Mean of first n Natural Numbers :

First n natural number are 1, 2, ..., n.

The mean, $\bar{x} = \frac{1+2+.....+n}{n} = \frac{1}{n}\frac{n(n+1)}{2} = \frac{n+1}{2}$

$\therefore \bar{x} = \frac{n+1}{2}$

Advantages of Arithmetic mean:

- It is rigidly defined.
- It is easy to calculate.
- It is based upon all the observations.
- It is suitable for further algebraic treatment.
- It is less affected by sampling fluctuations.

Disadvantages of Arithmetic mean:

- It is affected very much by extreme values.
- It cannot be calculated if the extreme class is open.
- It is not suitable for extremely skewed distribution.
- It cannot be used if we are dealing with qualitative characteristics; such as intelligence, honesty, beauty, etc.
- It cannot be obtained if a single observation is missing or lost.

Uses of Arithmetic Mean:

- It is widely used to calculate average age, average income, average price, average salary, average increment, average import and average consumption, etc.
- It is used to establish the various theories and formulas of Mathematics and also used as an aid in further statistical analysis.
- It is used in computation of index number.

3.2. Geometric Mean (GM) :

Geometric mean of a set of n non-zero positive observations is the nth root of their product. The GM of n non-zero positive values $x_1, x_2, \ldots\ldots\ldots, x_n$ of a variable x is given by

$$GM = \sqrt[n]{x_1.x_2.\ \ldots\ldots x_n} = (x_1.x_2.\ \ldots\ldots\ldots\ldots\ x_n)^{\frac{1}{n}}$$

$$\log(GM) = \log\ (x_1.x_2.\ \ldots\ldots\ldots x_n)^{\frac{1}{n}} = \frac{1}{n}\sum_{i=1}^{n}\log x_i$$

$$\therefore GM = \text{Anti log}\left\{\frac{1}{n}\sum_{i=1}^{n}\log x_i\right\}$$

In case of frequency distribution, when $f_1, f_2, \ldots\ldots, f_n$ be the frequencies of $x_1, x_2, \ldots\ldots, x_n$ respectively, then

$$GM = \sqrt[N]{x_1^{f_1}.x_2^{f_2}\ldots\ldots\ldots x_n^{f_n}}\ ;\ \text{where}\ N = \sum_{i=1}^{n} f_i$$

$$= \text{Anti log}\left[\frac{1}{N}\sum_{i=1}^{n} f_i \log x_i\right]$$

Advantages of Geometric Mean :

- It is rigidly defined.
- It is based upon all the observations.
- It is not affected much by sampling fluctuations.
- It is suitable for further algebrical treatments.
- In measuring rate of change it is the most suitable average.

Disadvantages of Geometric Mean :

- It cannot be computed where there is any negative or zero values in the series.
- It is not easy to understand and to calculate for persons having very weak mathematical skills.
- It cannot be computed when the extreme classes of the frequency distribution are open.
- The value of GM may not be found in the series.

Uses of Geometric Mean :

- Geometric mean is used to find the average of ratios, rate of population growth, rate of interest, average of percentages.
- It is used in the construction of index numbers.

Example 3.2 :

Rate of increase of yield of a new wheat variety compared with a local variety in 10 selected agricultural farms are given below -

Rate of increase of yield (%)	Number of farm
0-5	1
5-10	2
10-15	4
15-20	2
20-25	1

For computation of geometric mean, we construct the following table

Rate of change of yield (%)	Frequency f_i	Mid value x_i	log x_i	$f_i \log x_i$
0-5	1	2.5	0.39794	0.39794
5-10	2	7.5	0.87506	1.75012
10-15	4	12.5	1.09691	4.38764
15-20	2	17.5	1.24304	2.48608
20-25	1	22.5	1.35218	1.35218
	$\Sigma f_i = N = 10$			$\Sigma f_i \log x_i = 10.37396$

$$GM = \text{Anti log}\left\{\frac{1}{N}\Sigma f_i \log x_i\right\}$$

$$= \text{Anti log}\left\{\frac{1}{10}(10.373996)\right\} = \text{Anti log}\,(1.037396)$$

$$= 10.9 \text{ (Approx.)}$$

$\therefore$ The average rate of change of yield of the new variety of wheat is 10.9%

3.3 Harmonic Mean (HM) :

Harmonic mean of a set of non-zero observations is the reciprocal of the arithmetic mean of the reciprocals of the given values. Harmonic mean of n non-zero observations $x_1, x_2, \ldots\ldots, x_n$ is given by

$$HM = \frac{1}{\frac{1}{n}\left(\frac{1}{x_1} + \frac{1}{x_2} + \ldots\ldots + \frac{1}{x_n}\right)} = \frac{1}{\frac{1}{n}\sum_{i=1}^{n}\frac{1}{x_i}} = \frac{n}{\sum_{i=1}^{n}\frac{1}{x_i}}$$

$$\Rightarrow \frac{1}{HM} = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{x_i}$$

If $f_1, f_2, \ldots\ldots, f_n$ are respectively the frequencies of $x_1, x_2, \ldots\ldots, x_n$ non-zero observations; then

$$HM = \frac{1}{\frac{1}{N}\sum_{i=1}^{n}(f_i / x_i)} = \frac{N}{\sum_{i=1}^{n}(f_i / x_i)}$$

$$\Rightarrow \frac{1}{HM} = \frac{1}{N}\sum_{i=1}^{n}(f_i / x_i)$$

Advantages of Harmonic Mean :

- It is rigidly defined.
- It is based upon all the observations.
- Sampling fluctuation is less.
- It is not affected much by extreme values

Disadvantages of Harmonic Mean :

- It cannot be computed where there is any zero values in the series.
- It is not easily understood and difficult to compute.
- It is very complex for further algebrical treatments.
- It cannot be computed if the extreme classes of the frequency distribution are open.

Uses of Harmonic Mean :

- The harmonic mean is used when observations are made in terms of work done per hour, speeds (kilometers covered per hour), quantity of things purchased per taka etc.

Example 3.3 :

The frequency distribution of profit per share of 10 companies are given below -

Profit per share (Tk).	0-5	5-10	10-15	15-20	20-25
No. of companies	1	2	4	2	1

To calculate harmonic mean, we construct the following table :

Profit per share (Tk.)	Frequency f_i	Mid value x_i	f_i / x_i
0-5	1	2.5	0.4000
5-10	2	7.5	0.2667
10-15	4	12.5	0.3200
15-20	2	17.5	0.1143
20-25	1	22.5	0.0444
Total	10		1.1454

$$HM = \frac{N}{\Sigma f_i / x_i} = \frac{10}{1.1454} = 8.73$$

$\therefore$ The average profit per share is Tk. 8.73