Method.2 The backward Euler method

Backward differentiation:

$$y'(x) \approx \frac{y(x) - y(x-h)}{h}$$
$$y(x) \approx y(x-h) + h * y'(x)$$

i.e.,

$$y_{n+1} = y_n + h\, f(x_{n+1}, y_{n+1})$$

What does this mean? It means the "next step" depends on the past y_n and slope of your current state y_{n+1} that you do not have yet...

Note: The current state depends on the past AND the current state!

This Backward Euler method becomes implicit, meaning that we have to solve an equation to find the current state y_n, for each step. It's definitely more costly and we have something to gain, too.

The following is an illustration of the algorithm:

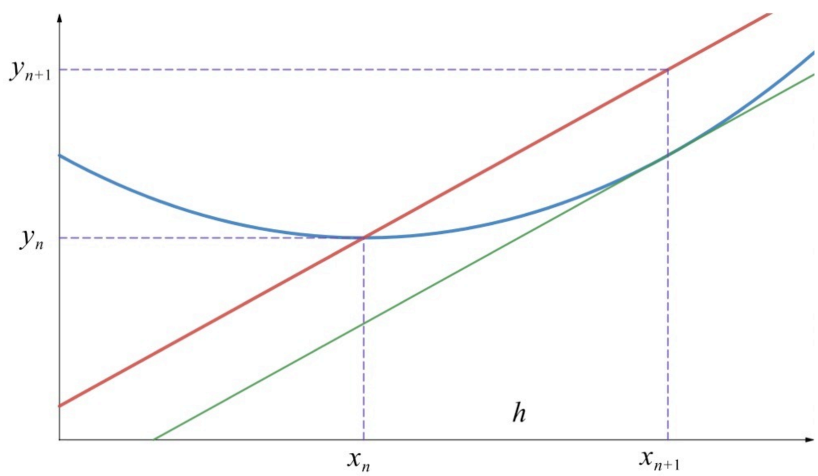

Figure 8. The backward Euler method.

Example 3: Using backward Euler method to solve the following IVP.

$$\begin{cases} y' = x + y + xy \\ y(0) = 1 \end{cases}$$

Solution:

As shown before, although this DE is first order linear, we can't find a closed-form solution. Let's outline a few steps for the solution:

Let's set $h = 0.1$ and we know when $x_0 = 0$, $y_0 = 1$. Solve the algebraic equation in Column-3 to generate results in Column-4. I did the following table manually.

	$x_0 = 0.0$		$y_0 = 1$