Summary of TLA$^+$

The Constant Operators

Logic

$\wedge$ $\vee$ $\neg$ $\Rightarrow$ $\equiv$

TRUE   FALSE   BOOLEAN [the set $\{\text{TRUE}, \text{FALSE}\}$]

$\forall\, x \in S : p$ [1] $\exists\, x \in S : p$ [1]

CHOOSE $x \in S : p$ [An x in S satisfying p]

Sets

$=$ $\neq$ $\in$ $\notin$ $\cup$ $\cap$ $\subseteq$ $\setminus$ [set difference]	
$\{e_1, \ldots, e_n\}$	[Set consisting of elements e_i]
$\{x \in S : p\}$ [2]	[Set of elements x in S satisfying p]
$\{e : x \in S\}$ [1]	[Set of elements e such that x in S]
SUBSET S	[Set of subsets of S]
UNION S	[Union of all elements of S]

Functions

$f[e]$	[Function application]
DOMAIN f	[Domain of function f]
$[x \in S \mapsto e]$ [1]	[Function f such that $f[x] = e$ for $x \in S$]
$[S \to T]$	[Set of functions f with $f[x] \in T$ for $x \in S$]
$[f \text{ EXCEPT } ![e_1] = e_2]$ [3]	[Function $\widehat{f}$ equal to f except $\widehat{f}[e_1] = e_2$]

Records

$e.h$	[The h-field of record e]
$[h_1 \mapsto e_1, \ldots, h_n \mapsto e_n]$	[The record whose h_i field is e_i]
$[h_1 : S_1, \ldots, h_n : S_n]$	[Set of all records with h_i field in S_i]
$[r \text{ EXCEPT } !.h = e]$ [3]	[Record $\widehat{r}$ equal to r except $\widehat{r}.h = e$]

Tuples

$e[i]$	[The ith component of tuple e]
$\langle e_1, \ldots, e_n \rangle$	[The n-tuple whose i^{th} component is e_i]
$S_1 \times \ldots \times S_n$	[The set of all n-tuples with i^{th} component in S_i]

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(1) $x \in S$ may be replaced by a comma-separated list of items $v \in S$, where v is either a comma-separated list or a tuple of identifiers.

(2) x may be an identifier or tuple of identifiers.

(3) $![e_1]$ or $!.h$ may be replaced by a comma separated list of items $!\, a_1 \cdots a_n$, where each a_i is $[e_i]$ or $.h_i$.

Miscellaneous Constructs

IF p THEN e_1 ELSE e_2 $\qquad\qquad\qquad\qquad$ [e_1 if p true, else e_2]

CASE $p_1 \to e_1 \;\square\; \ldots \;\square\; p_n \to e_n$ $\qquad\qquad$ [Some e_i such that p_i true]

CASE $p_1 \to e_1 \;\square\; \ldots \;\square\; p_n \to e_n \;\square\;$ OTHER $\to e$ $\quad$ [Some e_i such that p_i true, or e if all p_i are false]

LET $d_1 \triangleq e_1 \;\ldots\; d_n \triangleq e_n$ IN e $\quad$ [e in the context of the definitions]

$\wedge\; p_1$ [the conjunction $p_1 \wedge \ldots \wedge p_n$] $\qquad$ $\vee\; p_1$ [the disjunction $p_1 \vee \ldots \vee p_n$]

$\quad\vdots$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad\vdots$

$\wedge\; p_n$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad\;$ $\vee\; p_n$

Action Operators

e' $\qquad\qquad$ [The value of e in the final state of a step]

$[A]_e$ $\qquad\quad$ [$A \vee (e' = e)$]

$\langle A \rangle_e$ $\qquad\quad$ [$A \wedge (e' \neq e)$]

ENABLED A $\quad$ [An A step is possible]

UNCHANGED e $\quad$ [$e' = e$]

$A \cdot B$ $\qquad\quad$ [Composition of actions]

Temporal Operators

$\square F$ $\qquad\;$ [F is always true]

$\Diamond F$ $\qquad\;$ [F is eventually true]

$\text{WF}_e(A)$ $\quad$ [Weak fairness for action A]

$\text{SF}_e(A)$ $\quad$ [Strong fairness for action A]

$F \rightsquigarrow G$ $\quad$ [F leads to G]

User-Definable Operator Symbols

Infix Operators

$+$ [1]	$-$ [1]	$*$ [1]	$/$ [2]	$\circ$ [3]	$++$
$\div$ [1]	$\%$ [1]	$\hat{\ }$ [1,4]	$..$ [1]	$...$	$--$
$\oplus$ [5]	$\ominus$ [5]	$\otimes$	$\oslash$	$\odot$	$**$
$<$ [1]	$>$ [1]	$\leq$ [1]	$\geq$ [1]	$\sqcap$	$//$
$\prec$	$\succ$	$\preceq$	$\succeq$	$\sqcup$	$\hat{\ }\hat{\ }$
$\ll$	$\gg$	$<:$	$:>$ [6]	$\&$	$\&\&$
$\sqsubset$	$\sqsupset$	$\sqsubseteq$ [5]	$\sqsupseteq$	$\mid$	$\%\%$
$\subset$	$\supset$		$\supseteq$	$\star$	$@@$ [6]
$\vdash$	$\dashv$	$\models$	$=\mid$	$\bullet$	$\#\#$
$\sim$	$\simeq$	$\approx$	$\cong$	$\$$	$\$\$$
$\bigcirc$	$::=$	$\asymp$	$\doteq$	$??$	$!!$
$\propto$	$\wr$	$\uplus$			

Postfix Operators [7]

$\hat{\ }+$ $\hat{\ }*$ $\hat{\ }\#$

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(1) Defined by the *Naturals*, *Integers*, and *Reals* modules.
(2) Defined by the *Reals* module.
(3) Defined by the *Sequences* module.
(4) $x\hat{\ }y$ is printed as x^y.
(5) Defined by the *Bags* module.
(6) Defined by the *TLC* module.
(7) $e\hat{\ }+$ is printed as e^+, and similarly for $\hat{\ }*$ and $\hat{\ }\#$.

Precedence Ranges of Operators

The relative precedence of two operators is unspecified if their ranges overlap. Left-associative operators are indicated by (a).

Prefix Operators

¬	4–4	□	4–15	UNION	8–8
ENABLED	4–15	◇	4–15	DOMAIN	9–9
UNCHANGED	4–15	SUBSET	8–8	−	12–12

Infix Operators

⇒	1–1	≤	5–5	<:	7–7	⊖	11–11 (a)
+▷	2–2	≪	5–5	\	8–8	−	11–11 (a)
≡	2–2	≺	5–5	∩	8–8 (a)	−−	11–11 (a)
⤳	2–2	≼	5–5	∪	8–8 (a)	&	13–13 (a)
∧	3–3 (a)	∝	5–5	..	9–9	&&	13–13 (a)
∨	3–3 (a)	∼	5–5	...	9–9	⊙	13–13 (a)
≠	5–5	≃	5–5	!!	9–13	⊘	13–13
⊣	5–5	⊏	5–5	##	9–13 (a)	⊗	13–13 (a)
::=	5–5	⊑	5–5	$	9–13 (a)	∗	13–13 (a)
:=	5–5	⊐	5–5	$$	9–13 (a)	∗∗	13–13 (a)
<	5–5	⊒	5–5	??	9–13 (a)	/	13–13
=	5–5	⊂	5–5	⊓	9–13 (a)	//	13–13
⊨	5–5	⊆	5–5	⊔	9–13 (a)	◯	13–13 (a)
>	5–5	≻	5–5	⊎	9–13 (a)	•	13–13 (a)
≈	5–5	≽	5–5	≀	9–14	÷	13–13
≍	5–5	⊃	5–5	⊕	10–10 (a)	∘	13–13 (a)
≅	5–5	⊇	5–5	+	10–10 (a)	⋆	13–13 (a)
≐	5–5	⊢	5–5	++	10–10 (a)	^	14–14
≥	5–5	⊨	5–5	%	10–11	^^	14–14
≫	5–5	.[1]	5–14 (a)	%%	10–11 (a)	.[2]	17–17 (a)
∈	5–5	@@	6–6 (a)	|	10–11 (a)		
∉	5–5	:>	7–7	||	10–11 (a)		

Postfix Operators

^+	15–15	^*	15–15	^#	15–15	′	15–15

[1] Action composition (\cdot).
[2] Record field (period).

Operators Defined in Standard Modules.

Modules *Naturals*, *Integers*, *Reals*

$+$	$-$ [1]	$*$	$/$ [2]	$\wedge$ [3]	$..$	*Nat*	*Real* [2]
$\div$	$\%$	$\leq$	$\geq$	$<$	$>$	*Int* [4]	*Infinity* [2]

(1) Only infix $-$ is defined in *Naturals*.
(2) Defined only in *Reals* module.
(3) Exponentiation.
(4) Not defined in *Naturals* module.

Module *Sequences*

$\circ$	*Head*	*SelectSeq*	*SubSeq*
Append	*Len*	*Seq*	*Tail*

Module *FiniteSets*

IsFiniteSet   *Cardinality*

Module *Bags*

$\oplus$	*BagIn*	*CopiesIn*	*SubBag*
$\ominus$	*BagOfAll*	*EmptyBag*	
$\sqsubseteq$	*BagToSet*	*IsABag*	
BagCardinality	*BagUnion*	*SetToBag*	

Module *RealTime*

RTBound   *RTnow*   *now* (declared to be a variable)

Module *TLC*

$:>$ $@@$ *Print*   *Assert*   *JavaTime*   *Permutations*
SortSeq

ASCII Representation of Typeset Symbols

Symbol	ASCII	Symbol	ASCII	Symbol	ASCII
$\wedge$	`/\` or `\land`	$\vee$	`\/` or `\lor`	$\Rightarrow$	`=>`
$\neg$	`~` or `\lnot` or `\neg`	$\equiv$	`<=>` or `\equiv`	$\triangleq$	`==`
$\in$	`\in`	$\notin$	`\notin`	$\neq$	`#` or `/=`
$\langle$	`<<`	$\rangle$	`>>`	$\Box$	`[]`
$<$	`<`	$>$	`>`	$\Diamond$	`<>`
$\leq$	`\leq` or `=<` or `<=`	$\geq$	`\geq` or `>=`	$\rightsquigarrow$	`~>`
$\ll$	`\ll`	$\gg$	`\gg`	$\overset{+}{\rightarrow}$	`-+->`
$\prec$	`\prec`	$\succ$	`\succ`	$\mapsto$	`|->`
$\preceq$	`\preceq`	$\succeq$	`\succeq`	$\div$	`\div`
$\subseteq$	`\subseteq`	$\supseteq$	`\supseteq`	$\cdot$	`\cdot`
$\subset$	`\subset`	$\supset$	`\supset`	$\circ$	`\o` or `\circ`
$\sqsubset$	`\sqsubset`	$\sqsupset$	`\sqsupset`	$\bullet$	`\bullet`
$\sqsubseteq$	`\sqsubseteq`	$\sqsupseteq$	`\sqsupseteq`	$\star$	`\star`
$\vdash$	`|-`	$\dashv$	`-|`	$\bigcirc$	`\bigcirc`
$\models$	`|=`	$=\|$	`=|`	$\sim$	`\sim`
$\rightarrow$	`->`	$\leftarrow$	`<-`	$\simeq$	`\simeq`
$\cap$	`\cap` or `\intersect`	$\cup$	`\cup` or `\union`	$\asymp$	`\asymp`
$\sqcap$	`\sqcap`	$\sqcup$	`\sqcup`	$\approx$	`\approx`
$\oplus$	`(+)` or `\oplus`	$\uplus$	`\uplus`	$\cong$	`\cong`
$\ominus$	`(-)` or `\ominus`	$\times$	`\X` or `\times`	$\doteq$	`\doteq`
$\odot$	`(.)` or `\odot`	$\wr$	`\wr`	x^y	`x^y` [2]
$\otimes$	`(\X)` or `\otimes`	$\propto$	`\propto`	x^+	`x^+` [2]
$\oslash$	`(/)` or `\oslash`	"s"	`"s"` [1]	x^*	`x^*` [2]
$\exists$	`\E`	$\forall$	`\A`	$x^\#$	`x^#` [2]
$\exists\!\exists$	`\EE`	$\forall\!\forall$	`\AA`	$'$	`'`
$]_v$	`]_v`	$\rangle_v$	`>>_v`		
WF_v	`WF_v`	SF_v	`SF_v`		

`---------` [3]

`---------` [3]

`---------` [3]

`=========` [3]

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(1) s is a sequence of characters.

(2) x and y are any expressions.

(3) a sequence of four or more `-` or `=` characters.