

# Hypothesis Testing

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# Outlines

- Introduction
- Hypothesis
- Hypothesis Testing
- Calculation

# Expected outcomes

- Understand the concept of hypothesis testing as one of inference methods
- Understand the concepts of hypothesis and hypothesis testing
- Apply the concepts and steps in hypothesis testing to calculate relevant values and make decision

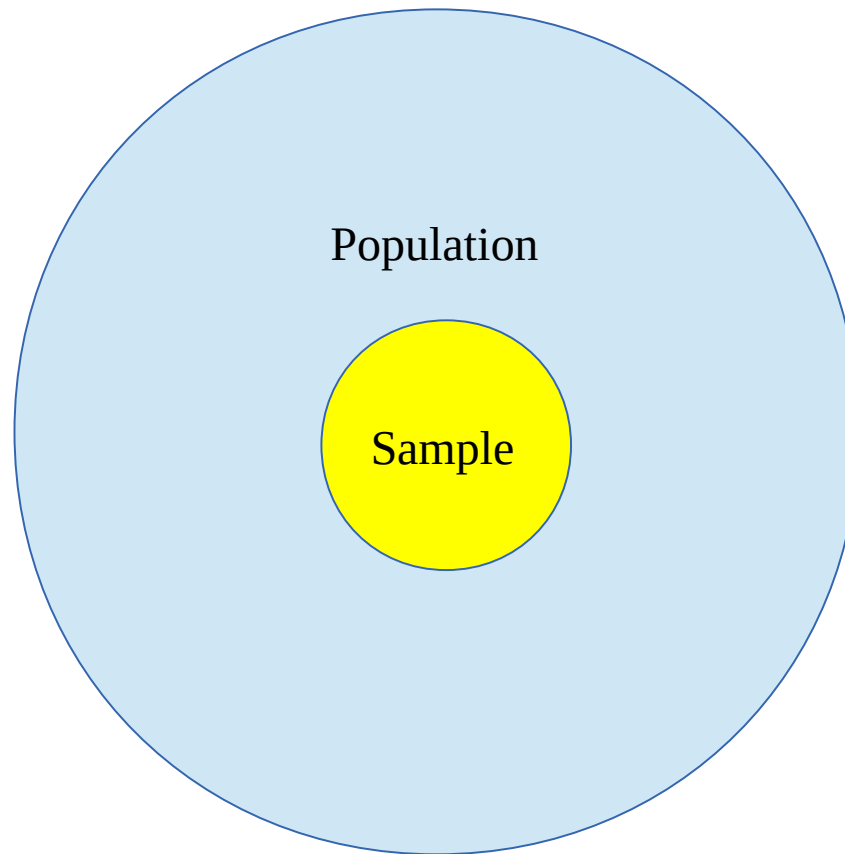
# Introduction

# Overview

- Statistics is a field of study dealing with (Daniel, 1995):
  1. Collection, organization, summarization and analysis of data.
  2. Making inference/conclusion about population data from sample data.

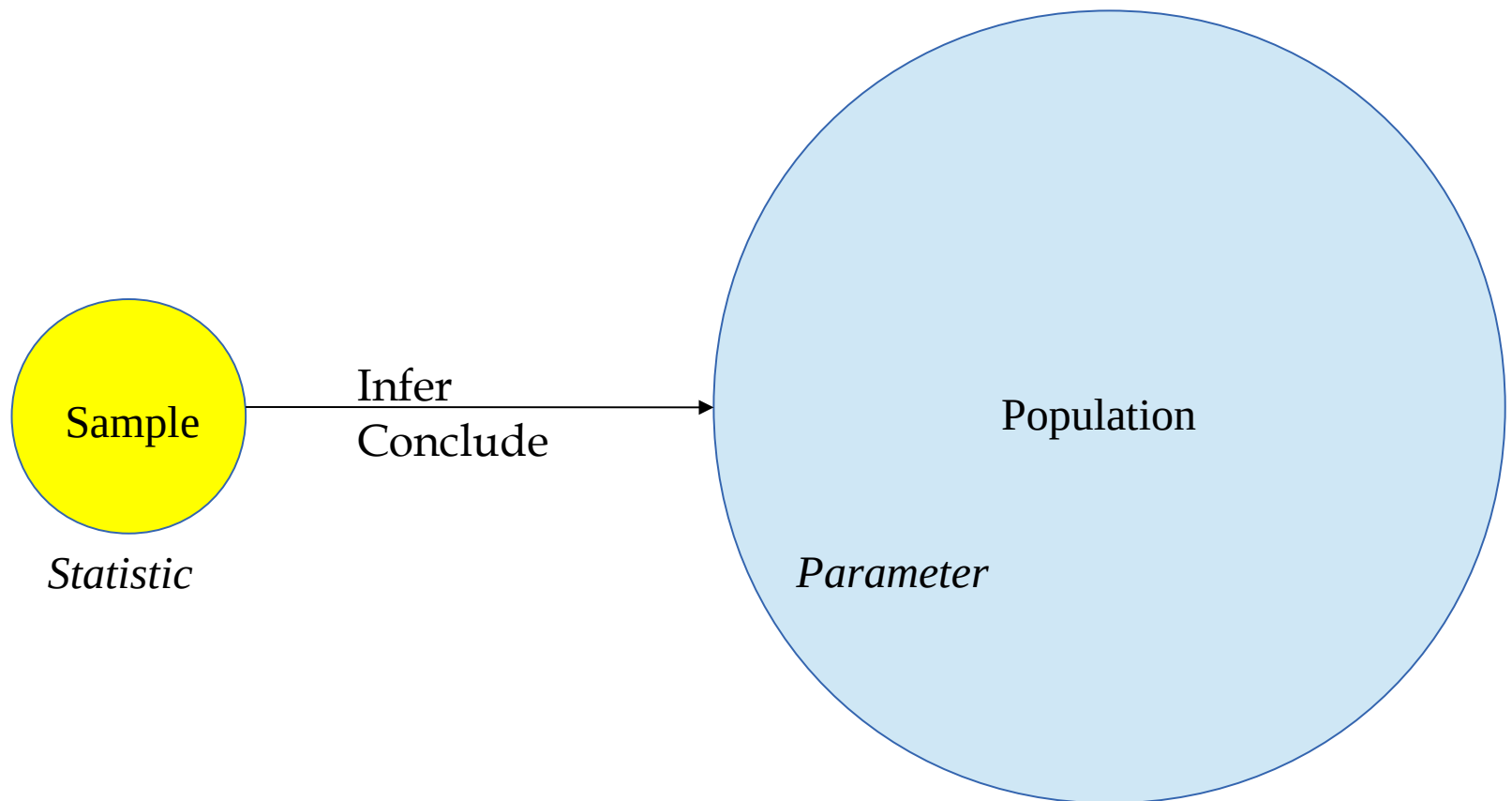
# Overview

- Population vs sample

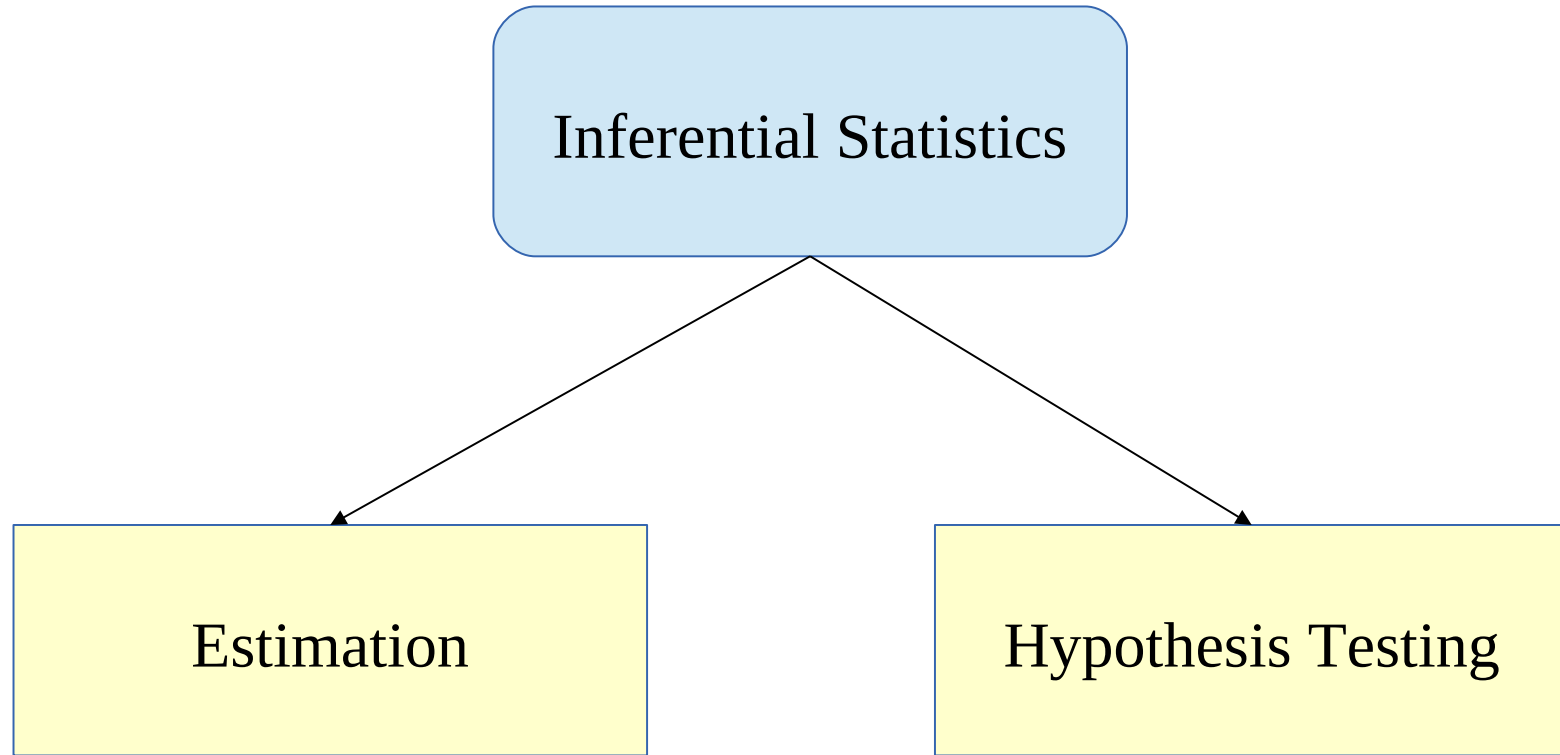


# Overview

- Inference:



# Overview





# Hypothesis

# Types of Hypothesis

- Research hypothesis “is the conjecture or supposition that motivates the research”
- Statistical hypothesis is the hypothesis that is stated in a way that is possible to evaluate by appropriate statistical analysis

# Types of Statistical Hypothesis

- Null hypothesis ( $H_0$ ) – It states a hypothesis of no difference/agreement with the population of interest
- Alternative hypothesis ( $H_A$ ) – The inverse of  $H_0$ . It states a hypothesis of disagreement with the population of interest. In most situations, it reflect the proposition in the research hypothesis

# Statistical Test & Hypothesis

- **Neyman-Pearson paradigm**

1. Acceptance region, the subset of the test statistic values for which  $H_0$  is accepted
2. Rejection region or critical region, the complement of the acceptance region, for which  $H_0$  is rejected and  $H_A$  is accepted

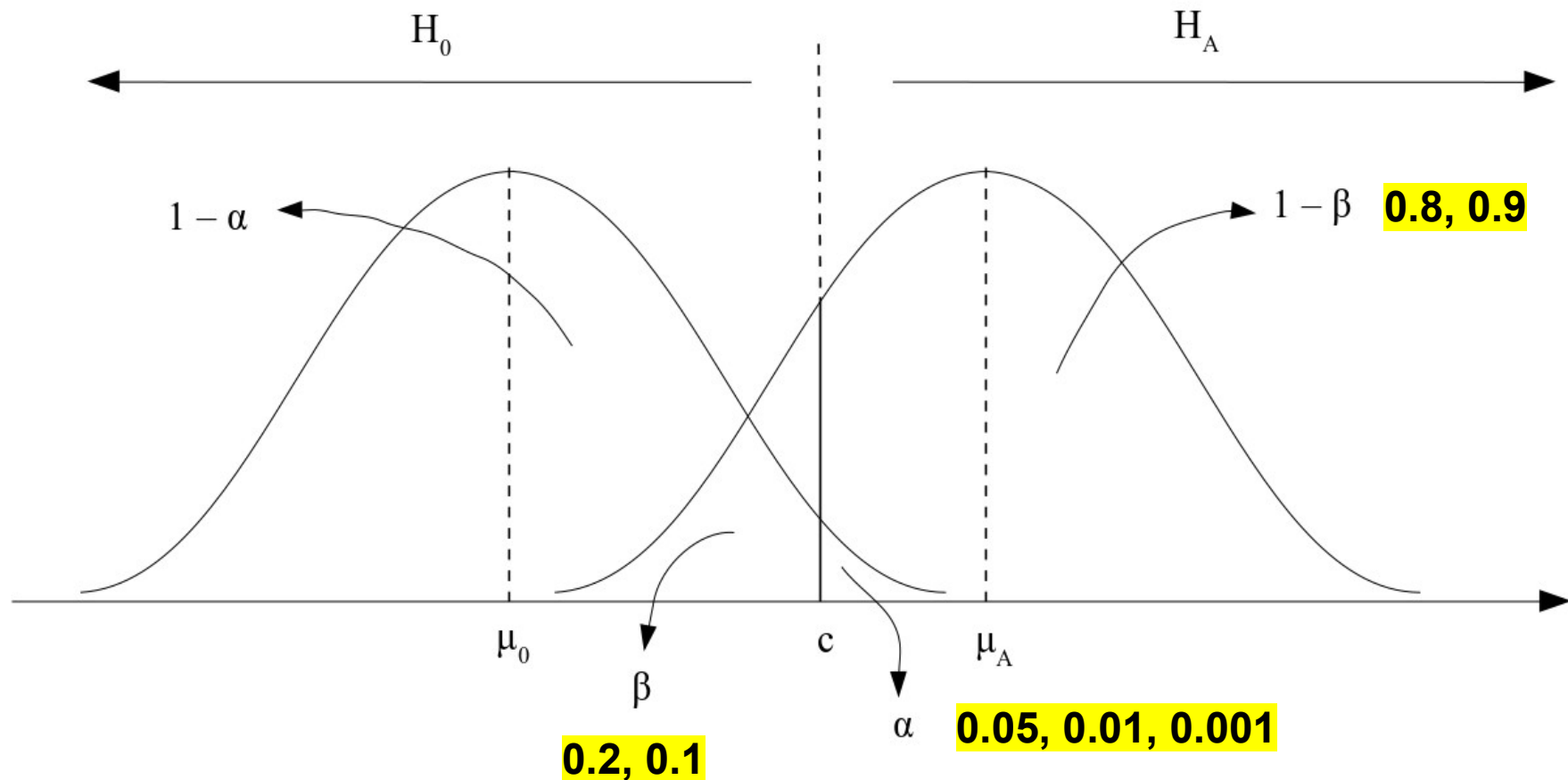
# Confidence Interval & Hypothesis Testing

- Acceptance region, region covered by a confidence interval,  $H_0$  is accepted
- Rejection region, region outside the confidence interval at both tails for a two-tailed test, or at either tail for a one-tailed test,  $H_0$  is rejected and  $H_A$  is accepted

# Statistical Test vs Hypothesis

		Hypothesis state (Truth)	
		True $H_A$ False $H_0$ (Difference +)	True $H_0$ False $H_A$ (Difference -)
Statistical test result	Significant (Test +)	True Positive $1 - \beta$ Sensitivity/ <b>Power</b>	False Positive $\alpha$ <b>Type I Error</b>
	No significant (Test -)	False Negative $\beta$ <b>Type II Error</b>	True Negative $1 - \alpha$ Specificity

# Statistical Test vs Hypothesis



# Test Statistic

- Statistic obtained from our calculation using appropriate formula
- General formula for hypothesis testing

$$\text{test statistic } W(\mathbf{X}) = \frac{\text{relevant statistic} - \text{hypothesized parameter}}{\text{standard error of relevant statistic}}$$

- Compared against a preset value OR
- Converted into probability value called *P*-value



# Significance level, $\alpha$

- The probability of rejecting a true null hypothesis
- Preset value of acceptable significance level (0.05, 0.01, 0.001), cutoff  $c$
- Acceptable level to commit type I error
- Compared to a test's  $P$ -value OR converted to test statistic value

# *P*-value

- The probability of getting a test statistic as extreme as the one that calculated
- Very low *P*-value means that the probability of getting a value similar or better is very unlikely
- When  $P\text{-value} < \alpha \rightarrow$  significant result, it would not happen easily by chance

# Hypothesis Testing

# Steps

1. Data description
2. Assumptions
3. Hypothesis statements
4. Test statistic used
5. Decision rule
6. Calculation of test statistic
7. Statistical decision
8. Conclusion

# Steps

1. Data description
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- Nature of data: numerical/categorical
- Present basic information:  $n$ , mean, variance/standard deviation

# Steps

1. Data description
  2. Assumptions
  3. Hypothesis statements
  4. Test statistic used
  5. Decision rule
  6. Calculation of test statistic
  7. Statistical decision
  8. Conclusion
- Distribution: normal/not normal
  - Random samples
  - Independence of samples:  
independent/paired
  - Population variance/standard deviation:  
known/not known

# Steps

1. Data description
2. Assumptions
3. Hypothesis statements
4. Test statistic used
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- $H_0$  and  $H_A$  can be stated in three ways based on  $H_A$

## **1) Difference (two-tailed test)**

Population mean SBP is not 120mmHg,

$$H_0: \mu = 120$$

$$H_A: \mu_A \neq 120$$

# Steps

1. Data description
2. Assumptions
3. Hypothesis statements
4. Test statistic used
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6. Calculation of test statistic
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8. Conclusion

- $H_0$  and  $H_A$  can be stated in three ways based on  $H_A$

## 2) More (one-tailed test)

Population mean SBP is more than 120mmHg,

$$H_0 : \mu_A \leq 120$$

$$H_A : \mu_A > 120$$



# Steps

1. Data description
2. Assumptions
3. Hypothesis statements
4. Test statistic used
5. Decision rule
6. Calculation of test statistic
7. Statistical decision
8. Conclusion

- $H_0$  and  $H_A$  can be stated in three ways based on  $H_A$

### **3) Less (one-tailed test)**

Population mean SBP is less 120mmHg,

$$H_0 : \mu_A \geq 120$$

$$H_A : \mu_A < 120$$

# Steps

1. Data description
2. Assumptions
3. Hypothesis statements
4. Test statistic used
5. Decision rule
6. Calculation of test statistic
7. Statistical decision
8. Conclusion

- Depending on the data and assumptions, state an appropriate statistic to use
- For example, for normally distributed data, when population variance is unknown  $\rightarrow t$  statistic instead of  $z$

# Steps

1. Data description
  2. Assumptions
  3. Hypothesis statements
  4. Test statistic used
  5. Decision rule
  6. Calculation of test statistic
  7. Statistical decision
  8. Conclusion
- Set the significance level, usually  $\alpha = 0.05$
  - Determine the cut-off value of the test statistic at the significance level
  - Specify rejection/acceptance region
  - One-tailed or two-tailed test based on the hypothesis statement → common two-tailed
  - If test statistic is outside the acceptance region/within rejection region → significant test, reject  $H_0$ , accept  $H_A$
  - If test statistic is within the acceptance region → test not significant, accept  $H_0$ , reject  $H_A$

# Steps

1. Data description
2. Assumptions
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4. Test statistic used
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6. Calculation of test statistic
7. Statistical decision
8. Conclusion

- Based on the chosen test statistic

# Steps

1. Data description
  2. Assumptions
  3. Hypothesis statements
  4. Test statistic used
  5. Decision rule
  6. Calculation of test statistic
  7. Statistical decision
  8. Conclusion
- Reject/accept  $H_0$
  - Significant/not significant given the preset significance level,  $\alpha$
  - State the  $P$ -value
  - For the two-tailed  $P$ -value, multiply the probability obtained from the probability distribution by two

# Steps

1. Data description
2. Assumptions
3. Hypothesis statements
4. Test statistic used
5. Decision rule
6. Calculation of test statistic
7. Statistical decision
8. Conclusion

- Conclude in relation to the research hypothesis

# Calculation

# One Population Mean

- Test statistic for one population mean

$$\text{test statistic} = \frac{\text{sample mean} - \text{hypothesized mean}}{\text{standard error of mean}}$$



# Standard normal distribution, z

- The test statistic for one population mean

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

given  $H_0$ , is distributed as z

- population variance  $\sigma^2$  is known
- data normally distributed

# Calculate

- A study was done in a sample of 30 adults to investigate the systolic blood pressure (SBP) in a population.
- It was hypothesized the mean SBP is 120mmHg.
- The data was normally distributed, with the mean SBP of 125mmHg and standard deviation  $s$  of 15mmHg.
- The sample  $s$  is treated as the population  $\sigma$ .
- Following the general hypothesis testing steps, based on the sample data, decide on the given hypothesis.

# Calculate

$$H_A: \mu \neq 120$$

For two-tailed, depending on value of sample mean (more or less than population mean), chose to calculate based on right/left tail

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

$n = 30$   
 $\bar{x} = 125$   
 $s = 15$   
 $\mu_0 = 120$

$$\begin{aligned} &P(SBP > 125) \\ &= P\left(Z > \frac{125 - 120}{15/\sqrt{30}}\right) \\ &= P(Z > 1.826) \\ &= 1 - P(Z < 1.826) \\ &= 1 - 0.966 \\ &= 0.034 \quad \text{Multiply by 2 for two tailed} \end{aligned}$$

Also refer to **hypothesis.R**

$$P\text{-value} = 0.034 \times 2 = 0.068 > 0.05, \text{ we do not reject Null } H_0$$

# Calculate

$$H_A: \mu \neq 120$$

For two-tailed, at significance level = 0.05,  
critical values between -1.96 to 1.96, as z

$$z = \frac{125 - 120}{15/\sqrt{30}} = 1.826$$

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \quad \begin{array}{l} n = 30 \\ \bar{x} = 125 \\ s = 15 \\ \mu_0 = 120 \end{array}$$

Therefore, 1.826 is within the range, we do  
not reject Null  $H_0$

# Calculate

$$H_A: \mu > 120$$

$$\begin{aligned} &P(SBP > 125) \\ &= P\left(Z > \frac{125 - 120}{15/\sqrt{30}}\right) \\ &= P(Z > 1.826) \\ &= 1 - P(Z < 1.826) \\ &= 1 - 0.966 \\ &= 0.034 \end{aligned}$$

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

$n = 30$   
 $\bar{x} = 125$   
 $s = 15$   
 $\mu_0 = 120$

Also refer to **hypothesis.R**

$P$ -value = 0.034 < 0.05, we reject Null  $H_0$

# Calculate

$$H_A: \mu > 120$$

For one-tailed, at significance level = 0.05,  
critical value at right tail is 1.65, as z

$$z = \frac{125 - 120}{15/\sqrt{30}} = 1.826$$

$$z = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

$n = 30$   
 $\bar{x} = 125$   
 $s = 15$   
 $\mu_0 = 120$

Therefore,  $1.826 > 1.65$ , we reject Null  $H_0$

# Calculate

$$H_A: \mu < 120$$

$$\begin{aligned} &P(SBP < 125) \\ &= P\left(Z < \frac{125 - 120}{15/\sqrt{30}}\right) \\ &= P(Z < 1.826) \\ &= 0.966 \end{aligned}$$

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

$n = 30$   
 $\bar{x} = 125$   
 $s = 15$   
 $\mu_0 = 120$

Also refer to **hypothesis.R**

$P$ -value = 0.966 > 0.05, we do not reject Null  $H_0$

# Calculate

$$H_A: \mu < 120$$

For one-tailed, at significance level = 0.05,  
critical value at left tail is -1.65, as z

$$z = \frac{125 - 120}{15 / \sqrt{30}} = 1.826$$

$$z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \quad \begin{array}{l} n = 30 \\ \bar{x} = 125 \\ s = 15 \\ \mu_0 = 120 \end{array}$$

Therefore,  $1.826 > -1.65$ , as we need z to be less than -1.65 (left tail) we do reject Null  $H_0$



# One Population Proportion

- Test statistic for one population proportion

$$\text{test statistic} = \frac{\text{sample proportion} - \text{hypothesized proportion}}{\text{standard error of proportion}}$$

# Standard normal distribution, $z$

- The test statistic for one population proportion

$$z = \frac{\hat{p} - p}{\sqrt{\left(\frac{p(1-p)}{n}\right)}}$$

given  $H_0$ , is distributed as  $z$  (provided  $np$  and  $n(1 - p) > 5$ )

# Calculate

- A study was done in a sample of 60 adults to investigate the proportion of diabetics in a population.
- It was hypothesized the proportion of diabetics in the population is 0.3.
- Based on the data, 25 participants were found to be diabetics.
- Following the general hypothesis testing steps, based on the sample data, decide on the given hypothesis.

# Calculate

$$H_A: p \neq 0.3$$

Complete the calculation. Also refer to **hypothesis.R**

# Calculate

$$H_A: p > 0.3$$

Complete the calculation. Also refer to **hypothesis.R**

# Calculate

$$H_A: p < 0.3$$

Complete the calculation. Also refer to **hypothesis.R**