

1.  $X \rightarrow$  甲投中的次数,  $Y \rightarrow$  乙投中的次数

$$\begin{aligned} (1) P(X=Y) &= \sum_{i=0}^3 P\{X=i \cap Y=i\} = \sum_{i=0}^3 P(X=i) P(Y=i) \\ &= C_3^0 (1-0.5)^3 (1-0.9)^3 + C_3^1 (1-0.5)^2 (0.5) C_3^1 (1-0.9)^2 (0.9) + C_3^2 0.5^2 (1-0.5) C_3^2 0.9^2 (1-0.9) + 0.5^3 \cdot 0.9^3 \\ &= 0.000125 + 0.010125 + 0.091125 + 0.091125 \\ &= 0.1925 \end{aligned}$$

$$\begin{aligned} (2) P(X>Y) &= P(X=1) \cap P(Y=0) + P(X=2) \cap P(Y \leq 1) + P(X=3) \cap P(Y \leq 2) \\ &= C_3^1 0.5 \cdot (1-0.5)^2 (1-0.9)^3 + C_3^2 0.5^2 (1-0.5) [C_3^0 (1-0.9)^3 + C_3^1 (1-0.9)^2 \cdot 0.9] + 0.5^3 (1-0.9^3) \\ &= 0.000375 + 0.0105 + 0.033875 \\ &= 0.04475 \end{aligned}$$

2.  $X \rightarrow$  外出时电话铃响的次数

$$P(X=k) = \frac{(2t)^k e^{-2t}}{k!}, \quad k=0,1,2,\dots$$

$$\begin{aligned} (1) t &= 30/60 = \frac{1}{2} \text{ 小时}, \quad X \sim \pi(2 \times \frac{1}{2}) \\ P(X=3) &= \frac{(2 \times \frac{1}{2})^3 e^{-2 \times \frac{1}{2}}}{3!} = \frac{e^{-1}}{3 \times 2 \times 1} = \frac{1}{6e} \end{aligned}$$

$$\begin{aligned} (2) P(X=0) &= e^{-2t} \geq 0.5 \\ \Rightarrow e^{2t} &\leq 2 \\ \Rightarrow t &\leq \frac{1}{2} \ln 2 = 0.3466 \text{ (h)} \end{aligned}$$

$$\begin{aligned} 3. P\{98 \leq X_i \leq 104\} &= \Phi\left(\frac{104-100}{2}\right) - \Phi\left(\frac{98-100}{2}\right) \\ &= \Phi(2) - \Phi(-1) \\ &= 0.5328 \end{aligned}$$

$$P(X_i \notin [98, 104]) = 1 - 0.5328 = 0.4672$$

$$P(Y=2) = C_5^2 (0.5328)^2 (0.4672)^3 = 0.2895$$

4. 电流  $I$  的概率密度为:  $f_I(i) = \begin{cases} \frac{1}{4} & 8A < i < 12 \\ 0 & \text{others} \end{cases}$

$W = 2I^2 \rightarrow W = g(i) = 2i^2$  在  $i > 0$  时,  $g(i)$  严格单增, 反函数  $i = h(W) = (\frac{W}{2})^{\frac{1}{2}}$   
 则  $h'(W) = \frac{1}{2\sqrt{2}} W^{-\frac{1}{2}}, g(8) = 128, g(12) = 288$

$$f_W(W) = \begin{cases} \frac{1}{4} \left( \frac{1}{2\sqrt{2}} W^{-\frac{1}{2}} \right) & 128 < W < 288 \\ 0 & \text{others} \end{cases}$$

$$5. F_X(x) = F(x, \infty) = \begin{cases} 1 - e^{-2x} & x > 0 \\ 0 & \text{others} \end{cases}$$

$$F_Y(y) = F(\infty, y) = \begin{cases} 1 - e^{-3y} & y > 0 \\ 0 & \text{others} \end{cases}$$

$$6. f_X(x) = \begin{cases} \int_{-x}^x 1 dy = 2x & 0 < x < 1 \\ 0 & \text{others} \end{cases} \quad f_Y(y) = \begin{cases} \int_{|y|}^1 1 dx = 1 - |y| & |y| < 1 \\ 0 & \text{others} \end{cases}$$

$$\text{if } |y| < 1 \text{ then } f_{X|Y}(x|y) = \begin{cases} \frac{1}{1-|y|} & |y| < x < 1 \\ 0 & \text{others (y)} \end{cases}$$

$$\text{if } 0 < x < 1 \text{ then } f_{Y|X}(y|x) = \begin{cases} \frac{1}{2x} & |y| < x \\ 0 & \text{others (x)} \end{cases}$$

$$7. (1) P(Y=2) = \sum_{i=0}^5 P\{X=i, Y=2\} = 0.03 + 0.04 + 0.05 + 0.05 + 0.05 + 0.06 = 0.28$$

$$P(X=1) = \sum_{i=0}^3 P\{X=1, Y=i\} = 0.03 + 0.02 + 0.04 + 0.02 = 0.11$$

$$P(X=4|Y=2) = \frac{P(X=4, Y=2)}{P(Y=2)} = \frac{0.05}{0.28} = \frac{5}{28}$$

$$P(Y=3|X=1) = \frac{P(X=1, Y=3)}{P(X=1)} = \frac{0.02}{0.11} = \frac{2}{11}$$

$$(2) \{V=i\} = \{\max\{X, Y\} = i\} \\ = \{X=i, Y < i\} \cup \{X=i, Y=i\} \cup \{X < i, Y=i\}$$

$$P(V=0) = 0.01$$

$$P(V=1) = 0.03 + 0.02 + 0.02 = 0.07$$

$$P(V=2) = 0.03 + 0.04 + 0.05 + 0.04 + 0.03 = 0.19$$

$$P(V=3) = 0.05 + 0.05 + 0.05 + 0.06 + 0.04 + 0.02 + 0.01 = 0.28$$

$$P(V=4) = 0.07 + 0.06 + 0.05 + 0.06 = 0.24$$

$$P(V=5) = 0.06 + 0.04 + 0.06 + 0.05 = 0.21$$

V	0	1	2	3	4	5
$P_V$	0.01	0.07	0.19	0.28	0.24	0.21

$$(3) P(U=0) = 0.01 + 0.02 + 0.03 + 0.01 + 0.03 + 0.03 + 0.05 + 0.07 + 0.06 = 0.31$$

$$P(U=1) = 0.02 + 0.04 + 0.02 + 0.04 + 0.05 + 0.06 + 0.04 = 0.27$$

$$P(U=2) = 0.04 + 0.05 + 0.05 + 0.05 + 0.06 = 0.25$$

$$P(U=3) = 0.06 + 0.06 + 0.05 = 0.17$$

U	0	1	2	3
$P_U$	0.31	0.27	0.25	0.17

$$(4) P(W=0) = 0.01$$

$$P(W=1) = 0.03 + 0.02 = 0.05$$

$$P(W=2) = 0.03 + 0.02 + 0.03 = 0.08$$

$$P(W=3) = 0.05 + 0.04 + 0.04 + 0.01 = 0.14$$

$$P(W=4) = 0.07 + 0.05 + 0.05 + 0.02 = 0.19$$

$$P(W=5) = 0.06 + 0.06 + 0.05 + 0.04 = 0.21$$

$$P(W=6) = 0.04 + 0.05 + 0.06 = 0.15$$

$$P(W=7) = 0.06 + 0.06 = 0.12$$

$$P(W=8) = 0.05$$

W	0	1	2	3	4	5	6	7	8
P <sub>k</sub>	0.01	0.05	0.08	0.14	0.19	0.21	0.15	0.12	0.05