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Electrochemical Impedance Spectroscopy Primer

This document presents an introduction to Electrochemical Impedance Spectroscopy (EIS) theory and has been keep as free from mathematics and electrical theory as is possible. If you still find the material presented here difficult to understand, don't stop reading. You will get useful information from this chapter even if you don't follow all of the discussions.

Four major topics are covered in this chapter.

- AC Circuit Theory and Representation of Complex Impedance Values
- Physical Electrochemistry and Circuit Elements
- Common Equivalent Circuit Models
- Extracting Model Parameters from Impedance Data

No prior knowledge of electrical circuit theory or electrochemistry is assumed. Each topic starts out at a quite elementary level. Each topic proceeds from this simple beginning to cover more advanced material.

AC Circuit Theory and Representation of Complex Impedance Values

Impedance definition: concept of complex impedance

Almost everyone knows about the concept of electrical resistance. It is the ability of a circuit element to resist the flow of electrical current. Ohm's law (Equation 0-1) defines resistance in terms of the ratio between voltage E and current I.

$$R \equiv \frac{E}{I} \tag{0-1}$$

While this is a well known relationship, it's use is limited to only one circuit element -- the ideal resistor. An ideal resistor has several simplifying properties:

- It follows Ohm's Law at all current and voltage levels.
- It's resistance value is independent of frequency.
- AC current and voltage signals though a resistor are in phase with each other.

The real world contains circuit elements that exhibit much more complex behavior. These elements force us to abandon the simple concept of resistance. In its place we use impedance, which is a more general

circuit parameter. Like resistance, impedance is a measure of the ability of a circuit to resist the flow of electrical current. Unlike resistance, impedance is not limited by the simplifying properties listed above.

Electrochemical impedance is usually measured by applying an AC potential to an electrochemical cell and measuring the current through the cell. Assume that we apply a sinusoidal potential excitation. The response to this potential is an AC current signal. This current signal can be analyzed as a sum of sinusoidal functions (a Fourier series).

Electrochemical Impedance is normally measured used a small excitation signal. This is done so that so that the cell's response is pseudo-linear. Linearity is described in more detail in the following section. In a linear (or pseudo-linear) system, the current response to a sinusoidal potential will be a sinusoid at the same frequency but shifted in phase. See Figure 0-1.

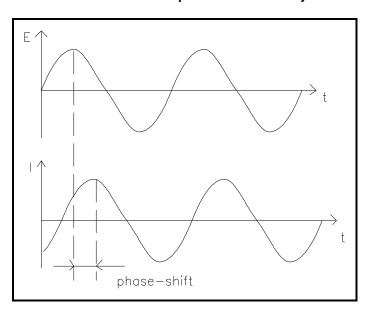


Figure 0-1
Sinusoidal Current Response in a Linear System

The excitation signal, expressed as a function of time, has the form

$$E_t = E_0 \sin(\omega t) \tag{0-2}$$

 E_t is the potential at time t, E_0 is the amplitude of the signal, and ω is the radial frequency. The relationship between radial frequency ω (expressed in radians/second) and frequency f (expressed in hertz) is:

$$\omega = 2\pi f \tag{0-3}$$

In a linear system, the response signal, I_t , is shifted in phase (ϕ) and has a different amplitude, I_0 .

$$I_{t} = I_{0} \sin \left(\omega t + \phi\right) \tag{0-4}$$

An expression analogous to Ohm's Law allows us to calculate the impedance of the system as:

$$Z = \frac{E_t}{I_t} = \frac{E_0 \sin(\omega t)}{I_0 \sin(\omega t + \phi)} = Z_0 \frac{\sin(\omega t)}{\sin(\omega t + \phi)}$$
(0-5)

The impedance is therefore expressed in terms of a magnitude, Z_0 , and a phase shift, ϕ .

If we plot the applied sinusoidal signal on the X-axis of a graph and the sinusoidal response signal I(t) on the Y-axis, an oval is plotted. See Figure 0-2. This oval is known as a "Lissajous figure". Analysis of Lissajous figures on oscilloscope screens was the accepted method of impedance measurement prior to the availability of FRAs.

Figure 0-2
Origin of Lissajous Figure

With Eulers relationship,

$$\exp(i\phi) = \cos\phi + i\sin\phi \tag{0-6}$$

it is possible to express the impedance as a complex function. The potential is described as,

$$E_t = E_0 \exp(j\omega t) \tag{0-7}$$

and the current response as,

$$I_{t} = I_{0} \exp(j\omega t - \phi) \tag{0-8}$$

The impedance is then represented as a complex number,

$$Z(\omega) = \frac{E}{I} = Z_0 \exp(j\phi) = Z_0(\cos\phi + j\sin\phi)$$
 (0-9)

Data Presentation

Look at Equation 0-9 in the previous section. The expression for $Z(\omega)$ is composed of a real and an imaginary part. If the real part is plotted on the Z axis and the imaginary part on the Y axis of a chart, we get a "Nyquist plot". See Figure 0-3. Notice that in this plot the y-axis is negative and that each point on the Nyquist plot is the impedance at one frequency. Figure 0-3 has been annotated to show that low frequency data are on the right side of the plot and higher frequencies are on the left.

On the Nyquist plot the impedance can be represented as an vector (arrow) of length |Z|. The angle between this vector and the x-axis is ϕ (=argZ).

Nyquist plots have one major shortcoming. When you look at any data point on the plot, you cannot tell what frequency was used to record that point.

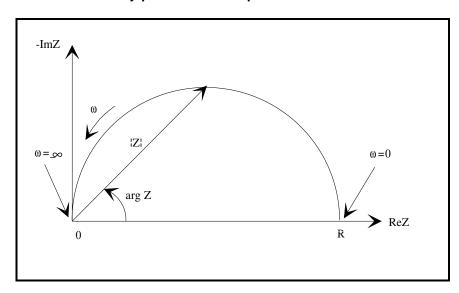
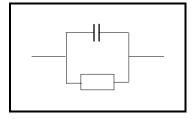


Figure 0-3
Nyquist Plot with Impedance Vector

The Nyquist plot in Figure 0-3 results from the electrical circuit of Figure 0-4. The semicircle is characteristic of a single "time constant". Electrochemical Impedance plots often contain several semicircles. Often only a portion of a semicircle is seen.

Figure 0-4
Simple Equivalent Circuit with One Time Constant



Another popular presentation method is the Bode-plot. The impedance is plotted with log frequency on the x-axis and both the absolute values of the impedance (|Z|=Z0) and the phase-shift on the y-axis.

The Bode plot for the electric circuit of Figure 0-4 is shown in Figure 0-5. Unlike the Nyquist plot, the Bode plot does show frequency information.

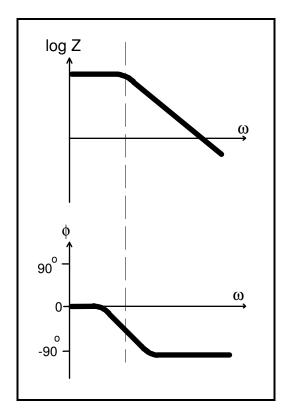


Figure 0-5
Bode Plot with One Time Constant

Electrochemistry - A Linear System?

Electrical circuit theory distinguishes between linear and non-linear systems (circuits). Impedance analysis of linear circuits is much easier than analysis of non-linear ones.

The following definition of a linear system is taken from Signals and Systems by Oppenheim and Willsky:

A linear system ... is one that possesses the important property of superposition: If the input consists of the weighted sum of several signals, then the output is simply the superposition, that is, the weighted sum, of the responses of the system to each of the signals. Mathematically, let $y_1(t)$ be the response of a continuous time system to $x_1(t)$ ant let $y_2(t)$ be the output corresponding to the input $x_2(t)$. Then the system is linear if:

- 1) The response to $x_1(t) + x_2(t)$ is $y_1(t) + y_2(t)$
- 2) The response to $ax_1(t)$ is $ay_1(t)$...

For a potentiostated electrochemical cell, the input is the potential and the output is the current. Electrochemical cells are not linear! Doubling the voltage will not necessarily double the current.

However, Figure 0-6 shows how electrochemical systems can be pseudo-linear. If you look at a small enough portion of a cell's current versus voltage curve, it appears to be linear.

In normal EIS practice, a small (1 to 10 mV) AC signal is applied to the cell. With such a small potential signal is the system is pseudo-linear. We don't see cell's large nonlinear response to the DC potential because we only measure the cell current at the excitation frequency.

Current

Voltage

Figure 0-6
Current versus Voltage Curve Showing Pseudo-Linearity

If the system is non-linear, the current response will contain harmonics of the excitation frequency.

Some researchers have made use of this phenomenon. Linear systems should not generate harmonics, so the presence or absence of significant harmonic response allows one to determine the systems linearity. Other researchers have intentionally used larger amplitude excitation potentials. They use the harmonic response to estimate the curvature in the cell's current voltage curve.

Steady State Systems

Measuring an EIS spectrum takes time (often many hours). The system being measured must be at a steady state throughout the time required to measure the EIS spectrum. A common cause of problems in EIS measurements and their analysis is drift in the system being measured.

In practice a steady state can be difficult to achieve. The cell can change through adsorption of solution impurities, growth of an oxide layer, build up of reaction products in solution, coating degradation, temperature changes, to list just a few factors.

Standard EIS analysis tools may give you wildly inaccurate results on a system that is not at steady state.

Time and Frequency Domains and Transforms

Signal processing theory refers to data representation domains. The same data can be represented in different domains. In EIS, we use two of these domains, the time domain and the frequency domain.

In the time domain, signals are represented as signal amplitude versus time. Figure 0-7 demonstrates this for a signal consisting of two superimposed sine waves.

Amplitude 0 -0.5 0 0.2 0.4 0.6 0.8 1 Time

Figure 0-7
Two Sine Waves in the Time Domain

Figure 0-8 shows the same data in the frequency domain. The data is plotted as amplitude versus frequency.

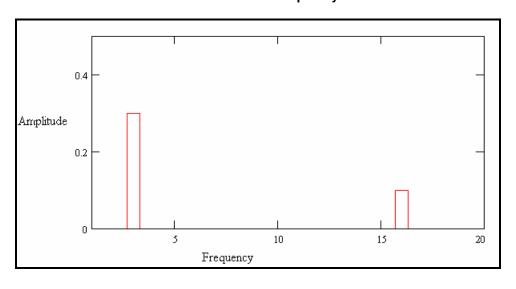


Figure 0-8
Two Sine Waves in the Frequency Domain

You use a transform to switch between the domains. The Fourier transform takes time domain data and generates the equivalent frequency domain data. The common term, FFT, refers to a fast, computerized implementation of the Fourier transform. The inverse Fourier transform changes frequency domain data into time domain data.

In modern EIS systems, lower frequency data are usually measured in the time domain. The controlling computer applies a digital approximation to a sine wave to the cell by means of a digital to analog

converter. The current response is measured using an analog to digital computer. The FFT is used to convert the current signal into the frequency domain.

Details of these transforms are beyond the scope of this manual.

Electrical Circuit Elements

EIS data is commonly analyzed by fitting it to an equivalent electrical circuit model. Most of the circuit elements in the model are common electrical elements such as resistors, capacitors, and inductors. To be useful, the elements in the model should have a basis in the physical electrochemistry of the system. As an example, most models contain a resistor that models the cell's solution resistance.

Some knowledge of the impedance of the standard circuit components is therefore quite useful. Table 0-1 lists the common circuit elements, the equation for their current versus voltage relationship, and their impedance.

Table 0-1
Common Electrical Elements

Component	Current Vs.Voltage	<u>Impedance</u>
resistor	E= IR	Z = R
inductor	E = L di/dt	Z = jωL
capacitor	I = C dE/dt	$Z = 1/j\omega C$

Notice that the impedance of a resistor is independent of frequency and has no imaginary component. With only a real impedance component, the current through a resistor stays in phase with the voltage across the resistor.

The impedance of an inductor increases as frequency increases. Inductors have only an imaginary impedance component. As a result, the current through an capacitor is phase shifted -90 degrees with respect to the voltage.

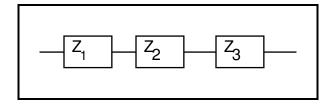
The impedance versus frequency behavior of a capacitor is opposite to that of an inductor. A capacitor's impedance decreases as the frequency is raised. Inductors also have only an imaginary impedance component. The current through an capacitor is phase shifted 90 degrees with respect to the voltage.

Serial and Parallel Combinations of Circuit Elements

Very few electrochemical cells can be modeled using a single equivalent circuit element. Instead, EIS models usually consist of a number of elements in a network. Both serial and parallel combinations of elements occur.

Fortunately, there are simple formulas the describe the impedance of circuit elements in both parallel and series combination.

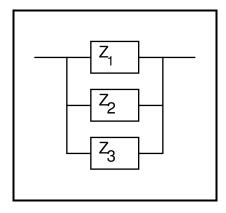
Figure 0-9 Impedances in Series



For linear impedance elements in series you calculate the equivalent impedance from:

$$Z_{eq} = Z_1 + Z_2 + Z_3 (0-10)$$

Figure 0-10 Impedances in Parallel



For linear impedance elements in parallel you calculate the equivalent impedance from:

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1 + Z_2 + Z_3} \tag{0-11}$$

We will calculate two examples to illustrate a point about combining circuit elements. Suppose we have a 1 Ω and a 4 Ω resistor is series. The impedance of a resistor is the same as its resistance (see Table 2-1). We thus calculate the total impedance as:

$$Z_{eq} = Z_1 + Z_2 = R_1 + R_2 = 1 \Omega + 4\Omega = 5 \Omega$$

Resistance and Impedance both go up when resistors are combined in series.

Now suppose that we connect two 2 μF capacitors in series. The total capacitance of the combined capacitors is 1 μF .

$$Z_{eq} = Z_1 + Z_2 = 1/j\omega C_1 + 1/j\omega C_2$$

= $1/j\omega(2e-6) + 1/j\omega(2e-6)$
= $1/j\omega(1e-6)$

Impedance goes up, but capacitance goes down when capacitors are connected in series. This is a consequence of the inverse relationship between capacitance and impedance.

Physical Electrochemistry and Equivalent Circuit Elements

Electrolyte resistance

Solution resistance is often a significant factor in the impedance of an electrochemical cell. A modern 3 electrode potentiostat compensates for the solution resistance between the counter and reference electrodes. However, any solution resistance between the reference electrode and the working electrode must be considered when you model your cell.

The resistance of an ionic solution depends on the ionic concentration, type of ions, temperature and the geometry of the area in which current is carried. In an bounded area with area A and length I carrying a uniform current the resistance is defined as,

$$R = \rho \frac{l}{A} \tag{0-12}$$

 ρ is the solution resistivity. The reciprocal of ρ (κ) is more commonly used. κ is called the conductivity of the solution and its relationship with solution resistance is:

$$R = \frac{1}{\kappa} \cdot \frac{l}{A} \Longrightarrow \kappa = \frac{l}{RA} \tag{0-13}$$

Standard chemical handbooks will often list κ values for specific solutions. For other solutions, you can calculate κ from specific ion conductances. The unit of κ is the Siemens per meter (S/m). The Siemen is the reciprocal of the ohm, so 1S=1/ohm.

Unfortunately, most electrochemical cells do not have uniform current distribution through a definite electrolyte area. The major problem in calculating solution resistance therefore concerns determination of the current flow path and the geometry of the electrolyte that carries the current. A comprehensive discussion of the approaches used to calculate practical resistances from ionic conductances is well beyond the scope of this manual.

Fortunately, you usually don't calculate solution resistance from ionic conductances Instead, you calculate it when you fit experimental EIS data to a model.

Double layer capacitance

A electrical double layer exists on the interface between an electrode and its surrounding electrolyte. This double layer is formed as ions from the solution "stick on" the electrode surface. Charges in the electrode are separated from ions charges. The separation is very small, often on the order of angstroms.

Charges separated by an insulator form a capacitor. On a bare metal immersed in an electrolyte, you can estimate that there will be 20 to 60 μ F of capacitance for every cm² of electrode area.

The value of the double layer capacitance depends on many variables. Electrode potential, temperature, ionic concentrations, types of ions, oxide layers, electrode roughness, impurity adsorption, etc. are all factors.

Polarisation Resistance

Whenever the potential of an electrode is forced away from it's value at open circuit, that is referred to as polarizing the electrode. When an electrode is polarized, it can cause current to flow through electrochemical reactions that occur at the electrode surface. The amount of current is controlled by the kinetics of the reactions and the diffusion of reactants both towards and away from the electrode.

In cells where an electrode undergoes uniform corrosion at open circuit, the open circuit potential is controlled by the equilibrium between two different electrochemical reactions. One of the reactions generates cathodic current and the other anodic current. The open circuit potential ends up at the potential where the cathodic and anodic currents are equal. It is referred to as a mixed potential. The value of the current for either of the reactions is known as the corrosion current.

Mixed potential control also occurs in cells where the electrode is not corroding. While this section discusses corrosion reactions, modification of the terminology makes it applicable in non-corrosion cases as well.

When there are two simple, kinetically controlled reactions occurring, the potential of the cell is related to the current by the following equation.

$$I = I_{corr} \left(e^{\frac{2.303(E - Eoc)}{\beta a}} - e^{\frac{-2.303(E - Eoc)}{\beta c}} \right)$$
 (0-14)

where

I is the measured cell current in amps,

I_{corr} is the corrosion current in amps,

E_{oc} is the open circuit potential in volts,

 β_a is the anodic Beta coefficient in volts/decade,

 β_c is the cathodic Beta coefficient in volts/decade.

If we apply a small signal approximation to equation 0-14, we get the following:

$$I_{corr} = \frac{\beta_a \beta_c}{2.303(\beta_a + \beta_c)} \cdot (\frac{1}{R_p})$$
 (0-15)

which introduces a new parameter, R_p , the polarization resistance. A you might guess from its name, the polarization resistance behaves like a resistor.

If the Tafel constants are known, you can calculate the I_{corr} from R_p using equation 0-15. I_{corr} in turn can be used to calculate a corrosion rate.

We will discuss the $R_{\rm p}$ parameter in more detail when we discuss cell models.

Charge Transfer Resistance

A similar resistance is formed by a single kinetically controlled electrochemical reaction. In this case we do not have a mixed potential, but rather a single reaction at equilibrium.

Consider a metal substrate in contact with an electrolyte. The metal molecules can electrolytically dissolve into the electrolyte, according to,

$$Me \Leftrightarrow Me^{n+} + ne^{-}$$
 (0-16)

or more generally

$$\operatorname{Re} d \Leftrightarrow Ox + ne^-$$
 (0-17)

In the forward reaction in the first equation, electrons enter the metal and metal ions diffuse into the electrolyte. Charge is being transferred.

This charge transfer reaction has a certain speed. The speed depends on the kind of reaction, the temperature, the concentration of the reaction products and the potential.

The general relation between the potential and the current (which is directly related with the amount of electrons and so the charge transfer via Faradays law) is:

$$i = i_0 \left(\frac{C_O}{C_0^{*_O}} \exp\left(\frac{\alpha n F \eta}{RT} \right) - \left(\frac{C_R}{C_0^{*_R}} \exp\left(\frac{-(1 - \alpha) n F \eta}{RT} \right) \right)$$
 (0-18)

with,

i₀ = exchange current density

C
 = concentration of oxidant at the electrode surface

 C_0^* = concentration of oxidant in the bulk

C_D = concentration of reductant at the electrode surface

F = Faradays constant

T = temperature R = gas constant

R = gas constant $\alpha = reaction order$

n = number of electrons involved

When the concentration in the bulk is the same as at the electrode surface, $C_O = C_O^*$ and $C_R = C_R^*$. This simplifies equation 0-18 into:

$$i = i_0 \left(\exp(\alpha \frac{nF}{RT} \eta) - \exp(-(1-\alpha) \frac{nF}{RT} \eta) \right)$$
 (0-19)

This equation is called the Butler-Volmer equation. It is applicable when the polarisation depends only on the charge transfer kinetics. Stirring the solution to minimize the diffusion layer thickness can help minimize concentration polarization.

When the overpotential, η , is very small and the electrochemical system is at equilibrium, the expression for the charge transfer resistance changes into,

$$R_{ct} = \frac{RT}{nFi_0} \tag{0-20}$$

From this equation the exchange current density can be calculated when R_{ct} is known.

Diffusion

Diffusion also can create an impedance called the Warburg-impedance. The impedance depends on the frequency of the potential perturbation. At high frequencies the Warburg impedance is small since diffusing reactants don't have to move very far. At low frequencies the reactants have to diffuse farther, increasing the Warburg-impedance.

The equation for the "infinite" Warburg impedance is:

$$Z_{W} = \sigma(\omega)^{-1/2} (1-j)$$
 (0-21)

On a Nyquist plot the Warburg impedance appears as a diagonal line with an slope of 45°. On a Bode plot, the Warburg impedance exhibits a phase shift of 45°.

In equation 0-21, σ is the Warburg coefficient defined as:

$$\sigma = \frac{RT}{n^2 F^2 A \sqrt{2}} \left(\frac{1}{C^{*_o} \sqrt{D_o}} + \frac{1}{C^{*_R} \sqrt{D_R}} \right)$$
 (0-22)

In which,

 ω = radial frequency

D_O = diffusion coefficient of the oxidant

D_R = diffusion coefficient of the reductant A = surface area of the electrode

n = number of electrons involved

This form of the Warburg impedance is only valid if the diffusion layer has an infinite thickness. Quite often however this is not the case. If the diffusion layer is bounded, the impedance at lower frequencies no longer obeys the equation above. Instead, we get the form:

$$Z_{0} = \sigma \omega^{-1/2} (1 - j) \tanh(\delta (\frac{j\omega}{D})^{1/2})$$
 (0-23)

with,

 δ = Nernst diffusion layer thickness

D = some average value of the diffusion coefficients of the diffusing species

This more general equation is called the "finite" Warburg. For high frequencies where $\omega \to \infty$, or for an infinite thickness of the diffusion layer where $\delta \to \infty$, $\tanh(\delta \, (\frac{j\omega}{D})^{1/2}) \to 1$ and equation 0-23 simplifies to the infinite Warburg impedance.

Coating capacitance

A capacitor is formed when two conducting plates are separated by a non-conducting media, called the dielectric. The value of the capacitance depends on the size of the plates, the distance between the plates and the properties of the dielectric. The relationship is,

$$C = \frac{\mathcal{E}_o \mathcal{E}_r A}{d} \tag{0-24}$$

With,

 ε_{o} = electrical permittivity

 ε_r = relative electrical permittivity

A = surface of one plate

d = distances between two plates

Whereas the electrical permittivity is a physical constant, the relative electrical permittivity depends on the material. Table 0-2 gives you some useful ε_r values.

Table 0-3
Typical Relative Electrical Permittivities

<u>Material</u>	<u> </u>
<u>vacuum</u>	<u>1</u>
water	80.1 (20°C)
organic coating	<u>4 - 8</u>

Notice the large difference between the electrical permittivity of water and that of an organic coating. The capacitance of a coated substrate changes as it absorbs water. EIS can be used to measure that change.

Constant Phase Element

Capacitors in EIS experiments often do not behave ideally. Instead they act like a constant phase element as defined below.

The impedance of a capacitor can be expressed as:

$$Z = A(j\omega)^{-\alpha} \tag{0-25}$$

where.

A = 1/C = The inverse of the capacitance α = An exponent which equals 1 for a capacitor For a constant phase element, the exponent α is less than one. The "double layer capacitor" on real cells often behaves like a CPE, not a capacitor. While several theories have been proposed to account for the non-ideal behavior of the double layer, it is probably best to treat α as an empirical constant with no real physical basis.

Virtual Inductor

The impedance of an electrochemical cell sometimes also appears to be inductive. Some workers have ascribed inductive behavior to the formation of a surface layer, like a passive layer or fouling. Others have claimed that inductive behavior results from errors in the measurement, including potentiostat non-idealities.

Common equivalent circuit models

In the following section we show some common equivalent circuits models. These models can be used to interpret simple EIS data. Many of these models have been included as standard models in the CMS300 analysis package.

To elements used in the following equivalent circuits are presented in table 0-4. Equations for both the admittance and impedance are given for each element.

Table 0-4
Circuit Elements Used in the Models

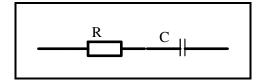
Equivalent element	<u>Admittance</u>	<u>Impedance</u>
R	1/R	R
С	jωC	$1/j\omega C$
L	$1/j\omega L$	jωL
W (infinite Warburg)	$Y_0\sqrt{(j\omega)}$	$1/Y_0\sqrt{(j\omega)}$
O (finite Warburg)	$1/Y_0\sqrt{(j\omega)}Coth(B\sqrt{(j\omega)})$	$Tanh(B\sqrt{(j\omega)})/Y \circ \sqrt{(j\omega)}$
Q (CPE)	$Y_0(j\omega)^{\alpha}$	$1/Y_0(j\omega)^{\alpha}$

The dependent variables used in these equations are R, C, L, Y_0 , B and α . The EIS300 uses these as fit parameters.

Purely Capacitive Coating

A metal covered with an undamaged coating generally has a very high impedance. The equivalent circuit for such a situation is in Figure 0-11.

Figure 0-11
Purely Capacitive Coating

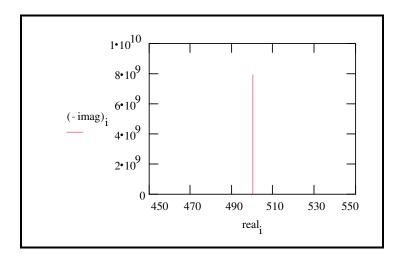


The model includes a resistor (due primarily to the electrolyte) and the coating capacitance in series.

A Nyquist plot for this model is shown in figure 0-12. In making this plot, the following values were assigned:

$R = 500 \Omega$	(a bit high but realistic for a poorly conductive solution)
C = 200 pF	(realistic for a 1 cm ² sample, a 25 μ M coating, and $\epsilon_r = 6$)
$F_i = 0.1 Hz$	(lowest scan frequency is a bit higher than typical)
$F_{\rm f} = 0.1 {\rm Hz}$	(highest scan frequency is at the CMS300 limit)

Figure 0-12
Typical Nyquist Plot for an Excellent Coating



The value of the capacitance cannot be determined from the Nyquist plot. It can be determined by a curve fit or from an examination of the data points. Notice that the intercept of the curve with the real axis gives an estimate of the solution resistance.

The highest impedance on this graph is close to $10^{10} \Omega$. This is close to the limit of measurement of most EIS systems.

The same data are shown in a Bode plot in Figure 0-13. Notice that the capacitance can be estimated from the graph but the solution resistance value does not appear on the chart. Even at 100 kHz, the impedance of the coating is higher than the solution resistance.

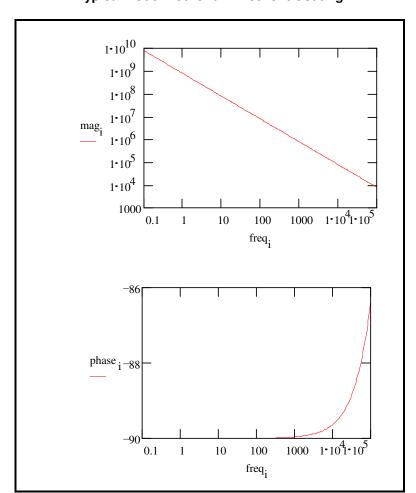


Figure 0-13
Typical Bode Plot for an Excellent Coating

Water uptake into the film is usually a fairly slow process. It can be measured by taking EIS spectra at set time intervals. An increase in the film capacitance can be attributed to water uptake.

Randles Cell

The Randles cell is one of the simplest cell models. It includes a solution resistance, a double layer capacitor and a charge transfer or polarization resistance. In addition to being a useful model in its own right, the Randles model is the starting point for other more complex models.

The equivalent circuit for the Randles Cell is shown in Figure 2-13. The double layer capacity is parallel with the impedance due to the charge transfer reaction.

Figure 0-13
Randles Cell Schematic Diagram

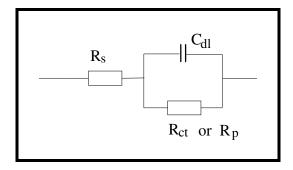
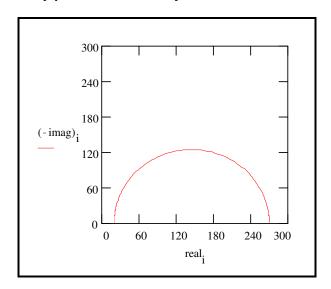


Figure 0-14 is the Nyquist plot for a typical Randles cell. The parameters in this plot were calculated assuming a 1 cm² electrode undergoing uniform corrosion at a rate of 1 mm/year. Reasonable assumptions were made for the β coefficients, metal density and equivalent weight. The polarization resistance under these conditions calculated out to 250 Ω . A capacitance of 40 $\mu\text{F/cm}^2$ and a solution resistance of 20 Ω were also assumed.

Figure 0-14
Nyquist Plot for 1 mm/year Corrosion Rate



The Nyquist plot for a Randles cell is always a semicircle. The solution resistance can found by reading the real axis value at the high frequency intercept. This is the intercept near the origin of the plot. Remember this plot was generated assuming that $R_s = 20~\Omega$ and $R_{p,=} = 250~\Omega$.

The real axis value at the other (low frequency) intercept is the sum of the polarization resistance and the solution resistance. The diameter of the semicircle is therefore equal to the polarization resistance (in this case 250 Ω).

Figure 0-15 is the Bode plot for the same cell.

1000 mag_i 100 10 1.1041.105 100 1000 0.1 10 freq; phase i -50 1.1041.10 10 100 1000 0.1 freq;

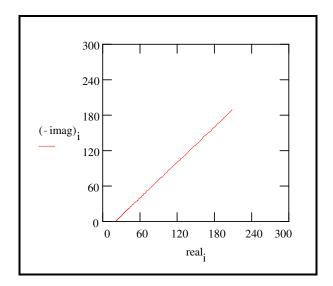
Figure 0-15
Nyquist Plot for 1 mm/year Corrosion Rate

Mixed Kinetic and Diffusion Control

First consider a cell where semi-infinite diffusion is the rate determining step, with a series solution resistance as the only other cell impedance.

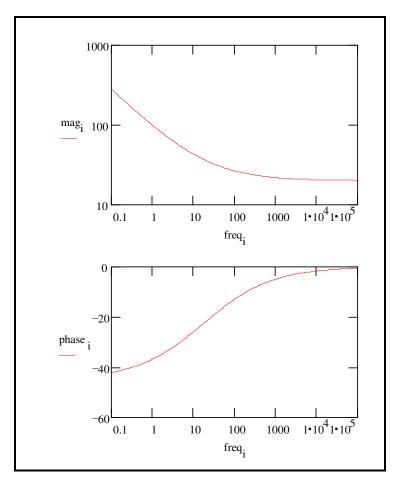
A Nyquist plot for this cell is shown in Figure 0-16. R_s was assumed to be 20 Ω . The Warbug coefficient calculated to be about 150 for the diffusion a species with a bulk concentration of 100 μ M and a typical diffusion coefficient of 1.6 x10⁻⁵ cm²/sec. Notice that the Warbug Impedance appears as a straight line with a slope of 45°.

Figure 0-16 Nyquist Plot for a Warburg Impedance



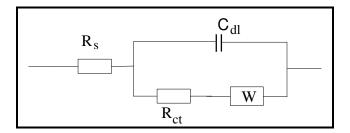
The same data is plotted in the Bode format in Figure 0-17. The phase angle of a Warburg impedance is 45°.

Figure 0-17
Bode Plot for a Warburg Impedance



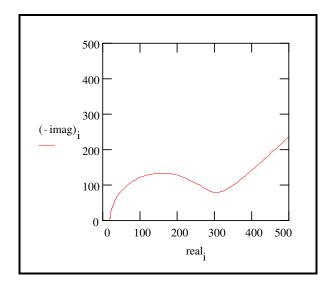
Adding a double layer capacitance and a charge transfer impedance, we get the equivalent circuit in figure 0-18.

Figure 0-18
Equivalent Circuit with Mixed Kinetic and Charge Transfer Control



This circuit models a cell where polarization is due to a combination of kinetic and diffusion processes. The Nyquist plot for this circuit is shown in Figure 0-19. As in the above example, the Warbug coefficient is assumed to be to be about 150. Other assumptions: $R_s = 20~\Omega$, $R_{ct} = 250~\Omega$, and $C_{dl} = 40~\mu F$.

0-19 Nyquist Diagram for Mixed Control Circuit



The Bode plot for the same data is shown in Figure 0-20. The lower frequency limit was moved down to 1mHz to better illustrate the differences in the slope of the magnitude and in the phase between the capacitor and the Warburg impedance.

1.104 1000 mag; 100 $0.001\,0.01$ 0.1 1 10 100 1000 1 101 10 freq; phase i -50 1000 1.101.10 0.001 0.01 0.1 10 100 freq;

0-20
Bode Plot for the Mixed Control Circuit

EIS of Coated Metals

The impedance behavior of a purely capacitive coating was discussed above. Most paint coatings degrade with time, resulting in more complex behavior.

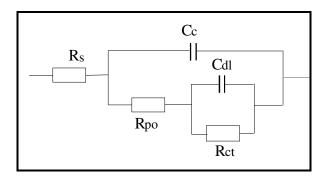
After a certain amount of time, water penetrates into the coating and forms a new liquid/metal interface under the coating. Corrosion phenomena can occur at this new interface.

The impedance of coated metals has been very heavily studied. The interpretation of impedance data from failed coatings can be very complicated. Only the simple equivalent circuit shown in Figure 0-21 will be discussed here. Even this simple model has been the cause of some controversy in the literature. Researchers do not agree on the assignment of impedances to physical processes that can occur in the cell. The discussion below is therefore only one of several interpretations of this model.

The capacitance of the intact coating is represented by C_c , Its value is much smaller than a typical double layer capacitance. Its units are pF or nF, not μ F. R_{po} (pore resistance) is the resistance of ion conducting paths the develop in the coating. These paths may not be physical pores filled with electrolyte. On the metal side of the pore, we assume that an area of the coating has delaminated and a pocket filled with an electrolyte solution has formed. This electrolyte solution can be very different that the bulk solution outside

of the coating. The interface between this pocket of solution and the bare metal is modeled as a double layer capacity in parallel with a kinetically controlled charge transfer reaction.

0-21 Equivalent Circuit for a Failed Coating



When you use EIS to test a coating, you fit a data curve to this type of model. The fit returns estimates for the model parameters, such as the pore resistance or the double layer capacitance. You then use these parameters to evaluate the degree to which the coating has failed.

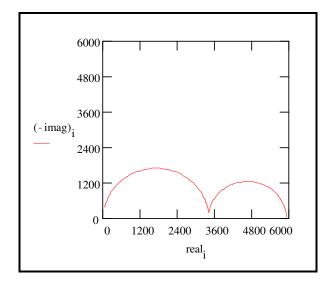
In order to show realistic data curves, we need to do this operation in reverse. Assume that we have a 10 cm 2 sample of metal coated with a 12 μ M film. Assume that we have 5 delaminated areas making up a total of 1% of the surface of the metal. The pores in the film that give access into these delaminated areas can be represented as being solution filled cylinders, with a 30 μ M diameter.

The parameters used to develop the curves are shown below:

C_{c}	= 4 n⊦	Calculated for 10 cm ² area , ε_r = 6 and 12 μ M thickness
R_{po}	$= 3400 \Omega$	Calculated assuming $\kappa = 0.01$ S/cm
R_s^{r}	$=20 \Omega$	Assumed
C _{dl}	$=4 \mu F$	Calculated for 1% of 10 cm ² area and assuming 40 μF/cm ² .
Rct	$= 2500 \Omega$	Calculated for 1% of 10 cm ² area and using Polarization
		Resistance assumptions from an earlier discussion.

With these parameters, the Nyquist plot for this model is shown in Figure 0-22. Notice that there are two time constants in this plot.

Figure 0-22 Nyquist Plot for a Failed Coating



The Bode pot of the same data is shown in Figure 0-23. The two time constants are not nearly as pronounced on this plot. The plot does not go high enough in frequency to measure the solution resistance. In practice this is not a problem, because the solution resistance is a property of the test solution and the test cell geometry, not a property of the coating. It is therefore not very interesting.

Figure 0-23
Bode Plot for a Failed Coating

Extracting Model Parameters from Data

Modeling Overview

EIS data is generally analyzed in terms of a equivalent circuit model. The analyst tries to find a model whose impedance matches the measured data.

The type of electrical components in the model and their interconnections controls the shape of the model's impedance spectrum. The model's parameters (i.e. the resistance value of a resistor) controls the size of each feature in the spectrum. Both these factors effect the degree to which the model's impedance spectrum matches a measured EIS spectrum.

In a physical model, each of the model's components is postulated to come from a physical process in the electrochemical cell. All of the models discussed earlier in this chapter are physical models. The choice of which physical model applies to a given cell is made from knowledge of the cell's physical characteristics.

Experienced EIS analysts can also use the shape of a cell's EIS spectrum to help choose a model for that cell.

Models can also be partially or completely empirical. The circuit components in this type of model are not assigned to physical processes in the cell. The model is chosen to given the best possible match between the model's impedance and the measured impedance.

An empirical model can be constructed by successively subtracting component impedances from a spectrum. If the subtraction of an impedance simplifies the spectrum, the component is added to the model, and the next component impedance is subtracted from the simplified spectrum. This process ends when the spectrum is completely gone.

As we shall see below, physical models are generally preferable to empirical models.

Non-linear Least Squares Fitting

Modern EIS analysis uses a computer to find the model parameters that cause the best agreement between a model's impedance spectrum and a measured spectrum. A non-linear least squares fitting (NLLS) algorithm is used.

NLLS starts with initial estimates for all the model's parameters. Starting from this initial point, the algorithm makes a change in one of the parameter values and evaluates the resulting fit. If the change improves the fit, the new parameter value is accepted. If the change worsens the fit, the old parameter value is retained. Next a different parameter value is changed and the test is repeated. Each trial with new values is called an iteration. Iterations continue until the goodness of fit exceeds an acceptance criterion, or until the number of iterations reaches a limit.

NLLS algorithms are not perfect. In some cases they do not converge on a useful fit. This can be the result of several factors including:

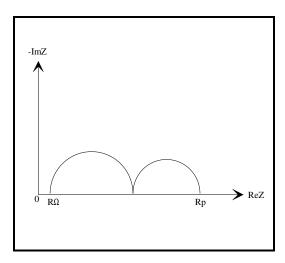
- An incorrect model for the data set being fitted.
- Poor estimates for the initial values.
- Noise

In addition, the fit from an NLLS algorithms can look poor when the fit's spectrum is superimposed on the data spectrum. It appears as though the fit ignores a region in the data. To a certain extent this is what happens. The NLLS algorithm optimizes the fit over the entire spectrum. It does not care that the fit looks poor over a small section of the spectrum.

Multiple Models

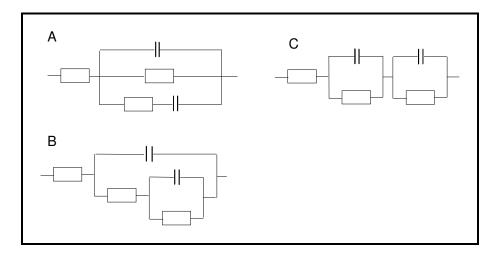
The impedance spectrum shown in Figure 0-24 shows two clearly defined time constants.

Figure 0-24
Two Time Constant Spectrum



This spectrum can be modeled by any of the equivalent circuits shown in Figure 0-25.

Figure 0-25
Equivalent Circuit Models with Two Time Constants



As you can see, there is not a unique equivalent circuit that describes the spectrum. You therefore can not assume that an equivalent circuit that produces a good fit to a data set represents an accurate physical model of the cell.

Even physical models are suspect in this regard. Whenever possible, the physical model should be verified before it is used. One way to verify the model is to alter a single cell component (for example increase a paint layer thickness) and see if you get the expected changes in the impedance spectrum.

Empirical models should be treated with even more caution. You can always get a good looking fit by adding additional circuit elements to a model. Unfortunately, these elements may have little relevance to the cell processes that you are trying to study. Empirical models should therefore use the fewest elements possible.

Literature

The following sources were used in preparing this chapter. The reader is encouraged to consult them for additional information.

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