AD Ü3 18.10

A1 $f(1)=1,\ f(2)=1 \quad f(n)=f(n-1)+f(n-2) \quad ,\forall n\geq 3$

und $f(n)=\dfrac{\phi^n-\hat{\phi}^n}{\sqrt{5}} \quad , \phi=\dfrac{1+\sqrt{5}}{2} \quad , \hat{\phi}\ \dfrac{1-\sqrt{5}}{2}$

IA: $n=3$

$$f(3)=f(2)+f(1)=1+1=2=\frac{\left(\frac{1+\sqrt{5}}{2}\right)^3-\left(\frac{1-\sqrt{5}}{2}\right)^3}{\sqrt{5}}=\frac{\frac{(1+\sqrt{5})^3}{8}-\frac{(1-\sqrt{5})^3}{8}}{\sqrt{5}}$$

$$=\frac{(1+\sqrt{5})^3}{8\sqrt{5}}-\frac{(1-\sqrt{5})^3}{8\sqrt{5}}=\frac{(1+\sqrt{5})^3-(1-\sqrt{5})^3}{8\sqrt{5}}$$

$$=\frac{(1+\sqrt{5})(1+2\sqrt{5}+5)-(1-\sqrt{5})(1-2\sqrt{5}+5)}{8\sqrt{5}}$$

$$=\frac{6+6\sqrt{5}+2\sqrt{5}+10-(6-6\sqrt{5}-2\sqrt{5}+10)}{8\sqrt{5}}$$

$$=\frac{\cancel{11}+8\sqrt{5}-6+6\sqrt{5}+2\sqrt{5}-20}{8\sqrt{5}}=\frac{16\sqrt{5}}{8\sqrt{5}}=2 \checkmark$$

IS: $n \mapsto n+1$

$$f(n+1)=\underbrace{f(n)}_{IV}+f(n-1)=\frac{\phi^{n+1}-\hat{\phi}^{n+1}}{\sqrt{5}}$$

$$=\frac{\phi^n-\hat{\phi}^n}{\sqrt{5}}+\frac{\phi^{n-1}-\hat{\phi}^{n-1}}{\sqrt{5}}=\frac{\phi^{n+1}-\hat{\phi}^{n+1}}{\sqrt{5}} \quad |\cdot\sqrt{5}$$

$$=\phi^n-\hat{\phi}^n+\phi^{n-1}-\hat{\phi}^{n-1}=\phi^{n+1}-\hat{\phi}^{n+1}$$

$$\left(\frac{1+\sqrt5}{2}\right)^n-\left(\frac{1-\sqrt5}{2}\right)^n+\left(\frac{1+\sqrt5}{2}\right)^{n-1}-\left(\frac{1-\sqrt5}{2}\right)^{n-1}$$

$$=\left(\frac{1+\sqrt5}{2}\right)^n-\left(\frac{1-\sqrt5}{2}\right)^n+\left(\frac{1+\sqrt5}{2}\right)^n\cdot\frac{2}{1+\sqrt5}-\left(\frac{1-\sqrt5}{2}\right)^n\cdot\frac{2}{1-\sqrt5}$$

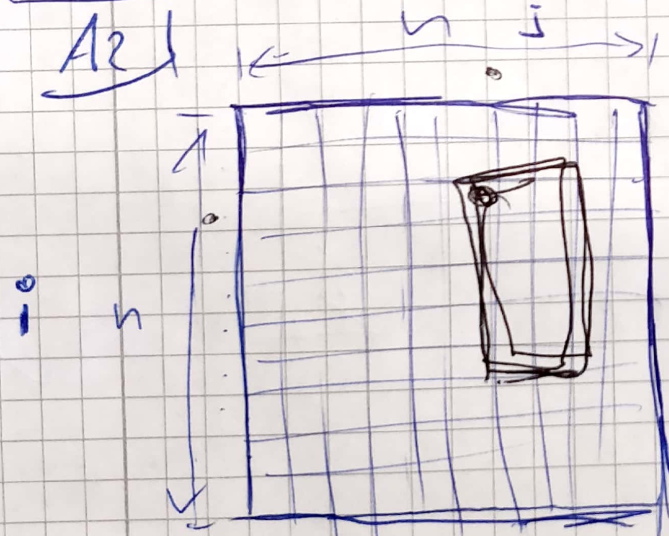
$$=\left(1+\frac{2}{1+\sqrt5}\right)\cdot\left(\frac{1+\sqrt5}{2}\right)^n-\left(1+\frac{2}{1-\sqrt5}\right)\left(\frac{1-\sqrt5}{2}\right)^n$$

$$=\frac{3+\sqrt5}{1+\sqrt5}\cdot\left(\frac{1+\sqrt5}{2}\right)^n-\frac{3-\sqrt5}{1-\sqrt5}\cdot\left(\frac{1-\sqrt5}{2}\right)^n$$

Z

6

A2)



3x3 Tafeln

$$=\underset{(1\times1)}{9}+\underset{(2\times2)}{4}+\underset{(3\times3)}{1}$$

$$+\underset{(1\times2)}{12}+\underset{(2\times3)}{4}+\underset{(1\times3)}{6}$$

$$=\cancel{38}\ 36$$

-7	15	32
-14	18	-8
5	-12	-24

$$15+32$$
$$+18-8$$
$$=57$$

AD Ü3

A3 | 1. $T(1) = 1,\ T(n) = T(n/2) + 1,\ \forall n > 1$

~~$T(n) = \Theta(\log n)$~~

$a = 1$
$b = 2$
$f(n) = 1$, ~~$\forall n \geq$~~ $\qquad n^{\log_2 1} = n^0 = 1$
~~$f(n) = \log n$~~

$\Rightarrow$ ~~Fall 1, da f(n) konstant~~ Fall 2

~~$\Rightarrow T(n) = \Theta(n^{\log_2 1} \log) f(n) = \Theta$~~

~~$f(n) = \Theta(\log n^0)$~~ $\quad f(n) = \Theta(n^{\log_b a})$

$\qquad 1 = n^{\log_2 1} = 1$ ✓

$\Rightarrow T(n) = \Theta(n^{\log_b a} \log n) = \Theta(\log n)$ ✓

A3.2 $T(1) = 1,\ T(n) = 3T(n/4) + n \log n,\ \forall n > 1$

$a = 3$
$b = 4$
$f(n) = n \log n$

$n^{\underbrace{\log_4 3}_{< 1}}$

$\Rightarrow$ Fall 3: ~~$f(n) \geq \Omega$~~

$f(n) = \Omega(n^{\log_4 3})$

① $\Rightarrow n \log n \geq c \cdot n^{\underbrace{\log_4 3}_{< 1}}$ ✓

② $a f\left(\frac{n}{b}\right) \leq c f(n)$, $\quad c < 1$

$\Rightarrow 3 \cdot \frac{n}{4} \log \frac{n}{4} \leq c \cdot n \log n$

?

$\Rightarrow T(n) = \Theta(f(n)) = \Theta(n \log n)$

13.3 $T(1) = 1, \quad T(n) = 7\,T(n/2) + n^2, \quad \forall n \geq 1$

$a = 7$
$b = 2$
$f(n) = n^2$

$\underbrace{n^{\log_2 7}}_{[2;\,3[}$

$\Rightarrow$ Fall 1:

$f(n) = O(n^{\log_2 7})$

$n^2 \leq c \cdot n^{\log_2 7}$

$n^2 \leq c \cdot n^{\log_2 4} = c \cdot n^2 \leq c \cdot n^{\log_2 7}$ ✓

$\Rightarrow T(n) = \Theta(n^{\log_2 7}) \approx \Theta(n^{2,81})$

A4 $-5, 13, -32, 7, -3, 17, 23, 12, -35, 19$

Insertion Sort:

$-5, 13, \ldots$

$-32, -5, 13, \ldots$

$\ldots, 7, 13, \ldots$

$\ldots, -3, 7, 13,$

$\ldots 13, 17, \ldots$

$\ldots, 17, 23, \ldots$

$\ldots, 12, 13, 17, 23, \ldots$

$-35, -32, -5, -3, 7, 12, 13, 17, 23, \ldots$

$\ldots, 19, 23$