Hierarchical Models for Crowdsourced Bicycle Route Ratings

A Thesis
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I want to thank a few people.

Preface

This is an example of a thesis setup to use the reed thesis document class.

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Abstract

The preface pretty much says it all.

Dedication

You can have a dedication here if you wish.

Introduction

Knock Software's *Ride Report* app combines a simple thumbs-up/thumbs-down rating system with GPS traces of bicycle rides to compile a crowdsourced data set of which routes are and are not stressful for urban bicyclists.

The app that collects the data is simple: *Ride Report* automatically detects when a user start riding their bike, records the GPS trace of the route, and then prompts the user at the end of the ride to give either a thumbs-up or thumbs-down rating. From this, they were able to create a simple "stress map" of Portland, OR, which displays the average ride rating of rides going through each discretized ride segment.

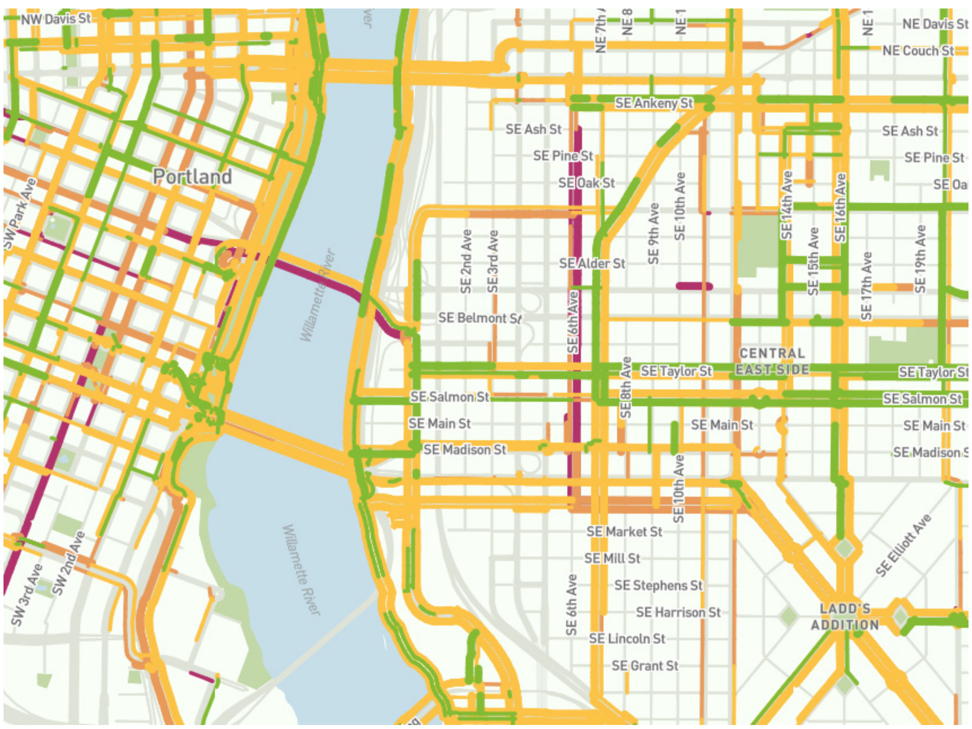


Figure 1: Ride Report's Stress Map for Portland. Greener road segments incidate less while more red segments indicate more stressful streets. "Stress" computed by taking the average rating for each segment.

The app privileges reducing barriers to response to increase sample size over ensuring quality and consistent responses. This presents the first problem: how can we analyze ratings when riders are likely rating rides inconsistently?

At the same time, we have another challenge. We have ratings associated with routes, but we would like to know the effect of particular road segments, for both inference (what effect does this road segment have on the rating?) and prediction (given a route, what do we expect the rating to be?) purposes.

Finally, there are interesting issues with missing data. First, the sample of rides we have are biased towards routes that riders percieve as better. Second, a significant portion of the rides are missing a response, and non response is unlikely to be independent of the response. The good news is that we have all the predictors for every ride, enabling us to build a model that is able to leverage the data with missing responses rather than listing the missing data as a liability.

0.1 Accounting for Rider Rating Variance

For ratings we are interested in modeling variance between riders (as we might expect riders to have different rating patterns than their peers) and within riders (as riders may not rate the same route and conditions the same every time). To model this, we propose using multilevel regression, with random effects from each rider. This approach has been used in similar situations, in one case to model sexual attraction[1].

In a multilevel model, we fit a regression where the intercept term varies by group but comes from a common distribution. For example, if we let r_i be the rating of the ith ride, X_i be the ride-level variables, then we can fit a regression:

$$\mathbb{P}(r_i = 1) = \text{logit}^{-1}\left(\alpha_{j[i]} + X_i\beta\right),$$

where α_j is an intercept specific to rider j. In addition, the rider intercepts come from a common distribution,

$$\alpha_j \sim N(\mu_\alpha, \sigma_\alpha^2),$$

where μ_α is the mean of all the α_js.

Using varying intercepts will allow our model to exhibit the property of varying rates of riders giving stressful ratings. This can be extended to model differences in how riders respond to different kinds of road conditions using varying slope models. We explore multilevel model further in Section Section 3.2 and multilevel models for riders in Section Section 4.2.

0.2 Addressing Road Segments as a Level

We examine multiple approaches to modeling road segments. In the first, we regard add regression term for the route, that is a weighted sum of terms for each road segment in the route, weighted by the lengths of the segments. The terms for the

the contribution of the route to be a weighted sum of the contributions of road segments in the route, weighted by the lengths of the segments.

[1]Mackaronis, Strassberg, Cundiff, & Cann (2013)

0.3 Approaching Missing Data at Multiple Levels

This data set contains missing data at two levels.

First, there are many routes without any ratings. The routes taken by riders are not a random sample of routes, but are often already chosen by the rider as the least stressful ride. As we might expect because of this bias, only a small proportion of rides are rated as "stressful."

One solution we explore to this problem is adding a segment popularity parameter as a segment-level predictor.

Second, not every ride is given a rating. We do have the route they chose, and all associated covariates, but the response variable, rating is missing. As we will discuss in chapter Section 2.3, the pattern of non-response is unlikely to be unbiased.

To address the second problem, we first build a separate probability model that predicts non response, and then integrate that model into our main model.

Chapter 1

Data Sources

Here is a brief introduction into using *R Markdown*. *Markdown* is a simple formatting syntax for authoring HTML, PDF, and MS Word documents. *R Markdown* provides the flexibility of *Markdown* with the implementation of **R** input and output. For more details on using *R Markdown* see `http://rmarkdown.rstudio.com`.

Be careful with your spacing in *Markdown* documents. While whitespace largely is ignored, it does at times give *Markdown* signals as to how to proceed. As a habit, try to keep everything left aligned whenever possible, especially as you type a new paragraph. In other words, there is no need to indent basic text in the Rmd document (in fact, it might cause your text to do funny things if you do).

1.1 Ride Report

It's easy to create a list. It can be unordered like

- Item 1
- Item 2

or it can be ordered like

1. Item 1
2. Item 2

Notice that I intentionally mislabeled Item 2 as number 4. *Markdown* automatically figures this out! You can put any numbers in the list and it will create the list. Check it out below.

To create a sublist, just indent the values a bit (at least four spaces or a tab). (Here's one case where indentation is key!)

1. Item 1
2. Item 2
3. Item 3
    - Item 3a
    - Item 3b

1.2 Weather Data

Make sure to add white space between lines if you'd like to start a new paragraph. Look at what happens below in the outputted document if you don't:

Here is the first sentence. Here is another sentence. Here is the last sentence to end the paragraph. This should be a new paragraph.

Now for the correct way:

Here is the first sentence. Here is another sentence. Here is the last sentence to end the paragraph.

This should be a new paragraph.

1.3 Road Data

When you click the **Knit** button above a document will be generated that includes both content as well as the output of any embedded **R** code chunks within the document. You can embed an **R** code chunk like this (`cars` is a built-in **R** dataset):

Chapter 2

Data Transformation

2.1 Working in Road Networks

2.2 Using Nearest Neighbor Search for Map Matching Data

2.3 Missing Data

Chapter 3

Methods

As an undergraduate thesis, a lot of research into methodology was done. Here I go through some of the essential methodology, while establishing the notation I will use for the rest of this paper.

3.1 Logistic Regression

With logistic regression, we seek to fit a model where the response variable is binary. We might consider the response variable, Y, a Bernoulli random variable,

$$Y = \text{Bernoulli}(p),$$

where p is the probability that an observation $y_i = 1$, for any i. (As a binary variable, the support of Y is $\{0, 1\}$, so $y_i = 0$ otherwise.) Thus, in predicting and making inference about a Bernoulli variable, we are concerned with p and how it varies with respect to other quantities.

Logistic regression is, as we will see, one form of regression generalized from linear regression.

Linear regression is the first form of regression most people learn: find the line

$$y_i = \beta_0 + \beta_1 x_1 + \ldots + \beta_j x_j + \epsilon,$$

based on data with response variable y_i and j predictor variables x_i, coefficients $\beta_0, \ldots, \beta_j$, and error term $\epsilon \sim N(0, \sigma^2)$. We can equivalently write,

$$Y \sim N(\beta_0 + \beta_1 x_1 + \ldots + \beta_j x_j, \sigma^2).$$

Generalized linear regression uses a "link function," g, to modify the regression:

$$g(y_i) = \beta_0 + \beta_1 x_1 + \ldots + \beta_j x_j + \epsilon.$$

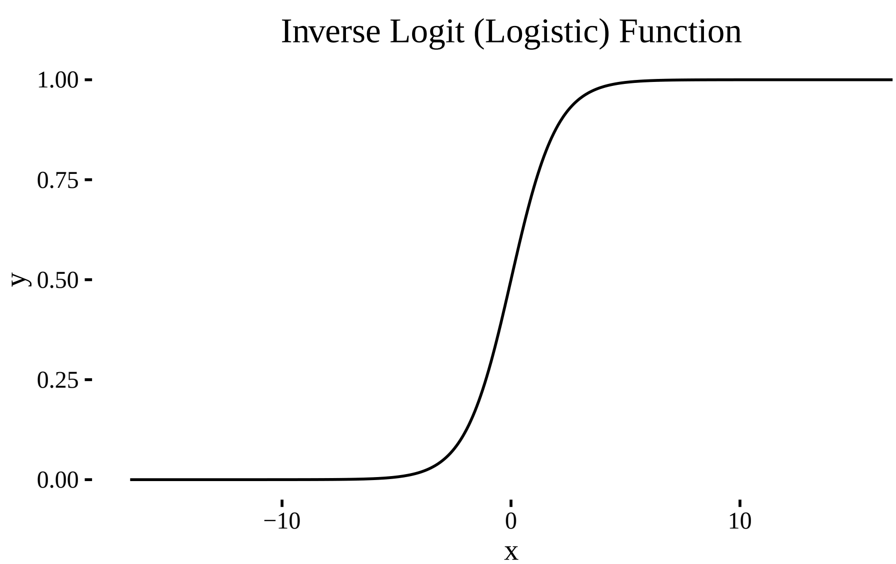
Logistic regression is a form of generalized regression where the 'link' function is the logit function, $\text{logit} : [0, 1] \to \mathbb{R}$:

$$\text{logit}(p) = \log\left(\frac{p}{1-p}\right),$$

also known as the log-odds, odds being $p/1-p$ for any probability p. So we can model this as a Bernoulli random variable where the probability of a 1 is:

$$\mathbb{P}(y_i = 1) = \text{logit}^{-1}(\beta_0 + \beta_1 x_1 + \ldots + \beta_j x_j).$$

Notice that the inverse logit function maps values from $\mathbb{R}$ to $[0, 1]$. The function provides a convenient way to map linear combinations of other variables onto values that are valid probabilities. Other such functions exist and are also used for regression of binary variables, such as the probit function.



Logistic regression is sometimes presented as a classification model

3.2 Hierarchical Models and Mixed Effects Models

Many data sets contain nested structures when viewed in some way. For example, a data set of student test scores may contain information about the schools and districts they are in. Or a dataset of soil samples may have multiple samples from each of a set of different sites. In the dataset we examine, rides can be grouped by rider.

Mutlilevel models allow us to address these kinds of relationships in regression models. They provide a number of computational advantages, as we shall describe later.

3.2.1 Description and Notation

These models of course work for other forms of regression, but we will focus on logistic regression, as it is the method we use in this paper. We will be using notation adapted from Gelman's description of multilevel models. Consider a data set composed of

- i observations of a binary response variable y_i,
- m observation level predictors $X_i = x_i^1, \ldots, x_i^m$,
- j groups in which the observations are split into,
- l group level predictors $U_{j[i]} = u_{j[i]}^1, \ldots, u_{j[i]}^l$, where $j[i]$ is the group of the ith observation,.

We could fit a model where the intercept varies by group:

$$\mathbb{P}(y_i = 1) = \text{logit}^{-1}(\alpha_{j[i]} + X_i\beta),$$

$$\alpha_{j[i]} \sim N(\gamma_0 + U_{j[i]}\gamma, \sigma_\alpha^2),$$

where $\alpha_{j[i]}$ is the intercept for the jth group, β are the coefficients for the observation-level predictors, γ_0 are the group-level intercepts, and γ are the coefficients for the group-level predictors. We could also imagine a similar model where there are no group level predictors, such that we simply have different intercepts for each group,

$$\alpha_{j[i]} \sim N(\gamma_0, \sigma_\alpha^2),$$

We can also consider a model that has slopes varying by group. For simplicity, let's consider just one observation level predictor, x_i, that will have varying slopes $\beta_{j[i]}$ as well as one group level predictor. We could specify the model as,

$$\mathbb{P}(y_i = 1) = \text{logit}^{-1}(\alpha_{j[i]} + \beta_{j[i]}x_i),$$

$$\begin{pmatrix} \alpha_j \\ \beta_j \end{pmatrix} = N\left(\begin{pmatrix} \gamma_0^\alpha + \gamma_1^\alpha u_j \\ \gamma_0^\beta + \gamma_1^\beta u_j \end{pmatrix}, \begin{pmatrix} \sigma_\alpha^2 & \rho\sigma_\alpha\sigma_\beta \\ \rho\sigma_\alpha\sigma_\beta & \sigma_\beta^2 \end{pmatrix}\right).$$

3.2.2 Examples and Advantages

Gelman puts forward a framework for thinking about multilevel models as a comprimise between no-pooling and complete pooling. For example, for the school example, one could fit a classical regression ignoring the schoolwide data, with students as the level of observation. (That would be "no pooling".) Alternatively, one could fit a separate regression for each school.

3.3 Tools for evaluating models

There are a couple ways we wish to evaluate our models. Most of the time, we will compare them to some other model.

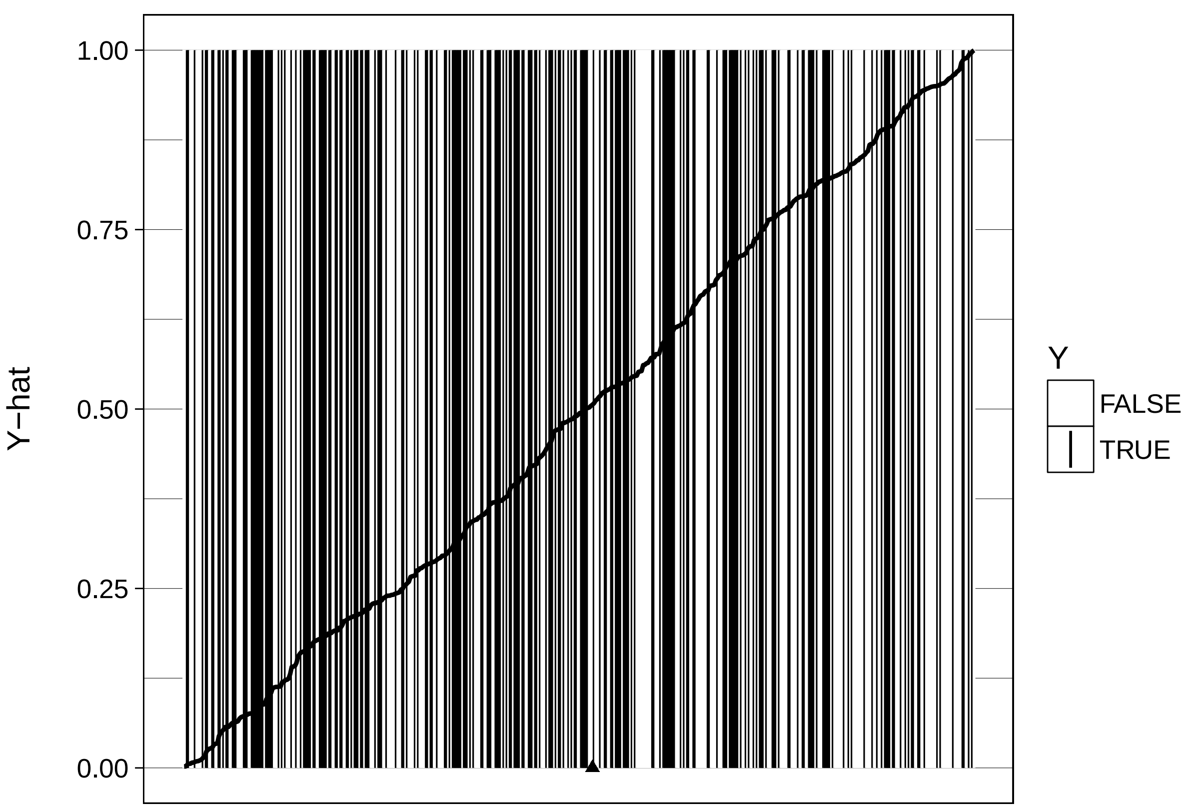
Predictive accuracy will be one aspect of our model we will want to evaluate. We will use cross-validation to evaluate accuracy, usually with K-fold cross-validation. Statistics such as misclassification rate, false-positive rate, and true-negative rate can be calculated for each validation. For a more comprehensive look at predictive accuracy, we use the separation plot.

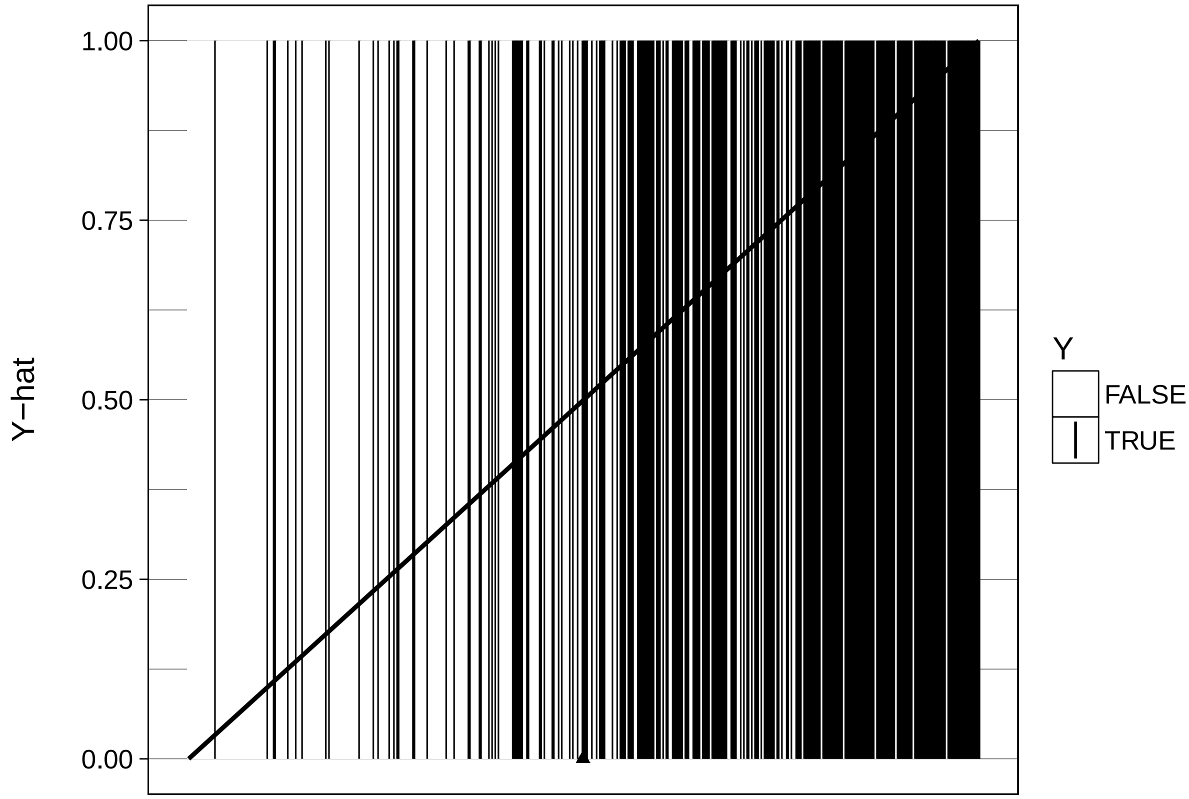
3.3.1 The Separation Plot

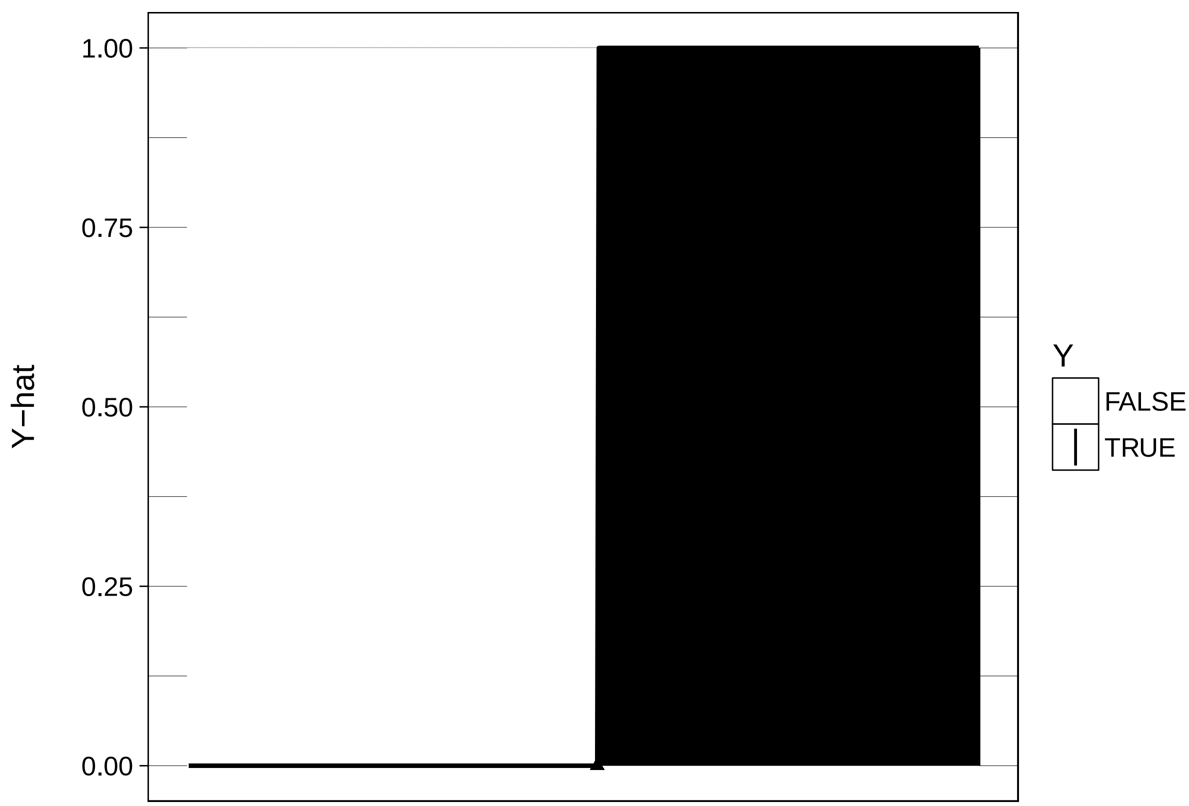
The separation plot, created by ______ in their paper ______, is designed to show how well a logistic regression model can distinguish between high and low probability events.

Creating a separation plot first requires a model fit to training data and testing data to evaluate predictive accuracy on. From the testing data, we need a vector Y of observed binary response data and a vector $\hat{Y}$ of predicted probabilities of a 1 for each observation, predicted using our model fitted to training data.

We plot the data $(Y, \hat{Y}$ as a sequence of vertical strips, colored according to observed outcome, Y, and ordered from low to high probability based on $\hat{Y}$. A curve is superimposed upon the stripes showing the $\hat{Y}$ as a line graph. And finally, a small triangle is placed indicated the point at which the two colors of lines would meet if all observations $Y = 0$ were placed to the left of all the $Y = 1$ observations; in other words showing where the boundary would be if the two classes were perfectly separated by the model.







Above we have examples of three separation plots. The first plot shows what it looks like when Y and $\hat{Y}$ are uncorrelated. The second plot

Chapter 4

Model 1: Rides and Riders

We start out model simply and then building up. The first problem to approach is handling rider variance. This sections describes how we do that using a random effects terms and demonstrates the improvement in the models fit and predictive accuracy over more classical models.

4.1 Choosing Ride-Level Parameters

4.1.1 Weather

4.1.2 Traffic

We would like to have some model for the effect of traffic. We don't have actual traffic data, but we do know that there are fluxuations in how riders rate rides based on time of day, with somewhat higher rates during the morning and evening. The simplist way to model this might be a rush hour indicator:

$$x_i^{\text{rush.hour}} = \mathbb{1}[x_i^{\text{time}} \text{during rush hour}],$$

where rush hour is defined as between 7 a.m. and 9 a.m. and 5 p.m. and 7 p.m. on weekdays. However this maybe a little too naive. We might also want to simply have a term that absorbed all time of day difference in ratings.

We might consider adding a term that is a sinusoidal function with a period of one day. We would be interested in fitting a term,

$$A \sin(T x^{\text{time}} + \phi),$$

where β and ϕ are coefficients estimated and $T = 2\pi/d$, where d is the period of one day (8.64×10^7 milliseconds.) This form isn't easy to estimate, but we can transform this expression into the sum of two trignometric functions:

$$
\begin{aligned}
A\sin(Tx+\phi) &= A\left(\sin(Tx)\cos(\phi)+\cos(Tx)\sin(\phi)\right) \\
&= (A+\cos(\phi))\sin(Tx)+\sin(\phi)\cos(Tx) \\
&= \beta_1\sin(Tx)+\beta_2\cos(Tx),
\end{aligned}
$$

where $\beta_1 = A + \cos(\phi)$ and $\beta_2 = \sin(\phi)$. This term won't model just traffic; it will probably absorb all effects that are cyclical over the day.

4.2 Adding Random Effects from Riders

4.3 Evaluating the Ride-Level Models

Chapter 5

Model 2: Regression Terms for Road Segments

Now we have the task of incorporating our knowledge of riders' routes into our regression. Our approach here will be to consider routes as sequences of discrete road segments, each of which have known properties. This is convenient because we have such data about roads that give us bike lanes, road size, etc. It is even possible for us to calculate popularity of particular segments easily.

Assume we have K total road segments in our road network and for each ride we have $\Omega_i \subseteq \{1, \ldots, K\}$, the set of road segments that are in the route of ride i. Let l_k be the length of the kth segment and define the length of ride i to be:

$$L_i = \sum_{k \in \Omega_i} l_k.$$

For the kth road segment, define the m-dimensional vector $W_k = W_k^1, W_k^2, \ldots, W_k^m$ road segment-level predictors. Then we shall define the term in our regression for the route of ride i as

$$R_i = \frac{1}{L_i} \sum_{k \in \Omega_i} l_k W_k \beta^{\text{road}},$$

Where β^{road} is a vector of coefficients for the road segment level predictors. When actually computing this value, it may be convenient to factor out the β^{road}

5.1 Choosing Segment-Level Parameters

5.2 Evaluating Segment-Level Models

Chapter 6

Model 3: A Spatial Model

Chapter 7

Comparative Evaluation

Conclusion

If we don't want Conclusion to have a chapter number next to it, we can add the `{.unnumbered}` attribute. This has an unintended consequence of the sections being labeled as 3.6 for example though instead of 4.1. The LaTeX commands immediately following the Conclusion declaration get things back on track.

More info

And here's some other random info: the first paragraph after a chapter title or section head *shouldn't be* indented, because indents are to tell the reader that you're starting a new paragraph. Since that's obvious after a chapter or section title, proper typesetting doesn't add an indent there.

References

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Gelman, A., & Hill, J. (2006). *Data analysis using regression and multilevel/Hierarchical models.* The Edinburgh Building, Cambridge CB2 8RU, UK: Cambridge University Press, New York.

Mackaronis, J. E., Strassberg, D. S., Cundiff, J. M., & Cann, D. J. (2013). Beholder and beheld: A multilevel model of perceived sexual appeal. *Archives of Sexual Behavior*.