Note on the paper

Background

This paper is aimed at solving **linear systems**, which can't be solved efficiently on classical computer. In this paper, the author proposed a **variational hybrid quantum-classical algrithm (VHQCA)** to figure out the **quantum linear systems problem (QLSP)**. They call it **Variational Quantum Linear Solver (VQLS)**

Goal

Given matrixd $A_{n\times n}$ and vector $b_{n\times 1}$, find a quantum state $|x\rangle$ such that $A|x\rangle = |b\rangle$.

Outline of ideas

1. A can be decomposed as $A = \sum_l A_l$.
2. Condition number κ of A can be used to analyze the problem theoretically, as well as to compare with HHL algorithm that aims at the same problem.
3. Define two kinds of cost function $C_G(|\psi\rangle)$ and $C_L(|\psi\rangle)$ that evaluates the distance between $|\psi\rangle$ and the exact solution globally or locally.
4. Extimating $C_G(|\psi\rangle)$ and $C_L(|\psi\rangle)$ by classical method is proven to be $DQC1 - hard$ by reduce in to an estimation of Hilbert-Schmidt inner-product magnitude.
5. The cost function can be seen as expection of H_G and H_L under the state $|0\rangle$, which can be reduced to computing three set of values: $\delta_{ll'}, \gamma_{ll'}, \beta_{ll'}$ and sum them up.
6. $|\psi\rangle$ is produced by $|\psi\rangle = V(\alpha)|0\rangle$, where $V(\alpha)$ is separated into p driver hamiltonion and p mixer hamiltonion alternatively (QAOA ansza) in certain order.

   $$V(\boldsymbol{\alpha}) = e^{-iH_M\alpha_{2p}}e^{-iH_D\alpha_{2p-1}}\dots e^{-iH_M\alpha_2}e^{-iH_D\alpha_1}$$

   The parameters to be optimized is the time period of each hamiltonion.
7. Hadamard test and Hadamard-overlap test are used to calculate the parameters. Hadamard-overlap benefits the experiment, for it avoid using much control gates. We can get the real and imaginary parts of the $\delta_{ll'}, \gamma_{ll'}, \beta_{ll'}$ by measuring $P(0)$ and $P(1)$ of the ancilla qubit.
8. Divide the algorithm into quantum part and classical part. The quantum part calculate the cost

functions and the gradients while the classical parts do control and feedback.

Two ansatz

1.Hardware-Efficient Ansatz: A is composed of the gates native to that hard-ware.

2.Quantum Alternating Operator Ansatz(QAOA): By alternatively acting driver hamiltonion H_D and mixer hamiltion H_M , one can simulate any unitary gates efficiently.

ALgorithm

> Input $A = \Sigma_{l=1}^{L} c_l A_l$ and U such that $U|0\rangle = |b\rangle$ to VLQS.
>
> Input parameters $\boldsymbol{\alpha}$ to the quantum computer, which prepares $|x(\boldsymbol{\alpha})\rangle$.
>
> > The quantum computer will estimate a cost function $C(\boldsymbol{\alpha})$.
> >
> > The value of $C(\boldsymbol{\alpha})$ from the quantum computer is returned to the classical computer, which adjusts $\boldsymbol{\alpha}$ to reduce the cost.
>
> The loop above will go on until the classical computer fint that $C(\boldsymbol{\alpha}) \leq \gamma$.
>
> VQLS outputs the final parameters $\boldsymbol{\alpha}_{opt}$.
>
> Using gate sequence $V(\boldsymbol{\alpha}_{opt})$ to obtain the quantum state $|x(\boldsymbol{\alpha}_{opt})\rangle$: $|x(\boldsymbol{\alpha}_{opt})\rangle = V(\boldsymbol{\alpha}_{opt})|0\rangle$.

Experiments&&Simulation

1. Numerically implemented VQLS to solve three different QLSPs of A with different degeneracy g with Hardware-Efficient Ansatz. Plot the dependence of running-per-success with condition number κ.

2. Numerically implemented VQLS and plot the Runs-per-success versus 1/ε for the QLSPs.

3. Implementation the VQLS using Rigetti's quantum chip 16Q Aspen-4.

Something more to explore

1. How to consider the impact of noise in the simulation? Why the algorithm is rubost to noise?

2. Does the resources we need depends largely on A or not?

3. Can the vector be encoded more efficiently ?