Multilevel Models and Missing Data Models for Crowdsourced Bicycle Route Ratings

---

A Thesis
Presented to
The Division of Mathematics and Natural Sciences
Reed College

In Partial Fulfillment
of the Requirements for the Degree
Bachelor of Arts

Will Jones

May 2016

Approved for the Division
(Mathematics - Statistics)

Andrew Bray

Preface

Given cyclists' commuting bicycle routes and their ratings of those routes, how can we determine which infrastructure features are not working for bicyclists? This is the question that first fascinated me when I started this thesis. As someone just getting into statistics, it was a mind boggling problem at the time, and even now I still don't have a complete answer; it turned out to be a larger research question than could fit in one undergraduate thesis. But here I make some substantial steps forward, refining the question ("how can we model how ride ratings are influenced by route, as well as weather, who is riding, and when they are riding?"), addressing part of the question (the non-route part), and coming up with some ideas on how to proceed further.

This thesis represents a year's worth of hard work and the most enlightening educational experience I've had yet. It was a deep dive into how to do statistical modeling for a rich and complex data set with many open questions. During this undertaking, I couldn't be more thankful for the guidance of Andrew Bray, my thesis adviser.

The story of how I got interested in this data was just as much a story of Reedies helping Reedies as it was of fascination with technical challenges. I first encountered *Ride Report* during Rennie Meyers' '15 presentation of her final project in the Introduction to Data Science class taught by Albert Kim in the spring of 2016. In a turn of luck, I found myself during the following summer interning at Switchboard (more formally known as Weathergram, Inc.), who shared an office with Knock Software, the creators of the *Ride Report* app. During that summer I got to talk with William Henderson '08 and Evan Heidtmann, the two developers at Knock, and became fascinated with the open questions in the data. I have to thank Rennie Meyers for sharing what she learned and Switchboard introducing me to William and Evan, but most of all thesis would not be possible without the generosity of William and Evan. They not only allowed me to examine the data they are obligated to keep as private as possible, but hosted me for hours in their office while I ran and debugged my models.

Few feats at Reed are possible without the support of loved ones, and I've been especially fortunate here. I got through this semester in no small part because of the times I spend sharing great food with my friend Xian; the loving reassurance and support of my partner Kiki; and, of course, the love and financial support of my mom Kathleen.

Table of Contents

List of Tables

List of Figures

Abstract

Crowdsourced ratings are an increasingly important data source, leveraging the abundance of internet connected consumer devices to boost sample sizes. In this paper, we examine a data set of crowdsourced bicycle route ratings in Portland, OR collected by the *Ride Report* app. We fit multilevel models that show ratings are best described by models with random intercepts by rider. We also show that the majority of variation in ride ratings across time of day is owed to patterns in who is riding, rather than any effect particular to that time of day, such as traffic. A brief exploration in clustering shows that some trends in cyclist's ride length and time of day routines can be picked out, but that these patterns do not provide much useful information for predicting rider ratings. Finally, we develop models that can adjust for non-ignorable missing ride ratings, but caution that their use for inference is inappropriate until the data quality of unrated rides can be assured.

Introduction

Knock Software's *Ride Report* app combines a simple thumbs-up/thumbs-down rating system with GPS traces to compile a crowdsourced data set of commuter bicycle rides. Knock's goal is to use this data to help cities identify the most problematic routes in their infrastructure and help cyclists identify the best routes in their area.

From the user's perspective, the app that collects the data is simple: *Ride Report* automatically detects when a user starts riding their bike, records the GPS trace of the route, and then prompts the user at the end of the ride to give either a thumbs-up or thumbs-down rating. From this, they were able to create a simple "stress map" of Portland, OR, which displays the average ride rating of rides going through each discrete ride segment.

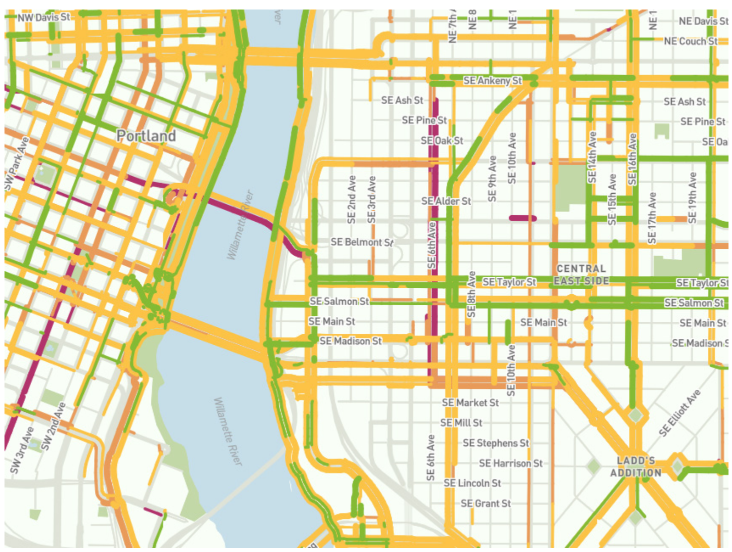


Figure 1: Ride Report's Stress Map for Portland, OR. Greener road segments incidate less stressful streets while more red segments indicate more stressful streets. "Stress" is computed by taking the average rating for each segment.

The app is designed to minimize barriers to response in order to maximize sample size, at the expense of ensuring quality and consistent responses. It automates all of the data collection except the rating, and the rating only requires (and allows) a binary response. There is no direct prompting from the app indicating what criteria cyclists should be using to evaluate their rides, other than the labels for the binary ratings. In addition, riders' rating their rides on *Ride Report* are volunteers, so they are under no obligation to rate their rides. In fact, most of the rides lack ratings, and we have no guarantee that the pattern of missing ratings is ignorable.

The end goal of collecting and studying this data is to be able to accurately map which roads are not serving bicycle commuters well. This paper makes steps toward this goal by building models that predict ride rating based on information other than route. (We did not create models that involve routes, but we do propose ways of modeling routes in Chapter 6.) The models we discuss, besides measuring the effect of weather, time of day, and ride length on the probability of a negative rating, address two nontrivial issues in this data: the variability in how different riders rate their rides and the problematic missing ratings.

We are not the first to worry about the issue of variability in riders interpretations of what a good or bad route is. As Meyers, a previous researcher examining the *Ride Report* data observed, "everyone has different standards for what a 'good' or 'bad' ride is, and the data might benefit from randomized IDs attached to each cell device."[1] Thankfully, *Ride Report* does keep track of which rides belong to each to rider. We model the varying overall tendencies of each rider to rate a ride negatively with random intercepts in a multilevel model. For example, if we let y_i be the rating of the ith ride and X_i be the ride-level variables, then we can fit a regression:

$$\mathbb{P}(y_i = 1) = \text{logit}^{-1}\left(\alpha_{j[i]} + X_i\beta\right),$$

where α_j is an intercept specific to rider j. In addition, the rider intercepts come from a common distribution,

$$\alpha_j \sim N(\mu_\alpha, \sigma_\alpha^2),$$

where μ_α is the mean of all the α_js. Similar models have been used in situations when data consist of subjective ratings, including one study examining how people rate sexual attraction[2].

Missing ratings are another important problem in this data set. While we have the route they chose and all associated covariates, the response variable (rating) is missing for many rides. As we will discuss in Chapter 5, the pattern of non-response is likely to be correlated with the rating the rider would have given, which may mean our parameter estimates are inaccurate. To address this problem, we implement a version of the expectation maximization algorithm for missing data, creating a model that simultaneously estimates the missing data mechanism and the ride rating model.

[1] Meyers (2015)
[2] Mackaronis, Strassberg, Cundiff, & Cann (2013)

Chapter 1

Data Sources

We combine several data sources to do our analysis. Information about individual rides, including the GPS trace, the rider, and start timestamp were provided by Ride Report. Weather data were collected from Weather Underground's archive of the KPDX weather station and a Portland Fire Bureau station.

Our goal in this chapter is to discuss these data and what considerations we should have in mind before exploring it in depth. This includes how and by whom the data were collected, who and what this data are representative of, and what samples were taken of the data.

Some of these considerations, such as the limited demographics represented in the Ride Report data, pose serious limitations to how our inferences can be generalized. Others, such as the large number of missing responses in the Ride Report data, motivate the analysis we are doing in this thesis. Finally, there are other considerations which we will acknowledge here, but addressing them is out of the scope of this thesis. This data set contains an abundance of potential research questions, only a fraction of which could be reasonably addressed in one thesis.

1.1 Ride Report

Ride Report's data is the focus of this paper. Knock Software created the app to collect large amounts of information about urban cyclists' routes and experiences on those routes. The hope is that this information will be valuable to city planners.[1]

Ride Report's approach to crowd sourcing these data is particularly important to understand. The app automates every piece of the data collection process except for the rating given by the rider. Thus, the app casts aside nuanced and (somewhat) reliable human input in favor of increasing sample size: one could imagine a similar app where users have more control over how the route is recorded, have the ability to rate on a more fine-grained scale, and are given more direction in what they are rating for. This trade off causes two problems with the reliability with the data.

Before we get into the potential issues in the data collection, though, let's examine

[1] Knock's other project is making a cheaper bicycle counter for cities to monitor traffic flow, again intended to be sold to cities wishing to improve bike infrastructure.

the data collection process itself. When installed on a person's phone, the Ride Report app attempts to automatically detect when the user starts riding their bicycle, based accelerometer data, when a user leaves a familiar Wi-Fi network, and some other pieces of information. When the app detects the start of a ride, it starts recording a GPS trace. At the end of the users ride, the app detects them getting of their bike (in a similar process to how it detected the start of a ride) and prompts them to give a rating of the ride. The ride data are saved then, even if the user does not provide a rating.

This automatic detection of when a ride starts and stops leads to two related and common errors in the dataset: first, one ride is often split into two or more rides at points, such as at a stoplight or a train crossing, where a cyclist stops for an extended period of time; second, car rides are sometimes misclassified as bicycle rides and vice-versa (car rides are not rated.) The app allows riders to correct the misclassification, but provides no way to join split rides back together.

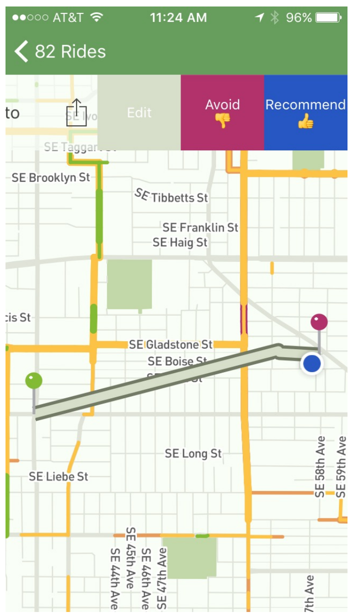


Figure 1.1: The Ride Report app's interface has changed significantly between versions, including the rating text displayed after a ride. This is the current version as of Februrary 2015.

The app only recently became publicly available and has undergone significant changes in the course of its life. In particular, while the ratings have always been binary, the labels have changed at various points in time. At one point the rating labels were "Stressful" and "Chill", while now they are labelled "Recommend" and "Avoid" (see Figure 1.1). Other fundamentals of the data collection process—such as the binary ratings, the automatic collection of GPS traces of routes—have remained constant.

The data collection method itself has some problems, but there also may be some biases in the population of riders using the app. The app is only available on iOS, so only iPhone owners could use this application which may imply a bias toward riders of higher socioeconomic status. At the time of the start of the thesis, the app was in private beta, meaning only people who actively sought out using the app were able to use it. Now the application is public and on the Apple App Store, making it more widely available. Due to these issues, many of the earlier rides may be people within the developer's personal network. Unfortunately, it's hard to make any solid conclusions about the users of the app because Ride Report doesn't collect any demographic data about their riders.

One other issue with the Ride Report data guided our analysis: privacy. Because the data involves time stamps and GPS locations of people's commutes, the data is very sensitive: one could easily infer someone's home and workplace based on their most common routes. In fact, this data is protected by an end-user license agreement (EULA) which prevents sharing of data, without the explicit permission of those involved. This presented a logistical challenge: how were we to do inference and data exploration without access to the data?

By agreement with Knock Software, identifying data must be kept private. With permission from five riders, Knock was able to give us a small subset consisting of all the rides from those five riders, to be kept confidential. That is the data set we used for prototyping models and some basic exploratory analysis. Knock also agreed to allow us to run models fitting scripts on larger samples of their data set, as long as they were performed on their computers, with no identifying data leaving their system.

While at first this set up seems like an inconvenience, it actually has some advantages. One of the pitfalls of having an entire data set, especially a high dimensional one, is that in performing exploratory analyses it is often too easy to find spurious "statistically significant" results. Instead, we must come up with our models before running them, greatly limiting the choices we can make in the "garden of forking paths."[2]

1.2 Weather Data

Slippery roads and formidable winds are no fun for anyone balancing on a two-wheeled vehicle. Weather is, then, one of the most obvious family of predictors for ride rating, at least intuitively. We use the time of a ride to join in data about the weather conditions during the ride, including

- the temperature,
- whether and how much it is raining,
- whether the roads are wet or have puddles,
- wind and gust speed.

We include the first two, temperature and precipitation, to account for rider

[2] "The garden of forking paths," a reference to the short story by Jorge Luis Borges, is a term coined by Andrew Gelman to refer to the infinite number of choices researchers have in analyzing a set of data, which often allows for enough flexibility to discover coincidences ("The garden of forking paths," 2013).

comfort. A sweltering, frigid, or stormy day could make an unpleasant experience for a bicyclist and thus could lead to more negative ratings.

On the other hand, we include the last two, wet road and gust speed, as factors that impact safety. During and after storms, puddles often accumulate in bike lanes before the center of the road, pushing cyclists into lanes shared by cars, which are often more dangerous.

Gust speeds impact the aerodynamics of a ride, which are particular important for bicyclists. It's one of the main reasons cyclists care about getting into lower (and more aerodynamic) rider positions. Thus, high wind or gust speeds may affect rider rating.

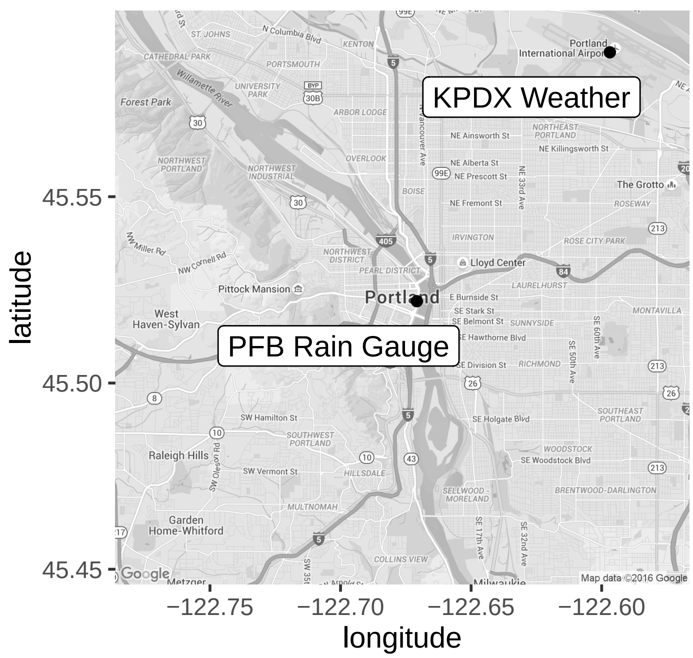


Figure 1.2: Locations of weather data collection sites. Daily weather information was collected at the KPDX weather station at Portland International Airport. Hourly precipitation data were collected at the Portland Fire Bureau's rain gauge in downtown Portland.

We are limiting our study to rides in Portland, Oregon. Given this, we can first assume that it may be reasonable to expect that riders are used to the same climate, and thus have somewhat similar responses to weather. This also makes it reasonable to use data from one nearby weather station, rather than attempting to collect from several stations and creating a spatial model for weather.

For daily summaries of weather conditions, we used weather history from the KPDX weather station at Portland International Airport downloaded from Weather Underground.[3] From this we were able to get daily weather data, including

- Average, minimum, maximum temperature for the day.

[3]Weather Underground (2016)

- Total precipitation.
- Mean wind speed, as well as gust speed (speed of brief, strong winds.)

We also got hourly rainfall data from a data stream at the Portland Fire Bureau Rain Gage at 55 SW Ash St.,[4] which is just about the geographic center of Portland. This just gives raw uncorrected rain gauge data, but gives us a fine grain look at how much rain there has been recently.

For daily weather data, such as temperature highs and average wind speed, we use information from the KPDX weather station. It is further from the geographic center of the rides we are examining, but because the weather is daily summary statistics, we don't expect closer weather stations to be much more informative. Figure 1.2 shows the geographic positions of these two stations.

1.3 Notation for the Joined Data Set

We combined the ride records with weather data by joining by start date stamp of the ride. We will denote our set of ride-level predictors, each of which is an n by 1 column vector as,

- x^{length}, log ride length, scaled to have mean 0 and standard deviation of 1
- x^{rain}, rainfall during hour of ride, in tenths of inches
- x^{rain4h}, rainfall during past four hours before ride, in tenths of inches
- x^{wind}, mean wind speed for the day, in miles per hour
- x^{gust}, max gust speed for the day, in miles per hour[5]
- x^{temp}, average temperature, in degrees Fahrenheit for the day.

We will often represent this set of predictors as the ride-level predictor matrix $X = (x^{\text{length}}\ x^{\text{rain}}\ x^{\text{rain4h}}\ x^{\text{wind}}\ x^{\text{gust}}\ x^{\text{temp}})$. We also have the predictor $t \in [0, 24]^n$, representing the time of day of the ride, measured by hours since midnight. (Because it must be modelled in a different fashion than the other variables, we use the simple notation of a single letter for it.)

Let $y_i = 1$ if the ith ride received a negative rating, and $y_i = 0$ if it received a positive rating, for $i = 1, \dots, n$. Choice of coding which events are 0's and which are 1's is arbitrary when making logistic regression models, though we made this choice because for the sake of our analysis, negatively rated rides are more interesting events. For urban planning applications, they define the areas that need attention.

[4] Portland Bureau of Environmental Science (2016)

[5] Gust speeds are the max speeds of winds that are fast, highly variable, and short-term. For METAR weather stations, which the KPDX station is, gust speeds report the maximum wind speed when there were rapid fluctuations in wind speed with at least 10 knots in the difference between the lows and highs.

Chapter 2

Methods

We use many statistical methodologies in this paper. We outline here the central methods used, both to familiarize the reader and to establish the notation we use throughout this paper. The models we present combine logistic regression, multilevel[1] models, additive models, and smoothing splines. We also make use of two recently developed graphical model evaluation tools: the separation plot and the heat map plot.

The one large methodology not covered here is the expectation maximization algorithm we use to model the missing data mechanism for missing ride ratings. The theory of that algorithm is outlined in Chapter 5 and an example of its implementation can be found in Appendix A.

2.1 Logistic Regression

Statistical models are often split into regression models—models with a quantitative response—and classification models—models with a categorical response. Thus, it may seem odd that we are using a regression model when our response variable, ride rating, is a binary outcome.

But, when modeling a binary variable Y, we consider it a Bernoulli random variable,

$$Y \sim \text{Bernoulli}(p),$$

where p is the probability the response is 1 and $1 - p$ is the probability the response is 0. So our response variable Y may be binary, but the primary quantity of interest behind that outcome is p, a quantitative variable. This is why we consider logistic regression a regression model.

Logistic regression is one form of a generalized linear model. Recall that in linear regression we use data with response variable y_i and j predictors $x_{i1}, \ldots, x_{ij}$ to fit the best-fitting linear function

$$y_i = \beta_0 + \beta_1 x_{i1} + \cdots + \beta_j x_{ij} + \epsilon,$$

[1]Multilevel models are often referred to as hierarchical models or mixed effects models.

where $\epsilon \sim N(0, \sigma^2)$, by estimating $\beta_0, \ldots, \beta_j$. We can equivalently write,

$$y_i \sim N(\beta_0 + \beta_1 x_{i1} + \ldots + \beta_j x_{ij}, \sigma^2).$$

We could, in fact, try to predict p with a linear regression, though such a model will always have the problem of predicting probabilities outside of the range of $[0, 1]$. That's not a recipe for simple interpretation or reliable predictions. A generalized linear model uses a "link function," g, to modify the regression so the range of the response more accurately reflects the practical range of the variable; i.e. model:

$$g(y_i) = \beta_0 + \beta_1 x_1 + \ldots + \beta_j x_j + \epsilon_i.$$

In logistic regression, the "link" function is the logit function, $\text{logit} : [0, 1] \to \mathbb{R}$,

$$\text{logit}(p) = \log\left(\frac{p}{1-p}\right).$$

This function is also known as the log-odds, because odds are defined as $p/1 - p$ for any probability p.

So in logistic regression, we model the probability of $y_i = 1$ as,

$$\mathbb{P}(y_i = 1) = \text{logit}^{-1}(\beta_0 + \beta_1 x_1 + \ldots + \beta_j x_j).$$

Notice that the inverse logit function[2] maps values from $\mathbb{R}$ to $[0, 1]$. Thus, the function provides a convenient way to map linear combinations of other variables onto values that are valid probabilities. Other such functions exist and are also used for regressions with binary responses, such as the probit function. Logistic regression, however, is easier to interpret—because of the odds connection—and more efficient to compute.

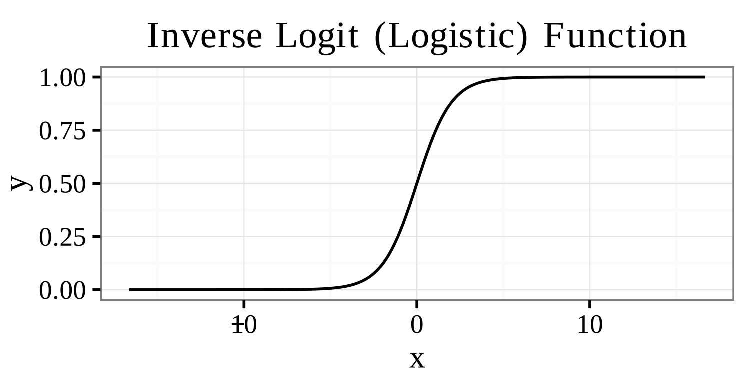


Figure 2.1: The inverse logit function gives a convenient way to map linear combinations of real numbers to valid probability values.

Though coefficients from a logistic regression can't be interpreted the same way as a linear model, they do have a convenient interpretation. Because the linear part

[2] sometimes called the logistic function

of the model represents log odds, the coefficients are log odds ratios; That is, the exponentiated coefficients $e^{\beta_1}, \ldots, e^{\beta_j}$ represent the multiplicative effect a one-unit increase in the corresponding predictor has on the odds. For example, if we fit a simple logistic regression with $\mathbb{P}(Y = 1) = \text{logit}^{-1}(\alpha + \beta + X)$, we could interpret $e^{\beta} = 1.1$ as meaning that a one unit increase in X gives a 10% increase in the odds that $Y = 1$.

2.2 Hierarchical Models and Mixed Effects Models

Data often contain hierarchies. For example, a set of students' test scores may contain the hierarchy of districts and schools those students attend. Or a set of soil samples may have been taken at several distinct sites, thus having a hierarchy of sites and samples. In the bike ride data we examine, there is the hierarchy of riders and rides.

We will talk about different "levels" of variables corresponding to places in this hierarchy. When we refer to "ride-level variables," we refer to variables that are specific to a ride, whereas we refer to "rider-level variables" as those specific to the rider, and thus also all the rides that rider takes. For example, we consider length a ride-level variable and total number of rides taken a rider-level variable.

We will also discuss road segment-level variables, which are variables that are specific to the road segments in the route of a ride (e.g. length, presence of bike lanes, etc). But there isn't a clear road segment-ride hierarchy: each ride contains multiple road segments and each road segment is contained by multiple rides. Thus, this isn't a case where multilevel modeling is applicable. (The ideas behind it, though, may be fruitfully adapted, as we discuss in Chapter 6)

Gelman describes two traditional ways of dealing with hierarchical data that multilevel models contrasts with: "complete pooling" and "no pooling."[3] In "complete pooling," we ignore the group-level variables, and give identical estimates for parameters for every group. In "no pooling," we do entirely separate regressions for each group. Multilevel models are a compromise between these extremes ("partial pooling", as Gelman calls it) where all the data are considered in a single regression with some parameters shared between groups and some different between groups.

These multilevel models work for other forms of regression, but we will focus on logistic regression, as it is the method we use in this paper. We will be using notation adapted from Gelman and Hill's description of multilevel models.[4] Consider a data set composed of

- n observations of a binary response variable y_i, $i \in 1, \ldots, n$,
- p observation-level predictors $X_i = x_i^1, \ldots, x_i^p$,
- j groups into which the observations are split,
- l group-level predictors $U_{j[i]} = u_{j[i]}^1, \ldots, u_{j[i]}^l$, where $j[i]$ is the group of the ith observation.

We could fit a model where the intercept varies by group:

[3] Gelman & Hill (2006), p. 7
[4] Gelman & Hill (2006), p. 251–252

$$y_i \sim \text{Bernoulli}\left(\text{logit}^{-1}\left(\alpha_{j[i]} + X_i\beta\right)\right), \tag{2.1}$$

$$\alpha_{j[i]} \sim N(\gamma_0 + U_{j[i]}\gamma, \sigma_\alpha^2), \tag{2.2}$$

where $\alpha_{j[i]}$ is the intercept for the jth group, β is a vector of coefficients for the observation-level predictors, γ_0 are the group-level intercepts, and γ is a vector of coefficients for the group-level predictors. We could also specify a similar model where there are no group-level predictors, such that we simply have different intercepts for each group,

$$\alpha_{j[i]} \sim N(\gamma_0, \sigma_\alpha^2). \tag{2.3}$$

We can also consider a model that has slopes varying by group. For simplicity, let's consider just one observation-level predictor, x_i, that will have varying slopes $\beta_{j[i]}$ as well as one group-level predictor, u_j. We could specify the model as,

$$y_i \sim \text{Bernoulli}\left(\text{logit}^{-1}(\alpha_{j[i]} + \beta_{j[i]}x_i)\right), \tag{2.4}$$

$$\begin{pmatrix} \alpha_j \\ \beta_j \end{pmatrix} = N\left(\begin{pmatrix} \gamma_0^\alpha + \gamma_1^\alpha u_j \\ \gamma_0^\beta + \gamma_1^\beta u_j \end{pmatrix}, \begin{pmatrix} \sigma_\alpha^2 & \rho\sigma_\alpha\sigma_\beta \\ \rho\sigma_\alpha\sigma_\beta & \sigma_\beta^2 \end{pmatrix}\right). \tag{2.5}$$

These models can be fit with maximum likelihood estimation using the `lme4` package in *R*(Bates, Mächler, Bolker, & Walker, 2015) or can be fit with Bayesian MCMC using *Stan*(Carpenter et al., 2016). The latter has the advantage of making it easy to estimate group-level uncertainty at the expense of more computation. We fit models using `lme4`, but make use of *Stan* when we have ride-level parameters we want to estimate, in Section 4.2.

2.3 Additive Models and Smoothing Splines

Often, it is helpful to allow more flexibility in the functional forms in the models. While parametric models, like logistic regression, assume a particular form for the relationship between the variables and response, nonparametric models use the data to determine both the functional form and values of the parameters in models. However, the curse of dimensionality (the more predictors that are in a model, the fewer similar observations there are to any observation) can impair nonparametric models. Additive models, however, are able to keep a lot of the flexibility of nonparametric methods while avoiding the curse of dimensionality. Additive models assume that the response is the sum of functions of each of the predictors:

$$\text{logit}(\mathbb{P}(y_i = 1)) = \alpha + \sum_{j=1}^{p} f_j(x_{ij}).$$

These functions can be linear, so generalized linear regression is a subset of additive models. But more interestingly, these functions can be non-parametric.[5] One of the most common types of functions fit are smoothing splines.

Smoothing splines are essentially cubic functions stitched together at points called "knots" such that the full piece-wise function is continuous and has continuous first and second derivatives. One can further define cyclic cubic splines, which simply have the constraint that the last knot and first knot be treated as the same knot, thus allowing a continuous cyclic function to be fit.[6]

Computation of multilevel additive models with splines is available in the `gamm4` package (Wood & Scheipl, 2014).

2.4 Tools for Evaluating Models

After fitting our models, we will want to know how each of our models compare. Did adding a particular term enhance or diminish the accuracy of our model? Instead of focusing on one measure of fit we use several. Log likelihood and AIC provide useful summaries of fits to the data based on the likelihood function. Separation plots—which we discuss in the next section—allow us to assess the predictive ability of a model; in particular, can the model identify high and low probability events? We also use the area under the ROC curve (AUC) measure popular for assessing logistic regression. In particular, we compute 10-fold cross validated ROC statistics to detect if models are overfitting to the data.

2.4.1 The Separation Plot

The separation plot, created by Greenhill, Ward, and Sacks[7], is designed to show how well a logistic regression model can distinguish between high and low probability events.

Let y be a vector of observed binary response and $\hat{y}$ a vector of predicted probabilities of a 1 for each observation, predicted by some model. Then we can construct the plot as follows: We plot the data $(y, \hat{y})$ as a sequence of vertical stripes, colored according to observed outcome, y, and ordered from low to high probability based on $\hat{y}$. A curve is superimposed upon the stripes showing the $\hat{y}$ as a line graph. And finally, a small triangle is placed indicated the point at which the two colors of lines would meet if all observations $y = 0$ were placed to the left of all the $y = 1$ observations; *i.e.* showing where the boundary would be if the two classes were perfectly separated by the model.

[5] How are these models fit? Using what's known as the Backfitting Algorithm. We define the kth partial residuals $Y^{(k)} = Y - \left(\alpha + \sum_{j \neq k} f_j(x_j)\right)$. (That is, define the portion of Y leftover for $f_k(x_k)$ to fit to after the other f_j's have had their share.) Then, iteratively fit each of the functions f_j on the partial residuals $Y^{(j)}$ until each of the functions converge. For a further quick look at additive models, check out Cosma Shalizi's lecture notes (Shalizi (2013a))

[6] For a brief and entertaining introduction to smoothing splines, see Shalizi (2013b). For a more in-depth look at splines, check out Wood (2006)

[7] Greenhill, Ward, & Sacks (2011)

Figure 2.2: Examples of three separation plots. The first plot shows what it looks like when y and $\hat{y}$ are uncorrelated. The second plot shows a fairly good model, where the y are generated as Bernoulli($\hat{y}$). The third plot shows a model where the responses are fully separated.

Separation plots don't do well with larger sample sizes: if there are too many observations, it becomes difficult to read. There are several ways around this, but we choose to randomly sample the observations.

Chapter 3

Modeling Rides and Riders

Complex statistical models can accurately model intricate processes. But they also run the risk of overfitting to the data. To avoid this, we build up our models from simple to complex, comparing the models with cross validation to make sure the complexities introduced add real value.

In this chapter we focus on building models that incorporate information about rider, weather conditions, time of day, and ride length. In brief, our models start with a logistic regression model considering only ride-level variables, and formulate more complex models by adding various terms. Table 3.1 describes each model briefly along with the models label.

Table 3.1: Brief descriptions of Models 1–6

Model	**Description**
Model 1	(Baseline) logistic regression
Model 2	Add rider intercepts
Model 3	Add trigonometric terms for time of day
Model 4	Additive model with cubic cyclic spline for time of day
Model 5	Additive model with spline for ride length
Model 6	Remove random rider intercepts from Model 4

3.1 Six Models for Probability of a Negative Ride Rating

Model 1, which we will use as the baseline for comparing further models, is a logistic regression model:

$$\mathbb{P}(Y_i = 1) = \text{logit}^{-1}(\alpha + X_i\beta),$$

where $\alpha \in \mathbb{R}$ and $\beta \in \mathbb{R}^p$ are parameters to be estimated. (X is the matrix of ride-level predictors specified at the end of Section 1.3.)

Riders appear to have different tendencies to rate rides negatively more often, as

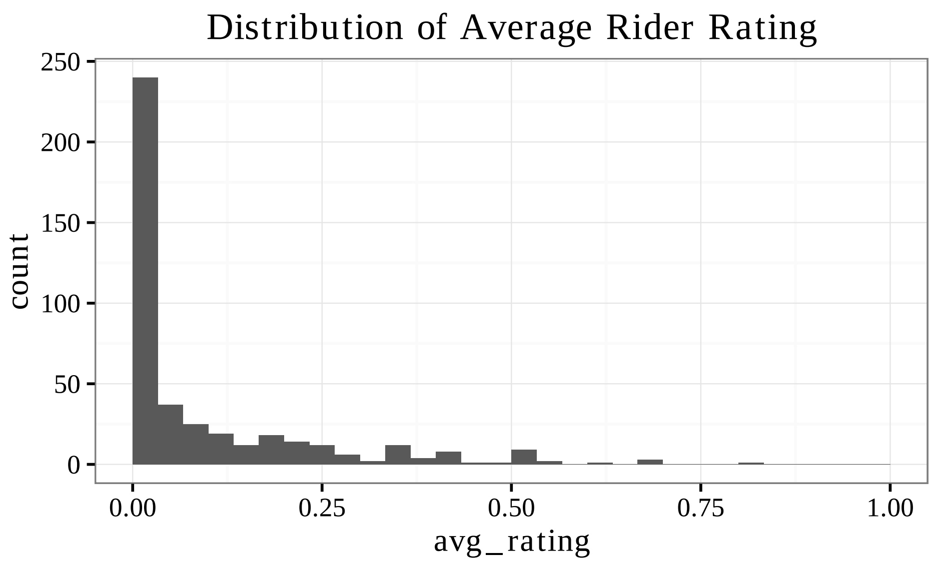


Figure 3.1: The overall rates at which each rider gives a negative rating for a ride varies greatly. This is our primary motivation for including rider intercepts and predictors.

we note in Figure 3.1. In fact, many riders give zero or nearly zero negative ratings. For **Model 2**, we account for this variability by adding intercepts that vary by rider:

$$y_i \sim \text{Bernoulli}\left(\text{logit}^{-1}\left(\alpha + \alpha_{j[i]} + X_i\beta\right)\right), \tag{3.1}$$

for $i = 1, \ldots, n$.

Rider intercepts themselves aren't as interesting as how they deviate from the mean, so we keep a fixed intercept α and constrain the rider intercepts, α_j, by specifying

$$\alpha_j \sim N(0, \sigma_\alpha^2).$$

Starting with **Model 3**, we address time of day, $t \in [0, 24)$ as a predictor. (We measure time of day in hours since midnight.) We use time of day to account for all the daily trends that may affect ratings, including as a simple way to model the overall traffic level, which is difficult to model on its own. These patterns are cyclic and very non-linear, so we can't model time as a linear term. One approach is to add sinusoidal terms with a period of one day. We would be interested in fitting a term,

$$\beta \sin(Tx^{\text{time}} + \phi).$$

Estimating β wouldn't be hard: we can easily estimate coefficents of transformed terms; it's more difficult to estimate T and ϕ. But, we know that we want to restrict our terms to fitting trends that happen over the course of one day, so we can set $T = 2\pi/d$, where d is 24 hours or some fraction of that.

As for ϕ, a trigonometric transformation reframes the estimation of a phase shift parameter into the estimation of two coefficients for trigonometric functions with no phase shift:

$$
\begin{aligned}
\beta \sin(Tx + \phi) &= \beta \left(\sin(Tx)\cos(\phi) + \cos(Tx)\sin(\phi)\right) \\
&= \beta \cos(\phi)) \sin(Tx) + \sin(\phi)\cos(Tx) \\
&= \beta_1 \sin(Tx) + \beta_2 \cos(Tx),
\end{aligned}
$$

where $\beta_1 = \beta\cos(\phi)$ and $\beta_2 = \sin(\phi)$. At this point, we are now just estimating the coefficients of a couple of transformed variables, which can easily be done in any package that does generalized linear regressions.

We also want to take into account that weekday hourly patterns may be different than weekend patterns. We use a variable X^{weekend} that serves as a weekend indicator. For Model 3, we add two sets of sinusoidal terms: one set for weekdays and one for weekends. More explicitly, we define the model,

$$
\begin{aligned}
\mathbb{P}(Y_i = 1) = \text{logit}^{-1}(\alpha &+ \alpha_{j[i]} + X_i\beta \\
&+ X^{\text{weekend}} \cdot [\beta^{t1}\sin(T\cdot t) + \beta^{t2}\cos(T\cdot t) \\
&+ \beta^{t3}\sin(T/2\cdot t) + \beta^{t4}\cos(T/2\cdot t)] \\
&+ (1 - X^{\text{weekend}}) \cdot [\beta^{t1}\sin(T\cdot t) + \beta^{t2}\cos(T\cdot t) \\
&+ \beta^{t3}\sin(T/2\cdot t) + \beta^{t4}\cos(T/2\cdot t))].
\end{aligned}
\tag{3.2}
$$

For **Model 4** we abandon parametric methods and use a cyclic non-parametric smoother to model time of day, making our model,

$$
y_i \sim \text{Bernoulli}\left(\text{logit}^{-1}\left(\alpha + \alpha_{j[i]} + X_i\beta + X^{\text{weekend}} \cdot f^{\text{time.w}}(t_i) + (1 - X^{\text{weekend}}) \cdot f^{\text{time}}(t_i)\right)\right), \tag{3.3}
$$

for $i = 1, \ldots, n$, where α_j is specified like Model 2 and $f^{\text{time.w}}$ and f^{time} are cyclic cubic spline terms for weekend and weekday rides, respectively, with knots at 0, 3, 6, 9, 12, 15, 18, 21, and 24 (0, again) hours.

Model 5 extends Model 4 by adding a cubic spline for ride_length:

$$
y_i \sim \text{Bernoulli}\left(\text{logit}^{-1}\left(\begin{array}{l}\alpha + \alpha_{j[i]} + X_i\beta + f^{\text{length}}(x_i^{\text{log.length}}) + \\ X^{\text{weekend}} \cdot f^{\text{time.w}}(t_i) + (1 - X^{\text{weekend}}) \cdot f^{\text{time}}(t_i)\end{array}\right)\right), \tag{3.4}
$$

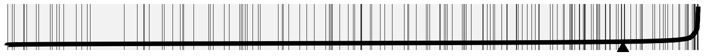
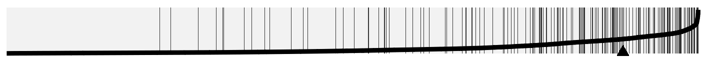
for $i = 1, \ldots, n$, where f^{length} is a cubic spline smoother.

Finally, **Model 6** is identical to Model 5, but without the rider intercepts:

$$
y_i \sim \text{Bernoulli}\left(\text{logit}^{-1}\left(\alpha + X_i\beta + f^{\text{time}}(t_i)\right)\right), \tag{3.5}
$$

for $i = 1, \ldots, n$, where f^{time} is a cyclic cubic spline term, with the same knots as in Model 4.

Table 3.2: Fit summaries for Models 1–6.

Model	**Separation Plot**	$\log(\mathcal{L})$	**AIC**	$\textbf{AUC}_{\text{CV}}$[1]
Model 1		-4,786	9,586	0.552
Model 2		-3,971	7,957	0.797
Model 3		-3,923	7,877	0.802
Model 4		-3,930	7,870	0.802
Model 5		-3,928	7,878	0.803
Model 6		-4,713	9,455	0.601

3.2 Model Evaluation

To fit the data, we got all of the rides in Portland, OR, from December 3, 2014 to February 8, 2016 for riders that had over 20 rated rides. (We only look at riders with a certain number of rides to make sure we get can get estimates for rider-level parameters, like rider random intercepts, that aren't too uncertain.) There were 25,397 rides, 14,032 of which were rated. Overall, 10.88 percent of these rides were given a negative rating. There were 138 riders in the data set.

The separation plots in Table 3.2 give a clear initial picture of how these model fits compare. Model 1 performs very poorly compared to those that include rider intercepts, assigning the same probability to most observations. Models that include the rider intercept perform similarly to each other. The log likelihoods and AIC,[2] shown in Table 3.2, corroborate this. Adding time dependency doesn't seem to impact predictive ability. We will see later, however, that it gives a fascinating result to interpret.

The gains from the rider intercepts are great, but we are compelled to ask: how much of that gain could have been achieved with randomly chosen groups? In other words, if riders were randomly assigned to rides, would the flexibility in the model created by allowing intercepts to vary increase predictive performance to the same degree? To test this, we ran a Model 4 after we randomly assign the rides a rider, by randomly permuting the rider column. This quick test nullified this skepticism, as you can see in the resulting separation plots in Figure 3.2.

3.3 Model Results

Table 3.3 presents the fixed effect estimates for our models. Length has a robust strong negative effect. This makes sense if we think of length as the only information

[1] Area under ROC curve estimated with 10-fold cross-validation.

[2] Akaike Information Criterion (AIC) is a metric that penalizes the $\log(\mathcal{L})$ with the number of parameters estimated k, with lower values being better. It is defined as $\text{AIC} = 2k - 2\log(\mathcal{L})$.

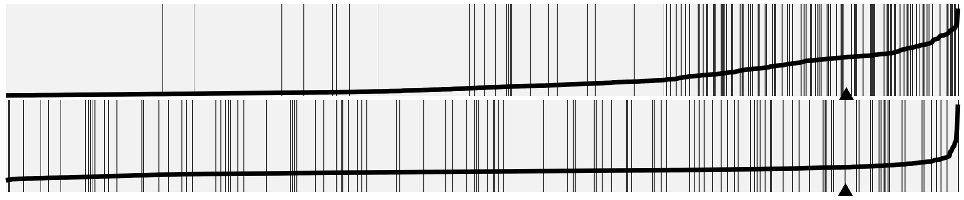


Figure 3.2: Separation plots for models 2 compared to a similar model where riders are randomly assigned to rides. This test demonstrates that the improvement in predictive performance provided by the rider intercepts was not coincidence.

we have about route in these models: it seems routes that are longer tend to be less likely to be rated negatively. Perhaps longer rides tend to be for leisure or sport rather than commuting, so and so are less likely to go along routes with high traffic and other dangers. Temperature also seems to have a small effect. It could be the case that the temperature itself is important, or perhaps season is a confounding factor. It could be the case that the type of rides taken during the warmer months are more likely to be rated negatively. Wind and gust speed don't seem to have robust effects. These models suggest that four hour cumulative rainfall was more important than rainfall during the hour of the ride. This supports our suspicion that weather effects that are more relevant to safety than comfort— like wet road and puddles rather than rain at the time of a ride—are important in determining a cyclist's rating of their ride. These coefficients, however, aren't nearly as enlightening as the time of day fits.

The marginal fits for time of day, shown in Figure 3.3, are predictable. On weekdays, the probability of a negative rating peaks in the afternoon from 4–6 p.m., around when we expect rush hour traffic, and on weekends it stays steady throughout the day. While Model 4 and Model 5 give similar fits for time of day, Model 3's predictions peak at different times on weekdays and exhibit much more variability on weekends. There are two probable reasons for these differences: first, the sinusoidal terms are less flexible than the splines; second, the splines, because they are non-parametric functions, penalize complexity of the fit while the parametric sinusoidal form does not, making the splines more conservative in their "curviness." The former explains the discrepancies in the weekday fits while the latter explains the discrepancies in the weekend fits. Given these differences, fitting time of day with splines is preferable; there is no motivation to constrain the functional form to any strict parametric form.

But these marginal time-of-day fits don't just tell a story about our time terms; they also reveal part of why the random rider intercepts are such powerful predictors. Notice that in comparing Model 6 to the other models in Figure 3.3, the scale at which the Model 6 time fitted probabilities vary is much larger than the scale at which the other models' predictions vary. Without allowing for varying rider intercepts, the time terms take on a significant role. Yet, interestingly, the time term has nowhere near the amount of information that the rider intercepts seem to encode, according to the separation plots in Table 3.2.

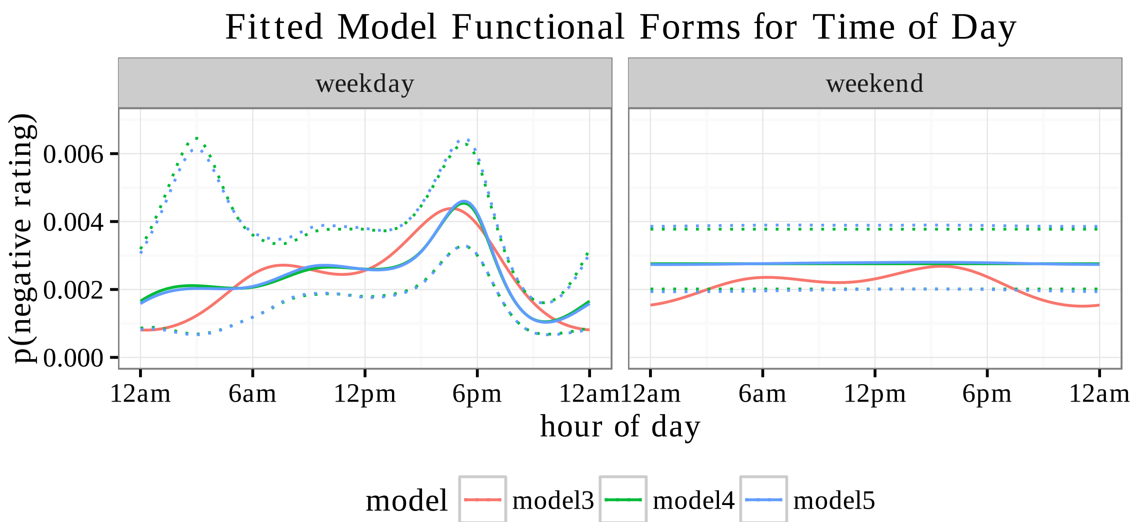
Figure 3.4 paints a clearer picture of what is going on. These models show two

Table 3.3: Regression coefficients for Model 1, Model 2, Model 4, and Model 6. 95% confidence intervals are given in parentheses.

Regression Term	Model 1	Model 2	Model 4	Model 6
Log(length)	-0.122	-0.100	-0.092	-0.114
	(-0.180, -0.063)	(-0.168, -0.032)	(-0.162, -0.022)	(-0.174, -0.054)
Mean Temp.	0.053	0.076	0.075	0.069
	(-0.0004, 0.110)	(0.005, 0.147)	(0.003, 0.147)	(0.012, 0.127)
Mean Wind speed	0.028	0.014	0.012	0.027
	(0.004, 0.052)	(-0.013, 0.041)	(-0.014, 0.039)	(0.002, 0.051)
Gust speed	-0.003	0.001	0.001	-0.003
	(-0.015, 0.008)	(-0.012, 0.013)	(-0.012, 0.013)	(-0.014, 0.009)
Rainfall	0.008	0.012	0.008	0.005
	(-0.015, 0.031)	(-0.015, 0.038)	(-0.019, 0.035)	(-0.019, 0.027)
Rainfall 4-Hour	0.013	0.016	0.017	0.014
	(0.005, 0.021)	(0.007, 0.025)	(0.008, 0.027)	(0.006, 0.022)
Intercept	-2.2868	-3.075	-3.127	-2.313
	(-2.428, -2.108)	(-3.386, -2.764)	(-3.436, -2.818)	(-2.475, -2.151)

different ways to look at the time of day pattern in ride rating: Model 2 suggests *who is riding* determines these patterns while Model 6 suggests there is something inherent in that time of day, such as traffic, that determines these patterns. The models between fit a combination of these, but as we saw in Figure 3.3, the time dependence is more than an order of magnitude weaker after accounting for the rider intercepts. The two black pillars of rides in the predictions of Models 2–5 line up with when we expect commuters to be riding, suggesting that that the riders with high rider intercepts are commuters. The converse, however, is not true: a great number of rides during commuting times are predicted to have almost zero probability of receiving a negative rating.

What explains this relationship between the temporal patterns and the rider intercepts? We suspect ride route is a confouding factor here. Figure 3.4 confirms our suspicion that riders have particular schedules they stick to; so it's also likely, given that these are mostly commuting cyclists, that most of their rides follow the same route as well. These models ignore ride route, so we suspect the typical ride route is encoded in the rider intercepts; *i.e.* a rider whose commuting route goes through many of the most stressful intersections and streets in Portland will likely have a higher intercept than most riders. This hypothesis can only be tested, however, when future research develops models with random rider intercepts and a model for how routes effect ratings.



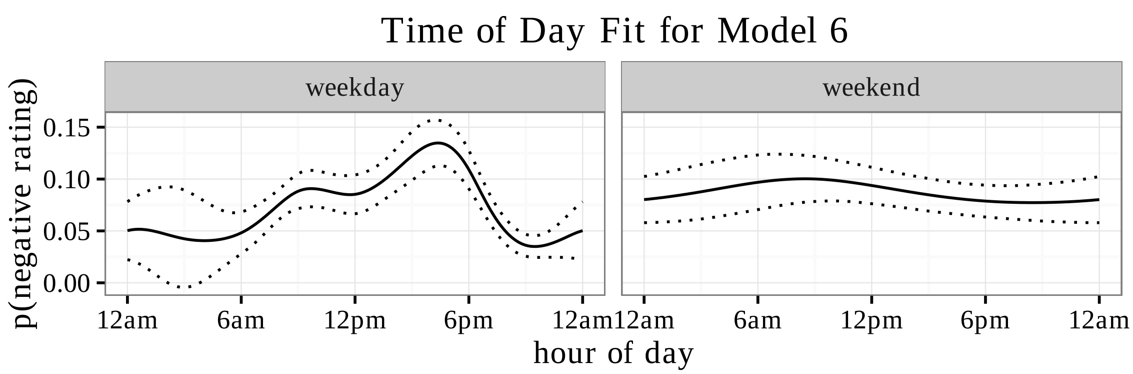


Figure 3.3: Predicted probabilities of a negative rating by time for a typical ride. The rider was chosen so the intercept was closest to the mean intercept for model 5. The median length and average mean temperature were used, and all other predictors were set to zero. The dotted lines show ± 2 standard errors from the predictions.

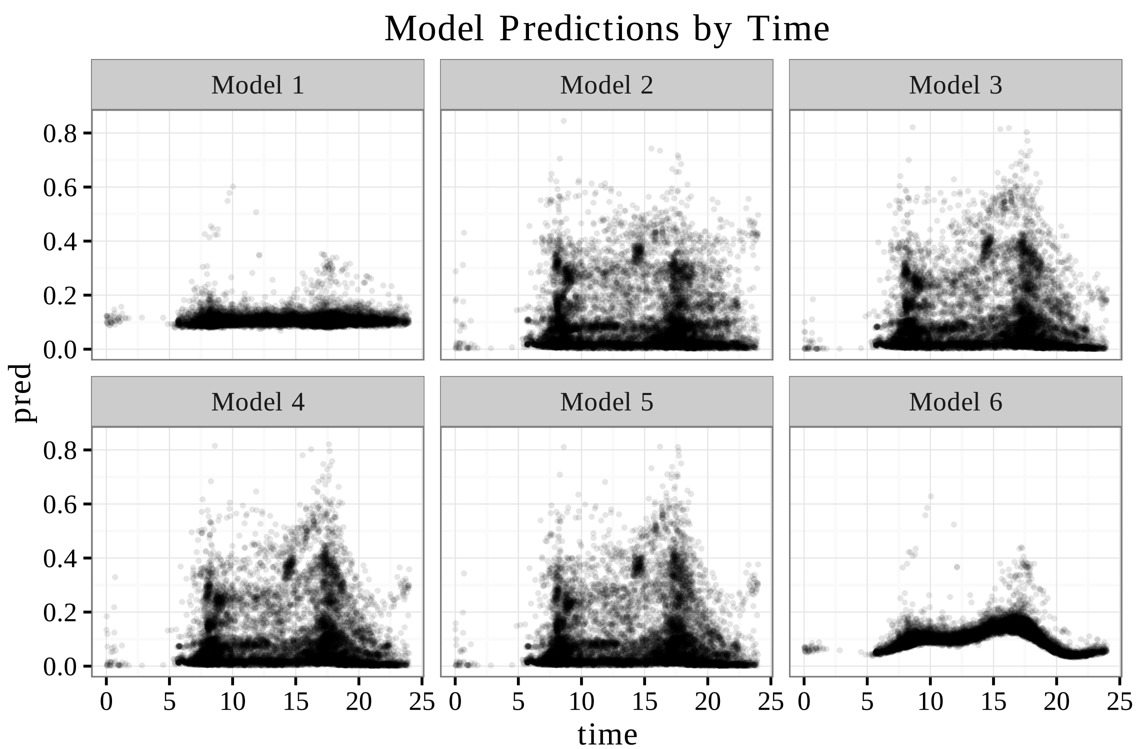


Figure 3.4: Predicted probabilities of negative rating by time of day for Models 3–6. Notice how starting with Model 2, daily trends start to emerge. This indicates that the rider intercepts are picking up on time of day trends, which must be reflected in a rider's typical ride.

Chapter 4

Characterizing Riders

Given the results from last chapter, there is a clear need to understand what kinds of riders are in this data set. To do that, we identify predictors that differentiate riders and then use these variables to identify clusters of riders. We assess how these predictors do as rider-level predictors for the rider intercepts.

We also compare random intercepts with riders to random intercept models done by cluster. Ride Report, because they need to respect the privacy of their users, cannot identify individual riders in the data they provide to clients, yet our model results show that differentiating riders is crucial to getting good estimates in our models. If we can identify clusters of riders in the data set that give us nearly the same information as grouping by individuals did, these could be provided by Ride Report without the same level of risks to user privacy as identifying individual riders.

4.1 Extracting Features and Determining Clusters

We characterized riders based on their rides because *Ride Report* does not collect data about their users besides their rides and email address. We limited our exploration to riders that had over 20 rated rides, in order to focus on riders who had been using the app for some time and had an identifiable pattern of rides.

Cyclists' patterns in their rides are complex, particularly their time patterns. Computing their mean ride length for weekends was useful, but mean time of day for their rides does not capture anything meaningful. So we took care in selecting features that distinguished different rider patterns we saw when exploring the data.

We define the following for each rider j: frequency of rides[1] (u_j^{freq}), proportion of rides on weekdays (u_j^{weekend}), median length of rides on weekdays ($u_j^{\text{med.len}}$) and weekends ($u_j^{\text{med.len.w}}$), variance of ride length on weekdays ($u_j^{\text{var.len}}$) and weekends ($u_j^{\text{var.len.w}}$), and proportion of weekday rides during morning rush (u_j^{morning}), lunch rush (u_j^{lunch}), and evening rush (u_j^{evening}). The time intervals that describe the morning, lunch, and evening rush are shown in Figure 4.1.

[1] We define frequency of a cyclist's rides as the number of rides divided by the difference between the time of the most recent ride and time of the first ride. (Units are arbitrary, because we standardized all of our rider-level variables.)

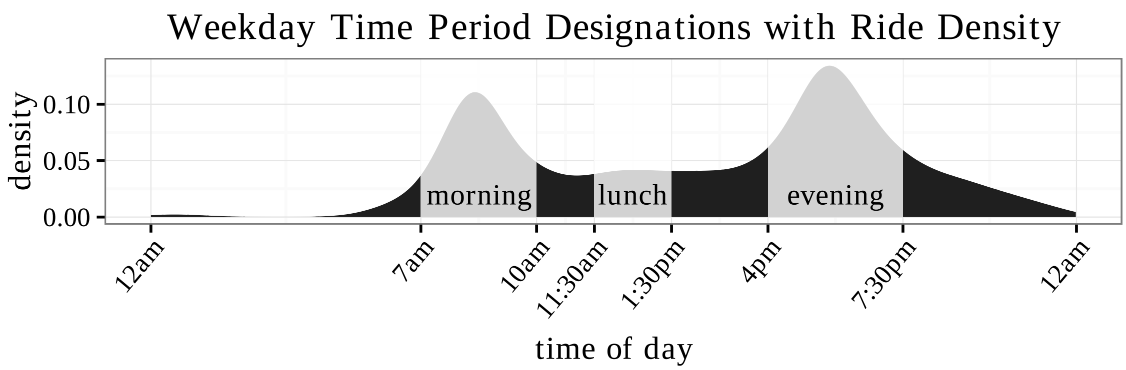


Figure 4.1: Time intervals used for morning, lunchtime, and evening in rider feature extraction. These intervals define the time designations we used in clustering. The proportion of each riders rides in each of these time intervals made up three of our features.

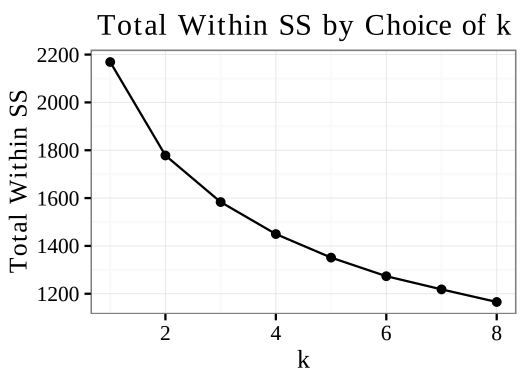


Figure 4.2: Comparing total within sum of squares for different k for k-means clustering of riders.

Selecting variables for a cluster analysis is difficult, for the reason that many choices about what to use are arbitrary. We have chosen these variables, but two important other choices remain: what scales and transformations should these variables have? We chose here to transform all variables to be approximately gaussian—eliminating the right skew that was present in most of these features with log and square root transformations—and standardizing them by subtracting their mean and dividing by their standard deviation. When clustering, the scaling of variables determines how much weight each of them has in determining the clusters. Future research may find more appropriate ways to select features for clustering, but in our approach here we stick to a naive and simple approach to see what we can learn.

With the rider-level predictors in hand, we clustered the riders using k-means clustering, which groups a set of points into the k clusters that minimize the within-cluster sum of squared distance to the cluster centroids.[2] To choose the number of

[2] k-means clustering is fit using a hueristic algorithm, which assigns points random to clusters, and then repeatedly recomputes the cluster centroids and reassigns the points to the cluster with the

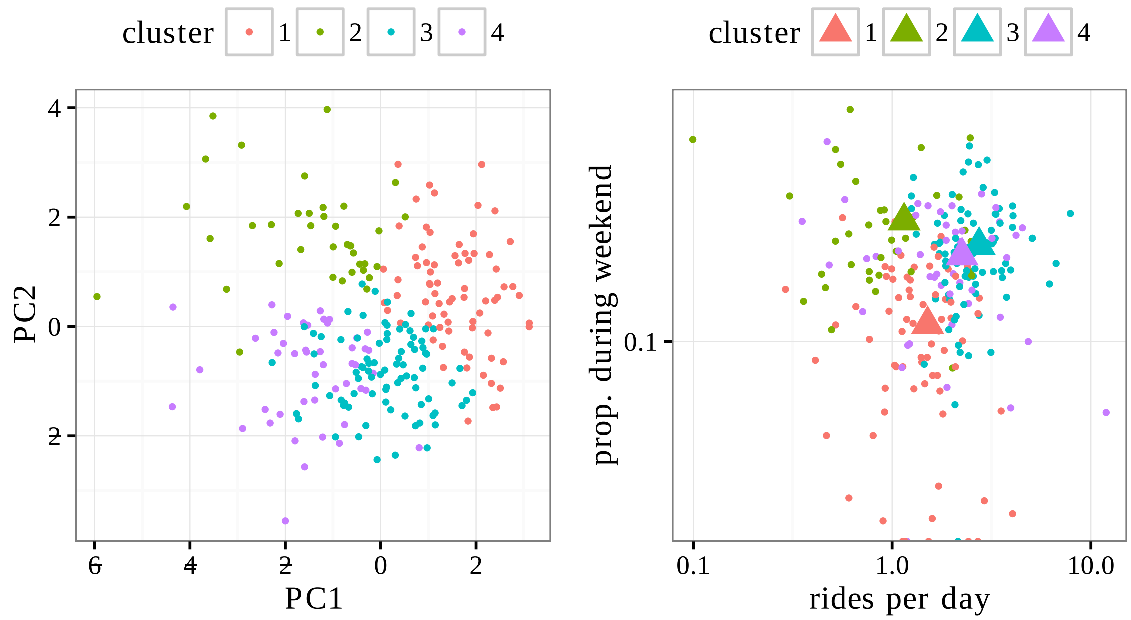


Figure 4.3: Rider clusters identified by k-means clustering. The triangles represent the centroids (computed as the mean) of the cluster members.

clusters, we assessed the total of the sum of squares within each cluster for different values of k, shown in Figure 4.2, and selected $k = 4$ as the point where we thought after which there was little value in having more clusters.

Looking at Figure 4.3, the clusters split the data into the four quadrants of the first two principal components of the rider data. These clusters appear to be less distinct groups than a partition of the space. Regardless, they still provide some useful information about riders. Cluster 2 seemed to pick out more casual riders, with fewer rides per week and more weekend rides than the other clusters, as shown in the visible in the right panel of Figure 4.3. This is seen more clearly in Figure 4.4, where there aren't strong weekday commuting patterns for cluster 2, but there are for clusters 1 and 3. Clusters 1 and 3 seem to be the groups that are the most consistent commuters, but are differentiated by the typical length of their weekend rides. Clusters 2 and 4 show much more variance in the timing of their weekday rides, with cluster 4 having more consistently long weekend rides.

nearest centroid, until the clusters stop changing. This algorithm is fast, but the result is sensitive to the initial random assignment, so it is run many times. See page 460 of Hastie, Tibshirani, & Friedman (2008) for more details of k-means clustering.

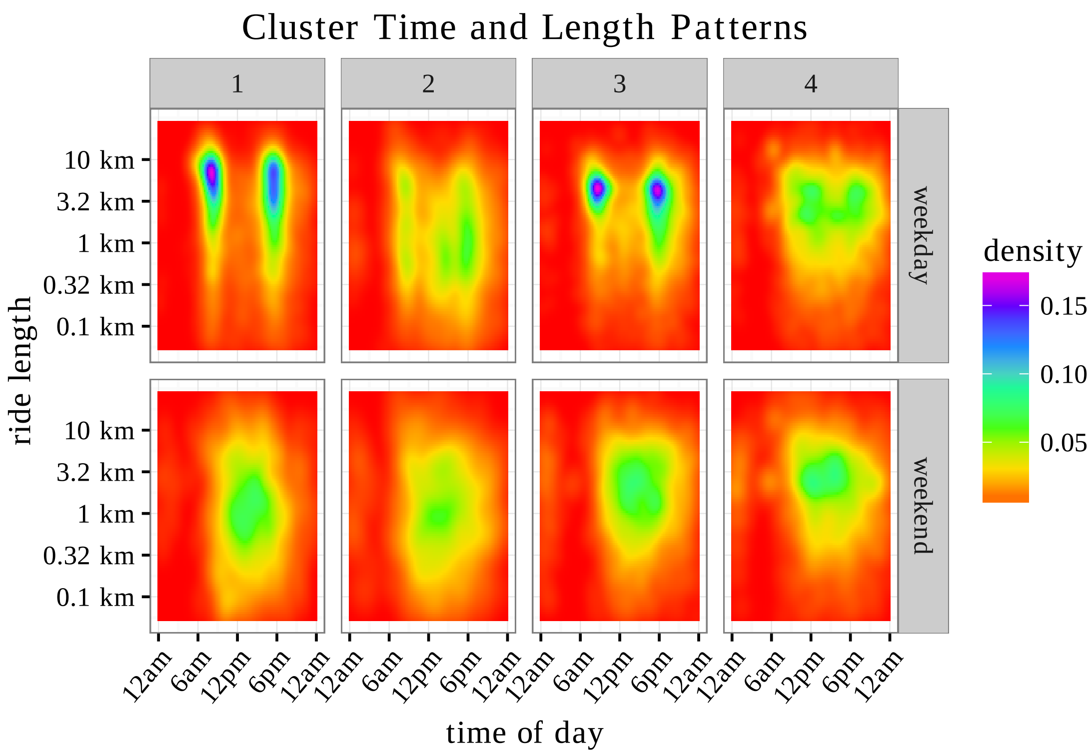


Figure 4.4: Patterns of ride length and ride time of day for each cluster.

4.2 Models with Rider-Level Predictors

Having several rider-level predictors, we set out to see how well they predict rider intercepts. Now let U_j be the vector of rider-level variables. Then our model will be

$$Y_i \sim \text{Bernoulli}\left(\text{logit}^{-1}(\alpha_{j[i]} + X_i\beta)\right), \tag{4.1}$$

where,

$$\alpha_j \sim N(\gamma_0 + U_j\gamma, \sigma_\alpha), \tag{4.2}$$

with (γ_0, γ) being the group-level parameters we estimate.

This model should be comparable to Model 2, though because we are trying to get estimates of the rider-level predictor parameters, we make use of *Stan* instead of `lme4`. Though we would prefer to use a model similar to Model 4 from the previous chapter (the one with smoothing splines for time of day) the current additive mixed models package `gamm4` (which uses `lme4` to fit the mixed models part) does not support estimating the variability in group-level estimates. Unfortunately, in *Stan*, smoothing splines would have to be coded by hand and we lacked the expertise to write the functions to fit smoothing splines ourselves.

The ride-level predictor coefficients from the fitted model, however, are unimpressive. The variance in the rider intercepts not captured by the predictors, quantified with σ_α, is high, and not one of the rider-level predictors have a 95% confidence

Table 4.1: Estimates of rider level predictors.

Parameter	**Estimate**	**2.5% percentile**	**97.5% percentile**
γ^{freq}	0.08	-0.19	0.35
γ^{weekend}	-0.13	-0.50	0.35
γ^{morning}	0.06	-0.22	0.34
$\gamma^{\text{afternoon}}$	0.13	-0.20	0.44
γ^{evening}	-0.02	-0.31	0.27
$\gamma^{\text{med.len}}$	0.01	-0.25	0.29
$\gamma^{\text{med.len.w}}$	0.08	-0.19	0.36
$\gamma^{\text{var.len}}$	0.07	-0.15	0.31
$\gamma^{\text{var.len.w}}$	-0.15	-0.47	0.17
γ_0	-2.99	-3.29	-2.69
σ_α	1.47	1.27	1.69

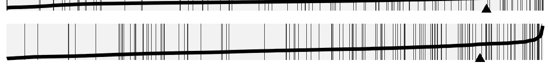
interval that does not contain zero. (In fact, many are centered near zero.) These features may differentiate riders, but they don't give much information about how they rate their rides.

4.3 Cluster Intercepts Versus Rider Intercepts

Do these clusters provide similarly useful information that we got from introducing rider intercepts? Because a model with only 4 random intercepts—as in the case of cluster intercepts—rather than several hundred—as in the case with rider intercepts—is much less flexible, we expect that the cluster intercept model will preform much worse. One still might suspect there is still a significant benefit over a fixed intercept model.

There isn't. We computed Model 7, which is identical to Model 4 from the previous chapter, but has random intercepts by cluster rather than rider. Model 7 performed slightly better than Model 6—which only had a fixed intercept—but nowhere near as well as Model 4. The separation plots, $\log(\mathcal{L})$, AIC, and AUC measures, shown in Table 4.2, all demonstrate this clearly. Given that the rider-level predictors did not seem to be predictive of the rider intercepts, this is not a surprise.

Table 4.2: Model fit summaries for fixed intercept, rider random intercept, and cluster random intercepts.

Model	**Separation Plot**	$\log(\mathcal{L})$	**AIC**	**AUC**[3]
Model 4		-4,266	8,549	0.805
Model 6		-5,089	10,205	0.597
Model 7		-4,973	9,965	0.646

[3] Area under ROC curve for training data.

Chapter 5

Modeling Missing Response

Of the 25,397 rides in the data set, 11,365 were not rated. With such a large amount of missing data, careful consideration should be made about what can be inferred from this data set. A common problem with missing responses in crowdsourced rating data sets is that the missingness of ratings is not independent of the ratings that the users would give. This worry motivated Ying, Feinberg, and Wedel's work on creating models for recomendation systems based on online ratings that explicitely modelled missing data[1]. In the case of rides, it's possible that cyclists are more likely to rate their ride if they had a bad experience than if their ride was uneventful. This kind of correlation between missingness and the response can cause strong biases in the estimates, as we will demonstrate.

In this chapter, we attempt to address the missing data issues by fitting a model that simultanesouly models the missing data mechanism and the ride ratings. However, with the current state of the ride data, these models may be unable to come up with accurate estimates because of another problem in the data collection. As mentioned in Chapter 1, rides are often misclassified as bike rides when they are actually car rides or rides on public transit. We suspect that many of the unrated rides are rides that were misclassified as bike rides, and thus were not rated by the rider. (We assume that riders don't often go through the effort of correcting the classification of rides and know not to rate rides that weren't bike rides.) If this is the case, then it would be inappropriate to make use of the data with missing responses. If, however, *Ride Report* is able to improve their classification enough to make this a non-issue, these methods could be vital to accurately modeling ride rating.

5.1 What could possibly go wrong?

We focus on the situation we have, where our response variable y_i has missing values. Define the vector $R = (r_1, r_2, \ldots, r_n)$ such that

$$r_i = \begin{cases} 1, & \text{if } y_i \text{ is missing;} \\ 0, & \text{if } y_i \text{ is observed;} \end{cases} \tag{5.1}$$

[1]Ying, Feinberg, & Wedel (2006)

for $i = 1, \ldots, n$

Rubin classifies missing data into three situations[2]:

1. **Missing Completely at Random (MCAR)**, where R is independent of Y and the predictors X. *i.e.* $\mathbb{P}(R = 1|Y, X) = \mathbb{P}(R = 1)$
2. **Missing at Random (MAR)**, where R is independent of Y, but may depend on X, *i.e.* $\mathbb{P}(R = 1|Y, X) = \mathbb{P}(R = 1|X)$
3. **Nonignorable, or not MCAR nor MAR**, where R is dependent on Y.

As discussed in the introduction, we believe that rider ratings may be correlated with nonresponse and thus the missing ratings are non-ignorable.

If missing data is nonignorable, what could go wrong with our models? Let's look at a toy example. Define the data set of n observations with $x \in \mathbb{R}^n$, $y \in \{0, 1\}^n$, and R defined as before, where

$$x_i \sim \text{Normal}(0, 1),$$

$$y_i \sim \text{Bernoulli}(\text{logit}^{-1}(4x_i)),$$

$$r_i \sim \text{Bernoulli}(0.3 + 0.4y_i),$$

for $i = 1, \ldots, n$.

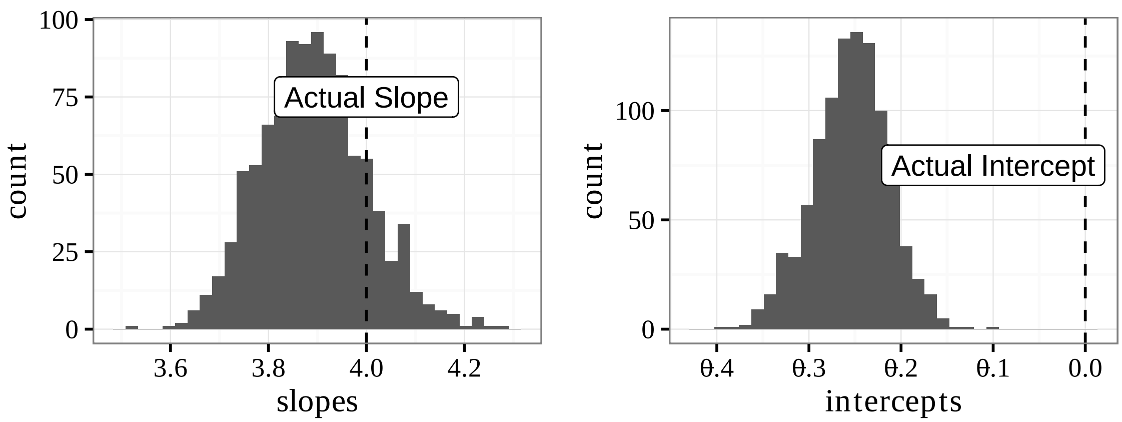


Figure 5.1: Simulated example of logistic regression fits to a model with nonignorable missing response. One data set of size $n = 10^4$ was computed from the toy data model. We then recomputed R 1,000 times, each time fitting a simple logistic regression model to y and X.

If we attempt to fit a logistic regression model to this data our estimate of the intercept will be inaccurate. Figure 5.1 shows the results of a simulation, computing the slope and intercepts for 1,000 different patterns of missing data for the same generated data set generated from our toy data model.

It makes sense that we are underestimating our intercept. The intercept can be interpreted as the base rate, and if values of $y_i = 1$ are more likely to be missing, the overall rate we observe will be lower.

[2]Little & Rubin (1987) (page 14)

Clearly, if we have nonignorable missing response, we are in a bad situation. Having missingness depend on Y leads to biased estimates of our intercepts when we fit models. But we do have all of our predictors of y, with no missingness. Could we leverage our understanding of how X predicts Y to understand the patterns of missing response?

5.2 Modeling the Missing Data Mechanism with Expectation Maximization

Here we perform the expectation maximization (EM) algorithm using the weighting method proposed by Ibrahim and Lipsitz[3]. Let y be our binary response and X be our predictors. With these we have our complete data logistic regression model $f(y \mid X, \beta)$, where β is a vector of parameters in the complete data model.

We then specify a logistic regression model for missingness (R): $f(R \mid X, y, \alpha)$, where α is the vector of parameters in the missingness model.

We begin the algorithm by getting our first estimates of α and β. We obtain $\beta^{(1)}$ by estimating β with only the non-missing data (*i.e.* fit the models as if there were no missing data). We can then estimate y for the missing data using $\beta^{(1)}$, and then use those estimates to compute $\alpha^{(1)}$.

For the E-step, we compute weights for each observation with missing response, representing the probability that the ith observation has response value y_i:

$$w_{i\,y_i}^{(t)} = f(y_i \mid r_i, x_i, \alpha^{(t)}, \beta^{(t)}) = \frac{f(y_i \mid x_i, \beta^{(t)}) f(r_i \mid x_i, y_i, \alpha^{(t)})}{\sum_{y_i \in \{0,1\}} f(y_i \mid x_i, \beta^{(t)}) f(r_i \mid x_i, y_i, \alpha^{(t)})}. \tag{5.2}$$

5.2 is essentially an application of Bayes' theorem. We can view $f(y_i \mid r_i, x_i, \alpha^{(t)}, \beta^{(t)})$ as the posterior density of y_i given observation i is missing, where $f(y_i \mid x_i, \beta^{(t)})$ is the prior distribution and $f(r_i \mid x_i, y_i, \alpha^{(t)})$ serves as the likelihood.

For observed responses, $w_{i\,y_i}^{(t)} = 1$. Note that for each observation i, $\sum_{y_i \in \{0,1\}} w_{i\,y_i} = 1$. We can compute $f(y_i \mid x_i, \beta^{(t)})$ and $f(r_i \mid x_i, y_i, \alpha^{(t)})$ by making use of predictions from regression models. So in `R`, we can fit models and use the `predict()` function to get our probabilities from each of these models.

For the M-step, we find our next estimates of the parameters, $\alpha^{(t+1)}$ and $\beta^{(t+1)}$, by maximizing

$$Q(\alpha, \beta \mid \alpha^{(t)}, \beta^{(t)}) = \sum_{i=1}^{n} \sum_{y_i \in \{0,1\}} w_{iy_i}^{(t)} \cdot l(\alpha, \beta | x_i, y_i, r_i). \tag{5.3}$$

We do this by first by estimating $\beta^{(t+1)}$ using weighted maximum likelihood for the complete data model, and then estimating $\alpha^{(t+1)}$ using the same method. To maximize $l(\alpha, \beta | x_i, y_i, r_i)$, we maximize the product of their likelihoods,

$$l(\alpha, \beta | x_i, y_i, r_i) = l(\beta | x_i, y_i) l(\alpha | r_i, x_i, y_i),$$

[3] Ibrahim & Lipsitz (1996)

which we can maximize by maximizing each of the likelihoods separately because our estimates of α and β are only dependent on each other through x and y. This allows us to use any package that can fit models by maximum likelihood estimation using weights for the observations, which includes all of the model fitting packages we used in Chapter 3.

In order to create the data to fit these models, we create an augmented data set where each observation missing the response is recorded as two rows. These duplicate rows represent the two possible values of the response, and also contain the weights computed in the E-step. Figure 5.2 describes this process graphically.

Figure 5.2: How to create augmented data for EM algorithm: duplicate rows that are missing the response variable, assigning to each row a possible value of the reponse and its associated weight.

Original Data				**Augmented Data**			
y_i	x_i	r_i		y_i	x_i	r_i	w_i
1	2.4	0		1	2.4	0	1
0	1.3	0	$\rightarrow$	0	1.3	0	1
NA	-0.4	0		1	-0.4	0	0.2
				0	-0.4	0	0.8

We repeat the E and M step until the joint loglikelihood converges to within some tolerance. An implementation of this algorithm can be found in Appendix A.

As an example, we simulated a dataset from the same model we presented earlier of size 10^4. Of those observations, 6,252 were missing. As shown in Table 5.1, the estimate for the intercept in the model that only considers the complete data is way off, but the model resulting from the EM algorithm is nearly as accurate as the model fit to the full data (with missing values filled in from the original data model.) The missing data model is also able to get accurate estimates of the parameters that define the missing data mechanism, but the estimates are quite uncertain.

Table 5.1: Coefficients for models fit to simulated data set ($\pm$ twice the standard error.)

Model	$\hat{\beta}_0$	$2 \cdot SE_{\hat{\beta}_0}$	$\hat{\beta}_X$	$2 \cdot SE_{\hat{\beta}_X}$
Actual	0	–	4	–
Full Data Model	-0.009	0.065	3.881	0.080
Complete Data Model	-0.278	0.106	3.819	0.259
EM Final Model	0.042	0.065	3.814	0.157

Table 5.2: Estimates for missing data mechanism for simulated model.

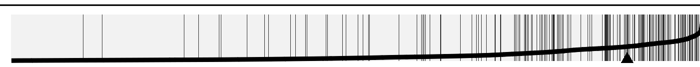
Model	$\hat{\alpha}_0$	$2 \cdot SE_{\hat{\alpha}_0}$	$\hat{\alpha}_Y$	$2 \cdot SE_{\hat{\alpha}_Y}$
Actual	0.3	–	0.4	–
EM Missing Data Model	0.263	0.132	0.530	0.268

5.3 EM Algorithm for the Ride Data

In order to perform the algorithm, we need to specify a model for nonresponse. We will use the same predictors that we do in Model 4 for ride rating—including a smoothing spline for time of day for weekdays and weekends—except we do not use random rider intercepts. For the EM algorithm, we use Model 4 as our ride rating model and use the following model for the rating nonresponse mechanism:

$$r_i \sim \text{Bernoulli}(\text{logit}^{-1}(\alpha_0 + y_i\alpha_y + X_i\alpha_x + X^{\text{weekend}} \cdot f^{\text{time.w}}(t_i) + (1 - X^{\text{weekend}}) \cdot f^{\text{time}}(t_i))). \tag{5.4}$$

Table 5.3: Fit summaries for Model 4 and the EM Model

Model	**Separation Plot**	**AUC**[4]
Model 4		0.802
EM Model		0.763

The fit for the EM algorithm seems to be worse. The AUC, shown in Table 5.3, which was computed on the complete data, was lower than that of Model 4.

There are two disagreements between the EM model and Model 4 for ride rating: the coefficients for x^{length} and x^{rain}. The former has flipped sign while the latter has much less uncertainty in its estimate.

The coefficients for the missing model, shown in Table 5.5 confirm our worry that many of the rides missing the rating are not bike rides. These model coefficients suggests that rides are much more likely to be missing if they have a negative rating. We hypothesized that there would be a weak negative effect of a negative rating on missingness; while any reasonable researcher wouldn't dismiss the estimates because the sign wasn't what was expected, the magnitude seems much more in line with the hypothesis that many of the missing ratings correspond to car rides.

It's tempting to suggest that longer rides tend to be missing, but they are also more likely to be rated negatively; the distribution of ride lengths are actually about the same for rated and non-rated rides. But does it make sense that we would have the same distribution of ride lengths for rated and non-rated rides, if we suspect many of the non-rated rides are actually car rides? Yes, so long as we keep in mind that these are rides that have been misclassified as bike rides; we expect the classifier already filtered out car rides that were too long and fast to be bike rides.

Table 5.4: Ride rating model estimates after EM algorithm

Parameter	**Model 4**	**EM Model**
Log(Length)	-0.147	0.205
	(-0.290, -0.005)	(0.106, 0.304)
Mean Temperature	0.142	0.100
	(0.004, 0.281)	(0.005, 0.196)
Mean Wind Speed	0.002	-0.026
	(-0.054, 0.057)	(-0.069, 0.016)
Max Gust Speed	-0.005	0.020
	(-0.031, 0.021)	(0.001, 0.039)
Rainfall	0.050	0.051
	(-0.017, 0.117)	(0.009, 0.093)
Rainfall 4-Hour	0.022	0.017
	(0.003, 0.041)	(0.003, 0.030)
Intercept	-2.792	-3.144
	(-3.334, -2.250)	(-3.604, -2.684)

Unfortunately, these models do not seem ready for use on the *Ride Report* data until the quality of data with missing ratings can be assured. Knock Software is planning on fixing this, so such an analysis may be viable within a year or two of collecting new data. (Because the accelorometer data is not saved, they cannot go back and attempt to reclassify old rides.)

Table 5.5: Estimates for ride rating nonresponse mechanism. The Basic Nonresponse Model is estimated based on the data with y predicted by Model 4. The EM Nonresponse Model is estimated with the EM algorithm, which uses the same model specifications.

Parameter	**Basic Nonresponse Model**	**EM Nonresponse Model**
y	0.730	1.035
	(0.235, 1.224)	(0.493, 1.577)
Log(Length)	-0.297	-0.327
	(-0.362, -0.232)	(-0.393, -0.262)
Mean Temperature	0.200	0.139
	(0.139, 0.262)	(0.077, -0.262)
Mean Wind Speed	0.032	0.031
	(0.003, 0.060)	(0.001, 0.061)
Max Gust Speed	-0.003	-0.007
	(-0.016, 0.010)	(-0.021, 0.006)
Rainfall	0.007	-0.024
	(-0.028, 0.041)	(-0.057, 0.009)
Rainfall 4-Hour	-0.002	0.010
	(-0.012, 0.009)	(-0.001, 0.021)
Intercept	-0.927	-0.967
	(-1.124, -0.729)	(-1.163, -0.771)

Chapter 6

Unfinished Work: Incorporating Routes

These models so far do not incorporate routes. Though our initial aim was to create models that use the routes, we were not able to transform the route to a state that was useful for modeling. I leave the models as they are, but here I explain some of my work toward the goal of incorporating routes and describe some potential modeling approaches. Throughout this work is the caveat that many of these results are hard to interpret without taking into account route. We hope future researchers will be able to accomplish this.

6.1 Data Structures for Routes

Before one can model routes, they need to be represented properly. Ride Report's approach has been to discretize the routes into road segments by map matching the GPS traces to road segments in the Open Street Map (OSM) data. Road segments are readily availible in the OSM data sets as well as in shapefiles published by cities. In particular, Civicapps.org, a public data portal for the Portland, OR area has a shapefile of the bicycle network in Portland, OR, with information about the type of roads, presence of bike lanes, and other useful information about the roads bikes have access to in Portland.

Road segments are not the only way to represent routes, however. One could also consider a route as a sequence of intersections or intersections and road segments. Though modeling intersections may be more difficult, it's likely they could be more interesting parts of the routes. In the bike accident data examined by Meyers, most of the accidents occurred at intersections[1], so it would make sense if these were often the most stressful and dangerous portion of riders' routes.

For now, however, the most readily available data are on road segments.

[1] Meyers (2015)

6.2 Regression Terms for Road Segments

How could we use our knowledge of riders' routes into our regression? The approach we present here will be to consider routes as sequences of discrete road segments, each of which have known properties. As mentioned before, there are data about roads that give information about bike lanes, road size, and other attributes of road segments.

Ideally we would like to estimate some parameter for each road segment that indicated its typical contribution to the probability of a negative ride rating. The most immediate hurdle is figuring out how to estimate all of those parameters, particularly when the number of rides per a particular road segment may be low.

Bayesian inference may be the best bet to get something for each road segment, but as a extremely simple example, we outline here a method that would be easy to fit, but likely not a very good model. Regardless, it gives a good idea of how the ideas of multilevel models could be adapted for use with road segments, despite the lack of a clear hierarchy. Assume we have K total road segments in our road network and for each ride we have $\Omega_i \subseteq \{1, \ldots, K\}$, the set of road segments that are in the route of ride i. Let l_k be the length of the kth segment and define the length of ride i to be:

$$L_i = \sum_{k \in \Omega_i} l_k.$$

For the kth road segment, define the m-dimensional vector $W_k = W_k^1, W_k^2, \ldots, W_k^m$ road segment-level predictors. Then we shall define the term in our regression for the route of ride i as

$$R_i = \frac{1}{L_i} \sum_{k \in \Omega_i} l_k W_k \beta^{\text{road}},$$

Where β^{road} is a vector of coefficients for the road segment level predictors. When actually computing this value, we can factor out the β^{road}, and then the rest of R_i is just a transformation of road-level variables.

Conclusion

By focusing on minimizing barriers to responding and automating as much of the data collection as possible, the designers of the *Ride Report* app created an infrastructure that could collect large numbers of ride ratings. But sample size isn't everything: the subjectivity of the ratings and the pattern of missing ratings make this a treacherous data set to model naively.

The subjective ratings pose a problem particularly when used to infer the quality of particular road segments; if most of the rides are by one particular rider, then the typical rating over that segment will reflect that particular rider's interpretation of the ratings more than others. Our models from Chapter 3 confirm that modeling ride rating with rider intercepts is essential. Adding rider intercepts to a multivariate regression model increased the cross-validated AUC from 0.552—little better than the null model—to 0.797. These intercepts turned out to encode much more than a rider's baseline tendency to rate a ride negatively; Figure 3.4 showed how much information rider intercepts had about riders' typical time of day, and it's likely that riders' typical routes are also encoded in these intercepts as well. Future research should pay special attention to how these intercepts change when routes are incorporated into these models.

Our missing data models showed some questionable results, though it's hard to know if those issues stem from the data quality of the unrated rides or a flaw in the model. If many of the unrated rides are actually not bicycle rides— which we suspect is the case—then these missing data models will not be appropriate until the misclassification of non-bike rides as bike rides is no longer a problem.

In some ways, this is an incomplete work. To leverage the insights from this paper in creating a map of good and bad routes, models that use ride route information need to be developed and implemented. Both the theoretical development and the technical implementation are difficult problems in and of themselves. There are many ways one could model the relationship between ride rating and route, and it's difficult to find any good theoretical justification for one particular model. And even if a good theoretical model can be formulated, such models will likely not be simple to implement, both because the models will probably not be supported by common model fitting packages and because matching the GPS traces to the road network model is a difficult inference problem in itself.

Appendix A

A Code Sample of the EM Algorithm

Despite its attractive features, there are few explicit explanation of how to actually program the EM algorithm for missing response using weights. The theoretical is contained in Section 5.2, but for the benefit of the reader, we lay out the practical implementation here.

For this example, we present the same code used for the simulation in Section 5.2. The data can be simulated with,

```
inv_logit <- function(x) 1 / (1 + exp(-x))
n <- 1e3
x <- rnorm(n, 0, 1)
y <- rbinom(n, 1, prob = inv_logit(4 * x))
r <- rbinom(n, 1, prob = inv_logit(0.3 + 0.4 * y))

simulated_data <- data.frame(x, y, r)
simulated_data$y <- ifelse(simulated_data$r == 1, NA, simulated_data$y)
```

For convenience, we define the models once as functions, so we can use them more than once.

```
fit_r <- function(data, weights = NULL) {
  gam(r ~ x + y_pred, data = data, family = binomial, weights = weights)
}
fit_y <- function(data, weights = NULL) {
  gam(y_pred ~ x, data = data, family = binomial, weights = weights)
}
```

First, we separate out the portions of the data that are complete and that are missing the response. To make our code clear and simple, we make use of the `dplyr` package.

```
data_complete <- simulated_data %>% filter(!is.na(y)) %>% mutate(weight = 1)
data_missing <- simulated_data %>% filter(is.na(y)) %>% mutate(weight = NA)
```

We then start the algorithm with our initial guesses at the model for `y` and `r`:

```
simulated_data$y_pred <- simulated_data$y
model_y <- fit_model_y(data_complete)
simulated_data$y_pred <- (predict(model_y,
                                  newdata = simulated_data,
                                  type = "response") > 0.5) %>% as.numeric()
model_r <- fit_model_r(simulated_data)
```

Finally, we perform the main loop of EM algorithm iterations. We have two stopping conditions here: when the algorithm reaches the maximum number of iterations or when the difference between the current model's AIC and the previous model's AIC is less than the tolerance.

```
last_aic <- AIC(model_y)

for (i in 1:1000) {
  # get prob of 1
  y_pred <- predict(model_y, newdata = data_missing, type="response")

  # get prob missing given y
  pred_r_y1 <- predict(model_r,
                       newdata = mutate(data_missing, y_pred = 1),
                       type="response")
  pred_r_y0 <- predict(model_r,
                       newdata = mutate(data_missing, y_pred = 0),
                       type="response")

  # Make weights
  denom <- (y_pred * pred_r_y1) + ((1-y_pred) * pred_r_y0)
  w_y1 <- y_pred * pred_r_y1 / denom
  w_y2 <- (1-y_pred) * pred_r_y0 / denom

 # print(pred)
  data_augmented <- bind_rows(data_complete,
                        mutate(data_missing,
                               weight = w_y1,
                               y_pred = 1),
                        mutate(data_missing,
                               weight = w_y2,
                               y_pred = 0))
```

```
  model_y <- fit_y(data_augmented,
                        data_augmented$weight)
  model_r <- fit_r(data_augmented,
                        data_augmented$weight)

  # Check Stopping Condition
  current_aic <- AIC(model_y)
  print(AIC(model_y))
  if ((i > 1) && (last_aic - current_aic < 0.0001)) break
  last_aic <- current_aic
}
```

References

Bates, D., Mächler, M., Bolker, B., & Walker, S. (2015). Fitting linear mixed-effects models using lme4. *Journal of Statistical Software*, *67*(1), 1–48.

Carpenter, B., Gelman, A., Hoffman, M., Lee, D., Goodrich, B., Betancourt, M., … Riddell, A. (2016). Stan: A probabilistic programming language. *Journal of Statistical Software*.

Cressie, N., & Wikle, C. K. (2011). *Statistics for spatio-temporal data*. John Wiley & Sons.

Gelman, A., & Hill, J. (2006). *Data analysis using regression and multilevel/Hierarchical models*. The Edinburgh Building, Cambridge CB2 8RU, UK: Cambridge University Press, New York.

Greenhill, B., Ward, M. D., & Sacks, A. (2011). The separation plot: A new visual method for evaluating the fit of binary models. *American Journal of Political Science*, *55*(4), 991–1002.

Hastie, T., Tibshirani, R., & Friedman, J. (2008). *The elements of statistical learning: Data mining, inference, and prediction* (Second edition). Springer.

Ibrahim, J. G., & Lipsitz, S. R. (1996). Parameter estimation from incomplete data in binomial regression when the missing mechanism is nonignorable. *Biometrics*, *52*(3), 1071–1078.

Little, R. J., & Rubin, D. B. (1987). *Statistical analysis with missing data*. John Wiley & Sons.

Mackaronis, J. E., Strassberg, D. S., Cundiff, J. M., & Cann, D. J. (2013). Beholder and beheld: A multilevel model of perceived sexual appeal. *Archives of Sexual Behavior*.

Meyers, R. (2015). Retrieved from `https://github.com/ReedCollegeMATH241/MATH241_PortlandBikeWorks`

Portland Bureau of Environmental Science, C. of. (2016, February). Portland fire bureau rain gage data table. Retrieved from `http://or.water.usgs.gov/non-usgs/bes/ankeny.rain`

Shalizi, C. (2013a, February). Additive models. Retrieved from `http://www.stat.`

cmu.edu/~cshalizi/uADA/13/lectures/ch09.pdf

Shalizi, C. (2013b, February). Splines. Retrieved from http://www.stat.cmu.edu/~cshalizi/uADA/13/lectures/ch08.pdf

Team, R. C. (2015). *R: A language and environment for statistical computing*. Vienna, Austria: R Foundation for Statistical Computing. Retrieved from https://www.R-project.org/

The garden of forking paths: Why multiple comparisons can be a problem, even when there is no "fishing expedition" or "p-hacking" and the research hypothesis was posited ahead of time. (2013, Nov). Retrieved from http://www.stat.columbia.edu/~gelman/research/unpublished/p_hacking.pdf

Weather Underground, I. (2016, February). Weather history for kPDX. Retrieved from https://www.wunderground.com/history/airport/KPDX/2015/9/20/CustomHistory.html?dayend=8&monthend=2&yearend=2016&req_city=&req_state=&req_statename=&reqdb.zip=&reqdb.magic=&reqdb.wmo=&MR=1

Wood, S. (2006). *Generalized additive models: An introduction with r*. CRC press.

Wood, S., & Scheipl, F. (2014). Gamm4: Generalized additive mixed models using mgcv and lme4. Retrieved from https://CRAN.R-project.org/package=gamm4

Ying, Y., Feinberg, F., & Wedel, M. (2006). Leveraging missing ratings to improve online recommendation systems. *Journal of Marketing Research*, *43*(3), 355–365.