

Recall the original (non-TLA<sup>+</sup>) definition:

$$R ** S \triangleq \{ \langle x, y \rangle : \exists z : (\langle x, z \rangle \in R) \wedge (\langle z, y \rangle \in S) \}$$

Since  $x$  is replacing  $\langle x, y \rangle$ , we can replace  $x$  and  $y$  by  $x[1]$  and  $x[2]$ . Hence, the definition becomes

$$R ** S \triangleq \{ x \in T : \exists z : (\langle x[1], z \rangle \in R) \wedge (\langle z, x[2] \rangle \in S) \}$$

We now have to decide what the set  $T$  should be. A little thought reveals that the elements of  $R ** S$  have to be pairs  $\langle r, s \rangle$  with  $r$  a node of  $R$  and  $s$  a node of  $S$ . Therefore, we can take  $T$  to be the Cartesian product  $NodesOf(R) \times NodesOf(S)$ , to obtain:

$$\begin{aligned} R ** S \triangleq \{ x \in NodesOf(R) \times NodesOf(S) : \\ \exists z : (\langle x[1], z \rangle \in R) \wedge (\langle z, x[2] \rangle \in S) \} \end{aligned}$$

This is a legal TLA<sup>+</sup> definition, but TLC can't evaluate it because it contains the unbounded quantifier  $\exists z : \dots$ . We need to restrict the range of the bound identifier  $z$ . The body of the quantified expression is satisfied only if  $z$  is an element of both  $NodesOf(R)$  and  $NodesOf(S)$ . So we could write this quantified expression in any of these ways:

$$\begin{aligned} \exists z \in NodesOf(R) : \dots \\ \exists z \in NodesOf(S) : \dots \\ \exists z \in NodesOf(R) \cap NodesOf(S) : \dots \end{aligned}$$

Although longer, I find the third to be a little clearer:

$$\begin{aligned} R ** S \triangleq \{ x \in NodesOf(R) \times NodesOf(S) : \\ \exists z \in NodesOf(R) \cap NodesOf(S) : \\ (\langle x[1], z \rangle \in R) \wedge (\langle z, x[2] \rangle \in S) \} \end{aligned}$$