Discretization and numerical schemes (Explicit)

Everything starts from Taylor series!

Considering a system of two dependent variables or the following Initial Valued System (IVS):

$$\begin{cases} \dfrac{dx}{dt} = X(x, y) \\ \dfrac{dy}{dt} = Y(x, y) \\ x(t=0) = x_0 \\ y(t=0) = y_0 \end{cases}$$

One of the most common examples is the **Predator–prey system:**

> Introduction: As an example of systems, the Lotka–Volterra (L-V) equations are a pair of first-order, non-linear, ODEs frequently used to describe the dynamics of biological systems in which two species interact, one a predator and one its prey. They evolve in time according to the pair of equations which was, independently in 1910, introduced by two American mathematicians:
>
> $$\begin{cases} \dfrac{dx}{dt} = x(\alpha_x - \beta_x y) \\ \dfrac{dy}{dt} = -y(\alpha_y - \beta_y x) \end{cases}$$

Let's use this example to illustrate several schemes to solve Systems (not a single DE).

Scheme 1:

Given x_0 and y_0, start iterations.

$$\begin{cases} x_{n+1} = x_n + \underbrace{x_n(\alpha_x - \beta_x y_n)}_{x_n'} \Delta t \\ y_{n+1} = y_n \underbrace{- y_n(\alpha_y - \beta_y x_n)}_{y_n'} \Delta t \end{cases}$$

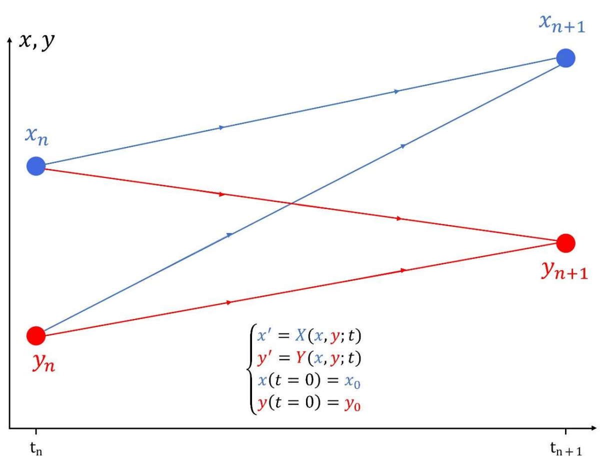

Figure 1. Synchronized advancing, explicitly.