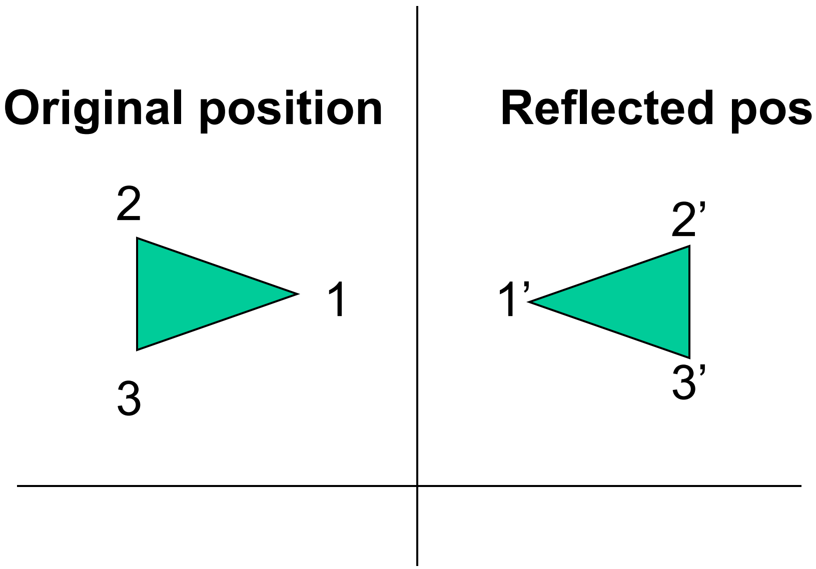
CSE- 4105

Lecture- 07

Transformation-III

2D and 3D Transformation

Reflection

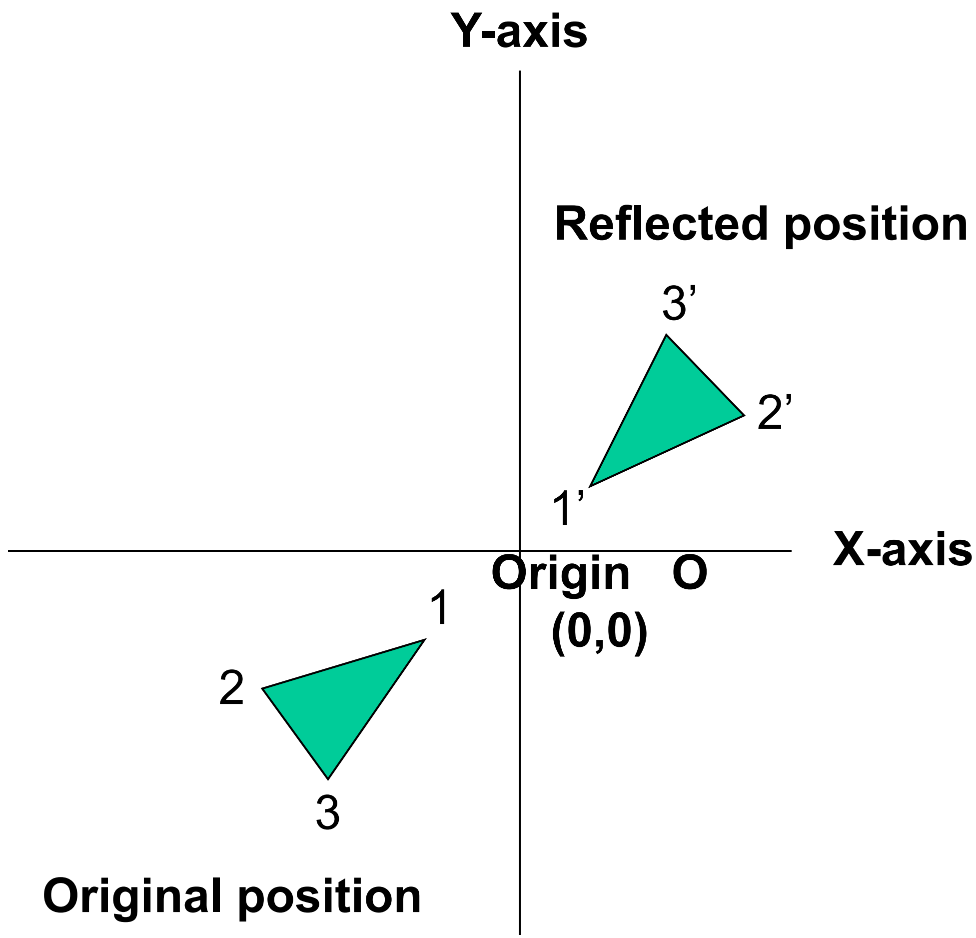


Reflection about the line x=0, the **Y- axis** , is accomplished with the transformation matrix

$$\begin{vmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Reflection

Reflection of an object relative to an axis perpendicular to the xy plane and passing through the coordinate origin



$$\begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

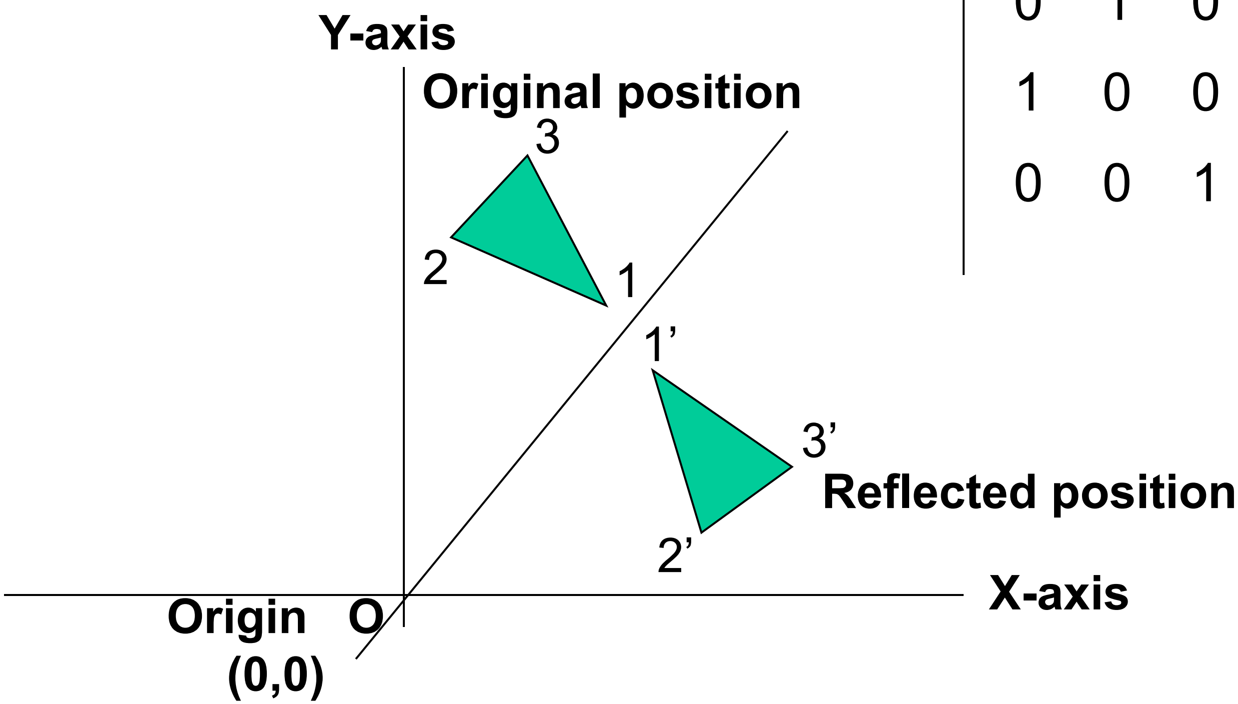
The above reflection matrix is the rotation matrix with angle=180 degree.

This can be generalized to any reflection point in the xy plane. This reflection is the same as a 180 degree rotation in the xy plane using the reflection point as the pivot point.

Reflection

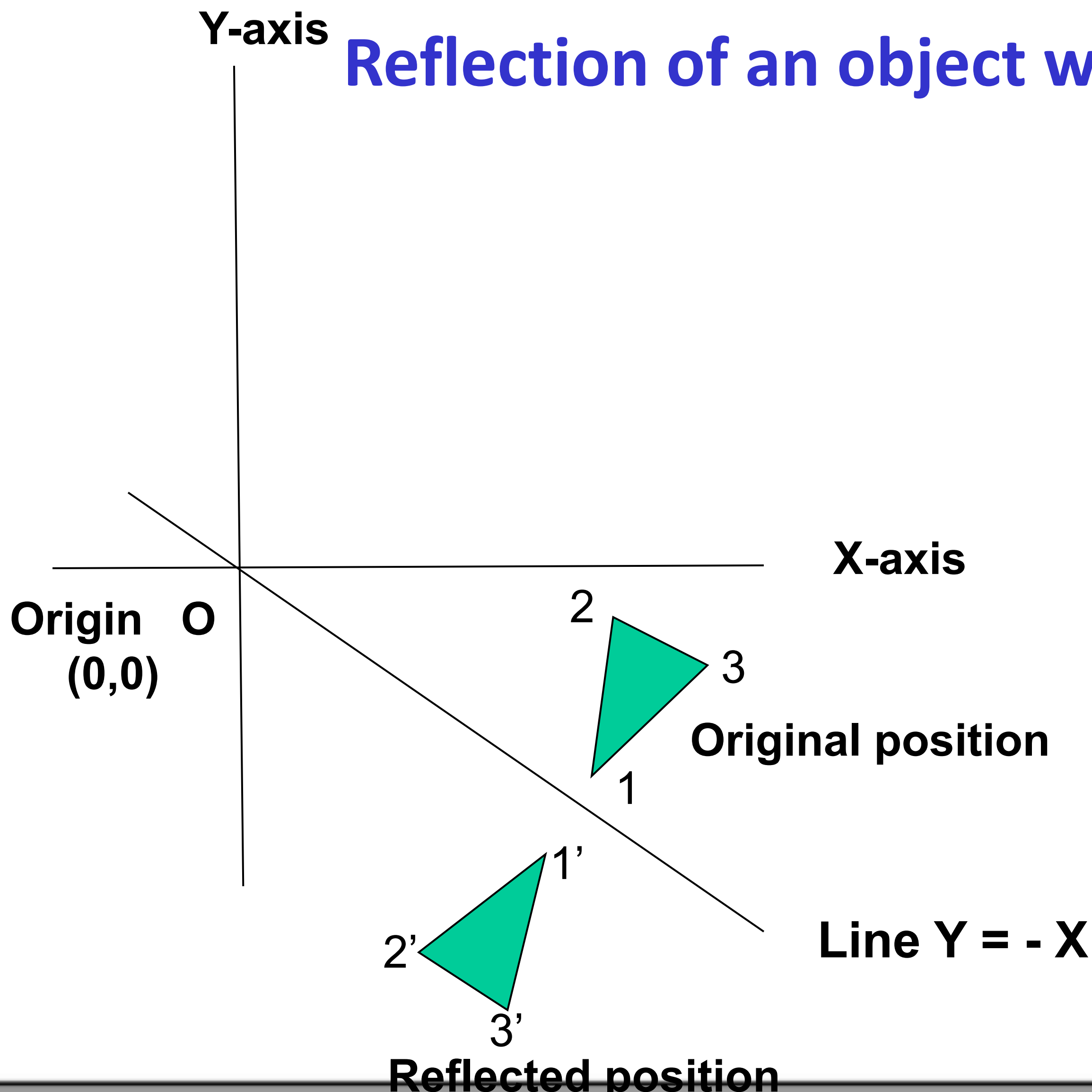
Reflection of an object w.r.t the straight line y=x

$$\begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$



Reflection

Reflection of an object w.r.t the straight line y=-x



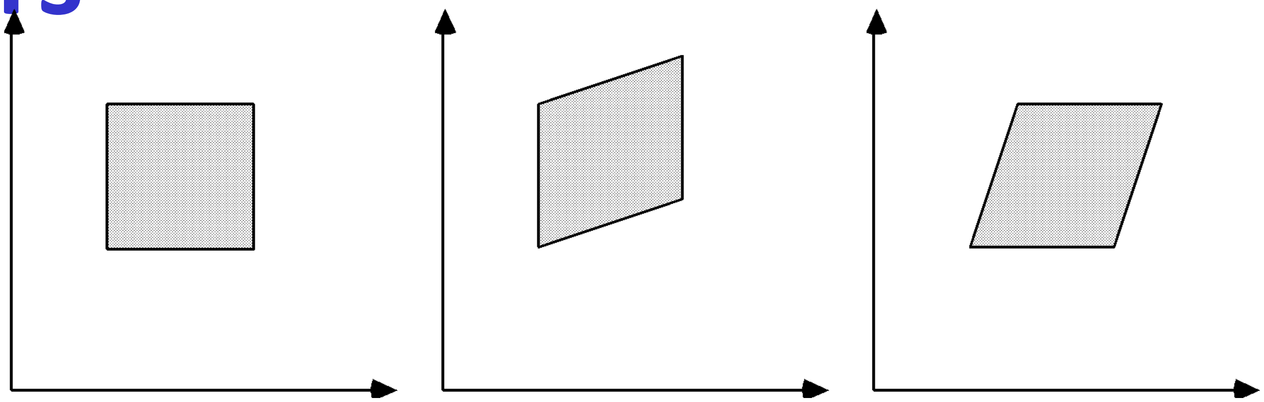
$$\begin{vmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Shear Transformations

- Shear is a transformation that distorts the shape of an object such that the transformed shape appears as if the object were composed of internal layers that had been caused to slide over each other.
- Two common shearing transformations are those that shift coordinate x values and those that shift y values.

Shearing

Shears

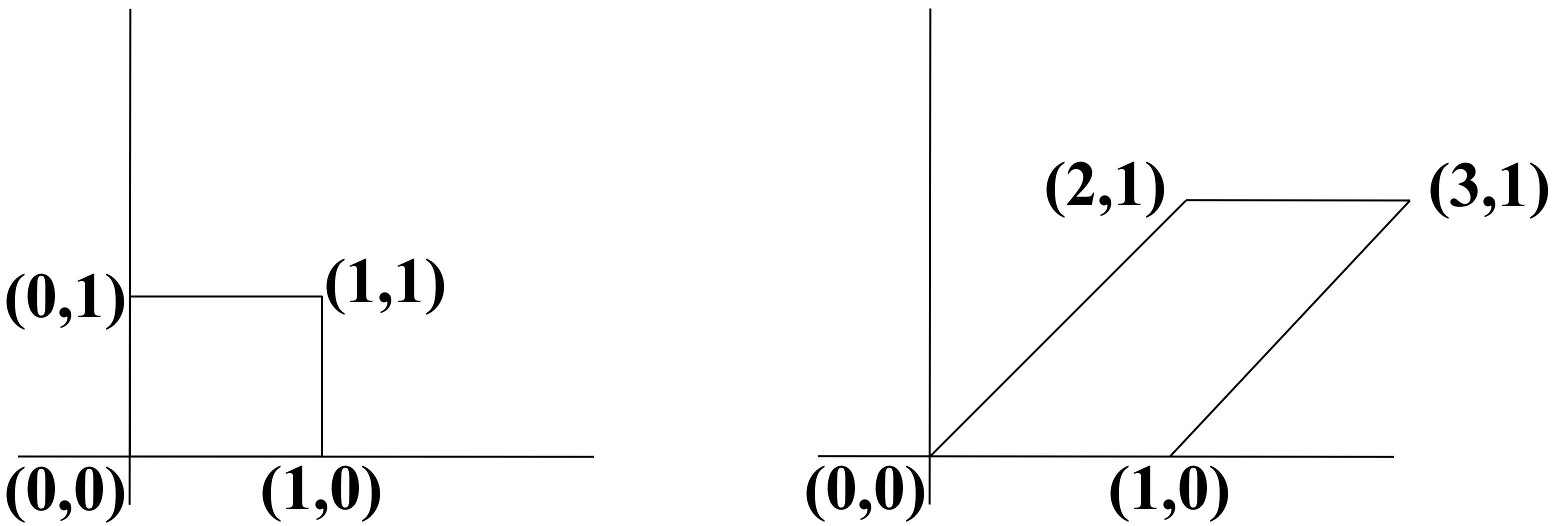


Original Data **y Shear** **x Shear**

Shearing

An X- direction Shear

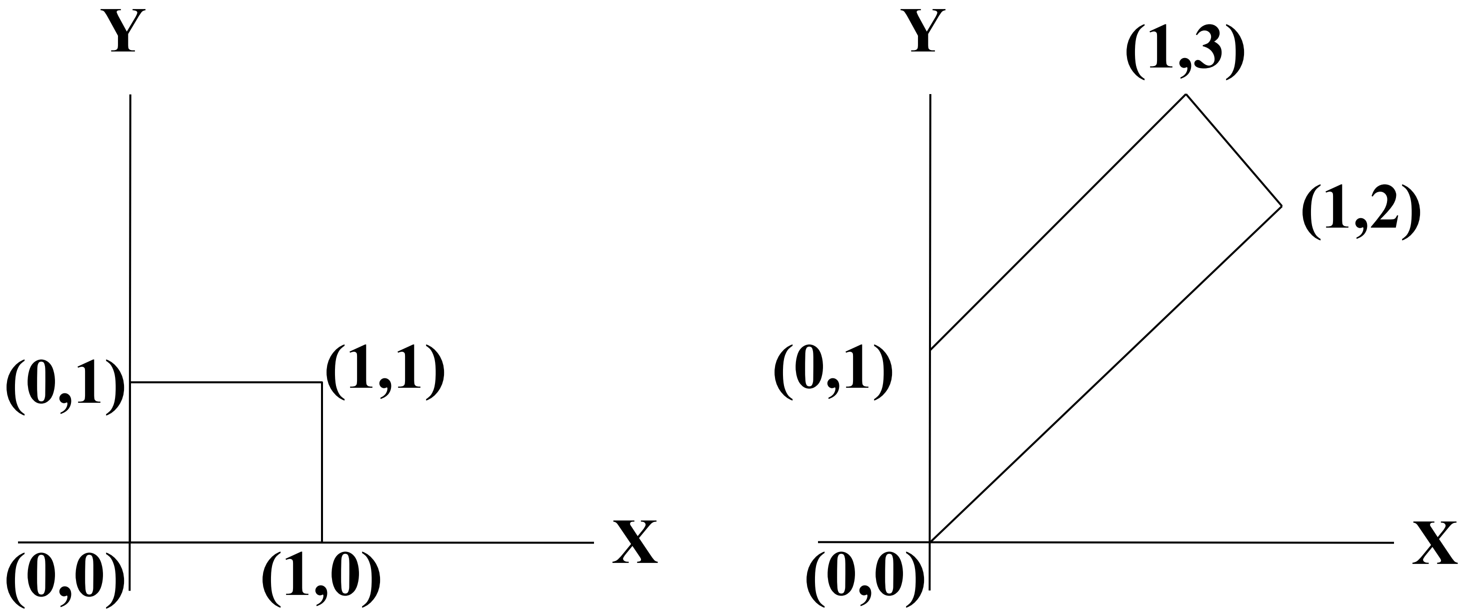
For example, $Sh_x=2$



Shearing

An Y- direction Shear

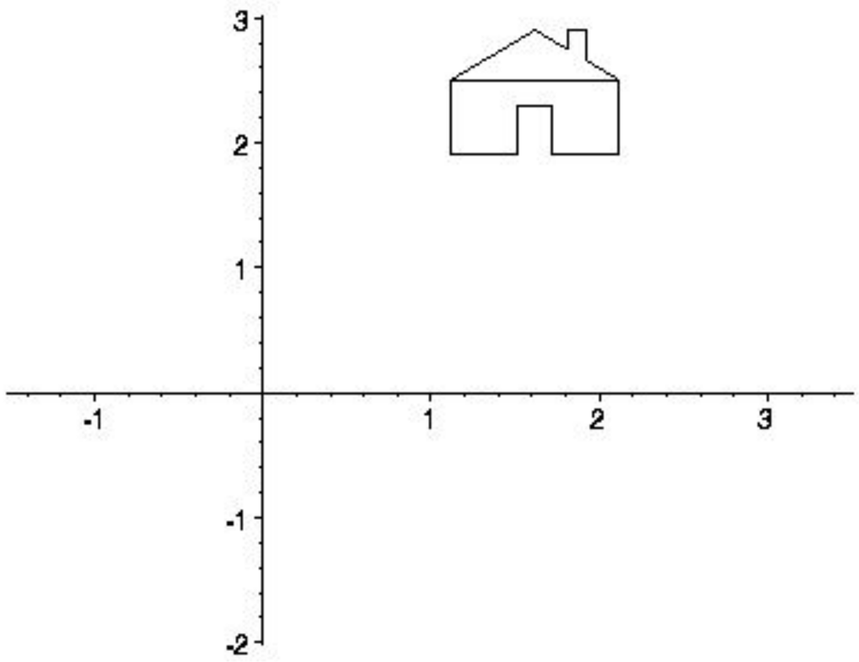
For example, $Sh_y=2$



3D TRANSFORMATION

Transformation

- Transformations are a fundamental part of the computer graphics. Transformations are the movement of the object in Cartesian plane .



Types of Transformation

- There are two types of transformation in computer graphics.
  1) 2D transformation
  2) 3D transformation
- Types of 2D and 3D transformation
  1) Translation
  2) Rotation
  3) Scaling
  4) Shearing
  5) Mirror reflection

Uses of Transformation

- Transformation are used to position objects , to shape object , to change viewing positions , and even how something is viewed.

- In simple words transformation is used for

  1) Modeling

  2) viewing

3D Transformation

- When the transformation takes place on a 3D plane, it is called 3D transformation.
- Generalize from 2D by including **z** coordinate

Straight forward for translation and scale, rotation more difficult

Homogeneous coordinates: 4 components

Transformation matrices: 4×4 elements

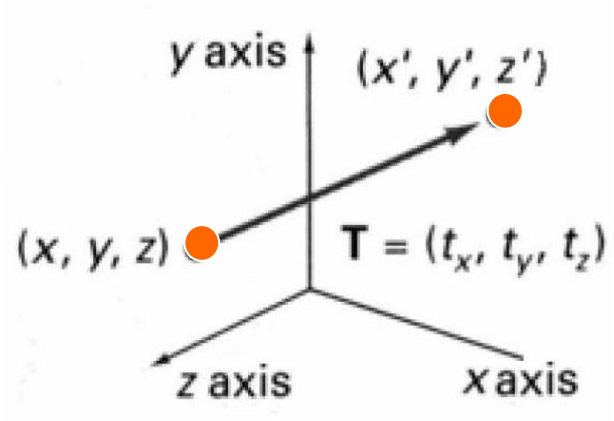
$$\begin{bmatrix} a & b & c & t_x \\ d & e & f & t_y \\ g & h & i & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3D Translation

- Moving of object is called translation.
- In 3 dimensional homogeneous coordinate representation , a point is transformed from position P = (x, y , z) to P’=(x’, y’, z’)
- This can be written as:-

Using **P’ = T . P**

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

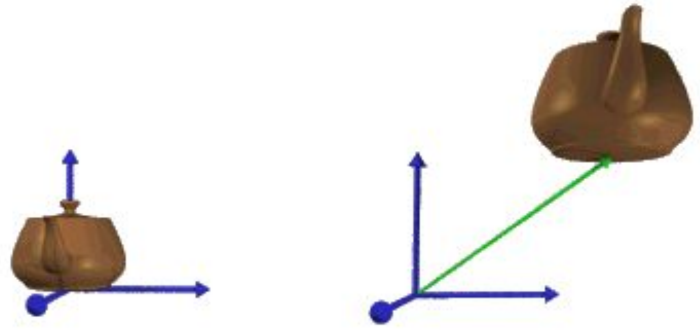


3D Translation

- The matrix representation is equivalent to the three equation.

$$x' = x + t_x, \quad y' = y + t_y, \quad z' = z + t_z$$

Where parameter t_x, t_y, t_z are specifying translation distance for the coordinate direction x , y , z are assigned any real value.

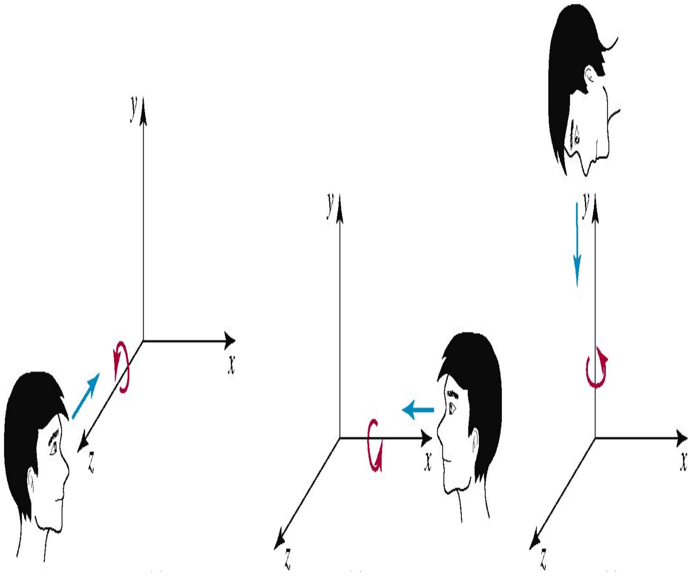


3D Rotation

Where an object is to be rotated about an axis that is parallel to one of the coordinate axis, we can obtain the desired rotation with the following transformation sequence.

Coordinate axis rotation

Z- axis Rotation(Roll)
Y-axis Rotation(Yaw)
X-axis Rotation(Pitch)



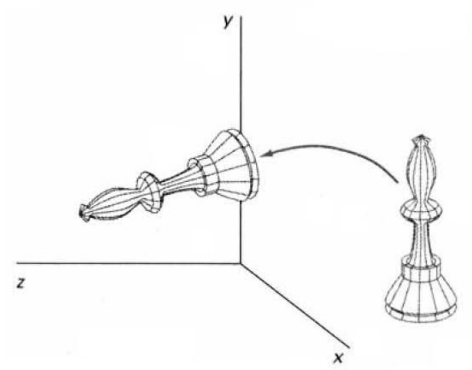
X-Axis Rotation

The equation for X-axis rotation

x' = x

y' = y cosθ – z sinθ

z' = y sinθ + z cosθ



$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Y-Axis Rotation

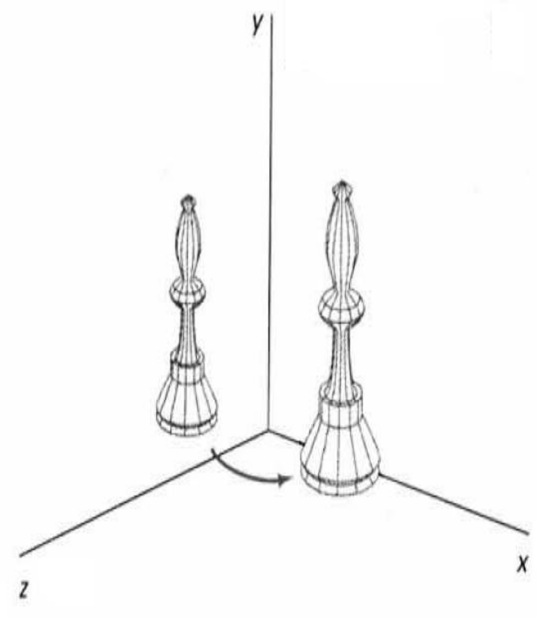
The equation for Y-axis rotation

$x' = x \cos\theta + z \sin\theta$

$y' = y$

$z' = z \cos\theta - x \sin\theta$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



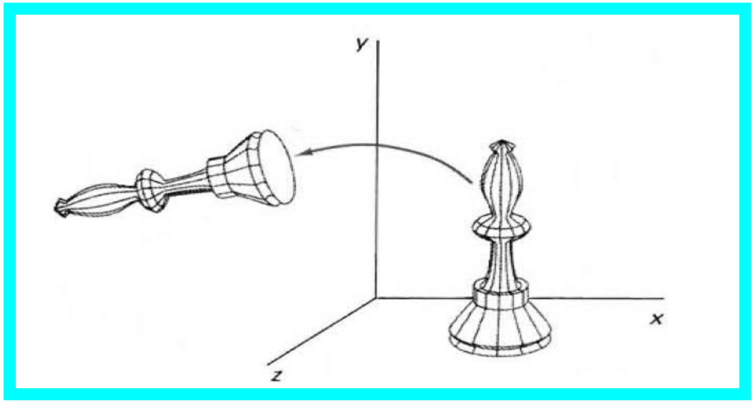
Z-Axis Rotation

The equation for Z-axis rotation

$x' = x \cos\theta - y \sin\theta$

$y' = x \sin\theta + y \cos\theta$

$z' = z$

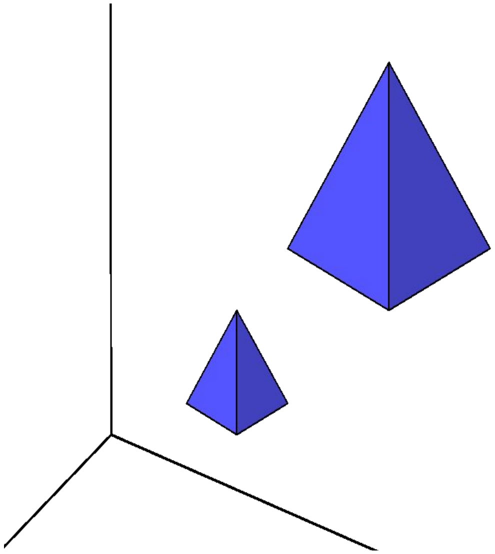


$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3D Scaling

- Changes the size of the object and repositions the object relative to the coordinate origin.

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



3D Scaling

- The equations for scaling

$$S_{sx,sy,sz} \Rightarrow \begin{cases} x' = x \cdot sx \\ y' = y \cdot sy \\ z' = z \cdot sz \end{cases}$$

3D Reflection

- Reflection in computer graphics is used to emulate reflective objects like mirrors and shiny surfaces
- Reflection may be an x-axis

y-axis , z-axis and also in
the planes xy-plane,yz-plane , and
zx-plane.

Reflection relative to a given
Axis are equivalent to 180
Degree rotations

3D Reflection

- Reflection about x-axis:-

x'=x y'=-y z'=-z

$$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

Reflection about y-axis:-

y'=y x'=-x z'=-z

3D Reflection

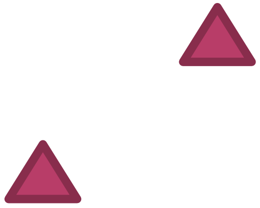
- The matrix for reflection about y-axis:-

$$\begin{matrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

- Reflection about z-axis:-

x’=-x y’=-y z’=z

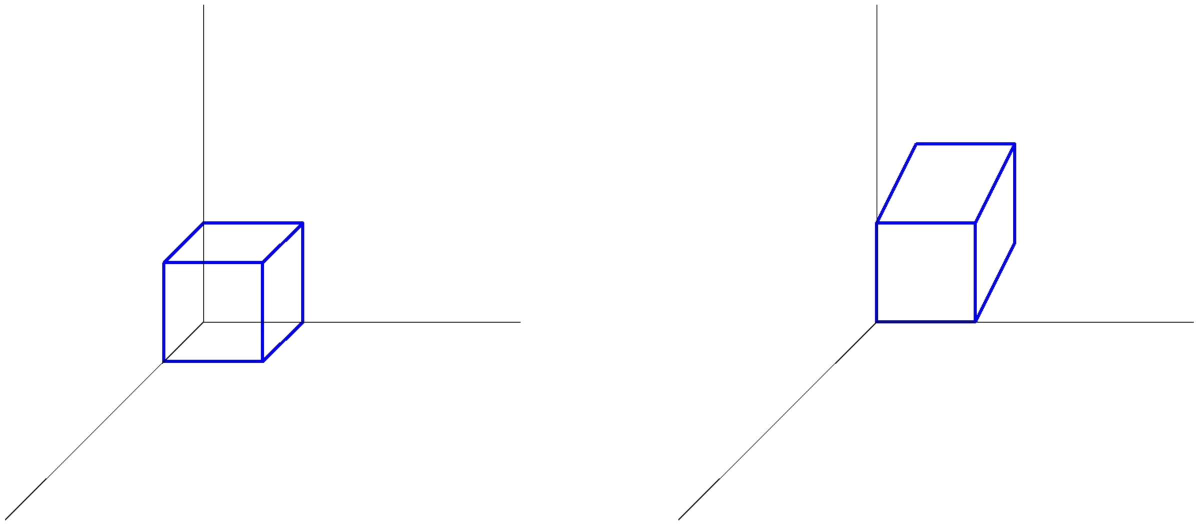
$$\begin{matrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$



3D Shearing

- **Modify object shapes**
- **Useful for perspective projections**
- When an object is viewed from different directions and at different distances, the appearance of the object will be different. Such view is called perspective view. Perspective projections mimic what the human eyes see.

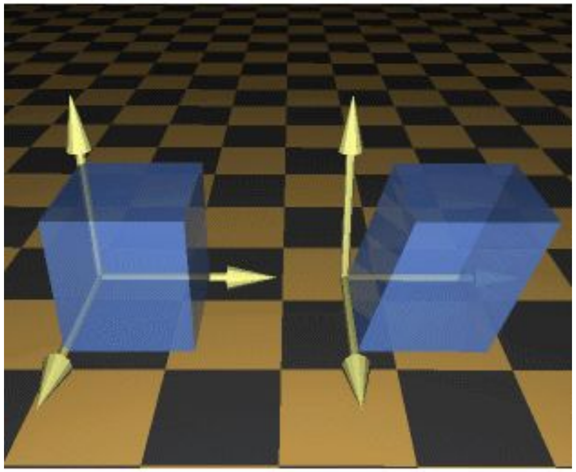
3D Shearing



3D Shearing

- Matrix for 3d shearing
- Where a and b can
  Be assigned any real
  Value.

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & a & 0 \\ 0 & 1 & b & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



3D Shearing

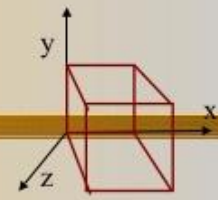
- In (y, z) w.r.t. x value $SH_{yz} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ sh_y & 1 & 0 & 0 \\ sh_z & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

- In (z, x) w.r.t. y value $SH_{xz} = \begin{bmatrix} 1 & sh_x & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & sh_z & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

- In (x, y) w.r.t. z value $SH_{xy} = \begin{bmatrix} 1 & 0 & sh_x & 0 \\ 0 & 1 & sh_y & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

3D Shearing

Shear along Z-axis



$$SH_{xy}(sh_x, sh_y) \quad * \quad P \quad = \quad P'$$

$$\Downarrow \qquad\qquad \Downarrow \qquad\qquad \Downarrow$$

$$\begin{bmatrix} 1 & 0 & sh_x & 0 \\ 0 & 1 & sh_y & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x + z * sh_x \\ y + z * sh_y \\ z \\ 1 \end{bmatrix}$$

Thank You