

$$1. \langle u, v \rangle = \langle v, u \rangle$$

$$2. \langle u+v, w \rangle = \langle u, w \rangle + \langle v, w \rangle$$

$$3. \langle ku, v \rangle = \langle u, kv \rangle = k \langle u, v \rangle$$

$$4. \langle u, u \rangle \geq 0 \quad \langle u, u \rangle = 0 \Leftrightarrow u = 0$$

ex) $u = (u_1, u_2), v = (v_1, v_2)$ \mathbb{R}^2 내 벡터 $\langle u, v \rangle = 3u_1v_1 + 2u_2v_2$ 내적공간?

$$1. \langle v, u \rangle \text{ 성립} \quad 2. \langle u+v, w \rangle = \langle u, w \rangle + \langle v, w \rangle$$

$$3(v_1+u_1)w_1 + 2(u_2+v_2)w_2 = (3u_1w_1 + 2u_2w_2) + (3v_1w_1 + 2v_2w_2)$$

$$\text{내적공간 두 벡터 } u \text{ 와 } v \text{ 직교 : } |u+v|^2 = |u|^2 + |v|^2$$

$$v \text{ 가 내적공간이면 } d(u, v) = |u-v| \text{ 로 정의}$$

$$\Rightarrow |u-v| = |u+v|$$

연습문제

$$1.4 \quad u = \langle 1, 1, 1 \rangle, v = \langle 2, 5, 2 \rangle \quad \langle u, v \rangle = u_1v_1 + 2u_2v_2 + u_3v_3$$

$$\langle u, v \rangle = 1 \times 2 + 2(1 \times 5) + 1 \times 2 = 14$$

$$|u| = \langle u, u \rangle^{\frac{1}{2}} = \sqrt{1+2+1} = 2$$

$$|v| = \langle v, v \rangle^{\frac{1}{2}} = \sqrt{4+50+4} = \sqrt{58}$$

$$d(u, v) = |u-v| = |\langle 1-2, 1-5, 1-2 \rangle| = \sqrt{1+32+1} = \sqrt{34}$$

7-2

$$\langle x, y \rangle = x_1^2 y_1^2 + x_2^2 y_2^2 + x_3^2 y_3^2$$

$$1. \langle x, y \rangle = \langle y, x \rangle \text{ 성립}$$

$$2. \langle x+y, w \rangle = \langle x, y \rangle + \langle y, w \rangle$$

$$z_1 = (x_1+y_1)^2 w_1^2 + (x_1+y_1)^2 w_2^2 + (x_1+y_1)^2 w_3^2$$

$$= x_1^2 y_1^2 + x_2^2 y_2^2 + x_3^2 y_3^2 + y_1^2 w_1^2 + y_2^2 w_2^2 + y_3^2 w_3^2$$

$\therefore z_1 \neq w$

$$3. \langle kX, Y \rangle = k \langle X, Y \rangle$$

$$(kx)^2 y$$

$$1. (a) \langle u, v \rangle, (b) |u|, (c) |v|, (d) d(u, v)$$

$$1-1 \quad u = \langle 3, 4 \rangle, v = \langle 5, -12 \rangle, \langle u, v \rangle = u \cdot v$$

$$(a) 15 - 48 = -33 \quad (b) = 5 \quad (c) \sqrt{25+104} = 13 \quad (d) \sqrt{4+16^2} = 2\sqrt{65}$$

$$1-2 \quad u = \langle -4, 3 \rangle, v = \langle 0, 5 \rangle, \langle u, v \rangle = 3u_1v_1 + u_2v_2$$

$$(a) 15 \quad (b) 5 \quad (c) 5 \quad (d) \sqrt{20}$$

$$1-3 \quad u = \langle 0, 9, 4 \rangle, v = \langle 9, -2, -4 \rangle, \langle u, v \rangle = u \cdot v$$

$$(a) -18 - 16 = -34 \quad (b) \sqrt{917} \quad (c) \sqrt{101} \quad (d) \sqrt{266}$$

$$2. \langle A, B \rangle = 2a_{11}b_{11} + a_{12}b_{12} + a_{21}b_{21} + 2a_{22}b_{22} \quad (1) \langle A, B \rangle \quad (2) |A| \quad (3) |B| \quad (4) d(A, B)$$

$$A = \begin{bmatrix} -1 & 3 \\ 4 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix}$$

$$\langle A, B \rangle = 0 - 6 + 4 - 4 = -6 \quad |A| = \langle A, A \rangle = 2(-1)^2 + 9 + 16 + 8 = 35$$

$$|B| = \langle B, B \rangle = 0 + 4 + 1 + 2 = 7 \quad d(A, B) = |A - B| = \langle A - B, A - B \rangle$$

$$A - B = \begin{bmatrix} -1 & 5 \\ 3 & -3 \end{bmatrix} = 2 \cdot 1 + 25 + 9 + 18 = 54 \quad \therefore \sqrt{54} = 3\sqrt{6}$$

$$3. \langle p, q \rangle = a_0 b_0 + a_1 b_1 + a_2 b_2 \quad (1) \langle p, q \rangle \quad (2) |p| \quad (3) |q| \quad (4) d(p, q)$$

$$p(x) = 1 - x + 3x^2 \quad q(x) = x - x^2$$

$$p = \langle 1, -1, 3 \rangle \quad q = \langle 0, 1, -1 \rangle$$

$$\langle p, q \rangle = -1 - 3 = -4 \quad |p| = \sqrt{1+1+9} = \sqrt{11} \quad |q| = \sqrt{2}$$

$$d(p, q) = |p - q| = |\langle 1, -2, 4 \rangle| = \sqrt{1+4+16} = \sqrt{21}$$

4. 두 벡터 사이의 각을 구하여라

$$u = \langle 3, 4 \rangle \quad v = \langle 5, -12 \rangle \quad \langle u, v \rangle = u \cdot v$$

$$\cos \theta = \frac{\langle u, v \rangle}{|u| |v|} = \frac{-33}{5 \times 13} = \frac{-33}{65}$$

$$4-2 \quad u = \langle 1, 1, 1 \rangle, v = \langle 2, -2, 2 \rangle, \langle u, v \rangle = u_1 v_1 + 2u_2 v_2 + u_3 v_3$$

$$\cos \theta = \frac{\langle u, v \rangle}{|u| |v|} = \frac{2-4+2}{\sqrt{3} \sqrt{8}} = 0$$

5. X와 Y 사이의 사이각 θ 구하라

$$5-1 \quad X = (1, -3) \quad Y = (2, 4) \quad \cos \theta = \frac{\langle X, Y \rangle}{|X| |Y|} = \frac{-10}{\sqrt{10} \sqrt{20}} = \frac{-10}{10\sqrt{2}} = -\frac{1}{\sqrt{2}}$$

$$5-2 \quad X = (4, 1, 8) \quad Y = (1, 0, -3) \quad \cos \theta = \frac{\langle X, Y \rangle}{|X| |Y|} = \frac{-20}{9 \cdot \sqrt{10}} = -\frac{20}{9} \times \frac{\sqrt{10}}{10} = -\frac{2\sqrt{10}}{9}$$

$$5-3 \quad X = (1, 0, 1, 0) \quad Y = (-3, -3, -3, -3)$$

$$\cos \theta = \frac{-6}{\sqrt{2} \sqrt{36}} = -\frac{6\sqrt{2}}{2 \times 6} = -\frac{\sqrt{2}}{2}$$

$$6. \quad X = (2, 1, -4, 0) \quad Y = (-1, -1, 2, 2) \quad Z = (3, 2, 5, 4)$$