin 2x$ ..... (ii)

where c_1 and c_2 are constants.

Let $y = A\cos 2x + B\sin 2x$ ..... (iii) be the general solution of (i) where A and B are functions of x.

D (iii) with r. to x,

$$\therefore y_1 = -2A\sin 2x + 2B\cos 2x$$

$$+ A_1 \cos 2x + B_1 \sin 2x \quad \text{---- (iv)}$$

Choose A and B such that

$$A_1 \cos 2x + B_1 \sin 2x = 0 \quad \text{---- (v)}$$

From (iv) and (v),

$$\text{(iv)} \Rightarrow y_1 = -2A\sin 2x + 2B\cos 2x \quad \text{---- (vi)}$$

D. (vi) w. r. to x,

$$\therefore y_2 = -4A\cos 2x - 4B\sin 2x$$

$$-2A_1 \sin 2x + 2B_1 \cos 2x \quad \text{---- (vii)}$$

From (i), (iii) and (vii) we get,

$$(i) \Rightarrow -4A\cos 2x - 4B\sin 2x - 2A_1\sin 2x$$

$$+2B_1\cos 2x + 4A\cos 2x + 4B\sin 2x = 4\tan 2x$$

$$\Rightarrow -A_1\sin 2x + B_1\cos 2x = 2\tan 2x \quad \text{--- (viii)}$$

From (v), $A_1 = -B_1 \dfrac{\sin 2x}{\cos 2x}$ --- (ix)

From (viii),

$$(viii) \Rightarrow B_1 \frac{\sin 2x}{\cos 2x} \cdot \sin 2x + B_1\cos 2x = 2\tan 2x$$

$$\Rightarrow \frac{B_1}{\cos 2x}(\sin^2 2x + \cos^2 2x) = 2\tan 2x$$

$$\Rightarrow B_1 = 2\frac{\sin 2x}{\cos 2x} \cdot \cos 2x$$

$$\Rightarrow B_1 = 2\sin 2x$$

Integrating we get,

$$B = -2 \cdot \frac{\cos 2x}{2} + C_2 = -\cos 2x + C_2$$

Putting the value of B_1 in (ix).

$$(ix) \Rightarrow A_1 = -2\sin 2x . \frac{\sin 2x}{\cos 2x}$$

$$= -\frac{2\sin^2 2x}{\cos 2x}$$

$$\therefore A_1 = \frac{-2(1-\cos^2 2x)}{\cos 2x}$$

$$\therefore A_1 = -2\sec 2x + 2\cos 2x$$

Integrating we get.

$$A = -\ln\left\{\tan\left(\frac{\pi}{4}+x\right)\right\} + \sin 2x + c_1$$

Putting the value of A and B in (iii) we get the general solution, is,

$$y = \left[-\ln\left\{\tan\left(\frac{\pi}{4}+x\right)\right\} + \sin 2x + c_1\right]\cos 2x$$
$$+ (-\cos 2x + c_2) . \sin 2x$$

$$= c_1 \cos 2x + c_2 \sin 2x - \ln\left\{\tan\left(\frac{\pi}{4}+x\right)\right\} \cos 2x$$

$$- \cos 2x . \sin 2x \quad \text{(Ans)}$$

$\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$

$\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$

- - - - - - - - - - -

$$= c_1 \cos 2x + c_2 \sin 2x - \ln\left\{\tan\left(\frac{\pi}{4}+x\right)\right\} \cos 2x$$

$$- \cos 2x . \sin 2x \quad \text{(Ans)}$$

$\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$

$\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$

Differential Equations (Lec 20)

9#
11/2/22

Pg: 87

Example: 3(a)

$$\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x} \quad \text{------ (i)}$$

Let $y = e^{mx}$ be the trial solution of

$$\frac{d^2y}{dx^2} - y = 0 \quad \text{------ (ii)}$$

$\therefore$ A.E is $m^2 - 1 = 0$, $e^{mx} \neq 0$

$m = \pm 1$

The C.F is $y_c = c_1 e^{x} + c_2 e^{-x}$

Let y_p be P.I.

$\therefore y_p = Ae^{x} + Be^{-x}$, where A and B are

functions x, Let $y_p = y$

$$\therefore \frac{dy}{dx} = Ae^{x} + A_1 e^{x} - Be^{x} + B_1 e^{x} \quad \text{------ (iii)}$$

choose A and B such that,

$$\therefore A_1 e^{x} + B_1 e^{-x} = 0 \quad \text{------ (iv)}$$

$\therefore$ (iii) $\Rightarrow \dfrac{dy}{dx} = Ae^{x} - Be^{-x}$

$$\therefore \frac{d^2y}{dx^2} = Ae^{x} + A_1e^{x} + Be^{-x} - B_1e^{-x}$$

Putting the value of $\frac{d^2y}{dx^2}$ and y in (i)

we get,

$$(i) \rightarrow Ae^{x} + A_1e^{x} + Be^{-x} - B_1e^{-x} - Ae^{x} - Be^{-x} = \frac{2}{1+e^{x}}$$

$$\Rightarrow A_1e^{x} - B_1e^{-x} = \frac{2}{1+e^{x}} \quad \text{---- (v)}$$

(iv) + (v), we get,

$$2A_1e^{x} = \frac{2}{1+e^{x}}$$

$$\Rightarrow A_1 = \frac{e^{-x}}{1+e^{x}}$$

$$\Rightarrow \frac{dA}{dx} = \frac{\cancel{e^x}\, e^{-x}}{1+e^x}$$

$$\Rightarrow dA = \frac{e^{-x}}{1+e^x}\, dx$$

$$\therefore A = \int \frac{e^{-x}}{1+e^x}\, dx$$

Let, $e^x = z$

$e^x dx = dz$

$dx = \frac{dz}{e^x}$

$= \frac{dz}{z}$

$$= \int \frac{dz}{z.\, z(1+z)}$$

~~Let~~

$$= \int \left(\frac{1}{z^2} - \frac{1}{z} + \frac{1}{z+1}\right) dz$$

Let $\frac{1}{z^2(1+z)} = \frac{C}{z} + \frac{D}{z^2} + \frac{E}{1+z}$

$1 = Cz(z+1) + D(1+z) + Ez^2$

$$= -\frac{1}{z} - \ln z + \ln(1+z) + c_1$$

$$= \ln \frac{1+z}{z} - \frac{1}{z} + c_1$$

$0 = C + E$

$0 = C + D$

$$= \ln \frac{1+e^x}{e^x} - \frac{1}{e^x} + c_1$$

$1 = D$

$C = -1$

$E = 1$

$$A = \ln\left(\frac{1+e^{x}}{e^{x}}\right) - e^{-x} + 4$$

from (4) $\Rightarrow B_1 e^{-x} = -A_1 e^{x}$

$$\Rightarrow B_1 = -A_1 \frac{e^{x}}{e^{-x}}$$

$$= -\frac{e^{-x}}{1+e^{x}} \cdot \frac{e^{x}}{e^{-x}}$$

$$= \frac{-e^{x}}{1+e^{x}}$$

$$\Rightarrow \frac{dB}{dx} = -\frac{e^{x}}{1+e^{x}}$$

$$\Rightarrow dB = -\frac{e^{x}}{1+e^{x}} dx$$

$$\therefore B = \int \frac{-e^{x}}{1+e^{x}} dx$$

$$= -\ln(1+e^{x}) + c_2$$

The general solution is,

$$y_P = y = Ae^{x} + Be^{-x}$$

$$\Rightarrow y = \left(\ln \frac{e^x}{1+e^x} - e^{-x} + c_1\right)e^x$$

$$+ \left\{-\ln(e^x+1) + c_2\right\}e^{-x}$$

$$= e^x \ln \frac{e^x}{1+e^x} - 1 + c_1 e^x - e^{-x}\ln(e^x+1)$$

$$+ c_2 e^{-x}$$

Ex-4

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2 e^x \quad \text{(i)}$$

Let $x = e^z$

$$x\frac{dy}{dx} = Dy$$

$D = \frac{d}{dz}$, $x = e^z$

$$x^2 \frac{d^2y}{dx^2} = D(D-1)\,y$$

(i) $\Rightarrow D(D-1)y + Dy - y = 0$

$\Rightarrow (D^2 - D + D - 1)y = 0$

$\Rightarrow (D^2 - 1)\,y = 0$

Let $y = e^{mz}$ be the trial solution. ~~of~~

A.E is $m^2 - 1 = 0$, $e^{mz} \neq 0$

$m = \pm 1$

$\therefore y_c = C_1 e^{z} + C_2 e^{-z}$

$= C_1 x + \frac{C_2}{x}$

P.I be $y_p = y = Ax + \frac{B}{x}$

where A and B are functions of x.

$$\therefore \frac{dy}{dx} = A + A_1 x - \frac{B}{x^2} + \frac{B_1}{x} \quad \text{---} (ii)$$

Choose A and B such that

$$A_1 x + \frac{B_1}{x} = 0 \quad \text{----} (iii)$$

$$(ii) \Rightarrow \frac{dy}{dx} = A - \frac{B}{x^2} \cancel{+ \frac{B_1}{x}}$$

$$\therefore \frac{d^2y}{dx^2} = A_1 + \frac{2B}{x^3} - \frac{B_1}{x^2}$$

Putting $\frac{d^2y}{dx^2} \cdot y$ in $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2 e^x$

$$\Rightarrow \frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} - \frac{1}{x^2} y = e^x$$

$$\Rightarrow A_1 + \frac{2B}{x^3} - \frac{B_1}{x^2} + \frac{1}{x}\left(A - \frac{B}{x^2}\right) - \frac{1}{x^2}\left(Ax + \frac{B}{x}\right) = e^x$$

$$\Rightarrow A_1 + \frac{2B}{x^3} - \frac{B_1}{x^2} + \frac{A}{x} - \frac{B}{x^3} - \frac{A}{x} - \frac{B}{x^3} = e^x$$

$$\Rightarrow A_1 + \frac{B_1}{x^2} = e^x \quad \text{----- (iv)}$$

from (iii) $\Rightarrow \quad A_1 = -\frac{B_1}{x^2}$

from (iv) $\Rightarrow -\frac{B_1}{x^2} \cancel{e^x} - \frac{B_1}{x^2} = e^x$

$$\Rightarrow -2B_1 = x^2 e^x$$

$$\therefore B_1 = \cancel{2} - \frac{1}{2} x^2 e^x$$

$$\Rightarrow B_1 = -\frac{1}{2} \int x^2 e^x \, dx \quad \text{..... (v)}$$

$$B = -\frac{1}{2}\left[x^2 e^x - \int 2x e^x \, dx\right]$$

$$= -\frac{1}{2}\left[x^2 e^x - 2x e^x + 2e^x + c_2\right]$$

$$= -\frac{1}{2} e^x \left(x^2 - 2x + 2 + c_2\right)$$

from (V), $A_1 = \frac{1}{2}x^2 e^x \cdot \frac{1}{x^2}$

$\Rightarrow A_1 = \frac{1}{2} e^x$

$\therefore A = \frac{1}{2} \int e^x \, dx$

$= \frac{1}{2} e^x + c_1$

$\therefore$ The complete solution is,

$$y_p = y = Ax + \frac{B}{x}$$

$$y = \left(\frac{1}{2} e^x + c_1\right)x + \frac{1}{x}\left[-\frac{1}{2} e^x (x^2 - 2x + 2 + c_2)\right]$$

$$= \frac{x}{2} e^x + c_1 x - \frac{1}{2} x e^x + e^x - \frac{1}{x} e^x + \frac{c_2}{x}$$

$$= c_1 x + \frac{c_2}{x} + e^x - \frac{e^x}{x} \quad \text{(Ans)}$$

Pg (1(a), 1(b))

Math (DE) (Lec 21) 18/2/2022

এর পরিবর্তে $\frac{dy}{dx}$

$$\boxed{\frac{dy}{dx} + py = q}$$

Integration factor $= e^{\int p\,dx}$

Pg: 116

Ex: 1

$$p^4 - (x+2y+1)p^3 + (x+2y+2xy)p^2 - 2xyp = 0$$

$$\Rightarrow p\{p^3 - (x+2y+1)p^2 + (x+2y+2xy)p - 2xy\} = 0$$

$$\Rightarrow p\{p^2(p-1) - xp(p-1) - 2yp(p-1) + 2xy(p-1)\} = 0$$

$$\Rightarrow p(p-1)\{p^2 - xp - 2yp + 2xy\} = 0$$

$$\Rightarrow p(p-1)\{p(p-x) - 2y(p-x)\} = 0$$

$$\Rightarrow p(p-1)(p-x)(p-2y) = 0$$

$$\therefore p = 0,\ p-1 = 0,\ p-x = 0,\ p-2y = 0$$

$\therefore \frac{dy}{dx}=0, \quad \frac{dy}{dx}=1, \quad \frac{dy}{dx}=x, \quad \frac{dy}{dx}=2y$

$dy=0, \quad dy=dx, \quad dy=x\,dx, \quad \frac{dy}{y}=2dx$

Integrating,

$y=c, \quad y=x+c, \quad y=\frac{x^2}{2}+4, \quad \log y=2x+c_2$

$y-c=0, \quad y-x-c=0 \quad 2y-x^2-c=0, \quad \log y=2x+c_2$

$\log y=\log e^{2x}+\log e^{c}$

$\Rightarrow \log y=\log ce^{2x}$

$\Rightarrow y=ce^{2x}$

$\Rightarrow y-ce^{2x}=0$

$\therefore$ General solution is

$(y-c)(y-x-c)(2y-x^2-c)(y-ce^{2x})=0$

Differential Equation (Lec 22)

24/2/2022

Linear differential function

Pg: 18 $$\frac{dy}{dx} + Py = Q$$

reduce to linear form.

$$\frac{dy}{dx} + Py = Qy$$

3.2

Exact theorem.

Homogenious linear equation,

Curtesian pair,

$$\boxed{\frac{dy}{dx} = -\frac{dx}{dy}} \rightarrow \text{For curtesian}$$

$$\boxed{\frac{dr}{d\theta} = -r^2 \frac{d\theta}{dr}} \rightarrow \text{For polar}$$

Chapter: 4

Pg: 60

Partical integral, complementary function

$$(D^3 - 2D^2 - 4D + 8)\, y = 0 \qquad \Big| \; D = \frac{d}{dx}$$

$$\Rightarrow \frac{d^3y}{dx^2} - 2 \cdot \frac{d^2y}{dx^2} - 4 + 8y = 0 \quad \ldots (i)$$

Let, $y = e^{mx}$ be the trial sol^n of (i)

$\therefore \frac{dy}{dx} = me^{mx}$, $\frac{d^2y}{dx^2} = m^2e^{mx}$, $\frac{d^3y}{dx^3} = m^3e^{mx}$

(i) $\Rightarrow (m^3 - 2m^2 - 4m + 8)e^{mx} = 0$

$\therefore$ A.E is $m^3 - 2m^2 - 4m + 8 = 0$, $e^{mx} \neq 0$

$\Rightarrow m^2(m-2) - 4(m-2) = 0$

$\Rightarrow \cancel{(m+2)}\ (m-2)(m^2-4) = 0$

$\Rightarrow (m-2)(m-2)(m+2) = 0$

$\therefore m = 2, 2, -2$

$\therefore$ The general solution is: . . . .

Math (DE) (Lec 23)

28/02/2022

Method of undetermine coefficients

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = x^2,\ \sin x,\ xe^x,\ x\sin x,\ xe^x\sin x$$

U.C function . U.C set

U.C function	U.C set
x	$\{x, 1\}$
x^2	$\{x^2, x, 1\}$
e^x	$\{e^x\}$
xe^x	$\{xe^x, e^x\}$
$\sin x$	$\{\sin x, \cos x\}$
$\cos x$	$\{\cos x, \sin x\}$

$x \sin x \quad \{x\sin x, x\cos x, \sin x, \cos x\}$

~~General~~ $y_p = c_1$

Solve the D.E by the method of U.C.

$$\frac{d^2y}{dx^2} - y = e^{2x} \quad \ldots\ldots (i)$$

$\Rightarrow$ Let $y = e^{mx}$ be the trial solution of

$$\frac{d^2y}{dx^2} - y = 0 \quad \ldots\ldots (ii)$$

$$\therefore \frac{dy}{dx} = me^{mx}, \quad \frac{d^2y}{dx^2} = m^2e^{mx}$$

$$(ii) \Rightarrow (m^2-1)e^{mx} = 0$$

$\therefore$ A. E is $m^2 - 1 = 0$, $e^{mx} \neq 0$

$\therefore m = \pm 1$

$\therefore$ C.F is $y_c = c_1 e^{x} + c_2 e^{-x}$.

Here, U.C function $= e^{2x}$

U.C set $= \{e^{2x}\}$

Let P.I be ~~Let~~ $y_p = y = Ae^{2x}$

$$\frac{dy}{dx} = 2Ae^{2x}$$

$$\frac{dy}{dx^2} = 4Ae^{2x}$$

$$(i) \Rightarrow \frac{d^2y}{dx^2} - y = e^{2x}$$

$$\Rightarrow 4Ae^{2x} - Ae^{2x} = e^{2x}$$

$$\Rightarrow 3Ae^{2x} = e^{2x}$$

$$\Rightarrow 3A = 1$$

$$\therefore A = \frac{1}{3}$$

$$\therefore y_p = \frac{1}{3} e^{2x}$$

$\therefore$ The G.S is

$$y = y_c + y_p$$

$$= c_1 e^{x} + c_2 e^{-x} + \frac{1}{3} e^{2x} \quad \text{(Ans).}$$

$(D^2 - 2D + 1)y = x\sin x$

$\Rightarrow$ . . . . . .

A.E is $m^2 - 2m + 1 = 0$, $e^{mx} \neq 0$

$$(m-1)^2 = 0$$

$$\Rightarrow m = 1, 1$$

$\therefore$ C.F is $y_c = (c_1 + c_2 x) e^{x}$

U.C function $= x\sin x$

U.C set $= \{x\sin x, x\cos x, \sin x, \cos x\}$

Let P.I be $y_p = y = Ax\sin x + Bx\cos x + E\sin x + F\cos x$

$= (Ax+E)\sin x + (Bx+F)\cos x$

$\therefore Dy = (Ax+E)\cos x + A\sin x - (Bx+F)\sin x$

$+ B\cos x$

$= (Ax+B+E)\cos x + (A - Bx - F)\sin x$

$\therefore D^2y = -(Ax+E)\sin x + A\cos x + A\cos x$

$- (Bx+F)\cos x - B\sin x - B\sin x$

$= (-Ax - E - 2B)\sin x + (2A - Bx - F)\cos x$

$\therefore (D^2 - 2D + 1)y = x\sin x$

$\Rightarrow -(Ax + E + 2B)\sin x + (2A - Bx - F)\cos x$

$-2\,(Ax + B + E)\cos x - 2\,(A - Bx - F)\sin x$

$+ (Ax + E)\sin x + (Bx + E)\cos x = x\sin x$

$\Rightarrow (-A + 2B + A)\,x\sin x + (-B - 2A + B)\,x\cos x$

$+ (E + 2B - 2A + 2F + E)\sin x + (2A - F - 2B - 2E + F)\cos x$

$= x\sin x$

Equating corresponding components,

$\therefore 2B = 1, \quad -2A = 0, \quad 2B - 2A + 2F + 2E = 0, 2A - 2B - 2E = 0$

$\therefore B = \frac{1}{2}, \quad A = 0. \quad 2.\frac{1}{2} - 0 + 2F + 2.\frac{1}{2} = 0 \quad \Rightarrow 0 - 2.\frac{1}{2} - 2E = 0$

$\Rightarrow 2E = -1$

$\Rightarrow 1 - 1 + 2F = 0, \quad \Rightarrow E = -\frac{1}{2}$

$\Rightarrow F = 0$

$\boxed{\text{Wrong calculation}}$

D.E (Lec 24)

7/3/2022

Ex-9

$\cancel{y}$

$$y'' \cancel{y} - 3y' = 8e^{3x} + 4\sin x \quad \ldots\ldots (i)$$

A.E is $m^2 - 3m = 0$, $e^{mx} \neq 0$

$\Rightarrow m(m-3) = 0$

$\therefore m = 0, 3$

$\therefore$ C.F is $y_c = C_1 + C_2 e^{3x}$

U.C function $= 8e^{3x} + 4\sin x$

$= 8x_1 + 4x_2$

U.C set for $x_1 = \{xe^{3x}\}$

U.C set for $x_2 = \{\sin x, \cos x\}$

$m = 2, 2$

$(C_1 + C_2 x)e^{2x})$

P.I be $y_p = y = Axe^{3x} + B\sin x + C\cos x$

$\therefore y' = Ae^{3x} + 3Axe^{3x} + B\cos x - C\sin x$

$y'' = 3Ae^{3x} + 3Ae^{3x} + 9Axe^{3x} - B\sin x - C\cos x$

$= 6Ae^{3x} + 9Axe^{3x} - B\sin x - C\cos x$

Putting the values of y'' and y' in (i) we get.

(i) $\Rightarrow 6Ae^{3x} + 9Axe^{3x} - B\sin x - C\cos x - 3Ae^{3x} - 9Axe^{3x}$

$-3B\cos x + 3C\sin x = 8e^{3x} + 4\sin x$

$\Rightarrow 3Ae^{3x} + (3C-B)\sin x - (C+3B)\cos x$

$= 8e^{3x} + 4\sin x$

Equating corresponding components

$\therefore 3A = 8$ | $3C - B = 4$

$\therefore A = \frac{8}{3}$ | $-C - 3B = 0$

$C = -3B$ | $\therefore 3C - B = 4$

$= \frac{6}{5}$ | $-3C - 9B = 0$

(+) $-10B = 4$

$\therefore B = -\frac{2}{5}$

$\therefore y_p = \frac{8}{3}xe^{3x} - \frac{2}{5}\sin x + \frac{6}{5}\cos x$

$\therefore$ The G.S is $y = y_c + y_p$.

$$= C_1 + C_2 e^{3x} + \frac{8}{3} x e^{3x}$$

$$- \frac{2}{3} \sin x + \frac{6}{5} \cos x.$$

$(D^2 - 3D + 2)y = 2x^2 + e^x + 2xe^x + 4e^{3x}$

A.E is $m^2 - 3m + 2 = 0$, $e^{mx} \neq 0$

$(m-2)(m-1) = 0$

$\therefore m = 1, 2$

$y_c = C_1 e^x + C_2 e^{2x}$

U.C function $= 2x^2 + e^x + 2xe^x + 4e^{3x}$

$= 2x_1 + x_2 + 2x_3 + 4x_4$

U.C set for $x_1 = \{x^2, x, 1\} = S_1$

U.C set for $x_2 = \{e^x\} = S_2$

U.C set for $x_3 = \{xe^x, e^x\} = S_3$

U.C set for $x_4 = \{e^{3x}\} = S_4$

Since e^x is a member ~~number~~ of y_c and S_2, S_3

So, $S_2' = \{xe^x\}$, $S_3' = \{x^2e^x, xe^x\}$

$$y_p = y = Ax^2 + Bx + C + Exe^x + Fx^2e^x + Gxe^x + He^{3x}$$

$$\therefore Dy = 2Ax + B + Ee^x + Exe^x + 2Fxe^x + Fx^2e^x + Ge^x + Gxe^x + 3He^{3x}$$

$$D^2y = 2A + Ee^x + Ee^x + Exe^x + 2Fe^x + 2Fxe^x + Ge^x + 2Fxe^x + Fx^2e^x + Ge^x + Gxe^x + 9He^{3x}$$

DE (Lec 26)

Modeling in Biology

14/03/2022

Pg: 109 :: Ex: 1

Let at a time t_1 the population is P. Then by the condition,

$$\frac{dP}{dt} \propto P$$

$$\Rightarrow \frac{dP}{dt} = kP$$

$$\Rightarrow \frac{dP}{P} = k\,dt$$

Integrating,

$$\int_{P_0}^{2P_0} \frac{dP}{P} = k\int_{t=0}^{t=50} dt$$

[initially $t=0$, $P=P_0$]

$t=50$, $P=2P_0$

$$\Rightarrow [\ln P]_{P_0}^{2P_0} = k[t]_0^{50}$$

$$\Rightarrow \ln 2P_0 - \ln P_0 = k(50-0)$$

$$\Rightarrow \ln \frac{2P_0}{P_0} = 50k$$

$$\Rightarrow \ln 2 = 50k$$

$$\Rightarrow k = \frac{\ln 2}{50}$$

Again, $P = P_0$, $P = 3P_0$

$t = 0$, $t = T$

$$\int_{P_0}^{3P_0} \frac{dP}{P} = k \int_{t=0}^{T} dt$$

$$\Rightarrow [\ln P]_{P_0}^{3P_0} = k[t]_0^T$$

$$\Rightarrow \ln 3P_0 - \ln P_0 = kT$$

$$\Rightarrow \ln \frac{3P_0}{P_0} = kT$$

$$\therefore T = \frac{\ln 3}{k}$$

$$= \frac{50 \ln 3}{\ln 2}$$

$$= 79.24$$

$$\therefore T \approx 79 \text{ years}$$

Pg : 110

Ex-2

$$\frac{dN}{dt} = kN \qquad t = 0, \; N = N_0$$

$$t = T, \; N = 3N_0$$

$$\frac{dN}{N} = k\,dt$$

$$\Rightarrow \int_{N_0}^{3N_0} \frac{dN}{N} = k \int_{t=0}^{T} dt$$

$$\Rightarrow [\ln N]_{N_0}^{3N_0} = k[t]_0^T$$

$$\Rightarrow \ln 3N_0 - \ln N_0 = kT$$

$$\Rightarrow \ln \frac{3N_0}{N_0} = kT$$

$$\Rightarrow T = \frac{\ln 3}{k}$$

$$\frac{dN}{N} = k\,dt$$

$$\Rightarrow \int \frac{dN}{N} = k \int dt$$

$$\Rightarrow \ln N = kt + \ln c$$

$$\Rightarrow \ln N = \ln e^{kt} + \ln c$$

$$\Rightarrow \ln N = \ln c e^{kt}$$

$$\therefore N = Ce^{kt}$$

Initially, $t = 0$, $N = N_0$

$$N_0 = Ce^0$$

$$c = N_0$$

$$\therefore N(t) = N_0 e^{kt}$$

~~Let~~ $\lim_{t\to\infty} N(t) = \lim_{t\to\infty} N_0 e^{kt}$

$= \infty$

Pg: 113

Ex-09

Let at a time t, the number of becteria N

α	
$3 \rightarrow$	400
$10 \rightarrow$	2000

$$\frac{dN}{dt} \propto N$$

$$\frac{dN}{dt} = kN$$

$$\frac{dN}{N} = k\,dt$$

when $t = 3$ hrs, then $N = 400$.

when, $t = 10$ hrs. then $N = 2000$

$$\Rightarrow \int_{400}^{2000} \frac{dN}{N} = k \int_{3}^{10} dt$$

$$\Rightarrow [\ln N]_{400}^{2000} = k[t]_3^{10}$$

$$\Rightarrow \ln 2000 - \ln 400 = k(10-3)$$

$$\Rightarrow \ln \frac{2000}{400} = 7k$$

$$\Rightarrow k = \frac{\ln 5}{7}$$

Again, initially

$t = 0, \quad N = N_0$

$t = 3, \quad N = 400$

$$\int_{N_0}^{400} \frac{dN}{N} = k \int_{t=0}^{3} dt$$

$$\Rightarrow [\ln N]_{N_0}^{400} = k[t]_0^3$$

$$\Rightarrow \ln 400 - \ln N_0 = 3k$$

$$\Rightarrow \ln \frac{400}{N_0} = \ln e^{3k}$$

$$\Rightarrow \frac{400}{N_0} = e^{3k}$$

$$\Rightarrow \frac{N_0}{400} = e^{-3k}$$

$$\therefore N_0 = 400 e^{-3k}$$

$$= 400 e^{-\frac{3\ln 5}{7}}$$

Differential Equations (Lec 27)

21/03/2022

$\cancel{\boxed{\frac{dy}{dx} = x+y}}$ Pichard's Method

$$\frac{dy}{dx} = f(x, y). \quad y(x_0) = y_0$$

$$x \longrightarrow x_0$$

$$y(x) = y$$

$$y_n = y_0 - \int_{x_0}^{x} f(x, y_{n-1})dx$$

$$y_1 = y_0 - \int_{x_0}^{x} f(x_1, y_0)dx$$

$$y_2 = y_0 - \int_{x_0}^{x} f(x_1, y_1)dx \quad \Bigg| \quad y_3 = y_0 - \int_{x_0}^{x} f(x_1, y_2)dx$$

$\frac{dy}{dx} = x+y; \quad y(0) = -1$

$$x_0 = 0, \quad y_0 = -1$$

$$f(x, y) = x+y$$

$$y_1 = y_0 + \int_{x_0}^{x} f(x, y_0)\,dx$$

$$f(x, y_0) = x + y_0 = x - 1$$

$$y_n = y_0 + \int_{x_0}^{x} f(x, y_{n-1})\,dx$$

$$= -1 + \int_0^x (x-1)\,dx$$

$$= -1 + \left[\frac{x^2}{2} - x\right]_0^x$$

$$= -1 + \frac{x^2}{2} - x$$

$$\therefore y_1 = \frac{x^2}{2} - x - 1$$

2nd approximation,

$$y_2 = y_0 + \int_{x_0}^{x} f(x, y_1)\,dx$$

$y_0 = -1,\ x_0 = 0$

$f(x, y_1) = x + y_1$

$= x + \frac{x^2}{2} - x - 1$

$$= -1 + \int_0^x \left(x + \frac{x^2}{2} - x - 1\right)dx$$

$$= -1 + \left[\frac{x^3}{6} - x\right]_0^x$$

$$= -1 + \frac{x^3}{6} - x$$

$$\therefore y_2 = -1 - x + \frac{x^3}{6}$$

3rd approximation,

$$y_3 = y_0 + \int_{x_0}^{x} f(x, y_2)\, dx$$

$$= -1 + \int_0^x \left(-1 + \frac{x^3}{6}\right) dx$$

$y_0 = -1, x_0 = 0$

$f(x, y_2) = x + y_2$

$= x - 1 - x + \frac{x^3}{6}$

$= -1 + \frac{x^3}{6}$

$$= -1 + \left[-x + \frac{x^4}{24}\right]_0^x$$

$$\therefore y_3 = -1 - x + \frac{x^4}{24}$$

Pg: 364

$$\frac{dy}{dx} = e^x + y^2, \quad y(0) = 0 \longrightarrow y(x_1) = y_0$$

1st approximation,

$$y_1 = y_0 + \int_{x_0}^{x} f(x, y_0)\, dx$$

$x_0 = 0, \ y_0 = 0$

$f(x, y) = e^x + y^2$

$f(x, y_1) = e^x + y_0^2$

$= e^x$

$$= 0 + \int_{x_0}^{x} f(x, y_0)\, dx$$

$$= 0 + \int_{0}^{x} e^x\, dx$$

$$= [e^x]_0^x$$

$$= e^x - e^0$$

$$\therefore y_1 = e^x - 1$$

2nd approximation,

$$y_2 = y_0 + \int_{x_0}^{x} f(x, y_1)\, dx$$

$$f(x, y_1) = e^x + y_1^2$$
$$= e^x + (e^x - 1)^2$$
$$= e^{2x} - e^x + 1$$

$$= 0 + \int_{0}^{x} (e^{2x} - e^x + 1)\, dx$$

$$= \left[\frac{e^{2x}}{2} - e^x + x\right]_0^x$$

$$= \frac{e^{2x}}{2} - e^{x} + x - \frac{e^{0}}{2} + e - 0$$

$$\therefore y_2 = \frac{1}{2} e^{2x} - e^{x} + x + \frac{1}{2}$$

3rd approximation,

$$y_3 = y_0 + \int_{x_0}^{x} f(x, y_2)\, dx$$

$$= 0 + \int_0^x \left(\frac{1}{4} e^{4x} - e^{3x} + \frac{3}{2} e^{2x} + xe^{2x} - 2x e^{x} + x^2 + x + \frac{1}{4}\right) dx$$

$$f(x, y_2) = e^{x} + y_2^{2}$$

$$= e^{x} + \left(\frac{1}{2} e^{2x} - e^{x} + x + \frac{1}{2}\right)^2$$

$$= e^{x} + \left(\frac{1}{2} e^{2x} - e^{x}\right)^2 + 2\left(\frac{1}{2} e^{2x} - e^{x}\right)\left(x + \frac{1}{2}\right) + \left(x + \frac{1}{2}\right)^2$$

$$= e^{x} + \frac{1}{4} e^{4x} - e^{2x}.e^{x} + e^{2x} + 2\left(\frac{1}{2} x e^{2x} + \frac{1}{4} e^{2x} - x e^{x} - \frac{1}{2} e^{x}\right)$$

$$= \cancel{e^{x} + \frac{1}{4} e^{4x} - e^{3x} + e^{2x}} + x^2 + x + \frac{1}{4}$$

$$= \left[\frac{1}{16} e^{4x} \cancel{\frac{e^{3x}}{3}} - \frac{e^{3x}}{3} + \frac{3}{4} e^{2x} + \frac{x}{2} e^{2x} - \frac{1}{4} e^{2x} - 2x e^{x} + 2e^{x} + \frac{x^3}{3} + \frac{x^2}{2} + \frac{x}{4}\right]_0^x$$

$$= \frac{1}{16} e^{4x} - \frac{1}{3} e^{3x}$$

$$+ \frac{3}{4} e^{2x} + \frac{x}{2} e^{2x}$$

$$- \frac{1}{4} e^{2x} - 2xe^{x} + 2e^{x}$$

$$+ \frac{x^3}{3} + \frac{x^2}{2} + \frac{x}{4}$$

$$- \frac{1}{16} + \frac{1}{3} - \frac{3}{4}$$

$$- 0 + \frac{1}{4} + 0$$

$$- 2 - 0 - 0 - 0)$$

$$= e^{x} + \frac{1}{4} e^{4x} - e^{3x} + e^{2x} + xe^{2x}$$

$$+ \frac{1}{2} e^{2x} - 2xe^{x} - e^{x} + x^2 + x + \frac{1}{4}$$

$$= \frac{1}{4} e^{4x} - e^{3x} + \frac{3}{2} e^{2x} + xe^{2x}$$

$$- 2xe^{x} + x^2 + x + \frac{1}{4}$$

$$\int xe^{2x}\,dx = \frac{x}{2} e^{2x} - \int \frac{1}{2} e^{2x}\,dx$$

$$= \frac{x}{2} e^{2x} - \frac{1}{4} e^{2x}$$

$$\boxed{\int uv\,dx = u \int v\,dx - \int \left\{ \frac{du}{dx} \int v\,dx \right\} dx}$$

$$\int xe^{x} = xe^{x} - \int e^{x}\,dx$$

$$= xe^{x} - e^{x}$$

$$\therefore y_3 = \frac{1}{16} e^{4x} - \frac{1}{3} e^{3x} + \frac{1}{2} e^{2x} + \frac{x}{2} e^{2x}$$

$$- 2x e^{x} + 2e^{x} + \frac{x^3}{3} + \frac{x^2}{2} + \frac{x}{4}$$

$$- \frac{3 - 16 + 36 - 12 + 96}{48}$$

Differential Equations (Lec 28) 24/03/2022

Initial value problem:

$$\frac{dy}{dx} = f(x,y) \qquad \boxed{y(x_0) = y_0} \longrightarrow \text{Initial value}$$

$$x = x_0$$
$$y = y_0$$

$\frac{dy}{dx} + y = 0$

$$y(\cancel{x}\underbrace{0}_{x_0}) = \underbrace{1}_{y_0}$$

$$\Rightarrow \frac{dy}{y} + dx = 0$$

$$\Rightarrow \int \frac{dy}{y} + \int dx = \cancel{a} \log c$$

$$\Rightarrow \ln y + x = \ln c$$

$$\Rightarrow \ln y = \ln c + \ln e^{-x}$$

$$\Rightarrow \cancel{to} \ln y = \ln c e^{-x} \qquad \Rightarrow y = ce^{-x}$$

$$1 = Ce^{-0}$$

$$\therefore \; C = 1$$

$$\therefore \; y = e^{-x}$$

$$2x^3 y' = y(y^2 + 3x^2) \quad ; \quad y(1) = 1$$

$$\Rightarrow 2x^3 \frac{dy}{dx} = y(y^2 + 3x^2)$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^3 + 3x^2 y}{2x^3}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^3}{2x^3} + \frac{3}{2} \cdot \left(\frac{y}{x}\right)$$

$$\Rightarrow \frac{dy}{dx} - \frac{3}{2} \cdot \frac{y}{x} = \frac{y^3}{2x^3}$$

$$\Rightarrow y^{-3} \frac{dy}{dx} - \frac{3}{2} \cdot \frac{y^{-2}}{x} = \frac{1}{2x^3}$$

Let $y^{-2} = t$

$$\Rightarrow -2y^{-3}\frac{dy}{dx} = \frac{dt}{dx}$$

$$\Rightarrow y^{-3}\frac{dy}{dx} = -\frac{1}{2}\frac{dt}{dx}$$

$$\Rightarrow -\frac{1}{2}\frac{dt}{dx} - \frac{3}{2}\cdot\frac{t}{x} = \frac{1}{2x^3}$$

$$\Rightarrow \frac{dt}{dx} + \frac{3}{x}\cdot t = \frac{-1}{x^3}$$ [which is linear eqn.] ------ (i)

$\therefore$ I.F $= e^{\int \frac{3}{x}dx}$

$= e^{3\ln x}$

$= e^{\ln x^3}$

$= x^3$

$\frac{dy}{dx} + Py = Q$

I.F $= e^{\int P dx}$

Multiplying I.F by (i)

$$(i) \Rightarrow x^3\frac{dt}{dx} + \frac{3t}{x}x^3 = -\frac{x^3}{x^3}$$

$\Rightarrow x^3 dt + 3x^2 t\, dx = -dx$ → $\frac{d}{dx}(uv)$

$\Rightarrow d(t x^3) = -dx$

Integrating, we get

$y.I.F = \int Q.I.F$

$t. x^3 = -x + c$

$\Rightarrow y^{-2}. x^3 + x = c$

$\Rightarrow x^3 + xy^2 = cy^2$ $\Rightarrow x^3 + xy^2 = 2y^2$.

$x = 1, \; y = 1$

$\Rightarrow 1^3 + 1.1^2 = c.1^2$

$\therefore c = 2$

$2\frac{d^2y}{dx^2} - \frac{dy}{dx} - 3y = 0$ ---(i) $y(0) = 2$, $y'(0) = -\frac{7}{2}$

Let $y = e^{mx}$ be the trial solution of (i)

$\therefore \cancel{(i) \Rightarrow}$ ~~A.E is $2m^2 - m - 3$~~

$(2m^2 - m - 3)e^{mx} = 0$

$\therefore$ A.E is $2m^2 - m - 3 = 0$

$\Rightarrow 2m^2 - 3m + 2m - 3 = 0$

$\Rightarrow m(2m-3) + 1(2m-3) = 0$

$\Rightarrow (m+1)(2m-3) = 0$

$\therefore \quad m = -1, \frac{3}{2}$

The G.S is

$y = c_1 e^{-x} + c_2 e^{3/2 x}$ ---- (ii)

$y'(x) = -c_1 e^{-x} + \frac{3}{2} c_2 e^{\frac{3}{2}x}$ ---- (iii)

Putting $x=0$, $y=2$ in (ii)

(ii) $\Rightarrow 2 = C_1 + C_2$ ---- (iv)

Putting $x=0$, $y=-\frac{7}{2}$ in (iii)

$$-\frac{7}{2} = -C_1 + \frac{3}{2} C_2 \quad \text{(v)}$$

Adding (iv) & (v) we get.

$$\boxed{2 - \frac{7}{2} = 0 + \left(1 + \frac{3}{2}\right) C_2}$$

$$C_1 + C_2 = 2$$

$$C_1 + \frac{3}{2} C_2 = -\frac{7}{2}$$

$$(+) \quad \left(1 + \frac{3}{2}\right) C_2 = 2 - \frac{7}{2}$$

$$\Rightarrow \frac{5}{2} C_2 = -\frac{3}{2}$$

$$\therefore C_2 = -\frac{3}{5}$$

$$(iv) \Rightarrow C_1 = 2 - C_2$$

$$= 2 + \frac{3}{5}$$

$$= \frac{13}{5}$$

$$(ii) \Rightarrow \boxed{y = \frac{13}{5} e^{-x} - \frac{3}{5} e^{3/2 x}}$$

(Ans)

DE (Lec 29) 28/03/2022

Cla.... Equation:

$$y = px + f(p) \qquad \frac{dy}{dx} = p$$

$$y = cx + f(c)$$

$$y = px + f(p) \longrightarrow y = cx + f(c)$$

D. w. r to

$$\frac{dy}{dx} = p + x\frac{dp}{dx} + f'(p)\cdot\frac{dp}{dx}$$

$$\Rightarrow p = p + x\frac{dp}{dx} + f'(p)\frac{dp}{dx}$$

$$\Rightarrow \{x + f'(p)\}\frac{dp}{dx} = 0$$

$$\therefore \frac{dp}{dx} = 0, \quad x + f'(p) = 0$$

$$\frac{dp}{dx} = 0$$

$$\Rightarrow dp = 0$$

Integrating

$$\int dp = 0$$

$$\Rightarrow p = c$$

$y = px - p - p^2 \quad \ldots\ldots (i)$

D. w. r. to

$\therefore \frac{dy}{dx} = p + x\frac{dp}{dx} + \frac{dp}{dx} \cdot 2p\frac{dp}{dx}$

$\Rightarrow p = p + x\frac{dp}{dx} + \frac{dp}{dx} \cdot 2p\frac{dp}{dx}$

$\Rightarrow (1 + x - 2p)\frac{dp}{dx} = 0$

$\Rightarrow \therefore 1 + x - 2p = 0 \,; \ldots (ii) \quad \frac{dp}{dx} = 0 \quad \ldots\ldots (iii)$

Putting $p = c$ in (i), we get

$\Rightarrow y = cx + c - c^2$

which is general solution.

From (ii), $2P = 1 + x$

$\therefore P = \frac{1+x}{2}$

Putting $p = \frac{x+1}{2}$ in (i)

$$(i) \Rightarrow y = \frac{x+1}{2} \cdot x + \frac{x+1}{2} - \left(\frac{x+1}{2}\right)^2$$

$$\Rightarrow y = \frac{1}{4}\left(2x + 2 + 2x + 2 - x^2 - 2x - 1\right)$$

$$\Rightarrow 4y = x^2 + 2x + 1$$

$$\Rightarrow 4y = (x+1)^2$$

which is singular solⁿ.

$4y = (x+1)^2$
[Parabola]

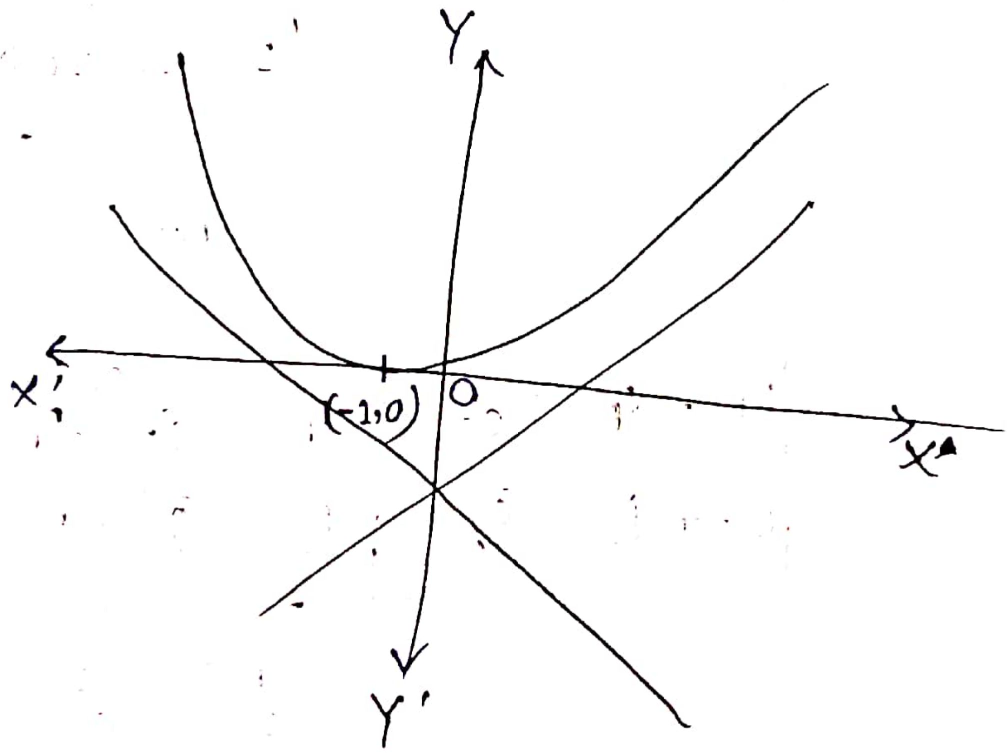
$x^2 = 4ay$

$y^2 = 4ax$

$(x+1)^2 = 4y$

$X^2 = 4.1.Y$

$x+1$
$x-(-1)$



Differential Equation (Lec 30)

Pg: 132

52// $e^{3x}(p-1) + p^3 e^{2y} = 0$ .... (i) $\quad p = \frac{dy}{dx}$

$\Rightarrow p - 1 = \frac{-p^3 e^{2y}}{e^{3x}}$

$\Rightarrow 1 - p = p^3 e^{2y} \cdot e^{-3x}$

$\Rightarrow 1 - p = p^3 e^{2y-3x}$

$\Rightarrow e^y(1-p) = p^3 e^{2y-3x} \cdot e^y$

$\Rightarrow e^y(1-p) = p^3 e^{3y-3x}$

$\Rightarrow e^y(1-p) = p^3 (e^{y-x})^3$

$\Rightarrow e^y(1-p) = (p \cdot e^{y-x})^3$ .... (ii)

~~Putting the va~~

Let, $e^x = v$

$\therefore e^x dx = dv$

$e^y = v$

$\Rightarrow e^y dy = dv$

$\therefore \frac{e^y}{e^x} \cdot \frac{dy}{dx} = \frac{dv}{du}$

$\Rightarrow e^{y-x} \cdot \frac{dy}{dx} = \frac{dv}{du}$

$\therefore e^{y-x} \cdot p = \frac{dv}{du}$;

$\frac{dv}{du} = q$

$\Rightarrow \frac{u}{v} \cdot p = q$

$\therefore p = \frac{u}{v} q$ .... (iii) from (i) and (iii)

$$u^3(p-1) + p^3 v^2 = 0$$

$$\Rightarrow u^3\left(\frac{u}{v}q - 1\right) + \frac{u^3}{v^3} q^3 v^2 = 0$$

$$\Rightarrow \frac{u}{v}q - 1 + \frac{1}{v} q^3 = 0$$

$$\Rightarrow uq - v + q^3 = 0$$

$$\Rightarrow v = uq + q^3$$

It's solution,

$$v = uc + c^3$$

$$\therefore e^y = uc + c^3 \text{ (Ans.)}$$

Cauchy Weller's

$$\frac{x^3 d^3y}{dx^3} + 3x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0 \quad \ldots (i)$$

Let, $x = e^z$

$\therefore z = \ln x$

$\therefore \frac{dy}{dz} = D_1 y$

$D_1 = \frac{d}{dz}$

$x = e^z$

$\therefore z = \ln x$

$\frac{dz}{dx} = \frac{1}{x}$

$\frac{dy}{dx} = \frac{dy}{dz}\left(\frac{dz}{dx}\right)$

$\Rightarrow \frac{dy}{dx} = \frac{1}{x} \frac{dy}{dz}$

$\Rightarrow x \cdot \frac{dy}{dx} = \frac{dy}{dz}$

$\Rightarrow x \cdot Dy = D_1 y$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dx}\left(\frac{1}{x} \cdot \frac{dy}{dz}\right)$$

$$= \frac{1}{x} \cdot \frac{d}{dx}\left(\frac{dy}{dz}\right) + \left(\frac{-1}{x^2}\right) \cdot \frac{dy}{dz}$$

$$= \frac{1}{x} \cdot \frac{d}{dz}\left(\frac{dy}{dz}\right)\frac{dz}{dx} - \frac{1}{x^2} \frac{dy}{dz}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{1}{x} \cdot \frac{d^2y}{dz^2} \cdot \frac{1}{x} - \frac{1}{x^2} \cdot \frac{dy}{dz}$$

$$\Rightarrow x^2 \frac{d^2y}{dx^2} = \frac{d^2y}{dz^2} - \frac{dy}{dz}$$

$$\Rightarrow x^2 \cdot D^2 y = D_1^2 y - D_1 y = D_1(D_1 - 1)y$$

$$\Rightarrow x \cdot \frac{dy}{dx} = xDy = D_1 y$$

$$\Rightarrow x^2 \cdot \frac{d^2y}{dx^2} = x^2 D^2 y = D_1 (D_1 - 1) y$$

$$\Rightarrow x^3 \frac{d^3y}{dx^3} = x^3 D^3 y = D_1 (D_1 - 1)(D_1 - 2) y$$

from eq (i),

$$\Rightarrow D_1 (D_1 - 1)(D_1 - 2) y + 3D_1 (D_1 - 1) y - 2D_1 y + 2y = 0$$

$$\Rightarrow D_1 (D_1^2 - 3D_1 + 2) y + 3D_1^2 y - 3D_1 y - 3D_1 y - 2D_1 y + 2y = 0$$

$$\Rightarrow (D_1^3 - 3D_1^2 + 2D_1 + 3D_1^2 - 3D_1^2 - 3D_1 - 2D_1 + 2) y = 0$$

$$\Rightarrow \{(D_1)^3 - 3D_1 + 2\} y = 0 \quad \text{------ (ii)}$$

Let, $y = e^{mz}$ be the trial solution of (i)

$\therefore$ A.E is.

$$m^3 - 3m + 2 = 0 \quad , \quad e^{mz} \neq 0$$

$$\Rightarrow m^2(m-1)+m(m-1)-2(m-1)=0$$

$$\Rightarrow (m-1)(m^2+m-2)=0$$

$$\Rightarrow (m-1)\{m(m-1)+2(m-1)\}=0$$

$$\Rightarrow (m-1)(m-1)(m+2)=0$$

$$\therefore m=1,1,-2.$$

$\therefore$ Sol^n: $y=(c_1+c_2 z)e^z+c_3 e^{-2z}$

$$=(c_1+c_2\ln x)x+c_3\frac{1}{x^2}$$

(Ans).