15.094J Robust Modeling, Optimization and Computation

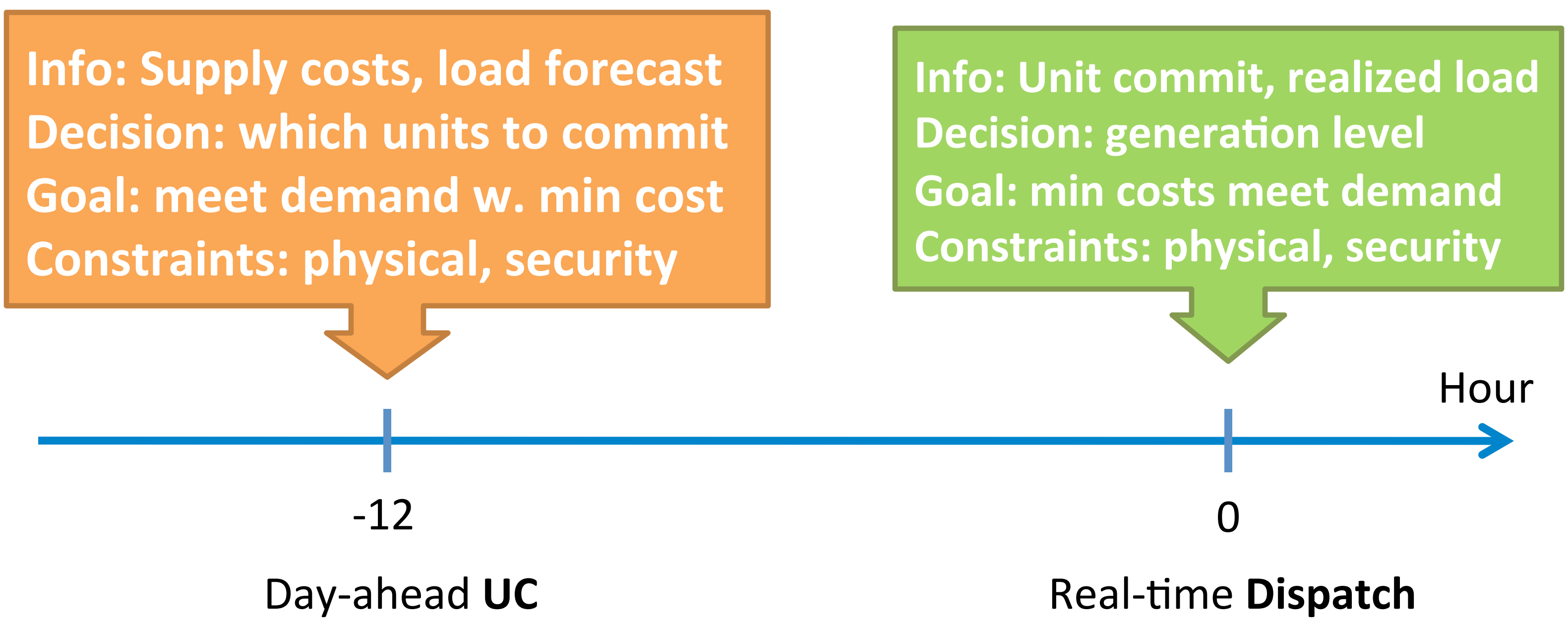
Lecture 14: RO in the unit commitment problem in electricity production

Outline

- Background
  - Unit commitment problem
- New challenges
  - Increasing uncertainty in supply/demand
- Adaptive Robust Optimization
- Conclusions

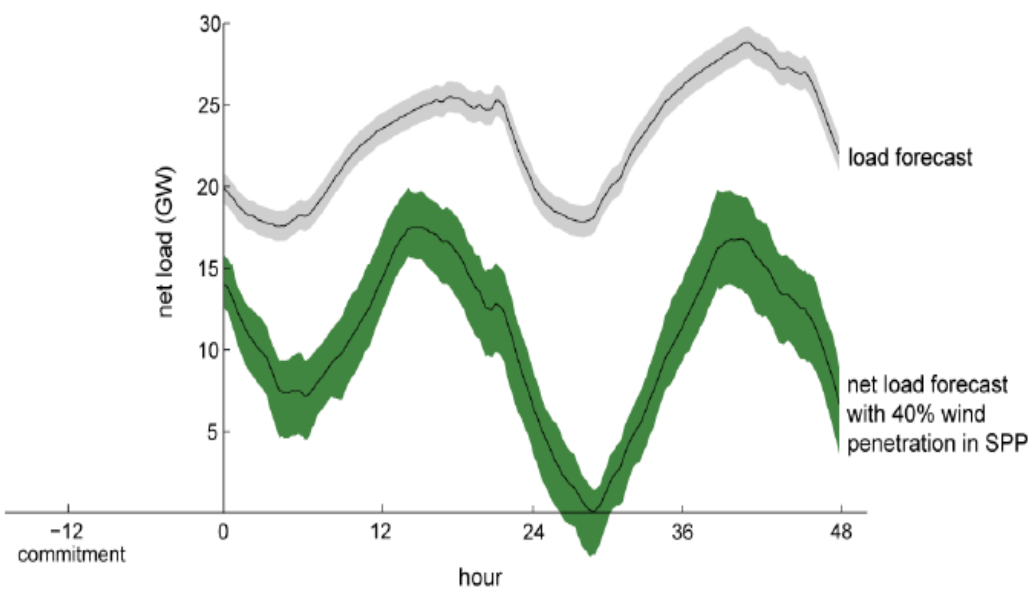
Electric Power System Operations

- Day-Ahead Decision Making: Unit Commitment
  - Generators must be committed before real-time operation (long startup time)



Challenges: Growing Uncertainty

- Supply uncertainty (Renewables like wind are exhibiting 40% annual growth)
- Demand uncertainty



Current Practice: Reserve Adjustment

- Deterministic Reserve adjustment approach

  Incorporating extra resources called reserve

  [Sen and Kothari 98] [Billinton and Fotuhi-Firuzabad 00]

Drawbacks:

1. Uncertainty not explicitly modeled
2. Both system and locational requirement are preset, heuristic, ad hoc

Current practice: MIO

Min c'x +b'y

s.t. Fx<f   (min-up/down times, start-up/shut-down)

Hy<h (energy balance, reserve requirement and capacity ransmission limit, and ramping constraints)

Ax+By<g (min-max generation capacity constraints)

I_u y=d

x binary (commitment variables)

y>0 (dispatch variables).

Stochastic Optimization

- Stochastic optimization approach

  Uncertainty modeled by distributions and scenarios

  [Takriti et. al. 96, 00] [Ozturk et. al. 05][Wong et. al. 07]

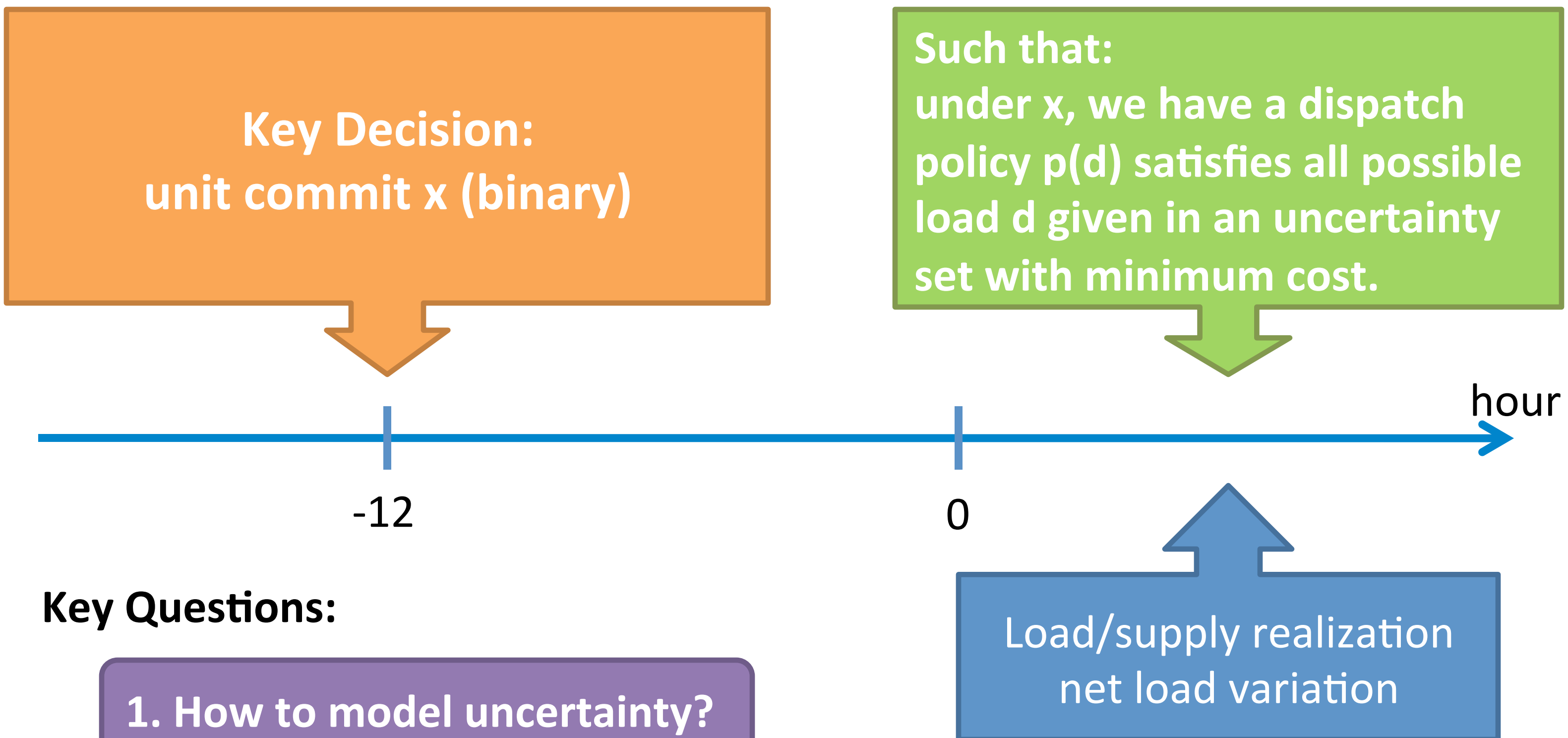
  [Wu et. al. 07]

Weakness:

1. Hard to select "right" scenarios in large systems
2. The huge number of scenarios needed make the problem intractable for large scale problems.

Adaptive Robust Optimization

- Two-stage robust optimization framework



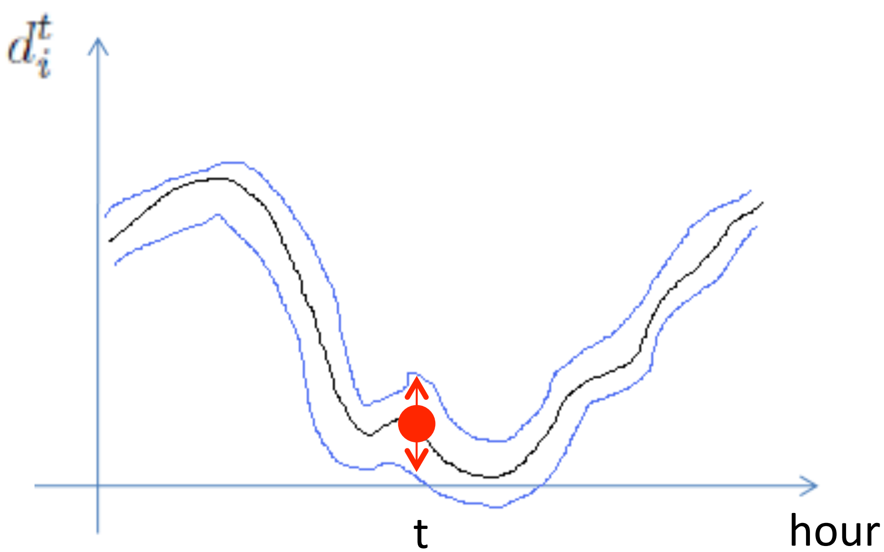
Key Questions:

1. How to model uncertainty?
2. What policy p(d)?

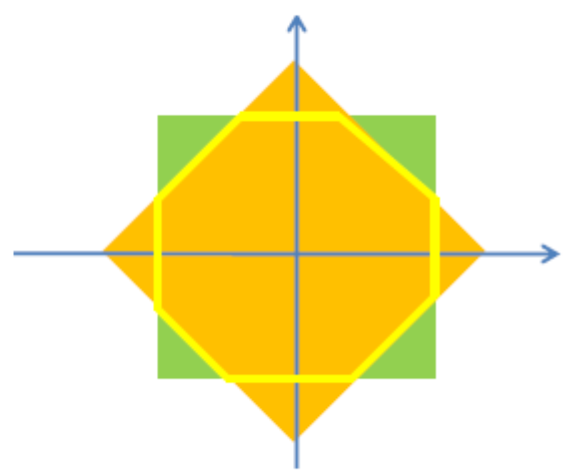
Model of Uncertainty

- Uncertainty model of net load variation

$$\mathcal{D}^t(\bar{\mathbf{d}}^t, \hat{\mathbf{d}}^t, \Delta^t) := \Big\{ \mathbf{d}^t \in \mathbb{R}^{N_d} : \sum_{i \in N_d} \frac{|d_i^t - \bar{d}_i^t|}{\hat{d}_i^t} \le \Delta^t, \\ d_i^t \in [\bar{d}_i^t - \hat{d}_i^t, \bar{d}_i^t + \hat{d}_i^t], \forall i \in N_d \Big\}$$



$d_i^t \in [\bar{d}_i^t - \hat{d}_i^t, \bar{d}_i^t + \hat{d}_i^t]$



Budgeted

ARO

$$\text{Min}_{x,\, y(d)} \quad c'x + \max_{d \text{ in } D} \quad b'y(d)$$

s.t. $Fx<f$

$H\, y(d)<h$ for all d in D

$Ax+By(d) <g$ for all d in D

$I_u\, y(d)=d$ for all d in D

x binary

$y(d)>0$

$d=(d^t, t=1,\ldots,T)$

$D=D^1 \times D^2 \times \ldots \times D^T$

Re-formulation of ARO

$\text{Min}_x \;(c'x + \max_{d \text{ in } D} \min_{y \text{ in } O(y,d)} \; b'y)$

s.t. $Fx<f$, x binary

$O(y,d)=\{y: H\,y(d)<h,\ Ax+By(d) <g,\ I_u\,y(d)=d\}$

By duality $\min_{y \text{ in } O(y,d)} \; b'y$ is the same as

$S(x,d)=\max_{\lambda,\,\phi,\,\eta} \lambda'(Ax-g)-\phi'h+\eta'd$

s.t. $-\lambda'B-\phi'H+\eta'I_u = b'$

$\phi>0,\ \lambda>0,\ \eta$ free

Re-formulation of ARO

$R(x)=\max_{d \text{ in } D} \min_{y \text{ in } O(y,d)} b'y$ can be rewritten:

$R(x)=\max_{d,\lambda,\phi,\eta} \lambda'(Ax-g)-\phi'h+\eta'd$ bilinear problem

s.t. $-\lambda'B-\phi'H+\eta'I_u = b'$

d in D

$\phi>0, \lambda>0, \eta$ free

Benders Decomposition Algorithm

- Initialization: Get feasible x_0, solve bilinear problem $R(x_0)$ and obtain $d_1, \lambda_1, \phi_1, \eta_1$. Set k=1.
- Iteration k:
  - Step 1: Solve MIO: $\min_{x,a} \; c'x+a$

    $$\text{s.t. } a > \lambda_\iota'(Ax-g)-\phi_\iota'h+\eta_\iota'd, \; i=1,\ldots,k$$

    $$Fx<f, \; x \text{ binary.}$$

    Let (x_k,a_k) optimal solution. Set $L=c'x_k+a_k$
  - Step 2: Solve $R(x_k)$ and obtain $d_{k+1}, \lambda_{k+1}, \phi_{k+1}, \eta_{k+1}$.

    Set $U=c'x_k+R(x_k)$

  -- Step 3: If $U-L<\varepsilon$, stop and return x_k

    otherwise k=k+1 , go to Step 1.

Inner Problem: Solving R(x)

- Recall $R(x)=\max_{d,\lambda,\phi,\eta} \lambda'(Ax-g)-\phi'h+\eta'd$

  $$\text{s.t.} \quad -\lambda'B-\phi'H+\eta'I_u = b'$$

  $$d \text{ in } D,\ \phi>0,\ \lambda>0,\ \eta \text{ free}$$

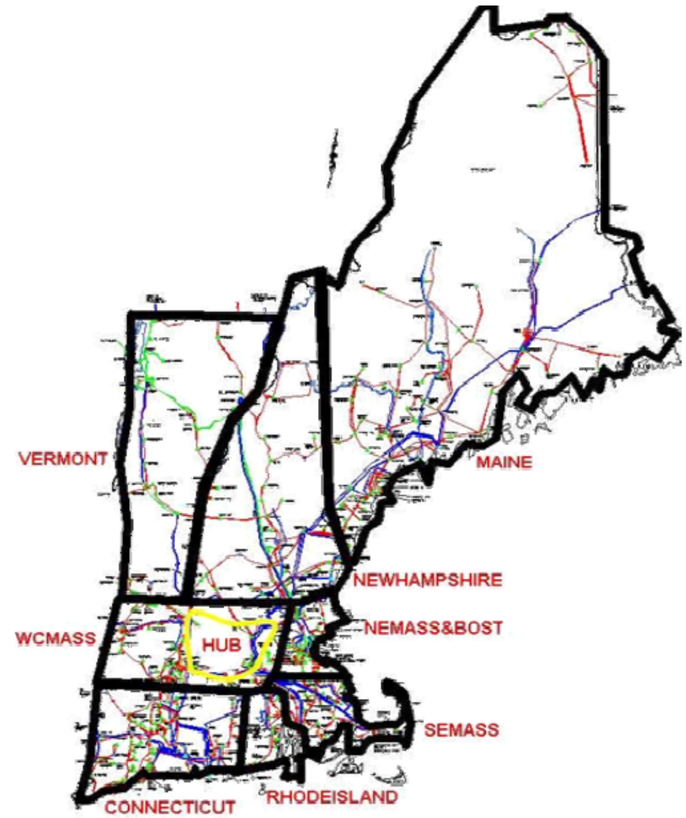
- Idea: Linearize: $\eta'd=\eta_j'd_j+(\eta-\eta_j)'d_j+(d-d_j)'\eta_j$

- Algorithm converges to stationary point

- In practice 2-3 iterations are needed.

A Real-World Example: ISO-NE Power System

- 312 Generators
- 174 Loads
- 2816 Nodes
- 4 representative trans lines
- 24-hr data: gen/load/reserve
- Total gen cap: 31.4GW
- Total forecast load: 14.1GW

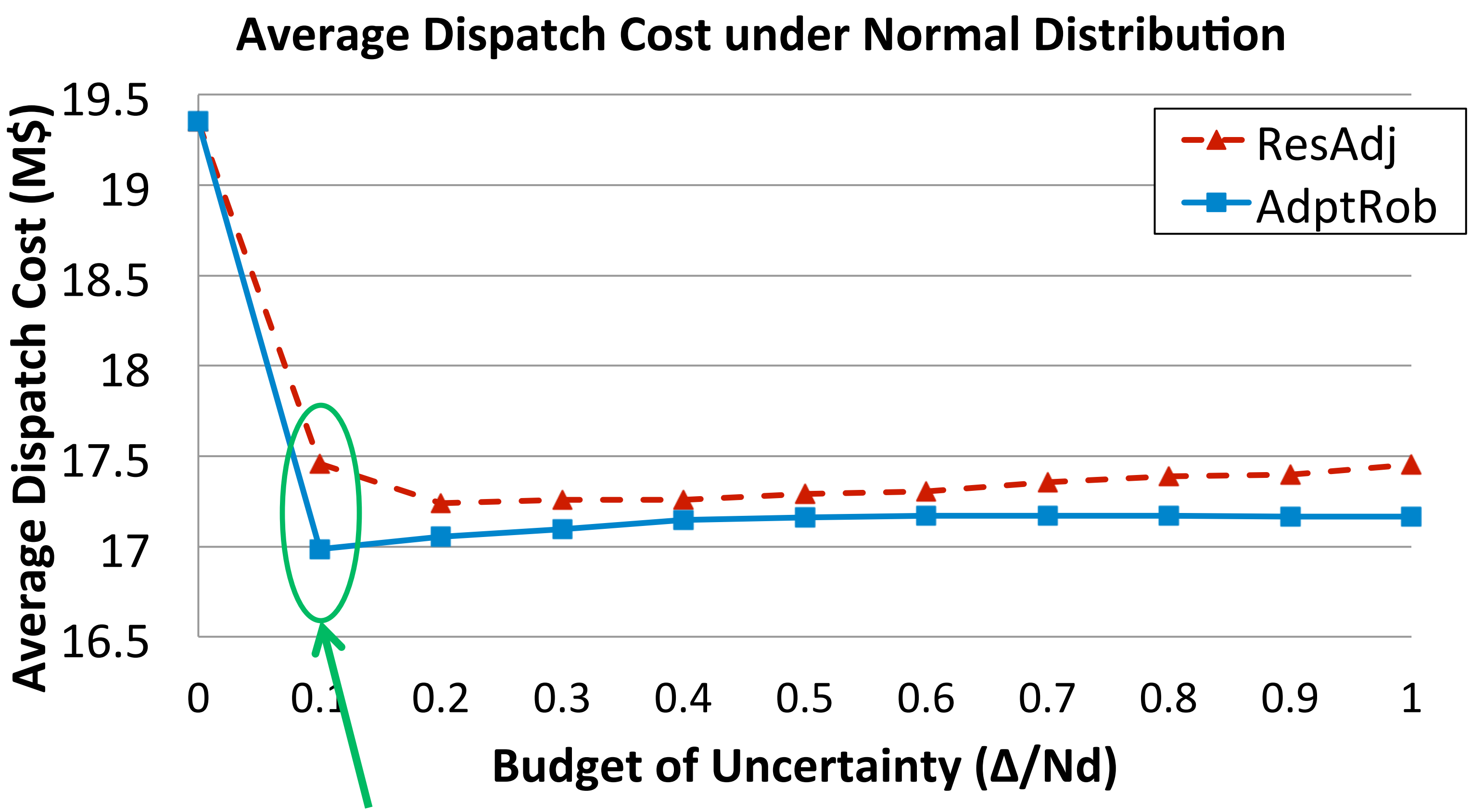


Computation Procedure and Measures

- Solve AdptRob and ResAdj UC solutions for
  Δ^t = 0, 0.1Nd,…,Nd for all t.
- Fix UC solutions, simulate dispatch over load samples
  - 1000 load samples from $[\bar{d}_i^t - \hat{d}_i^t, \bar{d}_i^t + \hat{d}_i^t]$
- Compute average dispatch cost and std.

- Avg dispatch cost: Economic efficiency
- Standard deviation: Price and Operation Stability
- Robustness to distributions

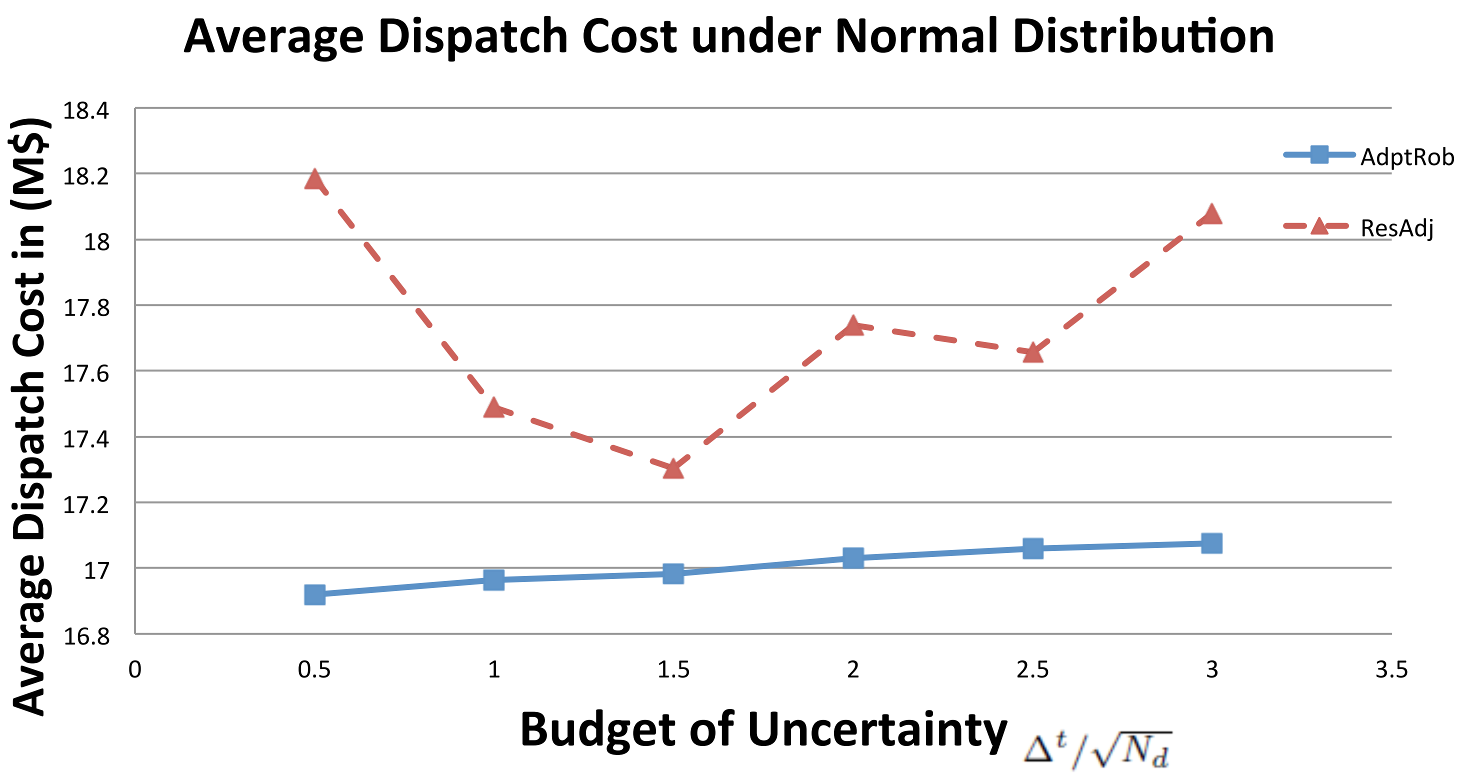
Computational Results (I-a): Average dispatch cost



2.7% relative saving or 472.9k$

Avg Dispatch Cost Relative Saving := (ResAdj – AdptRob)/ResAdj **0.65% - 2.7%**

Design using Probability Law: Average dispatch cost



Design the uncertainty set
By probability law: CLT

$\Delta^t/\sqrt{N_d}$ vs Δ^t/N_d

Relative Saving: 1.86% to 6.96%
Absolute Saving: $321.2k to $1.27Million

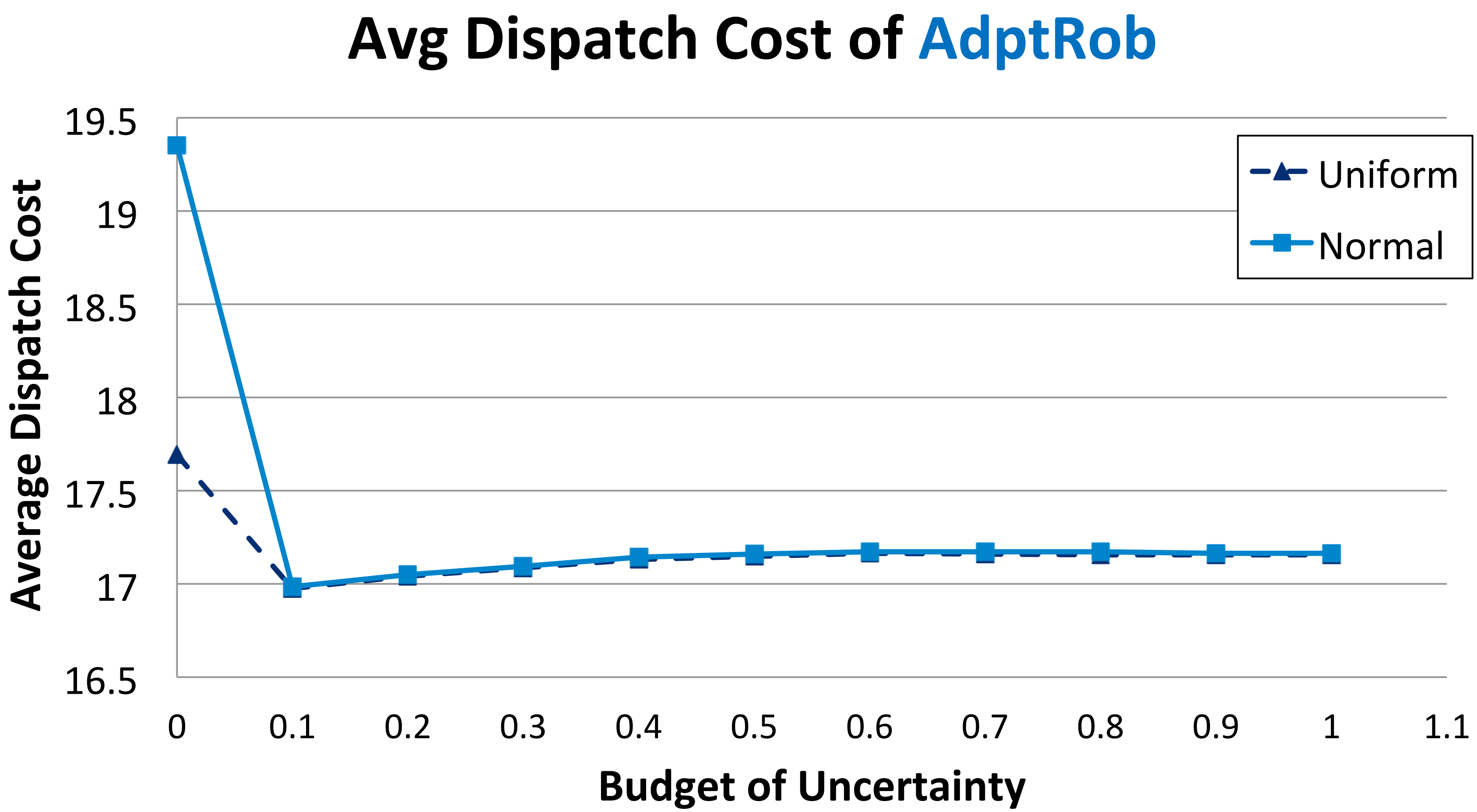
Computational Results (II): Volatility of Costs

Budget of Uncertainty	AdptRob Std disp cost ($k)	ResAdj Std disp cost ($k)	ResAdj/ AdptRob
0.1	47.5	687.5	**14.48**
0.2	46.4	687.5	**8.62**
0.3	45.4	377.8	**8.32**
0.4	44.2	366.7	**8.29**
0.5	44.1	377.2	**8.55**
0.6	44.0	370.9	**8.43**
0.7	44.0	377.1	**8.58**
0.8	43.9	370.7	**8.44**
0.9	43.9	357.9	**8.15**
1.0	43.9	361.0	**8.22**

Coeff Var: 44k/17.2M=0.25%
370k/17.3M=2.1%

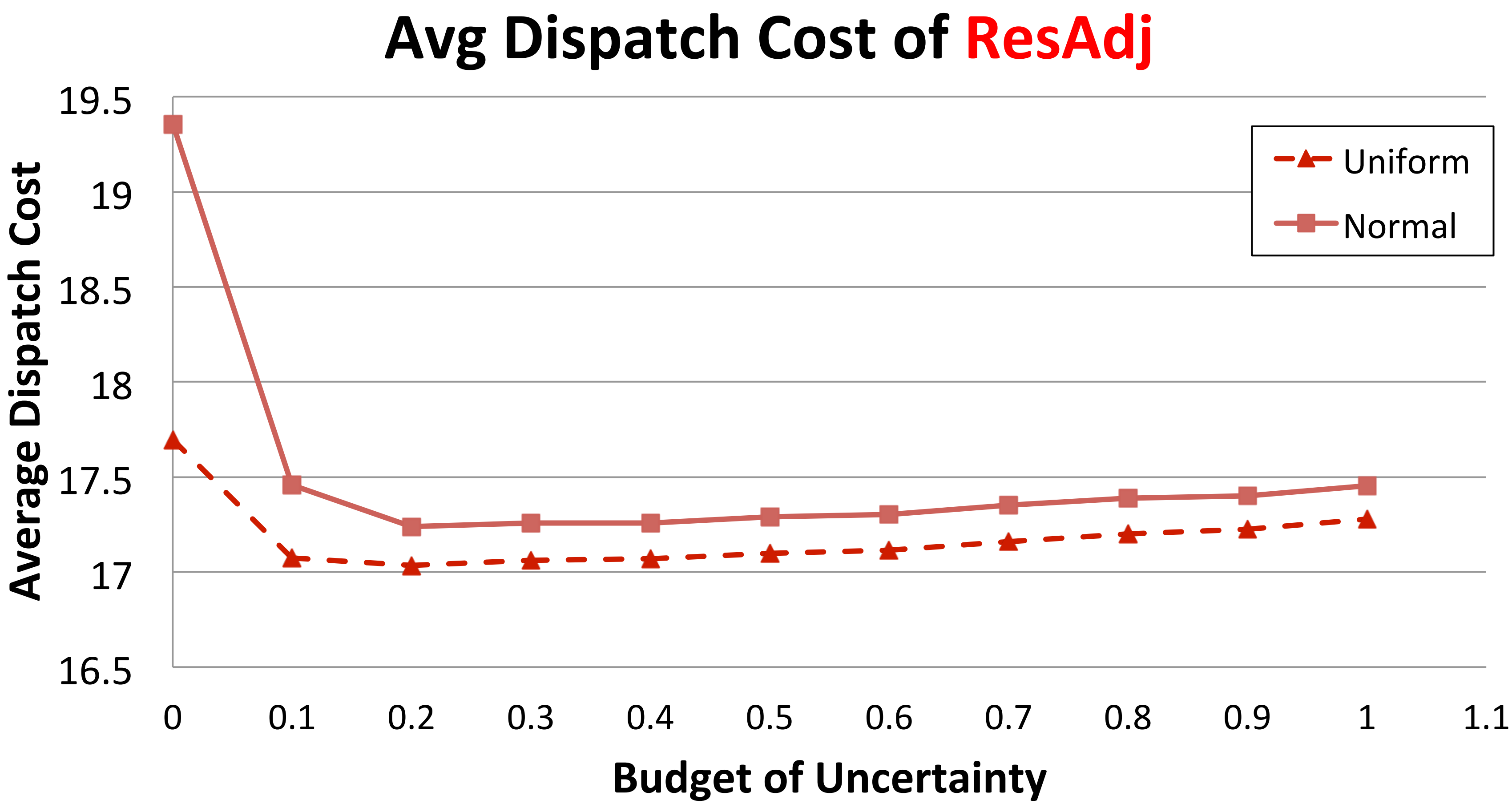
Significant reduction in cost volatility!

Computational Results (III): Robustness to Distribution



Relative difference: 0.0368% - 0.0920%
Absolute difference: \$6.3k – \$15.8k

Computational Results: Robustness to Distribution



Relative difference: 1.00% - 2.19%
Absolute difference: \$174.4k – \$382.2k

Concluding Remarks

- saves dispatch cost (6.92% $1.27M) → Economic Efficiency
- Significantly reduces cost volatility → Reduces Price & System Operation Volatility
- robust against load distributions → Data Driven Approach Demand Modeling

Reference: Adaptive Robust Optimization for Security Constrained Unit Commitment Problems, D. Bertsimas, E. Litvinov, A. Sun, J. Zhao, T. Zheng, *IEEE Transactions on Power Systems 2012*