11/6/21

Review Class (1st sem)

Physics (8 টা থেকে 5 টা Ans করতে হবে)

Waves and Oscillation (4 টা)

[SHM, Differential eq^n SHO, Total energy and average energy. ...... all from syllabus] (~~exc~~ বাদ যাবে Architectural acoustics, Reverberation and sabine's formula)

[Wave and Oscillation এ class এ যতটুকু ততটুকুই]

Propertikes of Matter (2টি ques)

- Crystaline and non-crystaline solids [Definition + Example]
- Unit cell [definition]
- Lattice and basis [definition]

(*) RC Circuit [ques must]

→ eq^n, current বের করা।।

charging, discharging

(*) Maxwell eq^n; (diff form, integral form, significance)
[প্রমাণ মুখস্থ না, কন্টিনিউটি, pointing vector]

(*) Induction, Induction এর ক্ষেত্রে Farad's Law

ছবি এঁকে explain

[1 টা para তে আছে]

- Crystal Planes and Miller Indices [ভালোভাবে]
- Interplaner spacing (Mathematical eq^n)
- NaCl এর Structure [Crystal structure]
- Bragg's Law
- Bonding Energy of NaCl [+ এ Type এর Math sheet]
- Band Theory-র মাধ্যমে Metal, semiconductor ও insulator এর মধ্যে পার্থক্য।

Electricity and Magnetism (2 টা ques)

* Definition

Fields, Potential, Electric Field, Electric Field intensity, Dielectrics, Capacitor

** Capacitor, Capacitance, Capacitance বের করা [ques must]

~~সিলি~~ সিলিন্ডার, সার্কেল, গোলক

Differential equation of wave motion (From general eqⁿ of a progressive wave):

The general equation of a progressive simple harmonic wave is

$$y = a \sin \frac{2\pi}{\lambda}(vt - x) \quad \text{——— (I)}$$

Differentiating eqⁿ (I) with respect to time

$$\frac{dy}{dt} = a \frac{2\pi v}{\lambda} \cos \frac{2\pi}{\lambda}(vt - x) \quad \text{——— (II)}$$

Again differentiating eqⁿ (II) with respect to time

$$\frac{d^2y}{dt^2} = -a \frac{2\pi v}{\lambda} \cdot \frac{2\pi v}{\lambda} \sin \frac{2\pi}{\lambda}(vt - x)$$

$$= -a \frac{4\pi^2 v^2}{\lambda^2} \sin \frac{2\pi}{\lambda}(vt - x) \quad \text{——— (III)}$$

To find the value of compression, differentiating eqⁿ (I) with respect to x

$$\frac{dy}{dx} = -a \frac{2\pi}{\lambda} \cos \frac{2\pi}{\lambda}(vt - x) \quad \text{——— (IV)}$$

differentiating eqⁿ (IV) with respect to x

$$\frac{d^2y}{dx^2} = -\frac{4\pi^2 a}{\lambda^2} \sin \frac{2\pi}{\lambda}(vt - x) \quad \text{——— (V)}$$

From eqⁿs (II) and (IV) we get

$$\frac{dy}{dt} = -v \frac{dy}{dx} \quad \text{——— (VI)}$$

From equations (III) and (V) we get

$$\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2} \quad \text{——— (VII)}$$

Equation (VII) represent the differential equation of wave motion.

The general differential equation of wave motion can be written as

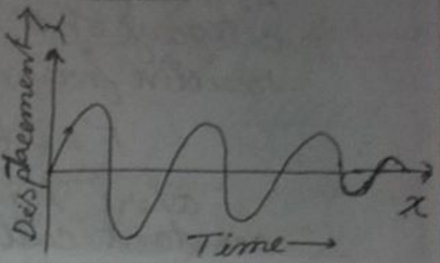
$$\frac{d^2y}{dt^2} = K \frac{d^2y}{dx^2} \quad \text{——— (VIII)}$$

where $K = v^2$ and $v = \sqrt{K}$

Thus knowing the value of K, the value of v can be calculated.

DAMPED VIBRATION / DAMPED OSCILLATIONS :

In actual particle a simple harmonic oscillator almost always vibrates in a ~~rest~~ resisting medium. Consequently when the oscillator vibrates in such a medium energy is dissipated in each vibration, therefore goes on decreasing progressively with time. Such forces, which are non-conservative in nature have thus a damping effect on the oscillation.



Damping Co-efficient and differential equation of a damping harmonic oscillator

A body executing simple harmonic oscillations in a damping medium will be simultaneously subjected to the following two opposing forces:

(i) The restoring force acting on the body which is proportional to the displacement of the body and acts in a direction opposite to the displacement. Let this force $-ay$, where a is a force constant.

(ii) a resistative or damping force show by Mayevski that at ordinary velocities, the opposing resistive or damping force is, to a first approximation, proportional to the velocity of the oscillating body.

Particle velocity and wave velocity :

The equation for a simple harmonic wave is given by

$$y = a \sin \frac{2\pi}{\lambda}(vt - x) \quad \text{——— (1)}$$

Differentiating equation (1) with respect to time, we get,

$$\frac{dy}{dt} = \text{Particle velocity} = u = \frac{2\pi a v}{\lambda} \cos \frac{2\pi}{\lambda}(vt - x) \quad \text{——— (2)}$$

The maximum particle velocity $u_{max} = \frac{2\pi a v}{\lambda}$ ——— (3)

Maximum particle velocity $= \frac{2\pi a}{\lambda} \times$ wave velocity.

Particle acceleration: To find the particle acceleration differentiating eqn (2) with respect to time we get,

$$f = \frac{d^2y}{dt^2} = -\frac{4\pi^2 a v^2}{\lambda^2} \sin \frac{2\pi}{\lambda}(vt - x)$$

$$= -\frac{4\pi^2 v^2}{\lambda^2}\left[a \sin \frac{2\pi}{\lambda}(vt - x)\right]$$

$$= -\frac{4\pi^2 v^2}{\lambda^2} y$$

Maximum acceleration will be when $y = a$

$$\therefore f_{max} = -\frac{4\pi^2 v^2}{\lambda^2} \cdot a$$

The negative sign shows that the acceleration of the particle is directed towards its mean position.

Prob: A simple harmonic wave of amplitude 8 units tranverses a line of particles in direction of the positive X-axis. At any instant of time, for a particle at a distance of 10 cm from the origin, the displacement is +6 units, and for a particle at a distance of 25 cm from the origin, the displacement is 4 units, calculate the wavelength.

Soln: We know $y = a \sin \frac{2\pi}{\lambda}(vt - x)$

or, $\frac{y}{a} = \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$

(1) In the first case, $\frac{y_1}{a} = \sin 2\pi\left(\frac{t}{T} - \frac{x_1}{\lambda}\right)$

Here $y_1 = +6$, $a = 8$, $x_1 = 10$ cm

$$\therefore \frac{6}{8} = \sin 2\pi\left(\frac{t}{T} - \frac{10}{\lambda}\right) \quad \text{——— (1)}$$

Energy of a body Executing SHM:

Total Energy:

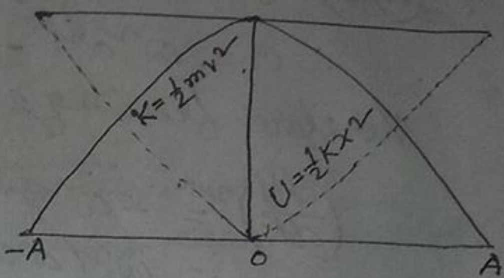


Fig: Energy of SHM

If K be the Kinetic energy and U be the potential energy of a particle executing SHM. Then the total energy

$$E = K+U \quad \text{------ (i)}$$

Let the displacement of a particle executing SHM at any instant of y. If the mass of the particle be m and velocity v, then its kinetic energy is $\frac{1}{2}mv^2$. The potential energy is the amount of work that must be done in over-coming the force through a displacement y and is given by the relation $\int_0^y F.dy$ where F is the force required to maintain the displace-ment and dy is a small displacement.

Now the displacement is given by the relation, $y = a\sin(\omega t+\phi)$

Therefore velocity $\frac{dy}{dt} = a\omega\cos(\omega t+\phi)$ and the acceleration

$$\frac{d^2y}{dt^2} = -a\omega^2\sin(\omega t+\phi) = -\omega^2 a\sin(\omega t+\phi)$$

$$\therefore \frac{d^2y}{dt^2} = -\omega^2 y$$

Then force F = mass × acceleration

$= m(-\omega^2 y) = -m\omega^2 y$

Then, the potential energy of the particle is –

$$P.E = \int_0^y F.dy = \int_0^y m\omega^2 y\, dy = m\omega^2\int_0^y y\,dy$$

$$= m\omega^2 . \frac{y^2}{2} = \frac{1}{2}m\omega^2 y^2 = \frac{1}{2}m\omega^2 a^2\sin^2(\omega t+\phi)$$

$$= \frac{1}{2}Ka^2\sin^2(\omega t+\phi) \quad \text{------ (ii)} \quad [\because \omega^2 = \frac{K}{m}]$$

② In the second case,

$$\frac{y_2}{a} = \sin 2\pi\left(\frac{t}{T} - \frac{x_2}{\lambda}\right)$$

$y_2 = +4$, $a = 8$, $x_2 = 25$

$$\therefore \frac{4}{8} = \sin 2\pi\left(\frac{t}{T} - \frac{25}{\lambda}\right) \text{ ——— } ②$$

From equation ①

$$0.75 = \sin 2\pi\left(\frac{t}{T} - \frac{10}{\lambda}\right)$$

But $0.75 = \sin\left(\frac{48.6\pi}{180}\right)$

$$2\pi\left(\frac{t}{T} - \frac{10}{\lambda}\right) = \frac{48.6\pi}{180}$$

$$\text{or, } \frac{t}{T} - \frac{10}{\lambda} = \frac{48.6}{360} \text{ ——— } ③$$

From equation ② we get

$$0.5 = \sin 2\pi\left(\frac{t}{T} - \frac{25}{\lambda}\right)$$

But $\sin\frac{\pi}{6} = 0.5$

$$\therefore \sin 2\pi\left(\frac{t}{T} - \frac{25}{\lambda}\right) = \sin\frac{\pi}{6}$$

$$\therefore 2\pi\left(\frac{t}{T} - \frac{25}{\lambda}\right) = \frac{\pi}{6}$$

$$\text{or, } \frac{t}{T} - \frac{25}{\lambda} = \frac{1}{12} \text{ ——— } ④$$

~~Substituting~~ ④ from ③ we get,
Substructing

$$\frac{25}{\lambda} - \frac{10}{\lambda} = \frac{48.6}{360} - \frac{1}{12}$$

$$\text{or, } \frac{25-10}{\lambda} = \frac{48.6-30}{360}$$

$$\text{or, } \frac{15}{\lambda} = \frac{18.6}{360}$$

$$\therefore \lambda = \frac{360 \times 15}{18.6} = 290.8 \text{ cm} \quad \text{Ans.}$$

So the average energy of the particle over a complete cycle

$$= \frac{1}{T}\int_0^T \frac{1}{2} m\omega^2 a^2 \sin^2(\omega t+\phi)\,dt$$

$$= \frac{1}{T}\cdot\frac{m\omega^2 a^2}{4}\int_0^T 2\sin^2(\omega t+\phi)\,dt$$

$$= \frac{m\omega^2 a^2}{4T}\int_0^T [1-\cos 2(\omega t+\phi)]\,dt$$

$$= \frac{m\omega^2 a^2}{4T}\left[\int_0^T dt - \int_0^T \cos 2(\omega t+\phi)\,dt\right]$$

The average value of both a sine and a cosine function for a complete cycle is zero.

Therefore $P.E = \frac{1}{4T} m\omega^2 a^2 [t]_0^T - 0$

$$= \frac{1}{4T} m\omega^2 a^2 T = \frac{1}{4} m\omega^2 a^2$$

$$= \frac{1}{4} K a^2 \quad \left[\because \omega^2 = \frac{K}{m}\right]$$

The kinetic energy of the particle at displacement y is given by. $KE = \frac{1}{2} m\left(\frac{dy}{dt}\right)^2 = \frac{1}{2} m\left[\frac{d}{dt}(a\sin(\omega t+\phi))\right]^2$

$$= \frac{1}{2} m\omega^2 a^2 \cos^2(\omega t+\phi)$$

The average KE of the particle for a complete cycle $= \frac{1}{T}\int_0^T \frac{1}{2} m\omega^2 a^2 \cos^2(\omega t+\phi)\,dt$

$$= \frac{1}{T}\frac{m\omega^2 a^2}{4}\int_0^T \{1+\cos 2(\omega t+\phi)\}\,dt$$

$$= \frac{m\omega^2 a^2}{4T}\left[\int_0^T dt + \int_0^T \cos 2(\omega t+\phi)\,dt\right]$$ The average value of both sine and cosine function over a complete cycle is zero.

Average $K.E = \frac{m\omega^2 a^2}{4T}\cdot T = \frac{1}{4} m\omega^2 a^2 = \frac{1}{4} K a^2$

10-A

[redacted] -BME
11.03.16

Prob: The scale of a spring balance reading from 0–10kg is 0.25m. A body suspended from the balance oscillates with a frequency of $\frac{10}{\pi}$ Hz. Calculate the mass of the body attached to the spring.

Sol^n: We know,

Here, mass $m = 10kg$
Length increase $x = 0.25m$
acceleration due to gravity $g = 9.8 m/s^2$

$$T = 2\pi\sqrt{\frac{Mx}{mg}}$$

Again $T = \frac{1}{n}$

$$\therefore \frac{1}{n} = 2\pi\sqrt{\frac{Mx}{mg}}$$

$$\text{or, } \frac{1}{n^2} = 4\pi^2\frac{Mx}{mg}$$

$$\therefore M = \frac{mg}{4\pi^2 n^2 x} = \frac{10\times 9.8}{4\times(3.14)^2\times\left(\frac{10}{3.14}\right)^2\times 0.25}$$

$$M = 0.98\,kg \text{ Ans.}$$

⊛ Calculation of Minimum Time period of compound Pendulum:

The Time period of a compound Pendulum

$$t = 2\pi\sqrt{\frac{K^2+l_1^2}{l_1 g}} \quad \text{—— (i)}$$

Here l_1 is the distance of the point of suspension from the centre of gravity and K is the radius of gyration about the C.G. Suppose the distance of the point of oscillation from the C.G is l_2

$$\therefore \frac{K^2}{l_1} = l_2$$

$$\text{or, } K^2 = l_1 l_2 \quad \text{—— (ii)}$$

For the time period to be minimum, the value of $\left(\frac{K^2}{l_1}+l_1\right)$ should be minimum,

Differentiating $\left(\frac{K^2}{l_1}+l_1\right)$ with respect to l_1

$$\frac{d}{dl_1}\left(\frac{K^2}{l_1}+l_1\right) = -\frac{K^2}{l_1^2}+1$$

Let the Periodic force to which a damped harmonic oscillator is subjected be $F = F_0 \sin pt$. F_0 is amplitude and frequency $P/2\pi$.

Now the damping and restoring force on the oscillator are $-b\frac{dy}{dt}$ and $-ay$ respectively where b and a have the same meanings. Hence the equation of motion may be written as

$$m\frac{d^2y}{dt^2} = -b\frac{dy}{dt} - ay + F$$

or, $$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ay = F_0 \sin pt$$

or, $$\frac{d^2y}{dt^2} + \frac{b}{m}\frac{dy}{dt} + \frac{a}{m}y = \frac{F_0}{m}\sin pt$$

or, $$\frac{d^2y}{dt^2} + 2\lambda\frac{dy}{dt} + \omega^2 y = f_0 \sin pt \quad \text{——— (1)}$$

Where $2\lambda = \frac{b}{m}$, $\omega^2 = \frac{a}{m}$ and $f_0 = \frac{F}{m}$

After the steady state has been attained be

$$Y = A \sin (Pt - \theta) \quad \text{——— (11)}$$

Where A is the amplitude and θ is the possible phase difference between the applied force and the displacement of the oscillator, Then we have

$$\frac{dy}{dt} = AP \cos (Pt - \theta)$$

and $$\frac{d^2y}{dt^2} = -AP^2 \sin (Pt - \theta)$$

Simple Harmonic Motion:

Whenever a force acting on a particle and hence the acceleration of the particle is proportional to its displacement from the equilibrium position or any other fixed point to its path, but it is always directed in a direction opposite to the direction of the displacement and if the maximum displacement of the particle is the same on either side of the mean position, the particle is said to execute a simple harmonic motion.

Differential Equation of a Simple Harmonic Motion:

If F be the force acting on a particle and y its displacement from its equilibrium position, then $F = -Ky$ —— (i)

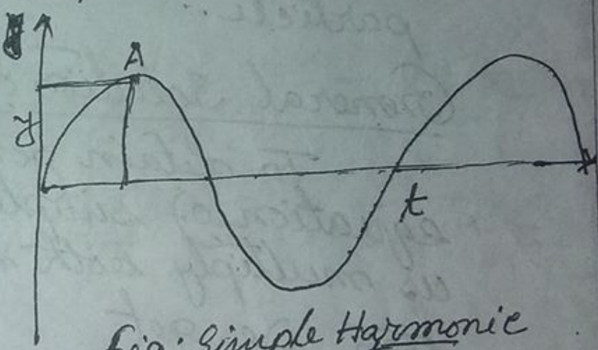


Fig: Simple Harmonic Motion.

According to Newton's second law of motion,

$F = ma'$ —— (ii)

From eqn (i) and (ii) we get

$ma' = -Ky$

or, $a' = \frac{-Ky}{m}$ —— (iii)

where K is a force constant.

The acceleration in differential form

$a' = \frac{d^2y}{dt^2}$, and hence we can write

$-Ky = m\frac{d^2y}{dt^2}$ $\quad (\because F = ma')$

Composition of two SHV at right angle to each other having equal frequencies but different phase and amplitude.

Let us consider two simple harmonic motions of the same frequencies but of amplitude of a and b, and having their vibrations mutually perpendicular to one another.

If ϕ is the phase difference between two motions, Then the equations can be written as

$$x = a \sin(\omega t + \phi) \quad \text{——— (1)}$$

$$y = b \sin \omega t \quad \text{——— (2)}$$

From eqn (1), $\frac{x}{a} = \sin(\omega t + \phi)$

$$= \sin \omega t \cos\phi + \cos \omega t \sin\phi$$

$$= \sin \omega t \cos\phi + \sqrt{1 - \sin^2 \omega t}\, \sin\phi \quad \text{——— (iii)}$$

From eqn (ii) $\Rightarrow \sin \omega t = \frac{y}{b}$ ——— (iv)

So from eqn (iii) $\Rightarrow$ $\frac{x}{a} = \frac{y}{b}\cos\phi + \sqrt{1 - \frac{y^2}{b^2}}\, \sin\phi$

$$\Rightarrow \left(\frac{x}{a} - \frac{y}{b}\cos\phi\right) = \sqrt{1 - \frac{y^2}{b^2}}\, \sin\phi$$

squaring both sides, we get

$$\frac{x^2}{a^2} + \frac{y^2}{b^2}\cos^2\phi - 2\cdot\frac{x}{a}\cdot\frac{y}{b}\cos\phi = \left(1 - \frac{y^2}{b^2}\right)\sin^2\phi$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2}\cos^2\phi - 2\frac{xy}{ab}\cos\phi = \sin^2\phi - \frac{y^2}{b^2}\sin^2\phi$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2}(\cos^2\phi + \sin^2\phi) - 2\frac{xy}{ab}\cos\phi = \sin^2\phi$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} - 2\frac{xy}{ab}\cos\phi = \sin^2\phi$$

This is the general equation of conic whose shape will depend upon the value of the phase difference between the two vibrations.

* Show that for a particle executing SHM, the instantaneous velocity is $\omega\sqrt{a^2-y^2}$ and instantaneous acceleration is $-\omega^2 y$.

Soln: For a particle executing SHM

we know, $y = a \sin(\omega t+\phi)$ ——— (I)

The instantaneous velocity

$$V = \frac{dy}{dt} = \frac{d}{dt}[a\sin(\omega t+\phi)] = a\cos(\omega t+\phi)\cdot\omega$$

$$= a\omega\cos(\omega t+\phi) \quad \text{——— (II)}$$

From eqn (I) we can write,

$$\sin(\omega t+\phi) = \frac{y}{a}$$

$$\sin^2(\omega t+\phi) = \frac{y^2}{a^2} \quad \text{——— (III)}$$

Again we can write,

$$\sin^2(\omega t+\phi)+\cos^2(\omega t+\phi) = 1$$

$$\Rightarrow \cos^2(\omega t+\phi) = 1-\sin^2(\omega t+\phi)$$

$$\Rightarrow \cos^2(\omega t+\phi) = 1-\frac{y^2}{a^2}$$

$$\Rightarrow \cos(\omega t+\phi) = \sqrt{1-y^2/a^2}$$

$$\Rightarrow \cos(\omega t+\phi) = \sqrt{\frac{a^2-y^2}{a^2}} = \frac{\sqrt{a^2-y^2}}{a}$$

$$\Rightarrow a\cos(\omega t+\phi) = \sqrt{a^2-y^2}$$

Since $V = a\omega\cos(\omega t+\phi)$

$\therefore V = \omega\cdot a\cos(\omega t+\phi) = \omega\cdot\sqrt{a^2-y^2}$ (showed)

Instantaneous acceleration,

$$\alpha = \frac{dv}{dt} = \frac{d}{dt}\{a\omega\cos(\omega t+\phi)\}$$

$$= -a\omega\sin(\omega t+\phi)\cdot\omega$$

$$= -a\omega^2\sin(\omega t+\phi)$$

$$\therefore a' = -\omega^2 a\sin(\omega t+\phi)$$

$$a' = -\omega^2 y \quad \because y = a\sin(\omega t+\phi)$$

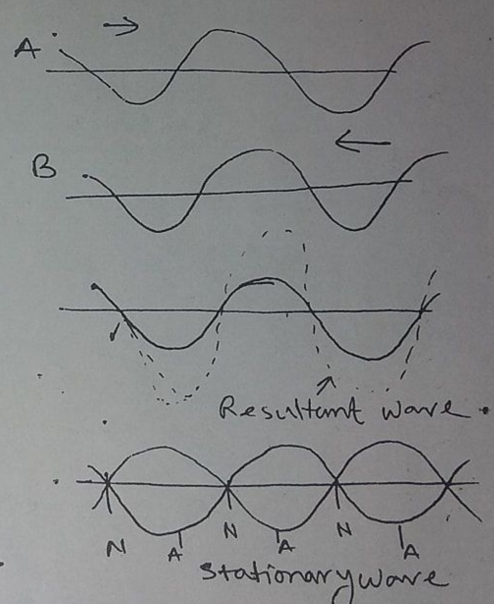
(showed)

Stationary waves :

When two simple harmonic waves of the same amplitude, frequency and time period travel in opposite directions in a straight line, the resultar wave obtained is called a stationary or a standing wave.

Stationary waves are formed in an open end organ pipe or a closed end organ pipe.

Stationary waves are also formed with a stretched string fixed at one end and free at the other end or fixed at the other end.



N → Node point
A → Anti node point

Fig → ①

Formation of Stationary wave :

Consider a simple harmonic wave given by the equation,

$$Y_1 = a \sin \frac{2\pi}{\lambda}(vt - x) \quad \text{——— (1)}$$

The displacement of the same particle at the same instant due to the reflected wave is given by

$$Y_2 = a \sin \frac{2\pi}{\lambda}(vt + x) \quad \text{——— (11)}$$

The resultant displacement of the wave

$$Y = Y_1 + Y_2$$

or, $$Y = a \sin \frac{2\pi}{\lambda}(vt - x) + a \sin \frac{2\pi}{\lambda}(vt + x)$$

$$= a\left\{ \sin \frac{2\pi}{\lambda}(vt - x) + \sin \frac{2\pi}{\lambda}(vt + x) \right\}$$

$$= a\left\{ 2 \sin \frac{2\pi}{\lambda}\left(\frac{vt - x + vt + x}{2}\right) \cos \frac{2\pi}{\lambda}\left(\frac{vt - x + vt + x}{2}\right) \right\}$$

$$= -a\left\{ 2 \sin \frac{2\pi}{\lambda} x \cos \frac{2\pi vt}{\lambda} \right\}$$

$$\cancel{= 2a \cos \frac{2\pi x}{\lambda} \sin \frac{2\pi vt}{\lambda}}$$

$$= -2a \sin \frac{2\pi x}{\lambda} \cos \frac{2\pi vt}{\lambda}$$

$$= A \cos \frac{2\pi vt}{\lambda}$$

Where A is the resultant amplitude of stationary wave, and $A = -2a \sin \frac{2\pi x}{\lambda}$

Now, eqn (2) divided by eqn (1) we get.

$$\frac{f_0 \sin\theta}{f_0 \cos\theta} = \cancel{\frac{A(\omega^2 - p^2)}{2\lambda A p}} = \frac{2\lambda A p}{A(\omega^2 - p^2)}$$

$$\Rightarrow \tan\theta = \frac{2\lambda A p}{A(\omega^2 - p^2)}$$

$$\theta = \tan^{-1}\left(\frac{2\lambda A p}{A(\omega^2 - p^2)}\right)$$

Thus the phase difference between the driven or forced oscillator and the applied force is —

$$\theta = \tan^{-1}\left(\frac{2\lambda A p}{A(\omega^2 - p^2)}\right) \quad \text{(vii)}$$

this value should be zero, for the time period to be minimum

$$-\frac{K^2}{l_1^2}+1=0$$

or, $\frac{K^2}{l_1^2}=1$

$\therefore K^2=l_1$

But $K^2=l_1 l_2$

$\therefore l_1^2=l_1 l_2$

$\therefore l_1=l_2$ ——— (IV)

It means that the time period will be minimum when the points of suspension and oscillation are equidistant from the centre of the gravity.

Ex: A uniform circular disc of radius 10cm vibrate about a horizontal axis perpendicular to its plane and at a distance of 5cm from the centre. Calculate the time period of oscillation and the equivalent length of the simple pendulum.

We Know

$$t=2\pi\sqrt{\frac{K^2+l_1^2}{l_1 g}}$$

$$=2\pi\sqrt{\frac{50+25}{5\times 980}}$$

$= 0.782$ sec

Equivalent length of simple pendulum $=L$

$$L=\frac{K^2+l_1^2}{l_1}=\frac{50+25}{5}$$

$=15$ cm.

Here,

$R=10$ cm

$K^2=\frac{R^2}{2}=50\ cm^2$

Distance of the point of suspension from C.G

$=l_1=5$ cm

$g=980\ cm/sec^2$

Putting the value of $\frac{dy}{dt}$ and $\frac{d^2y}{dt^2}$ in eqn (I), we get

$$-Ap^2 \sin(pt-\theta) + 2\lambda Ap\cos(pt-\theta) + \omega^2 A\sin(pt-\theta) = f_0 \sin pt$$
$$= f_0 \sin[(pt-\theta)+\theta]$$
$$= f_0 \sin(pt-\theta)\cos\theta + f_0\cos(pt-\theta)\sin\theta$$

or, $$A(\omega^2 - p^2)\sin(pt-\theta) + 2\lambda Ap\cos(pt-\theta)$$
$$= f_0\cos\theta \sin(pt-\theta) + f_0 \sin\theta\cos(pt-\theta) \quad \text{—— (IV)}$$

If this solution is to hold good for all values of t, the respective coefficients of $\sin(pt-\theta)$ and $\cos(pt-\theta)$ on either side of equn (IV) must be equal. Thus we must have

$$A(\omega^2 - p^2) = f_0\cos\theta \quad \text{—— (a)}$$

and $$2\lambda Ap = f_0 \sin\theta \quad \text{—— (b)}$$

Squaring and adding eqn (a) and (b) we have

$$A^2(\omega^2-p^2)^2 + 4\lambda^2A^2p^2 = f_0^2\cos^2\theta + f_0^2\sin^2\theta$$

a $$A^2\left[(\omega^2-p^2)^2 + 4\lambda^2p^2\right] = f_0^2$$

a $$A^2 = \frac{f_0^2}{(\omega^2-p^2)^2+4\lambda^2p^2} \quad \text{—— (V)}$$

Thus the amplitude of the driven or forced oscillator is

$$A = \frac{f_0}{\sqrt{(\omega^2-p^2)^2+4\lambda^2p^2}} \quad \text{—— (VI)}$$

~~The phase difference between the driven force and applied force is~~

(...)

Combination of SHM:

Composition of two SHM of same frequency but different phase and amplitude:

Let two equations of SHM are

$y_1 = a_1 \sin(\omega t + \phi_1)$ and $y_2 = a_2 \sin(\omega t + \phi_2)$

where y_1 and y_2 are the displacement of the particles due to the individual vibration of amplitude a_1 and a_2 respectively. If y be the resultant displacement; then

$$y = y_1 + y_2 = a_1 \sin(\omega t + \phi_1) + a_2 \sin(\omega t + \phi_2)$$

$$= a_1 \{\sin\omega t \cos\phi_1 + \cos\omega t \sin\phi_1\} + a_2 \{\sin\omega t \cos\phi_2 + \cos\omega t \sin\phi_2\}$$

$$= (a_1 \cos\phi_1 + a_2 \cos\phi_2) \sin\omega t + (a_1 \sin\phi_1 + a_2 \sin\phi_2) \cos\omega t$$

a_1, a_2, ϕ_1 and ϕ_2 are constant.

Hence putting, $a_1 \cos\phi_1 + a_2 \cos\phi_2 = A\cos\phi$

and $a_1 \sin\phi_1 + a_2 \sin\phi_2 = A\sin\phi$

$$\therefore y = A\cos\phi \sin\omega t + A\sin\phi \cos\omega t$$

$$= A\sin(\omega t + \phi)$$

As most case of interest to us fall in the category of ordinary velocities, the damping or resistive force may thus be represented by

$$F' = -bv = -b\frac{dy}{dt}$$

where b is the constant of proportionality, b is a constant called damping coefficient of the medium.

Thus the differential equation may be written as

$$m\frac{d^2y}{dt^2} = -ay - b\frac{dy}{dt}$$

or, $$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ay = 0$$

or, $$\frac{d^2y}{dt^2} + \frac{b}{m}\frac{dy}{dt} + \frac{a}{m}y = 0$$

or, $$\frac{d^2y}{dt^2} + 2\lambda\frac{dy}{dt} + \omega^2 y = 0 \quad \text{——— (1)}$$

where $2\lambda = \frac{b}{m}$ and $\omega^2 = \frac{a}{m}$

Eqn (1) is referred to as the differential eqn of a damping harmonic oscillator.

Waves and Oscillations.

$\therefore$ (VI) $\Rightarrow 0 = -\omega^2 a^2 + c$

$\therefore c = \omega^2 a^2$ ——— (VII)

Therefore from eqn (VI) we can write

$$\left(\frac{dy}{dt}\right)^2 = -\omega^2 y^2 + \omega^2 a^2$$
$$= \omega^2 (a^2 - y^2)$$

or, $\frac{dy}{dt} = \pm \omega \sqrt{a^2 - y^2} = \pm \sqrt{\frac{K}{m}} \cdot \sqrt{a^2 - y^2}$

$$\boxed{\frac{dy}{dt} = \pm \sqrt{\frac{K}{m}(a^2 - y^2)}}$$

or, $\frac{dy}{dt} = \omega \sqrt{a^2 - y^2}$

or, $\frac{dy}{\sqrt{a^2 - y^2}} = \omega dt$ ——— (VIII)

Integrating with respect to time we have

$\sin^{-1} \frac{y}{a} = \omega t + \phi$ $\quad \left\{ \frac{d}{dt}\left(\sin^{-1} \frac{y}{a}\right) = \frac{dy}{\sqrt{a^2 - y^2}} \right.$

or, $\frac{y}{a} = \sin(\omega t + \phi)$

$\therefore y = a \sin(\omega t + \phi)$ ——— (IX)

This is the general solution of the differential equation of simple harmonic motion.

Expression for a plane progressive wave:

A progressive wave is one which travels onward through the medium in a given direction without attenuation i.e with its amplitude constant.

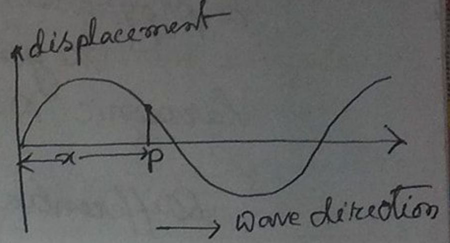


Fig: progressive wave

A typical wave form is shown in figure. Let a wave is originating at O, travel to the right along the x-axis. The equation of motion of this particle at O is obviously

$$y = a \sin \omega t \quad \text{——— (I)}$$

where y is the displacement of the particle at time t, a its amplitude and ω its angular velocity.

For a particle at P which is at a distance x away from O, let this phase difference be ϕ. Hence the equation of motion of the particle at P is

$$y = a \sin(\omega t - \phi) \quad \text{——— (II)}$$

For a path difference of λ, the difference in phase is 2π. Hence for a distance x, the corresponding phase difference is $\frac{2\pi}{\lambda}x$. Substituting this value in eqn (II) we get

$$y = a \sin\left(\omega t - \frac{2\pi}{\lambda}x\right)$$
$$= a \sin(\omega t - Kx) \quad \text{——— (III)}$$

where, $K = \frac{2\pi}{\lambda}$ is refferred to as the propagative constant.
Now $\omega = \frac{2\pi}{T}$, where T is a time period from complete oscillation, n is the frequency $v = n\lambda \Rightarrow n = \frac{v}{\lambda}$.
Again $n = \frac{1}{T}$ $\therefore \frac{1}{T} = \frac{v}{\lambda}$

$\therefore \omega = \frac{2\pi v}{\lambda}$, Then eqn (III) becomes

$$y = a \sin\left(\frac{2\pi v}{\lambda}t - \frac{2\pi}{\lambda}x\right)$$

$$\therefore y = a \sin \frac{2\pi}{\lambda}(vt - x)$$

The most commonly used equation of progassive wave.

Forced Vibration:

The time period of a body executing simple harmonic motion depends upon the dimensions of the body and its elastic proporties. When the body oscillates in a medium like air, its oscillations, as we know get damped.

If how ever, an external periodic force is applied to the oscillator, of a frequency not necessarily the same as the natural frequency of the oscillator, a sort of tussle ensues between the damping force and the applied force. The damping force ~~retard~~ tends to retard the motion of the body and the applied force tends to maintain it. Initially the amplitude of the oscillations increases, then decreases with time, becomes minimum and again increases. After some initial erratic movements, the body ultimately succumbs to the applied or driving force and settles down to oscillating with the frequency of the applied or driving force and a constant amplitude and phase so long as the applied force remain operative. Such vibration of the body are called forced vibrations. The amplitude of this vibration of the body depends on the difference between the natural and applied force frequency.

To determine the value of K. a small m is attached to the free end of the spring. Let the increase in length is x, $\therefore mg = Kx$

Then $K = \frac{mg}{x}$

There for from eqn (IV)

$$T = 2\pi\sqrt{\frac{Mx}{mg}} \quad \text{——— (V)}$$

1.

or, $m\frac{d^2y}{dt^2}+Ky=0$

or, $\frac{d^2y}{dt^2}+\frac{K}{m}y=0$ ——— (iv)

Equation (iv) is called the differential equation of motion of a body executing SHM.

Rearranging eqn (iv). We can write

$$\frac{d^2y}{dt^2}=-\frac{K}{m}y=-\omega^2 y \quad \text{——— (v)}$$

Where $\omega=\sqrt{\frac{K}{m}}$ is the angular velocity of the particle.

General Solution of differential equation of SHM:

To obtain general solution of differential equation of simple harmonic motion, let us multiply both sides of equation (v) by $2\frac{dy}{dt}$ then we get,

$$2\frac{dy}{dt}\cdot\frac{d^2y}{dt^2}=-\omega^2 y\cdot 2\frac{dy}{dt}$$

$$\Rightarrow 2\frac{dy}{dt}\cdot\frac{d^2y}{dt^2}=-\omega^2\, 2\frac{dy}{dt}\,y=-2\omega^2 y\frac{dy}{dt}$$

Integrating with respect to time we have

$$\left(\frac{dy}{dt}\right)^2=-\omega^2y^2+c \quad \text{——— (vi)}$$

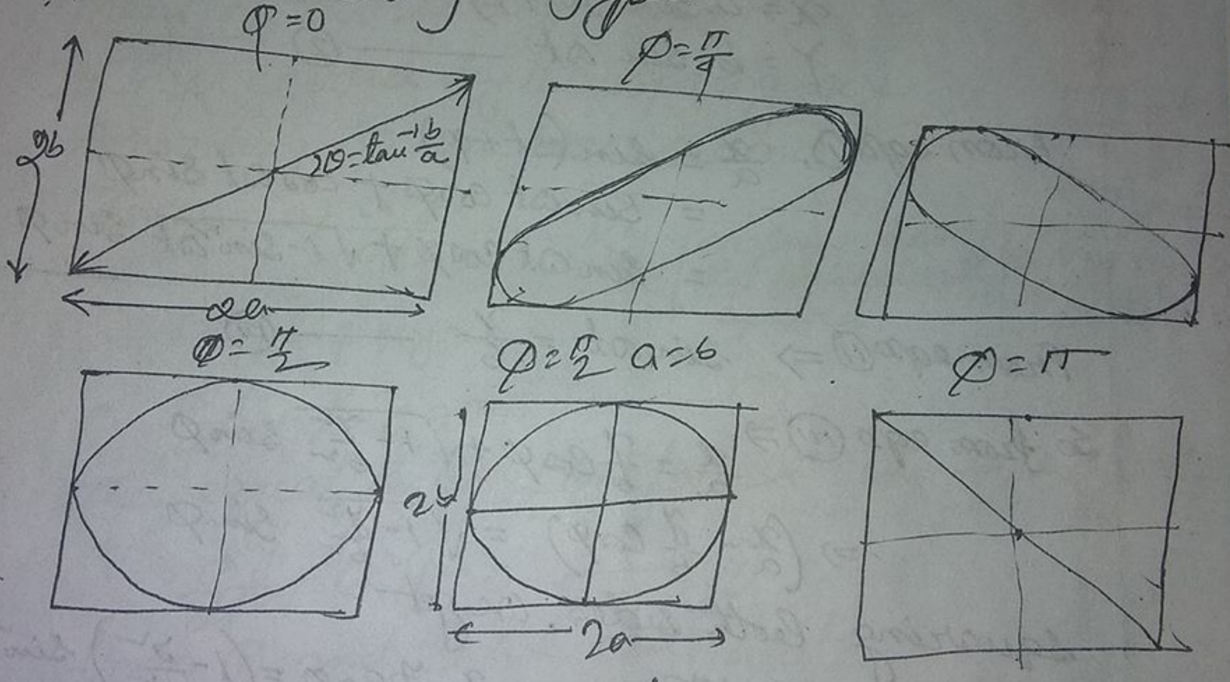
Where c is a constant of integation.

At maximum desplacement, (when $y=a$ amplitude)

$$\frac{dy}{dt}=0$$

LISSAJOUS' FIGURES:

The composition of two simple harmonic vibrations in mutually perpendicular directions give rise to an ellipse. The resultant motion is thus in general, along an elliptical path. the actual shape of the curve will, however depend upon the phase difference ϕ between the two vibrations and also on the ratio of the frequencies of the component vibrations. these figures are known as Lissajous' figures -



Lissajou's figures:

Spring Mass System

Simple Harmonic oscillations of a loaded spring:

Consider a spring S whose upper end is fixed to a rigid support and lower end is attached to a mass M. In the equilibrium position the mass is at A. Suppose at any instant the mass is at B. The distance AB = y. Let the tension per unit displacement of the spring be K.

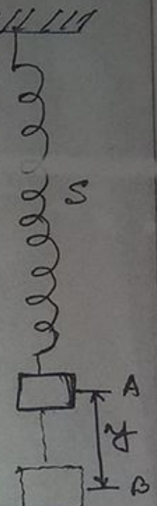


Fig: Spring with load.

∴ Force exerted by the spring = Ky

According to Newton's second law

$$\text{force} = M\frac{d^2y}{dt^2} = -Ky$$

[-ve sign shows that force is directly upward]

$$\therefore M\frac{d^2y}{dt^2} + Ky = 0$$

$$\text{or, } \frac{d^2y}{dt^2} + \frac{K}{M}y = 0 \quad \text{——— (i)}$$

This equation is similar to the eqn of SHM.

$$\frac{d^2y}{dt^2} + \omega^2 y = 0 \quad \text{——— (ii)}$$

from (i) and (ii) we can write

$$\omega^2 = \frac{K}{M}$$

$$\text{or } \omega = \sqrt{\frac{K}{M}} \quad \text{——— (iii)}$$

again we know Time period $T = \frac{2\pi}{\omega}$

$$\therefore T = 2\pi\sqrt{\frac{M}{K}} \quad \text{——— (iv)}$$

Now kinetic energy of the particle

$$K.E. = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{dy}{dt}\right)^2$$

$$= \frac{1}{2}m\left[\omega a\cos(\omega t+\phi)\right]^2$$

$$= \frac{1}{2}m\omega^2 a^2\cos^2(\omega t+\phi)$$

$$K.E. = \frac{1}{2}Ka^2\cos^2(\omega t+\phi) \quad \text{—— (iii)}$$

Therefore the total energy,

$$E = K+U = \frac{1}{2}Ka^2\sin^2(\omega t+\phi) + \frac{1}{2}Ka^2\cos^2(\omega t+\phi)$$

$$= \frac{1}{2}Ka^2\left[\sin^2(\omega t+\phi)+\cos^2(\omega t+\phi)\right]$$

$$= \frac{1}{2}Ka^2$$

$$= \frac{1}{2}m\omega^2a^2$$

$$= \frac{1}{2}m\left(\frac{2\pi}{T}\right)^2\cdot a^2$$

$$= \frac{2\pi^2 ma^2}{T^2}, \left[\because \omega = \frac{2\pi}{T}\right]$$

But $\frac{1}{T} = n$, the frequency of the oscillation.

$$E = \frac{1}{2}Ka^2 = 2\pi^2 ma^2n^2$$

= maximum value of potential energy

= maximum value of kinetic energy.

Average Energy : Average values of energies of Harmonic oscillator —

The potential energy of the particle at a displacement y is given by

$$PE = \frac{1}{2}m\omega^2y^2 \quad [\because y = a\sin(\omega t+\phi)]$$

$$= \frac{1}{2}m\omega^2a^2\sin^2(\omega t+\phi)$$