Nonlinear and Discrete Optimization—Homework Sets 3-4

1. Consider the optimization problem

$$\min_{x_1,x_2,x_3} \quad e^{x_1+1} + e^{-2x_1+1} + e^{x_2+1} + e^{-2x_2+1} + e^{x_3+1} + e^{-2x_3+1} + (x_1 + 4x_2 + 6x_3)^4$$

   Write a code to solve this problem using the Gradient method with your choice of the backtracking parameters. Draw $f(x^{(k)})$ verses k for $k = 0, 1, 2, \ldots, 100$ on a log-linear plot. Show the trajectory of points $x^{(0)}, x^{(1)}, ..., x^{(100)}$ in the 3-dimensional (x_1, x_2, x_3) plane.

2. Redo the previous problem with Newton's method.

3. Consider the optimization problem

$$\min_{x\in\mathbb{R}^n} \quad -\sum_{i=1}^{n} \log(5 - x_i^2) - \sum_{i=1}^{n} \log(1 - 3a_i^T x)$$

   where $n = 5000$ and $a_i \in \mathbb{R}^n$ are randomly generated vectors. Solve this problem using Newton's method with the backtracking line search. Draw $|f(x^{(k)})|$ versus k for $k = 0, 1, \ldots, 300$ on a log-linear plot.