## INTEROPERATION FOR LAZY AND EAGER EVALUATION

## A Thesis

### Presented to

the Faculty of California Polytechnic State University

San Luis Obispo

In Partial Fulfillment of the Requirements for the Degree

by

Master of Science in Computer Science

William Faught

May 2011

## AUTHORIZATION FOR REPRODUCTION OF MASTER'S THESIS

I reserve the reproduction rights of this thesis for a period	d of seven years from the
date of submission. I waive reproduction rights after the	e time span has expired.
Signature	
Data	•
Date	

## APPROVAL PAGE

TITLE: Interoperation for Lazy and Eag	er Evaluation
AUTHOR: William Faught	
DATE SUBMITTED: May 2011	
Dr. John Clements	
Advisor or Committee Chair	Signature
Dr. Gene Fisher	
Committee Member	Signature
Dr. Phillip Nico	
Committee Member	Signature

#### Abstract

Interoperation for Lazy and Eager Evaluation

by

#### William Faught

Programmers forgo existing solutions to problems in other programming languages where software interoperation proves too cumbersome; they remake solutions, rather than reuse them. To facilitate reuse, interoperation must resolve language incompatibilities transparently. To address part of this problem, we present a model of computation that resolves lazy and eager evaluation strategies using dual notions of evaluation contexts and values to mirror the lazy evaluation strategy in the eager one. This method could be extended to resolve incompatible evaluation contexts for expressions common to any pair of languages.

## Acknowledgements

I thank my family and friends, especially my parents, Jerry and Jo Ann, for their encouragement, advice, and support, without which this would not have been possible.

# Contents

Li	vi	
1	Introduction	1
2	Model of Computation	4
3	Conclusion	27
В	28	

# List of Figures

1.1	Scheme forces the conversion of list construction operands	2
1.2	Scheme does not force the conversion of list construction operands.	2
1.3	Scheme forces the conversion of arguments	3
1.4	Scheme does not force the conversion of arguments	3
2.1	Symbol names	7
2.2	Syntax names	8
2.3	Syntax names	9
2.4	Haskell syntax and evaluation contexts	10
2.5	Haskell typing rules	11
2.6	Haskell operational semantics	12
2.7	Haskell-ML operational semantics	13
2.8	Haskell-Scheme operational semantics	14
2.9	ML syntax and evaluation contexts	15
2.10	ML typing rules	16
2.11	ML operational semantics	17
2.12	ML-Haskell operational semantics	18
2.13	ML-Scheme operational semantics	19
2.14	Scheme syntax and evaluation contexts	20
2.15	Scheme typing rules	21
2.16	Scheme operational semantics	22
2.17	Scheme-Haskell operational semantics	23
2.18	Scheme-ML operational semantics	24

2.19	Unbrand function														25
2.20	Lump equality														26

# Chapter 1

## Introduction

Programmers forgo existing solutions to problems in other programming languages where software interoperation proves too cumbersome; they remake solutions, rather than reuse them. To facilitate reuse, interoperation must resolve language incompatibilities transparently. To address part of this problem, we present a model of computation that resolves lazy and eager evaluation strategies.

Matthews and Findler presented a method of safe interoperation between languages with incompatible polymorphic static and dynamic type systems [1]. We observe that their method is insufficient for safe interoperation between languages with incompatible lazy and eager evaluation strategies, then explain the underlying problem, and then finally present a method of interoperation that resolves this incompatibility.

The model of computation of Matthews and Findler comprises two eager languages based on ML and Scheme. We extend their model of computation with a third language that is based on Haskell and identical to their ML-like

```
sh \{N\} (cons (wrong N "Not a number") (nil N)) \rightarrow cons (sh N (wrong N "Not a number")) (sh \{N\} (nil N)) \rightarrow Error: "Not a number"
```

Figure 1.1: Scheme forces the conversion of list construction operands.

```
sh \{N\} (cons (wrong N "Not a number") (nil N)) \rightarrow cons (sh N (wrong N "Not a number")) (sh \{N\} (nil N)) \rightarrow cons (sh N (wrong N "Not a number")) (nil N)
```

Figure 1.2: Scheme does not force the conversion of list construction operands.

language, except it is lazy. We introduce lists to all three languages. Hereafter, we use the names of Haskell, ML, and Scheme to refer to their counterparts in our model of computation.

Unlike ML and Scheme, Haskell does not evaluate function arguments or list construction operands. These three evaluation contexts comprise the set of incompatible evaluation contexts between Haskell and ML, and Haskell and Scheme. Since Haskell permits unused erroneous or divergent expressions in these evaluation contexts and ML and Scheme do not, there are Haskell values that have no counterpart in ML and Scheme. Attempting to convert such values to ML and Scheme forces the evaluation of such expressions and breaks the transparency of interoperation.

Figure 1.1 demonstrates how a straightforward introduction of Haskell to the model of Matthews and Findler breaks the transparency of interoperation when converting a list construction from Haskell to Scheme. The Haskell list construction contains an erroneous operand that Scheme forces to evaluate in the process of converting the Haskell list construction. Figure 1.2 demonstrates Scheme correctly deferring the evaluation of the erroneous Haskell list construction operand

```
(hs (N \to N) (\lambda x_S.\overline{0})) (wrong N "Not a number") \to (\lambda x_H : N.\text{hs N} ((\lambda x_S.\overline{0}) (\text{sh N} x_H))) (wrong N "Not a number") \to hs N ((\lambda x_S.\overline{0}) (\text{sh N} (\text{wrong N "Not a number"}))) <math>\to Error: "Not a number"
```

Figure 1.3: Scheme forces the conversion of arguments.

```
(hs (N \rightarrow N) (\lambda x_S.\overline{0}) (wrong N "Not a number") \rightarrow (\lambda x_H: N.hs N ((\lambda x_S.\overline{0}) (sh N x_H))) (wrong N "Not a number") \rightarrow hs N ((\lambda x_S.\overline{0}) (sh N (wrong N "Not a number"))) \rightarrow hs N \overline{0} \rightarrow \overline{0}
```

Figure 1.4: Scheme does not force the conversion of arguments.

and producing as a result the counterpart Scheme list construction.

Moreover, since the conversion of functions from ML and Scheme to Haskell requires the application of the original function to the converted Haskell argument, ML and Scheme always force the evaluation of the converted Haskell argument, even if it is never used. The application of such converted functions effectively changes the order of evaluation of Haskell and breaks the transparency of interoperation.

Likewise, figure 1.3 demonstrates the conversion of a function from Haskell to Scheme. Scheme forces the evaluation of the erroneous Haskell argument in the process of applying the Scheme function, even though the Haskell argument is never used. From the perspective of the outermost Haskell application, the argument must have been used, but it was not. Figure 1.4 demonstrates Scheme not forcing the evaluation of the Haskell argument, which allows the Scheme function to produce a number.

## Chapter 2

# Model of Computation

To preserve the transparency of interoperation, ML and Scheme must not force Haskell to evaluate reducible expressions in Haskell boundaries in the incompatible evaluation contexts, and force their evaluation in all other evaluation contexts. Haskell boundaries must be a new kind of value that ML and Scheme can force to become a reducible expression in certain evaluation contexts, and thereby force the evaluation of the inner Haskell reducible expressions to Haskell values and the conversion of those values to ML or Scheme.

Since ML and Scheme do not force Haskell to evaluate in some evaluation contexts, we must factor Haskell boundaries out of ML and Scheme evaluation context nonterminals, E, into new evaluation context nonterminals. We name these new nonterminals F because they allow ML and Scheme to force Haskell to evaluate, and we rename the primary evaluation context nonterminals from E to U (unforced) because they do not. Likewise, we factor Haskell boundaries out of ML and Scheme value nonterminals, v, into new value nonterminals. We name these new nonterminals f (forced) and rename the old value nonterminals from v to u (unforced). We rename Haskell evaluation contexts and values to F and

f, respectively.

In ML and Scheme, we tie F and U together by replacing U with F in the syntax and operational semantics in all evaluation contexts except the incompatible ones. Likewise, we tie f and u together by replacing u with f in the syntax and operational semantics in those same evaluation contexts. F evaluation contexts produce f values, and f evaluation contexts produce f values. f only applies to incompatible evaluation contexts, and f applies to all others. ML and Scheme use f to evaluate expressions. We rename the meta evaluation context from f to f.

Transparency is restored for interoperation in all cases with our changes to the model of computation of Matthews and Findler.

**Theorem 1.** Interoperation is transparent:

1. 
$$e_H \equiv \min t_M t_H e_H \equiv \sinh t_H e_H$$

2. 
$$e_M \equiv \operatorname{hm} t_H t_M e_M \equiv \operatorname{sm} t_M e_M$$

3. 
$$e_S \equiv \text{hs } t_H \ e_S \equiv \text{ms } t_M \ e_S$$

where  $\equiv$  denotes contextual equivalence.

*Proof.* By structural induction.

Another complication with introducing Haskell to the model of computation is how to convert type abstractions between Haskell and ML. The application of a converted type abstraction cannot substitute the type argument into the nested expression because the type argument is meaningless in the nested expression's language. Instead, the application substitutes the type argument and a lump into

the boundary's outer and inner types, respectively. Since the natural embedding requires the boundary's outer and inner types to be equal [1], we use a new notion of equality called lump equality that allows lumps within the boundary's inner type to match any corresponding type in the boundary's outer type.

Legends of symbol and syntax names are presented in figures 2.1-2.3; Haskell is presented in figures 2.4-2.8; ML is presented in figures 2.9-2.13; Scheme is presented in figures 2.14-2.18; the unbrand function is presented in figure 2.19; and lump equality is presented in figure 2.20.

Symbol	Name
b	Brand
k	Conversion scheme
e	Expression
F	Forced evaluation context
f	Forced value
L	Lump
Ė	Lump equality relation
$\mathscr{F}$	Meta evaluation context
$\overline{n}$	Natural number
N	Natural number
$\rightarrow$	Reduction relation
t	Type
y	Type variable
$\Gamma$	Typing environment
$\vdash$	Typing relation
U	Unforced evaluation context
u	Unforced value
x	Variable

Figure 2.1: Symbol names

```
Syntax Name
                  Addition
          + e e
      if0 e e e
                  Condition
          nil t Empty list
           nil
                  Empty list
       null? e
                  Empty list predicate
wrong t string
                  Error
 wrong string
                  Error
          \mathtt{fix}\,e
                  Fixed-point operation
        \lambda x : t.e Function abstraction
        \lambda x_S.e_S
                  Function abstraction
       fun? e_S
                  Function abstraction predicate
                  Function application
 \mathtt{hm}\ t_H\ t_M\ e_M
                  Haskell-ML guard
     hs k_H e_S
                  Haskell-Scheme guard
      \cos e \, e
                  List construction
           {\tt hd}\; e
                  List head
     list? e_S
                  List predicate
           \mathtt{tl}\; e
                  List tail
  \mathtt{mh}\ t_{M}\ t_{H}\ e_{H}
                  ML-Haskell guard
     \mathtt{ms}\ k_M\ e_S
                  ML-Scheme guard
       num? e_S
                  Number predicate
     \mathtt{sh}\; k_H\; e_H
                  Scheme-Haskell guard
     \mathtt{sm}\ k_M\ e_M
                  Scheme-ML guard
          -ee
                  Subtraction
           \Lambda y.e
                  Type abstraction
                  Type application
```

Figure 2.2: Syntax names

# $\begin{array}{ccc} \mathbf{Syntax} & \mathbf{Name} \\ & b \diamond t & \mathbf{Branded \ type} \\ & \forall y.t & \mathbf{Universally \ quantified \ type} \\ & \forall y.k & \mathbf{Universally \ quantified \ conversion \ scheme} \\ & t \rightarrow t & \mathbf{Function \ abstraction} \\ & k \rightarrow k & \mathbf{Function \ abstraction} \\ & \{t\} & \mathbf{List} \\ & \{k\} & \mathbf{List} \end{array}$

Figure 2.3: Syntax names

```
\begin{array}{lll} e_{H} & = & x_{H} \mid u_{H} \mid e_{H} \mid e_{H} \mid e_{H} \mid t_{H} \rangle \mid \text{fix } e_{H} \mid a \mid e_{H} \mid e_{H} \mid e_{H} \mid e_{H} \mid c \mid e_{H} \\ & & \text{null? } e_{H} \mid \text{wrong } t_{H} \mid string \mid \text{hm } t_{H} \mid t_{M} \mid e_{M} \mid \text{hs } k_{H} \mid e_{S} \\ \\ u_{H} & = & \lambda x_{H} : t_{H} . e_{H} \mid \Lambda y_{H} . e_{H} \mid \overline{n} \mid \text{nill } t_{H} \mid \text{cons } e_{H} \mid e_{H} \mid \text{hm } \mathbf{L} \mid t_{M} \mid f_{M} \\ & & \text{hs } \mathbf{L} \mid f_{S} \\ \\ t_{H} & = & \mathbf{L} \mid \mathbb{N} \mid y_{H} \mid \{t_{H}\} \mid t_{H} \rightarrow t_{H} \mid \forall y_{H} . t_{H} \\ \\ k_{H} & = & \mathbf{L} \mid \mathbb{N} \mid y_{H} \mid \{k_{H}\} \mid k_{H} \rightarrow k_{H} \mid \forall y_{H} . k_{H} \mid b \diamond t_{H} \\ \\ a & = & + \mid - \\ \\ c & = & \text{hd} \mid \text{tl} \\ \\ F_{H} & = & []_{H} \mid F_{H} \mid e_{H} \mid F_{H} \langle t_{H} \rangle \mid \text{fix } F_{H} \mid a \mid F_{H} \mid e_{H} \mid a \mid u_{H} \mid F_{H} \\ & & \text{if 0 } F_{H} \mid e_{H} \mid c \mid F_{H} \mid \text{null 1? } F_{H} \mid \text{hm } t_{H} \mid t_{M} \mid F_{M} \mid \text{hs } k_{H} \mid F_{S} \\ \end{array}
```

Figure 2.4: Haskell syntax and evaluation contexts

Figure 2.5: Haskell typing rules

```
\begin{split} \mathscr{F}[(\lambda x_H:t_H.e_H)\ e'_H]_H &\to \mathscr{F}[e_H[e'_H/x_H]] \\ \mathscr{F}[(\Lambda y_H.e_H)\langle t_H\rangle]_H &\to \mathscr{F}[e_H[b\diamond t_H/y_H]] \\ \mathscr{F}[\operatorname{fix}\ (\lambda x_H:t_H.e_H)]_H &\to \mathscr{F}[e_H[\operatorname{fix}\ (\lambda x_H:t_H.e_H)/x_H]] \\ \mathscr{F}[+\overline{n}\ \overline{n}']_H &\to \mathscr{F}[\overline{n+n'}] \\ \mathscr{F}[-\overline{n}\ \overline{n}']_H &\to \mathscr{F}[\overline{max}(n-n',0)] \\ \mathscr{F}[\operatorname{if0}\ \overline{0}\ e_H\ e'_H]_H &\to \mathscr{F}[e_H] \\ \mathscr{F}[\operatorname{if0}\ \overline{n}\ e_H\ e'_H]_H &\to \mathscr{F}[e'_H]\ (n\neq 0) \\ \mathscr{F}[\operatorname{hd}\ (\operatorname{nil}\ t_H)]_H &\to \mathscr{F}[\operatorname{wrong}\ t_H\ \operatorname{"Empty}\ \operatorname{list"}] \\ \mathscr{F}[\operatorname{tl}\ (\operatorname{nil}\ t_H)]_H &\to \mathscr{F}[\operatorname{wrong}\ t_H\ \operatorname{"Empty}\ \operatorname{list"}] \\ \mathscr{F}[\operatorname{hd}\ (\operatorname{cons}\ e_H\ e'_H)]_H &\to \mathscr{F}[e'_H] \\ \mathscr{F}[\operatorname{null?}\ (\operatorname{nil}\ t_H)]_H &\to \mathscr{F}[\overline{0}] \\ \mathscr{F}[\operatorname{null?}\ (\operatorname{cons}\ e_H\ e'_H)]_H &\to \mathscr{F}[\overline{1}] \\ \mathscr{F}[\operatorname{wrong}\ t_H\ string]_H &\to \operatorname{Error:}\ string \end{split}
```

Figure 2.6: Haskell operational semantics

```
\begin{split} \mathscr{F}[\operatorname{hm} t_H \operatorname{L} (\operatorname{mh} \operatorname{L} t'_H e_H)]_H &\to \mathscr{F}[e_H] \quad (t_H = t'_H \text{ and } t_H \neq \operatorname{L}) \\ \mathscr{F}[\operatorname{hm} t_H \operatorname{L} (\operatorname{mh} \operatorname{L} t'_H e_H)]_H &\to \mathscr{F}[\operatorname{wrong} t_H \text{ "Type mismatch"}] \\ \quad (t_H \neq t'_H \text{ and } t_H \neq \operatorname{L}) \\ \mathscr{F}[\operatorname{hm} t_H \operatorname{L} (\operatorname{ms} \operatorname{L} f_S)]_H &\to \mathscr{F}[\operatorname{wrong} t_H \text{ "Bad value"}] \quad (t_H \neq \operatorname{L}) \\ \mathscr{F}[\operatorname{hm} \operatorname{N} \operatorname{N} \overline{n}]_H &\to \mathscr{F}[\overline{n}] \\ \mathscr{F}[\operatorname{hm} \{t_H\} \{t_M\} (\operatorname{nil} t'_M)]_H &\to \mathscr{F}[\operatorname{nil} t_H] \\ \mathscr{F}[\operatorname{hm} \{t_H\} \{t_M\} (\operatorname{cons} u_M u'_M)]_H &\to \\ \mathscr{F}[\operatorname{cons} (\operatorname{hm} t_H t_M u_M) (\operatorname{hm} \{t_H\} \{t_M\} u'_M)] \\ \mathscr{F}[\operatorname{hm} (t_H \to t'_H) (t_M \to t'_M) (\lambda x_M : t''_M . e_M)]_H &\to \\ \mathscr{F}[\lambda x_H : t_H . \operatorname{hm} t'_H t'_M ((\lambda x_M : t''_M . e_M) (\operatorname{mh} t_M t_H x_H))] \\ \mathscr{F}[\operatorname{hm} (\forall y_H . t_H) (\forall y_M . t_M) (\Lambda y'_M . e_M)]_H &\to \mathscr{F}[\Lambda y_H . \operatorname{hm} t_H t_M [\operatorname{L}/y_M] e_M [\operatorname{L}/y'_M]] \end{split}
```

Figure 2.7: Haskell-ML operational semantics

```
\begin{split} \mathscr{F}[\text{hs N}\,\overline{n}]_H &\to \mathscr{F}[\overline{n}] \\ \mathscr{F}[\text{hs N}\,f_S]_H &\to \mathscr{F}[\text{wrong N "Not a number"}] \; (f_S \neq \overline{n}) \\ \mathscr{F}[\text{hs }\{k_H\}\,\text{nil}]_H &\to \mathscr{F}[\text{nil }\lfloor k_H\rfloor] \\ \mathscr{F}[\text{hs }\{k_H\}\,\,(\text{cons }u_S\,u_S')]_H &\to \mathscr{F}[\text{cons }(\text{hs }k_H\,u_S)\,\,(\text{hs }\{k_H\}\,u_S')] \\ \mathscr{F}[\text{hs }\{k_H\}\,f_S]_H &\to \mathscr{F}[\text{wrong }\lfloor\{k_H\}\rfloor\,\,\text{"Not a list"}] \\ &\quad (f_S \neq \text{nil and }f_S \neq \text{cons }u_S\,u_S') \\ \mathscr{F}[\text{hs }(b \diamond t_H)\,\,(\text{sh }(b \diamond t_H)\,e_H)]_H &\to \mathscr{F}[e_H] \\ \mathscr{F}[\text{hs }(b \diamond t_H)\,\,f_S]_H &\to \mathscr{F}[\text{wrong }t_H\,\,\,\text{"Brand mismatch"}] \;\,(f_S \neq \text{sh }(b \diamond t_H)\,e_H) \\ \mathscr{F}[\text{hs }(k_H \to k_H')\,\,(\lambda x_S.e_S)]_H &\to \mathscr{F}[\lambda x_H:\lfloor k_H\rfloor.\text{hs }k_H'\,\,((\lambda x_S.e_S)\,\,(\text{sh }k_H\,x_H))] \\ \mathscr{F}[\text{hs }(k_H \to k_H')\,\,f_S]_H &\to \mathscr{F}[\text{wrong }\lfloor k_H \to k_H'\rfloor\,\,\,\,\text{"Not a function"}] \\ &\quad (f_S \neq \lambda x_S.e_S) \\ \mathscr{F}[\text{hs }(\forall y_H.k_H)\,\,f_S]_H &\to \mathscr{F}[\Lambda y_H.\text{hs }k_H\,\,f_S] \end{split}
```

Figure 2.8: Haskell-Scheme operational semantics

```
\begin{array}{lll} e_{M} & = & x_{M} \mid u_{M} \mid e_{M} \; e_{M} \mid e_{M} \langle t_{M} \rangle \; | \; \mathrm{fix} \; e_{M} \mid a \; e_{M} \; e_{M} \mid i \mathrm{fo} \; e_{M} \; e_{M} \; e_{M} \\ & & \mathrm{cons} \; e_{M} \; e_{M} \mid c \; e_{M} \mid \mathrm{null} \; ? \; e_{M} \mid \mathrm{wrong} \; t_{M} \; \mathrm{string} \mid \mathrm{ms} \; k_{M} \; e_{S} \\ u_{M} & = & f_{M} \mid \mathrm{mh} \; t_{M} \; t_{H} \; e_{H} \\ f_{M} & = & \lambda x_{M} : t_{M} . e_{M} \mid \Lambda y_{M} . e_{M} \mid \overline{n} \mid \mathrm{nil} \; t_{M} \mid \mathrm{cons} \; u_{M} \; u_{M} \mid \mathrm{mh} \; \mathrm{L} \; t_{H} \; e_{H} \\ & \mathrm{ms} \; \mathrm{L} \; f_{S} \\ t_{M} & = & \mathrm{L} \mid \mathrm{N} \mid y_{M} \mid \{t_{M}\} \mid t_{M} \to t_{M} \mid \forall y_{M} . t_{M} \\ k_{M} & = & \mathrm{L} \mid \mathrm{N} \mid y_{M} \mid \{k_{M}\} \mid k_{M} \to k_{M} \mid \forall y_{M} . k_{M} \mid b \diamond t_{M} \\ a & = & + \mid - \\ c & = & \mathrm{hd} \mid \mathrm{tl} \\ F_{M} & = & U_{M} \mid \mathrm{mh} \; t_{M} \; t_{H} \; F_{H} \\ U_{M} & = & \left[ \left| \right|_{M} \mid F_{M} \; e_{M} \mid f_{M} \; U_{M} \mid F_{M} \langle t_{M} \rangle \mid \mathrm{fix} \; F_{M} \mid a \; F_{M} \; e_{M} \mid a \; f_{M} \; F_{M} \\ & & \mathrm{if0} \; F_{M} \; e_{M} \; e_{M} \mid \mathrm{cons} \; U_{M} \; e_{M} \mid \mathrm{cons} \; u_{M} \; U_{M} \mid c \; F_{M} \mid \mathrm{null} \; ? \; F_{M} \\ & & \mathrm{ms} \; k_{M} \; F_{S} \\ \end{array}
```

Figure 2.9: ML syntax and evaluation contexts

Figure 2.10: ML typing rules

```
\begin{split} \mathscr{F}[(\lambda x_M:t_M.e_M)\;u_M]_M &\to \mathscr{F}[e_M[u_M/x_M]] \\ \mathscr{F}[(\Lambda y_M.e_M)\langle t_M\rangle]_M &\to \mathscr{F}[e_M[b \diamond t_M/y_M]] \\ \mathscr{F}[\operatorname{fix}\;(\lambda x_M:t_M.e_M)]_M &\to \mathscr{F}[e_M[\operatorname{fix}\;(\lambda x_M:t_M.e_M)/x_M]] \\ \mathscr{F}[+\overline{n}\;\overline{n}']_M &\to \mathscr{F}[\overline{n+n'}] \\ \mathscr{F}[-\overline{n}\;\overline{n}']_M &\to \mathscr{F}[\overline{max}(n-n',0)] \\ \mathscr{F}[\operatorname{if0}\;\overline{0}\;e_M\;e_M']_M &\to \mathscr{F}[e_M] \\ \mathscr{F}[\operatorname{if0}\;\overline{n}\;e_M\;e_M']_M &\to \mathscr{F}[e_M']\;(n\neq 0) \\ \mathscr{F}[\operatorname{hd}\;(\operatorname{nil}\;t_M)]_M &\to \mathscr{F}[\operatorname{wrong}\;t_M\;\operatorname{"Empty}\;\operatorname{list"}] \\ \mathscr{F}[\operatorname{tl}\;(\operatorname{nil}\;t_M)]_M &\to \mathscr{F}[\operatorname{wrong}\;\{t_M\}\;\operatorname{"Empty}\;\operatorname{list"}] \\ \mathscr{F}[\operatorname{hd}\;(\operatorname{cons}\;u_M\;u_M')]_M &\to \mathscr{F}[u_M] \\ \mathscr{F}[\operatorname{null}?\;(\operatorname{nil}\;t_M)]_M &\to \mathscr{F}[\overline{0}] \\ \mathscr{F}[\operatorname{null}?\;(\operatorname{cons}\;u_M\;u_M')]_M &\to \mathscr{F}[\overline{1}] \\ \mathscr{F}[\operatorname{wrong}\;t_M\;\operatorname{string}]_H &\to \operatorname{Error:}\;\operatorname{string} \end{split}
```

Figure 2.11: ML operational semantics

```
\begin{split} \mathscr{F}[\mathrm{mh}\ t_M\ \mathrm{L}\ (\mathrm{hm}\ \mathrm{L}\ t'_M\ f_M)]_M \to \mathscr{F}[f_M]\ (t_M = t'_M\ \mathrm{and}\ t_M \neq \mathrm{L}) \\ \mathscr{F}[\mathrm{mh}\ t_M\ \mathrm{L}\ (\mathrm{hm}\ \mathrm{L}\ t'_M\ f_M)]_M \to \mathscr{F}[\mathrm{wrong}\ t_M\ \mathrm{``Type}\ \mathrm{mismatch''}]\ (t_M \neq t'_M\ \mathrm{and}\ t_M \neq \mathrm{L}) \\ \mathscr{F}[\mathrm{mh}\ t_M\ \mathrm{L}\ (\mathrm{hs}\ \mathrm{L}\ f_S)]_H \to \mathscr{F}[\mathrm{wrong}\ t_M\ \mathrm{``Bad\ value''}]\ (t_M \neq \mathrm{L}) \\ \mathscr{F}[\mathrm{mh}\ \mathrm{N}\ \overline{n}]_M \to \mathscr{F}[\overline{n}] \\ \mathscr{F}[\mathrm{mh}\ \mathrm{N}\ \overline{n}]_M \to \mathscr{F}[\overline{n}] \\ \mathscr{F}[\mathrm{mh}\ \{t_M\}\ \{t_H\}\ (\mathrm{nil}\ t'_H)]_M \to \mathscr{F}[\mathrm{nil}\ t_M] \\ \mathscr{F}[\mathrm{mh}\ \{t_M\}\ \{t_H\}\ (\mathrm{cons}\ e_H\ e'_H)]_M \to \mathscr{F}[\mathrm{cons}\ (\mathrm{mh}\ t_M\ t_H\ e_H)\ (\mathrm{mh}\ \{t_M\}\ \{t_H\}\ e'_H)] \\ \mathscr{F}[\mathrm{mh}\ (t_M \to t'_M)\ (t_H \to t'_H)\ (\lambda x_H: t''_H.e_H)]_M \to \\ \mathscr{F}[\lambda x_M: t_M.\mathrm{mh}\ t'_M\ t'_H\ ((\lambda x_H: t''_H.e_H)\ (\mathrm{hm}\ t_H\ t_M\ x_M))] \\ \mathscr{F}[\mathrm{mh}\ (\forall y_M.t_M)\ (\forall y_H.t_H)\ (\Lambda y'_H.e_H)]_M \to \mathscr{F}[\Lambda y_M.\mathrm{mh}\ t_M\ t_H[\mathrm{L}/y_H]\ e_H[\mathrm{L}/y'_H]] \end{split}
```

Figure 2.12: ML-Haskell operational semantics

```
\begin{split} \mathscr{F}[\operatorname{ms} \operatorname{N} \overline{n}]_{M} &\to \mathscr{F}[\overline{n}] \\ \mathscr{F}[\operatorname{ms} \operatorname{N} f_{S}]_{M} &\to \mathscr{F}[\operatorname{wrong} \operatorname{N} \text{ "Not a number"}] \ (f_{S} \neq \overline{n}) \\ \mathscr{F}[\operatorname{ms} \{k_{M}\} \operatorname{nil}]_{M} &\to \mathscr{F}[\operatorname{nil} \lfloor k_{M} \rfloor] \\ \mathscr{F}[\operatorname{ms} \{k_{M}\} \ (\operatorname{cons} u_{S} u_{S}')]_{M} &\to \mathscr{F}[\operatorname{cons} (\operatorname{ms} k_{M} u_{S}) \ (\operatorname{ms} \{k_{M}\} u_{S}')] \\ \mathscr{F}[\operatorname{ms} \{k_{M}\} f_{S}]_{M} &\to \mathscr{F}[\operatorname{wrong} \lfloor \{k_{M}\} \rfloor \text{ "Not a list"}] \\ (f_{S} \neq \operatorname{nil} \ \operatorname{and} f_{S} \neq \operatorname{cons} u_{S} u_{S}') \\ \mathscr{F}[\operatorname{ms} (b \diamond t_{M}) \ (\operatorname{sm} (b \diamond t_{M}) u_{M})]_{M} &\to \mathscr{F}[u_{M}] \\ \mathscr{F}[\operatorname{ms} (b \diamond t_{M}) f_{S}]_{M} &\to \mathscr{F}[\operatorname{wrong} \lfloor b \diamond t_{M} \rfloor \text{ "Brand mismatch"}] \\ (f_{S} \neq \operatorname{sm} (b \diamond t_{M}) e_{M}) \\ \mathscr{F}[\operatorname{ms} (k_{M} \to k_{M}') (\lambda x_{S}.e_{S})]_{M} &\to \\ \mathscr{F}[\lambda x_{M} : \lfloor k_{M} \rfloor.\operatorname{ms} k_{M}' ((\lambda x_{S}.e_{S}) \ (\operatorname{sm} k_{M} x_{M}))] \\ \mathscr{F}[\operatorname{ms} (k_{M} \to k_{M}') f_{S}]_{M} &\to \mathscr{F}[\operatorname{wrong} \lfloor k_{M} \to k_{M}' \rfloor \text{ "Not a function"}] \\ (f_{S} \neq \lambda x_{S}.e_{S}) \\ \mathscr{F}[\operatorname{ms} (\forall y_{M}.k_{M}) f_{S}]_{M} &\to \mathscr{F}[\Lambda y_{M}.\operatorname{ms} k_{M} f_{S}] \end{split}
```

Figure 2.13: ML-Scheme operational semantics

```
\begin{array}{lll} e_S & = & x_S \mid u_S \mid e_S \, e_S \mid a \, e_S \, e_S \mid p \, e_S \mid \text{if0} \, e_S \, e_S \mid c \, \text{ons} \, e_S \, e_S \mid c \, e_S \\ & \text{wrong } string \mid \text{sm } k_M \, e_M \\ \\ u_S & = & f_S \mid \text{sh } k_H \, e_H \\ \\ f_S & = & \lambda x_S.e_S \mid \overline{n} \mid \text{nil} \mid c \, \text{ons} \, u_S \, u_S \mid \text{sh } (b \diamond t_H) \, e_H \mid \text{sm } (b \diamond t_M) \, f_M \\ \\ a & = & + \mid - \\ \\ c & = & \text{hd} \mid \text{tl} \\ \\ p & = & \text{fun?} \mid \text{list?} \mid \text{null?} \mid \text{num?} \\ \\ F_S & = & U_S \mid \text{sh } k_H \, F_H \\ \\ U_S & = & []_S \mid F_S \, e_S \mid f_S \, U_S \mid a \, F_S \, e_S \mid a \, f_S \, F_S \mid p \, F_S \mid \text{if0} \, F_S \, e_S \, e_S \\ \\ & & \text{cons } U_S \, e_S \mid \text{cons } u_S \, U_S \mid c \, F_S \mid \text{sm } k_M \, F_M \\ \end{array}
```

Figure 2.14: Scheme syntax and evaluation contexts

$$\overline{\vdash_S \mathsf{TST}}$$

$$\frac{\Gamma, x_S : \mathsf{TST} \vdash_S e_S : \mathsf{TST}}{\Gamma \vdash_S \lambda x_S.e_S : \mathsf{TST}} \xrightarrow{\vdash_S \overline{n} : \mathsf{TST}} \xrightarrow{\vdash_S \mathsf{nil} : \mathsf{TST}} \frac{\Gamma \vdash_S e_S : \mathsf{TST}}{\Gamma \vdash_S \mathsf{cons} \ e_S \ e_S' : \mathsf{TST}} \xrightarrow{\Gamma, x_S : \mathsf{TST} \vdash_S x_S : \mathsf{TST}} \frac{\Gamma \vdash_S e_S : \mathsf{TST}}{\Gamma \vdash_S e_S : \mathsf{TST}} \xrightarrow{\Gamma, x_S : \mathsf{TST} \vdash_S x_S : \mathsf{TST}} \frac{\Gamma \vdash_S e_S : \mathsf{TST}}{\Gamma \vdash_S e_S : \mathsf{TST}} \xrightarrow{\Gamma \vdash_S e_S : \mathsf{TST}} \frac{\Gamma \vdash_S e_S : \mathsf{TST}}{\Gamma \vdash_S e_S : \mathsf{TST}} \xrightarrow{\Gamma \vdash_S e_S : \mathsf{TST}} \frac{\Gamma \vdash_S e_S : \mathsf{TST}}{\Gamma \vdash_S e_S : \mathsf{TST}} \xrightarrow{\Gamma \vdash_S e_S : \mathsf{TST}} \frac{\Gamma \vdash_S e_S : \mathsf{TST}}{\Gamma \vdash_S e_S : \mathsf{TST}} \xrightarrow{\Gamma \vdash_S e_S : \mathsf{TST}} \frac{\Gamma \vdash_S e_S : \mathsf{TST}}{\Gamma \vdash_S \mathsf{if0} \ e_S \ e_S' : \mathsf{TST}} \xrightarrow{\Gamma \vdash_S e_S' : \mathsf{TST}} \xrightarrow{\vdash_S \mathsf{wrong} \ string : \mathsf{TST}} \frac{\Gamma \vdash_H [k_H] \Gamma \vdash_H e_H : t_H [k_H] = t_H}{\Gamma \vdash_S \mathsf{sh} \ k_H e_H : \mathsf{TST}} \xrightarrow{\Gamma \vdash_M [k_M] \Gamma \vdash_M e_M : t_M [k_M] = t_M} \xrightarrow{\Gamma \vdash_S \mathsf{sm} k_M e_M : \mathsf{TST}}$$

Figure 2.15: Scheme typing rules

```
\mathscr{F}[(\lambda x_S.e_S) \ u_S]_S \to \mathscr{F}[e_S[u_S/x_S]]
\mathscr{F}[f_S \ u_S]_S \to \mathscr{F}[\text{wrong "Not a function"}] \ (f_S \neq \lambda x_S.e_S)
\mathscr{F}[+\overline{n}\ \overline{n}']_S \to \mathscr{F}[\overline{n+n'}]
\mathscr{F}[-\overline{n}\ \overline{n}']_S \to \mathscr{F}[\overline{max(n-n',0)}]
\mathscr{F}[a\ f_S\ f_S']_S \to \mathscr{F}[\mathtt{wrong}\ \mathrm{``Not\ a\ number''}]\ (f_S \neq \overline{n}\ \mathrm{or}\ f_S' \neq \overline{n})
\mathscr{F}[\mathsf{if0}\ \overline{0}\ e_S\ e_S']_S \to \mathscr{F}[e_S]
\mathscr{F}[\mathsf{if0}\ \overline{n}\ e_S\ e_S']_S \to \mathscr{F}[e_S']\ (n \neq 0)
\mathscr{F}[\texttt{if0}\ f_S\ e_S\ e_S']_S \to \mathscr{F}[\texttt{wrong "Not a number"}]\ (f_S \neq \overline{n})
\mathscr{F}[c \text{ nil}]_S \to \mathscr{F}[\text{wrong "Empty list"}]
\mathscr{F}[\operatorname{hd}(\operatorname{cons} u_S u_S')]_S \to \mathscr{F}[u_S]
\mathscr{F}[\mathsf{tl}\ (\mathsf{cons}\ u_S\ u_S')]_S \to \mathscr{F}[u_S']
\mathscr{F}[c \ f_S]_S \to \mathscr{F}[\text{wrong "Not a list"}] \ (f_S \neq \text{nil and } f_S \neq \text{cons } u_S \ u_S')
\mathscr{F}[\operatorname{fun}?(\lambda x_S.e_S)]_S \to \mathscr{F}[\overline{0}]
\mathscr{F}[\operatorname{fun}? f_S]_S \to \mathscr{F}[\overline{1}] \ (f_S \neq \lambda x_S.e_S)
\mathscr{F}[\mathtt{list?\,nil}]_S \to \mathscr{F}[\overline{0}]
\mathscr{F}[\mathtt{list}? (\mathtt{cons}\ u_S\ u_S')]_S \to \mathscr{F}[\overline{0}]
\mathscr{F}[\mathtt{list}?\ f_S]_S \to \mathscr{F}[\overline{1}]\ (f_S \neq \mathtt{nil}\ \mathrm{and}\ f_S \neq \mathtt{cons}\ u_S\ u_S')
\mathscr{F}[\mathtt{null}?\,\mathtt{nil}]_S \to \mathscr{F}[\overline{0}]
\mathscr{F}[\text{null? } f_S]_S \to \mathscr{F}[\overline{1}] \ (f_S \neq \text{nil})
\mathscr{F}[\operatorname{num}? \overline{n}]_S \to \mathscr{F}[\overline{0}]
\mathscr{F}[\operatorname{num}? f_S]_S \to \mathscr{F}[\overline{1}] \ (f_S \neq \overline{n})
\mathscr{F}[\text{wrong } string]_S \to \mathbf{Error: } string
```

Figure 2.16: Scheme operational semantics

```
\begin{split} \mathscr{F}[\operatorname{sh} \mathsf{L} & (\operatorname{hm} \mathsf{L} \ k_M \ f_M)]_S \to \mathscr{F}[\operatorname{wrong "Bad value"}] \\ \mathscr{F}[\operatorname{sh} \mathsf{L} & (\operatorname{hs} \mathsf{L} \ f_S)]_S \to \mathscr{F}[f_S] \\ \mathscr{F}[\operatorname{sh} \mathsf{N} \ \overline{n}]_S \to \mathscr{F}[\overline{n}] \\ \mathscr{F}[\operatorname{sh} \ \{k_H\} \ (\operatorname{nil} \ t_H)]_S \to \mathscr{F}[\operatorname{nil}] \\ \mathscr{F}[\operatorname{sh} \ \{k_H\} \ (\operatorname{cons} \ e_H \ e'_H)]_S \to \mathscr{F}[\operatorname{cons} \ (\operatorname{sh} \ k_H \ e_H) \ (\operatorname{sh} \ \{k_H\} \ e'_H)] \\ \mathscr{F}[\operatorname{sh} \ (k_H \to k'_H) \ (\lambda x_H : t_H.e_H)]_S \to \\ \mathscr{F}[\lambda x_S.\operatorname{sh} \ k'_H \ ((\lambda x_H : t_H.e_H) \ (\operatorname{hs} \ k_H \ x_S))] \\ \mathscr{F}[\operatorname{sh} \ (\forall y_H.k_H) \ (\Lambda y'_H.e_H)]_S \to \mathscr{F}[\operatorname{sh} \ k_H[\mathsf{L}/y_H] \ e_H[\mathsf{L}/y'_H]] \end{split}
```

Figure 2.17: Scheme-Haskell operational semantics

```
\begin{split} \mathscr{F}[\operatorname{sm} \mathsf{L} \; (\operatorname{mh} \mathsf{L} \; k_H \; e_H)]_S &\to \mathscr{F}[\operatorname{wrong} \; \text{``Bad value''}] \\ \mathscr{F}[\operatorname{sm} \mathsf{L} \; (\operatorname{ms} \mathsf{L} \; f_S)]_S &\to \mathscr{F}[f_S] \\ \mathscr{F}[\operatorname{sm} \mathsf{N} \; \overline{n}]_S &\to \mathscr{F}[\overline{n}] \\ \mathscr{F}[\operatorname{sm} \; \{k_M\} \; (\operatorname{nil} \; t_M)]_S &\to \mathscr{F}[\operatorname{nil}] \\ \mathscr{F}[\operatorname{sm} \; \{k_M\} \; (\operatorname{cons} \; u_M \; u_M')]_S &\to \mathscr{F}[\operatorname{cons} \; (\operatorname{sm} \; k_M \; u_M) \; (\operatorname{sm} \; \{k_M\} \; u_M')] \\ \mathscr{F}[\operatorname{sm} \; (k_M \to k_M') \; (\lambda x_M : t_M.e_M)]_S &\to \\ \mathscr{F}[\lambda x_S.\operatorname{sm} \; k_M' \; ((\lambda x_M : t_M.e_M) \; (\operatorname{ms} \; k_M \; x_S))] \\ \mathscr{F}[\operatorname{sm} \; (\forall y_M.k_M) \; (\Lambda y_M'.e_M)]_S &\to \mathscr{F}[\operatorname{sm} \; k_M[\mathsf{L}/y_M] \; e_M[\mathsf{L}/y_M']] \end{split}
```

Figure 2.18: Scheme-ML operational semantics

```
\begin{bmatrix} \mathsf{L} \end{bmatrix} \ = \ \mathsf{L}
\lfloor \mathsf{N} \rfloor \ = \ \mathsf{N}
\lfloor y_H \rfloor \ = \ y_H
\lfloor y_M \rfloor \ = \ y_M
\lfloor \{k_H\} \rfloor \ = \ \{\lfloor k_H \rfloor\}
\lfloor \{k_M\} \rfloor \ = \ \{\lfloor k_M \rfloor\}
\lfloor k_H \to k_H \rfloor \ = \ \lfloor k_H \rfloor \to \lfloor k_H \rfloor
\lfloor k_M \to k_M \rfloor \ = \ \lfloor k_M \rfloor \to \lfloor k_M \rfloor
\lfloor \forall y_H \cdot k_H \rfloor \ = \ \forall y_H \cdot \lfloor k_H \rfloor
\lfloor \forall y_M \cdot k_M \rfloor \ = \ \forall y_M \cdot \lfloor k_M \rfloor
\lfloor b \diamond t_H \rfloor \ = \ t_H
\lfloor b \diamond t_M \rfloor \ = \ t_M
```

Figure 2.19: Unbrand function

$$x \doteq x$$

$$x \doteq y \Rightarrow y \doteq x$$

$$x \doteq y \text{ and } y \doteq z \Rightarrow x \doteq z$$

$$t_H \doteq L$$

$$t_M \doteq L$$

$$t_H = t_M \Rightarrow t_H \doteq t_M$$

Figure 2.20: Lump equality  $\mathbf{r}$ 

# Chapter 3

## Conclusion

Interoperation transparently resolves lazy and eager evaluation strategies by the eager evaluation strategy mirroring the lazy one for reducible expressions inside boundaries in evaluation contexts common to both languages where strictness is incompatible. Forced and unforced evaluation contexts and values comprise a simple framework that implements such a system. This method could be extended to resolve incompatible evaluation contexts for expressions common to any pair of languages.

# Bibliography

[1] J. Matthews and R. B. Findler. Operational semantics for multi-language programs. SIGPLAN Not., 42(1):3–10, 2007.