Problem set 3: Real and complex numbers, cardinality

Math 521 Section 001, UW-Madison, Spring 2024

February 5, 2024

Please solve the following problems in a clear, complete, and concise manner. You are welcome to work together, but your write-up must be your own. Use of outside internet resources is prohibited.

Due on paper at the beginning of class on **Wednesday, Feb. 14th**. Please be sure to staple your writeup.

1. Let $x \in \mathbb{R}$. Prove that
$$\sup\{q \in \mathbb{Q} \mid q < x\} = x.$$

2. Rudin 1.6-9, 1.12-14.

3. Rudin 2.2-4.

4. (Extra credit) Write down an explicit bijection between the sets $\{0, 1, 2, 3\}^{\mathbb{N}}$ and $\{0, 1\}^{\mathbb{N}}$.