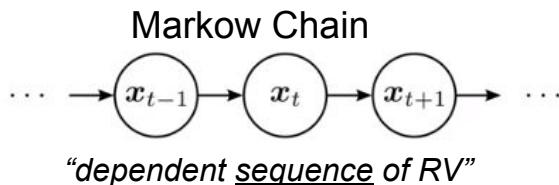# History of the Metropolis–Hastings Algorithm

Group Goliath, 07.04.2022
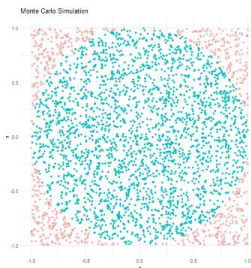
Jonas Füglistaler, Zhixuan Li, Lea Bührer

Markow-Chain Monte Carlo (MCMC) sampling motivation

1. Bayesians' quantity of interest: **Posterior distribution** $\quad p(\theta|y) = \dfrac{p(y|\theta)p(\theta)}{p(y)}$

2. Usually **no closed-form** for $\quad p(y) = \displaystyle\int p(y|\theta)p(\theta)d\theta$

3. **MCMC** allows for performing inference for such probability distributions via:

Markow Chain

$$\cdots \longrightarrow (x_{t-1}) \longrightarrow (x_t) \longrightarrow (x_{t+1}) \longrightarrow \cdots$$

"dependent sequence of RV"

+

Monte Carlo Simulation

Monte Carlo

"sampling of dependent sample"

4. **Metropolis-Hasting algorithm**: Generally applicable MCMC algorithm

Metropolis-Hastings algorithm name details

- Named after: **Nicholas Metropolis'** article (1953), some controversy regarding accreditation

- Uses the **Hastings ratios A** probability to accept/reject moves within **Markow chain**

$$A = \min \left(1, \frac{f(\theta' \mid \mathbf{y}) q(\theta \mid \theta')}{f(\theta \mid \mathbf{y}) q(\theta' \mid \theta)} \right) = \frac{f(\mathbf{y} \mid \theta') f(\theta') q(\theta \mid \theta')}{f(\mathbf{y} \mid \theta) f(\theta) q(\theta' \mid \theta)} \Bigg\} \quad \text{ind. of } f(y)$$

- ➢ No need to compute **marginal likelihood _f(y)_**
- ➢ Circumvents **_curse of dimensionality_** of numerical approximation of integral by drawing dependent samples

# Developing of MH: symmetric step

1953 **symmetric** step, based on phase transition phenomena

Equation of State Calculations by Fast Computing Machines (Nicholas Metropolis, Arianna W. Rosenbluth, Marshall Rosenbluth, Augusta H. Teller and Edward Teller)
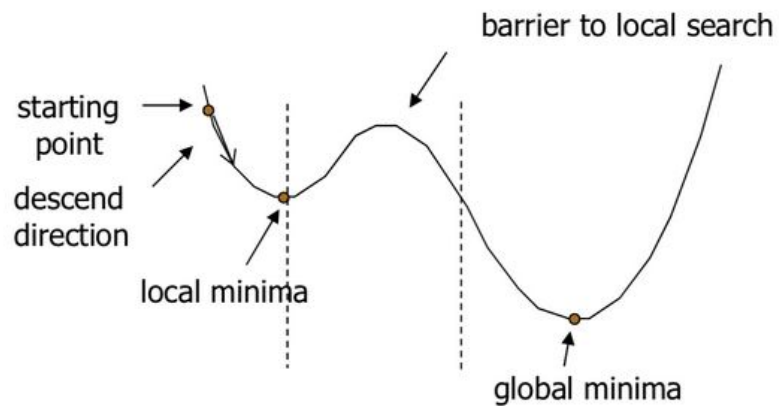
For iterations i=1,...,n:

- Select randomly a particle
- Select randomly a new place
- If is accepted (no overlap) change the place, else remain

1970 **nonsymmetric** proposal, Wilfred Keith Hastings extended it to the more general case
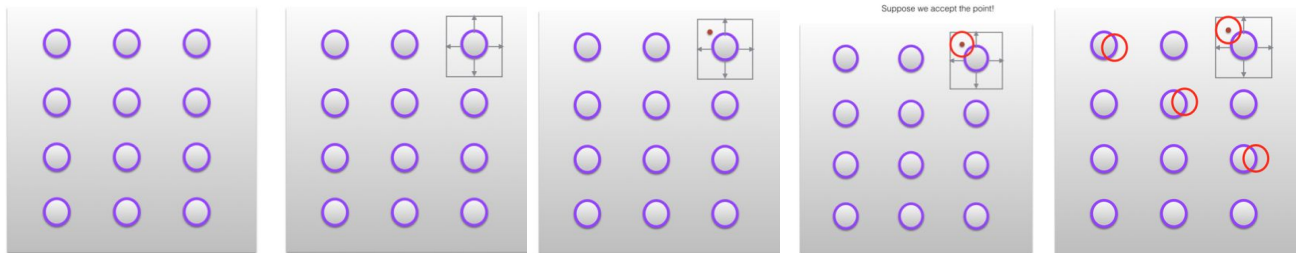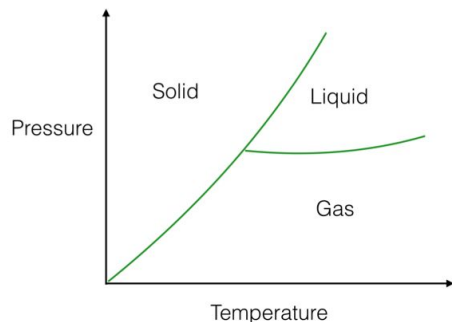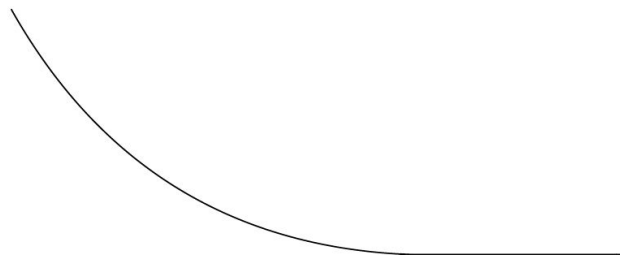
Simulated Annealing - A Metropolis-Hastings Application

Example of a problem with a local minima

Temperature Function

Simulated Annealing - Examples

Sudoku

Travelling Salesman Problem

Rubik Cube

References

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