

## Should it be $\exists x : P$ or $\exists x : P(x)$ ?

I often write  $\forall x : P$ , where  $P$  is an arbitrary formula that can contain  $x$ . To emphasize that  $P$  can contain  $x$ , I may instead write  $\forall x : P(x)$ . There is no significance to this difference when I'm discussing quantification in general. However, the exact formula  $\forall x : P$  will never appear in a specification for the following reason. Because  $\forall x : P$  can be a legal TLA<sup>+</sup> formula only in a context in which  $x$  has no meaning,  $P$  cannot depend on  $x$ . (In particular, if  $P$  is a user-defined symbol, then  $x$  cannot appear in its definition.) Since  $P$  does not depend on  $x$ , the formula  $\forall x : P$  is equivalent to  $P$ , so one would write simply  $P$  instead of  $\forall x : P$ . On the other hand, the exact formula  $\forall x : P(x)$  could very well appear in a specification.

With obvious modifications, everything I just wrote applies as well to  $\forall x \in S : P$  and with  $\forall$  replaced by  $\exists$ . (Note that, if  $P$  does not depend on  $x$ , then  $\forall x \in S : P$  equals  $P \vee (S = \{\})$  rather than  $P$ .) I will never write  $\forall x \in S(x) : P(x)$  because, in the formula  $\forall x \in S : P$ , the variable  $x$  may not appear in  $S$ .