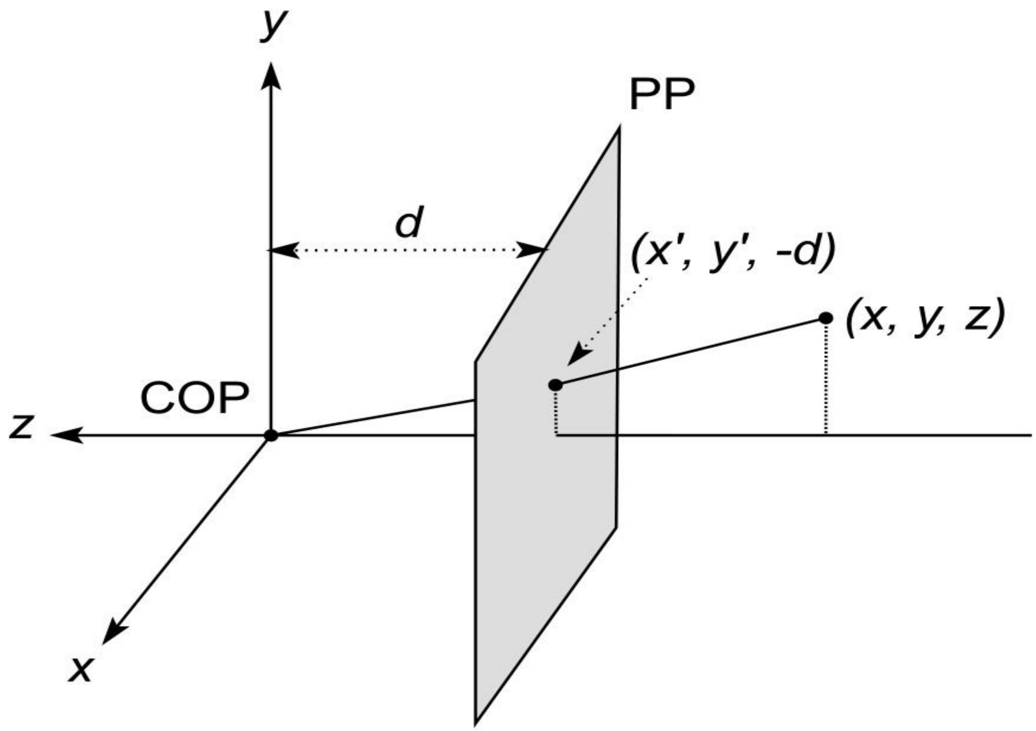

# CSE- 4105

Lecture- 10

Projection-II

# Derivation of perspective projection

- Assume projection plane is normal to z-axis.
- Projection point is at origin and projection plane is at distant d.



# Derivation of perspective projection

- Now we see two triangle namely ΔABC and ΔADE
- Projection point is at origin and projection plane is at distant d.

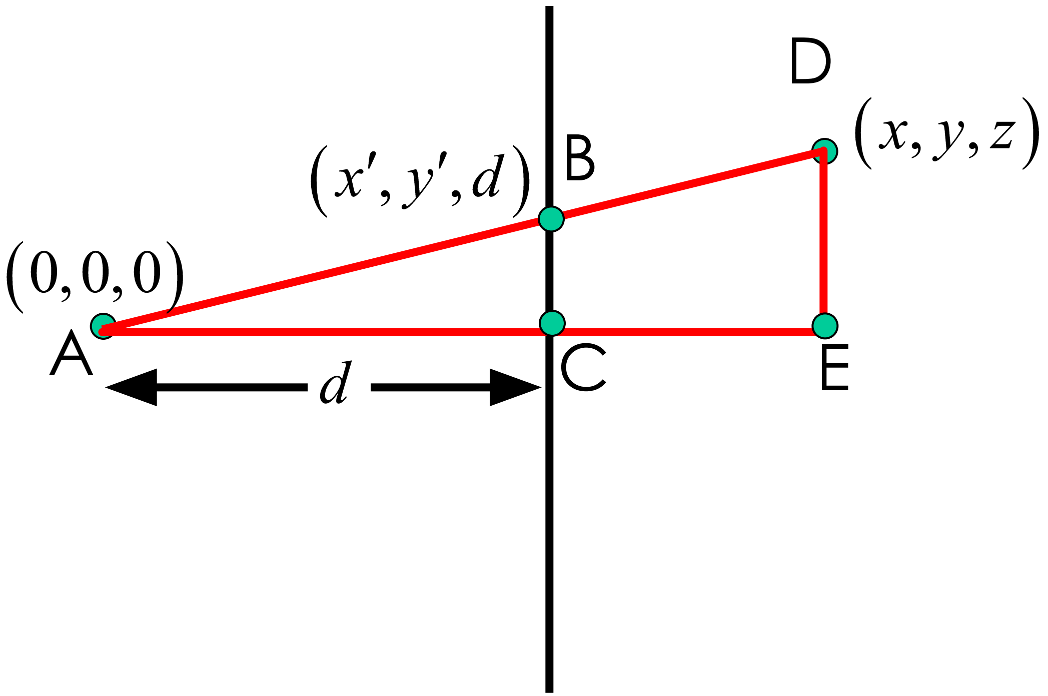
Apply similarity rule:

$$\frac{BC}{DE} = \frac{AC}{AE}$$

$$\Rightarrow BC = DE.\frac{AC}{AE}$$

$$\Rightarrow y' = y.\frac{d}{z}$$

$$\Rightarrow y' = \frac{y}{z/d}$$



# Derivation of perspective projection

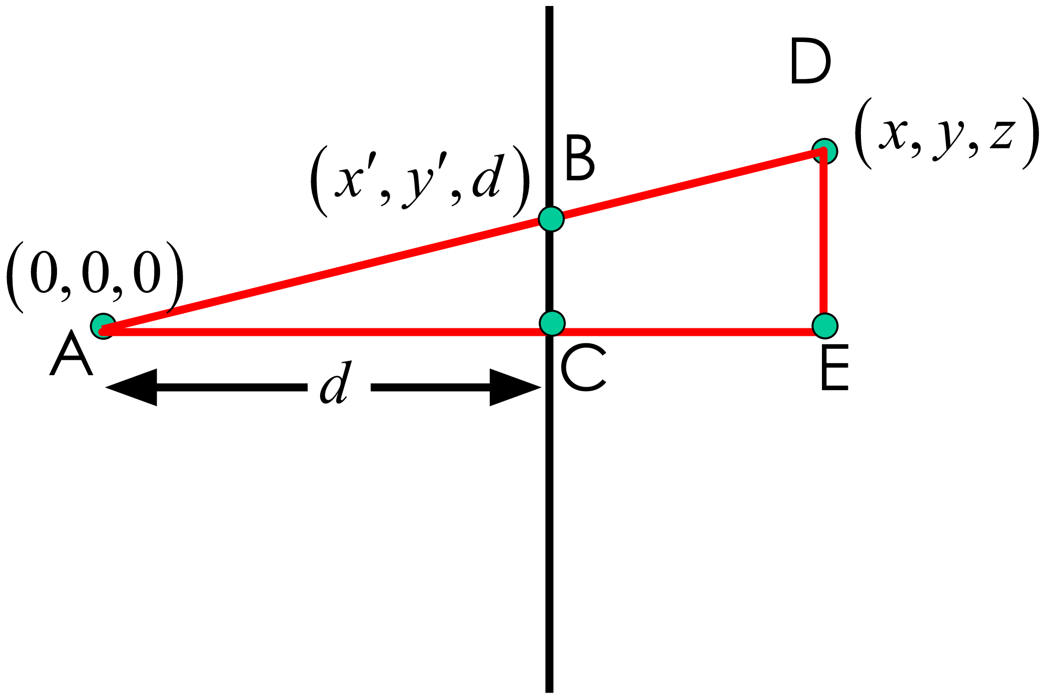
$$\Rightarrow y' = y.\frac{d}{z}$$

- Similarly

$$\Rightarrow x' = \frac{x}{z/d}$$

$$\Rightarrow z' = \frac{z}{z/d}$$

$$\Rightarrow z' = \frac{z}{z/d} = d$$



# Homogeneous Coordinate Form

- The transformation can be represented as

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix}$$

- If we multiply the homogeneous point P=(x,y,z,1) by M, to get the projected point P' we obtain:

$$\mathbf{P'} = MP = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix}$$

# Derivation of perspective projection

- If the projection plane is placed at *z=o*.

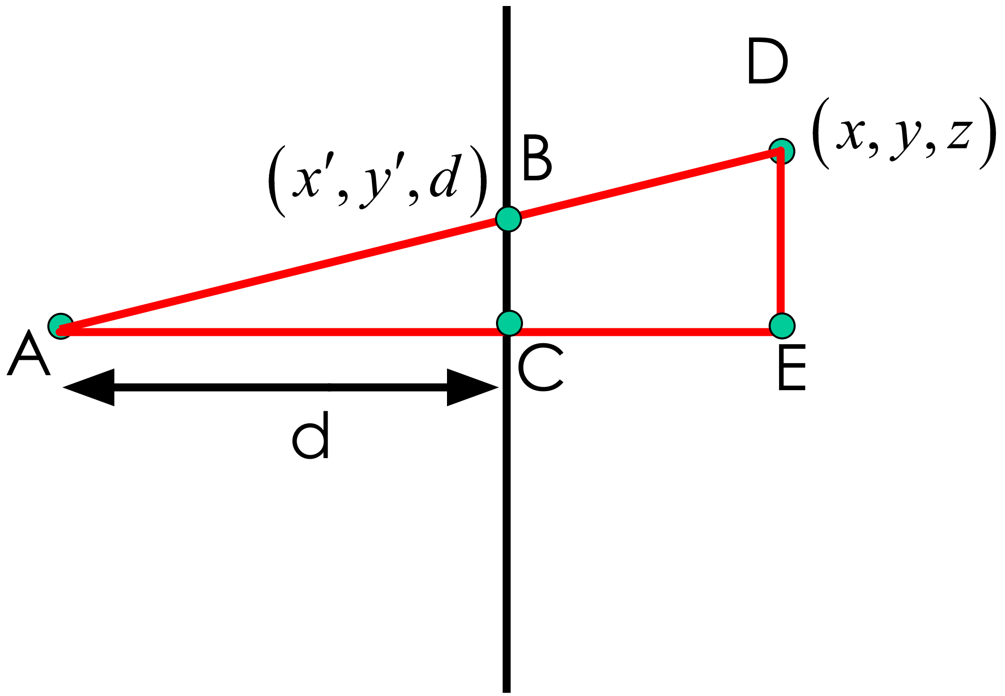
$$AE = AC + CE = z + d$$

Apply similarity rule:

$$\frac{BC}{DE} = \frac{AC}{AE}$$

$$\Rightarrow \frac{y'}{d} = \frac{y}{z+d}$$

$$\Rightarrow y' = \frac{y}{1 + z/d}$$



# Derivation of perspective projection

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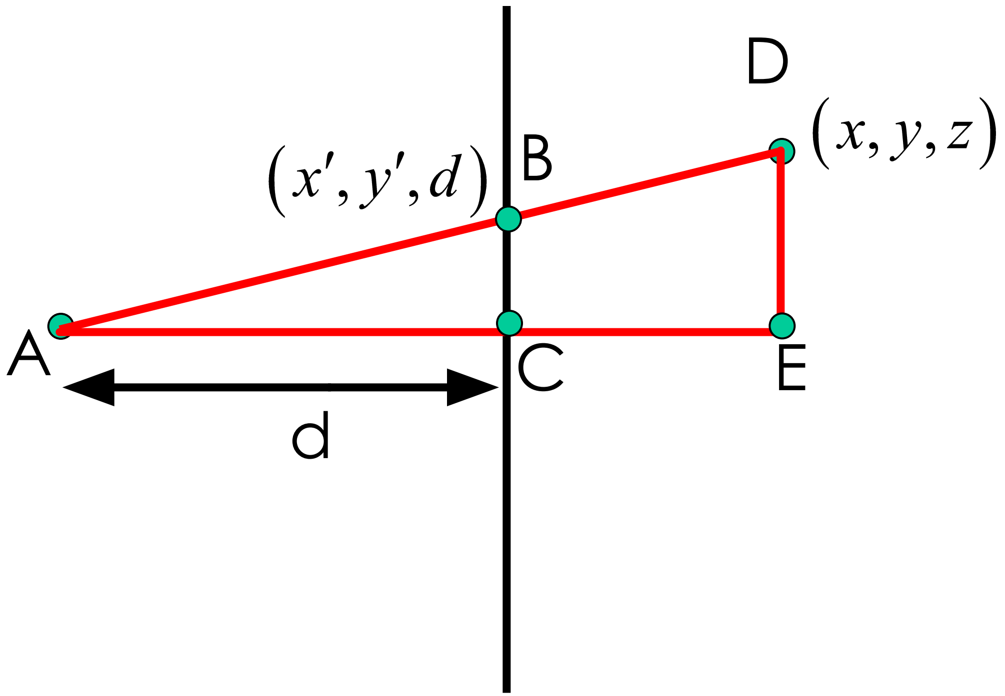


# Derivation of perspective projection

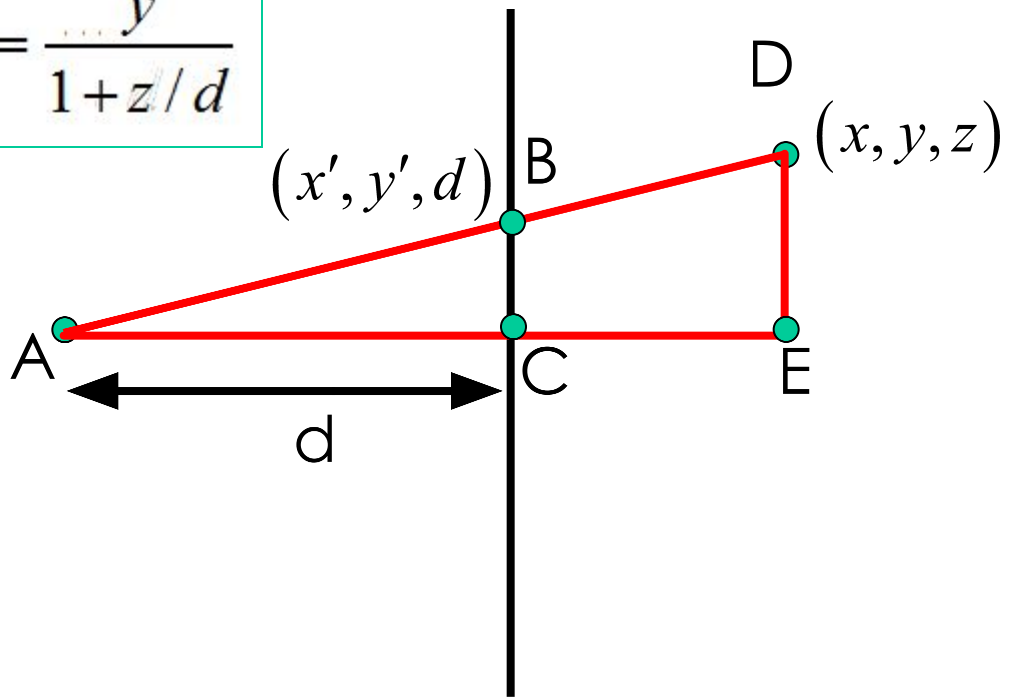
- If the projection plane is placed at $z=0$.
- Similarly

$$\boxed{\Rightarrow x' = \frac{x}{1+z/d}}$$

$$\boxed{\Rightarrow y' = \frac{y}{1+z/d}}$$

$$\boxed{z' = 0}$$

$$\mathbf{M'} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix}$$



# Derivation of perspective projection

- We get

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

# Thank You