Controllability of an Electrohydrodynamic Aerial Vehicle Using Angled Thrusters

Craig Schindler
Department of Electrical Engineering and Computer Sciences
University of California, Berkeley
Email: craig.schindler@berkeley.edu

Daniel Drew
Department of Electrical Engineering and Computer Sciences
University of California, Berkeley
Email: ddrew73@berkeley.edu

Kristofer S.J. Pister
Department of Electrical Engineering and Computer Sciences
University of California, Berkeley
Email: ksjp@berkeley.edu

***Abstract*—Design, dynamics, and controllability analysis of a novel electrohydrodynamic aerial vehicle (EHDAV) design is presented. Unlike a standard quadcopter which uses rotor torque to control yaw, an EHDAV does not have rotors. To remain completely controllable, the EHDAV must therefore generate yaw in a different way. A design using angled thrusters which provide full state controllability of the EHDAV is presented and analyzed.**

I. Introduction

DD: Intro needs more content. Maybe of form: EHD why an EHDAV that looks like a quadcoptor (not standard triangle model they MAY have seen) controlling yaw related work (unconventional MAV control)

An electrohydrodynamic aerial vehicle (EHDAV), also known as an ionocraft, is an aerial vehicle which generates thrust using ionic thrusters. An ionic thruster consists of an asymmetric capacitor, typically made using two wires of different radii as the electrodes. When a large voltage is applied across the two wires, a plasma sheath is generated in the local region near the wire with a smaller radius of curvature. Ionized air is then accelerated through the electric field created by the applied voltage and in the process transfers momentum to neutral air molecules. The ionized air is then collected in the electrode with a larger radius of curvature and the entire process creates a net thrust perpendicular to the direction of the current. The theory of ionic thrust can be found in [1], and it can be shown that the force generated by a thruster is given by

$$\vec{F}_{ion} = \frac{\vec{I}_{ion} d}{\mu} \tag{1}$$

where $\vec{I}_{ion}$ is the ionic current across the wires, d is the distance between the wires, and μ is the ion mobility of air (nominally $2 \times 10^{-4}[\frac{m^2}{Vs}]$).

There exist designs for triangularly shaped, tethered, and uncontrolled ionocrafts that have successfully generated enough thrust to lift themselves. However, these designs are not well studied, and do not lend themselves to easy dynamical modeling. We propose a new ionocraft design which uses modified quadcopter dynamics. Quadcopters are well studied, and much engineering research has been done on quadcopters (see [2] and [3]).

The four independently actuated rotors of a standard quadcopter allow for decoupled control of pitch, roll, and yaw. This is accomplished by modulating the speeds of each of the rotors. A standard quadcopter controls yaw by coupling the torque from each of the rotors to the torque of the entire system. An EHDAV is unable to independently control yaw since it does not have torque-producing rotors. Instead, we propose a method where the EHDAV generates yaw using angled thrusters. While the angled thrusters allow for complete controllability, they also couple pitch, roll, and yaw together.

II. Quadcopter Model

A diagram of a standard quadcopter is shown in Figure 1. It has 12 states, and the state vector $\vec{q}$ is

$$\vec{q} = \begin{bmatrix} \vec{\zeta} & \vec{\xi} \end{bmatrix}^T \tag{2}$$

where

$$\vec{\zeta} = \begin{bmatrix} x & \dot{x} & y & \dot{y} & z & \dot{z} \end{bmatrix}^T \tag{3}$$

and

$$\vec{\xi} = \begin{bmatrix} \theta & \dot{\theta} & \psi & \dot{\psi} & \phi & \dot{\phi} \end{bmatrix}^T \tag{4}$$

where x, y, and z are the global position vectors, and θ, ψ, and ϕ are pitch, roll, and yaw, respectively.

The quadcopter has its own local spatial coordinate system called the body frame, which can represented by a vector of the form $\begin{bmatrix} x_b & y_b & z_b \end{bmatrix}^T$. The following matrix

$$\boldsymbol{R} = \begin{bmatrix} c_\phi c_\theta & c_\phi s_\theta s_\psi - s_\phi c_\psi & c_\phi s_\theta c_\psi + s_\phi s_\psi \\ s_\phi c_\theta & s_\phi s_\theta s_\psi + c_\phi c_\psi & s_\phi s_\theta c_\psi - c_\phi s_\psi \\ -s_\theta & c_\theta s_\psi & c_\theta c_\psi \end{bmatrix} \tag{5}$$

is a z-y-x rotation matrix which maps a vector $\vec{v_b}$ in the body frame to a vector $\vec{v_g}$ in the global frame:

$$\vec{v_g} = \boldsymbol{R}\vec{v_b}. \tag{6}$$

In a standard quadcopter, the thrusts applied by the rotors are always in the body frame's $\hat{z}$ direction. Therefore, the total

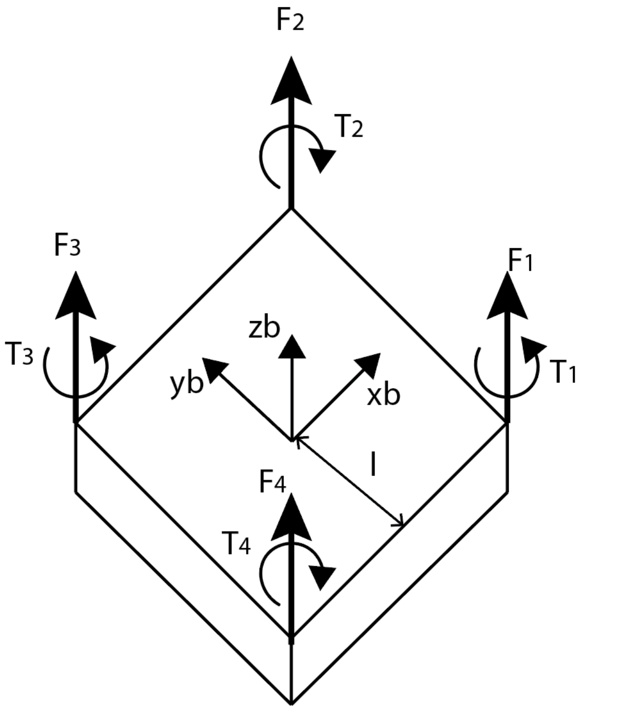

Fig. 1. An isometric diagram of a standard quadcopter. The full body dyanmics are derived in [2].

input force vector of a standard quadcopter from the body frame's perspective is always of the form

$$\vec{F}_b = \begin{bmatrix} 0 & 0 & \sum_{i=1}^{4} F_i \end{bmatrix}^T \tag{7}$$

where F_i is the total force provided by an individual rotor, and it follows that only the third column of $\boldsymbol{R}$ is needed for mapping input forces from the quadcopter's body frame to the global frame.

The final equations of motion of a standard quadcopter, which are fully derived in [2], are:

$$\ddot{x} = u_1(cos\phi sin\theta cos\psi + sin\phi sin\psi) - B_x\dot{x}/m \tag{8}$$
$$\ddot{y} = u_1(sin\phi sin\theta cos\psi - cos\phi sin\psi) - B_y\dot{y}/m \tag{9}$$
$$\ddot{z} = u_1(cos\theta cos\psi) - g - B_z\dot{z}/m \tag{10}$$
$$\ddot{\theta} = l(u_2 - B_\theta\dot{\theta}/J_1) \tag{11}$$
$$\ddot{\psi} = l(u_3 - B_\psi\dot{\psi}/J_2) \tag{12}$$
$$\ddot{\phi} = l(u_4 - B_\phi\dot{\phi}/J_3) \tag{13}$$

where l is the length from the quadcopter's center of mass to the quadcopter's edge, the B_i terms are damping coefficients, and u_1, u_2, u_3, and u_4 are functions of the individual rotor input forces F_1, F_2, F_3, F_4:

$$u_1 = (F_1 + F_2 + F_3 + F_4)/m \tag{14}$$
$$u_2 = (-F_1 - F_2 + F_3 + F_4)/J_1 \tag{15}$$
$$u_3 = (-F_1 + F_2 + F_3 - F_4)/J_2 \tag{16}$$
$$u_4 = C(F_1 - F_2 + F_3 - F_4)/J_3. \tag{17}$$

The mass of the system is m, the moments of inertia around the y, x, and z axes are J_1, J_2, and J_3, respectively, and C is the coupling factor from the rotor torques to yaw torque.

Consider a quadcopter hovering in the equilibrium state

$$\theta = \psi = \phi = \ddot{x} = \ddot{y} = \ddot{z} = 0. \tag{18}$$

This corresponds to input forces of

$$F_1 = F_2 = F_3 = F_4 = mg/4. \tag{19}$$

Neglecting the damping forces, which is a valid approximation for low velocity maneuvers such as hovering, a corresponding Jacobian linearization around this equilibrium state results in the $\tilde{A}$ and $\tilde{B}$ state space matrices in Eq. 20 and 21.

$$\tilde{A} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & g & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{20}$$

$$\tilde{B} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{m} & \frac{1}{m} & \frac{1}{m} & \frac{1}{m} \\ 0 & 0 & 0 & 0 \\ -\frac{l}{J_1} & -\frac{l}{J_1} & \frac{l}{J_1} & \frac{l}{J_1} \\ 0 & 0 & 0 & 0 \\ -\frac{l}{J_2} & \frac{l}{J_2} & \frac{l}{J_2} & -\frac{l}{J_2} \\ 0 & 0 & 0 & 0 \\ \frac{C}{J_3} & -\frac{C}{J_3} & \frac{C}{J_3} & -\frac{C}{J_3} \end{pmatrix} \tag{21}$$

The controllability matrix

$$\tilde{W_c} = \begin{bmatrix} \tilde{B} & \tilde{A}\tilde{B} & \dots & \tilde{A}^{n-1}\tilde{B} \end{bmatrix} \tag{22}$$

is full rank if and only if C from Eq. 17 is non-zero. In the case when C is equal to 0, then the controllability matrix $\tilde{W_c}$ of the Jacobian linearized model is of rank $n-2$, which means the overall system is not controllable. Specifically, ϕ and $\dot{\phi}$ (corresponding to yaw) are uncontrollable states because C is what couples the inputs to $\ddot{\phi}$. The latter situation is what occurs if an EHDAV is built like a quadcopter replica, but with rotors swapped out for ion thrusters. A new EHDAV design is required to regain controllability of ϕ and $\dot{\phi}$. This design and the dynamical theory are now discussed.

III. Ionocraft Model

The main difference between a quadcopter and an EHDAV is the inability of the EHDAV to directly control yaw. A quadcopter controls yaw by coupling torques from the rotors to the torque of the entire system; this can be seen in equation 17, where C represents the coupling factor. An EHDAV is shown in Figure 2, and to overcome this limitation uses four angled thrusters, one of which can be shown in Figure 3. These forces create a net force in the EHDAV body frame's $\hat{z}$ direction, and also the body frame's $\hat{x}$ and $\hat{y}$ directions. While only the third column of the matrix in equation 5 is needed for mapping forces in the standard quadcopter's body frame to the global frame, all three columns are needed for mapping forces in the EHDAV's body frame to the global frame. Each one of the EHDAV's thrusters produces a force like in equation 1.

The forces generated by the four angled thrusters in the EHDAV's body frame as shown in Figure 2 are:

$$\begin{aligned} F_{1y} &= F_1 sin(\alpha) \\ F_{1z} &= F_1 cos(\alpha) \\ F_{2x} &= F_2 sin(\alpha) \\ F_{2z} &= F_2 cos(\alpha) \\ F_{3y} &= -F_3 sin(\alpha) \\ F_{3z} &= F_3 cos(\alpha) \\ F_{4x} &= -F_4 sin(\alpha) \\ F_{4z} &= F_4 cos(\alpha). \end{aligned} \tag{23}$$

The total input force vector of an EHDAV from the body frame's perspective is therefore:

$$\vec{F_b} = \begin{bmatrix} \sum_{i=1}^4 F_{ix} & \sum_{i=1}^4 F_{iy} & \sum_{i=1}^4 F_{iz} \end{bmatrix}^T . \tag{24}$$

Using equation 6 to map the EHDAV's forces in the body frame's perspective to the global frame, the resulting EHDAV full body dynamics are:

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \frac{\boldsymbol{R}\vec{F_b} - \begin{bmatrix} B_x\dot{x} & B_y\dot{y} & B_z\dot{z} \end{bmatrix}^T - \begin{bmatrix} 0 & 0 & mg \end{bmatrix}^T}{m} \tag{25}$$

$$\ddot{\theta} = l\frac{(F_3 - F_1)cos(\alpha) - B_\theta\dot{\theta}}{J_1} \tag{26}$$

$$\ddot{\psi} = l\frac{(F_2 - F_4)cos(\alpha) - B_\psi\dot{\psi}}{J_2} \tag{27}$$

$$\ddot{\phi} = l\frac{(F_1 - F_2 + F_3 - F_4)sin(\alpha) - B_\phi\dot{\phi}}{J_3} \tag{28}$$

where the mass of the system is m, the moments of inertia around the y, x, and z axes respectively, are J_1, J_2, and J_3, and the B_i terms are damping coefficients.

The main difference between the EHDAV and standard quadcopter dynamics arises because of the difference in body frame forces, which can be seen by comparing equations 7 and 24. All three columns of $\boldsymbol{R}$ are needed for mapping input forces from the EHDAV's body frame to the global frame whereas only the last column of $\boldsymbol{R}$ is needed for mapping

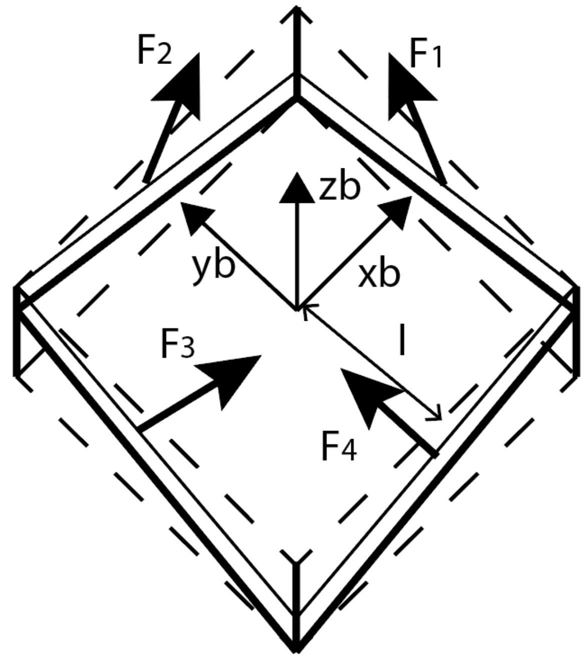

Fig. 2. An isometric view of an EHDAV made using four individually controllable ionic thrusters. Two of the EHDAV's ionic thrusters generate forces in the body frame's $\hat{z}$ and $\hat{x}$ directions, and two generate forces in the body frame's $\hat{z}$ and $\hat{y}$ directions. This allows yaw control to be possible without torque producing rotors.

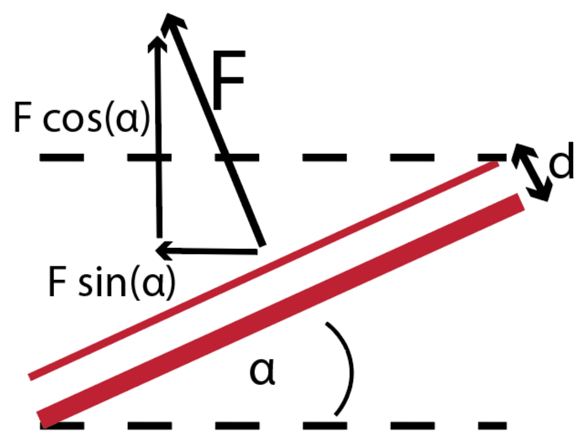

Fig. 3. An isometric diagram of a thruster. The red lines are wires which form an asymmetric capacitor. The thin red line represents the electrode with the smaller radius of curvature, and the thick red line represents the electrode with the larger radius of curvature. The force vector "F" is perpendicular to the two wires, and is the ionic force generated by equation 1. The decomposed force vectors are also shown. The dotted lines are only to give perspective with respect to the horizontal.

input forces from the quadcopter's body frame to the global frame.

Consider the EHDAV hovering in the equilibrium state

$$\theta = \psi = \phi = \ddot{x} = \ddot{y} = \ddot{z} = 0. \tag{29}$$

This corresponds to input forces of

$$F_1 = F_2 = F_3 = F_4 = \frac{mg}{4cos(\alpha)}. \tag{30}$$

Neglecting the damping forces, which is a valid approximation for low velocity maneuvers such as hovering, a Jacobian linearization around this equilibrium state results in the $\tilde{A}$ and $\tilde{B}$ state space matrices in equations 31 and 32.

$$\tilde{A} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & g & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{31}$$

$$\tilde{B} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\sin(\alpha)}{m} & 0 & -\frac{\sin(\alpha)}{m} \\ 0 & 0 & 0 & 0 \\ \frac{\sin(\alpha)}{m} & 0 & -\frac{\sin(\alpha)}{m} & 0 \\ 0 & 0 & 0 & 0 \\ \frac{\cos(\alpha)}{m} & \frac{\cos(\alpha)}{m} & \frac{\cos(\alpha)}{m} & \frac{\cos(\alpha)}{m} \\ 0 & 0 & 0 & 0 \\ -\frac{l\ \cos(\alpha)}{\mathrm{J1}} & 0 & \frac{l\ \cos(\alpha)}{\mathrm{J1}} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{l\ \cos(\alpha)}{\mathrm{J2}} & 0 & -\frac{l\ \cos(\alpha)}{\mathrm{J2}} \\ 0 & 0 & 0 & 0 \\ \frac{l\ \sin(\alpha)}{\mathrm{J3}} & -\frac{l\ \sin(\alpha)}{\mathrm{J3}} & \frac{l\ \sin(\alpha)}{\mathrm{J3}} & -\frac{l\ \sin(\alpha)}{\mathrm{J3}} \end{pmatrix} \tag{32}$$

The Jacobian linearized EHDAV controllability matrix $\tilde{W_c}$ (which can be seen in equation 22) is only full rank when α is non-zero. When α is equal to 0, then the controllability matrix $\tilde{W_c}$ of the Jacobian linearized model is of rank $n-2$, and ϕ and $\dot{\phi}$ are uncontrollable states. The $\tilde{B}$ matrix shows that pitch, roll, and yaw are coupled together, and the input forces F_i direcly affect both linear accelerations and angular acceleraions of the EHDAV. This is unlike the standard quadcopter in which the input forces only directly affect pitch, roll, and yaw.

It should also be noted that the $\tilde{A}$ matrices for the standard quadcopter and the EHDAV are exactly the same, and only the $\tilde{B}$ matrices have changed. The internal dynamics of the two systems are exactly the same; only the coupling between inputs and states has changed.

IV. Conclusion

In order to make a fully controllable electrohydrodynamic aerial vehicle (EHDAV), all system states must be controllable. A standard quadcopter couples the torque generated by its individual rotors to the torque of the total system. We show why an EHDAV is uncontrollable without these torque-generating rotors. A new design is presented to overcome this limitation. The controllablity of this design is analyzed and we show that the angled thrusters allow for complete control of the system. **DD: This kind of sounds like another Abstract**

References

[1] Masuyama K, Barrett SRH. 2013 On the performance of electrohydrodynamic propulsion. Proc R Soc A 469: 20120623.

[2] Altuğ, Erdinç and Ostrowski, James P and Mahony, Robert. Control of a Quadrotor Helicopter Using Visual Feedback. Proceedings of the 2002 IEEE International Conference on Robotics & Automation. Washington, DC. May 2002.

[3] G. Hoffmann, H. Huang, S. Waslander and C. Tomlin, "Precision flight control for a multi-vehicle quadrotor helicopter testbed", Control Engineering Practice, vol. 19, no. 9, pp. 1023-1036, 2011.