Problem set 4: Cardinality, metric spaces, limit points

Math 521 Section 001, UW-Madison, Spring 2024

February 12, 2024

Please solve the following problems in a clear, complete, and concise manner. You are welcome to work together, but your write-up must be your own. Use of outside internet resources is prohibited.

Due on paper at the beginning of class on **Wednesday, Feb. 21st**. Please be sure to staple your writeup.

1. Suppose that $g : A \to B$ is surjective and $f : B \to C$ is *not* injective. Show that $f \circ g$ is also *not* injective. (*Hint*: It may be helpful to start by drawing a picture.)

2. Let $f : S \to T$ be a function. Suppose that S is uncountable and for each $y \in T$, $f^{-1}(\{y\})$ is countable. Prove that T is uncountable.

3. Consider the following subsets of $\mathbb{R}^2$. Which ones are open? Which ones are closed? Please justify each claim briefly; you do not have to give a full proof. (*Note*: For this problem, you are welcome to use drawings as justification.)

   (a) $[0,1] \times (0,1)$

   (b) $\{0\} \times (0,1)$

   (c) $\{(n, n^2) \mid n \in \mathbb{Z}\}$.

4. Rudin 2.5, 10 (except the "...compact?" part), 11.

5. (Extra credit + 1) Let S be a nonempty set. Prove that S and its power set $\mathcal{P}(S)$ do not have the same cardinality.