

# Reactive Stepping to Prevent Falling for Humanoids

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**Abstract**— This paper proposes a reactive motion controller for a humanoid robot to maintain balance against a large disturbance, by relatively stepping. A reactive step is performed by the robot, so that it reduces the disturbance force. Several problems are addressed: first the motion is designed to ensure the respect of stepping constraints such as a dynamical stability, motion feasibility of the swing leg and so on. Moreover the stepping has to be generated in real-time and to be updated as quick as possible after the disturbance. To overcome these problems, we extend simultaneous the center of gravity (COG) and the zero-moment point (ZMP) planning based on a generic analytical solution of the linear inverted pendulum [8]. The ZMP fluctuation and the modification of foot placement are determined by numerical optimization according to the position and velocity error of the COG due to the disturbance. All these computations are performed at low cost. The proposed method is validated through several simulations.

## I. INTRODUCTION

Humanoid robots have to improve their stability before operating in a human environment. Although a robot can absorb a small disturbance without stepping, a reactive stepping is necessary when subject to a large disturbance (see Fig. 1). However, stepping reactively requires to be able to update a motion satisfying the dynamic stability constraints in real-time.

Because the dynamics of a humanoid robot becomes non linear and high dimension, it is difficult to obtain sufficient real-time performances for balance recovery. Therefore, most of previous researches in reactive stepping were based on the linear inverted pendulum model through a state feedback controller [1]-[6]. Miura discretized a linear inverted pendulum model by step cycles under instantaneous changing of the support leg, and determined an appropriate landing position by feedback on the COG position and velocity [1]. Komura presented a modification of the foot position to counteract a strong disturbance using the angular momentum induced by the inverted pendulum [2]. Nishiwaki proposes the *dynamically stable pattern method*, which updates the COG trajectory through the ZMP tracking control in a short cycle using the preview control [3]. Pratt presented the

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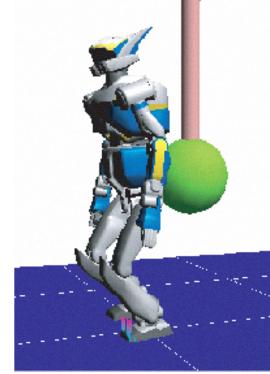


Fig. 1. Typical disturbance that may lead to stepping or falling.

*capture point*, which computes a stable region for the inverted pendulum with flywheel model [4]. Diedam proposes adaptive foot positioning under several constraints using a linear-model predictive control [5]. Tajima demonstrated a humanoid robot that can recover from human-pushing disturbances during running in place [6].

However the major part of these works assumed a constant ZMP position during the single support phase, and no detail were given about how to update the foot placement from sensor. Alternatively, it was considered that the balance recovery could be done within one step.

In this paper, we propose to extend the planning method of both the COG and ZMP proposed in [8]. Regarding foot placements as unknown parameters, a reactive stepping will be archived in a low computational cost. At first, a fast real-time gait planning method based on an analytical solution is formulated as shown in Section II. Then we extend a real-time gait-planning method and not only the COG and the ZMP but also the foot placement are determined by an numerical-optimization solver in Section III. The swing motion which can be modified immediately for a request is shown in Section IV. Several simulations of reactive motion are finally presented in Section V, before concluding.

## II. FAST SIMULTANEOUS THE COG AND THE ZMP PLANNING

We previously proposed online simultaneous planning of the COG and the ZMP which allows immediate modification of foot placement[9],[10] based on analytical approach[8]. In this method, to modify a foot placement immediately, the 3rd and 4th order polynomials are applied to the ZMP trajectory with a fluctuation. In a similar way, the faster calculation method for the unknown coefficients of analytical solution

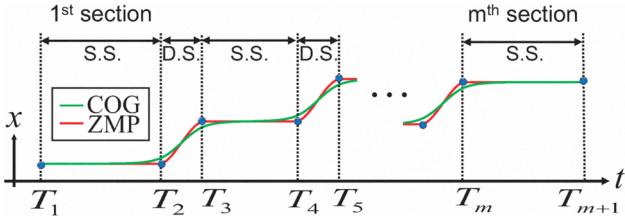


Fig. 2. The COG and The ZMP trajectories

was appeared by Hong[11]. This paper also derives fast COG and ZMP simultaneous planning using above formulation.

Let us focus on the COG motion on sagittal plane, and the COG dynamics of a humanoid robot in  $x$ -axis can be approximated by an inverted pendulum which is given as

$$\ddot{x}(t) = \omega^2(x(t) - p_x(t)), \quad (1)$$

$$\omega \equiv \sqrt{\frac{g + \ddot{z}(t)}{z(t) - p_z(t)}},$$

where  $x$  and  $z$  are horizontal and vertical position of the COG.  $p_x$  is the ZMP position.  $g$  means gravity constant. The COG motion on lateral plane can be also formulated as the same equation.

At first, a biped gait is divided into several sections every single support phase (S.S.) and double support phase (D.S.) shown in Fig. 2 and each section is applied to an analytical solution. Where  $T_1$  is current time, and  $m$  is number of sections. If  $N$  steps are planned in advance,  $m$  becomes  $2N + 1$ .

At  $j$ -th section, let us assume that the ZMP  $p_x^{(j)}$  can be represented by  $N_j$ -th order polynomial, that is

$$p_x^{(j)}(t) = \sum_{i=0}^N a_i^{(j)}(\Delta t_j)^i, \quad (2)$$

$$\Delta t_j \equiv t - T_j,$$

where  $a_i^{(j)}$  is coefficient of polynomials which is determined by boundary conditions and connectivity of the trajectory of the COG and the ZMP. We assume that the COG height is constant and is calculated at an average of initial and terminal value in each section.

$$\omega_j = \sqrt{\frac{g}{\dot{z}_j}}, \quad (3)$$

$$\dot{z}_j = \frac{(z(T_{j+1}) - p_z(T_{j+1})) - (z(T_j) - p_z(T_j))}{2}$$

From (1)-(3), the analytical solution with 4th order polynomial of the ZMP trajectory can be obtained.

$$x^{(j)}(t) = V^{(j)} \cosh(\omega_j \Delta t_j) + W^{(j)} \sinh(\omega_j \Delta t_j) + \left( a_0^{(j)} + \frac{2}{\omega_j^2} a_2^{(j)} + \frac{24}{\omega_j^4} a_4^{(j)} \right) + \left( a_1^{(j)} + \frac{6}{\omega_j^2} a_3^{(j)} \right) \Delta t_j + \left( a_2^{(j)} + \frac{12}{\omega_j^2} a_4^{(j)} \right) (\Delta t_j)^2 + a_3^{(j)} (\Delta t_j)^3 + a_4^{(j)} (\Delta t_j)^4 \quad (4)$$

Substituting the initial position and velocity of the COG  $x^{(j)}(T_j)$ ,  $\dot{x}^{(j)}(T_j)$  and the ZMP  $p_x^{(j)}(T_j)$ ,  $\dot{p}_x^{(j)}(T_j)$  at  $j$ -th section into (2), (4) and its derivatives, some coefficients can be calculated as

$$a_0^{(j)} = p_x^{(j)}(T_j), \quad (5)$$

$$a_1^{(j)} = \dot{p}_x^{(j)}(T_j), \quad (6)$$

$$V^{(j)} = x^{(j)}(T_j) - p_x^{(j)}(T_j) - \frac{2}{\omega_j^2} a_2^{(j)} - \frac{24}{\omega_j^4} a_4^{(j)}, \quad (7)$$

$$W^{(j)} = \frac{1}{\omega_j} \left( \dot{x}^{(j)}(T_j) - \dot{p}_x^{(j)}(T_j) - \frac{6}{\omega_j^2} a_3^{(j)} \right). \quad (8)$$

In term of continuity, the relation between the sections of the COG and the ZMP ( $j = 1 \dots m-1$ ) can be represented.

$$x(T_{j+1}) \equiv x^{(j)}(T_{j+1}) = x^{(j+1)}(T_{j+1}) \quad (9)$$

$$\dot{x}(T_{j+1}) \equiv \dot{x}^{(j)}(T_{j+1}) = \dot{x}^{(j+1)}(T_{j+1}) \quad (10)$$

$$p_x(T_{j+1}) \equiv p_x^{(j)}(T_{j+1}) = p_x^{(j+1)}(T_{j+1}) \quad (11)$$

$$\dot{p}_x(T_{j+1}) \equiv \dot{p}_x^{(j)}(T_{j+1}) = \dot{p}_x^{(j+1)}(T_{j+1}) \quad (12)$$

From (2) - (12), the terminal COG  $X(T_{j+1})$  can be expressed by the initial COG  $X(T_j)$ , the boundary condition of the ZMP  $p(T_j), p(T_{j+1})$  and unknowns  $a_4^{(j)}$  which is given as

$$X(T_{j+1}) = \mathbf{A}^{(j)} X(T_j) + \left[ \begin{array}{cc} \mathbf{B}_1^{(j)} & \mathbf{B}_2^{(j)} \end{array} \right] \left[ \begin{array}{c} \mathbf{p}(T_j) \\ \mathbf{p}(T_{j+1}) \end{array} \right] + \mathbf{C}^{(j)} a_4^{(j)}, \quad (13)$$

where

$$\mathbf{X}(T_j) = [x(T_j) \ \dot{x}(T_j)]^T, \quad \mathbf{p}(T_j) = [p_x(T_j) \ \dot{p}_x(T_j)]^T.$$

More details of (13) are given as follows.

$$\mathbf{A}^{(j)} = \mathbf{A}_{11}^{(j)} \in \mathbb{R}^{2 \times 2}$$

$$\mathbf{B}_1^{(j)} = \mathbf{A}_{12}^{(j)} + \mathbf{B}_{11}^{(j)} (\mathbf{B}_{21}^{(j)})^{-1} \mathbf{A}_{22}^{(j)} \in \mathbb{R}^{2 \times 2}$$

$$\mathbf{B}_2^{(j)} = \mathbf{B}_{11}^{(j)} (\mathbf{B}_{21}^{(j)})^{-1} \in \mathbb{R}^{2 \times 2}$$

$$\mathbf{C}^{(j)} = \mathbf{B}_{12}^{(j)} + \mathbf{B}_{11}^{(j)} (\mathbf{B}_{21}^{(j)})^{-1} \mathbf{B}_{22}^{(j)} \in \mathbb{R}^2$$

$$\mathbf{A}_{11}^{(j)} = \begin{bmatrix} c_j & \frac{s_j}{\omega_j} \\ \omega_j s_j & c_j \end{bmatrix}, \quad \mathbf{A}_{12}^{(j)} = \begin{bmatrix} 1 - c_j & -\frac{s_j}{\omega_j} + \Delta T_j \\ -\omega_j s_j & 1 - c_j \end{bmatrix},$$

$$\mathbf{A}_{22}^{(j)} = \begin{bmatrix} 1 & \Delta T_j \\ 0 & 1 \end{bmatrix},$$

$$\mathbf{B}_{11}^{(j)} = \begin{bmatrix} \frac{2}{\omega_j^2} (1 - c_j) + (\Delta T_j)^2 & \frac{6}{\omega_j^2} (\Delta T_j - \frac{s_j}{\omega_j}) + (\Delta T_j)^3 \\ 2(\Delta T_j - \frac{s_j}{\omega_j}) & \frac{6}{\omega_j^2} (1 - c_j) + 3(\Delta T_j)^2 \end{bmatrix},$$

$$\mathbf{B}_{12}^{(j)} = \begin{bmatrix} \frac{24}{\omega_j^4} (1 - c_j) + \frac{12}{\omega_j^2} (\Delta T_j)^2 + (\Delta T_j)^4 \\ \frac{24}{\omega_j^2} (\Delta T_j - \frac{s_j}{\omega_j}) + 4(\Delta T_j)^3 \end{bmatrix},$$

$$\mathbf{B}_{21}^{(j)} = \begin{bmatrix} (\Delta T_j)^2 & (\Delta T_j)^3 \\ 2\Delta T_j & 3(\Delta T_j)^2 \end{bmatrix}, \quad \mathbf{B}_{22}^{(j)} = \begin{bmatrix} (\Delta T_j)^4 \\ 4(\Delta T_j)^3 \end{bmatrix},$$

$$\Delta T_j \equiv T_{j+1} - T_j.$$

We assume a few future steps are preplanned to prevent the ZMP fluctuation largely. For instance, if  $n$ -th steps are

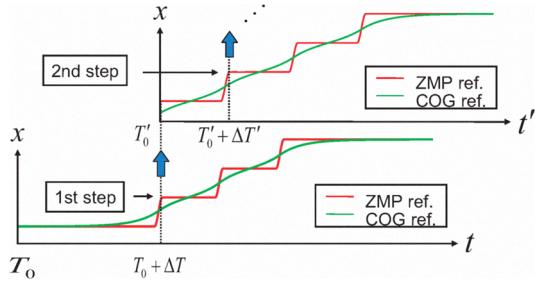


Fig. 3. Sequential gait planning

predesigned, the number of section  $m$  becomes  $2n+1$ . Generally, 3 steps are enough to suppress the ZMP fluctuation (see Fig. 3). When the future steps does not exist including stopping, these step length will be set as 0. Substituting (13) at  $j$ -th section into its at  $j+1$ -th section sequentially, the COG position and velocity at the end of  $m$ -th section  $\mathbf{X}(T_{m+1})$  can be obtained.

$$\mathbf{X}(T_{m+1}) = \hat{\mathbf{A}}^{(m)} \mathbf{X}(T_1) + \hat{\mathbf{B}}^{(m)} \hat{\mathbf{p}} + \hat{\mathbf{C}}^{(m)} \hat{\mathbf{a}}_4 \quad (14)$$

where

$$\hat{\mathbf{p}} = [ \mathbf{p}^T(T_1) \dots \mathbf{p}^T(T_m) \mathbf{p}^T(T_{m+1}) ]^T \quad (15)$$

$$\hat{\mathbf{a}}_4 = [ a_4^{(1)} \dots a_4^{(m-1)} a_4^{(m)} ]^T \quad (16)$$

$$\hat{\mathbf{A}}^{(m)} = \prod_{i=m}^1 \mathbf{A}^{(i)} \quad (17)$$

$$\begin{aligned} \hat{\mathbf{B}}^{(m)} &= \left[ \left( \prod_{i=m}^2 \mathbf{A}^{(i)} \right) \mathbf{B}_1^{(1)} \right. \\ &\quad \left( \prod_{i=m}^3 \mathbf{A}^{(i)} \right) \left( \mathbf{A}^{(2)} \mathbf{B}_2^{(1)} + \mathbf{B}_1^{(2)} \right) \\ &\quad \dots \mathbf{A}^{(m)} \mathbf{B}_2^{(m-1)} + \mathbf{B}_1^{(m)} \mathbf{B}_2^{(m)} \right] \quad (18) \end{aligned}$$

$$\begin{aligned} \hat{\mathbf{C}}^{(m)} &= \left[ \left( \prod_{i=m}^2 \mathbf{A}^{(i)} \right) \mathbf{C}^{(1)} \left( \prod_{i=m}^3 \mathbf{A}^{(i)} \right) \mathbf{C}^{(2)} \right. \\ &\quad \dots \mathbf{A}^{(m)} \mathbf{C}^{(m-1)} \mathbf{C}^{(m)} \left. \right] \quad (19) \end{aligned}$$

Then, unknown coefficients can be calculated as follows.

$$\hat{\mathbf{a}}_4 = (\hat{\mathbf{C}}^{(m)})^+ (\mathbf{X}(T_{m+1}) - \hat{\mathbf{A}}^{(m)} \mathbf{X}(T_1) - \hat{\mathbf{B}}^{(m)} \hat{\mathbf{p}}) \quad (20)$$

The value of an element of  $\hat{\mathbf{a}}_4$  is equivalent to the ZMP fluctuation. In fact, it is enough to consider the ZMP fluctuation only at the first and the last sections[10].

$$\begin{bmatrix} a_4^{(1)} \\ a_4^{(m)} \end{bmatrix} = \left[ \left( \prod_{i=m}^2 \mathbf{A}^{(i)} \right) \mathbf{C}^{(1)} \mathbf{C}^{(m)} \right]^{-1} (\mathbf{X}(T_{m+1}) - \hat{\mathbf{A}}^{(m)} \mathbf{X}(T_1) - \hat{\mathbf{B}}^{(m)} \hat{\mathbf{p}}) \quad (21)$$

In (21), only  $2 \times 2$  inverse matrix are required. Therefore it is suitable to calculate in real-time. Equation (20) and (21) can be connected any trajectories of the COG and the ZMP in principle, although the ZMP fluctuation will be caused.

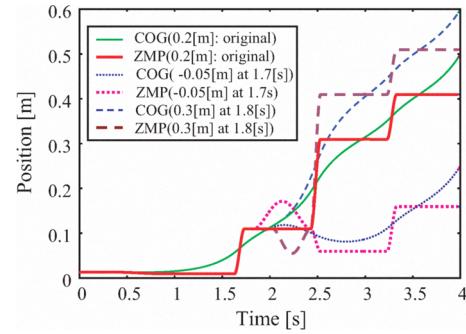


Fig. 4. The ZMP fluctuation in case of the modification of foot placement

Finally the foot placement can be changed at anytime. For examples, in Fig. 4, the maximum ZMP fluctuation becomes 0.071[m] when the foot placement is changed from 0.2[m] to -0.05[m] at 1.7[s] and 0.045[m] when the foot placement is changed from 0.2[m] to 0.3[m] at 1.8[s]. These ZMP fluctuation can be suppressed by preview control[9] (originally proposed by Nishiawaki[3]).

### III. REACTIVE STEPPING

#### A. Extention of Simultaneous Planning of the COG and the ZMP

In previous section, the desired ZMP pass points which are given from the desired foot placement are always satisfied in exchange for the ZMP fluctuation. In this section, the ZMP pass points are also regarded as unknown parameters and these are determined so that the desired foot placement can be satisfied as much as possible.

The ZMP pass points in (15) can be replaced as the current and the desired future foot placements, and the ZMP velocity. The second term in right side of (14) can be decomposed as

$$\begin{aligned} \hat{\mathbf{B}}^{(m)} \hat{\mathbf{p}} &\equiv \left[ \hat{\mathbf{B}}_c^{(m)} \hat{\mathbf{B}}_f^{(m)} \hat{\mathbf{B}}_v^{(m)} \right] \begin{bmatrix} \mathbf{u}_c \\ \mathbf{u}_f \\ \mathbf{p}_v \end{bmatrix}, \quad (22) \\ \mathbf{u}_c &= u^{(1)}, \mathbf{u}_f = [u^{(2)} \dots u^{(n)}]^T, \\ \mathbf{p}_v &= [\dot{p}_x(T_1), \dots, \dot{p}_x(T_m)]^T, \end{aligned}$$

where  $\bigcirc_c$  and  $\bigcirc_f$  denote the current and the future foot placement respectively.  $\mathbf{p}_v$  is a vector of the ZMP velocity at the ZMP pass points.  $n$  is number of steps ( $m = 2n+1$ ). Substituting (22) into (14),

$$\begin{aligned} &\hat{\mathbf{B}}_f^{(m)} \mathbf{u}_f + \hat{\mathbf{C}}^{(m)} \hat{\mathbf{a}}_4 \\ &= \mathbf{X}(T_{m+1}) - \hat{\mathbf{A}}^{(m)} \mathbf{X}(T_1) - \hat{\mathbf{B}}_c^{(m)} \mathbf{u}_c - \hat{\mathbf{B}}_v^{(m)} \mathbf{p}_v, \end{aligned} \quad (23)$$

can be obtained. Then this formula can be derived as a quadratic programming problem.

$$\begin{aligned} \min \frac{1}{2} &\{(\mathbf{u}_f - \mathbf{u}_f^{ref})^T \mathbf{Q}_1 (\mathbf{u}_f - \mathbf{u}_f^{ref}) \\ &+ (\mathbf{a}_4 - \mathbf{a}_4^{ref})^T \mathbf{Q}_2 (\mathbf{a}_4 - \mathbf{a}_4^{ref})\} \\ \text{subject to } &(23) \end{aligned} \quad (24)$$

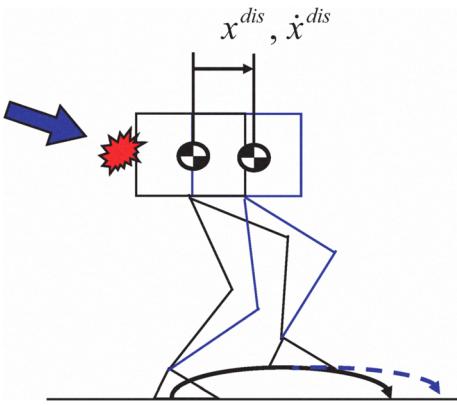


Fig. 5. The COG errors corresponding to disturbance

Where,  $\mathbf{u}_f^{ref}$  is the desired foot placement.  $\mathbf{a}_f^{ref}$  is the desired ZMP fluctuation which is obtained in previous section. Ideally this value is desired as 0.  $\mathbf{Q}_1$  and  $\mathbf{Q}_2$  are defined as positive definite symmetric matrix. Solution of (24) can be analytically calculated as

$$\begin{aligned} \mathbf{s} &= \mathbf{s}^{ref} \\ &- \mathbf{Q}^{-1} \mathbf{M}^T (\mathbf{M} \mathbf{Q}^{-1} \mathbf{M}^T)^{-1} (\mathbf{M} \mathbf{s}^{ref} - \mathbf{N}), \end{aligned} \quad (25)$$

where

$$\mathbf{s} = \begin{bmatrix} \mathbf{u}_f \\ \hat{\mathbf{a}}_4 \end{bmatrix}, \quad \mathbf{s}^{ref} = \begin{bmatrix} \mathbf{u}_f^{ref} \\ \hat{\mathbf{a}}_4^{ref} \end{bmatrix}, \quad (26)$$

$$\mathbf{M} = \begin{bmatrix} \hat{\mathbf{B}}_f^{(m)} & \hat{\mathbf{C}}^{(m)} \end{bmatrix}, \quad (27)$$

$$\mathbf{N} = \mathbf{X}(T_{m+1}) - \hat{\mathbf{A}}^{(m)} \mathbf{X}(T_1) - \hat{\mathbf{B}}_c^{(m)} \mathbf{u}_c - \hat{\mathbf{B}}_v^{(m)} \mathbf{p}_v, \quad (28)$$

$$\mathbf{Q} = \begin{bmatrix} \mathbf{Q}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}_2 \end{bmatrix}. \quad (29)$$

Equation (25) does not always generate the feasible foot placement which avoids a self collision, satisfies a joint limitation and so on. This problem can be avoided by setting an admissible disturbance in advance or recalculating this equation which next foot placement is fixed as the feasible maximum/minimum value.

### B. Disturbance estimation

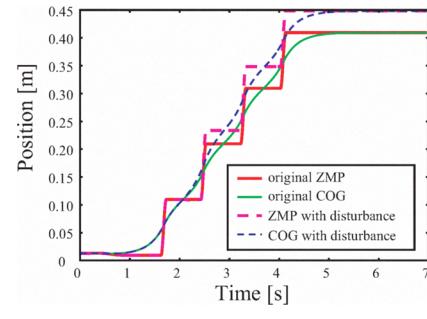
We assume that a stabilization controller keeps a reference attitude of the trunk. The disturbance will be estimated as a result of an attitude error through a stabilization (see Fig. 5).

$$\mathbf{x}^{dis} = (\mathbf{R}^{err} - \mathbf{I})(\mathbf{x}^{ref} - \mathbf{p}^{ref}) \quad (30)$$

$$\mathbf{R}^{err} = \mathbf{R}^{res}(\mathbf{R}^{ref})^T \quad (31)$$

$\mathbf{x}^{ref}$ ,  $\mathbf{p}^{ref}$  and  $\mathbf{R}^{ref}$  denote the reference COG position, the ZMP position and attitude of the trunk respectively.  $\mathbf{R}^{res}$  is the estimated attitude of the trunk.

An estimated disturbance includes not only an external force but also an internal modeling error, an attitude estimation error, a sensor measurement error and so on. Therefore



(a) sagittal plane

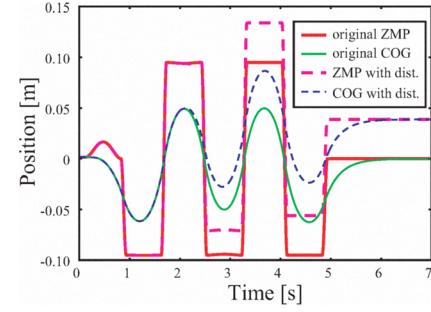


Fig. 6. The COG and ZMP trajectories with disturbance

we prepare a threshold  $x_{threshold}^{dis}$  for the compensation value.

$$\left\{ \begin{array}{ll} \text{if } x^{dis} > x_{threshold}^{dis} : & \bar{x}^{dis} = x^{dis} - x_{threshold}^{dis} \\ \text{if } x^{dis} < -x_{threshold}^{dis} : & \bar{x}^{dis} = x^{dis} + x_{threshold}^{dis} \\ \text{otherwise :} & \bar{x}^{dis} = 0 \end{array} \right.$$

Finally a reactive motion can be achieved by adding an estimated disturbance to the initial COG position and velocity. In Fig. 6, when an impulsive velocity disturbance  $\Delta \mathbf{V} = [0.02 \ 0.02]^T$  [m/s] is added to the COG at 1.95[s], the foot placement is modified.

### IV. SWING LEG TRAJECTORY

To modify a foot placement immediately, swing leg motion is desirable to generate sequentially as well as the COG motion. The trajectory of swing leg should also be satisfied the initial and the terminal conditions, and the continuity. 4-1-4th order polynomial is applied to the horizontal motion in  $x$  and  $y$  axes. The vertical motion in  $z$  axis is generated by two 4-1-4th order polynomials shown in Fig. 7.

This trajectory is composed of 3 parts, that is the acceleration period  $[T_1 : T_2]$ , the constant velocity  $[T_2 : T_3]$  and the deceleration period  $[T_3 : T_4]$ .

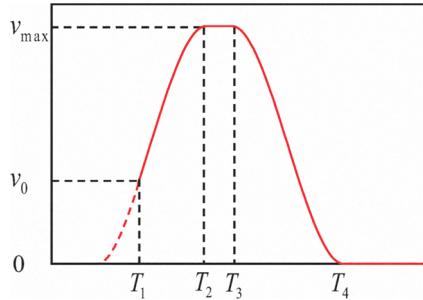
$$T_1 \leq t \leq T_2 :$$

$$x_1(t) = a_{10} + a_{11}(t - T_1) + a_{12}(t - T_1)^2 + a_{13}(t - T_1)^3 + a_{14}(t - T_1)^4 \quad (32)$$

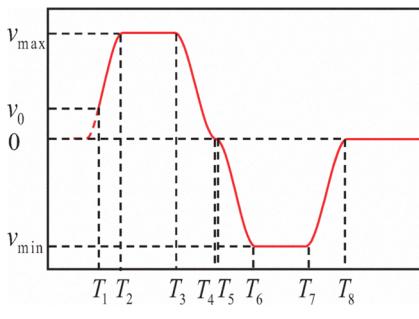
$$T_2 \leq t \leq T_3 :$$

$$x_2(t) = a_{20} + a_{21}(t - T_2) \quad (33)$$

$$T_3 \leq t \leq T_4 :$$



(a) horizontal axis



(b) vertical axis

Fig. 7. Velocity profiles of swing leg

$$\begin{aligned} x_3(t) = & a_{30} + a_{31}(t - T_4) + a_{32}(t - T_4)^2 \\ & + a_{33}(t - T_4)^3 + a_{34}(t - T_4)^4 \end{aligned} \quad (34)$$

We set the boundary condition as

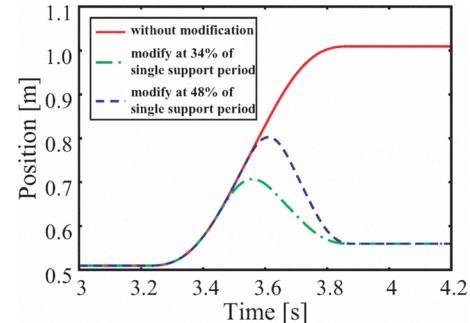
$$\begin{aligned} \text{Initial conditions: } & x_1(T_1), \dot{x}_1(T_1), \ddot{x}_1(T_1), \\ \text{Terminal conditions: } & x_3(T_4), \dot{x}_3(T_4), \ddot{x}_3(T_4), \\ \text{Continuity: } & x_1(T_2) = x_2(T_2), x_2(T_3) = x_3(T_3), \\ & \dot{x}_1(T_2) = \dot{x}_2(T_2), \dot{x}_2(T_3) = \dot{x}_3(T_3), \\ & \ddot{x}_1(T_2) = 0, \ddot{x}_3(T_3) = 0 \end{aligned}$$

From these conditions, the coefficients of polynomials in (32)-(34) can be obtained analytically. Second velocity profile of vertical motion at  $[T_5 : T_8]$  can be generated to move an opposite direction but same calculation way as the first velocity profile.

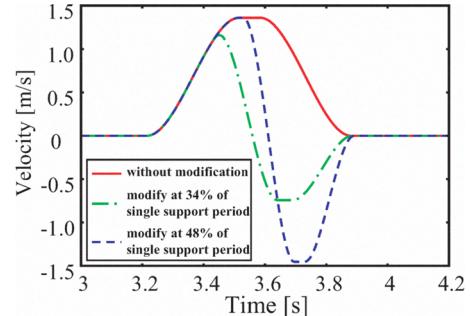
Examples of the position and the velocity of the foot trajectories with immediate modification are shown in Fig. 8 (a) and (b) respectively. The preplanned foot motion which moves from approximately 0.5[m] to 1[m] is shown in solid line. The step lengths are changed from 0.25[m] to -0.2[m] at 34% and 48% of single support phase respectively (dots and dashed line). Time parameters  $T_i (i = 1 \dots 4)$  of a horizontal foot motion in case of modification of foot placement are also distributed according to the remaining of traveling time of swing leg.

## V. SIMULATION

To confirm the effectiveness of proposed method, simulation results will be shown using HRP-2[12]. HRP-2 was hit from behind and side by a pendulum whose mass is 30[kg]



(a) Position



(b) Velocity

Fig. 8. Swing leg trajectory with immediate modification

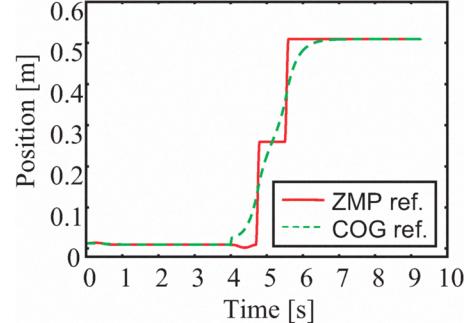


Fig. 9. Disturbance comes from behind

at 0.47[m/s]. The COG and the ZMP trajectories at sagittal plane in case of a backward disturbance is shown in Fig. 9. The COG and the ZMP trajectories at frontal plane in case of a side disturbance is shown in Fig. 10. Although robot have fallen over in case of without reactive stepping (see Fig. 11 and 13), it could keep a balance with reactive stepping by proposed method (see Fig. 12 and 14). In this simulation, HRP-2 could keep a balance against a hit of a pendulum until 0.47[m/s] at any time. To improve a stability, the landing position of the swing leg should be considered as an attitude error.

## VI. CONCLUSION AND FUTURE WORKS

This paper proposed a reactive motion controller for a humanoid robot to maintain balance against a large disturbance. We extended simultaneous the COG and the ZMP planning based on a generic analytical solution of the linear inverted

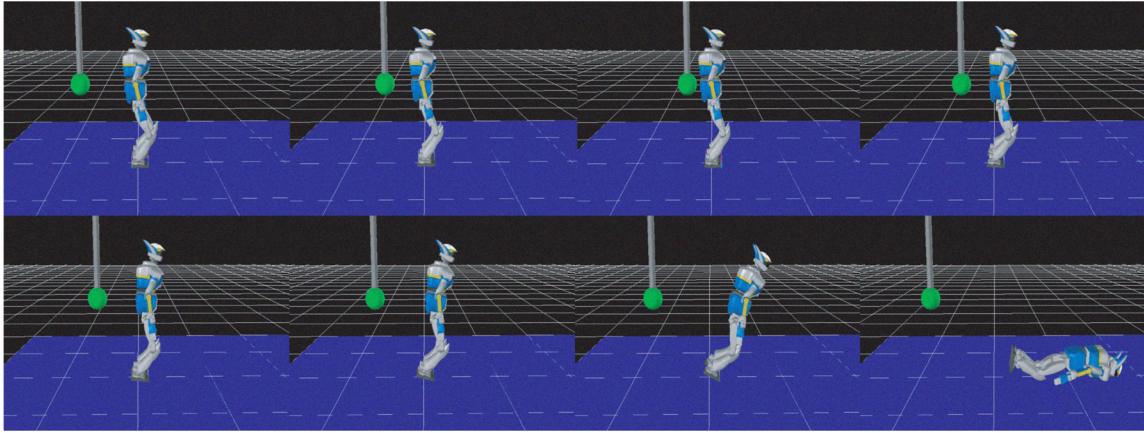


Fig. 11. Without modification of foot place

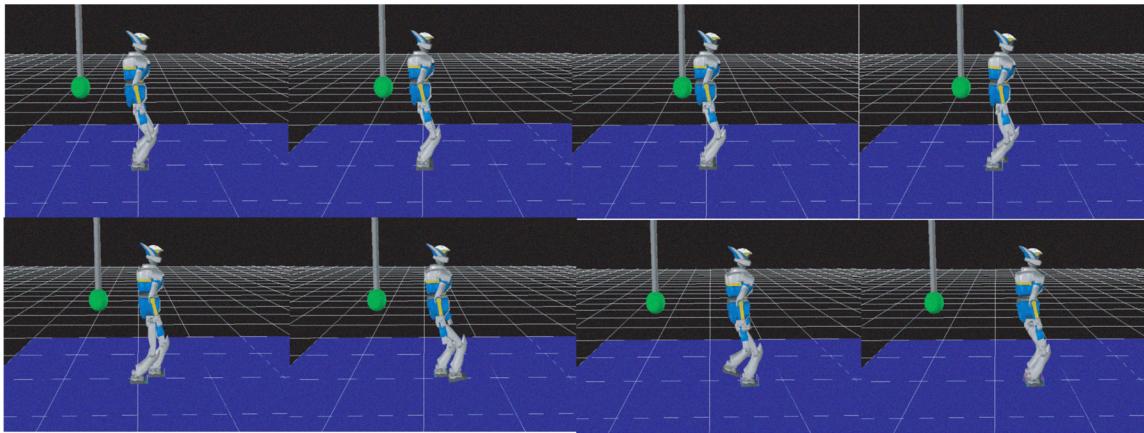


Fig. 12. With modification of foot place

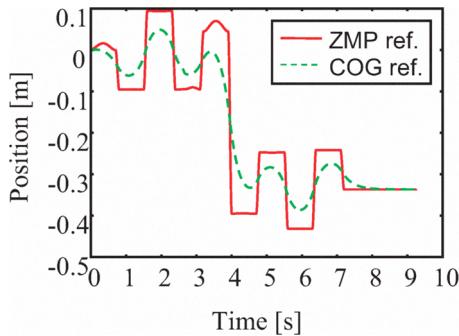


Fig. 10. Disturbance comes from side

pendulum. The ZMP fluctuation and the modification of foot placement were determined by numerical optimization according to the position and velocity error of the COG due to the disturbance. All these computations were performed at low cost. The proposed method was validated through several simulations.

## VII. ACKNOWLEDGMENTS

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## REFERENCES

- [1] H. Miura, and I. Shimoyama, "Dynamic walk of a biped," Int. Jour. of Robotics Research, Vol.3, No.2, pp.60-74, 1984
- [2] T. Komura, H. Leung, S. Kudoh, and J. Kuffner, "A Feedback Controller for Biped Humanoids that Can Counteract Large Perturbations During Gait," in Proc. of IEEE Int. Conf. on Robotics and Automation, pp.1989-1995, 2005.
- [3] K. Nishiwaki, and S. Kagami, "High Frequency Walking Pattern Generation based on Preview Control of ZMP," in Proc. of IEEE Int. Conf. on Robotics and Automation, pp.2667-2672, 2006.
- [4] J. Pratt, J. Carff, S. Drakunov, and A. Goswami, "Capture Point: A Step toward Humanoid Push Recovery," in Proc. of IEEE-RAS Int. Conf. on Humanoid Robots, pp.200-207, 2006.
- [5] H. Diedam, D. Dimitrov, P. B. Wieber, K. Mombaur, and M. Diehl, "Online Walking Gait Generation with Adaptive Foot Positioning through Linear Model Predictive Control," in Proc. of IEEE/RSJ Int. Conf. on Intelligent Robotics and Systems, pp.1121-1126, 2008.
- [6] R. Tajima, D. Honada, and K. Suga, "Fast Running Experiments Involving a Humanoid Robot," in Proc. of IEEE Int. Conf. on Robotics and Automation, pp.1571-1576, 2009
- [7] M. Vukobratovic, and D. Juricic, "Contribution to the Synthesis of Biped Gait," IEEE Trans. on Bio-Med. Eng., vol.BME-16, no.1, pp.1-6, 1969.

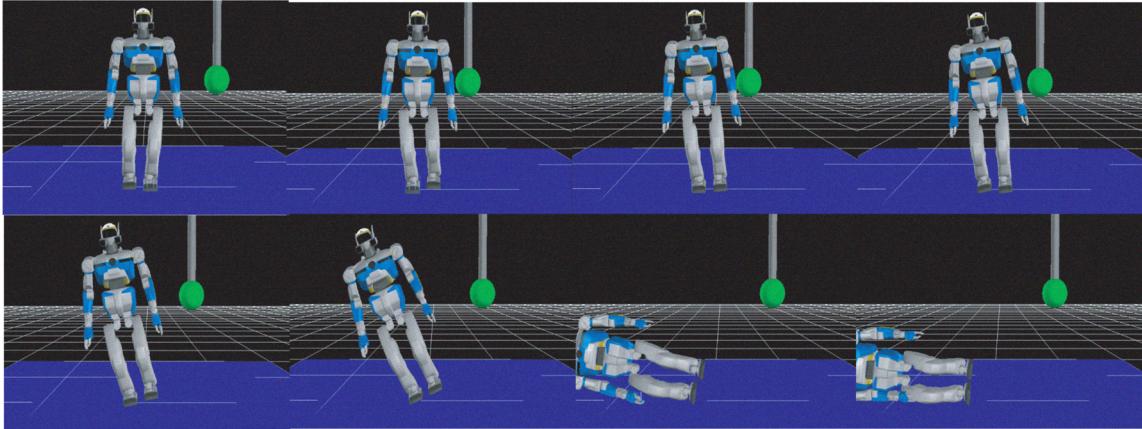


Fig. 13. Without modification of foot place

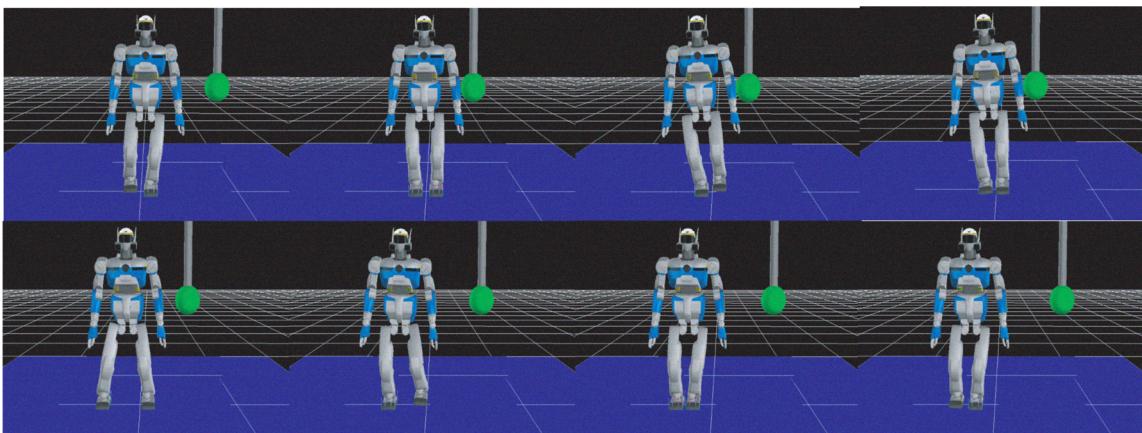


Fig. 14. With modification of foot place

- [8] K. Harada, S. Kajita, K. Kaneko, and H. Hirukawa, "An Analytical Method on Real-time Gait Planning for a Humanoid Robot," in Proc. of IEEE-RAS Int. Conf. Humanoid Robots, Paper #60, 2004.
- [9] M. Morisawa, K. Harada, S. Kajita, K. Kaneko, F. Kanehiro, K. Fujiwara, S. Nakaoka, and H. Hirukawa, "A Biped Pattern Generation Allowing Immediate Modification of Foot Placement in Real-Time," in Proc. of IEEE-RAS Int. Conf. Humanoid Robots, pp.581-586, 2006.
- [10] M. Morisawa, K. Harada, S. Kajita, S. Nakaoka, K. Fujiwara, F. Kanehiro, K. Kaneko, and H. Hirukawa, "Experimentation of Humanoid Walking Allowing Immediate Modification of Foot Place Based on Analytical Solution," in Proc. of IEEE Int. Conf. on Robotics and Automation, pp. 3989-3994, 2007.
- [11] S. Hong, Y. Oh, Y. H. Chang, and B. J. You, "An Omni-directional Walking Pattern Generator Method for Humanoid Robots with Quartic Polynomials," in Proc. of IEEE/RSJ Int. Conf. on Intelligent Robots and Systems, pp.4207-4213, 2007.
- [12] K. Kaneko, et al., "The Humanoid Robot HRP2," Proc. of IEEE Int. Conf. on Robotics and Automation, pp.1083-1090, 2004.