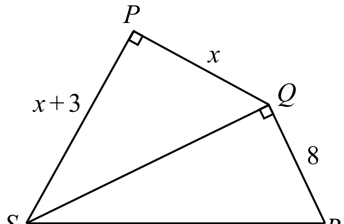
(b) Stephanie starts with a large number of soccer balls. She gives $\frac{2}{5}$ of them to Alphonso and $\frac{6}{11}$ of them to Christine. The number of balls that she is left with is a multiple of 9. What is the smallest number of soccer balls with which Stephanie could have started?

(c) Each student in a math club is in either the Junior section or the Senior section. No student is in both sections.
Of the Junior students, 60% are left-handed and 40% are right-handed.
Of the Senior students, 10% are left-handed and 90% are right-handed.
No student in the math club is both left-handed and right-handed.
The total number of left-handed students is equal to the total number of right-handed students in the math club.
Determine the percentage of math club members that are in the Junior section.

4. (a) Hexagon $ABCDEF$ has vertices $A(0,0)$, $B(4,0)$, $C(7,2)$, $D(7,5)$, $E(3,5)$, $F(0,3)$. What is the area of hexagon $ABCDEF$?

(b) In the diagram, $\triangle PQS$ is right-angled at $P$ and $\triangle QRS$ is right-angled at $Q$. Also, $PQ = x$, $QR = 8$, $RS = x + 8$, and $SP = x + 3$ for some real number $x$. Determine all possible values of the perimeter of quadrilateral $PQRS$.



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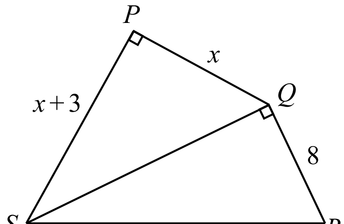
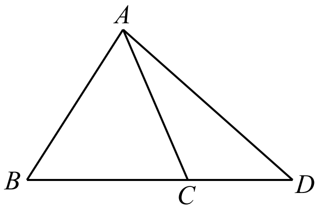


$x \neq 0$. What is the value of $f(4)$?

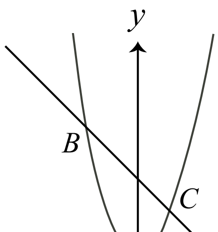
(b) Determine all real numbers $a$, $b$ and $c$ for which the graph of the function $y = \log_a(x+b) + c$ passes through the points $P(3,5)$, $Q(5,4)$ and $R(11,3)$.

7. (a) A computer is programmed to choose an integer between 1 and 99, inclusive, so that the probability that it selects the integer $x$ is equal to $\log_{100}\left(1+\frac{1}{x}\right)$. Suppose that the probability that $81 \leq x \leq 99$ is equal to 2 times the probability that $x = n$ for some integer $n$. What is the value of $n$?

(b) In the diagram, $\triangle ABD$ has $C$ on $BD$. Also, $BC = 2$, $CD = 1$, $\dfrac{AC}{AD} = \dfrac{3}{4}$, and $\cos(\angle ACD) = -\dfrac{3}{5}$. Determine the length of $AB$.




8. (a) Suppose that $a > \frac{1}{2}$ and that the parabola with equation $y = ax^2 + 2$ has vertex $V$. The parabola intersects the line with equation $y = -x + 4a$ at points $B$ and $C$, as shown. If the area of $\triangle VBC$ is $\frac{72}{5}$, determine the value of $a$.



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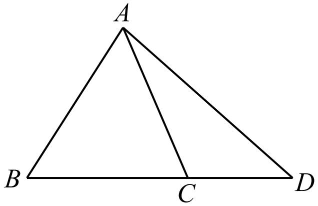


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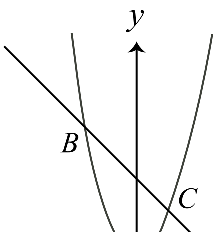

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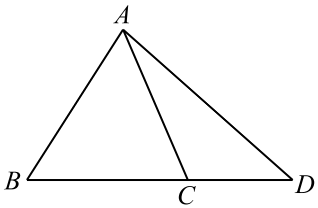


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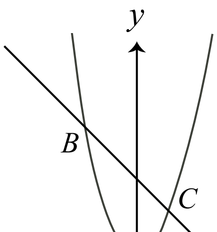

9. Suppose that $m$ and $n$ are positive integers with $m \geq 2$. The $(m, n)$-*sawtooth sequence* is a sequence of consecutive integers that starts with 1 and has $n$ teeth, where each tooth starts with 2, goes up to $m$ and back down to 1. For example, the $(3, 4)$-sawtooth sequence is

$$\begin{array}{ccccccccccccccccc}
 & & 3 & & & & 3 & & & & 3 & & & & 3 & & \\
 & 2 & & 2 & & 2 & & 2 & & 2 & & 2 & & 2 & & 2 & \\
1 & & & & 1 & & & & 1 & & & & 1 & & & & 1
\end{array}$$

The $(3, 4)$-sawtooth sequence includes 17 terms and the average of these terms is $\frac{33}{17}$.

(a) Determine the sum of the terms in the $(4, 2)$-sawtooth sequence.

(b) For each positive integer $m \geq 2$, determine a simplified expression for the sum of the terms in the $(m, 3)$-sawtooth sequence.

(c) Determine all pairs $(m, n)$ for which the sum of the terms in the $(m, n)$-sawtooth sequence is 145.

(d) Prove that, for all pairs of positive integers $(m, n)$ with $m \geq 2$, the average of the terms in the $(m, n)$-sawtooth sequence is not an integer.

10. At Pizza by Alex, toppings are put on

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