1. $X \to$ 甲投中的次数，$Y \to$ 乙投中的次数

(1) $P(X=Y) = \sum_{i=0}^{3} P\{(X=i)\cap(Y=i)\} = \sum_{i=0}^{3} P(X=i)\,P(Y=i)$

$= (1-0.5)^3(1-0.9)^3 + C_3^1(1-0.5)^2(0.5)\,C_3^1(1-0.9)^2\cdot 0.9 + C_3^2\, 0.5^2(1-0.5)\,C_3^2\, 0.9^2(1-0.9) + 0.5^3\cdot 0.9^3$

$= 0.000125 + 0.010125 + 0.091125 + 0.091125$

$= 0.1925$

(2) $P(X>Y) = P(X=1)\cap P(Y=0) + P(X=2)\cap P(Y\le 1) + P(X=3)\cap P(Y\le 2)$

$= C_3^1\, 0.5\cdot(1-0.5)^2(1-0.9)^3 + C_3^2\, 0.5^2(1-0.5)\left[(1-0.9)^3 + C_3^1(1-0.9)^2 0.9\right] + 0.5^3(1-0.9^3)$

$= 0.000375 + 0.0105 + 0.033875$

$= 0.04475$

2. $X \to$ 外出时电话铃响的次数

$P(X=k) = \frac{(2t)^k e^{-2t}}{k!}$，$k = 0, 1, 2 \cdots$

(1) $t = 30/60 = \frac{1}{2}$ 时，$X \sim \pi(2\times\frac{1}{2})$

$P(X=3) = \frac{(2\times\frac{1}{2})^3 e^{-2\times\frac{1}{2}}}{3!} = \frac{e^{-1}}{3\times 2\times 1} = \frac{1}{6e}$

(2) $P(X=0) = e^{-2t} \ge 0.5$

$\Rightarrow e^{2t} \le 2$

$\Rightarrow t \le \frac{1}{2}\ln 2 = 0.3466$ (h)

3. $P\{98 \le X_i \le 104\} = \phi\left(\frac{104-100}{2}\right) - \phi\left(\frac{98-100}{2}\right)$

$= \phi(2) - \phi(-1)$

$= 0.8185$

$P(X_i \notin [98, 104]) = 1 - 0.8185$

$P(Y=2) = C_5^2(0.5328)^2(0.4672)^3 = 0.1806$

4. 电流I的概率密度为：$f_I(i) = \begin{cases} \frac{1}{4} & 8A < i < 12 \\ 0 & \text{others} \end{cases}$

$W = 2I^2 \to w = g(i) = 2i^2$ 在 $i>0$ 时，$g(i)$ 严格单增，反函数 $i = h(w) = (\frac{w}{2})^{\frac{1}{2}}$

则 $h'(w) = \frac{1}{2\sqrt{2}} w^{-\frac{1}{2}}$，$g(8) = 128$，$g(12) = 288$

$f_W(w) = \begin{cases} \frac{1}{4}\left(\frac{1}{2\sqrt{2}} w^{-\frac{1}{2}}\right) & 128 < w < 288 \\ 0 & \text{others} \end{cases}$

5. $F_X(x) = F(x, \infty) = \begin{cases} 1 - e^{-2x} & x > 0 \\ 0 & \text{others} \end{cases}$

$F_Y(y) = F(\infty, y) = \begin{cases} 1 - e^{-3y} & y > 0 \\ 0 & \text{others} \end{cases}$

6. $f_X(x)=\begin{cases}\int_{-x}^{x} 1\,dy = 2x & 0<x<1 \\ 0 & \text{others}\end{cases}$ $\quad f_Y(y)=\begin{cases}\int_{|y|}^{1} 1\,dx = 1-|y| & |y|<1 \\ 0 & \text{others}\end{cases}$

当 $|y|<1$ 时 $f_{X|Y}(x|y)=\begin{cases}\frac{1}{1-|y|} & |y|<x<1 \\ 0 & \text{others}\end{cases}$ (y)

当 $0<x<1$ 时 $f_{Y|X}(y|x)=\begin{cases}\frac{1}{2x} & |y|<x \\ 0 & \text{others}\end{cases}$ (x)

7. (1) $P(Y=2)=\sum_{i=0}^{5} P\{X=i, Y=2\} = 0.03+0.04+0.05+0.05+0.05+0.06=0.28$

$P(X=1) = \sum_{i=0}^{3} P\{X=1, Y=i\} = 0.03+0.02+0.04+0.02=0.11$

$P(X=4|Y=2) = \frac{P(X=4, Y=2)}{P(Y=2)} = \frac{0.05}{0.28} = \frac{5}{28}$

$P(Y=3|X=1) = \frac{P(X=1, Y=3)}{P(X=1)} = \frac{0.02}{0.11} = \frac{2}{11}$

(2) $\{V=i\} = \{\max\{X, Y\}=i\}$

$= \{X=i, Y<i\} \cup \{X=i, Y=i\} \cup \{X<i, Y=i\}$

$P(V=0) = 0.01$

$P(V=1) = 0.03+0.02+0.02 = 0.07$

$P(V=2) = 0.03+0.04+0.05+0.04+0.03 = 0.19$

$P(V=3) = 0.05+0.05+0.05+0.06+0.04+0.02+0.01=0.28$

$P(V=4) = 0.07+0.06+0.05+0.06 = 0.24$

$P(V=5) = 0.06+0.04+0.06+0.05 = 0.21$

V	0	1	2	3	4	5
P_k	0.01	0.07	0.19	0.28	0.24	0.21

(3) $P(U=0) = 0.01+0.02+0.03+0.01+0.03+0.03+0.05+0.07+0.06=0.31$

$P(U=1) = 0.02+0.04+0.02+0.04+0.05+0.06+0.04 = 0.27$

$P(U=2) = 0.04+0.05+0.05+0.05+0.06 = 0.25$

$P(U=3) = 0.06+0.06+0.05 = 0.17$

U	0	1	2	3
P_k	0.31	0.27	0.25	0.17

(4) $P(W=0) = 0.01$

$P(W=1) = 0.03 + 0.02 = 0.05$

$P(W=2) = 0.03 + 0.02 + 0.03 = 0.08$

$P(W=3) = 0.05 + 0.04 + 0.04 + 0.01 = 0.14$

$P(W=4) = 0.07 + 0.05 + 0.05 + 0.02 = 0.19$

$P(W=5) = 0.06 + 0.06 + 0.05 + 0.04 = 0.21$

$P(W=6) = 0.04 + 0.05 + 0.06 = 0.15$

$P(W=7) = 0.06 + 0.06 = 0.12$

$P(W=8) = 0.05$

W	0	1	2	3	4	5	6	7	8
P_k	0.01	0.05	0.08	0.14	0.19	0.21	0.15	0.12	0.05