Topic 4D: Eigenvalues (e-values) and eigenvectors (e-vectors)

If A is a square matrix, a non-zero vector $\boldsymbol{v}$ is an eigenvector of A if there is a scalar λ such that

$$Av = \lambda v$$

The λ is the eigenvalue of A corresponding to eigenvector $\boldsymbol{v}$. An eigenspace of A is the set of all eigenvectors with the same eigenvalue.

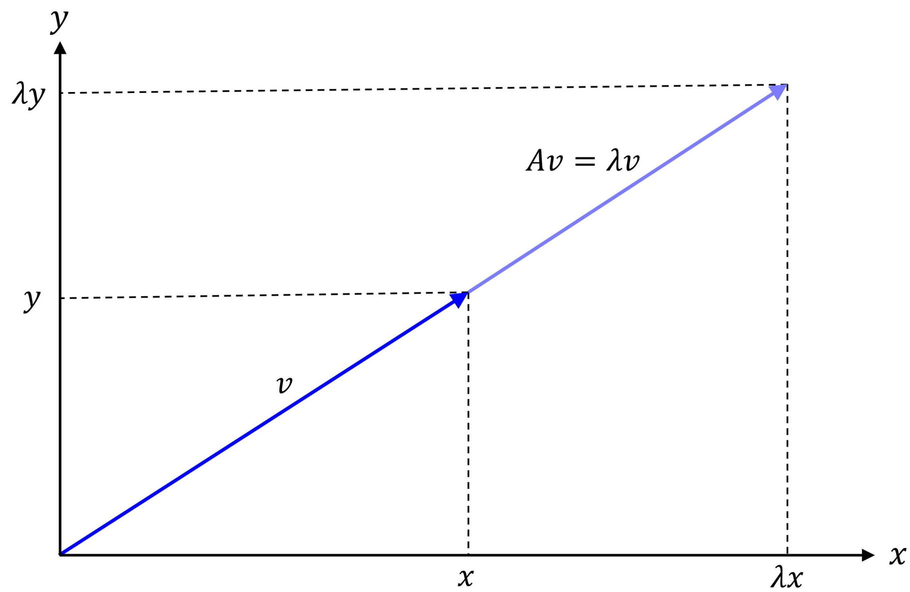

Figure 16. Definitions of eigenvectors and eigenvalues.

Eigenvalues can be defined by the following characteristic equation (the LHS=characteristic polynomial):

$$\det(A - \lambda I) = 0$$

Example 13: Compute e-values of

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Solution:

The characteristic equation (or eigen equation) of this matrix reads

$$\det(A - \lambda I) = \det \begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix} = 0$$

whose roots are $\lambda = 1$ and $\lambda = 3$, i.e., the e-values

$$\lambda_{1,2} = 1, 3$$

In general, the characteristic equation of a nxn matrix is

$$\det(A - \lambda I) = \lambda^n + c_{n-1}\lambda^{n-1} + \cdots + c_0 = 0$$

One must be quite careful in computing e-values: