

The Open Source CFD Toolkit

Programmer's Guide

Version 1.0 8th December 2004 P-2

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P-5

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P-6

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P-8

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P-7

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Contents

Copyright Notice P				
G	NU I	Free Doo	cumentation Licence	P-3
			BILITY AND DEFINITIONS	P-3
			M COPYING	P-4
			GIN QUANTITY	P-4
			ATIONS	P-5
			NG DOCUMENTS	P-6
			TIONS OF DOCUMENTS	P-7
			ATION WITH INDEPENDENT WORKS	P-7
			ATION	P-7
			ATION	P-7
			E REVISIONS OF THIS LICENSE	P-8
Tì	ader	narks		P-9
C	onter	nts		P-11
1	Ten	sor mat	hematics	P-15
	1.1	Coordin	ate system	P-15
	1.2	Tensors		P-15
		1.2.1	Tensor notation	P-17
	1.3		ic tensor operations	P-18
		1.3.1	The inner product	P-18
		1.3.2	The double inner product of two tensors	P-19
		1.3.3	The triple inner product of two third rank tensors	P-19
		1.3.4	The outer product	P-19
		1.3.5	The cross product of two vectors	P-20
		1.3.6	Other general tensor operations	P-20
			Geometric transformation and the identity tensor	P-21
		1.3.8 U	Useful tensor identities	P-21
		1.3.9	Operations exclusive to tensors of rank 2	P-22
		1.3.10	Operations exclusive to scalars	P-23
	1.4	OpenFC	DAM tensor classes	P-23
		1.4.1	Algebraic tensor operations in OpenFOAM	P-24
	1.5	Dimensi	onal units	P-26

P-12 Contents

Di	scretisation procedures
2.1	Differential operators
	2.1.1 Gradient
	2.1.2 Divergence
	2.1.3 Curl
	2.1.4 Laplacian
	2.1.5 Temporal derivative
2.2	Overview of discretisation
	2.2.1 OpenFOAM lists and fields
2.3	Discretisation of the solution domain
	2.3.1 Defining a mesh in OpenFOAM
	2.3.2 Defining a geometricField in OpenFOAM
2.4	Equation discretisation
	2.4.1 The Laplacian term
	2.4.2 The convection term
	2.4.3 First time derivative
	2.4.4 Second time derivative
	2.4.5 Divergence
	2.4.6 Gradient
	2.4.7 Grad-grad squared
	2.4.8 Curl
	2.4.9 Source terms
	2.4.10 Other explicit discretisation schemes
2.5	Temporal discretisation
	2.5.1 Treatment of temporal discretisation in OpenFOAM
2.6	Boundary Conditions
	2.6.1 Physical boundary conditions
Ev	amples of the use of OpenFOAM
3.1	Flow around a cylinder
5.1	3.1.1 Problem specification
	3.1.2 Note on potentialFoam
	3.1.3 Mesh generation
	3.1.4 Boundary conditions and initial fields
	3.1.5 Running the case
	3.1.6 Generating the analytical solution
	3.1.7 Exercise
3.2	Steady turbulent flow over a backward-facing step
3.2	
	*
	3.2.2 Mesh generation
9.9	
3.3	9.11
	3.3.1 Problem specification
	3.3.2 Mesh generation
	3.3.3 Running the case
	3.3.4 Exercise

Contents			P-13
3.4	Decon	appression of a tank internally pressurised with water	P-63
	3.4.1	Problem specification	P-64
	3.4.2	Mesh Generation	P-65
	3.4.3	Preparing the Run	P-67
	3.4.4	Running the case	P-68
	3.4.5	Improving the solution by refining the mesh	P-69
3.5	Magne	etohydrodynamic flow of a liquid	P-69
	3.5.1	Problem specification	P-69
	3.5.2	Mesh generation	P-71
	3.5.3	Running the case	P-72
Index			P-75

P-14 Contents

Chapter 1

Tensor mathematics

This Chapter describes tensors and their algebraic operations and how they are represented in mathematical text in this book. It then explains how tensors and tensor algebra are programmed in OpenFOAM.

1.1 Coordinate system

OpenFOAM is primarily designed to solve problems in continuum mechanics, *i.e.* the branch of mechanics concerned with the stresses in solids, liquids and gases and the deformation or flow of these materials. OpenFOAM is therefore based in 3 dimensional space and time and deals with physical entities described by tensors. The coordinate system used by OpenFOAM is the right-handed rectangular Cartesian axes as shown in Figure 1.1. This system of axes is constructed by defining an origin O from which three lines are drawn at right angles to each other, termed the Ox, Oy, Oz axes. A right-handed set of axes is defined such that to an observer looking down the Oz axis (with O nearest them), the arc from a point on the Ox axis to a point on the Oy axis is in a clockwise sense.

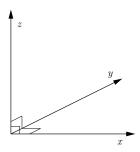


Figure 1.1: Right handed axes

1.2 Tensors

The term tensor describes an entity that belongs to a particular space and obeys certain mathematical rules. Briefly, tensors are represented by a set of *component values* relating

P-16 Tensor mathematics

to a set of unit base vectors; in OpenFOAM the unit base vectors \mathbf{i}_x , \mathbf{i}_y and \mathbf{i}_z are aligned with the right-handed rectangular Cartesian axes x, y and z respectively. The base vectors are therefore orthogonal, i.e. at right-angles to one another. Every tensor has the following attributes:

Dimension d of the particular space to which they belong, i.e. d = 3 in OpenFOAM;

Rank An integer r > 0, such that the number of component values $= d^r$.

While OpenFOAM 1.x is set to 3 dimensions, it offers tensors of ranks 0 to 3 as standard while being written in such a way to allow this basic set of ranks to be extended indefinitely. Tensors of rank 0 and 1, better known as scalars and vectors, should be familiar to readers; tensors of rank 2 and 3 may not be so familiar. For completeness all ranks of tensor offered as standard in OpenFOAM 1.x are reviewed below.

Rank 0 'scalar' Any property which can be represented by a single real number, denoted by characters in italics, e.q. mass m, volume V, pressure p and viscosity μ .

Rank 1 'vector' An entity which can be represented physically by both magnitude and direction. In component form, the vector $\mathbf{a} = (a_1, a_2, a_3)$ relates to a set of Cartesian axes x, y, z respectively. The *index notation* presents the same vector as a_i , i = 1, 2, 3, although the list of indices i = 1, 2, 3 will be omitted in this book, as it is intuitive since we are always dealing with 3 dimensions.

Rank 2 'tensor' or second rank tensor, T has 9 components which can be expressed in array notation as:

$$\mathbf{T} = T_{ij} = \begin{pmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{pmatrix}$$
(1.1)

The components T_{ij} are now represented using 2 indices since r=2 and the list of indices i, j=1,2,3 is omitted as before. The components for which i=j are referred to as the diagonal components, and those for which $i\neq j$ are referred to as the off-diagonal components. The transpose of **T** is produced by exchanging components across the diagonal such that

$$\mathbf{T}^{\mathrm{T}} = T_{ji} = \begin{pmatrix} T_{11} & T_{21} & T_{31} \\ T_{12} & T_{22} & T_{32} \\ T_{13} & T_{23} & T_{33} \end{pmatrix}$$
(1.2)

Note: a rank 2 tensor is often colloquially termed 'tensor' since the occurrence of higher order tensors is fairly rare.

Symmetric rank 2 The term 'symmetric' refers to components being symmetric about the diagonal, i.e. $T_{ij} = T_{ji}$. In this case, there are only 6 independent components since $T_{12} = T_{21}$, $T_{13} = T_{31}$ and $T_{23} = T_{32}$. OpenFOAM distinguishes between symmetric and non-symmetric tensors to save memory by storing 6 components rather than 9 if the tensor is symmetric. Most tensors encountered in continuum mechanics are symmetric.

1.2 Tensors P-17

Rank 3 has 27 components and is represented in index notation as P_{ijk} which is too long to represent in array notation as in Equation 1.1.

Symmetric rank 3 Symmetry of a rank 3 tensor is defined in OpenFOAM to mean that $P_{ijk} = P_{ikj} = P_{jik} = P_{jki} = P_{kij} = P_{kij} = P_{kji}$ and therefore has 10 independent components. More specifically, it is formed by the outer product of 3 identical vectors, where the outer product operation is described in Section 1.3.4.

1.2.1 Tensor notation

This is a book on computational continuum mechanics that deals with problems involving complex PDEs in 3 spatial dimensions and in time. It is vital from the beginning to adopt a notation for the equations which is compact yet unambiguous. To make the equations easy to follow, we must use a notation that encapsulates the idea of a tensor as an entity in the own right, rather than a list of scalar components. Additionally, any tensor operation should be perceived as an operation on the entire tensor entity rather than a series of operations on its components.

Consequently, in this book the *tensor notation* is preferred in which any tensor of rank 1 and above, *i.e.* all tensors other than scalars, are represented by letters in bold face, *e.g.* a. This actively promotes the concept of a tensor as a entity in its own right since it is denoted by a single symbol, and it is also extremely compact. The potential drawback is that the rank of a bold face symbol is not immediately apparent, although it is clearly not zero. However, in practice this presents no real problem since we are aware of the property each symbol represents and therefore intuitively know its rank, *e.g.* we know velocity **U** is a tensor of rank 1.

A further, more fundamental idea regarding the choice of notation is that the mathematical representation of a tensor should not change depending on our coordinate system, *i.e.* the vector ais the same vector irrespective of where we view it from. The tensor notation supports this concept as it implies nothing about the coordinate system. However, other notations, $e.g.\ a_i$, expose the individual components of the tensor which naturally implies the choice of coordinate system. The unsatisfactory consequence of this is that the tensor is then represented by a set of values which are not unique — they depend on the coordinate system.

That said, the index notation, introduced in Section 1.2, is adopted from time to time in this book mainly to expand tensor operations into the constituent components. When using the index notation, we adopt the *summation convention* which states that whenever the same letter subscript occurs twice in a term, the that subscript is to be given all values, *i.e.* 1, 2, 3, and the results added together, *e.q.*

$$a_i b_i = \sum_{i=1}^3 a_i b_i = a_1 b_1 + a_2 b_2 + a_3 b_3 \tag{1.3}$$

In the remainder of the book the symbol \sum is omitted since the repeated subscript indicates the summation.

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P-18 Tensor mathematics

1.3 Algebraic tensor operations

This section describes all the algebraic operations for tensors that are available in Open-FOAM. Let us first review the most simple tensor operations: addition, subtraction, and scalar multiplication and division. Addition and subtraction are both commutative and associative and are only valid between tensors of the same rank. The operations are performed by addition/subtraction of respective components of the tensors, *e.g.* the subtraction of two vectors **a** and **b** is

$$\mathbf{a} - \mathbf{b} = a_i - b_i = (a_1 - b_1, a_2 - b_2, a_3 - b_3) \tag{1.4}$$

Multiplication of any tensor \mathbf{a} by a scalar s is also commutative and associative and is performed by multiplying all the tensor components by the scalar. For example,

$$s\mathbf{a} = sa_i = (sa_1, sa_2, sa_3)$$
 (1.5)

Division between a tensor \mathbf{a} and a scalar is only relevant when the scalar is the second argument of the operation, *i.e.*

$$\mathbf{a}/s = a_i/s = (a_1/s, a_2/s, a_3/s) \tag{1.6}$$

Following these operations are a set of more complex products between tensors of rank 1 and above, described in the following Sections.

1.3.1 The inner product

The inner product operates on any two tensors of rank r_1 and r_2 such that the rank of the result $r = r_1 + r_2 - 2$. Inner product operations with tensors up to rank 3 are described below:

• The inner product of two vectors \mathbf{a} and \mathbf{b} is commutative and produces a scalar $\mathbf{s} = \mathbf{a} \cdot \mathbf{b}$ where

$$s = a_i b_i = a_1 b_1 + a_2 b_2 + a_3 b_3 \tag{1.7}$$

ullet The inner product of a tensor T and vector a produces a vector $b = T \cdot a$, represented below as a column array for convenience

$$b_i = T_{ij}a_j = \begin{pmatrix} T_{11}a_1 + T_{12}a_2 + T_{13}a_3 \\ T_{21}a_1 + T_{22}a_2 + T_{23}a_3 \\ T_{31}a_1 + T_{32}a_2 + T_{33}a_3 \end{pmatrix}$$

$$(1.8)$$

It is non-commutative if **T** is non-symmetric such that $\mathbf{b} = \mathbf{a} \cdot \mathbf{T} = \mathbf{T}^{\mathrm{T}} \cdot \mathbf{a}$ is

$$b_i = a_j T_{ji} = \begin{pmatrix} a_1 T_{11} + a_2 T_{21} + a_3 T_{31} \\ a_1 T_{12} + a_2 T_{22} + a_3 T_{32} \\ a_1 T_{13} + a_2 T_{23} + a_3 T_{33} \end{pmatrix}$$

$$(1.9)$$

• The inner product of two tensors T and S produces a tensor $P = T \cdot S$ whose components are evaluated as:

$$P_{ij} = T_{ik} S_{kj} \tag{1.10}$$

It is non-commutative such that $\mathbf{T} \cdot \mathbf{S} = (\mathbf{S}^{\mathrm{T}} \cdot \mathbf{T}^{\mathrm{T}})^{\mathrm{T}}$

1.3 Algebraic tensor operations P-19

• The inner product of a vector \mathbf{a} and third rank tensor \mathbf{P} produces a second rank tensor $\mathbf{T} = \mathbf{a} \cdot \mathbf{P}$ whose components are

$$T_{ij} = a_k P_{kij} \tag{1.11}$$

Again this is non-commutative so that $T = P \cdot a$ is

$$T_{ij} = P_{ijk}a_k \tag{1.12}$$

• The inner product of a second rank tensor T and third rank tensor P produces a third rank tensor $Q = T \cdot P$ whose components are

$$Q_{ijk} = T_{il}P_{ljk} \tag{1.13}$$

Again this is non-commutative so that $\mathbf{Q} = \mathbf{P} \cdot \mathbf{T}$ is

$$Q_{ijk} = P_{ijl}T_{lk} \tag{1.14}$$

1.3.2 The double inner product of two tensors

The double inner product of two second-rank tensors T and S produces a scalar s = T : S which can be evaluated as the sum of the 9 products of the tensor components

$$s = T_{ij}S_{ij} = T_{11}S_{11} + T_{12}S_{12} + T_{13}S_{13} + T_{21}S_{21} + T_{22}S_{22} + T_{23}S_{23} + T_{31}S_{31} + T_{32}S_{32} + T_{33}S_{33}$$

$$(1.15)$$

The double inner product between a second rank tensor T and third rank tensor P produces a vector a = T : P with components

$$a_i = T_{ik}P_{iki} \tag{1.16}$$

This is non-commutative so that $\mathbf{a} = \mathbf{P} \cdot \mathbf{T}$ is

$$a_i = P_{ijk}T_{jk} \tag{1.17}$$

1.3.3 The triple inner product of two third rank tensors

The triple inner product of two third rank tensors **P** and **Q** produces a scalar $s = \mathbf{P} \cdot \mathbf{Q}$ which can be evaluated as the sum of the 27 products of the tensor components

$$s = P_{ijk}Q_{ijk} \tag{1.18}$$

1.3.4 The outer product

The outer product operates between vectors and tensors as follows:

• The outer product of two vectors \mathbf{a} and \mathbf{b} is non-commutative and produces a tensor $\mathbf{T} = \mathbf{a}\mathbf{b} = (\mathbf{b}\mathbf{a})^{\mathrm{T}}$ whose components are evaluated as:

$$T_{ij} = a_i b_j = \begin{pmatrix} a_1 b_1 & a_1 b_2 & a_1 b_3 \\ a_2 b_1 & a_2 b_2 & a_2 b_3 \\ a_3 b_1 & a_3 b_2 & a_3 b_3 \end{pmatrix}$$
(1.19)

OpenVFOAM-1.0

P-20 Tensor mathematics

• An outer product of a vector \mathbf{a} and second rank tensor \mathbf{T} produces a third rank tensor $\mathbf{P} = \mathbf{a}\mathbf{T}$ whose components are

$$P_{ijk} = a_i T_{jk} \tag{1.20}$$

This is non-commutative so that P = T a produces

$$P_{ijk} = T_{ij}a_k \tag{1.21}$$

1.3.5 The cross product of two vectors

The cross product operation is exclusive to vectors only. For two vectors \mathbf{a} with \mathbf{b} , it produces a vector $\mathbf{c} = \mathbf{a} \times \mathbf{b}$ whose components are

$$c_i = e_{ijk}a_jb_k = (a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1)$$

$$(1.22)$$

where the *permutation symbol* is defined by

$$e_{ijk} = \begin{cases} 0 & \text{when any two indices are equal} \\ +1 & \text{when } i,j,k \text{ are an even permutation of } 1,2,3 \\ -1 & \text{when } i,j,k \text{ are an odd permutation of } 1,2,3 \end{cases}$$
 (1.23)

in which the even permutations are 123, 231 and 312 and the odd permutations are 132, 213 and 321.

1.3.6 Other general tensor operations

Some less common tensor operations and terminology used by OpenFOAM are described below.

Square of a tensor is defined as the outer product of the tensor with itself, e.g. for a vector \mathbf{a} , the square $\mathbf{a}^2 = \mathbf{a}\mathbf{a}$.

*n*th power of a tensor is evaluated by *n* outer products of the tensor, *e.g.* for a vector \mathbf{a} , the 3rd power $\mathbf{a}^3 = \mathbf{a}\mathbf{a}\mathbf{a}$.

Magnitude squared of a tensor is the rth inner product of the tensor of rank r with itself, to produce a scalar. For example, for a second rank tensor \mathbf{T} , $|\mathbf{T}|^2 = \mathbf{T} \cdot \mathbf{T}$.

Magnitude is the square root of the magnitude squared, e.g. for a tensor \mathbf{T} , $|\mathbf{T}| = \sqrt{\mathbf{T} \cdot \mathbf{T}}$ Vectors of unit magnitude are referred to as unit vectors.

Component maximum is the component of the tensor with greatest value, inclusive of sign, i.e. not the largest magnitude.

Component minimum is the component of the tensor with smallest value.

Component average is the mean of all components of a tensor.

Scale As the name suggests, the scale function is a tool for scaling the components of one tensor by the components of another tensor of the same rank. It is evaluated as the product of corresponding components of 2 tensors, e.g., scaling vector **a** by vector **b** would produce vector **c** whose components are

$$c_i = \text{scale}(\mathbf{a}, \mathbf{b}) = (a_1 b_1, a_2 b_2, a_3 b_3)$$
 (1.24)

1.3 Algebraic tensor operations P-21

1.3.7 Geometric transformation and the identity tensor

A second rank tensor ${\bf T}$ is strictly defined as a linear vector function, i.e. it is a function which associates an argument vector ${\bf a}$ to another vector ${\bf b}$ by the inner product ${\bf b} = {\bf T} \cdot {\bf a}$. The components of ${\bf T}$ can be chosen to perform a specific geometric transformation of a tensor from the x, y, z coordinate system to a new coordinate system x^*, y^*, z^* ; ${\bf T}$ is then referred to as the transformation tensor. While a scalar remains unchanged under a transformation, the vector ${\bf a}$ is transformed to ${\bf a}^*$ by

$$\mathbf{a}^* = \mathbf{T} \cdot \mathbf{a} \tag{1.25}$$

A second rank tensor S is transformed to S^* according to

$$\mathbf{S}^* = \mathbf{T} \cdot \mathbf{S} \cdot \mathbf{T}^{\mathrm{T}} \tag{1.26}$$

The $identity\ tensor\ {f I}$ is defined by the requirement that it transforms another tensor onto itself. For all vectors ${f a}$

$$\mathbf{a} = \mathbf{I} \cdot \mathbf{a} \tag{1.27}$$

and therefore

$$\mathbf{I} = \delta_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{1.28}$$

where δ_{ij} is known as the Kronecker delta symbol.

1.3.8 Useful tensor identities

Several identities are listed below which can be verified by under the assumption that all the relevant derivatives exist and are continuous. The identities are expressed for scalar s and vector \mathbf{a} .

$$\nabla \cdot (\nabla \times \mathbf{a}) \equiv 0$$

$$\nabla \times (\nabla s) \equiv \mathbf{0}$$

$$\nabla \cdot (s\mathbf{a}) \equiv s \nabla \cdot \mathbf{a} + \mathbf{a} \cdot \nabla s$$

$$\nabla \times (s\mathbf{a}) \equiv s \nabla \times \mathbf{a} + \nabla s \times \mathbf{a}$$

$$\nabla (\mathbf{a} \cdot \mathbf{b}) \equiv \mathbf{a} \times (\nabla \times \mathbf{b}) + \mathbf{b} \times (\nabla \times \mathbf{a}) + (\mathbf{a} \cdot \nabla) \mathbf{b} + (\mathbf{b} \cdot \nabla) \mathbf{a}$$

$$\nabla \cdot (\mathbf{a} \times \mathbf{b}) \equiv \mathbf{b} \cdot (\nabla \times \mathbf{a}) - \mathbf{a} \cdot (\nabla \times \mathbf{b})$$

$$\nabla \times (\mathbf{a} \times \mathbf{b}) \equiv \mathbf{a}(\nabla \cdot \mathbf{b}) - \mathbf{b}(\nabla \cdot \mathbf{a}) + (\mathbf{b} \cdot \nabla) \mathbf{a} - (\mathbf{a} \cdot \nabla) \mathbf{b}$$

$$\nabla \times (\nabla \times \mathbf{a}) \equiv \nabla(\nabla \cdot \mathbf{a}) - \nabla^2 \mathbf{a}$$

$$(\nabla \times \mathbf{a}) \times \mathbf{a} \equiv \mathbf{a} \cdot (\nabla \mathbf{a}) - \nabla(\mathbf{a} \cdot \mathbf{a})$$

It is sometimes useful to know the $e-\delta$ identity to help to manipulate equations in index notation:

$$e_{ijk}e_{irs} = \delta_{jr}\delta_{ks} - \delta_{js}\delta_{kr} \tag{1.30}$$

Open VFOAM-1.0

P-22 Tensor mathematics

1.3.9 Operations exclusive to tensors of rank 2

There are several operations that manipulate the components of tensors of rank 2 that are listed below:

Transpose of a tensor $T = T_{ij}$ is $T^{T} = T_{ii}$ as described in Equation 1.2.

Symmetric and skew (antisymmetric) tensors As discussed in section 1.2, a tensor is said to be symmetric if its components are symmetric about the diagonal, i.e. $\mathbf{T} = \mathbf{T}^{\mathrm{T}}$. A skew or antisymmetric tensor has $\mathbf{T} = -\mathbf{T}^{\mathrm{T}}$ which intuitively implies that $T_{11} = T_{22} = T_{33} = 0$. Every second order tensor can be decomposed into symmetric and skew parts by

$$\mathbf{T} = \underbrace{\frac{1}{2}(\mathbf{T} + \mathbf{T}^{\mathrm{T}})}_{summetric} + \underbrace{\frac{1}{2}(\mathbf{T} - \mathbf{T}^{\mathrm{T}})}_{skew} = \operatorname{symm} \mathbf{T} + \operatorname{skew} \mathbf{T}$$
(1.31)

Trace The trace of a tensor T is a scalar, evaluated by summing the diagonal components

$$\operatorname{tr} \mathbf{T} = T_{11} + T_{22} + T_{33} \tag{1.32}$$

 ${\bf Diagonal}$ returns a vector whose components are the diagonal components of the second rank tensor ${\bf T}$

$$\operatorname{diag} \mathbf{T} = (T_{11}, T_{22}, T_{33}) \tag{1.33}$$

Deviatoric and hydrostatic tensors Every second rank tensor \mathbf{T} can be decomposed into a deviatoric component, for which $\operatorname{tr} \mathbf{T} = 0$ and a hydrostatic component of the form $\mathbf{T} = s\mathbf{I}$ where s is a scalar. Every second rank tensor can be decomposed into deviatoric and hydrostatic parts as follows:

$$\mathbf{T} = \underbrace{\mathbf{T} - \frac{1}{3} (\operatorname{tr} \mathbf{T}) \mathbf{I}}_{deviatoric} + \underbrace{\frac{1}{3} (\operatorname{tr} \mathbf{T}) \mathbf{I}}_{hudrostatic} = \operatorname{dev} \mathbf{T} + \operatorname{hyd} \mathbf{T}$$
(1.34)

Determinant The determinant of a second rank tensor is evaluated by

$$\det \mathbf{T} = \begin{vmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{vmatrix} = T_{11}(T_{22}T_{33} - T_{23}T_{32}) - T_{12}(T_{21}T_{33} - T_{23}T_{31}) + T_{13}(T_{21}T_{32} - T_{22}T_{31})$$

$$= \frac{1}{6}e_{ijk}e_{pqr}T_{ip}T_{jq}T_{kr}$$
(1.35)

Cofactors The *minors* of a tensor are evaluated for each component by deleting the row and column in which the component is situated and evaluating the resulting entries as a 2×2 determinant. For example, the minor of T_{12} is

$$\begin{vmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{vmatrix} = \begin{vmatrix} T_{21} & T_{23} \\ T_{31} & T_{33} \end{vmatrix} = T_{21}T_{33} - T_{23}T_{31}$$

$$(1.36)$$

1.4 OpenFOAM tensor classes P-23

The cofactors are *signed minors* where each minor is component is given a sign based on the rule

+ve if
$$i + j$$
 is even
-ve if $i + j$ is odd (1.37)

The cofactors of T can be evaluated as

$$\operatorname{cof} \mathbf{T} = \frac{1}{2} e_{jkr} e_{ist} T_{sk} T_{tr} \tag{1.38}$$

Inverse The inverse of a tensor can be evaluated as

$$\operatorname{inv} \mathbf{T} = \frac{\operatorname{cof} \mathbf{T}^{\mathrm{T}}}{\det \mathbf{T}} \tag{1.39}$$

Hodge dual of a tensor is a vector whose components are

$$*\mathbf{T} = (T_{23}, -T_{13}, T_{12}) \tag{1.40}$$

1.3.10 Operations exclusive to scalars

OpenFOAM supports most of the well known functions that operate on scalars, e.g. square root, exponential, logarithm, sine, cosine etc..., a list of which can be found in Table 1.2. There are 3 additional functions defined within OpenFOAM that are described below:

Sign of a scalar s is

$$sgn(s) = \begin{cases} 1 & \text{if } s \ge 0, \\ -1 & \text{if } s < 0. \end{cases}$$
 (1.41)

Positive of a scalar s is

$$pos(s) = \begin{cases} 1 & \text{if } s \ge 0, \\ 0 & \text{if } s < 0. \end{cases}$$
 (1.42)

Limit of a scalar s by the scalar n

$$limit(s,n) = \begin{cases} s & \text{if } s < n, \\ 0 & \text{if } s \ge n. \end{cases}$$
(1.43)

1.4 OpenFOAM tensor classes

OpenFOAM contains a C++ class library primitive that contains the classes for the tensor mathematics described so far. The basic tensor classes that are available as standard in OpenFOAM are listed in Table 1.1. The Table also lists the functions that allow the user to access individual components of a tensor, known as access functions.

We can declare the tensor

$$\mathbf{T} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \tag{1.44}$$

in OpenFOAM by the line:

Open VFOAM-1.0

P-24 Tensor mathematics

Rank	Common name	Basic class	Access functions
0	Scalar	scalar	
1	Vector	vector	x(), y(), z()
2	Tensor	tensor	xx(), xy(), xz()

Table 1.1: Basic tensor classes in OpenFOAM

tensor T(1, 2, 3, 4, 5, 6, 7, 8, 9);

We can then access the component T_{13} , or T_{xz} using the ${\tt xz}$ () access function. For instance the code

outputs to the screen:

Txz = 3

1.4.1 Algebraic tensor operations in OpenFOAM

The algebraic operations described in Section 1.3 are all available to the OpenFOAM tensor classes using syntax which closely mimics the notation used in written mathematics. Some functions are represented solely by descriptive functions, e.g. symm(), but others can also be executed using symbolic operators, e.g. *. All functions are listed in Table 1.2.

Operation	Comment	Mathematical	Description
		Description	in OpenFOAM
Addition		$\mathbf{a} + \mathbf{b}$	a + b
Subtraction		a - b	a - b
Scalar multiplication		$s\mathbf{a}$	s * a
Scalar division		\mathbf{a}/s	a/s
Outer product	$\operatorname{rank} \mathbf{a}, \mathbf{b} > = 1$	ab	a * b
Inner product	$\operatorname{rank} \mathbf{a}, \mathbf{b} > = 1$	a•b	a & b
Double inner product	$\operatorname{rank} \mathbf{a}, \mathbf{b} >= 2$	a:b	a && b
Cross product	$\operatorname{rank} \mathbf{a}, \mathbf{b} = 1$	$\mathbf{a} \times \mathbf{b}$	a ^ b
Square		\mathbf{a}^2	sqr(a)
Magnitude squared		$ \mathbf{a} ^2$	magSqr(a)
Magnitude		a	mag(a)
Power	n = 0, 1,, 4	\mathbf{a}^n	pow(a,n)
Component average	i = 1,, N	$\overline{a_i}$	cmptAv(a)
Component maximum	i = 1,, N	$\max(a_i)$	max(a)
Component minimum	i = 1,, N	$\min(a_i)$	min(a)
Scale		$scale(\mathbf{a}, \mathbf{b})$	scale(a,b)
Geometric transformation	transforms \mathbf{a} u	sing tensor T	transform(T,a)

Operations exclusive to tensors of rank 2

Continued on next page

1.4 OpenFOAM tensor classes P-25

Comment	Mathematical	Description
	Description	in OpenFOAM
	\mathbf{T}^{T}	T.T()
	$\operatorname{diag} \mathbf{T}$	diag(T)
	$\operatorname{tr} \mathbf{T}$	tr(T)
	$\operatorname{dev} \mathbf{T}$	dev(T)
	$\operatorname{symm} \mathbf{T}$	symm(T)
	skew T	skew(T)
	$\det \mathbf{T}$	det(T)
	$\operatorname{cof}\mathbf{T}$	cof(T)
	$\operatorname{inv} \mathbf{T}$	inv(T)
	$*\mathbf{T}$	*T
	Comment	$\begin{array}{c} {\rm Description} \\ {\bf T}^{\rm T} \\ {\rm diag} {\bf T} \\ {\rm tr} {\bf T} \\ {\rm dev} {\bf T} \\ {\rm symm} {\bf T} \\ {\rm skew} {\bf T} \\ {\rm det} {\bf T} \\ {\rm cof} {\bf T} \\ {\rm inv} {\bf T} \end{array}$

Operations exclusive to scalars

Sign (boolean)		sgn(s)	sign(s)
Positive (boolean)		s >= 0	pos(s)
Negative (boolean)		s <= 0	neg(s)
Limit	n scalar	limit(s, n)	limit(s,n)
Square root		\sqrt{s}	sqrt(s)
Exponential		$\exp s$	exp(s)
Natural logarithm		$\ln s$	log(s)
Base 10 logarithm		$\log_{10} s$	log10(s)
Sine		$\sin s$	sin(s)
Cosine		$\cos s$	cos(s)
Tangent		$\tan s$	tan(s)
Arc sine		asin s	asin(s)
Arc cosine		$a\cos s$	acos(s)
Arc tangent		a tan s	atan(s)
Hyperbolic sine		$\sinh s$	sinh(s)
Hyperbolic cosine		$\cosh s$	cosh(s)
Hyperbolic tangent		$\tanh s$	tanh(s)
Hyperbolic arc sine		a s inh s	asinh(s)
Hyperbolic arc cosine		$a\cosh s$	acosh(s)
Hyperbolic arc tangent		$\operatorname{atanh} s$	atanh(s)
Error function		$\operatorname{erf} s$	erf(s)
Complement error function		$\operatorname{erfc} s$	erfc(s)
Logarithm gamma function		$\ln \Gamma s$	lgamma(s)
Type 1 Bessel function of order 0		$J_0 s$	j0(s)
Type 1 Bessel function of order 1		$J_1 s$	j1(s)
Type 2 Bessel function of order 0		$Y_0 s$	y0(s)
Type 2 Bessel function of order 1		$Y_1 s$	y1(s)
1 C 1:4 1	1 (1 .	1	·

a, b are tensors of arbitrary rank unless otherwise stated

Table 1.2: Algebraic tensor operations in OpenFOAM

Open VFOAM-1.0

P-26 Tensor mathematics

1.5 Dimensional units

In continuum mechanics, properties are represented in some chosen units, e.g. mass in kilograms (kg), volume in cubic metres (m³), pressure in Pascals (kg m s⁻²). Algebraic operations must be performed on these properties using consistent units of measurement; in particular, addition, subtraction and equality are only physically meaningful for properties of the same dimensional units. As a safeguard against implementing a meaningless operation, OpenFOAM encourages the user to attach dimensional units to any tensor and will then perform dimension checking of any tensor operation.

Units are defined using the dimensionSet class, e.g.

dimensionSet pressureDims(1, -1, -2, 0, 0, 0, 0);

No.	Property	Unit	Symbol
1	Mass	kilogram	k
2	Length	metre	\mathbf{m}
3	Time	second	S
4	Temperature	Kelvin	K
5	Quantity	moles	mol
6	Current	ampere	A
7	Luminous intensity	candela	cd

Table 1.3: S.I. base units of measurement

where each of the values corresponds to the power of each of the S.I. base units of measurement listed in Table 1.3. The line of code declares pressureDims to be the dimensionSet for pressure kg m s $^{-2}$ since the first entry in the pressureDims array, 1, corresponds to k^1 , the second entry, -1, corresponds to m^{-1} etc.. A tensor with units is defined using the dimensioned<Type> template class, the <Type> being scalar, vector, tensor, etc.. The dimensioned<Type> stores a variable name of class word,the value <Type> and a dimensionSet

```
dimensionedTensor sigma
  (
     "sigma",
     dimensionSet(1, -1, -2, 0, 0, 0, 0),
     tensor(1e6,0,0,0,1e6,0,0,0,1e6),
  );
```

creates a tensor with correct dimensions of pressure, or stress

$$\mathbf{\sigma} = \begin{pmatrix} 10^6 & 0 & 0\\ 0 & 10^6 & 0\\ 0 & 0 & 10^6 \end{pmatrix} \tag{1.45}$$

s is a scalar, N is the number of tensor components

Chapter 2

Discretisation procedures

So far we have dealt with algebra of tensors at a point. The PDEs we wish to solve involve derivatives of tensors with respect to time and space. We therefore need to extend our description to a tensor field, i.e. a tensor that varies across time and spatial domains. In this Chapter we will first present a mathematical description of all the differential operators we may encounter. We will then show how a tensor field is constructed in OpenFOAM and how the derivatives of these fields are discretised into a set of algebraic equations.

2.1 Differential operators

Before defining the spatial derivatives we first introduce the nabla vector operator ∇ , represented in index notation as ∂_i :

$$\nabla \equiv \partial_i \equiv \frac{\partial}{\partial x_i} \equiv \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}\right) \tag{2.1}$$

The nabla operator is a useful notation that obeys the following rules:

- it operates on the tensors to its right and the conventional rules of a derivative of a product, e.g. ∂_iab = (∂_ia) b + a (∂_ib);
- otherwise the nabla operator behaves like any other vector in an algebraic operation.

2.1.1 Gradient

If a scalar field s is defined and continuously differentiable then the gradient of s, ∇s is a vector field

$$\nabla s = \partial_i s = \left(\frac{\partial s}{\partial x_1}, \frac{\partial s}{\partial x_2}, \frac{\partial s}{\partial x_3}\right) \tag{2.2}$$

The gradient can operate on any tensor field to produce a tensor field that is one rank higher. For example, the gradient of a vector field \mathbf{a} is a second rank tensor field

$$\nabla \mathbf{a} = \partial_i a_j = \begin{pmatrix} \partial a_1 / \partial x_1 & \partial a_2 / \partial x_1 & \partial a_3 / \partial x_1 \\ \partial a_1 / \partial x_2 & \partial a_2 / \partial x_2 & \partial a_3 / \partial x_2 \\ \partial a_1 / \partial x_3 & \partial a_2 / \partial x_3 & \partial a_3 / \partial x_3 \end{pmatrix}$$

$$(2.3)$$

P-28 Discretisation procedures

2.1.2 Divergence

If a vector field \mathbf{a} is defined and continuously differentiable then the divergence of \mathbf{a} is a scalar field

$$\nabla \cdot \mathbf{a} = \partial_i a_i = \frac{\partial a_1}{\partial x_1} + \frac{\partial a_2}{\partial x_2} + \frac{\partial a_3}{\partial x_3} \tag{2.4}$$

The divergence can operate on any tensor field of rank 1 and above to produce a tensor that is one rank lower. For example the divergence of a second rank tensor field \mathbf{T} is a vector field (expanding the vector as a column array for convenience)

$$\nabla \cdot \mathbf{T} = \partial_i T_{ij} = \begin{pmatrix} \partial T_{11}/\partial x_1 + \partial T_{12}/\partial x_1 + \partial T_{13}/\partial x_1 \\ \partial T_{21}/\partial x_2 + \partial T_{22}/\partial x_2 + \partial T_{23}/\partial x_2 \\ \partial T_{31}/\partial x_3 + \partial T_{32}/\partial x_3 + \partial T_{33}/\partial x_3 \end{pmatrix}$$
(2.5)

2.1.3 Curl

If a vector field \mathbf{a} is defined and continuously differentiable then the curl of \mathbf{a} , $\nabla \times \mathbf{a}$ is a vector field

$$\nabla \times \mathbf{a} = e_{ijk} \partial_j a_k = \left(\frac{\partial a_3}{\partial x_2} - \frac{\partial a_2}{\partial x_3}, \frac{\partial a_1}{\partial x_3} - \frac{\partial a_3}{\partial x_1}, \frac{\partial a_2}{\partial x_1} - \frac{\partial a_1}{\partial x_2} \right)$$
(2.6)

The curl is related to the gradient by

$$\nabla \times \mathbf{a} = 2 \,(* \,\mathrm{skew} \,\nabla \mathbf{a}) \tag{2.7}$$

2.1.4 Laplacian

The Laplacian is an operation that can be defined mathematically by a combination of the divergence and gradient operators by $\nabla^2 \equiv \nabla \cdot \nabla$. However, the Laplacian should be considered as a single operation that transforms a tensor field into another tensor field of the same rank, rather than a combination of two operations, one which raises the rank by 1 and one which reduces the rank by 1.

In fact, the Laplacian is best defined as a *scalar operator*, just as we defined nabla as a vector operator, by

$$\nabla^2 \equiv \partial^2 \equiv \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2} \tag{2.8}$$

For example, the Laplacian of a scalar field s is the scalar field

$$\nabla^2 s = \partial^2 s = \frac{\partial^2 s}{\partial x_1^2} + \frac{\partial^2 s}{\partial x_2^2} + \frac{\partial^2 s}{\partial x_3^2} \tag{2.9}$$

2.1.5 Temporal derivative

There is more than one definition of temporal, or time, derivative of a tensor. To describe the temporal derivatives we must first recall that the tensor relates to a property of a volume of material that may be moving. If we track an infinitesimally small volume of material, or

2.2 Overview of discretisation P-29

particle, as it moves and observe the change in the tensorial property ϕ in time, we have the *total*, or *material* time derivative denoted by

$$\frac{D\phi}{Dt} = \lim_{\Delta t \to 0} \frac{\Delta\phi}{\Delta t} \tag{2.10}$$

However in continuum mechanics, particularly fluid mechanics, we often observe the change of a ϕ in time at a fixed point in space as different particles move across that point. This change at a point in space is termed the *spatial* time derivative which is denoted by $\partial/\partial t$ and is related to the material derivative by:

$$\frac{D\phi}{Dt} = \frac{\partial\phi}{\partial t} + \mathbf{U} \cdot \nabla\phi \tag{2.11}$$

where **U** is the velocity field of property ϕ . The second term on the right is known as the convective rate of change of ϕ .

2.2 Overview of discretisation

The term discretisation means approximation of a problem into discrete quantities. The FV method and others, such as the finite element and finite difference methods, all discretise the problem as follows:

Spatial discretisation Defining the solution domain by a set of points that fill and bound a region of space when connected;

Temporal discretisation (For transient problems) dividing the time domain into into a finite number of time intervals, or steps;

Equation discretisation Generating a system of algebraic equations in terms of discrete quantities defined at specific locations in the domain, from the PDEs that characterise the problem.

2.2.1 OpenFOAM lists and fields

OpenFOAM frequently needs to store sets of data and perform functions, such as mathematical operations, on the data. OpenFOAM therefore provides an array template class List<Type>, making it possible to create a list of any object of class Type that inherits the functions of the Type. For example a List of vector is List<vector>.

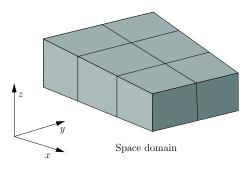
Lists of the tensor classes are defined as standard in OpenFOAM by the template class Field<Type>. For better code legibility, all instances of Field<Type>, e.g. Field<vector>, are renamed using typedef declarations as scalarField, vectorField, tensorField, symmTensorField, tensorThirdField and symmTensorThirdField. Algebraic operations can be performed between Fields subject to obvious restrictions such as the fields having the same number of elements. OpenFOAM also supports operations between a field and single tensor, e.g. all values of a Field U can be multiplied by the scalar 2 with the operation U = 2.0 * U.

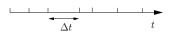
Open∇FOAM-1.0

P-30 Discretisation procedures

2.3 Discretisation of the solution domain

Discretisation of the solution domain is shown in Figure 2.1. The space domain is discretised into computational mesh on which the PDEs are subsequently discretised. Discretisation of time, if required, is simple: it is broken into a set of time steps Δt that may change during a numerical simulation, perhaps depending on some condition calculated during the simulation.





Time domain

Figure 2.1: Discretisation of the solution domain

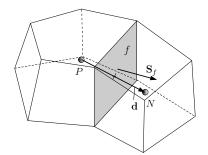


Figure 2.2: Parameters in finite volume discretisation

On a more detailed level, discretisation of space requires the subdivision of the domain into a number of cells, or control volumes. The cells are contiguous, *i.e.* they do not overlap one another and completely fill the domain. A typical cell is shown in Figure 2.2. Dependent variables and other properties are principally stored at the cell centroid P although they

2.3 Discretisation of the solution domain P-31 P-32 Discretisation procedures

may be stored on faces or vertices. The cell is bounded by a set of flat faces, given the generic label f. In OpenFOAM there is no limitation on the number of faces bounding each cell, nor any restriction on the alignment of each face. This kind of mesh is often referred to as "arbitrarily unstructured" to differentiate it from meshes in which the cell faces have a prescribed alignment, typically with the coordinate axes. Codes with arbitrarily unstructured meshes offer greater freedom in mesh generation and manipulation in particular when the geometry of the domain is complex or changes over time.

Whilst most properties are defined at the cell centroids, some are defined at cell faces. There are two types of cell face.

Internal faces Those faces that connect two cells (and it can never be more than two).
For each internal face, OpenFOAM designates one adjoining cell to be the face owner and the other to be the neighbour:

Boundary faces Those belonging to one cell since they coincide with the boundary of the domain. These faces simply have an owner cell.

2.3.1 Defining a mesh in OpenFOAM

There are different levels of mesh description in OpenFOAM, beginning with the most basic mesh class, named polyMesh since it is based on polyhedra. A polyMesh is constructed using the minimum information required to define the mesh geometry described below and presented in Figure 2.3:

Points A list of cell vertex point coordinate vectors, *i.e.* a vectorField, that is renamed pointField using a typedef declaration;

Faces A list of cell faces List<face>, or faceList, where the face class is defined by a list of vertex numbers, corresponding to the pointField:

Cells a list of cells List<cell>, or cellList, where the cell class is defined by a list of face numbers, corresponding to the faceList described previously.

Boundary a polyBoundaryMesh decomposed into a list of patches, polyPatchList representing different regions of the boundary. The boundary is subdivided in this manner to allow different boundary conditions to be specified on different patches during a solution. All the faces of any polyPatch are stored as a single block of the faceList, so that its faces can be easily accessed using the slice class which stores references to the first and last face of the block. Each polyPatch is then constructed from

- a slice;
- a word to assign it a name.

FV discretisation uses specific data that is derived from the mesh geometry stored in polyMesh. OpenFOAM therefore extends the polyMesh class to fvMesh which stores the additional data needed for FV discretisation. fvMesh is constructed from polyMesh and stores the data in Table 2.1 which can be updated during runtime in cases where the mesh moves, is refined etc..

OpenVFOAM-1.0

Patch 1

Patch 3

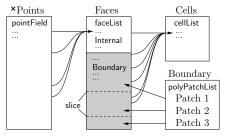


Figure 2.3: Schematic of the basic mesh description used in OpenFOAM

2.3.2 Defining a geometricField in OpenFOAM

So far we can define a field, *i.e.* a list of tensors, and a mesh. These can be combined to define a tensor field relating to discrete points in our domain, specified in OpenFOAM by the template class <code>geometricField<Type></code>. The Field values are separated into those defined within the internal region of the domain, *e.g.* at the cell centres, and those defined on the domain boundary, *e.g.* on the boundary faces. The <code>geometricField<Type></code> stores the following information:

Internal field This is simply a Field<Type>, described in Section 2.2.1;

BoundaryField This is a GeometricBoundaryField, in which a Field is defined for the faces of each patch and a Field is defined for the patches of the boundary. This is then a field of fields, stored within an object of the FieldField<Type> class. A reference to the fvBoundaryMesh is also stored [**].

Mesh A reference to an fvMesh, with some additional detail as to the whether the field is defined at cell centres, faces, etc..

Dimensions A dimensionSet, described in Section 1.5.

Old values Discretisation of time derivatives requires field data from previous time steps.

Class	Description	Symbol	Access function
volScalarField	Cell volumes	V	V()
surfaceVectorField	Face area vectors	\mathbf{S}_f	Sf()
surfaceScalarField	Face area magnitudes	$ \mathbf{S}_f $	magSf()
volVectorField	Cell centres	\mathbf{C}	C()
surfaceVectorField	Face centres	\mathbf{C}_f	Cf()
surfaceScalarField	Face motion fluxes **	ϕ_a	phi()

Table 2.1: fvMesh stored data.

The geometricField<Type> will store references to stored fields from the previous, or old, time step and its previous, or old-old, time step where necessary.

Previous iteration values The iterative solution procedures can use under-relaxation which requires access to data from the previous iteration. Again, if required, geometricField<Type> stores a reference to the data from the previous iteration.

As discussed in Section 2.3, we principally define a property at the cell centres but quite often it is stored at the cell faces and on occasion it is defined on cell vertices. The geometricField<Type> is renamed using typedef declarations to indicate where the field variable is defined as follows:

volField<Type> A field defined at cell centres;

surfaceField<Type> A field defined on cell faces;

pointField<Type> A field defined on cell vertices.

These typedef field classes of geometricField<Type>are illustrated in Figure 2.4. A geometricField<Type> inherits all the tensor algebra of Field<Type> and has all operations subjected to dimension checking using the dimensionSet. It can also be subjected to the FV discretisation procedures described in the following Section. The class structure used to build geometricField<Type> is shown in Figure 2.5¹.

2.4 Equation discretisation

Equation discretisation converts the PDEs into a set of algebraic equations that are commonly expressed in matrix form as:

$$[A][x] = [b] \tag{2.12}$$

where [A] is a square matrix, [x] is the column vector of dependent variable and [b] is the source vector. The description of [x] and [b] as 'vectors' comes from matrix terminology rather than being a precise description of what they truly are: a list of values defined at locations in the geometry, i.e. a geometricField<Type>, or more specifically a volField<Type> when using FV discretisation.

OpenVFOAM-1.0

P-34 Discretisation procedures

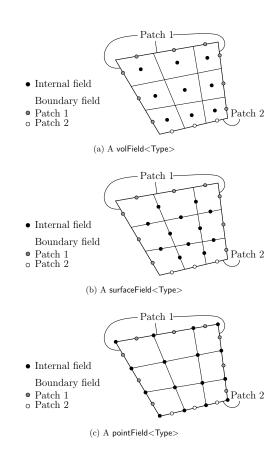


Figure 2.4: Types of $\mathsf{geometricField} < \mathsf{Type} > \mathsf{defined}$ on a mesh with 2 boundary patches (in 2 dimensions for simplicity)

¹The diagram is not an exact description of the class hierarchy, rather a representation of the general structure leading from some primitive classes to geometric<Type>Field.

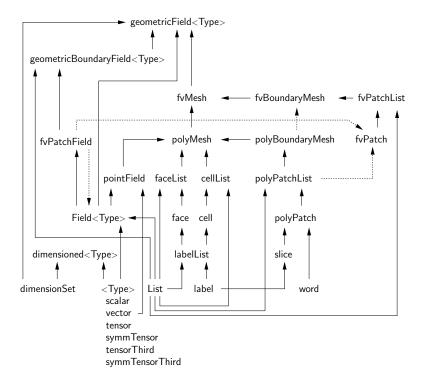


Figure 2.5: Basic class structure leading to geometricField<Type>

OpenVFOAM-1.0

P-36 Discretisation procedures

[A] is a list of coefficients of a set of algebraic equations, and cannot be described as a geometricField<Type>. It is therefore given a class of its own: fvMatrix. fvMatrix<Type> is created through discretisation of a geometric<Type>Field and therefore inherits the <Type>. It supports many of the standard algebraic matrix operations of addition +, subtraction - and multiplication *.

Each term in a PDE is represented individually in OpenFOAM code using the classes of static functions finiteVolumeMethod and finiteVolumeCalculus, abbreviated by a typedef to fvm and fvc respectively. fvm and fvc contain static functions, representing differential operators, e.g. ∇^2 , $\nabla \cdot$ and $\partial/\partial t$, that discretise geometricField<Type>s. The purpose of defining these functions within two classes, fvm and fvc, rather than one, is to distinguish:

- functions of fvm that calculate implicit derivatives of and return an fvMatrix<Type>
- some functions of fvc that calculate explicit derivatives and other explicit calculations, returning a geometricField<Type>.

Figure 2.6 shows a geometric Field<Type> defined on a mesh with 2 boundary patches and illustrates the explicit operations merely transform one field to another and drawn in 2D for simplicity.

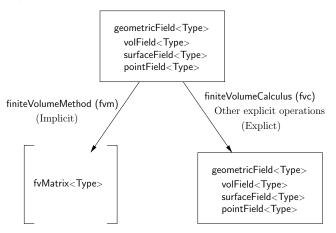


Figure 2.6: A geometricField<Type> and its operators

Table 2.2 lists the main functions that are available in fvm and fvc to discretise terms that may be found in a PDE. FV discretisation of each term is formulated by first integrating the term over a cell volume V. Most spatial derivative terms are then converted to integrals over the cell surface S bounding the volume using Gauss's theorem

$$\int_{V} \nabla \star \phi \ dV = \int_{S} d\mathbf{S} \star \phi \tag{2.13}$$

where **S** is the surface area vector, ϕ can represent any tensor field and the star notation \star is used to represent any tensor product, *i.e.* inner, outer and cross and the respective

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Term description	Implicit /	Text	fvm::/fvc:: functions
	Explicit	expression	*
Laplacian	Imp/Exp	$\nabla^2 \phi$	laplacian(phi)
		$\nabla \cdot \Gamma \nabla \phi$	laplacian(Gamma, phi)
Time derivative	$\mathrm{Imp}/\mathrm{Exp}$	$\frac{\partial \phi}{\partial t}$	ddt(phi)
		$\frac{\partial \rho \phi}{\partial t}$	ddt(rho,phi)
Second time derivative	Imp/Exp	$\frac{\partial}{\partial t} \left(\rho \frac{\partial \phi}{\partial t} \right)$	d2dt2(rho, phi)
Convection	Imp/Exp	$\nabla \cdot (\psi)$	$\mathtt{div}(\mathtt{psi},\mathtt{scheme})^*$
		$\nabla \cdot (\psi \phi)$	$\mathtt{div}(\mathtt{psi},\ \mathtt{phi},\ \mathtt{word})^*$
			div(psi, phi)
Divergence	Exp	$\nabla \cdot \chi$	div(chi)
Gradient	Exp	$\nabla \chi$	grad(chi)
		$\nabla \phi$	gGrad(phi)
			lsGrad(phi)
			<pre>snGrad(phi)</pre>
			${\tt snGradCorrection(phi)}$
Grad-grad squared	Exp	$ \nabla\nabla\phi ^2$	sqrGradGrad(phi)
Curl	Exp	$\nabla \times \phi$	curl(phi)
Source	Imp	$\rho\phi$	Sp(rho,phi)
10 0 0	Imp/Exp†	1: :: 1:	SuSp(rho,phi)

†fvm::SuSp source is discretised implicit or explicit depending on the sign of rho. †An explicit source can be introduced simply as a vol<Type>Field, e.g. rho*phi. Function arguments can be of the following classes:

phi: vol<Type>Field

Gamma: scalar volScalarField, surfaceScalarField, volTensorField, surfaceTensorField,

rho: scalar, volScalarField psi: surfaceScalarField.

chi: surface<Type>Field, vol<Type>Field.

Table 2.2: Discretisation of PDE terms in OpenFOAM

Open VFOAM-1.0

P-38 Discretisation procedures

derivatives: divergence $\nabla \cdot \phi$, gradient $\nabla \phi$ and $\nabla \times \phi$. Volume and surface integrals are then linearised using appropriate schemes which are described for each term in the following Sections. Some terms are always discretised using one scheme, a selection of schemes is offered in OpenFOAM for the discretisation of other terms. The choice of scheme is either made by a direct specification within the code or it can be read from an input file at job run-time and stored within an fvSchemes class object.

2.4.1 The Laplacian term

The Laplacian term is integrated over a control volume and linearised as follows:

$$\int_{V} \nabla \cdot (\Gamma \nabla \phi) \ dV = \int_{S} d\mathbf{S} \cdot (\Gamma \nabla \phi) = \sum_{f} \Gamma_{f} \mathbf{S}_{f} \cdot (\nabla \phi)_{f}$$
(2.14)

The face gradient discretisation is implicit when the length vector \mathbf{d} between the centre of the cell of interest P and the centre of a neighbouring cell N is orthogonal to the face plane, *i.e.* parallel to \mathbf{S}_f :

$$\mathbf{S}_{f} \cdot (\nabla \phi)_{f} = |S_{f}| \frac{\phi_{N} - \phi_{P}}{|\mathbf{d}|} \tag{2.15}$$

In the case of non-orthogonal meshes, an additional explicit term is introduced [?] which is evaluated by interpolating cell centre gradients, themselves calculated by central differencing cell centre values.

2.4.2 The convection term

The convection term is integrated over a control volume and linearised as follows:

$$\int_{V} \nabla \cdot (\rho \mathbf{U} \phi) \ dV = \int_{S} d\mathbf{S} \cdot (\rho \mathbf{U} \phi) = \sum_{f} \mathbf{S}_{f} \cdot (\rho \mathbf{U})_{f} \phi_{f} = \sum_{f} F \phi_{f}$$
(2.16)

The face field ϕ_f can be evaluated using a variety of schemes:

Central differencing (CD) is second-order accurate but unbounded

$$\phi_f = f_x \phi_P + (1 - f_x) \phi_N \tag{2.17}$$

where $f_x \equiv \overline{fN}/\overline{PN}$ where \overline{fN} is the distance between f and cell centre N and \overline{PN} is the distance between cell centres P and N.

Upwind differencing (UD) determines ϕ_f from the direction of flow and is bounded at the expense of accuracy

$$\phi_f = \begin{cases} \phi_P & \text{for } F \ge 0\\ \phi_N & \text{for } F < 0 \end{cases}$$
 (2.18)

Blended differencing (BD) schemes combine UD and CD in an attempt to preserve boundedness with reasonable accuracy,

$$\phi_f = (1 - \gamma) \left(\phi_f\right)_{UD} + \gamma \left(\phi_f\right)_{CD} \tag{2.19}$$

OpenFOAM has several implementations of the Gamma differencing scheme to select the blending coefficient γ [?] but it offers other well-known schemes such as van Leer, SUPERBEE, MINMOD etc.

2.4.3 First time derivative

The first time derivative $\partial/\partial t$ is integrated over a control volume as follows:

$$\frac{\partial}{\partial t} \int_{V} \rho \phi \ dV$$
 (2.20)

The term is discretised by simple differencing in time using:

new values $\phi^n \equiv \phi(t + \Delta t)$ at the time step we are solving for;

old values $\phi^o \equiv \phi(t)$ that were stored from the previous time step;

old-old values $\phi^{oo} \equiv \phi(t - \Delta t)$ stored from a time step previous to the last.

One of two discretisation schemes can be declared using the timeScheme keyword in the appropriate input file, described in detail in section 4.4 of the User Guide.

Euler implicit scheme, timeScheme EulerImplicit, that is first order accurate in time:

$$\frac{\partial}{\partial t} \int_{V} \rho \phi \ dV = \frac{(\rho_P \phi_P V)^n - (\rho_P \phi_P V)^o}{\Delta t}$$
(2.21)

Backward differencing scheme, timeScheme BackwardDifferencing, that is second order accurate in time by storing the old-old values and therefore with a larger overhead in data storage than EulerImplicit:

$$\frac{\partial}{\partial t} \int_{V} \rho \phi \ dV = \frac{3 \left(\rho_{P} \phi_{P} V\right)^{n} - 4 \left(\rho_{P} \phi_{P} V\right)^{o} + \left(\rho_{P} \phi_{P} V\right)^{oo}}{2\Delta t} \tag{2.22}$$

2.4.4 Second time derivative

The second time derivative is integrated over a control volume and linearised as follows:

$$\frac{\partial}{\partial t} \int_{V} \rho \frac{\partial \phi}{\partial t} dV = \frac{(\rho_{P} \phi_{P} V)^{n} - 2(\rho_{P} \phi_{P} V)^{o} + (\rho_{P} \phi_{P} V)^{oo}}{\Delta t^{2}}$$
(2.23)

It is first order accurate in time.

2.4.5 Divergence

The divergence term described in this Section is strictly an explicit term that is distinguished from the convection term of Section 2.4.2, *i.e.* in that it is not the divergence of the product of a velocity and dependent variable. The term is integrated over a control volume and linearised as follows:

$$\int_{V} \nabla \cdot \phi \ dV = \int_{S} d\mathbf{S} \cdot \phi = \sum_{f} \mathbf{S}_{f} \cdot \phi_{f}$$
(2.24)

The fvc::div function can take as its argument either a surface<Type>Field, in which case ϕ_f is specified directly, or a vol<Type>Field which is interpolated to the face by central differencing as described in Section 2.4.10:

Open FOAM-1.0

P-40 Discretisation procedures

2.4.6 Gradient

The gradient term is an explicit term that can be evaluated in a variety of ways. The scheme can be evaluated either by selecting the particular grad function relevant to the discretisation scheme, e.g. fvc::gGrad, fvc::lsGrad etc., or by using the fvc::grad function combined with the appropriate timeScheme keyword in an input file

Gauss integration is invoked using the fvc::grad function with timeScheme Gauss or directly using the fvc::gGrad function. The discretisation is performed using the standard method of applying Gauss's theorem to the volume integral:

$$\int_{V} \nabla \phi \ dV = \int_{S} d\mathbf{S} \ \phi = \sum_{f} \mathbf{S}_{f} \phi_{f} \tag{2.25}$$

As with the fvc::div function, the Gaussian integration fvc::grad function can take either a surfaceField<Type> or a volField<Type> as an argument.

Least squares method is based on the following idea:

- a value at point P can be extrapolated to neighbouring point N using the gradient at P:
- the extrapolated value at N can be compared to the actual value at N, the difference being the error;
- if we now minimise the sum of the square of weighted errors at all neighbours of P with the respect to the gradient, then the gradient should be a good approximation.

Least squares is invoked using the fvc::grad function with timeScheme leastSquares or directly using the fvc::lsGrad function. The discretisation is performed as by first calculating the tensor G at every point P by summing over neighbours N:

$$\mathbf{G} = \sum_{N} w_{N}^{2} \mathbf{dd} \tag{2.26}$$

where **d** is the vector from P to N and the weighting function $w_N = 1/|\mathbf{d}|$. The gradient is then evaluated as:

$$(\nabla \phi)_P = \sum_N w_N^2 \mathbf{G}^{-1} \cdot \mathbf{d} (\phi_N - \phi_P)$$
(2.27)

Surface normal gradient The gradient normal to a surface $\mathbf{n}_f \cdot (\nabla \phi)_f$ can be evaluated at cell faces using the scheme

$$(\nabla \phi)_f = \frac{\phi_N - \phi_P}{|\mathbf{d}|} \tag{2.28}$$

This gradient is called by the function fvc::snGrad and returns a surfaceField<Type>. The scheme is directly analogous to that evaluated for the Laplacian discretisation scheme in Section 2.4.1, and in the same manner, a correction can be introduced to improve the accuracy of this face gradient in the case of non-orthogonal meshes. This correction is called using the function fvc::snGradCorrection [Check**].

2.4.7 Grad-grad squared

The grad-grad squared term is evaluated by: taking the gradient of the field; taking the gradient of the resulting gradient field; and then calculating the magnitude squared of the result. The mathematical expression for grad-grad squared of ϕ is $|\nabla(\nabla\phi)|^2$.

2.4.8 Curl

The curl is evaluated from the gradient term described in Section 2.4.6. First, the gradient is discretised and then the curl is evaluated using the relationship from Equation 2.7, repeated here for convenience

$$\nabla \times \phi = 2 * (\text{skew } \nabla \phi)$$

2.4.9 Source terms

Source terms can be specified in 3 ways

Explicit Every explicit term is a volField<Type>. Hence, an explicit source term can be incorporated into an equation simply as a field of values. For example if we wished to solve Poisson's equation $\nabla^2 \phi = f$, we would define phi and f as volScalarField and then do

Implicit An implicit source term is integrated over a control volume and linearised by

$$\int_{V} \rho \phi \ dV = \rho_P V_P \phi_P \tag{2.29}$$

Implicit/Explicit The implicit source term changes the coefficient of the diagonal of the matrix. Depending on the sign of the coefficient and matrix terms, this will either increase or decrease diagonal dominance of the matrix. Decreasing the diagonal dominance could cause instability during iterative solution of the matrix equation. Therefore OpenFOAM provides a mixed source discretisation procedure that is implicit when the coefficients that are greater than zero, and explicit for the coefficients less than zero. In mathematical terms the matrix coefficient for node P is $V_P \max(\rho_P, 0)$ and the source term is $V_P \phi_P \min(\rho_P, 0)$.

2.4.10 Other explicit discretisation schemes

There are some other discretisation procedures that convert volField < Type > s into surface < Type > Fields and visa versa.

Surface integral fvc::surfaceIntegrate performs a summation of surface
-Type>Field face values bounding each cell and dividing by the cell volume, i.e.
 $(\sum_f \phi_f)/V_P$. It returns a volField
-Type>.

Surface sum fvc::surfaceSum performs a summation of surface<Type>Field face values bounding each cell, i.e. $\sum_{t} \phi_{f}$ returning a volField<Type>.

OpenVFOAM-1.0

P-42 Discretisation procedures

Average fvc::average produces an area weighted average of surface<Type>Field face values, i.e. $(\sum_f S_f \phi_f)/\sum_f S_f$, and returns a volField<Type>.

Reconstruct

Face interpolate The geometric<Type>Field function faceInterpolate() interpolates volField<Type> cell centre values to cell faces using central differencing, returning a surface<Type>Field.

2.5 Temporal discretisation

Although we have described the discretisation of temporal derivatives in Sections 2.4.3 and 2.4.4, we need to consider how to treat the spatial derivatives in a transient problem. If we denote all the spatial terms as $\mathcal{A}\phi$ where \mathcal{A} is any spatial operator, e.g. Laplacian, then we can express a transient PDE in integral form as

$$\int_{t}^{t+\Delta t} \left[\frac{\partial}{\partial t} \int_{V} \rho \phi \ dV + \int_{V} \mathcal{A} \phi \ dV \right] \ dt = 0$$
(2.30)

Using the Euler implicit method of Equation 2.21, the first term can be expressed as

$$\int_{t}^{t+\Delta t} \left[\frac{\partial}{\partial t} \int_{V} \rho \phi \ dV \right] \ dt = \int_{t}^{t+\Delta t} \frac{(\rho_{P} \phi_{P} V)^{n} - (\rho_{P} \phi_{P} V)^{o}}{\Delta t} \ dt$$

$$= \frac{(\rho_{P} \phi_{P} V)^{n} - (\rho_{P} \phi_{P} V)^{o}}{\Delta t} \Delta t$$
(2.31)

The second term can be expressed as

$$\int_{t}^{t+\Delta t} \left[\int_{V} \mathcal{A}\phi \ dV \right] \ dt = \int_{t}^{t+\Delta t} \mathcal{A}^{*}\phi \ dt \tag{2.32}$$

where \mathcal{A}^* represents the spatial discretisation of \mathcal{A} . The time integral can be discretised in three ways:

Euler implicit uses implicit discretisation of the spatial terms, thereby taking current values ϕ^n .

$$\int_{t}^{t+\Delta t} \mathcal{A}^{*}\phi \ dt = \mathcal{A}^{*}\phi^{n}\Delta t \tag{2.33}$$

It is first order accurate in time, guarantees boundedness and is unconditionally stable.

Explicit uses explicit discretisation of the spatial terms, thereby taking old values ϕ^o .

$$\int_{t}^{t+\Delta t} \mathcal{A}^* \phi \ dt = \mathcal{A}^* \phi^o \Delta t \tag{2.34}$$

It is first order accurate in time and is unstable if the Courant number Co is greater than 1. The Courant number is defined as

$$Co = \frac{\mathbf{U}_f \cdot \mathbf{d}}{|\mathbf{d}|^2 \Delta t} \tag{2.35}$$

where \mathbf{U}_f is a characteristic velocity, e.g. velocity of a wave front, velocity of flow.

2.6 Boundary Conditions P-43

Crank Nicholson uses the trapezoid rule to discretise the spatial terms, thereby taking a mean of current values ϕ^n and old values ϕ^o .

$$\int_{t}^{t+\Delta t} \mathcal{A}^{*} \phi \ dt = \mathcal{A}^{*} \left(\frac{\phi^{n} + \phi^{o}}{2} \right) \Delta t \tag{2.36}$$

It is second order accurate in time, is unconditionally stable but does not guarantee boundedness.

2.5.1 Treatment of temporal discretisation in OpenFOAM

At present the treatment of the temporal discretisation is controlled by the implementation of the spatial derivatives in the PDE we wish to solve. For example, let us say we wish to solve a transient diffusion equation

$$\frac{\partial \phi}{\partial t} = \kappa \nabla^2 \phi \tag{2.37}$$

An Euler implicit implementation of this would read

```
solve(fvm::ddt(phi) == kappa*fvm::laplacian(phi))
```

where we use the fvm class to discretise the Laplacian term implicitly. An explicit implementation would read

```
solve(fvm::ddt(phi) == kappa*fvc::laplacian(phi))
```

where we now use the fvc class to discretise the Laplacian term explicitly. The Crank Nicholson scheme can be implemented by the mean of implicit and explicit terms:

```
solve
  (
  fvm::ddt(phi)
  ==
  kappa*0.5*(fvm::laplacian(phi) + fvc::laplacian(phi))
  )
```

2.6 Boundary Conditions

Boundary conditions are required to complete the problem we wish to solve. We therefore need to specify boundary conditions on all our boundary faces. Boundary conditions can be divided into 2 types:

Dirichlet prescribes the value of the dependent variable on the boundary and is therefore termed 'fixed value' in this guide;

Neumann prescribes the gradient of the variable normal to the boundary and is therefore termed 'fixed gradient' in this guide.

Open∇FOAM-1.0

P-44 Discretisation procedures

When we perform discretisation of terms that include the sum over faces \sum_f , we need to consider what happens when one of the faces is a boundary face.

Fixed value We specify a fixed value at the boundary ϕ_b

- We can simply substitute φ_b in cases where the discretisation requires the value on a boundary face φ_f, e.g. in the convection term in Equation 2.16.
- In terms where the face gradient $(\nabla \phi)_f$ is required, e.g. Laplacian, it is calculated using the boundary face value and cell centre value,

$$\mathbf{S}_{f} \cdot (\nabla \phi)_{f} = |S_{f}| \frac{\phi_{b} - \phi_{P}}{|\mathbf{d}|} \tag{2.38}$$

Fixed gradient The fixed gradient boundary condition g_b is a specification on inner product of the gradient and unit normal to the boundary, or

$$g_b = \left(\frac{\mathbf{S}}{|\mathbf{S}|} \bullet \nabla \phi\right)_f \tag{2.39}$$

• When discretisation requires the value on a boundary face ϕ_f we must interpolate the cell centre value to the boundary by

$$\phi_f = \phi_P + \mathbf{d} \cdot (\nabla \phi)_f$$

$$= \phi_P + |\mathbf{d}| q_h$$
(2.40)

• ϕ_b can be directly substituted in cases where the discretisation requires the face gradient to be evaluated.

$$\mathbf{S}_f \bullet (\nabla \phi)_f = |S_f| \, g_b \tag{2.41}$$

2.6.1 Physical boundary conditions

The specification of boundary conditions is usually an engineer's interpretation of the true behaviour. Real boundary conditions are generally defined by some physical attributes rather than the numerical description as described of the previous Section. In incompressible fluid flow there are the following physical boundaries

Inlet The velocity field at the inlet is supplied and, for consistency, the boundary condition on pressure is zero gradient.

Outlet The pressure field at the outlet is supplied and a zero gradient boundary condition on velocity is specified.

No-slip impermeable wall The velocity of the fluid is equal to that of the wall itself, *i.e.* a fixed value condition can be specified. The pressure is specified zero gradient since the flux through the wall is zero.

In a problem whose solution domain and boundary conditions are symmetric about a plane, we only need to model half the domain to one side of the symmetry plane. The boundary condition on the plane must be specified according to

Symmetry plane The symmetry plane condition specifies the component of the gradient normal to the plane should be zero. [Check**]

Chapter 3

Examples of the use of OpenFOAM

In this section we shall describe several test cases supplied with the OpenFOAM distribution. The intention is to provide example cases, including those in the tutorials in chapter 2 of the User Guide, for every standard solver. The examples are designed to introduce certain tools and features of OpenFOAM, e.g. within pre-/post-processing, numerical schemes, algorithms. They also provide a means for validation of solvers although that is not their principal function.

Each example contains a description of the problem: the geometry, initial and boundary conditions, a brief description of the equations being solved, models used, and physical properties required. The solution domain is selected which may be a portion of the original geometry, e.g. if we introduce symmetry planes. The method of meshing, usually blockMesh, is specified; of course the user can simply view the mesh since every example is distributed with the polyMesh directory containing the data files that describe the mesh.

The examples coexist with the tutorials in the *tutorials* subdirectory of the OpenFOAM installation. They are organised into a set of subdirectories by solver, *e.g.* all the <code>icoFoam</code> cases are stored within a subdirectory <code>icoFoam</code>. Before running a particular example, the user is urged to copy it into their user account. We recommend that the user stores all OpenFOAM cases in a directory we recommend that the tutorials are copied into a directory <code>\$FOAM_RUN</code>. If this directory structure has not yet been created in the user's account, it can be created with

```
mkdir -p $FOAM_RUN
```

The tutorials can then be copied into this directory with

cp -r \$FOAM_TUTORIALS/* \$FOAM_RUN

3.1 Flow around a cylinder

In this example we shall investigate potential flow around a cylinder using potentialFoam. This example introduces the following OpenFOAM features:

- non-orthogonal meshes;
- generating an analytical solution to a problem in OpenFOAM.

P-46

Examples of the use of OpenFOAM

3.1.1 Problem specification

The problem is defined as follows:

Solution domain The domain is 2 dimensional and consists of a square domain with a cylinder collocated with the centre of the square as shown in Figure 3.1.

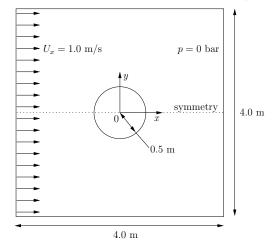


Figure 3.1: Geometry of flow round a cylinder

Governing equations

• Mass continuity for an incompressible fluid

$$7 \cdot \mathbf{U} = 0 \tag{3.1}$$

 Pressure equation for an incompressible, irrotational fluid assuming steady-state conditions

$$\nabla^2 p = 0 \tag{3.2}$$

$Boundary\ conditions$

- Inlet (left) with fixed velocity $\mathbf{U} = (1, 0, 0) \text{ m/s}$.
- Outlet (right) with a fixed pressure p = 0 Pa.
- No-slip wall (bottom);
- Symmetry plane (top).

Initial conditions U=0 m/s, p=0 Pa — required in OpenFOAM input files but not necessary for the solution since the problem is steady-state.

Solver name potential Foam: a potential flow code, *i.e.* assumes the flow is incompressible, steady, irrotational, inviscid and it ignores gravity.

Case name cylinder case located in the \$FOAM_TUTORIALS/potentialFoam directory.

3.1.2 Note on potentialFoam

potentialFoam is a useful solver to validate OpenFOAM since the assumptions of potential flow are such that an analytical solution exists for cases whose geometries are relatively simple. In this example of flow around a cylinder an analytical solution exists with which we can compare our numerical solution. potentialFoam can also be run more like a utility to provide a (reasonably) conservative initial U field for a problem. When running certain cases, this can useful for avoiding instabilities due to the initial field being unstable. In short, potentialFoam creates a conservative field from a non-conservative initial field supplied by the user.

3.1.3 Mesh generation

Mesh generation using blockMesh has been described in tutorials in the User Guide. In this case, the mesh consists of 10 blocks as shown in Figure 3.2. Remember that all meshes

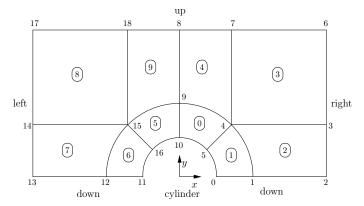


Figure 3.2: Blocks in cylinder geometry

are treated as 3 dimensional in OpenFOAM. If we wish to solve a 2 dimensional problem, we must describe a 3 dimensional mesh that is only one cell thick in the third direction that is not solved. In Figure 3.2 we show only the back plane of the geometry, along z=-0.5, in which the vertex numbers are numbered 0-18. The other 19 vertices in the front plane, z=+0.5, are numbered in the same order as the back plane, as shown in the mesh description file below:

OpenVFOAM-1.0

Examples of the use of OpenFOAM

```
13
14
                                  ""
15
            case
                                  ....
16
17
            instance
                                  dictionary
19
            class
                                  blockMeshDict:
            object
21
22
                                      23
24
25
      convertToMeters 1:
27
      vertices
            (0.5 \ 0 \ -0.5)
29
            (1 0 -0.5)
(2 0 -0.5)
(2 0.707107 -0.5)
(0.707107 0.707107 -0.5)
31
32
            (0.353553 0.353553 -0.5)
(2 2 -0.5)
            (0.707107 2 -0.5)
            (0 2 -0.5)
(0 1 -0.5)
            (0 0.5 -0.5)
(-0.5 0 -0.5)
(-1 0 -0.5)
(-2 0 -0.5)
            (-2 0 -0.5)

(-2 0.707107 -0.5)

(-0.707107 0.707107 -0.5)

(-0.353553 0.353553 -0.5)

(-2 2 -0.5)

(-0.707107 2 -0.5)
\frac{46}{47}
            (0.5 0 0.5)
(1 0 0.5)
49
            (1 0 0.5)
(2 0 0.5)
(2 0.707107 0.5)
(0.707107 0.707107 0.5)
50
51
52
53
            (0.707107 0.707107 0.5)
(0.353553 0.353553 0.5)
(2 2 0.5)
(0.707107 2 0.5)
(0 2 0.5)
(0 1 0.5)
(0 0.5 0.5)
54
55
56
57
58
59
60
61
            (-0.5 0 0.5)
(-1 0 0.5)
(-2 0 0.5)
            (-2 0.707107 0.5)
(-0.707107 0.707107 0.5)
63
             (-0.353553 0.353553 0.5)
64
65
            (-0.707107 2 0.5)
66
67
     );
68
69
70
      blocks
71
           hex (5 4 9 10 24 23 28 29) (10 10 1) simpleGrading (1 1 1)
72
           hex (0 1 4 5 19 20 23 24) (10 10 1) simpleGrading (1 1 1)
           hex (1 2 3 4 20 21 22 23) (20 10 1) simpleGrading (1 1 1)
73
           hex (4 3 6 7 23 22 25 26) (20 20 1) simpleGrading (1 1 1)
74
           hex (9 4 7 8 28 23 26 27) (10 20 1) simpleGrading (1 1 1)
75
           hex (15 16 10 9 34 35 29 28) (10 10 1) simpleGrading (1 1 1)
           hex (12 11 16 15 31 30 35 34) (10 10 1) simpleGrading (1 1 1)
           hex (13 12 15 14 32 31 34 33) (20 10 1) simpleGrading (1 1 1)
78
           hex (14 15 18 17 33 34 37 36) (20 20 1) simpleGrading (1 1 1)
79
           hex (15 9 8 18 34 28 27 37) (10 20 1) simpleGrading (1 1 1)
81
82
83
      edges
84
85
                 0 5 (0.469846 0.17101 -0.5)
           arc 5 10 (0.17101 0.469846 -0.5)
arc 1 4 (0.939693 0.34202 -0.5)
           arc 4 9 (0.34202 0.939693
88
           arc 19 24 (0.469846 0.17101 0.5)
arc 24 29 (0.17101 0.469846 0.5)
arc 20 23 (0.939693 0.34202 0.5)
```

Open FOAM-1.0

P-48

```
arc 23 28 (0.34202 0.939693 0.5)
          arc 11 16 (-0.469846 0.17101 -0.5)
arc 16 10 (-0.17101 0.469846 -0.5)
           arc 12 15 (-0.939693 0.34202 -0.5)
                       (-0.34202 0.939693 -0.5)
(-0.469846 0.17101 0.5)
           arc 15 9
arc 30 35
          arc 35 29 (-0.17101 0.469846 0.5)
arc 31 34 (-0.939693 0.34202 0.5)
           arc 34 28 (-0.34202 0.939693 0.5)
100
101
103
      natches
104
           symmetryPlane down
105
106
                (0 1 20 19)
(1 2 21 20)
107
                 (12 11 30 31)
(13 12 31 32)
109
110
111
           patch right
112
                 (2 3 22 21)
114
                 (3 6 25 22)
115
           symmetryPlane up
117
118
119
                (6 7 26 25)
(8 18 37 27)
121
                (18 17 36 37)
122
123
          patch left
124
125
                 (14 13 32 33)
126
127
128
           symmetryPlane cylinder
129
130
                 (10 5 24 29)
(5 0 19 24)
131
132
133
                 (16 10 29 35
134
     );
\frac{136}{137}
      mergePatchPairs
139
141
```

3.1.4 Boundary conditions and initial fields

Using FoamX or editing case files by hand, set the boundary conditions in accordance with the problem description in Figure 3.1, i.e. the left boundary should be an Inlet, the right boundary should be an Outlet and the down and cylinder boundaries should be symmetryPlane. The top boundary conditions is chosen so that we can make the most genuine comparison with our analytical solution which uses the assumption that the domain is infinite in the y direction. The result is that the normal gradient of \mathbf{U} is small along a plane coinciding with our boundary. We therefore impose the condition that the normal component is zero, i.e. specify the boundary as a symmetryPlane, thereby ensuring that the comparison with the analytical is reasonable.

3.1.5 Running the case

No fluid properties need be specified in this problem since the flow is assumed to be incompressible and inviscid. In the *system* subdirectory, the *controlDict* specifies the control

OpenVFOAM-1.0

Examples of the use of OpenFOAM

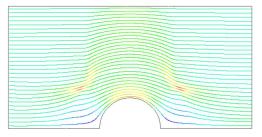
parameters for the run. Note that since we assume steady flow, we only run for 1 time step:

```
OpenFOAM: The Open Source CFD Toolbox
              F ield
              O peration
                             Version: 1.0
      11
              A nd
                             Web:
                                      http://www.openfoam.org
              M anipulation
   FoamFile
10
11
       version
      format
                     ascii;
       case
       instance
       class
                     dictionary
21
   applicationClass potentialFoam;
   startFrom
                 startTime;
   startTime
                 0;
                 endTime:
   stopAt
                 1;
   endTime
                 1;
   writeControl
                 timeStep;
   writeInterval
                 1;
   cvcleWrite
   writeFormat
                 ascii:
   writePrecision 6:
   writeCompression uncompressed;
                 general;
   timeFormat
   timePrecision
                 6;
   runTimeModifiable ves:
```

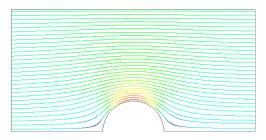
potentialFoam executes an iterative loop around the pressure equation which it solves in order that explicit terms relating to non-orthogonal correction in the Laplacian term may be updated in successive iterations. The number of iterations around the pressure equation is controlled by the nNonOrthogonalCorrectors keyword in controlDict. In the first instance we can set nNonOrthogonalCorrectors to 0 so that no loops are performed, i.e. the pressure equation is solved once, and there is no non-orthogonal correction. The solution is shown in Figure 3.3(a) (at t=1, when the steady-state simulation is complete). We expect the solution to show smooth streamlines passing across the domain as in the analytical solution in Figure 3.3(c), yet there is clearly some error in the regions where there is high non-orthogonality in the mesh, e.g. at the join of blocks 0, 1 and 3. The case can be run a second time with some non-orthogonal correction by setting nNonOrthogonalCorrectors to 3. The solution shows smooth streamlines with no significant error due to non-orthogonality as shown in Figure 3.3(b).

OpenVFOAM-1.0

P-50



(a) With no non-orthogonal correction



(b) With non-orthogonal correction

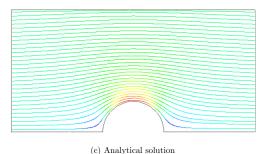


Figure 3.3: Streamlines of potential flow

Open VFOAM-1.0

P-52 Examples of the use of OpenFOAM

3.1.6 Generating the analytical solution

Source code is included in the $FOAM_TUTORIALS/potentialFoam/analyticalCylinder$ directory to generate the analytical solution for the potential flow case. The velocity at any point at a distance d and angle θ from the cylinder centre is described analytically as

$$U_x = U_{\infty} \left[1 - \left(\frac{r}{d} \right)^2 \cos 2\theta \right]$$

$$U_y = U_{\infty} \left(\frac{r}{d} \right)^2 \sin 2\theta \quad (3.3)$$

where r is the cylinder radius and U_{∞} is the inlet flow velocity. Here, θ describes the angle subtended from the x-axis.

Let us examine some details of the source code in the <code>analyticalCylinder</code> directory. In <code>createFields.H</code>, the velocity field is read in using the <code>IOobject::NO_WRITE</code> option to ensure that the field data can never be overwritten during execution of <code>analyticalCylinder</code>. The inlet velocity and cylinder radius are taken from data read from the mesh and a field <code>UA</code> is set up to store the analytical solution:

```
Info<< "Reading field U\n" << endl;
yolVectorField U</pre>
            IOobject
                  runTime.timeName(),
                  runTime,
IOobject::MUST_READ,
IOobject::NO_WRITE
11
12
13
14
15
      Info<< "Reading inlet velocity uInfX\n" << endl;</pre>
16
17
      dimensionedScalar uInfX
18
19
            "uInfx",
dimensionSet(0, 1, -1, 0, 0),
U.boundaryField()[3][0].x()
22
23
24
25
26
27
28
      Info << "U at inlet = " << uInfX.value() << " m/s" << endl;</pre>
      dimensionedScalar radius
             "radius"
           dimensionSet(0, 1, 0, 0, 0),
mag(U.mesh().boundary()[4].Cf()[0])
31
32
33
34
35
      Info << "Cylinder radius = " << radius.value() << " m" << endl;</pre>
      volVectorField UA
36
             IOobject
37
38
39
40
                  runTime.timeName(),
                  runTime,
IOobject::NO_READ,
IOobject::AUTO_WRITE
41
42
43
44
     );
```

Thea main code analyticalCylinder. C performs the following tasks:

• increments the time step by runTime++;

• generates the analytical solution for field UA using tensor arithmetic;

```
• writes the solution to file by runTime.writeObjects().
                    F ield
                                         OpenFOAM: The Open Source CFD Toolbox
                    O peration
                                        Copyright (C) 1991-2004 OpenCFD Ltd.
                    A nd
           \\/
                    M anipulation
     License
          This file is part of OpenFOAM.
          {\tt OpenFOAM} is free software; you can redistribute it and/or modify it under the terms of the GNU General Public License as published by the
          Free Software Foundation; either version 2 of the License, or (at your
13
          option) any later version.
14
          OpenFOAM is distributed in the hope that it will be useful, but WITHOUT ANY WARRANTY; without even the implied warranty of MERCHANTABILITY or FINNESS FOR A PARTICULAR PURPOSE. See the GNU General Public License
          for more details.
          You should have received a copy of the GNU General Public License along with OpenFOAM; if not, write to the Free Software Foundation, Inc., 59 Temple Place, Suite 330, Boston, MA 02111-1307 USA
     Application
25
          analyticalCylinder
     Description
28
          Generates an analytical solution for potential flow around a cylinder.
          Can be compared with the solution from the potentialFlow/cylinder example.
33
     #include "fvCFD.H"
     int main(int argc, char *argv[])
39
          include "setRootCase.H"
          include "createDatabase.H' include "createMesh.H"
45
          include "createFields.H"
        Info << "\nEvaluating analytical solution" << endl;
          volVectorField centres = UA.mesh().C();
52
          volScalarField magCentres = mag(centres);
          volScalarField theta = acos((centres & vector(1,0,0))/magCentres);
          volVectorField cs2theta =
            cos(2*theta)*vector(1,0,0)
+ sin(2*theta)*vector(0,1,0);
59
          UA = uInfX*(dimensionedVector(vector(1,0,0))
61
            pow((radius/magCentres),2)*cs2theta);
          runTime.writeObjects();
          Info<< "end" << endl;</pre>
          return(0);
68
```

The utility must be compiled with wmake as normal. It can then be run by typing

analyticalCylinder \$FOAM_RUN/potentialFoam cylinder

Open∇FOAM-1.0

P-54

The analytical solution is plotted as streamlines as shown in Figure 3.3(c). Note that differences in the analytical and numerical solutions at the top plane are due to the fact that the analytical solution assumes an infinite boundary and the numerical solution specifies a zeroGradient boundary condition at that boundary.

3.1.7 Exercise

Investigate the accuracy of the numerical solution by implementing some measure of comparison between the numerical and analytical in analyticalCylinder.

3.2 Steady turbulent flow over a backward-facing step

In this example we shall investigate steady turbulent flow over a backward-facing step. The problem description is taken from one used by Pitz and Daily in an experimental investigation [**] against which the computed solution can be compared. This example introduces the following OpenFOAM features for the first time:

- generation of a mesh using blockMesh using full mesh grading capability;
- steady turbulent flow.

3.2.1 Problem specification

The problem is defined as follows:

Solution domain The domain is 2 dimensional, consisting of a short inlet, a backward-facing step and converging nozzle at outlet as shown in Figure 3.4.



Figure 3.4: Geometry of backward-facing step

Governing equations

• Mass continuity for incompressible flow

$$\nabla \cdot \mathbf{U} = 0 \tag{3.4}$$

Steady flow momentum equation

$$\nabla \cdot (\mathbf{U}\mathbf{U}) + \nabla \cdot \mathbf{R} = -\nabla p \tag{3.5}$$

where p is kinematic pressure and (in slightly over-simplistic terms) $\mathbf{R} = \nu_{eff} \nabla \mathbf{U}$ is the viscous stress term with an effective kinematic viscosity ν_{eff} , calculated from selected transport and turbulence models.

Initial conditions U = 0 m/s, p = 0 Pa — required in OpenFOAM input files but not necessary for the solution since the problem is steady-state.

Boundary conditions

- Inlet (left) with fixed velocity $\mathbf{U} = (10, 0, 0) \text{ m/s}$;
- Outlet (right) with fixed pressure p = 0 Pa;
- · No-slip walls on other boundaries.

Transport properties

• Kinematic viscosity of air $\nu = \mu/\rho = 18.1 \times 10^{-6}/1.293 = 14.0 \ \mu m^2/s$

Turbulence model

- Standard $k \epsilon$;
- Coefficients: $C_{\mu} = 0.09$; $C_1 = 1.44$; $C_2 = 1.92$; $\alpha_k = 1$; $\alpha_{\epsilon} = 0.76923$.

Solver name simpleFoam: an implementation for steady incompressible flow.

Case name pitzDaily, located in the \$FOAM_TUTORIALS/simpleFoam directory.

The problem is solved using simpleFoam, so-called as it is an implementation for steady flow using the SIMPLE algorithm [**]. The solver has full access to all the turbulence models in the incompressibleTurbulenceModels library and the non-Newtonian models incompressibleTransportModels library of the standard OpenFOAM release.

3.2.2 Mesh generation

We expect that the flow in this problem is reasonably complex and an optimum solution will require grading of the mesh. In general, the regions of highest shear are particularly critical, requiring a finer mesh than in the regions of low shear. We can anticipate where high shear will occur by considering what the solution might be in advance of any calculation. At the inlet we have strong uniform flow in the x direction and, as it passes over the step, it generates shear on the fluid below, generating a vortex in the bottom half of the domain. The regions of high shear will therefore be close to the centreline of the domain and close to the walls.

The domain is subdivided into 12 blocks as shown in Figure 3.5.

The mesh is 3 dimensional, as always in OpenFOAM, so in Figure 3.5 we are viewing the back plane along z=-0.5. The full set of vertices and blocks are given in the mesh description file below:

OpenVFOAM-1.0

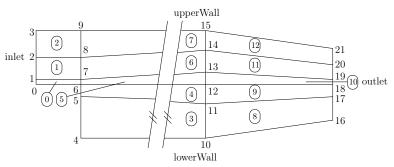
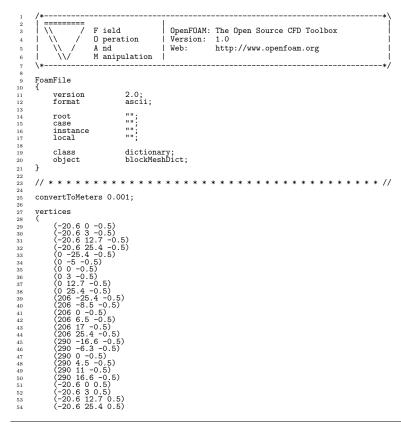


Figure 3.5: Blocks in backward-facing step



OpenVFOAM-1.0

P-56

P-58

```
(0 -25.4 0.5)
(0 -5 0.5)
(0 0 0.5)
(0 3 0.5)
(0 12.7 0.5)
(0 25.4 0.5)
(206 -25.4 0.5)
(206 -8.5 0.5)
(206 0.5)
              (206 0 0.5)
(206 6.5 0.5)
             (206 17 0.5)
(206 25.4 0.5)
(290 -16.6 0.5)
(290 -6.3 0.5)
(290 0 0.5)
             (290 4.5 0.5)
(290 11 0.5)
              (290 16.6 0.5)
 72
73
       blocks
             hex (0 6 7 1 22 28 29 23) (18 7 1) simpleGrading (0.5 1.8 1)
             hex (1 7 8 2 23 29 30 24) (18 10 1) simpleGrading (0.5 4 1)
             hex (2 8 9 3 24 30 31 25) (18 13 1) simpleGrading (0.5 0.25 1)
 79
             hex (4 10 11 5 26 32 33 27) (180 18 1) simpleGrading (4 1 1)
 80
             hex (5 11 12 6 27 33 34 28) (180 9 1) edgeGrading (4 4 4 4 0.5 1 1 0.5 1 1 1 1) hex (6 12 13 7 28 34 35 29) (180 7 1) edgeGrading (4 4 4 4 1.8 1 1 1.8 1 1 1 1)
 81
             hex (7 13 14 8 29 35 36 30) (180 10 1) edgeGrading (4 4 4 4 4 1 1 4 1 1 1 1)
             hex (8 14 15 9 30 36 37 31) (180 13 1) simpleGrading (4 0.25 1)
 84
             hex (10 16 17 11 32 38 39 33) (25 18 1) simpleGrading (2.5 1 1) hex (11 17 18 12 33 39 40 34) (25 9 1) simpleGrading (2.5 1 1)
 85
             hex (12 18 19 13 34 40 41 35) (25 7 1) simpleGrading (2.5 1 1)
             hex (13 19 20 14 35 41 42 36) (25 10 1) simpleGrading (2.5 1 1)
             hex (14 20 21 15 36 42 43 37) (25 13 1) simpleGrading (2.5 0.25 1)
 90
 93
       patches
 97
             patch inlet
                    (0 22 23 1)
(1 23 24 2)
101
                    (2 24 25 3)
102
             patch outlet
104
105
                         17 39 38
106
                   (17 18 40 39)
(18 19 41 40)
(19 20 42 41)
(20 21 43 42)
108
109
111
112
             wall upperWall
113
                    (3 25 31 9)
(9 31 37 15)
114
115
                    (15 37 43 21)
116
117
             wall lowerWall
118
119
                    (0 6 28 22)
(6 5 27 28)
(5 4 26 27)
120
121
122
                    (4 10 32 26)
(10 16 38 32)
124
125
126
             empty frontAndBack
127
                   (22 28 29 23)
(23 29 30 24)
(24 30 31 25)
(26 32 33 27)
120
130
131
```

OpenVFOAM-1.0

```
(27 33 34 28)
(28 34 35 29)
(29 35 36 30)
(30 36 37 31)
(32 38 39 33)
(33 39 40 34)
132
133
134
135
136
137
138
                        (33 39 40 34)
(34 40 41 35)
(35 41 42 36)
(36 42 43 37)
(0 1 7 6)
(1 2 8 7)
(2 3 9 8)
140
141
143
                        (4 5 11 10)
(5 6 12 11)
(6 7 13 12)
144
145
146
147
                        (8 9 15 14)
(10 11 17 16
(11 12 18 17
\frac{149}{150}
                        (12 13 19 18)
(13 14 20 19)
151
152
153
                        (14 15 21 20)
154
155
        );
156
        mergePatchPairs
157
158
159
160
```

A major feature of this problem is the use of the full mesh grading capability of blockMesh that is described in section 6.3.1 of the User Guide. The user can see that blocks 4,5 and 6 use the full list of 12 expansion ratios. The expansion ratios correspond to each edge of the block, the first 4 to the edges aligned in the local x_1 direction, the second 4 to the edges in the local x_2 direction and the last 4 to the edges in the local x_3 direction. In blocks 4, 5, and 6, the ratios are equal for all edges in the local x_1 and x_3 directions but not for the edges in the x_2 direction that corresponds in all blocks to the global y. If we consider the ratios used in relation to the block definition in section 6.3.1 of the User Guide, we realize that different gradings have been prescribed along the left and right edges in blocks 4,5 and 6 in Figure 3.5. The purpose of this differential grading is to generate a fine mesh close to the most critical region of flow, the corner of the step, and allow it to expand into the rest of the domain.

The mesh can be generated using blockMesh from the command line or from within FoamX and viewed as described in previous examples.

3.2.3 Boundary conditions and initial fields

The case files can be viewed, or edited from within FoamX or by hand. In this case, we are required to set the initial and boundary fields for velocity \mathbf{U} , pressure p, turbulent kinetic energy k and dissipation rate ε . The boundary conditions can be specified by setting the physical patch types in FoamX: the upper and lower walls are set to Wall, the left patch to Inlet and the right patch to Outlet. These physical boundary conditions require us to specify a fixedValue at the inlet on \mathbf{U} , k and ε . \mathbf{U} is given in the problem specification, but the values of k and ϵ must be chosen by the user in a similar manner to that described in section 2.1.8.1 of the User Guide. We assume that the inlet turbulence is isotropic and estimate the fluctuations to be 5% of \mathbf{U} at the inlet. We have

$$U'_x = U'_y = U'_z = \frac{5}{100} 10 = 0.5 \text{ m/s}$$
 (3.6)

$$k = \frac{3}{2}(0.5)^2 = 0.375 \text{ m}^2/\text{s}^2$$
 (3.7)

If we estimate the turbulent length scale l to be 10% of the width of the inlet then

$$\varepsilon = \frac{C_{\mu}^{0.75}k^{1.5}}{l} = \frac{0.09^{0.75}0.375^{1.5}}{0.1 \times 25.4 \times 10^{-3}} = 14.855 \,\mathrm{m}^2/\mathrm{s}^3$$
(3.8)

At the outlet we need only specify the pressure p = 0Pa.

3.2.4 Case control

The choices of fvSchemes are as follows: the timeScheme should be SteadyState; the gradScheme and laplacianScheme should be set as default to Gauss; and, the divScheme should be set to UD to ensure boundedness.

Special attention should be paid to the settings of *tvTolerances*. Although the top level simpleFoam code contains only equations for p and \mathbf{U} , the turbulent model solves equations for k, ε and \mathbf{R} , and tolerance settings are required for all 5 equations. A solverTolerance of 10^{-5} and solverRelativeTolerance of 0.1 are acceptable for all variables with the exception of p when 10^{-6} and 0.01 are recommended. Under-relaxation of the solution is required since the problem is steady. A relaxationFactor of 0.7 is acceptable for \mathbf{U} , k, ε and \mathbf{R} but 0.3 is required for p to avoid numerical instability.

Finally, in controlDict, the time step deltaT should be set to 1 since in steady state cases such as this is effectively an iteration counter. With benefit of hindsight we know that the solution requires 1000 iterations reach reasonable convergence, hence endTime is set to 1000. Ensure that the writeFrequency is sufficiently high, e.g. 50, that you will not fill the hard disk with data during run time.

3.2.5 Running the case and post-processing

Run the case and post-process the results. After a few iterations, e.g. 50, a vortex develops beneath the corner of the step that is the height of the step but narrow in the x-direction as shown by the vector plot of velocities is shown Figure 3.6(a). Over several iterations the vortex stretches in the x-direction from the step to the outlet until at 1000 iterations the system reaches a steady-state in which the vortex is fully developed as shown in Figure 3.6(b-c).

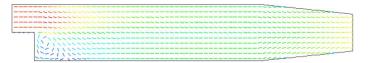
3.3 Supersonic flow over a forward-facing step

In this example we shall investigate supersonic flow over a forward-facing step. The problem description involves a flow of Mach 3 at an inlet to a rectangular geometry with a step near the inlet region that generates shock waves.

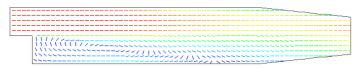
This example introduces the following OpenFOAM features for the first time:

supersonic flow;

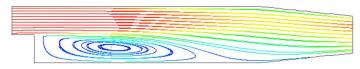
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(a) Velocity vectors after 50 iterations



(b) Velocity vectors at 1000 iterations



(c) Streamlines at 1000 iterations

Figure 3.6: Development of a vortex in the backward-facing step.

3.3.1 Problem specification

The problem is defined as follows:

Solution domain The domain is 2 dimensional and consists of a short inlet section followed by a forward-facing step of 20% the height of the section as shown in Figure 3.7

$Governing\ equations$

P-60

Mass continuity

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0 \tag{3.9}$$

• Ideal gas

$$p = \rho RT \tag{3.10}$$

• Momentum equation for Newtonian fluid

$$\frac{\partial \rho \mathbf{U}}{\partial t} + \nabla \cdot (\rho \mathbf{U} \mathbf{U}) - \nabla \cdot \mu \nabla \mathbf{U} = -\nabla p \tag{3.11}$$

P-61

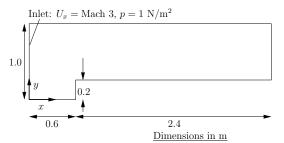


Figure 3.7: Geometry of the forward step geometry

• Energy equation for fluid (ignoring some viscous terms), $e = C_v T$, with Fourier's Law $\mathbf{q} = -k \nabla T$

$$\frac{\partial \rho e}{\partial t} + \nabla \cdot (\rho \mathbf{U} e) - \nabla \cdot \left(\frac{k}{C_v}\right) \nabla e = p \nabla \cdot \mathbf{U}$$
(3.12)

Initial conditions U = 0 m/s, p = 1 Pa, T = 1 K.

Boundary conditions

- Inlet (left) with fixed Value for velocity $U=3~\mathrm{m/s}=\mathrm{Mach}\ 3,$ pressure $p=1~\mathrm{Pa}$ and temperature $T=1~\mathrm{K};$
- Outlet (right) with zeroGradient on U, p and T;
- No-slip adiabatic wall (bottom);
- Symmetry plane (top).

Transport properties

• Dynamic viscosity of air $\mu = 18.1 \mu Pa s$

Thermodynamic properties

- Specific heat at constant volume $C_v = 1.78571 \text{ J/kg K}$
- Gas constant R = 0.714286 J/kg K
- Conductivity $k = 32.3 \, \mu \text{W/m K}$

Case name forwardStep case located in the \$FOAM_TUTORIALS/sonicFoam directory.

 $Solver\ name\ sonic$ Foam: an implementation for compressible trans-sonic/supersonic laminar gas flow.

The case is designed such that the speed of sound of the gas $c = \sqrt{\gamma RT} = 1$ m/s, the consequence being that the velocities are directly equivalent to the Mach number, e.g. the inlet velocity of 3 m/s is equivalent to Mach 3. This speed of sound calculation can be verified using the relationship for a perfect gas, $C_n - Cv = R$, i.e. the ratio of specific heats

$$\gamma = C_p/C_v = \frac{R}{C_v} + 1 \tag{3.13}$$

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3.3.2 Mesh generation

P-62

The mesh used in this case is relatively simple, specified with uniform rectangular cells of length $0.06~\mathrm{m}$ in the x direction and $0.05~\mathrm{m}$ in the y direction. The geometry can simply be divided into 3 blocks, one below the top of the step, and two above the step, one either side of the step front. The full set of vertices and blocks are given in the mesh description file below:

```
OpenFOAM: The Open Source CFD Toolbox
                      0 peration
                                             Version:
                      A nd
                                                         http://www.openfoam.org
            11/
                      M anipulation
     FoamFile
10
11
          version
                               2.0:
                               ascii;
12
13
          format
          root
          instance
                               m m .
          class
                               dictionary;
blockMeshDict;
21
22
23
24
25
     convertToMeters 1;
26
27
     vertices
28
29
          (0 0 -0.05)
(0.6 0 -0.05)
(0 0.2 -0.05)
(0.6 0.2 -0.05)
37
38
39
40
41
42
43
           (0.6 0.2 0.05)
           (0 1 0.05)
(0.6 1 0.05)
44
45
46
47
48
     blocks
          hex (0 1 3 2 8 9 11 10) (25 10 1) simpleGrading (1 1 1)
          hex (2 3 6 5 10 11 14 13) (25 40 1) simpleGrading (1 1 1)
          hex (3 4 7 6 11 12 15 14) (100 40 1) simpleGrading (1 1 1)
51
52
53
54
     edges
55
56
57
58
     );
     patches
59
          patch inlet
                (0 8 10 2)
(2 10 13 5)
           patch outlet
                (4 7 15 12)
```

OpenVFOAM-1.0

3.3.3 Running the case

The case approaches a steady-state at some time after 5 s. The results for pressure at 10 s are shown in Figure 3.8. The results clearly show discontinuities in pressure, *i.e.* shock waves, emanating from ahead of the base of the step.

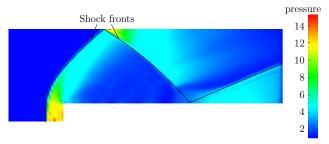


Figure 3.8: Shock fronts in the forward step problem

3.3.4 Exercise

The user can examine the effect on the solution of increasing the inlet velocity.

3.4 Decompression of a tank internally pressurised with water

In this example we shall investigate a problem of rapid opening of a pipe valve close to a pressurised liquid-filled tank. The prominent feature of the result in such cases is the propagation of pressure waves which must therefore be modelled as a compressible liquid.

This tutorial introduces the following OpenFOAM features for the first time:

• Mesh refinement

OpenVFOAM-1.0

• Pressure waves in liquids

P-64

3.4.1 Problem specification

Solution domain The domain is 2 dimensional and consists of a tank with a small outflow pipe as shown in Figure 3.9

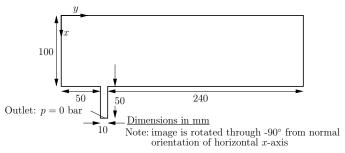


Figure 3.9: Geometry of a tank with outflow pipe

Governing equations This problem requires a model for compressibility ψ in the fluid in order to be able to resolve waves propagating at a finite speed. A barotropic relationship is used to relate density ρ and pressure p are related to ψ .

Mass continuity

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0 \tag{3.14}$$

• The barotropic relationship

$$\frac{\partial \rho}{\partial p} = \frac{\rho}{K} = \psi \tag{3.15}$$

where K is the bulk modulus

• Equation 3.15 is linearised as

$$\rho \approx \rho_0 + \psi \left(p - p_0 \right) \tag{3.16}$$

where ρ_0 and p_0 are the reference density and pressure respectively such that $\rho(p_0) = \rho_0$.

• Momentum equation for Newtonian fluid

$$\frac{\partial \rho \mathbf{U}}{\partial t} + \nabla \cdot (\rho \mathbf{U} \mathbf{U}) - \nabla \cdot \mu \nabla \mathbf{U} = -\nabla p \tag{3.17}$$

 $Boundary\ conditions$ Using FoamX the following physical boundary conditions can be set:

- outerWall is specified the wall condition;
- axis is specified as the symmetryPlane;
- nozzle is specified as a pressureOutlet where p = 0 bar.
- front and back boundaries are specified as empty.

Initial conditions U = 0 m/s, p = 100 bar.

Transport properties

• Dynamic viscosity of water $\mu = 1.0 \text{ mPa s}$

Thermodynamic properties

- Density of water $\rho = 1000 \text{ kg/m}^3$
- Reference pressure $p_0 = 1$ bar
- Compressibility of water $\psi = 4.54 \times 10^{-7} \text{ s}^2/\text{m}^2$

Solver name sonicLiquidFoam: a compressible sonic laminar liquid flow code.

 $\label{located} \textit{Case name decompressionTank} \ \text{case located in the $FOAM_TUTORIALS/sonicLiquidFoam directory.}$

3.4.2 Mesh Generation

The full geometry is modelled in this case; the set of vertices and blocks are given in the mesh description file below:

```
F ield
                                            OpenFOAM: The Open Source CFD Toolbox
                      O peration
                                            Version: 1.0
                                                          http://www.openfoam.org
                      A nd
                      M anipulation
      FoamFile
           version
                                2.0;
           instance
                               dictionary;
blockMeshDict;
           class
20
21
23
24
25
     convertToMeters 0.1;
     vertices
           (0 0 -0.1)
(1 0 -0.1)
(0 0.5 -0.1)
(1 0.5 -0.1)
(1.5 0.5 -0.1)
           (0 0.6 -0.1)
(1 0.6 -0.1)
```

Open VFOAM-1.0

```
(0 3 -0.1)

(1 3 -0.1)

(0 0 0.1)

(1 0 0.1)

(0 0.5 0.1)

(1 0.5 0.1)

(1.5 0.5 0.1)

(1 0.6 0.1)

(1 0.5 0.6 0.1)

(1 0.6 0.1)
37
38
39
40
41
42
43
            (0 3 0.1)
(1 3 0.1)
48
49
50
51
52
      );
      blocks
            hex (0 1 3 2 10 11 13 12) (30 20 1) simpleGrading (1 1 1)
53
            hex (2 3 6 5 12 13 16 15) (30 5 1) simpleGrading (1 1 1)
            hex (3 4 7 6 13 14 17 16) (25 5 1) simpleGrading (1 1 1)
55
            hex (5 6 9 8 15 16 19 18) (30 95 1) simpleGrading (1 1 1)
56
57
58
59
       edges
60
61
62
63
      patches
64
65
             wall outerWall
                  (0 1 11 10)
(1 3 13 11)
(3 4 14 13)
(7 6 16 17)
(6 9 19 16)
                   (9 8 18 19)
             symmetryPlane axis
75
76
77
78
79
80
                  (0 10 12 2)
(2 12 15 5)
(5 15 18 8)
            patch nozzle
                  (4 7 17 14)
82
            empty back
84
                   (0 2 3 1)
87
            empty front
                   (10 11 13 12)
                  (10 11 13 12)
(12 13 16 15)
(13 14 17 16)
(15 16 19 18)
      );
99
100
      mergePatchPairs
\frac{101}{102}
```

In order to improve the numerical accuracy, we shall use the reference level of 1 bar for the pressure field. Note that both the internal field level and the boundary conditions are offset by the reference level.

P-66

3.4.3 Preparing the Run

Before we commence the setup of the calculation, we need to consider the characteristic velocity of the phenomenon we are trying to capture. In the case under consideration, the fluid velocity will be very small, but the pressure wave will propagate with the speed of sound in water. The speed of sound is calculated as:

$$c = \sqrt{\frac{1}{\psi}} = \sqrt{\frac{1}{4.54 \times 10^{-7}}} = 1483.2 \text{m/s}.$$
 (3.18)

For the mesh described above, the characteristic mesh size is approximately 2 mm (note the scaling factor of 0.1 in the *blockMeshDict* file). Using

$$Co = \frac{U \,\Delta t}{\Delta x} \tag{3.19}$$

a reasonable time step is around $\Delta t = 5 \times 10^{-7} \mathrm{s}$, giving the Co number of 0.35, based on the speed of sound. Also, note that the reported Co number by the code (associated with the convective velocity) will be two orders of magnitude smaller. As we are interested in the pressure wave propagation, we shall set the simulation time to 0.25 ms. For reference, the *controlDict* file is quoted below.

```
OpenFOAM: The Open Source CFD Toolbox
                  F ield
                 0 peration
                                   Version:
                                             1.0
                                              http://www.openfoam.org
                  M anipulation
    FoamFile
10
        version
        instance
        class
                         dictionary;
20
21
23
    applicationClass sonicLiquidFoam;
25
27
    startFrom
                     startTime;
29
    startTime
    stopAt
                     endTime;
    endTime
                    0.0001;
                     5e-07;
    writeControl
                    timeStep;
    writeInterval
                    20:
    cycleWrite
                    0;
    writeFormat
                    ascii;
    writePrecision 6;
    writeCompression compressed;
```

OpenVFOAM-1.0



3.4.4 Running the case

P-68

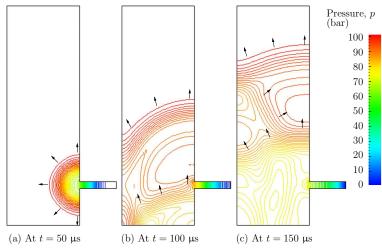


Figure 3.10: Propagation of pressure waves

The user can run the case and view results in dxFoam. The liquid flows out through the nozzle causing a wave to move along the nozzle. As it reaches the inlet to the tank, some of the wave is transmitted into the tank and some of it is reflected. While a wave is reflected up and down the inlet pipe, the waves transmitted into the tank expand and propagate through the tank. In Figure 3.10, the pressures are shown as contours so that the wave fronts are more clearly defined than if plotted as a normal isoline plot.

If the simulation is run for a long enough time for the reflected wave to return to the pipe, we can see that negative absolute pressure is detected. The modelling permits this and has some physical basis since liquids can support tension, *i.e.* negative pressures. In reality, however, impurities or dissolved gases in liquids act as sites for cavitation, or vapourisation/boiling, of the liquid due to the low pressure. Therefore in practical situations, we generally do not observe pressures falling below the vapourisation pressure of the liquid; not at least for longer than it takes for the cavitation process to occur.

Figure 3.11: Propagation of pressure waves with refined mesh

3.4.5 Improving the solution by refining the mesh

Looking at the evolution of the resulting pressure field in time, we can clearly see the propagation of the pressure wave into the tank and numerous reflections from the inside walls. It is also obvious that the pressure wave is smeared over a number of cells. We shall now refine the mesh and reduce the time step to obtain a sharper front resolution. Simply edit the *blockMeshDict* and increase the number of cells by a factor of 4 in the x and y directions, *i.e.* block 0 becomes (120 80 1) from (30 20 1) and so on. Run blockMesh on this file. In addition, in order to maintain a Courant number below 1, the time step must be reduced accordingly to $\Delta t = 10^{-7}$ s. The second simulation gives considerably better resolution of the pressure waves as shown in Figure 3.11.

3.5 Magnetohydrodynamic flow of a liquid

In this example we shall investigate an flow of an electrically-conducting liquid through a magnetic field. The problem is one belonging to the branch of fluid dynamics known as magnetohydrodynamics (MHD) that uses mhdFoam.

3.5.1 Problem specification

The problem is known as the Hartmann problem, chosen as it contains an analytical solution with which mhdFoam can be validated. It is defined as follows:

Solution domain The domain is 2 dimensional and consists of flow along two parallel plates as shown in Fig. 3.12.

Governing equations

Open VFOAM-1.0

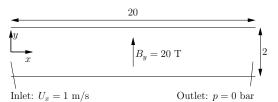


Figure 3.12: Geometry of the Hartmann problem

· Mass continuity for incompressible fluid

$$\nabla \cdot \mathbf{U} = 0 \tag{3.20}$$

• Momentum equation for incompressible fluid

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \bullet \left(\mathbf{U} \mathbf{U} \right) + \nabla \bullet \left(2 \mathbf{B} \Gamma_{\mathbf{B} \mathbf{U}} \mathbf{B} \right) + \nabla \bullet \left(\nu \mathbf{U} \right) + \nabla \left(\Gamma_{\mathbf{B} \mathbf{U}} \mathbf{B} \mathop{\vdots} \mathbf{B} \right) = - \nabla p \ \, (3.21)$$

where **B** is the magnetic flux density, $\Gamma_{\mathbf{B}\mathbf{U}} = (2\mu\rho)^{-1}$.

• Maxwell's equations

P-70

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{3.22}$$

where E is the electric field strength.

$$\nabla \cdot \mathbf{B} = 0 \tag{3.23}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J} \tag{3.24}$$

assuming $\partial \mathbf{D}/\partial t \ll \mathbf{J}$. Here, \mathbf{H} is the magnetic field strength, \mathbf{J} is the current density and \mathbf{D} is the electric flux density.

• Charge continuity

$$\nabla \cdot \mathbf{J} = 0 \tag{3.25}$$

• Constitutive law

$$\mathbf{B} = \mu \mathbf{H} \tag{3.26}$$

• Ohm's law

$$\mathbf{J} = \sigma \left(\mathbf{E} + \mathbf{U} \times \mathbf{B} \right) \tag{3.27}$$

• Combining Equation 3.22, Equation 3.24, Equation 3.27, and taking the curl

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot (\mathbf{U}\mathbf{B}) - \nabla \cdot (\phi_{\mathbf{B}}\mathbf{U}) - \nabla \cdot (\Gamma_{\mathbf{B}}\mathbf{B}) = 0$$
(3.28)

Boundary conditions

- inlet is specified the inlet condition with fixed velocity U = (1, 0, 0) m/s;
- outlet is specified as the outlet with with fixed pressure p = 0 Pa;
- upperWall is specified as a wall where $\mathbf{B} = (0, 20, 0) \mathrm{T}$.
- lowerWall is specified as a wall where $\mathbf{B} = (0, 20, 0) \mathrm{T}$.
- front and back boundaries are specified as empty.

Initial conditions U = 0 m/s, p = 100 Pa, B = (0, 20, 0) T.

Transport properties

- Kinematic viscosity $\nu = 1$ Pas
- Density $\rho = 1 \text{ kg m/s}$
- Electrical conductivity $\sigma = 1 \ (\Omega \, m)^{-1}$
- Permeability $\mu = 1 \text{ H/m}$

Solver name mhdFoam: an incompressible laminar magneto-hydrodynamics code.

Case name hartmann case located in the \$FOAM_TUTORIALS/mhdFoam directory.

3.5.2 Mesh generation

The geometry is simply modelled with 100 cells in the x-direction and 40 cells in the y-direction; the set of vertices and blocks are given in the mesh description file below:

```
F ield
                                            OpenFOAM: The Open Source CFD Toolbox
         //
                      O peration
                                            Version: 1.0
           //
                      A nd
                                                         http://www.openfoam.org
                      M anipulation
      FoamFile
           version
                                2.0:
11
           instance
           class
                               dictionary; blockMeshDict:
21
23
24
25
     convertToMeters 1;
27
     vertices
           (0 -1 0)
(20 -1 0)
(20 1 0)
(0 1 0)
(0 -1 0.1
           (20 -1 0.1)
(20 1 0.1)
```

OpenVFOAM-1.0

```
37
   );
39
40
41
   blocks
       hex (0 1 2 3 4 5 6 7) (100 40 1) simpleGrading (1 1 1)
42
43
44
    edges
45
46
47
    );
   patches
48
       patch inlet
50
           (0 4 7 3)
52
       patch outlet
55
           (2651)
       patch lowerWall
59
           (1540)
       patch upperWall
62
           (3 7 6 2)
65
66
        empty frontAndBack
67
68
69
70
71
72
73
   mergePatchPairs
74
75
```

3.5.3 Running the case

P-72

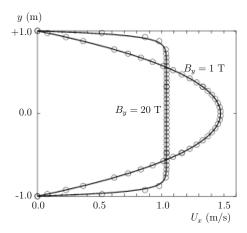
The user can run the case and view results in dxFoam. It is also useful at this stage to run the Ucomponents utility to convert the U vector field into individual scalar components. MHD flow is governed by, amongst other things, the Hartmann number which is a measure of the ratio of electromagnetic body force to viscous force

$$M = BL\sqrt{\frac{\sigma}{\rho\nu}} \tag{3.29}$$

where L is the characteristic length scale. In this case with $B_y=20$ T, M=20 and the electromagnetic body forces dominate the viscous forces. Consequently with the flow fairly steady at t=2 s the velocity profile is almost planar, viewed at a cross section midway along the domain x=10 m. The user can plot a graph of the profile of U_x in dxFoam. Now the user should reduce the magnetic flux density ${\bf B}$ to 1 Tand re-run the code and Ucomponents. In this case, M=1 and the electromagnetic body forces no longer dominate. The velocity profile consequently takes on the parabolic form, characteristic of Poiseuille flow as shown in Figure 3.13. To validate the code the analytical solution for the velocity profile U_x is superimposed in Figure 3.13, given by:

$$\frac{U_x(y)}{U_x(0)} = \frac{\cosh M - \cosh M(y/L)}{\cosh M - 1} \tag{3.30}$$

where the characteristic length L is half the width of the domain, i.e. 1 m.



3.5 Magnetohydrodynamic flow of a liquid

Figure 3.13: Velocity profile in the Hartmann problem for $B_y=1~\mathrm{T}$ and $B_y=20~\mathrm{T}$.

Index P-75 P-76 Index

keyword entry, U-117

Backward differencing, P-39

Index

Symbols

Symbols Numbers A B C D E F G H I J K L M N O P Q R S T U V W X Z

access functions P-23

*	Actor color button, U-171
tensor member function, P-25	adiabaticFlameT utility, U-95
+	adjustableRunTime
tensor member function, P-25	keyword entry, U-62, U-110
_	adjustPhi tools, U-97
tensor member function, P-25	adjustTimeStep keyword, U-62
/	algebraic multi-grid, U-120
tensor member function, P-25	algorithms tools, U-96
/**/	Allow Rerendering button, U-176
C++ syntax, U-80	allTime
//	menu entry, U-131
C++ syntax, U-80	AMG
OpenFOAM file syntax, U-104	keyword entry, U-119
# include	analytical solution, P-45
C++ syntax, U-74, U-80	anisotropicFilter model, U-100
&	Annotate window panel, U-27, U-173
tensor member function, P-25	APIfunctions model, U-99
&&	applicationClass keyword, U-109
tensor member function, P-25	applications, U-71
•	arbitrarily unstructured, P-31
tensor member function, P-25	arc
<lesmodel>Coeffs keyword, U-203</lesmodel>	keyword entry, U-48, U-155
<delta>Coeffs keyword, U-203</delta>	arc keyword, U-154
<turbulencemodel>Coeffs keyword, U-203</turbulencemodel>	ascii
0.000000e+00 directory, U-104	keyword entry, U-110
1-dimensional mesh, U-144	attachMesh utility, U-92
1D mesh, U-144	autoPatch utility, U-92
2-dimensional mesh, U-144	axes
2D mesh, U-144	right-handed, U-153
3D View button, U-172, U-173	right-handed rectangular Cartesian, P-15
3D view Properties	U-21
menu entry, U-27, U-172-U-174	axi-symmetric cases, U-150, U-159
,	axi-symmetric mesh, U-144
Numbers	-
0 directory, U-104	В
	background
\mathbf{A}	process, U-28, U-84
Accept button, U-171	backward

```
basicThermophysicalModels
                                                   setup, U-23
                                                   slip, U-152
    library, U-98
                                                   supersonicFreeStream, U-152
BDCG
                                                   surfaceNormalFixedValue, U-152
    keyword entry, U-119
BICCG
                                                   symmetryPlane, P-65, U-149
    keyword entry, U-119
                                                   totalPressure, U-152
binary
                                                   turbulentInlet, U-152
    keyword entry, U-110
                                                   wall, U-42
BirdCarreau model, U-101
                                                   wall, P-65, P-71, U-149
blended differencing, P-38
                                                   wallBuoyantPressure, U-152
block
                                                   wedge, U-146, U-150, U-159
    expansion ratio, U-156
                                                   zeroGradient, U-151
block keyword, U-154
                                               boundary conditions, P-43
blockMesh solver, P-47
                                                   Dirichlet, P-43
blockMesh utility, U-39, U-92, U-153
                                                   inlet. P-44
                                                   Neumann, P-43
blockMesh
                                                   no-slip impermeable wall, P-44
    menu entry, U-22, U-33
blockMesh executable
                                                   outlet, P-44
                                                   physical, P-44
    vertex numbering, U-156
blockMeshDict
                                                   symmetry plane, P-44
    dictionary, U-21, U-22, U-37, U-48, U-153, boundary type
        U-160
                                                   empty, U-136
blocks keyword, U-22, U-155
                                                   wall, U-42
bound tools, U-97
                                               boundaryField keyword, U-108
boundaries, U-146
                                               boundaryFoam solver, U-89
boundary, U-146
                                               bounded
boundary
                                                   keyword entry, U-115, U-116
    dictionary, U-143, U-153
                                               boxTurb utility, U-91
boundary condition
                                               breaking of a dam, U-56
    calculated, U-151
                                               bubbleFoam solver, U-90
    cyclic, U-150
                                               buoyantFoam solver, U-91
    directionMixed, U-151
                                               buoyantSimpleFoam solver, U-91
    empty, P-65, P-71, U-21, U-146, U-150
                                               button
    fixedGradient, U-151
                                                   3D View, U-172, U-173
    fixedValue, U-151
                                                   Accept, U-171
    fluxCorrectedVelocity, U-152
                                                   Actor color, U-171
    gammaContactAngle, U-58
                                                   Allow Rerendering, U-176
    inlet, P-71
                                                   Cell, U-181
    inletOutlet, U-152
                                                   Close Case, U-33
    mixed, U-151
                                                   Color Range, U-178
    movingWallVelocity, U-152
                                                   Compact, U-132
    outlet, P-71
                                                   Contour, U-181
                                                   Delete, U-171
    outletInlet, U-152
    partialSlip, U-152
                                                   Disable, U-181
                                                   Display Orientation Axes, U-173
    patch, U-149
    pressureDirectedInletVelocity, U-152
                                                   Info. U-132
    pressureInletVelocity, U-152
                                                   Map Plane, U-178
    pressureOutlet, P-65
                                                   My Jobs, U-132
```

pressureTransmissive, U-152

processor, U-150

Index P-77 P-78 Index

Normalise glyphs, U-182	environment variable, U-192
Orientation Axes, U-27	Cell button, U-181
Refresh Case Browser, U-41	cell
Reset Range, U-28	expansion ratio, U-156
Reset, U-171, U-177	cell class, P-31
Rubbersheet, U-181	cell
Select, U-176	keyword entry, U-194
Sequencer, U-176	cellDecompFiniteElement
Start Calculation Now, U-28	library, U-97
Start Calculation, U-35	cellPoint
Stepper, U-176	keyword entry, U-194
Undo, U-177	cellPointFace
Update Times, U-176	keyword entry, U-194
Use parallel projection, U-27, U-172	cells
Volume, U-181	dictionary, U-143, U-153
cont, U-132	cellSet utility, U-92
endNow, U-132	central differencing, P-38
end, $U-132$	cfdTools
kill, U-132	library, U-97
purge, U-132	cfxToFoam utility, U-92, U-160
read, U -132	cGamma keyword, U-64
status, U-132	channelOodles solver, U-90
suspend, U-132	checkMesh utility, U-92, U-162
\mathbf{C}	checkYPlus utility, U-95
· ·	chemistryModel
C++ syntax	library, U-99
/**/, U-80	chemistryModel model, U-99
//, U-80	chemistrySolver model, U-99
# include, U-74, U-80	chemkinMixture model, U-98, U-200
calculated	Class menu, U-129
boundary condition, U-151	class
Camera window panel, U-173	cell, P-31
Camera	dimensionSet, P-26, P-32, P-33
menu entry, U-177	face, P-31
Camera Controls window panel, U-173	finiteVolumeCalculus, P-36
Camera Orientation window panel, U-173	finiteVolumeMethod, P-36
Case menu, U-89, U-176 case	fvMesh, P-31, U-143
	fvSchemes, P-38
browser, U-127 server, U-133	fvc, P-36
case keyword, U-105	fvm, P-36
case manager	pointField, P-31
FoamX, U-123	polyBoundaryMesh, P-31
Case Name text box, U-129	polyMesh, P-31, U-141, U-143
Case Root text box, U-129	polyPatchList, P-31
caseRoots keyword, U-19	polyPatch, P-31
cases, U-103	scalarField, P-29
cavity flow, U-19	scalar, P-24
CELARCH	slice, P-31
environment variable, U-192	symmTensorField, P-29
CELHOME	symmTensorThirdField, P-29
CLILITOWIL	symminenson minument, 1-29

tensorField, P-29	convertToMeters keyword, U-153, U-154
tensorThirdField, P-29	coordinate
tensor, P-24	system, P-15
vectorField, P-29	coordinate system, U-21
vector, P-24, U-107	CORBA, U-97, U-123
word, P-26, P-31	corrected
class keyword, U-105	keyword entry, U-115, U-116
clockTime	couplePatches utility, U-92
keyword entry, U-110	Courant number, P-42, U-25
Close Case button, U-33	cpuTime
cloud keyword, U-195	keyword entry, U-110
cmptAv	Crank Nicholson
tensor member function, P-25	temporal discretisation, P-43
Co utility, U-94	CrankNicholson
cofactors	keyword entry, U-117
tensor member function, P-25	createPatch utility, U-92
coldEngineFoam solver, U-90	cross product, see tensor, vector cross product
Color by menu, U-171	CrossPowerLaw
Color Range button, U-178	keyword entry, U-61
Color Range window, U-178	CrossPowerLaw model, U-100
combustionThermophysicalModels	cubeRootVolDelta model, U-100
library, U-98	cubic
comments, U-80	keyword entry, U-114, U-117
Compact button, U-132	curl, P-37
compressed	curl
keyword entry, U-110	fvc member function, P-37
compressible tools, U-97	curve keyword, U-195
compressibleLESmodels	cycleWrite keyword, U-110
library, U-100	cyclic
compressible Turbulence Models	boundary condition, U-150
library, U-99	cyclic
Connection menu, U-177, U-188	keyword entry, U-149
constant directory, U-103, U-199	cylinder
constLaminarFlameSpeed model, U-98	flow around a, P-45
constTransport model, U-99, U-200	D
cont button, U-132	d2dt2
contactStressFoam solver, U-91	fvc member function, P-37
containers tools, U-96	fvm member function, P-37
continuum	dam
mechanics, P-15	breaking of a, U-56
Contour button, U-181	db tools, U-96
Contour window panel, U-181	DCG
control	keyword entry, U-119
of time, U-109	ddt
controlDict	fvc member function, P-37
dictionary, P-67, U-25, U-34, U-43, U-52,	fvm member function, P-37
U-62, U-103, U-167	DeardorffDiffStress model, U-100
controlDict file, P-49	decomposePar utility, U-84, U-85, U-95
convection, see divergence, P-38	decomposeParDict
convergence, U-41	dictionary, U-84

Index P-79 P-80 Index

decomposition	Gamma, P-38
of field, U-84	MINMOD, P-38
of mesh, U-84	SUPERBEE, P-38
decompression of a tank, P-63	upwind, P-38
deformedGeom utility, U-92	van Leer, P-38
Delete button, U-171	dimension
delta keyword, U-86, U-203	checking in OpenFOAM, P-26
deltaT keyword, U-109	dimensioned <type> template class, P-26</type>
dependencies, U-74	dimensionedTypes tools, U-96
dependency lists, U-74	dimensions keyword, U-108
det	dimensionSet class, P-26, P-32, P-33
tensor member function, P-25	dimensionSet tools, U-96
determinant, see tensor, determinant	diMethylEther model, U-98
dev	direct numerical simulation, U-63
tensor member function, P-25	directionMixed
diag	boundary condition, U-151
tensor member function, P-25	directory
Dictionaries dictionary tree, U-136	0.000000e+00, U-104
dictionary	0, U-104
blockMeshDict, U-21, U-22, U-37, U-48	
U-153, U-160	constant, U-103, U-199
boundary, U-143, U-153	fluentInterface, U-189
cells, U-143, U-153	polyMesh, U-103, U-143
controlDict, P-67, U-25, U-34, U-43, U-52	
U-62, U-103, U-167	run, U-103
decomposeParDict, U-84	system, P-49, U-103
faces, U-143, U-153	tutorials, P-45, U-19
fvSchemes, U-63, U-103, U-111, U-112 fvSolution, U-103, U-118	Disable button, U-181 discretisation
mechanicalProperties, U-51	
points, U-143, U-153	equation, P-33
thermalProperties, U-51	Display window, U-175, U-178
thermophysicalProperties, U-199	Display window panel, U-27, U-28, U-170, U-171
transportProperties, U-24, U-41, U-43	Display Orientation Axes button, U-173 distance
turbulenceProperties, U-43, U-202	
dictionary tree	keyword entry, U-195 distributed keyword, U-86, U-88
Dictionaries, U-136	div
	 -
Fields, U-23, U-136 Mesh, U-23	fvc member function, P-37 fvm member function, P-37
	,
Patches, U-23 dieselEngineFoam solver, U-90	divergence, P-37, P-39 divSchemes keyword, U-112
dieselMixture model, U-98, U-200	divU utility, U-94
dieselSpray	dnsFoam solver, U-90
library, U-97	double inner product, see tensor, double inner
diEthylEther model, U-98	product
differencing	dxFoam, U-175
Backward, P-39	dxFoamExec utility, U-93
blended, P-38	DXMEMORY
central, P-38	environment variable, U-188
Euler implicit, P-39	dynamicMesh
Euror Impuote, 1 -55	dynamicivic311

library, U-97	WM_LINK_LANGUAGE, U-78
dynMixedSmagorinsky model, U-100	WM_MPLIB, U-78
dynOneEqEddy model, U-100	WM_OPTIONS, U-78
dynSmagorinsky model, U-100	WM_PROJECT_DIR, U-78
_	WM_PROJECT_INST_DIR, U-78
${f E}$	WM_PROJECT_LANGUAGE, U-78
edgeGrading keyword, U-156	WM_PROJECT_USER_DIR, U-78
edgeMesh	WM_PROJECT_VERSION, U-78
library, U-97	WM_PROJECT, U-78
edges keyword, U-154	WM_SHELL, U-78
electrostaticFoam solver, U-91	wmake, U-78
empty	environmentalProperties file, U-61
boundary condition, P-65, P-71, U-21	equilibriumCO utility, U-95
U-146, U-150	equilibriumFlameT utility, U-95
empty boundary type, U-136	errorEstimation
empty	library, U-97
keyword entry, U-149	estimateScalarError utility, U-96
end button, U-132	Euler
endNow button, U-132	keyword entry, U-117
endTime keyword, U-25, U-109	Euler implicit
engine	differencing, P-39
library, U-97	temporal discretisation, P-42
engineCompRatio utility, U-95	*
engineFoam solver, U-90	examples
engineSwirl utility, U-91	decompression of a tank, P-63
ensight74FoamExec utility, U-93, U-192	flow around a cylinder, P-45
ensight76FoamExec utility, U-93	flow over backward step, P-54
ENSIGHT7_INPUT	Hartmann problem, P-69
environment variable, U-192	supersonic flow over forward step, P-59
ENSIGHT7_READER	Execute menu, U-177
environment variable, U-192	Execute on Change
enstrophy utility, U-94	menu entry, U-177
environment variable	explicit
	temporal discretisation, P-42
CELLOME II 102	exponential model, U-99
CEI_HOME, U-192	\mathbf{F}
DXMEMORY, U-188	=
ENSIGHT7_INPUT, U-192	face class, P-31
ENSIGHT7_READER, U-192	face keyword, U-195
FOAMX_PATH, U-139	faceDecompFiniteElement
FOAMX_SYSTEM_CONFIG, U-139	library, U-97
FOAMX_USER_CONFIG, U-139	faces
FOAM_RUN, U-103, U-139	dictionary, U-143, U-153
JAVA_HOME, U-139	faceSet utility, U-92
WM_ARCH, U-78	field
WM_COMPILER_BIN, U-78	U, U-25
WM_COMPILER_DIR, U-78	p, U-25
WM_COMPILER_LIB, U-78	decomposition, U-84
WM_COMPILER, U-78	FieldField <type> template class, P-32</type>
WM_COMPILE_OPTION, U-78	Fields dictionary tree, U-23, U-136
WM_DIR, U-78	Fields window, U-28
WM_JAVAC_OPTION, U-78	fields, P-29

Index P-81 P-82 Index

monning II 167	anninanment regioble II 102 II 120
mapping, U-167 fields tools, U-96	environment variable, U-103, U-139 Foam Utilities menu, U-22, U-33, U-34
fields keyword, U-194	foamConvert21To22 utility, U-96
,	0,
fieldToCellSet utility, U-92	foamCorrectVrt script/alias, U-165
Field <type> template class, P-29</type>	foamDataToFluent utility, U-93, U-189
File menu, U-176	foamDebugSwitches utility, U-96
file	FoamFile keyword, U-105
FoamX.cfg, U-139	foamInfoExec utility, U-96
FoamXClient.cfg, U-124, U-138	foamJob script/alias, U-196
Make/files, U-77	foamLog script/alias, U-196
controlDict, P-49	foamMeshToFluent utility, U-92, U-189
environmentalProperties, U-61	foamToDX utility, U-93
files, U-75	foamToEnsight utility, U-93
options, U-75	foamToFieldview utility, U-93
transportProperties, U-61	foamToFieldview9 utility, U-94
file format, U-104	foamToVTK utility, U-94
files file, U-75	foamUser
financialFoam solver, U-91	library, U-83
finite volume	FoamX
discretisation, P-27	case browser, U-127
mesh, P-31	case manager, U-123
finiteVolume tools, U-96	case server, U-133
finiteVolumeCalculus class, P-36	OpenFOAM case manager, U-123
finiteVolumeMethod class, P-36	host browser, U-124
firstTime	JAVA GUI, U-125
menu entry, U-131	name server, U-124
firstTime keyword, U-109	FoamX utility, U-91
fixed	FoamX.cfg file, U-139
keyword entry, U-110	FOAMX_PATH
fixedGradient	environment variable, U-139
boundary condition, U-151	FOAMX_SYSTEM_CONFIG
fixedValue	environment variable, U-139
boundary condition, U-151	FOAMX_USER_CONFIG
flattenMesh utility, U-92	environment variable, U-139
flow	FoamXClient.cfg file, U-124, U-138
free surface, U-56	foreground
laminar, U-19	process, U-28
steady, turbulent, P-54	format keyword, U-105
supersonic, P-59	fourth
turbulent, U-20	keyword entry, U-115, U-116
flow around a cylinder, P-45	fvc class, P-36
flow over backward step, P-54	fvc member function
fluentInterface directory, U-189	curl, P-37
fluentMeshToFoam utility, U-92, U-160	d2dt2, P-37
fluxCorrectedVelocity	ddt, P-37
boundary condition, U-152	div, P-37
fluxRequired keyword, U-112	gGrad, P-37
OpenFOAM	grad, P-37
cases, U-103	laplacian, P-37
FOAM_RUN	lsGrad, P-37
	,

snGrad, P-37	fvc member function, P-37
snGradCorrection, P-37	(Grad Grad) squared, P-37
sqrGradGrad, P-37	gradient, P-37, P-40
vm class, P-36	Gauss scheme, P-40
vm member function	Gauss's theorem, U-53
d2dt2, P-37	least square fit, U-53
ddt, P-37	least squares method, P-40, U-53
div, P-37	surface normal, P-40
laplacian, P-37	gradSchemes keyword, U-112
Su, P-37	Graph window, U-183
SuSp, P-37	graphFormat keyword, U-110
vMatrix template class, P-36	Grid Point text box, U-183
vMesh class, P-31, U-143	Grid point text box, U-182
fvSchemes	Grid vectors text box, U-182
dictionary, U-63, U-103, U-111, U-112	Gstream
vSchemes class, P-38	library, U-97
EvSchemes	guldersLaminarFlameSpeed model, U-98
menu entry, U-53	TT
fvSolution	H
dictionary, U-103, U-118	hConstThermo model, U-98, U-199
C	hhuMixtureThermo model, U-98, U-200
G	hierarchical
gambitToFoam utility, U-92, U-160	keyword entry, U-85, U-86
Gamma differencing, P-38	hMixtureThermo model, U-98, U-200
Gamma2	homogeneousMixture model, U-98, U-200
keyword entry, U-114 Gamma201	host, U-20
keyword entry, U-114	browser, U-124
Gamma2V	hThermo model, U-98, U-200
keyword entry, U-114	Ī
gammaContactAngle	I
boundary condition, U-58	tensor member function, P-25
Gauss	ICCG
keyword entry, U-115	keyword entry, U-119
Gauss's theorem, P-36	icoErrorEstimate utility, U-96
GaussSeidel	icoFoam solver, U-19, U-24, U-25, U-28, U-8
keyword entry, U-119	icoFoamAutoMotion solver, U-90
General window panel, U-172	icoMomentError utility, U-96
general model, U-99	icoTopoFoam solver, U-90
general	ideasToFoam utility, U-92, U-160
keyword entry, U-110	identities, see tensor, identities
GeometricBoundaryField template class, P-32	identity, see tensor, identity
geometricField <type> template class, P-32</type>	Image window, U-175, U-176
gGrad	incompressible tools, U-97
fvc member function, P-37	incompressibleLESmodels
global tools, U-96	library, U-100
Glyph Attributes window panel, U-182	incompressiblePostProcessing
gmshToFoam utility, U-92	library, U-97
gnuplot	incompressible Transport Models
keyword entry, U-110, U-194	library, P-55, U-100
grad	incompressibleTurbulenceModels
=	

library, P-55, U-99	dimensions, U-108
index	distributed, U-86, U-88
notation, P-16, P-17	divSchemes, U-112
Info button, U-132	edgeGrading, U-156
Information window panel, U-170	edges, U-154
inhomogeneousMixture model, U-98, U-200	endTime, U-25, U-109
inlet	face, U-195
boundary condition, P-71	fields, U-194
inletOutlet	firstTime, U-109
boundary condition, U-152	fluxRequired, U-112
inner product, see tensor, inner product	format, U-105
insideCells utility, U-92	gradSchemes, U-112
instance keyword, U-105	graphFormat, U-110
interFoam solver, U-90	instance, U-105
internalField keyword, U-108, U-136	internalField, U-108, U-136
interpolationScheme keyword, U-194	interpolationSchemes, U-112
interpolations tools, U-96	interpolationScheme, U-194
interpolationSchemes keyword, U-112	kappa, U-203
inv	laplacianSchemes, U-112
tensor member function, P-25	latestTime, U-41
isoOctane model, U-98	leastSquares, U-53
_	local, U-105
J	manualCoeffs, U-86
janafThermo model, U-98, U-199	maxCo, U-62
JAVA_HOME	maxDeltaT, U-62
environment variable, U-139	method, U-86
jplot	metisCoeffs, U-86
keyword entry, U-110, U-194	midPointAndFace, U-195
K	midPoint, U-195
	nFaces, U-144
kappa keyword, U-203	nGammaSubCycles, U-64
kEpsilon model, U-99, U-100	numberOfSubdomains, U-86
keyword	n, U-86
FoamFile, U-105	object, U-105
LESmodel, U-203	order, U-86
adjustTimeStep, U-62	
applicationClass, U-109	outputFormat, U-194 patchMap, U-167
arc, U-154	patches, U-154, U-156, U-157
blocks, U-22, U-155	physicalType, U-144, U-146
block, U-154	processorWeights, U-86
boundaryField, U-108	refGradient, U-151
cGamma, U-64	referenceLevel, U-108, U-136
caseRoots, U-19	
case, U-105	roots, U-86, U-88
class, U-105	root, U-105
cloud, U-195	runTimeModifiable, U-111
convertToMeters, U-153, U-154	sampleSets, U-194
curve, U-195	simpleGrading, U-156
cycleWrite, U-110	snGradSchemes, U-112
deltaT, U-109	solvers, U-118
delta, U-86, U-203	spline, U-154

T. 144
startFace, U-144
startFrom, U-25, U-109
startTime, U-25, U-109
stopAt, U-109
thermoType, U-199
timeFormat, U-110
timePrecision, U-110
timeScheme, U-112
turbulenceModel, U-203
turbulence, U-203
type, U-146
uniform, U-195
valueFraction, U-151
value, U-151
version, U-105
vertices, U-22, U-154
wallFunctionCoeffs, U-203
writeCompression, U-110
$\mathtt{writeControl},\ U\text{-}25,\ U\text{-}62,\ U\text{-}109$
$\mathtt{writeFormat},\ U55,\ U110$
$\mathtt{writeInterval},\ U\text{-}25,\ U\text{-}35,\ U\text{-}110$
writePrecision, U-110
<LESmodel $>$ Coeffs, U - 203
<delta $>$ Coeffs, U - 203
<turbulenceModel $>$ Coeffs, U - 203
keyword entry
AMG, U-119
BDCG, U-119
BICCG, U-119
CrankNicholson, U-117
CrossPowerLaw, U-61
DCG, U-119
Euler, U-117
Gamma201, U-114
${\tt Gamma2V},\ U\text{-}114$
${\tt Gamma2},\ {\tt U-114}$
GaussSeidel, U-119
${\tt Gauss, U-115}$
ICCG, U-119
MUSCL, U-114
Newtonian, U - 61
QUICK, U-114, U-117
SFCD, U-114, U-117
UMIST, U-113
VanLeer, U-114
adjustableRunTime, U-62, U-110
arc, U-48, U-155
ascii, U-110
backward, U-117
binary, $U-110$

bounded, U-115, U-116 $\mathtt{cellPointFace},\, \textcolor{red}{U\text{-}194}$ cellPoint, U-194 cell, U-194 clockTime, U-110 compressed, U-110 corrected, U-115, U-116 cpuTime, U-110 cubic, U-114, U-117 cyclic, U-149distance, U-195 empty, U-149fixed, U-110 fourth, U-115, U-116 ${\tt general},\, {\color{red} U\text{-}110}$ gnuplot, U-110, U-194 hierarchical, U-85, U-86 jplot, U-110, U-194 latestTime, U-109 leastSquares, U-115 limitedCubic, U-114 limited, U-115, U-116 linearUpwind, U-114, U-117 linear, U-114, U-117 line, U-155 manual, U-85, U-86 metis, U-85, U-86 nextWrite, U-109 noWriteNow, U-109 none, U-113 patch, U-149 polyLine, U-155 polySpline, U-155processor, U-149 raw, U-110, U-194 runTime, U-35, U-110 scientific, U-110 ${\tt simpleSpline}, \, {\color{red}U\textbf{-}155}$ simple, U-85, U-86 skewLinear, U-114, U-117 startTime, U-25, U-109 steadyState, U-117 ${\tt symmetryPlane},\, {\color{red}U\textbf{-}149}$ timeStep, U-25, U-35, U-110 uncompressed, U-110 uncorrected, U-115, U-116 upwind, U-114, U-117 wall, U-149 wedge, U-149 writeControl, U-109

Index P-85 P-86 Index

II 100	DVFDd II 100
writeNow, U-109	PVFoamReader, U-169
xmgr, U-110, U-194	basicThermophysicalModels, U-98
xyz, U-195	cellDecompFiniteElement, U-97
x, U-195	cfdTools, U-97
у, U-195	chemistryModel, U-99
z, U-195	combustionThermophysicalModels, U-98
kill button, U-132	compressibleLESmodels, U-100
kivaToFoam utility, U-92	compressibleTurbulenceModels, U-99
Kronecker delta, P-21	dieselSpray, U-97
\mathbf{L}	dynamicMesh, U-97
lagrangian	edgeMesh, U-97
library, U-97	engine, U-97
LAM	errorEstimation, U-97
	faceDecompFiniteElement, U-97
message passing interface, U-86 MPI, U-86	foamUser, U-83
· · · · · · · · · · · · · · · · · · ·	incompressibleLESmodels, U-100
Lambda2 utility, U-94	incompressiblePostProcessing, U-97
LamBremhorstKE model, U-99	incompressibleTransportModels, P-55, U-100
laminar model, U-99, U-100	incompressibleTurbulenceModels, P-55, U-99
laminarFlameSpeedModels	lagrangian, U-97
library, U-98	laminarFlameSpeedModels, U-98
laplaceFilter model, U-100	liquids, U-98
Laplacian, P-38	meshTools, U-97
laplacian, P-37	mico-2.3.11, U-97
laplacian	mpich-1.2.4, U-97
fvc member function, P-37	pdf, U-99
fvm member function, P-37	primitive, P-23
laplacianFoam solver, U-89	randomProcesses, U-97
laplacianSchemes keyword, U-112	sampling, U-97
latestTime	shapeMeshTools, U-97
keyword entry, U-109	specie, U-98
menu entry, U-131	thermophysicalFunctions, U-99
latestTime keyword, U-41	thermophysical, U-199
LaunderGibsonRSTM model, U-99, U-100	triSurface, U-97
LaunderSharmaKE model, U-99, U-100	vtkFoam, U-169
leastSquares	zlib-1.2.1, U-97
keyword entry, U-115	,
leastSquares keyword, U-53	lid-driven cavity flow, U-19
LESdeltas	LienCubicKE model, U-99
library, U-100	LienCubicKELowRE model, U-99
LESfilters	LienLeschzinerLowRE model, U-99
library, U-100	liftDrag utility, U-95
lesInterFoam solver, U-90	limited
LESmodel keyword, U-203	keyword entry, U-115, U-116
libraries, U-71	limitedCubic
library	keyword entry, U-114
Gstream, U-97	line
LESdeltas, U-100	keyword entry, U-155
LESfilters, U-100	linear
ODE, U-97	keyword entry, U-114, U-117
OpenFOAM, U-96	linearUpwind

keyword entry, U-114, U-117	Connection, U-177, U-188
iquid	Execute, U-177
electrically-conducting, P-69	File, U-176
iquids	Foam Utilities, U-22, U-33, U-34
library, U-98	Mesh, $U-50$
ists, P-29	Mode:, U-177
List <type> template class, P-29</type>	Options, U-177
Local keyword, U-105	Projection:, U-177
ocDynOneEqEddy model, U-100	Root, U-176
Lower and Upper Times text box, U-172	Set view, U-177
owReOneEqEddy model, U-100	View, U-28, U-172, U-173
_RDDiffStress model, U-100	Windows, U-177
_RR model, U-99, U-100	name, U-178
LsGrad	type, U-178
fvc member function, P-37	menu entry
7. 6	3D view Properties, U-27, U-172-U-174
M	Camera, U-177
Mach utility, U-94	Execute on Change, U-177
nag	Navigate, U-177
tensor member function, P-25	Orthographic, U-177
magGradU utility, U-94	Pan/Zoom, U-177
nagnetohydrodynamics, P-69	Perspective, U-177
nagSqr	Pick, U-177
tensor member function, P-25	Print, U-176
nagU utility, U-35, U-94	Property, U-171
Make directory, U-75	1 0,
make script/alias, U-73	Read Mesh & Fields, U-23
Make/files file, U-77	Read Mesh, U-45, U-50
makePolyMesh utility, U-92	Refresh Case Browser, U-41
nanual	Rendering Options, U-177
keyword entry, U-85, U-86	Roam, U-177
nanualCoeffs keyword, U-86	Rotate, U-177
Map Plane button, U-178	Save, U-176
Map Plane window, U-178	Source, U-28, U-173
$mapFields\ \mathrm{utility},\ \mathrm{U\text{-}}33,\ \mathrm{U\text{-}}40,\ \mathrm{U\text{-}}43,\ \mathrm{U\text{-}}55,\ \mathrm{U\text{-}}92,$	View Control, U-177
U-167	Wireframe, U-171
napFields	Zoom, U-177
menu entry, U-34	allTime, U-131
napping	blockMesh, U-22, U-33
fields, U-167	firstTime, U-131
matrices tools, U-97	fvSchemes, U-53
nax	latestTime, U-131
tensor member function, P-25	mapFields, U-34
naxCo keyword, U-62	noTime, U-131
maxDeltaT keyword, U-62	preProcessing, U-34
mechanicalProperties	sample, U-54
dictionary, U-51	mergeMeshes utility, U-92
nenu	Mesh dictionary tree, U-23
Case, U-89, U-176	Mesh window, U-184
Class, U-129	Mesh menu, U-50
Color by, U-171	mesh

Index P-87 P-88 Index

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	I D I WE HAD
1-dimensional, U-144	LamBremhorstKE, U-99
1D, U-144	LaunderGibsonRSTM, U-99, U-100
2-dimensional, U-144 2D, U-144	LaunderSharmaKE, U-99, U-100 LienCubicKELowRE, U-99
axi-symmetric, U-144	LienCubicKE, U-99
basic, P-31	
block structured, U-153	LienLeschzinerLowRE, U-99 NSRDSfunctions, U-99
decomposition, U-84	Newtonian, U-100
description, U-141	NonlinearKEShih, U-99
finite volume, P-31	PrandtlDelta, U-100
generation, U-153	QZeta, U-99
grading, U-153, U-156	RNGkEpsilon, U-99, U-100
grading, example of, P-54	RosinRammler, U-99
non-orthogonal, P-45	Smagorinsky2, U-100
refinement, P-63	Smagorinsky, U-100
resolution, U-33	SpalartAllmaras, U-99, U-100
specification, U-141	anisotropicFilter, U-100 chemistryModel, U-99
validity constraints, U-141 meshes tools, U-97	chemistryNodel, U-99
meshTools	chemkinMixture, U-98, U-200
library, U-97	constLaminarFlameSpeed, U-98
Message window, U-175, U-177	constTransport, U-99, U-200
message passing interface	cubeRootVolDelta, U-100
LAM, U-86	diEthylEther, U-98
MPICH, U-205	diMethylEther, U-98
	,
method keyword, U-86 metis	dieselMixture, U-98, U-200 dynMixedSmagorinsky, U-100
	dynOneEqEddy, U-100
keyword entry, U-85, U-86 metisCoeffs keyword, U-86	dynSmagorinsky, U-100
,	exponential, U-99
mhdFoam solver, P-71, U-91 mico-2.3.11	general, U-99
library, U-97	9 ,
• /	guldersLaminarFlameSpeed, U-98 hConstThermo, U-98, U-199
midPoint keyword, U-195 midPointAndFace keyword, U-195	
min	hMixtureThermo, U-98, U-200 hThermo, U-98, U-200
tensor member function, P-25	hhuMixtureThermo, U-98, U-200
MINMOD differencing, P-38	homogeneousMixture, U-98, U-200
mirrorMesh utility, U-93	inhomogeneousMixture, U-98, U-200
mixed	isoOctane, U-98
boundary condition, U-151	janafThermo, U-98, U-199
mixedSmagorinsky model, U-100	kEpsilon, U-99, U-100
mixtureAdiabaticFlameT utility, U-95	laminar, U-99, U-100
Mode: menu, U-177	laplaceFilter, U-100
model	locDynOneEqEddy, U-100
APIfunctions, U-99	lowReOneEqEddy, U-100
BirdCarreau, U-101	mixedSmagorinsky, U-100
CrossPowerLaw, U-100	multiComponentMixture, U-98, U-200
DeardorffDiffStress, U-100	nDecane, U-98
LRDDiffStress, U-100	nDodecane, U-98
LRR, U-99, U-100	nHeptane, U-98

nOctane, U-98	keyword entry, U-109
normal, U-99	nFaces keyword, U-144
oneEqEddy, U-100	nGammaSubCycles keyword, U-64
perfectGas, U-98, U-199	nHeptane model, U-98
pureMixture, U-98, U-200	nOctane model, U-98
scaleSimilarity, U-100	non-orthogonal mesh, P-45
simpleFilter, U-100	none
smoothDelta, U-100	keyword entry, U-113
specieThermo, U-99, U-199	NonlinearKEShih model, U-99
spectEddyVisc, U-100	nonNewtonianIcoFoam solver, U-90
sutherland Transport, U-99, U-200	normal model, U-99
uniform, U-99	Normalise glyphs button, U-182
veryInhomogeneousMixture, U-98, U-200	noTime
water, U-98	menu entry, U-131
momentScalarError utility, U-96	noWriteNow
moveEngineMesh utility, U-93	keyword entry, U-109
moveMesh utility, U-93	NSRDSfunctions model, U-99
movingWallVelocity	numberOfSubdomains keyword, U-86
=	
boundary condition, U-152 MPI	numerical diffusion, U-63
	0
LAM, U-86	object keyword, U-105
MPICH, U-205	objToVTK utility, U-93
MPICH	ODE
message passing interface, U-205	library, U-97
MPI, U-205	oneEqEddy model, U-100
mpich-1.2.4	oodles solver, U-90
library, U-97	Opacity text box, U-171
mshToFoam utility, U-92	openDX
multi-grid	openDX
algebraic, U-120	installation problems, U-187
multiComponentMixture model, U-98, U-200	OpenFOAM
MUSCL	*
keyword entry, U-114	applications, U-71
MUST_READ, U-59	file format, U-104
My Jobs button, U-132	libraries, U-71
N	OpenFOAM
- '	library, U-96
n keyword, U-86	OpenFOAM file syntax
nabla D. 27	//, U-104
operator, P-27	operator
name	scalar, P-28
server, U-124	vector, P-27
name menu, U-178	Options window panel, U-181
Navigate	Options menu, U-177
menu entry, U-177	options file, U-75
nDecane model, U-98	order keyword, U-86
nDodecane model, U-98	Orientation Axes button, U-27
Newtonian	Orthographic
keyword entry, U-61	menu entry, U-177
Newtonian model, U-100	outer product, see tensor, outer product
nextWrite	outlet

P-89 P-90 Index Index

boundary condition, P-71	postChannel utility, U-95
outletInlet	potentialFoam solver, P-46, U-89
boundary condition, U-152	pow
outputFormat keyword, U-194	tensor member function, P-25
	PrandtlDelta model, U-100
P	preProcessing
p field, U-25	menu entry, U-34
Pan/Zoom	pressure waves
menu entry, U-177	in liquids, P-64
paraFoam, U-26, U-169	pressureDirectedInletVelocity
parallel	boundary condition, U-152
running, U-84	pressureInletVelocity
Parameters window panel, U-28, U-170, U-171	boundary condition, U-152
partialSlip	pressureOutlet
boundary condition, U-152	boundary condition, P-65
patch	pressureTransmissive
boundary condition, U-149	boundary condition, U-152
patch	primitive
keyword entry, U-149	library, P-23
Patches dictionary tree, U-23	primitives tools, U-97
patches keyword, U-154, U-156, U-157	Print
patchMap keyword, U-167	menu entry, U-176
pdf	process
library, U-99	background, U-28, U-84
Pe utility, U-94	foreground, U-28
perfectGas model, U-98, U-199	processor
permutation symbol, P-20	boundary condition, U-150
Perspective	processor
menu entry, U-177	keyword entry, U-149
physicalType keyword, U-144, U-146	processorN directory, U-86
Pick	processorWeights keyword, U-8
menu entry, U-177	Projection: menu, U-177
pointField class, P-31	Property
pointField <type> template class, P-33</type>	menu entry, U-171
points	ptot utility, U-95
dictionary, U-143, U-153	pureMixture model, U-98, U-200
pointSet utility, U-93	purge button, U-132
polyBoundaryMesh class, P-31	PVFoamReader
polyLine	library, U-169
keyword entry, U-155	•
polyMesh directory, U-103, U-143	${f Q}$
polyMesh class, P-31, U-141, U-143	Q utility, U-94
polyPatch class, P-31	QUICK
polyPatchList class, P-31	keyword entry, U-114, U-117
polySpline	QZeta model, U-99
keyword entry, U-155	ъ
post-processing, U-169	${f R}$
post-processing	R utility, U-94
dxFoam, U-175	randomProcesses
post-processing	library, U-97
naraFoam II-169	rasInterFoam solver II-90

raw	sample utility, U-95, U-193
keyword entry, U-110, U-194	sample
Rcomponents utility, U-94	menu entry, U-54
read button, U-132	sampleSets keyword, U-19
Read Data window, U-175, U-176	sampleSurface utility, U-95
Read Mesh	sampling
menu entry, U-45, U-50	library, U-97
Read Mesh & Fields	Save
menu entry, U-23	menu entry, U-176
reconstructPar utility, U-89, U-95	scalar, P-16
referenceLevel keyword, U-108, U-136	operator, P-28
refGradient keyword, U-151	scalar class, P-24
refineMesh utility, U-93	scalarField class, P-29
refineShapeMesh utility, U-93	scalarTransportFoam solver,
Refresh Case Browser button, U-41	scale
Refresh Case Browser	tensor member function
menu entry, U-41	scalePoints utility, U-93, U-
Region window, U-28	scaleSimilarity model, U-100
relative tolerance, U-119	scientific
Rendering Options	keyword entry, U-110
menu entry, U-177	script/alias
renumberMesh utility, U-93	foamCorrectVrt, U-165
Reset button, U-171, U-177	foamJob, U-196
Reset Range button, U-28	foamLog, U-196
restart, U-41	make, U-73
Reynolds number, U-20, U-24	rmdepall, U-79
rhopSonicFoam solver, U-90	runFoamXHB, U-123, U
rhoSonicFoam solver, U-90	runFoamX, U-123-U-12
rmdepall script/alias, U-79	wclean, U-78
RNGkEpsilon model, U-99, U-100	wmake, U-73
Roam	second time derivative, P-3
menu entry, U-177	Seed window, U-175
Root menu, U-176	Select button, U-176
root keyword, U-105	Selection Window window,
roots keyword, U-86, U-88	Sequencer button, U-176
RosinRammler model, U-99	Sequencer window, U-176, U
Rotate	Set view menu, U-177
menu entry, U-177	set GammaDamBreak solver,
Rubbersheet button, U-181	
Rubbersheet Attributes window panel, U-181	settlingFoam solver, U-90 SFCD
run	
parallel, U-84	keyword entry, U-114,
run directory, U-103	shape, U-156
runFoamX script/alias, U-123-U-125	shapeMeshTools
runFoamXHB script/alias, U-123, U-124	library, U-97
runTime	simple
keyword entry, U-35, U-110	keyword entry, U-85, U
runTimeModifiable keyword, U-111	simpleFilter model, U-100
C	simpleFoam solver, P-55, U-
S	simpleGrading keyword, U
sammToFoam utility, U-92	simpleSpline

sampie
menu entry, U-54
sampleSets keyword, U-194
sampleSurface utility, U-95
sampling
library, U-97
Save
menu entry, U-176
scalar, P-16
operator, P-28
scalar class, P-24
scalarField class, P-29
scalarTransportFoam solver, U-89
scale
tensor member function, P-25
scalePoints utility, U-93, U-164
scaleSimilarity model, U-100
scientific
keyword entry, U-110
script/alias
foamCorrectVrt, U-165
foamJob, U-196
foamLog, U-196
make, U-73
rmdepall, U-79
runFoamXHB, U-123, U-124
runFoamX, U-123–U-125
wclean, U-78
wmake, U-73
second time derivative, P-37
Seed window, U-175
Select button, U-176
Selection Window window, U-27, U-170
Sequencer button, U-176
Sequencer window, U-176, U-184
Set view menu, U-177
setGammaDamBreak solver, U-60
settlingFoam solver, U-90
SFCD
keyword entry, U-114, U-117
shape, U-156
shapeMeshTools
library, U-97
simple
keyword entry, U-85, U-86
simpleFilter model, U-100
simpleFoam solver, P-55, U-90
simpleGrading keyword, U-156
simpleSpline
PIMPIENPIINE

Index P-91 P-92 Index

	keyword entry, U-155	simpleFoam, P-55, U-90
ske	ew	sonicFoamAutoMotion, U-90
	tensor member function, P-25	sonicFoam, P-61, U-90
ske	ewLinear	sonicLiquidFoam, P-65, U-90
	keyword entry, U-114, U-117	sonicTurbFoam, U-90
SΙι	itility, U-95	stressFemFoam, U-91
slice	e class, P-31	stressedFoam, U-51, U-91
slip		turbFoam, U-20, U-90
	boundary condition, U-152	solver relative tolerance, U-119
Sm	agorinsky model, U-100	solver tolerance, U-119
Sm	agorinsky2 model, U-100	solvers keyword, U-118
sma	apToFoam utility, U-94	sonicFoam solver, P-61, U-90
smo	oothDelta model, U-100	sonicFoamAutoMotion solver, U-90
snG	Frad	sonicLiquidFoam solver, P-65, U-90
	fvc member function, P-37	sonicTurbFoam solver, U-90
snG	GradCorrection	Source
	fvc member function, P-37	menu entry, U-28, U-173
snG	GradSchemes keyword, U-112	source, P-37
solv	ver	SpalartAllmaras model, U-99, U-100
	XiFoam, U-90	specie
	Xoodles, U-91	library, U-98
	blockMesh, P-47	specieThermo model, U-99, U-199
	boundaryFoam, U-89	spectEddyVisc model, U-100
	bubbleFoam, U-90	spline keyword, U-154
	buoyantFoam, U-91	splitMesh utility, U-93
	buoyantSimpleFoam, U-91	splitMeshRegions utility, U-93
	channelOodles, U-90	sqr
	coldEngineFoam, U-90	tensor member function, P-25
	contactStressFoam, U-91	sqrGradGrad
	dieselEngineFoam, U-90	fvc member function, P-37
	dnsFoam, U-90	Standard Views window panel, U-173
	electrostaticFoam, U-91	Start Calculation button, U-35
	engineFoam, U-90	Start Calculation Now button, U-28
	financialFoam, U-91	startFace keyword, U-144
	icoFoamAutoMotion, U-90	startFrom keyword, U-25, U-109
	icoFoam, U-19, U-24, U-25, U-28, U-89	starToFoam utility, U-92, U-160
	icoTopoFoam, U-90	startTime
	interFoam, U-90	keyword entry, U-25, U-109
	laplacianFoam, U-89	startTime keyword, U-25, U-109
	lesInterFoam, U-90	status button, U-132
	mhdFoam, P-71, U-91	steady flow
	nonNewtonianIcoFoam, U-90	turbulent, P-54
	oodles, U-90	steadyState
	potentialFoam, P-46, U-89	keyword entry, U-117
	rasInterFoam, U-90	Stepper button, U-176
	rhoSonicFoam, U-90	stitchMesh utility, U-93
	rhopSonicFoam, U-90	stopAt keyword, U-109
	scalarTransportFoam, U-89	Stored Camera Position window panel, U-173
	setGammaDamBreak, U - 60	streamFunction utility, U-94
	settlingFoam, U-90	Streamlines window, U-182

algebraic operations, P-18
algebraic operations in OpenFOAM, P-24
antisymmetric, see tensor, skew
calculus, P-27
classes in OpenFOAM, P-23
cofactors, P-22
component average, P-20
component maximum, P-20
component minimum, P-20
determinant, P-22
deviatoric, P-22
diagonal, P-22
dimension, P-16
double inner product, P-19
geometric transformation, P-21
Hodge dual, P-23
hydrostatic, P-22
identities, P-21
identity, P-21
inner product, P-18
inverse, P-23
magnitude, P-20
magnitude, 1-20 magnitude squared, P-20
mathematics, P-15
notation, P-17
nth power, P-20
outer product, P-19
rank, P-16
rank 3, P-17
scalar division, P-18
scalar multiplication, P-18
scale function, P-20
second rank, P-16
skew, P-22
square of, P-20
subtraction, P-18
symmetric, P-22
symmetric rank 2, P-16
symmetric rank 3, P-17
trace, P-22
transformation, P-21
transpose, P-16, P-22
triple inner product, P-19
vector cross product, P-20
tensor class, P-24
tensor member function
*, P-25
+, P-25
-, P-25
/, P-25

Index P-93 P-94 Index

&, P-25	time derivative, P-37
&&, P-25	first, P-39
^, P-25	second, P-37, P-39
cmptAv, P-25	Time step text box, U-172
cofactors, P-25	time step, U-25
det, P-25	timeFormat keyword, U-110
dev, P-25	timePrecision keyword, U-110
diag, P-25	times text box, U-33
I, P-25	timeScheme keyword, U-112
inv, P-25	timeStep
mag, P-25	keyword entry, U-25, U-35, U-110
magSqr, P-25	tolerance
max, P-25	solver, U-119
min, P-25	solver relative, U-119
pow, P-25	tools
scale, P-25	adjustPhi, U-97
skew, P-25	algorithms, U-96
sqr, P-25	bound, U-97
symm, P-25	compressible, U-97
T(), P-25	containers, U-96
tr, P-25	db, U-96
transform, P-25	dimensionSet, U-96
tensorField class, P-29	dimensionedTypes, U-96
tensorThirdField class, P-29	fields, U-96
tetDecomposition utility, U-93	finiteVolume, U-96
tetgenToFoam utility, U-92	global, U-96
text box	incompressible, U-97
Case Name, U-129	interpolations, U-96
Case Root, U-129	matrices, U-97
Grid Point, U-183	meshes, U-97
Grid point, U-182	primitives, U-97
Grid vectors, U-182	wallDist, U-97
Lower and Upper Times, U-172	totalPressure
Opacity, U-171	boundary condition, U-152
Time step, U-172	tr
Vector, U-183	tensor member function, P-25
View Angle, U-177	trace, see tensor, trace
times, U-33	transform
thermalProperties	tensor member function, P-25
dictionary, U-51	transportProperties
thermophysical	dictionary, U-24, U-41, U-43
library, U-199	transportProperties file, U-61
thermophysicalFunctions	triple inner product, P-19
library, U-99	triSurface
thermophysicalProperties	library, U-97
dictionary, U-199	turbFoam solver, U-20, U-90
thermoType keyword, U-199	turbulence
Time window, U-28	dissipation, U-42
time	kinetic energy, U-42
control, U-109	length scale, U-42

model, U-43	attachMesh, U-92
turbulence keyword, U-203	autoPatch, U-92
turbulence model, U-42	blockMesh, U-39, U-92, U-153
turbulenceModel keyword, U-203	boxTurb, U-91
turbulenceProperties	cellSet, U-92
dictionary, U-43, U-202	cfxToFoam, U-92, U-160
turbulent flow	checkMesh, U-92, U-162
steady, P-54	checkYPlus, U-95
turbulentInlet	couplePatches, U-92
boundary condition, U-152	createPatch, U-92
tutorials	decomposePar, U-84, U-85, U-95
breaking of a dam, U-56	deformedGeom, U-92
lid-driven cavity flow, U-19	divU, U-94
stress analysis of plate with hole, U-45	dxFoamExec, U-93
tutorials directory, P-45, U-19	engineCompRatio, U-95
type keyword, U-146	engineSwirl, U-91
type menu, U-178	ensight74FoamExec, U-93, U-192
•	ensight76FoamExec, U-93
\mathbf{U}	enstrophy, U-94
U field, U-25	equilibriumCO, U-95
Ucomponents utility, P-72, U-35, U-94	equilibriumFlameT, U-95
UMIST	estimateScalarError, U-96
keyword entry, U-113	
uncompressed	faceSet, U-92
keyword entry, U-110	fieldToCellSet, U-92
uncorrected	flattenMesh, U-92
keyword entry, U-115, U-116	fluentMeshToFoam, U-92, U-160
Undo button, U-177	foamConvert21To22, U-96
uniform model, U-99	foamDataToFluent, U-93, U-189
uniform keyword, U-195	foamDebugSwitches, U-96
units	foamInfoExec, U-96
of measurement, P-26	foamMeshToFluent, U-92, U-189
S.I. base, P-26	foamToDX, U-93
Update Times button, U-176	foamToEnsight, U-93
uprime utility, U-94	foamToFieldview9, U-94
upwind	foamToFieldview, U-93
keyword entry, U-114, U-117	foamToVTK, U-94
upwind differencing, P-38, U-63	gambitToFoam, U-92, U-160
Use parallel projection button, U-27, U-172	gmshToFoam, U-92
utility	icoErrorEstimate, U-96
Co, U-94	icoMomentError, U-96
FoamX, U-91	ideasToFoam, U- 92 , U- 160
Lambda2, U-94	insideCells, U-92
Mach, U-94	kivaToFoam, U - 92
Pe, U-94	liftDrag, U - 95
Q, U-94	magGradU, U - 94
Rcomponents, U-94	$magU,\ U\text{-}35,\ U\text{-}94$
R, U-94	makePolyMesh, U-92
SI, U-95	mapFields, U-33, U-40, U-43, U-55, U-95
Ucomponents, P-72, U-35, U-94	U-167
adiabaticFlameT, U-95	mergeMeshes, U-92

Index P-95 P-96 Index

mirrorMesh, U-93	vertices keyword, U-22, U-154
mixtureAdiabaticFlameT, U-95	veryInhomogeneousMixture model, U-98, U-200
momentScalarError, U-96	View menu, U-28, U-172, U-173
moveEngineMesh, U-93	View Angle text box, U-177
moveMesh, U-93	View Control
mshToFoam, U-92	
objToVTK, U-93	menu entry, U-177 viscosity
pointSet, U-93	kinematic, U-24, U-43
postChannel, U-95	Visualisation Type window panel, U-181
ptot, U-95	volField <type> template class, P-33</type>
reconstructPar, U-89, U-95	Volume button, U-181
refineMesh, U-93	vorticity utility, U-94
refineShapeMesh, U-93	vtkFoam
renumberMesh, U-93	library, U-169
sammToFoam, U-92	
sampleSurface, U-95	\mathbf{W}
sample, U-95, U-193	wall
scalePoints, U-93, U-164	boundary condition, P-65, P-71, U-149
smapToFoam, U-94	wall boundary type, U-42
splitMeshRegions, U-93	wall
splitMesh, U-93	keyword entry, U-149
starToFoam, U-92, U-160	wall function, U-99, U-100
stitchMesh, U-93	wallBuoyantPressure
streamFunction, U-94	boundary condition, U-152
stressComponents, $U-95$	wallDist tools, U-97
subsetMesh, U-93	wallFunctionCoeffs keyword, U-203
tetDecomposition, U-93	wallGradU utility, U-95
tetgenToFoam, U - 92	wallShearStress utility, U-95
uprime, U-94	water model, U-98
vorticity, U-94	wclean script/alias, U-78
wallGradU, U-95	wdot utility, U-95 wedge
wallShearStress, U-95	boundary condition, U-146, U-150, U-159
wdot, U-95	wedge
writeMeshObj, U-92	keyword entry, U-149
yPlusLES, U-95	window
zipUpMesh, U-93	Color Range, U-178
\mathbf{V}	Display, U-175, U-178
value keyword, U-151	Fields, U-28
valueFraction keyword, U-151	Graph, U-183
van Leer differencing, P-38	Image, U-175, U-176
VanLeer	Map Plane, U-178
keyword entry, U-114	Mesh, U-184
Vector text box, U-183	Message, U-175, U-177
vector, P-16	Read Data, U-175, U-176
operator, P-27	Region, U-28
unit, P-20	Seed, U-175
vector class, P-24, U-107	Selection Window, U-27, U-170
vector product, see tensor, vector cross product	Sequencer, U-176, U-184
vectorField class, P-29	Streamlines, U-182
version keyword, U-105	Time, U-28

Window Control, U-175, U-177	environment variable, U-78
Window Control window, U-175, U-177	WM_PROJECT_LANGUAGE
window panel	environment variable, U-78
Annotate, U-27, U-173	WM_PROJECT_USER_DIR
Camera Controls, U-173	environment variable, U-78
Camera Orientation, U-173	WM_PROJECT_VERSION
Camera, U-173	environment variable, U-78
Contour, U-181	WM_SHELL
Display, U-27, U-28, U-170, U-171	environment variable, U-78
General, U-172	wmake
Glyph Attributes, U-182	platforms, U-75
Information, U-170	wmake script/alias, U-73
Options, U-181	word class, P-26, P-31
Parameters, U-28, U-170, U-171	writeCompression keyword, U-110
Rubbersheet Attributes, U-181	writeControl
Standard Views, U-173	keyword entry, U-109
Stored Camera Position, U-173	writeControl keyword, U-25, U-62, U-109
Visualisation Type, U-181	writeFormat keyword, U-55, U-110
Windows menu, U-177	writeInterval keyword, U-25, U-35, U-110
Wireframe	writeMeshObj utility, U-92
menu entry, U-171	writeNow
WM_ARCH	keyword entry, U-109
environment variable, U-78	writePrecision keyword, U-110
WM_COMPILE_OPTION	#110011001210H hoj word, 0 110
environment variable, U-78	${f X}$
WM_COMPILER	x
environment variable, U-78	keyword entry, U-195
WM_COMPILER_BIN	XiFoam solver, U-90
environment variable, U-78	xmgr
WM_COMPILER_DIR	keyword entry, U-110, U-194
	Xoodles solver, U-91
environment variable, U-78 WM_COMPILER_LIB	xyz
	keyword entry, U-195
environment variable, U-78 WM_DIR	
	Y
environment variable, U-78 WM_JAVAC_OPTION	У
	keyword entry, U-195
environment variable, U-78 WM_LINK_LANGUAGE	yPlusLES utility, U-95
	${f z}$
environment variable, U-78	-
WM_MPLIB	Z
environment variable, U-78	keyword entry, U-195
WM_OPTIONS	zeroGradient
environment variable, U-78	boundary condition, U-151
WM_PROJECT	zipUpMesh utility, U-93
environment variable, U-78	zlib-1.2.1
WM_PROJECT_DIR	library, U-97
environment variable, U-78	Zoom
WM_PROJECT_INST_DIR	menu entry, U-177