Contents

1 Function Notation & Inverse Functions

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1.1 Goals

SWBAT use function notation to write functions

SWBAT perform operations on functions using function notation

SWBAT perform compositions of functions

SWBAT identify inverse functions by composing one into another

SWBAT create inverse functions

1.2 Standards

Interpreting Functions **F-IF**

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).

2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Building Functions **F-BF**
Build new functions from existing functions

3. Identify the effect on the graph of replacing $f(x)$ by $f(x)+k$, $kf(x)$, $f(kx)$, and $f(x+k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

4. Find inverse functions.

a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ for $x > 0$ or $f(x) = (x+1)/(x1)$ for $x1$.

b. (+) Verify by composition that one function is the inverse of another.

c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.

d. (+) Produce an invertible function from a non-invertible function by restricting the domain.

1.3 Before & After

Before we learned about exponential functions and their applications.

Now we are learning about function notation so that we can learn about inverse functions, specifically the inverses of exponential functions (i.e. logarithms).

After we will learn about exponential functions, and logarithms, and how logarithms are the inverse functions of exponential functions.

2 What is a Function?

A function is a machine that turns one thing into another.

A toaster is a "function" that turns bread into toast.

A mathematical function is a rule that changes some number or expression into a different number or expression.

The input of a function is called the **DOMAIN**, and the output is called the **RANGE**.

Functions are written with ordered pairs.

3 Using Function Notation

Another perk of using function notation is that you can plug things into the function really easily. If we want to find out when $x = 3$ we will write $f(3)$

$$\mathbf{f(x) = 3x - 1}$$

$f(1) = 3(1) - 1$

$f(2) = 3(2) - 1$

$f(3) = 3(3) - 1$

You can also plug in other variables. To plug these

1. Put a set of parenthesis wherever x was

2. Then plug your variable/expression into the parenthesis

3. Simplify

$f(c) =$

$f(5r) =$

Plug in expressions

$f(x + 1) =$

$f(y^2) =$

You Try:

$$\mathbf{f(x) = x^2 - 2x - 1}$$

1. $f(1) =$
2. $f(-1) =$
3. $f(7) =$
4. $f(0) =$

4 Function Notation & Operations

Function notation is a common way to write equations. Function notation is written $f(x)$ and read "f of x." When we are dealing with functions it is enough to say $y = f(x)$. y and $f(x)$ are pretty much interchangeable, but there are few things that function notation allows us to do that a regular y doesn't.

$$f(x) = 2x \text{ and } g(x) = x + 5$$

Operation	**Definition**	**Example**
Addition	$[f+g](x) = f(x) + g(x)$	$2x + (x+5)$
Subtraction	$[f-g](x) = f(x) - g(x)$	$2x - (x+5)$
Multiplication	$[f \cdot g](x) = f(x) \cdot g(x)$	$2x \cdot (x+5)$
Division	$[f \div g](x) = \frac{f(x)}{g(x)}$	$\frac{2x}{x+5}$

Example:
$f(x) = 3x + 4$ and $g(x) = 5x - 8$

1. $[f+g](x) =$
2. $[f-g](x) =$
3. $[g-f](x) =$
4. $[f \cdot g](x) =$
5. $[f \div g](x) =$

You Try Perform the indicated operations on the two functions.

$$h(x) = x - 8$$

$$k(x) = 5x$$

1. $[h + k](x) =$
2. $[h - k](x) =$
3. $[k - h](x) =$
4. $[h \cdot k](x) =$
5. $\left[\frac{h}{k}\right](x) =$

You Try: Perform the operations on functions.

$$p(x) = x + 10$$

$$q(x) = 2x - 10$$

1. $[p + q](x) =$
2. $[p \cdot q](x) =$
3. $\left[\frac{p}{q}\right](x) =$
4. $[q - p](x) =$
5. $[p - q](x) =$

5 Composition of Functions

Composition of functions means that we can put a function inside a function. There are two equivalent ways to write composition of functions.

$$f(x) = 2x \qquad g(x) = x^3$$

$$[f \circ g](x) = f[g(x)] = f(x^3)$$

3-step process

Step 1) Write out $f(x)$ with parenthesis around where the x would be

$$2(\quad)$$

Step 2) Fill in the blank spots with $g(x)$

$$2(x^3)$$

Step 3) Combine like-terms and simplify

$$2x^3$$

These are not commutative. Therefore it's going to be different if we write

$$[g \circ f](x)$$

Step 1) Write $g(x)$ with parenthesis around where the x would be

$$(\quad)^3$$

Step 2) Fill in the blank spots with $f(x)$

$$(2x)^3$$

Step 3) Combine like-terms and simplify

$$2^3x^3 = 8x^3$$

Example 2 Compose the functions. $h(x) = x + 2$ and $k(x) = 2x^2$

$$[h \circ k](x) =$$

$$[k \circ h](x) =$$

You Try: Compose the Functions. $p(x) = 5x + 5$ and $q(x) = x - 10$

$$[p \circ q](x) =$$

$$[q \circ p](x) =$$

Directions: Compose the functions as indicated.

$$f(x) = x + 3$$

$$g(x) = x^2$$

$$h(x) = 5x$$

1. $[f \circ g](x) =$

2. $[g \circ h](x) =$

3. $[f \circ h](x) =$

4. $[g \circ f](x) =$

5. $[h \circ g](x) =$

6. $[h \circ f](x) =$

6 Function Notation Review

NAME:________________________________

Review day for function notation. This worksheet is due at the end of class.

6.1 Function Notation & Operations

Directions: Complete the following operations on the above listed functions.
Use $f(x) = 3x - 10$ and $h(x) = 3x$

1. $[f + h](x) =$
2. $[f - h](x) =$
3. $[h - f](x) =$
4. $[f \cdot h](x) =$

6.2 Using Function Notation

Directions: Plug the numbers or expressions into function notation
Use $f(x) = 3x - 10$ and $h(x) = 3x$

5. $f(3) =$
6. $f(16) =$
7. $h(-12) =$
8. $h(0) =$
9. $f(2y) =$
10. $h(2y) =$
11. $f(p + 14) =$
12. $h(2b^2) =$

6.3 Composition of Functions

Directions: Use the following functions and compose them as each question indicates.

$$p(x) = 5x - 10$$
$$r(x) = 9x$$
$$t(x) = x^2$$

13. $[p \circ r](x) =$

14. $[p \circ t](x) =$

15. $[r \circ p](x) =$

16. $[r \circ t](x) =$

17. $[t \circ p](x) =$

18. $[p \circ p](x) =$

Accelerated Objective: A new car dealer is discounting all new cars by 12%. At the same time, the manufacturer is offering a \$1500 rebate on all new cars. Mr. Slaughter is buying a car that is priced \$24,500. Will the final price be lower if the discount is applied before the rebate or if the rebate is applied before the discount?

Let x represent the original price of a new car, $d(x)$ represent the price of a car after the discount, and $r(x)$ represent the price of the car after the rebate.

Making Connections

<table>
<tr><th>Math</th><th>Python</th></tr>
<tr><td>$f(x) = x + 10$

$g(x) = x^2$</td><td><pre>>>> def f(x):
        return x + 10

>>> def g(x):
      return x**2</pre></td></tr>
<tr><td>$f(7) = 7 + 10 = 17$

$g(7) = 7^2 = 49$</td><td><pre>>>> f(7)
17

>>> g(7)
49</pre></td></tr>
<tr><td>$f \circ g(7) = f(g(7))$

$= f(7^2) = f(49) = 59$</td><td><pre>>>> f(g(7))

>>> f(g(7))
59</pre></td></tr>
</table>

7 Domain & Range

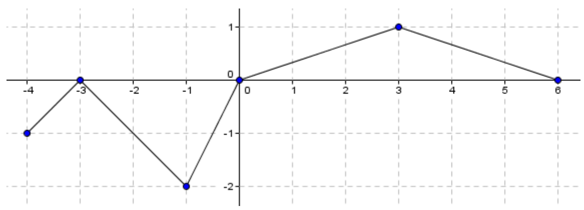
Domain: x values.

Range: y values.

simple!

7.1 Domain and Range from a Graph

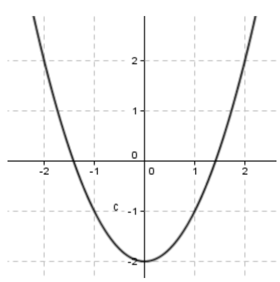
Example 1: Use the graph to determine the *domain* and *range*.



	Which values?	Where does the graph go	Answer
Domain	x –values	from -4 to +6	Domain: [-4,6]
Range:	y–values	from -2 to 1	Range: [-2,1]

Example 2: Find the domain and range of the graph. $f(x) = x^2 - 2$

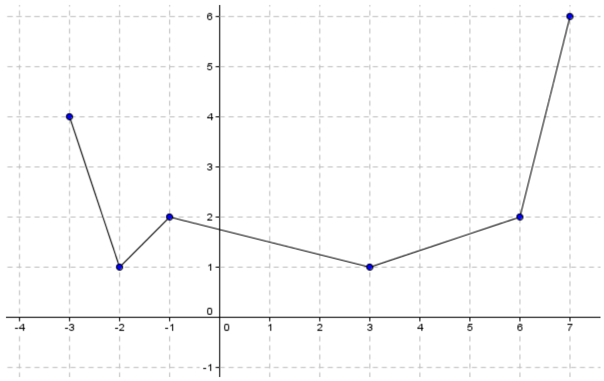
The hard part of this one is thinking about the *end behavior* of the graph.



Domain:

Range:

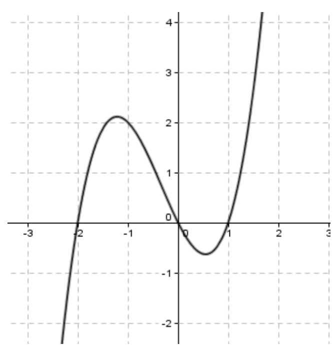
You Try: Find the domain and range of the graph.



Domain:

Range:

You Try: Find the domain and range of the graph.



Domain:

Range:

More Practice

`https://www.khanacademy.org/math/algebra2/functions_and_graphs/domain_range/e/range_of_a_function`

7.2 Domain and Range from Number Sets

If we just have a set of numbers we can still determine the *domain* and *range*.

Example 1) Determine the domain and range of the data set.

$$(-3, 18), (7, 3), (8, 10), (17, -1)$$

smallest x	-3
biggest x	17

smallest y	-1
biggest y	18

So the **domain** is $[-3, 17]$

and the **range** is $[-1, 18]$

You Try: Find the domain and range of the data set.

$$(-2, 9), (4, 2), (3, 3), (10, -3), (-4, 11)$$

smallest x	
biggest x	

smallest y	
biggest y	

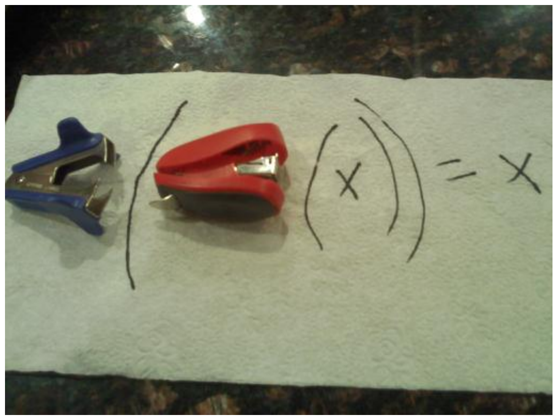
Domain =

Range =

8 Inverse Functions

Inverse functions are just functions that are *opposites*. If you compose two functions and the result is just x then they are called *inverse functions*. It need to go both ways though.

Inverse functions are another way of just saying that they undo one another



The stapler puts things together with a staple. The staple remover separates things by removing a staple. So they are inverse functions.

For one function to be the opposite of the other it must have the opposite *operations*

Function	Inverse
$+$	$-$
$\times$	$\div$
x^2	$\sqrt{x}$
x^3	$\sqrt[3]{x}$

If $f(x)$ and $g(x)$ are inverses, then

$$[f \circ g](x) = x$$

$$[g \circ f](x) = x$$

English: If two functions are inverses, then when composed the answer is x. But to be sure we must compose both ways.

Example 1: $d(x) = x + 5$ and $b(x) = x - 5$. Are they inverses?

$[d \circ b](x) =$

$[b \circ d](x) =$

Example 2: $j(x) = 2x - 1$ and $k(x) = 2x + 1$. Are they inverses?

$[j \circ k](x) =$

$[k \circ j](x) =$

Example 3: $p(x) = x^2 + 2$ and $q(x) = \sqrt{x + 2}$. Are they inverses?

$[p \circ q](x) =$

$[q \circ p](x) =$

Example 4: $f(x) = \frac{x}{2} - 4$ and $g(x) = 2x + 8$

$[f \circ g](x) =$

$[g \circ f](x) =$

8.1 Creating Inverse Functions

We know that when a function is composed into its inverse the result is just x. How do we create inverse functions though?

Step 1: Change $f(x)$ into a y.

Step 2: Switch x and y.

Step 3: Solve for y. You should get $y =$ .....

Step 4: replace y with $f^{-1}(x)$

Example 1: Find the inverse of $f(x) = 7x - 3$

1. $y = 7x - 3$

2. $x = 7y - 3$

3. $x + 3 = 7y \longrightarrow y = \frac{x+3}{7}$

4. $f^{-1}(x) = \frac{x+3}{7}$

Example 2: Find the inverse of $p(x) = x^2 - 9$

You Try: Find the inverse of $h(x) = 2x - 10$

Calculating Inverse Functions

NAME:____________________________________

DATE: ________________________

Directions: Determine if the functions are inverses.

1. $a(x) = 2x + 1$

   $b(x) = \frac{x}{2} - 1$

2. $c(x) = \sqrt{x + 10}$

   $d(x) = x^2 - 10$

3. $t(x) = 8x^2 + 13$

   $u(x) = \sqrt{\frac{x-13}{8}}$

Directions: Create the inverse function of each function given.

4. $f(x) = 24x$

5. $g(x) = 18x + 13$

6. $h(x) = \frac{x}{15}$

7. $j(x) = \frac{-5}{x}$

8. $k(x) = \frac{2x}{-3} + -7$

9. $m(x) = \sqrt{-5x}$

10. $n(x) = \sqrt{-5x + -3}$

11. $p(x) = \sqrt{-6x + 0} + 17$

12. $q(x) = x^2 + 16$

13. $r(x) = (2x)^2 + 6$

Challenge Problem: $f(x) = \frac{3x^2+1}{2} - 15$

8.2 Graphs of Inverse Functions

Graphs of inverse functions are always *reflections* over the line $y = x$ (yes, the same x that's a solution when we find they're inverses).

Another way of saying this is that if (a, b) is a point

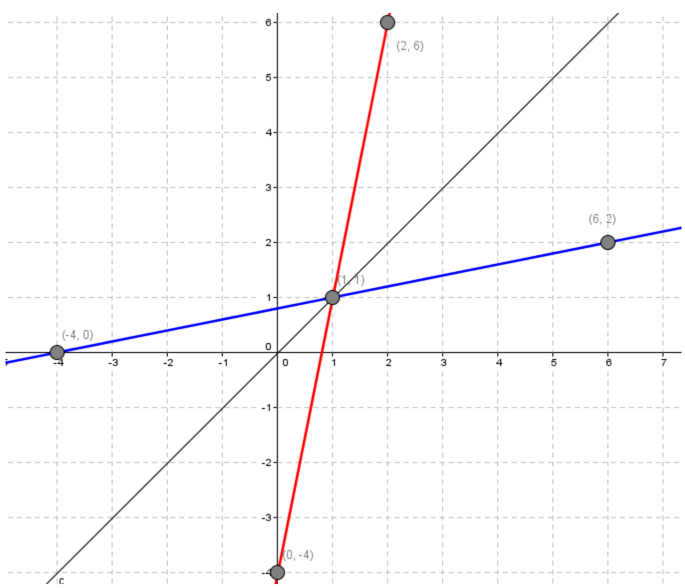
$$(a, b) \in f(x) \Leftrightarrow (b, a) \in f^{-1}(x)$$

That's another way of saying: switch the x and y.

Example 1)

$$f(x) = 5x - 4$$

$$f^{-1}(x) = \frac{x + 4}{5}$$



x	y (or $f(x)$)
0	-4
1	1
2	6

x	y (or $f^{-1}(x)$)
-4	0
1	1
6	2

The values of the points are reversed. There is also a reflection over the line $y = x$

Geogebra demo. Download Geogebra!
Example 2) No graph necessary

$$g(x) = 2x - 1$$

$$g^{-1}(x) = \frac{x+1}{2}$$

x	y (or $g(x)$)
-2	-5
-1	-3
0	-1
1	1
2	3

x	y (or $g^{-1}(x)$)
-5	
-3	
-1	
1	
3	

You Try: Fill in the t-chart with the values of the inverse function based on the information of the regular function.

$$p(x) = 3x + 4$$
$$p^{-1}(x) = \frac{x-4}{3}$$

x	y (or $g(x)$)
-2	-2
-1	1
0	4
1	5
2	8

x	y (or $g^{-1}(x)$)

9 Review

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Pre-Calc
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2014-2015

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DATE:______________

Operations with Function Notation

$$a(x) = 4x - 13 \qquad b(x) = 9x \qquad c(x) = x^2$$

Complete the following operations on the functions (2 points each)

1. $[a + b](x) =$
2. $[a + c](x) =$
3. $[b - a](x) =$
4. $[b \cdot c](x) =$
5. $c(5) =$
6. $c(-5) =$
7. $a(3h - 2) =$

$$k(x) = 7x + 7 \qquad h(x) = \frac{x}{7} - 1$$

8. $[k \circ h](x) =$
9. $[h \circ k](x) =$
10. are h and k inverses?

11. Write the inverse of $p(x) = 2x - 1$

12. Fill out the t-chart for $p(x)$

x	y (or $p(x)$)
-2	
-1	
0	
1	
2	

13. List the domain and range of the t-chart

14. Fill the t-chart for $p^{-1}(x)$

x	y (or $p(x)$)

Quiz Review 2

Mr. Wolf NAME:________________________________

Operations with Function Notation

$$a(x) = 3x - 10 \qquad b(x) = 2x \qquad c(x) = x^2$$

Complete the following operations on the functions (2 points each)

1. $[a + b](x) =$
2. $[a + c](x) =$
3. $[b - a](x) =$
4. $[b \cdot c](x) =$
5. $c(10) =$
6. $c(-7) =$
7. $a(2k + 15) =$

$$k(x) = \frac{x}{2} - 3 \qquad\qquad h(x) = 2x + 6$$

8. $[k \circ h](x) =$
9. $[h \circ k](x) =$
10. are h and k inverses?

11. Write the inverse of $p(x) = 5x - 10$

12. Fill out the t-chart for $p(x)$

x	y (or $p(x)$)
-2	
-1	
0	
1	
2	

13. List the domain and range of the t-chart

14. Fill the t-chart for $p^{-1}(x)$ (from answer to 11)

x	y (or $p^{-1}(x)$)

Quiz

Mr. Wolf NAME:______________________________
Pre-Calc
CMSD-JFK DATE:____________________
2014-2015

Operations with Function Notation

$$a(x) = 3x - 10 \qquad b(x) = 5x \qquad c(x) = x^2$$

Complete the following operations on the functions (2 points each)

1. $[a + b](x) =$
2. $[a + c](x) =$
3. $[b - a](x) =$
4. $[b \cdot c](x) =$
5. $c(2) =$
6. $b(-3) =$
7. $a(7r) =$

$$k(x) = 2x + 6 \qquad h(x) = 2x - 6$$

8. $[k \circ h](x) =$
9. $[h \circ k](x) =$
10. are h and k inverses?

11. Calculate the inverse of $p(x) = 3x + 2$

12. Fill out the t-chart for $p(x)$

x	y (or $p(x)$)
-2	
-1	
0	
1	
2	

13. List the domain and range of the t-chart

14. Fill the t-chart for $p^{-1}(x)$

x	y (or $p(x)$)

Extra Credit: What is the answer to the extra credit question?

1. $(x^2 + 5x - 3) + (4x^2 - 9x + 12)$

2. $(xy^3 + 2x - 7y + 3) - (x^2 + 2xy^3 + 2x - y + 10)$

3. $(2x - 1)(3y + 3)$

4. $(3x - 2)^2$