

Course notes for EE394V

Restructured Electricity Markets: Locational Marginal Pricing

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Microeconomics

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6.1 Nomenclature

- We will need to discuss the functional relationship between demand and prices in a market.
- In contrast to the sciences and engineering, standard practice in economics is to graph price on the vertical axis, whether price is being considered the dependent or the independent variable:
 - the **demand function** is a function with price as its argument (the independent variable) and quantity demanded (or bought, or rented) as the dependent variable,
 - the **inverse demand function** is a function (if it exists) with quantity demanded (or bought, or rented) as its argument (the independent variable) and **willingness-to-pay** as its dependent variable, and
 - the phrase **demand curve** is sometimes used to refer to either or both of these relations without explicitly specifying whether price or quantity is the independent variable.

Nomenclature, continued

- Similarly, the **supply curve** shows the relationship between supply and prices in a market.
 - the **supply function** is a function with price as its argument (the independent variable) and the quantity produced (or sold, or rented) as the dependent variable,
 - the **inverse supply function** or **offer function** is a function (if it exists) with quantity produced (or sold, or rented) as its argument (the independent variable) and the required price as its dependent variable, and
 - the phrase supply curve is used to refer to the relation without explicitly specifying whether price or quantity is the independent variable.
- We first discussed the offer function in Section 5.3.4.
- Finally, note that the convention in economics (rather than in power engineering) is to use the symbols p or P for price and q or Q for quantity:
 - we will use lower case π for price, and
 - we will use x for decision variables generally, and
 - p or P for (average) real power and q or Q reactive power.

6.2 Example of apartment rental

- We will first consider a simple example based on apartment rental.
- We will subsequently generalize various aspects of this example to the case of an electricity market.

6.2.1 Renters

- Consider a number of possible renters in an area.
- Ask each of them their **willingness-to-pay** for an apartment in this area and graph this **reservation price** versus number of apartments, ordered from highest to lowest reservation price.
- The **demand curve** shows the number of renters willing to rent at a given price:
 - if I am willing to pay \$1000 per month then I would be happy to rent at any price at or below \$1000 per month.
- Figure 6.1 (based on Figure 1.1 of Hal R. Varian, *Intermediate Microeconomics*) shows the case where there are just a few renters and (strictly speaking) the inverse demand function does not exist:
 - we define the **benefit** (strictly speaking, the benefit per unit time) to the renters to be the sum, over the renters actually renting apartments, of their willingness-to-pay.

Renters, continued

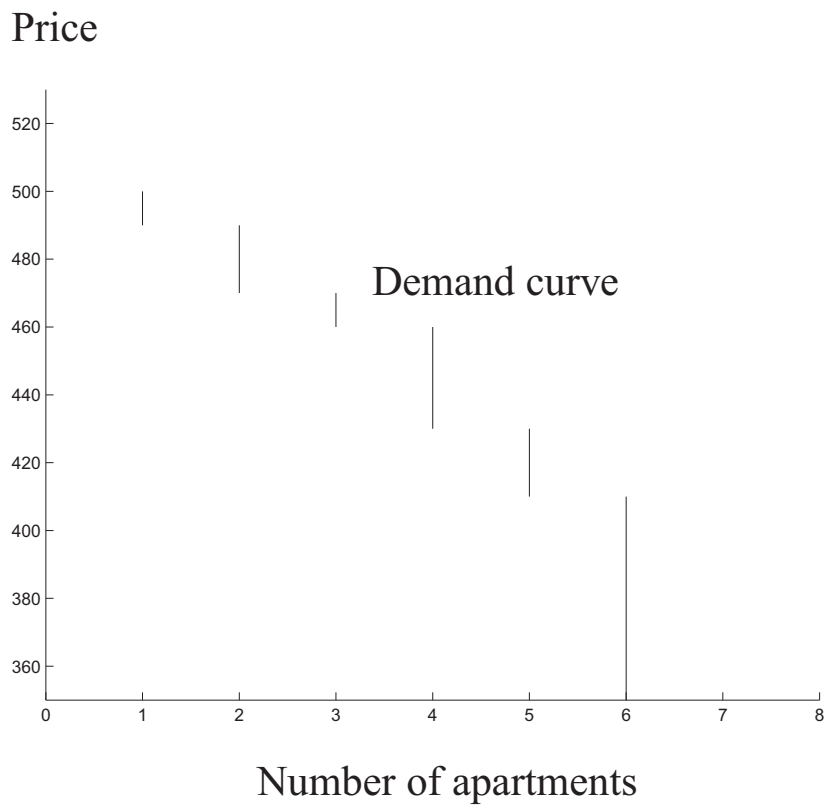


Fig. 6.1. The demand curve for apartments.
Source: This is based on Figure 1.1 of Hal R. Varian, *Intermediate Microeconomics*.

Renters, continued

- Figure 6.2 (based on Figure 1.2 of *Intermediate Microeconomics*) shows the case where there are enough renters so that we can think of the demand curve as being a continuous curve, so that both the demand function and the inverse demand function exists.
 - when we generalize to continuous-valued consumption, as in electricity, we will think of the benefit as being the integral of the willingness-to-pay over the range of consumption.
- We will also assume that there are arbitrarily many potential renters, so that the curve extends arbitrarily far to the right.

Renters, continued

Price

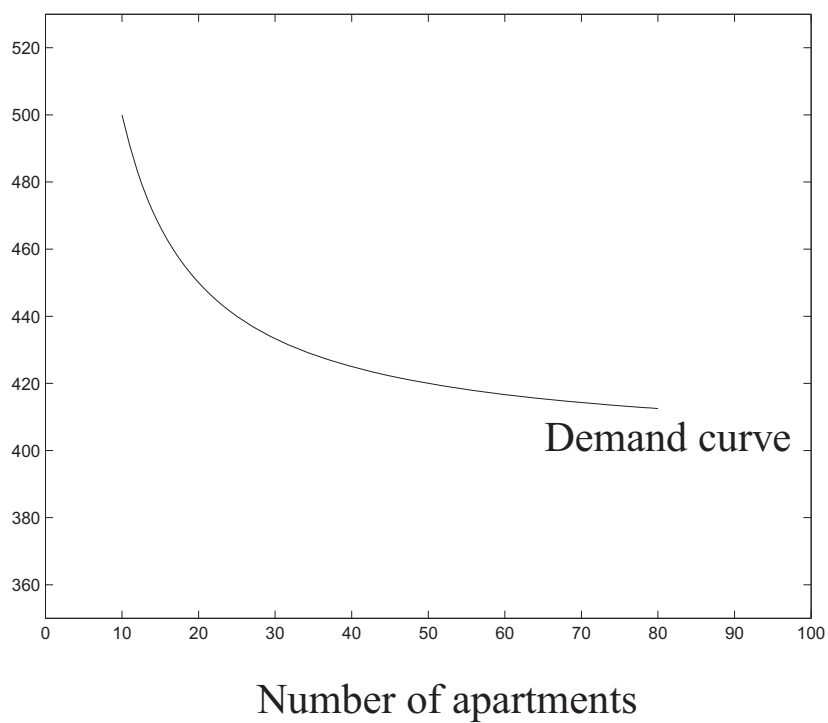


Fig. 6.2. The demand curve for apartments with many “demanders.”

Source: This is based on Figure 1.2 of Hal R. Varian, *Intermediate Microeconomics*.

6.2.2 Supply

- Consider a number, \bar{S} , of independent landlords offering to rent identical apartments on a month-to-month basis:
 - in the “short-run” this supply is fixed,
 - the supply is **inelastic**, meaning that the total number of apartments for rent stays fixed despite variations in price.
- Assume that there are no “operating costs” of leasing an apartment.
- At what price or prices would they rent?
 - If any two apartments are renting at different prices then the person paying the higher price would have an incentive to cut a deal with the owner of the lower priced apartment to rent at an intermediate price.
 - So, if any rental prices are different, some landlord and renter want to change the rental arrangements to their mutual benefit.
- So, all prices paid by the renters must be same in the “equilibrium,” defined to be where no group of renters and landlords could change their situation for mutual benefit:
 - despite the lack of any centralized coordination, price in the market for apartments will tend towards an equilibrium of uniform prices.

Supply, continued

- Consider operating profit maximization by a landlord facing a price $\pi > 0$:
 - if the landlord does not rent the apartment then the operating profit is zero,
 - if the landlord does rent the apartment then the operating profit is $\pi \times 1$.
- So, if $\pi > 0$ then all \bar{S} apartments will be rented.
- Conversely, if the supply of apartments rented is \bar{S} then any positive price could occur.
- Figure 6.3 (based on Figure 1.3 of *Intermediate Microeconomics*) shows the supply curve as a vertical line that represents the number, \bar{S} , of apartments that could be rented:
 - (strictly speaking) the inverse supply function does not exist in this case.

Supply, continued

Price

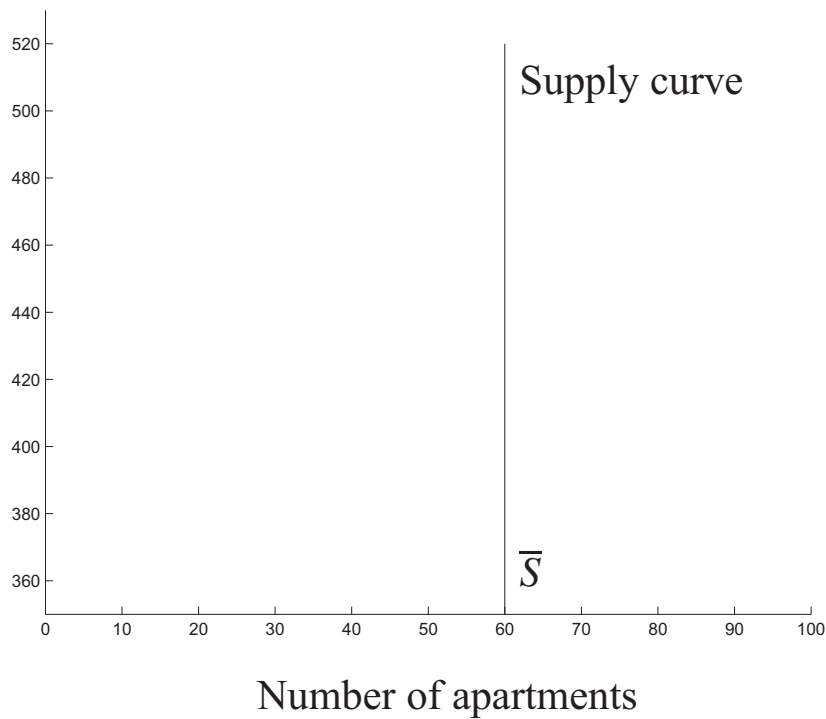


Fig. 6.3. The supply of apartments is fixed in the short-run.

Source: This is based on Figure 1.3 of Hal R. Varian, *Intermediate Microeconomics*.

6.2.3 Equilibrium

- Which possible renters will actually get to rent?
- What price and allocation of apartments to renters would result in no one wanting to change their situation:
 - the equilibrium price and quantity.
- Consider the intersection of demand curve and the vertical line representing the number of apartments.
- The intersection defines a price $\pi = \pi^*$ at the quantity $x = \bar{S}$.
- What price will the renters pay?
- See Figure 6.4 (based on Figure 1.4 of *Intermediate Microeconomics*).

Equilibrium, continued

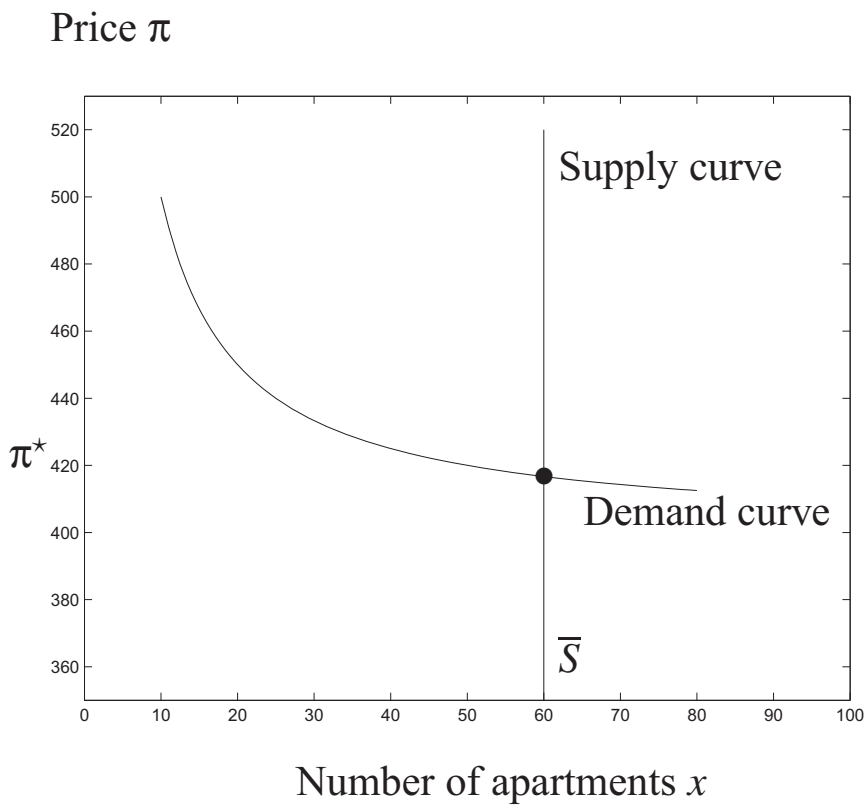


Fig. 6.4. The equilibrium price π^* is determined by the intersection of supply and demand.

Source: This is based on Figure 1.4 of Hal R. Varian, *Intermediate Microeconomics*.

Equilibrium, continued

- We have already argued that there will be a single price in the market that every renter will pay.
- Suppose that the single price π is above π^* :
 - only the people with willingness-to-pay at least π , which is above π^* will rent,
 - so less than \bar{S} apartments would be rented at the price π ,
 - one of the landlords of the unrented apartments would offer to lease at a cheaper price than π to “capture” one of the renters,
 - so not in equilibrium.
- Suppose that the single price π is below π^* :
 - more than \bar{S} renters would want to rent at the price π ,
 - only \bar{S} apartments could be rented,
 - some of the unsatisfied renters with willingness-to-pay that is more than π would be willing to pay more than π to “capture” one of the apartments,
 - so not in equilibrium.
- So, the price must be π^* in equilibrium with all \bar{S} apartments rented.

6.3 Market clearing price and surplus

- The price π^* is called the **market clearing price**, since it is the price that equates supply to demand.
- At this price:
 - neither renters, potential renters, nor landlords have any desire to change their situation,
 - the potential renters with the highest willingness-to-pay actually rent the apartments,
 - summed across the renters, the total willingness-to-pay, or benefit (strictly speaking, the increase in benefit per unit time) is maximized.
- The benefit minus the operating costs is called the **surplus** or **welfare**.
- In this example the operating costs of the apartments are zero:
 - we will consider non-zero operating costs in Section 6.5.
- At the market clearing price π^* , surplus is maximized:
 - the price π^* is the same as the Lagrange multiplier on supply-demand balance in the equality-constrained problem of maximizing benefit minus operating costs subject to supply-demand balance.
 - will also be true in the case of non-zero operating costs.

Market clearing price and surplus, continued

- Using sensitivity analysis from Theorem 4.11, the market clearing price π^* is the sensitivity of surplus to changes in supply or demand:
 - in this context, it is sometimes called the **marginal surplus**.
- The price π^* provides the correct incentive for actions by individual renters and landlords to result in maximizing the surplus:
 - if a central agent announced this price then the actions of individual renters and landlords in response to this price would result in maximizing the surplus.
- We can divide the surplus into:
 - **consumer surplus**, which is the benefit minus the total paid by the renters, and
 - **producer surplus**, which is the total paid by the renters to the landlords.

6.4 Longer term issues

- In the longer term, new investors may decide to build new apartments (or new generating stations):
 - If the price π^* is high enough to support the investment (and is expected to stay high enough to support the investment) then new apartments will be built.
- In the longer term, there may be more potential renters:
 - this will tend to increase the equilibrium price,
 - but subsequent new investment will tend to decrease the price again,
 - amount of new entry and new equilibrium price depends on relation between willingness-to-pay of new renters and construction costs of new apartments.

6.5 Operating costs

- So far, we have assumed that the operating cost is zero.
- What would happen if there were operating costs of renting the apartments were non-zero:
 - apartment 1 has operating cost $c_1 = \$150$ per month,
 - apartment 2 has a slightly higher operating cost $c_2 = \$155$ per month,
 - and so on.
- In this case, instead of a vertical supply curve, the supply curve would equal c_1 at a supply of one apartment, c_2 at a supply of two apartments, etc.
- There would still typically be an intersection of demand and supply:
 - but there would be no intersection if c_1 were above the highest willingness-to-pay of the renters.
- At the intersection, the difference between the benefit of renting the apartments minus the cost of supplying apartments is maximized:
 - that is, the surplus is maximized.
- There are also operating costs of supply in electricity markets.

Operating costs, continued

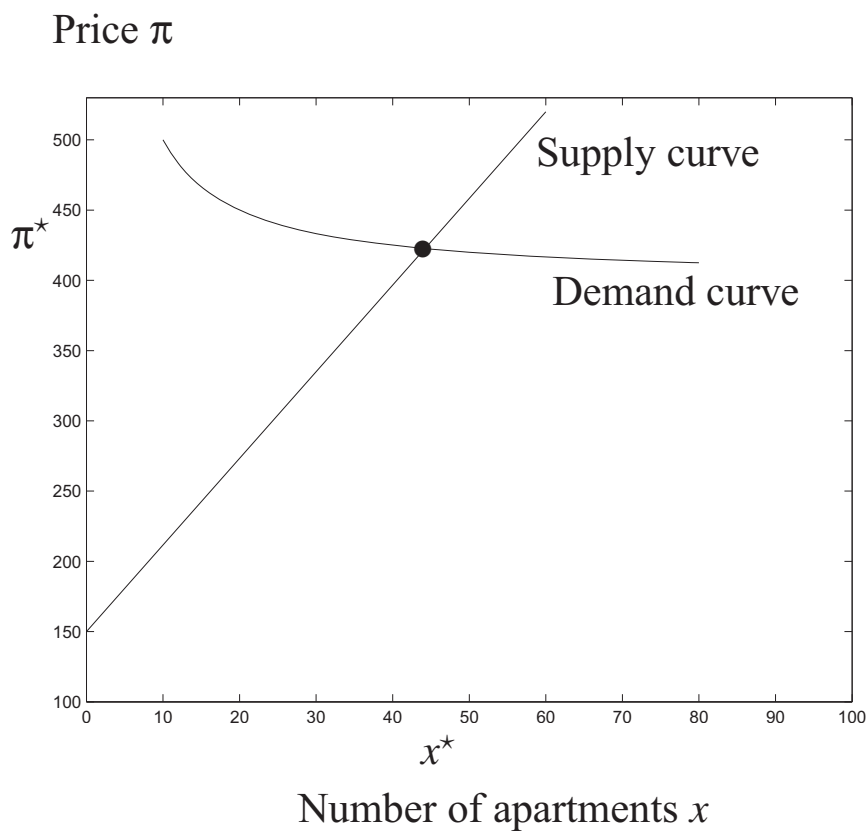


Fig. 6.5. The supply curve and the equilibrium with variable costs.

Operating costs, continued

- The intersection of supply and demand is at an equilibrium price $\pi = \pi^*$ and quantity $x = x^*$.
- The surplus is the area between the supply and demand curves up to the quantity x^* supplied.
- We can again divide the surplus into:
 - **consumer surplus**, which is the benefit minus the total paid by the consumers to the producers, and
 - **producer surplus**, or operating profit, which is the total paid to the producers minus the operating cost of production.
- In the electricity context, operating costs are primarily due to the cost of fuel, as we have discussed in the context of economic dispatch.
- “New” generators tend to have higher efficiency than “old” generators:
 - ignoring changes in fuels and fuel costs, in the long-term, the supply curve tends to shift “down” as new capacity is introduced (and old capacity is retired),
 - changes in fuels and fuel costs can either reinforce or can act in opposition to this trend.

6.6 Inelastic demand

- In electricity markets particularly, the demand may not change much with price:
 - demand is said to be **inelastic**,
- Several reasons for demand inelasticity:
 - consumers may not care about price, since their willingness-to-pay for electricity may be far higher than the price, and
 - consumers (residential retail) may not be exposed (directly) to wholesale prices.
- In this case, instead of a “downward sloping” demand curve, the demand curve is vertical.
- There will still usually be an intersection of supply and demand.
- However, for example, in extreme weather conditions there may be no (apparent) intersection of supply and demand:
 - key question: what is appropriate rule for setting price,
 - we will re-visit this issue in the context of defining the “value of lost load.”

Inelastic demand, continued

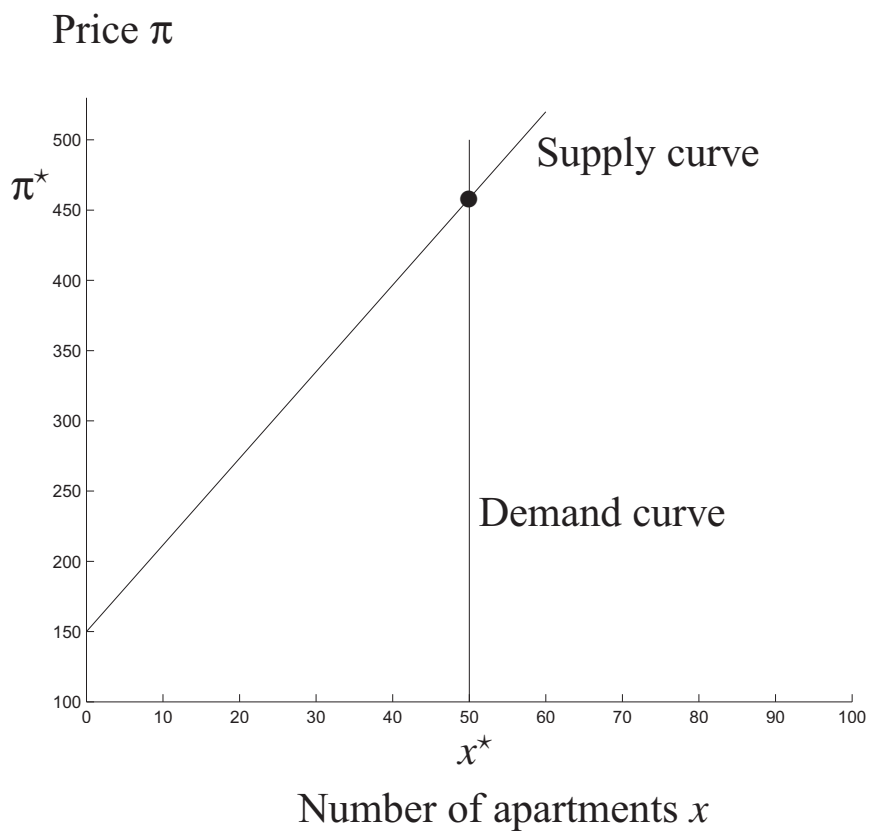


Fig. 6.6. The supply curve and equilibrium with variable costs and inelastic demand.

6.7 Spot and forward markets

- The description so far is most applicable to a **spot market** where producers and consumers interact to trade:
 - in electricity, this is called a **real-time market**.
- The actual price (and amount traded) will depend on the interaction between supply and demand at the time of trading and is uncertain.
- In a **forward contract** or **futures contract** parties decide on an amount to trade and a price in advance:
 - enables a contracted quantity to be sold at a fixed price, avoiding the variability of the uncertain “spot” price,
 - deviation of actual from contracted quantity is still sold at the spot price,
 - in electricity, the **day-ahead market** is a forward market.
- Futures and forward trading is especially important in electricity because the prices are very variable or **volatile**.
- By “locking in” prices, future and forward trading helps with investment:
 - investor or bank typically require evidence that investment will be repaid,
 - forward contract provides more certainty than spot prices.

6.8 Summary

- (i) Example of apartment rental,
- (ii) Renters,
- (iii) Supply,
- (iv) Equilibrium,
- (v) Market clearing price,
- (vi) Longer term issues,
- (vii) Operating costs,
- (viii) Inelastic demand,
- (ix) Spot and forward markets.

This chapter is based on:

- Chapter 1 of Hal R. Varian, *Intermediate Microeconomics*, Norton, 2006,
- Chapter 2 of Daniel S. Kirschen and Goran Strbac, *Power System Economics*, Wiley, 2004.

Homework exercises

6.1 Consider the following supply functions $S : \mathbb{R} \rightarrow \mathbb{R}$ and the demand functions $D : \mathbb{R} \rightarrow \mathbb{R}$. For each pair, find the equilibrium price π^* and corresponding quantity x^* .

- (i) $\forall \pi, S(\pi) = \pi, D(\pi) = 10 - \pi,$
- (ii) $\forall \pi, S(\pi) = 60, D(\pi) = 1000/(\pi - 400),$
- (iii) $\forall \pi, S(\pi) = 60(\pi - 150)/(520 - 150), D(\pi) = 1000/(\pi - 400).$

6.2 In Section 6.2.2 it was assumed that each of the \bar{S} apartments were owned by different landlords. Without any knowledge of the demand curve, except for assuming that there were arbitrarily many renters, we showed that all \bar{S} apartments were rented.

- (i) How would the profit maximization problem change for a landlord who owned two apartments? Can we determine how many apartments are rented in equilibrium without knowing the demand curve?
- (ii) How would the profit maximization problem change if there were only a single landlord owning all \bar{S} apartments? Can we determine how many apartments are rented in equilibrium without knowing the demand curve?