Flow Nets

Introduction

Steady-state groundwater flow is determined by the Laplace equation, $\nabla^2 h = 0$, where h is the total hydraulic head and ∇ is the Del operator. To convince yourself of this solution, take the groundwater flow equation (Dingman eq. 8-14),

$$\frac{\partial}{\partial x}\left(K_{hx}\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_{hy}\frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_{hz}\frac{\partial h}{\partial z}\right) = S_s\frac{\partial h}{\partial t}$$

,

and set the time derivative of head to zero – at steady-state, the head does not change – assuming the soil is homogeneous and isotropic (i.e., the saturated conductivity does not depend on spatial location or the direction of flow).

Laplace's equation is a relatively simple partial differential equation, but is difficult to solve analytically, especially when you have irregular boundary conditions as are found in regional groundwater flow problems. Numerical approximations to Laplace's equation are often used, but today we'll skip the finite differences and use flow nets.

A flow net is a two-dimensional plot of streamlines and equipotential surfaces which allows you to solve Laplace's equation graphically. Because we're solving this equation graphically, there is an element of trial-and-error in the solution. Simply work at it, following the rules provided below, and you'll get there in the end. There are six options, detailed below, each worth a certain number of points. This is a ten point lab, but you can choose which flownets you want to work on to get there.

Rules for flow nets

1. Streamlines are perpendicular to equipotential surfaces
2. Equipotential surfaces are perpendicular to impermeable surfaces
3. Lines of constant head are by definition equipotential surfaces
4. The area between the equipotential surfaces and the streamlines should be a curvilinear square. This requirement is not essential to the solution of Laplace's equation by the flow net, but is essential if you want to calculate the groundwater flow from your flow net.

Getting started

1. Define your boundary conditions. Where are the impermeable surfaces? Where are the free water surfaces?

2. The water table represents a surface with zero pressure head; therefore, the hydraulic head on the water table, h, is entirely due to gravity. Because you know the height of the water table above an arbitrary datum, you should be able determine the hydraulic head along the water table.
3. Mark off some intervals of head along the water table then and try to draw equipotential surfaces from those lines. Remember, head remains constant along equipotential surfaces, and they intersect impermeable boundaries at right angles.
4. When you have some equipotential surfaces, draw flow lines at right angles to them, making sure to form curvilinear squares in between the two sets of lines.
5. Does your flow net obey the four rules above? Does it make sense? If the answer is no to either of those questions, adjust your lines until you can answer yes.

Using a flow net to calculate discharge

The discharge per unit width, $q[L^2/T]$, into the stream for a flow net is given by the following relation:

$$q = \frac{N_f}{N_H} K \cdot H = N_f K \Delta h$$

where N_f is the number of flow channels (the tubes between your flow lines), N_H is the number of drops in equipotential you cross in each streamtube, K is the saturated hydraulic conductivity (make a good guess based on your hypotheses about the location of your imaginary hillslope), and $H = \Delta h \cdot N_H$ is the total drop in hydraulic head. Δh is the contour interval of your equipotential surfaces.

Assignment

The lab is out of ten points. Choose the flow nets you want to do to get to ten points. Any points more than that count as extra credit. Each flow net has a question or two on it. Answer on the back.

1. Hillslope (5 pts; requires calculation. Be careful with your curvilinear squares!)
2. Dam (5 pts; requires calculation. Be careful with your curvilinear squares!)
3. High conductivity (3 pts)
4. Regional groundwater flow (2 pts)
5. Riparian discharge (2 pts)
6. Recharge and discharge (2 pts)