연습문제 1-1

3. $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 6 & -2 \\ 0 & 1 & 1 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

1) $A + 2B + 3C$

$= \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} -2 & 12 & -4 \\ 0 & 2 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 6 & 9 \\ 12 & 15 & 18 \end{bmatrix}$

$= \begin{bmatrix} 3 & 19 & 5 \\ 13 & 17 & 20 \end{bmatrix}$

2) $5A + 3B - C$

$= \begin{bmatrix} 10 & 5 & 0 \\ 5 & 0 & 0 \end{bmatrix} + \begin{bmatrix} -3 & 18 & -6 \\ 0 & 3 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

$= \begin{bmatrix} 6 & 21 & -9 \\ 1 & -2 & -3 \end{bmatrix}$

3) $B + C - A$

$= \begin{bmatrix} -1 & 6 & -2 \\ 0 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} - \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} -2 & 7 & 1 \\ 3 & 6 & 7 \end{bmatrix}$

연습문제 1-2

5. $C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $D = \begin{bmatrix} 5 & 6 & 7 \\ 8 & 9 & 10 \end{bmatrix}$ DC 정의가능?

⇒ 아니다. $C = 2\times2$ 행렬 $D = 2\times3$ 행렬로

CD 는 2×3 행렬로 정의가 가능하지만 DC는 불가능

8. $a_0, a_1, a_2 \cdots$ 는 $k \geq 3$ 일때

어떠한 수 a_k는 세수의 합

즉, $a_{k+3} = a_{k+2} + a_{k+1} + a_k$ 이다.

$\begin{bmatrix} a_{k+3} \\ a_{k+2} \\ a_{k+1} \end{bmatrix} = A \begin{bmatrix} a_{k+2} \\ a_{k+1} \\ a_k \end{bmatrix}$ 행렬 A?

3×1 행렬 $= [n \times m][3\times1]$ ∴ $A = 3\times3$ 행렬

$\begin{bmatrix} a_{k+2}+a_{k+1}+a_k \\ a_{k+2} \\ a_k \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} a_{k+2} \\ a_{k+1} \\ a_k \end{bmatrix}$

⇒ $a \cdot a_{k+2} + b a_{k+1} + c a_k = a_{k+2} + a_{k+1} + a_k$

$a=b=c=1$ $d=f=0$ $e=1$ $g=h=0$, $i=1$

∴ $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

11. (문제 생각)

$\begin{bmatrix} A_{t+1} \\ B_{t+1} \end{bmatrix} = \begin{bmatrix} 0.6 & 0.7 \\ 0.1 & 1.2 \end{bmatrix} \begin{bmatrix} A_t \\ B_t \end{bmatrix}$

1년후: $\begin{bmatrix} A_1 \\ B_1 \end{bmatrix} = \begin{bmatrix} 0.6 & 0.7 \\ 0.1 & 0.2 \end{bmatrix} \begin{bmatrix} 50000 \\ 100000 \end{bmatrix} = \begin{bmatrix} 100000 \\ 125000 \end{bmatrix}$

2년후: $\begin{bmatrix} A_2 \\ B_2 \end{bmatrix} = \begin{bmatrix} 0.6 & 0.7 \\ 0.1 & 0.2 \end{bmatrix} \begin{bmatrix} 100000 \\ 125000 \end{bmatrix} = \begin{bmatrix} 147500 \\ 150000 \end{bmatrix}$

⋮

n년후 $\begin{bmatrix} A_n \\ B_n \end{bmatrix} = \begin{bmatrix} 0.6 & 0.7 \\ 0.1 & 0.2 \end{bmatrix} \begin{bmatrix} A_{n-1} \\ B_{n-1} \end{bmatrix}$

$= \begin{bmatrix} 0.6 & 0.7 \\ 0.1 & 0.2 \end{bmatrix}^n \begin{bmatrix} A_0 \\ B_0 \end{bmatrix}$

14번 n×n 행렬 $A=(a_{ij})$ $tr(A)=\sum_{i=1}^{n} a_{ij}$

1) $tr(A+B) = tr(A)+tr(B)$

$A=\begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}$ $tr(A)=x_1+x_4$, $tr(B)=y_1+y_4$

$B=\begin{bmatrix} y_1 & y_2 \\ y_3 & y_4 \end{bmatrix}$ $A+B=\begin{bmatrix} x_1+y_1 & x_2+y_2 \\ x_3+y_3 & x_4+y_4 \end{bmatrix}$

$\therefore tr(A+B)=tr(A)+tr(B) = x_1+y_1+x_4+y_4$

2) $tr(AB) = tr(BA)$

$AB = \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}\begin{bmatrix} y_1 & y_2 \\ y_3 & y_4 \end{bmatrix} = \cancel{x_1+y_1}$

$= \begin{bmatrix} x_1y_1+x_2y_3 & x_1y_2+x_2y_4 \\ x_3y_1+x_4y_3 & x_3y_2+x_4y_4 \end{bmatrix}$

$= tr(AB) = x_1y_1+x_2y_3+x_3y_2+x_4y_4$

$BA = \begin{bmatrix} x_1y_1+x_3y_2 & x_2y_1+x_4y_2 \\ x_1y_3+x_2y_4 & x_2y_3+x_4y_4 \end{bmatrix}$

$= tr(BA) = x_1y_1+x_3y_2+x_2y_3+x_4y_4$

$\therefore tr(AB) = tr(BA)$

연습문제 1-3

5. $x=\begin{bmatrix} x \\ y \end{bmatrix}$ $x^TAx = 5x^2+6xy+y^2$

$A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $a=5,\ d=1,\ b=3,\ c=3$ $\therefore A=\begin{bmatrix} 5 & 3 \\ 3 & 1 \end{bmatrix}$

6. $x=\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$ $x^TAx = ax^2+2hxy+by^2+2gx+2fy+c$

$\therefore A=\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$

연습문제 2-1

2.3 $x_1+x_2-5x_3=3$

$x_1 \quad -2x_3=5$

$2x_1-x_2-x_3=0$

$\left[\begin{array}{ccc:c} 1 & 1 & -5 & 3 \\ 1 & 0 & -2 & 5 \\ 2 & -1 & -1 & 0 \end{array}\right]$

$R_1-R_2 \to R_2$ $\begin{bmatrix} 1 & 1 & -5 & 3 \\ 0 & 1 & -3 & -2 \\ 2 & -1 & -1 & 0 \end{bmatrix}$

$2R_1-R_3 \to R_3$ $\begin{bmatrix} 1 & 1 & -5 & 3 \\ 0 & 1 & -3 & -2 \\ 0 & 3 & -9 & 6 \end{bmatrix}$

$-3R_2+R_3 \to R_3$ $\begin{bmatrix} 1 & 1 & -5 & 3 \\ 0 & 1 & -3 & -2 \\ 0 & 0 & 0 & 12 \end{bmatrix}$

∴ 해가 없다.

2.4 $3x+3y+12z=6$

$x+y+4z=2$

$-x+2y+8z=4$

$2x+5y+20z=10$

$\left[\begin{array}{ccc:c} 3 & 3 & 12 & 6 \\ 1 & 1 & 4 & 2 \\ -1 & 2 & 8 & 4 \\ 2 & 5 & 20 & 10 \end{array}\right]$

$\frac{1}{3}R_1 \to R_1$ $\begin{bmatrix} 1 & 1 & 4 & 2 \\ 1 & 1 & 4 & 2 \\ -1 & 2 & 8 & 4 \\ 2 & 5 & 20 & 10 \end{bmatrix}$

$R_1-R_2 \to R_2$ $\begin{bmatrix} 1 & 1 & 4 & 2 \\ 0 & 0 & 0 & 0 \\ -1 & 2 & 8 & 4 \\ 2 & 5 & 20 & 10 \end{bmatrix}$

$R_1+R_3 \to R_3$ $\begin{bmatrix} 1 & 1 & 4 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 3 & 12 & 6 \\ 2 & 5 & 20 & 10 \end{bmatrix}$

$2R_1-R_4 \to R_4$ $\begin{bmatrix} 1 & 1 & 4 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 3 & 12 & 6 \\ 0 & -3 & -12 & -6 \end{bmatrix}$

$R_3+R_4 \to R_4$ $\begin{bmatrix} 1 & 1 & 4 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 3 & 12 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\therefore x=0 \quad y=2-4t \quad z=t$

3-1 $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$x_3 = t \quad x_1 = 0 \quad x_2 = -t$

$\therefore (0, -t, t)$

3-2 $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $x_1 + x_4 = 0$, $x_3 = 0$

$x_2 = s \quad x_4 = t \quad x_1 = -t \quad x_3 = 0$

$\therefore (-t, s, 0, t)$

연습문제 2-2

(1) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 5 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} -4 & 2 \\ 5 & 3 \end{bmatrix}$

$-2R_1 \to R_1 \quad \underbrace{\begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}}_{E}\begin{bmatrix} 2 & -1 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} -4 & 2 \\ 5 & 3 \end{bmatrix}$

(2) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 3 \\ -2 & 4 & 5 \\ 3 & 1 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 4 \\ -2 & 4 & 5 \end{bmatrix}$

$R_2 \leftrightarrow R_3 \quad \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}}_{E}\begin{bmatrix} 2 & 1 & 3 \\ -2 & 4 & 5 \\ 3 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 4 \\ -2 & 4 & 5 \end{bmatrix}$

(3) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 4 & -2 & 3 \\ 1 & 0 & 2 \\ -2 & 3 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 4 & -2 & 3 \\ 1 & 0 & 2 \\ 0 & 3 & 5 \end{bmatrix}$

$2R_2 + R_3 \to R_3 \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}\begin{bmatrix} 4 & -2 & 3 \\ 1 & 0 & 2 \\ -2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 3 \\ 1 & 0 & 2 \\ 0 & 3 & 5 \end{bmatrix}$

$\therefore E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$

4-1 $E = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 \\ 0 & 0 & 0 & \lambda_4 \end{bmatrix}$

$\frac{1}{\lambda_1}R_1 \to R_1$

$\frac{1}{\lambda_2}R_2 \to R_2$

$\frac{1}{\lambda_3}R_3 \to R_3$

$\frac{1}{\lambda_4}R_4 \to R_4$

$E^{-1} = \begin{bmatrix} \frac{1}{\lambda_1} & 0 & 0 & 0 \\ 0 & \frac{1}{\lambda_2} & 0 & 0 \\ 0 & 0 & \frac{1}{\lambda_3} & 0 \\ 0 & 0 & 0 & \frac{1}{\lambda_4} \end{bmatrix}$

4-2

$E = \begin{bmatrix} 0 & 0 & 0 & \lambda_1 \\ 0 & 0 & \lambda_2 & 0 \\ 0 & \lambda_3 & 0 & 0 \\ \lambda_4 & 0 & 0 & 0 \end{bmatrix}$

$\frac{1}{\lambda_1}R_4 \to R_1$

$\frac{1}{\lambda_2}R_3 \to R_2$

$\frac{1}{\lambda_3}R_2 \to R_3$

$\frac{1}{\lambda_4}R_1 \to R_4$

$\therefore E^{-1} = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{\lambda_1} \\ 0 & 0 & \frac{1}{\lambda_2} & 0 \\ 0 & \frac{1}{\lambda_3} & 0 & 0 \\ \frac{1}{\lambda_4} & 0 & 0 & 0 \end{bmatrix}$

연습문제 2-3

5. $\begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix}$

$\sin^2\theta + \cos^2\theta = 1$ 이므로

역 행렬이 존재하고 정칙 행렬이다.

$\frac{1}{\sin\theta}R_1 \to R_1 \quad \left[\begin{array}{cc|cc} 1 & \frac{\cos\theta}{\sin\theta} & \frac{1}{\sin\theta} & 0 \\ -\cos\theta & \sin\theta & 0 & 1 \end{array}\right]$

$\cos\theta R_1 + R_2 \to R_2 \quad \left[\begin{array}{cc|cc} 1 & \frac{\cos\theta}{\sin\theta} & \frac{1}{\sin\theta} & 0 \\ 0 & \frac{\cos^2\theta}{\sin\theta} + \sin\theta & \frac{\cos\theta}{\sin\theta} & 1 \end{array}\right]$

이때, $\frac{\cos^2\theta}{\sin\theta} + \frac{\sin^2\theta}{\sin\theta} = \frac{1}{\sin\theta}$

$\left[\begin{array}{cc|cc} 1 & \frac{\cos\theta}{\sin\theta} & \frac{1}{\sin\theta} & 0 \\ 0 & \frac{1}{\sin\theta} & \frac{\cos\theta}{\sin\theta} & 1 \end{array}\right]$

$\sin\theta R_2 \to R_2 \quad \left[\begin{array}{cc|cc} 1 & \frac{\cos\theta}{\sin\theta} & \frac{1}{\sin\theta} & 0 \\ 0 & 1 & \cos\theta & 1 \end{array}\right]$

$R_1 - \frac{\cos\theta}{\sin\theta}R_2 \to R_1 \quad \left[\begin{array}{cc|cc} 1 & 0 & \frac{1-\cos^2\theta}{\sin\theta} & -\frac{\cos\theta}{\sin\theta}\cdot\sin\theta \\ 0 & 1 & \cos\theta & 1 \end{array}\right]$

$\therefore A^{-1} = \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

10-3 $x+2y-4z=3$

$x-2y+3z=-1$

$2x+3y-z=5$

$$\begin{pmatrix} 1 & 2 & -4 \\ 1 & -2 & 3 \\ 2 & 3 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 2 & -4 \\ 1 & -2 & 3 \\ 2 & 3 & -1 \end{pmatrix}^{-1}\begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix}$$

$$\left(\begin{array}{ccc:ccc} 1 & 2 & -4 & 1 & 0 & 0 \\ 1 & -2 & 3 & 0 & 1 & 0 \\ 2 & 3 & -1 & 0 & 0 & 1 \end{array}\right)$$

$R_1 - R_2 \to R_2$ $\left(\begin{array}{ccc:ccc} 1 & 2 & -4 & 1 & 0 & 0 \\ 0 & 4 & -7 & 1 & -1 & 0 \\ 2 & 3 & -1 & 0 & 0 & 1 \end{array}\right)$

$2R_1 - R_3 \to R_3$ $\left(\begin{array}{ccc:ccc} 1 & 2 & -4 & 1 & 0 & 0 \\ 0 & 4 & -7 & 1 & -1 & 0 \\ 0 & 1 & -7 & 2 & 0 & -1 \end{array}\right)$

$\frac{1}{4}R_2 - R_3 \to R_3$ $\left(\begin{array}{ccc:ccc} 1 & 2 & -4 & 1 & 0 & 0 \\ 0 & 4 & -7 & 1 & -1 & 0 \\ 0 & 0 & \frac{11}{2} & -\frac{7}{4} & -\frac{1}{4} & 1 \end{array}\right)$

$\frac{2}{11}R_3 \to R_3$ $\left(\begin{array}{ccc:ccc} 1 & 2 & -4 & 1 & 0 & 0 \\ 0 & 4 & -7 & 1 & -1 & 0 \\ 0 & 0 & 1 & -\frac{7}{22} & -\frac{1}{22} & \frac{2}{11} \end{array}\right)$

$R_2 - 2R_1 \to R_1$ $\left(\begin{array}{ccc:ccc} -2 & 0 & 1 & -1 & -1 & 0 \\ 0 & 4 & -7 & 1 & -1 & 0 \\ 0 & 0 & 1 & -\frac{7}{22} & -\frac{1}{22} & \frac{2}{11} \end{array}\right)$

$\frac{1}{4}R_2 \to R_2$ $\left(\begin{array}{ccc:ccc} -2 & 0 & 1 & -1 & -1 & 0 \\ 0 & 1 & -\frac{7}{4} & \frac{1}{4} & -\frac{1}{4} & 0 \\ 0 & 0 & 1 & -\frac{7}{22} & -\frac{1}{22} & \frac{2}{11} \end{array}\right)$

$R_3 - R_1 \to R_1$ $\left(\begin{array}{ccc:ccc} 2 & 0 & 0 & \frac{15}{22} & \frac{21}{22} & \frac{2}{11} \\ 0 & 1 & -\frac{7}{4} & \frac{1}{4} & -\frac{1}{4} & 0 \\ 0 & 0 & 1 & -\frac{7}{22} & -\frac{1}{22} & \frac{2}{11} \end{array}\right)$

$\frac{7}{4}R_3 + R_2 \to R_2$ $\left(\begin{array}{ccc:ccc} 2 & 0 & 0 & \frac{15}{22} & \frac{21}{22} & \frac{2}{11} \\ 0 & 1 & 0 & -\frac{27}{88} & -\frac{29}{88} & \frac{7}{22} \\ 0 & 0 & 1 & -\frac{7}{22} & -\frac{1}{22} & \frac{2}{11} \end{array}\right)$

$\frac{1}{2}R_1 \to R_1$ $\left(\begin{array}{ccc:ccc} 1 & 0 & 0 & \frac{15}{44} & \frac{21}{44} & \frac{1}{11} \\ 0 & 1 & 0 & -\frac{27}{88} & -\frac{29}{88} & \frac{7}{22} \\ 0 & 0 & 1 & -\frac{7}{22} & -\frac{1}{22} & \frac{2}{11} \end{array}\right)$

$$\begin{pmatrix} 1 & 2 & -4 \\ 1 & -2 & 3 \\ 2 & 3 & -1 \end{pmatrix}^{-1} = \frac{1}{11}\begin{pmatrix} \frac{15}{4} & \frac{21}{4} & 1 \\ -\frac{27}{8} & -\frac{29}{8} & \frac{7}{2} \\ -\frac{7}{2} & -\frac{1}{2} & 2 \end{pmatrix} = A^{-1}$$

$$A^{-1}\begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$x = \frac{45-21+20}{4} \times \frac{1}{11} = 1$

$y = \frac{1}{11} \times \frac{-81+29+140}{8} = 1$

$z = \frac{1}{11}\left(-\frac{21}{2} + \frac{1}{2} + 10\right) = 0$

$\therefore x=1 \quad y=1 \quad z=0$

연습문제 2-4

4-1 $I_1 + I_3 = I_2$ ... 키르히호프 1법칙

$R_1 I_1 + R_2 I_2 = I_1 + 2I_2 = 5$

$R_2 I_2 + R_3 I_3 = 2I_2 + 4I_3 = 8$ } 2법칙

$$\begin{pmatrix} 1 & -1 & 1 & 0 \\ 1 & 2 & 0 & 5 \\ 0 & 2 & 4 & 8 \end{pmatrix}$$

$R_1 - R_2 \to R_2$ $\begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & -3 & 1 & -5 \\ 0 & 2 & 4 & 8 \end{pmatrix}$

$\frac{1}{2}R_3 \to R_3$ $\begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & -3 & 1 & -5 \\ 0 & 1 & 2 & 4 \end{pmatrix}$

$\frac{1}{3}R_2 + R_3 \to R_3$ $\begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & -3 & 1 & -5 \\ 0 & 0 & \frac{7}{3} & \frac{7}{3} \end{pmatrix}$

$\therefore I_3 = 1 \qquad I_1 - I_2 = -1$

$-3I_2 = -6 \qquad I_2 = 2, I_1 = 1$

$\therefore I_1 = 1, \ I_2 = 2 \ \ I_3 = 1$

4-2 $I_1 + I_3 = I_2$

$I_1 + 2I_2 = 2$ $\Rightarrow$ ~~1 -1 1 0~~ ~~1 -3 1 -5~~

$2I_2 + 4I_3 = 6$

$$\Rightarrow \begin{pmatrix} 1 & -1 & 1 & 0 \\ 1 & 2 & 0 & 2 \\ 0 & 2 & 4 & 6 \end{pmatrix}$$

$R_1 - R_2 \to R_2 \quad \begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & -3 & 1 & -2 \\ 0 & 2 & 4 & 6 \end{pmatrix}$

$\frac{1}{2}R_3 \to R_3 \quad \begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & -3 & 1 & -2 \\ 0 & 1 & 2 & 3 \end{pmatrix}$

$\frac{1}{3}R_2 + R_3 \to R_3 \quad \begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & -3 & 1 & -2 \\ 0 & 0 & \frac{7}{3} & \frac{7}{3} \end{pmatrix}$

$I_3 = 1 \qquad -3I_2 + 1 = -2 \qquad I_2 = 1$

$I_1 - 1 + 1 = 0 \qquad I_1 = 0$

$\therefore I_1 = 0, \ I_2 = 1, \ I_3 = 1$

~~5-4~~ $\cancel{X = \begin{pmatrix} 1 & -5 \\ 1 & 10 \\ 1 & 8 \\ 1 & 3 \\ 1 & 6 \end{pmatrix} \quad Y = \begin{pmatrix} 10 \\ 8 \\ 6 \\ 4 \\ 5 \end{pmatrix}}$

$\cancel{X^TX = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ -5 & 10 & 8 & 3 & 6 \end{pmatrix} X = \begin{pmatrix} 5 & 22 \\ 22 & 234 \end{pmatrix}}$

$\cancel{X^TY = \begin{bmatrix} 33 \\ 120 \end{bmatrix}}$

$\cancel{\hat{b} = (X^TX)^{-1}X^TY = \begin{pmatrix} 5 & 22 \\ 22 & 234 \end{pmatrix}^{-1} \begin{pmatrix} 33 \\ 120 \end{pmatrix}}$

5-4 $\quad X = \begin{bmatrix} 1 & -5 \\ 1 & -1 \\ 1 & 3 \\ 1 & 7 \\ 1 & 5 \end{bmatrix} \quad Y = \begin{bmatrix} 10 \\ 8 \\ 6 \\ 4 \\ 5 \end{bmatrix}$

$X^TX = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -5 & -1 & 3 & 7 & 5 \end{bmatrix} X = \begin{bmatrix} 5 & 9 \\ 9 & 109 \end{bmatrix}$

$X^TY = \begin{bmatrix} 33 \\ 13 \end{bmatrix}$

$\hat{b} = (X^TX)^{-1}X^TY = \begin{bmatrix} 5 & 9 \\ 9 & 109 \end{bmatrix}^{-1} \begin{bmatrix} 33 \\ 13 \end{bmatrix}$

$\begin{bmatrix} 5 & 9 \\ 9 & 109 \end{bmatrix}^{-1} = \frac{1}{464} \begin{bmatrix} 109 & -9 \\ -9 & 5 \end{bmatrix}$

$\frac{1}{464} \begin{bmatrix} 109 & -9 \\ -9 & 5 \end{bmatrix} \begin{bmatrix} 33 \\ 13 \end{bmatrix} = \begin{bmatrix} 7.5 \\ -0.5 \end{bmatrix}$

$\therefore y = -0.5x + 7.5$

6(1) $(1, 450), \ (1.25, 375), \ (1.5, 330)$

$\hat{b} = (X^TX)^{-1}X^TY$

$X = \begin{pmatrix} 1 & 1 \\ 1 & 1.25 \\ 1 & 1.5 \end{pmatrix} \quad Y = \begin{pmatrix} 450 \\ 375 \\ 330 \end{pmatrix}$

$X^TX = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1.25 & 1.5 \end{pmatrix} X = \begin{pmatrix} 3 & 3.75 \\ 3.75 & 4.8125 \end{pmatrix}$

$X^TY = \begin{pmatrix} 1155 \\ 1413.75 \end{pmatrix}$

$\cancel{\begin{pmatrix} 3 & 3.75 \\ 3.75 & 4.8125 \end{pmatrix}^{-1} = -\frac{2}{3} \begin{pmatrix} \frac{67}{16} & -3.75 \\ -3.75 & 3 \end{pmatrix}}$

$\cancel{-\frac{2}{3} \begin{pmatrix} \frac{67}{16} & -3.75 \\ -3.75 & 3 \end{pmatrix} \begin{pmatrix} 1155 \\ 1413.75 \end{pmatrix} = -\frac{2}{3} \begin{bmatrix} -465 \\ -90 \end{bmatrix}}$

$\cancel{= \begin{bmatrix} 310 \\ 60 \end{bmatrix}}$

$\cancel{\therefore y =}$

$(X^TX)^{-1} = \frac{1}{0.375} \begin{pmatrix} 4.8125 & -3.75 \\ -3.75 & 3 \end{pmatrix}$

$\frac{1}{0.375} \begin{pmatrix} 4.8125 & -3.75 \\ -3.75 & 3 \end{pmatrix} \begin{pmatrix} 1155 \\ 1413.75 \end{pmatrix}$

$= \begin{bmatrix} 685 \\ -240 \end{bmatrix}$

$\therefore y = -240x + 685$

6-2 $\quad x = 1.4$

$y = -240 \times 1.4 + 685$

$y = 349$