Sub-Saharan Violence and Human Development

William G. Foote

Manhattan College
(mailto:#)wfoote01@manhattan.edu (mailto:wfoote01@manhattan.edu)

2021-10-13

Abstract

The intermittent very high and very low counts of political violence can severely bias inference about the influence of violence on human development. The paucity and potential unreliability of reporting of national level data further occludes analysis. We use the generalized Pareto distribution within a multi-level model of the joint distribution of country experiences with political violence to account for extreme bursts of violence and to begin to address data quality. Provisional measures of transnational violence become immediately available from this analysis. While levels of casualties widely affects GDP per capita and years of life expected, it is the interaction of development measures that impounds the influence of political violence.

1 Some Problems with the Analysis of Violence

The intensity and sporadic character of acts of political violence interrupts the stability of human development. Krug et al. (2002) documents the many health and societal effects of individual and collective violence. Among its many findings the UN75 Office reports (UN75-Office 2021) that reducing conflict is a high priority among respondents in Eastern and South-eastern Asia, Northern Africa and Western Asia, and Sub-Saharan Africa. The 2010 global burden of disease report by Rafael Lozano (2012) documents the strong, rippling effect of violence on years of life lost, especially in tropical countries, most notably Sub-Saharan Africa.

We thus focus in this preliminary study on a subset of the 49 countries of Sub-Saharan Africa for two reasons. First, a disproportionately large number of incidents of, and severity stemming from political violence plagues this region. Second, the region historically has been the donor of large masses of commodity wealth fueling in part their growth, but certainly accelerating the growth of the recipient nations around the world. These two drivers of growth and violence coexist in a divergent milieu of human beings, many of whose lives are arrested and detained by intense forces sporadically occurring.

Seminal work by Richardson (1944) dissected such intense forces including interstate conflict using the Poisson distribution for incidences of war and a power law distribution for severity, measured in battle dead. Clauset (2020) validated Richardson's approach with a much expanded data set. Juan Camilo Bohorquez (2009) explains high-frequency, intra-conflict behavior across human insurgencies, again with power law dynamics, outfitted with epoch detection of some major switch points in human behavior. The use of power laws and their relaively thick distributional tails highlights the stylized fact of sporadic and intense acts of violence.

Regarding extractive resources, Merrill and Orlando (2020) finds that oil producing nations actually increase production, thus increase external payments, with rising political violence. Bandyopadhyay, Sandler, and Younas (2013) show that political violence depresses foreign direct investment, an engine of development, which can be mitigated by bilateral aid. On another research trajectory, Marc Helbling (2020) find that a greater exposure to transnational terrorism is associated with stricter migration controls. Tobias Böhmelt (2020) provide evidence for the weakening of terrorism diffusion cross borders when host and migrant cultures diverge. None of these studies employ the modeling of political violence with power law distributions.

This paper builds on work which attempts to characterize the impact of political violence in several ways to answer a richer set of initial questions ranging from Richardson's frequency and severity to Bohorquez' timing and extent of violence on to further questions about the spatial distribution of violence. This last desideratum focuses on the transnational character of violence. The cited studies also move the horizon from the acts and resulting initial mortality to a conditioning of some the major features of economic development including the components of the human development, colonization, foreign direct investment, and, as flawed a measure as it is, gross domestic product (GDP).[1]

In a completely different context we can profit from the insights of Belikov (2017) who studied over 20 million cancer cases, a very different form of violence on the person and in society. The classical Gamma probability distribution function has shape and scale parameters which measure the average number of key events to develop to a stage of an outbreak of cancerous incidents. The scale parameter can indicate the average time interval between such events.

Mixing the Gamma function with the exponential function results in a power law distribution called the Generalized Pareto Distribution (Johnson and Kotz (1970)). The location parameter in the GPD can help us measure the threshold beyond which excess violence probably erupts. Computing the joint probability of parameters across each country and type of political violence curses this study with a dizzying number of parameters.[2] If we were to further condition violent events by socio-economic indicators, notably gross domestic product (GDP), foreign direct investment (FDI), and the years of life expectancy as constitutive of human development, more parameters abound. Subsets of countries can alleviate this brand of the curse of dimensionality to some extent.

This study will attempt to perform the following tasks:

1. Provide a rationale for the use of the GPD and its interpretation;[3]
2. Fit the parameters of the GPD across just 5 candidate sub-Saharan countries chosen for their diversity of violent incidents and markers of human development using a Bayesian inference framework;[^to-bayes]
3. Transform the GPD parameters into Gamma parameters to understand more concretely the timing and threshold levels of violence in each country;[^belikov]
4. Condition violent incidents with jointly occurring measures of human development;
5. Measure the co-relationships of conditioned violent incidents among the sample countries.

The paper concludes with caveats and a short discussion.

[^to-bayes]; Bayesian methods have become widely used in machine learning and research regarding human development. So-called traditional frequentist statistics frames probability as the limit of relative frequencies of events as the number of experimental trials increases, while assuming a fixed set of distribution parameters. Bayesian inference adapts a model's parameters to observed outcomes and can update the model as more data becomes available. So, while the former approach is well suited for testing of a priori formulated hypotheses, the latter approach, with its evolving model estimations can adapt a model to what is known and improve it by means of what becomes known over time.

2 Violent Episodes by Country

This study utilizes datasets, one comprised of incidents and casualties of political violence and the other composed of some component indices of human development.

The Global Terrorism Data Base (GTD) contains over 191,000 events of political violence by country, region, province, and city, from 1970 through 2018. The GTD further categorizes the number killed and wounded by type of attack, targets of attack, perpetrators, and mode of operation, for example, insurgency and civil war. This study reviews events only in several sub-Saharan Africa countries from 1990 through 2018. We will review countries that are contiguous geographically, exhibit varying degrees of violence, and inhabit areas of recent high demand for natural resources, that is, Cameroon, Central African Republic, Chad, Ghana, Niger, and Nigeria.

We use annual data from the United Nations which publishes an annual composite index measuring average achievement in three basic dimensions of human development: years of life, measured in years of life expectancy,[4] knowledge, measured in years of schooling,[5] and a decent standard of living, measured in gross national income per capita.[6][7] In addition, included is foreign direct investment inflows.[8]

Table 2.1: Part I: Incidents and Casualties, 1990-2018

country	incidents	casualties	q_0.05	median	q_0.90
Swaziland	11	3	0.00	0.0	0.0
Comoros	7	4	0.00	1.0	1.0
Lesotho	6	4	0.00	0.0	2.0
Equatorial Guinea	2	5	1.15	2.5	3.7
Botswana	2	7	1.25	3.5	5.5
Gabon	7	7	0.00	0.0	2.8
Benin	8	8	0.00	0.0	3.3
Gambia	3	15	0.20	2.0	10.8
Malawi	7	34	0.00	1.0	13.2
Ghana	20	39	0.00	1.0	4.3
Guinea-Bissau	9	44	0.40	3.0	8.6
Zambia	27	54	0.00	0.0	3.4
Mauritania	15	69	0.00	1.0	8.8
Togo	45	107	0.00	0.0	6.0

country	incidents	casualties	q_0.05	median	q_0.90
Eritrea	10	108	0.00	8.0	23.5
Namibia	32	163	0.00	1.5	8.8
Zimbabwe	42	189	0.00	0.0	8.0
Liberia	34	213	0.00	0.5	14.1
Madagascar	27	216	0.00	0.0	26.6
Republic of the Congo	36	236	0.00	2.0	14.5
Guinea	25	270	0.00	1.0	40.8
Tanzania	59	314	0.00	1.0	9.4
Djibouti	17	377	0.00	7.0	65.2
Ivory Coast	76	441	0.00	1.5	12.0
Zaire	41	502	0.00	2.0	39.0

Table 2.1: Part II: Incidents and Casualties, 1990-2018

country	incidents	casualties	q_0.05	median	q_0.90
Zaire	41	502	0	2	39.0
Burkina Faso	121	599	0	1	8.0
Senegal	117	664	0	3	14.4
Sierra Leone	101	962	0	2	25.0
Mozambique	294	1576	0	2	14.4
Niger	171	1919	0	5	27.0
Ethiopia	162	2605	0	4	35.0
Chad	99	2899	0	5	52.6
Uganda	322	3459	0	4	25.9
Central African Republic	327	3535	0	3	30.0
Mali	732	3555	0	2	12.0
Cameroon	583	3951	0	2	15.0
Angola	434	3980	0	0	21.7
South Sudan	264	4102	0	2	43.7
South Africa	1016	4157	0	1	9.0
Rwanda	164	4179	0	6	36.0
Sudan	971	5787	0	1	15.0
Democratic Republic of the Congo	938	6702	0	2	16.0

country	incidents	casualties	q_0.05	median	q_0.90
Burundi	623	6775	0	3	27.0
Kenya	722	8120	0	2	11.0
Somalia	4656	21090	0	1	10.0
Nigeria	4549	36491	0	2	20.0
NA	NA	NA	NA	NA	NA
NA	NA	NA	NA	NA	NA
NA	NA	NA	NA	NA	NA

As much as this table reveals the high level of violence across decades, but it also hides the intensity of violence across relative short periods of time. The pattern seems to favor a threshold of violent events and casualties, persons killed or wounded, far below the relatively infrequent extreme levels of violence many of the people in these countries have experienced. Arithmetic means and standard deviations do not accurately depict the range and extent of violent activity. Thus we use quantiles alone to tell the story in a robust manner.

In Figure \ref{fig:plot-inc) we plot incidents alone. In some cases, there were no casualties reported.

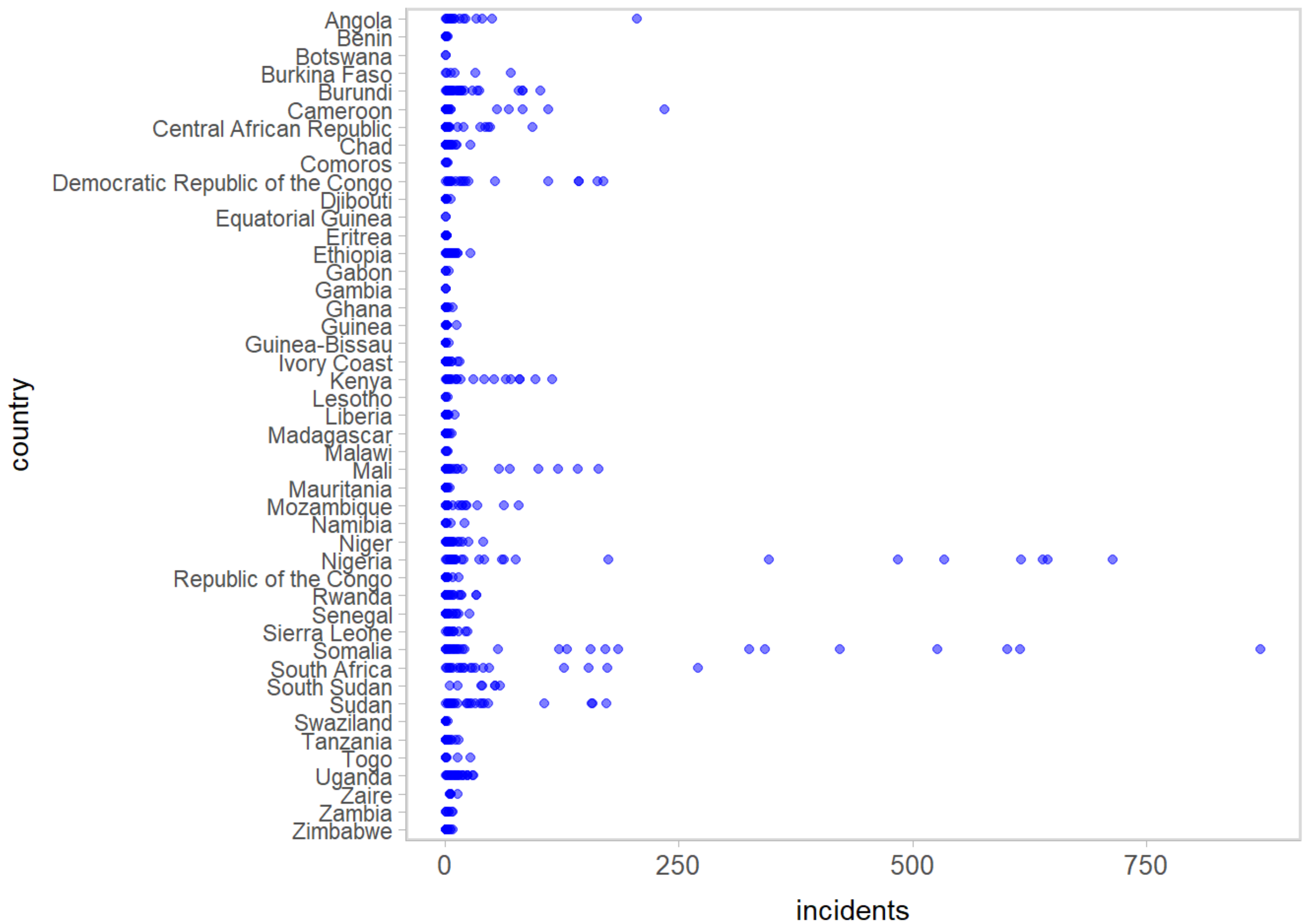


Figure 2.1: Incidents of Political Violence: Sub-Saharan Africa 1990-2018

The frequency of events of political violence exhibits a *bursty* pattern. A notable threshold number of events seems to stretch into the lower 10%ile,1, with median, 4, reaching an upper 90%tile, 58. Across countries, the pattern is highly variable with some very low levels of incidence, for example, Benin, and others, for example Somalia, with very high rates across this time period.

Figure plots the raw number of casualties across Sub-Saharan Africa.

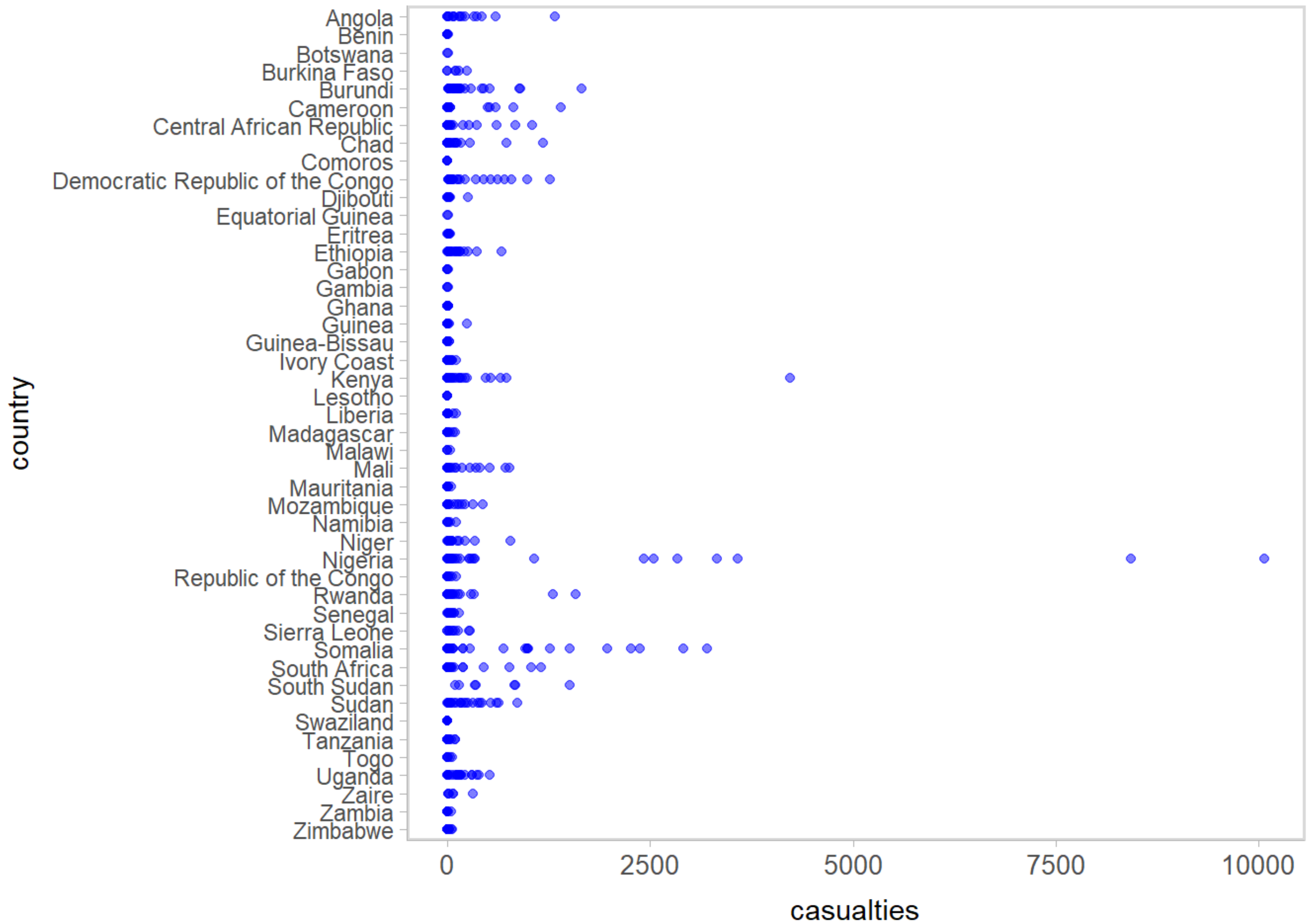


Figure 2.2: Number of Killed or Wounded by Political Violence: Sub-Saharan Africa 1990-2018

The number of persons killed or wounded again exhibits a similar pattern similar to incidents across countries and within countries. A notable threshold number of events also seems to stretch into the lower 10%ile, \(n=\), with median, \(n=\), reaching an upper 90%tile, \(n=\).

Figure \ref{fig:plot-nkw-inc) further examines casualties scaled by incidents.

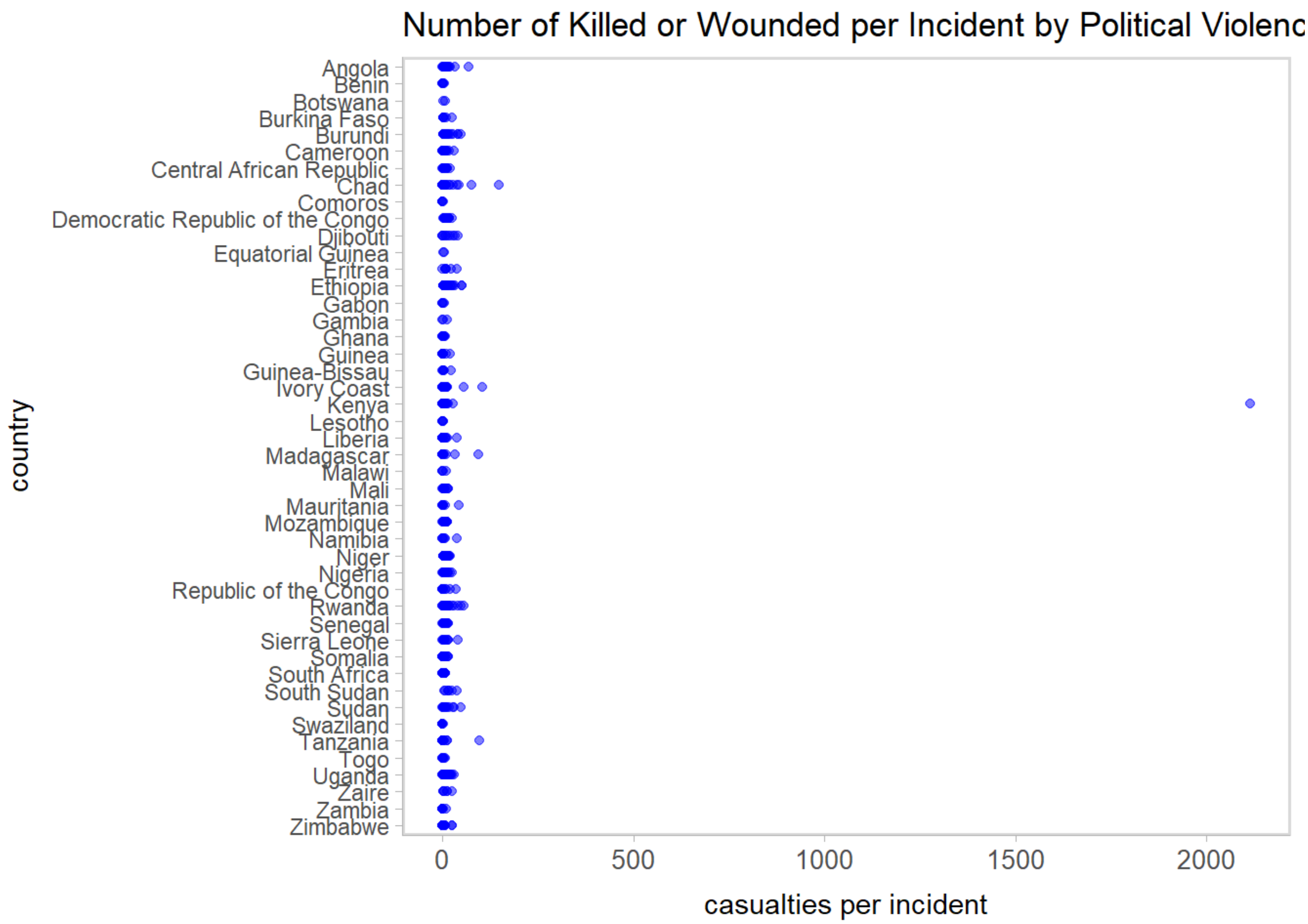
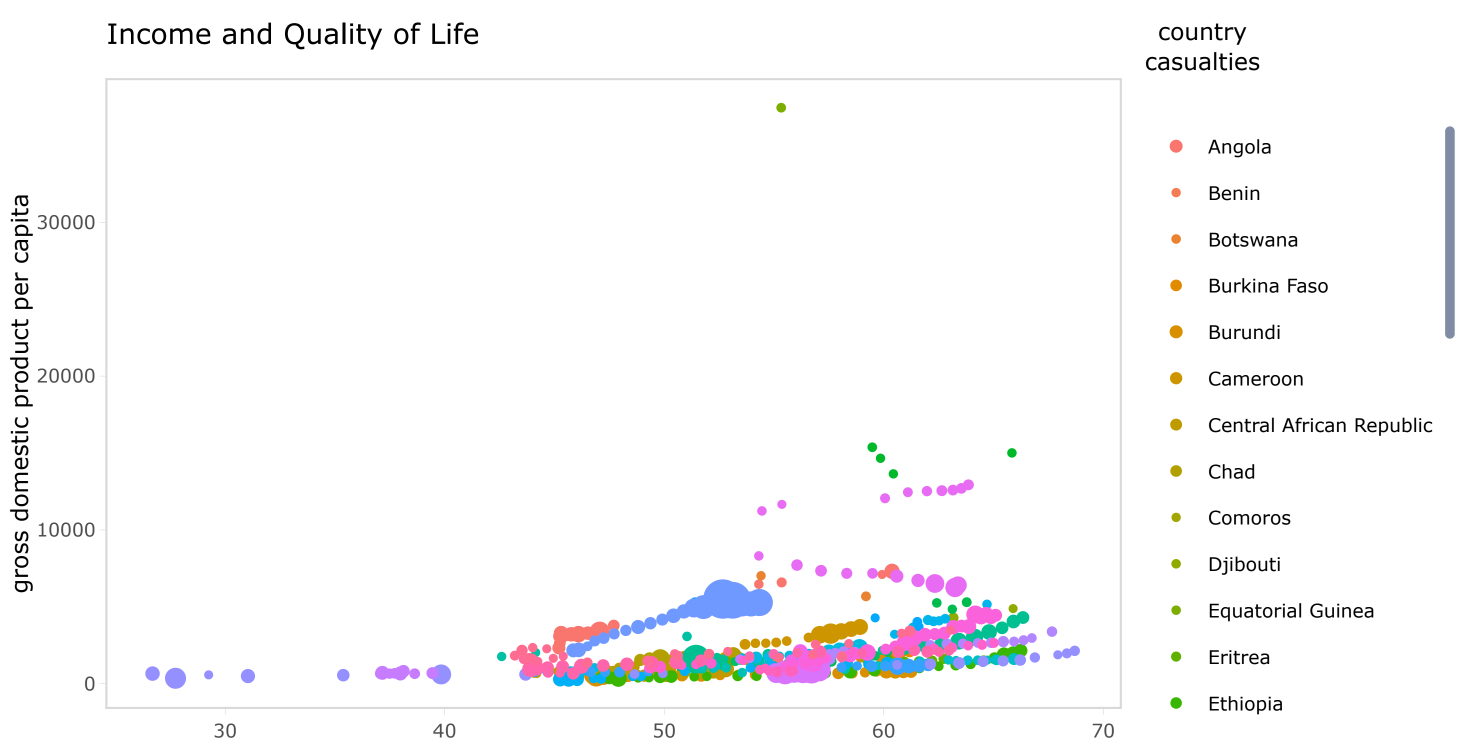


Figure 2.3: Number of Killed or Wounded Per Incident by Political Violence: Sub-Saharan Africa 1990-2018

The relative impact of each incident tells a similar story again with thresholds and extreme years of strife, for example, the civil war in Cote d'Ivoire (Ivory Coast). A threshold number of events seems to stretch into the lower 10%ile, \(n=\), with median, \(n=\) 5, reaching an upper 90%tile, \(n=\) 20. Experience of extreme events is rarer for this impact variable and again highly variable across countries and years.

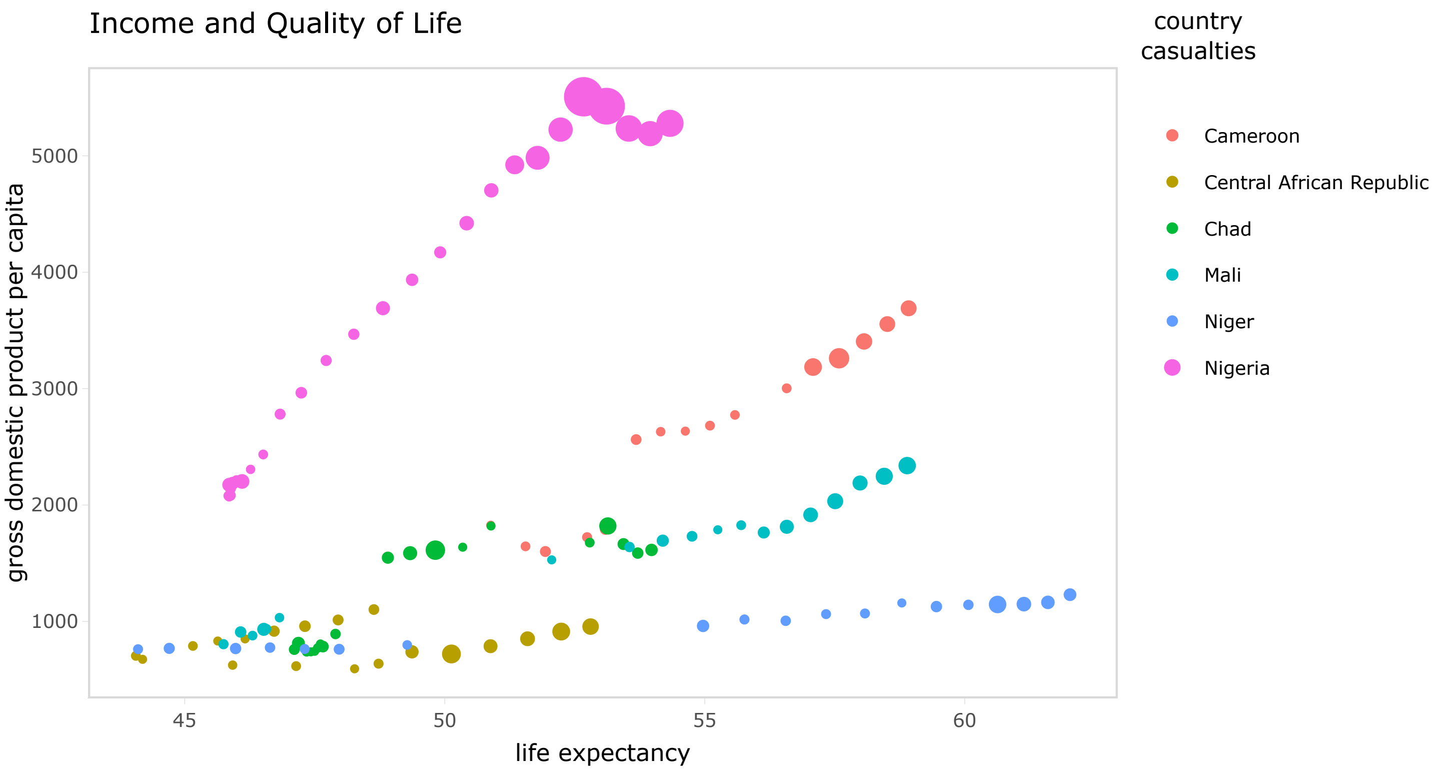
We can review the relationship between GDP per capita and life expectancy further conditioned by levels of violence levels in Figure .



life expectancy

Figure 2.4: Income and Quality of Life by Levels of Violence: Sub-Saharan Africa 1990-2018

Some of most violent, and geographically contiguous, countries include those around the Gulf of Guinea. On the coast are Nigeria and Cameroon. Inland are Niger, Mali, Chad, and Central African Republic. Figure \ref{fig:plot-gppd-life-vio-gulf) depicts the various possibilities.



(#fig:plot-gppd-life-vio-gulf,)Income and Quality of Life by Levels of Violence: Gulf of Guinea 1990-2018

The data naturally stratifies into separate country trajectories for income and life expectancy. Nigeria seems a likely outlier in terms of size of violent casualties as well as a low life expectancy with high GDP per capital. The high GDP per capital derives from its massive hydrocarbon reserves offshore. For the empirical experiments to follow we will focus on three geographically contiguous nations: Cameroon, Nigeria, and Niger.

3 Building an Observational Model

To examine the facts of political violence, namely intermittent extreme incidents and casualities, We need an observational model with these qualities:

1. The model generates a regularly occurring range of outcomes which acts like a threshold or background level of violence.
2. On occasion high concentrations of violent events beyond the threshold level.
3. It will be relatively easy to implement, compute, and interpret.

We always hope for 3) but will get by with 1) and 2). Such a distribution is the generalized Pareto distribution (GPD) (https://en.wikipedia.org/wiki/Generalized_Pareto_distribution) built on ideas developed by the Italian civil engineer, sociologist and economist Vilfredo Pareto (https://en.wikipedia.org/wiki/Vilfredo_Pareto) at the turn of the last century. He observed very high concentrations of wealth in the hands of a few individuals. For us, we observe very high concentrations of incidents and casualties in a small number of years and countries. A rule of thumb estimate is that about 85% of casualties from political violence occur during less than 20% of recorded incidents. The GPD has been used in the same way in quality control for catastrophic failure, epidemic outbreaks, severe weather, finance collapse, and previous studies of political violence and war, both civil and interstate.

To address disideratum 3 above, we would like the ability to answer several questions latent in the data. The time between events and the number of events it takes to produce an outbreak of violence are among the more policy prone queries. It turns out that the GPD is a mixture of two distributions, a compound of gamma distributed exponential parameters. We can thus express political violence \(V\) as an exponential random variable, with a Gamma distributed rate parameter.

\[\begin{align} V|\Lambda &\sim \operatorname{Exp}(\Lambda) \\ \Lambda &\sim \operatorname{Gamma}(\alpha , \, \beta) \end{align} \]

then \(V \sim \operatorname{GPD}(\xi =1/\alpha ,\ \sigma =\beta /\alpha)\). This formulation builds in the richer interpretation of GPD parameters we need for further consideration especially as indicated by Belikov (2017). Here is a simulation to illustrate the role of the gamma parameters in the GPD.

Figure \ref{fig:plot-gpd-gamma) overlays the gamma scenarios that generate exponential rates into the GPD.

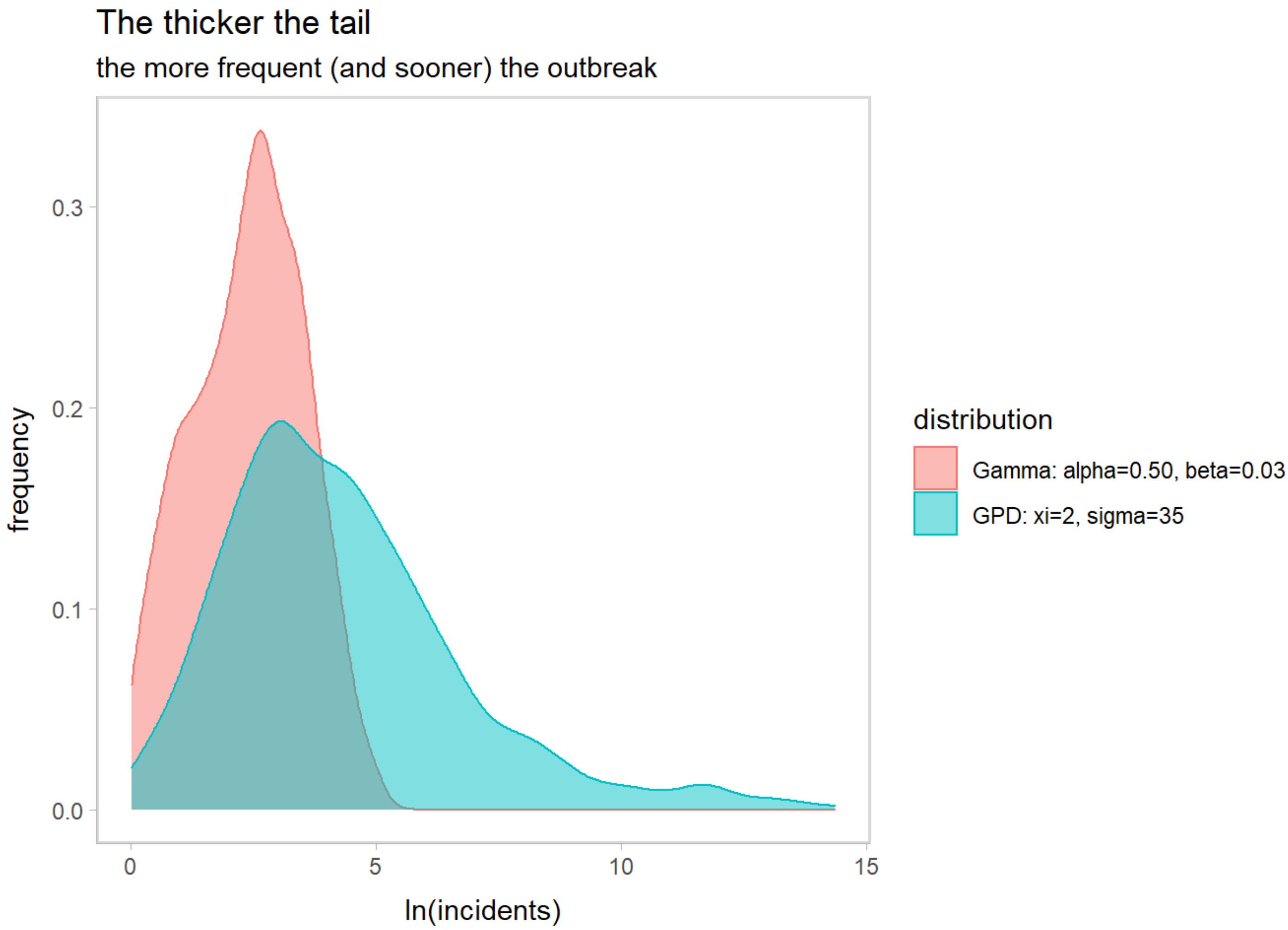
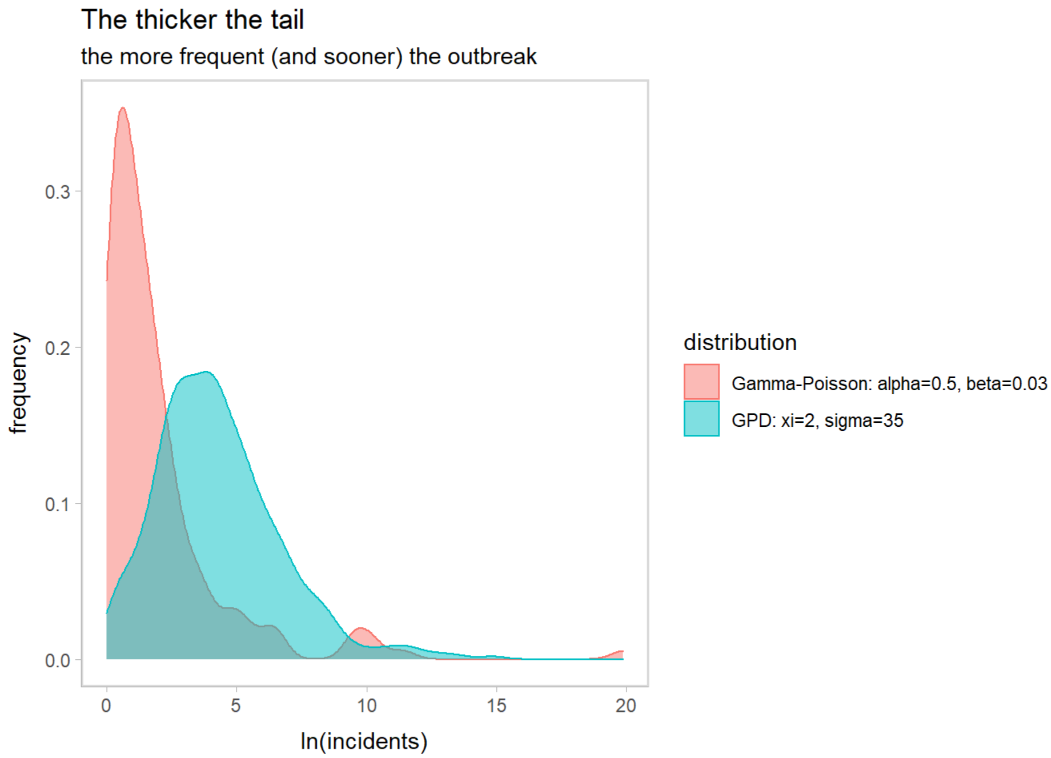


Figure 3.1: The Gamma in the Generalized Pareto Distribution

In this simulation of 1000 draws we compare a GPD with its underlying Gamma distribution. Gamma generated frequent outbreaks of violence with little time between outbreaks compound themselves into concentrations of violent bursts in the GPD.

Richardson (1944) noticed this same phenomenon by compounding Gamma incident severity into a Poisson rate of arrival. But he did not get the desired concentration of incidents into the tail of the distribution, well, almost. Here is a simulation of the Gamma-Poisson description of incidents.

Figure compares a gamma-Poisson distribution with the GPD.



The story is the same, even with the Poisson layering of Gamma generated rates. GPD provides more tail coverage than the Gamma-Poisson. The reason why this works is that the compounded distributions result from a flurry of underlying Poisson, and in the case of the GPD, the special Poisson case, exponential rates, not just one rate. They literally bunch up into high incident concentrations, in a word, they diverge. Exponential distributions will diverge more frequently due to higher kurtosis and thus the thicker GPD tail can evolve.

3.1 A sketch

Here is a sketch.[9] We begin with an exponentially distributed instance $z \in Z$ with rate parameter λ. We then calculate the probability that any of the other values of Z are greater than the conjectured value z given the exponential rate parameter. This calculation is also known as the survival function of z.

But first, to be complete, we start with the Poisson distribution. In a Poisson process if there are λ events per time segment, then there with be λz casualties in z time segments, perhaps we can think of these segments as years. The probability of w events is the Poisson distribution.

$$ Pr(w \mid \lambda) = e^{-\lambda z} \frac{(\lambda z)^w}{w!} $$

In the special case of $w = 0$, $Pr(w=0 \mid \lambda) = e^{\lambda t}$. This is also the probability of the first time an incident occurs at a time z beyond some threshold Z.

$$ Pr(Z > z \mid \lambda) = exp(-\lambda x) $$

This probability is the survival function $S(z)=1-F(z)$, where $F(z)$ is the cumulative probability distribution. It is well known that the excess kurtosis of the exponential distribution is 6. It is fact that drives the thick tails of any distribution that might be mixed with the exponential distribution.

We parameterize the bridge between Gamma exponential rates and exponentially evolving variate x with the variable t. We then use λ as the rate parameter for the Gamma distribution.

$$ f(t \mid \alpha, \lambda) = \frac{\lambda^{\alpha}t^{\alpha-1}exp(-\lambda t)}{\Gamma(\alpha)} $$

We can now express the compound survival function of the mixture distribution as this expression.

$$ S(z) = \int_0^{\infty}S(z \mid t)f(t)dt= \frac{\lambda^{\alpha}}{\Gamma(\alpha)}\int_0^{\infty}t^{\alpha-1}exp(-(\lambda+z)t)dt $$

Wethen transform \$S(z)@ with $u=(\lambda + z)t$.

$$ \begin{align} S(z) = \frac{\lambda^{\alpha}}{\Gamma(\alpha)} \int_0^{\infty} &(u\lambda + z)^{\alpha-1}exp(-u)d(u\lambda + z) \\ &=\frac{\lambda^{\alpha}}{\Gamma(\alpha)} (\lambda + z)^{-\alpha}\int_0^{\infty}u^{\alpha-1}exp(-u)du \end{align} $$

We know by definition of the Gamma function that

$$ \Gamma(\alpha) = \int_0^{\infty}u^{\alpha-1}exp(-u)du. $$

With a little bit of rearrangement we arrive then at this expression.

$$ S(z)=\lambda^{\alpha} (\lambda+z)^{-\alpha}=\left(1+\frac{z}{\lambda}\right)^{-\alpha} $$

We let $\alpha + 1 = \xi$, If we interpret λ as a scale parameter then we replace every λ with $1/\lambda$, finally we align this pdf with the standard GPD so that $\alpha = 1/\xi$ and $\xi= \lambda$ as scale (not rate) parameter we get the standardized cumulative density function $F(z)=1-S(z)$

$$ F(z) = 1- (1+\xi z)^{-1/\xi} $$

The first derivative of the cumulative density function $F(x)$ delivers the probability density function.

$$ F'(z)= f(x)=(1+\xi z)^{-\frac{\xi+1}{\xi}} $$

Here we will work with the Gamma function’s support, that is, with $z \geq 0$ and this also means that $\xi > 0$.

We can perform one more transformation by letting $z = (x - \mu) / \sigma$ which allows us to interpret the z variate as a standardized variable with mean μ and standard deviation σ. Since $z \geq 0$ and $\sigma > 0$ then we will only consider positive deviations of x about the location parameter μ. Integer counts of events of political violence, x, will consider only those equal to or above the location parameter μ. Thus we can interpret μ as a threshold parameter for the distribution.

$$ F(x) = 1- \left(1+\xi\left(\frac{x - \mu}{\sigma}\right)\right)^{-1/\xi} $$

Here is the probability distribution function. There are three parameters, location μ is a threshold of events of violence v, σ is the scale or dispersion of events, ξ is the shape of events that produces the thick or thin tails in the distribution. That last quality is crucial.

$$ \begin{align} F'(x) &= f(x) \\ &= \operatorname{Pr}(x \mid \mu, \sigma, \xi) = \frac{1}{\sigma}\left(1 + \xi \left(\frac{x-\mu}{\sigma}\right) \right)_{+}^{- (1+ \xi)/ \xi} \end{align} $$

where we use the short hand notation $h_+ = \max(h,0)$.

The interpretation of this distribution as a Gamma distributed Poisson rate mixture allows us one more pass at a further interpretation directly in terms of the Poisson distribution. Since the Poisson distribution with zero events can be interpreted as the probability of the first time an event occurs after a threshold time the GPD is a special case of Poisson arrivals with Gamma distributed rates, also known as the Negative Binomial distribution.

Here we isolate z to get an idea of what z can take on before the model becpmes singular.

$$ \begin{align} 1 - \xi (x-\mu) / \sigma &< 0 \\ 1 &< \xi (x-\mu) / \sigma &< 0 \\ \frac{\sigma}{\xi} + \mu &< x \\ x &> \frac{\sigma}{\xi} + \mu \end{align} $$

In other words, x definitely needs to be greater than the μ threshold. In general, the support is $x>\mu$ for $\xi>0$, and $\mu < x < \mu-\sigma / \xi$ for $\xi<0$. This will be our observational model, either directly as the GPD, or as a special case of the Negative Binomial. that is, the Gamma-Poisson mixture. Such a model takes a $\xi,\,\sigma$ combination as a conjecture, for a given level of threshold μ, a hypothesis as a given, a condition.

4 Empirical Results

We derive the power law GPD from a compound mixture of Gamma generated exponential functions. This allows us to develop answers to these questions.

1. What is the probable range of incidents of political violence within and across countries?
2. What is the relationship between incidents and casualties?
3. What is the average time to an outbreak of violent incidents?
4. How large a burst can we expect?

Power law distributions notoriously do not possess first, second, third, or even fourth moments analytically across the GPD parameter space, especially for ξ (see Embrechts (2000) for examples). This divergent feature means we should rely on the more robust median, mean absolute deviation, and inter-quartile ranges (probability intervals) to summarize the outcomes of power law distributions.

In what follows we use the Carpenter (2017) probabilistic program library to produce estimates of the GPD function. There are more direct ways possible. Using Bayesian methods Zhang and Stephens (2009) modify Maximum Likelihood estimators of the GPD parameters to manage known computational issues using direct Maximum Likelihood optimization and Method of Moments estimators. Due to singularities in the GPD, such as reported for mixtures by Watanabe (2010), which result in only semi-positive definite Fisher Information Matrices (Mahmoud and El-Ghafour (2015)), only asymptotic efficiency results are available for $-0.5 < \xi < 0.5$ as developed initially by Hosking and Wallis (1987). Most of the data we will confront will have shape parameters $\xi > 0.5$.

The modified estimation technique is also deployed by the Stan development team to produce the Pareto-Smooth Importance Sampling (PSIS) measure used with a Leave-One-Out (LOO) cross-validation for out-of-sample inference worked out by G. Vehtari A. (2017). Yet another reason for subjecting the analysis to a Stan treatment is that we will eventually want to include power law distributions as regressors.

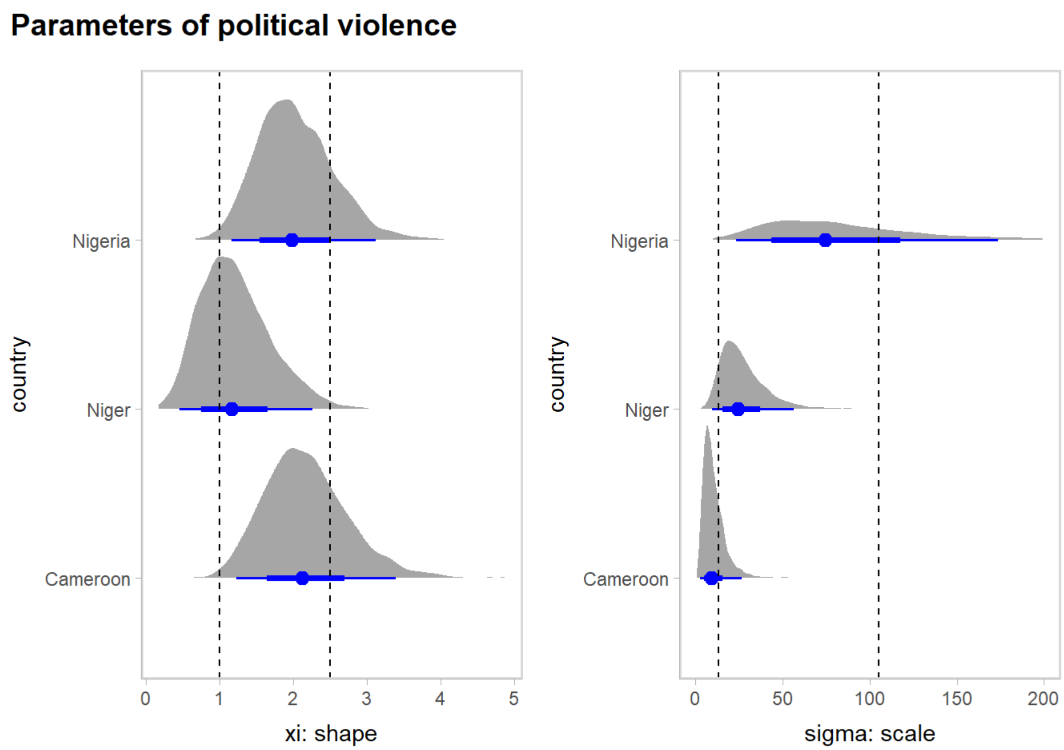
4.1 Casualty Violence

Our first stop is the very simple ensemble of univariate casualty (V for violent incidents of killing and wounding) models. Causally this is $C \rightarrow U \rightarrow V$, where C represents the country stratification of violence, and U are unobserved confounders such as the sources of information about casualties. We implement this causal model with the following distributional structure.

$$\begin{align} V_i &\sim \operatorname{GPD}(\xi_i, \sigma_i) \\ \xi_i &= \alpha_i\\ \alpha_i &\sim \operatorname{Normal}(0,\, 1) \\ \sigma_i &\sim \operatorname{Exponential}(1) \end{align}$$

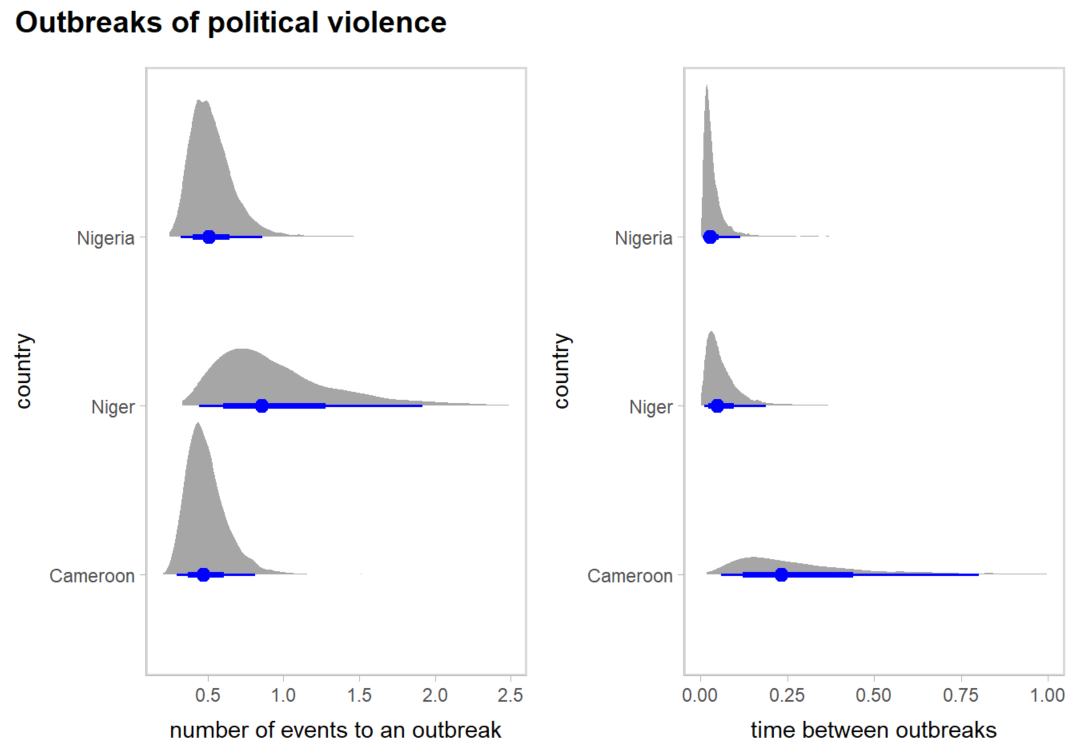
Here V are annual counts of those persons killed and wounded and $i=1\ldots 3$ for each of the countries Cameroon, Niger, and Nigeria.

Figure reports means and 90% credibility intervals of marginal distributions of 1,000 draws of casualty parameters across the five countries.



A wide range of shapes from $\xi < 1$ to $\xi = 4$, with the highest associated with the thickest, that is the most highly concentrated extreme values of political violence. Nigeria and Cameroon have this distinction, with Niger considerably less tail concentrations of killed and wounded.

In Figure we further interpret GPD shape and scale through the compounding effect of the Gamma distribution inside the GPD. We can characterize the Gamma shape parameter $\alpha = 1/\xi$ as the number of events per year that drive a burst of violence and scale parameter $\beta = \alpha / \sigma$ the mean time in years between bursts of violence. Here a burst is a threshold level of violence, especially a level that is not sustained.



The number of events to an outbreak are measured per year in this data. If we were to multiply the per year number of events (\(\alpha\)) by a rolling 5 years, while Niger can withstand 5. For all three countries the time between outbreaks of casualties is within 3 months, very short indeed. Cameroon exhibits a wide dispersion of time to outbreaks stretching to a year, thus very much more uncertain than either Niger or Nigeria.

4.2 Is violence any different when impacting human development?

We suppose that violence possesses power law characteristics in this paper. Other studies have used, essentially, Gaussian characterizations of violence in regressions on variates such as gross domestic product and life expectancy. We specify the causal model using this generative model.

$$\begin{aligned} \xi &\sim \operatorname{Normal}(\mu_{\xi}, \sigma_{\xi}) \\ \sigma_v &\sim \operatorname{Exponential}(\lambda_v) \\ V &\sim \operatorname{GPD}(\xi, \sigma_v) \\ \alpha &\sim \operatorname{Normal}(\mu_{alpha} , \sigma_{alpha}) \\ \beta &\sim \operatorname{Normal}(\mu_{beta} , \sigma_{beta}) \\ \mu \mid V &= \alpha + \beta\,V \\ \sigma_g &\sim \operatorname{Exponential}(\lambda_g) \\ G &\sim \operatorname{Normal}(\mu, \sigma_g) \end{aligned}$$

Table details the impact of casualties on GDP per capita.[10]

Table 4.1: Does violence influence per capita GDP?

	mean	sd	5.5%	94.5%	n_eff	Rhat4
a[1]	6.3990503	0.0695572	6.2899560	6.5110104	3593.923	1.0002286
a[2]	1.0141732	1.9644355	-2.1398815	4.1171220	4215.779	0.9995409
a[3]	1.0267683	1.9670482	-2.0876746	4.1755589	4414.452	0.9995702
b[1]	0.0422765	0.0028079	0.0378337	0.0466694	3701.180	1.0001309
b[2]	0.5197659	0.9831381	-1.0638763	2.0905615	4094.329	0.9998284

Note:

Countries 1, 2, and 3 are Cameroon, Niger, and Nigeria. Parameters a are intercepts, b are slopes, sigma are standard errors of the regression of log(GPD per capita) on log(casualties). k and sigma_x are shape and scale parameters of the GPD for log(casualties). Means and standard deviations, and 89% quantiles are averaged from Markov Chain Monte Carlo samplings (2,000 draws each from 4 chains). N_eff are the effective number of variables and Rhat4 reports sampling stability.

	mean	sd	5.5%	94.5%	n_eff	Rhat4
b[3]	0.5107105	0.9796052	-1.0698634	2.0906640	4622.001	0.9992913
sigma[1]	0.3215621	0.0639685	0.2367001	0.4344669	3048.854	1.0004654
sigma[2]	0.1996574	0.0360497	0.1506572	0.2629707	2969.799	1.0002853
sigma[3]	0.4003242	0.0599403	0.3168123	0.5023623	3273.329	0.9996014
sigma_X[1]	12.9670545	9.6013411	2.8462841	30.4356019	3887.906	0.9995589
sigma_X[2]	30.6581471	13.8579585	13.8389880	54.6043092	5281.786	0.9995298
sigma_X[3]	97.2453283	46.4836629	37.7318041	179.0766397	3225.193	0.9991597
k[1]	2.6810053	0.9852827	1.4288273	4.4323928	3895.524	0.9996099
k[2]	1.4338574	0.4457048	0.9879723	2.2743096	5504.204	0.9994279
k[3]	2.1588617	0.6565788	1.2890552	3.2900391	3050.476	0.9993567

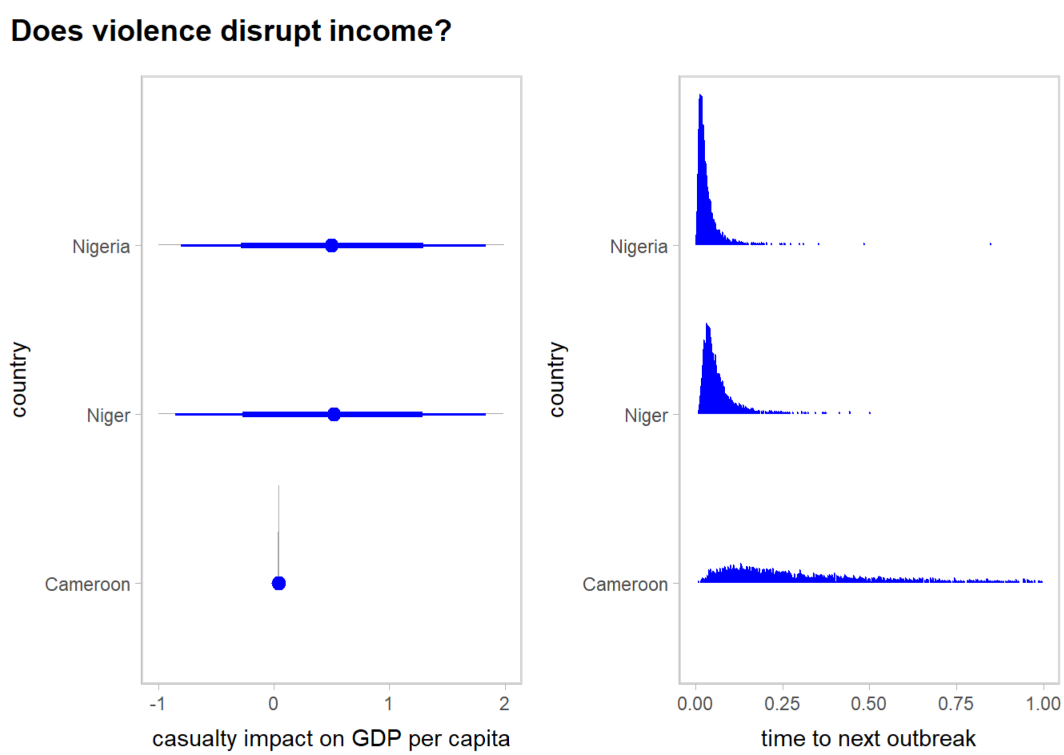
Note:

Countries 1, 2, and 3 are Cameroon, Niger, and Nigeria. Parameters a are intercepts, b are slopes, sigma are standard errors of the regression of log(GPD per capita) on log(casualties). k and sigma_x are shape and scale parameters of the GPD for log(casualties). Means and standard deviations, and 89% quantiles are averaged from Markov Chain Monte Carlo samplings (2,000 draws each from 4 chains). N_eff are the effective number of variables and Rhat4 reports sampling stability.

The size of the shapes and scale of the GDP regressor are consistent with univariate results. The estimation reports elasticities. While wide ranges of plausible values of regression parameters result from this causal experiment, we can interpret these credibility intervals (see Gelman and Greenland (2019)) as degrees of uncertainty. For both Niger and Nigeria, the sampling draws a wide range of scenarios from a preponderance of positive and highly sensitive elasticity to some negative, and also very sensitive responses of per capital GDP to sporadic and intense violence. In Cameroon, violence seems to elicit little response from per capital GDP.

Figure compares the impact of casualtieson GDP per capital (the regression slope) with the time to outbreak of political violence.

```
## Warning: Ignoring unknown parameters: quantile
```



The answer seems to be that the relationship between casualties and per capita Gross Domestic Product, stratified by country effects, is a draw. There are some explanatory scenarios where Niger and Nigeria might experience some impact. But for all intents and purposes, with Niger and Nigeria expecting nearly monthly occurrences of intense and intermittent violence, apparently, the data acts as if both countries have neutralized the impact on income per capita.

Cameroon exhibits little or no variation in any sensitivity to income. But this disjunction of income and violence comes with a wide variability in the probable time between outbreaks. For perhaps different reasons than for Niger and Nigeria, Cameroon seems to have just as heavily discounted the influence of violence on national income as consumption, investment, and government spending.

4.3 Does violence impact life expectancy?

The causal model to answer this question is similar to that for the influence of violence on GDP per capita.

$$
\begin{align} \xi &\sim \operatorname{Normal}(\mu_{\xi}, \sigma_{\xi}) \\ \sigma_v &\sim \operatorname{Exponential}(\lambda_v) \\ V &\sim \operatorname{GPD}(\xi, \sigma_v) \\ \alpha &\sim \operatorname{Normal}(\mu_{alpha} , \sigma_{alpha}) \\ \beta &\sim \operatorname{Normal}(\mu_{beta} , \sigma_{beta}) \\ \mu \mid V &= \alpha + \beta\,V \\ \sigma_l &\sim \operatorname{Exponential}(\lambda_l) \\ L &\sim \operatorname{Normal}(\mu, \sigma_l) \end{align}
$$

Table presents the results of the simple regression of life expectancy against casualties, again with the novel GPD regressor distribution.

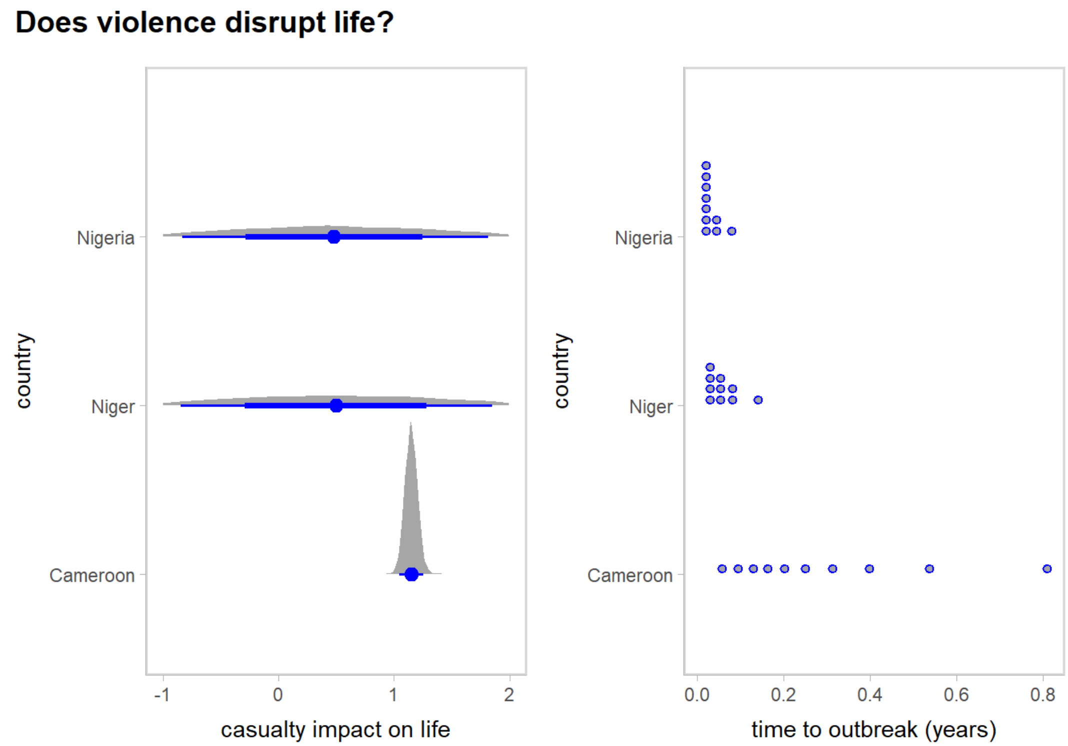
Table 4.2: Does violence influence life expectancy?

	mean	sd	5.5%	94.5%	n_eff	Rhat4
a[1]	5.0780298	2.0526962	1.8272147	8.364136	3318.287	0.9994117
a[2]	1.0022329	1.9994986	-2.2320910	4.162032	5299.333	0.9997876
a[3]	1.0179466	1.9393468	-2.0798685	4.084188	6540.905	1.0008835
b[1]	1.1503883	0.0556343	1.0640798	1.238628	3260.571	0.9993892
b[2]	0.5014143	1.0094502	-1.1094771	2.128532	5976.548	0.9994550
b[3]	0.4827870	1.0007410	-1.1211209	2.114251	5979.616	0.9994662
sigma[1]	7.6930858	1.1947673	5.9612473	9.767390	3082.969	0.9996641
sigma[2]	25.0535946	2.5610961	21.3094910	29.342308	4062.454	1.0004760
sigma[3]	3.1121581	0.4542586	2.4868598	3.916300	3663.500	1.0001936
sigma_X[1]	13.0205273	9.9103982	3.1087542	30.234143	3809.518	1.0000470
sigma_X[2]	30.4713210	13.7568102	13.3385698	55.787819	4608.837	0.9998184
sigma_X[3]	96.1012668	47.6777653	37.1112384	182.759422	3895.365	0.9997395
k[1]	2.6710261	0.9906193	1.4350157	4.404624	3511.680	0.9992689
k[2]	1.4389762	0.4462513	0.9891245	2.272848	5134.968	1.0004697
k[3]	2.1744383	0.6702828	1.2998358	3.301048	4324.337	1.0007634

Note:

Countries 1, 2, and 3 are Cameroon, Niger, and Nigeria. Parameters a are intercepts, b are slopes, sigma are standard errors of the regression of years of life expectancy on casualties. k and sigma_x are shape and scale parameters for casualties. Means and standard deviations, and 89% quantiles are averaged from Markov Chain Monte Carlo samplings (2,000 draws each from 4 chains). N_eff are the effective number of variables and Rhat4 reports sampling stability.

This figure depicts the country-specific casualty impact on life expectancy relative to the time to an outbreak in political violence (in years).



A pattern similar to casualty impact on per capita GDP emerges with life expectancy. Here each dot represents a 10th of the sampled parameters. Both Niger and Nigeria can expect monthly violence but this does not seem to phase life expectancy. Have the people become so used to violence? Perhaps this is so, but we may think of the time series of data as already conditioned to high frequencies of casualties.

In a seeming reversal of this pattern Cameroon's life expectancy is very much influenced by casualties. This sensitivity seems influenced by the higher dispersion of outbreaks of violence. The Cameroonian experience might seem to be one of reaction to violence rather than its assimilation.

4.4 Violence as disruptor of development

In this causal relationship we suppose tha GDP per capita is dependent on life expectancy. In turn violence directly influences life expectancy. Years of life expected literally pipes violence into GDP per capita. Here is the generative model.

\[\begin{align} G &\sim \operatorname{Normal}(\mu_G, \sigma_G) \\ \mu_G &= a_G + b_{GL}L \\ L &\sim \operatorname{Normal}(\mu_L, \sigma_L) \\ \mu_L &= a_L + b_{LV}V \\ V &\sim \operatorname{Gamma-Poisson}(\lambda, \phi) \\ \operatorname{log}(\lambda) &= a_V + \tau U \\ U &\sim \operatorname{Normal}(0,\, 1) \\ a_G,\,a_L,\,a_V &\sim \operatorname{Normal}(0,\, 1) \\ b_{GL},\, b_{GV}, \, b_{LG}, \, b_{LV}, \, \tau &\sim \operatorname{Normal}(0,\, 1) \\ \sigma_G, \sigma_L, \phi &\sim \operatorname{Exponential}(1) \end{align} \] We add a new variable \(U\) to the model which represents any number of reasons why casualties (V) might be misreported, not reported at all, or might occur for reasons we have yet to specify. In the model the parameter \(\tau\) measures the amount of unobserved arrivals of violence in a country.

For this rendering we replace the GPD with its explicit mixture distribution Gamma-Poisson, recalling that the exponential distirbution is one case of the Poisson. In this setting, and with the proper interpretation of parameters, the use of this distribution proves more computationally stable than the use of the GPD in terms of mixing Markov chains and realizing Rhat4 values near to 1.

We perform one more maneuver by transforming casualties into their absolute deviations. This technique effectively computes the point-wise volatility of casualties.

Table **??** presents the results of the not so simple regression of life expectancy against casualties, which in turn mediates the transmission of violence to GDP per capita.

(#tab:table-gdppc-on-l-on-v)Does violence influence life expectancy?

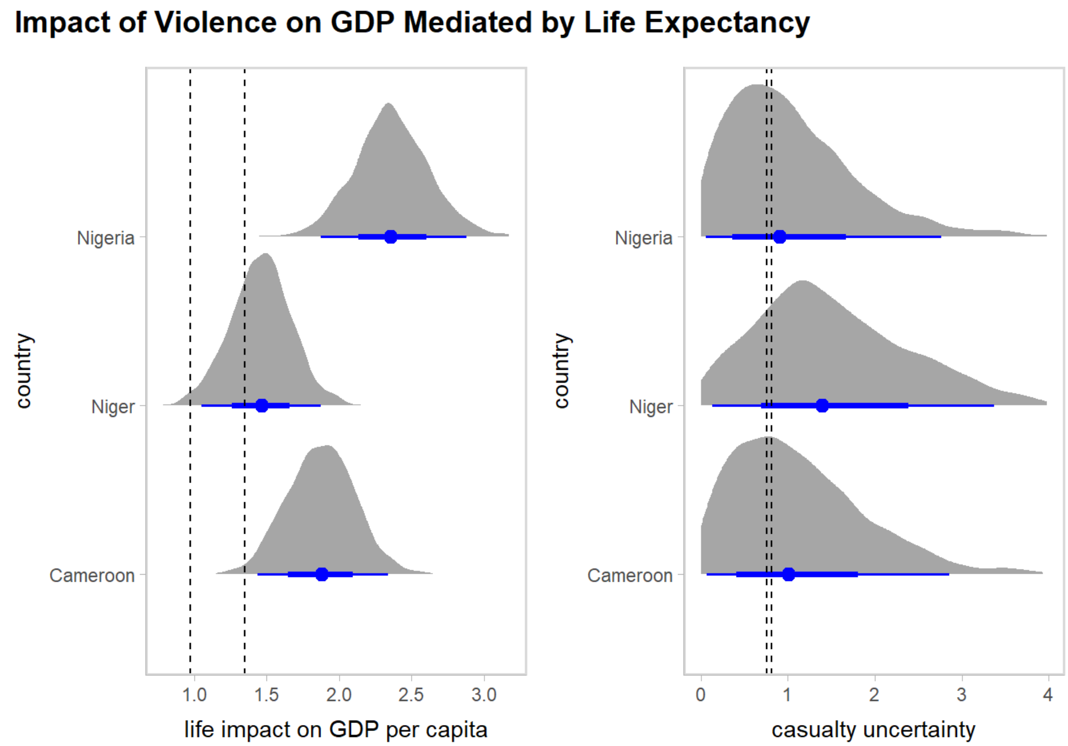
	mean	sd	5.5%	94.5%	n_eff	Rhat4

	mean	sd	5.5%	94.5%	n_eff	Rhat4
GDP per capita						
aG[1]	0.3211145	0.9265744	-1.1221497	1.8207858	1840.758	1.0012899
aG[2]	1.0436987	0.8269100	-0.2434360	2.4035533	1420.491	0.9999131
aG[3]	-1.0759093	0.9826868	-2.6799392	0.4788252	1439.438	0.9999760
bGL[1]	1.8757447	0.2311637	1.5038488	2.2375048	1847.200	1.0011583
bGL[2]	1.4612674	0.2076762	1.1209282	1.7803972	1405.021	0.9999820
bGL[3]	2.3646873	0.2522731	1.9622767	2.7795057	1447.607	1.0001080
sigma_G	0.1982812	0.0205983	0.1675395	0.2321637	1719.345	1.0003047
Life Expectancy						
aL[1]	4.0042092	0.0265776	3.9607695	4.0464349	1945.542	1.0000518
aL[2]	3.9800767	0.0201513	3.9476007	4.0124990	2085.354	0.9988982
aL[3]	3.8606402	0.0194094	3.8303509	3.8909363	1863.558	0.9987546
bLV[1]	0.0037965	0.0430250	-0.0646341	0.0716555	2178.013	1.0010431
bLV[2]	0.0477559	0.0519074	-0.0326026	0.1324467	2081.454	1.0003239
bLV[3]	0.0635003	0.0252324	0.0232259	0.1040889	1941.296	0.9983496
sigma_L	0.0814698	0.0081553	0.0694455	0.0947542	2139.383	1.0006988
Casualties						
aV[1]	0.4003661	0.8412818	-0.7809269	1.8878830	1453.001	1.0000935
aV[2]	0.1701730	0.9849013	-1.3387097	1.7598003	1375.006	1.0006103
aV[3]	0.4198669	0.7919705	-0.6450722	1.8516446	1627.895	0.9996674
tau[1]	1.1492986	0.8034107	0.1458726	2.5506856	1808.697	1.0004138
tau[2]	1.6032068	1.0176775	0.2852357	3.2823119	1769.242	0.9991260
tau[3]	1.0569990	0.7792434	0.1198499	2.4631401	1896.772	0.9982158
U	-1.2432721	0.5657446	-2.2349233	-0.4497850	1455.542	1.0030309
phi	2.1012479	1.1855304	0.7315069	4.3713687	3275.101	0.9986579

Note:

Countries 1, 2, and 3 are Cameroon, Niger, and Nigeria. Parameters a are intercepts, b are slopes, sigma are standard errors of the regression of GDP percapita (G) on years of life expectancy (L) conditional on casualties (V) with tau casualty uncertainty of unobserved causes (U). Means and standard deviations, and 89% quantiles are averaged from Markov Chain Monte Carlo samplings (2,000 draws each from 4 chains). N_eff are the effective number of variables and Rhat4 reports sampling stability.

In Figure \ref{fig:table-gdppc-on-l-on-v) we single out a comparison of the impact of life expectancy on GDP per capita with uncertainty around the causes of casualty volatility.



4.4.1 Interactive violence

In this model we allow GPD pe capita and life expectancy to cause one another. We use casualties to identify the model with differential impacts on life expectancy and GDP per capita. We make use of this version of the generative model.

$$\begin{align} G &\sim \operatorname{Normal}(\mu_G, \sigma_G) \\ \mu_G &= a_G + b_{GL}L \\ L &\sim \operatorname{Normal}(\mu_L, \sigma_L) \\ \mu_L &= a_L + b_{LG}G + b_{LV}V \\ V &\sim \operatorname{Gamma-Poisson}(\lambda, \phi) \\ \operatorname{log}(\lambda) &= a_V + \tau U \\ U &\sim \operatorname{Normal}(0,\, 1) \\ a_G,\,a_L,\,a_V &\sim \operatorname{Normal}(0,\, 1) \\ b_{GL},\, b_{GV}, \, b_{LG}, \, b_{LV}, \, \tau &\sim \operatorname{Normal}(0,\, 1) \\ \sigma_G, \sigma_L, \phi &\sim \operatorname{Exponential}(1) \end{align}$$

Figure \ref{fig:dag-lgvsimul) develops a directed acyclic graph (DAG) view of the generative model. The causal paths from country effects (C) through unobserved casualties (U) on to casualties (V) then would confound GDP per capita (G) and life expectancy (L) if allowed to influence both outcomes. Using the logic of Pearl, Glymour, and Jewell (2016), the backdoor confounding of G and L is averted by eliminating an arrow from V to G, just we effectively did in using L to mediate the influence of V on G. L and G are left to mutually influence one another. V shares information only with L. This allows for a fairly clean, as much as can be expected in probability, computation of the relationship between G and L.

Table presents the results of the more complex interactive relationship between GDP per capital and life expectancy. Again life expectancy mediates casualties, but this time in the simultaneous variations of GDP per capita with life expectancy.

(#tab:table-gdppc-to-from-l-on-v)Violence, life expectancy and GDP interactions

	mean	sd	5.5%	94%	n_eff	Rhat4
GDP per capita						
aG[1]	0.28	0.98	-1.26	1.79	1624	1
aG[2]	1.06	0.80	-0.20	2.34	1333	1
aG[3]	-1.07	0.97	-2.61	0.51	1498	1

Note:

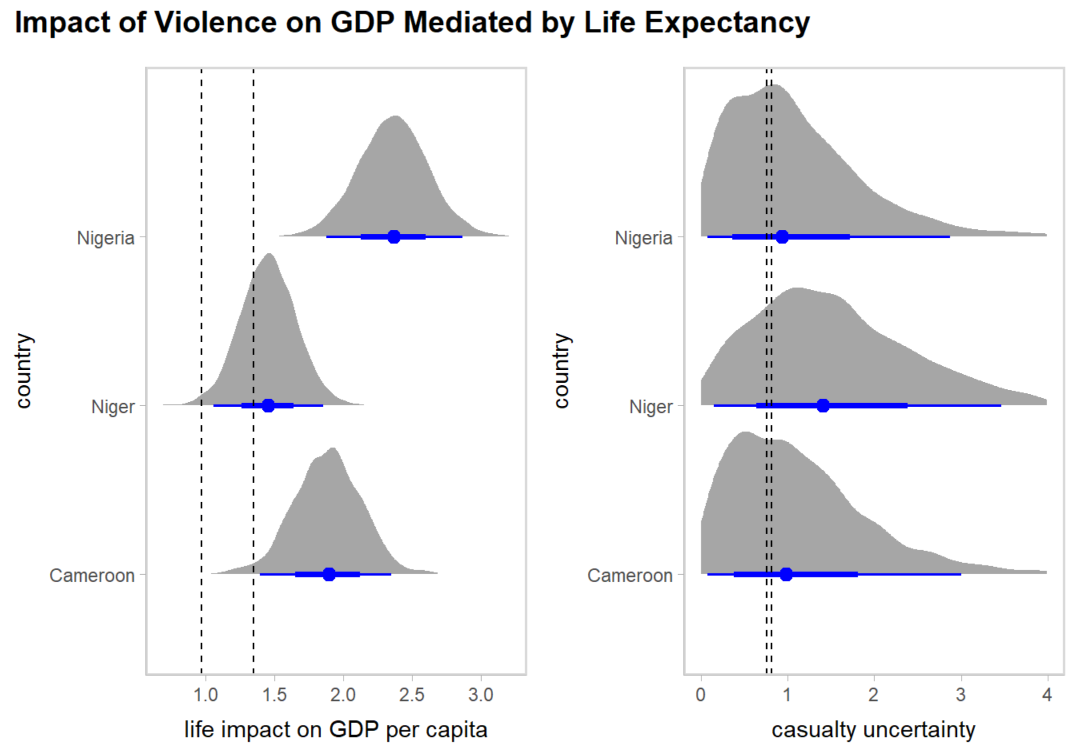
Countries 1, 2, and 3 are Cameroon, Niger, and Nigeria. Parameters a are intercepts, b are slopes, sigma are standard errors of the simultaneous regression of GDP percapita (G) on years of life expectancy (L), and L on G and conditional on casualties (V) with tau casualty uncertainty of unobserved causes (U). Means and standard deviations, and 89% quantiles are averaged from Markov Chain Monte Carlo samplings (2,000 draws each from 4 chains). N_eff are the effective number of variables and Rhat4 reports sampling stability.

	mean	sd	5.5%	94%	n_eff	Rhat4
bGL[1]	1.89	0.24	1.51	2.27	1626	1
bGL[2]	1.46	0.20	1.14	1.78	1314	1
bGL[3]	2.36	0.25	1.96	2.76	1503	1
sigma_G	0.20	0.02	0.17	0.23	2051	1
Life Expectancy						
aL[1]	2.75	0.13	2.54	2.94	1672	1
aL[2]	-0.42	0.16	-0.67	-0.18	1246	1
aL[3]	2.72	0.09	2.58	2.86	1516	1
bLG[1]	0.16	0.02	0.14	0.19	1645	1
bLG[2]	0.64	0.02	0.61	0.68	1238	1
bLG[3]	0.14	0.01	0.13	0.16	1509	1
bLV[1]	-0.01	0.01	-0.03	0.00	2818	1
bLV[2]	0.00	0.01	-0.02	0.02	3162	1
bLV[3]	0.01	0.01	0.00	0.02	2094	1
sigma_L	0.02	0.00	0.02	0.02	1770	1
Casualties						
aV[1]	0.38	0.86	-0.82	1.85	1441	1
aV[2]	0.13	0.97	-1.30	1.73	1645	1
aV[3]	0.45	0.81	-0.66	1.91	1415	1
tau[1]	1.14	0.82	0.15	2.66	1832	1
tau[2]	1.60	1.01	0.29	3.38	1800	1
tau[3]	1.09	0.78	0.14	2.52	1637	1
U	-1.22	0.54	-2.18	-0.47	1935	1
phi	2.12	1.15	0.72	4.22	2256	1

Note:

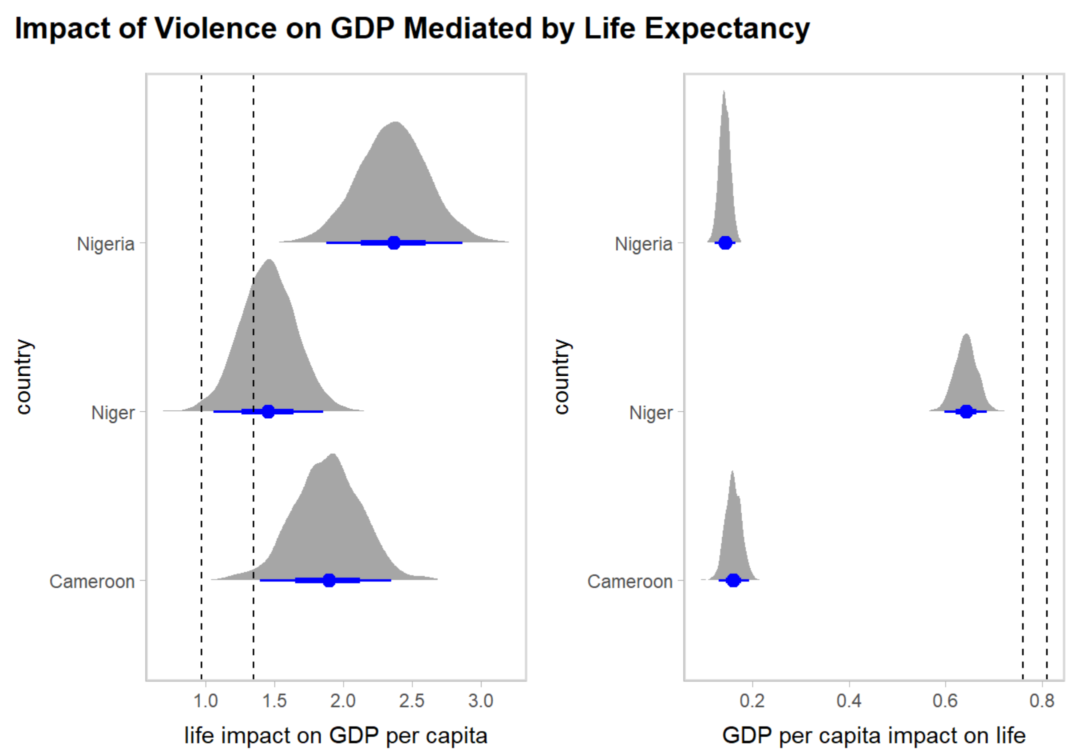
Countries 1, 2, and 3 are Cameroon, Niger, and Nigeria. Parameters a are intercepts, b are slopes, sigma are standard errors of the simultaneous regression of GDP percapita (G) on years of life expectancy (L), and L on G and conditional on casualties (V) with tau casualty uncertainty of unobserved causes (U). Means and standard deviations, and 89% quantiles are averaged from Markov Chain Monte Carlo samplings (2,000 draws each from 4 chains). N_eff are the effective number of variables and Rhat4 reports sampling stability.

In Figure \ref{fig:table-gdppc-to-from-l-on-v) we again, but for this interactive model, single out a comparison of the impact of life expectancy on GDP per capita with uncertainty around the causes of casualty volatility.



The sensitivity of life expectancy on GDP percapita is starker for Nigeria than in previous models. Casualty uncertainty seems to cluster together.

In Figure we set the joint impacts of GDP per capita and life expectancy on each other side by side.



It appears that GDP per capita has little impact on life expectancy. However, life has a much stronger influence on GDP per capita for each country. Perhaps we might say that the causation more probably runs from life to GDP per capita, than the other way around.

4.5 Model selection

Here are two criteria for selecting a model. The PSIS-LOO (Pareto Smoothing Importance Sampling and Leave-One-Out Cross Validation) develops GPD measures of the uncertainty associated with each observation. It uses the cross-validation procedure of leave one observation out for out-of-sample predictions (A. G. Vehtari A. and Gabry (2015)).

```
compare( m2.1 , m2.2, func=PSIS )
```

```
##      PSIS  SE dPSIS  dSE pPSIS weight
## m2.1  -22 8.3  0.00   NA   3.4   0.54
## m2.2  -22 8.3  0.36 0.13   3.7   0.46
```

The WAIC (Watanable-Aikaike or Widely Available Information Criterion) is an information divergence measure which allows for singularities in the parameter space (Watanabe (2010)).

```
compare( m2.1 , m2.2, func=WAIC )
```

```
##      WAIC  SE dWAIC  dSE pWAIC weight
## m2.1  -23 8.2  0.00   NA   3.4   0.54
## m2.2  -22 8.3  0.34 0.13   3.7   0.46
```

We obtain nearly identical results for predictive power between the two models with both life and GDP. Life mediation has slightly less information leakage than full interactive model.

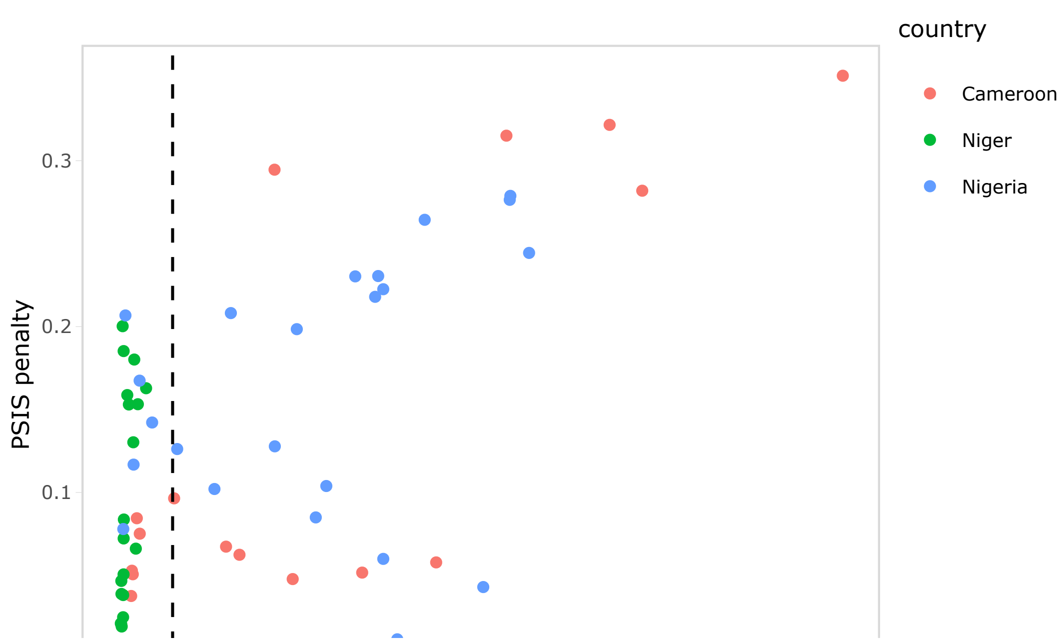
PSIS exploits the distribution of potentially outlying and influential observations using the GPD to model and measure the data point-wise with the shape parameter \(k=\xi\). Any point with \(k>0.5\) will have infinite variance and thus contribute to a concentration of points – the thick tail.

WAIC is the *log-posterior-predictive density* (lppd, that is, the Bayesian deviance) and a penalty proportional to the variance in posterior predictions:

\[WAIC(y, \Theta) = −2(lppd − \underbrace{\Sigma_i var_{\theta}\,log \,\,p(y_i|\theta))}_{penalty} \]

The penalty is related to the number of free parameters in the simulations.

```
## R code 7.34 McElreath2020
d <- d_all %>%
  filter( country %in% c( "Cameroon", "Niger", "Nigeria"))
cid <- as.factor(d$country)
set.seed(4284)
PSIS_m2.2 <- PSIS(m2.2,pointwise=TRUE)
PSIS_m2.2 <- cbind( PSIS_m2.2, country=d$country, casualties=d$casualties)
set.seed(4284)
#WAIC_m2.2 <- WAIC(m2.2,pointwise=TRUE)
p1 <- PSIS_m2.2 %>%
  ggplot( aes( x=penalty, y=k, group=casualties, color=country) ) +
  geom_point( ) +
  xlab("PSIS Pareto k") + ylab("PSIS penalty") +
  geom_vline( xintercept = 0.03, linetype = "dashed")
ggplotly( p1 )
```



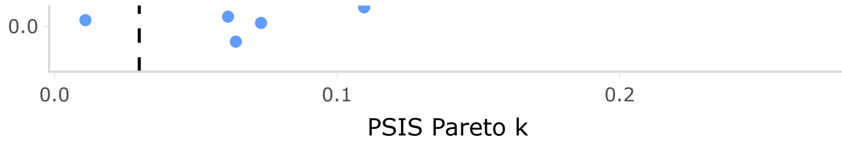


Figure 4.1: Highly influential observations and out-of-sample prediction. Cameroon inhabits the NE quadrant with high penalty and Pareto k values. These observations are highly unpredictable.

Cameroon is the outlying country as can be seen in the often flat profile of estimated parameters. This country is much more uncertainty and less predictable than Nigeria and Niger. Nigeria seems to settle right into the model with low Pareto k and PSIS penalties.

4.6 Statistical modeling

We use the generative stochastic models described above as Bayesian statistical models (McElreath (2020)) coded in Stan (Carpenter (2017); Team (2020b)) and fit using R (R Core Team) and rstan (Team (2020a)). The complete workflow will be maintained on GitHub at https://github.com/wgfoote/developmentandviolence (https://github.com/wgfoote/developmentandviolence).

5 Discussion

While levels of casualties widely affects GDP per capita and years of life expected, it is the interaction of development measures that impounds the influence of political violence. Reporting of violent incidents which result in persons killed or wounded exhibit varying ranges of uncertainty. These ranges vary widely among state actors. It appears that GDP per capita is not causative of years of life expected. On the contrary, country specific violence causes changes in GDP per capita by way of its more or less obvious influence on years of life expected. Longer life strongly impacts GDP per capita and seems to be one channel through which violence is transmitted to economic well-being.

A major caveat of this study is the highly aggregated measurement of all variates. This is unavoidable for years of life expected and GDP per capita. These measures are reported only annually and nationally, which leads to another caveat. Many observers do not believe that the nation-state is an appropriate model for many of the sub-Saharan countries. Rather a more disaggregated approach may be more yielding of relationships among social and economic development and disruptions to well-being fomented by political violence. Violence is reported by latitude and longitude on a daily frequency. Developing a daily model of political violence, and if that is too noisy perhaps a weekly or monthly version, and then aggregating the salient features of violence into aggregated economic and social measures may prove a fruitful avenue of research.

References

Bandyopadhyay, Subhayu, Todd Sandler, and Javed Younas. 2013. “Foreign Direct Investment, Aid, and Terrorism.” *Oxford Economic Papers* 66 (1): 25–50.

Belikov, Aleksey V. 2017. “The Number of Key Carcinogenic Events Can Be Predicted from Cancer Incidence.” *Nature Scientific Reports* 7: 12170. https://doi.org/DOI:10.1038/s41598-017-12448-7 (https://doi.org/DOI:10.1038/s41598-017-12448-7).

Carpenter, Gelman, B. 2017. “Stan: A Probabilistic Programming Language.” *Journal of Statistical Software* 76 (1).

Clauset, Aaron. 2020. “On the Frequency and Severity of Interstate Wars.” In *Lewis Fry Richardson: His Intellectual Legacy and Influence in the Social Sciences*, edited by Nils Petter Gleditsch, 113–27. Cham: Springer International Publishing. https://doi.org/10.1007/978-3-030-31589-4_10 (https://doi.org/10.1007/978-3-030-31589-4_10).

Embrechts, Haan, Paul. 2000. “Modellinf Multivariate Extremes.” In *Extremes and Integrated Risk Management*, edited by Paul Embrechts, 59–67. RISK Publications.

Gelman, Andrew, John B Carlin, Hal S Stern, David B Dunson, Aki Vehtari, and Donald B Rubin. 2013. *Bayesian Data Analysis*. CRC press. https://stat.columbia.edu/~gelman/book/ (https://stat.columbia.edu/~gelman/book/).

Gelman, Andrew, and Sander Greenland. 2019. “Are Confidence Intervals Better Termed ‘Uncertainty Intervals?’.” *BMJ*, September, l5381. https://doi.org/10.1136/bmj.l5381 (https://doi.org/10.1136/bmj.l5381).

Hosking, J. R. M., and J. R. Wallis. 1987. “Parameter and Quantile Estimation for the Generalized Pareto Distribution.” *Technometrics* 29 (3): 339–49. http://www.jstor.org/stable/1269343 (http://www.jstor.org/stable/1269343).

Johnson, N. L., and S. Kotz. 1970. *Continuous Univariate Distributions*. Continuous Univariate Distributions 1. Hougton Mifflin. https://books.google.com/books?id=-wPvAAAAMAAJ (https://books.google.com/books?id=-wPvAAAAMAAJ).

Juan Camilo Bohorquez, Alexander R. Dixon, Sean Gourley. 2009. “Common Ecology Quantifies Insurgencies.” *Nature* 462 (17). doi:10.1038/nature08631 (https://doi:10.1038/nature08631).

Keynes, John Maynard. 2010. “How to Pay for the War.” In *Essays in Persuasion*, 367–439. London: Palgrave Macmillan UK. https://doi.org/10.1007/978-1-349-59072-8_27 (https://doi.org/10.1007/978-1-349-59072-8_27).

Krug, Etienne G, James A Mercy, Linda L Dahlberg, and Anthony B Zwi. 2002. “The World Report on Violence and Health.” *The Lancet* 360 (9339): 1083–88. https://doi.org/https://doi.org/10.1016/S0140-6736(02)11133-0 (https://doi.org/10.1016/S0140-6736(02)11133-0).

Mahmoud, Mahmoud Riad, and Amani Shaheen Abd El-Ghafour. 2015. “Fisher Information Matrix for the Generalized Feller-Pareto Distribution.” *Communications in Statistics - Theory and Methods* 44 (20): 4396–4407.

Marc Helbling, Daniel Meierrieks. 2020. "Transnational Terrorism and Restrictive Immigration Policies." *Journal of Peace Research* 57 (4). https://doi.org/10.1177/0022343319897105 (https://doi.org/10.1177/0022343319897105).
McElreath, Richard. 2020. *Statistical Rethinking: A Bayesian Course with Examples in R and Stan*. Second edition. CRC Press. https://xcelab.net/rm/statistical-rethinking/ (https://xcelab.net/rm/statistical-rethinking/).
Merrill, Ryan K., and Anthony W. Orlando. 2020. "Oil at Risk: Political Violence and Accelerated Carbon Extraction in the Middle East and North Africa." *Energy Economics* 92: 104935. https://doi.org/https://doi.org/10.1016/j.eneco.2020.104935 (https://doi.org/10.1016/j.eneco.2020.104935).
Pearl, J., M. Glymour, and N. P. Jewell. 2016. *Causal Inference in Statistics: A Primer*. Wiley. https://books.google.com/books?id=L3G-CgAAQBAJ (https://books.google.com/books?id=L3G-CgAAQBAJ).
Rafael Lozano, et al., Mohsen Naghavi. 2012. "Global and Regional Mortality from 235 Causes of Death for 20 Age Groups in 1990 and 2010: A Systematic Analysis for the Global Burden of Disease Study 2010." *Lancet* 380: 2095–2128. file:///C:/Users/wgfoo/OneDrive/Documents/0-research/terrorism-fdi/gpd-individual-effects/elsev-article/violence/lazano2012.pdf (file:///C:/Users/wgfoo/OneDrive/Documents/0-research/terrorism-fdi/gpd-individual-effects/elsev-article/violence/lazano2012.pdf).
Richardson, Lewis F. 1944. "The Distribution of Wars in Time." *Journal of the Royal Statistical Society* 107 (3/4): 242–50. http://www.jstor.org/stable/2981216 (http://www.jstor.org/stable/2981216).
Team, Stan Development. 2020a. "RStan: The r Interface to Stan (r Package Version 2.17.3) [Computer Software]." http://mc-stan.org/ (http://mc-stan.org/).
———. 2020b. "The Stan Core Library (Version 2.18.0)." http://mc-stan.org/ (http://mc-stan.org/).
Tobias Böhmelt, Vincenzo Bove. 2020. "Does Cultural Proximity Contain Terrorism Diffusion?" *Journal of Peace Research* 57 (2). https://doi.org/10.1177/0022343319864425 (https://doi.org/10.1177/0022343319864425).
UN75-Office. 2021. "Shaping Our Furure Together." https://www.un.org/sites/un2.un.org/files/un75_final_report_shapingourfuturetogether.pdf (https://www.un.org/sites/un2.un.org/files/un75_final_report_shapingourfuturetogether.pdf).
Vehtari, A. Gelman, A., and J. Gabry. 2015. "Efficient Implementation of Leave-One-Out Cross-Validation and WAIC for Evaluating Fitted Bayesian Models."
Vehtari, Gelman, A. 2017. "Practical Bayesian Model Evaluation Using Leave-One-Out Cross-Validation and WAIC." *Statistical Computing* 27: 1413–32.
Watanabe, S. 2010. "Asymptotic Equivalence of Bayes Cross Validation and Widely Applicable Information Criterion in Singular Learning Theory." *Journal of Machine Learning Research* 11: 3571–94.
Zhang, J., and M. A. Stephens. 2009. "A New and Efficient Estimation Method for the Generalized Pareto Distribution." *Technometrics* 51: 316–25.

---

1. In 1940, Keynes (2010) in his essay "How to Pay for the War." argued that, for war-time purposes, gross domestic product as consumption, investment, and government spending (not income) offered the most relevant way to measure war-time procurement trends and shortfalls. However, aggregate measures such as GDP fail to take into account externalities such as environmental and social costs, including the impact of intense and sporadic violence on generations of persons. This lack of inclusion magnifies the disproportionate focus on production and domestic consumption to the detriment of valuing such goals as good governance, equity, environmental conservancy, and stability all summed up in a human good of order.↩

2. We would be estimating across 49 countries 5 distinct types of political violence using 3 base parameters for the GPD for a total of \(49 \times 5 \times 3=735\) combinations of impact parameters. Why subject ourselves to this? In this way we can make statistical statements about the transmission of particular types of political violence across borders and across types of violence.↩

3. Embrechts (2000) analyzes the divergent nature of thick tail distributions.↩

4. Definition: Number of years a newborn infant could expect to live if prevailing patterns of age-specific mortality rates at the time of birth stay the same throughout the infant's life. Source: UNDESA (2019a). World Population Prospects: The 2019 Revision. Rev 1. New York. (https://population.un.org/wpp/). Accessed 30 April 2020.↩

5. Definition: Average number of years of education received by people ages 25 and older, converted from education attainment levels using official durations of each level.
   Source: UNESCO Institute for Statistics (2020), Barro and Lee (2018), ICF Macro Demographic and Health Surveys, UNICEF Multiple Indicator Cluster Surveys and OECD (2019b).↩

6. Definition: GDP in a particular period divided by the total population in the same period. Source: World Bank (2020a). World Development Indicators database. Washington, DC (http://data.worldbank.org). Accessed 22 July 2020.↩

7. See Technical Note 1 at this site for further aspects of the collection and computation of variables. (http://hdr.undp.org/sites/default/files/hdr2020_technical_notes.pdf) and for details on how the Human Developent Index is calculated.↩

8. Definition: Sum of equity capital, reinvestment of earnings, other longterm capital and short-term capital, expressed as a percentage of GDP. Source: World Bank (2020a). World Development Indicators database. Washington, DC. (http://data.worldbank.org). Accessed 22 July 2020.↩

9. For a similar derivation of a slightly different mixture see Johnson and Kotz (1970), an encyclopedic reference for continuous distributions, both convergent like the Gaussian distribution, and divergent in moments like the power law and generalized Pareto distirbutions.↩

10. The Rhat4 measured is based on Gelman et al. (2013). A number close to 1 indicates convergent mixing of samples across the 4 Markov chains.↩