Formulation for Two-stage Stochastic Planning Model

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Nomenclature

Power System Sets

N^e	Set of electric power buses (nodes)
Ω	Set of power generators
Ω^{gf}	Set of gas-fired power generators, $\Omega^g \subseteq \Omega$
Ω^{ngf}	Set of non-gas-fired power generators, $\Omega^{ng} \subseteq \Omega$
Γ_i	Set of generators connected to bus i
N_i^e	Set of buses connected to bus i by an edge
A^e	Set of power transmission lines

Power System Parameters

δ_0	Index of the reference bus
P_i^l	Nodal active power load at bus i
$\underline{P_i^e}, \overline{P_i^e}$	Active power generation limits of generator i
C_1^i, C_2^i	Cost coefficients of power generator i
H_i	Heat rate coefficient of gas-fired power generators
X_{ij}	Reactance of a transmission line
$\overline{F_{ij}}$	Thermal limit/capacity of a transmission line
M	A large penalty constant for Big M method

Power System Variables

$p_{ij,k}$	Active power of a transmission line in event k
$p_{j,k}^e$	Active power output of generator j in event k
$\theta_{i,k}$	Phase angle at bus i in event k
$use_{i,k}$	Unserved electricity demand of node i in event k

Natural Gas System Sets

N^g	Set of natural gas junctions (nodes)
A^g	Set of all links joining a pair of junctions
T_i	Set of gas-fired power plants connected to node i

A^p Set of base pipelines, subset of A^g

A^c Set of base compressors, subset of A^g

Natural Gas System Parameters

W_a Pipeline resistance (Weymouth) factor

$\underline{PD_{ij}}, \overline{PD_{ij}}$ Pressure drop limits from junction i to juntion j

$\underline{\pi_i}, \overline{\pi_i}$ Squared pressure limits at junction i

$\underline{\alpha^c_{ij}}, \overline{\alpha^c_{ij}}$ Compression limits squared at compressor station

D_i Firm gas consumption at junction i???

$\underline{D_i}, \overline{D_i}$ Gas consumption limits at junction i

$\underline{S_i}, \overline{S_i}$ Gas product limits at junction i

Y_i Cost coefficient of gas production at junction i

Natural Gas System Variables

$\pi_{i,k}$ Squared pressure of gas node i in event k

$x_{ij,k}$ Gas flow on pipelines and compressors in event k

$\lambda_{ij,k}$ Auxiliary relaxation variable in event k

$d_{i,k}$ Gas consumption at junction i in event k

$s_{i,k}$ Gas production at junction i in event k

$y^+_{ij,k}, y^-_{ij,k}$ Binary flow direction for links in event k

Extreme Events and Expansion Sets

$k \in K$ Set of extreme weather and climate events

Λ^t Set of transmission expansion candidates

Λ^p Set of pipeline expansion candidates

Φ^t_k Set of transmission lines impacted in event k

Φ^p_k Set of pipelines impacted in event k

Extreme Events and Expansion Parameters

β^p_{ij} Expansion cost of a pipeline

β^t_{ij} Expansion cost of a transmission line

η Penalty cost for 1 MWh of unserved energy

$ST_{ij,k}$ Binary status of an existing line in event k

$SP_{ij,k}$ Binary status of existing pipelines in event k

Extreme Events and Expansion Variables

$use_{i,k}$ Unserved electricity demand of node i in event k

$usg_{i,k}$ Unserved gas demand of junction i in event k

z^p_{ij} Binary expansion decision for pipeline candidates

z^t_{ij} Binary expansion decision for transmission line candidates

1 Model Formulation

In this section, we provided the model formulation for the two-stage stochastic optimization model of joint power and gas system. The constraints for the operation stage will hold for each scenario k within the set of extreme weather and climate events K.

1.1 Constraints for Gas System

The gas flow mass balance is preserved at gas junctions by constraint in (1).

$$\begin{aligned} & s_{i,k} - d_{i,k} + \sum_{(j,i)\in A^g} x_{ji,k} - \sum_{(i,j)\in A^g} x_{ij,k} \\ & + \sum_{(j,i)\in \Lambda^p} x_{ji,k} - \sum_{(i,j)\in \Lambda^p} x_{ij,k} = 0 \qquad \forall i \in N^g \end{aligned} \tag{1}$$

Natural gas flows in either direction in pipelines and compressors. The binary flow direction variable is defined as 1 when gas flows from junction i to j. Gas flow variables can be positive or negative depending on whether gas flows from junction i to j or j to i.

$$y^+_{ij,k} + y^-_{ij,k} = SP_{ij,k} \qquad \forall (i,j) \in A^p \tag{2}$$

$$y^+_{ij,k} + y^-_{ij,k} = z^p_{ij} \qquad \forall (i,j) \in \Lambda^p \tag{3}$$

$$y^+_{ij,k} + y^-_{ij,k} = 1 \qquad \forall (i,j) \in A^c \tag{4}$$

The sign of gas flow variable should align with the binary flow direction in pipelines. Equations (5) and (6) represent those constraints that apply to both existing pipelines and new pipelines. The pressure drop across pipelines are consistent with the sign of the flow as in equation (7) and (8) for both existing pipeline set and pipeline candidate set.

$$-(1 - y^+_{ij,k}) \sum_{i\in N^g} \overline{S_i} \le x_{ij,k} \tag{5}$$

$$x_{ij,k} \le (1 - y^-_{ij,k}) \sum_{i\in N^g} \overline{S_i} \tag{6}$$

$$(1 - y^+_{ij,k})(\underline{\pi_i} - \overline{\pi_i}) \le \pi_{i,k} - \pi_{j,k} \tag{7}$$

$$\pi_{i,k} - \pi_{j,k} \le (1 - y^-_{ij,k})(\underline{\pi_i} - \overline{\pi_i}) \tag{8}$$

Weymouth equation ensures that the gas flow in a pipeline satisfy the physical constraints in fluid mechanics. Because the Weymouth equation is not convex, we use bilinear relaxations introduced by McCormick and the second-order cone relaxation which is usually tight for expansion planning problems. The relaxation variable $\lambda_{ij,k}$ is in $\langle y^+_{ij,k} - y^-_{ij,k}, \pi_{i,k} - \pi_{j,k}\rangle$, which are represented by equations (9)-(12). The relaxation applies to both existing pipelines and pipeline candidates.

$$\lambda_{ij,k} \ge \pi_{j,k} - \pi_{i,k} + (\underline{\pi_i} - \overline{\pi_j})(y^+_{ij,k} - y^-_{ij,k} + 1) \tag{9}$$

$$\lambda_{ij,k} \ge \pi_{i,k} - \pi_{j,k} + (\overline{\pi_i} - \underline{\pi_j})(y^+_{ij,k} - y^-_{ij,k} - 1) \tag{10}$$

$$\lambda_{ij,k} \le \pi_{j,k} - \pi_{i,k} + (\overline{\pi_i} - \underline{\pi_j})(y^+_{ij,k} - y^-_{ij,k} + 1) \tag{11}$$

$$\lambda_{ij,k} \le \pi_{i,k} - \pi_{j,k} + (\underline{\pi_i} - \overline{\pi_j})(y^+_{ij,k} - y^-_{ij,k} - 1) \tag{12}$$

$$SP_{ij,k}\lambda_{ij,k} \ge W_{ij} x^2_{ij,k} \qquad \forall (i,j) \in A_p \tag{13}$$

$$z^p_{ij,k}\lambda_{ij,k} \ge W_{ij} x^2_{ij,k} \qquad \forall (i,j) \in \Lambda_p \tag{14}$$

The compressors allow gas flow in two directions. We assume that the gas flow get compressed flowing in one direction through the compressors and does not get compressed in the other direction. Compressors' compression limits are ensured by Eqs.(15)-(18).

$$\pi_{j,k} - \overline{\alpha^c_{ij}}\pi_{i,k} \le (1 - y^+_{ij,k})\overline{\pi_j} \tag{15}$$

$$\underline{\alpha^c_{ij}}\pi_{i,k} - \pi_{j,k} \le (1 - y^+_{ij,k})\overline{\pi_i} \tag{16}$$

$$\pi_{i,k} - \pi_{j,k} \le (1 - y^-_{ij,k})\overline{\pi_i} \tag{17}$$

$$\pi_{j,k} - \pi_{i,k} \le (1 - y^-_{ij,k})\overline{\pi_j} \tag{18}$$

Additional constrains (19) are supposed to ensure no gas flows through the pipeline if that candidate pipeline is not built.

$$-z_{ij}^p \sum_{i \in N^g} \overline{S_i} \leq x_{ij,k} \leq z_{ij}^p \sum_{i \in N^g} \overline{S_i} \qquad \forall (i,j) \in \Lambda^p \tag{19}$$

Consumption, production and pressure should fall within the limits for each gas junction. For junctions with fixed gas consumption, the difference between fixed gas consumption and the amount of gas demand that is met is the quantity of unserved gas demand as in Eq.(20). For junctions with flexible gas consumption, which are junctions connected to gas-fired power plants, the variable gas consumption is between the nodal gas consumption bounds as in Eq.(21).

$$\underline{D_i} \leq d_{i,k} \leq \overline{D_i} \tag{20}$$

$$d_{i,k} + usg_{i,k} = D_i \tag{21}$$

$$\underline{\pi_i} \leq \pi_{i,k} \leq \overline{\pi_i} \tag{22}$$

$$\underline{S_i} \leq s_{i,k} \leq \overline{S_i} \tag{23}$$

1.2 Constraints for Power System

The power system is represented by a DC power flow model which follows Kirchhoff's law and Ohm's law. Eq.(24) enforces that the total power flow into a node equals the total power flow out of the node for all power nodes. The electricity demand that is not satisfied is represented by the unserved electricity to balance the nodal flow. Eqs.(25)(26) and Eqs.(27)(28) impose Ohm's law on existing transmission lines and transmission line candidates respectively. Ohm's law is implemented via the Big M method and we use 10^6 as the large penalty constant.

$$\sum_{j \in \Gamma_i} p_{j,k}^e = \sum_{(i,j) \in A^e} p_{ij,k} + P_i^l - use_{i,k} + \sum_{(i,j) \in \Lambda^t} p_{ij,k} \tag{24}$$

$$-(1 - ST_{ij,k})M \leq p_{ij,k} - \frac{\theta_{i,k} - \theta_{j,k}}{X_{ij}} \tag{25}$$

$$p_{ij,k} - \frac{\theta_{i,k} - \theta_{j,k}}{X_{ij}} \leq (1 - ST_{ij,k})M \tag{26}$$

$$-(1 - z_{ij}^t)M \leq p_{ij,k} - \frac{\theta_{i,k} - \theta_{j,k}}{X_{ij}} \tag{27}$$

$$p_{ij,k} - \frac{\theta_{i,k} - \theta_{j,k}}{X_{ij}} \leq (1 - z_{ij}^2)M \tag{28}$$

Eqs. (29)-(32) describe the bounds on the power system variables. Eq.(30) ensures that no power flows through a transmission line if the transmission line is down during the extreme event k. Eq.(31) ensures that there is no power flowing through a new transmission line if it is not being built. Eq.(33) sets the phase angle of a slack bus to be 0 as a reference.

$$\underline{P_i^e} \leq p_{i,k}^e \leq \overline{P_i^e} \qquad \forall i \in \Omega \tag{29}$$

$$-ST_{ij,k}V_{ij} \leq p_{ij,k} \leq ST_{ij,k}V_{ij} \qquad \forall (i,j) \in A^e \tag{30}$$

$$-z_{ij}^t S_{ij} \leq p_{ij,k} \leq z_{ij}^t S_{ij} \qquad \forall (i,j) \in \Lambda^t \tag{31}$$

$$0 \leq use_{i,k} \leq P_i^l \qquad \forall i \in N^e \tag{32}$$

$$\theta_{\delta_0,k} = 0 \qquad \forall k \in K \tag{33}$$

1.3 Coupling Constraint between Power System and Gas System

The gas system and electricity system are connected because gas system supplies fuel to gas-fired power plants. The connection is reflected in the formulation by the constraints of heat-rate coefficients of gas-fired power plants that specifies their efficiency of consuming gas to produce electricity. The equality in heat-rate constraint results in non-convexity. Therefore, we relax the equality condition of gas consumption by power plants into an inequality.

$$d_{i,k} = \sum_{j \in \Gamma_i} H_j \times p^e_{j,k} \qquad \forall i \in N^g \tag{34}$$

1.4 Extreme Weather and Climate Events

The binary status variables represent whether components fail during an extreme weather and climate event. The status of energy system components in a contingency scenario is determined by the location of system component and the EWCE event impacted region. If the component is within the region impacted by EWCE, the pipelines or transmission lines fail and will not allow power or gas flow to go through. Otherwise, the status parameter equals one representing that the component fully functions. We assume that the pipeline and transmission line candidates are designed to be robust and resilient to both geographically correlated failures and uncorrelated failures. They will always function in the operation stage if they are chosen to be built in the investment stage.

$$ST_{ij,k} = 0 \qquad if \forall (i,j) \in \phi^t_k, \forall k \in K \tag{35}$$

$$SP_{ij,k} = 0 \qquad if \forall (i,j) \in \phi^p_k, \forall k \in K \tag{36}$$

1.5 Objective Function

Objective function for the stochastic optimization model is stated in Eq.(37). It composes the expected operating cost and expected penalty cost for unserved energy of all possible scenarios in the operation stage and the expansion cost in the investment stage.

$$\begin{aligned} min \sum_{k \in K} \Bigg[& \sum_{i \in \Omega^{ngf}} (C^i_1 p^e_{i,k} + C^i_2 {p^e_{i,k}}^2) + \sum_{i \in N^g} Y_i s_{i,k} \\ & + \sum_{i \in N^e} 24\eta \times use_{i,k} + \sum_{i \in N^g} 86400\eta \times usg_{i,k} \Bigg] \\ & + \sum_{(i,j) \in \Lambda^t} \beta^t_{ij} z^t_{ij} + \sum_{(i,j) \in \Lambda^p} \beta^p_{ij} z_{ij^p} \end{aligned} \tag{37}$$