Optical Pumping and the Nuclear Spin of Rubidium

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Abstract

Quantum mechanics predicts several energy level splittings due to the inner structure of the atom, as well as energy level splittings due to external manipulations, like an external magnetic field. Combinations of these energy level splittings can be used to pump electrons into higher energy level states, which is the essence of the optical pumping process. This experiment uses optical pumping to measure the nuclear spin of a couple of Rubidium isotopes and then uses those spins to estimate the strength of the Earth's magnetic field at UC Berkeley. The values of the nuclear spins of ^{85}Rb and ^{87}Rb were measured to be 5/2 and 3/2 respectively and the strength of the Earth's magnetic field was measured to be 0.333 ± 0.002 Gauss.

1 Introduction

1.1 Theory

In quantum mechanics, the study of energy level splittings is of high importance. The framework of quantum mechanics is very applicable to the study of atomic structure. Several natural energy level splittings related to atomic structure (see Figure 1) that occur include the Coulomb interaction between the electron and nuclear charge, the spin-orbit interaction, and an interaction from the nuclear spin. These splittings are included in the Hamiltonian of the system, but they cannot be manipulated by an experimenter. While splittings related to the structure cannot be touched, a further splitting of the energy levels can be induced by applying an external magnetic field to the electrons, and this splitting as a quantum number m_F. Splitting from an external magnetic field is called Zeeman splitting.

A relationship between the Zeeman splitting frequency and an external magnetic field is given by the Breit-Rabi formula [1],

$$\frac{\nu}{B_{ext}} = \left(\frac{2.799}{2I+1}\right) \ MHz/gauss \tag{1}$$

where I is the nuclear spin. When the direction used for measuring spin is aligned with the Earth's magnetic field and if no external field is applied, then the strength of the Earth's magnetic field can determined from measuring the splitting frequency, assuming that the nuclear spin is known. In fact, the Earth's magnetic field is measured in this experiment.

The optical pumping experiment is done with the Rubidium element in gas form and a rough visualization of the optical pumping process can be seen in Figure 2. The general principle behind optical pumping is to raise electrons into higher m_F states. This is done by shining circularly polarized light onto a bulb of Rubidium gas. Because of the polarization of the light, the electron

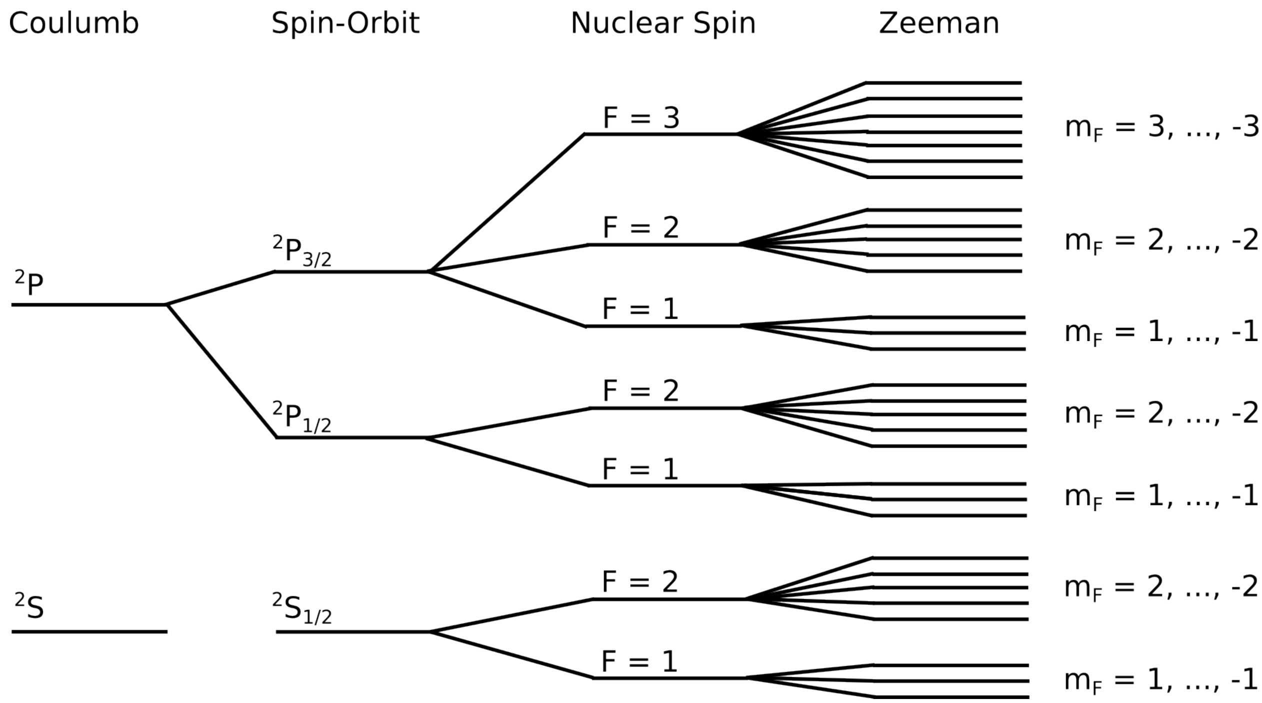

Figure 1: Diagram of the energy level splittings studied in the optical pumping experiment. Energy levels are not to scale.

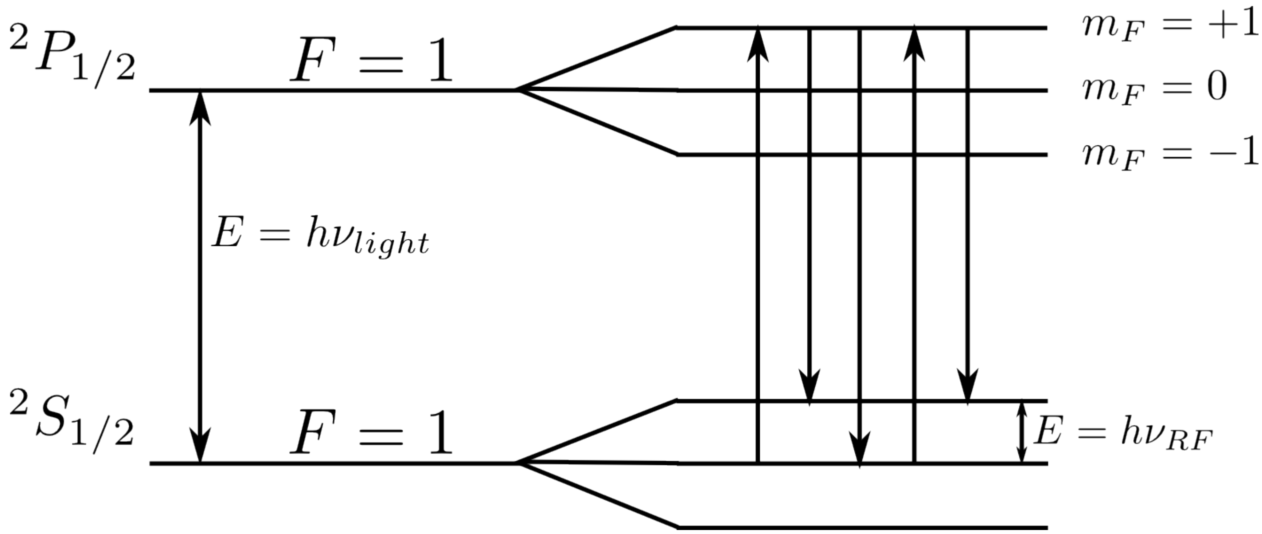

Figure 2: Simplified diagram of the process of optical pumping using only one large energy jump. In the actual experiment, transitions from all of the other energy levels will occur.

can only make $\Delta m_F = +1$ jumps during when it absorbs a photon. After a while, the electron emits a photon and makes a jump where $\Delta m_F = 0, \pm 1$. Once all of the electrons are pumped up, the Rubidium gas cannot absorb any more light. In order for the pumping process to be repeated, a radio frequency (RF) field must be applied to cause the electrons to transition back down to a lower m_F state.

Figure 2 illustrates an example to demonstrate why the optical pumping technique forces electrons into higher m_F states. In the figure, electrons are able to move from subsets of two $F = 1$ states involved in the experiment. Suppose that an electron is in the $m_F = 0$ state for the $^2S_{1/2}$ level. When it absorbs a photon, it will move up to the $m_F = 1$ state in the $^2P_{1/2}$ level. When losing a photon, the electron will either move to the $m_F = 1$ state or the $m_F = 0$ state, with roughly 50% probability for going into either state. If the electron goes to the $m_F = 1$ state, it is done, as the P level doesn't have an $m_F = 2$ state. When $m_F = 0$, the process just repeats from the beginning. Clearly, this process leads to electrons going to the higher m_F states.

1.2 Experimental Setup

The experimental setup for the optical pumping lab is pretty straightforward, and can be seen in Figure 3 [1]. Rubidium gas contained in a bulb is placed in a box, which contains circuitry to measure the temperature, shine circularly polarized light at the Rubidium (see the Theory section for description of the optical pumping process), oscillate a radio frequency, and measure emitted light from the Rubidium. The Rubidium gas contains both ^{85}Rb and ^{87}Rb, as will be seen in the Experimental Methods section.

To induce Zeeman splitting, an external magnetic field is produced from Helmholtz coils as well as the Earth's magnetic field. The Earth's magnetic field determines the direction that spin is measured on, and thus the field from the Helmholtz coils must point in the same direction of the Earth's field. The field inside of a Helmholtz coil is relatively uniform, which makes it a good choice for this experiment. The total magnetic field applied to the Rubidium is [1]

$$\vec{B}_{ext} = \vec{B}_E \pm 0.9 \times 10^{-6} \left(\frac{tesla \cdot meter}{ampere} \right) \frac{Ni}{a} \hat{n} \tag{2}$$

where $\vec{B}_E$ is the Earth's magnetic field, N is the number of turns in the coil, i is the current in the coil, a is the radius of the coil, and $\hat{n}$ is the direction normal to the planes made from the coil. The $\pm$ depends on the polarity of the current running through the coil.

The other main components of the experiment are the AC modulation unit and the heater control unit. The AC modulator takes in a DC current source and sends it to the coils. This is necessary because some parts of the experiment require a modulated field from the Helmholtz coil, which will be described in the Experimental Method section. The temperature control is important because the optical signal strength of the Rubidium depends on how hot the gas is. The ideal temperature to operate this experiment can be seen in Figure 4 [**?**]. It should be noted that the heater unit should not be turned on while making measurements. This is because it has its own coil, which produces a magnetic field when turned on and disturbs the experiment. Fortunately, the temperature of the bulb drops slowly enough so that many measurements can be made before needing to reheat the bulb.

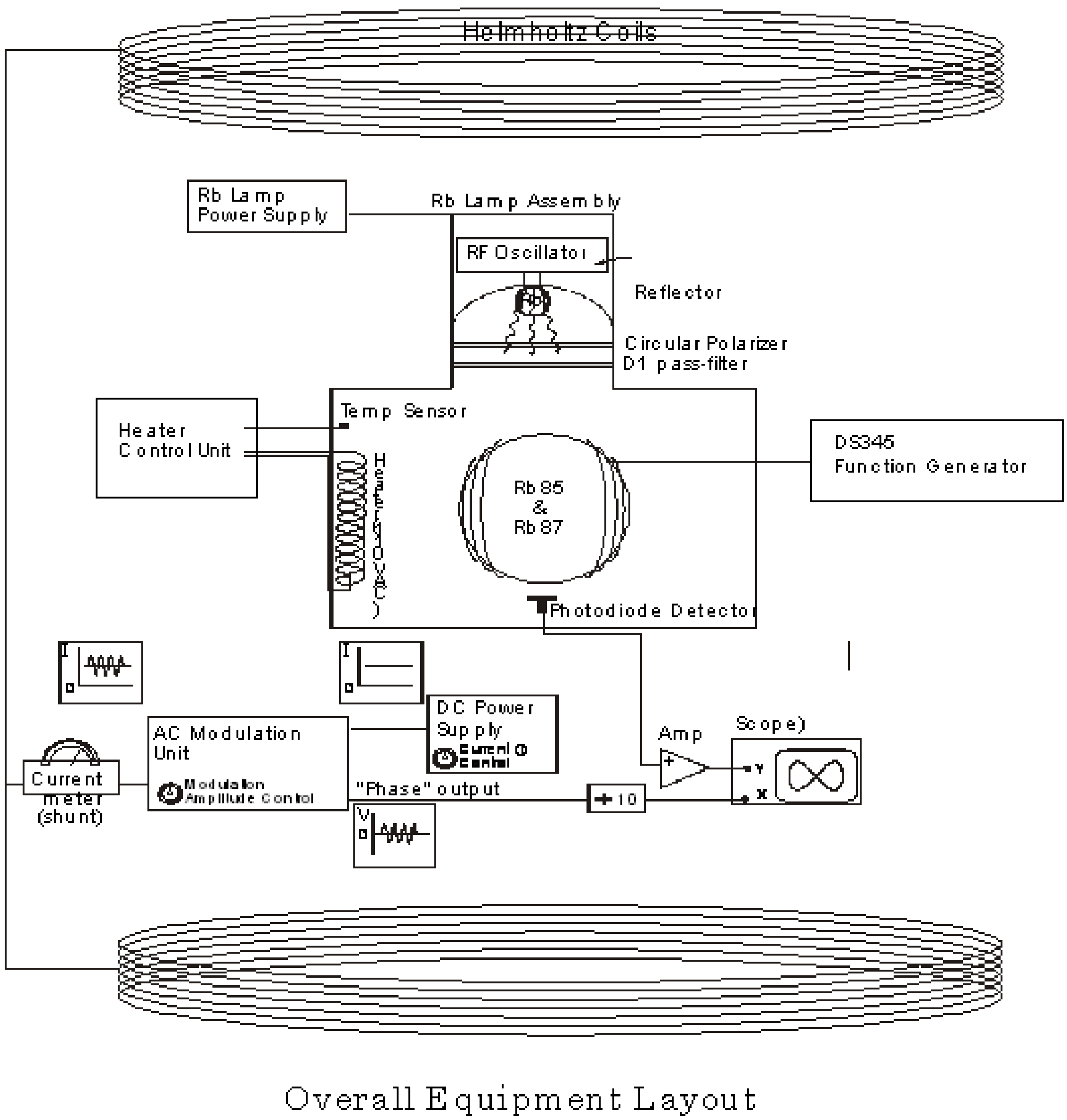

Figure 3: Block diagram of the optical pumping experiment [1].

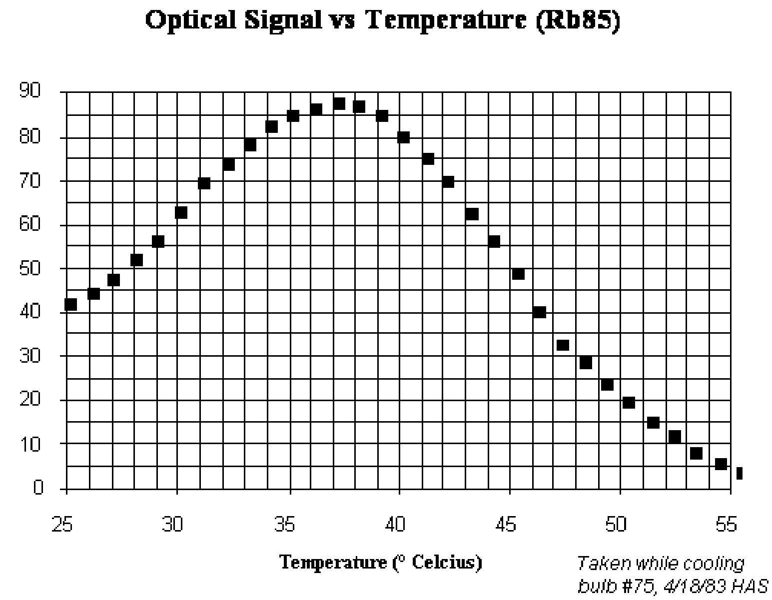

Figure 4: The optical signal strength for ^{85}Rb is relatively flat between $35°C$ and $40°C$ [1].

2 Experimental Method

2.1 Opacity of Rubidium

One method of measuring the resonance frequency for a given current is to set the current in the Helmholtz coil to some constant value and to ramp the RF from the function generator controlling it [1]. The way to do this is choose a range of frequencies as the span of the function generator and set the carrier frequency as about half of the span. In order for this to work, the span must be larger than the resonance frequencies of the isotopes, else the isotopes will not be seen. The physics of what is going on is that the function generator is attempting to match the splitting frequency for some fixed magnetic field, as governed by the Breit-Rabi formula (1). The actual resonance frequency is acquired by sending the modulation output of the function generator to the first channel on the oscilloscope, and the output of the photodetector through and amplifier and to the second channel on the scope. When the scope is set to X-Y mode, the resonance frequency for a specific isotope by finding the peak signal for that isotope. An example of this for a Rubidium bulb containing only ^{87}Rb can be seen in Figure 5. Since the resonance frequency of the Rubidium really only depends on the strength of the magnetic field, the measured resonance is pretty much invariant on the span and carrier frequency from the function generator, and that was observed in the experiment.

2.2 Increasing precision of the resonance frequency

The above method for determining the resonance frequency is very imprecise and should not be used, as more precise methods are available. The method used to determine the resonance frequency goes as follows. First, modulation is turned off for the RF and on for the external magnetic field. The modulation frequency of the magnetic field is set to 60 Hz and the modulation amplitude is set to roughly 30 mA. The phase output and photodetector signals can now be displayed in X-Y mode on the oscilloscope and their shape forms a Lissajous figure, which can be used to determine the resonance frequency. The resonance frequency for a magnetic field strength is the frequency

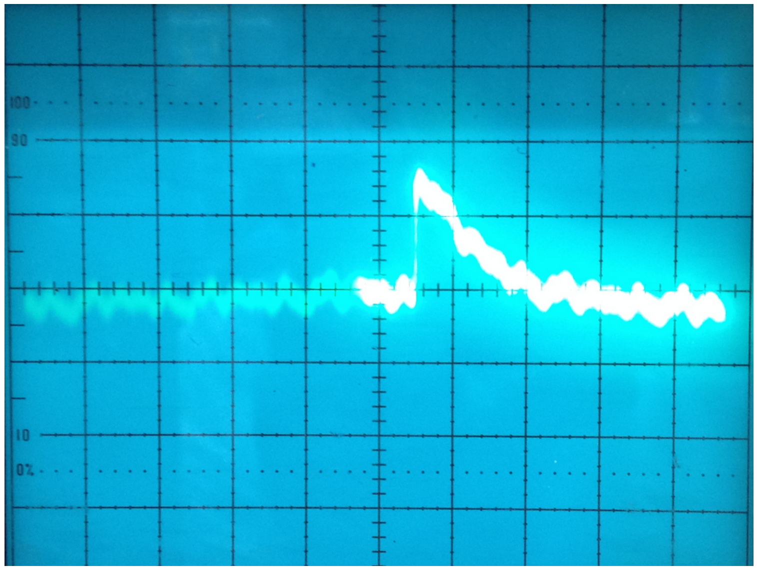

Figure 5: Opacity measurement featuring the bulb that only contained ^{87}Rb. Since the X axis is the modulation output, it can be easily translated to a frequency measurement.

that makes the Lissajous figure the most symmetric about the Y axis. An example of this can be seen in Figure 6.

The reason that the above method works in finding the resonance frequency is as follows and a visualization of it can be seen in Figure 7. Suppose that the frequency from the function generator is set to some value near the resonance frequency for the magnetic field given by the input current. Because of the modulation of the field, the strength of the field will oscillate to the strength for which the applied RF is at resonance. If that strength is far from the magnetic field strength of the center of modulation, then the time differences between when the field reaches the strength of the field for resonance alternate between being a longer period and a shorter period, which will result in an asymmetry on the Lissajous figure. As can be seen from Figure 7, when the field that will cause RF to be the resonance frequency is closer to the center field strength of the modulation, the periods between when the field is at that resonant strength become more consistent and even, which results in a symmetric Lissajous figure. By setting the RF to a value where the Lissajous figure becomes symmetric, the resonance frequency for the magnetic field strength can be found. Of course, there will be some range in the frequencies that give a symmetric Lissajous pattern, and that is a systematic error in the measurement process that must be noted and accounted for. Ideally, that systematic error should be small and the resonance frequency for a given magnetic field strength can be easily found.

3 Analysis

3.1 Nuclear spin

Equations (1) and (2) can easily be manipulated to give the following relationship between current and frequency:

$$\nu = \left(\frac{2.799}{2I+1}\frac{0.9\times 10^{-2}N}{a}\right)i + \left(\frac{2.799}{2I+1}\right)B_E \tag{3}$$

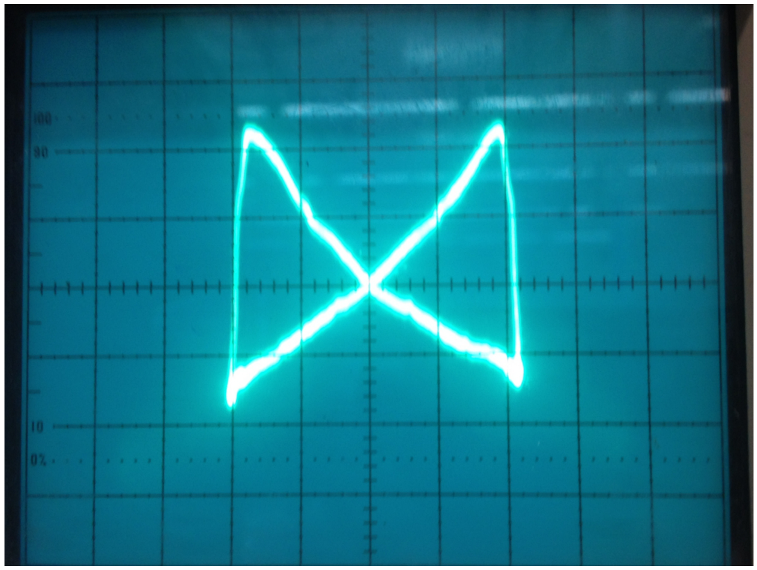

Figure 6: The resonance frequency can be found by making a symmetric Lissajous figure in X-Y mode of the scope. The X axis is the phase output of the AC modulation unit, whereas the Y axis is the amplified output of the photodetector.

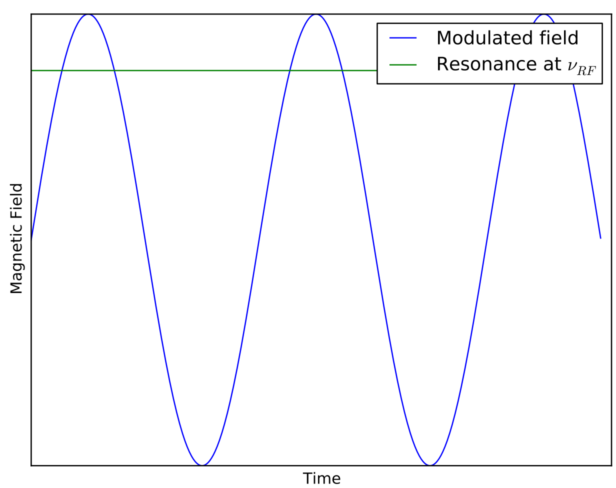

Figure 7: Demonstration for why the symmetry of the Lissajous figure is a good way to determine the resonance frequency.

This formula is useful because it provides a method of calculating the nuclear spin from a linear fit of the data. Using a fit of $y = mx + b$, it can shown that I can be calculated as

$$I = \frac{1}{2}\left(\frac{2.5191 \times 10^{-2} N}{ma} - 1\right) \tag{4}$$

where m is the slope of the linear fit, a is the radius of the Helmholtz coil, and N is the number of turns of the coil. Of course, I must be a half integer and there is really no guarantee that (4) will give out such a number. However, the value for I computed from (4) has an error associated with it, and the nuclear spin should be the closest half integer to the value of I computed from (4) within some error associated with the measurement.

3.2 Earth's magnetic field

Equation (3) might suggest that the Earth's magnetic field can be calculated using the intercept of a linear fit. While this is true, there is a better way to do calculate it. Suppose that for every time the Helmholtz coil has a positive current, a measurement is made where the polarity on the current is flipped. This will cause a slightly different resonance frequency for the Rubidium. These two resonance frequency pairs have the following values [1]:

$$\nu^{+} = \left(\frac{2.799}{2I+1}\right)|B_H + B_E| \tag{5}$$

$$\nu^{-} = \left(\frac{2.799}{2I+1}\right)|-B_H + B_E| \tag{6}$$

In the above equations, ν^{-} can be thought of as a negative frequency, but in reality, all frequency measurements are positive. Clearly, these two frequencies can be added together to get the Earth's magnetic field, which is given as

$$B_E = \frac{1}{2}\left(\frac{\nu^{+} - \nu^{-}}{2.799}\right)(2I+1) \tag{7}$$

and it should be stressed that the value used for ν^{-} is the positive measurement. Since this can be done for every $\nu^{\pm}$ pair, the Earth's magnetic field can be computed by taking the average of all pairs, which makes calculating the uncertainty of the Earth's field incredibly straightforward.

3.3 Error analysis

Since the value of the nuclear spin is exactly a half integer, a full error analysis for the spin is not needed. However, the measurements of the resonance frequency do have errors and these must be accounted for. There are two ways to get the error of the frequency measurements. The first way is to measure the range over which the Lissajous figure looks symmetric. In the actual experiment, this error was approximately 1 kHz for currents less than 1.6 A and about 5 kHz for currents greater than 1.6 A.

The second way to calculate the error of the resonance frequency is to determine what error would be needed for a linear fit to have $\chi^2/ndf = 1$. This method can be quickly derived from the definition of χ^2, which is [2]

$$\chi^2 = \sum_{i=1}^{n}\left(\frac{y_i - mx_i - b}{\sigma_i}\right)^2 \tag{8}$$

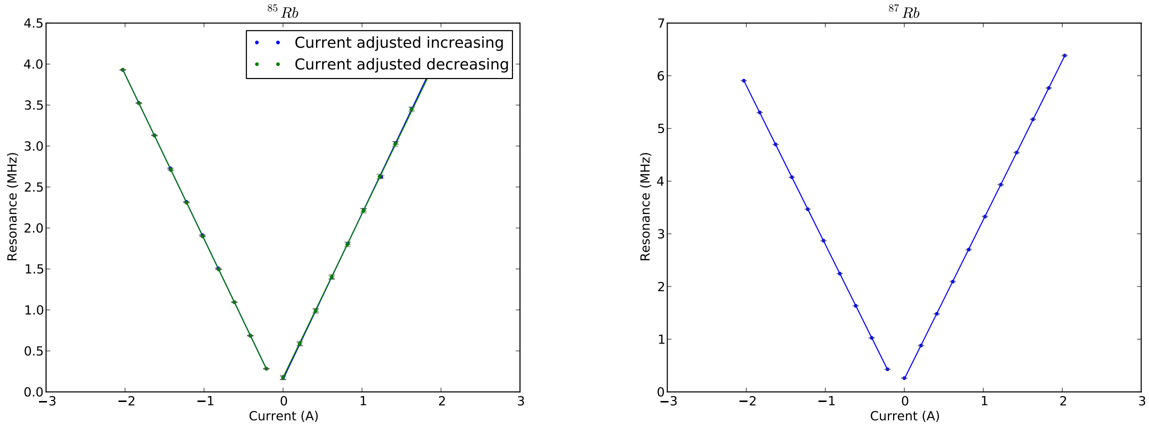

Figure 8: Nuclear spin can be calculated with a linear fit of the data. Error bars are printed on the plot, but the errors are small. (left) The plot for ^{85}Rb shows that hysteresis effects are relatively minimal and can be neglected. (right) Plot for ^{87}Rb.

If it assumed that σ_i is the same for every y_i, and that the value of χ^2 is equal to ndf, one can solve for σ. This yields

$$\sigma_{RF} = \sqrt{\frac{1}{n-2}\sum_{i=1}^{n}(y_i - mx_i - b)^2} \tag{9}$$

where n is the number of data points used in the least squares fit.

The uncertainty of the Earth's magnetic field is very straightforward to calculate, as the field is computed by taking a mean. The error of the field from individual measurements only really depends on the combined error of ν^+ and ν^-. Since this error is the same for all individual field measurements, there is no difference between taking a weighted mean or an unweighted mean, and the error on the mean strength of the earth's magnetic field is the desired uncertainty.

4 Results

The nuclear spins of both Rubidium isotopes was calculated by fitting the slopes of the lines fit to the Breit-Rabi formula, which can be seen in Figure 8. It should be noted that there are two curves in the left figure. That was to test for hysteresis effects in the AC modulation unit. If hysteresis was large, then there would be some notable separation between the blue and green lines, and a correction would have to have been made for it. However, hysteresis was small, so no correction needed to be made and this effect therefore was not measured with measuring the spin of ^{87}Rb. The uncertainties on RF were 0.018 Hz for positive current on ^{85}Rb, 0.003 Hz for negative current on ^{85}Rb, 0.008 Hz for positive current on ^{87}Rb, and 0.003 Hz for negative current on ^{87}Rb. The spins measured from the linear fits were 2.4 and 1.4 for ^{85}Rb and ^{87}Rb respectively, which implies that the spins of these isotopes are 5/2 and 3/2, which are the accepted values for these isotopes. The value of the Earth's magnetic field was calculated to be 0.333 ± 0.002 Gauss, which is a reasonable measurement. Details of the analysis code used to compute these results can be found at https://bitbucket.org/domagalski/physics111-advanced-lab/ [3].

References

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