No. Date.

5-1

2(4)

① $a(u+v) = au+av$

$a(x_1+x_2, y_1+y_2, z_1+z_2) = (ax_1, y_1, z_1) + (ax_2, y_2, z_2)$

$= (a(x_1+x_2), y_1+y_2, z_1+z_2) = (a(x_1+x_2), y_1+y_2, z_1+z_2)$

성립

② $(a+b)u = au+bu$

$(a+b)(x, y, z) = a(x, y, z) + b(x, y, z)$

$((a+b)x, y, z) = ((a+b)x, y, z)$ 성립

$\therefore$ 벡터공간

2(5)

① $a(u+v) = au+av$

$a(x_1+x_2, y_1+y_2) = (2ax_1, 2ay_1) + (2ax_2, 2ay_2)$

$(2a(x_1+x_2), 2a(y_1+y_2)) = (2a(x_1+x_2), 2a(y_1+y_2))$

성립

② $(a+b)u = au+bu$

$(2(a+b)x, 2(a+b)y) = (2ax, 2ay) + (2bx, 2by)$

$(2(a+b)x, 2(a+b)y) = (2(a+b)x, 2(a+b)y)$

5-2

3(1) $\begin{pmatrix}2\\3\\1\end{pmatrix}x_1 + \begin{pmatrix}3\\2\\1\end{pmatrix}x_2 + \begin{pmatrix}1\\4\\2\end{pmatrix}x_3 + \begin{pmatrix}5\\2\\4\end{pmatrix}x_4 = \begin{pmatrix}2\\3\\1\end{pmatrix}$

$$\begin{bmatrix}2&3&1&5\\3&2&4&2\\1&1&2&4\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}2\\3\\1\end{bmatrix}$$

$x_4 = t$

가우스소거법 $\begin{bmatrix}1&-1&3&-3&-1\\0&-1&-2&-10&0\\1&1&2&4&1\end{bmatrix}$

$\rightarrow \begin{bmatrix}1&-1&3&-3&-1\\0&1&2&10&0\\0&-2&1&-7&-2\end{bmatrix} \rightarrow \begin{bmatrix}1&-1&3&-3&-1\\0&1&2&10&0\\0&0&5&13&0\end{bmatrix}$

$x_3 = -\frac{13}{5}t \quad x_2 - \frac{26}{5}t + 10t = 0$

$x_2 = -\frac{24}{5}t \quad x_1 = 1+6t$

5-3

1(3)

$u = \begin{bmatrix}x_1\\x_2\\-2x_1-x_2\end{bmatrix} \quad v = \begin{bmatrix}x_1'\\x_2'\\-2x_1'-x_2'\end{bmatrix}$

$u+v = \begin{bmatrix}x_1+x_1'\\x_2+x_2'\\-2(x_1+x_1')-(x_2+x_2')\end{bmatrix} \in W$

$\alpha u = \begin{bmatrix}\alpha x_1\\\alpha x_2\\\alpha(-2x_1-x_2)\end{bmatrix} \in W$

부분공간임

5. $x_1(1,3,4) + x_2(4,0,1) + x_3(3,1,2) = (1,1,1)$

$\begin{bmatrix}1&4&3&1\\3&0&1&1\\4&1&2&1\end{bmatrix} \rightarrow \begin{bmatrix}1&4&3&1\\3&0&1&1\\0&15&10&3\end{bmatrix} \rightarrow \begin{bmatrix}1&4&3&1\\0&12&8&2\\0&15&10&3\end{bmatrix}$

$\rightarrow \begin{bmatrix}1&4&3&1\\0&12&8&2\\0&3&2&1\end{bmatrix} \rightarrow \begin{bmatrix}1&4&3&1\\0&0&0&-3\\0&3&2&1\end{bmatrix}$

$\therefore$ 해 없음. 부분공간 X

5-4

3.

$x_1[2,1,1,1] + x_2[3,-2,1,0] + x_3[a,-1,2,0] = 0$

$\rightarrow x_1 = 0 \quad -2x_2 - x_3 = 0$

$3x_2 + ax_3 = 0$

$-6x_2 - 3x_3 = 0$

$+\ 6x_2 + 2ax_3 = 0$

$(2a-3)x_3 = 0$

독립이기 위해선 $x_3 = 0$

$\therefore$ a는 모든 실수

11-4

$$\left\{ \begin{vmatrix} x \\ y \\ z \end{vmatrix} \mid z = x+y \right\} = \begin{vmatrix} x \\ y \\ x+y \end{vmatrix}$$

$$\Rightarrow x \begin{vmatrix} 1 \\ 0 \\ 1 \end{vmatrix} + y \begin{vmatrix} 0 \\ 1 \\ 1 \end{vmatrix} \quad \therefore 2$$

12-6

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$x_1 = t$, $x_2 = s$ $\quad 0x_1 + 0x_2 = 0$

$$\therefore t \begin{bmatrix} 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \therefore 1$$

12-7

$x_2 - x_3 = 0 \qquad x_3 = t$

$x_1 + x_3 = 0 \qquad x_2 = t$

$2x_1 + x_2 + x_3 = 0 \qquad x_1 = -t$

$$\therefore t \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \quad \therefore 1$$

5-5

1-4

$$A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 3 & 4 \\ 0 & 4 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 0 \\ 0 & 5 & 4 \\ 0 & 4 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ 행공간 기저 : $\langle [1,2,0],[0,1,1],[0,0,1] \rangle$

$$A^T = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 4 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 4 \\ 0 & 4 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 13 \end{bmatrix}$$

전치 $\to \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 4 & 13 \end{bmatrix}$ 열공간 기저

$\hookrightarrow \left\langle \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ 13 \end{bmatrix} \right\rangle$

$\therefore$ Rank(A) = 3

3-1 (문제 착각)

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \therefore \text{기저 } \langle [1\ 1\ 1] \rangle$$

4-3

$$\begin{bmatrix} 1 & 2 & 1 \\ 4 & 5 & 6 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 4 & 5 & 6 \\ 1 & -1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 1 \\ 4 & 5 & 6 \\ 0 & 3 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 3 & -2 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \quad \therefore x_3 = x_2 = x_1 = 0$$

$\therefore$ 기저 : $\emptyset$

4-2

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$z = t \quad x - t = 0 \quad x = t$

$2t + y + \cancel{z} = 0 \quad y = -3t$

$$t \begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix} \quad \therefore \text{기저 } \left(\begin{bmatrix} 1 \\ -3 \\ 1 \end{bmatrix} \right)$$

6-3

$$\begin{bmatrix} 0 & 1 & 2 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$x_3 = 1 \quad x_1 = -\frac{1}{3} \quad x_2 = -2$

$$\therefore \text{기저는 } \left\langle \begin{bmatrix} -\frac{1}{3} \\ -2 \\ 1 \end{bmatrix} \right\rangle$$

No. Date.

5-6

1-4

$$\begin{bmatrix} 2 & 5 \\ 3 & 6 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 2 & 5 \\ 3 & 6 \\ 4 & 7 \end{bmatrix} \to \begin{bmatrix} 2 & 5 \\ 1 & 1 \\ 4 & 7 \end{bmatrix} \to \begin{bmatrix} 2 & 5 \\ 1 & 1 \\ 0 & -3 \end{bmatrix} \to \begin{bmatrix} 0 & 3 \\ 1 & 1 \\ 0 & -3 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & 1 \\ 0 & 3 \\ 0 & -3 \end{bmatrix} \to \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \quad x_1 = x_2 = 0$$

∴ 기저 $\emptyset$, 영차원 $=0$

3-1

$$\begin{bmatrix} 2 & 1 & -3 & 1 \\ 3 & -1 & 1 & -1 \\ 5 & 1 & 1 & 9 \\ 10 & 1 & -1 & 9 \end{bmatrix} \xrightarrow{6 \cdot 5} \begin{bmatrix} 1 & 0 & 0 & \frac{8}{13} \\ 0 & 1 & 0 & \frac{57}{13} \\ 0 & 0 & 1 & \frac{20}{13} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow x_4 = t, \quad x_3 = -\frac{20}{13}t$

$x_2 = -\frac{57}{13}t \quad x_1 = -\frac{8}{13}t$

∴ 자명하지 않은 해 존재

4-3

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 1 \\ 1 & 1 & 1 & 1 & 2 \\ 2 & 3 & 4 & 5 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 1 & 1 & 2 \\ 2 & 3 & 4 & 5 & 1 \\ 0 & 1 & 2 & 3 & 3 \\ 2 & 3 & 4 & 5 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & 3 & 3 \\ 2 & 3 & 4 & 5 & 1 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 2 & 3 & -3 \\ 0 & 1 & 2 & 3 & 3 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -1 & -2 & -1 \\ 0 & 1 & 2 & 3 & 3 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

~~$x_4 = t,\ x_2 = s,\ x_5 = 0\ \ x_3 = \frac{-4t-5}{2}$~~

$x_5 = 0 \quad x_4 = t \quad x_3 = s \quad x_2 = -2s - 3t \quad x_1 = 2t + s$

$$\begin{bmatrix} 2t+s \\ -3t-2s \\ s \\ t \\ 0 \end{bmatrix} = t\begin{bmatrix} 2 \\ -3 \\ 0 \\ 1 \\ 0 \end{bmatrix} + s\begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

∴ 기저 $\left\langle \begin{bmatrix} 2 \\ -3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\rangle$

6-1

1-3

$\langle u, v \rangle = 1\times2 + 2(1\times5) + 1\times2 = 14$

$|u| = \langle u, u \rangle^{\frac{1}{2}} = \sqrt{1+2+1} = 2$

$|v| = \langle v, v \rangle^{\frac{1}{2}} = \sqrt{4+50+4} = \sqrt{58}$

$d(u,v) = |u-v| = |\langle -1\ -4\ -1 \rangle| = 34$

7-2

1. $\langle X, Y \rangle = \langle Y, X \rangle$ 성립

2. $\langle X+Y, w \rangle = \langle X, Y \rangle + \langle Y, w \rangle$

좌변 : $(x_1+y_1)^2 w_1^2 + (x_2+y_2)^2 w_2^2 + (x_3+y_3)^2 w_3^2$

우변 : $x_1^2 y_1^2 + x_2^2 y_2^2 + x_3^2 y_3^2 + y_1^2 w_1^2 + y_2^2 w_2^2 + y_3^2 w_3^2$

∴ 좌변 $\neq$ 우변 2번 성립 X

3. $\langle kX, Y \rangle = k\langle X, Y \rangle$

좌변 : $(kx_1)^2 y_1^2 + (kx_2)^2 y_2^2 + (kx_3)^2 y_3^2$

우변 : $kx_1^2 y_1^2 + kx_2^2 y_2^2 + kx_3^2 y_3^2$

∴ 좌변 $\neq$ 우변 3번 성립 X

4. $\langle X, X \rangle \geq 0$, $\langle X, X \rangle = 0 \Leftrightarrow X = 0$ 성립

7-4

1. $\langle X, Y \rangle = \langle Y, X \rangle$ 성립

2. $\langle X+Y, w \rangle = \langle X, Y \rangle + \langle Y, w \rangle$

$(x_1+y_1)w_1 - (x_2+y_2)w_2 + (x_3+y_3)w_3$

$= x_1 y_1 - x_2 y_2 + x_3 y_3 + y_1 w_1 - y_2 w_2 + y_3 w_3$

성립

3. $\langle kX, Y \rangle = k\langle X, Y \rangle$ 성립

4. $\langle X, X \rangle \geq 0$

$x_1^2 - x_2^2 + x_3^2$ if $x_2^2 > x_1^2 + x_3^2$ 부정

∴ 4번 성립 X

6-2

2-3

$v_1 = \langle 4\ -3\ 0\rangle,\ v_2 = \langle 1, 2, 0\rangle,\ v_3 = \langle 0\ 0\ 4\rangle$

$v_2 \cdot v_3 = 0 \quad v_1 \cdot v_3 = 0$ ∴ v_3 직교기저

$w_1 = v_1 = \langle 4\ -3\ 0\rangle$

$w_2 = v_2 - \frac{\langle w_1, v_2\rangle}{\langle w_1, w_1\rangle} w_1$

$= \langle 1\ 2\ 0\rangle - \frac{-2}{25}\langle 4\ -3\ 0\rangle = \langle \frac{33}{25}\ \frac{44}{25}\ 0\rangle$

∴ $u_1 = \frac{1}{5}\langle 4\ -3\ 0\rangle$

$u_2 = \frac{5}{11}\langle \frac{33}{25}\ \frac{44}{25}\ 0\rangle = \langle \frac{3}{5}\ \frac{4}{5}\ 0\rangle$

$u_3 = \frac{1}{16}\langle 0\ 0\ 4\rangle = \langle 0\ 0\ \frac{1}{4}\rangle$

∴ $\{\langle \frac{4}{5}\ -\frac{3}{5}\ 0\rangle, \langle \frac{3}{5}\ \frac{4}{5}\ 0\rangle, \langle 0\ 0\ \frac{1}{4}\rangle\}$

4-1

$$\begin{bmatrix} 2 & 2 & -6 & 4 \\ 1 & 2 & -3 & 4 \\ 1 & 1 & -3 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & -3 & 2 \\ 1 & 1 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & -3 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_3 = t \quad x_4 = s \quad x_2 = 0 \quad x_1 - 3t + 2s = 0$

$x_1 = 3t - 2s \quad \begin{bmatrix} 3t-2s \\ 0 \\ t \\ s \end{bmatrix} = t\begin{bmatrix} 3 \\ 0 \\ 1 \\ 0 \end{bmatrix} + s\begin{bmatrix} -2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

$w_1 = v_1 = \langle 3\ 0\ 1\ 0\rangle$

$w_2 = v_2 - \frac{\langle v_2\ w_1\rangle}{\langle w_1\ w_1\rangle} w_1 = \langle 0\ -2\ 0\ 1\rangle$

$u_1 = \langle \frac{3}{\sqrt{10}}, 0, \frac{1}{\sqrt{10}}, 0\rangle$

$u_2 = \langle 0, -\frac{2}{\sqrt{5}}, 0, \frac{1}{\sqrt{5}}\rangle$

6-3

2-2

$A^TAx = A^Tb$

$$\begin{bmatrix} 2 & -1 & 0 \\ 3 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}\begin{bmatrix} 2 & -1 & 0 \\ 3 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 0 \\ 3 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}\begin{bmatrix} -1 \\ 0 \\ 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 15 & -1 & 5 \\ -1 & 22 & 30 \\ 5 & 30 & 45 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 9 \\ 13 \end{bmatrix}$$

3-1

$A^TA = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -2 & 6 \end{bmatrix}$

$A^Tb = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} 7 \\ 0 \\ -7 \end{bmatrix} = \begin{bmatrix} 14 \\ -7 \end{bmatrix}$

$(A^TA)^{-1} = \frac{1}{14}\begin{bmatrix} 6 & 2 \\ 2 & 3 \end{bmatrix}$

$(A^TA)^{-1}A^Tb = \frac{1}{2}\begin{bmatrix} 6 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ \frac{1}{2} \end{bmatrix}$

$\hat{x}_1 = 5 \quad \hat{x}_2 = \frac{1}{2}$

$A\hat{x} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{11}{2} \\ -\frac{9}{2} \\ -4 \end{bmatrix}$

6.

$A = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 5 & 25 \\ 1 & 6 & 36 \end{bmatrix} \quad y = \begin{bmatrix} 0 \\ -10 \\ -48 \\ -76 \end{bmatrix}$

$A^TA = \begin{bmatrix} 4 & 16 & 74 \\ 16 & 74 & 376 \\ 74 & 376 & 2018 \end{bmatrix}$

$A^Tb = \begin{bmatrix} -134 \\ -726 \\ -4026 \end{bmatrix}$

$(A^TA)^{-1}A^Tb = \begin{bmatrix} 2 \\ 5 \\ -3 \end{bmatrix}$

∴ $2 + 5x - 3x^2$

3-3

$$A^T A = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ -1 & -2 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & -2 \\ 1 & 1 & 0 \\ 1 & 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 4 & -6 \\ 4 & 3 & -3 \\ -6 & -3 & 6 \end{bmatrix}$$

$$A^T b = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ -1 & -2 & 0 & -1 \end{bmatrix} \begin{bmatrix} 6 \\ 0 \\ 9 \\ 3 \end{bmatrix} = \begin{bmatrix} 18 \\ 12 \\ -9 \end{bmatrix}$$

$$(A^T A)^{-1} A^T b = \begin{bmatrix} 12 \\ -3 \\ 9 \end{bmatrix}$$ $\hat{x}_1 = 12$ $\hat{x}_2 = -3$ $\hat{x}_3 = 9$

$$A\hat{x} = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & -2 \\ 1 & 1 & 0 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 12 \\ -3 \\ 9 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 9 \\ 0 \end{bmatrix}$$