

Instantiating Fairness

The formulas $\overline{\text{WF}_{\text{vars}_A}(\text{Consumer}_A)}$ and $\text{WF}_{\text{vars}_A}(\overline{\text{Consumer}_A})$ need not be equivalent. This is because $\text{WF}_{\text{vars}_A}(\text{Consumer}_A)$ is defined in terms of the enabling of action $\langle \text{Consumer}_A \rangle_{\text{vars}_A}$, and the following two assertions need not be equivalent

- $\overline{\langle \text{Consumer}_A \rangle_{\text{vars}_A}}$ is enabled
- $\overline{\langle \text{Consumer}_A \rangle_{\text{vars}_A}}$ (which equals $\overline{\langle \text{Consumer}_A \rangle_{\text{vars}_A}})$ is enabled

More precisely, WF is defined in terms of the ENABLED operator, where $\text{ENABLED } C$ is true in a state s iff action C is enabled in s . The formula $\overline{\text{ENABLED } C}$ is true in state s iff action C is enabled in state \bar{s} . The formula $\text{ENABLED } \bar{C}$ is true in state s iff \bar{C} is enabled in state s , where \bar{C} is the action that is true of the transition $s \rightarrow t$ iff C is true of $\bar{s} \rightarrow \bar{t}$. The state predicates $\overline{\text{ENABLED } C}$ and $\text{ENABLED } \bar{C}$ are not necessarily equivalent. If $\overline{\text{ENABLED } \langle \text{Consumer}_A \rangle_{\text{vars}_A}}$ is not equivalent to $\text{ENABLED } \overline{\langle \text{Consumer}_A \rangle_{\text{vars}_A}}$, then $\overline{\text{WF}_{\text{vars}_A}(\text{Consumer}_A)}$ and $\text{WF}_{\text{vars}_A}(\overline{\text{Consumer}_A})$ are not equivalent.

As an example, let C be the action $x' \neq y'$, where x and y are variables. Since for any state s there is a state t in which $x \neq y$, this action is always enabled. Hence, $\text{ENABLED } C$ equals TRUE. Since the expression TRUE does depend on any variables, $\overline{\text{TRUE}} = \text{TRUE}$ and hence $\overline{\text{ENABLED } C} = \text{TRUE}$ for any refinement mapping. Now consider the refinement mapping defined by

$$\bar{x} = z \quad \bar{y} = z$$

We have

$$\begin{aligned} \bar{C} &= \overline{x' \neq y'} && \text{By definition of } C. \\ &= \bar{x'} \neq \bar{y'} && \text{By the meaning of } \bar{F} \text{ for a formula } F. \\ &= z' \neq z' && \text{By definition of } \bar{x} \text{ and } \bar{y}. \\ &= \text{FALSE} && \text{No value is unequal to itself.} \end{aligned}$$

Therefore, $\overline{\text{ENABLED } C}$ equals ENABLED FALSE , which equals FALSE, so it is not equal to $\overline{\text{ENABLED } C}$, which equals TRUE.