# AD Ü5

| $i$ | max |
|---|---|
| 0 | $n=1$ |
| 1 | $n=2$ |
| 2 | $n=4$ |

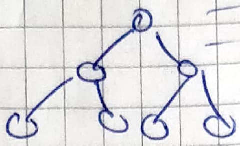
$n = 2^i$ | log

$\Rightarrow i \cdot \log 2 = \log n$

$\Leftrightarrow i = \log n$

$i$ = Schicht index

5.2 1.] $n=7$

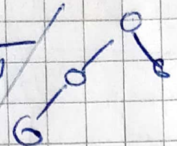


$\Rightarrow h=2$

$\lfloor \log n \rfloor = \lfloor \log 7 \rfloor = 2$

~~$\Rightarrow$ Höhe Baum = $\lfloor \log$ (Summe Anzahl Knoten) $\rfloor$~~

~~Behauptung~~



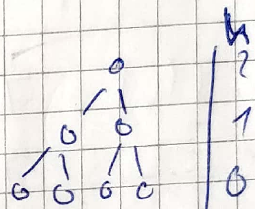
$\Rightarrow 2^h \leq n \leq \cancel{2^{h+1} - 1} < 2^{h+1}$ | log

$\Rightarrow h \log 2 \leq \log n < h+1 \log 2$

$\Rightarrow h \leq \log_2 n < h+1$

$\Rightarrow h = \lfloor \log n \rfloor$

2.] Heap $n = 7$



IA: $h = 0$

$\left\lceil \frac{7}{2^{0+1}} \right\rceil = \left\lceil \frac{7}{2} \right\rceil = \lceil 3,. \rceil = \underline{\underline{4}}$

~~IS: $h \rightarrow h+1$~~

~~$\left\lceil \frac{n}{2^{h+1+1}} \right\rceil = \left\lceil \frac{7}{2^{h+2}} \right\rceil = \left\lceil \frac{7}{2^{h+1} \cdot 2} \right\rceil$~~

~~IA: $h = 0$~~

~~$\left\lceil \frac{n}{2^{0+1}} \right\rceil = \left\lceil \frac{n}{2} \right\rceil$~~

~~IS: $h \rightarrow h+1$~~

~~$\left\lceil \frac{n}{2^{h+1+1}} \right\rceil =$~~

2

$\theta$

AD Ü5

2.3 Tipp:

$$\sum_{k=0}^{\infty} x^k = \frac{1}{1-x} \quad |x| < 1$$

$$\Rightarrow 1 + x^1 + x^2 + x^3 + x^4 + x^5 \dots = \frac{1}{1-x} \quad \Big| \frac{d}{dx}$$

$$1 + 2x + 3x^2 + 4x^3 + 5x^4 + \dots = \frac{0 - (-1)}{(1-x)^2}$$

$$\Rightarrow \sum_{k=0}^{\infty} k \cdot x^{(k-1)} = \frac{1}{(1-x)^2} \quad \Big| \cdot x$$

$$\Rightarrow x \sum_{k=0}^{\infty} \cdot k \cdot x^{(k-1)} = \frac{x}{(1-x)^2}$$

$$\Rightarrow \sum_{k=0}^{\infty} k \cdot x^k = \frac{x}{(1-x)^2}$$

4. Nein BuildHeap kann nicht vertauscht werden, da sonst ~~Werte~~ max Werte nicht zum Root kommen.

5.3 | Standard $\Rightarrow O_{ij} = \sum_{k=1}^{n} M_{ik} \cdot N_{kj}$

$$O = \begin{pmatrix} M_{11} \cdot N_{11} + M_{12} \cdot N_{21} & M_{11} \cdot N_{12} + M_{12} \cdot N_{22} \\ M_{21} \cdot N_{11} + M_{22} \cdot N_{21} & M_{21} \cdot N_{12} + M_{22} \cdot N_{22} \end{pmatrix}$$

$\Rightarrow$ Variante 1 $\hat{=}$ Standard

Laufzeit Variante 1

$$T(n) = 8T\left(\frac{n}{2}\right) + 4 \cdot \left(\frac{n}{2}\right)^2 = 8T\left(\frac{n}{2}\right) + \cancel{\frac{4n^2}{4}}\, n^2 = \Theta(n^3)$$

8 Multiplikationen
4 Additionen

$$\begin{array}{cc} & 1 \dots \frac{n}{2} \dots n \\ M & \left(\begin{array}{c|c} M_{11} & M_{12} \\ \hline M_{21} & M_{22} \end{array}\right) \end{array}$$

Variante 2: Laufzeit:

7 Multiplikationen und 18 Additionen

$$\Rightarrow T(n) = 7T\left(\frac{n}{2}\right) + 18\left(\frac{n}{2}\right)^2 = 7T\left(\frac{n}{2}\right) + \frac{18n^2}{4}$$

$n^{\log_2 7}$ with $\underbrace{\log_2 7}_{<3,\ >2}$ $\rightarrow n^{\log_2 7} > \frac{18n^2}{4}$

$\Rightarrow$ 1. Fall

$$T(n) = \Theta\left(n^{\log_2 7}\right) = \Theta\left(n^{2{,}80735\dots}\right)$$

Beweis Variante 1

IA: $n = 2$ $2 \times 2$ Matrix ✓

IV: Annahme: gilt für $\frac{n}{2} \times \frac{n}{2}$

IS: ~~n → n+1~~ $\frac{n}{2} \rightarrow n$

Nach Definition von Var 1:

$$O_{11} = M_{11} \cdot N_{11} + M_{12} \cdot N_{21}$$

$\overset{IV}{=}$ ~~$O_{11} \sum_{k=1}^{n/2}$~~

↳

$$O_{11}^{ij} \overset{IV}{=} \sum_{k=1}^{\frac{n}{2}} m_{11}^{ik} \cdot n_{11}^{kj} + \sum_{k=1}^{\frac{n}{2}} m_{12}^{ik} \cdot n_{21}^{kj}$$

$$= \text{"} \quad + \sum_{k=\frac{n}{2}+1}^{n} m^{ik} \cdot n^{kj}$$

$$= \sum_{k=1}^{n} m^{ik} \cdot n^{kj} .$$

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Beweis Variante 2

$$O_{11} = H_1 + H_4 - H_5 + H_7$$

$$H_1 = (M_{11} + M_{22}) \cdot (N_{11} + N_{22})$$
$$H_4 = M_{22} \cdot (N_{21} - N_{11})$$
$$H_5 = (M_{11} + M_{12}) \cdot N_{22}$$
$$H_7 = (M_{12} - M_{22}) \cdot (N_{21} + N_{22})$$

$$O_{11} = (M_{11} + M_{22}) \cdot (N_{11} + N_{22}) + M_{22} \cdot N_{21} - M_{22} \cdot N_{11}$$
$$- (M_{11} N_{22} + M_{12} N_{22}) + (M_{12} - M_{22}) \cdot (N_{21} + N_{22})$$

$$= M_{11} N_{11} + \overset{2}{\cancel{M_{11} N_{22}}} + \overset{1}{\cancel{M_{22} N_{11}}} + \overset{4}{\cancel{M_{22} N_{22}}} + \overset{4}{\cancel{M_{22} N_{21}}} - \overset{1}{\cancel{M_{22} N_{11}}}$$
$$- \overset{2}{\cancel{M_{11} N_{22}}} - \overset{3}{\cancel{M_{12} N_{22}}} + M_{12} N_{21} + \overset{3}{\cancel{M_{12} N_{22}}} - \overset{4}{\cancel{M_{22} N_{21}}} - \overset{4}{\cancel{M_{22} N_{22}}}$$

$$= M_{11} N_{11} + M_{12} N_{21}$$

$$= O_{11} \text{ aus Variante 1}$$