

행렬곱

$$Q_1) \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \stackrel{\text{sol}}{=} 1 \times 4 + 2 \times 5 + 3 \times 6 = [32] \quad Q_2) \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

$$Q_3) \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{bmatrix} \stackrel{\text{sol}}{=} \begin{bmatrix} 6 & 12 & 18 & 24 \\ 15 & 30 & 45 & 60 \end{bmatrix}$$

(2x3) (3x4)

$$Q_4) AB \neq BA \text{ 증명 } A = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 6 & 3 \end{bmatrix} \quad BA = \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 2 & 3 \end{bmatrix} \quad \therefore AB \neq BA$$

행렬곱이 정의되어 있는 경우 성립

- 1) $A(B+C) = AB+AC$
- 2) $(A+B)C = AC+BC$
- 3) $(AB)C = A(BC)$
- 4) $(rA)C = r(AC) = A(rC)$

연립 1차 방정식

$$\Rightarrow \begin{cases} 5x - 3y + 2z = 14 \\ x + y - 4z = -7 \\ 7x \quad \quad - 3z = 1 \end{cases} \Rightarrow A = \begin{bmatrix} 5 & -3 & 2 \\ 1 & 1 & -4 \\ 7 & 0 & -3 \end{bmatrix} \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

행렬

$$b = \begin{bmatrix} 14 \\ -7 \\ 1 \end{bmatrix} \quad \therefore Ax = b$$

$$Q_1) A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \quad A^2? \quad \text{sol)} \quad AA = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$Q_2) A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \quad A^3? \quad A^n? \quad \text{sol)} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 27 \end{bmatrix} \quad A^n = \begin{bmatrix} 1^n & 0 & 0 \\ 0 & 2^n & 0 \\ 0 & 0 & 3^n \end{bmatrix}$$

연습문제

$$Q_{1-1}) \begin{bmatrix} -1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = [-2] \quad Q_{1-2}) \begin{bmatrix} -2 & 3 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix} = [-2+15+10] = [23]$$

$$Q_{1-3}) \begin{bmatrix} 0 & 2 & 1 & -2 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \end{bmatrix} = [-4+1] = [-3]$$

$$Q_{2-1}) \begin{bmatrix} 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 0 \\ -1 \end{bmatrix}$$

$$= [4] + [2] + [3] = [9]$$

$$Q_{2-2}) \begin{bmatrix} 1 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 1 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 6 \\ 2 \\ 0 \end{bmatrix}$$

$$= [4] + [1] + [-1] = [4]$$

$$Q_{3-1}) \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \quad \begin{matrix} x-y=2 \\ 2x-y=0 \end{matrix} \quad \therefore x=2, y=0$$

$$Q_{3-2}) \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \end{bmatrix} \quad \begin{matrix} 2x+3y=5 \\ -x+2y=-1 \end{matrix} \Rightarrow \begin{matrix} 2x+3y=5 \\ -2x+4y=-2 \end{matrix} \quad \begin{matrix} x = -\frac{13}{7} \\ y = \frac{3}{7} \end{matrix}$$

$$Q_{3-3}) \begin{bmatrix} 1 & 3 & -1 \\ 2 & 5 & 1 \\ 1 & -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix} \quad \begin{matrix} x+3y-z=1 \\ 2x+5y+z=5 \\ x-y+3z=2 \end{matrix} \Rightarrow \begin{matrix} 4y-4z=1 \rightarrow 4y=4z+1 \\ 3x+8y=6 \rightarrow 3x+8z=5 \\ 4x+8z=7 \rightarrow 4x+8z=7 \end{matrix} \Rightarrow \begin{matrix} y = \frac{1}{8} \\ x = -1, z = \frac{3}{8} \end{matrix}$$

$$Q_{3-4}) \begin{bmatrix} 1 & 1 & -3 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{cases} x+y-3z=2 \\ 2x+z=1 \end{cases} \quad z=t \text{ (자유변수)} \quad \begin{cases} 2x = -t+1 \\ x = -\frac{t+1}{2} \end{cases} \quad \begin{cases} y = \frac{t+1}{2} + 3t+2 \\ y = \frac{7}{2}t + \frac{5}{2} \end{cases}$$

$$Q_4) A = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 1 & 1 \\ -1 & -2 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 5 & 7 \\ 9 & 11 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad AB? \quad BA?$$

$$AB = \begin{bmatrix} 28 & 38 & 8 \\ 33 & 21 & 16 \\ -21 & -27 & -9 \end{bmatrix} \quad BA = \begin{bmatrix} 3 & 5 & 7 \\ 9 & 11 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 0 \\ 2 & 1 & 1 \\ -1 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 5 \\ 30 & 36 & 11 \\ -1 & -2 & 0 \end{bmatrix}$$

$$Q_5) C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad D = \begin{bmatrix} 5 & 6 & 7 \\ 8 & 9 & 10 \end{bmatrix} \quad DC \text{ 정의됨?}$$

$$C = 2 \times 2 \quad D = 2 \times 3 \quad \therefore DC \text{ 는 정의 } X$$

$$Q_6) A \neq 0, B \neq 0, AB=0 \quad 2 \times 2 \quad A, B?$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{pmatrix} \quad \begin{cases} ae = -bg & ce = -dg \\ af = -bh & cf = -dh \end{cases} \quad \begin{cases} e+h=0 \\ g+f=0 \\ a+d=0 \\ b+c=0 \end{cases} \quad \begin{matrix} A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \\ \Rightarrow B = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \end{matrix}$$

$$Q_7) \begin{cases} x+2y+3z=1 \\ 5x+y+6z=2 \end{cases} \Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 5 & 1 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad Q_{7-2}) \begin{cases} x_1 - 2x_2 + 3x_3 + 4x_4 = 1 \\ 2x_1 - x_3 + x_4 = 2 \\ 5x_1 + 6x_2 + 7x_3 + 8x_4 = 9 \end{cases} \Rightarrow \begin{bmatrix} 1 & -2 & 3 & 4 \\ 0 & 2 & -1 & 1 \\ 5 & 6 & 7 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix}$$

$$Q_{7-3}) \begin{cases} 2u+3v-5w=1 \\ u+v-7w=6 \end{cases} \Rightarrow \begin{bmatrix} 2 & 3 & -5 \\ 1 & 1 & -7 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \end{bmatrix}$$

$$Q_8) a_0, a_1, a_2, \dots \text{ 은 } k \geq 3 \text{ 일 때 } a_k \text{ 는 } a_{k-2} + a_{k-1} + a_{k-3} \text{ 의 합, 즉 } a_{k+3} = a_{k+2} + a_{k+1} + a_k$$

$$\begin{bmatrix} a_{k+3} \\ a_{k+2} \\ a_{k+1} \end{bmatrix} = A \begin{bmatrix} a_{k+2} \\ a_{k+1} \\ a_k \end{bmatrix} \quad \text{행렬 } A \text{ 는?}$$

$$3 \times 1 = [n \times m] [3 \times 1] \quad \therefore A = 3 \times 3 \text{ 행렬} \quad \begin{bmatrix} a_{k+2} + a_{k+1} + a_k \\ a_{k+2} \\ a_{k+1} \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} a_{k+2} \\ a_{k+1} \\ a_k \end{bmatrix}$$

$$\Rightarrow a \cdot a_{k+2} + b \cdot a_{k+1} + c \cdot a_k = a_{k+2} + a_{k+1} + a_k \quad a=b=c=1$$

$$\therefore A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{matrix} e=1 & d=f=0 \\ g=h=0 & i=1 \end{matrix}$$

$$Q_9) k \geq 1, a_{k+1} = a_k + 2a_{k-1} \quad \text{if } a_0 = 0, a_1 = 1, \quad \begin{bmatrix} a_{k+1} \\ a_k \end{bmatrix} = A \begin{bmatrix} a_k \\ a_{k-1} \end{bmatrix} \quad A?$$

$$\begin{bmatrix} 2 \times 1 \end{bmatrix} = [m \times n] [2 \times 1] \quad A = 2 \times 2 \text{ 행렬} \quad \begin{bmatrix} a_k + 2a_{k-1} \\ a_k \end{bmatrix} = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} \begin{bmatrix} a_k \\ a_{k-1} \end{bmatrix}$$

$$\Rightarrow x_{11} a_k + x_{12} a_{k-1} = a_k + 2a_{k-1} \quad x_{11}=1, x_{12}=2$$

$$\Rightarrow x_{21} a_k + x_{22} a_{k-1} = a_k \quad x_{21}=1, x_{22}=0$$

$$\therefore A = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

$$a_2 = a_1 + 2a_0 = 1$$

$$a_3 = a_2 + 2a_1 = 3$$

$$a_4 = a_3 + 2a_2 = 5$$

$$Q_{10}) a_{k+1} = a_k + a_{k-1} \quad \begin{bmatrix} a_{k+1} \\ a_k \end{bmatrix} = A \begin{bmatrix} a_k \\ a_{k-1} \end{bmatrix} \quad A?$$

$$\begin{bmatrix} a_k + a_{k-1} \\ a_k \end{bmatrix} = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} \begin{bmatrix} a_k \\ a_{k-1} \end{bmatrix}$$

$$\therefore A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$x_{11} a_k + x_{12} a_{k-1} = a_k + a_{k-1} \quad x_{11} = x_{12} = 1 = x_{21}, x_{22} = 0$$

$$Q_{11}) t \text{ 년도 인구: } A_t, B_t \quad t+1 \text{ 년도 인구: } 0.6A_t + 0.7B_t, 0.1A_t + 1.2B_t \quad t=0 \text{ 일 때, } A_0 = 50000, B_0 = 100000$$

$$\begin{bmatrix} A_{t+1} \\ B_{t+1} \end{bmatrix} = \begin{bmatrix} 0.6 & 0.7 \\ 0.1 & 1.2 \end{bmatrix} \begin{bmatrix} A_t \\ B_t \end{bmatrix}$$

$$1 \text{ 년 후 } \begin{bmatrix} A_1 \\ B_1 \end{bmatrix} = \begin{bmatrix} 0.6 & 0.7 \\ 0.1 & 1.2 \end{bmatrix} \begin{bmatrix} 50000 \\ 100000 \end{bmatrix} = \begin{bmatrix} 100000 \\ 125000 \end{bmatrix}$$

$$2 \text{ 년 후 } \begin{bmatrix} A_2 \\ B_2 \end{bmatrix} = \begin{bmatrix} 0.6 & 0.7 \\ 0.1 & 1.2 \end{bmatrix} \begin{bmatrix} 100000 \\ 125000 \end{bmatrix} = \begin{bmatrix} 147500 \\ 150000 \end{bmatrix}$$

$$\vdots$$

$$n \text{ 년 후 } \begin{bmatrix} A_n \\ B_n \end{bmatrix} = \begin{bmatrix} 0.6 & 0.7 \\ 0.1 & 1.2 \end{bmatrix} \begin{bmatrix} A_{n-1} \\ B_{n-1} \end{bmatrix} = \begin{bmatrix} 0.6 & 0.7 \\ 0.1 & 1.2 \end{bmatrix}^n \begin{bmatrix} A_0 \\ B_0 \end{bmatrix}$$

$$Q_{12}) i \text{ 년 말에 불고기 개체군 } S_i, F_i \quad k = A \text{ 여의 개수 } B \text{ 의 사망률}$$

$$\begin{cases} S_{i+1} = 0.7S_i + 0.4F_i \\ F_{i+1} = -kS_i + 1.2F_i \end{cases} \quad S_0 = 1000, F_0 = 50000 \quad \textcircled{1} k = 0.02 \text{ 일 때 } i=5,6 \Rightarrow S_i, K_i? \quad \textcircled{2} k = 0.2 \text{ 일 때 } i=5,6 \Rightarrow S_i, K_i?$$

$$\begin{bmatrix} S_{i+1} \\ F_{i+1} \end{bmatrix} = \begin{bmatrix} 0.7 & 0.4 \\ -k & 1.2 \end{bmatrix} \begin{bmatrix} S_i \\ F_i \end{bmatrix} \quad A_n = \begin{bmatrix} 0.7 & 0.4 \\ -k & 1.2 \end{bmatrix}^n \begin{bmatrix} 1000 \\ 50000 \end{bmatrix}$$

$$\textcircled{1} k = 0.02 \quad \begin{bmatrix} S_5 \\ F_5 \end{bmatrix} = \begin{bmatrix} 0.7 & 0.4 \\ -0.02 & 1.2 \end{bmatrix}^5 \begin{bmatrix} 1000 \\ 50000 \end{bmatrix} = \begin{bmatrix} 91460 \\ 119758 \end{bmatrix}$$

$$\begin{bmatrix} S_6 \\ F_6 \end{bmatrix} = \begin{bmatrix} 0.7 & 0.4 \\ -0.02 & 1.2 \end{bmatrix}^6 \begin{bmatrix} 1000 \\ 50000 \end{bmatrix} = \begin{bmatrix} 111925 \\ 141881 \end{bmatrix}$$

$$\textcircled{2} k = 0.2 \quad \begin{bmatrix} S_5 \\ F_5 \end{bmatrix} = \begin{bmatrix} 0.7 & 0.4 \\ -0.2 & 1.2 \end{bmatrix}^5 \begin{bmatrix} 1000 \\ 50000 \end{bmatrix} = \begin{bmatrix} 77944 \\ 79413 \end{bmatrix}$$

$$\begin{bmatrix} S_6 \\ F_6 \end{bmatrix} = \begin{bmatrix} 0.7 & 0.4 \\ -0.2 & 1.2 \end{bmatrix}^6 \begin{bmatrix} 1000 \\ 50000 \end{bmatrix} = \begin{bmatrix} 86326 \\ 79706 \end{bmatrix}$$

$$Q_{13}) A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix} \quad B = \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}$$

$$AB = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix} \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}$$