EE5111/EE5061 Selected Topics in Industrial Control & Instrumentation

Advanced PID Control and Tuning

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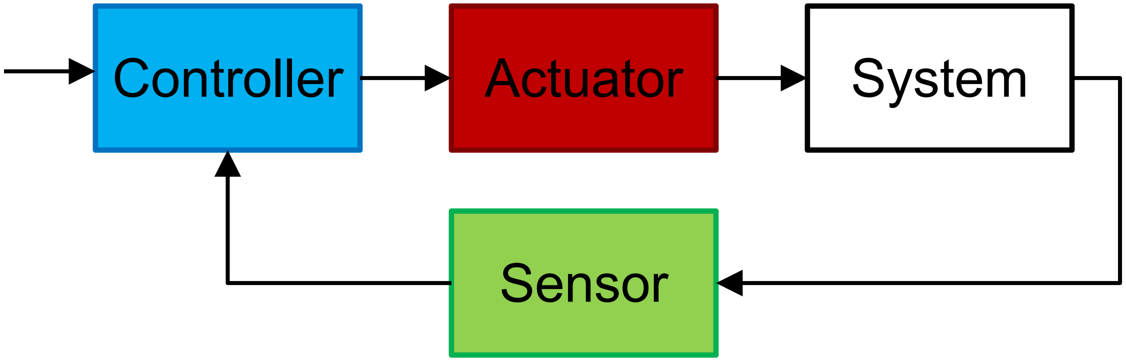
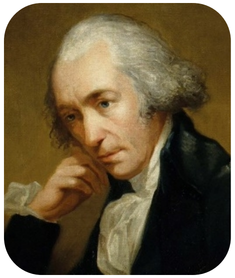
Recap

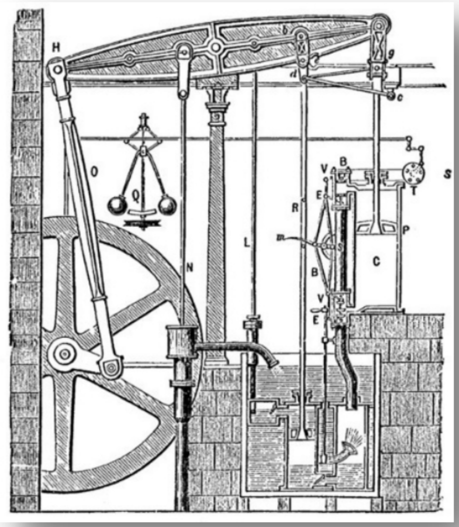
- **Feedback Control**
  - Its power:
    - A Feedback Control System (K. J. Åströma and P.R. Kumar)
      - *makes good systems from bad components*
      - *makes a system insensitive to disturbances and component variations*
      - *stabilizes an unstable system*
      - *creates desired behavior*
  - How?
    - **Controller**
    - Actuator
    - Sensor

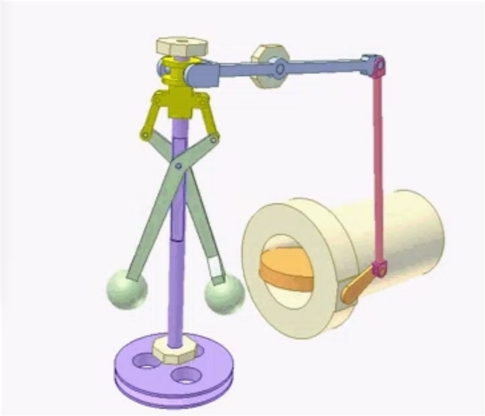


Overview

- **Controller (Feedback)**
  - History
    - **Christiaan Huygens** (pendulum clock, centrifugal force, 17th century)
    - **James Watt** (centrifugal governor, steam engine 1788)
  - Basic Idea
    - **Feedback loop**
  - Types
    - ON-OFF Control
      (Bang-Bang Control)
    - PID Control
    - Adaptive Control
    - Robust Control
    - Optimal Control
    - Nonlinear Control
    - Fuzzy Logic, Neural Network…







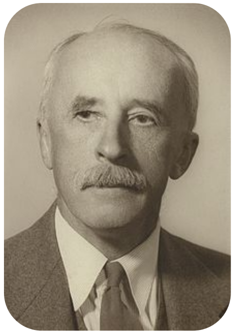
Topics to Be Covered

- **PID Controller**
  - Basic concepts
  - Common tuning methods
  - Applications
- **Optimal PID Controller**
  - Linear-Quadratic Regulator (LQR)
  - Main method
  - Some extensions

PID Controller

PID Controller

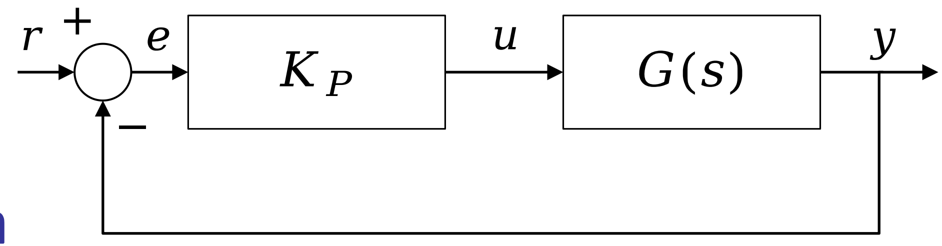
- PID: Proportional-Integral-Derivative
- History
  - An early mathematical analysis of a steam engine with a governor was made by James Clerk Maxwell in 1868
  - PID control was first developed using theoretical analysis by a Russian American engineer Nicolas Minorsky in 1922
    - based on observations of a helmsman
      - *current course error*
      - *past error*
      - *current rate of change*



(https://en.wikipedia.org/wiki/PID_controller)

- Type
  - It can be considered as a kind of Model-free Controller, which is very very widely used in various applications!!!

PID Controller



- **Proportional (P) Action**
  - The control action is simply proportional to the control error

$$u_P(t) = K_P e(t)$$

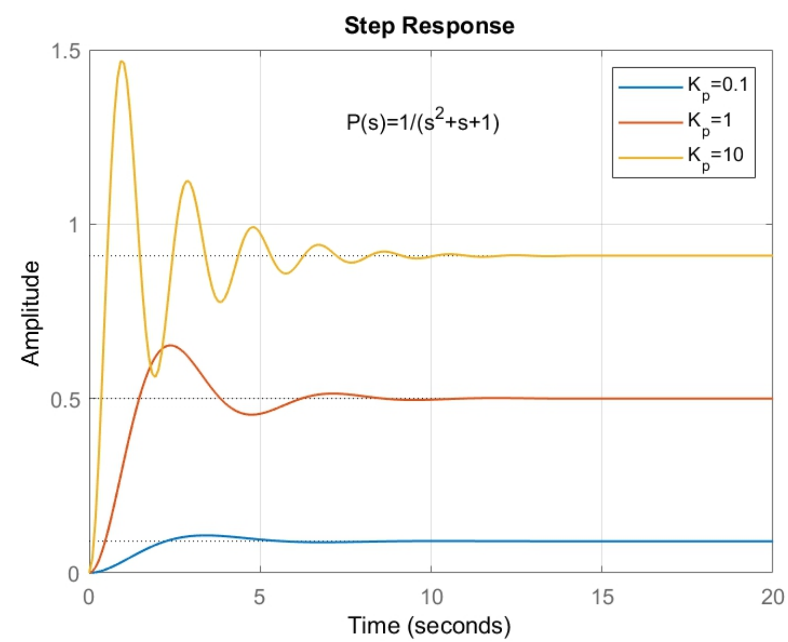
$$U_P(s) = K_P E(s)$$

  - High proportional gain results in small errors
  - Stead-state error e_{ss} exists
    - Why?

$$e_{ss} = e(\infty) = \lim_{s \to 0} sE(s)$$

$$E(s) = \frac{R(s)}{1 + K_P G(s)}$$

    - How to solve?



PID Controller

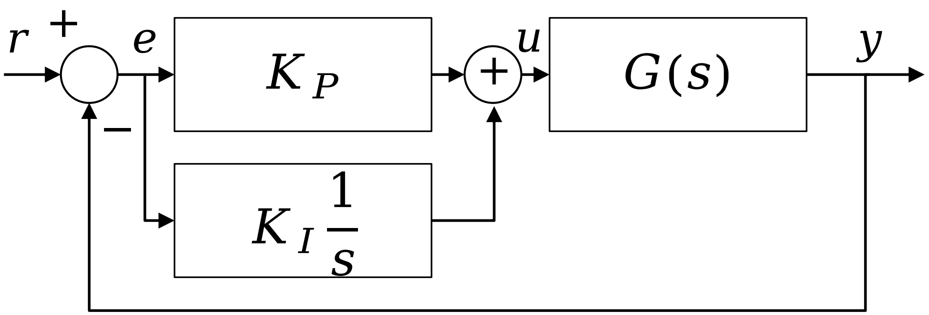
- **Integral Action**
  - The controller action is proportional to the integral of the error with respect to time

$$u_I(t) = K_I \int_0^t e(\tau)\, d\tau$$

$$U_I(s) = \frac{K_I}{s} E(s)$$

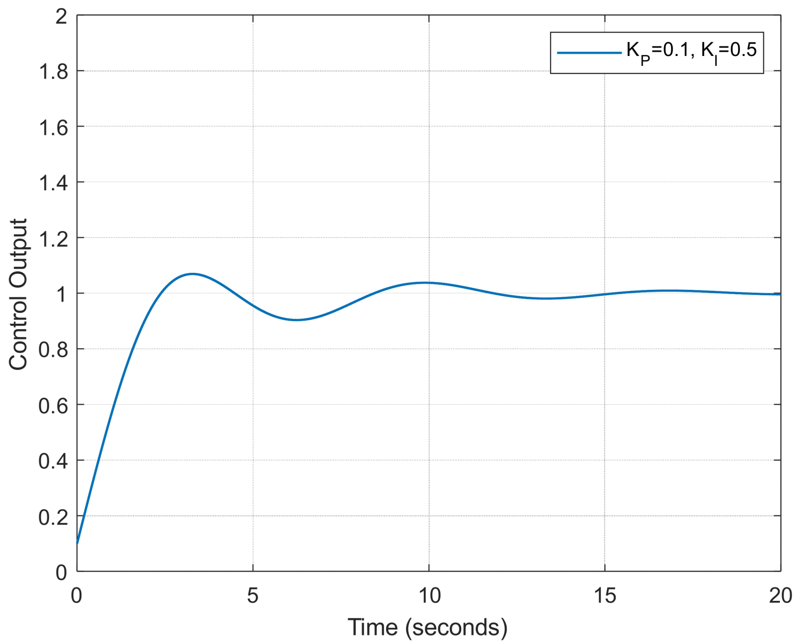
  - To make sure that the process output agrees with the set point in steady state

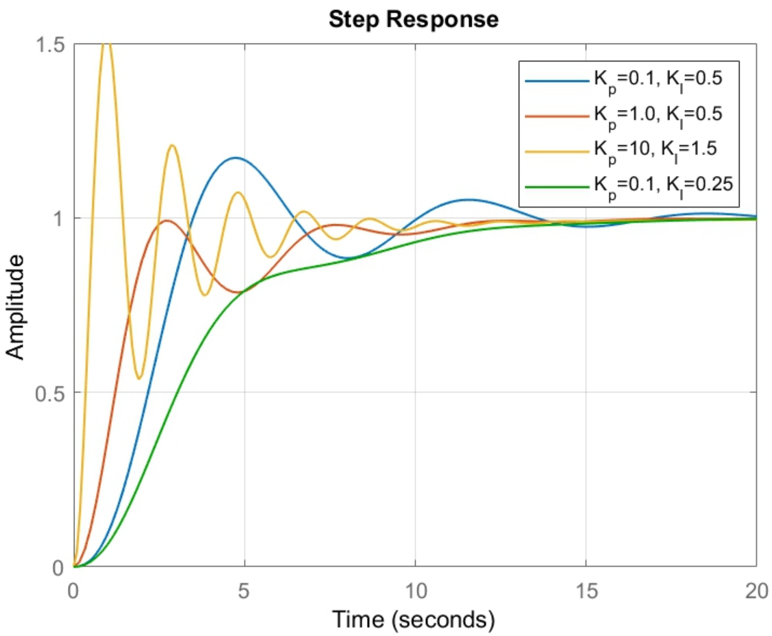
$$\dot{u}_I(t) = K_I e(t)$$



PID Controller

- **Integral Action**
  - For small values of the integral gain, the response creeps slowly towards the set point.
  - The approach is faster for larger values of the integral gain but it is also more oscillatory





PID Controller

- **Derivative Action**
  - The controller action depends on the rate of error change

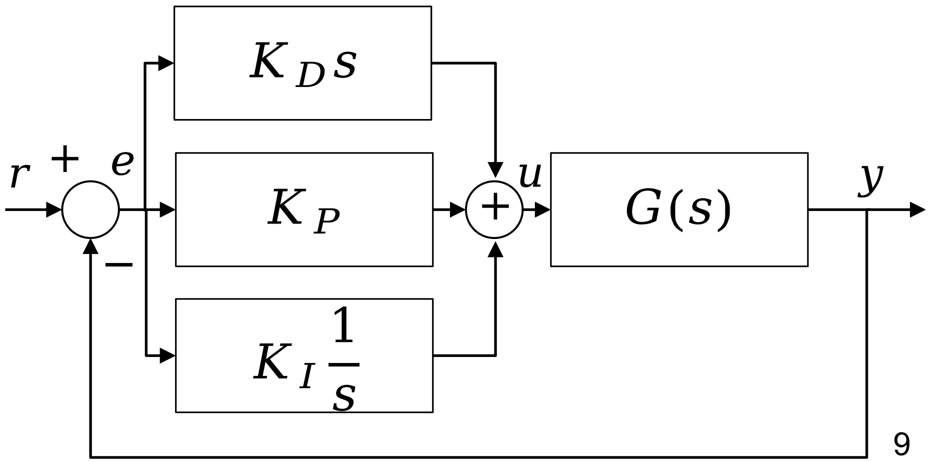
$$u_D(t) = K_D \frac{de(t)}{dt} = K_D \dot{e}(t)$$

$$U_D(s) = K_D s E(s)$$

  - To improve the closed-loop stability
  - A Taylor series expansion of $e(t + T_D)$, we have

$$e(t + T_D) \approx e(t) + T_D \frac{de(t)}{dt}$$

- The control signal is thus proportional to an estimate of the control error at time T_D ahead
- Like a predictive control



PID Controller

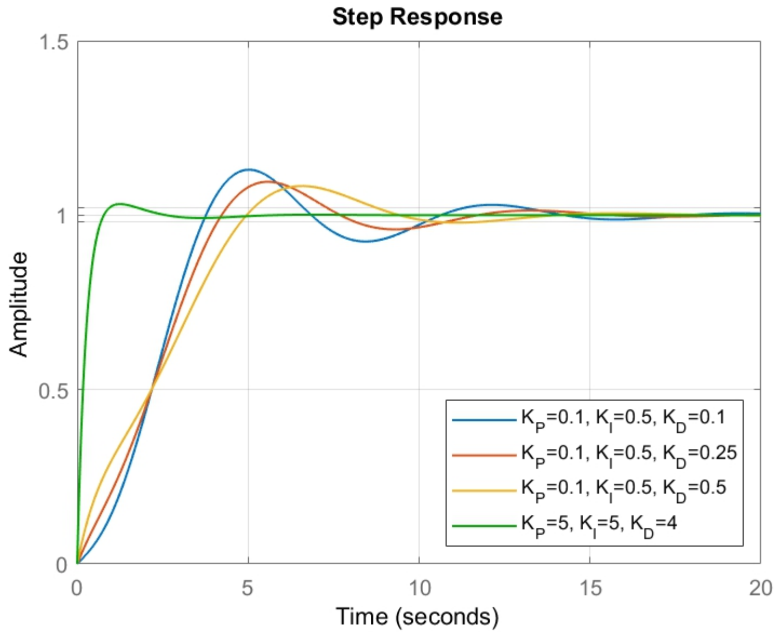
- **Derivative Action**
  - Derivative action predicts system behavior and thus it can improve the settling time and stability of the system
  - To further reduce the undesirable noise amplification brought by the pure derivative action, the pure derivative action is combined with a low-pass filter
    - First-order filter

$$K_D \frac{s}{T_f s + 1} E(s)$$

or

$$\frac{sKT_D}{sT_D/N + 1} E(s)$$

N is typically use 2 to 20



PID Controller

- **Composite Control Mode**
  - PI Controller

$$u_{PI}(t) = K_P e(t) + K_I \int_0^t e(\tau)\, d\tau$$

  - PD Controller

$$u_{PD}(t) = K_P e(t) + K_D \frac{de(t)}{dt}$$

  - PID Controller

$$u_{PID}(t) = K_P e(t) + K_I \int_0^t e(\tau)\, d\tau + K_D \frac{de(t)}{dt}$$

PID Controller

- **Basic Forms**
  - Parallel

$$u_{PID}(t) = K_P e(t) + K_I \int_0^t e(\tau)\, d\tau + K_D \frac{de(t)}{dt}$$

  - Series

$$u_{PID}(t) = K_P (e(t) + \frac{1}{T_I} \int_0^t e(\tau)\, d\tau + T_D \frac{de(t)}{dt})$$

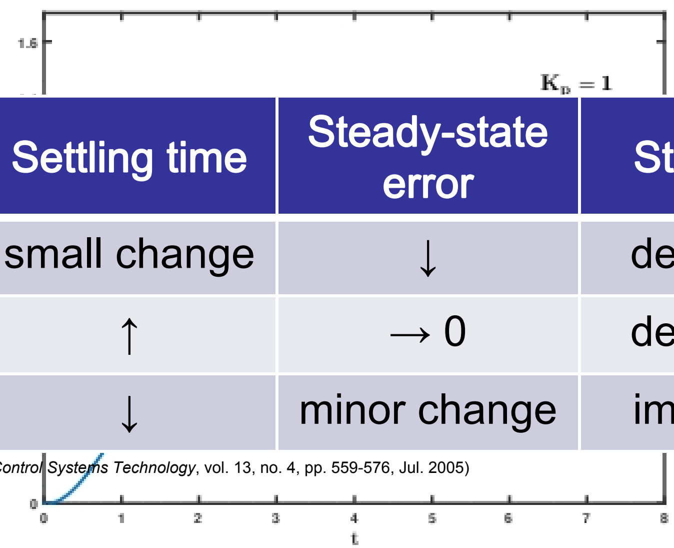
- **Controller Terms**
  - Proportional: basic continuous **feedback** control
  - Integral: ensures steady-state error becomes **zero**
  - Derivative: allows **prediction** of the future error

PID Controller

- **How to tune the controller gains (parameters)?**
  - Experience
  - Experiment (Data-based)
  - Model-based
- **Common Tuning Methods**
  - Trial-and-Error
  - Ziegler-Nichols (1942)
  - Cohen-Coon
  - Relay Auto-tuning
  - Iterative Feedback Tuning

PID Controller

- **Trial-and-Error Tuning Method (Manual Tuning)**
  - Based on some rules of thumb and experience
    - Set K_I and K_D to be zero
    - Increase K_P until the output is oscillating, then set K_P to be about half
    - Increase K_I if the steady-state error is not zero
    - Increase K_D if required



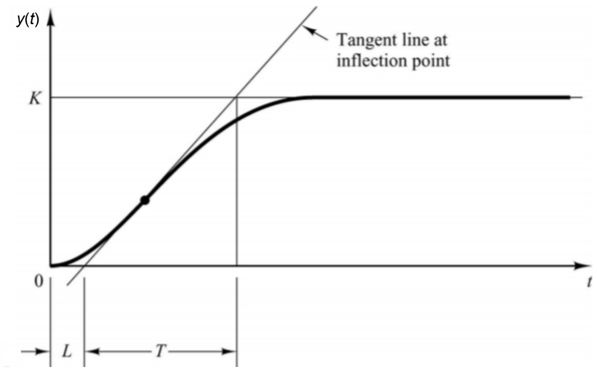
Parameter	Rise time	Overshoot	Settling time	Steady-state error	Stability
$K_P \uparrow$	$\downarrow\downarrow$	$\uparrow$	small change	$\downarrow$	degrade
$K_I \uparrow$	small $\downarrow$	$\uparrow$	$\uparrow$	$\rightarrow 0$	degrade
$K_D \uparrow$	small $\downarrow$	$\downarrow$	$\downarrow$	minor change	improve

(K. H. Ang; G. Chong and Y. Li, PID control system analysis, design, and technology, *IEEE Transactions on Control Systems Technology*, vol. 13, no. 4, pp. 559-576, Jul. 2005)

(https://en.wikipedia.org/wiki/PID_controller)

PID Controller

- **Ziegler-Nichols (Z-N) Tuning Method: 1st Method**
  - The Step Response Method
    - based on the process information in the form of the open-loop step response
    - How?
      - *Capture the step response of the system*
      - *Find out K, T and L*
      - *Use both parameters and the following rule to calculate controller gains*



Controller	K_P	T_I	T_D
P	$\overline{a} = \dfrac{T}{KL}$	∞	0
PI	$0.9\overline{a}$	$3L$	0
PID	$1.2\overline{a}$	$2L$	$L/2$

PID Controller

- **Ziegler-Nichols (Z-N) Tuning Method: 2nd Method**
  - The Frequency Response Method
    - based on a simple characterization of the process dynamics
    - How?
      - *Set $K_{\boldsymbol{I}}$ and $K_{\boldsymbol{D}}$ to be zero*
      - *Increase K_P slowly until the process starts to oscillate*
      - *The gain when this occurs is the ultimate gain $K_{\boldsymbol{u}}$, and the period of the oscillation is the ultimate period T_u*
      - *Use both parameters and the following rule to calculate controller gains*

Controller	K_P	K_I	K_D
P	$0.50K_u$	0	0
PI	$0.45K_u$	$0.54K_u/T_u$	0
PID	$0.60K_u$	$1.20K_u/T_u$	$3K_uT_u/40$

PID Controller

- **Cohen-Coon (C-C) Tuning Method**
  - based on first-order time-delay (FOTD) process model
  - How?
    - exactly the same way as the Z-N step response method but the calculation is different

$$a = 1/\bar{a} = KL/T \qquad \tau = L/(L+T)$$

Controller	K_P	T_I	T_D
P	$(1 + \frac{0.35\tau}{1-\tau})/a$	∞	0
PI	$0.9(1 + \frac{0.092\tau}{1-\tau})/a$	$\frac{3.3-3.0\tau}{1+1.2\tau}L$	0
PD	$1.24(1 + \frac{0.13\tau}{1-\tau})/a$	∞	$\frac{0.27-0.36\tau}{1-0.87\tau}L$
PID	$1.35(1 + \frac{0.18\tau}{1-\tau})/a$	$\frac{2.5-2.0\tau}{1-0.39\tau}L$	$\frac{3.7-0.37\tau}{1-0.81\tau}L$

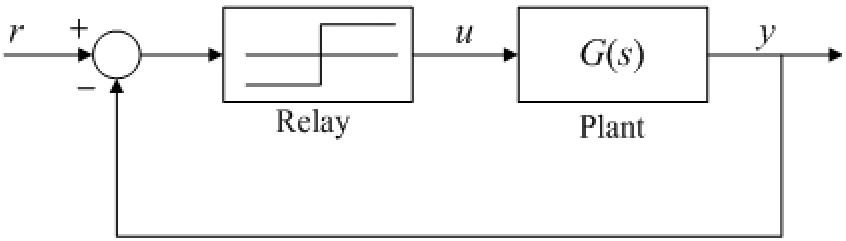
PID Controller

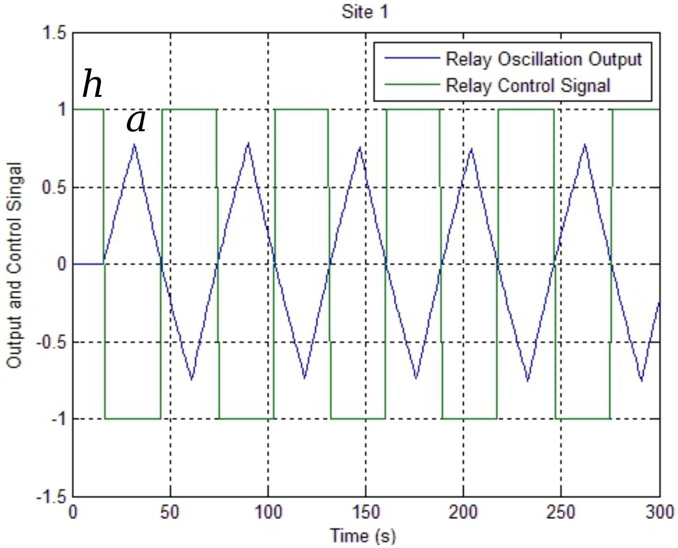
- **Relay Auto-tuning Method**
  - The basic idea of relay auto-tuning is the observation that many processes have limit cycle oscillations under relay feedback
  - The **ultimate gain** K_u, and the **ultimate period** T_u can be also obtained via the relay feedback

$$K_u = \frac{4h}{a\pi}$$

$$\boxed{\begin{aligned} &\arg(G(j\omega_u)) = -\pi \\ &|G(j\omega_u)| = \frac{a\pi}{4h} = \frac{1}{K_u} \end{aligned}}$$

  - Then, the Z-N tuning second method can be applied to calculate the PID controller gains





PID Controller

- **Iterative Feedback Tuning (IFT) Method**
  - IFT is an iterative data-based method for adjusting controller parameters
    - to minimize the loss function

$$J(\rho) = \frac{1}{2N}\sum_{t=1}^{N} \tilde{y}(t,\rho)^2$$

  - How?

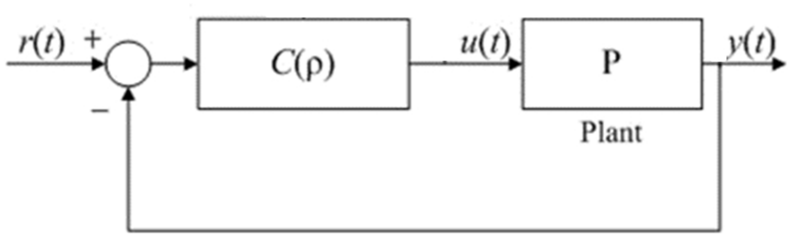
$$\frac{\partial J(\rho)}{\partial \rho} = \frac{1}{N}\sum_{t=1}^{N}\left(\tilde{y}(t,\rho)\frac{\partial \tilde{y}(t,\rho)}{\partial \rho}\right) = \frac{1}{N}\sum_{t=1}^{N}\left(\tilde{y}(t,\rho)\frac{\partial y(t,\rho)}{\partial \rho}\right)$$

$$\frac{\partial y(t,\rho)}{\partial \rho} = Pu'(t,\rho) = P\frac{1}{1+C(\rho)P}\frac{\partial C(\rho)}{\partial \rho}(r - y(\rho)) = \frac{1}{C(\rho)}\frac{\partial C(\rho)}{\partial \rho}\left[\frac{C(\rho)P}{1+C(\rho)P}(r-y(\rho))\right]$$

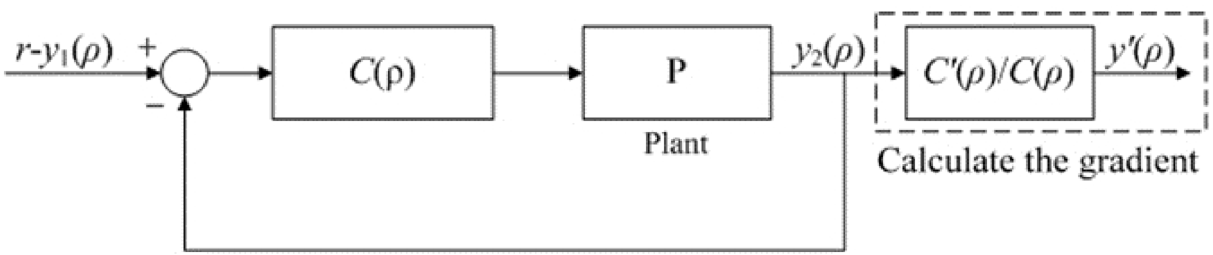
$$\frac{\partial y(t,\rho_i)}{\partial \rho} = \frac{1}{C(\rho_i)}\frac{\partial C}{\partial \rho}(\rho_i)\, y_2(\rho_i)$$

PID Controller

- Iterative Feedback Tuning (IFT) Method
  - How?
    - First experiment (normal experiment) to get y_1



    - Second experiment (gradient experiment) to get y_2



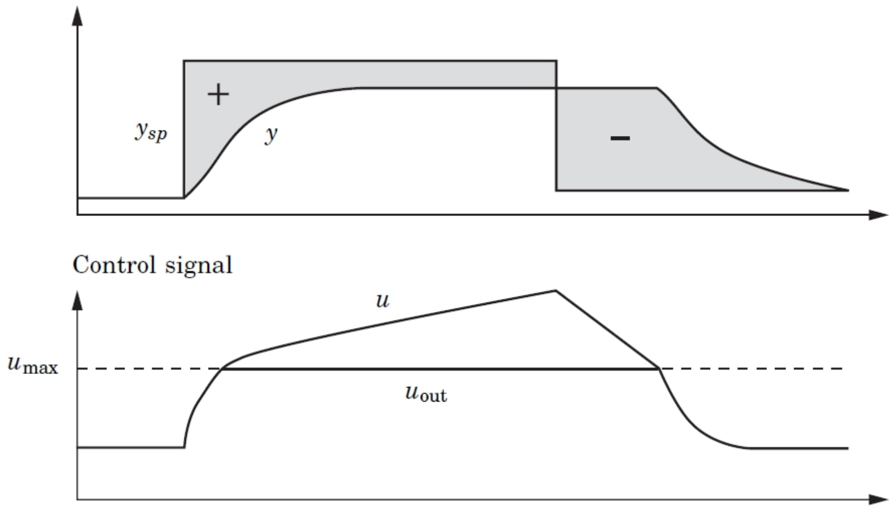
    - Update the controller parameters via gradient-based minimization

$$\rho_{i+1} = \rho_i - \gamma R_i^{-1} J'(\rho_i)$$

    - Repeat until the requirement is met

PID Controller

- Integrator windup



(K. J. Åström; T. Hägglund, Advanced PID control (vol. 461), Research Triangle Park, NC: ISA-The Instrumentation, Systems, and Automation Society, 2006)

- Anti-windup

$$u_i(t) = \int_0^t [\frac{K_c}{T_i} e(\tau) + \frac{1}{T_{aw}} e_s(\tau)] d\tau$$

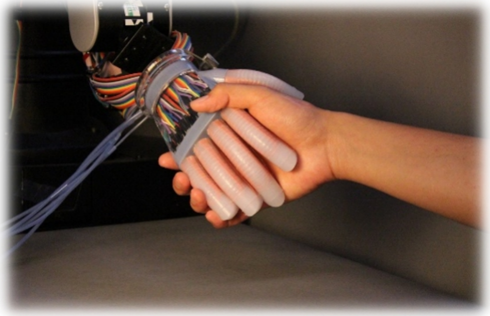
  - e_s is the difference between the controller output and the saturated output

PID Controller - Applications

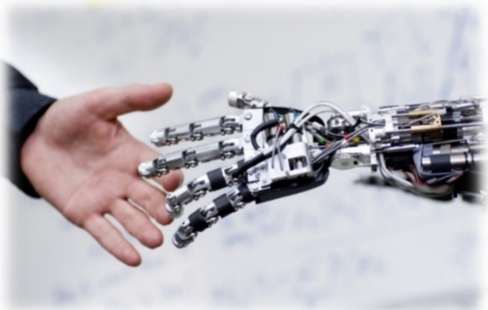
Modelling and Control of A Soft Inchworm Robot

Soft Robotics

- What?
  - Made up of compliant materials
  - Continuously deformable structure with muscle-like actuation
  - Results in amazing resilient motion



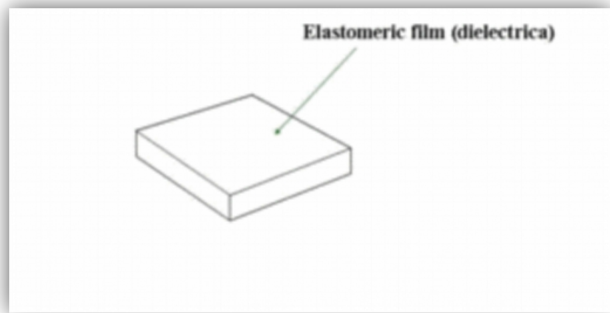
VS

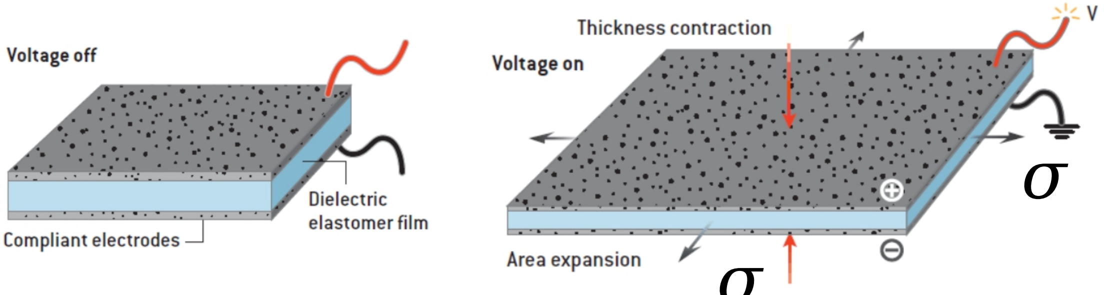


	Soft robots	Rigid robots
Materials	KPa ~ MPa	> 100 GPa
Actuation	Various soft actuators	Electric motors
Environment	Unstructured	Determined
Advantages	Adaptability, compliance, interaction	Load, precision, speed

Soft Robotics

- Soft actuators are essential to soft robots
  - Among the explored soft actuators, Dielectric Elastomer Actuators (DEAs), as a kind of electroactive polymer actuators, stand out
    - due to their muscle-like properties
      - *large deformation*
      - *fast response*
      - *high energy density*





(S. Ashley, 2003)

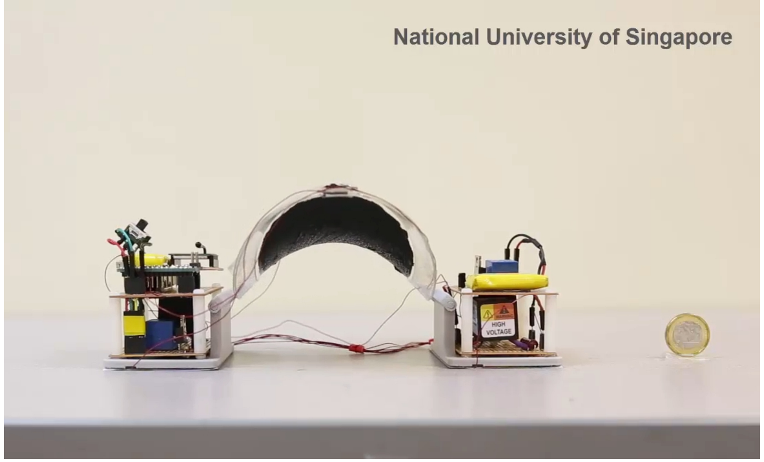
- Lightweight (~ water density)
- High energy density (~ 0.4 J/g)
- Giant strain (158% to 1692%)
- Fast response (~0.1ms)
- Low noise
- Self-sensing capability

Soft Robotics

- Applications

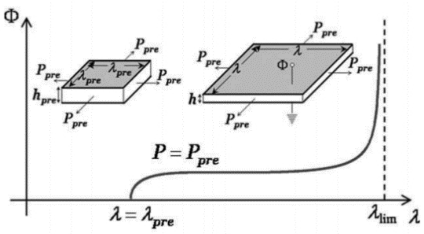
A frog swimming robot
(Tang et al. 2017)

Jaw movement
(Wang et al. 2015)



An inchworm crawling robot
(Cao et al. 2018)

Soft Robotics

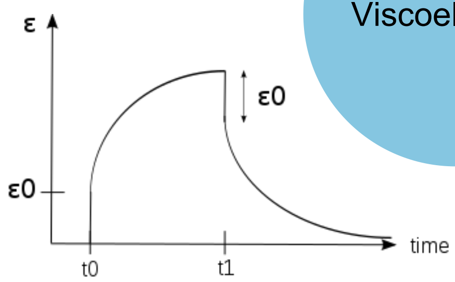


Material Nonlinearity

Viscoelasticity

Challenges

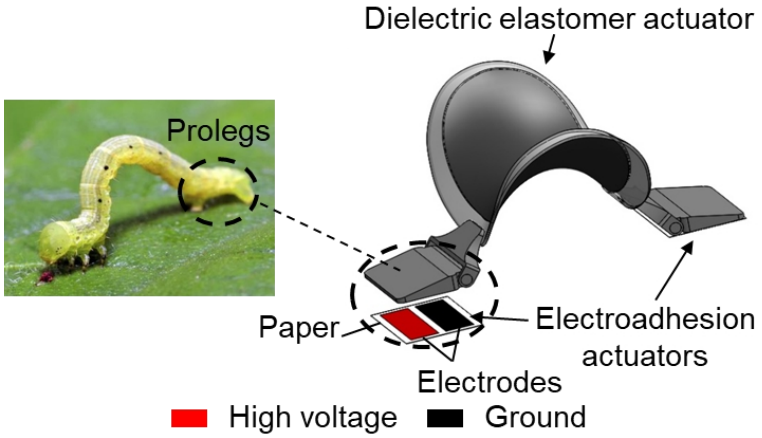
Geometric Complexity

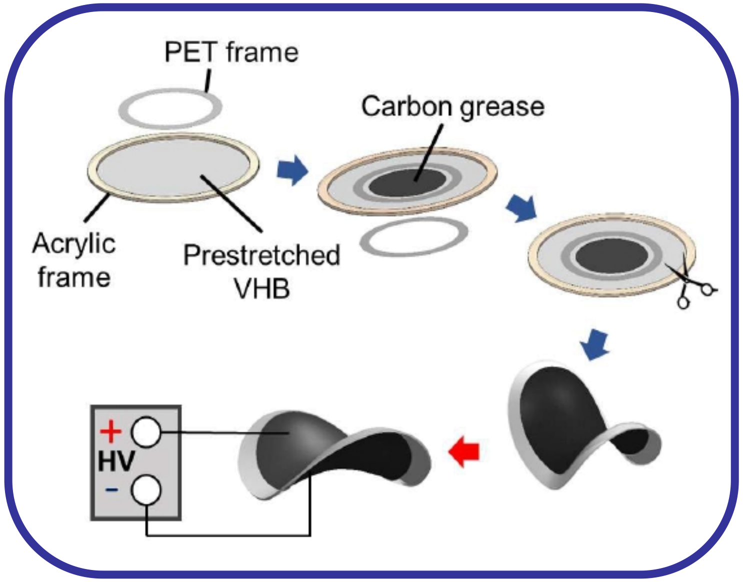


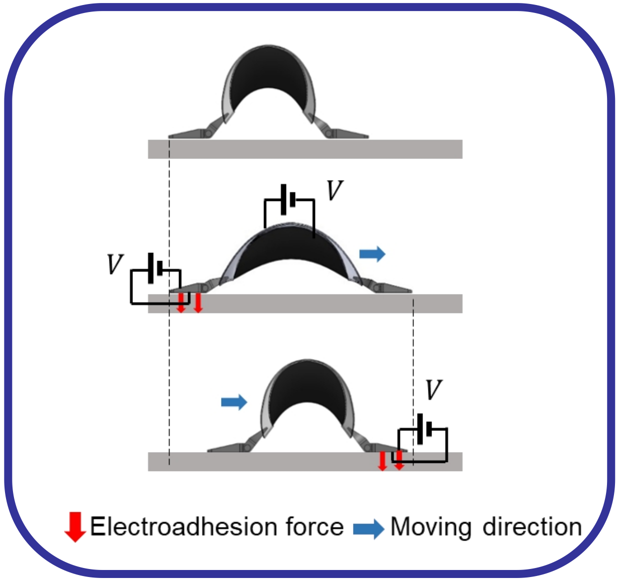
Environmental Interactions

Soft Robotics

- **A DEA-based Soft Inchworm Robot**
  - A bioinspired robot (inchworm-like)
  - Two VHB 4910 films are attached between two annular polyethylene terephthalate (PET) frames

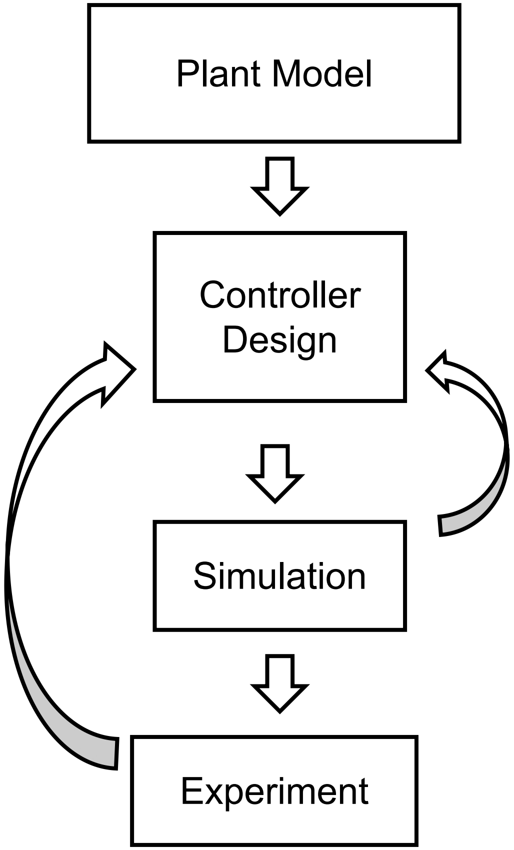






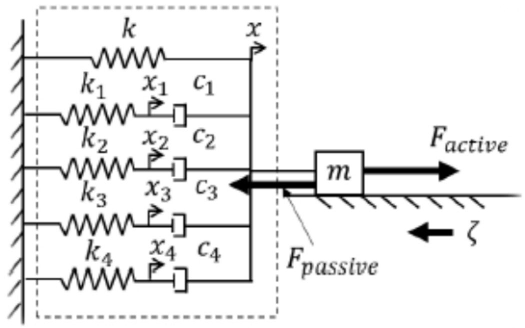
Soft Robotics

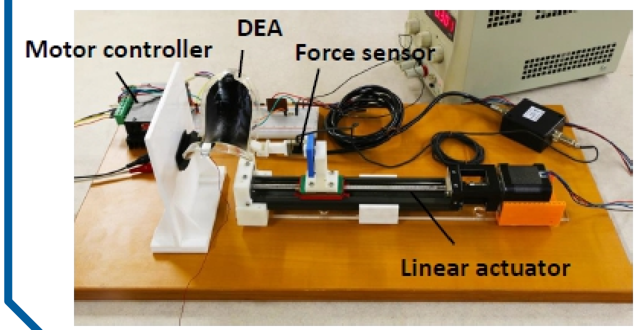
- **Control System Design Process**
  - Modelling
    - First-principle
    - System Identification
  - Controller Design
  - Simulation & Experiment
    - Verification
    - Validation



Soft Robotics

- Dynamic Model





Model:

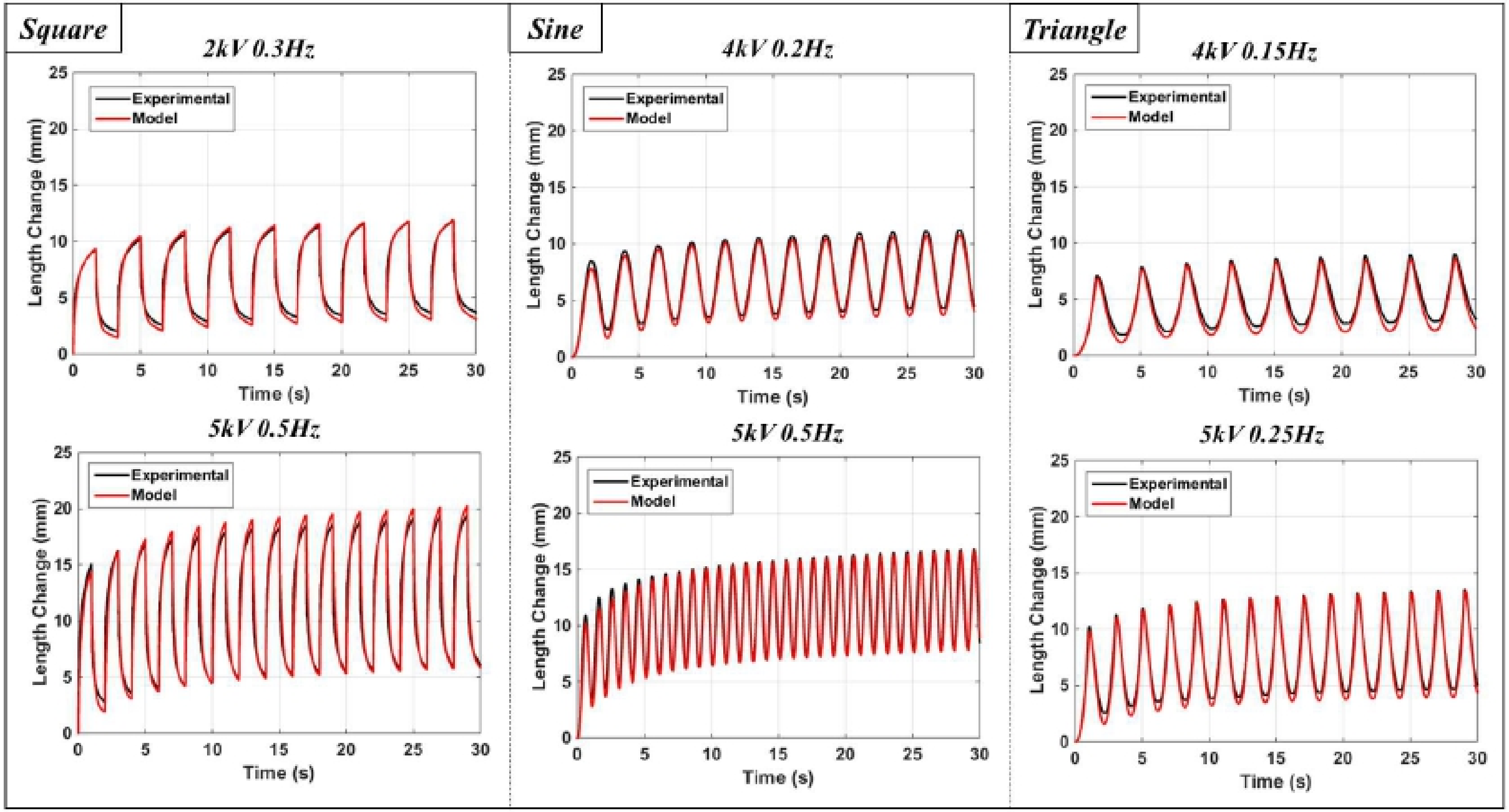
$$F_{active} = \Phi^2 \Omega(x)$$

$$\tilde{m}\ddot{x} = -kx - \sum_{i=1}^{4} k_i x_i + \Phi^2 \Omega(x) - \zeta \mathrm{sign}(\dot{x})$$

$$k_i x_i = c_i(\dot{x} - \dot{x}_i), \quad i = 1, 2, 3, 4$$

Soft Robotics

- System Identification & Validation



Soft Robotics

- Controller Design
  - Overall controller

$$u = u_{ff} + u_{fb} + u_{fc}$$

  - Linearization

$$\begin{cases} u = \Phi^2(\alpha x + \beta)/\widetilde{m} \\ u_{fc} = B^{-1}\zeta \mathrm{sign}(\dot{x}) \end{cases} \rightarrow \begin{matrix} \dot{X} = AX + Bu \\ y = CX \end{matrix}$$

$$X = [x_1 \quad x_2 \quad x_3 \quad x \quad \dot{x}]$$

$$A = \begin{bmatrix} -\frac{k_1}{c_1} & 0 & 0 & 0 & 1 \\ 0 & -\frac{k_2}{c_2} & 0 & 0 & 1 \\ 0 & 0 & -\frac{k_3}{c_3} & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ -\frac{k_1}{m} & -\frac{k_2}{m} & -\frac{k_3}{m} & -\frac{k}{m} & 0 \end{bmatrix}$$

$$B = [0 \quad 0 \quad 0 \quad 0 \quad 1]^T$$
$$C = [0 \quad 0 \quad 0 \quad 1 \quad 0]$$

  - Feedforward controller

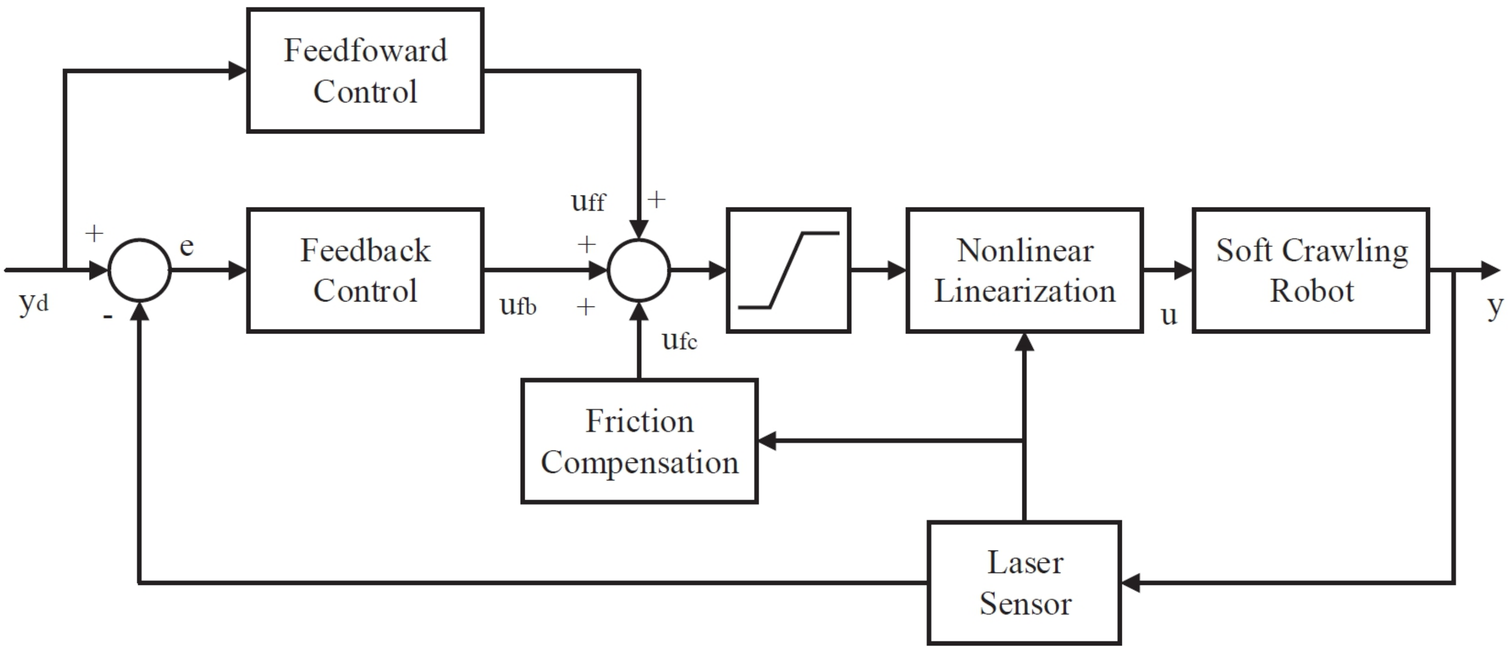
$$G(s) = C(sI - A)^{-1}B \quad C_{ff} = Q_f(s)G(s)^{-1}$$

  - Feedback controller
    - PI controller
    - Relay auto-tuning & Ziegler-Nichols rule
    - Back calculation anti-windup scheme

$$u_{fb} = K_p(e(e) + \int_0^t \left[\frac{1}{T_i}e(\tau) + \frac{1}{K_p T_{aw}} e_s(\tau)\right] d\tau)$$

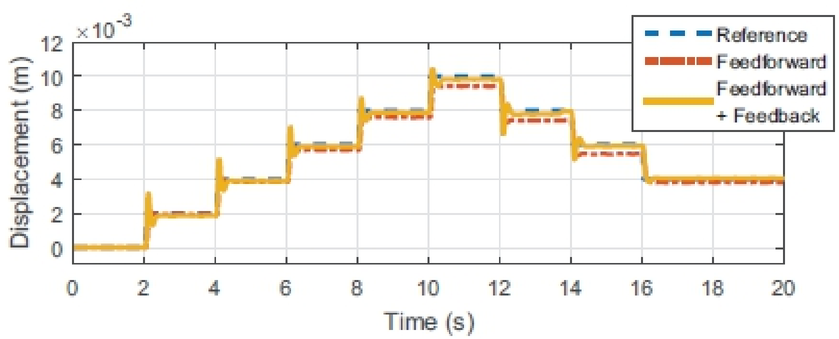
Soft Robotics

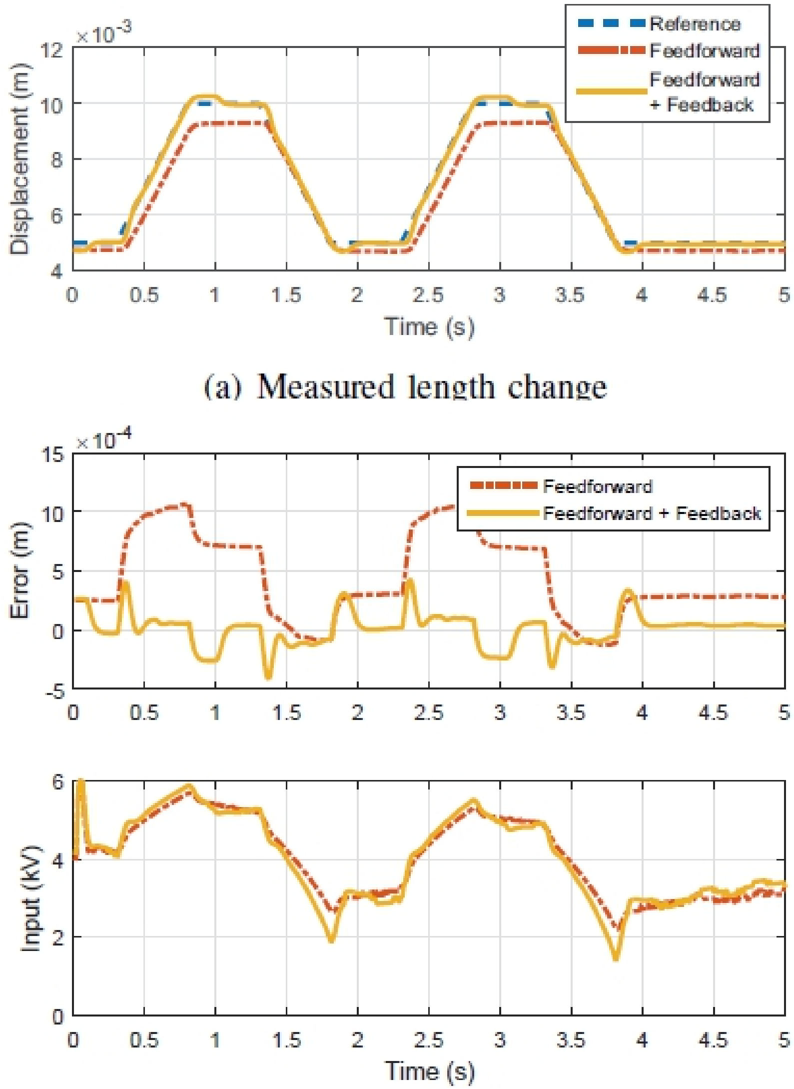
- **Overall Control System**
  - 2-DOF Control



Soft Robotics

- **Experiments**
  - Point-to-point movement
  - Trajectory tracking
- **Results**
  - Fast response ensured by the feedforward controller
  - Good tracking performances benefiting from the feedback controller



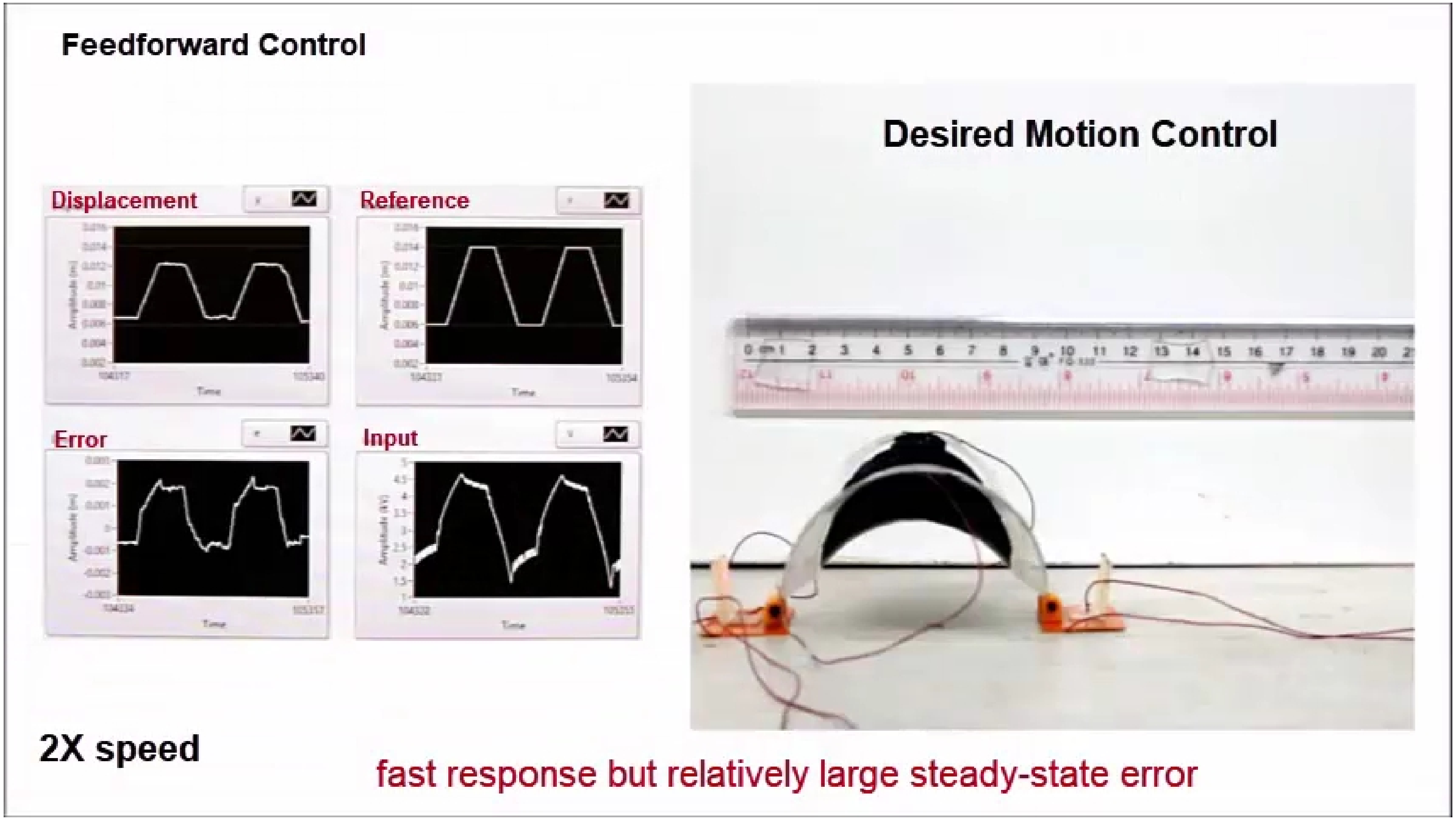


(a) Measured length change

(b) Error and input voltage

Soft Robotics

- Soft Inchworm Robot - demo video



Soft Robotics

- Soft Inchworm Robot demo video

IEEE Transactions on Industrial Electronics

Control of A Soft Inchworm Robot with Environment Adaptation

<u>Videos of Robot Motion in Different Environments</u>

Break

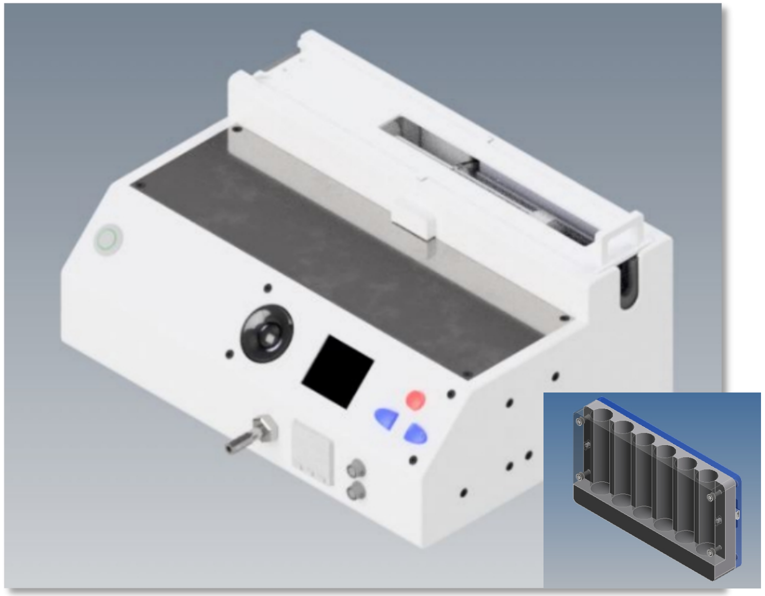
(continue at 19:30)

PID Controller - Applications

Other Examples

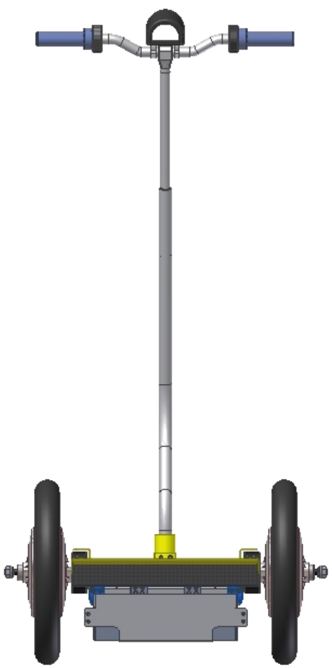
PID Controller - Applications

- **Medical Device for Oocyte Retrieval in In Vitro Fertilization (IVF) Process**
  - An advanced medical device for oocyte retrieval aims to improve the success rate
  - Temperature control



PID Controller - Applications

- **Personal transportation**
  - A coaxial two-wheel electric vehicle which provides a convenient way to travel



PID Controller - Applications

- **Inverted Pendulum**
  - An **inverted pendulum** is a pendulum that has its center of mass above its pivot point
  - It is an “unstable” system without any control

Optimal PID Controller

Optimal PID Controller

- **Optimal Control**
  - What?
    - A control law for a given system, which is to minimize the cost function (based on the specifications)

$$\dot{X}(t) = AX(t) + Bu(t)$$

$$Y(t) = CX(t)$$

$$X(0) \neq 0$$

- **Linear-Quadratic Regulator (LQR)**
  - minimum mixed error and energy
  - The cost function takes the *quadratic* form

$$J = \frac{1}{2}\int_0^{\infty} (X^T Q X + u^T R u) dt$$

  $Q > 0$ and $R > 0$ are the weighting matrices (normally are chosen as diagonal matrices)

  - The optimal control law is in the form of linear state feedback

$$u = -KX$$

  - Now, let's develop an algorithm for computing the optimal K

Optimal PID Controller

- **LQR Solution**
  - The closed-loop system is

$$\dot{X} = (A - BK)X$$

  - Assume that K is such that the system is stable
  - Let P an symmetric matrix satisfy the following ***Algebraic Riccati Equation (ARE)***

$$A^T P + PA + Q = PBR^{-1}B^T P$$

  - Then, we have

$$\frac{d}{dt}(X^T P X) = \dot{X}^T P X + X^T P \dot{X}$$
$$= X^T(A^T P + PA)X + u^T B^T P X + X^T P B u$$
$$= (X^T P B + u^T R)R^{-1}(B^T P X + Ru) - X^T Q X - u^T R u$$

  - Integrate both sides, the LHS becomes

$$X^T(t) P X(t)|_0^\infty = X^T(\infty) P X(\infty) - X^T(0) P X(0) =- X^T(0) P X(0)$$

as a stable system, i.e., $X(\infty) = 0$

Optimal PID Controller

$$J = \frac{1}{2}\int_0^\infty (X^T Q X + u^T R u)dt$$

- **LQR Solution**
  - The RHS becomes
    $$\int_0^\infty (X^T P B + u^T R) R^{-1} (B^T P X + R u) dt - \int_0^\infty (X^T Q X + u^T R u) dt$$
  - Therefore, the cost function becomes
    $$J = \frac{1}{2} X^T(0) P X(0) + \frac{1}{2}\int_0^\infty (X^T P B + u^T R) R^{-1} (B^T P X + R u) dt$$
  - It is clear that the minimum occurs at
    $$B^T P X + R u = 0$$
    i.e.,
    $$u = -R^{-1} B^T P X$$
  - Thus, if we select $K = R^{-1} B^T P$, we can have the optimal control law,

    where P is the solution of the *Algebraic Riccati Equation*

Optimal PID Controller

- **Error State-Space Model**
  - Consider a second-order system,

$$G(s) = \frac{X(s)}{U(s)} = \frac{b}{s^2 + a_1 s + a_0}$$

or

$$\ddot{x} = -a_1 \dot{x} - a_0 x + bu$$

  - In the state-space form, it gives

$$\dot{X}(t) = AX(t) + Bu(t) = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix} u$$

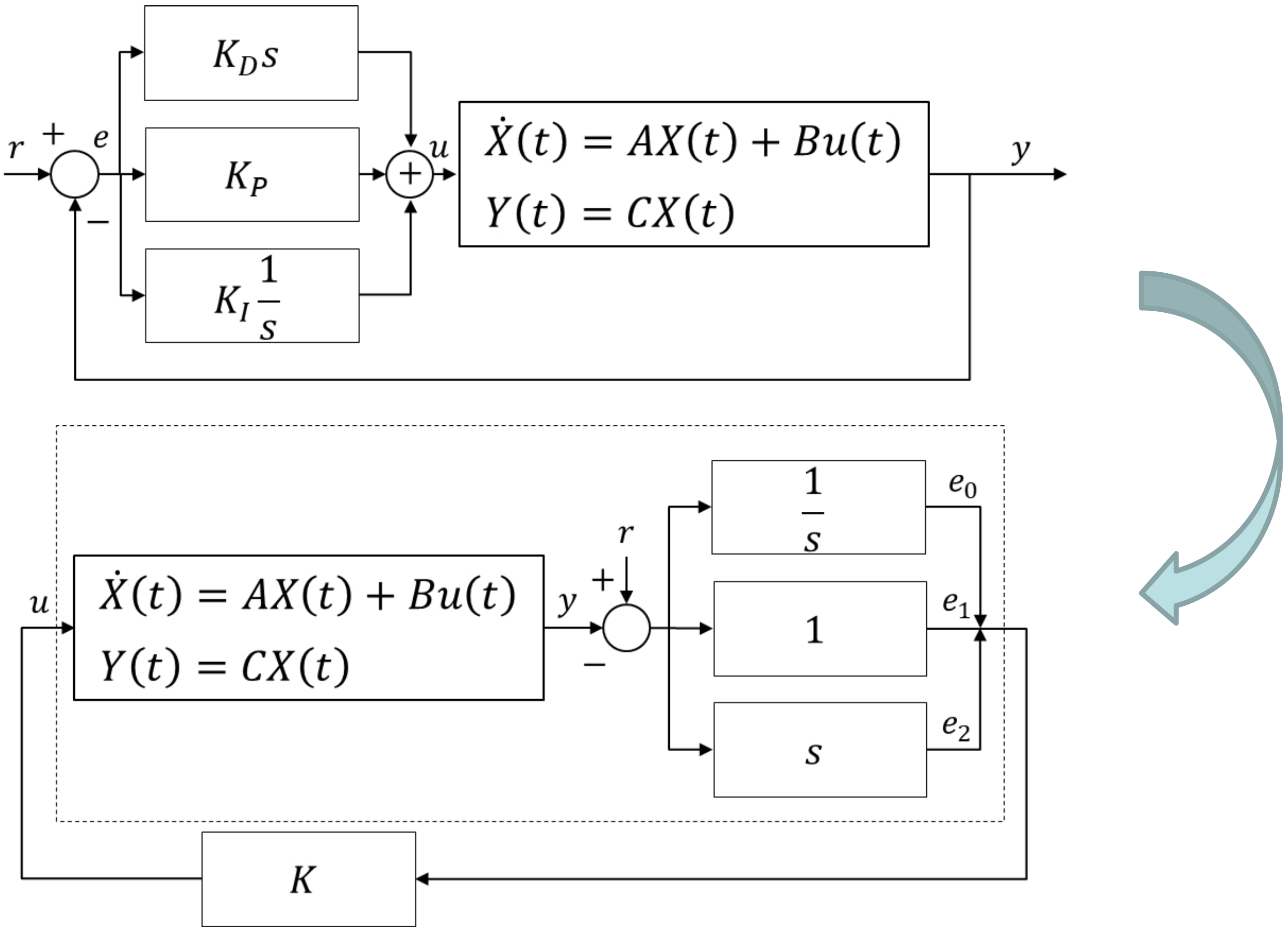
  - If x_d is constant or very slowly varying, we can have the error state-space model

$$\dot{E}(t) = \overline{A}E(t) + \overline{B}u(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -a_0 & -a_1 \end{bmatrix} \begin{bmatrix} e_0 \\ e_1 \\ e_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -b \end{bmatrix} u$$

where $e_0 = \int_0^\infty e(\tau) d\tau$, $e_1 = e$, $e_2 = \dot{e}$

Optimal PID Controller

- Error State-Space Model



Optimal PID Controller

- **LQR-assisted PID Tuning Method**
  - With the error model, define the following cost function

$$J = \frac{1}{2}\int_0^{\infty} (E^T Q E + u^T R u) dt$$

  - The PID tuning problem is now converted to be the LQR problem
    - The PID controller is equivalent to the error state feedback control
  - Then, the LQR technique can be applied to determine the PID controller gains, where the gains are the optimal ones

$$u = KE$$

$$K = -R^{-1}\overline{B}^T P$$

  P is the solution of the following new Riccati equation

$$\overline{A}^T P + P\overline{A} + Q = P\overline{B}R^{-1}\overline{B}^T P$$

Optimal PID Controller

- **LQR-assisted PID Tuning Method**
  - How to choose the weighting matrices?
    - Trial-and-Error
    - Bryson's rule (E. Bryson, Jr. and Yu-Chi Ho)
      - *Choose diagonal matrices such that*

$$q_{ii} = 1/\text{maximum acceptable value of } x_{\mathrm{i}}^2$$

$$r_{ii} = 1/\text{maximum acceptable value of } u_{\mathrm{i}}^2$$

  - The relationship between Q and R
  - Note: a model-based feedforward controller is needed for trajectory tracking problem

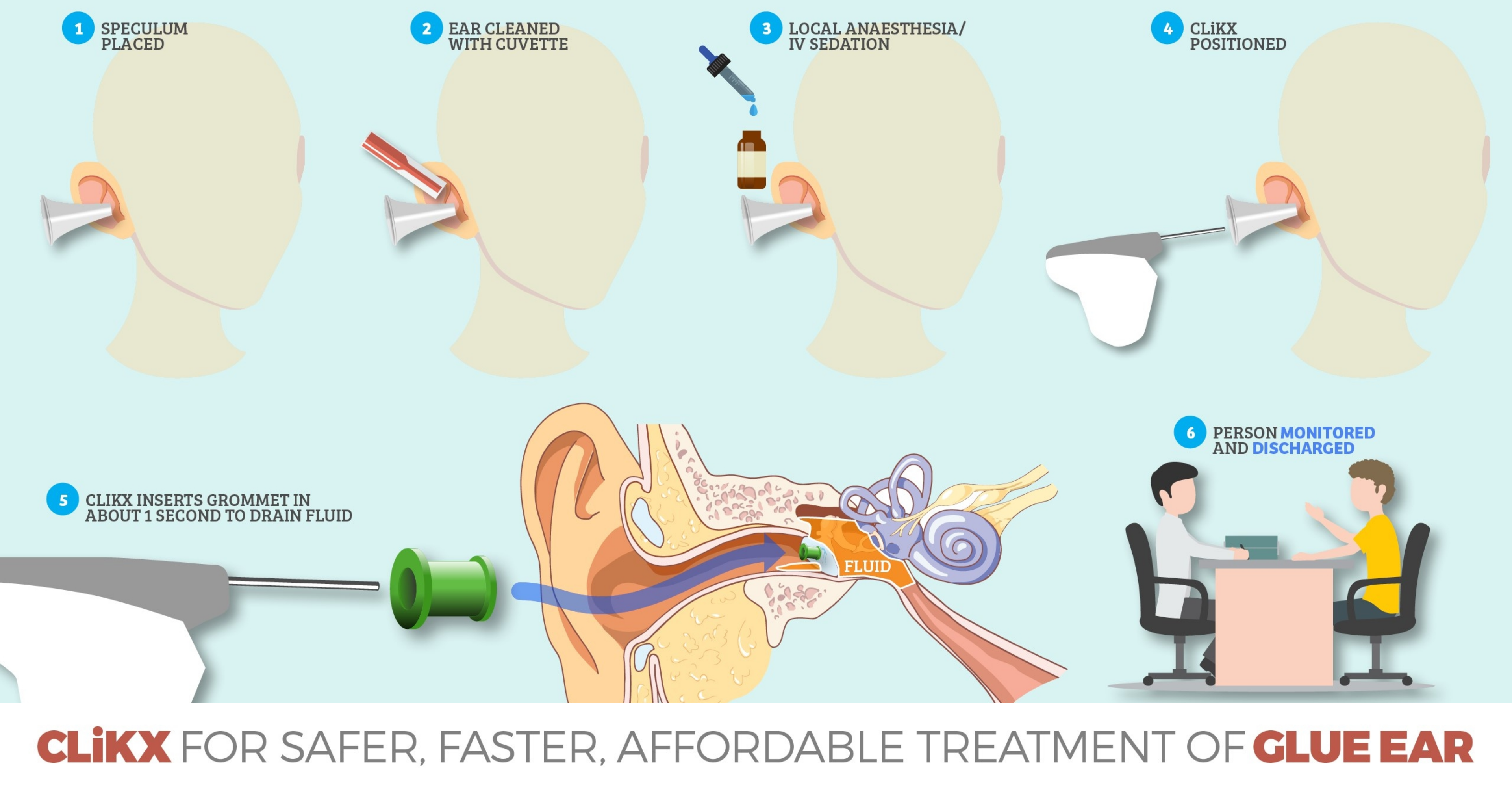
J. B. He; Q.-G. Wang; and T. H. Lee, PI/PID controller tuning via LQR approach, *Chemical Engineering Science*, vol. 55, no. 13, pp. 2429-2439, Jul. 2000

Optimal PID Controller - Applications

Precision Motion Control for An Ear Surgical Device

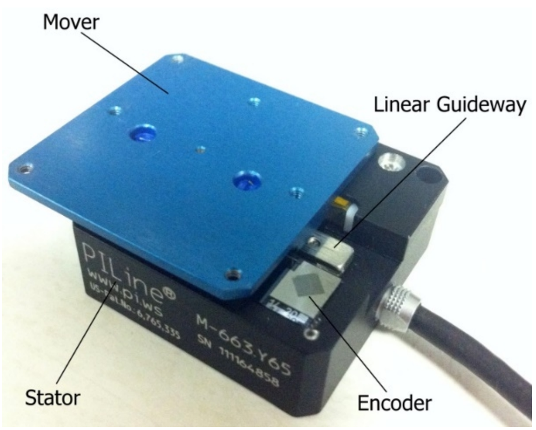
Ear Surgical Device

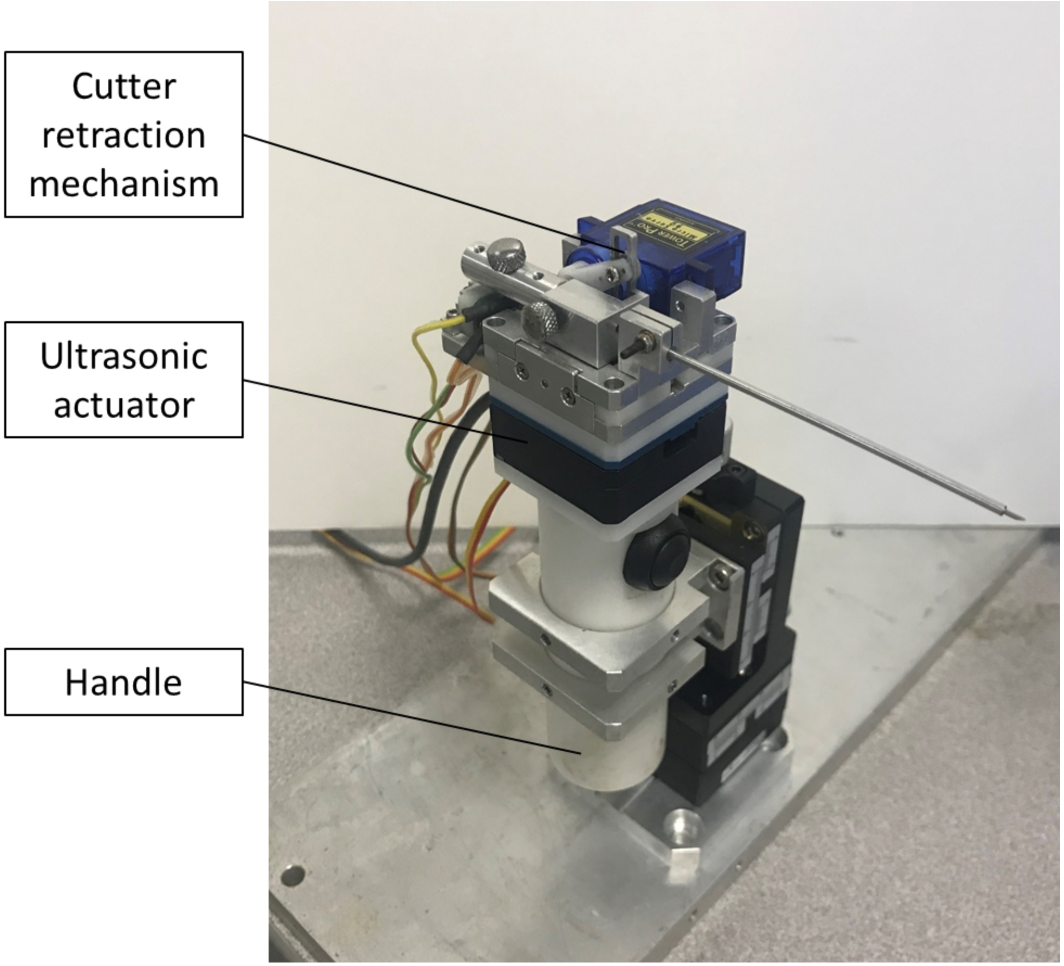
- Ventilation Tube Applicator (VTA): CLiKX



Ear Surgical Device

- **Mechanical System**
  - Its motion is driven by an Ultrasonic Motor (USM) Stage





Ear Surgical Device

- **Motion Control System**
  - The focus is to develop a precision motion controller for the USM stage
    - To learn and build the model of the USM and identify its parameters for controller design purpose
    - To achieve high precision motions by a linear USM stage and compensate the nonlinearity of the USM stage
  - Solution
    - LQR-PID

      +

      Model-based friction compensation

      +

      Sliding Mode Control

Ear Surgical Device

- **Motion Control System: Modeling**
  - Consists of two parts: **a linear term** and **a nonlinear term**

$$\ddot{x}(t) = F_{linear}(t) + F_{nonlinear}(t)$$

- **Linear term:**

  Second-order system

$$F_{linear}(t) = -a_1 x(t) - a_2 \dot{x}(t) + bu(t)$$

- **Nonlinear term:**

  Coulomb friction + uncertain component

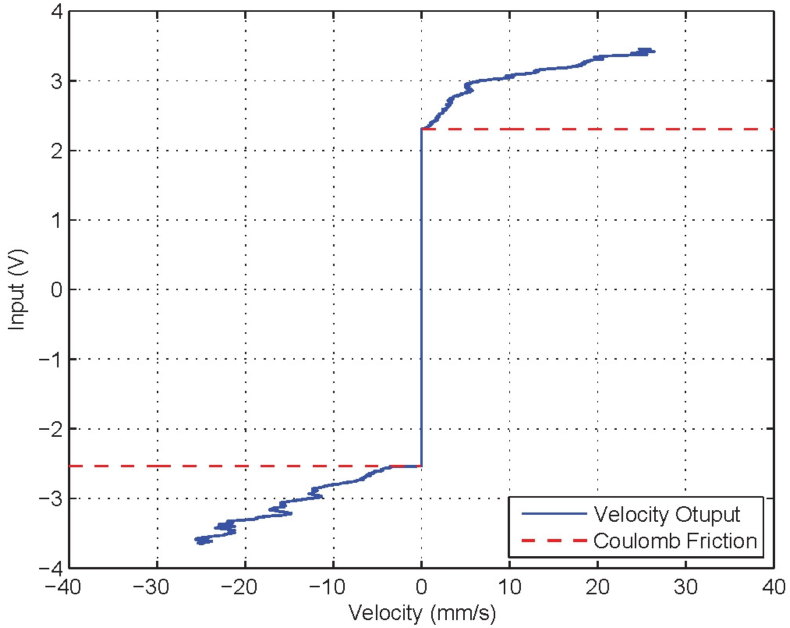
$$F_{nonlinear}(t) = -f_c(\dot{x}) - \Delta f(\dot{x})$$

unknown but bounded

$$\ddot{x}(t) = -a_1 x(t) - a_2 \dot{x}(t) + bu(t) - f_c(\dot{x}) - \Delta f(\dot{x})$$

Ear Surgical Device

- **Motion Control system: Parameter Estimation**
  - **Nonlinear term** (Coulomb friction)
    - **A slow sine wave with low frequency of 0.5 Hz as input signal**
    - **Coulomb friction model:**
      - *Not symmetric*



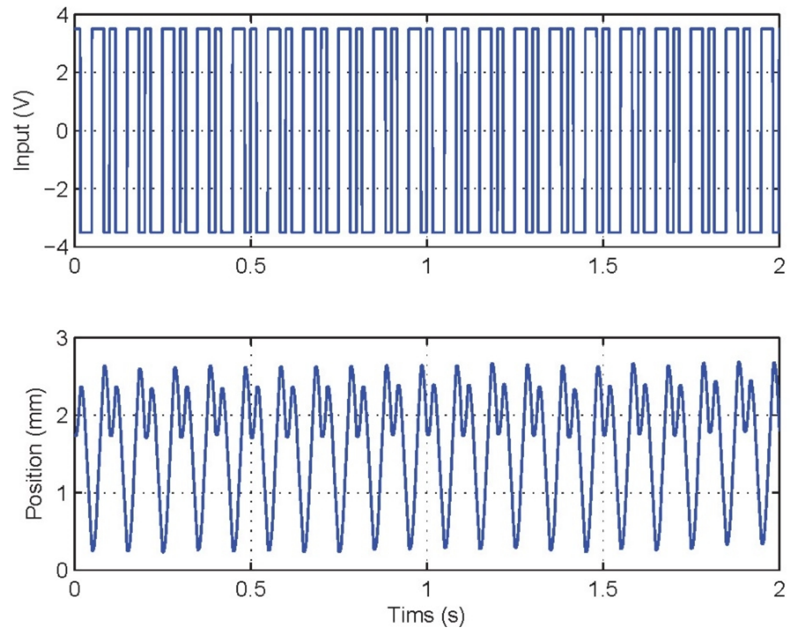
$$u_{fc}(\dot{x}) = \sigma \operatorname{sign}(\dot{x}) - \delta|\operatorname{sign}(\dot{x})|$$

$$f_c(\dot{x}) = b u_{fc}(\dot{x})$$

$$F_{nonlinear} = -2.420 \operatorname{sign}(\dot{x}) b + 0.115|\operatorname{sign}(\dot{x})| b - \Delta f(\dot{x})$$

Ear Surgical Device

- Motion Control System: Parameter Estimation
  - Linear term
    - Friction compensator is applied
    - Multi-frequency input signal: 10 Hz, 20 Hz, and 30 Hz

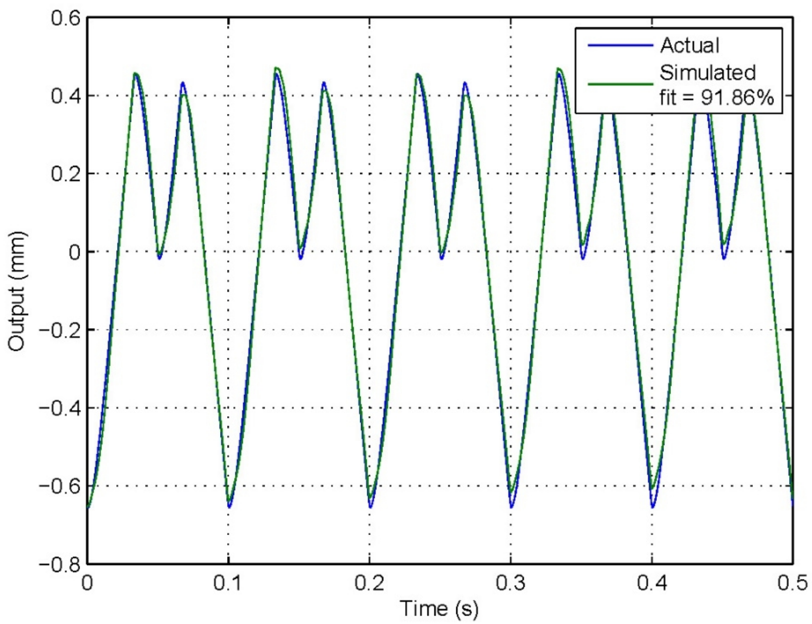


    - Model: $F_{linear}(t) = -248.4x(t) - 202\dot{x}(t) + 4940u(t)$

Ear Surgical Device

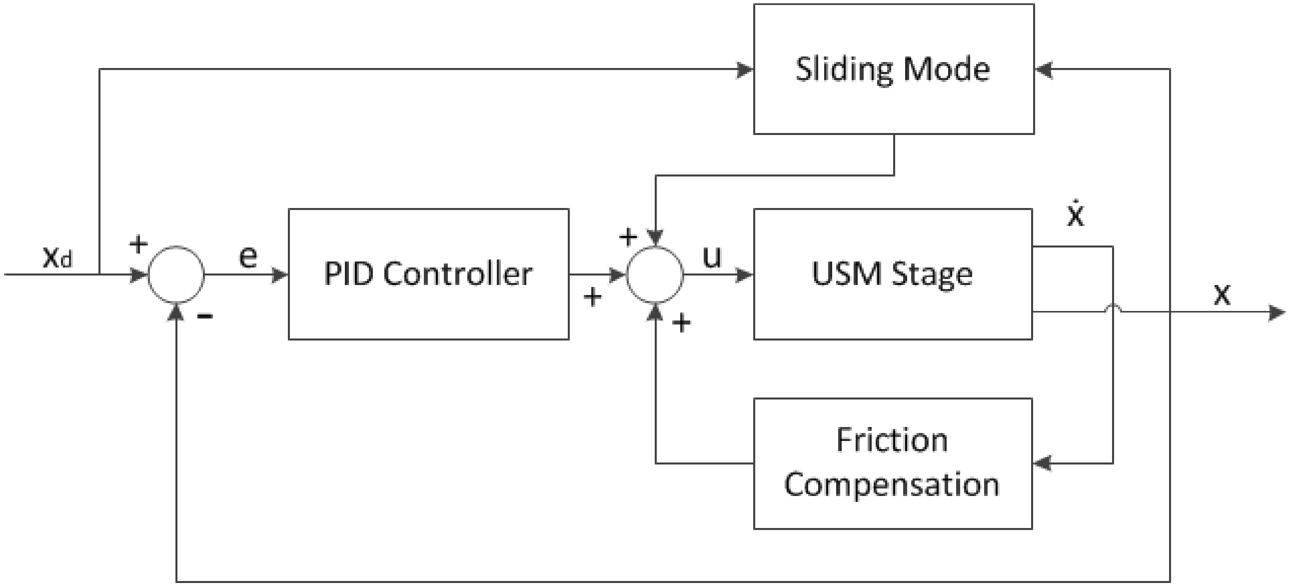
- Motion Control System: Model Validation
  - Full model

$$\ddot{x}(t) = -248.4x(t) - 202\dot{x}(t) + 4940u(t) - 2.420\,\text{sign}(\dot{x})b + 0.115|\text{sign}(\dot{x})|b - \Delta f(\dot{x})$$



Ear Surgical Device

- **Motion Control System: Control Scheme**
  - Linear term: **PID controller** (as main controller)
  - Nonlinear term
    - Structured component: Friction compensation
    - Uncertain component: Sliding mode controller



Ear Surgical Device

- **Motion Control System: LQR assisted PID Controller**
  - Linear term: Error model

$$X(t) = [e_0(t), e_1(t), e_2(t)]^T$$

$$\boxed{\dot{\boldsymbol{X}}(\boldsymbol{t}) = \boldsymbol{A}\boldsymbol{X}(\boldsymbol{t}) - \boldsymbol{B}\boldsymbol{u}(\boldsymbol{t})} + B[\frac{\sigma(x)\text{sign}(\dot{x})b - \delta(x)|\text{sign}(\dot{x})|b + \Delta f(\dot{x}) + bu_d}{b}]$$

  - LQR technique

$$J = \int_0^\infty [X(\tau)^T Q X(\tau) + r u(\tau)^T u(\tau)] d\tau$$

  - PID controller → State feedback

$$u_l(t) = K X(t)$$

$$K = [k_1, k_2, k_3]^T = -r^{-1} B^T P$$

Riccati equation:

$$A^T P + P A - P B r^{-1} B^T P + Q = 0$$

Diagonal weighting matrix:

$$Q = \text{diag}\{q_1, q_2, q_3\}$$

$$K_p = k_2 = -r^{-1} b p_{32}$$
$$K_i = k_1 = -r^{-1} b p_{31}$$
$$K_d = k_3 = -r^{-1} b p_{33}$$

Ear Surgical Device

- **Motion Control System: Nonlinear Compensation**
  - Nonlinear term

$$B[\frac{\sigma(x)sign(\dot{x})b - \delta(x)|\mathrm{sign}(\dot{x})|b + \Delta f(\dot{x})}{b}]$$

structured component uncertain component

  - Structured component compensation
    - A sign function

$$u_{fc}(t) = \sigma \mathrm{sign}(\dot{x}) - \delta|\mathrm{sign}(\dot{x})|$$

  - Uncertain component compensation
    - A sliding mode control law

$$u_s(t) = \hat{k}_s \mathrm{sign}(X^T P B)$$

      - *Adaptive law*

$$\dot{\hat{k}}_s = \rho_1 |X^T P B|$$

Ear Surgical Device

- **Motion Control System: Nonlinear Compensation***
  - Uncertain component compensation
    - A sliding mode control law
      - *Proof*

$$V = X^T P X + \rho_1^{-1}\tilde{k}_s^2 \longleftarrow \left(\tilde{k}_s = \frac{\Delta f_M}{b} - \hat{k}_s\right)$$

$$\begin{aligned}\dot V =& X^T(\bar A^T P + P\bar A)X - 2X^T PB\hat{k}_s \mathrm{sign}(X^T PB) \\ &+ 2X^T PB\frac{\Delta f}{b} - 2\rho^{-1}\tilde{k}_s\dot{\hat{k}}_s \\ =& X^T(A^T P + PA - PBr^{-1}B^T P)X - X^T PBr^{-1}B^T PX \\ &- 2|X^T PB|\hat{k}_s + 2|X^T PB||\frac{\Delta f}{b}| - 2\rho^{-1}\tilde{k}_s\dot{\hat{k}}_s \\ \le& X^T(A^T P + PA - PBr^{-1}B^T P)X - X^T PBr^{-1}B^T PX \\ &- 2|X^T PB|\hat{k}_s + 2|X^T PB||\frac{\Delta f_M}{b}| - 2\rho^{-1}\tilde{k}_s\dot{\hat{k}}_s\end{aligned}$$

$$\begin{aligned}\dot V \le& -X^T(Q + PBr^{-1}B^T P)X + 2\tilde{k}_s|X^T PB| - 2\rho^{-1}\tilde{k}_s\dot{\hat{k}}_s \\ =& -\lambda_{min}(Q + PBr^{-1}B^T P)||X||^2 + 2\tilde{k}_s|X^T PB| \\ &- 2\rho^{-1}\tilde{k}_s\dot{\hat{k}}_s\end{aligned}$$

$$\boxed{\dot V \le -\lambda_{min}(Q + PBr^{-1}B^T P)||X||^2}$$

X and $\tilde{k}_s$ are bounded

$$\begin{aligned}&\lim_{t\to\infty}\int_0^t \lambda_{min}(Q + PBr^{-1}B^T P)||X||^2 d\tau \\ &\le \; V(0) - V(\infty) \\ &\le \; V(0)\end{aligned}$$

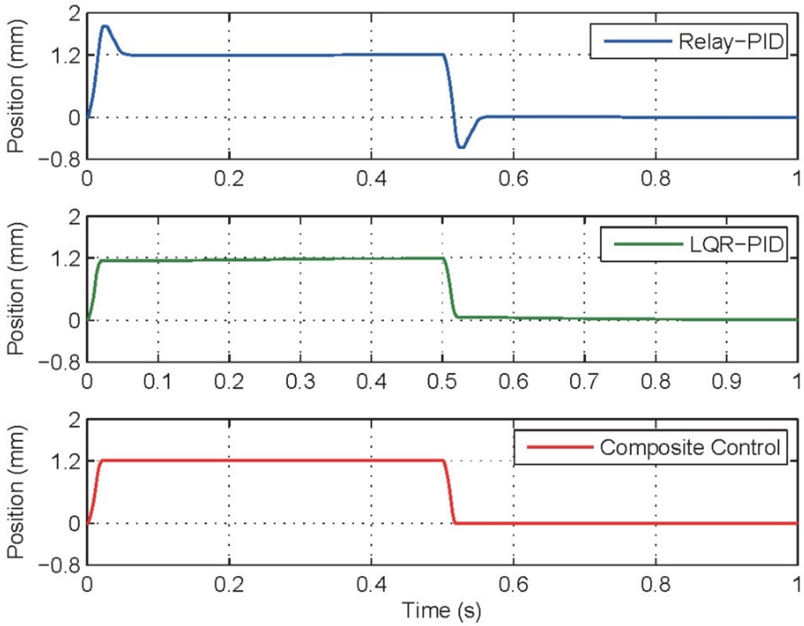
$$\lim_{t\to\infty}||X|| = 0$$

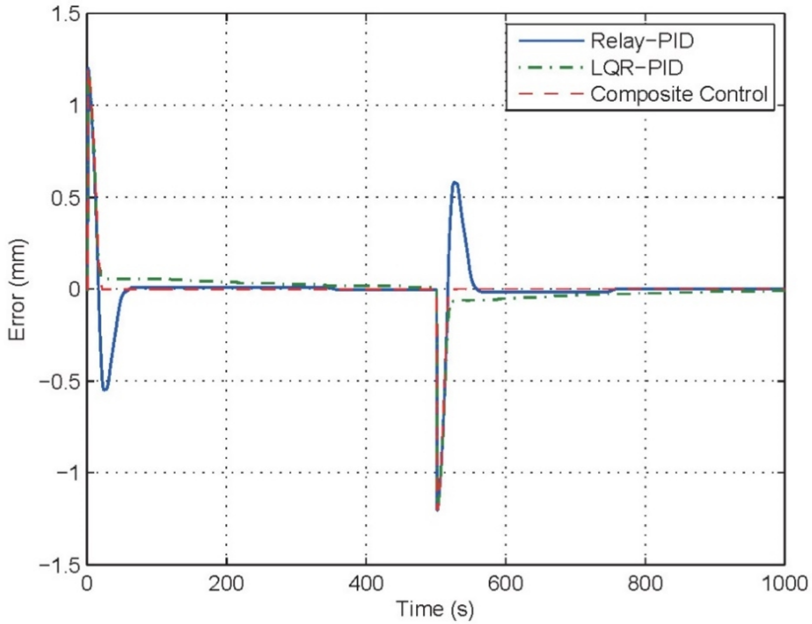
unknown nonlinear term $\Delta f(\dot x)$ can be rejected

- *A modification:* $\dot{\hat{k}}_s = \rho_1|X^T PB| - \rho_0\hat{k}_s$

Ear Surgical Device

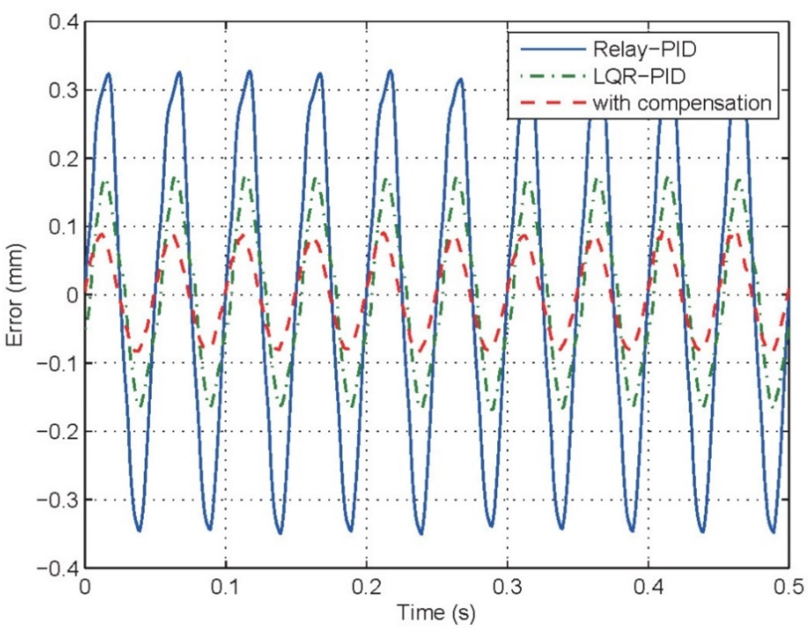
- Experimental Results
    - Point-to-Point Movements
        - A square wave (amplitude: 1.2 mm, frequency: 1 Hz)
        - Comparing with Relay-tuned PID controller

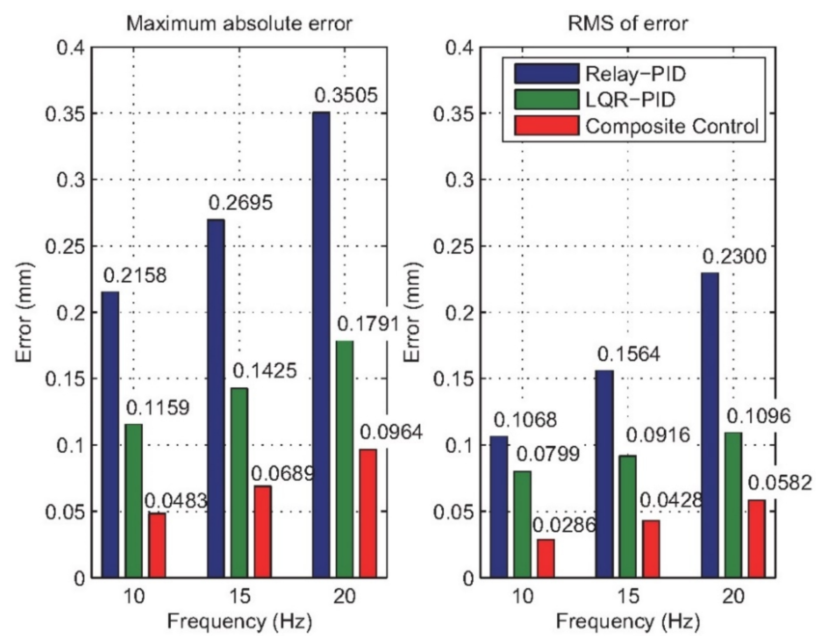




Ear Surgical Device

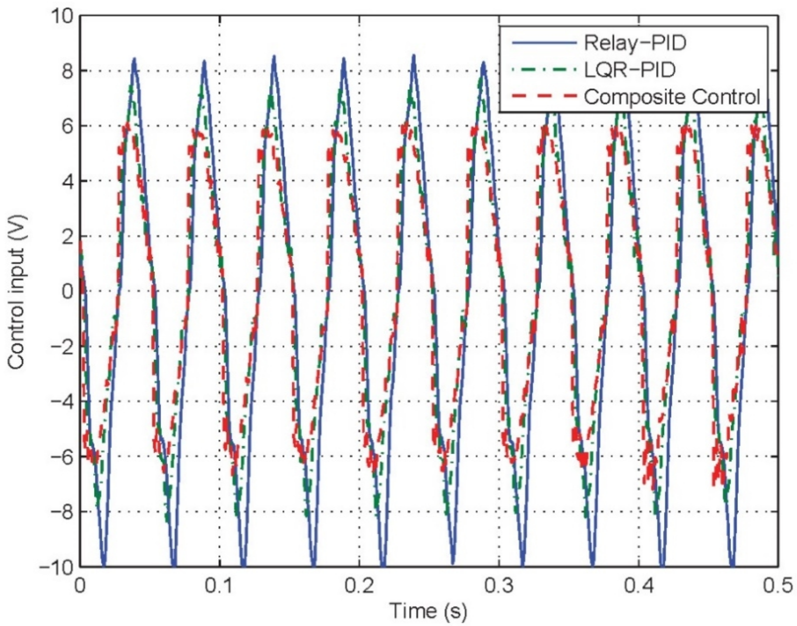
- Experimental Results
  - Trajectory Tracking (without Disturbance)
    - Three sine waves (amplitude: ±0.6 mm, frequencies: 10, 15, 20 Hz)
    - Errors: Max. absolute error; RMS (Root-Mean-Square) of error

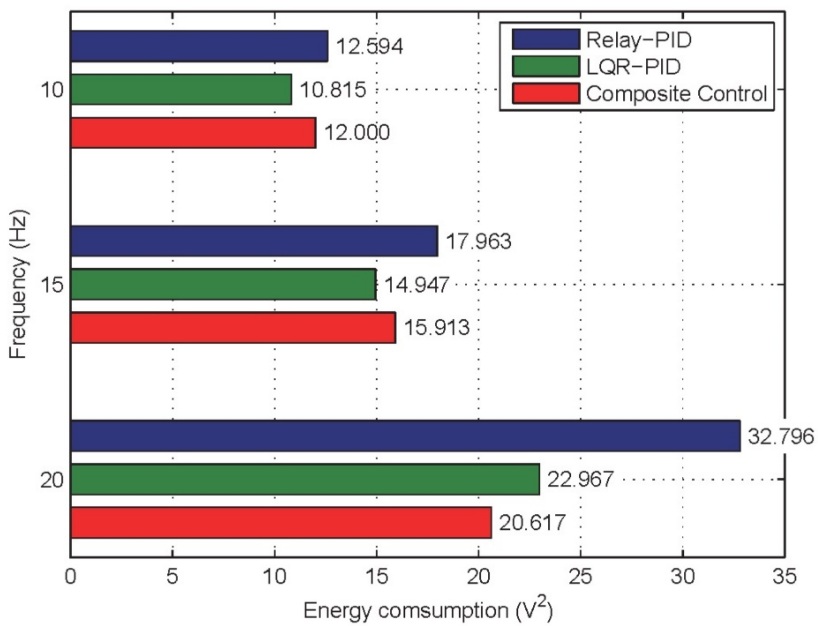




Ear Surgical Device

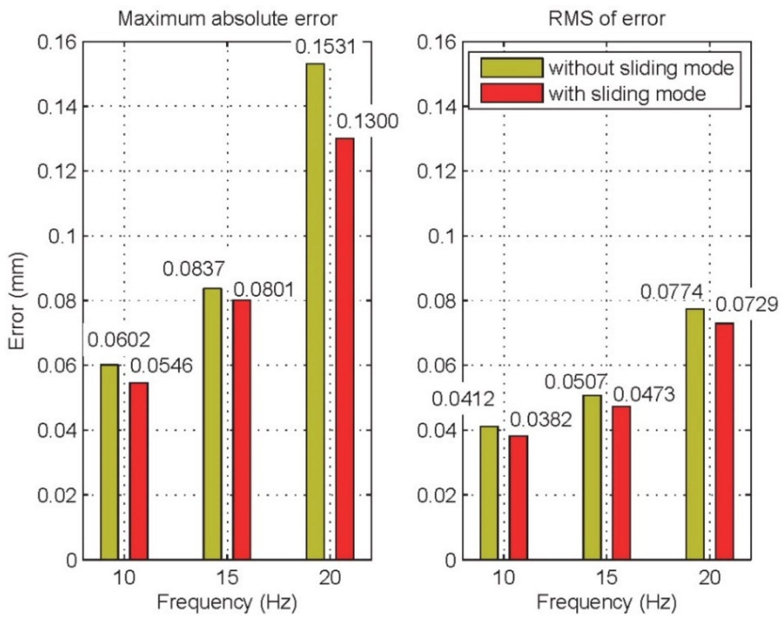
- Experimental Results
  - Trajectory Tracking (without Disturbance)
    - Three sine waves (amplitude: ±0.6 mm, frequencies: 10, 15, 20 Hz)
    - Control effort: Input; Energy





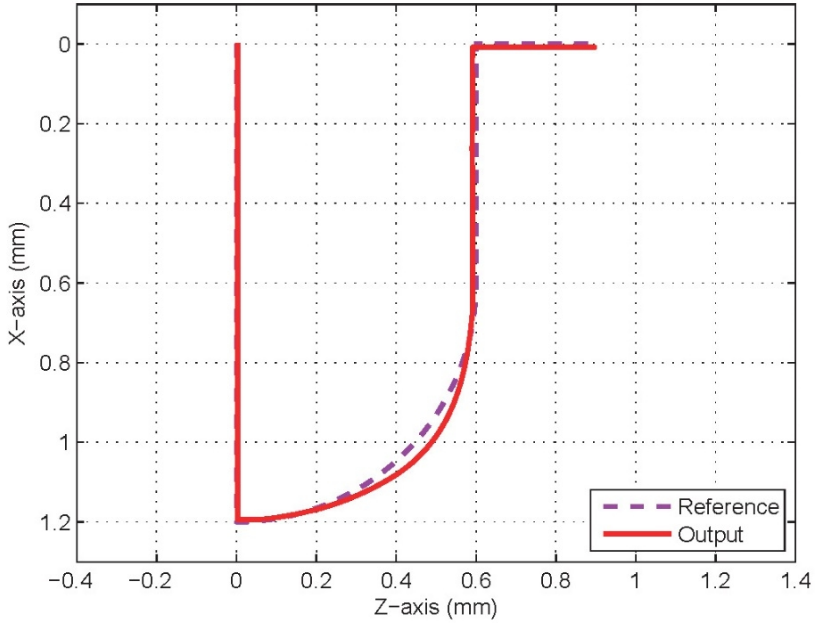
Ear Surgical Device

- Experimental Results
  - Trajectory Tracking (with Disturbance)
    - Three sine waves (amplitude: $\pm$0.6 mm, frequencies: 10, 15, 20 Hz)
    - Disturbance by mock membrane
    - Max. absolute error: reduced by **4.30% to 15.09%**
    - RMS of error: reduced by **5.81% to 7.28%**



Ear Surgical Device

- Experimental Results
  - Trajectory Tracking (2-DOF)
    - Quarter circle path for insertion: radius of 0.6 mm



The LQR-assisted PID tuning method can be a better and more flexible way for PID tuning than the common tuning method

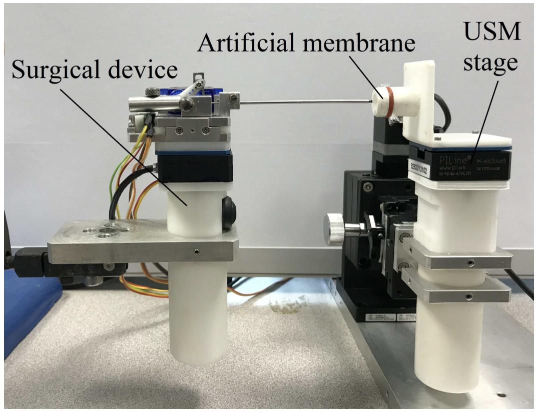
Optimal PID Controller - Applications

Contact Force Control on Soft Membrane

Robotic Surgery

- Contact Force Control
  - Surgical Robots are not only required to generate precise motions to complete their tasks but also required to handle the interactions between the environment or human and themselves in the sophisticated tasks

Steady tissue contact
(Latt et al. 2011)

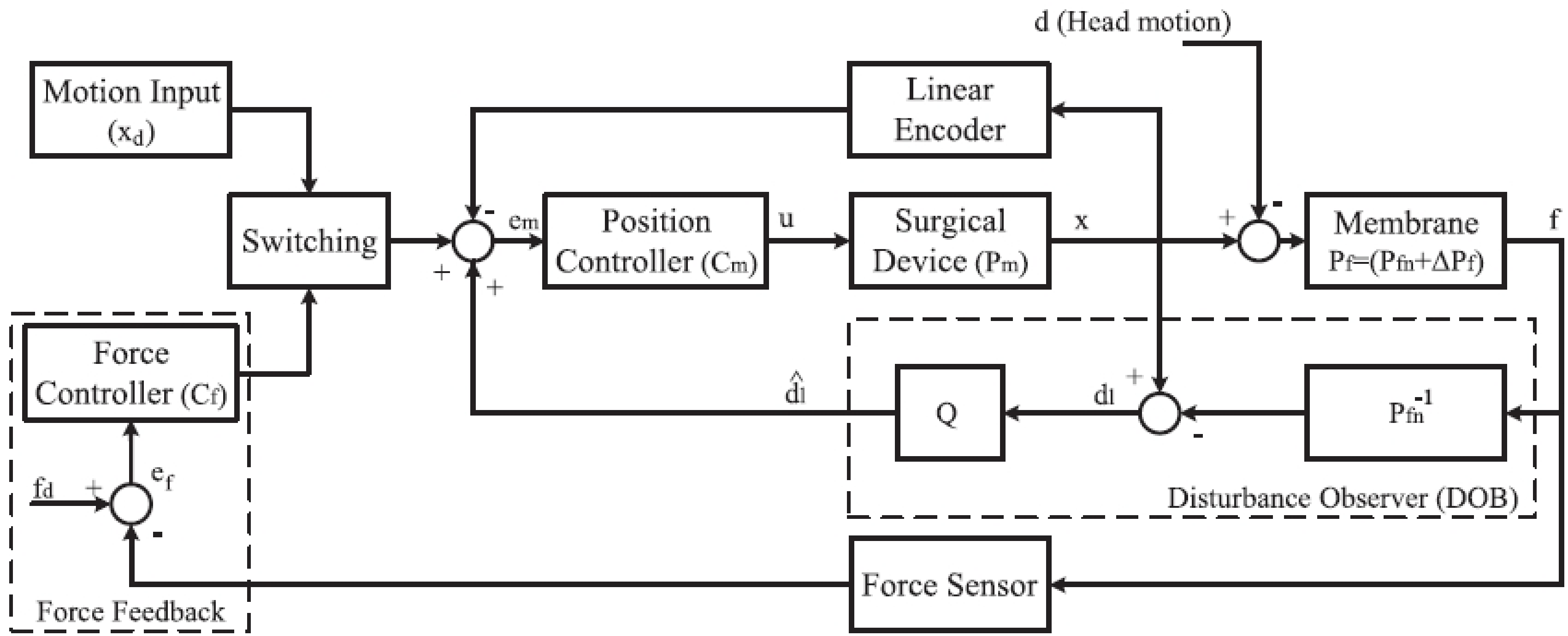


Soft membrane contact
(Liang et al. 2018)

  - One main objective is to maintain the contact force within an acceptable range

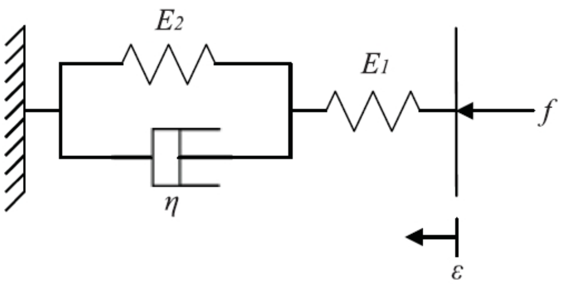
Robotic Surgery

- ## Contact Force Control
  - Cascade Control



Robotic Surgery

- **Contact Modelling**
  - Displacement-Force Relationship
    - **Viscoelastic Behavior**
      - *Standard Linear Solid (SLS) model*



$$P_v(s) = \frac{F(s)}{\mathcal{E}(s)} = \frac{\gamma_v s + \beta_v}{\alpha_v s + 1}$$

    - **Contact Effect**
      - *Second-order low-pass filter*

$$P_c(s) = \frac{\mathcal{E}(s)}{X(s)} = \frac{\beta_c}{s^2 + \alpha_c s + \beta_c}$$

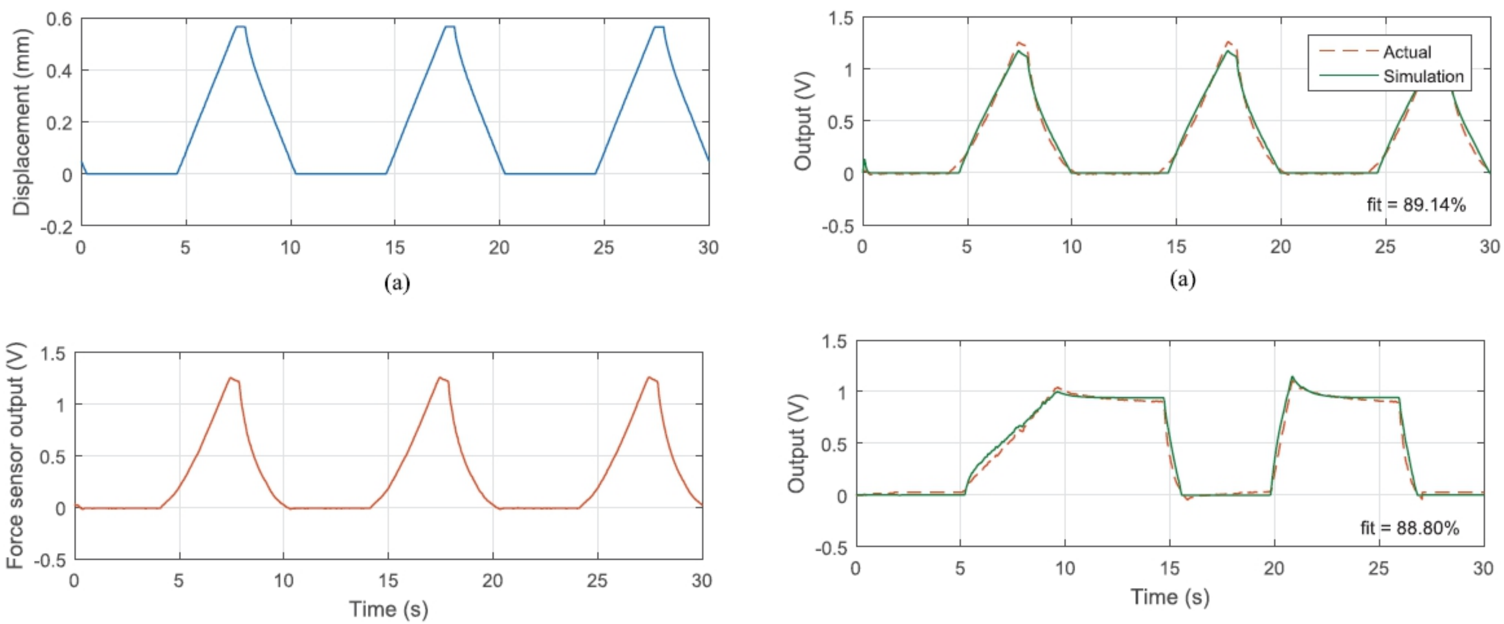
    - **Combined Model**

$$P_{fn}(s) = \frac{F(s)}{X(s)} = P_c(s)P_v(s) = \frac{\beta_c}{s^2 + \alpha_c s + \beta_c} \cdot \frac{\gamma_v s + \beta_v}{\alpha_v s + 1}$$

$$= \frac{b_1 s + b_0}{s^3 + a_2 s^2 + a_1 s + a_0}$$

Robotic Surgery

- **Contact Modelling**
  - System Identification



$$P_{fn}(s) = \frac{4101s + 4276}{s^3 + 34.52s^2 + 1520s + 2287}$$

Robotic Surgery

- **Contact Modelling**
  - State-space Model

$$\dot{\boldsymbol{X}}_f = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \end{bmatrix} \boldsymbol{X}_f + \begin{bmatrix} 0 \\ b_1 \\ b_0 - a_2 b_1 \end{bmatrix} u_f$$

$$Y_f = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \boldsymbol{X}_f = f$$

$$\boldsymbol{X}_f = \begin{bmatrix} f & \dot{f} & \bar{\ddot{f}} \end{bmatrix}^T \qquad \bar{\ddot{f}} = \ddot{f} - b_1 u_f$$

  - Error State-space Model

$$\dot{\boldsymbol{E}}_f = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -a_0 & -a_1 & -a_2 \end{bmatrix} \boldsymbol{E}_f + \begin{bmatrix} 0 \\ 0 \\ -b_1 \\ -b_0 + a_2 b_1 \end{bmatrix} u_f \triangleq A\boldsymbol{E}_f + B u_f$$

$$\boldsymbol{E}_f = \begin{bmatrix} \int_0^t e_f d\tau & e_f & \dot{e}_f & \bar{\ddot{f}} \end{bmatrix}^T$$

Robotic Surgery

- **LQR-PID Controller Design**

$$\boldsymbol{E}_f = \begin{bmatrix} \int_0^t e_f d\tau & e_f & \dot{e}_f & \bar{\ddot{f}} \end{bmatrix}^T$$

  - State feedback control

$$u_f = K_f \boldsymbol{E}_f \qquad K_f = \begin{bmatrix} K_{if} & K_{pf} & K_{df} & K_4 \end{bmatrix}$$

  - If we want to convert this state feedback control to PID controller, we have the following constrain

$$K_4 = 0$$

  - However, there is no standard closed-form solution when there are constraints in the gain matrix
    - i.e., it can not be solved by the Riccati equation
  - To solve the constrain LQ problem, the Projection-gradient-based Optimization Algorithm is utilized

$$J_f = \int_0^\infty (\boldsymbol{E}_f^T Q_f \boldsymbol{E}_f + u_f^T r_f u_f) d\tau$$

Robotic Surgery

- LQR-PID Controller Design
  - Gradient-based Optimization Algorithm

Algorithm 1: Gradient-based Optimization Algorithm for Constrained LQ Problem.

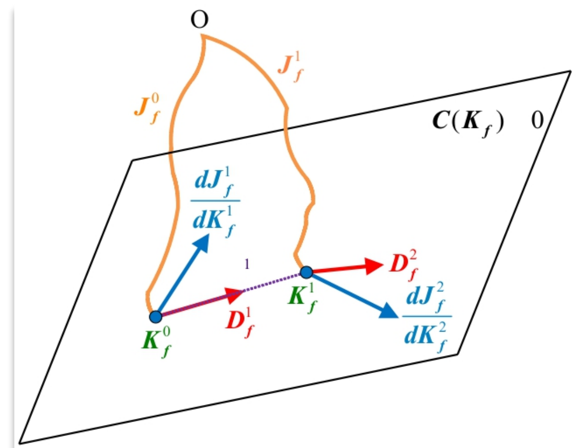
- Step 1: Set $i = 0$ and define an initial stable gain such that $C(K_f^0) = 0$.
- Step 2: Set $i = i + 1$ and determine the projection gradient matrix D_f^i by Lemma 1.
- Step 3: Optimize the step size α^i to minimize the functional cost after the iteration, written as $\min_{\alpha^i} J(K_f^{i-1} - \alpha^i D_f^i)$.
- Step 4: Update the gain matrix as $K_f^i = K_f^{i-1} - \alpha^i D_f^i$.
- Step 5: Go back to Step 2 to continue the iterations until the stopping criterion is met.

$$C(K_f) = K_f M = 0$$

where $M = \mathrm{diag}\{0, 0, 0, 1\}$

$$\min_{D_f} \frac{1}{2} \left\| \frac{dJ_f}{dK_f} - D_f \right\|^2$$

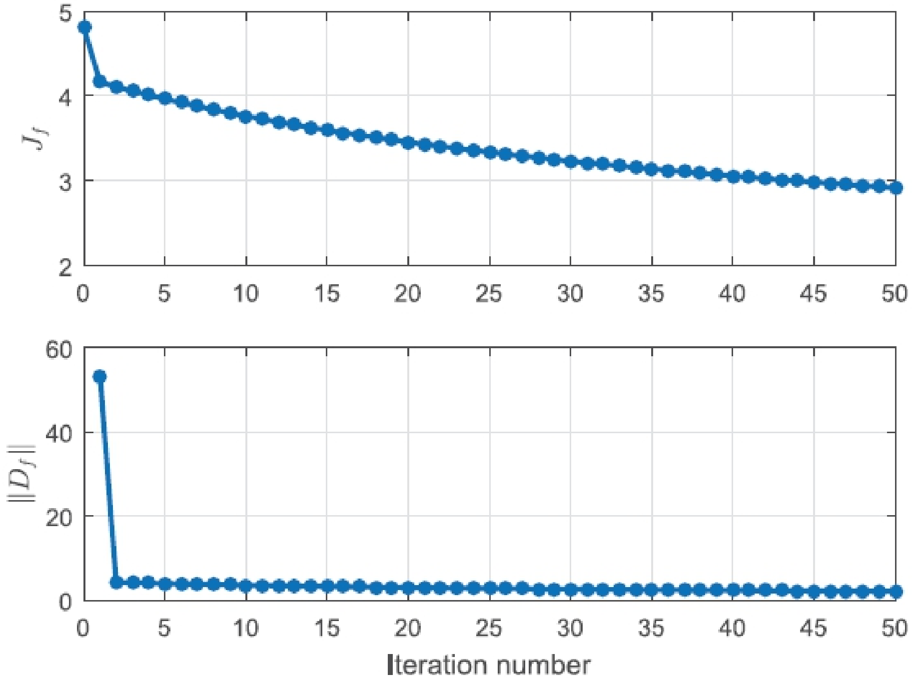
$$C(D_f) = 0$$



Lemma 1: The projection gradient matrix is given by $D_f = \frac{dJ_f}{dK_f}(I - M)$ where $\frac{dJ_f}{dK_f} = 2\left(r_f K_f + B^T P\right) V$, P and V are the solutions of $(A + BK_f)^T P + P(A + BK_f) + Q_f + K_f^T r_f K_f = 0$ and $(A + BK_f)V + V(A + BK_f)^T + X_0 = 0, X_0 = x_0 x_0^T$ where the initial state value can be assumed to be uniformly distributed over the surface of a unit sphere.

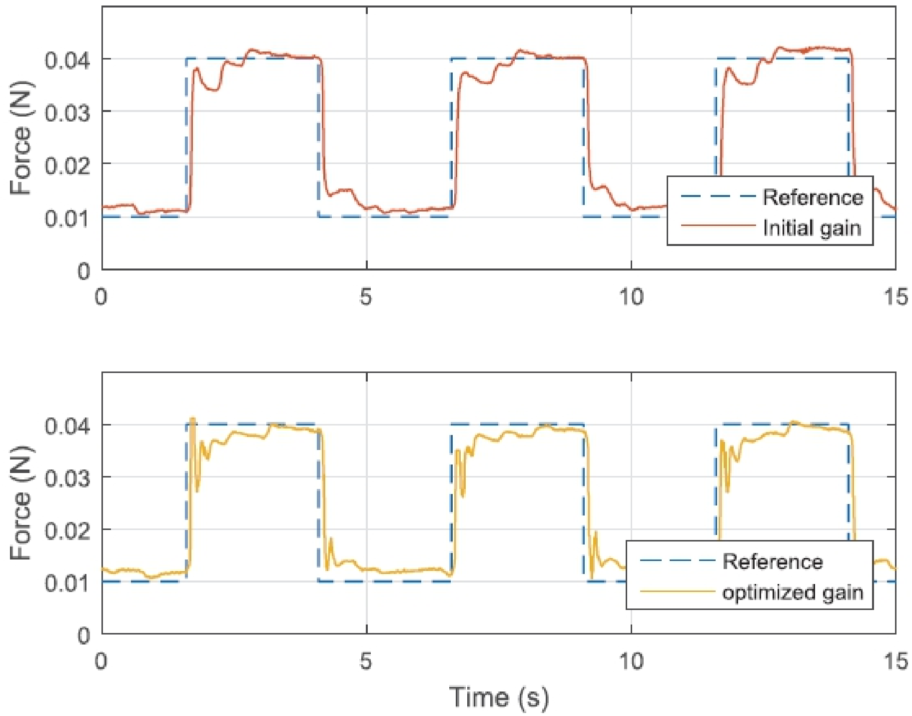
Robotic Surgery

- LQR-PID Controller Design
  - Gradient-based Optimization Algorithm for Constrained LQ Problem
    - Change of cost and norm of projection gradient matrix



Robotic Surgery

- Experimental Results
  - Point-to-Point Movements
    - A square wave (peak-to-peak amplitude: 0.03 N, frequency: 0.2 Hz)
    - Initial gain vs Optimized gain



Robotic Surgery

- **Experimental Results**
  - **Point-to-Point Movements**
    - The controller with the optimized gain can achieve a faster rise time and smaller error
    - Max. Absolute Errors and RMSEs of them are
      - *0.0318 N and 0.00616 N for the initial gain*
      - *0.0290 N and 0.00589 N for the optimized gain*
    - The Max. Absolute Error is improved by 8.81% with the optimized gain
    - The RMSE is reduced by 4.38% while the optimized gain is applied
      - *It implies that the controller with the optimized gain can have a better regulation performance*

Optimal PID Controller

Some Extensions

Optimal PID Controller - Some Extensions

- **Data-based LQR-PID Controller**
  - Consider this cost function

$$F(K) = \mathbb{E}\left[\int_0^\infty \left[x_a'(t,K)Qx_a(t,K) + u'(t,K)Ru(t,K)\right] \mathrm{d}t\right]$$

  - Calculate the Gradient of cost function:

$$J(\rho) = \frac{1}{N}E[\sum_{t=1}^{N} x^T Q \frac{\partial x}{\partial K} + \sum_{t=1}^{N} u^T(\rho) R \frac{\partial u}{\partial K}]$$

$$= \frac{1}{N}E[\sum_{t=1}^{N} x_1 q_1 \frac{\partial x_1}{\partial K} + \sum_{t=1}^{N} x_2 q_2 \frac{\partial x_2}{\partial K} + \sum_{t=1}^{N} x_3 q_3 \frac{\partial x_3}{\partial K} + \sum_{t=1}^{N} u(\rho) r \frac{\partial u}{\partial K}]$$

  - If the gradient of the cost function with respect to the controller parameters is obtained, the Gauss-Newton Method (GNM) can be used to tune the parameters

Z. Cheng et al., Data-Driven Tuning Method for LQR Based Optimal PID Controller, IECON 2019

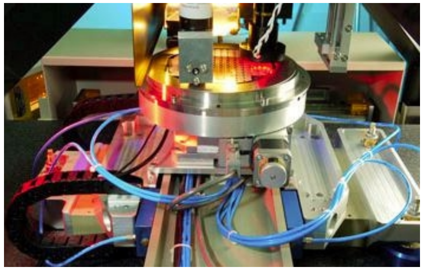
Optimal PID Controller - Some Extensions

- Constrained LQ Problem
  - Possible Solutions
    - Projection-gradient-based Optimization Algorithm
    - Linear Matrix Inequality (LMI)
    - Sequential Convex Programming (SCP)
    - …

S. Boyd; L. Vandenberghe, Convex optimization, Cambridge University Press, Mar. 2004

Summary

- **PID Controller**
  - Basic concepts
    - PID
  - Common tuning methods
    - Manual, Z-N, C-C, Relay auto-tuning, IFT…
  - Applications
- **Optimal PID Controller**
  - Basic idea of optimal control
  - LQR-assisted PID tuning method
  - Some extensions
    - Data-based LQR-PID
    - Constrained LQR-PID



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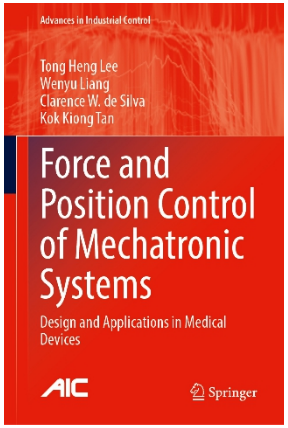


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mochchakkeram
tapadh leat
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Assessment

- **Please check the "CAs (EE5111/5061)" folder in LumiNUS**
  - The second part (CA3b) of my assignment will be uploaded on next Friday (Oct. 8, 2021)
- **Submission Method**
  - Soft copy (Word or .pdf)
- **Submission deadline:**
  - Friday (Nov. 19, 2021) of the reading week

Please Do A Good Time Control!