

행렬식

$$2 \times 2 \begin{bmatrix} a & b \\ c & d \end{bmatrix} \rightarrow \underline{ad-bc} \quad \det(A) \text{ 라고 표현}$$

$$3 \times 3 \quad A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \rightarrow \det(A) = (-1)^{1+1} a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

ex 1. $\det \begin{vmatrix} 1 & 2 & 1 \\ -2 & 0 & 7 \\ 3 & 0 & 7 \end{vmatrix} ?$

$$(-1)^{1+2} 2 \begin{vmatrix} -2 & 7 \\ 3 & 7 \end{vmatrix} = 70$$

행렬식 성질

1. 한 행이나 한 열이 모두 0이면 $\det(A) = 0$ $A = \begin{bmatrix} 0 & 0 \\ 2 & 3 \end{bmatrix} \det(A) = 0$

2. 한 행렬에서 두개의 행이나 열이 교환되면 행렬식은 부호가 바뀐다 $A = \begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix} B = \begin{bmatrix} 5 & 6 \\ 2 & 3 \end{bmatrix} \det(A) = -3, \det(B) = 3$

3. 한 행이나 두개의 열 동일하면 행렬식은 0이 된다 $A = \begin{bmatrix} 5 & 6 \\ 5 & 6 \end{bmatrix} \det(A) = 0$

4. $\det B = a \det A$ $A = \begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix} B = \begin{bmatrix} 10 & 15 \\ 25 & 30 \end{bmatrix} \det(B) = 5 \det(A)$

5. 행(열)에 다른 행(열)에 상수배해서 더한 건 $\det B = \det A$ $A = \begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix} B = \begin{bmatrix} 2 & 3 \\ 11 & 16 \end{bmatrix} \det(A) = \det(B)$

6. $\det A = \det A^T$

7. 삼각 행렬의 행렬식은 대각원소들의 곱 $A = \begin{bmatrix} 3 & 5 & 8 \\ 0 & 2 & 7 \\ 0 & 0 & 5 \end{bmatrix} \det A = 3 \times 2 \times 5$

8. $\det(AB) = \det(A) \det(B)$

연습문제

$$1-1 \quad \begin{bmatrix} 3 & 0 & 2 \\ -1 & 5 & 0 \\ 1 & 9 & 6 \end{bmatrix} \quad (-1)^{11} \times 3 \begin{vmatrix} 5 & 0 \\ 9 & 6 \end{vmatrix} + (-1)^{13} \times 2 \begin{vmatrix} -1 & 5 \\ 1 & 9 \end{vmatrix}$$

$$\Rightarrow 3 \times 30 + -28 = 62$$

$$1-2 \quad \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 2 & 6 & 0 \end{bmatrix} \Rightarrow (-1)^{2+2} \begin{vmatrix} 1 & -1 \\ 2 & 0 \end{vmatrix} = 2$$

$$4. \det(A - xI) = 0 \text{ 이가 되는 } x \text{는? } A = \begin{bmatrix} 0 & -3 & 4 \\ 0 & 5 & 0 \\ 1 & -2 & 0 \end{bmatrix}$$

$$A - xI = \begin{bmatrix} -x & -3 & 4 \\ 0 & 5-x & 0 \\ 1 & -2 & -x \end{bmatrix} \rightarrow (-1)^{11} \times -x \begin{vmatrix} 5-x & 0 \\ -2 & -x \end{vmatrix} + (-1)^{31} \begin{vmatrix} -3 & 4 \\ 5-x & 0 \end{vmatrix}$$

$$-x(x^2 - 5x) + 20 - 4x = 0$$

$$-x^3 + 5x^2 + 20 - 4x = 0 \quad x^3 - 5x^2 + 4x - 20 = 0$$

$$\begin{array}{r|rrrr} & 1 & -5 & 4 & -20 \\ 5 & & 5 & 0 & 20 \\ \hline & 1 & 0 & 4 & 0 \end{array}$$

$$\Rightarrow (x-5)(x-2)(x+2) \quad \therefore x = 5 \text{ or } \pm 2$$