Name: _____ Student ID: _____

1. (16 points) (Combining Factors)

   (a) Given the factors $P(A|C)$ and $P(B|A,C)$ what is the resulting factor after joining over C?

   ☐ $P(A,B,C)$ ☐ $P(A|B,C)$ ☐ $P(A,B|C)$ ☐ None of these.

   (b) Given the factors $P(A|B)$ and $P(B|C)$ and $P(C)$ which factor will be created after joining on C and summing out over C?

   ☐ $P(B,C)$ ☐ $P(B)$ ☐ $P(C)$ ☐ None of these

   (c) Given the factors $P(A|C)$ and $P(B|A,C)$ what is the resulting factor after joining over A and summing over A?

   ☐ $P(C)$ ☐ $P(B)$ ☐ $P(B,C)$ ☐ $P(A|C)$ ☐ $P(B|C)$

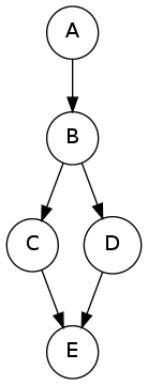
   ☐ None of the above.

   (d) Given the factors $P(C|A)$, $P(D|A,B,C)$, $P(B|A,C)$, what is the resulting factor after joining over C and summing over C?

   ☐ $P(D|A)$ ☐ $P(C,D|A)$ ☐ $P(B,C,D|A)$ ☐ $P(B,D|A)$ ☐ $P(C,B|A,D)P(A|D)$

   ☐ None of the above.

2. (20 points) (Variable Elimination Tables) Assume the following Bayes Net and corresponding CPTs. In this exercise, we are given the query $P(C|e=1)$, and we will complete the tables for each factor generated during the elimination process.



After introducing evidence, we have the following probability tables.

A	$P(A)$
0	0.100
1	0.900

B	A	$P(B\|A)$
0	0	0.500
1	0	0.500
0	1	0.400
1	1	0.600

C	B	$P(C\|B)$
0	0	0.400
1	0	0.600
0	1	0.300
1	1	0.700

D	B	$P(D\|B)$
0	0	0.600
1	0	0.400
0	1	0.900
1	1	0.100

C	D	$P(e=1\|C,D)$
0	0	0.600
1	0	0.200
0	1	0.600
1	1	0.200

(a) Three steps are required for elimination, with the resulting factors listed below:
Step 1: eliminate A. We get the factor $f_1(B) = \sum_a P(a)P(B|a)$
Step 2: eliminate B. We get the factor $f_2(C,D) = \sum_b P(C|b)P(D|b)f_1(b)$
Step 3: eliminate D. We get the factor $f_3(C, e=1) = \sum_d P(e=1|C,d)f_2(C,d)$.
Fill in the missing quantities. (Some quantities are computed for you.)

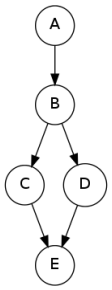
$f_1(B=0) =$ _____
$f_1(B=1) =$ _____
$f_2(C=0, D=0) =$ _____ $\qquad f_2(C=0, D=1) = 0.083$
$f_2(C=1, D=0) =$ _____ $\qquad f_2(C=1, D=1) = 0.14$
$f_3(C=0, e=1) =$ _____ $\qquad f_3(C=1, e=1) = 0.132$

(b) After getting the final factor $f_3(C, e=1)$, a final renormalization step needs to be carried out to obtain the conditional probability $P(C|e=1)$. Fill in the final conditional probabilities below.

$P(C=0|e=1) =$ _____ $\qquad P(C=1|e=1) =$ _____

3. (16 points) (Rejection Sampling) We will work with a Bayes' net of the following structure.

0.320	0.037	0.303	0.318	0.032	0.969	0.018	0.058	0.908	0.249



A	$P(A)$
0	0.200
1	0.800

B	A	$P(B\|A)$
0	0	0.800
1	0	0.200
0	1	0.400
1	1	0.600

C	B	$P(C\|B)$
0	0	0.600
1	0	0.400
0	1	0.400
1	1	0.600

D	B	$P(D\|B)$
0	0	0.800
1	0	0.200
0	1	0.600
1	1	0.400

E	C	D	$P(E\|C,D)$
0	0	0	0.800
1	0	0	0.200
0	1	0	0.600
1	1	0	0.400
0	0	1	0.400
1	0	1	0.600
0	1	1	0.400
1	1	1	0.600

In this question, we will perform rejection sampling to estimate $P(C = 1|B = 1, E = 1)$. Perform one round of rejection sampling, using the random samples given in the table below. Variables are sampled in the order A, B, C, D, E. In the boxes below, choose the value (0 or 1) that each variable gets assigned to. **Note**. *The sampling attempt should stop as soon as you discover that the sample will be rejected. In that case mark the assignment of that variable and write 'none' for the rest of the variables.*

When generating random samples, use as many values as needed from the table below, which we generated independently and uniformly at random from $[0, 1)$. Use numbers from left to right. To sample a binary variable W with probability $P(W = 0) = p$ and $P(W = 1) = 1 - p$ using a value a from the table, choose $W = 0$ if $a < p$ and $W = 1$ if $a \geq p$.

Enter either a 0 or 1 for each variable that you assign a value to. Upon rejecting a sample, enter its assigned value, and enter 'none' for the remaining variables. For example, if C gets rejected, fill in 'none' for D and E.

A: _____ **B:** _____ **C:** _____

D: _____ **E:** _____

Which variable will get rejected? ☐ A ☐ B ☐ C ☐ D ☐ E ☐ None is rejected

4. (16 points) (Estimating Probabilities from Samples) Below are a set of samples obtained by running rejection sampling for the Bayes' net from the previous question. Use them to estimate $P(C = 1|B = 1, E = 1)$. The estimation cannot be made whenever all samples were rejected. In this case, input -1 into the box below.

Sample 1

	0	1	rejected
A		x	
B	x		x
C			
D			
E			

Sample 2

	0	1	rejected
A	x		
B		x	
C	x		
D	x		
E		x	

Sample 3

	0	1	rejected
A		x	
B		x	
C		x	
D	x		
E		x	

Sample 4

	0	1	rejected
A		x	
B	x		x
C			
D			
E			

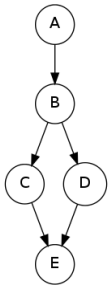
Sample 5

	0	1	rejected
A	x		
B		x	
C		x	
D	x		
E	x		x

Estimation: _____

5. (16 points) (Likelihood Weighting) We will work with a Bayes' net of the following structure.

0.249	0.052	0.299	0.773	0.715	0.550	0.703	0.105	0.236	0.153



A	$P(A)$
0	0.200
1	0.800

B	A	$P(B\|A)$
0	0	0.400
1	0	0.600
0	1	0.200
1	1	0.800

C	B	$P(C\|B)$
0	0	0.600
1	0	0.400
0	1	0.600
1	1	0.400

D	B	$P(D\|B)$
0	0	0.800
1	0	0.200
0	1	0.600
1	1	0.400

E	C	D	$P(E\|C,D)$
0	0	0	0.200
1	0	0	0.800
0	1	0	0.600
1	1	0	0.400
0	0	1	0.800
1	0	1	0.200
0	1	1	0.800
1	1	1	0.200

In this question, we will perform likelihood weighting to estimate $P(C = 1|B = 1, E = 1)$. Generate a sample and its weight, using the random samples given in the table below. Variables are sampled in the order A, B, C, D, E. In the table below, select the assignments to the variables you sampled.

When generating random samples, use as many values as needed from the table below, which we generated independently and uniformly at random from $[0, 1)$. Use numbers from left to right. To sample a binary variable W with probability $P(W = 0) = p$ and $P(W = 1) = 1 - p$ using a value a from the table, choose $W = 0$ if $a < p$ and $W = 1$ if $a \geq p$.

A: _____ **B:** _____ **C:** _____

D: _____ **E:** _____

What is the weight for the sample you obtained above? _____

6. (16 points) (Estimating Probabilities from Weighted Samples)

Below are a set of weighted samples obtained by running likelihood weighting for the Bayes' net from the previous question. Use them to estimate $P(C = 1|B = 1, E = 1)$. Input -1 in the box below if the estimation cannot be made.

Sample 1

	0	1
A		x
B		x
C		x
D		x
E		x

Weight = 0.64

Sample 2

	0	1
A		x
B		x
C		x
D		x
E		x

Weight = 0.64

Sample 3

	0	1
A		x
B		x
C	x	
D		x
E		x

Weight = 0.32

Sample 4

	0	1
A		x
B		x
C	x	
D	x	
E		x

Weight = 0.16

Sample 5

	0	1
A	x	
B		x
C		x
D		x
E		x

Weight = 0.48

Estimation: _____