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Online Decision of Foot Placement using Singular LQ Preview Regulation

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Abstract— **In this paper, we introduce an online decision method of foot placement for legged robots that manages unknown external forces. It consists of a fast trajectory generation method and an optimization method of foot placement. The fast trajectory generation method is based on an explicit solution for singular LQ preview regulation problem of an inverted pendulum model. By using the regulator problem, the conditions of target ZMP trajectory that will not make the COM trajectory diverge and a fast generation method of the COM trajectory which satisfies the target ZMP trajectory are obtained. An online optimization of foot placement is realized by using the fast trajectory generation method. An experiment of the online decision method of foot placement that manages unknown external force shows the performance of the proposed method.**

I. Introduction

Online replanning of footstep placement is required for a humanoid robot so that is will not tip off even when an unexpected significant external force is applied. A simple approach is to adjust next landing foot position continuously[1]. To determine modified position, some simple rules were used. This method was able to respond to disturbance continuously.

As an approach for balancing against unknown external forces, Hyon et al. proposed a balancing method exploiting full-body compliance[2]. Replanning of step position was not mainly utilized.

An alternative approach is a footstep generation using framework of dynamical system. Sugihara proposed a regulator which unifies standing stability and stepping stability[3].

Several works involving footstep replanning against unknown external forces have been proposed. Morisawa et al. utilized an analytical solution of the linear invert pendulum model by limiting the ZMP trajectory as polynomial. Preview control of the linear invert pendulum model is a fast method to generate COM trajectory that realizes target ZMP trajectory[5]. Nishiwaki et al. proposed a method to modify footstep placement using characteristics of the preview control[7].

In many of previous methods, ZMP and COM trajectory are sought simultaneously. Because ZMP or COM trajectory depends on each other, it is difficult to obtain trajectory which satisfies required conditions and has desired property. For bipedal walking, a significant required condition of a ZMP and COM trajectory pair is that COM will not diverge relative to ZMP. In our method, ZMP and COM trajectory are obtained in two step.

- In the first step, the set of ZMP trajectories, the paired COM trajectory of which will not diverge is obtained.
- In the second step, desired ZMP and COM pair is chosen from the set.

Since the first step ensures the nondivergence condition, we only have to care abount other properties in the second step. To obtain the set in the first step, we utilize the preview control of the linear invert pendulum model. Explicit solution is obtained by using the form in which input weight is set to zero. The conditions of ZMP trajectory that the COM trajectory will not diverge is obtained by the solution. The characteristics of the obtained form also enable a fast trajectory generation. Online optimization of footstep replacement is realized by using the fast trajectory generation method.

This paper is organized as follows. In Section III, we propose a pattern generation method based on the singular LQ preview regulation problem. The nondivergence condition of ZMP and COM pair and fast COM trajectory generation method is obtained by the explicit solution of the singular LQ preview regulation problem obtained in Section III. In Section IV, an optimization method of foot placement for online decision of foot placement is described. The optimization method utilizes the nondivergence condition and fast COM trajectory generation method. The results from an experiment using the proposed method are shown in Section V. Finally, our conclusions are presented in Section VI.

II. Online Decision of Foot Placement

III. ZMP-COM Pattern Generation based on Singular LQ Preview Regulation

The proposed method in this paper is a modified version of the preview control method. In the conventional methods, the optimal regulator has no general explicit solution, the approach proposed here will get an explicit solution by using a limiting form in which the input weight is set to zero. By using this method, followings are obtained:

1) The conditions of target ZMP trajectory that the COM trajectory will not diverge.
2) Fast generation method of the COM trajectory which satisfies the target ZMP trajectory.

From the perspective of the search of the gait trajectory, 1) gives the search space of the target ZMP trajectories, and 2) is suitable for using in the evaluation function for search, which is repetitively used.

Let the differential of ZMP be input of the system, the motion equation of the Linear Inverted Pendulum Model is expressed as follows:

$$\frac{d}{dt}\begin{bmatrix} p \\ x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ -\frac{g}{z_h} & \frac{g}{z_h} & 0 \end{bmatrix} \begin{bmatrix} p \\ x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u, \tag{1}$$

$$p = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ x \\ \dot{x} \end{bmatrix}. \tag{2}$$

Here, p is the ZMP and x is the displacement of the center of mass. In this paper, the system is discretized by Euler's method for simplicity. The discretized system (A, B, C) is following:

$$x_{k+1} = Ax_k + u_k \tag{3}$$

$$y_k = cx_k \tag{4}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \Delta t \\ -a^2\Delta t & a^2\Delta t & 1 \end{bmatrix} \tag{5}$$

$$b = \begin{bmatrix} \Delta t \\ 0 \\ 0 \end{bmatrix} \tag{6}$$

$$c = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} a \equiv \sqrt{\frac{g}{z_h}}, \tag{7}$$

In the preview control theory, the evaluation function in terms of output weight Q and input weight R is described as follows:

$$J = \sum_{j=1}^{\infty} Q(p_j^{ref} - p_j)^2 + \ Ru_j^2. \tag{8}$$

This function is minimized by a control input expressed as:

$$u_k = -\ Kx_k + \sum_{i=1}^{\infty}\ f_i p_{k+i}^{ref}, \tag{9}$$

where, K and f_i include the solution of Riccati equation expressed in the following. In general, explicit solution of the equation is difficult to be obtained.

$$K \equiv \left(R + b^t P b\right)^{-1} b^t P A, \tag{10}$$

$$f_i \equiv \left(R + b^t P b\right)^{-1} b^t \left(A - bK\right)^{(i-1)t} c^t Q, \tag{11}$$

$$P = A^t P A + C^t Q C - A^t - A^t P b (R + b^t P b)^{-1} b^t P A. \tag{12}$$

A. Optimal Preview Control by Limiting as $R \to 0$

While the ZMP must strictly track the reference ZMP: p_k^{ref} in the support polygon, control input, which is the velocity of ZMP, has not always severe restrictions. Therefore, the input weight R is set to be a very small value compared to the output weight Q in common case.

Given this characteristic, analyzing the limit system in which R approaches 0 is useful. According to the literature[8][9], K^*, which is K at $R \to 0$, is given by the following equation:

$$K^* = \left(\hat{C}B\right)^{-1} \hat{C}A. \tag{13}$$

Here, $\hat{C}$ is a constant matrix which satisfies the following equation while system $\left(A, B, \hat{C}\right)$ is the minimum phase system:

$$\begin{aligned} &B^t(z^{-1}I_n - A^t)^{-1}C^t Q C(zI_n - A)^{-1}B \\ &= B^t(z^{-1}I_n - A^t)^{-1}\hat{C}^t Q \hat{C}(zI_n - A)^{-1}B \end{aligned} \tag{14}$$

Since the following $\hat{C}$ satisfies the condition, we obtain an explicit expression of K^* as follows:

$$\hat{C} = \left[-(1 + a\Delta t) \ \ 2 + a\Delta t \ \ \tfrac{(2+a\Delta t)}{a} \right], \tag{15}$$

$$a \equiv \sqrt{\frac{g}{z_h}},$$

$$K^* = \left[\tfrac{1+a\Delta t}{\Delta t} + \tfrac{a}{1+a\Delta t} \ \ -\tfrac{2+a\Delta t}{\Delta t} \ \ -\tfrac{2+a\Delta t}{a\Delta t} \right]. \tag{16}$$

The Riccati equation **eq.** (12) is deformed by using K as follows:

$$P = C^t C + (A - BK)^t P (A - BK) + K^t R K. \tag{17}$$

Let $R \to 0$, P^*, the limit of P, is:

$$P^* = C^t C + (A - BK^*)^t P^* (A - BK^*). \tag{18}$$

Since K^* is already obtained, we get a symmetric matrix P^* which is the solution of the equation as follows:

$$P^* = \begin{bmatrix} (1 + a\Delta t)^2 & -3a\Delta t - a^2\Delta t^2 - 2 & -3\Delta t - a\Delta t^2 - \frac{2}{a} \\ -3a\Delta t - a^2\Delta t^2 - 2 & \frac{2}{a\Delta t} + 4a\Delta t + a^2\Delta t^2 + 5 & \frac{2}{\Delta t a^2} + 4\Delta t + a\Delta t^2 + \frac{5}{a} \\ -3\Delta t - a\Delta t^2 - \frac{2}{a} & \frac{2}{\Delta t a^2} + 4\Delta t + a\Delta t^2 + \frac{5}{a} & \frac{2}{a^3\Delta t} + \frac{4\Delta t}{a} + \frac{5}{a^2} + \Delta t^2 \end{bmatrix} \tag{19}$$

Limit of the preview feed forward gain f_i^* and the output ZMP error to the reference ZMP at $R \to 0$ are obtained by using the derived P^*:

$$f_i^* = \frac{\delta_{i1}}{\Delta t} - a(2 + a\Delta t)\left(\frac{1}{1 + a\Delta t}\right)^{i+1}, \tag{20}$$

$$\frac{e_{k+1}^*}{e_k^*} = \frac{1}{1 + a\Delta t} < 1, \tag{21}$$

$$e_k^* \equiv p_k^* - p_{k-1}^{ref}.$$

From **eq.** (21), e_k^* is maximized at $k = 1$. If $e_1^* = 0$, $e_k^* = 0$ at $k > 1$, and the output ZMP tracks the reference ZMP in one sampling delay. $e_1^* = 0$ can be realized by modifying the reference ZMP as follows:

$$e_1^* = \left| p_0 - p_0^{ref} + \Delta t \left(-K^* \begin{bmatrix} p_0 \\ x_0 \\ \dot{x}_0 \end{bmatrix} + \sum_{i=1}^{\infty} f_i^* p_i^{ref} \right) \right|. \tag{22}$$

Because the difference of p_k^{ref} is a sparse vector for the case in which p_k^{ref} is expressed as a step function, using differential form is advantageous.

$$\begin{aligned} e_1^* = & \left| p_0 - p_0^{ref} + \Delta t \left(-K^* \begin{bmatrix} p_0 \\ x_0 \\ \dot{x}_0 \end{bmatrix} \right.\right. \\ & \left.\left. + \sum_{i=1}^{\infty} (F_\infty^* - F_i^*) \Delta p_i^{ref} + F_\infty^* p_1^{ref} \right) \right|, \end{aligned} \tag{23}$$

where,

$$\begin{aligned} F_n^* &\equiv \sum_{k=1}^{n} f_k^*, \\ \Delta p_k^{ref} &\equiv p_{k+1}^{ref} - p_k^{ref}. \end{aligned}$$

eq. (22) or **eq.** (23) are the conditions for the target ZMP trajectory so that the COM trajectory will not diverge.

Then the method of fast COM trajectory generation that realizes given target ZMP trajectory is discussed in the following. Convolution of the preview control gain and reference ZMP sequence in **eq.** (9) will be:

$$u_k^{ff} \equiv \sum_{i=1}^{\infty} f_i p_{k+i}^{ref}. \tag{24}$$

It is deformed by using **eq.** (20) to the following recurrence form, provided that R approaches 0:

$$S_{k-1} = p_k^{ref} + \frac{S_k}{1 + a\Delta t}, \tag{25}$$

$$u_k^{ff*} = \frac{p_k^{ref}}{\Delta t} - \frac{a(2 + a\Delta t) S_k}{(1 + a\Delta t)^2}. \tag{26}$$

In special cases that $p_k^{ref} = p_{k+1}^{ref} (k \geq N)$, S_N will be:

$$S_N = p_k^{ref} \frac{(1 + a\Delta t)}{a\Delta t}. \tag{27}$$

Recurrence form is advantageous in terms of reduced computational complexity without the need for convolution calculation.

When **eq.** (22) or **eq.** (23) is satisfied, COM trajectory is simply calculated by following formula.

$$\begin{aligned} x_{k+1} &= x_k + \dot{x}_k \Delta t \\ \dot{x}_{k+1} &= a^2 \Delta t (x_k - p_k^{ref}) + \dot{x}_k \end{aligned} \tag{28}$$

This formula is equivalent to integral of motion equation of LIPM. Because **eq.** (22) is satisfied, COM will not diverge by the integral.

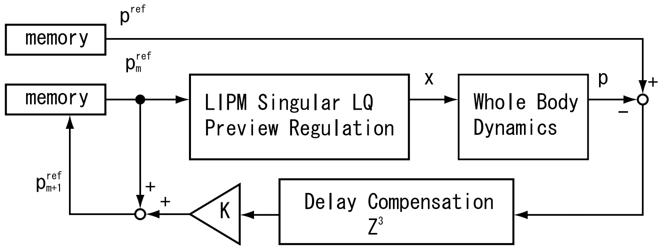

Fig. 1. Iterative Learning Control

B. Linear Inverted Pendulum model to Multi Link Model

To obtain the actual robot motion by using a whole-body dynamics model, it is necessary to generate the whole body motion trajectory that satisfies the reference ZMP trajectory. That trajectory is calculated by an iterative method based on the trajectory obtained by the linear inverted pendulum model. **Fig.** 1 shows the block diagram of the iterative learning control method. In each iteration, the ZMP error is fed back to the reference ZMP. As noted previously, in case of $R \to 0$, ZMP tracks the reference ZMP in exactly one sampling delay. The joint velocity and the ZMP is calculated by numerical differentiation method. Therefore, the total delay from the reference ZMP to the calculated ZMP will be exactly three sampling. The three sampling delay is compensated in the feedback calculation. In contrast, in the case without the limiting form, it is difficult to guarantee the feedback delay. To achieve fast calculation, computations of the inverse kinematics, the momentum, and the angular momentum in each time frame are parallelized by utilizing the independent nature. Trajectories of the swing leg and the body z-axis are designed in advance and the inputs of this iteration.

IV. Optimization of the Foot Placement

The goal of deciding the foot placement is to find a feasible pair of the ZMP and COM trajectories that are connected to the current robot state. Here, the current robot state is given as a pair of position and velocity of the COM and the ZMP: $\begin{bmatrix} p_0 & x_0 & \dot{x}_0 \end{bmatrix}^t$. The ZMP trajectory needs to be piecewise continuous.

Robots have many constraints, such as, joint range of motion, maximum torque of joints, and collisions between links. The trajectory that minimize an heuristic evaluation function, which approximates these constraints, is obtained by an optimization method.

For simplicity, we fix first two frames of target ZMP to the initial ZMP value as it has little effect to the optimization:

$$p_0^{ref} = p_1^{ref} = p_0.$$

The condition of the target ZMP trajectory that will not make COM trajectory diverge is derived from **eq.** (23).

$$0 = -K^* \begin{bmatrix} p_0 \\ x_0 \\ \dot{x}_0 \end{bmatrix} + \sum_{i=1}^{\infty} (F_\infty^* - F_i^*) \Delta p_i^{ref} + F_\infty^* p_0 \tag{29}$$

Applying **eq.** (20), we obtain:

$$F_\infty^* - F_k^* = -\frac{2 + a\Delta t}{\Delta t(1 + a\Delta t)^{k+1}},$$
$$F_\infty^* = \frac{1}{\Delta t} - \frac{2 + a\Delta t}{\Delta t(1 + a\Delta t)}.$$

K^* is given in **eq.** (16). Substituting these for **eq.** (29), we get;

$$0 = \begin{bmatrix} 1 & -1 & \frac{-1}{a} \end{bmatrix} \begin{bmatrix} p_0 \\ x_0 \\ \dot{x}_0 \end{bmatrix} + \sum_{i=1}^{\infty} \frac{\Delta p_i^{ref}}{(1 + a\Delta t)^{k+1}}. \quad (30)$$

When the initial state $\begin{bmatrix} p_0 & x_0 & \dot{x}_0 \end{bmatrix}^t$ is given, Δp_i^{ref} is restricted to the space expressed by **eq.** (30). Trajectory optimization is carried out in this space. Note that the Δp_i^{ref} is a sparce vector when the ZMP pattern is a step function.

While various optimization method can be applied, DIRECT (DIviding RECTangle) method[11] is adopted in this paper. DIRECT is a robust optimization method, but not fast. Because the evaluation function is lightweight, enough execution speed is realized even by this technique. The evaluation function consists of the following steps:

- Generate p^{ref} sequence from the optimization parameters.
- Generate the swing foot trajectory.
- Generate the COM trajectory using **eq.** (28).
- Evaluate the COM trajectory and the swing foot trajectory.

A cycloid pattern is used for the swing foot trajectory. As the evaluation function of the COM trajectory and the swing foot trajectory, a heuristic function is used. In our implementation, the maximum of the relative acceleration from COM to each foot is minimized. Any evaluation functions including non-linear functions or functions involved with external environments of the robot can be used.

TABLE I
CALCULATION TIME ($\Delta t = 0.005s$, $N = 300$)

	Time [us]
Generation of Swing Foot Trajectory	0.78
Generation of COM Trajectory	0.48
Evaluation of Trajectory	0.60
Total (Including Overheads)	1.9

Table I shows the calculation time of each step using a Intel E8600 4.0GHz core. Evaluation count in an optimization is about 175 in case of the experiment described in the following section. Total calculaion time for an optimization is about 330 us. It is sufficiently short because the sampling time of our controller is 1 ms.

V. EXPERIMENT

In the following experiment, a standing robot is pushed by an unknown external force. The height of COM is 0.62 m. The height of position where the body is pushed is about 0.803 m. Three steps are used to react for the force, because larger number of steps has little effect to the balance recovery from the disturbance. Five parameters, x and y position of the first two steps and the landing time of the first step, are selected for the optimization parameters. Under the constraint of **eq.** (30), optimizations are carried out in three dimensional space.

Replanning of the footstep placement is triggered by a heuristic criterion. When the shift of the body link from its original position exceeds 0.03 m, optimization for deciding the foot placement and the joint trajectory generation for succeeding five seconds are carried out within the next 1 ms, which is the sampling period of the controllers.

A. *Configuration of the Biped Robot Hardware*

TABLE II
SPECIFICATION OF HRP3L-JSK

Weight	53kg
Degree of Freedom	12DOF
Link Length	0.3m
Voltage	80V
Motor Drivers	200A(max)/50A(cont.)
Power Source(Capacitor)	13.5F 51.1mΩ
Sensors	6-Axis Force Sensor, Accelerometer, FOG

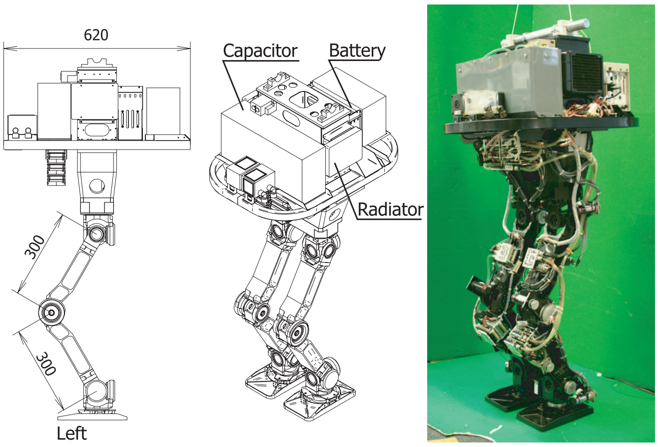

Fig. 2. Overview of HRP3L-JSK

We developed a biped robot HRP3L-JSK[10] (**Fig.** 2). **Table** II shows its hardware specification. It has 12 degrees of freedom in legs. All motors are 200W AC motors and motors for the pitch joints at the hip and the knees are liquid-cooled in order to achieve higher continuous torque. By high voltage and high current liquid-cooled motor drivers and a capacitor, instantaneous joint output range is experimentally over 1000 deg/s of rate and 300 Nm of torque.

B. *An Experiment of Reaction to External Force*

The iteration number in the optimization process is empirically determined beforehand for this experiment. 174 evaluations was done in eight iterations. Three axis external forces were measured for the evaluation purpose. Figure 3 shows the measured force when the robot was kicked. The peak force was 597 N. Figure 4 shows the planned and measured ZMP and COM trajectory. At 0.5 s in the plot, the gait pattern generation is triggered. It shows that COM did not diverge and the planned COM is continuously connected to precedent COM trajectory. The error of the measured ZMP from the planned one is considered to be due to the bounce at landing. Figure 5 shows the sequential photographs of the experiment.

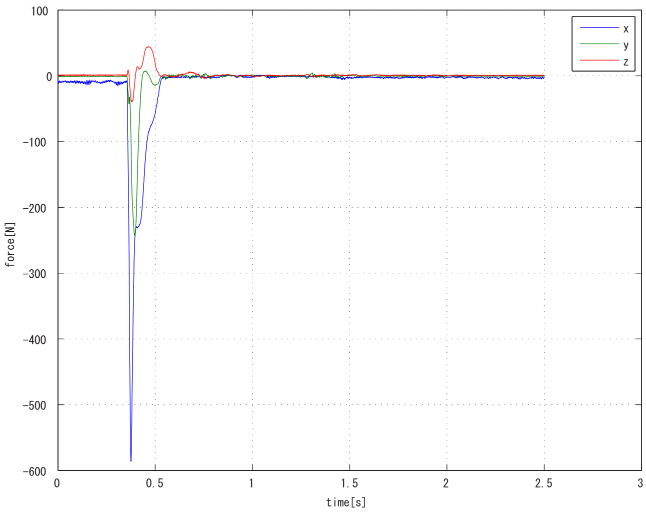

Fig. 3. Measured External Force

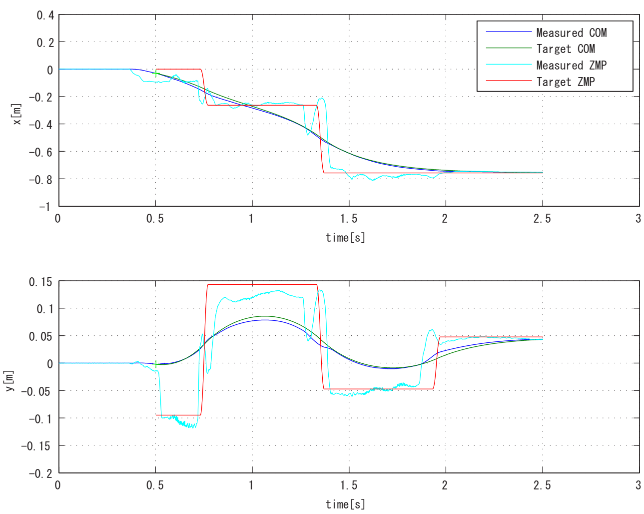

Fig. 4. The COM and ZMP Trajectory

VI. Conclusions

In this paper, an online decision method of footstep placement by using singular LQ preview regulation of an inverted pendulum model was introduced. The explicit solution of the regulation problem gives the conditions of target ZMP trajectory that will not make the COM trajectory diverge. The fast generation method of the COM trajectory which satisfies the target ZMP trajectory was also developed.

By utilizing the proposed methods, fast online gait optimization was performed. In the experiment, the proposed method was implemented and the online decision of the footstep placement under unknown external force was shown.

VII. Future Works

The proposed online decision method of the footstep placement can be naturally applied to the avoidance of tip-off during walking. A future work is to design stable controllers which uses the footstep placement generation successively.

In this paper, we used a heuristic cost function. Ideally, actual constraints of robots, such as, joint range of motion, maximum torque of joints, and collisions between links, are directly utilized. To involve these constraints, we will seek for more efficient optimization techniques and utilization of more computing resource.

References

[1] Raibert M.H.: Legged robots that balance, MIT Press, 1986.
[2] Sang-Ho Hyon, Hale J.G. and Cheng G.: Full-Body Compliant Human-Humanoid Interaction: Balancing in the Presence of Unknown External Forces, IEEE Transactions on Robotics, Volume 23, Issue 5, pp. 884-898, 2007.
[3] Sugihara T.: Standing stabilizability and stepping maneuver in planar bipedalism based on the best COM-ZMP regulator, Proceedings of the IEEE International Conference on Robotics and Automation, ICRA 2009., pp. 1966-1971, 2009.
[4] Morisawa M., Kanehiro F., Kaneko K., Mansard N., Sola J., Yoshida E., Yokoi K., Laumond J.: Combining suppression of the disturbance and reactive stepping for recovering balance, IEEE/RSJ International Conference on Intelligent Robots and Systems, IROS 2010., pp. 3150-3156, 2010.
[5] S. Kajita, F. Kanehiro, K Kaneko, K Fujiwara, K Harada, K Yokoi, and H. Hirukawa, Biped Walking Pattern Generation by using Preview Control of Zero-Moment Point, in Proc. IEEE Int Conf. on Robotics and Automation, pp. 1620-1626, 2003.
[6] Richard L. Knoblauch1, Martin T. Pietrucha, Marsha Nitzburg, Field Studies of Pedestrian Walking Speed and Start-Up Time, Transportation Research Record: Journal of the Transportation Research Board, Volume 1538, pp. 27-38, 1996.
[7] Koichi Nishiwaki, Satoshi Kagami: Online Walking Control System for Humanoids with Short Cycle Pattern Generation, International Journal of Robotics Research , Vol.28, No.6, pp.729-742, May, 2009.
[8] U. Shaked, Explicit Solution to the Singular Discrete-Time Stationary Linear Filtering Problem, IEEE Transactions on Automatic Control, AC-30-1, pp. 34-47, 1985.
[9] Y. Peng and M. Kinnaert, Explicit Solution to the Singular LQ Regulation Problem, IEEE Transactions on Automatic Control, AC-37-5, pp. 633-636, 1992.
[10] Junichi Urata, Yuto Nakanishi, Kei Okada, Masayuki Inaba: Design of High Torque and High Speed leg Module for High Power Humanoid, in Proceedings of The 2010 IEEE/RSJ International Conference on Intelligent Robots and Systems, pp.4497–4502, 2010.
[11] C.D. Perttunen Jones, D.R. and B.R. Stuckman. Lipschitzian optimization without the Lipschitz constant. Joural of Optimization Theory and Applications, Vol.79,No.1,pp.157-181, 1993.

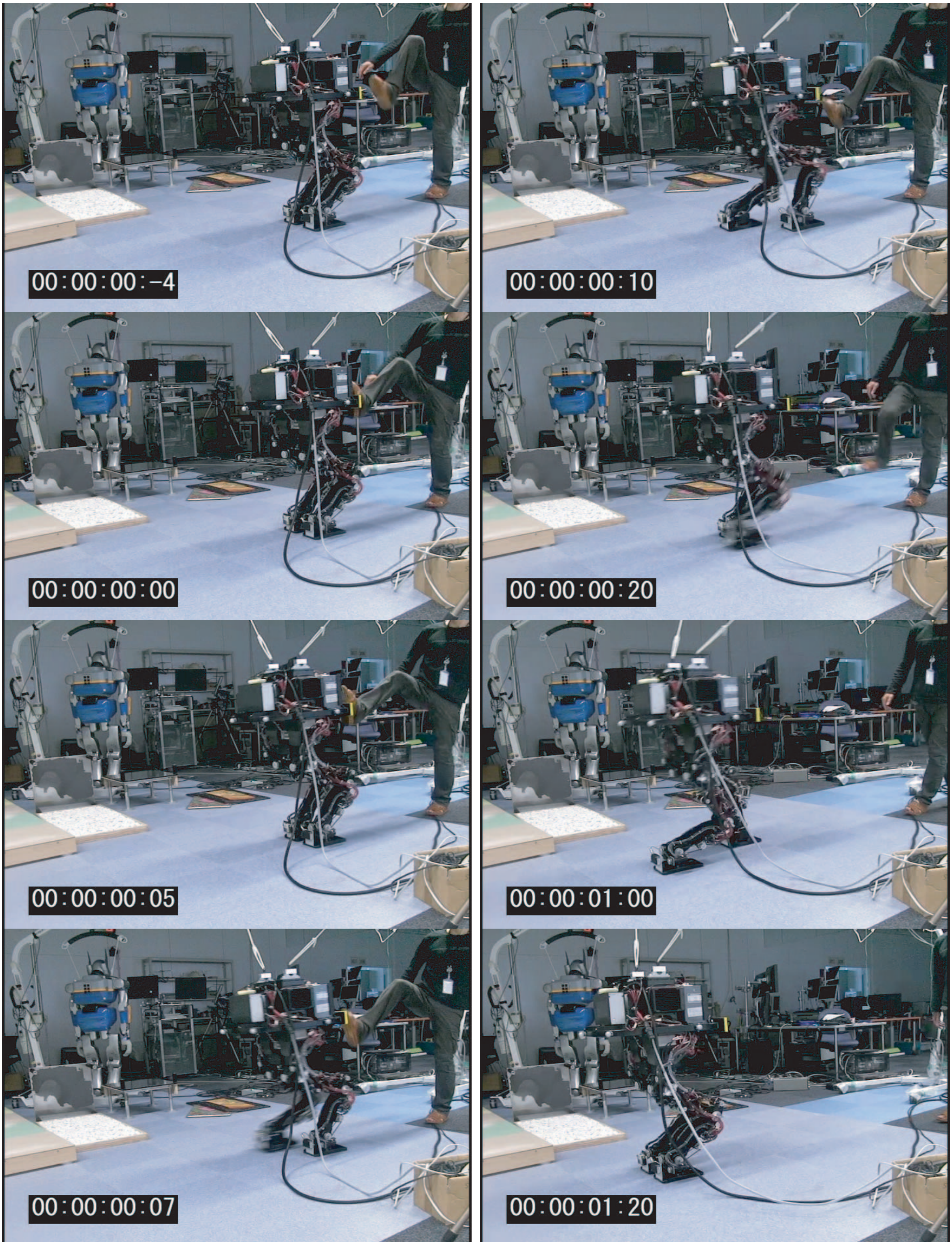

Fig. 5. Photographs of the Experiment (30fps)