# Biped Walking Pattern Generation Using an Analytic Method for a Unit Step With a Stationary Time Interval Between Steps

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***Abstract*—In this paper, an analytic method for generating a unit step pattern using a center of mass (COM) constraint is presented. Using a unit step pattern with the COM position constrained in the supporting polygon at the end of each step, a robot can have a stationary time interval between steps to control and stabilize its posture. The walking pattern is based on the linear inverted pendulum model. Assuming radical zero moment point (ZMP) trajectories, a simple solution form of the COM trajectories is formulated. Three types of case studies are presented for an analytic solution, which is based on different constraints on ZMP references, COM, and time differences. The unknown parameters of the COM trajectories in the solution form are formulated for the different cases. One of the cases was tested for long-stride walking with the DRC-HUBO robot developed at the Korea Advanced Institute of Science and Technology for the Defense Advanced Research Project Agency Robotics Challenge. Experiments of long-stride walking on bricks were successfully performed using the unit step pattern with several previous controllers of HUBO.**

***Index Terms*—Analytic solution, humanoid robot, linear inverted pendulum model (LIPM), walking pattern.**

## I. Introduction

FOR humanoid robots, bipedal locomotion is a perpetual problem that has proven to be the ultimate challenging objective. Humanoid robots need to walk and even run [1], [5] as stably as humans in all conditions and at all times. Furthermore, not only researchers but also the public expect humanoid robots to always walk well, despite unknown obstacles. As humanoid robot research spreads throughout the world, the numerous categories within humanoid robot research are diverging. However, the biped walking problem has been, unquestionably, the most important problem among the numerous areas for years and therefore the most improved area to date. For this purpose, walking patterns are widely used and are a fundamental method for enabling the robot to walk. Since the start of research on walking patterns, many methods have been proposed using various models and structures. Many have been verified and are considered reliable under certain conditions. Nevertheless, the exploration of walking patterns continues, with the goal of eventually having a bipedal robot able to walk adaptively and efficiently in any environment. Due to countless unmodeled factors involving the robot and environment, it is very difficult to achieve the perfect walk. To overcome the differences between the models and the real world, some researchers focus on sensor problems [2], [4], while others work to refine the control schemes that must be used in conjunction with the walking pattern. Those controllers require the right conditions and enough time to properly settle, or stabilize the robot. Consequently, inserting a stationary phase between walking steps could be a good solution to the settling time problem.

To generate walking patterns, the concept of zero moment point (ZMP) is commonly used [3], [6]–[8], [10]–[12], [16]–[18]. In order for a robot to be stationary and stable after stepping, the robot's center of mass (COM) must be inside its supporting polygon. This is a new constraint on the generation of a walking pattern. Typically, the COM trajectory is formulated from the ZMP trajectory, which is a constraint of walking. However, this study focuses on the COM constraint and the ZMP trajectory, in order to correctly locate the COM at the end of a step.

"Walking pattern" may imply that the walking is a continuous and periodic motion based on ZMP, and many walking pattern generation methods take this approach. However, this study proposes dividing the periodic pattern into a "unit step," which extends from one single-support phase to the next single-support phase of walking. Dividing a walking pattern into a unit step and a stable and constrained COM position after the unit step can be very useful and beneficial for coping with various environments.

For example, we can imagine a person crossing a stream using stepping stones, as shown in Fig. 1. The stepping stones, which are located intermittently, require using long strides or jumping from stone to stone. The person cannot just step anywhere because there is not always a stone available at every potential step. In some cases, the traverser may be forced to balance on just one supporting leg. The areas on which a person can step are narrow and have predetermined positions. In this case, having enough stationary time during the single-supporting phase on a stone is helpful to confirm stability and ensure a good posture for stepping to the next stone. To use this strategy, the COM is constrained to be on the stone during each static supporting phase.

For another example, the bricks shown on the right in Fig. 1 have been selected as one of the scenarios that robots should

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Fig. 1. Stepping-stone examples and DRC obstacle (right).

overcome in a disaster situation, for the Defense Advanced Research Project Agency (DARPA) Robotics Challenge (DRC). DRC is a competition of robots funded by the U.S. DARPA. The brick obstacle chosen for the DRC is much similar to the stepping-stone problem. Ultimately, a walking pattern that is able to overcome stepping stones must be devised to advance humanoid robot locomotion.

In this paper, a new approach for generating walking patterns is proposed, specifically using a unit step for the purpose of inserting a sufficient stationary time interval between each step. To generate the unit step pattern, simple and radical conditions that allow the solution to be formulated simply and analytically are used. In addition, three analytic solution sets for walking under different constraints are provided and compared.

Experiments on basic robot walking were performed using the generated unit step pattern. To realize bipedal walking and cope with unmodeled factors, several controllers were used together with the walking pattern. Particularly in the case of the long stride step, the robot is greatly affected by unmodeled dynamics. However, with just simple controllers, even the long stride forward and side unit steps were performed successfully due to the sufficient time interval after each unit step. In addition, walking on bricks using the unit step pattern was achieved.

## II. Related Research

In conventional walking pattern generation research, walking patterns are understandably based on the stability of the robot. Here, the ZMP is a useful index to achieve robot stability [6]. Takanishi *et al.* explored walking pattern generation with a multimass robot model that had nonlinear dynamics and was vertically and horizontally coupled [7]. They overcame this complexity and achieved a stable walking pattern for the multimass model after iterative calculations of the upper body trajectories that stabilized all walking patterns with linearization. Furthermore, Kajita *et al.* realized dynamic walking in a biped robot based on the linear inverted pendulum model (LIPM) [8]. This model is currently the simplest and most influential model in humanoid robot locomotion research. It is a linearized and constrained model that constructs the robot in a 1-D linear dynamics system. Unlike previous model-based methods, Kim *et al.* realized dynamic walking through experimentally generated walking patterns with simple parameters [9]. The empirically obtained pattern was well realized in a real robot, while the model-based methods have unmodeled dynamics.

As robot walking using patterns generated offline is becoming standard, researchers have begun to focus on controlling the robot walking patterns while the robot is walking. Thus, real-time walking pattern generation has become the next stage in the development of humanoid robot walking patterns. Lim *et al.* realized this problem using a combination of unit gait patterns generated offline [10]. In contrast to that method, Nishiwaki *et al.* proposed the online generation of walking motions based on the ZMP trajectory of a multimass model with three steps [11]. Furthermore, Kajita *et al.* proposed a walking pattern generation method that uses preview control based on the ZMP reference of the LIPM [12]. The preview control is one of the optimization methods. Although many other optimization methods [13]–[15] could provide an optimal walking pattern to the ZMP, users are unable to handle all of the parameters for a specific case, such as the position of COM. These two groups do share a common area, however, which is to use future walking information to connect with the current state of walking.

Similar to the approach taken in this paper, some researchers are interested in developing walking patterns using analytic methods. Harada *et al.* further improved the online generation method, in order to calculate a pattern quickly, using an analytic solution based on the LIPM [16]. Their method could be the more generous case than this study because they proposed the analytic solution to a polynomial form of ZMP trajectory. However, they did not make the solution closed form or easily usable. In addition, the COM is not able to be constrained. In this paper, a closed-form analytic solution is formulated in special cases, which constrained the COM. Kim proposed a method of creating walking patterns using a convolution sum in a discrete domain [17]. These analytic solution methods are efficient at calculating the walking pattern quickly, but the COM constraint is not considered. In the case of the stepping stone, the COM position before and after the step must be inside the supporting polygon to achieve a stable supporting phase of the robot's step. However, as of yet, no one has considered this COM constraint at the beginning and end of the unit step.

In this paper, a new and simple analytic solution is proposed for the unit step walking pattern. Analytic methods commonly share the disadvantage that the analytically obtained walking pattern cannot be changed during the unit step because it is already formulated in closed-form analytic solutions. In spite of this drawback, analytic solutions are still used for walking pattern generation because they do not require future information, as presented earlier, and they can be easily implemented since solutions are formulated in a closed form. In Section III, new analytic solutions of the walking pattern are presented. Using the proposed solution, inserting a stationary interval between steps is available since ZMP, COM constraints, and time limits are concurrently considered in this paper. This is remarkably different from other analytic solutions. For ease of understanding, three different analytic solutions are formulated and compared in Section III. Among the three cases, Case 3 is the proposed solution to implement the special case of walking on stepping stones.

Here, a simple solution form of the COM trajectory is assumed, using the radical ZMP conditions of the LIPM. The proposed walking pattern is thus not for a general case of walking, but rather is suitable for a special case. However, as long as ZMP is constrained in the supporting polygon, stability of the walking pattern will be guaranteed.

### A. LIPM

In the multimass robot model, the ZMP equation [1] in the $y$-direction is calculated as follows:

$$y_{\text{zmp}} = \frac{\sum_{i=0}^{n} m_i(\ddot{z}_i + g)y_i - \sum_{i=0}^{n} m_i(\ddot{y}_i)z_i + \sum_{i=0}^{n} I_{xi}\ddot{\phi}_{xi}}{\sum_{i=0}^{n} m_i(\ddot{z}_i + g)} \tag{1}$$

where $I_{xi}$ is the moment of inertia with respect to the $x$-direction. In this paper, only the $yz$ plane is considered, which is the so-called frontal plane, since the $xz$ plane is separated from the $yz$ plane. Equation (1) is obtained using the full-body dynamics equation of the robot. It can be complicated with an infinite number of mass. It is a more accurate model, and a more accurate walking pattern, but the accuracy induces too much complexity. Thus, in this study, a simplified model is used to generate the walking pattern. For the articulated robot system, the number of mass ($n$) can be a finite value of the degrees of freedom that the robot has. However, if $n = 1$ is set as a single mass and $z_1$ as a constant without angular momentum, then the multimass model becomes the LIPM [3], as follows:

$$y_{\text{zmp}} = y - \frac{l}{g}\ddot{y} \tag{2}$$

where $l$ is a constant height of the mass. For simplicity, if one assumes that the COM of a robot only moves on the horizontal axis, then vertical and rotational accelerations do not exist. This assumption causes (1) to become (2), which is a second-order differential equation that can be solved easily. Naturally, there are numerous types of methods that can generate COM trajectories based on (2). In order to solve the differential equation in the simplest way, a couple of constraints can be assumed, such as initial, final, and continuous conditions of a unit step. In addition, the ZMP value is assumed to be constant during a single-support phase. From these conditions, the simplest solution form can be formulated for the walking pattern based on the ZMP equation of LIPM as follows.

### B. Simple Analytic Solution Form of Unit Step Pattern

Here, a simple analytic solution form of unit step pattern is proposed. Fig. 2 presents a COM trajectory and a ZMP during one step. In Fig. 2, $t_0$, $t_1$, and $t_2$, represent the initial time of a step, the supporting phase change time, and the final time of the step (step time), respectively. $LF_i, RF_i$ are positions in the $y$-direction, where $i = l, u, o$ denote lower, upper, and origin.

The ZMP value is assumed to be constant, using $zmp_1$ during the first supporting phase and $zmp_2$ during the second supporting phase. This assumption is a radical condition of ZMP to solve (2). From this assumption, the second-order nonhomogeneous differential equation in (2) has a simple solution

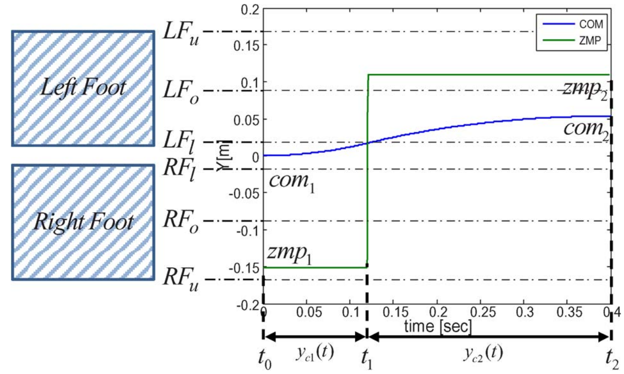



Fig. 2. Walking pattern of unit step.

form, as presented in (3). Moreover, the solution can be solved separately for each unit step. In (3), the exponential terms indicate the homogeneous solution of the differential equation, and the constant term is the particular solution. Hence,

$$\begin{cases} y_{c1}(t) = a_1 + b_1 e^{-\lambda(t-t_0)} + c_1 e^{\lambda(t-t_0)}, & t_0 \le t \le t_1 \\ y_{c2}(t) = a_2 + b_2 e^{-\lambda(t-t_1)} + c_2 e^{\lambda(t-t_1)}, & t_1 \le t \le t_2. \end{cases} \tag{3}$$

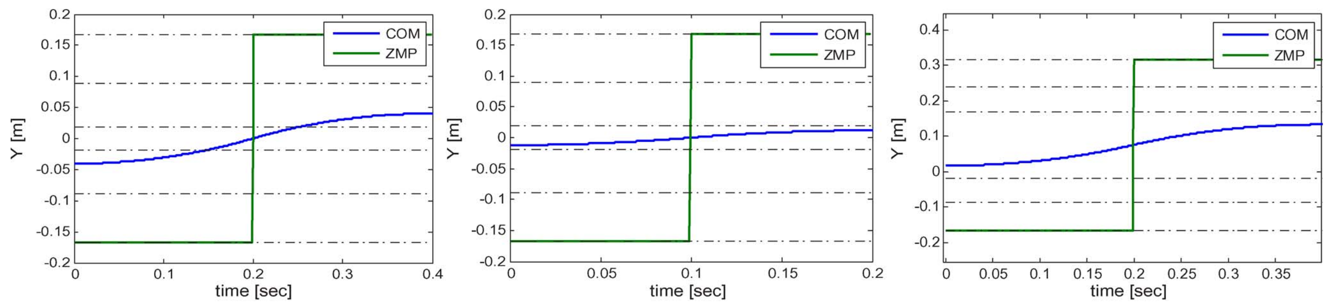
Here, $\lambda$ is $\sqrt{g/l}$, which is the natural frequency of the inverted pendulum. If constraints are imposed on the COM velocities so that they are zero at the initial and final step times, then it is possible to easily obtain the COM trajectories from the analytic solution. However, this solution is not a general form for walking pattern generation. Since the ZMP is not continuous during a step, this assumption induces acceleration discontinuity.

However, if the ZMP is designed as the maximum value in the allowable range, then the obtained COM trajectory can become a walking pattern with maximum acceleration because robots walk stably as long as the ZMP is within the supporting polygon, despite its discontinuity. Then, the walking pattern is calculated using a very simple procedure with few constraints. Next, using the analytic solution form of the ZMP equation, this walking pattern is interpreted in three case studies, and the relationships between the allowable ZMP region, COM, and step time are investigated.

## III. Case Study of Unit Step Pattern

Generating walking patterns is an inverse problem to estimating acceleration using the governing equation of the model. If the dynamic equation of a model and the external forces are known, then the movement of the model masses can be calculated. In contrast, if the model moves on a trajectory, then resultant forces can also be predicted by the model. The walking patterns are the same as the trajectory when the resultant forces are constrained well.

For example, the ZMP is involved in the constrained resultant forces. During robot walking, if the allowable ZMP reference is forced in the LIPM, then the COM is required to move on a specific trajectory in order to meet the ZMP reference. That trajectory is the walking pattern. (This will be Case 1.) In contrast, if the mass of the LIPM moves according to the specified trajectories, then the ZMP will be the result of the




$$\begin{cases} t_0 = 0,\ t_1 = 0.2,\ t_2 = 0.4 \\ zmp_1 = RF_u,\ zmp_2 = LF_u \end{cases} \qquad \begin{cases} t_0 = 0,\ t_1 = 0.1,\ t_2 = 0.2 \\ zmp_1 = RF_u,\ zmp_2 = LF_u \end{cases} \qquad \begin{cases} t_0 = 0,\ t_1 = 0.2,\ t_2 = 0.4 \\ zmp_1 = RF_u,\ zmp_2 = LF_u + 0.15 \end{cases}$$

Fig. 3. Examples of the unit step walking pattern for CASE 1: ZMP first and COM later.

robot system. The trajectory would be also a walking pattern as long as the ZMP is within the allowable range. (This will be Case 2.)

Thus, both directions can be adopted to generate walking patterns for bipedal robots on the understanding that the ZMP is always within the supporting polygon. In this manner, various constraints can be applied to obtain unknown variables of analytic solution (3), as desired. Here, three kinds of cases are discussed to obtain the analytic solution of the unit step case by case.

### A. Case 1: ZMP First and COM Later

The first case is titled ZMP first and COM later. This means that the COM trajectory is obtained from a predesigned ZMP for a specified fixed step time. The $zmp_1$ and $zmp_2$ are set to be within the support polygon. In addition, the initial, final, and support change times are already determined, as desired. In this case, there are two boundary conditions at $t_0$ and $t_2$ and two continuity conditions at $t_1$, as shown in

$$\text{conditions}\begin{cases} y_{c1}(t_1) = y_{c2}(t_1) \\ \dot{y}_{c1}(t_0) = 0,\ \dot{y}_{c1}(t_1) = \dot{y}_{c2}(t_1),\ \dot{y}_{c2}(t_2) = 0. \end{cases} \tag{4}$$

In (3), the constant terms of $\{a_1, a_2\}$ are $zmp_1$ and $zmp_2$, respectively, because the terms are the particular solution of the homogeneous differential equation. Thus, (3) has four unknown coefficients, i.e., $\{b_1, c_1, b_2, c_2\}$, and these can be obtained by solving simultaneous equations using the four conditions presented in (4), as follows:

$$\begin{aligned} a_1 &= zmp_1 \quad a_2 = zmp_2 \\ c_2 &= \frac{\left(T_{10}^2 - 1\right)(a_1 - a_2)}{2T_{10}^2 T_{21}^2 - 2}, \quad b_2 = T_{21}^2 c_2 \\ c_1 &= \frac{-T_{10}\left(T_{21}^2 - 1\right)(a_1 - a_2)}{2T_{10}^2 T_{21}^2 - 2}, \quad b_1 = c_1. \end{aligned} \tag{5}$$

Here, the substitution of $T_{10} \triangleq e^{\lambda(t_1 - t_0)}$ and $T_{21} \triangleq e^{\lambda(t_2 - t_1)}$ is used. Generally, at a gait planning level, the walking frequency of the robot and the footprints, where the robot steps, are determined first. Then, the ZMP is designed to pass through the predetermined footprints. Using this ZMP reference, the COM trajectory is obtained from the ZMP equation. Without stationary intervals during the walking, it does not matter whether the COM position is inside the support polygon or not: the obtained COM trajectory will result in the ZMP that the robot should have. This procedure of generating walking patterns, i.e., ZMP first and COM later with a fixed time, is similar to the conventional method. Although other researchers do not use analytic solution forms such as that in (3), this order is the same as other methods for obtaining walking patterns because stability is the most important feature for robots during walking. Nevertheless, walking patterns using this method have a demerit in that the COM trajectory cannot be controlled. Sometimes, the COM of the robot is intended to move in a specific manner; thus, this method is not the only one to generate walking patterns with stability.

In Fig. 3, there are three examples of a unit step walking pattern for Case 1 with different conditions. Those are the right candidates for the walking pattern. However, to constrain the COM in the supporting region and insert a stationary time interval at the end of the unit step, only the first example in Fig. 3 is the desired solution. Therefore, it is necessary to constrain COM.

### B. Case 2: COM First and ZMP Later

The second case is titled COM first and ZMP later. This indicates that the ZMP is obtained from a predetermined COM trajectory that has the form of (3) in a specified fixed step time. The COM positions, i.e., $com_1$ and $com_2$, are set to be at specific points such as footprints. In addition, the initial, final, and supporting phase change times have already been determined, as required. In this case, there are four boundary conditions at $t_0$ and $t_2$ and two continuity conditions at $t_1$, as shown in

$$\text{conditions}\begin{cases} y_{c1}(t_0) = com_1, y_{c1}(t_1) = y_2(t_1), y_{c2}(t_2) = com_2 \\ \dot{y}_{c1}(t_0) = 0, \qquad \dot{y}_{c1}(t_1) = \dot{y}_2(t_1), \dot{y}_{c2}(t_2) = 0. \end{cases} \tag{6}$$

Using these six conditions, the six unknown coefficients, i.e., $\{a_1, b_1, c_1, a_2, b_2, c_2\}$ in (3), can be easily derived, as

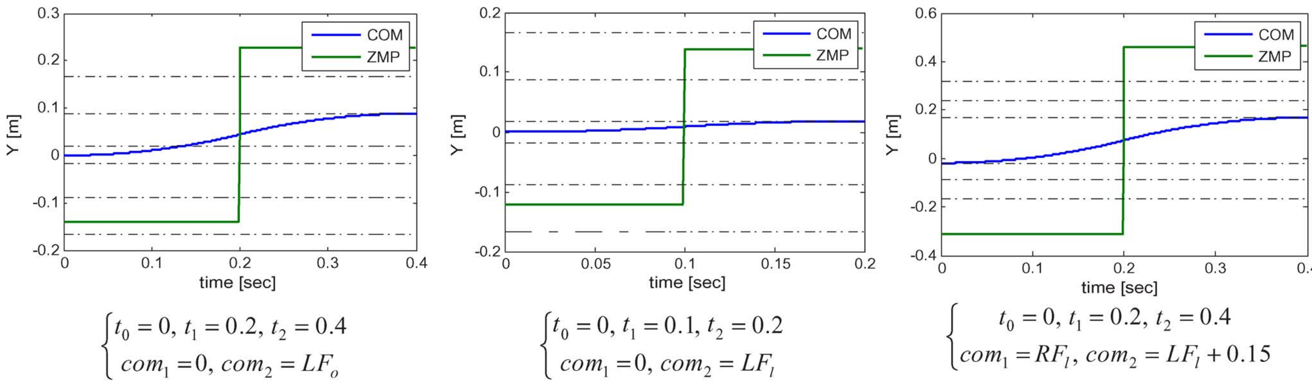



Fig. 4. Examples of the unit step walking pattern for CASE 2: COM first and ZMP later.

shown in (7). Again, $a_1$ and $a_2$ indicate $zmp_1$ and $zmp_2$, as stated for Case 1. Hence,

$$c_1=\frac{-T_{10}(com_1-com_2)(T_{21}+1)}{2(T_{10}T_{21}-1)(T_{10}-1)},\ b_1=c_1,\ a_1=com_1-2c_1$$
$$c_2=\frac{(com_1-com_2)(T_{10}+1)}{2(T_{10}T_{21}-1)(T_{21}-1)},\ b_2=T_{21}^2c_2,\ a_2=com_2-2T_{21}c_2. \tag{7}$$

Fig. 4 shows examples of the unit step walking pattern for Case 2 with different conditions. Since COM is constrained first, the resultant ZMP trajectories are not in the allowable range, except for the middle graph in Fig. 4. Therefore, when someone uses this solution, they have to keep in mind that the available step time and COM difference exist.

One more thing that can be mentioned about Case 2 is that it is possible to verify the foot size design criteria. If (3) is solved using a predefined COM position in the initial and final times of a step, then the ZMP values in Fig. 2 can be derived using (7), and those are rearranged as follows. In (8), the expressions of $f_{c_1}(T_{10}, T_{21})$ and $f_{c_2}(T_{10}, T_{21})$ indicate the positive functions of $T_{10}$ and $T_{21}$, respectively. This is because $T_{10}$ and $T_{21}$ are always larger than 1 due to $t_0 < t_1 < t_2$. This derives the inequality results described as follows:

$$a_1 = com_1 + 2f_{c_1}(T_{10}, T_{21})(com_1 - com_2) = zmp_1$$
$$a_2 = com_2 - 2f_{c_2}(T_{10}, T_{21})(com_1 - com_2) = zmp_2 \tag{8}$$
$$\begin{cases} zmp_1 > com_1, zmp_2 < com_2 & \text{where } com_1 > com_2 \\ zmp_1 < com_1, zmp_2 > com_2 & \text{where } com_1 < com_2. \end{cases} \tag{9}$$

From (9), it is possible to guarantee the relationship of the ZMP and COM difference. The $zmp_i$ is always outside of $com_i$ for the robot in the frontal plane. If the COM difference $(com_1 - com_2)$ is increased, then the ZMP range $(zmp_1 - zmp_2)$ will also be increased. Although these relationships are common understandings about the relationship between the ZMP and COM difference, this relation is easily verified using this case. Furthermore, the size of the foot required is determined using the ZMP that is calculated using (8) because the COM trajectory, when the ZMP is constant during a support phase, indicates movement with a maximum acceleration for the phase. Therefore, (8) must be a minimum design condition for the foot size in predefined COM positions with fixed time. This is a criteria of foot size design, i.e., to let the ZMP be in the supporting polygon.

### C. Case 3: Symmetric ZMP With Constrained COM Considering Minimum Step Time $(t_2)_{\min}$

As mentioned earlier, walking patterns generated by Cases 1 and 2 are sometimes not an appropriate solution for inserting a stationary time interval between steps. If the ZMP and COM are overconstrained, then it will not result in a solution for the step time. In order to avoid this situation, it is appropriate to consider a more specific case, and this case could be more normal. It is normal for humanoid robots to have symmetric foot sizes. Thus, a symmetrical ZMP for each supporting phase in a single step is a better strategy for walking with stability and safety. Furthermore, the step time is usually fixed. In this case, it is considered that the ZMP is the constant $com_i \pm \delta$, where $\delta$ is the ZMP offset from the COM position and is unknown. In order to formulate a solution for the unknown variable, eight variables of the second nonlinear simultaneous equations must be solved. The unknown variable set is $\{t_1, \delta, a_1, b_1, c_1, a_2, b_2, c_2\}$. Equation (10) is the solution of the unknowns.

Here, the substitution of $T_i \triangleq e^{\lambda t_i}$ is used. The first equation in (10) means that $t_1$ is always a half step time $t_2 - t_0$ because the ZMP reference is symmetric during each step. Fig. 5 is an example of the unit step walking pattern generated by Case 3. It shows the symmetrical shape of COM, ZMP, and time. Hence,

$$T_1 = \sqrt{T_0T_2}$$
$$\delta = \frac{com_2 - com_1}{\frac{T_0^2+T_1^2}{2T_0T_1} + \frac{T_1^2+T_2^2}{2T_1T_2} - 2}$$
$$a_1 = com_1 - \delta, \quad a_2 = com_2 + \delta$$
$$b_1 = \frac{\delta}{2}, \quad b_2 = -\frac{\delta T_2}{2T_1}$$
$$c_1 = b_1, \quad c_2 = b_2\frac{T_1^2}{T_2^2}. \tag{10}$$

With this solution, the ZMP offset $\delta$ can be compared in different step times. Fig. 6 illustrates the relationship between the ZMP offset $\delta$ and $t_2$ with $com_2 - com_1$. In this case, a

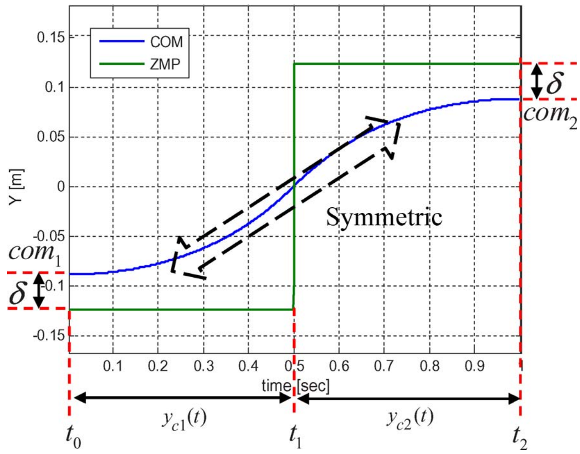



Fig. 5. Example of the unit step walking pattern for Case 3.

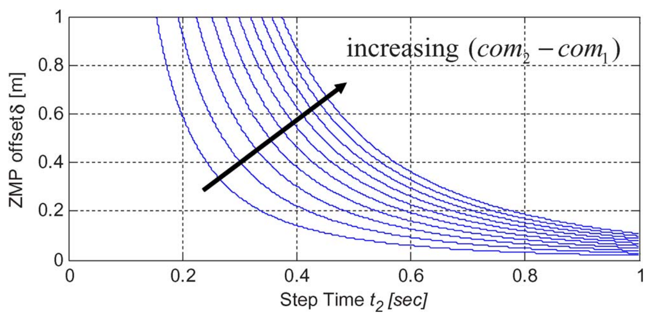



Fig. 6. Relationship between the ZMP offset $\delta$ and $t_2$ with $com_2 - com_1$.

shorter step time results in a bigger ZMP offset. Then, if the ZMP offset $\delta$ is set with an allowable ZMP range of $\delta_{\max}$, then the step time in this condition is the minimum value $(t_2)_{\min}$ that the robot can walk using the walking pattern from the LIPM. The solution for $t_2$ can be obtained for the same condition with (6). However, $t_2$ is not fixed, and the ZMP is fixed as $com_i \pm \delta_{\max}$ in this case. This causes also six variables of second nonlinear simultaneous equation because of $e^{\lambda t_i}$. The solution for $t_2$ with maximum ZMP $com_i \pm \delta_{\max}$ is shown as (12) with substitution of (11).

Equation (12) is a discriminant of the walking speed of a robot that has a limited ZMP allowable range of $\delta_{\max}$ and a COM difference of $\Delta com$. As shown in (12), the term inside the square root is always positive, and $t_2$ is deterministic. Finally, Case 3 induces a minimum time limit, and the COM position can be designed with a guaranteed ZMP range, as desired. Hence,

$$\begin{aligned}\Delta com &\triangleq com_2 - com_1\\ \delta_{\max} &\triangleq zmp_2 - com_2 = com_1 - zmp_1\end{aligned} \tag{11}$$

$$\begin{aligned}T_2 =& \frac{T_0}{2(\delta_{\max})^2}\\ &\times\Big(\Delta com^2 + 4\delta_{\max}\Delta com + 2(\delta_{\max})^2\\ &+\sqrt{(\Delta com+2\delta_{\max})^2(\Delta com+4\delta_{\max})(\Delta com)}\Big)\\ (t_2)_{\min} =& \frac{1}{\lambda}\ln(T_2).\end{aligned} \tag{12}$$

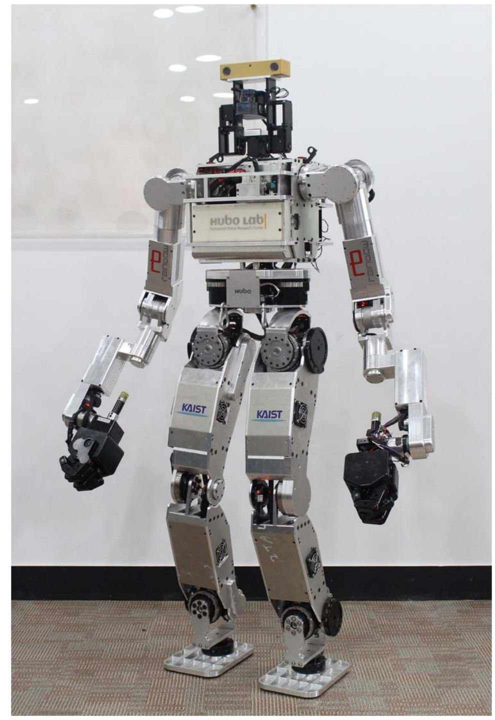


Fig. 7. DRC-HUBO developed in KAIST (2013).

## IV. Realization

### A. Brief Overview of DRC-HUBO

The formulated unit step walking was realized with a real humanoid robot, i.e., DRC-HUBO, as shown in Fig. 7. DRC-HUBO is the upgraded version of HUBO, a bipedal humanoid robot originally developed in 2005 for the DRC. HUBO has been constantly improved by the Korea Advanced Institute of Science and Technology (KAIST). Its total weight, including a battery and computers, is about 50 kg. For performing various tasks for the DRC, the legs and arms of the present version are longer than previous versions of HUBO. The height of the robot is 170 cm. Each of the robot's joints has a local servo controller that uses a high-gain proportional–derivative position control with a control frequency of 1 kHz. Actuators of main joints, such as legs and arms, are 200-/100-W brushless dc (BLDC) or dc motors. They provide sufficient power for this robot to do various tasks since each actuator is used with a Harmonic Drive of 100:1 or 200:1 gear ratio. Moreover, the DRC-HUBO is designed to have reduced weight, with the result that this robot is very light and powerful compared to the other robots of its size. DRC-HUBO has an inertial measurement unit (IMU) sensor on the pelvis, for estimating angle and angular velocity, and a force/torque sensor in each ankle. Specifications of the DRC-HUBO are shown in Table I.

### B. Kinematics and Controllers

To realize the walking pattern on a real robot, not only the COM trajectories but also the foot trajectories are needed. For a forward step, sinusoidal foot trajectories in the $xz$ plane were designed. For every step, the supporting foot is alternated at

TABLE I
SPECIFICATION OF DRC-HUBO

| Item | Description |
|---|---|
| Weight | 50 kg |
| Height | 170 cm |
| Actuators | 200W, 100W BLDC/DC motor |
| Power | 48V (7.8Ah, 380 Wh Battery) |
| Degrees of Freedom | Total 30 DOF (7 DOF arms, 1 DOF grippers) |
| Remarks | Legs are 6 cm longer and arms are 26 cm longer than previous HUBOs |

$t_0$ and $t_2$ in Fig. 2, except on the initial and final steps of any walking sequence. For the initial and final steps, the supporting foot is changed at $t_1$ in Fig. 2. The COM trajectories generated in the previous section can be applied to the center of the robot's pelvis (a good estimation for the robot's COM). To realize a more exact COM trajectory, whole-body kinematics using the inverse Jacobian [18] must be solved. However, in this demonstration, whole-body kinematics were not applied, and instead, a very simple kinematic condition in the $y$-direction was used, as shown in (13) and Fig. 8. This is the same meaning as two-link inverse kinematics with the condition of upright upper body since the robot's upper body is required to be upright all the time. For the $x$-direction, it was assumed that the robot's COM was at the center of the pelvis since the two legs are spread out symmetrically forward and backward. In (13), $m_{\text{upper}}$ is a mass of the upper body including the swing leg, and $m_{\text{lower}}$ is a mass of the supporting leg. Hence,

$$\begin{aligned} y_{\text{com}} &= \frac{m_{\text{upper}} y_{m_{\text{upper}}} + m_{\text{lower}} y_{m_{\text{lower}}}}{m_{\text{upper}} + m_{\text{lower}}} \\ y_{m_{\text{upper}}} &= y_{\text{pelvis}} \\ L_1 y_{\text{pelvis}} &= l_1 y_{m_{\text{lower}}} \\ y_{\text{pelvis}} &= \frac{(m_{\text{upper}} + m_{\text{lower}}) y_{\text{com}}}{m_{\text{upper}} + m_{\text{lower}} L_1 / l_1}. \end{aligned} \tag{13}$$

Although this considers a simple kinematic condition for the COM trajectory, there should be a COM movement error, since it does not apply whole-body inverse kinematics and even does not control the COM by a feedback. In addition, the COM movement error, combined with the compliance of the robot, environmental external disturbances, and kinematical mismatch, must be compensated. For this reason, various walking controllers must be used on top of the walking patterns.

However, in this paper, the primary focus is about generating a unit step walking pattern with a stationary time interval between steps. Since several controllers are necessary to verify a generated walking pattern, several previous controllers of HUBO were utilized to stabilize the robot.

One of the main referenced controllers is a compliance controller. The robot's compliance is an important factor when

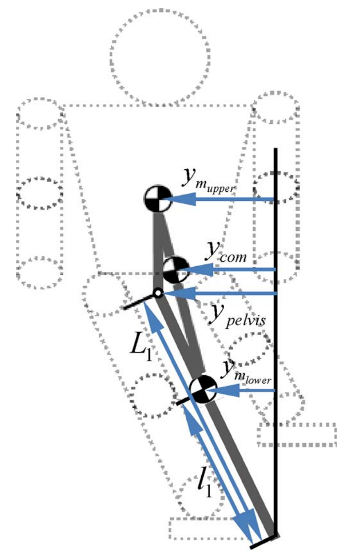



Fig. 8. Kinematic condition for the COM trajectory in the $y$-direction.

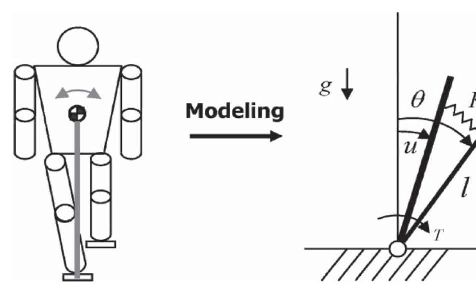



Fig. 9. Compliant single inverted pendulum model [16], [17].

walking. A humanoid robot is not a single rigid object. It is an articulated system, and each joint is compliant despite the high-gain position control of the actuators. The aluminum frame assembly of a robot also has compliant factors. The compliant factors in the robot and the link masses can be expressed as a spring-mass single inverted pendulum model, as shown in Fig. 9. This model was originally introduced by Kim and Oh [19] and has been repeatedly applied to the posture and walking control of HUBO [20]. From an IMU sensor on the pelvis and force and torque (F/T) sensors on the ankles, one can estimate or directly measure angular information of the COM as in a spring-mass single inverted pendulum [9], [18]–[21]. This controller plays an important role for the control of compliance during walking. It also can be used to stabilize the robot, when it walks on slightly uneven terrain, because excitation of the compliant mass is caused by disturbances from the ground. To realize the unit step walking pattern, this controller mentioned earlier was applied during walking.

One of the important controllers is an upright pose controller. The upright pose controller for uneven terrain walking proposed by Kim *et al.* [21] has been performed well to the HUBO robot, particularly during single-support phases. Since a static time interval between steps exists in this experiment, enough time can be secured for the robot to settle its torso to a reference angle, which is commonly zero.

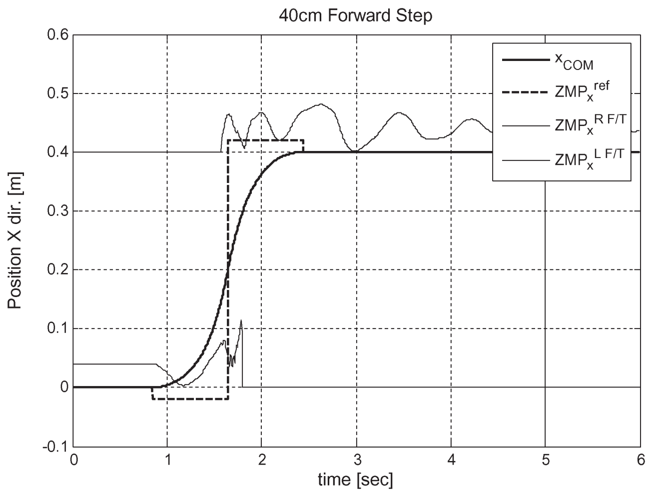



Fig. 10. Unit step walking with a 40-cm forward step. (Snapshots in Fig. 13).

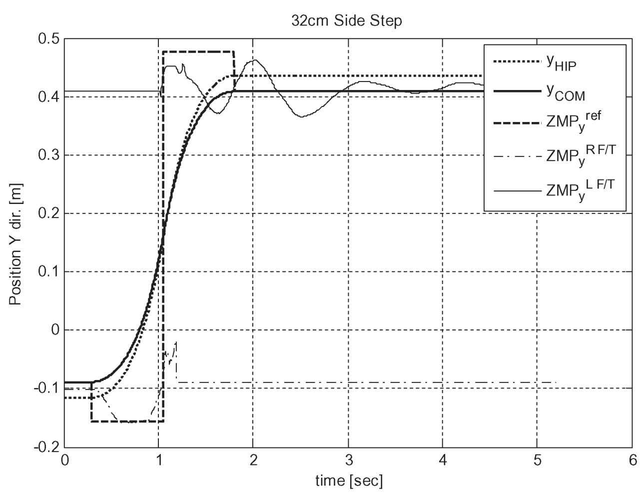



Fig. 11. Unit step walking with a 32-cm side step. (Snapshots in Fig. 14).

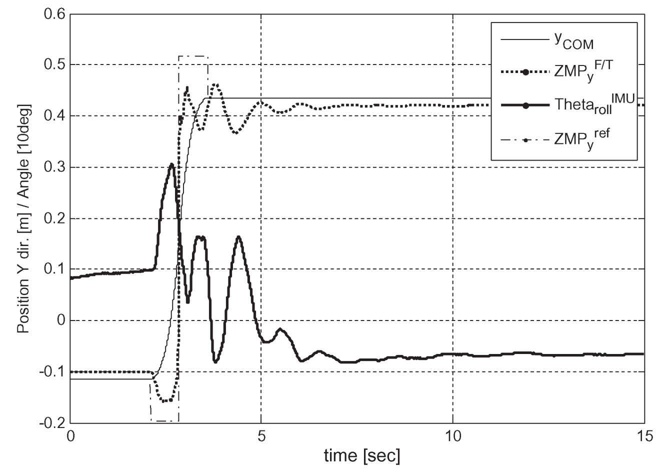



Fig. 12. Estimated angle of the torso from IMU during the 32-cm side step. (Snapshots in Fig. 14).

## C. Experiments

We performed experiments using unit step walking for several cases using the humanoid robot DRC-HUBO. Among the cases of unit step constraints, Case 3 was used: symmetric ZMP from COM considering the minimum step time $(t_2)_{\min}$. It is clear that the other cases would bring the same result, if the same COM or ZMP or time parameter was set as in Case 3. However, Cases 1 and 2 were not adopted to realize walking with a stationary time interval between steps because they do not guarantee the robot's stability in this specific objective, as discussed in Section IV. For a stationary time interval between steps, the COM and ZMP have to be inside the support polygon at the end of a unit step. However, if only the ZMP is considered for the walking pattern as in Case 1, then the COM at the end of the unit step can be outside the support polygon. If the COM is considered first as in Case 2, the ZMP can be also outside the support polygon.

Fig. 10 is a result of a 40-cm unit step forward. The graph only shows the $x$-direction because this is remarkable in a forward step. Fig. 11 shows a result of a 32-cm side unit step. The graph only shows the $y$-direction because of the side step. Indeed, the whole step length from the first supporting foot to the next supporting foot in Fig. 11 is 48 cm because the original width between the feet is 16 cm. These big steps clearly show the advantage of the unit step with a stationary time interval since a longer stride evokes a bigger disturbance by deflection of the compliant model. In particular, $y_{\mathrm{HIP}}$, which is the hip trajectory, and $y_{\mathrm{COM}}$, which is the generated COM trajectory, are not identical since the kinematic condition is applied, as shown in (13).

Fig. 12 shows the result of the estimated torso angle from the IMU during the 32-cm side step. Figs. 13–15 show snapshots of video for several step conditions. Finally, experiments of walking on bricks were performed. The bricks were placed at 30-cm forward intervals alternating left to right with a 20-cm distance. In Fig. 15, video snapshots of two-unit-step walking sequences on bricks are shown with a 30-cm forward/20-cm side-to-side stride with a static interval between steps. The diagonal step length is almost 36 cm, which should be noted as a step length never before attempted with any robot in the HUBO series. The positions of the bricks are known and not visually estimated. These videos are linked as follows. (http://www.youtube.com/channel/UC4R1heJq6gtqd7fSUP6ALow).

## V. Conclusion

In this paper, the unit step walking patterns of humanoid robots have been investigated. We obtained the analytic solution of the unit step walking pattern. From the obtained results, this study shows that a special case of walking, inserting a stationary time interval between steps, can be achieved using the proposed analytic solution.

Inserting static time intervals between steps will be helpful to allow sufficient time for walking controllers to settle after a large step. To achieve this, the COM at the end of a step was constrained to remain inside the supporting polygon of the single-support phase. While the ZMP constraint is essential for walking, this COM constraint is new. We have also shown that there exist a step time limit and a ZMP limit for the designed feet size. Considering these three conditions (ZMP, COM, and step time) simultaneously, the special case of walking was

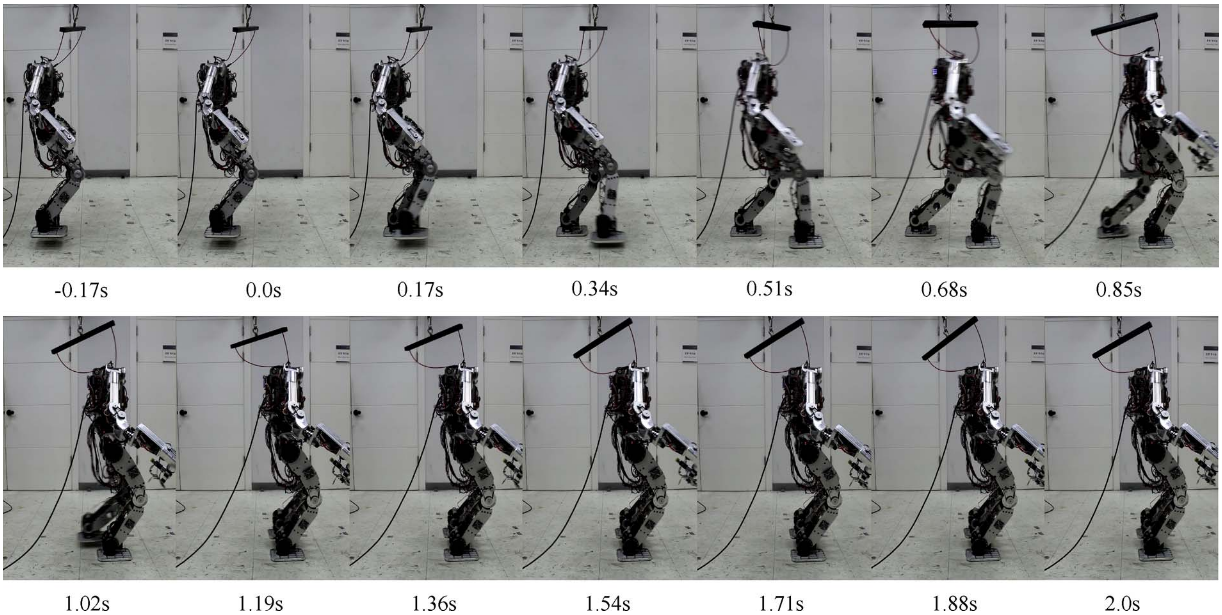



Fig. 13. Video snapshots of unit step walking with a 40-cm forward stride.

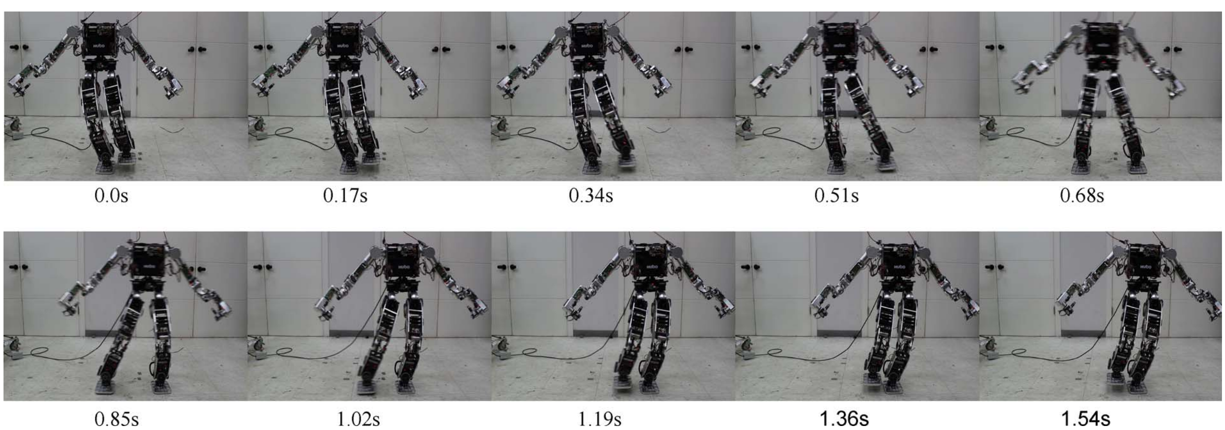



Fig. 14. Video snapshots of unit step walking with a 32-cm side stride (or 48 cm with the original width of 16 cm between two feet).

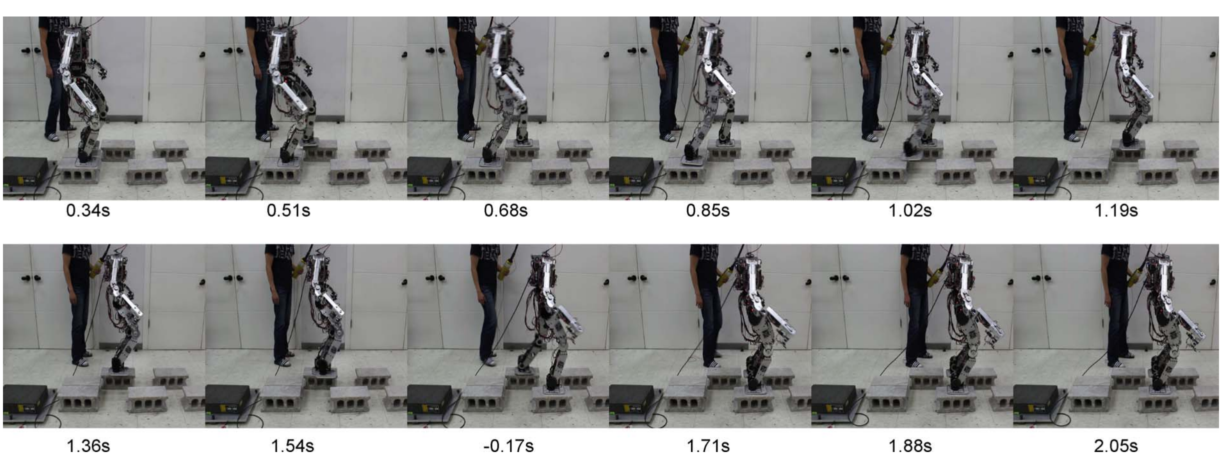



Fig. 15. Video snapshots of two-unit-step walking sequences on bricks with 30-cm stride/20-cm side step and a static interval between steps.

implemented well. Although the proposed solution is not a general form, it can be easily adopted to special walking.

The analysis in this paper was conducted under the simplest assumptions. In particular, a discontinuous ZMP is not a normal case: in a real system, the ZMP is not a digital output. Rather, it always has a continuous value. The discontinuity of the ZMP reference does not have a great effect on real walking, particularly for stepping-stone walking, since the supporting foot has to be changed quickly with a short double-support phase. In future research, the authors intend to design a new walking pattern with a long double-support phase and a new controller, in order to reduce the length of the static time interval.

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