AD Ü1

A2:

1. $17 + 22 + 45 = O(1)$

$84 = c \cdot 1$

$c = \underline{\underline{84}} = O(1)$

2. $\underbrace{5n^3 + 12n^2 + 3n + 5}= \Omega(n^3)$

$\cancel{5n^3 = \Omega(n^3)}$

$\cancel{= \Omega(n^3)}$

$\Rightarrow 5n^3 + 12n^2 + 3n + 5 \geq c \cdot n^3$

$\Rightarrow 5 + \frac{12}{n} + \frac{3}{n^2} + \frac{5}{n^3} \geq c$

3. $2^{n+1} = \underbrace{O(2^n)}$ $\wedge$ $2^{2n} = O(2^n)$ $\Rightarrow$ falsch

$\Rightarrow 2^{n+1} = 2 \cdot 2^n \leq c \cdot 2^n \quad | : 2^n$

$\Rightarrow c \geq 2$

bei $n \geq 2$ $\Rightarrow$ ~~wahr~~ wahr

$\Rightarrow 2^{2n} \leq c \cdot 2^n$

$\log(c \cdot 2^n)$

$\log(2^n)$

$\log c + n \cdot \log 2$

$= \log c + n$

$2n \cdot \log 2 \leq \cancel{\log c \cdot n \cdot \log 2}$

$2n \leq \log c + n \quad \cancel{|{-n}}\ |{-n}$

$\Rightarrow n \leq \log c$

$\cancel{2 \leq \log c}$

$\Rightarrow$ falsch ↯

$$\left\{ \begin{array}{l} \log(a^b) = b \log a \\ \log\left(\frac{a}{b}\right) = \log a - \log b \end{array} \right\}$$

4) $\log(n!) = \Theta(n \log n)$

Variante 1 (Stirling)

$$\log(n!) = \log\left(\sqrt{2\pi n} \cdot \left(\frac{n}{e}\right)^n\right)$$

$$= \underbrace{\frac{1}{2}\log(2\pi)}_{\Theta(1)} + \underbrace{\frac{1}{2}\log(n)}_{\Theta(\log n)} + \underbrace{n \cdot \log n}_{\Theta(n \log n)} - \underbrace{n \log e}_{\Theta(n)}$$

$$= \Theta(n \log n)$$

Variante 2:

$$\log(n!) = \log(1 \cdot 2 \cdot 3 \cdot \ldots \cdot n)$$
$$= \log(1) + \log 2 + \log 3 + \ldots + \log n$$
$$\leq n \cdot \log n = O(n \log n)$$

$$\log(n!) = \log 1 + \ldots + \log n$$
$$\geq \log\left(\frac{n}{2}\right) + \log n$$
$$\geq \frac{n}{2} \cdot \log\left(\frac{n}{2}\right) \in \Omega(n \log n)$$

$$\Rightarrow \Theta(n \log n)$$

5) $2^n = O(n!)$ und $n! = O(n^n)$

$\cancel{2^n = O(n^n) \neq O(n!)}$ ↯

$2^n = \Theta(2^n) < O(n!)$ ✓

$n! = \Theta(n!) < O(n^n)$ ✓

AD ~~2022~~ Ü2

2.5 $2^n = O(n!) \wedge n! = O(n^n)$

$2^n \leq c \cdot n!$

$n=1 \quad 2 \leq 1$ ↯

$n=2 \quad 4 \leq 1 \cdot 2$ ↯

$n=3 \quad 8 \leq 1 \cdot 2 \cdot 3 = 6$ ↯

$n=4 \quad 16 \leq 1 \cdot 2 \cdot 3 \cdot 4 = 24$ ✓

$\Rightarrow n \geq 4$

IA:

$n=4 \quad 16 \leq 24$ ✓

IV: $2^n \leq n!$

$n \Rightarrow n+1$

$2^{n+1} \leq (n+1)!$

$2^{n+1} \leq n! \cdot (n+1)$

$2 \cdot 2^n \leq n! \cdot (n+1) \quad | : (n+1)$

$\underbrace{\frac{2}{n+1}}_{<1} \cdot 2^n \leq n!$

$\Rightarrow \frac{2}{n+1} \cdot 2^n < 2^n \leq n!$

$1 \leq 1^1$ ✓

$2 \leq 2^2 = 4$ ✓

$6 \leq 3^3 = 27$ ✓

$n \in N$

IA: $1 \leq 1^1$ ✓
$n=1$

IV: $n! < n^n$

$n \to n+1$:

$(n+1)! \leq (n+1)^{(n+1)}$

$n! \cdot (n+1) \leq (n+1)^{(n+1)}$

$n! \cdot \cancel{(n+1)} \leq \cancel{(n+1)} \cdot (n+1)^n$

$n! \leq (n+1)^n$ ✓

da $n^n < (n+1)^n$

AD Ü2

3.:

~~Fall 1 $n < m$~~

~~IA: $f(n,m) = f(0, m-n)$~~

$f(2,3) = f(2-1, f(2, 3-1)) = f(1, f(2,2))$

$= f(1, f(1, f(1, 2-1))) = f(1, f(1, f(1,1)))$

$= f(1, f(1, f(0, f(1, 0))))$

~~$= f(1, f(1, f(f(0, f(0, f(1$~~

$= f(1, f(1, f(0, f(0, 1))))$

$= f(1, f(1, f(0, 2)))$

$= f(1, f(1, 3))$

$= f(1, f(0, f(1, 2)))$

$= f(1, f(0, f(0, f(1, 1))))$

$= f(1, f(0, f(0, f(0, f(1, 0)))))$

$= f(1, f(0, f(0, f(0, f(0, 1)))))$

$= f(1, f(0, f(0, f(0, 2))))$

$= f(1, f(0, f(0, 3)))$

$= f(1, f(0, 4))$

$= f(1, 5)$

$= f(0, f(1, 4))$

AD ÜZ

4.1 $T(1) = 1$, $T(n) = 4T(n/2) + n$, $\forall n > 1$

$$T(n) = 4T(\tfrac{n}{2}) + n$$
$$= 4(4T(\tfrac{n}{4}) + \tfrac{n}{2}) + n$$
$$= 4(4(4T(\tfrac{n}{8}) + \tfrac{n}{4}) + \tfrac{n}{2}) + n$$
$$= 4^i \cdot T(\tfrac{n}{2^i}) + \sum_{k=0}^{i-1} \frac{n}{2^k} \cdot 4^k$$
$$= 4^i \cdot T(\tfrac{n}{2^i}) + n \sum_{k=0}^{i-1} 2^k$$
$$\Rightarrow 4^{\log n} \cdot \underbrace{T\left(\frac{n}{2^{\log n}}\right)}_{1} + n \sum_{k=0}^{\log n - 1} 2^k$$
$$\Rightarrow n^2 + n \cdot \left(\frac{1 - 2^{\log n}}{1-2}\right)$$
$$\Rightarrow n^2 + n \cdot (-1 + n)$$
$$\Rightarrow n^2 - n + n^2$$
$$= 2n^2 - n$$
$$T(n) = \Theta(n^2)$$

NR:
$$\frac{n}{2^i} = 1$$
$$\Rightarrow n = 2^i$$
$$\Rightarrow \log n = i \log 2$$
$$i = \log n$$

4.2 $T(1) = 1$, $T(n) = 2T(n/4) + \sqrt{n}$, $\forall n > 1$

$$T(n) = 2T(\tfrac{n}{4}) + \sqrt{n}$$
$$= 2(2T(\tfrac{n}{16}) + \sqrt{\tfrac{n}{4}}) + \sqrt{n}$$
$$= 2(2(2T(\tfrac{n}{4 \cdot 16}) + \sqrt{\tfrac{n}{16}}) + \sqrt{\tfrac{n}{4}}) + \sqrt{n}$$
$$= \dots$$
$$= 2^i \cdot T\left(\frac{n}{4^i}\right) + \sum_{k=0}^{i-1} \sqrt{\frac{n}{4^k}} \cdot 2^k$$
$$\Rightarrow \cancel{2^{\log_4 n}} \cdot \underbrace{T\left(\frac{n}{4^{\log_4 n}}\right)}_{1} + \underbrace{\sum_{k=0}^{\log_4 n - 1} \sqrt{\frac{n}{4^k}} \cdot 2^k}$$
$$\underbrace{2^{\frac{\log n}{2}}}$$
$$\underbrace{\sqrt{2^{\log n}}} \qquad + \underbrace{\sum \frac{\sqrt{n}}{\sqrt{4^k}} \cdot 2^k}$$
$$\sqrt{n} \qquad + \frac{\sqrt{n}}{2^k} \cdot \cancel{2^k} \Rightarrow \Theta(\sqrt{n})$$

NR:
$$\frac{n}{4^i} = 1$$
$$n = 4^i$$
$$\log n = i \log 4$$
~~$\log n = 2i$~~
$$i = \frac{\log n}{\cancel{\log 4}}$$
~~$i = \log_4 n$~~
$$\Rightarrow i = \frac{\log n}{2}$$

4.3) $T(1)=1$, $T(2)=1$, $T(3)=1$

$T(n) = 2T(n-1)+n^2$, $\forall\ n>3$

$$\Rightarrow T(n) = 2T(n-1)+n^2$$
$$= 2(2T(n-2)+(n-1)^2)+n^2$$
$$= 2(2(2T(n-3)+(n-2)^2)+(n-1)^2)+n^2$$
$$= \dots$$
$$= 2^i\, T(\underbrace{n-i}_{=3}) + \sum_{k=0}^{i-1} (n-k)^2 \cdot 2^k$$

$\Rightarrow i = n-3$

$$T(n) = \underbrace{2^{n-3} \cdot T(\underbrace{n-(n-3)}_{1})}_{\in \Theta(2^n)} + \underbrace{\sum_{k=0}^{n-4} 2^k \cdot (n-k)^2}_{\in O(2^n \cdot n^2)}$$

$$\sum_{k=0}^{n-4} 2^k \cdot (n-k)^2 = 2^0 n^2 + 2^1 (n-1)^2 + \dots + 2^{n-4} 4^2$$

$$= \sum_{k=4}^{n} 2^{(n-k)} \cdot k^2$$

$$= \sum_{k=4}^{n} 2^n \cdot \frac{k^2}{2^k} = 2^n \cdot \underbrace{\sum_{k=4}^{n} \frac{k^2}{2^k}}_{\in \Theta(1)}$$

mit Quotientenkriterium

$\Rightarrow$

$T(n) = \Theta(2^n)$