Example 8: In a refrigerated warehouse that's kept at a constant temperature of 34.44F, a corpse was found at midnight with a temperature of 91.11F. Three hours (180 minutes) later, its temperature dropped to 55.55F. At the time of death, the person's body temperature was at the normal 97.88F.

Write a program to estimate when the person died?

Also, compute the time when the corpse's temperature will reach 44.44F. The corpse's temperature changes according to the modified Newton's Law of Cooling invented by I. Newton in 1701:

$$\begin{cases} \dfrac{dT}{dt} = k(A - T^{0.999}) \\ T(t=0) = T_0 \end{cases}$$

where A is the temperature of the warehouse, $T(t)$ is the temperature of the corpse at time t, and $T(t=0) = T_0$ is the initial temperature of the corpse (the instance when the person died.)

Solution: (I wish to use this example to show some "machine learning"):

A few important parameters for this IVP:

The situation has the following sequence of events:

(1) Temperature of warehouse, which does not change with time (constant). Well, I can change this problem to make it happen in a closed apartment room (scary!)

$$A = 34.44\ F$$

(2) Temperature of live body at the time $t = 0$:

$$T(t=0) = 97.88\ F$$

(3) Temperature of cadaver at the time t_x it was first found (unsure how long since death):

$$T(t_x) = 91.11\ F$$

(4) Temperature of cadaver after 3 more hours at the time $t_x + 180$ since death:

$$T(t_x + 180) = 55.55\ F$$

The key issue of this problem is that of computing the cooling constant k. The problem is designed to allow you a taste of the big fever of machine learning. How to determine cooling constant k? As soon as we find "k", the rest is trivial.

In the IVP:

$$\begin{cases} \dfrac{dT}{dt} = k(A - T^{0.999}) \\ T(t=0) = T_0 \end{cases}$$

Solution 1: Shooting method to "shoot" for k.
(By the way, we will learn shotting method for BVP in a different setting).

This method "the most common, but most ugly"

How does it work?

Using Euler's method or Runge-Kutta method to solve the given IVP with

$$T(t=0) = T_0 = 91.11$$
$$T(t=180) = 55.55$$
$$A = 34.44$$

What's missing here is the cooling constant k.

The best option is to "guess" a value of "k" and then solve the IVP (numerically) and then repeat a few 100s or 1000s times to, hopefully, achieve (shoot for) the ending temperature $T_{180} = 55.55$F.

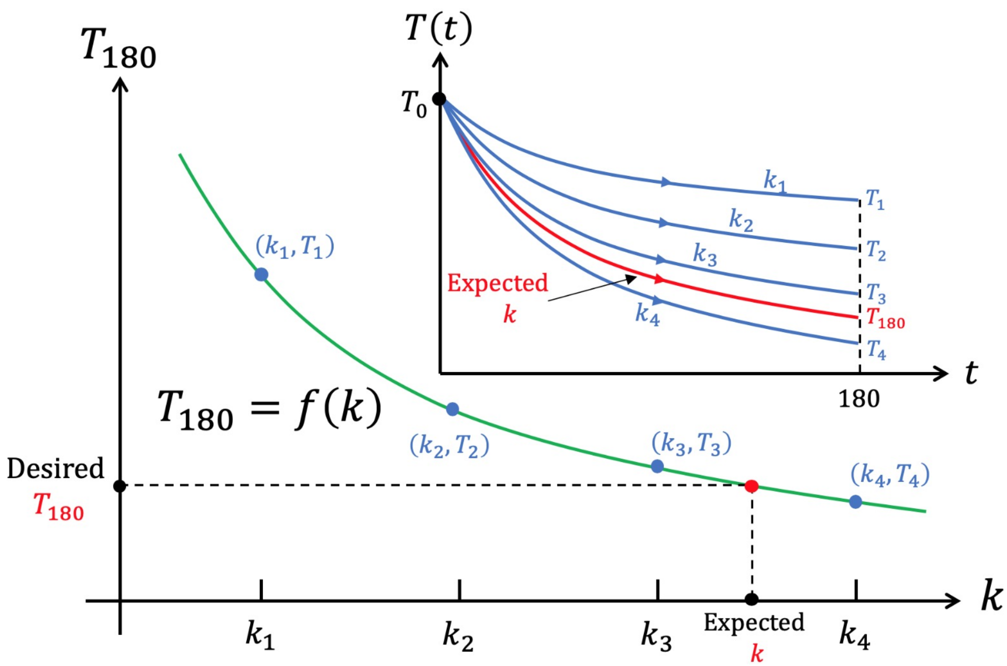

Figure 38. Shooting method to obtain a parameter in a DE.

I made a few experiments:

For $k = 0.010, 0.008, 0.006, 0.04$