# CSER 2207_8: Numerical Analysis-I

## Lecture-1
## Solution of equation in single variable

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# References

1. **Introduction to Numerical Analysis- S. S. Sastray.**
2. **Numerical Analysis- Burden & J.D. Faires.**
3. Numerical Methods & Calculus- S.S Kuo.
4. Numerical Method –E. Balagurusamy.
5. Numerical Analysis-Timothy Sauer.

# Outlines

- Bisection method
- Method of false-position
- Fixed point iteration method
- Newton-Raphson method

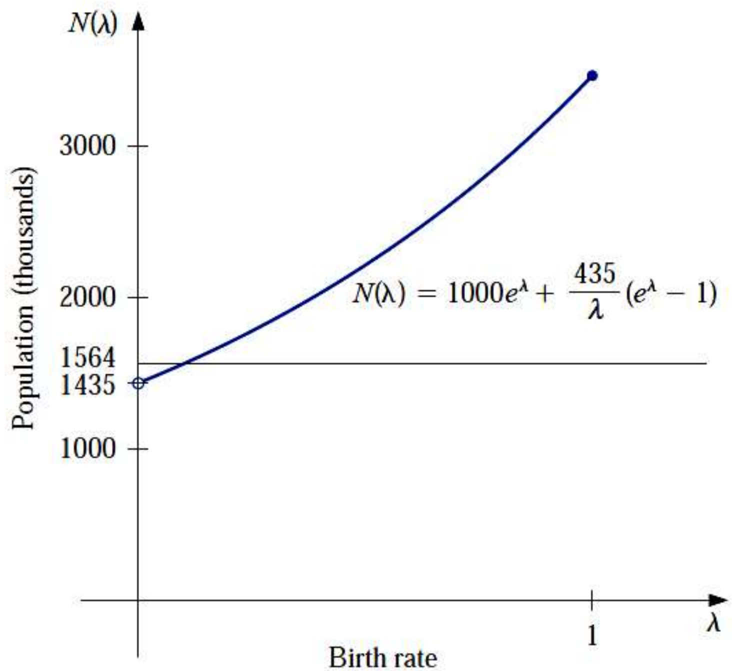
- Error analysis for iterative method
- Accelerating limit of convergence
- Algorithms of above Methods.

# Introduction [2]

The growth of a population can often be modeled over short periods of time by assuming that the population grows continuously with time at a rate proportional to the number present at that time. Suppose that $N(t)$ denotes the number in the population at time $t$ and $\lambda$ denotes the constant birth rate of the population. Then the population satisfies the differential equation

$$\frac{dN(t)}{dt} = \lambda N(t),$$

whose solution is $N(t) = N_0 e^{\lambda t}$, where $N_0$ denotes the initial population.



# Cont…

This exponential model is valid only when the population is isolated, with no immigration. If immigration is permitted at a constant rate $v$, then the differential equation becomes

$$\frac{dN(t)}{dt} = \lambda N(t) + v,$$

whose solution is

$$N(t) = N_0 e^{\lambda t} + \frac{v}{\lambda}(e^{\lambda t} - 1).$$

Suppose a certain population contains $N(0) = 1{,}000{,}000$ individuals initially, that 435,000 individuals immigrate into the community in the first year, and that $N(1) = 1{,}564{,}000$ individuals are present at the end of one year. To determine the birth rate of this population, we need to find $\lambda$ in the equation

$$1{,}564{,}000 = 1{,}000{,}000 e^{\lambda} + \frac{435{,}000}{\lambda}(e^{\lambda} - 1).$$

It is not possible to solve explicitly for $\lambda$ in this equation, but numerical methods discussed in this chapter can be used to approximate solutions of equations of this type to an arbitrarily high accuracy. The solution to this particular problem is considered in Exercise 24 of Section 2.3.

# Bisection method [2]

The first technique, based on the Intermediate Value Theorem, is called the **Bisection**, or **Binary-search, method**.

Suppose $f$ is a continuous function defined on the interval $[a, b]$, with $f(a)$ and $f(b)$ of opposite sign. The Intermediate Value Theorem implies that a number $p$ exists in $(a, b)$ with $f(p) = 0$. Although the procedure will work when there is more than one root in the interval $(a, b)$, we assume for simplicity that the root in this interval is unique. The method calls for a repeated halving (or bisecting) of subintervals of $[a, b]$ and, at each step, locating the half containing $p$.

To begin, set $a_1 = a$ and $b_1 = b$, and let $p_1$ be the midpoint of $[a, b]$; that is,

$$p_1 = a_1 + \frac{b_1 - a_1}{2} = \frac{a_1 + b_1}{2}.$$

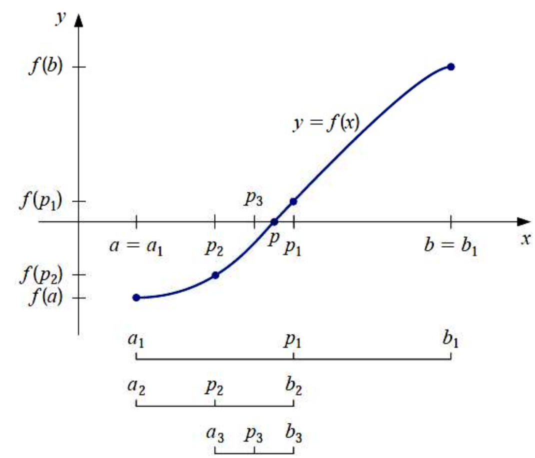
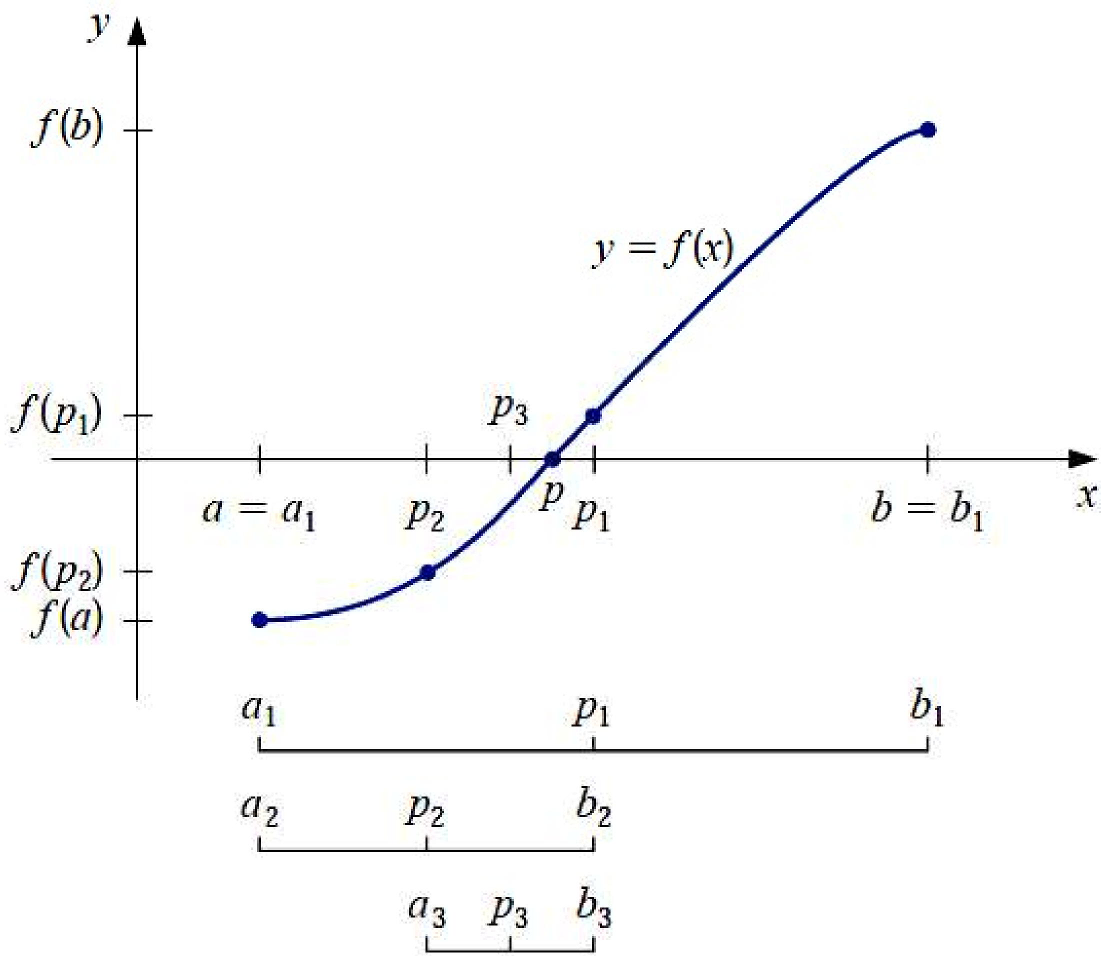



Figure 2.1

- If $f(p_1) = 0$, then $p = p_1$, and we are done.
- If $f(p_1) \neq 0$, then $f(p_1)$ has the same sign as either $f(a_1)$ or $f(b_1)$.
  - If $f(p_1)$ and $f(a_1)$ have the same sign, $p \in (p_1, b_1)$. Set $a_2 = p_1$ and $b_2 = b_1$.
  - If $f(p_1)$ and $f(a_1)$ have opposite signs, $p \in (a_1, p_1)$. Set $a_2 = a_1$ and $b_2 = p_1$.

Then reapply the process to the interval $[a_2, b_2]$. This produces the method described in Algorithm 2.1. (See Figure 2.1.)

# Figure 2.1



# Thank You