Model Transformation with Triple Graph Grammars and Non-terminal Symbols

William da Silva[1,2] and Max Bureck[1]

[1] Fraunhofer Fokus, Berlin, Germany
{william.bombardelli.da.silva, max.bureck}@fokus.fraunhofer.de
[2] Technische Universität Berlin, Berlin, Germany

Abstract. **Keywords:** First keyword · Second keyword · Another keyword.

1 Introduction

Quality of service is a very common requirement for software and engineering projects, specially for safety-critical systems. A technique that aims to assure and enhance quality of software is the model-based engineering (MDE) approach, which consists of the use of abstract models to specify aspects of the system under construction. The use of such models often allows for cheaper tests and verification as well as easier discussions about the system.

The construction of a system with MDE commonly requires the creation of various models in different levels of abstraction, in which case we are interested in generating models automatically from other models. One example of such a situation is the transformation of a UML diagram into source-code or the compilation of source-code into machine-code. This problem is known as model transformation and several approaches to solve it have been proposed so far. Some of them consist of using the theory of graph grammars to formalize models and describe relations between them. One of which is the triple graph grammar (TGG) approach [5], which consists of building context-sensitive-like grammars of so-called triple graphs.

Triple graphs are composed of three graphs, the source and the target graphs, representing two models, and the correspondence graph that connects the source and the target through morphisms. A triple graph can be used to express the relationship between two graphs through the morphisms between their vertices. In this sense, a TGG describes a language of pairs of graphs which vertices have a certain relationship. For the context of model transformation, in which one is interested in defining a translator from a source model to a target model, a TGG can be used to describe the set of all correctly translated source models and its correspondents target models, in form of a language of triple graphs.

Despite the various positive aspects of TGGs, like a well-founded theory and a reasonable tool support [1], they may sometimes get too big or too difficult to be constructed correctly. We judge, this downside stems from the absence of the concept of non-terminal symbols in the TGG formalism. This concept allows, in the theory of formal languages, for a very effective representation of abstract entities in string grammars.

So, motivated by this benefit, we present in this paper a novel formalism that redefines the standard triple graph grammars and introduces the notion of non-terminal symbols to create a context-free-like triple graph grammar formalism, that has in some cases a smaller size and with which we could describe one transformation that we could not with standard TGG.

Our approach consists of (1) mixing an already existent context-free-like graph grammar formalism, called BNCE graph grammar from [2], with the standard TGG formalism from [5], to create the BNCE TGG and (2) showing an argumentation of how it can be used to solve the model transformation problem.

The remainder of this paper is as follows, in Section 2, we present the research publications related to the topic, in Section 3, we give the main definitions necessary to build our approach, in Section 4, we finally propose the our modified version of TGG, the BNCE TGG, in Section 5 we argument that our approach can be used for model transformation, in Section 6 we evaluate our results and, finally, in Section 7 we summarize and close our discussion.

2 Related Works

3 Graph Grammars and Triple Graph Grammars

In this section, we introduce important definitions that are used throughout this paper. First, we present the definitions, taken mainly from [4], regarding graphs, second, we introduce the families of graph grammars NCE and BNCE taken from [2, 3] and then, we express our understanding of TGG, backed by [5].

Definition 1. *A directed labeled graph G over the set of symbols Σ, $G = (V, E, \phi)$ consists of a finite set of vertices V, a set of labeled directed edges $E \subseteq V \times \Sigma \times V$ and a total vertex labeling function $\phi : V \to \Sigma$.*

Directed labeled graphs are often referred to simply as graphs. For a fixed graph G we refer to its components as V_G, E_G and ϕ_G. Moreover, we define the special empty graph as $\varepsilon := (\emptyset, \emptyset, \emptyset)$ and we denote the set of all graphs over Σ by $\mathcal{G}_\Sigma$. Two graphs G and H are disjoint iff $V_G \cap V_H = \emptyset$. If $\phi_G(v) = a$ we say v is labeled by a. In special, we do not allow loops (vertices of the form (v, l, v)), but multi-edges with different labels are allowed.

Definition 2. *Two vertices v and w are neighbors (also adjacent) iff there is one or more edges between them, that is, $(v, l, w) \in E_G \vee (w, l, v) \in E_G$. And function $\mathrm{neigh}_G : 2^{V_G} \to 2^{V_G}$, applied to U gives the set of neighbors of the vertices in U minus U. That is $\mathrm{neigh}_G(U) = \{v \in V_G \setminus U \mid$ exists a $(v, l, u) \in E_G$ or a $(u, l, v) \in E_G$ with $u \in U\}$*

Definition 3. *A morphism of graphs G and H is a total mapping $m : V_G \to V_H$.*

Definition 4. *An isomorphism of directed labeled graphs G and H is a bijective mapping $m : V_G \to V_H$ that maintains the connections between vertices and their labels, that is, $(v, l, w) \in E_G$ if, and only if, $(m(v), l, m(w)) \in E_H$ and $\phi_G(v) = \phi_H(m(v))$. In this case, G and H are said to be isomorphic, we write $G \cong H$, and we denote the equivalence class of all graphs isomorphic to G by $[G]$.*

Notice that, contrary to isomorphisms, morphisms do not require bijectivity nor label or edge-preserving properties.

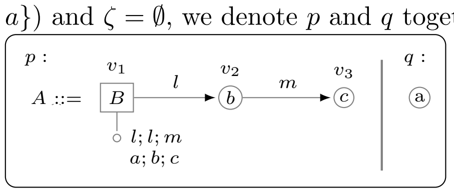
Definition 5. *A Γ-boundary graph G is such that vertices labeled with any symbol from Γ are not neighbors. That is, the graph G is Γ-boundary iff, $\nexists (v, l, w) \in E_G.\ \phi_G(v) \in \Gamma \wedge \phi_G(w) \in \Gamma$.*

Definition 6. *A graph grammar with neighborhood-controlled embedding (NCE graph grammar) $GG = (\Sigma, \Delta \subseteq \Sigma, S \in \Sigma, P)$ consists of a finite set of symbols Σ that is the alphabet, a subset of the alphabet $\Delta \subseteq \Sigma$ that holds the terminal symbols (we define the complementary set of non-terminal symbols as $\Gamma := \Sigma \setminus \Delta$), a special symbol of the alphabet $S \in \Sigma$ that is the start symbol, and a finite set of production rules P of the form $(A \to R, \omega)$ where $A \in \Gamma$ is the so-called left-hand side, $R \in \mathcal{G}_\Sigma$ is the right-hand side and $\omega : V_R \nrightarrow 2^{\Sigma \times \Sigma}$ is the partial embedding function from the R's vertices to pairs of edge and label symbols.*

Notice that, in the original definition of NCE graph grammars [2], the left-hand side of the productions were allowed to contain any connected graph. So, strictly speaking, the definition above characterizes actually a 1-NCE graph grammar, that contains only one element in the left-hand side. Nevertheless, for simplicity, we use the denomination NCE graph grammar, or simply graph grammar, to refer to a 1-NCE graph grammar along this paper. Moreover, vertices from the right-hand sides of rules labeled by non-terminal (terminal) symbols are said to be non-terminal (terminal) vertices. And finally, define, for convenience, the start graph of GG as $Z_{GG} := (\{v_s\}, \emptyset, \{v_s \mapsto S\})$.

Definition 7. *A boundary graph grammar with neighborhood-controlled embedding (BNCE graph grammar) GG is such that non-terminal vertices of the right-hand sides of rules are not neighbors. That is, the graph grammar GG is boundary iff all its rules' right-hand sides are Γ-boundary graphs.*

In the following, we present our concrete syntax inspired by the well-known backus-naur form to denote BNCE graph grammar rules. Let $GG = (\{A, a, b, c\}, \{a, b, c\}, A, \{p, q\})$ be a graph grammar with production rules $p = (A \to G, \omega)$ and $q = (A \to H, \zeta)$ where $G = (\{v_1, v_2, v_3\}, \{(v_1, l, v_2), (v_2, m, v_3)\}, \{v_1 \mapsto B, v_2 \mapsto b, v_3 \mapsto c\})$, $\omega = \{v_1 \mapsto \{(l, a), (l, b), (m, c)\}\}$, and $H = (\{u_1\}, \emptyset, \{u_1 \mapsto a\})$ and $\zeta = \emptyset$, we denote p and q together as



Notice that, we use squares for non-terminal vertices, circles for terminal vertices, position the respective label inside the shape and the (possibly omitted) identifier over it. Over each edge is positioned its respective label. To depict the embedding function, we place near the respective vertex a small circle labeled with the image pairs of the embedding function for this node aligned vertically and separated by semi-colons, which in certain circumstances may also be omitted.

With these syntactic notions of the formalism presented, we introduce below its semantics by means of the concepts of derivation step, derivation and language.

Definition 8. *Let $GG = (\Sigma, \Delta, S, P)$ be a graph grammar and G and H be two graphs over Σ that are disjoint to all right-hand sides from P, G concretely derives in one step into H with rule r and vertex v, we write $G \overset{r,v}{\Rrightarrow}_{GG} H$ and call it a concrete derivation step, if, and only if, the following holds:*

$$\begin{aligned} r &= (A \to R, \omega) \in P \text{ and } A = \phi_G(v) \text{ and} \\ V_H &= (V_G \setminus \{v\}) \cup V_R \text{ and} \\ E_H &= (E_G \setminus \{(w,l,t) \in E_G \mid v = w \vee v = t\}) \\ &\quad \cup E_R \\ &\quad \cup \{(w,l,t) \mid (w,l,v) \in E_G \wedge (l, \phi_G(w)) \in \omega(t)\} \\ &\quad \cup \{(t,l,w) \mid (v,l,w) \in E_G \wedge (l, \phi_G(w)) \in \omega(t)\} \text{ and} \\ \phi_H &= (\phi_G \setminus \{(v,x)\}) \cup \phi_R \end{aligned}$$

Notice that, without loss of generalization, we set $\omega(t) = \emptyset$ for all vertices t without an image defined in ω. Furthermore, let H' be isomorphic to H, if G *concretely derives* in one step into H, we say it *derives* in one step into H' and write $G \overset{r,v}{\Rightarrow}_{GG} H'$.

When GG, r or v are clear in the context or irrelevant we might omit them and simply write $G \Rrightarrow H$ or $G \Rightarrow H$. Moreover, we denote the reflexive transitive closure of $\Rightarrow$ by $\Rightarrow^*$ and, for $G \Rightarrow^* H'$, we say G derives in one or more steps into H', or simply G derives into H'.

A concrete derivation can be informally understood as the replacement of a non-terminal vertex v and all its adjacent edges in G by a graph R plus edges e from former neighbors w of v to some vertices t of R, provided e's label and w's label are in the embedding specification $\omega(t)$. That is, the embedding function ω of a rule specifies which neighbors of v are to be connected with which vertices of R, according to their labels and the adjacent edges' labels. The process that governs the creation of these edges is called embedding and can occur in various forms in different graph grammar formalisms. We opted for a rather simple approach, in which the edges' directions and labels are maintained and cannot be used to define embedding.

Definition 9. *A derivation D in GG is a sequence of derivation steps and is written as*

$$D = (G_0 \overset{r_0,v_0}{\Rightarrow} G_1 \overset{r_1,v_1}{\Rightarrow} G_2 \overset{r_2,v_2}{\Rightarrow} \dots \overset{r_{n-1},v_{n-1}}{\Rightarrow} G_n)$$

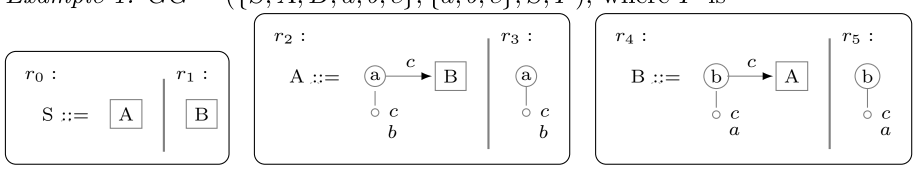
Definition 10. *The language $L(GG)$ generated by the grammar GG is the set of all graphs containing only terminal vertices derived from the start graph Z_{GG}, that is*

$$L(GG) = \{H \text{ is a graph over } \Delta \text{ and } Z_{GG} \Rightarrow^* H\}$$

Notice that for every graph $G \in L(GG)$, there is at least one finite derivation $(Z_{GG} \overset{r_0,v_0}{\Rightarrow} \dots \overset{r_{n-1},v_{n-1}}{\Rightarrow} G)$, but it is not guaranteed that this derivation be unique. In the case that there are more than one derivation for a G, we say that the grammar GG is ambiguous.

Below we give one example of a grammar whose language consists of all chains of one or more vertices with interleaved vertices labeled with a and b.

Example 1. $GG = (\{S, A, B, a, b, c\}, \{a, b, c\}, S, P)$, where P is



The graph $G = a \xrightarrow{c} b \xrightarrow{c} a$ belongs to $L(GG)$ because it contains only terminal vertices and Z_{GG} derives into it using the following derivation:

$$Z_{GG} \overset{r_0,v_0}{\Rightarrow} \overset{v_1}{\boxed{A}} \overset{r_2,v_1}{\Rightarrow} \overset{v_2}{a} \xrightarrow{c} \overset{v_3}{\boxed{B}} \overset{r_4,v_3}{\Rightarrow} \overset{v_2}{a} \xrightarrow{c} \overset{v_4}{b} \xrightarrow{c} \overset{v_5}{\boxed{A}} \overset{r_3,v_5}{\Rightarrow} \overset{v_2}{a} \xrightarrow{c} \overset{v_4}{b} \xrightarrow{c} \overset{v_6}{a}$$

Building upon the concepts of graphs and graph grammars, we present, in the following, our understanding over triple graphs and triple graph grammars (TGGs), supported by the TGG specification from [5].

Definition 11. *A directed labeled triple graph $TG = G_s \overset{m_s}{\leftarrow} G_c \overset{m_t}{\rightarrow} G_t$ over Σ consists of three disjoint directed labeled graphs over Σ (see 1), respectively, the source graph G_s, the correspondence graph G_c and the target graph G_t, together with two injective morphisms (see 3) $m_s : V_{G_c} \rightarrow V_{G_s}$ and $m_t : V_{G_c} \rightarrow G_{G_t}$.*

Directed labeled triple graphs are often referred to simply as triple graphs in this paper and we might omit the morphisms' names in the notation. Moreover, we denote the set of all triple graphs over Σ as $\mathcal{TG}_\Sigma$. We might refer to all vertices of TG by $V_{TG} := V_s \cup V_c \cup V_t$, all edges by $E_{TG} := E_s \cup E_c \cup E_t$ and the complete labeling function by $\phi_{TG} := \phi_{G_s} \cup \phi_{G_c} \cup \phi_{G_t}$. We also advise that in literature, TGGs are often modeled as typed graphs, but we judge that for our circumstance labeled graphs fit better and we are convinced that such divergence does not threat the validity of our approach.

Definition 12. *A Γ-boundary triple graph $TG = G_s \leftarrow G_c \rightarrow G_t$ is such that G_s, G_c and G_t are Γ-boundary graphs.*

Below we introduce the standard definition of TGG. As the reader should notice, this definition of TGG does not fit our needs optimally, because it defines a context-sensitive-like graph grammar whilst we wish a context-free-like graph grammar to use together with the NCE graph grammar formalism. Hence, after presenting the conventional TGG definition, we refine it, in the next Section, to create a NCE TGG, that fits our context best.

Definition 13. *A triple graph grammar* $TGG = (\Sigma, \Delta \subseteq \Sigma, S \in \Sigma, P)$ *consists of, analogously to graph grammars (see 6), an alphabet* Σ*, a set of terminal symbols* Δ *(also define* $\Gamma := \Sigma \setminus \Delta$*), a start symbol* S *and a set of production rules* P *of the form* $L \to R$ *with* $L = L_s \leftarrow L_c \to L_t$ *and* $R = R_s \leftarrow R_c \to R_t$ *and* $L \subseteq R$.

4 BNCE TGG: A TGG with Non-terminal Symbols

In this section, we put forward our first contribution, that is the result of mixing the NCE and the TGG grammars.

Definition 14. *A triple graph grammar with neighborhood-controlled embedding (NCE TGG)* $TGG = (\Sigma, \Delta \subseteq \Sigma, S \in \Sigma, P)$ *consists of, an alphabet* Σ*, a set of terminal symbols* Δ *(also define* $\Gamma := \Sigma \setminus \Delta$*), a start symbol* S *and a set of production rules* P *of the form* $(A \to (R_s \leftarrow R_c \to R_t), \omega_s, \omega_t)$ *with* $A \in \Gamma$ *being the left-hand side,* $(R_s \leftarrow R_c \to R_t) \in \mathcal{TG}_\Sigma$ *the right-hand side and* $\omega_s : V_{R_s} \nrightarrow 2^{\Sigma\times\Sigma}$ *and* $\omega_t : V_{R_t} \nrightarrow 2^{\Sigma\times\Sigma}$ *the partial embedding functions from the right-hand side's vertices to pairs of edge and label symbols.*

We might refer to the complete embedding function of a rule by $\omega := \omega_s \cup \omega_t$. For convenience, define also the start triple graph of TGG as $Z_{TGG} := Z_s \overset{ms}{\leftarrow} Z_c \overset{mt}{\rightarrow} Z_t$ where $Z_s = (\{s_0\}, \emptyset, \{s_0 \mapsto S\})$, $Z_c = (\{c_0\}, \emptyset, \{c_0 \mapsto S\})$, $Z_t = (\{t_0\}, \emptyset, \{t_0 \mapsto S\})$, $ms = \{c_0 \mapsto s_0\}$ and $mt = \{c_0 \mapsto t_0\}$.

Definition 15. *A boundary triple graph grammar with neighborhood-controlled embedding (BNCE TGG) is such that non-terminal vertices of the right-hand sides of rules are not neighbors. That is, the triple graph grammar* TGG *is boundary iff all its rules' right-hand sides are* Γ*-boundary triple graphs.*

The most important difference between the traditional TGG and the NCE TGG, is that the former allows any triple graph to occur in the left-hand sides, whereas the latter only one symbol. In addition to that, traditional TGG requires that the whole left hand side occur also in the right-hand side, that is to say, the rules are monotonic crescent. Therewith, embedding is not an issue, because an occurrence of the left-hand side is not effectively replaced by the right-hand side, instead, only new vertices are added. On the other hand, NCE TGG has to deal with embedding through the embedding function.

In the following, the semantics for NCE TGG is presented analogously to the semantics for NCE graph grammars.

Definition 16. *Let* $TGG = (\Sigma, \Delta, S, P)$ *be a NCE TGG and* $G = G_s \overset{g_s}{\leftarrow} G_c \overset{g_t}{\rightarrow} G_t$ *and* $H = H_s \overset{h_s}{\leftarrow} H_c \overset{h_t}{\rightarrow} H_t$ *be two triple graphs over* Σ *that are disjoint to all right-hand sides from* P*,* G *concretely derives in one step into* H *with rule* r *and distinct vertices* v_s, v_c, v_t*, we write* $G \overset{r,v_s,v_c,v_t}{\Rightarrow}_{TGG} H$ *if, and only if, the following holds:*

$$\begin{aligned}
r &= (A \to (R_s \overset{r_s}{\leftarrow} R_c \overset{r_t}{\rightarrow} R_t), \omega_s, \omega_t) \in P \text{ and} \\
A &= \phi_{G_s}(v_s) = \phi_{G_c}(v_c) = \phi_{G_t}(v_t) \text{ and} \\
V_{H_s} &= (V_{G_s} \setminus \{v_s\}) \cup V_{R_s} \text{ and} \\
V_{H_c} &= (V_{G_c} \setminus \{v_c\}) \cup V_{R_c} \text{ and} \\
V_{H_t} &= (V_{G_t} \setminus \{v_t\}) \cup V_{R_t} \text{ and} \\
E_{H_s} &= (E_{G_s} \setminus \{(w,l,t) \in E_{G_s} \mid w \in \{v_s\} \vee t \in \{v_s\}\}) \cup E_{R_s} \\
&\quad \cup \{(w,l,t) \mid (w,l,v) \in E_{G_s} \wedge (l, \phi_{G_s}(w)) \in \omega_s(t)\} \\
&\quad \cup \{(t,l,w) \mid (v,l,w) \in E_{G_s} \wedge (l, \phi_{G_s}(w)) \in \omega_s(t)\} \text{ and} \\
E_{H_c} &= (E_{G_c} \setminus \{(w,l,t) \in E_{G_c} \mid w = v_c \vee t = v_c\}) \cup E_{R_c} \text{ and} \\
E_{H_t} &= (E_{G_t} \setminus \{(w,l,t) \in E_{G_t} \mid w = v_t \vee t = v_t\}) \cup E_{R_t} \\
&\quad \cup \{(w,l,t) \mid (w,l,v) \in E_{G_t} \wedge (l, \phi_{G_t}(w)) \in \omega_t(t)\} \\
&\quad \cup \{(t,l,w) \mid (v,l,w) \in E_{G_t} \wedge (l, \phi_{G_t}(w)) \in \omega_t(t)\} \text{ and} \\
h_s &= (g_s \setminus \{(v_c, x)\}) \cup r_s \\
h_t &= (g_t \setminus \{(v_c, x)\}) \cup r_t \\
\phi_{H_s} &= (\phi_{G_s} \setminus \{(v_s, x)\}) \cup \phi_{R_s} \text{ and} \\
\phi_{H_c} &= (\phi_{G_c} \setminus \{(v_c, x)\}) \cup \phi_{R_c} \text{ and} \\
\phi_{H_t} &= (\phi_{G_t} \setminus \{(v_t, x)\}) \cup \phi_{R_t}
\end{aligned}$$

Notice that, without loss of generalization, we set $\omega(t) = \emptyset$ for all vertices t without an image defined in ω. And, analogously to graph grammars, if $G \overset{r,v_s,v_c,v_t}{\Rightarrow}_{TGG} H$ and $H' \in [H]$, then $G \overset{r,v_s,v_c,v_t}{\Rightarrow}_{TGG} H'$, moreover the reflexive transitive closure of $\Rightarrow$ is denoted by $\Rightarrow^*$ and we call these relations by the same names as before, namely, derivation in one step and derivation. We might also omit identifiers along this paper.

A concrete derivation of a triple graph $G = G_s \overset{g_s}{\leftarrow} G_c \overset{g_t}{\rightarrow} G_t$ can de informally understood as concrete derivations (see 8) of G_s, G_c and G_t according to the right-hand sides R_s, R_c and R_t. The only remark is the absence of an embedding mechanism for the correspondence graph, which edges are not important for our application. Nevertheless, the addition of such a mechanism for the correspondence graph should not be a problem if it is desired.

Definition 17. *A derivation* D *in* TGG *is a sequence of derivation steps*

$$D = (G_0 \overset{r_0,s_0,c_0,t_0}{\Rightarrow} G_1 \overset{r_1,s_1,c_1,t_1}{\Rightarrow} G_2 \overset{r_2,s_2,c_2,t_2}{\Rightarrow} \ldots \overset{r_{n-1},s_{n-1},c_{n-1},t_{n-1}}{\Rightarrow} G_n)$$

Definition 18. *The language $L(TGG)$ generated by the triple grammar TGG is the set of all triple graphs containing only terminal vertices derived from the start triple graph Z_{TGG}, that is*

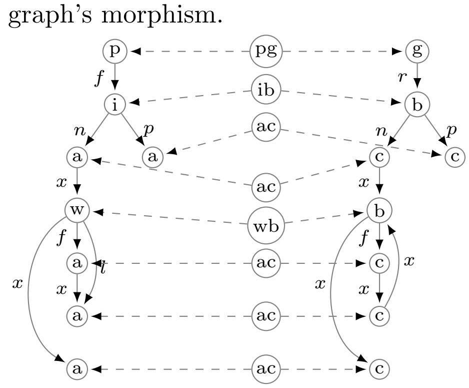
$$L(TGG) = \{H \text{ is a triple graph over } \Delta \text{ and } Z_{TGG} \Rightarrow^* H\}$$

Our concrete syntax for NCE TGG is similar to the one for NCE graph grammars and is presented below by means of the Example 2. The only difference is at the right-hand sides, that include the morphisms between the correspondence graph and source and target graphs depicted with dashed lines.

Example 2. This example illustrates the definition of a BNCE TGG that characterizes the language of all $Pseudocode$ graphs together with their respective $Controlflow$ graphs. A $Pseudocode$ graph is an abstract representation of a program written in a pseudo-code where vertices refer to *actions*, *ifs* or *whiles* and edges connect these items together according to how they appear in the program. A $Controlflow$ graph is a more abstract representation of a program, where vertices can only be either a *command* or a *branch*.

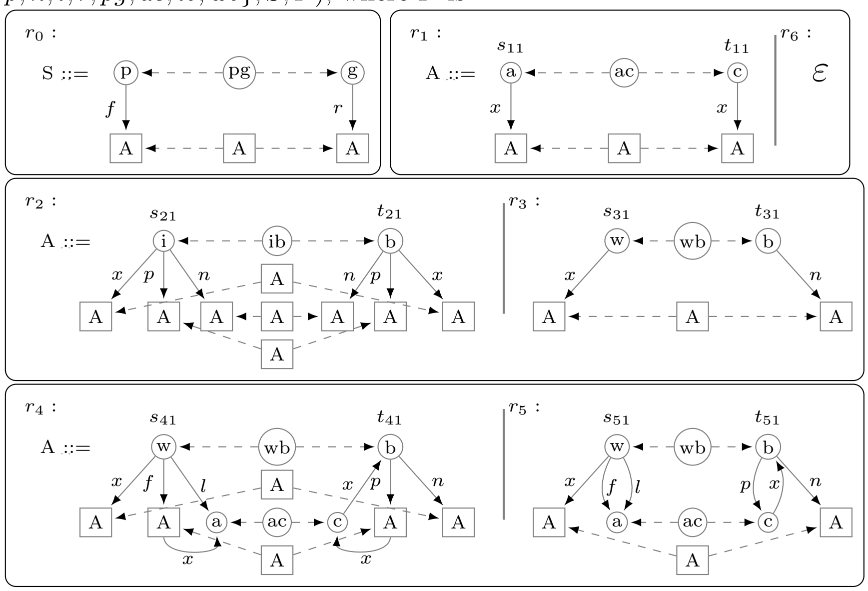
Consider, for instance, the program *main* below on the left, written in a pseudo-code. The triple graph TG on the right consists of the $Pseudocode$ graph of *main* connected to the $Controlflow$ graph of the same program through the correspondence graph in the middle of them. In such graph, the vertex labels of the $Pseudocode$ graph p, i, a, w correspond to the concepts of *program*, *if*, *action* and *while*, respectively. The edge label f is given to the edge from the vertex p to the program's first statement, x stands for *next* and indicates that a statement is followed by another, p and n stand for *positive* and *negative* and indicate which assignments correspond to the positive of negative case of the *if*'s evaluation, finally l stands for *last* and indicates the last action of a loop. In the $Controlflow$ graph, the vertex labels g, b, c stand for the concepts of *graph*, *branch* and *command*, respectively. The edge label r is given to the edge from the vertex g to the first program's statement, x, p and n mean, analogous to the former graph, *next*, *positive* and *negative*. In the correspondence graph, the labels pg, ib, ac, wb serve to indicate which labels in the source and target graphs are being connected through the triple graph's morphism.

program $main(n)$
if $n < 0$ **then**
  return *nothing*
else
  $f \leftarrow 1$
  while $n > 0$ **do**
    $f \leftarrow f * n$
    $n \leftarrow n - 1$
  end while
  return $Just f$
end if



The main difference between the two graphs is the absence of the w label in the $Controlflow$ graph, what makes it encode loops through the combination of b-labeled vertices and x-labeled edges.

The TGG that specifies the relation between these two types of graphs is $TGG = (\{S, A, p, a, i, w, g, b, c, f, x, p, n, l, r, pg, ac, ib, wb\}, \{p, a, i, w, g, b, c, f, x, p, n, l, r, pg, ac, ib, wb\}, S, P)$, where P is



with $\sigma_1(s_{11}) = \sigma_2(s_{21}) = \sigma_3(s_{31}) = \sigma_4(s_{41}) = \sigma_5(s_{51}) = \{(f, p), (x, a), (x, i), (x, w), (p, i), (n, i), (l, w), (f, w)\}$ and $\tau_1(t_{11}) = \tau_2(t_{21}) = \tau_3(t_{31}) = \tau_4(t_{41}) = \tau_5(t_{51}) = \{(r, g), (x, c), (x, b), (p, b), (n, b)\}$ being the complete definition of the source and target embedding functions of the rules, respectively.

The rule r_0 relates programs to graphs, r_1 actions to commands, r_2 ifs to branches, r_3 empty whiles to simple branches, r_4 filled whiles to filled loops with branches, r_5 whiles with one action to loops with branches with one command and, finally, r_6 produces an empty graph from a symbol A, what allows any derivation in the grammar to finish.

The aforementioned triple graph TG is in $L(TGG)$, because the derivation $Z_{TGG} \overset{r_0}{\Rightarrow} G_1 \overset{r_2}{\Rightarrow} G_2 \overset{r_6}{\Rightarrow} G_3 \overset{r_1}{\Rightarrow} G_4 \overset{r_6}{\Rightarrow} G_5 \overset{r_1}{\Rightarrow} G_6 \overset{r_4}{\Rightarrow} G_7 \overset{r_1}{\Rightarrow} G_8 \overset{r_6}{\Rightarrow} G_9 \overset{r_1}{\Rightarrow} G_{10} \overset{r_6}{\Rightarrow} TG$ is a derivation in TGG with appropriate G_i for $1 \leq i \leq 10$.

5 Model Transformation with BNCE TGG

As already introduced, TGGs can be used to characterize languages of triple graphs holding correctly transformed models. That is, one can interpret a TGG as the description of the correctly-transformed relation between two sets of models $\mathcal{S}$ and $\mathcal{T}$, where two models $G \in \mathcal{S}$ and $T \in \mathcal{T}$ are in the relation if and only if G and T are respectively, source and target graphs of any triple graph of the language $L(TGG)$. That being said, we are interested in this Section on defining a model transformation algorithm that interprets a BNCE TGG TGG to transform a source model G into one of its correspondent target models T according to the correctly-transformed relation defined by TGG.

For that end, let $TGG = (\Sigma = \Sigma_s \cup \Sigma_t, \Delta, S, P)$ be a triple graph grammar defining the correctly-transformed relation between two arbitrary sets of graphs $\mathcal{S}$ over Σ_s and $\mathcal{T}$ over Σ_t. And let $G \in \mathcal{S}$ be a source graph. We want to find a target graph $T \in \mathcal{T}$ such that $G \leftarrow C \rightarrow T \in L(TGG)$. To put in words, we wish to find a triple graph holding G and T that is in the language of all correctly transformed models. Hence, the model transformation problem is reduced— according to the definition of triple graph language (see Definition 18)— to the problem of finding a derivation $Z_{TGG} \Rightarrow^*{}_{TGG}\ G \leftarrow C \rightarrow T$.

Our strategy to solve this problem is, first, to parse G with the source part of TGG and then, with the derivation resultant of this parsing, construct the derivation $Z_{TGG} \Rightarrow^*{}_{TGG}\ G \leftarrow C \rightarrow T$. But before going into the solution, consider the definitions of the s and t functions, that extract the source and the target part of production rules.

Definition 19. *Let* $r = (A \rightarrow (G_s \leftarrow G_c \rightarrow G_t), \omega_s, \omega_t)$ *be a production rule of a triple graph grammar,* $\mathrm{s}(r) = (A \rightarrow G_s, \omega_s)$ *gives the source part of* r *and* $\mathrm{t}(r) = (A \rightarrow G_t, \omega_t)$ *gives the target part. Moreover,* $\mathrm{s}^- 1((A \rightarrow G_s, \omega_s)) = r$ *and* $\mathrm{t}^- 1((A \rightarrow G_t, \omega_t)) = r$ *are the inverse of these functions.*

Definition 20. *Let* $TGG = (\Sigma, \Delta, S, P)$ *be a triple graph grammar,* $\mathrm{S}(TGG) = (\Sigma, \Delta, S, \mathrm{s}(P))$ *gives the source grammar of* TGG *and* $\mathrm{T}(TGG) = (\Sigma, \Delta, S, \mathrm{t}(P))$ *gives the target grammar of* TGG.

Furthermore, consider the definition of the non-terminal consistent (NTC) property of TGGs, which assures that non-terminal vertices of the correspondent graph are connected to vertices with the same label in the source and target graphs.

Definition 21. *A triple graph grammar* $TGG = (\Sigma, \Delta, S, P)$ *is non-terminal consistent (NTC) if and only if, for all rules* $(A \rightarrow (G_s \overset{ms}{\leftarrow} G_c \overset{mt}{\rightarrow} G_t), \omega_s, \omega_t) \in P$*, the following holds:*

1. $\forall c \in V_{G_c}$. *if* $\phi_{G_c}(c) \in \Gamma$ *then* $\phi_{G_c}(c) = \phi_{G_s}(ms(c)) = \phi_{G_t}(mt(c))$ *and*
2. *For the sets* $N_s = \{v \mid \phi_{G_s}(v) \in \Gamma\}$ *and* $N_t = \{v \mid \phi_{G_t}(v) \in \Gamma\}$*, the range-restricted functions* $(ms \rhd N_s)$ *and* $(mt \rhd N_t)$ *are bijective.*

Theorem 1. *Let* $TGG = (\Sigma, \Delta, S, P)$ *be a NTC TGG,* $SG = S(TGG) = (\Sigma, \Delta, S, SP)$ *be its source grammar and* $D = Z \overset{r_0, s_0, c_0, t_0}{\Rightarrow} G^1 \overset{r_1, s_1, c_1, t_1}{\Rightarrow} \ldots \overset{r_{k-1}, s_{k-1}, c_{k-1}, t_{k-1}}{\Rightarrow} G^k$ *be a derivation in* TGG*,* D *can equivalently be rewritten as the derivation in* SG*,* $\overline{D} = Z \overset{s(r_0), s_0}{\Rightarrow} G_s^1 \overset{s(r_1), s_1}{\Rightarrow} \ldots \overset{s(r_{k-1}), s_{k-1}}{\Rightarrow} G_s^k$.

Proof. We want to show that if D is a derivation in TGG, then $\overline{D}$ is a derivation in SG, and vice-versa. We prove it by induction in the following.

First, if $Z_{TGG} \overset{r_0,s_0,c_0,t_0}{\Rightarrow}_{TGG} G^1$, that is $Z_s \leftarrow Z_c \rightarrow Z_t \overset{r_0,s_0,c_0,t_0}{\Rightarrow}_{TGG} G^1_s \leftarrow G^1_c \rightarrow G^1_t$, then, by Definition 16, $r_0 = (S \rightarrow (R_s \leftarrow R_c \rightarrow R_t), \omega_s, \omega_t) \in P$, and by Definition 19, $\mathrm{s}(r_0) = (S \rightarrow R_s, \omega_s) \in SP$, hence, using it, the configuration of $\phi_{Z_s}(s_0)$, $V_{G^1_s}$, $E_{G^1_s}$ and $\phi_{G^1_s}$ and the equality $Z_s = Z_{S(TGG)}$, we have $Z_{S(TGG)} \overset{\mathrm{s}(r_0),s_0}{\Rightarrow}_{SG} G^1_s$.

In the other direction, we choose c_0, t_0 from the definition of Z_{TGG}, with $\phi_{Z_c}(c_0) = S$ and $\phi_{Z_t}(t_0) = S$. In this case, if $Z_{SG} \overset{s(r_0),s_0}{\Rightarrow}_{SG} G^1_s$, then by Definition 8, we have $\mathrm{s}(r_0) = (S \rightarrow R_s, \omega_s) \in SP$ and using the bijectivity of s, we get $r_0 = \mathrm{s}^{-1}(s(r_0)) = (S \rightarrow (R_s \leftarrow R_c \rightarrow R_t), \omega_s, \omega_t) \in P$. Hence, using it, the configuration of $\phi_{Z_{SG}}(s_0)$, $V_{G^1_s}$, $E_{G^1_s}$ and $\phi_{G^1_s}$, the equality $Z_s = Z_{SG}$ and constructing $V_{G^1_c}$, $V_{G^1_t}$, $E_{G^1_c}$, $E_{G^1_t}$, $\phi_{G^1_c}$, $\phi_{G^1_t}$ from Z_c and Z_t according to the Definition 16, we have $Z_{TGG} \overset{r_0,s_0,c_0,t_0}{\Rightarrow}_{TGG} G^1_s \leftarrow G^1_c \rightarrow G^1_t$

Now, for the induction step, we want to show that if $Z_{TGG} \Rightarrow^*{}_{TGG} G^i \overset{r_i,s_i,c_i,t_i}{\Rightarrow}_{TGG} G^{i+1}$ is a derivation in TGG, then $Z_{TGG} \Rightarrow^*{}_{SG} G^i_s \overset{s(r_i),s_i}{\Rightarrow}_{SG} G^{i+1}_s$ is a derivation in SG and vice-versa. By induction hypothesis it holds for the first i steps, so we just have to show it for the step $i+1$.

So, if $G^i \overset{r_i,s_i,c_i,t_i}{\Rightarrow}_{TGG} G^{i+1}$, that is $G^i_s \overset{ms_i}{\leftarrow} G^i_c \overset{mt_i}{\rightarrow} G^i_t \overset{r_i,s_i,c_i,t_i}{\Rightarrow}_{TGG} G^{i+1}_s \leftarrow G^{i+1}_c \rightarrow G^{i+1}_t$, then, by Definition 16, $r_i = (S \rightarrow (R_s \leftarrow R_c \rightarrow R_t), \omega_s, \omega_t) \in P$, and by Definition 19, $\mathrm{s}(r_i) = (S \rightarrow R_s, \omega_s) \in SP$, hence, using it and the configuration of $\phi_{G^i_s}(s_i)$, $V_{G^{i+1}_s}$, $E_{G^{i+1}_s}$ and $\phi_{G^{i+1}_s}$, we have $G^i \overset{\mathrm{s}(r_i),s_i}{\Rightarrow}_{SG} G^{i+1}_s$.

In the other direction, we choose, using the bijectivity from the range-restricted s stemming from the NTC property, $c_i = ms_i^{-1}(s_i), t_i = mt_i(c_i)$. Moreover, since TGG is NTC, and because $Z_{TGG} \Rightarrow^*{}_{TGG} G^i$ is a derivation in TGG it is clear that G^i is NTC, thus $\phi_{G^i_s}(s_0) = \phi_{G^i_c}(c_0) = \phi_{G^i_t}(t_i)$.

In this case, if $G^i_s \overset{s(r_i),s_i}{\Rightarrow}_{SG} G^{i+1}_s$, then by Definition 8, we have $\mathrm{s}(r_i) = (A \rightarrow R_s, \omega_s) \in SP$ and using the bijectivity of s, we get $r_i = \mathrm{s}^{-1}(s(r_i)) = (A \rightarrow (R_s \leftarrow R_c \rightarrow R_t), \omega_s, \omega_t) \in P$.

Hence, using, additionally, the configuration of $\phi_{G^i_s}(s_i)$, $\phi_{G^i_c}(c_i)$, $\phi_{G^i_t}(t_i)$ $V_{G^{i+1}_s}$, $E_{G^{i+1}_s}$ and $\phi_{G^{i+1}_s}$ and constructing $V_{G^{i+1}_c}$, $V_{G^{i+1}_t}$, $E_{G^{i+1}_c}$, $E_{G^{i+1}_t}$, $\phi_{G^{i+1}_c}$, $\phi_{G^{i+1}_t}$ from G^i_c and G^i_t according to the Definition 16, we have $G^i_s \leftarrow G^i_c \rightarrow G^i_t \overset{r_i,s_i,c_i,t_i}{\Rightarrow}_{TGG} G^{i+1}_s \leftarrow G^{i+1}_c \rightarrow G^{i+1}_t$.

This finishes the proof. □

Therefore, by Theorem 1, the problem of finding a derivation $D = Z_{TGG} \Rightarrow^*{}_{TGG} G \leftarrow C \rightarrow T$ is reduced to finding a derivation $\overline{D} = Z_{S(TGG)} \Rightarrow_{S(TGG)} G$, what can be done with the already presented parsing algorithm **??**. The final construction of the triple graph $G \leftarrow C \rightarrow T$ becomes then just a matter of creating D out of $\overline{D}$.

The complete transformation procedure is presented in the Algorithm 1. Thereby it is required that the TGG be neighborhood preserving, what poses no problem to our procedure, once any TGG can be transformed into the neighborhood preserving normal form.

Algorithm 1 Transformation Algorithm for NP NTC BNCE TGGs

Require: TGG is a valid NP NTC BNCE triple graph grammar
Require: G is a valid graph over Σ
  function $transform(TGG = (\Sigma, \Delta, S, P), G = (V_G, E_G, \phi_G))$: $Graph$
    $GG \leftarrow S(TGG)$ ▷ see 19
    $\overline{D} \leftarrow parse(GG, G)$ ▷ use algorithm **??**
    if $\overline{D} = Z_{S(TGG)} \Rightarrow^{*}{}_{S(TGG)} G$ **then** ▷ if parsed successfully
      from $\overline{D}$ construct $D = Z_{TGG} \Rightarrow^{*}{}_{TGG} G \leftarrow C \rightarrow T$
      return T
    else
      return $nothing$ ▷ no T satisfies $(G \leftarrow C \rightarrow T) \in L(TGG)$
    end if
  end function
Ensure: $return$ is either $nothing$ or T, such that $(G \leftarrow C \rightarrow T) \in L(TGG)$

6 Evaluation and Discussion

7 Conclusion

References

1. Anjorin, A., Leblebici, E., Schürr, A.: 20 years of triple graph grammars: A roadmap for future research. Electronic Communications of the EASST **73** (2016)
2. Janssens, D., Rozenberg, G.: Graph grammars with neighbourhood-controlled embedding. Theoretical Computer Science **21**(1), 55–74 (1982)
3. Kim, C.: Efficient recognition algorithms for boundary and linear ence graph languages. Acta informatica **37**(9), 619–632 (2001)
4. Rozenberg, G., Welzl, E.: Boundary nlc graph grammarsbasic definitions, normal forms, and complexity. Information and Control **69**(1-3), 136–167 (1986)
5. Schürr, A.: Specification of graph translators with triple graph grammars. In: International Workshop on Graph-Theoretic Concepts in Computer Science. pp. 151–163. Springer (1994)