Д/з. Матан. с 7.02.2023

① $\int \frac{dx}{2x^2} = \frac{1}{2}\cdot\int\frac{dx}{x^2} = \frac{1}{2}\cdot\left(-\frac{1}{x}\right) = \boxed{-\frac{1}{2x}}$

② $\int \frac{dx}{\sqrt{4-x^2}} = \int\frac{dx}{(2^2-x^2)^{\frac{1}{2}}} = \boxed{\arcsin\left(\frac{x}{2}\right) + C}$

$\boxed{* \; |x|<|a|}$

③ $\int \frac{dx}{\sqrt{3+3x^2}} = \frac{1}{\sqrt{3}}\cdot\int\frac{dx}{\sqrt{1+x^2}} = \boxed{\frac{1}{\sqrt{3}}\cdot\ln\left|x+\sqrt{x^2+1}\right| + C}$

$\boxed{* \int\frac{dx}{\sqrt{x^2\pm a^2}} = \ln\left|x+\sqrt{x^2\pm a^2}\right| + C}$

④ $\int\frac{dx}{1-\cos 2x} = \cancel{\int\frac{d(2x)}{2\cdot(1-\cos 2x)} = \frac{1}{2}\cdot\int\frac{d(2x)}{1-\cos 2x}} =$

$= \int\frac{dx}{\cancel{1}+2\sin^2 x\cancel{-1}} = \int\frac{dx}{2\cdot\sin^2 x} = \frac{1}{2}\cdot\int\frac{dx}{\sin^2 x} = \boxed{-\frac{ctg x}{2} + C}$

⑤ $\int\frac{dx}{\cos^2 x\cdot\sin^2 x} = \int\frac{dx}{\frac{1}{4}\cdot\sin 2x^2} = 4\cdot\int\frac{dx}{\sin 2x^2} = 4\cdot\int\frac{d(2x)}{2\cdot\sin 2x^2} =$

$= 2\cdot\int\frac{d(2x)}{\sin 2x^2} = \boxed{-2\cot 2x + C}$

⑥ $\int\cos^2\frac{x}{2}\,dx = \int\cos^2\frac{x}{2}\cdot 2\,d\left(\frac{x}{2}\right) = 2\cdot\int\cos^2\frac{x}{2}\,d\left(\frac{x}{2}\right) =$

$= 2\cdot\int\frac{1+\cos x}{2}\,d\left(\frac{x}{2}\right) = \int 1+\cos x\,d\left(\frac{x}{2}\right) = \int 1\,d\frac{x}{2} + \int\cos x\,d\frac{x}{2} =$

$= \frac{x}{2} + \int\frac{1}{2}\cdot\cos x\,dx = \frac{x}{2} + \frac{1}{2}\cdot\sin x = \boxed{\frac{x+\sin x}{2} + C}$

⑦ $\int \frac{(\sqrt{x}-1)^3}{x}\,dx = \int \frac{(x-2\sqrt{x}+1)(\sqrt{x}-1)}{x}\,dx = \int \frac{(x\sqrt{x}-x-2x+2\sqrt{x}+\sqrt{x}-1)}{x}\,dx$

$= \int \frac{x\sqrt{x}-3x+3\sqrt{x}-1}{x}\,dx = \int \sqrt{x}-3+\frac{3}{\sqrt{x}}-\frac{1}{x}\,dx =$

$= \int x^{\frac12}\,dx - \int 3\,dx + \int 3x^{-\frac12}\,dx - \int \frac{1}{x}\,dx = \boxed{\frac{2}{3}x^{\frac32} - 3x + 6x^{\frac12} - \ln|x| + C}$

⑧ $\int x^3 \exp(x^4)\,dx = \int \cancel{x^3}\cdot e^t \cdot \frac{1}{4\cancel{x^3}}\,dt = \frac{1}{4}\int e^t\,dt = \boxed{\frac{e^{x^4}}{4} + C}$

⑨ $\int \frac{\ln x}{x}\,dx = \int t\frac{\cancel{x}}{\cancel{x}}\,dt = \int t\,dt = \frac{t^2}{2} = \boxed{\frac{\ln(x)^2}{2}}$

] $t = \ln x \Rightarrow t' = \frac{1}{x}$

⑩ $\int \frac{x}{\sqrt{1+x^4}}\,dx = \int \frac{1}{2\cancel{x}}\cdot\frac{\cancel{x}}{\sqrt{1+t^2}}\,dt = \frac{1}{2}\cdot\int\frac{1}{\sqrt{1+t^2}}\,dt = \boxed{\frac{1}{2}\operatorname{arctg} x^2 + C}$

] $t = x^2 \Rightarrow t' = 2x$

⑪ $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = \int \frac{\cancel{e^{\sqrt{x}}}\cdot 2\cancel{\sqrt{x}}}{\cancel{e^{\sqrt{x}}}\cdot\cancel{\sqrt{x}}}\,dt = \int 2\,dt = 2t = \boxed{2e^{\sqrt{x}} + C}$

] $t = e^{\sqrt{x}} \Rightarrow t' = e^{\sqrt{x}}$

⑫ $\int a^x e^x\,dx = \cancel{\int a^x e^x \, dt} = \int (ae)^x\,dx = \frac{(ae)^x}{\ln ae} + C = \boxed{\frac{a^x e^x}{\ln a + 1} + C}$

~~] $t = e^x \Rightarrow t' = e^x$~~

⑬ $\int \frac{dx}{x^2-10} = \boxed{\frac{1}{2\sqrt{10}}\cdot\ln\left(\left|\frac{x-\sqrt{10}}{x+\sqrt{10}}\right|\right) + C}$

⑭ $\int \frac{dx}{2x^2+9} = \int \frac{dx}{(\sqrt{2}x)^2+(3)^2} = \boxed{\frac{1}{3}\cdot\operatorname{arctg}\frac{\sqrt{2}x}{3} + C}$

⑮ $\int e^x\left(1-\frac{e^{-x}}{x^2}\right)dx = \int e^x - \frac{1}{x^2}\,dx = \int e^x\,dx - \int \frac{1}{x^2}\,dx =$

$\boxed{= e^x + \frac{1}{x} + C}$

Правило подстановки под дифференциал:

$$\boxed{dx = \frac{1}{t'} \cdot dt}$$

Пример: ① $\int x^3 \cdot e^{x^4}\, dx \;\circled{=}\; \Big| \Rightarrow \circled{=} \int x^3 \cdot e^{t} \cdot \frac{1}{4x^3}\, dt = \boxed{\frac{1}{4}\int e^{t} dt}$

$\exists\, t = x^4 \Rightarrow t' = 4x^3$

② $\int \cos^2 \frac{x}{2}\, dx \;\circled{=}\; \Big| \Rightarrow \circled{=} \int \frac{1}{\frac{1}{2}} \cdot \cos^2(t)\, dt$

$\exists\, t = \frac{x}{2} \Rightarrow t' = \frac{1}{2}$

Табличные значения интегралов:

1) $\int 0\, dx = C$; $\int dx = \int 1\, dx = x + C$

2) $\int x^n\, dx = \frac{x^{n+1}}{n+1} + C$; $\int \frac{1}{x}\, dx = \ln|x| + C$

3) $\int a^x\, dx = \frac{a^x}{\ln a} + C$; $\int e^x\, dx = e^x + C$

4) $\int \sin x\, dx = -\cos x + C$; $\int \cos x\, dx = \sin x + C$

5) $\int \frac{dx}{\sin^2 x} = -\operatorname{ctg} x + C$; $\int \frac{dx}{\cos^2 x} = \operatorname{tg} x + C$

6) $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C,\ |x| < |a|$; $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \cdot \ln\left|\frac{a+x}{a-x}\right| + C,\ |x| \neq a$

7) $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \cdot \operatorname{arctg} \frac{x}{a} + C$; $\int \frac{1}{x^2 - a^2}\, dx = \frac{1}{2a} \cdot \ln\left(\left|\frac{x-a}{x+a}\right|\right) + C$

8) $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$

(16) $\int \frac{3\cdot 2^x - 2\cdot 3^x}{2^x}\,dx = \int 3\,dx - \int 2\cdot\left(\frac{3}{2}\right)^x dx = 3x - 2\cdot\int\left(\frac{3}{2}\right)^x dx = \boxed{3x - \frac{2\cdot\left(\frac{3}{2}\right)^x}{\ln\frac{3}{2}} + C}$

(17) $\int \frac{\sqrt[3]{x^2} + 2 - 3\sqrt{x}}{\sqrt{x}}\,dx = \int x^{\frac{1}{6}}\,dx + \int 2x^{-\frac{1}{2}}\,dx - \int 3\,dx =$

$\boxed{= \frac{6}{7}x^{\frac{7}{6}} + 4x^{\frac{1}{2}} - 3x + C}$

(18) $\int \frac{4x+1}{2x^2+x}\,dx = \int \frac{\cancel{4x+1}}{\cancel{4x+1}\cdot t}\,dt = \int \frac{1}{t}\,dt = \ln|t| = \boxed{\ln|2x^2+x| + C}$

$\exists t = 2x^2 + x \Rightarrow t' = 4x+1$

(19) $\int \frac{\sqrt{\arcsin x}}{\sqrt{1-x^2}}\,dx = \int \sqrt{\arcsin x}\,dt = \int t^{\frac{1}{2}}\,dt = \frac{2}{3}\cdot t^{\frac{3}{2}} \ominus$

$\exists t = \sqrt{\arcsin x} \Rightarrow t' = \cancel{\tfrac{1}{2}}\cdot\cancel{\arcsin x}\cdot\frac{1}{\sqrt{1-x^2}}$ $\ominus \boxed{\frac{2}{3}\cdot\arcsin x^{\frac{3}{2}} + C}$

(20) $\int (2-3x)^{100}\,dx = \int t^{100}\cdot\left(-\frac{1}{3}\right)dt = -\frac{1}{3}\int t^{100}\,dt = -\frac{1}{303}t^{101} + C =$

$\exists t = 2-3x \Rightarrow t' = -3$ $= \boxed{-\frac{(2-3x)^{101}}{303} + C}$

(21) $\int \frac{1}{x^2}\cdot\cos\frac{1}{x}\,dx = \int \cos t\cdot\cancel{x^2}\cdot -\frac{\cancel{x^2}}{\cancel{x^2}}\,dt = \int -\cos t\,dt = -\sin t + C =$

$\exists t = \frac{1}{x} \Rightarrow t' = -\frac{1}{x^2}$ $= \boxed{-\sin\frac{1}{x} + C}$