

True Formulas versus Theorems

A *theorem* is a formula that is provable in a logic. A logic consists of a set of legal formulas (called well-formed formulas) and a collection of rules for proving that certain of those formulas are theorems. There is a simple procedure for checking if a sequence of rule applications constitutes a proof of a formula, though one seldom writes such a low-level, detailed proof.

Truth is a semantic concept. A *semantics* for a logic is a mapping Σ from formulas of the logic to formulas of “ordinary math”, where $\Sigma(F)$ is called the *meaning* of the formula F of the logic. A formula F is said to be *true* iff its meaning $\Sigma(F)$ is a true formula of ordinary math.

Gödel showed that it’s impossible to define formally what truth means for ordinary math. However, for the formulas that we encounter when describing and reasoning about algorithms and systems, truth and provability turn out to be equivalent. (This seems to be because the systems we study are ones that can be implemented by computers.) I will therefore not make any attempt to distinguish between the two concepts in this hyperbook, and I will consider theorems and true formulas to be the same.