Method.5 Runge-Kutta Methods

A long story on this method and its many variants! But the concept is quite remarkable, and this is also what leads to the Yoshida algorithms.

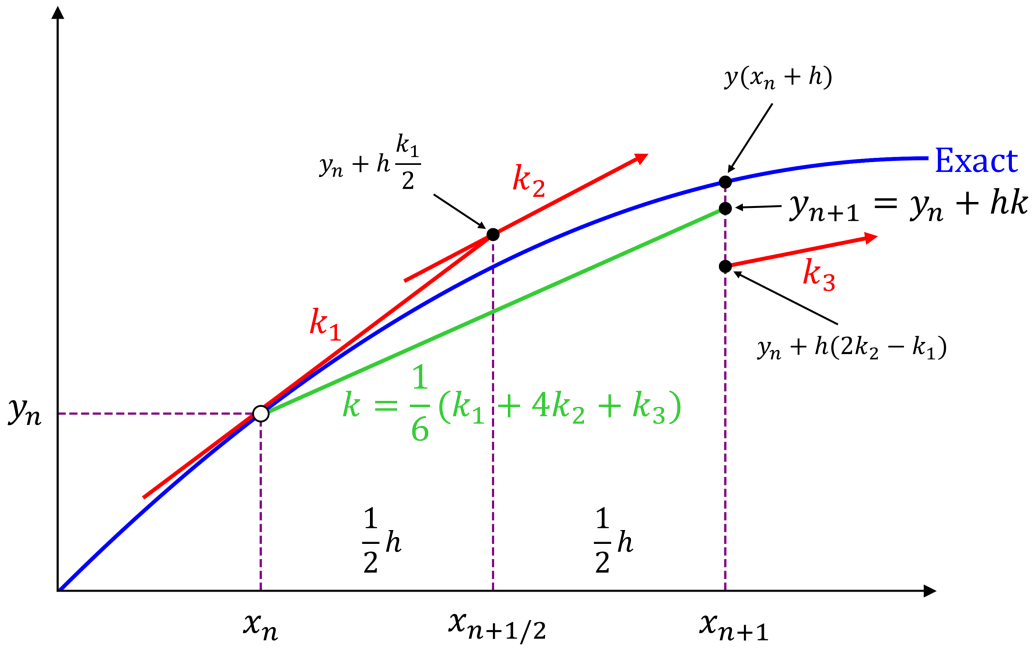

Figure 27. RK3.

RK3 method:

$$y_{n+1} = y_n + h\left(\frac{k_1}{6} + \frac{4k_2}{6} + \frac{k_3}{6}\right)$$

where

$$k_1 = f(x_n, y_n)$$
$$k_2 = f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}k_1\right)$$
$$k_3 = f(x_n + h, y_n - hk_1 + 2hk_2)$$

The average slope is,

$$k = h\left(\frac{k_1}{6} + \frac{4k_2}{6} + \frac{k_3}{6}\right)$$

In fact, we can derive this formula from Taylor series.

To start, we must set the following,

$$y_{n+1} = y_n + h(a_1k_1 + a_2k_2 + a_3k_3)$$