INTEROPERATION FOR LAZY AND EAGER EVALUATION

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Abstract

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by

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Programmers forgo existing solutions to problems in other programming languages where software interoperation proves too cumbersome; they remake solutions, rather than reuse them. To facilitate reuse, interoperation must resolve language incompatibilities transparently. To address part of this problem, we present a model of computation that resolves lazy and eager evaluation strategies using dual notions of evaluation contexts and values to mirror the lazy evaluation strategy in the eager one. This method could be extended to resolve incompatible evaluation strategies for evaluation contexts common to any pair of languages.

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Chapter 1

Introduction

Programmers forgo existing solutions to problems in other programming languages where software interoperation proves too cumbersome; they remake solutions, rather than reuse them. To facilitate reuse, interoperation must resolve language incompatibilities transparently. To address part of this problem, we present a model of computation that resolves lazy and eager evaluation strategies.

Matthews and Findler presented a method of safe interoperation between languages with incompatible polymorphic static and dynamic type systems [1]. We observe that their method is insufficient for safe interoperation between languages with incompatible lazy and eager evaluation strategies, then explain the underlying problem, and then finally present a method of interoperation that resolves this incompatibility.

The model of computation of Matthews and Findler comprises two eager languages based on ML and Scheme. We extend their model of computation with a third language that is based on Haskell and identical to their ML-like language, except it is lazy. We introduce lists to all three languages. Hereafter, we use the names of Haskell, ML, and Scheme to refer to their counterparts in our model of computation.

Unlike ML and Scheme, Haskell does not evaluate function arguments or list construction operands. These three evaluation contexts comprise the set of incompatible evaluation contexts between Haskell and ML, and Haskell and Scheme. Since Haskell permits unused erroneous or divergent expressions in these evaluation contexts and ML and Scheme do not, there are Haskell values that have no counterpart in ML and Scheme. Attempting to convert such values to ML and Scheme forces the evaluation of such expressions and breaks the transparency of interoperation.

Figure 1.1 demonstrates how a straightforward introduction of Haskell to the model of Matthews and Findler breaks the transparency of interoperation when converting a list construction from Haskell to Scheme. The Haskell list construction contains an erroneous operand that Scheme forces to evaluate in the process of converting the Haskell list construction. Figure 1.2 demonstrates Scheme correctly deferring the evaluation of the erroneous Haskell list construction operand and producing as a result the counterpart Scheme list construction.

Moreover, since the conversion of functions from ML and Scheme to Haskell requires the application of the original function to the converted Haskell argument, ML and Scheme always force the evaluation of the converted Haskell argument, even if it is never used. The application of such converted functions effectively changes the order of evaluation of Haskell and breaks the transparency of interoperation.

Likewise, figure 1.3 demonstrates the conversion of a function from Haskell to Scheme. Scheme forces the evaluation of the erroneous Haskell argument in the process of applying the Scheme function, even though the Haskell argument is never used. From the perspective of the outermost Haskell application, the argument must have been used, but it was not. Figure 1.4 demonstrates Scheme not forcing the evaluation of the Haskell argument, which allows the Scheme function to produce a number.

Figure 1.1: Transparency broken for list construction operands.

$\texttt{sh}\ \{\texttt{N}\}\ (\texttt{cons}\ (\texttt{wrong}\ \texttt{N}\ \text{“Not a number”})\ (\texttt{nil}\ \texttt{N})) \quad \rightarrow$

$\texttt{cons}\ (\texttt{sh}\ \texttt{N}\ (\texttt{wrong}\ \texttt{N}\ \text{“Not a number”}))\ (\texttt{sh}\ \{\texttt{N}\}\ (\texttt{nil}\ \texttt{N})) \quad \rightarrow$

Error: “Not a number”

Figure 1.2: Transparency not broken for list construction operands.

$\texttt{sh}\ \{\texttt{N}\}\ (\texttt{cons}\ (\texttt{wrong}\ \texttt{N}\ \text{“Not a number”})\ (\texttt{nil}\ \texttt{N})) \quad \rightarrow$

$\texttt{cons}\ (\texttt{sh}\ \texttt{N}\ (\texttt{wrong}\ \texttt{N}\ \text{“Not a number”}))\ (\texttt{sh}\ \{\texttt{N}\}\ (\texttt{nil}\ \texttt{N})) \quad \rightarrow$

$\texttt{cons}\ (\texttt{sh}\ \texttt{N}\ (\texttt{wrong}\ \texttt{N}\ \text{“Not a number”}))\ (\texttt{nil}\ \texttt{N})$

Figure 1.3: Transparency broken for function arguments.

$(\texttt{hs}\ (\texttt{N} \rightarrow \texttt{N})\ (\lambda x_S.\overline{0}))\ (\texttt{wrong}\ \texttt{N}\ \text{“Not a number”}) \quad \rightarrow$

$(\lambda x_H : \texttt{N}.\texttt{hs}\ \texttt{N}\ ((\lambda x_S.\overline{0})\ (\texttt{sh}\ \texttt{N}\ x_H)))\ (\texttt{wrong}\ \texttt{N}\ \text{“Not a number”}) \quad \rightarrow$

$\texttt{hs}\ \texttt{N}\ ((\lambda x_S.\overline{0})\ (\texttt{sh}\ \texttt{N}\ (\texttt{wrong}\ \texttt{N}\ \text{“Not a number”}))) \quad \rightarrow$

Error: “Not a number”

Figure 1.4: Transparency not broken for function arguments.

$(\texttt{hs}\ (\texttt{N} \rightarrow \texttt{N})\ (\lambda x_S.\overline{0}))\ (\texttt{wrong}\ \texttt{N}\ \text{“Not a number”}) \quad \rightarrow$

$(\lambda x_H : \texttt{N}.\texttt{hs}\ \texttt{N}\ ((\lambda x_S.\overline{0})\ (\texttt{sh}\ \texttt{N}\ x_H)))\ (\texttt{wrong}\ \texttt{N}\ \text{“Not a number”}) \quad \rightarrow$

$\texttt{hs}\ \texttt{N}\ ((\lambda x_S.\overline{0})\ (\texttt{sh}\ \texttt{N}\ (\texttt{wrong}\ \texttt{N}\ \text{“Not a number”}))) \quad \rightarrow$

$\texttt{hs}\ \texttt{N}\ \overline{0} \quad \rightarrow$

$\overline{0}$

Chapter 2

Model of Computation

To preserve the transparency of interoperation, ML and Scheme must not force Haskell to evaluate reducible expressions in Haskell boundaries in the incompatible evaluation contexts, and force their evaluation in all other evaluation contexts. Haskell boundaries must be a new kind of value that ML and Scheme can force to become a reducible expression in certain evaluation contexts, and thereby force the evaluation of the inner Haskell reducible expressions to Haskell values and the conversion of those values to ML or Scheme.

Since ML and Scheme do not force Haskell to evaluate in some evaluation contexts, we must factor Haskell boundaries out of ML and Scheme evaluation context nonterminals, E, into new evaluation context nonterminals. We name these new nonterminals F because they allow ML and Scheme to force Haskell to evaluate, and we rename the primary evaluation context nonterminals from E to U (unforced) because they do not. Likewise, we factor Haskell boundaries out of ML and Scheme value nonterminals, v, into new value nonterminals. We name these new nonterminals f (forced) and rename the old value nonterminals from v to u (unforced). We rename Haskell evaluation contexts and values to F and

f, respectively.

In ML and Scheme, we tie F and U together by replacing U with F in the syntax and operational semantics in all evaluation contexts except the incompatible ones. Likewise, we tie f and u together by replacing u with f in the syntax and operational semantics in those same evaluation contexts. F evaluation contexts produce f values, and U evaluation contexts produce u values. U only applies to incompatible evalation contexts, and F applies to all others. ML and Scheme use F to evaluate expressions. We rename the meta evaluation context from $\mathscr{E}$ to $\mathscr{F}$.

Transparency is restored for interoperation in all cases with our changes to the model of computation of Matthews and Findler.

Theorem 1. *Interoperation is transparent:*

1. $e_H \simeq \mathtt{hm}\ t_H\ t_M\ (\mathtt{mh}\ t_M\ t_H\ e_H) \simeq \mathtt{ms}\ t_M\ (\mathtt{sh}\ t_H\ e_H)$
2. $\simeq \mathtt{hs}\ t_H\ (\mathtt{sh}\ t_H\ e_H)$
3. $e_M \simeq \mathtt{mh}\ t_M\ t_H\ (\mathtt{hm}\ t_H\ t_M\ e_M) \simeq \mathtt{ms}\ t_M\ (\mathtt{sm}\ t_M\ e_M)$
4. $e_S \simeq \mathtt{hs}\ t_H\ e_S \simeq \mathtt{ms}\ t_M\ e_S$

where $\simeq$ denotes observational equivalence ???CITE???.

Proof. By structural induction. □

Another complication with introducing Haskell to the model of computation is how to convert type abstractions between Haskell and ML. The application of a converted type abstraction cannot substitute the type argument into the nested expression because the type argument is meaningless in the nested expression's language. Instead, the application substitutes the type argument and a lump into the boundary's outer and inner types, respectively. Since the natural embedding requires the boundary's outer and inner types to be equal [1], we use a new notion of equality called lump equality that allows lumps within the boundary's inner type to match any corresponding type in the boundary's outer type.

Legends of symbol and syntax names are presented in figures 2.1-2.5; Haskell is presented in figures 2.6-2.10; ML is presented in figures 2.11-2.15; Scheme is presented in figures 2.16-2.20; the unbrand function is presented in figure 2.2; and lump equality is presented in figure 2.3.

Figure 2.1: Symbol names

b	Brand
k	Conversion scheme
e	Expression
F	Forced evaluation context
f	Forced value
$\mathtt{L}$	Lump
$\doteq$	Lump equality relation
$\mathscr{F}$	Meta evaluation context
$\overline{n}$	Natural number
$\mathtt{N}$	Natural number
$\rightarrow$	Reduction relation
t	Type
y	Type variable
Γ	Typing environment
$\vdash$	Typing relation
U	Unforced evaluation context
u	Unforced value
x	Variable

Figure 2.2: Convert conversion schemes to types.

$$
\begin{array}{rcl}
\lfloor \mathtt{L} \rfloor & = & \mathtt{L} \\
\lfloor \mathtt{N} \rfloor & = & \mathtt{N} \\
\lfloor y_H \rfloor & = & y_H \\
\lfloor y_M \rfloor & = & y_M \\
\lfloor \{k_H\} \rfloor & = & \{\lfloor k_H \rfloor\} \\
\lfloor \{k_M\} \rfloor & = & \{\lfloor k_M \rfloor\} \\
\lfloor k_H \to k_H \rfloor & = & \lfloor k_H \rfloor \to \lfloor k_H \rfloor \\
\lfloor k_M \to k_M \rfloor & = & \lfloor k_M \rfloor \to \lfloor k_M \rfloor \\
\lfloor \forall y_H . k_H \rfloor & = & \forall y_H . \lfloor k_H \rfloor \\
\lfloor \forall y_M . k_M \rfloor & = & \forall y_M . \lfloor k_M \rfloor \\
\lfloor b \diamond t_H \rfloor & = & t_H \\
\lfloor b \diamond t_M \rfloor & = & t_M
\end{array}
$$

Figure 2.3: Lump equality

$$x \doteq x$$

$$x \doteq y \Rightarrow y \doteq x$$

$$x \doteq y \text{ and } y \doteq z \Rightarrow x \doteq z$$

$$t_H \doteq \mathtt{L}$$

$$t_M \doteq \mathtt{L}$$

$$t_H = t_M \Rightarrow t_H \doteq t_M$$

Figure 2.4: Syntax names

$+\ e\ e$	Addition
$\texttt{if0}\ e\ e\ e$	Condition
$\texttt{nil}\ t$	Empty list
$\texttt{nil}$	Empty list
$\texttt{null?}\ e$	Empty list predicate
$\texttt{wrong}\ t\ \mathit{string}$	Error
$\texttt{wrong}\ \mathit{string}$	Error
$\texttt{fix}\ e$	Fixed-point operation
$\lambda x : t.e$	Function abstraction
$\lambda x_S.e_S$	Function abstraction
$\texttt{fun?}\ e_S$	Function abstraction predicate
$e\ e$	Function application
$\texttt{hm}\ t_H\ t_M\ e_M$	Haskell-ML guard
$\texttt{hs}\ k_H\ e_S$	Haskell-Scheme guard
$\texttt{cons}\ e\ e$	List construction
$\texttt{hd}\ e$	List head
$\texttt{list?}\ e_S$	List predicate
$\texttt{tl}\ e$	List tail
$\texttt{mh}\ t_M\ t_H\ e_H$	ML-Haskell guard
$\texttt{ms}\ k_M\ e_S$	ML-Scheme guard
$\texttt{num?}\ e_S$	Number predicate
$\texttt{sh}\ k_H\ e_H$	Scheme-Haskell guard
$\texttt{sm}\ k_M\ e_M$	Scheme-ML guard
$-\ e\ e$	Subtraction
$\Lambda y.e$	Type abstraction
$e\ \langle t \rangle$	Type application

Figure 2.5: Syntax names

$b \diamond t$	Branded type
$\forall y.t$	Forall type
$\forall y.k$	Forall conversion scheme
$t \to t$	Function abstraction
$k \to k$	Function abstraction
$\{t\}$	List
$\{k\}$	List

Figure 2.6: Haskell syntax and evaluation contexts.

$$
\begin{array}{rcl}
e_H & = & x_H \mid f_H \mid e_H\ e_H \mid e_H\ \langle t_H \rangle \mid \texttt{fix}\ e_H \mid a\ e_H\ e_H \mid \texttt{if0}\ e_H\ e_H\ e_H \mid c\ e_H \\
& & \texttt{null?}\ e_H \mid \texttt{wrong}\ t_H\ \mathit{string} \mid \texttt{hm}\ t_H\ t_M\ e_M \mid \texttt{hs}\ k_H\ e_S \\
f_H & = & \lambda x_H : t_H.e_H \mid \Lambda y_H.e_H \mid \overline{n} \mid \texttt{nil}\ t_H \mid \texttt{cons}\ e_H\ e_H \mid \texttt{hm}\ \texttt{L}\ t_M\ f_M \\
& & \texttt{hs}\ \texttt{L}\ f_S \\
t_H & = & \texttt{L} \mid \texttt{N} \mid y_H \mid \{t_H\} \mid t_H \to t_H \mid \forall y_H.t_H \\
k_H & = & \texttt{L} \mid \texttt{N} \mid y_H \mid \{k_H\} \mid k_H \to k_H \mid \forall y_H.k_H \mid b \diamond t_H \\
a & = & + \mid - \\
c & = & \texttt{hd} \mid \texttt{tl} \\
F_H & = & [\,]_H \mid F_H\ e_H \mid F_H\ \langle t_H \rangle \mid \texttt{fix}\ F_H \mid a\ F_H\ e_H \mid a\ f_H\ F_H \\
& & \texttt{if0}\ F_H\ e_H\ e_H \mid c\ F_H \mid \texttt{null?}\ F_H \mid \texttt{hm}\ t_H\ t_M\ F_M \mid \texttt{hs}\ k_H\ F_S
\end{array}
$$

Figure 2.7: Haskell typing rules

$$\frac{}{\vdash_H \mathtt{L}} \quad \frac{}{\vdash_H \mathtt{N}} \quad \frac{}{\Gamma, y_H \vdash_H y_H}$$

$$\frac{\Gamma \vdash_H t_H}{\Gamma \vdash_H \{t_H\}} \quad \frac{\Gamma \vdash_H t_H \quad \Gamma \vdash_H t'_H}{\Gamma \vdash_H t_H \to t'_H} \quad \frac{\Gamma, y_H \vdash_H t_H}{\Gamma \vdash_H \forall y_H.t_H}$$

$$\frac{\Gamma \vdash_H t_H \quad \Gamma, x_H : t_H \vdash_H e_H : t'_H}{\Gamma \vdash_H (\lambda x_H : t_H.e_H) : t_H \to t'_H} \quad \frac{\Gamma, y_H \vdash_H e_H : t_H}{\Gamma \vdash_H \Lambda y_H.e_H : \forall y_H.t_H} \quad \frac{}{\vdash_H \overline{n} : \mathtt{N}}$$

$$\frac{\Gamma \vdash_H t_H :}{\Gamma \vdash_H \mathtt{nil}\ t_H : \{t_H\}} \quad \frac{\Gamma \vdash_H e_H : t_H \quad \Gamma \vdash_H e'_H : \{t_H\}}{\Gamma \vdash_H \mathtt{cons}\ e_H\ e'_H : \{t_H\}} \quad \frac{}{\Gamma, x_H : t_H \vdash_H x_H : t_H}$$

$$\frac{\Gamma \vdash_H e_H : t_H \to t'_H \quad \Gamma \vdash_H e'_H : t_H}{\Gamma \vdash_H e_H\ e'_H : t'_H} \quad \frac{\Gamma \vdash_H e_H : t_H \to t_H}{\Gamma \vdash_H \mathtt{fix}\ e_H : t_H}$$

$$\frac{\Gamma \vdash_H t_H \quad \Gamma \vdash_H e_H : \forall y_H.t'_H}{\Gamma \vdash_H e_H\ \langle t_H \rangle : t'_H[t_H/y_H]} \quad \frac{\Gamma \vdash_H e_H : \{t_H\}}{\Gamma \vdash_H \mathtt{hd}\ e_H : t_H} \quad \frac{\Gamma \vdash_H e_H : \{t_H\}}{\Gamma \vdash_H \mathtt{tl}\ e_H : \{t_H\}}$$

$$\frac{\Gamma \vdash_H e_H : \mathtt{N} \quad \Gamma \vdash_H e'_H : \mathtt{N}}{\Gamma \vdash_H a\ e_H\ e'_H : \mathtt{N}} \quad \frac{\Gamma \vdash_H e_H : \{t_H\}}{\Gamma \vdash_H \mathtt{null?}\ e_H : \mathtt{N}} \quad \frac{\Gamma \vdash_H \lfloor k_H \rfloor \quad \Gamma \vdash_S e_S : \mathtt{TST}}{\Gamma \vdash_H \mathtt{hs}\ k_H\ e_S : \lfloor k_H \rfloor}$$

$$\frac{\Gamma \vdash_H e_H : \mathtt{N} \quad \Gamma \vdash_H e'_H : t_H \quad \Gamma \vdash_H e''_H : t_H}{\Gamma \vdash_H \mathtt{if0}\ e_H\ e'_H\ e''_H : t_H} \quad \frac{\Gamma \vdash_H t_H}{\Gamma \vdash_H \mathtt{wrong}\ t_H\ \mathit{string} : t_H}$$

$$\frac{\Gamma \vdash_H t_H \quad \Gamma \vdash_M t_M \quad \Gamma \vdash_M e_M : t'_M \quad t_H \doteq t_M \quad t_M = t'_M}{\Gamma \vdash_H \mathtt{hm}\ t_H\ t_M\ e_M : t_H}$$

Figure 2.8: Haskell operational semantics

$\mathscr{F}[(\lambda x_H : t_H.e_H)\ e'_H]_H \to \mathscr{F}[e_H[e'_H/x_H]]$

$\mathscr{F}[(\Lambda y_H.e_H)\ \langle t_H\rangle]_H \to \mathscr{F}[e_H[b \diamond t_H/y_H]]$

$\mathscr{F}[\texttt{fix}\ (\lambda x_H : t_H.e_H)]_H \to \mathscr{F}[e_H[\texttt{fix}\ (\lambda x_H : t_H.e_H)/x_H]]$

$\mathscr{F}[+\ \overline{n}\ \overline{n}']_H \to \mathscr{F}[\overline{n+n'}]$

$\mathscr{F}[-\ \overline{n}\ \overline{n}']_H \to \mathscr{F}[\overline{max(n-n',0)}]$

$\mathscr{F}[\texttt{if0}\ \overline{0}\ e_H\ e'_H]_H \to \mathscr{F}[e_H]$

$\mathscr{F}[\texttt{if0}\ \overline{n}\ e_H\ e'_H]_H \to \mathscr{F}[e'_H]\ (n \neq 0)$

$\mathscr{F}[\texttt{hd}\ (\texttt{nil}\ t_H)]_H \to \mathscr{F}[\texttt{wrong}\ t_H\ \text{“Empty list”}]$

$\mathscr{F}[\texttt{tl}\ (\texttt{nil}\ t_H)]_H \to \mathscr{F}[\texttt{wrong}\ \{t_H\}\ \text{“Empty list”}]$

$\mathscr{F}[\texttt{hd}\ (\texttt{cons}\ e_H\ e'_H)]_H \to \mathscr{F}[e_H]$

$\mathscr{F}[\texttt{tl}\ (\texttt{cons}\ e_H\ e'_H)]_H \to \mathscr{F}[e'_H]$

$\mathscr{F}[\texttt{null?}\ (\texttt{nil}\ t_H)]_H \to \mathscr{F}[\overline{0}]$

$\mathscr{F}[\texttt{null?}\ (\texttt{cons}\ e_H\ e'_H)]_H \to \mathscr{F}[\overline{1}]$

$\mathscr{F}[\texttt{wrong}\ t_H\ \mathit{string}]_H \to \textbf{Error: } \mathit{string}$

Figure 2.9: Haskell-ML operational semantics

$\mathscr{F}[\texttt{hm}\ t_H\ t_M\ (\texttt{mh}\ t'_M\ t'_H\ e_H)]_H \to \mathscr{F}[e_H]\quad (t_H \neq \texttt{L} \wedge t_H = t'_H)$

$\mathscr{F}[\texttt{hm}\ t_H\ t_M\ (\texttt{mh}\ t'_M\ t'_H\ e_H)]_H \to$

$\quad\mathscr{F}[\texttt{wrong}\ t_H\ \text{“Type mismatch”}]\quad (t_H \neq \texttt{L} \wedge t_H \neq t'_H)$

$\mathscr{F}[\texttt{hm}\ t_H\ \texttt{L}\ (\texttt{ms}\ \texttt{L}\ f_S)]_H \to \mathscr{F}[\texttt{wrong}\ t_H\ \text{“Bad value”}]\quad (t_H \neq \texttt{L})$

$\mathscr{F}[\texttt{hm}\ \texttt{N}\ \texttt{N}\ \overline{n}]_H \to \mathscr{F}[\overline{n}]$

$\mathscr{F}[\texttt{hm}\ \{t_H\}\ \{t_M\}\ (\texttt{nil}\ t'_M)]_H \to \mathscr{F}[\texttt{nil}\ t_H]$

$\mathscr{F}[\texttt{hm}\ \{t_H\}\ \{t_M\}\ (\texttt{cons}\ u_M\ u'_M)]_H \to$

$\quad\mathscr{F}[\texttt{cons}\ (\texttt{hm}\ t_H\ t_M\ u_M)\ (\texttt{hm}\ \{t_H\}\ \{t_M\}\ u'_M)]$

$\mathscr{F}[\texttt{hm}\ (t_H \to t'_H)\ (t_M \to t'_M)\ (\lambda x_M : t''_M.e_M)]_H \to$

$\quad\mathscr{F}[\lambda x_H : t_H.\texttt{hm}\ t'_H\ t'_M\ ((\lambda x_M : t''_M.e_M)\ (\texttt{mh}\ t_M\ t_H\ x_H))]$

$\mathscr{F}[\texttt{hm}\ (\forall y_H.t_H)\ (\forall y_M.t_M)\ (\Lambda y'_M.e_M)]_H \to \mathscr{F}[\Lambda y_H.\texttt{hm}\ t_H\ t_M[\texttt{L}/y_M]\ e_M[\texttt{L}/y'_M]]$

Figure 2.10: Haskell-Scheme operational semantics.

$\mathscr{F}[\texttt{hs}\ t_H\ (\texttt{sh}\ t'_H\ e_H)]_H \rightarrow \mathscr{F}[e_H]\ (t_H = t'_H)$

$\mathscr{F}[\texttt{hs N}\ \overline{n}]_H \rightarrow \mathscr{F}[\overline{n}]$

$\mathscr{F}[\texttt{hs N}\ f_S]_H \rightarrow \mathscr{F}[\texttt{wrong N}\ \text{“Not a number”}]\ (f_S \neq \overline{n})$

$\mathscr{F}[\texttt{hs}\ \{k_H\}\ \texttt{nil}]_H \rightarrow \mathscr{F}[\texttt{nil}\ \lfloor k_H \rfloor]$

$\mathscr{F}[\texttt{hs}\ \{k_H\}\ (\texttt{cons}\ u_S\ u'_S)]_H \rightarrow \mathscr{F}[\texttt{cons}\ (\texttt{hs}\ k_H\ u_S)\ (\texttt{hs}\ \{k_H\}\ u'_S)]$

$\mathscr{F}[\texttt{hs}\ \{k_H\}\ f_S]_H \rightarrow \mathscr{F}[\texttt{wrong}\ \lfloor\{k_H\}\rfloor\ \text{“Not a list”}]$

$(f_S \neq \texttt{nil}\ \text{and}\ f_S \neq \texttt{cons}\ u_S\ u'_S)$

$\mathscr{F}[\texttt{hs}\ (b \diamond t_H)\ (\texttt{sh}\ (b \diamond t_H)\ e_H)]_H \rightarrow \mathscr{F}[e_H]$

$\mathscr{F}[\texttt{hs}\ (b \diamond t_H)\ f_S]_H \rightarrow \mathscr{F}[\texttt{wrong}\ t_H\ \text{“Brand mismatch”}]\ (f_S \neq \texttt{sh}\ (b \diamond t_H)\ e_H)$

$\mathscr{F}[\texttt{hs}\ (k_H \rightarrow k'_H)\ (\lambda x_S.e_S)]_H \rightarrow \mathscr{F}[\lambda x_H : \lfloor k_H \rfloor.\texttt{hs}\ k'_H\ ((\lambda x_S.e_S)\ (\texttt{sh}\ k_H\ x_H))]$

$\mathscr{F}[\texttt{hs}\ (k_H \rightarrow k'_H)\ f_S]_H \rightarrow \mathscr{F}[\texttt{wrong}\ \lfloor k_H \rightarrow k'_H \rfloor\ \text{“Not a function”}]$

$(f_S \neq \lambda x_S.e_S)$

$\mathscr{F}[\texttt{hs}\ (\forall y_H.k_H)\ f_S]_H \rightarrow \mathscr{F}[\Lambda y_H.\texttt{hs}\ k_H\ f_S]$

Figure 2.11: ML syntax and evaluation contexts.

$$
\begin{array}{rcl}
e_M & = & x_M \mid u_M \mid e_M\ e_M \mid e_M\ \langle t_M \rangle \mid \texttt{fix}\ e_M \mid a\ e_M\ e_M \mid \texttt{if0}\ e_M\ e_M\ e_M \\
 & & \texttt{cons}\ e_M\ e_M \mid c\ e_M \mid \texttt{null?}\ e_M \mid \texttt{wrong}\ t_M\ \mathit{string} \mid \texttt{ms}\ k_M\ e_S \\
u_M & = & f_M \mid \texttt{mh}\ t_M\ t_H\ e_H \\
f_M & = & \lambda x_M : t_M.e_M \mid \Lambda y_M.e_M \mid \overline{n} \mid \texttt{nil}\ t_M \mid \texttt{cons}\ u_M\ u_M \mid \texttt{mh}\ \texttt{L}\ t_H\ e_H \\
 & & \texttt{ms}\ \texttt{L}\ f_S \\
t_M & = & \texttt{L} \mid \texttt{N} \mid y_M \mid \{t_M\} \mid t_M \to t_M \mid \forall y_M.t_M \\
k_M & = & \texttt{L} \mid \texttt{N} \mid y_M \mid \{k_M\} \mid k_M \to k_M \mid \forall y_M.k_M \mid b \diamond t_M \\
a & = & + \mid - \\
c & = & \texttt{hd} \mid \texttt{tl} \\
F_M & = & U_M \mid \texttt{mh}\ t_M\ t_H\ F_H \\
U_M & = & [\,]_M \mid F_M\ e_M \mid f_M\ U_M \mid F_M\ \langle t_M \rangle \mid \texttt{fix}\ F_M \mid a\ F_M\ e_M \mid a\ f_M\ F_M \\
 & & \texttt{if0}\ F_M\ e_M\ e_M \mid \texttt{cons}\ U_M\ e_M \mid \texttt{cons}\ u_M\ U_M \mid c\ F_M \mid \texttt{null?}\ F_M \\
 & & \texttt{ms}\ k_M\ F_S
\end{array}
$$

Figure 2.12: ML typing rules.

$$\frac{}{\vdash_M \mathtt{L}} \quad \frac{}{\vdash_M \mathtt{N}} \quad \frac{}{\Gamma, y_M \vdash_M y_M}$$

$$\frac{\Gamma \vdash_M t_M}{\Gamma \vdash_M \{t_M\}} \quad \frac{\Gamma \vdash_M t_M \quad \Gamma \vdash_M t'_M}{\Gamma \vdash_M t_M \to t'_M} \quad \frac{\Gamma, y_M \vdash_M t_M}{\Gamma \vdash_M \forall y_M.t_M}$$

$$\frac{\Gamma \vdash_M t_M \quad \Gamma, x_M : t_M \vdash_M e_M : t'_M}{\Gamma \vdash_M (\lambda x_M : t_M.e_M) : t_M \to t'_M} \quad \frac{\Gamma, y_M \vdash_M e_M : t_M}{\Gamma \vdash_M \Lambda y_M.e_M : \forall y_M.t_M} \quad \frac{}{\vdash_M \overline{n} : \mathtt{N}}$$

$$\frac{\Gamma \vdash_M t_M}{\Gamma \vdash_M \mathtt{nil}\ t_M : \{t_M\}} \quad \frac{\Gamma \vdash_M e_M : t_M \quad \Gamma \vdash_M e'_M : \{t_M\}}{\Gamma \vdash_M \mathtt{cons}\ e_M\ e'_M : \{t_M\}} \quad \frac{}{\Gamma, x_M : t_M \vdash_M x_M : t_M}$$

$$\frac{\Gamma \vdash_M e_M : t_M \to t'_M \quad \Gamma \vdash_M e'_M : t_M}{\Gamma \vdash_H e_M\ e'_M : t'_M} \quad \frac{\Gamma \vdash_M e_M : t_M \to t_M}{\Gamma \vdash_M \mathtt{fix}\ e_M : t_M}$$

$$\frac{\Gamma \vdash_M t_M \quad \Gamma \vdash_M e_M : \forall y_M.t'_M}{\Gamma \vdash_M e_M\ \langle t_M \rangle : t'_M[t_M/y_M]} \quad \frac{\Gamma \vdash_M e_M : \{t_M\}}{\Gamma \vdash_M \mathtt{hd}\ e_M : t_M} \quad \frac{\Gamma \vdash_M e_M : \{t_M\}}{\Gamma \vdash_M \mathtt{tl}\ e_M : \{t_M\}}$$

$$\frac{\Gamma \vdash_M e_M : \mathtt{N} \quad \Gamma \vdash_M e'_M : \mathtt{N}}{\Gamma \vdash_M a\ e_M\ e'_M : \mathtt{N}} \quad \frac{\Gamma \vdash_M e_M : \{t_M\}}{\Gamma \vdash_M \mathtt{null?}\ e_M : \mathtt{N}} \quad \frac{\Gamma \vdash_M \lfloor k_M \rfloor \quad \Gamma \vdash_S e_S : \mathtt{TST}}{\Gamma \vdash_M \mathtt{ms}\ k_M\ e_S : \lfloor k_M \rfloor}$$

$$\frac{\Gamma \vdash_M e_M : \mathtt{N} \quad \Gamma \vdash_M e'_M : t_M \quad \Gamma \vdash_M e''_M : t_M}{\Gamma \vdash_M \mathtt{if0}\ e_M\ e'_M\ e''_M : t_M} \quad \frac{\Gamma \vdash_M t_M}{\Gamma \vdash_M \mathtt{wrong}\ t_M\ \mathit{string} : t_M}$$

$$\frac{\Gamma \vdash_M t_M \quad \Gamma \vdash_H t_H \quad \Gamma \vdash_H e_H : t'_H \quad t_M \doteq t_H \quad t_H = t'_H}{\Gamma \vdash_M \mathtt{mh}\ t_M\ t_H\ e_H : t_M}$$

Figure 2.13: ML operational semantics.

$\mathscr{F}[(\lambda x_M : t_M.e_M)\ u_M]_M \to \mathscr{F}[e_M[u_M/x_M]]$

$\mathscr{F}[(\Lambda y_M.e_M)\ \langle t_M\rangle]_M \to \mathscr{F}[e_M[b \diamond t_M/y_M]]$

$\mathscr{F}[\texttt{fix}\ (\lambda x_M : t_M.e_M)]_M \to \mathscr{F}[e_M[\texttt{fix}\ (\lambda x_M : t_M.e_M)/x_M]]$

$\mathscr{F}[+\ \overline{n}\ \overline{n}']_M \to \mathscr{F}[\overline{n+n'}]$

$\mathscr{F}[-\ \overline{n}\ \overline{n}']_M \to \mathscr{F}[\overline{max(n-n',0)}]$

$\mathscr{F}[\texttt{if0}\ \overline{0}\ e_M\ e'_M]_M \to \mathscr{F}[e_M]$

$\mathscr{F}[\texttt{if0}\ \overline{n}\ e_M\ e'_M]_M \to \mathscr{F}[e'_M]\ (n \neq 0)$

$\mathscr{F}[\texttt{hd}\ (\texttt{nil}\ t_M)]_M \to \mathscr{F}[\texttt{wrong}\ t_M\ \text{“Empty list”}]$

$\mathscr{F}[\texttt{tl}\ (\texttt{nil}\ t_M)]_M \to \mathscr{F}[\texttt{wrong}\ \{t_M\}\ \text{“Empty list”}]$

$\mathscr{F}[\texttt{hd}\ (\texttt{cons}\ u_M\ u'_M)]_M \to \mathscr{F}[u_M]$

$\mathscr{F}[\texttt{tl}\ (\texttt{cons}\ u_M\ u'_M)]_M \to \mathscr{F}[u'_M]$

$\mathscr{F}[\texttt{null?}\ (\texttt{nil}\ t_M)]_M \to \mathscr{F}[\overline{0}]$

$\mathscr{F}[\texttt{null?}\ (\texttt{cons}\ u_M\ u'_M)]_M \to \mathscr{F}[\overline{1}]$

$\mathscr{F}[\texttt{wrong}\ t_M\ \mathit{string}]_H \to$ **Error:** *string*

Figure 2.14: ML-Haskell operational semantics.

$\mathscr{F}[\texttt{mh}\ t_M\ t_H\ (\texttt{hm}\ t'_H\ t'_M\ f_M)]_M \rightarrow \mathscr{F}[f_M]\ (t_M \neq \texttt{L} \wedge t_M = t'_M)$

$\mathscr{F}[\texttt{mh}\ t_M\ t_H\ (\texttt{hm}\ t'_H\ t'_M\ f_M)]_M \rightarrow$

$\quad \mathscr{F}[\texttt{wrong}\ t_M\ \text{“Type mismatch”}]\ (t_M \neq \texttt{L} \wedge t_M \neq t'_M)$

$\mathscr{F}[\texttt{mh}\ t_M\ \texttt{L}\ (\texttt{hs}\ \texttt{L}\ f_S)]_H \rightarrow \mathscr{F}[\texttt{wrong}\ t_M\ \text{“Bad value”}]\ (t_M \neq \texttt{L})$

$\mathscr{F}[\texttt{mh}\ \texttt{N}\ \texttt{N}\ \overline{n}]_M \rightarrow \mathscr{F}[\overline{n}]$

$\mathscr{F}[\texttt{mh}\ \{t_M\}\ \{t_H\}\ (\texttt{nil}\ t'_H)]_M \rightarrow \mathscr{F}[\texttt{nil}\ t_M]$

$\mathscr{F}[\texttt{mh}\ \{t_M\}\ \{t_H\}\ (\texttt{cons}\ e_H\ e'_H)]_M \rightarrow \mathscr{F}[\texttt{cons}\ (\texttt{mh}\ t_M\ t_H\ e_H)\ (\texttt{mh}\ \{t_M\}\ \{t_H\}\ e'_H)]$

$\mathscr{F}[\texttt{mh}\ (t_M \rightarrow t'_M)\ (t_H \rightarrow t'_H)\ (\lambda x_H : t''_H.e_H)]_M \rightarrow$

$\quad \mathscr{F}[\lambda x_M : t_M.\texttt{mh}\ t'_M\ t'_H\ ((\lambda x_H : t''_H.e_H)\ (\texttt{hm}\ t_H\ t_M\ x_M))]$

$\mathscr{F}[\texttt{mh}\ (\forall y_M.t_M)\ (\forall y_H.t_H)\ (\Lambda y'_H.e_H)]_M \rightarrow \mathscr{F}[\Lambda y_M.\texttt{mh}\ t_M\ t_H[\texttt{L}/y_H]\ e_H[\texttt{L}/y'_H]]$

Figure 2.15: ML-Scheme operational semantics.

$\mathscr{F}[\texttt{ms N}\ \overline{n}]_M \to \mathscr{F}[\overline{n}]$

$\mathscr{F}[\texttt{ms N}\ f_S]_M \to \mathscr{F}[\texttt{wrong N}\ \text{“Not a number”}]\ (f_S \neq \overline{n})$

$\mathscr{F}[\texttt{ms}\ \{k_M\}\ \texttt{nil}]_M \to \mathscr{F}[\texttt{nil}\ \lfloor k_M \rfloor]$

$\mathscr{F}[\texttt{ms}\ \{k_M\}\ (\texttt{cons}\ u_S\ u'_S)]_M \to \mathscr{F}[\texttt{cons}\ (\texttt{ms}\ k_M\ u_S)\ (\texttt{ms}\ \{k_M\}\ u'_S)]$

$\mathscr{F}[\texttt{ms}\ \{k_M\}\ f_S]_M \to \mathscr{F}[\texttt{wrong}\ \lfloor \{k_M\} \rfloor\ \text{“Not a list”}]$

$(f_S \neq \texttt{nil}\ \text{and}\ f_S \neq \texttt{cons}\ u_S\ u'_S)$

$\mathscr{F}[\texttt{ms}\ (b \diamond t_M)\ (\texttt{sm}\ (b \diamond t_M)\ u_M)]_M \to \mathscr{F}[u_M]$

$\mathscr{F}[\texttt{ms}\ (b \diamond t_M)\ f_S]_M \to \mathscr{F}[\texttt{wrong}\ \lfloor b \diamond t_M \rfloor\ \text{“Brand mismatch”}]$

$(f_S \neq \texttt{sm}\ (b \diamond t_M)\ e_M)$

$\mathscr{F}[\texttt{ms}\ (k_M \to k'_M)\ (\lambda x_S.e_S)]_M \to$

$\mathscr{F}[\lambda x_M : \lfloor k_M \rfloor.\texttt{ms}\ k'_M\ ((\lambda x_S.e_S)\ (\texttt{sm}\ k_M\ x_M))]$

$\mathscr{F}[\texttt{ms}\ (k_M \to k'_M)\ f_S]_M \to \mathscr{F}[\texttt{wrong}\ \lfloor k_M \to k'_M \rfloor\ \text{“Not a function”}]$

$(f_S \neq \lambda x_S.e_S)$

$\mathscr{F}[\texttt{ms}\ (\forall y_M.k_M)\ f_S]_M \to \mathscr{F}[\Lambda y_M.\texttt{ms}\ k_M\ f_S]$

Figure 2.16: Scheme syntax and evaluation contexts.

$$
\begin{array}{lcl}
e_S & = & x_S \mid u_S \mid e_S\ e_S \mid a\ e_S\ e_S \mid p\ e_S \mid \texttt{if0}\ e_S\ e_S\ e_S \mid \texttt{cons}\ e_S\ e_S \mid c\ e_S \\
 & & \texttt{wrong}\ \textit{string} \mid \texttt{sm}\ k_M\ e_M \\
u_S & = & f_S \mid \texttt{sh}\ k_H\ e_H \\
f_S & = & \lambda x_S.e_S \mid \overline{n} \mid \texttt{nil} \mid \texttt{cons}\ u_S\ u_S \mid \texttt{sh}\ (b \diamond t_H)\ e_H \mid \texttt{sm}\ (b \diamond t_M)\ f_M \\
a & = & + \mid - \\
c & = & \texttt{hd} \mid \texttt{tl} \\
p & = & \texttt{fun}? \mid \texttt{list}? \mid \texttt{null}? \mid \texttt{num}? \\
F_S & = & U_S \mid \texttt{sh}\ k_H\ F_H \\
U_S & = & [\,]_S \mid F_S\ e_S \mid f_S\ U_S \mid a\ F_S\ e_S \mid a\ f_S\ F_S \mid p\ F_S \mid \texttt{if0}\ F_S\ e_S\ e_S \\
 & & \texttt{cons}\ U_S\ e_S \mid \texttt{cons}\ u_S\ U_S \mid c\ F_S \mid \texttt{sm}\ k_M\ F_M
\end{array}
$$

Figure 2.17: Scheme typing rules.

$$\frac{}{\vdash_S \texttt{TST}}$$

$$\frac{\Gamma, x_S : \texttt{TST} \vdash_S e_S : \texttt{TST}}{\Gamma \vdash_S \lambda x_S.e_S : \texttt{TST}} \qquad \frac{}{\vdash_S \overline{n} : \texttt{TST}} \qquad \frac{}{\vdash_S \texttt{nil} : \texttt{TST}}$$

$$\frac{\Gamma \vdash_S e_S : \texttt{TST} \quad \Gamma \vdash_S e'_S : \texttt{TST}}{\Gamma \vdash_S \texttt{cons}\ e_S\ e'_S : \texttt{TST}} \qquad \frac{}{\Gamma, x_S : \texttt{TST} \vdash_S x_S : \texttt{TST}}$$

$$\frac{\Gamma \vdash_S e_S : \texttt{TST} \quad \Gamma \vdash_S e'_S : \texttt{TST}}{\Gamma \vdash_H e_S\ e'_S : \texttt{TST}} \qquad \frac{\Gamma \vdash_S e_S : \texttt{TST}}{\Gamma \vdash_S c\ e_S : \texttt{TST}}$$

$$\frac{\Gamma \vdash_S e_S : \texttt{TST} \quad \Gamma \vdash_S e'_S : \texttt{TST}}{\Gamma \vdash_S a\ e_S\ e'_S : \texttt{TST}} \qquad \frac{\Gamma \vdash_S e_S : \texttt{TST}}{\Gamma \vdash_S p\ e_S : \texttt{TST}}$$

$$\frac{\Gamma \vdash_S e_S : \texttt{TST} \quad \Gamma \vdash_S e'_S : \texttt{TST} \quad \Gamma \vdash_S e''_S : \texttt{TST}}{\Gamma \vdash_S \texttt{if0}\ e_S\ e'_S\ e''_S : \texttt{TST}} \qquad \frac{}{\vdash_S \texttt{wrong}\ \mathit{string} : \texttt{TST}}$$

$$\frac{\Gamma \vdash_H \lfloor k_H \rfloor \quad \Gamma \vdash_H e_H : t_H \quad \lfloor k_H \rfloor = t_H}{\Gamma \vdash_S \texttt{sh}\ k_H\ e_H : \texttt{TST}} \qquad \frac{\Gamma \vdash_M \lfloor k_M \rfloor \quad \Gamma \vdash_M e_M : t_M \quad \lfloor k_M \rfloor = t_M}{\Gamma \vdash_S \texttt{sm}\ k_M\ e_M : \texttt{TST}}$$

Figure 2.18: Scheme operational semantics.

$\mathscr{F}[(\lambda x_S.e_S)\ u_S]_S \to \mathscr{F}[e_S[u_S/x_S]]$

$\mathscr{F}[f_S\ u_S]_S \to \mathscr{F}[\texttt{wrong}\ \text{“Not a function”}]\ (f_S \neq \lambda x_S.e_S)$

$\mathscr{F}[+\ \overline{n}\ \overline{n}']_S \to \mathscr{F}[\overline{n+n'}]$

$\mathscr{F}[-\ \overline{n}\ \overline{n}']_S \to \mathscr{F}[\overline{\mathit{max}(n-n',0)}]$

$\mathscr{F}[a\ f_S\ f'_S]_S \to \mathscr{F}[\texttt{wrong}\ \text{“Not a number”}]\ (f_S \neq \overline{n} \text{ or } f'_S \neq \overline{n})$

$\mathscr{F}[\texttt{if0}\ \overline{0}\ e_S\ e'_S]_S \to \mathscr{F}[e_S]$

$\mathscr{F}[\texttt{if0}\ \overline{n}\ e_S\ e'_S]_S \to \mathscr{F}[e'_S]\ (n \neq 0)$

$\mathscr{F}[\texttt{if0}\ f_S\ e_S\ e'_S]_S \to \mathscr{F}[\texttt{wrong}\ \text{“Not a number”}]\ (f_S \neq \overline{n})$

$\mathscr{F}[c\ \texttt{nil}]_S \to \mathscr{F}[\texttt{wrong}\ \text{“Empty list”}]$

$\mathscr{F}[\texttt{hd}\ (\texttt{cons}\ u_S\ u'_S)]_S \to \mathscr{F}[u_S]$

$\mathscr{F}[\texttt{tl}\ (\texttt{cons}\ u_S\ u'_S)]_S \to \mathscr{F}[u'_S]$

$\mathscr{F}[c\ f_S]_S \to \mathscr{F}[\texttt{wrong}\ \text{“Not a list”}]\ (f_S \neq \texttt{nil} \text{ and } f_S \neq \texttt{cons}\ u_S\ u'_S)$

$\mathscr{F}[\texttt{fun?}\ (\lambda x_S.e_S)]_S \to \mathscr{F}[\overline{0}]$

$\mathscr{F}[\texttt{fun?}\ f_S]_S \to \mathscr{F}[\overline{1}]\ (f_S \neq \lambda x_S.e_S)$

$\mathscr{F}[\texttt{list?}\ \texttt{nil}]_S \to \mathscr{F}[\overline{0}]$

$\mathscr{F}[\texttt{list?}\ (\texttt{cons}\ u_S\ u'_S)]_S \to \mathscr{F}[\overline{0}]$

$\mathscr{F}[\texttt{list?}\ f_S]_S \to \mathscr{F}[\overline{1}]\ (f_S \neq \texttt{nil} \text{ and } f_S \neq \texttt{cons}\ u_S\ u'_S)$

$\mathscr{F}[\texttt{null?}\ \texttt{nil}]_S \to \mathscr{F}[\overline{0}]$

$\mathscr{F}[\texttt{null?}\ f_S]_S \to \mathscr{F}[\overline{1}]\ (f_S \neq \texttt{nil})$

$\mathscr{F}[\texttt{num?}\ \overline{n}]_S \to \mathscr{F}[\overline{0}]$

$\mathscr{F}[\texttt{num?}\ f_S]_S \to \mathscr{F}[\overline{1}]\ (f_S \neq \overline{n})$

$\mathscr{F}[\texttt{wrong}\ \mathit{string}]_S \to \textbf{Error: } \mathit{string}$

Figure 2.19: Scheme-Haskell operational semantics.

$\mathscr{F}[\texttt{sh}\ k_H\ (\texttt{hs}\ k'_H\ f_S)]_S \rightarrow \mathscr{F}[f_S]$

$\mathscr{F}[\texttt{sh}\ \texttt{L}\ (\texttt{hm}\ \texttt{L}\ k_M\ f_M)]_S \rightarrow \mathscr{F}[\texttt{wrong}\ \text{“Bad value”}]$

$\mathscr{F}[\texttt{sh}\ \texttt{N}\ \overline{n}]_S \rightarrow \mathscr{F}[\overline{n}]$

$\mathscr{F}[\texttt{sh}\ \{k_H\}\ (\texttt{nil}\ t_H)]_S \rightarrow \mathscr{F}[\texttt{nil}]$

$\mathscr{F}[\texttt{sh}\ \{k_H\}\ (\texttt{cons}\ e_H\ e'_H)]_S \rightarrow \mathscr{F}[\texttt{cons}\ (\texttt{sh}\ k_H\ e_H)\ (\texttt{sh}\ \{k_H\}\ e'_H)]$

$\mathscr{F}[\texttt{sh}\ (k_H \rightarrow k'_H)\ (\lambda x_H : t_H.e_H)]_S \rightarrow$

$\quad \mathscr{F}[\lambda x_S.\texttt{sh}\ k'_H\ ((\lambda x_H : t_H.e_H)\ (\texttt{hs}\ k_H\ x_S))]$

$\mathscr{F}[\texttt{sh}\ (\forall y_H.k_H)\ (\Lambda y'_H.e_H)]_S \rightarrow \mathscr{F}[\texttt{sh}\ k_H[\texttt{L}/y_H]\ e_H[\texttt{L}/y'_H]]$

Figure 2.20: Scheme-ML operational semantics.

$\mathscr{F}[\texttt{sm}\ \texttt{L}\ (\texttt{mh}\ \texttt{L}\ k_H\ e_H)]_S \rightarrow \mathscr{F}[\texttt{wrong}\ \text{“Bad value”}]$

$\mathscr{F}[\texttt{sm}\ \texttt{L}\ (\texttt{ms}\ \texttt{L}\ f_S)]_S \rightarrow \mathscr{F}[f_S]$

$\mathscr{F}[\texttt{sm}\ \texttt{N}\ \overline{n}]_S \rightarrow \mathscr{F}[\overline{n}]$

$\mathscr{F}[\texttt{sm}\ \{k_M\}\ (\texttt{nil}\ t_M)]_S \rightarrow \mathscr{F}[\texttt{nil}]$

$\mathscr{F}[\texttt{sm}\ \{k_M\}\ (\texttt{cons}\ u_M\ u'_M)]_S \rightarrow \mathscr{F}[\texttt{cons}\ (\texttt{sm}\ k_M\ u_M)\ (\texttt{sm}\ \{k_M\}\ u'_M)]$

$\mathscr{F}[\texttt{sm}\ (k_M \rightarrow k'_M)\ (\lambda x_M : t_M.e_M)]_S \rightarrow$

$\quad \mathscr{F}[\lambda x_S.\texttt{sm}\ k'_M\ ((\lambda x_M : t_M.e_M)\ (\texttt{ms}\ k_M\ x_S))]$

$\mathscr{F}[\texttt{sm}\ (\forall y_M.k_M)\ (\Lambda y'_M.e_M)]_S \rightarrow \mathscr{F}[\texttt{sm}\ k_M[\texttt{L}/y_M]\ e_M[\texttt{L}/y'_M]]$

Chapter 3

Conclusion

Interoperation transparently resolves lazy and eager evaluation strategies by the eager evaluation strategy mirroring the lazy one for reducible expressions inside boundaries in evaluation contexts common to both languages where strictness is incompatible. Forced and unforced evaluation contexts and values comprise a simple framework that implements such a system. This method could be extended to resolve incompatible evaluation strategies for evaluation contexts common to any pair of languages.

Bibliography

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