

Introduction to Quadratics

Goals

SWBAT multiply monomials into quadratics by using the FOIL method

SWBAT multiply the square of a binomial and the difference of squares without using the FOIL method by understanding the pattern of these multiplication problems.

SWBAT factor quadratics into the product of two binomials.

SWBAT solve a quadratic equation by factoring.

Standards

Seeing Structure in Expressions

A-SSE

Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
 - a. Factor a quadratic expression to reveal the zeros of the function it defines

Reasoning with Equations and Inequalities

A-REI

Understand solving equations as a process of reasoning and explain the reasoning

4. Solve quadratic equations in one variable.
 - a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
 - b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b .

Connections

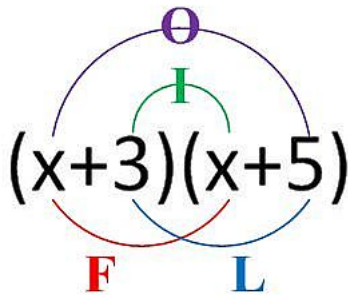
Before: We were learning about linear functions. When two linear functions multiply one another we get a Quadratic function. We are now learning to multiply binomials into quadratics, and factor quadratics into binomials.

After: The class will explore completing the square, the quadratic formula, quadratic graphs, and complex numbers (time dependent).

Introduction to Quadratics

Quadratic Function: a polynomial of degree 2. A quadratic is the result of the product of two linear functions (monomials).

Multiplying Monomials: Use the FOIL technique. **F**irst **O**utside **I**nside **L**ast



Example: $(x + 3)(x + 5)$

First: $x \cdot x = x^2$

Outside: $x \cdot 5 = 5x$

Inside: $3 \cdot x = 3x$

Last: $3 \cdot 5 = 15$

$$= x^2 + 5x + 3x + 15 = x^2 + 8x + 15$$

Your Try: $(x + 2)(x + 9)$

You Try: $(x - 12)(x + 7)$ (**hint:** the 12 should be treated as a -12)

Coefficients on x : The process is the same, but there's a little more to wrangle.

Example: $(3x + 10)(2x + 1)$

You Try: $(2x + 18)(x + 3)$

Independent Practice

1. $(x + 1)(x + 1)$

5. $(3x - 4)(x + 1)$

2. $(2x + 4)(x - 7)$

6. $(8x - 5)(3x + 2)$

3. $(2x - 1)(x - 4)$

7. $(5x + 3)(2x - 3)$

4. $(x - 9)(x - 4)$

8. $(3x + 10)(2x + 2)$

Multiplying Special Cases

Square of a Binomial

The Square of a binomial looks like

$$(a + b)^2$$

$$(a + b)^2 \neq (a^2 + b^2)$$

Think Pythagorean Theorem for the reason.

To obtain the correct value we need to FOIL. The meaning of $(x + 5)^2$ is $(x + 5)(x + 5)$

$$(x + 5)^2 = x^2 + 10x + 25$$

General Rule: $(a + b)^2 = a^2 + 2ab + b^2$

Example: $(2q - 9)^2 = 4q^2 - 36q + 81$

Use this pattern to simply convert the square of a binomial without actually FOILING.

You Try: Expand the problems without using FOIL.

1. $(f + 2)^2 =$

6. $(h - k)^2 =$

2. $(h - 3)^2 =$

7. $(2m + n)^2 =$

3. $(3p + 7)^2 =$

8. $(3u - v)^2 =$

4. $(4g - 5)^2 =$

9. $(3a + 6b)^2 =$

5. $(x + y)^2 =$

10. $(5r - 10s)^2 =$

Difference of Squares

$$a^2 - b^2$$

This is called the **difference of squares** because the a is squared, the b is also squared, and they are subtracted (difference).

This happens when you multiply 2 special binomials.

$$(a + b)(a - b) = a^2 - b^2$$

Example: Foil the following $(x + 2)(x - 2)$

$$(x^2 - 2x + 2x - 4)$$

Notice the middle terms $(-2x + 2x)$ cancel out. The first term x^2 and last term 4 are both perfect squares. They have subtraction in between them, hence the "difference" in **difference of squares**. Thus, our answer is

$$x^2 - 4$$

You Try: Rewrite the expression as a difference of squares.

1. $(h - 3)(h + 3) =$

3. $(2k - 5)(2k + 5)$

2. $(j + 10)(j - 10)$

4. $(9g - 9)(9g + 9)$

You can go in the opposite direction too. If you have a difference of squares you can write a product of binomials.

$$25x^2 - 16 = (5x - 4)(5x + 4)$$

You Try: Convert from a difference of squares to the product of binomials.

1. $n^2 - 100$

3. $4r^2 - 16$

2. $p^2 - 64$

4. $9m^2 - 81w^2$

Factoring Trinomials

We know how to multiply 2 binomials into a trinomial by using the FOIL method. Now we're going to *factor* the trinomial back into two binomials.

Example 1:

Factor $x^2 + 8x + 12$

Factors of 12	Sum of factors
1 and 12	13
2 and 6	8
3 and 4	7

Which set of factors add up to the middle term?

So the factored form will be

$$(x + \quad)(x + \quad)$$

Example 2:

Factor $x^2 + 9x + 20$

$$(x + \quad)(x + \quad)$$

You Try:

Factor $x^2 + 11x + 18$

Negatives in Trinomials

When there is a negative in the *second spot* or in *both spots* that means that there is **1 negative**.

$x^2 + 5x - 50$ – Bigger factor is **negative**

$$50 = 2 \cdot 25$$

$$5 \cdot 10$$

Which two of these subtracted from each other = +5?

$$(x + 10)(x - 5)$$

You Try: Factor $x^2 + 5x - 14$

$x^2 - 4x - 12$ – Smaller factor is **negative**

$$12 = 2 \cdot 6$$

$$3 \cdot 4$$

Which two of these subtracted from each other = -4?

$$(x + 2)(x - 4)$$

You Try: Factor $x^2 - x - 20$

When the first sign is **negative** then that means **BOTH FACTORS** are negative.

$$x^2 - 6x + 8$$

$$8 = 2 \cdot 4$$

$$(x - 2)(x - 4)$$

You Try: Factor $x^2 - 12x + 36$

Quiz Review – Trinomials

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Algebra 2
CMSD-JFK
2014-2015

NAME: _____

DATE: _____

Multiply the binomials into a trinomial by using the FOIL technique

1. $(x + 5)(x + 3)$

4. $(3p + 4)(p + 2)$

2. $(h - 4)(h + 6)$

5. $(4y + 1)(2y + 3)$

3. $(k - 2)(k - 8)$

6. $(2r + s)(3r - s)$

Multiply the binomials into trinomials without using the FOIL technique. Notice the *square of a binomial* and the *difference of squares*.

7. $(x + 4)^2$

14. $(10p - 10)(10p + 10)$

8. $(g + 5)^2$

15. $(d - 4)(d + 4)$

9. $(k - 7)^2$

16. $(r + 4)(r - 4)$

10. $(2p + 3)^2$

17. $(2v - 9)(2v + 9)$

11. $(3t - 3)^2$

18. $(7i - j)(7i + j)$

12. $(5y - 4f)^2$

19. $(m + n)(m - n)$

13. $(q + 1)(q - 1)$

Factor the trinomials into the product of two binomials

20. $x^2 + 21x + 20$

23. $x^2 + 11x + 30$

21. $x^2 + 9x + 20$

24. $x^2 - 8x + 12$

22. $x^2 + 13x + 30$

25. $x^2 - 7x + 12$

Quiz Trinomials – OPEN NOTE

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Multiply the binomials into a trinomial by using the FOIL technique

1. $(x + 4)(x + 2)$

4. $(2p + 5)(p + 3)$

2. $(h - 3)(h + 5)$

5. $(5y + 2)(3y + 4)$

3. $(k - 3)(k - 9)$

6. $(3r + s)(4r - s)$

Multiply the binomials into trinomials without using the FOIL technique. Notice the *square of a binomial* and the *difference of squares*.

7. $(x + 54)^2$

14. $(10p - 11)(10p + 11)$

8. $(g + 6)^2$

15. $(d - 5)(d + 5)$

9. $(k - 8)^2$

16. $(r + 6)(r - 6)$

10. $(2p + 4)^2$

17. $(2v - 8)(2v + 8)$

11. $(3t - 4)^2$

18. $(6i - j)(6i + j)$

12. $(4y - 5f)^2$

19. $(m + 2n)(m - 2n)$

13. $(q + 2)(q - 2)$

Factor the trinomials into the product of two binomials

20. $x^2 + 17x + 16$

23. $x^2 + 22x + 21$

21. $x^2 + 10x + 16$

24. $x^2 - 11x + 24$

22. $x^2 + 10x + 21$

25. $x^2 - 10x + 24$

Factoring Trinomials $a > 1$

$$ax^2 + bx + c$$

We know how to factor a trinomial when $a = 1$. That means that there is no coefficient in front of the a^2 term. However, it can be factored but it takes a little bit more effort.

When there is a common factor: If there is a common factor between all of the terms of the trinomial, then it can be factored out.

Example 1: Factor $2x^2 + 10x + 8$

The coefficients are 2, 10, and 8. Since 2 can go into all of those numbers it can be pulled out.

$$2(x^2 + 5x + 4)$$

Now we notice that $(x^2 + 5x + 4)$ can be further factored. So our end result is.

$$2(x + 4)(x + 1)$$

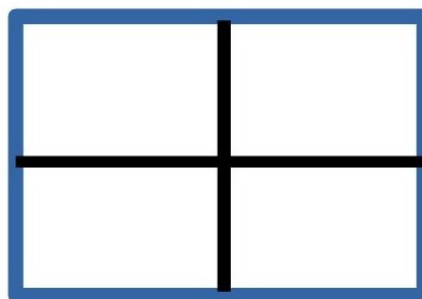
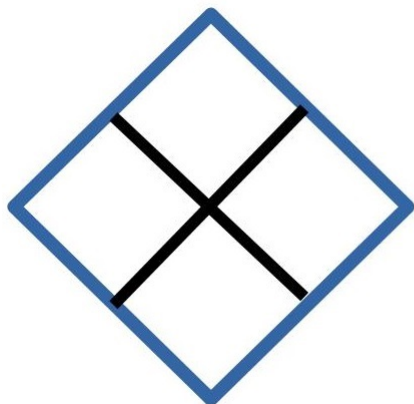
Example 2: Factor $3x^2 + 15x + 18$

You Try: Factor $2x^2 + 16x + 30$

Factor $5x^2 - 2x - 12$

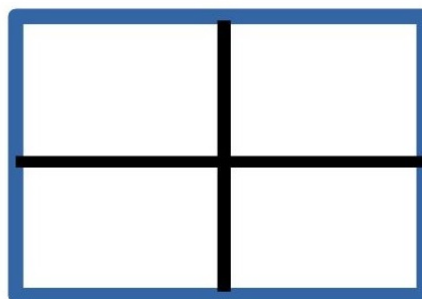
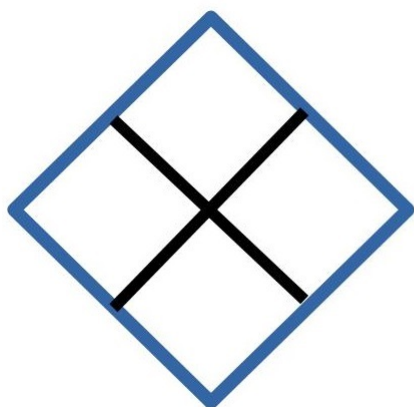
Diamond Method of Factoring

Use the diamond method to factor $2x^2 + 7x + 3$



Answer:

Factor: $3x^2 + 8x + 4$



Solving Quadratics by Factoring

We know how to solve equations like $5x + 7 = 52$

You may even know how to solve an equation like $x^2 = 9$

But how do you solve an equation like $x^2 + 21x = -20$? There are no like-terms to combine, you can't isolate an x to solve it! The answer is to *FACTOR* (everyone's favorite past-time).

Follow these 5 steps to solve

1. Make one side of the equals sign 0
2. Factor
3. Solve each of the factors alone
4. You have 1 *or* 2 answers
5. Check your answer

Example 1: $x^2 + 21x = -20$

1. $x^2 + 21x + 20 = 0$
2. $(x + 20)(x + 1) = 0$
3. (a) $x + 20 = 0$
(b) $x + 1 = 0$
4. (a) $x = -20$
(b) $x = -1$
5. (a) $(-20)^2 + 21(-20) + 20 = 0$
(b) $(-1)^2 + 21(-1) + 20 = 0$

Example 2: $x^2 + 9x = -20$

Example 3: $-x^2 = 13x + 30$

You Try: $x^2 + 11x + 30 = 0$

$$x^2 - 8x = -12$$

Solving equations practice

$$(x + 1)(x + 1) = 0$$

$$(2x + 4)(x - 7) = 0$$

$$x^2 - 8x = -12$$

$$x^2 = +7x - 12$$

$$2x^2 + 16x + 30 = 0$$

$$5x^2 - 2x = 12$$

$$2x^2 + 7x = -3$$

$$3x^2 + 13x + 10 = 5x + 6$$

Quiz Review – Factoring & Solving Quadratics

Mr. Wolf

NAME: _____

Factor the Trinomials $a = 1$

1. $x^2 + 6x + 5$

3. $x^2 - 99x - 100$

2. $x^2 + 8x + 12$

4. $x^2 - 28x + 75$

Factor the Trinomials $a > 1$ – Factor out a common factor

5. $3x^2 + 9x + 6$

7. $5x^2 - 75x - 500$

6. $2x^2 + 14x + 20$

8. $3x^2 - 60x + 225$

Factor the Trinomials $a > 1$ – Diamond Method or Factor by grouping

9. $2x^2 + 7x + 5$

11. $2x^2 + 13x - 7$

10. $6x^2 + 11x + 3$

12. $5x^2 + 18x + 9$

Solve the Quadratic Equations

13. $(x - 2)(x + 5) = 0$

14. $(3x - 9)(x + 5) = 0$

15. $x^2 + 6x + 5 = 0$

16. $x^2 - 99x = 100$

17. $x^2 + 6x + 105 = 100$

18. $2x^2 + 7x + 5 = 0$

19. $6x^2 + 11x + 9 = 6$

20. $2x^2 + 13x = 7$

Quiz – Factoring & Solving Quadratics

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OPEN NOTE

Factor the Trinomials $a = 1$

1. $x^2 + 3x + 2$

3. $x^2 - 15x - 100$

2. $x^2 + 7x + 10$

4. $x^2 - 20x + 75$

Factor the Trinomials $a > 1$ – Factor out a common factor

5. $2x^2 + 6x + 4$

7. $4x^2 - 60x - 400$

6. $3x^2 + 21x + 30$

8. $2x^2 - 40x + 150$

Factor the Trinomials $a > 1$ – Diamond Method or Factor by grouping

9. $2x^2 + 5x + 3$

11. $3x^2 + 13x - 10$

10. $3x^2 + 5x - 2$

12. $2x^2 - 11x + 14$

Solve the Quadratic Equations – BOOM!

13. $(x - 1)(x + 1) = 0$

14. $(2x - 8)(x + 10) = 0$

15. $x^2 + 3x + 2 = 0$

16. $x^2 + 7x = -10$

17. $x^2 = 15x + 100$

18. $x^2 - 20x + 90 = 15$

19. $2x^2 + 5x + 3 = 0$

20. $3x^2 + 5x = 2$

Now you're done :-)