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General Angles, Radian Measure, and The Unit Circle

Goals

I will be able to define general angles.

I will be able to convert from degrees to radians.

I will be able to evaluate trigonometric functions of any angle.

I will be able to use the unit circle.

I will be able to identify all of the reference angles and use them to my benefit.

Standards

Trigonometric Functions **F-TF**

Extend the domain of trigonometric functions using the unit circle

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\frac{\pi}{3}$, $\frac{\pi}{4}$ and $\frac{\pi}{6}$, and use the unit circle to express the values of sine, cosines, and tangent for x, $+x$, and $2x$ in terms of their values for x, where x is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Prove and apply trigonometric identities

8. Prove the Pythagorean identity $\sin^2(\theta) + cos^2(\theta) = 1$ and use it to calculate trigonometric ratios.
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Connections

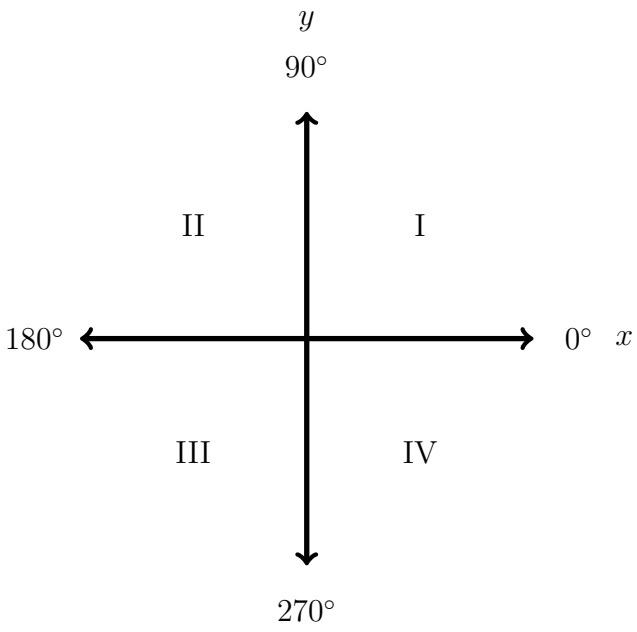
Before we learned about how trigonometry applied to triangles, and general graphing of circles.

Now we are learning about trigonometry on the coordinate plane and circles.

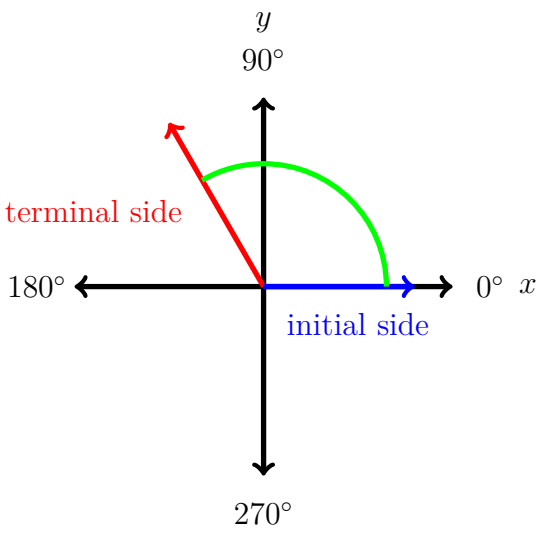
Later we will learn about graphing periodic trigonometric functions.

General Angles

Putting trig on the coordinate plane is called *analytic geometry*. The starting point on the coordinate plane in the *origin*, where the *x-axis* and *y-axis* meet $(0, 0)$. The rotation starts from the right side of the *x-axis* and opens counter-clockwise.



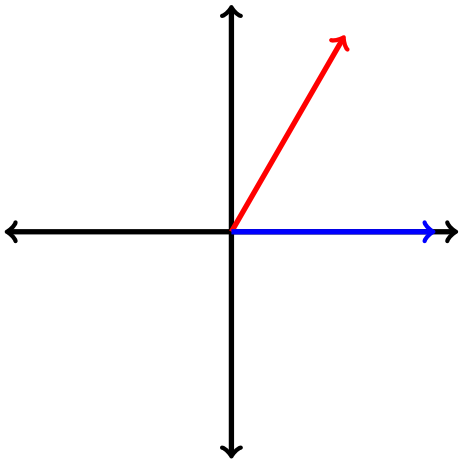
The *initial side* comes from the origin along the *x-axis* towards the right (positives). The *terminal side* is drawn **counter-clockwise** around the origin. The angle is between the initial side and the terminal side.



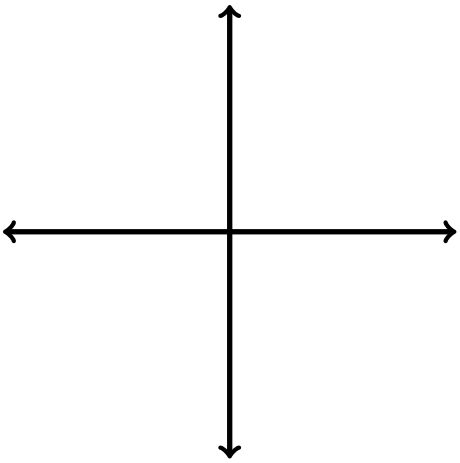
This is called **Standard Position** of an angle.

Standard Position

Example 1) Approximate the angle

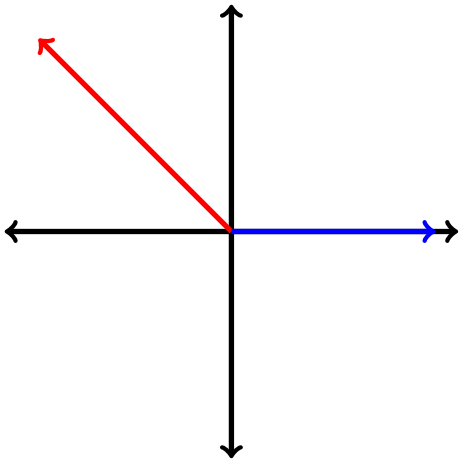


Example 2) Draw a 210° angle

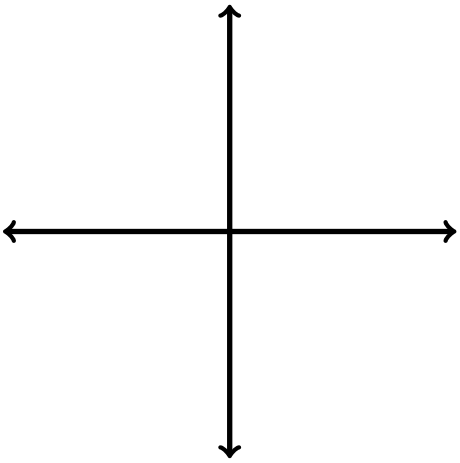


You Try:

Approximate the angle



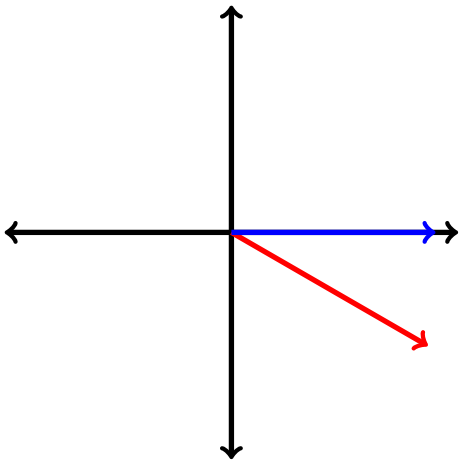
Draw a 100° angle.



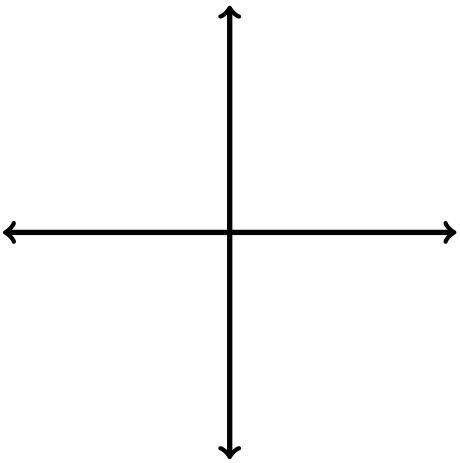
Negative Angles

A negative angle will go *clockwise* instead of *counter-clockwise*

Example 3) Approximate the angle

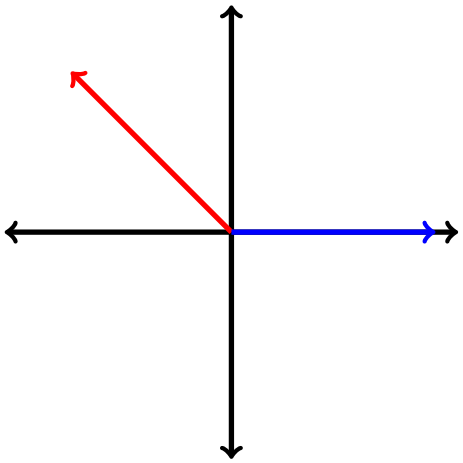


Example 4) Draw a $-100°$ angle;

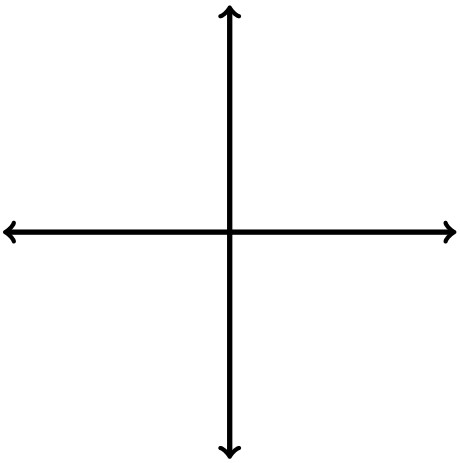


You Try:

Approximate the angle



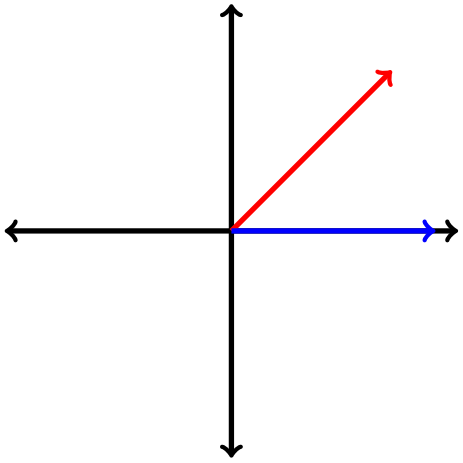
Draw a $-80°$ angle



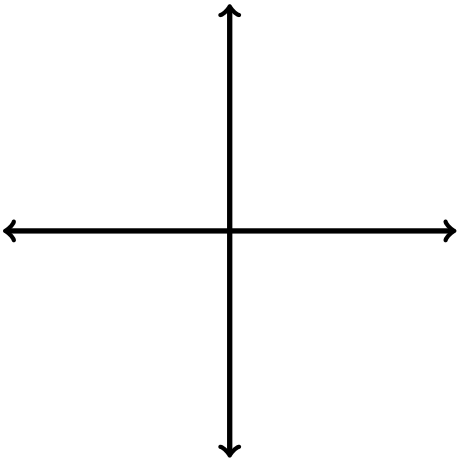
Co-Terminal Angles

Two angles with the same terminal side are called **co-terminal angles**. ($0° = 360°$ or $90° = -270°$)

Example 5) Two ways to write the angle.

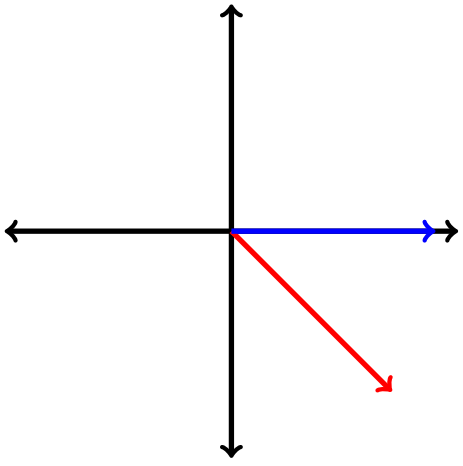


Example 6) What is coterminal to 210°?

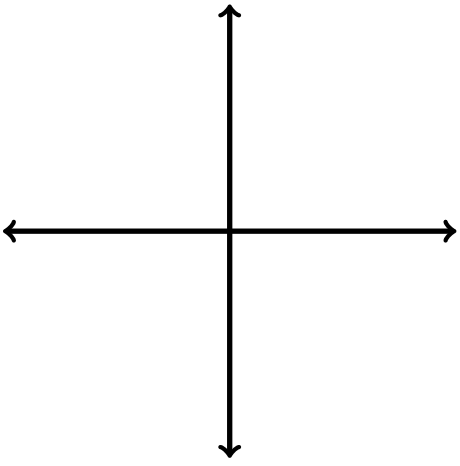


You Try:

Two ways to write the angle.



What is coterminal to 500°?

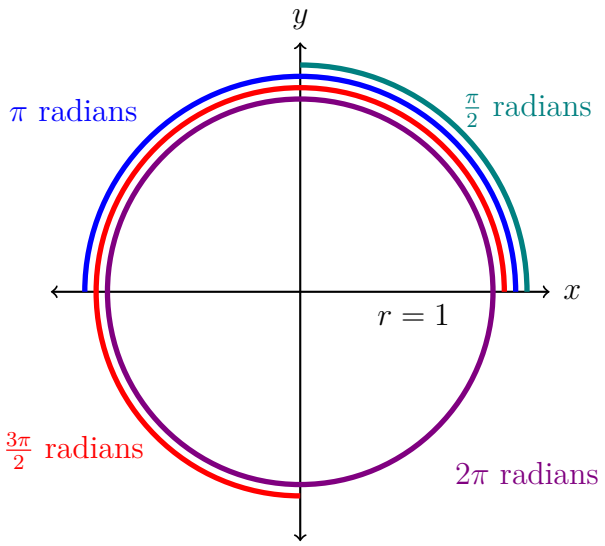


Radians

There are two main ways to measure angles. The first one that you know is called *degree angle measure*. The new type that we're learning is called *radian angle measure* (which is what "rad" stands for on your calculator).

The circumference of a circle is $2\pi r$. So a circle with a radius of 1 has a circumference of 2π.

$360^\circ = 2\pi$ radians $\qquad\qquad$ π radians $= 180^\circ$



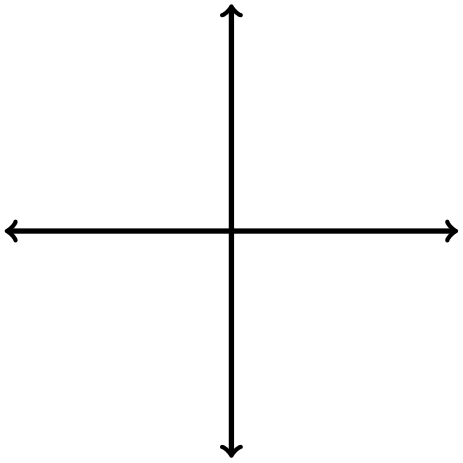
To convert from degree to radian $\times \frac{\pi}{180}$

To convert from radian to degree $\times \frac{180}{\pi}$

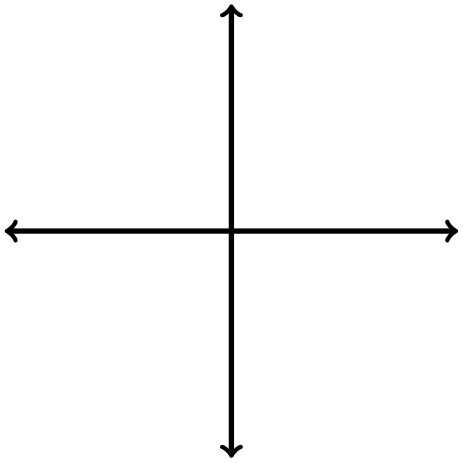
1. 190° is how many radians?
2. 2 radians is how many degrees?
3. 270° is how many radians?
4. $\frac{\pi}{6}$ radians is how many degrees?
5. 45° is how many radians?
6. $\frac{\pi}{3}$ is how many degrees?
7. 100° is how many radians?
8. $\frac{3\pi}{2}$ is how many degrees?

1 Practice

Draw a $\frac{3\pi}{4}$ radians



Draw a $-\frac{5\pi}{6}$ radians



Find the angle between 0 and 2π that is coterminal to $-\frac{23\pi}{10}$

Tips on Radians

Multiples of 90°

$90^\circ = \frac{\pi}{2}$ radians

$180^\circ = \frac{2\pi}{2}$ radians $= \pi$ radians

$270^\circ = \frac{3\pi}{2}$ radians

$360^\circ = \frac{4\pi}{2}$ radians $= 2\pi$ radians

Multiples of 60°

$60^\circ = \frac{\pi}{3}$ radians

$120^\circ = \frac{2\pi}{3}$ radians

$180^\circ = \frac{3\pi}{3}$ radians $= \pi$ radians

$240^\circ = \frac{4\pi}{3}$ radians

$300^\circ = \frac{5\pi}{3}$ radians

$360^\circ = \frac{6\pi}{3}$ radians $= 2\pi$ radians

Multiples of 45°

$45^\circ = \frac{\pi}{4}$ radians

$90^\circ = \frac{2\pi}{4}$ radians $= \frac{\pi}{2}$ radians

$135^\circ = \frac{3\pi}{4}$ radians

$180^\circ = \frac{4\pi}{4}$ radians $= \pi$ radians

$225^\circ = \frac{5\pi}{4}$ radians

$270^\circ = \frac{6\pi}{4}$ radians $= \frac{3\pi}{2}$ radians

$315^\circ = \frac{7\pi}{4}$ radians

$360^\circ = \frac{8\pi}{4}$ radians $= 2\pi$ radians

Multiples of 30°

$30^\circ = \frac{\pi}{6}$ radians

$30^\circ = \frac{2\pi}{6}$ radians $= \frac{\pi}{3}$ radians

$90^\circ = \frac{3\pi}{6}$ radians $= \frac{\pi}{2}$ radians

$120^\circ = \frac{4\pi}{6}$ radians $= \frac{2\pi}{3}$ radians

$150^\circ = \frac{5\pi}{6}$ radians

$180^\circ = \frac{6\pi}{6}$ radians $= \pi$ radians

$210^\circ = \frac{7\pi}{6}$ radians

$240^\circ = \frac{8\pi}{6}$ radians $= \frac{4\pi}{3}$ radians

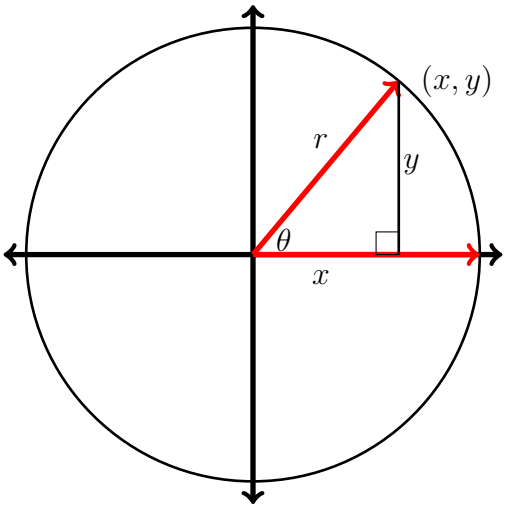
$270^\circ = \frac{9\pi}{6}$ radians $= \frac{3\pi}{2}$ radians

$300^\circ = \frac{10\pi}{6}$ radians $= \frac{5\pi}{3}$ radians

$330^\circ = \frac{11\pi}{6}$ radians

$360^\circ = \frac{12\pi}{6}$ radians $= 2\pi$ radians

Evaluating Trig Functions of Any Angle
Extending the Trig Ratios

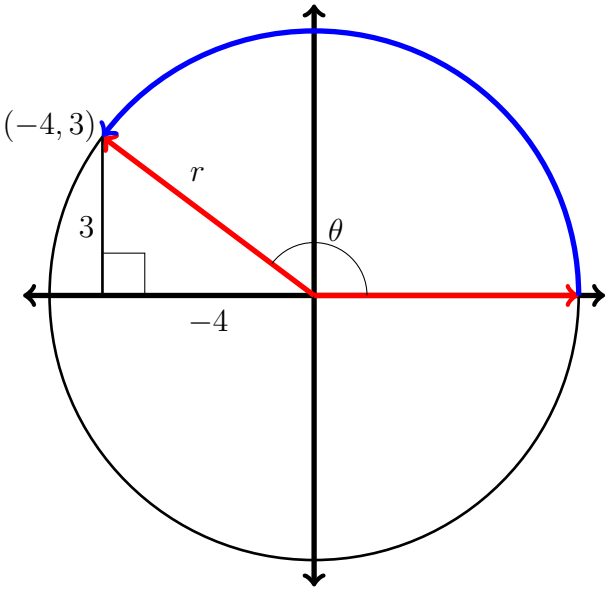


$$\sin(\theta) = \frac{y}{r}$$

$$\cos(\theta) = \frac{x}{r}$$

$$\tan(\theta) = \frac{y}{x}$$

Use the Pythagorean Theorem to find r, $r =$



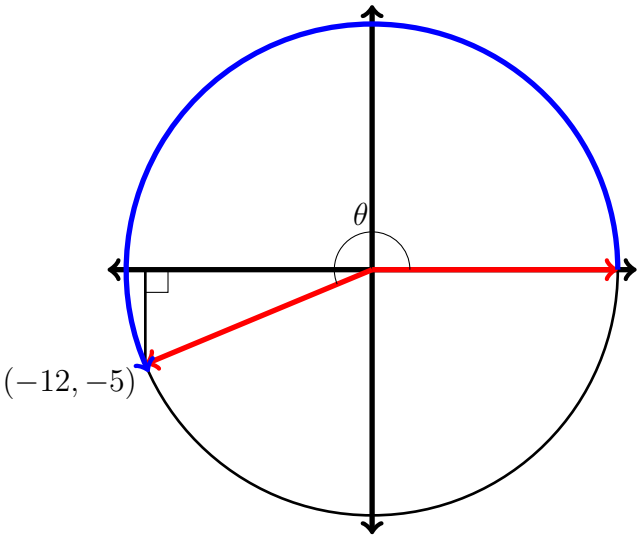
$x = -4, y = 3, r = 5$

$\sin(\theta) =$ —

$\cos(\theta) =$ —

$\tan(\theta) =$ —

Use the Pythagorean Theorem to find r, $r =$



$x =$ _____, $y =$ _____, $r =$ _____

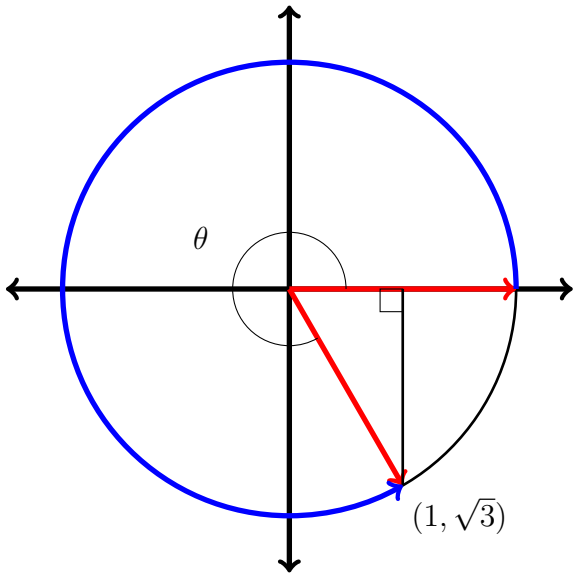
$$\sin(\theta) = \frac{y}{r} =$$

$$\cos(\theta) = \frac{x}{r} =$$

$$\tan(\theta) = \frac{y}{x} =$$

$\theta =?$

Use the Pythagorean Theorem to find r, $r =$



$x =$ _____, $y =$ _____, $r =$ _____

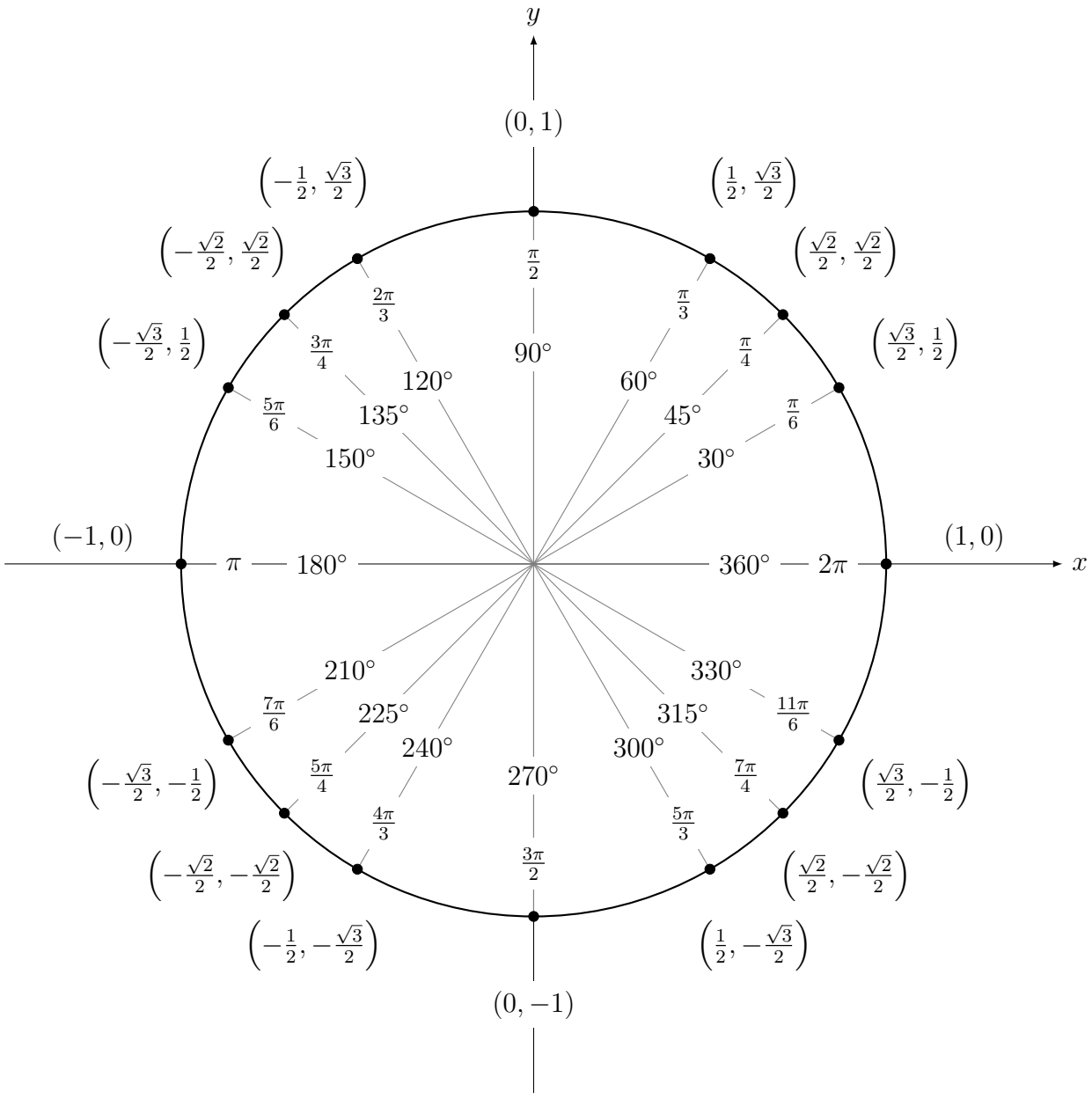
$$\sin(\theta) =$$

$$\cos(\theta) =$$

$$\tan(\theta) =$$

The Unit Circle

The unit circle is a circle whose radius is 1 and has a center about the origin. The values of $\sin(\theta)$ and $\cos(\theta)$ are just the y-values and x-values respectively.



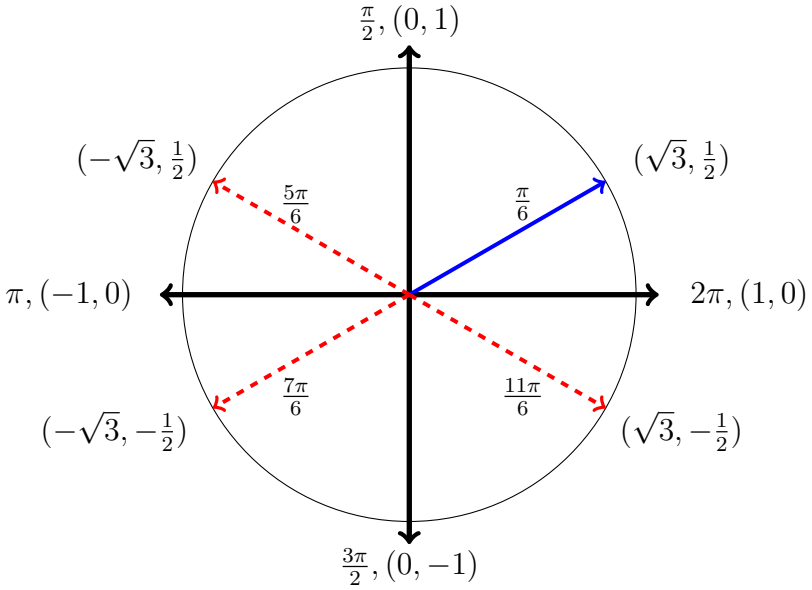
Reference Angles

Reference help us remember certain trig values. As can been seen from the unit circle, the points on the circle are the same all around except for the negative signs. Basically, knowing the first quadrant values will you to know all of the angles.

Quadratal Angles

These are angles whose terminal sides are on the axis, multiples of $90°$ or $\frac{\pi}{2}$.

Reference Agnles



The x and y values are the same on each of these

$\sin \frac{\pi}{6} =$	$\sin \frac{5\pi}{6} =$	$\sin \frac{7\pi}{6} =$	$\sin \frac{11\pi}{6} =$
$\cos \frac{\pi}{6} =$	$\cos \frac{5\pi}{6} =$	$\cos \frac{7\pi}{6} =$	$\cos \frac{11\pi}{6} =$
$\tan \frac{\pi}{6} =$	$\tan \frac{5\pi}{6} =$	$\tan \frac{7\pi}{6} =$	$\tan \frac{11\pi}{6} =$

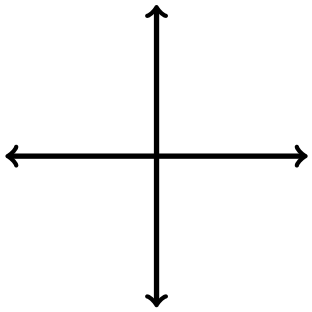
Unit Review

NAME:_____

General Angles

Draw each angle, name the quadrant, list one positive and one negative coterminal angle. (4 points each)

1. Draw a 210° angle

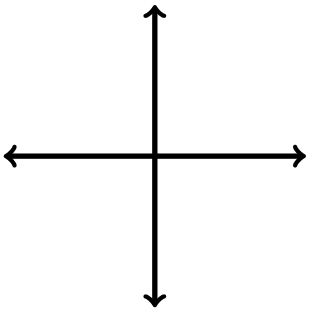


Quadrant:_____

+ coterminal:_____

- coterminal:_____

2. Draw a 66° angle

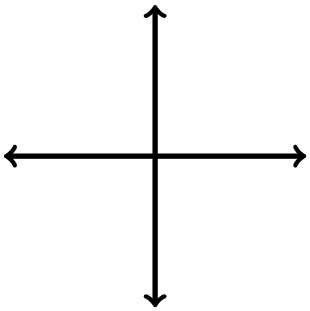


Quadrant:_____

+ coterminal:_____

- coterminal:_____

3. Draw a 345° angle



Quadrant:_____

+ coterminal:_____

- coterminal:_____

Radians

Convert from radians to degrees, or from degrees to radians. (2 points each).

4. 30°
5. 80°
6. 225°
7. 110°
8. 330°
9. $\frac{\pi}{9}$
10. $\frac{2\pi}{9}$
11. $\frac{7\pi}{4}$
12. $\frac{9\pi}{10}$
13. $\frac{3\pi}{2}$

Evaluating Trig functions

Evaluate sin, cos, and tan at the given point. (4 points each)

$$\sin\theta = \frac{y}{r} \quad cos\theta = \frac{x}{r} \quad \tan\theta = \frac{y}{x}$$

14. $(12, 5)$, r=_____

    $\sin\theta$ =_____

    $\cos\theta$ =_____

    $\tan\theta$ =_____

15. $(-6, 8)$, r=_____

    $\sin\theta$ =_____

    $\cos\theta$ =_____

    $\tan\theta$ =_____

16. $(-2, -2)$, r=_____

    $\sin\theta$ =_____

    $\cos\theta$ =_____

    $\tan\theta$ =_____

17. $(4, -3)$, r=_____

    $\sin\theta$ =_____

    $\cos\theta$ =_____

    $\tan\theta$ =_____

Unit Circle & Reference Angles

Use your unit circle (2 points each)

18. What is the reference angle for 300°?

19. Name 2 angles whose reference angle is 45°

20. How many reference angles are there for 30°

21. What is the sin and cos value at 150°?

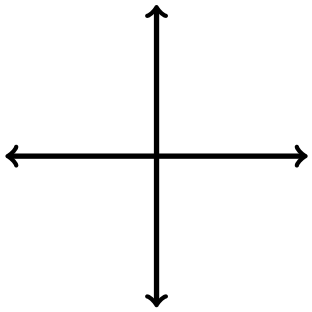
Unit Test.

NAME:_____

General Angles

Draw each angle, name the quadrant, list one positive and one negative coterminal angle. (4 points each)

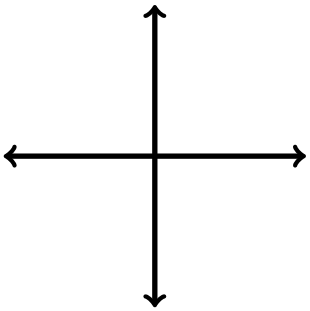
1. Draw a $110°$ angle
2. Draw a $-56°$ angle
3. Draw a $45°$ angle



Quadrant:_____

+ coterminal:_____

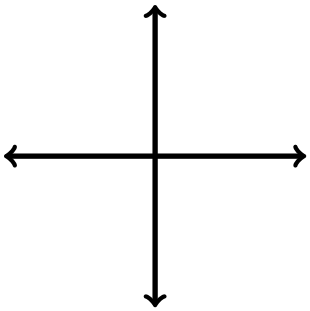
- coterminal:_____



Quadrant:_____

+ coterminal:_____

- coterminal:_____



Quadrant:_____

+ coterminal:_____

- coterminal:_____

Radians

Convert from radians to degrees, or from degrees to radians. (2 points each).

4. $45°$
5. $210°$
6. $160°$
7. $18°$
8. $600°$
9. $\frac{\pi}{18}$
10. $\frac{5\pi}{18}$
11. $\frac{7\pi}{9}$
12. $\frac{3\pi}{4}$
13. $\frac{\pi}{6}$

Evaluating Trig functions

Evaluate sin, cos, and tan at the given point. (4 points each)

$$\sin\theta = \frac{y}{r} \quad cos\theta = \frac{x}{r} \quad \tan\theta = \frac{y}{x}$$

14. $(5, -12)$, r=_____

    $\sin\theta$ =_____

    $\cos\theta$ =_____

    $\tan\theta$ =_____

15. $(8, -6)$, r=_____

    $\sin\theta$ =_____

    $\cos\theta$ =_____

    $\tan\theta$ =_____

16. $(1, 1)$, r=_____

    $\sin\theta$ =_____

    $\cos\theta$ =_____

    $\tan\theta$ =_____

17. $(-4, 3)$, r=_____

    $\sin\theta$ =_____

    $\cos\theta$ =_____

    $\tan\theta$ =_____

Unit Circle & Reference Angles

Use your unit circle (2 points each)

18. What is the reference angle for 210°?

19. Name 2 angles whose reference angle is 30°

20. How many reference angles are there for 60°

21. What is the sin and cos value at 120°?

    $\sin(120°)$ = $\cos(120°)$ =

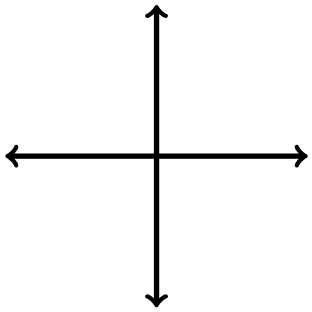
Unit Test:

NAME:_____

General Angles

Draw each angle, name the quadrant, list one positive and one negative coterminal angle. (4 points each)

1. Draw a 70° angle

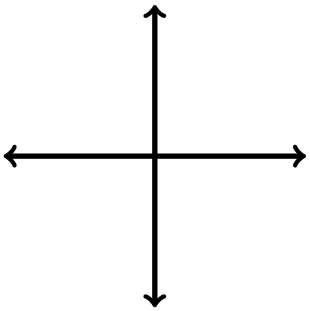


Quadrant:_____

+ coterminal:_____

- coterminal:_____

2. Draw a 190° angle

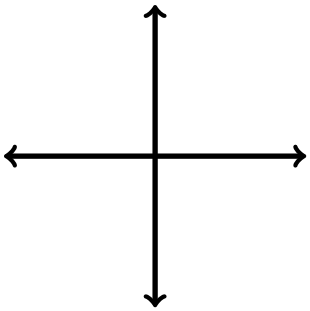


Quadrant:_____

+ coterminal:_____

- coterminal:_____

3. Draw a 315° angle



Quadrant:_____

+ coterminal:_____

- coterminal:_____

Radians

Convert from radians to degrees, or from degrees to radians. (2 points each).

4. 25°
5. 150°
6. 240°
7. 10°
8. 315°
9. $\frac{\pi}{90}$
10. $\frac{2\pi}{90}$
11. $\frac{\pi}{5}$
12. $\frac{2\pi}{3}$
13. $\frac{3\pi}{4}$

Evaluating Trig functions

Evaluate sin, cos, and tan at the given point. (4 points each)

$$\sin\theta = \frac{y}{r} \quad cos\theta = \frac{x}{r} \quad \tan\theta = \frac{y}{x}$$

14. $(3, 4)$, r=_____

    $\sin\theta$ =_____

    $\cos\theta$ =_____

    $\tan\theta$ =_____

15. $(-8, 6)$, r=_____

    $\sin\theta$ =_____

    $\cos\theta$ =_____

    $\tan\theta$ =_____

16. $(-1, -1)$, r=_____

    $\sin\theta$ =_____

    $\cos\theta$ =_____

    $\tan\theta$ =_____

17. $(-7, 24)$, r=_____

    $\sin\theta$ =_____

    $\cos\theta$ =_____

    $\tan\theta$ =_____

Unit Circle & Reference Angles

Use your unit circle (2 points each)

18. What is the reference angle for 330°?

19. Name 2 angles whose reference angle is 30°

20. How many reference angles are there for 60°

21. What is the sin and cos value at 240°?

    $\sin(240°)$ = $\cos(240°)$ =

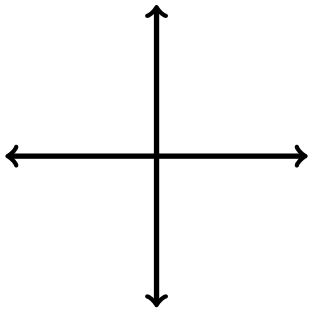
Unit Test::

NAME:_____

General Angles

Draw each angle, name the quadrant, list one positive and one negative coterminal angle. (4 points each)

1. Draw a 100° angle

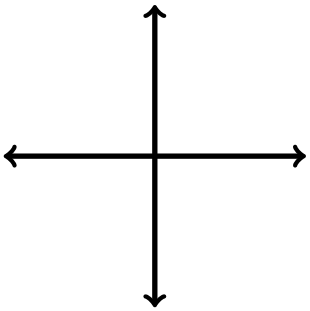


Quadrant:_____

+ coterminal:_____

- coterminal:_____

2. Draw a 200° angle

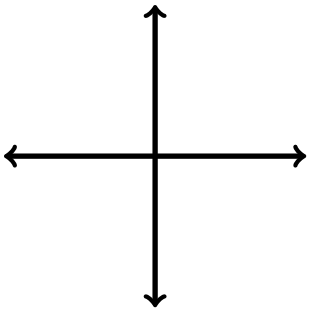


Quadrant:_____

+ coterminal:_____

- coterminal:_____

3. Draw a 300° angle



Quadrant:_____

+ coterminal:_____

- coterminal:_____

Radians

Convert from radians to degrees, or from degrees to radians. (2 points each).

4. 100°
5. 150°
6. 200°
7. 250°
8. 300°
9. $\frac{\pi}{10}$
10. $\frac{\pi}{5}$
11. $\frac{5\pi}{4}$
12. $\frac{7\pi}{3}$
13. $\frac{3\pi}{5}$

Evaluating Trig functions

Evaluate sin, cos, and tan at the given point. (4 points each)

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14. $(24, 7)$, r=_____

    $\sin\theta$ =_____

    $\cos\theta$ =_____

    $\tan\theta$ =_____

15. $(-5, 12)$, r=_____

    $\sin\theta$ =_____

    $\cos\theta$ =_____

    $\tan\theta$ =_____

16. $(-3, 3)$, r=_____

    $\sin\theta$ =_____

    $\cos\theta$ =_____

    $\tan\theta$ =_____

17. $(-3, 4)$, r=_____

    $\sin\theta$ =_____

    $\cos\theta$ =_____

    $\tan\theta$ =_____

Unit Circle & Reference Angles

Use your unit circle (2 points each)

18. What is the reference angle for 135°?

19. Name 2 angles whose reference angle is 30°

20. How many reference angles are there for 45°

21. What is the sin and cos value at 210°?

    $\sin(210°)$ = $\cos(210°)$ =

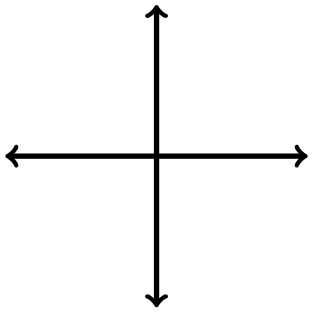
Unit Test

NAME:_____

General Angles

Draw each angle, name the quadrant, list one positive and one negative coterminal angle. (4 points each)

1. Draw a 10° angle

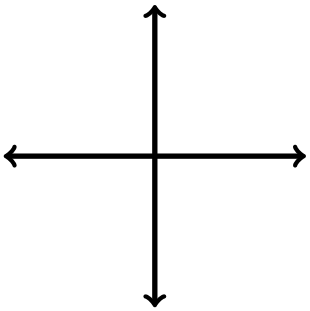


Quadrant:_____

+ coterminal:_____

- coterminal:_____

2. Draw a 100° angle

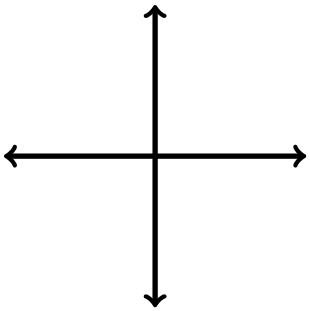


Quadrant:_____

+ coterminal:_____

- coterminal:_____

3. Draw a 177° angle



Quadrant:_____

+ coterminal:_____

- coterminal:_____

Radians

Convert from radians to degrees, or from degrees to radians. (2 points each).

4. 30°
5. 60°
6. 90°
7. 120°
8. 81°
9. $\frac{\pi}{3}$
10. $\frac{2\pi}{3}$
11. $\frac{3\pi}{2}$
12. $\frac{\pi}{30}$
13. $\frac{6\pi}{5}$

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$$\sin\theta = \frac{y}{r} \quad cos\theta = \frac{x}{r} \quad \tan\theta = \frac{y}{x}$$

14. $(-5, 5)$, r=_____

    $\sin\theta$ =_____

    $\cos\theta$ =_____

    $\tan\theta$ =_____

15. $(6, -8)$, r=_____

    $\sin\theta$ =_____

    $\cos\theta$ =_____

    $\tan\theta$ =_____

16. $(-3, -4)$, r=_____

    $\sin\theta$ =_____

    $\cos\theta$ =_____

    $\tan\theta$ =_____

17. $(24, 7)$, r=_____

    $\sin\theta$ =_____

    $\cos\theta$ =_____

    $\tan\theta$ =_____

Unit Circle & Reference Angles

Use your unit circle (2 points each)

18. What is the reference angle for 225°?

19. Name 2 angles whose reference angle is 30°

20. How many reference angles are there for 60°

21. What is the sin and cos value at 180°?

    $\sin(180)$ = $\cos(180)$ =

Graphing Trig Functions

$y = a\sin(bx + c) + d$

a = amplitude
b = period (horizontal strech/compression)
c = phase (horizontal) shift
d = vertical shift

$$\text{period} = \frac{2\pi}{b}$$

$$\text{phase shift} = -\frac{c}{b}$$