Практика. 07.02.2023

> Первообразная ф-ции: $dF(x) = f'(x)\,dx$

$f(x) = x^3 \Rightarrow f'(x) = 3x^2$

$x^2 = \frac{f'(x)}{3} = \boxed{\left(\frac{f(x)}{3}\right)'}$ - первообразная $F(x)$

$f(x) = F'(x)$

o Первообразной ф-ей $^{\vee}$ такая ф-я, производная которой = исходной ф-ции

~ Любая непр. на промежутке ф-я имеет на нём первообразную

o $\int f(x)\,dx = F(x) + C$ $\Bigg\}$ $\mathcal{E}_x$: $f(x) = x^6$

$\int x^5\,dx = \frac{x^6}{6} + C$

Первое табл. значение: $x^n \to \frac{x^{n+1}}{n+1}$, $n \neq -1$

а что если $n = -1$?

$f(x) = x^{-1} = \frac{1}{x}$

$\int x^{-1}\,dx = \ln(x) + C$

Табличка:

$\int \sin\,dx = -\cos x + C$

$\int \cos x\,dx = \sin x + C$

$\int \frac{1}{\cos^2 x}\,dx = tg\,x + C$

$\int \frac{1}{\sin^2 x}\,dx = -ctg + C$

$\int e^x\,dx = e^x + C$

$\int a^x\,dx = \frac{a^x}{\ln a} + C$

$\int 0\,dx = C$

$\int 2\,dx = 2x + C$

$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} arctg \frac{x}{a} + C$

$\int \frac{dx}{1 + x^2} = arctg\,x + C$

$$\int \sqrt{x}\, dx = \int x^{\frac{1}{2}}\, dx = \frac{2}{3} \cdot x^{\frac{3}{2}} + C$$

$$\int \frac{dx}{5x} = \frac{1}{5} \cdot \int \frac{dx}{x} = \frac{\ln(x)}{5} + C$$

$$\int \frac{3\, dx}{\sqrt[4]{x}} = 3 \cdot \int \frac{dx}{\sqrt[4]{x}} = 3 \cdot x^{\frac{3}{4}} \cdot \frac{4}{3} = 4 \cdot x^{\frac{3}{4}} + C$$

$$\int \sqrt[3]{x^5}\, dx = \int x^{\frac{5}{3}}\, dx = \frac{3}{8} x^{\frac{8}{3}} + C$$

$$\int \frac{dx}{x\sqrt{x}} = \int \frac{dx}{x^{\frac{3}{2}}} = -2\, x^{-\frac{1}{2}} + C$$

$\frac{15}{8} \cdot \frac{8}{15}$

! $$\int \sqrt{x\sqrt{x\sqrt{x}}}\, dx = \int x^{\frac{1}{2}} \cdot x^{\frac{1}{4}} \cdot x^{\frac{1}{6}}\, dx = \int x^{\frac{6}{12} + \frac{3}{12} + \frac{2}{12}}\, dx = \int x^{\frac{11}{12}}\, dx$$

$$\int \frac{2^{3x}}{3^{2x+1}}\, dx = \cancel{\frac{1}{3} \cdot \frac{\ln 3 \cdot 2^{3x}}{3^{2x} \ln 2}}$$

$$= \frac{1}{3} \cdot \int \frac{2^x \cdot 2^x \cdot 2^x}{3^x \cdot 3^x}\, dx = \frac{1}{3} \cdot \int \left(\frac{8}{9}\right)^x dx = \frac{1}{3} \cdot \frac{\left(\frac{8}{9}\right)^x}{\ln\left(\frac{8}{9}\right)} + C$$

$$\int 4^{2x} \cdot e^x \cdot dx = \int 16^x e^x\, dx = \int (16e)^x\, dx = \frac{(16e)^x}{\ln(16e)} + C$$

$$\int \frac{dx}{x^2 + 7} = \frac{1}{\sqrt{7}} \cdot \operatorname{arctg} \frac{1}{\sqrt{7}} + C$$

$$\int \operatorname{tg} x \cdot \cos x\, dx = \int \sin x\, dx = -\cos x + C$$

~ Свойства линейности интеграла

$$\int (\alpha f(x) + \beta g(x))\,dx = \alpha\int f(x)\,dx + \beta\int g(x)\,dx \;;\ \alpha, \beta = const$$

Ex:
$$\int (x^7 + 2x^5 - x^3 + x - 2)\,dx = \frac{x^8}{8} + \frac{x^6}{3} - \frac{x^4}{4} + \frac{x^2}{2} - 2x + C$$

$$\int \frac{x^2 - 3x + 1}{x}\,dx = \int x - 3 + \frac{1}{x}\,dx = \frac{x^2}{2} - 3x + \ln|x| + C$$

$$\int x(x+1)(x-2)\,dx = \int (x^2+x)(x-2)\,dx = \int x^3 - x^2 - 2x\,dx =$$

$$= \frac{x^4}{4} - \frac{x^3}{3} - x^2 + C$$

$$\int \frac{x^2}{1+x^2}\,dx = \int \frac{1+x^2}{1+x^2} - \frac{1}{1+x^2}\,dx = \int 1\,dx - \int \frac{1}{1+x^2}\,dx =$$

$$= x - \operatorname{arctg} x + C$$

$$\int \frac{\cos 2x}{\cos^2 x \cdot \sin^2 x}\,dx = \int \frac{\cos^2 x - \sin^2 x}{\cos^2 x \cdot \sin^2 x}\,dx = \int \frac{1}{\sin^2 x} - \frac{1}{\cos^2 x}\,dx =$$

$$= \int \frac{1}{\sin^2 x}\,dx - \int \frac{1}{\cos^2 x}\,dx = -\operatorname{ctg} x + \operatorname{tg} x + C$$

$$\int \frac{(1+x)^2}{x(1+x^2)}\,dx = \int \frac{1+2x+x^2}{x+x^3}\,dx = \int \frac{\cancel{1+x^2}}{x(\cancel{1+x^2})}\,dx + \int \frac{2x}{x(1+x^2)}\,dx =$$

$$= \int \frac{1}{x}\,dx + \int \frac{2}{1+x^2}\,dx = \ln(|x|) + 2\operatorname{arctg} x + C$$

$$\int \frac{x^4}{1+x^2}\,dx = \int \frac{x^4 - 1 + 1}{1+x^2}\,dx = \int \frac{(x^2-1)(x^2+1)+1}{1+x^2}\,dx =$$

$$= \int x^2 - 1\,dx + \int \frac{1}{1+x^2}\,dx = \frac{x^3}{3} - x + \operatorname{arctg}(x) + C$$

Интеграл сложной ф-ции:

$$\int \sin(5x+4)\,dx = \int \sin y\,dy =$$

$\cancel{y = 5x+4}$ $$= \frac{1}{5}\int \sin(5x+4)\cdot d(5x+4) =$$

$$= -\frac{1}{5}\cdot\cos(5x+4) + C$$

$$\int \cos x \cdot e^{\sin x}\,dx = \int e^{\sin x}\,d(\sin x) = e^{\sin x} + C$$

$$\int \frac{dx}{\cos^2 3x} = \int \frac{1}{\cos^2 3x}\,dx = \int \frac{1}{\cos^2 3x}\,d$$

$\cancel{3x = y}$

$$= \frac{1}{3}\cdot\int \frac{d(3x)}{\cos^2 3x} = \frac{1}{3}\cdot \operatorname{tg} x + C$$

$$\int \frac{dx}{1+25x^2} = \frac{1}{5}\int \frac{d(5x)}{1+25x^2} = \frac{1}{5}\operatorname{arctg} 5x + C$$

$$\int x^2\sqrt{2x^3+3}\,dx = \frac{x^3}{3}\int \sqrt{2x^3+3}\,d\left(\frac{x^3}{3}\right) = \cancel{\frac{x^3}{3}}$$

$$= \frac{1}{6}\int \sqrt{2x^3+3}\,d(2x^3+3) = \frac{1}{6}\,\frac{(2x^3+3)^{\frac{3}{2}}}{1,5}$$

14.02 | Матан - практика 2

$$\int \frac{2x+1}{x^2+x+3}\,dx = \int \frac{\cancel{2x+1}}{\cancel{2x+1}\cdot t}\,dt = \int \frac{1}{t}\,dt = \cancel{\ln|2x+1|+C}$$

$\exists t = x^2+x+3 \Rightarrow t' = 2x+1$

$$\boxed{= \ln|x^2+x+3| + C}$$

$$\int \sin^3 x \cos x\,dx = \int -t^4\,dt = \cancel{-\frac{1}{5}t^5 + C} = \boxed{-\frac{\sin^4}{4} + C}$$

$\exists t = \sin x$ ~~cos x~~ $\Rightarrow t' = -\sin x$

$$\int x^2 \sqrt[7]{2x^3+3}\,dx = \int \frac{x^2 \cdot \sqrt[7]{t}}{6x}\,dt = \frac{1}{6}\int t^{\frac{1}{7}}\,dt = \cancel{\frac{t^{\frac{3}{2}}}{3}} = t$$

$\exists t = 2x^3+3 \Rightarrow t' = 6x^2$

$$\boxed{= \frac{7t^{\frac{8}{7}}}{8\cdot 6} + C}$$

$$\int \sin(1-5x)\,dx = \int \sin t \cdot \left(-\frac{1}{5}\right)dt =$$

$\exists t = 1-5x \Rightarrow t' = -5$

$$\cancel{\int (3x+5)^{20}\,dx} \qquad \int \frac{x^4}{x^{10}+1}\,dx = \int \frac{1}{4x^3}$$

$\exists t = x^4 \Rightarrow t' = 4x^3$

$$\int \frac{e^{\sqrt{2x-1}}}{\sqrt{2x-1}} = \cancel{\frac{1}{2}\int \frac{e^{\sqrt{t}}}{\sqrt{t}}\,dt} = \int \frac{e^t\,\cancel{t}}{\cancel{t}}\,dt = \int e^t\,dt = e^t = \boxed{e^{\sqrt{2x-1}} + C}$$

$\exists t = \sqrt{2x-1} \Rightarrow t' = \frac{1}{2\sqrt{2x-1}} \cdot 2 = \frac{1}{\sqrt{2x-1}}$

$$\int \frac{e^{-2x}}{e^{-4x}+3}\,dx = \int \frac{\cancel{t}}{t^2+3}\cdot\frac{1}{-2\cancel{t}}\,dt = -\frac{1}{2}\cdot\frac{1}{\sqrt3}\cdot \operatorname{arctg}\frac{t}{\sqrt3}+C =$$

$\exists t = e^{-2x} \Rightarrow t' = -2e^{-2x}$ $= \boxed{-\frac{1}{2}\cdot\frac{1}{\sqrt3}\cdot \operatorname{arctg}\frac{e^{-2x}}{\sqrt3}+C}$

$$\int x\sqrt{x-1}\,dx = \int (t+1)\cdot\sqrt{t}\,dt = \int t^{\frac{3}{2}}dt + \int \sqrt{t}\,dt =$$

$\exists t = x-1 \Rightarrow t' = 1$ $= \frac{2}{5}t^{\frac{5}{2}} + \frac{2}{3}t^{\frac{3}{2}} + C =$

$$= \frac{2}{5}(x-1)^{\frac{5}{2}} + \frac{2}{3}(x-1)^{\frac{3}{2}} + C$$

Интегрирование по частям

$$\boxed{\int u\,dv = uv - \int v\,du}$$

$u \to du$ #произв.
$dv \to v$ #первообр.

$$\int \ln x\,dx = \left| \begin{array}{l} u = \ln x \\ v = x \\ du = \frac{dx}{x} \\ dv = dx \end{array} \right| = x\ln x - \int \cancel{x}\frac{dx}{\cancel{x}} =$$

$u \to du$ – ~~первообр. от u~~ произв.
$dv \to v$ – первообр.

$$= x\ln x - \int dx = x\ln x - x + C$$

$$\int (5-x^2)e^{2x}\,dx = \left| \begin{array}{l} u = 5-x^2 \\ v = \frac{1}{2}e^{2x} \\ du = -2x\,dx \\ dv = e^{2x}\,dx \end{array} \right| = (5-x^2)\cdot\frac{1}{2}e^{2x} - \int \frac{1}{\cancel{2}}e^{2x}\cancel{2}x\,dx =$$

$$= (5-x^2)e^{2x}\cdot\frac{1}{2} + \int e^{2x}x\,dx = \left| \begin{array}{l} u = x \\ v = \frac{1}{2}e^{2x} \\ du = 1dx \\ dv = e^{2x}\,dx \end{array} \right| =$$

$$= (5-x^2)e^{2x}\cdot\frac{1}{2} + x\frac{1}{2}e^{2x} - \int \frac{1}{2}e^{2x}\,dx =$$

$$= \boxed{(5-x^2)e^{2x}\cdot\frac{1}{2} + \frac{1}{4}x e^{2x} - \frac{e^{2x}}{4} + C}$$

$$\int (x^2+2x)\cos 2x \, dx = \left| \begin{array}{l} u = x^2+2x \\ v = \frac{1}{2}\sin 2x \\ du = 2x+2\, dx \\ dv = \cos 2x \, dx \end{array} \right| =$$

$$= (x^2+2x)\frac{1}{2}\cdot \sin 2x - \int \frac{1}{\cancel{2}} \sin 2x \, (\cancel{2}x+\cancel{2})\, dx =$$

$$= \frac{(x^2+2x)\sin 2x}{2} - \int \sin 2x (x+1) dx = \left| \begin{array}{l} \cancel{u = \sin 2x} \\ \cancel{v = \frac{1}{2}x^2+x} \\ \cancel{du = 2\cos 2x \, dx} \\ \cancel{dv = (x+1)dx} \end{array} \right| =$$

$$\cancel{= \frac{(x^2+2x)\sin 2x}{2} + \sin 2x\left(\frac{x^2}{2}+x\right) - \int\left(\frac{1}{2}x^2+x\right)2\cos 2x \, dx =}$$

$$\cancel{= \frac{(x^2+2x)\sin 2x}{2} + \sin 2x \cdot \left(\frac{x^2}{2}+x\right)}$$

$$= \left| \begin{array}{l} u = x+1 \\ v = -\frac{1}{2}\cos 2x \\ du = dx \\ dv = \sin 2x \, dx \end{array} \right| = \frac{(x^2+2x)\sin 2x}{2} - (x+1)\left(-\frac{1}{2}\cos 2x\right) + \int -\frac{1}{2}\cos 2x \, dx =$$

$$= \frac{(x^2+2x)\sin 2x}{2} + (x+1)\frac{1}{2}\cos 2x - \frac{1}{2}\cdot\frac{1}{2}\cdot \sin 2x + C$$

$$\int x \operatorname{arctg} x \, dx = \left| \begin{array}{l} u = x \\ v = \frac{1}{x^2+1} \\ du = 1 \, dx \\ dv = \operatorname{arctg} x \, dx \end{array} \right| = \frac{x}{x^2+1} - \int \frac{1}{x^2+1} dx =$$

$$= \frac{x}{x^2+1} - \operatorname{arctg} x + C$$