$$(11)_{10} = (13)_8 = (1011)_2$$

decimal	binary
1	1
2	10
3	11
4	100
5	101
6	110
7	111
8	1000
9	1001
10	1010
11	1011

$$(900)_{10} = (1110000100)_2$$

388

132

4

0

0	0	1	1	1	0	0	0	0	1	0	0
2048	1024	512	256	128	64	32	16	8	4	2	1
									$= 2^2$	$= 2^1$	$= 2^0$

$$\begin{array}{r} 900 \\ -\,512 \\ \hline 388 \end{array} \qquad \begin{array}{r} 388 \\ -\,256 \\ \hline 132 \end{array} \qquad \begin{array}{r} 132 \\ -\,128 \\ \hline 4 \end{array} \qquad \begin{array}{r} 4 \\ -\,4 \\ \hline 0 \end{array}$$

$$(900)_{10} = (1\,1\,1\,0000100)_2$$

$$900 \div 2 = 450 \text{ R } 0$$

$$450 \div 2 = 225 \text{ R } 0$$

$$225 \div 2 = 112 \text{ R } 1$$

$$112 \div 2 = 56 \text{ R } 0$$

$$56 \div 2 = 28 \text{ R } 0$$

$$28 \div 2 = 14 \text{ R } 0$$

$$14 \div 2 = 7 \text{ R } 0$$

$$7 \div 2 = 3 \text{ R } 1$$

$$3 \div 2 = 1 \text{ R } 1$$

$$1 \div 2 = \boxed{0} \text{ R } 1$$

$(900)_{10} = (1604)_8$

$$900 \div 8 = 112 \text{ R } 4$$
$$112 \div 8 = 14 \text{ R } 0$$
$$14 \div 8 = 1 \text{ R } 6$$
$$1 \div 8 = 0 \text{ R } 1$$

$900 = \underbrace{112}_{(160)_8} \cdot 8 + 4$

$(1600)_8 + 4 = 160\boxed{4}$

$\frac{1}{8^2}\ \frac{1}{8^1}\ \frac{5}{8^0}$

$\Downarrow$

$(1 \cdot 8^2 + 1 \cdot 8^1 + 5 \cdot 8^0) \cdot 8$

$=$

$1 \cdot 8^3 + 1 \cdot 8^2 + 5 \cdot 8^1$

$\frac{1}{8^3}\ \frac{1}{8^2}\ \frac{5}{8^1}\ \frac{0}{8^0}$

$(356)_{10} \cdot 10 = (3560)_{10}$

$(115)_8 \cdot 8 = (1150)_8$

$$(-52)_{10} = (-110100)_2$$

$$(-52)_{10} = (\underline{1}\,\underline{1}\,\underline{0}\,\underline{0}\,\underline{1}\,\underline{1}\,\underline{0}\,\underline{0})_{\text{8-bit 2's comp}}$$

$52 \div 2 = 26 \; R0$

$26 \div 2 = 13 \; R0$

$13 \div 2 = 6 \; R1$

$6 \div 2 = 3 \; R0$

$3 \div 2 = 1 \; R1$

$1 \div 2 = \underline{0} \; R1$

#1

$$\begin{array}{rcccccccc}
 & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & & {\scriptstyle 1} & {\scriptstyle 1} & {\scriptstyle 1} & \\
 & \underline{0} & \underline{0} & \underline{1} & \underline{1} & \underline{0} & \underline{1} & \underline{0} & \underline{0} \\
+ & \underline{1} & \underline{1} & \underline{0} & \underline{0} & \underline{1} & \underline{1} & \underline{0} & \underline{0} \\
\hline
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}$$

52

-52

2^8

$$\begin{array}{r}
{\scriptstyle 1} \\
37 \\
+\;23 \\
\hline
60
\end{array}
\qquad
\begin{array}{r}
{\scriptstyle 1} \\
(1\,1)_2 \\
+\;(0\,1)_2 \\
\hline
(\quad 0)_2
\end{array}$$

#2

52: $\boxed{0\,0\,1\,1\,0}\;\boxed{1\,0\,0}$

-52: $1\,1\,0\,0\,1\;\;0\,0$

Sets:

* A set is an unordered collection of objects

* ways to describe sets:

1) Roster method:

$$A = \{10, 11, 12, 13, 14, 15\}$$

2) Set-builder method

$$A = \{x \,/\, (x \text{ is an integer}) \; (10 \le x \le 15)\}$$

$$\left\{ x \,/\, \left(\begin{matrix} x \text{ is a square} \\ \text{of an integer} \end{matrix}\right) \text{ and } (x < 100) \right\} =$$

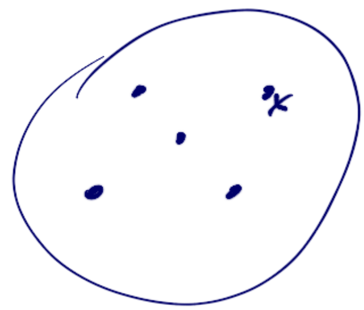
$$\{0, 1, 4, 9, 16, 25, 36, 49, 64, 81\}$$

$$\{-3, -2, -1, 0, 1, 2, 3\} = \left\{ x \,/\, \begin{matrix} (-3 \leq x \leq 3) \text{ and} \\ (x \text{ is an itger}) \end{matrix} \right\} =$$

$$= \left\{ x \,/\, \begin{matrix} (x \text{ is an integer}) \text{ and} \\ |x| \leq 3 \end{matrix} \right\}$$

Definitions:

1) membership: $x \in A$



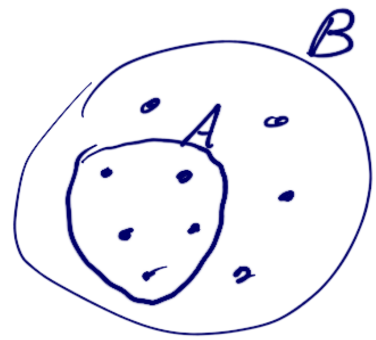
$A = \{1, \text{"abc"}, 5.6\}$

$1 \in A \equiv \text{true}$

$2 \notin A \equiv \text{true}$

$u \in A \equiv \text{false}$

2) subset: $A \subseteq B$



$A = \{1, 2, 3\}$

$B = \{1, 2, 3, 4, 5, 6\}$

$A \subseteq B$

$A = \{1, 2, 7\}$

$B = \{1, 2, 3, 4, 5, 6\}$

$A \nsubseteq B$

$A \subseteq B$ if $\forall_x (x \in A \to x \in B)$

$$A = \{1, 2, \{1\}, \{2,3\}, u\}$$

$1 \in A$ true

$3 \in A$ false

$\{1\} \in A$ true

$\{1,2\} \in A$ false

$\{1,2\} \subseteq A$ true

$\{2,3\} \subseteq A$ false

$\{2,3\} \in A$ true

$\{\{2,3\}\} \subseteq A$ true

$u \subseteq A$ false

$u \in A$ true

$\{u\} \subseteq A$ true

Determine if these statements $\emptyset = \{\}$ are true or false

$0 \in \emptyset$ false

$\emptyset \in \{0\}$ false

$\emptyset \subseteq \{0\}$ true

$\{0\} \subseteq \emptyset$ false

$\{0\} \subseteq \{0\}$ true

$\in \{0\}$ false

$\emptyset \in \{1, 0, \{\}, 7\}$

$\{1,2\} \subseteq \{1,2,3,4\}$

$\{\} \subseteq \{1,2,3,4\}$

$\emptyset \subseteq A$

Definitions:

① $|A| = \#$ of elements in A

$$A = \{1, \{\}, \{2,3\}, \{1\}\} \Longrightarrow |A| = 4$$

② We say that two sets $A = B$ if $(A \subseteq B)$ and $(B \subseteq A)$

✓ $\{1,2,3,4\} = \{1,2,3,4\}$

✓ $\{1,2,4,3\} = \{1,2,3,4\}$

✓ $\{1,2,2,3,3,3,4,4,4,4\} = \{1,2,3,4\}$

$|\{1,2,2,3,3,3,4,4,4,4\}| = 4$

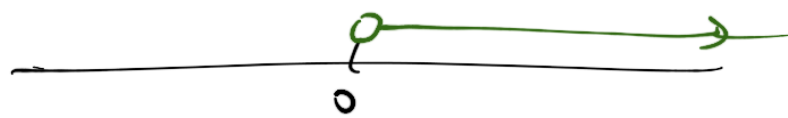
(3) Important sets

$\mathbb{N}$ - set of natural numbers $\left(\mathbb{N} = \{1, 2, 3, 4, \ldots\}\right)$

$\mathbb{Z}$ - set of integers $\left(\mathbb{Z} = \{0, 1, -1, 2, -2, \ldots\}\right)$

$\mathbb{R}$ - set of real numbers

$\mathbb{R}^+$ - set of positive real numbers

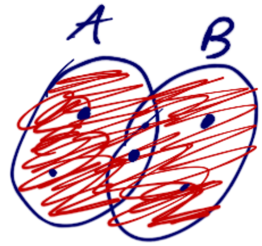


$\mathbb{Q}$ - set of rational numbers

4 set operations

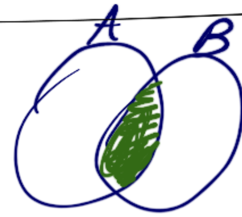
$A \cup B = \{x \,/\, x \in A \text{ or } x \in B\}$

$\{1,2\} \cup \{2,3\} = \{1,2,3\}$



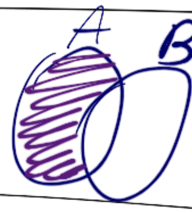
$A \cup B$

$A \cap B = \{x \,/\, x \in A \text{ and } x \in B\}$



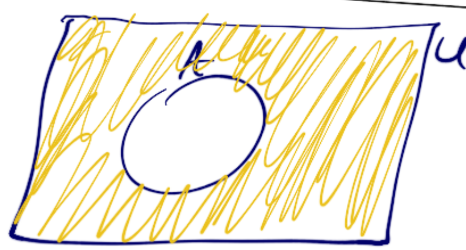
$A \cap B$

$A - B = \{x \,/\, x \in A \text{ and } x \notin B\}$



$A - B$

$\bar{A} = \{x \,/\, x \in U \text{ and } x \notin A\}$



$\bar{A}$

Important identities:

1. $\overline{\overline{A}} = A$

2. $\overline{A \cap B} = \overline{A} \cup \overline{B}$

$\overline{A \cup B} = \overline{A} \cap \overline{B}$

3. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

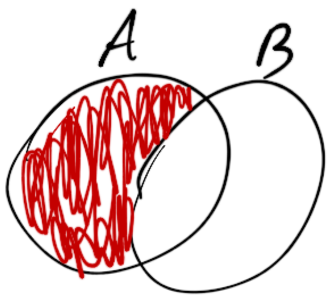
4. $A \cap (B \cap C) = (A \cap B) \cap C$

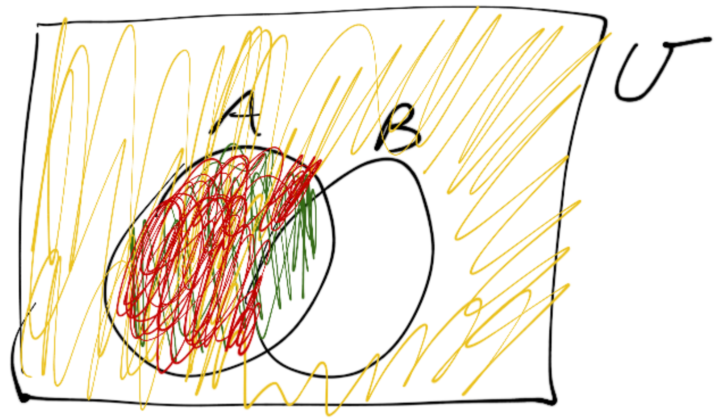
$A \cup (B \cup C) = (A \cup B) \cup C$

5. $A - B = A \cap \overline{B}$

$A - B = A \cap \overline{B}$

1) Venn Diagrams



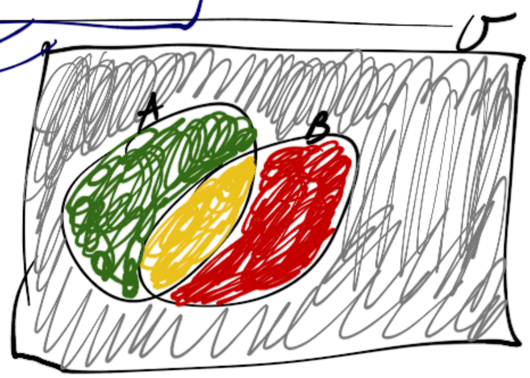


$A - B$ $\qquad$ $A \cap \overline{B}$

$$A - B = A \cap \overline{B}$$

II We orginize the different zones that elements can be in, inside a membership table

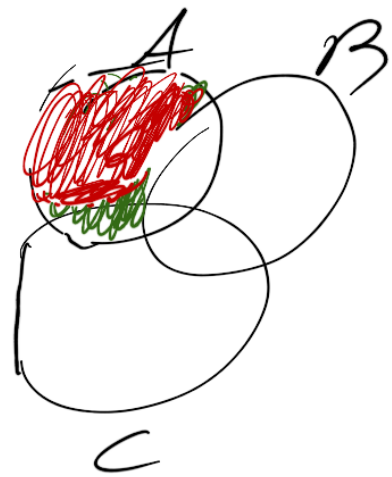
A	B	$A-B$	$\overline{B}$	$A \cap \overline{B}$
1	1	0	0	0
1	0	1	1	1
0	1	0	0	0
0	0	0	1	0



Since elements from all zones have the same membership behaviour to $A-B$ and to $A \cap \overline{B}$ we can conclude that $A-B = A \cap \overline{B}$

$$A-(B\cup C)=(A-B)-C$$

1) Venn diagrams



$$A-(B\cup C) = (A-B)-C$$

(2) $A-(B\cup C) = (A-B)-C$

We orginize the membership of elements in 8 possible zones, in a membership table

A	B	C	$B\cup C$	$A-(B\cup C)$	$A-B$	$(A-B)-C$
1	1	1	1	0	0	0
1	1	0	1	0	0	0
1	0	1	1	0	1	0
1	0	0	0	1	1	1
0	1	1	1	0	0	0
0	1	0	1	0	0	0
0	0	1	1	0	0	0
0	0	0	0	0	0	0

Since we have the same behavior in all zones we get $A-(B\cup C) = (A-B)-C$

③ $A-(B\cup C)=(A-B)-C$

$X-Y=X\cap\overline{Y}$ $\qquad$ $\overline{X\cup Y}=\overline{X}\cap\overline{Y}$

$$A-(B\cup C)=A\cap\overline{(B\cup C)}=A\cap(\overline{B}\cap\overline{C})=$$

$X\cap Y\cap Z=(X\cap Y)\cap Z$ $\qquad$ $X-Y=X\cap\overline{Y}$

$$=(A\cap\overline{B})\cap\overline{C}=(A-B)\cap\overline{C}=(A-B)-C$$

$X-Y=X\cap\overline{Y}$

$$\Downarrow$$

$$A-(B\cup C)=(A-B)-C$$

Let $A_i = \{1, 2, 3, \dots, i\}$ for $i = 1, 2, 3, \dots$

$A_4 = \{1, 2, 3, 4\}$

$A_7 = \{1, 2, 3, 4, 5, 6, 7\}$

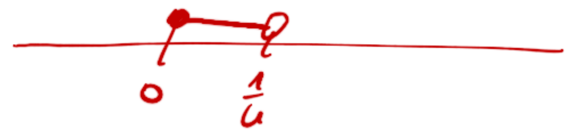
Find:

① $\underbrace{\bigcup_{i=1}^{\infty} A_i}_{A_1 \cup A_2 \cup A_3 \cup \dots} = \underline{\mathbb{N}}$

② $\bigcap_{i=1}^{\infty} A_i = \{1\}$

Let $A_i = [0, \frac{1}{i})$ for $i = 1, 2, 3, \ldots$

$A_4 = [0, \frac{1}{4})$



$A_7 = [0, \frac{1}{7})$

Find

① $\bigcup_{i=1}^{\infty} A_i = [0, 1)$

② $\bigcap_{i=1}^{\infty} A_i = \{0\}$

$A = \{1, 2, 3, 4\}$

$A \cup B$