概率论与数理统计(一)

1. (1) $D_1 = A\bar{B}\bar{C} = A-B-C$

(2) $D_2 = AB\bar{C} = AB-C$

(3) $D_3 = A\cup B\cup C = A\bar{B}\bar{C}\cup\bar{A}B\bar{C}\cup\bar{A}\bar{B}C\cup AB\bar{C}\cup A\bar{B}C\cup\bar{A}BC\cup ABC$

(4) $D_4 = ABC$

(5) $D_5 = \bar{A}\bar{B}\bar{C}$

(6) $D_6 = \bar{A}\bar{B}\bar{C}\cup AB\bar{C}\cup A\bar{B}C\cup\bar{A}BC = \bar{A}\bar{B}\cup\bar{B}\bar{C}\cup\bar{C}\bar{A}$

$= \overline{AB\cup BC\cup CA} = \bar{A}\bar{B}\cap\bar{B}\bar{C}\cap\bar{C}\bar{A}$

(7) $D_7 = \bar{A}\bar{B}\bar{C}\cup AB\bar{C}\cup A\bar{B}C\cup\bar{A}BC\cup AB\bar{C}\cup A\bar{B}C\cup\bar{A}BC$.

$= \bar{A}\cup\bar{B}\cup\bar{C}$

(8) $D_8 = AB\cup BC\cup CA = ABC\cup\bar{A}BC\cup A\bar{B}C\cup AB\bar{C}$

2. (1) $P(A\cup B\cup C) = P(A)+P(B)+P(C)-P(AB)-P(BC)-P(AC)+P(ABC)$

$= 1-\frac{1}{4}+P(ABC) = \frac{3}{4}+P(ABC) = \frac{3}{4}$

(2) ① $P(A\cup B) = P(A)+P(B)-P(AB) = \frac{1}{2}+\frac{1}{4}-\frac{1}{10} = \frac{13}{20}$

② $P(\bar{A}\bar{B}) = P(\overline{A\cup B}) = 1-P(A\cup B) = \frac{7}{20}$

③ $P(A\cup B\cup C) = P(A)+P(B)+P(C)-P(AB)-P(AC)-P(BC)+P(ABC)$

$= \frac{1}{2}+\frac{1}{4}+\frac{1}{5}-\frac{1}{10}-\frac{1}{20}-\frac{1}{30}+\frac{1}{60} = \frac{95}{120} = \frac{19}{24}$

$=$ [illegible]

④ $P(\bar{A}\bar{B}\bar{C}) = P(\overline{A\cup B\cup C}) = 1-P(A\cup B\cup C) = \frac{5}{24}$

⑤ $P(\bar{A}\bar{B}C) = P(\bar{A}\bar{B}-\bar{A}\bar{B}\bar{C}) = P(\bar{A}\bar{B})-P(\bar{A}\bar{B}\bar{C}) = \frac{7}{20}-\frac{5}{24} = \frac{17}{120}$

⑥ $P(\bar{A}\bar{B}\cup C) = P(\bar{A}\bar{B})+P(C)-P(\bar{A}\bar{B}C) = \frac{7}{20}+\frac{1}{5}-\frac{17}{120} = \frac{49}{120}$

(3). (i) $P(A\bar{B}) = P(A-AB) = P(A)-\underbrace{P(AB)}_{=0} = \frac{1}{2}$

(ii) $P(A\bar{B}) = P(A)-P(AB) = \frac{1}{2}-\frac{1}{4} = \frac{1}{4}$

3. (1) 假设 $A\bar{B}=\bar{A}B \Rightarrow (A\bar{B})\cup(AB)=(\bar{A}B)\cup(AB)$

$\Rightarrow A(\bar{B}\cup B)=(\bar{A}\cup A)B$

$\Rightarrow AS=SB \Rightarrow A=B$.

(2). ~~$P(A)+P(B)=2P(AB)$~~

$P(A\bar{B}\cup\bar{A}B)=P(A\bar{B})+P(\bar{A}B)=P(A-AB)+P(B-AB)$

$=P(A)-P(AB)+P(B)-P(AB)$

$=P(A)+P(B)-2P(AB)$

(中) 4. $P(AB\bar{C})=P(\bar{C}BA)=P(\bar{C}|BA)P(BA)$

$=P(\bar{C}|BA)P(B|A)P(A)$

$=0.4\times0.5\times0.4=0.08=\frac{2}{25}$

5. (1) $P(A_1A_2)=P(A_2|A_1)P(A_1)=\frac{5}{9}\times\frac{6}{10}=\frac{1}{3}$

(2) $P(\bar{A}_1\bar{A}_2)=\frac{3}{9}\times\frac{6}{10}=\frac{2}{15}$

(3) $P(A_1\bar{A}_2\cup\bar{A}_1A_2)=\frac{6}{10}\times\frac{4}{9}+\frac{4}{10}\times\frac{6}{9}=\frac{4}{15}+\frac{4}{15}=\frac{8}{15}$

(4) $P(\bar{A}_2)=P(A_1\bar{A}_2)+P(\bar{A}_1\bar{A}_2)=\frac{6}{10}\times\frac{4}{9}+\frac{2}{15}=\frac{6}{15}=\frac{2}{5}$