1. 设第i只蛋糕的价格为 X_i，则 X_i 有分布律为：

X_i	1	2	3
P_k	0.2	0.5	0.3

$E(X_i)=0.2\times1+0.5\times2+0.3\times3=2.1$

$E(X_i^2)=0.2\times1^2+0.5\times2^2+0.3\times3^2=4.9$

$D(X_i)=4.9-(2.1)^2=0.49$.

(1) 总收入 $X=\sum^{300}X_i$

$P(X\geq500)=P(500\leq X<\infty)=P\left\{\frac{500-300\times2.1}{\sqrt{300}\cdot\sqrt{0.49}}\leq\frac{\sum^{300}X_i-300\times2.1}{\sqrt{300}\cdot\sqrt{0.49}}<\frac{\infty-300\times2.1}{\sqrt{300}\sqrt{0.0489}}\right\}$

$\approx\phi(\infty)-\phi(1.65)$

$=1-\phi(1.65)=1-0.9505=0.0495$

(2). 以Y记为300只蛋糕中售价为3元的蛋糕的只数，于是 $Y\sim b(300,0.3)$

$E(Y)=300\times0.3=90\quad D(Y)=300\times0.3\times0.7=63$

$P(Y>60)=1-P(Y\leq100)=1-P\left\{\frac{Y-90}{\sqrt{63}}\leq\frac{100-90}{\sqrt{63}}\right\}=1-\phi(+1.26)$

$=1-0.8962=0.1038$

2. 由题意：$D(X_i)=400$，$\sigma=\sqrt{400}$，提 $\frac{\bar{X}-\mu}{\sqrt{400}/\sqrt{n}}=\frac{\bar{X}-\mu}{20/\sqrt{n}}\sim N(0,1)$

$P(|\bar{X}-\mu|<1)=P(-1<\bar{X}-\mu<1)$

$=P\left\{\frac{-1}{20/\sqrt{n}}<\frac{\bar{X}-\mu}{20/\sqrt{n}}<\frac{1}{20/\sqrt{n}}\right\}$

$\approx2\phi\left(\frac{1}{20/\sqrt{n}}\right)-1$

则. $2\phi\left(\frac{1}{20/\sqrt{n}}\right)-1\geq0.8\Rightarrow\phi\left(\frac{1}{20/\sqrt{n}}\right)\geq0.9$ 查表知：$\left(\frac{1}{20/\sqrt{n}}\right)\geq1.29$

$\Rightarrow n\geq665.64\quad$ 取 $n=666$

3. 由药厂断言. 100人中治愈人数 $X\sim b(100,0.75)$

(1) 若治愈率为0.75. 则 接受的概率为 $P(X>70)$.

由中心极限定理，有 $X\sim N(100\times0.75,\ 100\times0.75\times0.25)=N(75,18.75)$

$P_1=P(X>70)\approx1-\phi\left(\frac{70-75}{\sqrt{18.75}}\right)=\phi\left(\frac{5}{\sqrt{18.75}}\right)=\phi(1.15)=0.8749$

(2). 若治愈率为0.6. 即 $X\sim b(100,0.6)$ 则有 $X\sim N(100\times0.6,\ 100\times0.6\times0.4)$

$P_2=P(X>70)\approx1-\phi\left(\frac{70-60}{\sqrt{24}}\right)=1-\phi(2.04)=1-0.9793=0.0207$

4. 将这两个样本分别记作 X Y，均值分别记作 $\bar{X}$ $\bar{Y}$

则 $\bar{X} \sim N(20, 3/12)$　$\bar{Y} \sim N(20, 3/15)$

$\bar{X}-\bar{Y} \sim N(20-20, \frac{3}{12}+\frac{3}{15})$

$\Rightarrow \bar{X}-\bar{Y} \sim N(0, 0.45)$

$P = P(|\bar{X}-\bar{Y}|>0.5) = 1-P(|\bar{X}-\bar{Y}| \le 0.5) = 1-P\left(\frac{-0.5}{\sqrt{0.45}} \le \frac{\bar{X}-\bar{Y}}{\sqrt{0.45}} \le \frac{0.5}{\sqrt{0.45}}\right)$

$= 2-2\phi(0.745) = 2-2\times 0.7734 = 0.4532$

5. (1) 因为 $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$，$n=16$ $\Rightarrow \frac{15S^2}{\sigma^2} \sim \chi^2(15)$

故有：$P = P(S^2/\sigma^2 \le 2.041) = P(15S^2/\sigma^2 \le 15\times 2.041)$

$= 1-P(15S^2/\sigma^2 > 30.615) = 0.99$

(2) $D(15S^2/\sigma^2) = 2\times 15 = 30$

$D(S^2) = \frac{2\sigma^4}{15}$