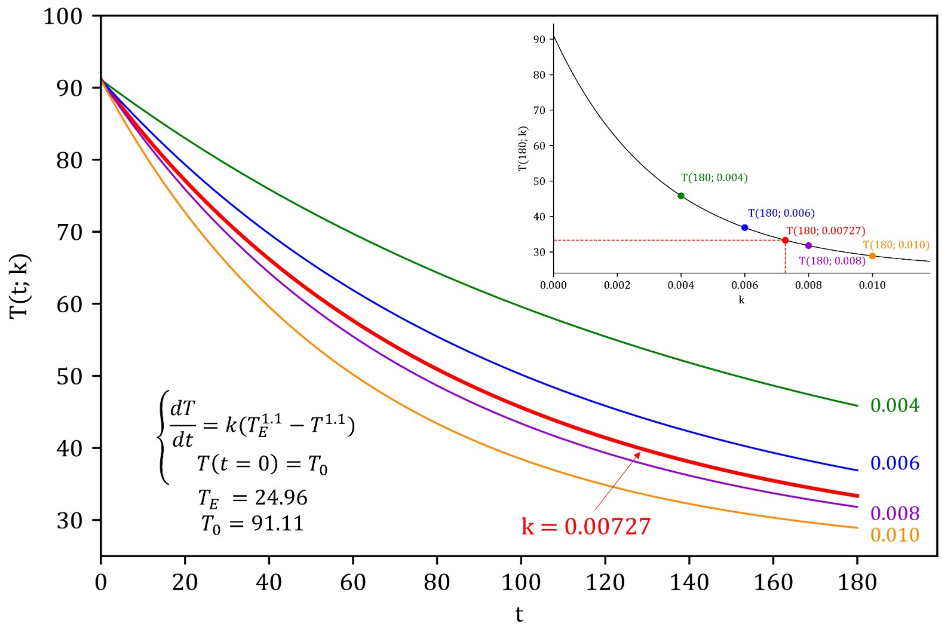
In a refrigerated warehouse that's kept at a constant temperature of $T_E = 24.96F$, a corpse was found at midnight with a temperature of $T(t = 0) = T_0 = 91.11F$. Three hours (180 minutes) later, its temperature dropped to $T(t = 180) = 33.33F$. At the time of death, the person's body temperature was at the normal 97.88F. Write a program to estimate when the person died?

The corpse's temperature changes according to the modified Newton's Law of Cooling invented by I. Newton in 1701:

$$\begin{cases} \dfrac{dT}{dt} = k(T_E^{1.1} - T^{1.1}) \\ T(t=0) = T_0 \end{cases}$$

where T_E is the temperature of the warehouse, $T(t)$ is the temperature of the corpse at time t, and $T(t = 0) = T_0$ is the initial temperature of the corpse when it's found.

Solution: The key issue of this problem is that of computing the cooling constant k. The problem is designed to allow you a taste of the big fever of machine learning. How to determine cooling constant k?



Example 8C:

In a warehouse of an initial temperature $T_{w0} = 33F$ in a hot area of constant temperature $T_e = 111F$, a corpse of temperature of $T_0 = 98F$ is found.

The warehouse's cooling system broke at this very moment, and 240 minutes later, the temperatures of the warehouse and the corpse change to 55F and 49F, respectively. Naturally, the corpse drops (and the warehouse raises) the temperature by 98-55= 43F (49-33=16F for warehouse).

It is reasonable to assume the warehouse is big enough that its temperature is not affected by the corpse's temperature change but that of the corpse does. We also make another assumption that T_w

changes so slowly that the Newton's heat equation applies, approximately, although the equation requires constant ambient temperature.

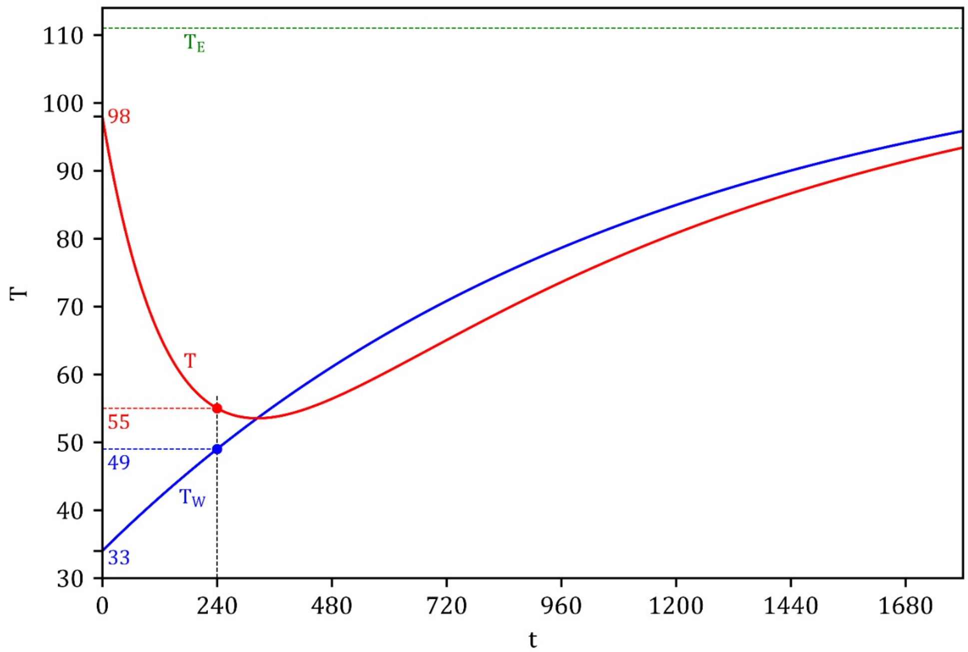
The warehouse's temperature follows

$$\begin{cases} \dfrac{dT_w}{dt} = k_w(T_e - T_w) \\ T(t=0) = T_{w0} \end{cases}$$

The corpse's temperature follows

$$\begin{cases} \dfrac{dT}{dt} = k(T_w - T) \\ T(t=0) = T_0 \end{cases}$$

Please compute their temperatures from $t = 0$ to 1680.



In this figure created by Andrew Bae (2021), the red (blue) curve shows the temperature of corpse T (warehouse T_w) varying with time. Three important observations: (1) T_w monotonically increases while T decreases and then, when approaching T_w, start to increase; (2) The corpse T overshoots below T_w, a defect resulting from using the Newton's eq approximately; (3) The T and T_w curves will overlap as time increases; (4) The T and T_w will both approach T_E when time is sufficiently large, as expected.