

## A Better Proof of GCD3

1. SUFFICES ASSUME:  $m$ ,  $n$ , and  $d$  are integers

PROVE:  $d$  divides both  $m$  and  $n$  iff  $d$  divides both  $m$  and  $n - m$

PROOF: Since the gcd of two numbers is the largest integer that divides both of them, it suffices to show that  $m$  and  $n$  have the same common divisors as  $m$  and  $n - m$ .

2. ASSUME:  $d$  divides both  $m$  and  $n$

PROVE:  $d$  divides both  $m$  and  $n - m$

PROOF: That  $d$  divides  $m$  follows by the assumptions; that it divides  $n - m$  follows from the assumptions and Lemma Div.

3. ASSUME:  $d$  divides both  $m$  and  $n - m$

PROVE:  $d$  divides both  $m$  and  $n$

PROOF: That  $d$  divides  $m$  follows from the assumptions; that it divides  $n$  follows from the assumptions, Lemma Div, and the simple algebraic relation:  $n = m + (n - m)$ .

4. Q.E.D.

PROOF: By 1, 2, and 3.