A Better Proof of GCD3 with Comments

1. SUFFICES ASSUME: m, n, and d are integers
   PROVE: d divides both m and n iff d divides both m and $n - m$

   This step introduces the symbols m, n, and d, and it assumes them to be integers for the remainder of the proof. It asserts that to prove the desired result (GCD3), it suffices to prove the statement of the PROVE clause. This statement becomes the goal of the proof.

   PROOF: Since the gcd of two numbers is the largest integer that divides both of them, it suffices to show that m and n have the same common divisors as m and $n - m$.

   We are implicitly also using the fact that to prove that an assertion is true for all positive integers m and n, it suffices to introduce two new symbols m and n, assume them to be positive integers, and then prove the assertion for those particular positive integers m and n. The m and n introduced in the ASSUME are logically different from the m and n in the statement of GCD3. We could replace m and n throughout the proof by other symbols. However, using the same symbols as in GCD3 makes the proof easier to understand.

2. ASSUME: d divides both m and n
   PROVE: d divides both m and $n - m$

   This step asserts that the ASSUME clause implies the PROVE clause.
   In the step's proof, we assume that the ASSUME clause is true and we must show that the PROVE clause is true.

   PROOF: That d divides m follows by the assumptions; that it divides $n - m$ follows from the assumptions and Lemma Div.
   The assumptions being used are the ASSUME clause of step 1 (that m and n are positive integers) and the ASSUME clause of the current step (step 2).

3. ASSUME: d divides both m and $n - m$
   PROVE: d divides both m and n

   PROOF: That d divides m follows by the assumptions; that it divides n follows from the assumptions, Lemma Div, and the simple algebraic relation: $n = m + (n - m)$.

4. Q.E.D.
   Q.E.D. stands for the current goal of the proof, which in this case is the PROVE clause of step 1.

   PROOF: By 1, 2, and 3.
   The current goal (the PROVE clause of step 1) follows from steps 2 and 3. Although step 1 is logically not part of the proof of the current goal, so it's not needed in the proof, mentioning it reminds the reader of what the current goal is.

CLOSE