여인자

$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 5 \\ 6 & 0 & 1 \end{bmatrix}$ $A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 1 \\ 3 & 5 \end{vmatrix} = -2$

ex) $A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & -3 \\ 4 & 0 & 1 \end{bmatrix}$ det (A) ?

$(-1)^{3+1} \times 4 \begin{vmatrix} 2 & 1 \\ 1 & -3 \end{vmatrix} + (-1)^{3+3} \begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} = -28 - 1 = -29$

$$A^{-1} = \frac{1}{\det A} \underbrace{\begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ \vdots & \vdots & \vdots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nn} \end{bmatrix}^T}_{adj(A)} \Rightarrow \frac{\text{여인자 행렬}^T}{\det A}$$

$\therefore A^{-1} = \frac{1}{\det(A)} adj(A)$

ex) $A = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 6 & 3 \\ 2 & -4 & 0 \end{bmatrix}$ 수반행렬 이용 A^{-1}!

$A_{11} = (-1)^{1+1} \times 12 = 12$ $A_{12} = 6$ $A_{13} = -16$

$A_{21} = 4$ $A_{22} = 2$ $A_{23} = 16$ $A_{31} = 12$ $A_{32} = -10$ $A_{33} = 16$

$adj(A) = \begin{bmatrix} 12 & 4 & 12 \\ 6 & 2 & -10 \\ -16 & 16 & 16 \end{bmatrix}$ $\det(A) = 3 \times 12 + 2 \times 6 + 16 = 64$

$\therefore A^{-1} = \frac{1}{64} \begin{bmatrix} 12 & 4 & 12 \\ 6 & 2 & -10 \\ -16 & 16 & 16 \end{bmatrix}$

크래머 법칙

ex) $2x_1 + 3x_2 - x_3 = 2$
$x_1 + 2x_2 + x_3 = -1$
$2x_1 + x_2 - 6x_3 = 4$
$\Rightarrow A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 2 & 1 \\ 2 & 1 & -6 \end{bmatrix}$ $b = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$ $\det A = 1$

$x_1 = \frac{1}{\det A} \begin{vmatrix} 2 & 3 & -1 \\ -1 & 2 & 1 \\ 4 & 1 & -6 \end{vmatrix} = -23$ $x_2 = \frac{1}{\det A} \begin{vmatrix} 2 & 2 & -1 \\ 1 & -1 & 1 \\ 2 & 4 & 6 \end{vmatrix} = 14$ $x_3 = \frac{1}{\det A} \begin{vmatrix} 2 & 3 & 2 \\ 1 & 2 & -1 \\ 2 & 1 & 4 \end{vmatrix} = -6$

연습문제

1-1

$$5x - y = 9$$
$$3x - 3y + z = 20$$
$$x + y + z = 2$$

$A = \begin{bmatrix} 5 & -1 & 0 \\ 3 & -3 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ $b = \begin{bmatrix} 9 \\ 20 \\ 2 \end{bmatrix}$ $\det(A) = (-1)^{2+3} 6 + (-1)^{3+3}(-12) = -18$

$$x = -\frac{1}{18}\begin{vmatrix} 9 & -1 & 0 \\ 20 & -3 & 1 \\ 2 & 1 & 1 \end{vmatrix} \rightarrow \frac{-18}{-18} = 1$$

$$y = -\frac{1}{18}\begin{vmatrix} 5 & 9 & 0 \\ 3 & 20 & 1 \\ 1 & 2 & 1 \end{vmatrix} \quad \frac{-1+73}{-18} = -4$$

$$z = -\frac{1}{18}\begin{vmatrix} 5 & -1 & 9 \\ 3 & -3 & 20 \\ 1 & 1 & 2 \end{vmatrix} \quad \frac{-14-3-73}{-18} = 5$$

1-2

$$2x + y - z = 5$$
$$4x - 2y + 4z = 10$$
$$x - y + z = -6$$

$\Rightarrow A = \begin{bmatrix} 2 & 1 & -1 \\ 4 & -2 & 4 \\ 1 & -1 & 1 \end{bmatrix}$ $b = \begin{bmatrix} 5 \\ 10 \\ -6 \end{bmatrix}$ $\det(A) = (-1)^{3+1} \times 2 + (-1)^{3+2}(-1) \times 12 + (-1)^{3+3}(-8)$

$= 2 + 12 - 8 = 6$

$$x = \frac{1}{6}\begin{vmatrix} 5 & 1 & -1 \\ 10 & -2 & 4 \\ -6 & -1 & 1 \end{vmatrix} \Rightarrow \frac{(-1)^4(-1)(-22) + 4 \times (-1)^5 + (-1)^6(-20)}{6} = -\frac{1}{3}$$

$$y = \frac{1}{6}\begin{vmatrix} 2 & 5 & -1 \\ 4 & 10 & 4 \\ 1 & -6 & 1 \end{vmatrix} \Rightarrow \frac{(-1)^4(-1)(-34) + -4 \times (-17) + (-1) \cdot 1(-10)}{6} = 22$$

$$z = \frac{1}{6}\begin{vmatrix} 2 & 1 & 5 \\ 4 & -2 & 10 \\ 1 & -1 & -6 \end{vmatrix} \Rightarrow \frac{(-1)^3(-34) + (-3)(-1)^4 2 + -10}{6} = 3$$