

$A = (a_{ij})_{m \times n}$ A 의 행공간은 A 의 행에 의해 생성된 \mathbb{R}^n (행의 계수)의 부분공간

A 의 열공간은 A 의 치역

A 의 행공간의 차원은 행 계수, A 의 열공간의 차원은 열 계수

ex) $A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 3 & 5 & 8 & -2 \\ 1 & 1 & 2 & 0 \end{bmatrix}$ A 의 행공간의 기저와 열공간의 기저는?

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 3 & 5 & 8 & -2 \\ 1 & 1 & 2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & -1 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

행공간 $\langle [1, 2, 3, 1], [0, -1, -1, 1] \rangle \sim$ 일차독립 \therefore 행 계수 2

$$A^T = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 5 & 1 \\ 3 & 8 & 2 \\ -1 & -2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & 1 \\ 0 & -1 & 1 \\ 0 & -1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \sim \text{Transpose} \left(\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right)$$

\therefore 열 계수 2

모든 행렬은 열 계수와 행 계수가 같다.

ex) $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ -2 \end{bmatrix}$ 기저 찾아라

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & -2 \\ 0 & 1 & -2 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 1 \\ -2 & -2 & -2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow T \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \text{ 열공간 기저 } \textcircled{2}$$

A 의 계수 ($\text{Rank}(A)$) = A 의 행공간의 차원의 계수

$Ax = b$ 에서 $[A | b]$ 의 계수가 같으면 $Ax = b$ 해 존재

ex) $\left(\begin{array}{ccc|c} 1 & 2 & 3 & b_1 \\ 0 & 1 & 4 & b_2 \\ 0 & 0 & 0 & b_3 \end{array} \right) \quad \text{Rank}(A) = 2 \quad \text{Rank}([A|b]) = 3 \quad \therefore \text{해 존재 } X$
 $\quad \quad \quad \hookrightarrow$ 행공간 차원

연습문제

1-4 $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 3 & 4 \\ 0 & 4 & 3 \end{bmatrix}$ 행과 열 공간의 기저 개수?

$$\begin{bmatrix} 1 & 2 & 0 \\ -1 & 3 & 4 \\ 0 & 4 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 0 \\ 0 & 5 & 4 \\ 0 & 4 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 4 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad \therefore \text{행 공간 기저 : 3}$$

$$A^T = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 4 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 4 \\ 0 & 4 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 13 \end{bmatrix} \rightarrow \text{Transpose} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 4 & 13 \end{bmatrix} \quad \text{열 공간 기저 : 3}$$

$$\therefore \text{RANK}(A) = 3$$

4-3 $x_1 + 2x_2 + 3x_3 = 0$
 $4x_1 + 5x_2 + 6x_3 = 0$
 $x_1 - x_2 + x_3 = 0$

$$\begin{bmatrix} 1 & 2 & 1 \\ 4 & 5 & 6 \\ 1 & -1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 4 & 5 & 6 \\ 0 & 3 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 2 \\ 0 & 3 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 3 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$x_2 = 0 \quad x_3 = 0 \quad x_1 = 0 \quad \therefore \emptyset$$

1. 행과 열 공간의 기저와 계수를 찾아라

1-1 $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}$ 행 공간 기저: $\{[1, 2], [0, -2]\}$
 $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ 열 공간 기저: $\{[1, 3]^T, [0, 2]^T\}$
 $\therefore \text{RANK}(A) = 2$

1-2 $A = \begin{bmatrix} 1 & 0 & 1 \\ 3 & 2 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -2 & 2 \end{bmatrix} \rightarrow \text{행 공간 기저 } \{[1, 0, 1], [0, -2, 2]\}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \text{열 공간 기저 } \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\} \quad \therefore \text{RANK}(A) = 2$$

1-3 $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 6 & 8 \end{bmatrix}$

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 3 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 3 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \rightarrow \text{행 공간의 기저 } \{[2, 3], [0, 1]\}$$

$$\begin{bmatrix} 2 & 4 & 6 \\ 3 & 5 & 8 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 4 & 6 \\ 1 & 1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix} \rightarrow \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} \right\} \quad \therefore \text{RANK}(A) = 2$$

1-5 $\begin{bmatrix} 5 & 6 & 7 \\ 0 & 5 & 6 \\ 0 & 0 & 5 \end{bmatrix}$

$$\begin{bmatrix} 5 & 6 & 7 \\ 0 & 5 & 6 \\ 0 & 0 & 5 \end{bmatrix} \rightarrow \text{행 } \{[5, 6, 7], [0, 5, 6], [0, 0, 5]\}$$

$$\begin{bmatrix} 5 & 0 & 0 \\ 6 & 5 & 0 \\ 7 & 6 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 6 & 5 & 0 \\ 7 & 6 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 6 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \therefore \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} \quad \text{RANK}(A) = 3$$

$$2-1 \quad A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 0 & 1 & 5 & 6 \\ 0 & 0 & 7 & 8 \\ 0 & 0 & 5 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 & 4 & 5 \\ 0 & 1 & 5 & 6 \\ 0 & 0 & 7 & 8 \\ 0 & 0 & 5 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 3 & 4 & 5 \\ 0 & 1 & 5 & 6 \\ 0 & 0 & 2 & -2 \\ 0 & 0 & 1 & -7 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 3 & 4 & 5 \\ 0 & 1 & 5 & 6 \\ 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 19 \end{bmatrix} \rightarrow \left\{ \begin{bmatrix} 2 & 2 & -1 & -1 \\ 0 & 1 & 5 & 6 \\ 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 19 \end{bmatrix} \right\}$$

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 4 & 5 & 7 & 5 \\ 5 & 6 & 8 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 4 & 5 & 7 & 5 \\ 5 & 6 & 8 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 4 & 7 & 5 \\ 0 & 1 & 1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 3 & 6 & 3 \\ 0 & 1 & 1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 1 & 1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{RANK}(A) = 4$$

$$\therefore \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\} \downarrow^T \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

$$2-2 \quad B = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 5 & 7 & 9 \\ 4 & 6 & 8 & 10 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 5 & 7 & 9 \\ 4 & 6 & 8 & 10 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \therefore \left\{ \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \right\}$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 5 & 7 & 9 \\ 4 & 6 & 8 & 10 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\} \text{ RANK}(B)=2$$

3. 기저 찾아라

$$1-1 \quad \left\{ \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \right\} \rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \text{ 행기저 } \left\{ \begin{bmatrix} 1 & 1 \end{bmatrix} \right\}$$

$$3-2 \quad \left\{ \begin{bmatrix} 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 4 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \right\} \rightarrow \begin{bmatrix} 3 & 2 & 1 \\ 4 & 3 & 2 \\ 1 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 2 & 1 \\ 1 & 1 & 1 \\ 4 & 3 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \left\{ \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \right\}$$

$$3-3 \quad \left\{ \begin{bmatrix} 2 \\ 7 \\ 0 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\} \rightarrow \begin{bmatrix} 2 & 7 & 0 \\ 3 & 5 & 1 \\ 1 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

4-1 $x_1 + x_2 - x_3 = 0$ 의 행벡터 기저

$$\begin{bmatrix} 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad x_1 = s \quad x_2 = t \quad x_3 = s + t \quad \begin{bmatrix} -s-t \\ s \\ t \end{bmatrix} = s \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \therefore \text{기저 } \left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

4-2 $2x + y + z = 0$
 $x - z = 0$

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \quad z = t \quad x = t \quad y = -3t \quad \left\{ \begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix} \right\}$$

$$4-4 \quad \begin{bmatrix} 1 & 2 & 0 & 3 \\ 2 & 5 & 2 & 1 \\ 1 & 3 & 1 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 1 & 2 & -5 \\ 0 & 1 & 1 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 1 & 2 & -5 \\ 0 & 0 & -1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 1 & 2 & -5 \\ 0 & 0 & -1 & 1 \end{bmatrix} \quad x_1 = t \quad x_2 = 3t \quad \left\{ \begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix} \right\}$$

5. LS의 답들을 구성하는 복원공간

$$\begin{aligned} x_1 + 2x_2 &= 0 \\ 2x_1 - x_2 + 3x_3 &= 0 \end{aligned} \quad \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 0 \\ 0 & 5 & 3 \end{bmatrix} \quad x_3 = t \quad 5x_2 + 3t = 0 \quad x_2 = -\frac{3}{5}t$$

$$\begin{aligned} x_1 + 2x_2 &= 0 \\ x_1 + \frac{6}{5}t &= 0 \\ x_1 &= -\frac{6}{5}t \end{aligned}$$

$$\text{기저} \left\{ \begin{bmatrix} -\frac{6}{5} \\ -\frac{3}{5} \\ 1 \end{bmatrix} \right\}$$

6. $Ax=0$ 의 해집합의 기저

1) $A = \begin{bmatrix} 1 & 5 \\ 2 & 6 \end{bmatrix} \quad \begin{bmatrix} 1 & 5 \\ 2 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 5 \\ 0 & 2 \end{bmatrix} \quad x_1 = 0 \quad x_2 = 0 \quad \text{기저 } \emptyset$

2) $A = \begin{bmatrix} 2 & 3 & 5 \\ 4 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 2 & 3 & 5 \\ 4 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 3 & 5 \\ 0 & 2 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 1 & 2 \\ 0 & 2 & 3 \end{bmatrix} \quad x_3 = t \quad x_1 = \frac{t}{4}$

$$2x_1 + 3t = 0 \quad x_1 = -\frac{3}{2}t$$

$$\therefore \left\{ \begin{bmatrix} \frac{1}{4} \\ -\frac{3}{2} \\ 1 \end{bmatrix} \right\}$$

3) $A = \begin{bmatrix} 0 & 1 & 2 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 3 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \quad x_1 = x_2 = x_3 = 0 \quad \text{기저 } \emptyset$

4) $A = [1 \ 2 \ 3 \ 4] \quad x_2 = t \quad x_3 = s \quad x_4 = n \quad x_1 + 2t + 3s + 4n = 0$

$$x_1 = -2t - 3s - 4n \quad \begin{bmatrix} -2t - 3s - 4n \\ t \\ s \\ n \end{bmatrix} \therefore \left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$