벡터의 길이 : $\vec{a}=\overrightarrow{AB}$ 의 길이는 $|\vec{a}|$, $|\overrightarrow{AB}|$ 로 나타냄

ex) $A=(1,1)$, $B=(3,4)$ 일 때 $|\overrightarrow{AB}|$?

$\sqrt{(3-1)^2+(4-1)^2} = \sqrt{4+9} = \sqrt{13}$

ex) $A(1,3)$, $B(4,1)$, $C(7,5)$ 가 있다.

1) $\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}$ 가 영벡터가 되는 점 P 구하라

$P(x,y)$ 라 하고 $(1-x,3-y)+(4-x,1-y)+(7-x,5-y)=(0,0)$

$12-3x=0$, $9-3y=0$ $\therefore P(4,3)$

2) $\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}$ 의 크기가 3이 되는 점 P의 자취의 방정식을 구하라

$P(x,y)$ $\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}=(12-3x,\ 9-3y)$

$\therefore \sqrt{(12-3x)^2+(9-3y)^2} = 3^2$ $\therefore (x-4)^2+(y-3)^2=1$

벡터의 스칼라곱

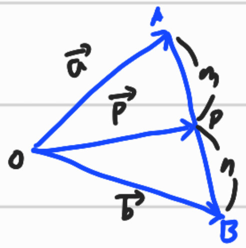
① $0\vec{a}=\vec{0}$, $1\vec{a}=\vec{a}$, $m\vec{0}=\vec{0}$

② 실수 m,n 에 대하여 $(m+n)\vec{a}=m\vec{a}+n\vec{a}$

③ $m(\vec{a}+\vec{b})=m\vec{a}+n\vec{b}$

④ $(mn)\vec{a}=m(n\vec{a})=n(m\vec{a})=mn\vec{a}$

ex) $\overrightarrow{OP}=\dfrac{m\overrightarrow{OB}+n\overrightarrow{OA}}{m+n}$ 증명하라 (선분 AB를 $m:n$으로 내분하는 점이 P)



$\overrightarrow{AP}=\dfrac{m}{m+n}\overrightarrow{AB}$

$\therefore \overrightarrow{OP}=\overrightarrow{OA}+\dfrac{m}{m+n}\overrightarrow{AB}=\overrightarrow{OA}+\dfrac{m}{m+n}(\overrightarrow{OB}-\overrightarrow{OA})=\dfrac{m\vec{b}+n\vec{a}}{m+n}$

위치 벡터 : 공간에 한 정점 O를 정하면 O를 시점으로 하는

점 A까지의 벡터

ex) $\vec{a}=(1,1)$, $\vec{b}=(-2,1)$, $\vec{c}=(-1,5)$ $x\vec{a}+y\vec{b}$ 꼴로 나타내라

$\vec{c}=x\vec{a}+y\vec{b}$ $(-1,5)=(x,x)+(-2y+y)$

$=(x-2y,x+y)$

$\begin{cases} x-2y=-1 \\ x+y=5 \end{cases}$ $-3y=-6$ $y=2$, $x=3$ $\therefore \vec{c}=3\vec{a}+2\vec{b}$

ex) $\vec{a}=(1,2)$ $\vec{b}=(x,1)$ 인 두 벡터 $\vec{a},\vec{b}$ 에대해 $\vec{a}+2\vec{b}$, $2\vec{a}-\vec{b}$ 평행

실 수 x는?

$\vec{a}+2\vec{b}=(1+2x,4)$ $2\vec{a}-\vec{b}=(2-x,3)$

$\vec{a}+2\vec{b}\,//\,2\vec{a}-\vec{b}$ $\Rightarrow$ $(1+2x,4)=m(2-x,3)$

$\Rightarrow$ $\begin{matrix} 1+2x=2m-mx \\ 4=3m \end{matrix}$ $\therefore m=\frac{4}{3}$, $x=\frac{1}{2}$

ex) $\vec{a}=(2,-3,1)$, $\vec{b}=(-1,2,1)$, $\vec{c}=(3,-2,4)$

1) $2\vec{a}+3\vec{b}=(4,-6,2)+(-3,6,3)=(1,0,5)$

2) $2(\vec{a}-\vec{b})-3(2\vec{a}+\vec{c})=-4\vec{a}-2\vec{b}-3\vec{c}=(-8,12,-4)+(2,-4,-2)+(-9,6,-12)$

$=(-15,14,-18)$

ex) $A(2,2,-1)$, $B(2,5,3)$ $\vec{a}=\vec{AB}$ 라고 할 때 $\vec{a}$ 의 크기는?

$\vec{AB}=\vec{OB}-\vec{OA}=(0,3,4)$ $\therefore |\vec{a}|=\sqrt{0+9+16}=5$

벡터를 기본 단위 벡터로 표현 : $a_1 i + a_2 j + a_3 k$

ex) $\vec{a} = 2i+3j+4k$ 를 성분으로 $\vec{a} = (2,3,4)$

$\vec{b} = 3k-4j+5i$ $\vec{b} = (5,-4,3)$

ex) $\vec{a} = -2i+j+3k$, $\vec{b} = i+3j+4k$ $2\vec{a}+\vec{b}$?

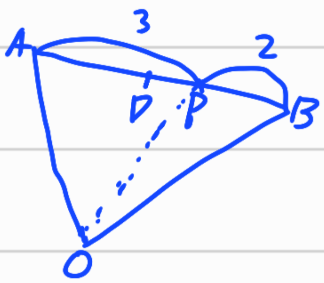
$\vec{a} = (-2,1,3)$ $\vec{b} = (1,3,4)$ $2\vec{a}+\vec{b} = (-3,5,10)$

연습문제

1-1 $2\vec{a}+3\vec{b}-4\vec{a}+\vec{b} = -2\vec{a}+4\vec{b}$ 2) $2\vec{a}-\vec{b}+3\vec{a}+2\vec{b} = 5\vec{a}+\vec{b}$

2. 선분 AB를 3:2로 내분하는 점 P, 외분하는점 Q, 중점 : D

각각의 위치 벡터를 $\vec{a}, \vec{b}, \vec{p}, \vec{q}, \vec{d}$ 로 할때 $\vec{p}, \vec{q}, \vec{d}$를 $\vec{a}, \vec{b}$로?



$\vec{AB} = \vec{b} - \vec{a}$ $\therefore \vec{p} = \frac{3\vec{b}+2\vec{a}}{3+2} = \frac{2\vec{a}+3\vec{b}}{5}$

$\vec{d} = \vec{OA} + \vec{AD} = \frac{\vec{a}}{2} + \frac{\vec{b}}{2}$

$\vec{AD} = \frac{\vec{b}-\vec{a}}{2}$

$\vec{q} = \frac{3\vec{b}-2\vec{a}}{3-2}$

3. $\vec{a} = (3,2)$, $\vec{b} = (-2,3)$ 일때 , $m\vec{a}+n\vec{b} = \vec{0}$ m, n ?

$3m-2n=0$
$2m+3n=0$ $m=\frac{2}{3}n$
$m=-\frac{3}{2}n$ $\frac{2}{3}n = -\frac{3}{2}n$ $\therefore m, n = 0$

4. $\vec{a} = (1,1)$, $\vec{b} = (1,-1)$ $\vec{a}, \vec{b}$로 나타내라

1) $\vec{p} = (2,3)$ $m+n=2$, $m-n=3$ $2m=5$ $m=\frac{5}{2}$ $n=-\frac{1}{2}$ $\vec{p} = \frac{5}{2}\vec{a} - \frac{1}{2}\vec{b}$

2) $\vec{q} = (-3,2)$ $m+n=-3$, $m-n=2$ $m=-\frac{1}{2}$ $n=\frac{3}{2}$ $\vec{q} = -\frac{1}{2}\vec{a} + \frac{3}{2}\vec{b}$

3) $\vec{r} = (-1,2)$ $m+n=-1$, $m-n=2$ $m=\frac{1}{2}$ $n=-\frac{3}{2}$ $\vec{r} = \frac{1}{2}\vec{a} - \frac{3}{2}\vec{b}$

5. $\vec{OA} = i - 2j$ $\vec{OB} = -4i + 2j$

1) $3\vec{OA} - 2\vec{OB} = 3(1,-2) - 2(-4,2) = (11,-10)$

2) $-\vec{OA} - 2\vec{OB} = -(1,-2) - 2(-4,2) = (7,-2)$

3) $2\vec{BA} = -\vec{b} + \vec{a} = (4,-2) + (-1,2) = (3,0)$

7. 시초점 P_1 과 종점 P_2를 가지는 벡터의 성분은?

1) $P_1(4,8)$, $P_2(3,7)$ $\Rightarrow P_2 - P_1 = (-1,-1)$

2) $P_1(3,-7,2)$, $P_2(-2,5,-4) \Rightarrow P_2 - P_1 = (-5,12,-6)$

8. $u = (-3,1,2)$, $v = (4,0,-8)$, $w = (6,-1,-4)$

1) $v - w = (4,0,-8) - (6,-1,-4) = (-2,1,-4)$

2) $-v + u = (-4,0,8) + (-3,1,2) = (-7,1,10)$

3) $-3(v - 8w) = -3((4,0,-8) + (16,-8+32)) = -3(20,-8,24) = (-60,24,-72)$

9. 시점 P_1 이고 종점 P_2 벡터 v의 길이 $|v|$ 구하라

1) $P_1(1,0,0)$, $P_2(4,2,0)$ $|(3,2,0)| = \sqrt{13}$

2) $P_1(3,-2,1)$, $P_2(1,2,-4)$ $v = (-2,4,-5)$ $|v| = \sqrt{4+16+25} = 3\sqrt{5}$

3) $P_1(8,6,1)$, $P_2(-8,6,1)$ $v = (-16,0,0)$ $|v| = 16$

10. 두 점의 길이 구해라

1) $(2,3)$, $(4,5)$ $\sqrt{4+4} = 2\sqrt{2}$ 2) $(-3,2)$, $(0,1)$ $\sqrt{3+1} = 2$

11. 길이 구하라

1) $(1,-1,2)$, $(3,0,2)$ $\sqrt{4+1+0} = \sqrt{5}$

2) $(-3,2)$, $(0,1)$ $\sqrt{9+1} = \sqrt{10}$

12. $u=(1,-3,2)$ $v=(1,1,0)$ $w=(2,2,-4)$

1) $|u+v|$ $\Rightarrow$ $u+v=(2,-2,2)$ $|2,-2,2|=2\sqrt{3}$

2) $|-2u|+2|v|$ $\Rightarrow$ $2|v-u|$ $2|(0,4,-2)|=2\times 2\sqrt{5}=4\sqrt{5}$

3) $\boxed{\frac{w}{|w|}}$ $\Rightarrow$ $\frac{(2,2,-4)}{\sqrt{4+4+16}}=\left(\frac{1}{\sqrt{6}},\frac{1}{\sqrt{6}},-\frac{2}{\sqrt{6}}\right)$

단위 벡터라고 함

13. $u=(1,2,3)$, $v=(2,-3,1)$, $w=(3,2,-1)$

1) $u-v$ $\Rightarrow$ $(-1,5,2)$

2) $2u-(v+w)$ $\Rightarrow$ $2u-v-w=(2,4,6)-(2,-3,1)-(3,2,-1)=(-3,5,6)$

3) $c_1u+c_2v+c_3w=(6,14,-2)$ 만족하는 c_1, c_2, c_3 ?

$(c_1, 2c_1, 3c_1)+(2c_2, -3c_2, c_2)+(3c_3\ 2c_3\ -c_3)$

$$\Rightarrow \begin{aligned} c_1+2c_2+3c_3&=6\\ 2c_1-3c_2+2c_3&=14\\ 3c_1+c_2-c_3&=-2 \end{aligned}$$

$2c_1+4c_2+6c_3=12$
$2c_1-3c_2+2c_3=14$

$3c_1+6c_2+9c_3=18$
$3c_1+c_2-c_3=-2$

$7c_2+4c_3=-2$
$7c_2+14c_3=28$
$-10c_3=-30$

$5c_2+10c_3=20$
$c_2+2c_3=4$

$c_3=3$ $c_2=-2$

$c_1=1$

$\therefore c_1=1$, $c_2=-2$ $c_3=3$

14. $P_1(1,1,2)$ $P_2(6,-7,3)$

1) P_1과 P_2 사이의 거리는? $\sqrt{25+64+1}=\sqrt{90}=3\sqrt{10}$

2) P_1이 시점 P_2가 종점인 벡터? $P_2-P_1=(5,-8,1)$

3) $u=(5,-8,1)$ 일때 $|3u|$? $9\sqrt{10}$

4) $\frac{u}{|u|}$ 길이 계산, $\frac{u}{|u|}$의 norm이 1인것을 증명 $\left|\frac{1}{3\sqrt{10}}(5,-8,1)\right|$ $\Rightarrow$ $\frac{3\sqrt{10}}{3\sqrt{10}}=1$

5) $|ku|=3$인 k? $3k\sqrt{10}=3$ $k\sqrt{10}=1$ $k=\pm\frac{\sqrt{10}}{10}$

6) $v=(1,1,1)$과 같은 방향 갖는 $\frac{v}{|v|}$ (단위벡터)는? $|v|=\sqrt{3}$ $\frac{v}{|v|}=\frac{1}{\sqrt{3}}(1,1,1)$

15. u- 방향의 단위벡터는?

1) $u = (1,2,1)$ $\frac{u}{|u|} = \frac{1}{\sqrt{6}}(1,2,1) = \left(\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$

2) $u = (0,-1,2,-1)$ $|u| = \sqrt{6}$ $\therefore \left(0, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}\right)$