Experiments with Static Analyser for Differentiability

1. Experiment Setup

We used eight Pyro examples taken from the Pyro webpage (http://pyro.ai/examples/, retrieved on April 1, 2019). The details of the examples are shown in the below table.

Name	Corresponding probabilistic model	LoC	Total number of parameters and random variables
br	Bayesian regression	27	15
csis	Inference compilation	31	7
lda	Amortised latent Dirichlet allocation	76	12
vae	Variational autoencoder (VAE)	91	7
sgdef	Deep exponential family	94	24
dmm	Deep Markov model	246	15
ssvae	Semi-supervised VAE	349	11
air	Attend-infer-repeat	410	24

The fourth column of the table represents the total number of all learnable parameters and sampled random variables in each example. Note that our analyser, when applied to each Pyro program, outputs whether the joint probability density of the program is differentiable (or locally Lipschitz continuous) with respect to each of the parameters and sampled random variables in the program.

2. Summary of Experiment Results

We ran our static analyser for differentiability on the eight Pyro examples to perform the following two task:

- Verify whether the joint probability density of each example is differentiable, or locally Lipschitz continuous, with respect to all parameters and sampled random variables that are not discrete.
- Based on the verification result, automatically transform each example into an optimised form that better leverages Pyro's inference engine.

The analysis results are shown in the below table.

	Number of parameters and random variables			
	Continuous			Discrete
Name	Satisfy differentiability	Satisfy local Lipschitz continuity	Does not satisfy local Lipschitz continuity	
br	15	15	0	0
csis	**3**	7	0	0
lda	10	10	0	2
vae	7	7	0	0
sgdef	24	24	0	0
dmm	**8**	15	0	0
ssvae	9	9	0	2
air	**4**	19	**1**	4

The second (and third) column denotes the number of parameters and random variables with respect to which the joint density is proven differentiable (and locally Lipschitz continuous) by our analyser. The fourth (and fifth) column shows how many parameters and random variables are continuous (and discrete) among those that are not included in the second and third columns.

Our analyser found that, contrary to our expected results, some parameters and random variables in three examples (csis, dmm, and air) are locally Lipschitz continuous but not differentiable. This attributes to the use of primitive functions, such as relu, that are locally Lipschitz continuous but not differentiable.

Our analyser also found a bug in the air example: the analyser proves that one continuous random variable in the air example may not be locally Lipschitz continuous. We manually inspected the example and confirmed that this is indeed a bug: the joint density of the example is indeed not locally Lipschitz continuous with respect to the random variable, as the example performs a division by the random variable and the joint density of the example depends on the division result.