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Robust stabilization of uncertain input-delay systems by sliding mode control with delay compensation☆

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Abstract

In this paper, a sliding mode control is proposed for the robust stabilization of uncertain linear input-delay systems with nonlinear parametric perturbations. The proposed sliding surface includes a predictor to compensate for the input delay of the system. A robust control law is derived to ensure the existence of a sliding mode and to overcome the effects of the delay and uncertainty in the sliding mode. A numerical example is given to illustrate the design procedure and results of the proposed approach. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Sliding mode control; Robust stabilization; Uncertain time-delay systems; Input-delay compensation; Predictor

1. Introduction

Time delays can be found in various engineering systems such as chemical processes, pneumatic/hydraulic systems, biological systems, and economic systems. Compared to the systems without delay, the presence of delay makes it more difficult to achieve the satisfactory performance and stability of the systems. Many authors deal with the control problem of the time-delay systems via predictor-based controllers (Fiagbedzi & Pearson, 1986; Furukawa & Shimemura, 1983). Predictor-based controllers includes a predictor to compensates for the time delay, and so is well known as a remedy to overcome the effect of the time delay. Under a predictor-based controller, therefore, a time-delay system can be transformed into a delay-free system in which the delay is eliminated from the closed-loop system.

Another major problem in real-world systems is the robust control to minimize the effects of uncertainty. One of the solutions to this problem for uncertain time-delay systems is to use the memoryless state feedback control. Many results can be found in the literature; the Reccati equation approaches (Choi & Chung, 1995, Phoojaruenchanachai & Furuta, 1992; Shen, Chen & Kung, 1991), the linear matrix inequality (LMI) approaches (Dugard & Verriest, 1997; Li & de Souza, 1997) and the Lyapunov min–max approach (Cheres, Gutman & Palmor, 1989). These approaches do not consider compensation for input delay. Moreover, they are sensitive to the uncertainty, which directly affects the control systems. Kojima, Uchida, Shimemura & Ishijima, 1994 investigated a robust stabilization problem for uncertain input-delay systems by using H_∞ control theory.

A sliding mode control (SMC) has attractive features such as fast response and good transient response (Hung, Gao & Hung, 1993; Ryan, 1983). It is also insensitive to variations in system parameters, and external disturbances. Other SMC schemes are proposed for uncertain linear systems with state delay (Koshkouei & Zinober, 1996; Shyu & Yan, 1993). Their methods have the same benefits, but cannot be applied to systems with input delay. In these types of systems, the control input can be generated without delay to cancel out the uncertainty. Thus, the sliding mode exists for $t_s > t_0$, where t_s is the reaching time of sliding mode and t_0 is the initial time. Their controllers do not use any predictor to compensate for input delay.

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In this paper, a new sliding mode control is proposed for the robust stabilization of uncertain input-delay systems with nonlinear parameter perturbations. The proposed sliding surface includes a predictor which consists of not only the current state but also the past control input during the period of delay. The predictor is applied to compensate for the input delay of the system. A control law is derived to ensure the existence of a sliding mode and to minimize the effects of the delay and uncertainty in the sliding mode. We also examine the system behavior in the sliding mode through the reduced order dynamics of the transformed delay-free system under the proposed control. A guideline is given for the design of the sliding surface. A numerical simulation is given to illustrate the results.

2. System description

Let us consider a linear input-delay system with uncertainties described by

$$\dot{x}(t) = Ax(t) + Bu(t-\tau) + f_0(x(t),t) + f_1(x(t-\tau),t), \tag{1}$$

where $x \in \Re^n$, $u(t) \in \Re^m$ and $\tau \in ([0,\infty),\Re)$ are the state vector, the input vector and the known delay time, respectively, and A and B are constant matrices with appropriate dimensions. The uncertainties, $f_0(x(t),t)$ and $f_1(x(t-\tau),t)$, represent the nonlinear parameter perturbations with respect to the current state and the delayed state of the system, respectively. In addition to Eq. (1), the initial conditions are given by

$$x(0) = x^0, \quad x_0(\theta) = \phi(\theta), \quad u_0(\theta) = v(\theta), \quad -\tau \le \theta \le 0, \tag{2}$$

where $x_t(\theta) = x(t+\theta)$ and $u_t(\theta) = u(t+\theta)$. It is assumed that the pair (A,B) is stabilizable, i.e., $\mathrm{rank}[A, \exp(s\tau)B] = n$, and the states are available for feedback. It is also assumed that the unknown functions $f_0, f_1 : \Re^n \times \Re_+ \to \Re^n$ satisfy the matching conditions, i.e.,

$$\begin{aligned} f_0(x(t),t) &= Be_0(x(t),t), \\ f_1(x(t-\tau),t) &= Be_1(x(t-\tau),t), \end{aligned} \tag{3}$$

where

$$\begin{aligned} \|e_0(x(t),t)\| &\le \rho_0\|x(t)\| + k, \\ \|e_0(x(t-\tau),t)\| &\le \rho_1\|x(t-\tau)\| \end{aligned} \tag{4}$$

for $\rho_0, \rho_1, k > 0$. In SMC, the matching condition is a suitable condition for which the system in the sliding mode is insensitive to uncertainty. Input-delay systems are not controllable by the control input for initial time, $t+\tau \le 0$. During the initial time, non-zero initial conditions and uncertainties can affect the stability of the system. In these types of systems, it is usually assumed that the initial condition $v(\theta) \in L^1((-\tau,0),\Re^m)$ exists.

3. Design of sliding mode controller

We consider a predictor (predictive state), $\bar{x} \in \Re^n$, as

$$\bar{x}(t) = e^{A\tau}x(t) + \int_{-\tau}^{0} e^{-A\theta}Bu(t+\theta)\,d\theta, \tag{5}$$

where $u_0(\theta) = v(\theta) \in L^1((-\tau,0),\Re^m)$. The sliding surface is defined as

$$\sigma(\bar{x}) = S\bar{x} = 0 \tag{6}$$

for $\sigma = [\sigma_1, \ldots, \sigma_m]^{\mathrm{T}} \in \Re^m$ and the sliding matrix, $S = [S_1^T, \ldots, S_m^T]^{\mathrm{T}} \in \Re^{m\times n}$. It is assumed that the matrix S is of full rank and the matrix SB is non-singular. The sliding matrix S is chosen so that the dynamics on the sliding surface has the desired closed-loop behaviors irrespective of input delay. Since the sliding surface includes the predictor which compensates for the input delay of the system on the surface, it depends on the size of the delay. When the predictor is bounded for initial time, $t+\tau \le 0$, the sliding surface is also bounded. This condition is needed to ensure the existence of sliding mode dynamics during the initial time.

After selecting the sliding surface, the next step is to choose a control law such that it satisfies the condition for the existence of the sliding mode; $\sigma^{\mathrm{T}}\dot{\sigma} < 0$. This condition ensures that the control law will force system trajectories toward the sliding surface in finite time and maintain them on the surface after then. We consider the following control structure of the form:

$$u(t) = u_{\mathrm{eq}} + u_{\mathrm{N}}, \tag{7}$$

where u_{eq} is an equivalent control for the nominal system of Eq. (1) without the uncertainty and u_{N} is a switching control to overcome the uncertainties of the system. The equivalent control law u_{eq} is derived by $\dot{\sigma} = 0$ for the nominal system of Eq. (1). Differentiating σ with respect to time gives

$$\dot{\sigma} = S\left[e^{A\tau}\dot{x}(t) + A\int_{-\tau}^{0} e^{-A\theta}Bu(t+\theta)\,d\theta + Bu(t) - e^{A\tau}Bu(t-\tau)\right]. \tag{8}$$

Substituting the nominal system of Eq. (1) without the uncertainty into Eq. (8) and setting $\dot{\sigma} = 0$, we have

$$\dot{\sigma} = S\left[e^{A\tau}Ax(t) + A\int_{-\tau}^{0} e^{-A\theta}Bu(t+\theta)\,d\theta + Bu_{\mathrm{eq}}(t)\right] = 0. \tag{9}$$

Then, the equivalent control is obtained by

$$u_{eq} = -[SB]^{-1}SA\left[e^{A\tau}x(t) + \int_{-\tau}^{0} e^{-A\theta}Bu(t+\theta)\,d\theta\right]$$
$$= -[SB]^{-1}SA\bar{x}. \tag{10}$$

Eq. (10) represents the predictive state feedback control that overcomes the effect of the delay of the nominal system of Eq. (1). Now, we need to eliminate the effect of the uncertainties in spite of the delays, and also to force the system trajectories toward the designed sliding surface. Then, the switching control U_N is chosen by

$$u_N = \begin{cases} -\dfrac{(SB)^{-1}\sigma S e^{A\tau}B}{\|\sigma\|}\hat{\delta}(x,t) & \text{if } \|\sigma\| \neq 0, \\ 0 & \text{otherwise,} \end{cases} \tag{11}$$

where $\hat{\delta}(x,t) = \rho\|x\| + k + \beta$, for $\rho = \rho_0 + \rho_1 q$, $q > 1$, $\beta > 0$, is the upper bound on the norm of the total uncertainty of the system. The switching control (11) is determined in advance to compensate for input delay by the sliding surface, and then enters the system after delay, τ. The uncertainty at time t is cancelled at $t + \tau$ by the switching control. It implies that the uncertainties given in the system are sequentially cancelled with each delay interval by the corresponding switching control.

Remark 1. Since the control input enters the system with the interval of delay, τ, the reaching motion of the sliding mode is generated after the delay, and the sliding mode exists for $t_s > t_0 + \tau$.

In order to ensure the existence of the sliding mode of system (1), we consider the time derivative of σ along uncertain input-delay system (1) as

$$\dot{\sigma} = S\left[e^{A\tau}Ax(t) + A\int_{-\tau}^{0} e^{-A\theta}Bu(t+\theta)\,d\theta + Bu(t)\right.$$
$$\left. + \, e^{A\tau}\{f_0(x(t),t) + f_1(x(t-\tau),t)\}\right]. \tag{12}$$

Substituting Eqs. (7) and (3) into Eq. (12) yields

$$\dot{\sigma} = SBu_N + (Se^{A\tau}B)\{e_0(x(t),t) + e_1(x(t-\tau),t)\}. \tag{13}$$

Now we are ready for the following:

Theorem. *If the control law* (7) *is used for system* (1), *then the sliding mode always exists*, *i.e.*, *dynamics* (13) *is asymptotically stable.*

Proof. We choose the Lyapunov function

$$V(\sigma,t) = \tfrac{1}{2}\sigma^T\sigma. \tag{14}$$

Differentiating V with respect to time along the trajectories of dynamics (13) gives

$$\dot{V} = \sigma^T[SBu_N + Se^{A\tau}B\{e_0(x(t),t) + e_1(x(t-\tau),t)\}]. \tag{15}$$

Substituting Eq. (11) into Eq. (15) yields

$$\dot{V} \leq -\|\sigma^T\|\|Se^{A\tau}B\|(\rho\|x\| + k + \beta)$$
$$+ \|\sigma^T\|\|Se^{A\tau}B\|\|e_0(x(t),t) + e_1(x(t-\tau),t.)\|. \tag{16}$$

From the Razumikhin theorem (Hale & Lunel, 1993), $\|x_t(\theta)\| < q\|x(t)\|$, $q > 1$, $-\tau \leq \theta \leq 0$, then

$$\|e_1(x(t-\tau),t)\| \leq \rho_1\|x(t-\tau)\| \leq \rho_1 q\|x(t)\|. \tag{17}$$

Thus

$$\|e_0(x(t),t) + e_1(x(t-\tau),t)\| \leq \|e_0\| + \|e_1\|$$
$$\leq \rho\|x(t)\| + k, \tag{18}$$

where $\rho = \rho_0 + \rho_1 q > 0$. We can finally obtain the following inequality:

$$\dot{V} \leq -\beta\|\sigma^T\|\|Se^{A\tau}B\| < 0 \tag{19}$$

for $\sigma \neq 0$, where $\det(Se^{A\tau}B) \neq 0$, since the matrix SB is nonsingular. Since β is positive, $\sigma \to 0$ as $t \to \infty$. □

From the above theorem the sliding mode along the sliding surface $\sigma = 0$ always exists in the finite time interval $[t_0 + \tau, t_s]$. In the next section, the behavior of the reduced order dynamics in the sliding mode is investigated.

4. Dynamics in the sliding mode

We first transform the input-delay system into a delay-free system so that both system share the same poles and provide the equivalent input/output mapping (Fiagedz & Pearson 1986; Kojima et al., 1994). Then, we can investigate the behavior of the reduced order dynamics in the sliding mode by using the transformed delay-free system.

Input-delay system (1) is transformed into a delay-free system as follows:

$$\dot{\bar{x}}(t) = A\bar{x}(t) + Bu(t) + e^{A\tau}\{f_0(x(t),t) + f_1(x(t-\tau),t)\}. \tag{20}$$

Let us consider a transformation matrix T given by

$$TB = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}, \tag{21}$$

where $B_2 \in \Re^{m\times m}$ is non-singular. Define

$$z = T\bar{x} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, \tag{22}$$

where $z_1 \in \Re^{n-m}$, $z_2 \in \Re^m$. Then, the reduced order dynamics in the sliding mode is obtained by

$$\dot{z}_1(t) = A_{11}z_1(t) + A_{12}z_2(t), \tag{23}$$

where

$$TAT = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}.$$

The sliding surface can be rewritten as

$$\sigma = \hat{S}z = 0, \tag{24}$$

where

$$\hat{S} = ST^{-1} = [\hat{S}_1 \quad \hat{S}_2] \tag{25}$$

for $\hat{S}_1 \in \Re^{m\times(n-m)}$ and $\hat{S}_2 \in \Re^{m\times m}$. From Eq. (24), we obtain the following constraint:

$$z_2 = -Kz_1, \tag{26}$$

where the matrix $K \in \Re^{m\times(n-m)}$ is given by

$$K = \hat{S}_2^{-1}\hat{S}_1, \tag{27}$$

where $\hat{S}_2$ is chosen to be non-singular. Combining Eqs. (23) and (26) gives the reduced order dynamics in the sliding mode as

$$\dot{z}_1(t) = (A_{11} - A_{12}K)z_1(t). \tag{28}$$

The dynamics of Eq. (28) exhibits a state feedback structure in which K and A_{12} represent the state feedback and an input matrix, respectively. Since (A, B) is a controllable pair, the matrix pair (A_{11}, A_{12}) is also controllable. Thus, the matrix K can be determined by any control scheme prescribing a desired closed-loop behavior of the reduced order dynamics (28).

Remark 2. The sliding surface matrix, S, can be found as follows:

(i) Find K so that assigns eigenvalues of the reduced order dynamics to the left-half plane,
(ii) Choose $\hat{S}_2 = I_m$,
(iii) $\hat{S}_1 = K$, $\hat{S} = [K \ I]$,
(iv) From Eq. (25)

$$S = \hat{S}T. \tag{29}$$

5. Numerical example

In order to illustrate the procedure of the proposed SMC scheme, we consider an unstable system as follows:

$$\dot{x}(t) = \begin{pmatrix} 0 & 1 \\ 3 & -2 \end{pmatrix} x(t) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(t-\tau) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} (e_0(x(t), t) + e_1(x(t-\tau), t)), \tag{30}$$

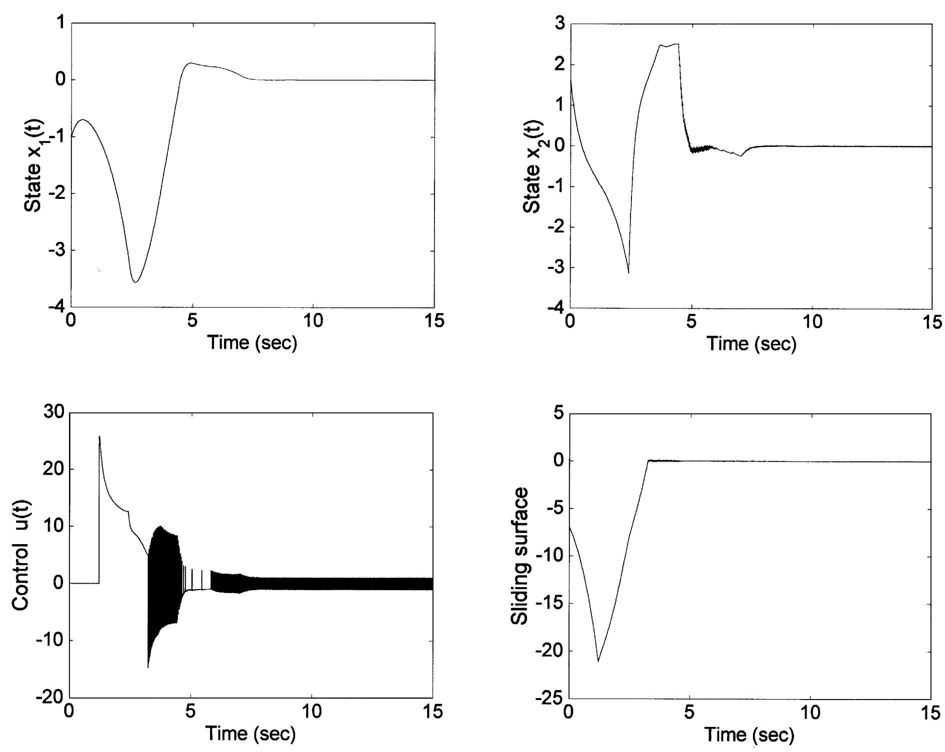

Fig. 1. States (x_1, x_2), control input (u) and sliding surface (σ).

where $x(0) = [-1.6\ \ 1]^{\mathrm{T}}$, $\tau = 1.2$, and the non-linear parameter perturbations and external disturbance are given by

$$e_0(x(t), t) = 0.3x_1(t)\sin(x_1(t)) + 0.2\sin(2\pi\omega t), \quad \omega = 30,$$

$$e_1(x(t-\tau), t) = 0.2x_2(t-\tau)\sin(x_2(t-\tau)). \tag{31}$$

It is easy to show $\rho_0 = 0.3$, $\rho_1 = 0.2$ and $k = 0.2$. The objectives are to determine the sliding surface and the control law that robustly stabilize the uncertain input-delay system in the sliding mode. According to the design procedure, we first choose $K = 5$ so that the assigned eigenvalue of the reduced order dynamics is -5. Then, choosing $T = I_n$ and $\hat{S}_2 = I_m$ results in $\hat{S}_1 = 5$. Thus, we have $S = [5\ 1]$. By choosing $q = 1.1$ in Eq. (11), the robust control law (7) is obtained. Fig. 1 shows that the system is asymptotically stable against uncertainties in the sliding mode. It is also shown that the reaching motion of the sliding mode is generated after delay, τ. Accordingly, the sliding mode exits at $t_s > t_0 + \tau$. Note that the small peaks in Fig. 1 are the effects of the delayed perturbations.

6. Conclusions

A sliding mode control with delay compensation is proposed for robust stabilization of uncertain linear input-delay systems. The sliding surface is designed to compensate for the input delay of the system. The robust control law is derived to ensure the existence of sliding mode and to yield sliding mode dynamics to overcome the effect of the delay of the system in the sliding mode. The proposed scheme depends on the size of delay. A guideline is also given for the design of the sliding surface. The simulation results imply that our method effectively controls the input-delayed system with nonlinear parameter perturbations and disturbance.

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