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Law of Sines & Law of Cosines

Goals

I will be able to use the Law of Sines to solve **Angle-Angle-Side**, **Angle-Side-Angle**, and **Side-Side-Angle** triangles for their unknown sides.

I will be able to determine if there is 1 solution, 2 solutions, or no solutions to a triangle.

I will be able to use the Law of Cosines to solve **Side-Angle-Side** and **Side-Side-Side** triangles.

I will be able to determine the correct law to solve any triangle.

I will be able to use both the Law of Sines and the Law of Cosines in the same triangle to solve that triangle.

Standards

Similarity, Right Triangles, and Trigonometry **G-SRT**

Apply trigonometry to general triangles

11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Connections

Before we learned about how trigonometry is used to solve for right triangles.

Now we are learning how trigonometry is applied to any triangle.

After we will be learning the unit circle and the way trigonometry is applied to the coordinate plane. This is called *analytic geometry*.

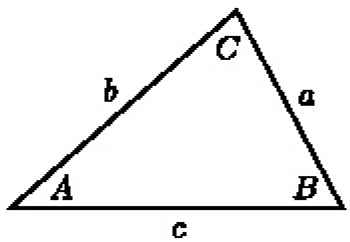
The Law of Sines

The **Law of Sines** helps us solve for **non**-right triangles. It is simply a proportion that we need to solve for.

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

or

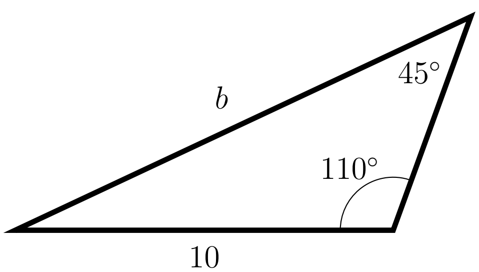
$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$



Capital letters always represent *angles*.
Lowercase letters always represent *sides*.

Angle A is always opposite side a.
Angle B is always opposite side b.
Angle C is always opposite side c.

Solving for a Side: Side - Angle - Side

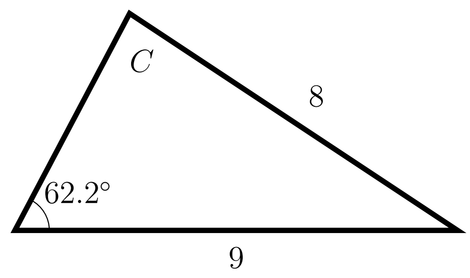


Since we're solving for a side, put the unknown side on the top-left fraction, then solve for b.

$$\frac{b}{\sin(110)} = \frac{10}{\sin(45)}$$

$$b = \frac{10\sin(110)}{\sin(45)} \approx 13.3$$

Solving for an Angle: Side - Side - Angle



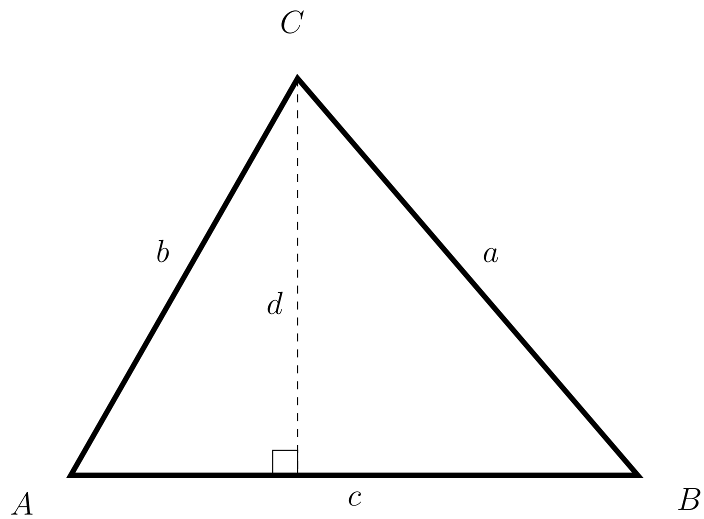
Since we're solving for an angle we must use an *inverse sine* to finish the problem. Just like above, we put the unknown in the top left of the fraction.

$$\frac{\sin(C)}{9} = \frac{\sin(62.2)}{8}$$

$$\sin(C) = \frac{9\sin(62.2)}{8}$$

$$C = \sin^{-1}\left(\frac{9\sin(62.2)}{8}\right) \approx 84.36$$

Law of Sines Proof



$\sin(A) = \frac{d}{b}$	$\sin(B) = \frac{d}{a}$
$d = b\sin(A)$	$d = a\sin(B)$

$$b\sin(A) = a\sin(B)$$

$$\frac{b}{\sin(B)} = \frac{a}{\sin(A)}$$

Law of Cosines

The **Law of Cosines** is the Pythagorean Theorem of all triangles, not just right triangles.

$$
\begin{aligned}
a^2 &= b^2 + c^2 - 2bc\cos(A) \\
b^2 &= a^2 + c^2 - 2ac\cos(B) \\
c^2 &= a^2 + b^2 - 2ab\cos(C)
\end{aligned}
$$

Which one of these you use depends on what you're trying to solve for.

Solving for a side: Side-Angle-Side

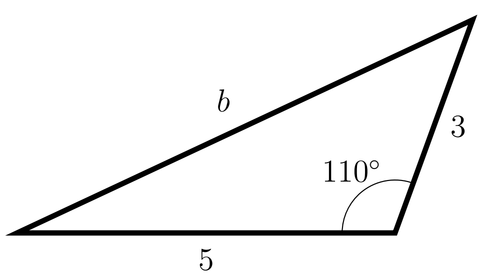
4 - step process

1. Label all of your points – This keeps them straight in your head.

2. Plug the pieces into the proper places. $b^2 = a^2 + c^2 - 2ac\cos(B)$

3. Evaluate.

4. Take square root.



$$
\begin{aligned}
\text{Step 2:}\quad b^2 &= 5^2 + 3^2 - 2(5)(3)\cos(110) \\
b^2 &= 25 + 9 - 30\cos(110) \\
\text{Step 3:}\quad b^2 &\approx 44.2606043 \\
\text{Step 4:}\quad b &= \sqrt{44.2606043} \approx 6.5
\end{aligned}
$$

Solving for an angle: Side - Side - Side

To solve for A, $a^2 = b^2 + c^2 - 2bc\cos(A)$ could be used. Instead we will use an alternate version of the same equation. This is the same equation, except that we're looking to solve and angle, so it's automatically solved for the $\cos(x)$. You can still use the above Law of Cosines to set up and solve.

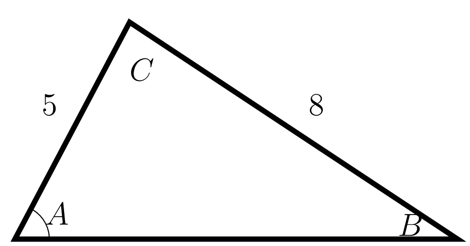
$$\cos(A) = \frac{a^2 - b^2 - c^2}{-2bc}$$

$$\cos(B) = \frac{b^2 - a^2 - c^2}{-2ac}$$

$$\cos(C) = \frac{c^2 - a^2 - b^2}{-2ab}$$

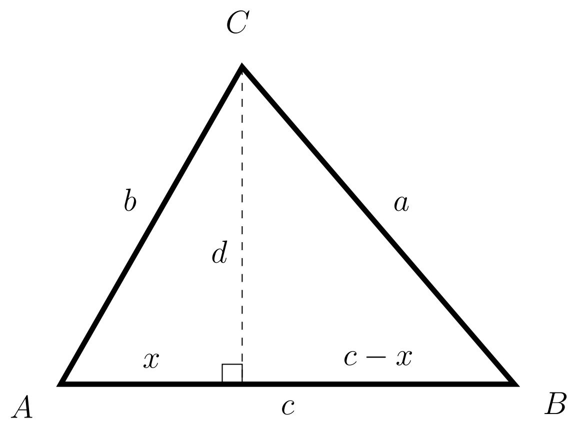
4 - step process to solve for an angle

1. Label all of your sides and angles.
2. Plug your values into the equation.
3. Evaluate.
4. Calculate $\cos^{-1}$ to determine the angle.



$$\begin{array}{lccl} \text{Step 2:} & \cos(A) & = & \frac{8^2-5^2-9^2}{-2(5)(9)} \\ & \cos(A) & = & \frac{-42}{-90} \\ \\ \text{Step 3:} & \cos(A) & \approx & .466 \\ \\ \text{Step 4:} & A & = & \cos^{-1}(.466) \approx 62 \end{array}$$

Law of Cosines Proof



$$\cos(A) = \frac{x}{b} \implies x = b\cos(A)$$

$$h^2 = b^2 - x^2 \qquad\qquad h^2 = a^2 - (c-x)^2$$

set h equal to itself

$$b^2 - x^2 = a^2 - (c-x)^2$$

expand the square of a binomial

$$b^2 - x^2 = a^2 - c^2 + 2cx - x^2$$

x^2 cancells out

$$b^2 = a^2 - c^2 + 2cx$$

$x = b\cos(A)$ from above

$$b^2 = a^2 - c^2 + 2cb\cos(A)$$

solving for a^2

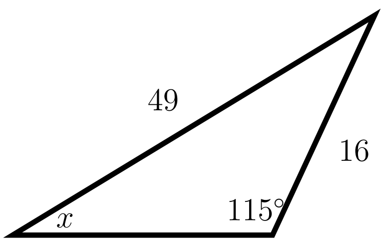
$$a^2 = b^2 + c^2 - 2cb\cos(A)$$

Law of Sines vs. Cosines

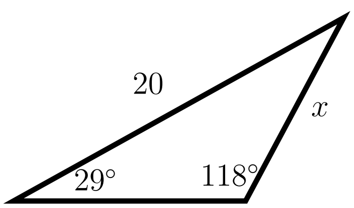
Which law do you use? The law of sines is used for side-side-angle and side-angle-side, while the law of cosines is used for s

Law of Sines

2 sides, 1 opposite angle

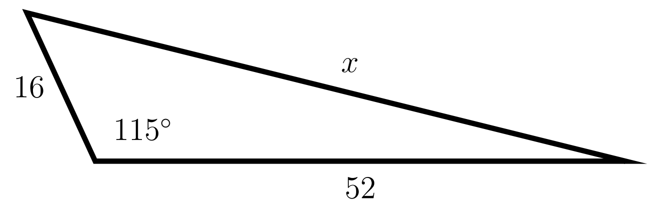


2 angles, 1 opposite side



Law of Cosines

2 sides, 1 angle inbetween



3 sides, no angles