No. Date.

기말고사

1. $\begin{vmatrix} \lambda-2 & -2 & 2 \\ -2 & \lambda+1 & -4 \\ 2 & -4 & \lambda+1 \end{vmatrix} = 0$

$(\lambda-3)^2(\lambda+6)=0$

$\therefore D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

$\lambda=3 \quad \begin{bmatrix} 1 & -2 & 2 \\ -2 & 4 & -4 \\ 2 & -4 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$x_3=t \quad x_2=s \quad x_1-2s+2t=0$

$x_1 = 2s-2t$

$s\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \therefore \left\{\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}\right\}$

$\lambda=-6 \quad \begin{bmatrix} -8 & -2 & 2 \\ -2 & 5 & -4 \\ 2 & -4 & -5 \end{bmatrix} \to \begin{bmatrix} 4 & 1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$\to \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$

$P = \begin{bmatrix} 2 & -2 & 1 \\ 1 & 0 & -2 \\ 0 & 1 & 2 \end{bmatrix}$

2. $\begin{vmatrix} \lambda+1 & 0 & -1 \\ 0 & \lambda-2 & 0 \\ 0 & -3 & \lambda-1 \end{vmatrix} = (\lambda+1)(\lambda-2)(\lambda-1)$

$\lambda = -1$ or 1 or 2

$\lambda=-1 \quad \begin{bmatrix} 0 & 0 & -1 \\ 0 & -3 & 0 \\ 0 & -3 & -2 \end{bmatrix} \to \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right\}$

$\lambda=1 \quad \begin{bmatrix} 2 & 0 & -1 \\ 0 & -1 & 0 \\ 0 & -3 & 0 \end{bmatrix} \to \left\{\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}\right\}$

$\lambda=2 \quad \begin{bmatrix} 3 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} \to \left\{\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}\right\}$

$D = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 2 & 3 \end{bmatrix}$

$PD^{17}P^{-1} = A^{17}$

$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2^{17} \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{3}{2} & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} -1 & 1 & 2^{17} \\ 0 & 0 & 2^{17} \\ 0 & 2 & 3\cdot2^{17} \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{3}{2} & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} -1 & 2^{17}-2 & 1 \\ 0 & 2^{17} & 0 \\ 0 & -3+3\cdot2^{17} & 1 \end{bmatrix} = A^{17}$

3. $A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 3 & 1 \\ 2 & 4 & 1 \end{bmatrix}$

$\det(A) = 1$ 1대1 대응

$A^{-1} = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 6 & -2 & -3 \end{bmatrix}$

$T^{-1}(x_1, x_2, x_3)$

$= \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 6 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$(-x_1 + x_2, -x_1 + x_3, 6x_1 - 2x_2 - 3x_3)$

---

4. ~~A → 3×2 행렬~~

~~A → 2~~

~~ker(T) = 2~~

a) $A = \begin{bmatrix} 5 & -3 \\ 1 & 1 \\ 1 & -1 \end{bmatrix} \to \begin{bmatrix} 5 & -3 \\ \cancel{0} & 1 \\ 0 & 2 \end{bmatrix} \to \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$

$\to \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ $\therefore \ker(T) = \{0\}$ (a)

b) $\text{nullity}(T) = 0$

c) $\begin{bmatrix} 5 & 1 & 1 \\ -3 & 1 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & \frac{4}{5} & -\frac{1}{5} \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -\frac{1}{4} \end{bmatrix}$

$R(T) = \{(1\ 1\ 0), (0, 1, -\frac{1}{4})\}$

d) $\text{rank}(T) = 2$

5.

$\begin{bmatrix} 1 & 1 & -3 & 2 \\ 1 & 2 & -3 & 4 \\ 1 & 1 & -3 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & -3 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$x_4 = t$ $x_3 = s$ $x_2 = -2t$

$x_1 = 3s$

$\left\{ \begin{bmatrix} 3 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 0 \\ 1 \end{bmatrix} \right\}$

$\quad\ \ \| \qquad\quad \|$

$\quad\ v_1 \qquad\ \ v_2$

$w_1 = (3\ 0\ 1\ 0)$

$w_2 = (0\ -2\ 0\ 1) - \frac{0}{10} = 0$

$\therefore u_1 = (\frac{3}{\sqrt{10}}, 0, \frac{1}{\sqrt{10}}, 0)$

$u_2 = (0, -\frac{2}{\sqrt{5}}, 0, \frac{1}{\sqrt{5}})$

6-1

$y = -x$ 대칭 ± $T(x, y) = (-y, -x)$

a) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ b) ~~(-3,~~ $(4, 3)$

6-2

$\begin{bmatrix} \cos 330 & -\sin 330 \\ \cos 330 & \sin 330 \end{bmatrix}$

$\|$

$\begin{bmatrix} \cos 30 & \sin 30 \\ \cos 30 & -\sin 30 \end{bmatrix}$

$\|$

a) $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}$ ~~b) (5/2 ...)~~

b) $(\frac{3}{2}\sqrt{3} + 2, \frac{3}{2}\sqrt{3} - 2)$

No. Date.

7.

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\hookrightarrow [x\ y]\begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{vmatrix} \lambda-1 & -1 & 0 \\ -1 & \lambda-2 & -1 \\ 0 & -1 & \lambda-1 \end{vmatrix} = \lambda(\lambda-3)(\lambda-1)$$

$\lambda = 0, 3, 1$

$$\lambda=0 \quad \begin{bmatrix} -1 & -1 & 0 \\ -1 & -2 & -1 \\ 0 & -1 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$\lambda=1 \quad \begin{bmatrix} 0 & -1 & 0 \\ -1 & -1 & -1 \\ 0 & -1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$\lambda=3 \quad \begin{bmatrix} 2 & -1 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$\therefore$ $\lambda=0$ 일때 $\begin{bmatrix} \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \\ 1/\sqrt{3} \end{bmatrix}$ Min 값

$\lambda=3$ 일때 $\begin{bmatrix} \frac{1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{bmatrix}$

8.

$$[x\ y]\begin{bmatrix} 4 & -10 \\ -10 & 25 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + [-15\ -6]\begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$\begin{vmatrix} \lambda-4 & 10 \\ 10 & \lambda-25 \end{vmatrix} = \lambda(\lambda-29) = 0 \quad \lambda = 0 \text{ or } 29$$

$$\lambda=0 \quad \begin{bmatrix} -4 & 10 \\ 10 & -25 \end{bmatrix} \rightarrow \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\lambda=29 \quad \begin{bmatrix} 25 & 10 \\ 10 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} -2 \\ 5 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 29 \end{bmatrix} \quad P = \begin{bmatrix} 5 & -2 \\ 2 & 5 \end{bmatrix}$$

$$\therefore [x'\ y']\begin{bmatrix} 0 & 0 \\ 0 & 29 \end{bmatrix}\begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$+ [-15\ -6]\begin{bmatrix} 5 & -2 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} x' \\ y' \end{bmatrix} = 0$$

$29y'^2 - 87x' = 0$

$y'^2 = 3x'$ 포물선