CSER 2207_8: Numerical Analysis-I

Lecture-3
Solution of equation in single variable

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2.3 THE METHOD OF FALSE POSITION

This is the oldest method for finding the real root of a nonlinear equation $f(x)=0$ and closely resembles the bisection method. In this method, also known as *regula falsi* or the *method of chords*, we choose two points a and b such that $f(a)$ and $f(b)$ are of opposite signs. Hence, a root must lie in between these points. Now, the equation of the chord joining the two points $[a, f(a)]$ and $[b, f(b)]$ is given by

$$\frac{y-f(a)}{x-a}=\frac{f(b)-f(a)}{b-a}. \tag{2.6}$$

The method consists in replacing the part of the curve between the points $[a, f(a)]$ and $[b, f(b)]$ by means of the *chord* joining these points, and taking the point of intersection of the chord with the x-axis as an *approximation* to the root. The point of intersection in the present case is obtained by putting $y=0$ in (2.6). Thus, we obtain

$$x_1=a-\frac{f(a)}{f(b)-f(a)}(b-a)=\frac{af(b)-bf(a)}{f(b)-f(a)}, \tag{2.7}$$

which is the *first approximation* to the root of $f(x)=0$. If now $f(x_1)$ and $f(a)$ are of opposite signs, then the root lies between a and x_1, and we replace b by x_1 in (2.7), and obtain the *next* approximation. Otherwise, we replace a by x_1 and generate the next approximation. The procedure is repeated till the root is obtained to the desired accuracy. Figure 2.2 gives a graphical representation of the method. The error criterion (2.5) can be used in this case also.

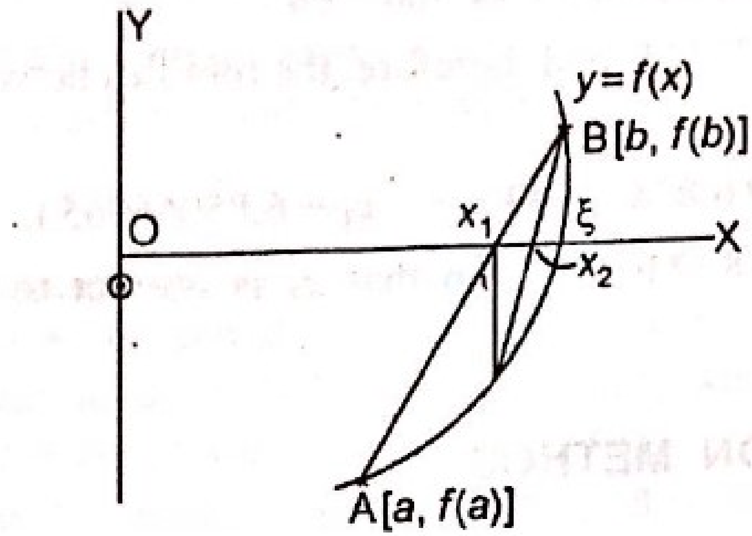

Figure 2.2 Method of false position.

Example 2.4 Find a real root of the equation :

$$f(x) = x^3 - 2x - 5 = 0.$$

We find $f(2) = -1$ and $f(3) = 16$. Hence $a = 2$, $b = 3$, and a root lies between 2 and 3. Equation (2.7) gives

$$x_1 = \frac{2(16) - 3(-1)}{16 - (-1)} = \frac{35}{17} = 2.058823529.$$

Now, $f(x_1) = -0.390799917$ and hence the root lies between 2.058823529 and 3.0. Using formula (2.7), we obtain

$$x_2 = \frac{2.058823529(16) - 3(-0.390799917)}{16.390799917} = 2.08126366.$$

Since $f(x_2) = -0.147204057$, it follows that the root lies between 2.08126366 and 3.0. Hence, we have

$$x_3 = \frac{2.08126366(16) - 3(-0.147204057)}{16.147204057} = 2.089639211.$$

Proceeding in this way, we obtain successively:

$$x_4 = 2.092739575, \qquad x_5 = 2.09388371,$$

$$x_6 = 2.094305452, \qquad x_7 = 2.094460846, \ldots$$

The correct value is 2.0945..., so that x_7 is correct to five significant figures.

Solve $x\log_{10} x = 1.2$ by Regula–Falsi method.

Solution. We have $f(x) = x\log_{10} x - 1.2 = 0$. Then $f(2) = -0.60$ and $f(3) = 0.23$. Therefore, the root lies between 2 and 3. Then

$$x_2 = \frac{x_1 f(x_0) - x_0 f(x_1)}{f(x_0) - f(x_1)} = \frac{3(-0.6) - 2(0.23)}{-0.6 - 0.23} = 2.723.$$

Now $f(2.72) = 2.72\log(2.72) - 1.2 = -0.01797$. Since we are getting closer to the root, we calculate $f(2.75)$ and have

$$f(2.75) = 2.75\log(2.75) - 1.2 = 2.75\,(0.4393) - 1.2 = 0.00816.$$

Therefore,

$$x_3 = \frac{2.75\,(-0.01797) - 2.72\,(0.00816)}{-0.01797 - 0.00816} = \frac{-0.04942 - 0.02219}{-0.02613} = 2.7405.$$

Now $f(2.74) = 2.74\log(2.74) - 1.2 = 2.74(0.43775) - 1.2 = -0.00056.$

Thus, the root lies between 2.74 and 2.75 and it is more close to 2.74. Therefore,

$$x_4 = \frac{2.75\,(-0.00056) - 2.74\,(0.00816)}{-0.00056 - 0.00816} = 2.7408.$$

Thus the root is 2.740 correct up to three decimal places.

False Position

To find a solution to $f(x) = 0$ given the continuous function f on the interval $[p_0, p_1]$ where $f(p_0)$ and $f(p_1)$ have opposite signs:

INPUT initial approximations p_0, p_1; tolerance TOL; maximum number of iterations N_0.

OUTPUT approximate solution p or message of failure.

Step 1 Set $i = 2$;
$q_0 = f(p_0)$;
$q_1 = f(p_1)$.

Step 2 While $i \leq N_0$ do Steps 3–7.

Step 3 Set $p = p_1 - q_1(p_1 - p_0)/(q_1 - q_0)$. (*Compute p_i.*)

Step 4 If $|p - p_1| < TOL$ then
OUTPUT (p); (*The procedure was successful.*)
STOP.

Step 5 Set $i = i + 1$;
$q = f(p)$.

Step 6 If $q \cdot q_1 < 0$ then set $p_0 = p_1$;
$q_0 = q_1$.

Step 7 Set $p_1 = p$;
$q_1 = q$.

Step 8 OUTPUT ('Method failed after N_0 iterations, $N_0 =$', N_0);
(*The procedure unsuccessful.*)
STOP.

Thank You