ex 1. (2,5), (3,2), (4,5) 를 지나는 다항식 $f(x) = a_0 + a_1 x + a_2 x^2$ 구하라.

$$f(2) = a_0 + 2a_1 + 4a_2 = 5$$
$$f(3) = a_0 + 3a_1 + 9a_2 = 2$$
$$f(4) = a_0 + 4a_1 + 16a_2 = 5$$

$$\begin{bmatrix} 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 5 \end{bmatrix} \quad R_2 - R_1 \to R_2 \quad \left[\begin{array}{ccc:c} 1 & 2 & 4 & 5 \\ 0 & 1 & 5 & -3 \\ 1 & 4 & 16 & 5 \end{array}\right]$$

$$R_3 - R_1 \to R_3 \quad \left[\begin{array}{ccc:c} 1 & 2 & 4 & 5 \\ 0 & 1 & 5 & -3 \\ 0 & 2 & 12 & 0 \end{array}\right] \quad \frac{R_3}{2} \to R_3 \quad \left[\begin{array}{ccc:c} 1 & 2 & 4 & 5 \\ 0 & 1 & 5 & -3 \\ 0 & 1 & 6 & 0 \end{array}\right] \quad \begin{array}{c} R_3 - R_2 \\ \to R_3 \end{array} \quad \left[\begin{array}{ccc:c} 1 & 2 & 4 & 5 \\ 0 & 1 & 5 & -3 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$\therefore a_2 = 3,\ a_1 = -18 \quad a_0 = 29 \qquad f(x) = 29 - 18x + 3x^2$

ex2. 3차 함수 그래프가 $(1,-2), (-1,2)$ 에서 수평 접선 가짐. 삼차함수 $f(x)$?

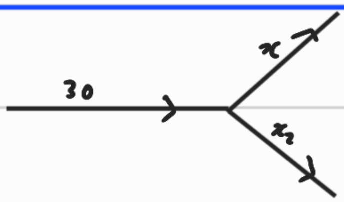
$f'(1) = 0 \quad f'(-1) = 0 \qquad f(x) = ax^3 + bx^2 + cx + d$

$f(1) = -2 \qquad f'(x) = 3ax^2 + 2bx + c$

$$\begin{array}{l} 3a + 2b + c = 0 \\ 3a - 2b + c = 0 \\ \hline \boxed{b = 0} \end{array} \qquad \begin{array}{l} a + b + c + d = -2 \\ -a + b - c + d = 2 \end{array} \qquad \begin{array}{l} a + c + d = -2 \\ -a - c + d = 2 \\ \hline \boxed{d = 0} \end{array}$$

$$\begin{array}{l} 3a + c = 0 \\ a + c = -2 \\ \hline 2a = -2 \end{array} \qquad a = -1 \quad c = -1 \qquad \therefore f(x) = -x^3 - x$$

회로분석

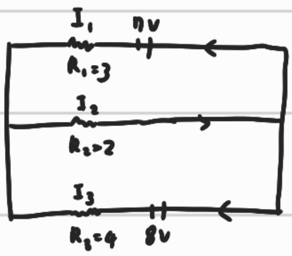


$\Rightarrow x_1 + x_2 = 30$

회로 분석 2

1. 교차점에 유입되는 전류는 반드시 유출된다.
2. $V = IR$

ex 3.



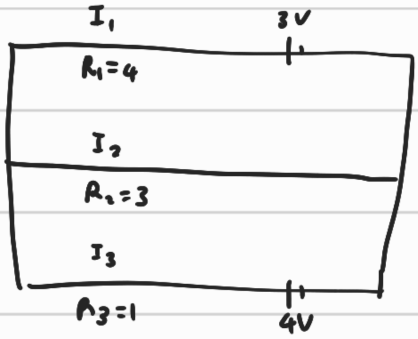
I_1, I_2, I_3 ?

① $I_1 + I_3 = I_2$ (제 1법칙) ② $R_1 I_1 + R_2 I_2 = 7$, $R_2 I_1 + R_3 I_3 = 8$

$3I_1 + 2I_2 = 7$
$2I_1 + 4I_3 = 8$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 3 & 2 & 0 & 7 \\ 2 & 0 & 4 & 8 \end{bmatrix} \to \quad I_1 = 1 \quad I_2 = 2 \quad I_3 = 1$$

ex4



I_1, I_2, I_3 ?

$I_1+I_3=I_2$, $R_1I_1+R_2I_2=3$, $R_2I_2+R_3I_3=4$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 4 & 3 & 0 & 3 \\ 0 & 3 & 1 & 4 \end{array}\right] \rightarrow I_1=0,\ I_2=1,\ I_3=1$$

최소제곱 회귀분석

$\hat{b} = (X^TX)^{-1}X^TY$, 오차제곱합 $= E^TE$

ex5) (1,1), (2,2), (3,4), (4,4), (5,6) 에 대한 최소 자승 회귀 직선 ?

$$X = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \end{bmatrix} \quad Y = \begin{bmatrix} 1 \\ 2 \\ 4 \\ 4 \\ 6 \end{bmatrix} \qquad \hat{b} = (X^TX)^{-1}X^TY$$

bias

$$X^TX = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 5 & 15 \\ 15 & 55 \end{bmatrix} \xRightarrow{\text{역행렬}} \frac{1}{50}\begin{bmatrix} 55 & -15 \\ -15 & 5 \end{bmatrix}$$

$$X^TY = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 4 \\ 4 \\ 6 \end{bmatrix} = \begin{bmatrix} 17 \\ 63 \end{bmatrix}$$

$$\hat{b} = \frac{1}{50}\begin{bmatrix} 55 & -15 \\ -15 & 5 \end{bmatrix}\begin{bmatrix} 17 \\ 63 \end{bmatrix} = \begin{bmatrix} -0.2 \\ 1.2 \end{bmatrix} \Rightarrow y = -0.2+1.2x$$

연습문제

1-1) (2,4), (3,6), (5,10) 을 지나는 직선 구하라

$f(x) = a_0 + a_1 x + a_2 x^2$

$f(2) = 4a_2 + 2a_1 + a_0 = 4$

$f(3) = 9a_2 + 3a_1 + a_0 = 6$

$f(5) = 25a_2 + 5a_1 + a_0 = 10$

$5a_2 + a_1 = 2$

$16a_2 + 2a_1 = 4$

$$\begin{array}{r} 10a_2 + 2a_1 = 4 \\ - \; 16a_2 + 2a_1 = 4 \\ \hline \end{array}$$

$\underline{a_2 = 0}$ $a_1 = 2$ $a_0 = 0$

$\therefore f(x) = 2x$

1-2 (2001,5), (2002,7), (2003,12)

$f(x) = a_0 + a_1 \underbrace{(x-2000)}_{z} + a_2 \underbrace{(x-2000)^2}_{z}$

$$\begin{cases} a_2 + a_1 + a_0 = 5 \\ 4a_2 + 2a_1 + a_0 = 7 \\ 9a_2 + 3a_1 + a_0 = 12 \end{cases}$$

$3a_2 + a_1 = 2$

$5a_2 + a_1 = 5$

$2a_2 = 3$

$a_2 = \frac{3}{2}$

$a_1 = -\frac{5}{2}$ $a_0 = 6$

$\therefore f(z) = 6 - \frac{5}{2}z + \frac{3}{2}z^2$

2-1 1) x_i 에 대한 방정식 풀어라

$$\left.\begin{array}{l} x_1 + x_3 = 600 \\ x_2 + x_4 = x_1 \\ 500 + x_5 = x_2 \\ x_5 + x_7 = 500 \\ x_3 + x_6 = 600 \\ x_4 + x_7 = x_6 \end{array}\right) \rightarrow$$

$x_7 = t$ $x_6 = x$

$\therefore x_1 = s,\ x_2 = t,\ x_3 = 600 - s,\ x_4 = s - t,\ x_5 = 500 - t,\ x_6 = s,\ x_7 = t$

2) $x_4 = x_7 = 0$ 일 때 물의 흐름은?

$s = t,\ t = 0$

$\therefore x_1 = 0, x_2 = 0, x_3 = 600, x_4 = 0, x_5 = 500, x_6 = 0, x_7 = 0$

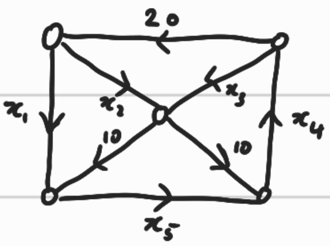
3) $x_5 = 1000, x_6 = 0$

$t = -500,\ s = 0$

$\therefore x_1 = 0, x_2 = -500, x_3 = 600, x_4 = 500, x_5 = 1000, x_6 = 0,$

$x_7 = -500$

3.



$x_1 + x_2 = 20$

$x_3 + 20 = x_4$

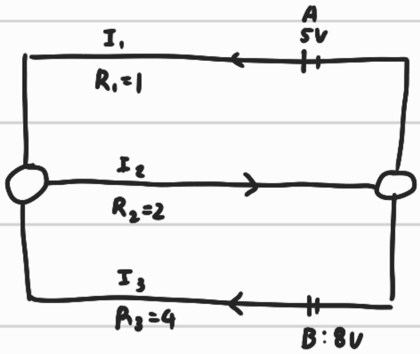
$x_2 + x_3 = 20$

$x_1 + 10 = x_5$

$x_5 + 10 = x_4$

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 20 \\ 0 & 0 & 1 & -1 & 0 & -20 \\ 0 & 1 & 1 & 0 & 0 & 20 \\ 1 & 0 & 0 & 0 & -1 & -10 \\ 0 & 0 & 0 & -1 & 1 & -10 \end{array}\right] \rightarrow \boxed{\begin{array}{l} x_5 = t \\ x_1 = t - 10 \\ x_2 = -t + 30 \\ x_3 = t - 10 \\ x_4 = t + 10 \end{array}}$$

4.



$I_1+I_3=I_2$

$R_1I_1+R_2I_2= \quad I_1+2I_2=5$

$R_2I_2+R_3I_3=2I_2+4I_3=8$

$$\begin{bmatrix}1&-1&1&0\\1&2&0&5\\0&2&4&8\end{bmatrix} \xrightarrow{\frac{R_3}{2}\to R_3} \begin{bmatrix}1&-1&1&0\\1&2&0&5\\0&1&2&4\end{bmatrix} \xrightarrow{R_1-R_2\to R_1} \begin{bmatrix}1&-1&1&0\\0&-3&1&-5\\0&1&2&4\end{bmatrix}$$

$$\frac{-R_2}{3}\to R_2 \begin{bmatrix}1&-1&1&0\\0&1&-\frac{1}{3}&\frac{5}{3}\\0&1&2&4\end{bmatrix} \quad R_2-R_3\to R_3 \begin{bmatrix}1&-1&1&0\\0&1&-\frac{1}{3}&\frac{5}{3}\\0&0&-\frac{7}{3}&-\frac{7}{3}\end{bmatrix}$$

$\therefore I_3=1,\ I_2=2,\ I_1=1$

5-1 (0,0), (1,2), (2,4) 최소 제곱 회귀 직선?

$$\hat{b}=(X^TX)^{-1}X^TY \quad X=\begin{bmatrix}1&0\\1&1\\1&2\end{bmatrix} \quad Y=\begin{bmatrix}0\\2\\4\end{bmatrix}$$

$$X^TX=\begin{bmatrix}1&1&1\\0&1&2\end{bmatrix}\begin{bmatrix}1&0\\1&1\\1&2\end{bmatrix}=\begin{bmatrix}3&3\\3&5\end{bmatrix} \xrightarrow{-1} \frac{1}{6}\begin{bmatrix}5&-3\\-3&3\end{bmatrix}$$

$$X^TY=\begin{bmatrix}1&1&1\\0&1&2\end{bmatrix}\begin{bmatrix}0\\1\\4\end{bmatrix}=\begin{bmatrix}5\\9\end{bmatrix}$$

$$\frac{1}{6}\begin{bmatrix}5&-3\\-3&3\end{bmatrix}\begin{bmatrix}5\\9\end{bmatrix}=\frac{1}{6}\begin{bmatrix}-2\\12\end{bmatrix}=\begin{bmatrix}-\frac{1}{3}\\2\end{bmatrix} \Rightarrow y=-\frac{1}{3}+2x$$

5-2 (-2,0), (-1,1), (0,1), (1,2) $X=\begin{bmatrix}1&-2\\1&-1\\1&0\\1&1\end{bmatrix}$ $Y=\begin{bmatrix}0\\1\\1\\2\end{bmatrix}$

$$X^TX=\begin{bmatrix}1&1&1&1\\-2&-1&0&1\end{bmatrix}\begin{bmatrix}1&-2\\1&-1\\1&0\\1&1\end{bmatrix}=\begin{bmatrix}4&-2\\-2&6\end{bmatrix} \xrightarrow{-1} \frac{1}{20}\begin{bmatrix}6&2\\2&4\end{bmatrix}=\frac{1}{10}\begin{bmatrix}3&1\\1&2\end{bmatrix}$$

$$X^TY=\begin{bmatrix}1&1&1&1\\-2&-1&0&1\end{bmatrix}\begin{bmatrix}0\\1\\1\\2\end{bmatrix}=\begin{bmatrix}4\\1\end{bmatrix} \qquad \frac{1}{10}\begin{bmatrix}3&1\\1&2\end{bmatrix}\begin{bmatrix}4\\1\end{bmatrix}=\frac{1}{10}\begin{bmatrix}13\\6\end{bmatrix} \quad y=1.3+0.6x$$

6-1 $x=(1,\ 1.25,\ 1.5)$, $y=(450, 375, 330)$

$$X=\begin{bmatrix}1&1\\1&1.25\\1&1.5\end{bmatrix} \quad Y=\begin{bmatrix}450\\375\\330\end{bmatrix} \xRightarrow{\text{생략}} y=685-240x$$

6-2 가격(x) = 1.4 일때 수요는? $685-240\times1.4=349$