

Popular numerical methods (chosen from a few dozens of methods and their variants).

Method.1 The Forward Euler method (introduced in 1768):

The basic idea: From any point on a curve, you can find an approximation of a nearby point on the curve by moving a short distance along a line [tangent](#) to the curve.

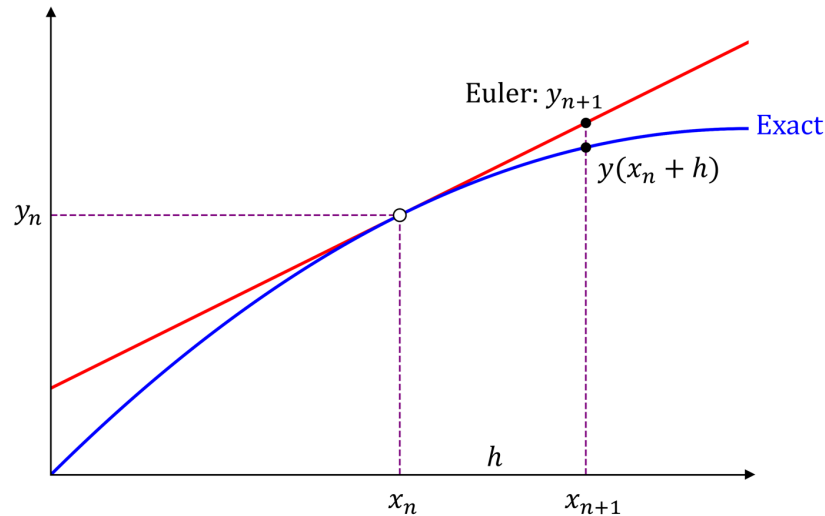


Figure 3. The forward Euler method.

Starting with the differential equation, we replace the derivative y' by the forward difference approximation,

$$y'(x) \approx \frac{y(x+h) - y(x)}{h}$$

Easily, we can write it as

$$y(x+h) \approx y(x) + hy'(x)$$

Since we know (from the original DE) $y' = f(x, y(x))$, we get,

$$y(x+h) \approx y(x) + hf(x, y(x))$$

With a step size h , we construct the sequence $x_0, x_1 = x_0 + h, x_2 = x_0 + 2h, \dots$ and the numerical solutions,

$$y_{n+1} = y_n + hf(x_n, y_n)$$

Note: The current state depends on the past and the past only!