offline : lec-1 Date: 10-03-22

Measures of Dispersion: Dispersion (বিচ্যুতি) :

* পার্থক্য Measures of Dispersion & Measures of tendency এর মাঝে:

(i) The Range:

(ii) The Quartile Deviation

(iii) The Mean Deviation

(iv) The Standard Deviation

Range: Range is the simplest method of studying variation. It defined as the difference between the value of the smallest observation and the value of the largest observation included in the distribution.

Symbolically, Range $= L - S$

where, S = Smallest observation

L = Largest Observation.

প্র) প্রত্যেকটা Measures এর Merits, Demerits & uses পড়তে হবে। (Math করতে হবে)।

Quartile Deviation: A measure similar to the above measure is the inter-quartile range (Q). It is the difference between the third quartile (Q_3) and the first quartile (Q_1).

Thus,

$$Q = Q_3 - Q_1$$

Inter Quart-tile range: ~~It is the di~~ It is the difference between Q_3 & Q_1. The inter-quartile range is frequently reduced to the semi quartile range interquartile range, also known as the quartile deviation (Q_D), by

$$Q_D = \frac{Q_3 - Q_1}{2}$$

Box and Whisker plot: A box plot is a graphical display based on quartiles, that helps us to picture a set of data. To ~~constant~~ construct a box plot we need only five statistics: the minimum value, Q_1, the median, Q_3 and the maximum value. These quantities are known as the five number summary of distribution.

* What is five number summary?

Let,

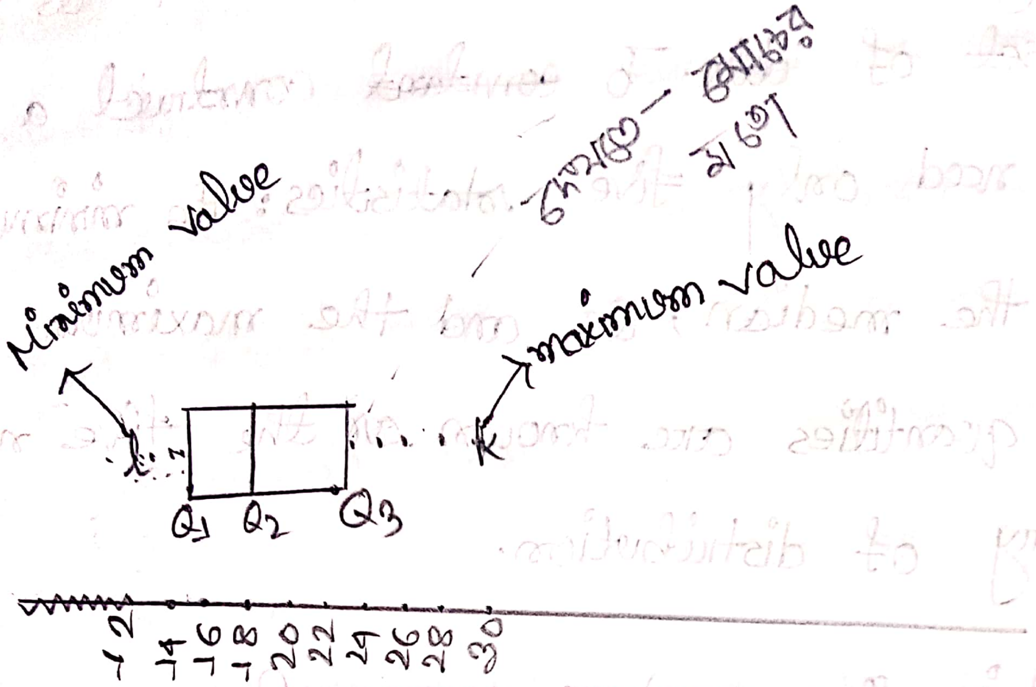
minimum value = 13 minutes

Q_1 = 15 minutes

Median, Q_2 = 18 minutes

Q_3 = 22 minutes

and maximum value = 30 minutes



The box extends from Q_1 to Q_3 representing the inter-quartile range and so encloses the 50% of the values. The whiskers are the lines that extend from the box to the heighest and lowest values and thus illustrate the range. A line across the box indicates the median (Q_2). The edges of the box are known as hinges which are approximated by Q_1 and Q_3 values. The points lying beyond 1.5 times the inter quartile ranges are known as outliers.

Lecture: 01

Online:

Importance of Statistics in Engineering: For any structure to be built properly and safely, the plan has to be ~~cole~~ accurate down to the last detail. this can only be achieved with perfect math.

Therefore without statistics and math in engineering, it would be almost impossible to build to the same standard and to the same time-frame. Everything is worked out by statistics from the quality control to the number of items needed to fulfill the project it is really at the heart of everything.

Engineering is a very logical process and statistics help back any decision up and so risks are never taken due to it being built on facts and solid maths.

Statistics: Statistics is concerned with scientific methods for collecting, organizing, summerizing presenting and analyzing sample data as well as drawing valid conclusions about population characteristics and making reasonable decesions on the basis of such analysis.

According to Fisher, the science of statistics is essentially a branch of applied mathmetics and may regarded as mathmetics applied to observational data.

Scope of Statistics: The scope of statistics is so vast and ever increasing that not only it is difficult to define but also onvoise to do so. It is a tool of all sciences indisperable to research and intelligent judgement and has become a recognized discipline in its own right. There is hardly any field whether it be ~~trade~~, industry or commerce, economics, biology, botany, astronomy, physics chemistry, education, medecine, sociology, phychology or technology where statistical tools are not applicable. The application of statistics are so numerous that is it is often remarked "Statistics is what statisticians do." A few fields in which statics is applied are given below:

Final এ [illegible] ব্যাপারে
লিখতে হবে

- Statistics and State
- Statistics in Business and Management.
- Statistics and Economics
- Statistics and physical sciences.
- Statistics and Natural sciences.
- Statistics and Research*** (details পড়তে হবে)
- Statistics in agriculture*** (details পড়তে হবে).
- Statististics in socio-economic study.
- Statististics in environment.
- Statistics in psychology and education****
- Statistics in production industry.
- Statistics in astronomy etc.

Lecture: 02 (online)

সমষ্টি

Population: An aggregate of all individuals or items (actual / possible) defined on some common characteristics is called a population. Conclusion draw করি Population-এর

Example: 2nd year honours students of statistics (session: 2019-20) of JnU Constitute a population.

Here, the common characteristics are:

(i) Students of JnU

(ii) Students of first year in statistics

(iii) Students of the session 2019-20

It is important word.

Sample: A representative small part of population is called a sample. For example, A group of students representing the first year statistics is called sample.

[illegible] - Sample নিবো।

Difference between Population & Sample:

Population	Sample
1. Defination	1. Defination
2. It is always larger than sample.	2. The size of the sample is always less than the total size of the population.

Population	Sample
3. Each and every unit of the group is included here	3. Only a handful of units of population is included here.
4. characteristic – Parameter	4. Characteristic – statistic.
5. Identifying the characteristic	5. Making inference about population.

Random sample: When a sample comes from a random experiment, then the sample is a random sample (24 min)

Variable: A variable is a measurable quantity, which can vary within its domain.

For example, family size is a variable, because it is a measureable quantity within its domain. All possible values of a variable will constitutes its domain. For the variable family size it if the lowest value in the measurements is considered to be 1 and the highest value is 30, then the domain of family size is obviously (1–30).

Random Variable: If each of the values of a variable is asociated with probability then it is called a random variable.

For example, the variable family size can be conseidered as a random variable as it takes each of the values with certain probability.

Parameter: A constant which is a function of population values, can characterise the variable of the underlying population to some extent and is usually unknown, is called a parameter.

Statistic: Any function of sample values which is an estimate of the parameter and which is a known value is called a statistic.

Difference between Parameter and Statistic:

Parameter	Statistic
Defination	Defination
2. Parameter refers to a measure which describes population.	2. Statistic is a measure which describes a fraction of population (sample)
3. It is fixed and unknown	3. It is variable and known.
4. μ = Population mean (statistical notation)	4. $\bar{x}$ = Sample mean.

Lecture: 13-03-22 (02)

Mean Deviation: The mean deviation is an average of absolute deviations of individual observations from the central value of a series.

If $x_1, x_2, \ldots x_n$ from a sample of observation the formula for completing the average or mean deviation from arithmatic mean is —

$$MD(\bar{x}) = \frac{\sum_{i=1}^{n} |x_i - \bar{x}|}{n} = \frac{\sum_{i=1}^{n} |d_i|}{n}$$

where, $d_i = x_i - \bar{x}$, which stands for the deviation of the individual observations from the mean.

যদি Median এর জন্য,

$$MD(Me) = \frac{\sum_{i=1}^{n} |x_i - Me|}{n}$$

যদি Mode এর জন্য

$$MD(Mo) = \sum_{i=1}^{n} \left|\frac{x_i - Mo}{n}\right|$$

If a grouped data frequency distribution is constructed as is usually done with large samples, the average deviation is,

$$MD(\bar{x}) = \frac{\sum_{i=1}^{k} f_i |x_i - \bar{x}|}{n}$$

where,

$MD(\bar{x})$ = average deviation about mean.

k = number of classes.

x_i = mid-point of the ith class.

f_i = frequency of the ith class

$n = \sum_{i=1}^{k} f_i$

Date: 15-03-22 (03)

Standrad deviation: The arithmetic mean of the squares of the deviation of the given observation from their arithmetic mean is known as variance. The positive square root of variance is the standard Deviation.

If $x_1, x_2 \cdots x_n$ be n observation of a variable then standrad deviation is defined by-

$$S = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2}$$

In case of frequency distribution or grouped data —

$$S = \sqrt{\frac{1}{n}\sum_{i=1}^{k} f_i (x_i-\bar{x})^2}$$

where,

$n = \sum_{i=1}^{k} f_i$ and $\bar{x}$ is the arithmetic mean.

যেখানে unit ভিন্ন থাকে- এখানে cv use করা হয়

Co-efficient of variation: The ratio of the standrad deviation of the arithmatic mean expressed a percent. In term of a formula for a sample:

$$CV = \frac{s \text{ (Standrad deviation)}}{\bar{x} \text{ (arithmatic mean)}} \times 100$$

It is very useful measure when:

(i) The data are in different units (much as dollars and days absent)

(ii) The data are in the same unit but the means are far apart (such as the incomes of the top executive and the incomes of the unskilled employee)

Class: 20-03-22. 2nd Mid

Moments: A set of descriptive measures, which can provide a unique characteristics of a distribution and ~~the~~ hence can determine the distribution uniq is called moments.

If $x_1, x_2 \dots x_n$ be n observation of a variante then the r th raw moment is defined by—

$$\mu_r' = \frac{1}{n}\sum_{i=1}^{n}(x_i - A)^r$$ where

A is any arbitory value.

The r the central moment is defined by

$$\mu_r = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^r$$

where $\bar{x}$ is Arithmatic Mean.

If $x_1, x_2 \dots x_k$ occur with frequencies $f_1, f_2 \dots f_k$ ~~Frepre~~ respectively the r th raw moment is —

$$\mu_r' = \frac{1}{n}\sum_{i=1}^{k} f_i(x_i - A)^r \text{ when, } n = \sum_{i=1}^{k} f_i$$

and the rth central moment is defined μ_1', μ_2'

by,

$$\mu_r = \frac{1}{n}\sum_{i=1}^{k} f_i(x_i - \bar{x})^r \text{ when, } n = \sum_{i=1}^{k} f_i.$$

Relation Between raw moments and ~~constant~~

Central moments:

We know,

rth raw moment is —

$$\mu_r' = \frac{1}{n}\sum_{i=1}^{n}(x_i - A)^r$$

putting $r = 1, 2, 3, 4 \ldots$

$$\mu_1' = \frac{1}{n}\sum_{i=1}^{n}(x_i - A)$$

$$\mu_2' = \frac{1}{n}\sum_{i=1}^{n}(x_i - A)^2$$

$$\mu_3' = \frac{1}{n}\sum_{i=1}^{n}(x_i - A)^3.$$

$$\mu_4' = \frac{1}{n}\sum_{i=1}^{n}(x_i - A)^4$$

rth central moment is —

$$\mu_r = \frac{1}{n} \Sigma (x_i - \bar{x})^r .$$

putting $r = 1, 2, 3, 4 \ldots$ etc.

$$\mu_1 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})$$

$$\mu_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

$$\mu_3 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^3$$

$$\mu_4 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^4$$

$$\mu_1' = \frac{1}{n} \sum_{i=1}^{n} (x_i - A)$$

$$= \frac{\sum_{i=1}^{n} x_i}{n} - \frac{\cancel{n} A}{\cancel{n}}$$

$$= \bar{x} - A$$

$(a-b)^3 =$

First central value is always zero:

$$\mu_1 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})$$

$$= \frac{\sum x_i}{n} - \frac{n\bar{x}}{n}$$

$$= \bar{x} - \bar{x}$$

$$= 0$$

We know that,

$$\mu_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$= \frac{1}{n}\sum_{i=1}^{n}\{(x_i - A) - (\bar{x} - A)\}^2$$

$$= \frac{1}{n}\sum_{i=1}^{n}\{(x_i - A)^2 - 2(x_i - A)(\bar{x} - A) + (\bar{x} - A)^2\}$$

$$= \frac{1}{n}\sum_{i=1}^{n}(x_i - A)^2 - 2(\bar{x} - A)\frac{1}{n}\sum_{i=1}^{n}(x_i - A)$$

$$+ \frac{1}{n}\cdot n(\bar{x} - A)^2$$

$$= \mu_2' - 2\mu_1' \cdot \mu_1' + \mu_1'^2$$

$$= \mu_2' - 2\mu_1'^2 + \mu_1'^2$$

$$\therefore \mu_2 = \mu_2' - \mu_1'^2$$

Higher order formula

μ_3, μ_4, μ_5 এর মান নির্ণয় করতে হবে।

We know that,

$$\mu_3 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^3$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left\{(x_i - A) - (\bar{x} - A)\right\}^3$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left\{(x_i - A)^3 - (\bar{x} - A)^3 - 3(x_i - A)(\bar{x} - A)(x_i - \cancel{A} - \bar{x} + \cancel{A})\right\}$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left\{(x_i - A)^3 - (\bar{x} - A)^3 - 3(x_i - A)(\bar{x} - A)(x_i - \bar{x})\right\}$$

$$= \mu_3' - \mu_1'^3 - 3\mu_1' \cdot \mu_1'$$

We know that,

$$\mu_3 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^3$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left\{(x_i - A) - (\bar{x} - A)\right\}^3$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left\{(x_i - A)^3 - 3.(x_i - A)^2(\bar{x} - A) + 3(x_i - A)(\bar{x} - A)^2 - (\bar{x} - A)^3\right\}$$

$$= \mu_3' - 3.\mu_2'.\mu_1' + 3\mu_1'\mu_1'^2 - \mu_1'^3$$

$$= \mu_3' - 3\mu_2'.\mu_1' + 3\mu_1'^3 - \mu_1'^3$$

$$= \boxed{\mu_3' - 3\mu_2'.\mu_1' + 2\mu_1'^3}$$

We know that,

$$\mu_4 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^4$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left\{(x_i - A) - (\bar{x} - A)\right\}^4$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left[\left\{(x_i - A) - (\bar{x} - A)\right\}^2\right]^2$$

$$= \cancel{\left(\mu_2' - \mu_1'^2\right)^2 = \mu_2^2} \quad \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$$

Class: 22-03-22

V.V.I

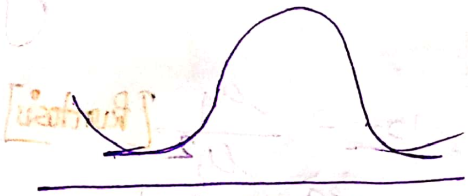
Shape Characteristics of a Distribution: Skewness

Skewness (বক্রতা): Skewness means "lack of symmetry" i.e departure from symmetry of distribution. The skewness may be either positive or negative. When the skewness is positive, the associated distribution is called positively swked distribution. When the skewness is negative, the associate distribution is called negatively swked distribution. Absence of swkeness makes the distribution symmetrical.

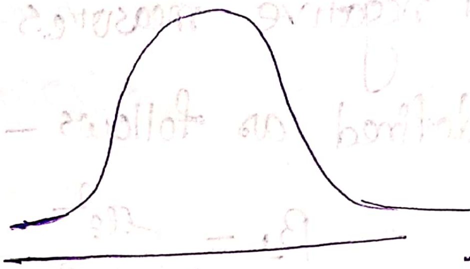
(a) Symetrical Distribution: Height, weight, examination score

(b) Positively Skewed Distributions: Family size, female age at first marriage, wages of the employess.

(c) Negatively Skewed Distribution: Reaction times of an experiment daily maximum temparature.

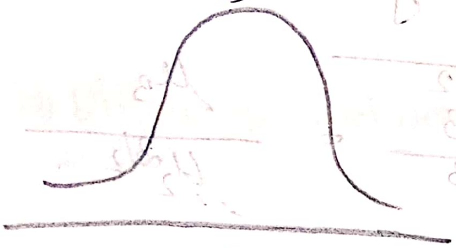


(a)

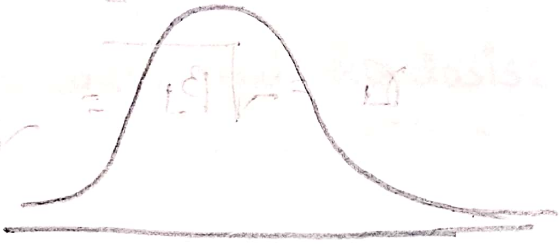


(b)

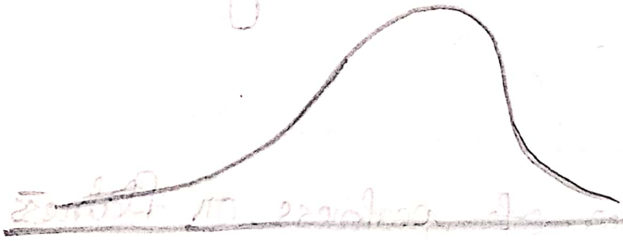
(c)



(a)



(b)



(c)

Measures of ~~the~~ skewness: Pearsons's Coefficient of

$$\text{Skewness} = \frac{\text{mean} - \text{mode}}{\text{standard deviation}}$$

If mean > mode, the skew is positive

If mean < mode, the skew is negative

If mean = mode, the skew is zero in which case the distribution is symmetrical.

A negative measures of skewness denoted by β_1, is defined as follows —

$$\beta_2 = \frac{\mu_4}{\mu_2^2} \text{ [kurtosis]}$$

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$ [generally এটাই use করব]

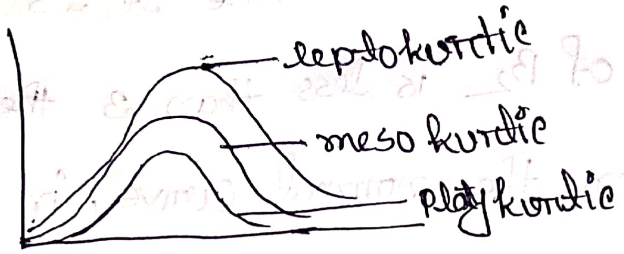
$$\gamma_1 = \sqrt{\beta_1} = \sqrt{\frac{\mu_3^2}{\mu_2^3}} = \frac{\mu_3}{\mu_2^{3/2}}$$

γ_1 measures the swkeness directly as compered to β_1.

Class: 24.03.22.

kurtosis: The degree of peakness or flatness of a distribution relative to a normal distribution is called kurtosis.

A curtosis having relatively higher peak than ~~the~~ normal curve is known as leptokurtic. If the curve is more flat topped than than the normal curve is called platykurtic. A normal curve itself is called mesokurtic



■ Difference between skewness and kurtosis:

■ Measures of kurtosis: The most important measure of kurtosis based in second and forth moments is β_2 defined as

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

For normal distribution $\beta_2 = 3$. When the value of β_2 is greater than three (3). the curve is more peaked than normal curve in which case it is lepokurtic.

When the value of β_2 is less than 3 the curve is less peaked than the normal curve, in which case it is playkurtic.

Example: for a distribution the four central moments were found to be as follow:

$\mu_1 = 0$, $\mu_2 = 2.5$, $\mu_3 = 0.7$ and $\mu_4 = 18.7$.

Find β_1 and β_2 hence comment on the nature of the distribution

$$\beta_1 = \frac{0.7}{0.7^{3/2}}$$

$$= \cancel{1.195}\, 0.031$$

$$\beta_2 = \frac{18.7}{(2.5)^2}$$

$$= 2.96 \simeq 3$$

$\beta_1 > 0$ & $\beta_2 = 3$.

So, the distribution is positively swkesed and it is mesocardic.

Date: 27.03.22 (Measures of Association)

Correlation and Regression থেকে 1 set ques আসবে (Math + theory)

Co-relation:

Quantitive এর জন্য হবে

Correlation coefficient is a quantitative measure of the direction and strength of linear relationship between two numerically quantitative measured variables.

$$r = \frac{\overset{\text{sum product}}{SP}(x,y)}{\sqrt{SS(x)\cdot SS(y)}}$$

SS mean sum of square

$$= \frac{\sum_{i}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_{i}^{n}(x_i-\bar{x})^2\sum_{i=1}^{n}(y_i-\bar{y})^2}}$$

$$= \frac{\sum x_iy_i - \frac{\sum x_i y_i}{N}}{\sqrt{\left\{\sum_{i=1}^{n} x_i^2 - \frac{(\sum x_i)^2}{N}\right\}\left\{\sum_{i=1}^{n} y_i^2 - \frac{(\sum y_i)^2}{N}\right\}}}$$

Math করার জন্য এটা use করব।

Co-relation is described or classified is several different ways. There of the most important are:

(i) Positive → direction same দিকে and negative → opposite direction দিকে

(ii) Simple, partial and Multiple.

(iii) linear and non-linear.

Class: 29.03.22.

Method of studying Co-relation:

(i) Scatter Diagram.

(ii) Karl Pearson's cofecient of corelation.

(iii) Spearman's Rank co-relation coefficient and

(iv) Method of least squares.

Properties of r:

(i) The co-relation coefficient is a symmetric measure i.e $r_{xy} = r_{yx}$

(ii) The correlation coefficient will be negative or positive depending on whether the sign of the numerator.

(iii) The co-relation coefficient lies between -1 and +1. (prove করতে হবে)

(iv) The co-relation coefficient is a dimensionless quantity implying that it is not expressed in any units of measurement.

(v) The co-efficient of correlation is independent of origin and scale of measurement (Prove করতে হবে)

Coefficient of Determination: One very convenient and useful way of interpreting the value of coefficient of correlation between two variables is to use the square of coefficient of correlation, which is called co-efficient of determination. It is denoted by r^2.

For example if $r^2 = 0.81$ this would mean that 81 % of variation of dependent variable has been explained by the independent variable.

☐ Interpret the $r^2 = 0.81$.

Scatter diagram / dot diagram:

যখনই কোন straight line আঁকা যাবে না তখন $r = 0$.

∴ there is no linear co-relation ~~relati~~

☐ Interpret these with scatter diagram.

(i) $r = -1$, (ii) $r = +1$ (iii) $r = 0.89$ (iv) $r = -0.79$

$\downarrow$ perfect neg. $\downarrow$ perfect positive. $\downarrow$ strong +ve $\downarrow$ strong −

(v) $r = 0$ (no relation)

.5 এর মধ্যে হলে moderate. .5 এর নিচে হলে weak.

Rank correlation Coefficient: This measure is specially useful when quantitative measure of certain factors (such as in the evaluation of leadership ability or the judgement of female beauty) can not be fixed but the individuals in the group can be arranged in order thereby obtaining for each individual a number indicating this rank in the group.

The rank correlation coffecient is -

$$R = 1 - \frac{6 \sum_{i=1}^{n} d_i^2}{n(n^2-1)}.$$

When d_i refers to the difference of ranks between paired items in two series.

R lies between +1 and -1 i.e $-1 \leq R \leq 1$

Class: 08.04.22

Example: Two housewives Mina and Rita asked to express their performance for different kinds to detergents gave the following replies:
To what extent the preformoners of these two ladies go tog

Detergents	Greeta (R_1)	Rita (R_2)	$d_i^2 = (R_1 - R_2)^2$
A	4	4	0
B	2	1	1
C	1	2	1
D	3	3	0
E	7	8	1
F	8	7	1
G	6	5	1
H	5	6	1
I	9	9	0
J	10	10	0
			$\Sigma d_i^2 = 6$.

$$\therefore R = 1 - \frac{6 \times 6}{10(10^2 - 1)}$$

$$= 0.964$$

Thus the preferences of these two ladies agree very closely as far as their option on detergents is concerned

Example of correlation:

Find the coficient of correlation and interpret the result

X:	57	42	40	33	42	45	42	44	40	56	44	43
Y:	10	60	30	41	29	27	27	19	18	19	31	29

we know that,

$$r = \frac{\sum x_i y_i - \frac{\sum x_i y_i}{N}}{\sqrt{\left\{\sum_{i=1}^{n} x_i^2 - \frac{(\sum x_i)^2}{N}\right\}\left\{\sum_{i=1}^{n} y_i^2 - \frac{(\sum y_i)^2}{N}\right\}}}$$

$= -0.53$

Therefore, it is a case of moderate degree of negative correlation.

Regressions Analysis

After having establish the fact two variables are closely related we may be interested in estimating (prediciting) the value of one variable given the value of another. For example, if we know that advertising and sales are correlated we may find out the expected amount of sales for a given advertising expenditure or the required amount of expenditure for acheiving a fixed sales ~~target~~ target.

Regression Analysis: The regression analysis is a technique of studying the dependence of one variable (called dependent variables) on one or more variables (called explanatory or independent variable) with a view to establishing or prediciting the average value of the dependent variable in term of the known or fixed values of the independent variables.

Difference between co-relation and Regression:

Co-relation	Regression
1. It indicates only the nature and extent of linear relationship.	1. It is the study about the impact of the independent variable on the dependent variable. It is used for predictions.
2. If the linear correlation is cofficient is positive/negative, then the two variables are positively/or negatively correlated.	2. The regression coefficient is positive, then for every unit increase in x, the corresponding average increase in y is b_{yx}. similarly, if the regression coefficent is negative, then for the every unit increase in x, the corresponding average decrease in b_{yx}.
3. One of the variables can be taken as x and the another can be taken as the variable y.	3. Care must be taken the choice of independent variable and dependent variable. We can't assign arbitrarily x as independent variable and y as dependent variable.
4. It is symmetric in x and y. $r_{xy} = r_{yx}$	4. It is not symmetric in x and y, that is b_{xy} and b_{yx} have different meaning and interpretations.

Date: 05-04-22

Regression equation of Y on X:

The regression equation of Y on X is expressed as follows:

$$Y = a + bx.$$

when, Y is the dependent variable to be estimated and x is the independent variable. In this equation a & b are two unknown constants which determine the position of the line completely. The constants are called the parameter of the line. The parameter 'a' determine the level of the fitted line. The parameter "b" determine the slope of the line.

Let us ~~supp~~ suppose that (x_i, y_i) $i = 1, 2, \dots n$ be a random sample from a bi-variant distribution, y is dependent and x is independent variable. Let the regression line of y

on x be,

$$y = a+bx \quad \cdots \quad (*)$$

Adding random error term the model become-

$$y_i = a+bx_i + e_i \quad \cdots \quad (1)$$

where a is the intercept and b is the slope usually called the regression coefficient of y and x and e_i's are random error component which are independently and normally distributed with 0 mean and variance σ^2.

From (1) we have -

$$e_i = y_i - a - bx_i$$

$$\Rightarrow \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_i - a - bx_i)^2$$

$$\Rightarrow \sum_{i=1}^{n} e_i^2 = E = \sum_{i=1}^{n} (y_i - a - bx_i)^2$$

Now, $\frac{\delta E}{\delta a} = 0$.

$\Rightarrow -2\Sigma(y_i - a - bx_i) = 0$.

$\Rightarrow \sum_{i=1}^{n} y_i = na + b\sum_{i=1}^{n} x_i$ -- (2)

and $\frac{\delta E}{\delta b} = 0$

$\Rightarrow \sum_i x_i y_i = a\sum_{i=} x_i + b\sum_i x_i^2$ -- (III)

These two equation are known as normal equation. Considering (2) and (3) and divided by n, we get,

$a + b\bar{x} = \bar{y}$

estimate value of a

$\Rightarrow \hat{a} = \bar{y} - b\bar{x}$ -- (4)

and, $\frac{\Sigma x_i y_i}{n} = a\bar{x} + b\frac{\sum_i x_i^2}{n}$

$\Rightarrow \frac{\Sigma x_i y_i}{n} = (\bar{y} - b\bar{x})\bar{x} + b\frac{\Sigma x_i^2}{n}$

$$\Rightarrow \frac{\sum x_i y_i}{n} = \bar{x}\bar{y} - b\bar{x}^2 + b\frac{\sum x_i^2}{n}$$

$$\Rightarrow \frac{\sum x_i y_i}{n} - \bar{x}\bar{y} = b\left(\frac{\sum x_i^2}{n} - \bar{x}^2\right)$$

$$\Rightarrow b = \frac{\frac{\sum x_i y_i - n\bar{x}\bar{y}}{n}}{\frac{\sum x_i^2 - n\bar{x}^2}{n}}$$

$$= \frac{\sum x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}$$

$$\therefore \hat{b} = \frac{\sum_i x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}$$

This is estimated values of a & b are

$$\hat{a} = \bar{y} - \hat{b}\bar{x}$$

$$\hat{b} = \frac{\sum_i x_i y_i - \frac{\sum x_i \sum y_i}{n}}{\sum x_i^2 - (\sum x_i)^2/n}$$

07.04.22

Problem: Calculate the regression equation of x and y

x:	1	2	3	4	5
y:	2	5	3	8	7

$x = \hat{a} + \hat{b} y$

$x = 1.10 + 1.30 y$

Properties of regression coefficient:

* Regression co-efficient is independent of origin but dependent on the scale of measurement.

* The regression coefficient lies between $-\infty$ to ∞

* The regression coefficient $b_{y/x}$ & $b_{x/y}$ for the same set of data can not have opposite signs.

Example:

A departmental store has the following statistics of sales (y) for a period of least one year of 10 salesman, who have varying year of experience (x).

(i) Find the regression line of Y on X.

(ii) Predict the annual sales volume of person who have 12 & 15 years of sales experience.

Salesman	Years of experience	Annual sales (Tk' 0000)	x_i^2	x_iy_i
1	1	80		
2	3	97		
3	4	92		
4	4	102		
5	6	103		
6	8	111		
7	10	119		
8	10	123		
9	11	117		
10	13	136		

$$Y = \hat{a} + \hat{b}x \quad \text{— (i)}$$

$$\hat{a} = \bar{y} - \hat{b}\bar{x} \quad \text{— (ii)}$$

$$\hat{b} = \text{formula } \frac{\sum x_iy_i - \frac{\sum x_i \sum y_i}{N}}{\sum x_i^2 - \frac{(\sum x_i)^2}{N}}$$

$a = 80$, $b = 4$

$$\hat{y}_i = 80 + 4x_i \quad \text{— Q}$$

(11) The estimate sales for $x = 12$ is $y_i = 128$

$x = 15$ Tk $\quad y_i = 80 + 4 \times 15 =$ Tk 140.

* The value $b = 4$ means that for an average increase of one year sales volume would increase on the average by Tk 4 thousand.

The value of $a = 80$ means that if the store employs a person without any experience (i.e. $x = 0$) the average increase in sales volume will be almost it 80 thousant.

Class: 10-04-22

Theory of Probability

Sample space: The collection or probability of all possible outcomes of a random experiment is called sample space.

Sample space is usually denoted by Ω or S.

Sample point: An element of the sample space is called a sample point.

Example: বই দেখো।

Event: An event is a subset of the sample space and is usually denoted by capital letter A, B, X, Y, Z etc. like set. Event is denoted by E.

There come two types of events –

(i) Sample event: An event is called simple event if it contain only one sample point.

(ii) Compound event: An event is called compound event if it contains more than one sample point or it is the union of sample events.

Example: If the outcome of an experiment consists in the determination of the sex of a newborn child, then sample space is

$S = \{g, b\}$ [$g \to$ girl, $b \to$ boy]

Here g sample point and b is another sample point.

In the preceding example if $E = \{g\}$, then E is the simple event that the child = girl.

Compound event Example $E = \{x : 0 < x \leq 5\}$

then E is the event that the transistor does not last longer than 5 hour.

Class: 12.04.22

Experiment: Experiment is an act that can be repeated under given conditions.

Unit experiment is known as trial. This means that trial is a special case of experiment. Experiment may be a trial or two or more trails.

Example: Tossing a coin is a trial and getting head and tail are outcomes.

Equally likely outcome: Outcomes of a trail are said to be equally likely if we have no reason to expect any one rather than the other.

Examples

(i) In tossing a fair coin, the outcomes head and tail are equally likely.

(ii) In throwing a balanced die, all the six faces the equally likely.

Mutually Exclusive Outcomes: Outcomes or cases are said to be mutually exclusive if the happening of any of them excludes the happening of all others.

~~Example:~~

(i) ~~In~~ tossing a

Example:

(i) In tossing a coin, the outcomes head and tail are mutually exclusive.

(ii) The newly born child will be a male and female are mutually exclusive.

Class: 19-04-22

Favourable outcomes: The outcomes of an experiment are said to be favourable outcomes to an event if they entrait the happening of the event.

Example: In throwing a die, the favourable outcomes of the even numbers on the faces of the die will be 2, 4 & 6.

Exhaustive Outcomes: Outcomes of an experiment are said to be exhaustive if they include all posible outcome.

Example: (i) In throwing a die, exhaustive number of outcomes are 6.

(ii) In tossing a coin, exhaustive number of outcomes are 2.

Classical or Mathmetical Priori Probality: If there are n mutually exclusive, equally likely and exhaustive outcomes of an experiment & if m of these out comes are favourable to an event. A. Then the probability of the event A which is denoted by P(A) is defined by:

$$P(A) = \frac{\text{favourable outcomes of an event A}}{\text{total numbers of outcomes of the experiment.}}$$

$$= \frac{m}{n}$$

Sure event: An event is called same event when it always happens. The probability of sure event is 1.

Impossible event: A event is called impossible when it never happens.

Example: A card is drawn from a part of 52 cards find the probability that it is (a) a red card (b) a spade (c) an ace (d) not a spade and (e) a king or a queen.

(a) $\frac{26}{52} = \frac{1}{2}$ (b) $\frac{13}{52} = \frac{1}{4}$

(c) $\frac{4}{52} = \frac{1}{13}$ (d) $\frac{8}{52}$ (e) $1 - \frac{1}{4} = 3/4$

Class: 21.04.22.

15 May Mid

Shyllabus:

Measures of shape charteristics to Probability.

Baye's Theorem: Statement

Let E_1 and E_2 be partitions of S. Let F be an avent with $P(F) > 0$.

Let $\left.\begin{matrix} P(E_1), P(F/E_1) \\ P(E_2), P(F/E_2) \end{matrix}\right\}$ be known.

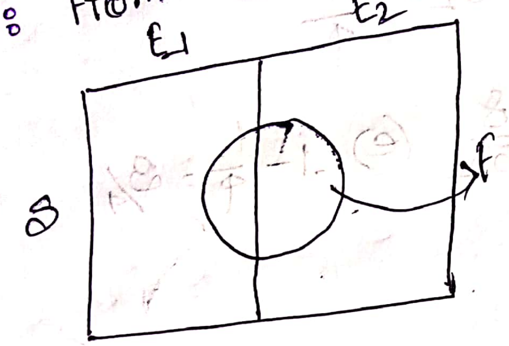
Then (i) $P(F) = P(E_1)\,P(F/E_1) + P(E_2) \cdot P(F/E_2)$

(ii) $P(E_1/F) = \dfrac{P(E_1)\;P(F|E_1)}{P(F)}$

conditional probability থেকে Proof করা হয়েছে

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Proof: From venn-diagram we see that.



(i) $F = (E_1 \cap F) \cup (E_2 \cap F)$

(ii) $P(F) = P(E_1 \cap F) + P(E_2 \cap F)$

$= P(E_1)\, P(F|E_1) + P(E_2) \cdot P(F|E_2)$.

[from conditional probability]

(iii) $P(E_1|F) = \dfrac{P(E_1 \cap F)}{P(F)} = \dfrac{P(E_1)\, P(F|E_1)}{P(F)}$.

(Proved).

* Conditional Probability + Math (H.W).

Math (3217.5) Proof (3217.5)

Independent and Dependent Events: Events are said to be independent if the happening or non-happening of an event is not affected by the happening of any number or remaining events. (1) Otherwise the events are said to be dependent.

Two events A and B are said to be independent if any one of the following conditions are satisfied-

(i) $P(AB) = P(A) \cdot P(B)$

(ii) $P(A|B) = P(A)$

(iii) $P(B|A) = P(B)$

Example: Three coins are tossed. Show that the events "heads on the first coin" and the event "tails on the last two." are independent

Soln: We construct a sample space S for the above experiment.

$\therefore S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

Let, A denote the event 'head' on the first coin" and B denote the events " tails on the last two coins."

Then,

$A = \{HHH, HHT, HTH, HTT\}$.

and B

$= \{$

$\therefore P(A) = \frac{4}{8}$

$= \frac{2}{4}$

$\therefore P(B) = \frac{2}{8} = \frac{1}{4}$.

$\therefore P(A \cap B) = P(A) \cdot P(B)$

$= \frac{2}{\cancel{4}} \cdot \frac{1}{4}$

class: 08 May, 2022.

Probality function: The probality $P(x_i)$ associated with value of discrete random variable x_i is called probability function $P(x_i)$ if it satisfies the following condition:

(i) $P(x_i) \geq 0$ for all associate i.

(ii) $\sum_{i=1}^{n} P(x_i) = 1$, where the random variable assumes n values $x_1, x_2 \ldots x_n$.

Probality Density Function: For continious random variable x the function $f(x)$ is usually known as probibility density function (p.d.f) which satisfies the following conditions:

(i) $f(x) \geq 0$ for all x within the ~~rangi~~ range.

(ii) $\int_{-\infty}^{+\infty} f(x)\,dx = 1$.

(iii) The probabilty that the continious variable x

with p.d.f $f(x)$ fall in any interval (a, b)

is given by,

$$P[a \le x \le b] = \int_a^b f(x)\,dx.$$

Example: A random variable x has the following probability density function.

$$f(x) = cx(2-x), \quad 0 \le x \le 2.$$

(i) determine c

(ii) find the probability that $0 \le x \le 1$?

(i)

Since we know

$$\int f(x)\,dx = 1.$$

The value of c can be obtained as.

(i) $\int f(x)\,dx = 1$

$\Rightarrow \int_0^2 cx(2-x)\,dx = 1$

$\Rightarrow c\int_0^2 2x - x^2\,dx = 1$

$\Rightarrow c\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_0^2 = 1$

$\Rightarrow c\left[2\frac{2^2}{2}-\frac{2^3}{3}\right] = 1$

$\Rightarrow c\left[4-\frac{8}{3}\right] = 1$

$\Rightarrow c = \frac{3}{4}$

(ii)

$P(0 \le x \le 1) = \int_0^1 f(x)\,dx$

$= \int_0^1 \frac{3}{4}x(2-x)\,dx$

$= \frac{3}{4}\int_0^1 2x - x^2\,dx$

$= \frac{3}{4}\left[\frac{x^2}{2}-\frac{x^3}{3}\right]$

$= \frac{3}{4}\left[1-\frac{1}{3}\right]$

$= \frac{\cancel{3}}{4}\times\frac{2}{\cancel{3}}$

$= \frac{1}{2}$

Distribution Function: Let, x be a random variable discrete or continuous and F(x) be the probability that the random variable x takes values less than or equal to k then F(x) is called the cumulative distribution function (c.d.f) or simply distribution function of x.

$$F(x) = P[X \le x] = \int_{-\infty}^{x} f(x)\,dx$$

Properties of a Distribution function:

(i) $F'(x) = \frac{d}{dx} F(x) = f(x) \ge 0$ so that $F(x)$ is a non decreasing function.

(ii) $F(-\infty) = 0$.

(iii) $F(\infty) = 1$

(iv) $Prob[a \le x \le b] = F(b) - F(a)$

properties (ii) and (iii) imply that $0 \le F(x) \le 1$.

Class: 10 May, 2022.

Example:

Let, $F(x) = \begin{bmatrix} 0 & \text{for} & x \leq 0 \\ x^2 & \text{for} & 0 \leq x \leq 1 \\ 0 & \text{for} & x \geq 1 \end{bmatrix}$

be the commulative distribution function of the random variable x. Find $P[x \leq -1]$, $P[x \leq 0.5]$, $P[x \leq 3]$, $P[x > 0.4]$ and $P[0.2 \leq x \leq 0.5]$

Soln: The probability density function of x is—

$f(x) = F'(x) = \begin{cases} 2x & , \text{for } 0 \leq x \leq 1 \\ 0 & \text{for } x \geq 1 \end{cases}$

Therefore,

$$P[x \leq -1] = \int_{-\infty}^{-1} f(x)\,dx.$$

$$= \int_{-\infty}^{-1} 0\,dx = 0.$$

$$P[x \leq 0.5] = \int_{-\infty}^{0.5} f(x)\,dx.$$

$$= \int_{\infty}^{0.5} 2x\,dx.$$

$$= 0.25.$$

$$P[x \le 3] = \int_{-\infty}^{3} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx.$$

$$= 0 + \int_{0}^{1} 2x\,dx + 0$$

$$= 1.$$

$$P[x > 0.4] = \int_{0.4}^{1} f(x)\,dx$$

$$= \int_{0.4}^{1} 2x\,dx = 0.84$$

$$P[0.2 \le x \le 0.5] = \int_{0.2}^{0.5} f(x)\,dx = \int_{0.2}^{0.5} 2x\,dx = 0.21$$

Verify the following function are probability function.

(a) $f(x) = \frac{2x-1}{8}$, $x = 0, 1, 2, 3$

(b) $f(x) = \frac{x+1}{16}$, $x = 0, 1, 2, 3$

Solutions: Summing the function over the entire range of x.

(a) $\sum_{x=0}^{3} = f(0) + f(1) + f(2) + f(3)$

$= -\frac{1}{8} + \frac{1}{8} + \frac{3}{8} + \frac{5}{8} = 1$.

$\therefore \sum_{x=0}^{3} f(x) = 1$

but $P(x=0) = f(0) = -\frac{1}{8}$ which contradicts the condition (ii)

Hence the function $f(x)$ is not a probability function

x^{-2}
$-2x^{-3}$

Example: Let x be a continues random variable with the following density function.

$$f(x) = \begin{cases} x/2 , & 0 < x < 2 . \\ 0 , & \text{otherwise} . \end{cases}$$

find the distribution function and $P[X < 1]$:

Soln:

$$\therefore f(x) = F'(x) = \begin{cases} \cancel{-2x^{-3}} \tfrac{1}{2} , & 0 < x < 2 . \\ 0 , & \text{otherwise} . \end{cases}$$

Therefore, $P[X < 1]$

$$= \int_{-\infty}^{1} f(x)\, dx$$

$$= \int_{-\infty}^{0} f(x)\, dx + \int_{0}$$

$\therefore F(x) = x^2/4$.

Therefore, $F(x) = \begin{cases} 0 & , x < 0 \\ x^2/4 & , 0 < x < 2 \\ 0 & , x \geq 2 \end{cases}$.

$\therefore F(x) = P(x < 1) = F(1) = \frac{1}{4}$.

Class: 12 May, 2022

Joint Probability function:

If we consider only two discrete random variables x and y, we have a joint probability function $P(x_i, y_j)$, called bivariate probability function which follows the following condition.

(i) $P(x_i, y_j) \geq 0$ for all admissible i and j.

(ii) $\sum_i \sum_j P(x_i, y_j) = 1$

Marginal Probability function: Let $P(x_i, y_j)$ be the probability function of the discrete random variable x and y, then the marginal probability function of x, $P(x_i)$ is given by—

$$P(x_i) = \sum_j P(x_i, y_j) \text{ for all } i$$

Simillarly, the marginal probability function of y, $P(y_i)$ is given by—

$$P(y_i) = \sum_i P(x_i, y_i) \text{ for all } i.$$

Continious Probility function:

$f(x_i, y_j)$

(i) $f(x_i, y_j) = 0$

(ii) $\int_{x}\int_{y} f(x_i, y_j)\, dy\, dx = 1.$

Marginal density function:

$$f(x) = \int_y f(x,y)\, dy$$

$$f(y) = \int_x f(x,y)\, dx$$

Conditional Probability function: The conditional probability function the discrete random variable x for a given value of y $P(x_i | y_j)$ is given by –

$$P(x_i | y_j) = \frac{P(x_i, y_j)}{P(y_j)}$$

Similarly, conditional probability of y for given x,

$$P(y_j / x_i) = \frac{P(x_i, y_j)}{P(x_i)}$$ [যেইটা given এ টা নিচে হবে]

Two continuous random variable x, y are said to independent if

(i) $f(x,y) = f(x) \cdot f(y)$

(ii) $f(x|y) = f(x)$

(iii) $f(y|x) = f(y)$

Example: A joint propability d function of two random variables x and y is given by -

$$f(x,y) = 12xy(1-y) \quad ; 0 < x < 1 .$$
$$0 < y < 1$$

Are x and y independent?

$$f(x) = \int_y 12xy \,~~dy~~ - 12xy^2 \, dy .$$

$$= \left[~~12x - 24xy~~ \right]_0^1 ~~= 2x~~ \quad ; 0 < x < 1 .$$

$$f(y) = \int_x 12xy - 12xy^2 \, dx .$$

$$= \left[12y - 12y^2 \right] = 6y(1-y) \; ; 0 < y < 1$$

Example :- The joint density function of the random variables x and y is given by-

$$f(x,y) = \begin{cases} 8xy ; & 0 < x \leq 1 \\ & 0 \leq y \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

find (a) The marginal density of $X \rightarrow 4x^3$

(b) " " " " $Y \rightarrow 4y(1-y^2)$

(c) " conditional " " $X \rightarrow \frac{2x}{1-y^2}$

(d) " " " " $Y \rightarrow 2y/x^2$

(e) determine wheter the random variables x and y are independendent? $\rightarrow$ Not independend.

Mid Syllebus

Class 16 May, 2022

Mathematical Expectation of a discrete random variable

If x is a discrete random variable which can take finite or infinite sequence of different variable values $x_1, x_2, \ldots, x_n \ldots$ which corresponding probabilities $f(x_1), f(x_2) \ldots f(x_n) \ldots$ then the mathematical expectation of the random variable x. denoted by μ is defined by—

$$\mu = E(x) = \sum_{i=1}^{n} x_i f(x_i)$$

Example: Suppose that x is a discrete random variable which can take values −2, 0, 3 and 5 with probabilities $P[x=-2] = 0.1$

$P[x=0] = 0.3$, $P[x=3] = 0.4$ and $P[x=5] = 0.2$.

Find the mean of x.

7, 8, 9 Batch এর Question solve →

Ans:

$\mu = -2\times0.1 + 0\times0.3 + 3\times0.4 + 5\times0.2$

$= 2.$

Mathmatecial Expection of a continious variable:

If x is

$\mu = E(x) = \int x f(x)\, dx$

Example: Suppose that x is a continious r.v with probability density function

$$f(x) = \begin{cases} 2x & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

find the expected value of x.

$E(x) = \int_0^1 x f(x)\, dx$

$= \int_0^1 (x \cdot 2x)\, dx$

$= \int_0^1 2x^2\, dx$ $=2$

$= 2\int_0^1 \left[\frac{x^3}{3}\right]$

$= 2\cdot\frac{1}{3}$

$= 2/3$ Ans.

* ~~expected~~ / E(x) বললেই mean বের করতে হবে।

$$E(x) = \int x\, f(x)\, dx$$

$$E(x^2) = \int x^2 f(x)\, dx.$$

$$E(x^2) = 2\int_1^0 (x^2 \cdot x)\, dx$$

$$= 2\left[\frac{x^4}{4}\right]_0^1$$

$$= 2 \cdot \frac{1}{4}$$

$$= \frac{1}{2}.$$

variant of x $V(x) = E(x^2) - [E(x)]^2$.

$\mu_r' = E(x^r) \rightarrow$ raw moment এর formula.

$$\therefore V(x) = \frac{1}{2} - \left(2/3\right)^2$$

$$= \frac{1}{18}$$ Ans.

Example: If (x, y) is a two dimensional random variable with probability density function:

$$f(x,y) = x+y \;,\; 0 < x \leq 1$$

find $E(x|y)$, $0 \leq y \leq 1$ and $var(x|y)$

$$\therefore E(x|y) = \int_0^1 x f(\cancel{x|y})\, d\cancel{y}\, dx$$

$\therefore$ ~~$f(y)$~~

$$\therefore f(x) = \int_0^1 x(x+y)\, dy.$$

$$= \int_0^1 x^2 + xy\, dy.$$

$$\therefore E(x|y) = \int_0^1 x f(x|y)\, dx \quad \text{—(i)}.$$

$$f(x|y) = \frac{f(x,y)}{f(y)}$$

$$f(y) = \int_x (x+y)\, dx$$

$$V = E(x^2|y) - \{E(x|y)\}^2$$

17 May, 2022:

Moment Generating function: let X be a random variable with probability function $f(x)$, then the function $M_X(t)$ called the moment generating function (MGF) is denoted by—

$$M_X(t) = E(e^{tx}) = \sum_x e^{tx} f(x); \text{ if } x \text{ is denoted.}$$

$$M_X(t) = E(e^{tx}) = \int_x e^{tx} f(x)\,dx; \text{ if } x \text{ is continuous.}$$

Since $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

We can expand $M_X(t)$ as follows:

$$M_X(t) = E(e^{tx}) = E\left(1 + tx + \frac{(tx)^2}{2!} + \frac{(tx)^3}{3!} + \cdots + \frac{(tx)^n}{n!} + \cdots\right]$$

$$= 1 + tE(x) + \frac{t^2}{2!}E(x^2) + \frac{t^3}{3!}E(x^3) + \cdots + \frac{t^n}{n!}E(x^n) + \cdots$$

$$= 1 + \frac{t}{1!}\mu_1' + \frac{t^2}{2!}\mu_2' + \frac{t^3}{3!}\mu_3' + \cdots + \frac{t^r}{r!}\mu_r' + \cdots$$

$$= \sum_{r=0}^{\infty} \frac{t^r}{r!}\mu_r'$$

The above expression clearly denotes that the coefficient of $\frac{t^r}{r!}$ in the expression of $M_X(t)$ is nothing but the rth raw moment.

An alternative way of deriving the moments from $M_X(t)$ is to differentiate $M_X(t)$ is to differentiate $M_X(t)$ with respect to t and then setting,

$t = 0$

Specifically,

$$\mu_r' = \frac{d^r}{dt^r} M_X(t) \Big|_{t=0}$$

Then differentiating $M_X(t)$ once we get,

$$\frac{d}{dt} M_X(t) = \mu_1' + t\mu_2' + \frac{t^2}{2!}\mu_3' + \cdots \frac{t^{r-1}}{(r-1)!}\mu_r' + \cdots$$

Setting $t=0$

$$\frac{d}{dt} M_x(0) = \mu_1' = E(x)$$

Differentiating again,

$$\frac{d^2}{dt^2} M_x(t) = \mu_2' + t\mu_3' + \cdots + \frac{t^2}{2!}\mu_4' + \cdots$$

Setting, $t=0$.

$$\frac{d^2}{dt^2} M_x(0) = \mu_2'$$

$\therefore$ In general $\frac{d^r}{dt^r} M_x(0) = \mu_r'$

Example: Suppose x is a discrete random variable having the following probability function:

$$f(x) = \frac{e^{-m} m^x}{x!}, \quad x = 0, 1, 2, \ldots$$

Find the moment generating function of x and its mean and variance.

Ans: By defination,

$$M_X(t) = E[e^{tx}]$$

$$= \Sigma e^{tx} f(x)$$

$$= \sum_{x=0}^{\infty} e^{tx} \frac{e^{-m} m^x}{x!}$$

$$= e^{-m} \sum_{x=0}^{\infty} \frac{(me^t)^x}{x!} \quad \left[1 + \frac{me^t}{1!} + \frac{(me^t)^2}{2!} + \cdots\right] = e^{me^t}$$

$$= e^{-m} \cdot e^{me^t}$$

$$= e^{m(e^t - 1)} \quad \text{—(i)}$$

∴ Hence the mean is,

$$E(x) = \frac{d}{dt} M_X(t) \Big|_{t=0}$$

$$\frac{d}{dt} e^{m(e^t-1)}$$

$$= \frac{d}{dt}\left(e^{m(e^t-1)}\right)$$

$$= e^{m(e^t-1)} \cdot me^{t}(t-1)$$

$$= \frac{d}{dt}\left(e^{me^t - m}\right)$$

$$= e^{m} me^t \quad m$$

$$= e^{m(e^t-1)} \cdot (me^t - 1)$$

$= me^t \cdot e^{m(e^t-1)}$

$= m.$

Simillary differentiating twice,

$$E(x^2) = \left[\frac{d^2}{dt^2}\right]\left\{e^{m(e^t-1)}\right\}\Big|_{t=0}$$

$=$

$= m + m^2$

Then the variance is,

$$V(x) = E(x^2) - \{E(x)\}^2$$

$= m + \cancel{m^2} - \cancel{m^2}$

$= m.$

Cumulents ~~an~~

Cumulants and Cumulant Generating function:

If the logarithm of moment generating function of a distⁿ can be expanded as a convergent series of t,

$$k_x(t) = \log M_x(t).$$

$$= k_1 t + k_2 \frac{t^2}{2!} + \cdots + k_r \frac{t^r}{r!} + \cdots$$

then the co-efficient k_r is called the rth cumulant of the distⁿ and $K_x(t)$ is called the cumulanting function $(K_{(x)}(t!))$

$$k_r = \frac{d^r}{dt^r} \log_e M_x(t) \Big|_{t=0}$$

$$= \left[\frac{d^r}{dt^r} k_x(t)\right]_{t=0}$$

Class: 19 May, 2022.

Binomial Distribution: A discrete random variable x is said to have a binomial distribution if its probability function is defined by,

$$f(x; n, p) = \begin{cases} \binom{n}{x} p^x q^{n-x} & \text{for } x = 0, 1, 2, 3 \ldots n \\ 0 & ; \text{otherwise.} \end{cases}$$

Where the two parameters n and p satisfy satisfy $0 \leq p \leq 1$ and n is a positive integer and $q = 1 - p$.

The following conditions must be satisfied for the binomial distribution —

(i) there should be a fixed number of trials

(ii) the trails are independent.

(iii) there are only two outcomes for each trail.

(iv) The probability of success and hence the probability of failure remain same or constant for trial to trial.

Math: Mean, variance, moment generation function.

probability of Unity:

* for Binomial distribution show that total probability is 1. [বই থেকে পড়তে হবে]

$$f(x) = \sum_{x=0}^{n} {}^{n}C_{x}\, p^{x} . q^{n-x}$$

$$= {}^{n}C_{0}\, p^{0} . q^{n-0} + {}^{n}C_{1}\, p^{1} q^{n-1} + {}^{n}C_{2}\, p^{2} q^{n-2} + \dots + {}^{n}C_{n}\, p^{n} . q^{n-n}.$$

$$= q^{n} + {}^{n}C_{1}\, p^{1} q^{n-1} + {}^{n}C_{2} . p^{2} q^{n-2} + \dots + p^{n}$$

$$= (q+p)^{n} \quad [\text{By Binomial } q+p=1]$$

$$= (1)^{n}$$

$$= 1.$$

Mean and variance: Let x be a random variable which can take values $0, 1, 2 \ldots n$ and n and p be the parameters of binomial distribution. We know that probability function of binomial distribution is –

$$f(x; n, p) = {}^nC_x p^x q^{n-x}$$

Now,

$$E(x) = \mu_1' = \mu = \sum_{x=0}^{n} x \cdot f(x; n, p)$$

$$= \sum_{x=0}^{n} x \cdot {}^nC_x \, p^x \cdot q^{n-x}$$

$$= \sum_{x=0}^{n} x \cdot \frac{n!}{x!(n-x)!} p^x \cdot q^{n-x}$$

$$= \sum_{x=1}^{n} \frac{n(n-1)}{(x-1)!(n-x)!} p^x \cdot q^{n-x}$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!\{(n-1)-(x-1)\}!} p^{x-1} q^{\{(n-1)-(x-1)\}}$$

$$= np \times 1$$

$$= np$$

$$\therefore V(x) = E(x^2) - \{E(x)\}^2$$

or $E[x(x-1)+x]$

$= E[x.(x-1)] + E(x)$ — (1)

Now

$$E(x)^2 = \sum_{x=0}^{n} x^2 f(x, n, p)$$

$$= \sum_{x=0}^{n} x^2 . {}^nC_x p^x . q^{n-x}$$

$$E[x(x-1)] = \sum_{x=0}^{n} x(x-1) f(x; n, p)$$

$$= \sum_{x=0}^{n} x(x-1) \frac{n!}{x!(n-x)!} p^x . q^{n-x}$$

$$= n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!(n-x)!} p^{x-2} q^{n-x}$$

$$= n(n-1)p^2$$

$$\therefore E(x^2) = n(n-1)p^2 + np - n^2p^2 = np(1-p) = npq.$$

Example: In a community, the probability that a newly born child will be boy is 2/5. Among the 4 newly born children in that community what is the probability that

(i) all the 4 boys (ii) at least 2 boys

(iii) no boys (iv) exactly one boy and

(v) at most two boys.

Solution: Let us consider the event that a newly born child is a boy as sucess in Bernoulli trial with probability of success 2/5. Let the number of boys be random variable x. Then x can take values 0, 1, 2, 3 and 4.

According to Binomial law, the probability function of x is -

$f(x; 4, 2/5) = \binom{4}{x} (2/5)^{x} (3/5)^{4-x}$ ——— (i)

(i) $P(\text{all boys}) = \binom{4}{4} (2/5)^{4} (3/5)^{4-4}$

$= (2/5)^{4}$

$= \frac{16}{625}$.

(ii) $P(\text{at least 2 boys}) = P(x \geq 2)$

$= 1 - P[X < 2]$

$= 1 - P[X=0] \cancel{+} P[X=1]$

poisson distribution is a limit case of the binomial distn under the following condition:

(i) The probability of success or failure in Bernoulli trail is very small. This is $p \to 0$ or $q \to 0$.

(ii) n, the number of trials is very large

(iii) $np = \lambda$ (say) is a finite constant.

Poisson Distribution:

$$P(x,\lambda) = \begin{cases} \frac{e^{-\lambda}\cdot\lambda^{x}}{x!}, & x = 0,1,\ldots\infty \\ 0, & \text{otherwise} \end{cases}$$

Where, λ is the parameter of the distribution which is the mean number of success and $\lambda > 0$.

Home work:

Prove:

(i) $\sum\limits_{x=0}^{p} P(x) = 1$

(ii) $E(x) = \sum x P(x) = \sum x \frac{e^{-\lambda}\lambda^{x}}{x!}$

$$= \lambda e^{-\lambda}\left[\sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!}\right]$$

$$= \lambda \cdot e^{-\lambda} \cdot e^{\lambda}$$

$$= \lambda.$$

Distribution থেকে ১ টা Ques আসবে.

(iii) $V(x) = \lambda$.

(iv) positively skewossed (v) Leptrocadic.

☒ Properties of a poisson Distrubution

☒ " " a binomial distribution.

☒ Practical example of poission.

Example: If the probability that a car accident happen in a very busy road in an hour is 0.001 If 2000 cars passed in one hour by that road, what is the probability that (i) exactly 0, 3 (ii) more than 2 car accidents happened on that hour of the road.

Solution:

Let, x be the number of car accident which follows poission distribution

$$\lambda = 2000 \times .001 = 2.$$

as the probability of accident is very small

(i) $P[X=3] = \frac{e^{-2} 2^3}{3!} = 0.180$

(ii) $P[x>2] = 1 - P[X \leq 2]$

$= 1 - [P[X=0] + P[X=1] + P[X=2]]$

$= 0.32$.