15.094J: Robust Modeling, Optimization, Computation

Lecture 16: Robust Option Pricing

Outline

What are options and why they are important?

- A call option on a stock of strike \$50 with maturity 3 months, gives the right to buy the stock 3 months from now at \$50. So, if the price at that time is \$90, then there is a profit of \$40.
- A put option gives the right to sell the stock.
- American versus European options.
- Call options are a widespread method of compensation for executives.
- Put options provide insurance.
- The derivatives industry: 10 trillion dollar industry

Basics

- An option is a contract defined on a set of *predetermined underlying securities*

$$\mathbf{S} = \{S_i\}_{i=1,\dots,M},$$

and is associated with a *payoff function*

$$P_f(\mathbf{S}, \mathbf{K}, T),$$

where $\mathbf{K}$ and T are a set of parameters specified in the contract.

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Example. European Call Option

The option holder has the right to buy a unit of stock at a price of K, at time T, even when the market price is S_T. The payoff, then, is

$$P_f(S, K, T) = \max(S_T - K, 0).$$

Dynamics

- Given a stock of price S_t

$$S_{t+1} = \begin{cases} u \cdot S_t & \text{with probability } q \\ d \cdot S_t & \text{with probability } 1-q \end{cases}$$

- Bond with price \$1

$$\$1 \rightarrow r, \quad r \text{ Riskless Return}$$

Payoff of an option

Consider an option (also called derivative security):

$$C_{t+1} = \begin{cases} C_u(S_t) & \text{with probability } q \\ C_d(S_t) & \text{with probability } 1-q \end{cases}$$

- What should the price of the option be?
- What should it depend on?

European call option

- $f(S) = \begin{cases} 0 & S \leq K \\ S - K & S > K \end{cases}$
- $C_u = C_u(S) = \max(u \cdot S - K, 0)$
- $C_d = C_d(S) = \max(d \cdot S - K, 0)$.

The Idea of no-arbitrage

- The story of the Princeton professor....
- Consider a portfolio that has x \$ worth of the stock and B \$ worth of the bond that pay off $B \cdot r$.
- Cost of portfolio: $x + B$
- Return of portfolio:

$$\begin{cases} x \cdot u + B \cdot r & \text{with probability } q \\ x \cdot d + B \cdot r & \text{with probability } 1 - q \end{cases}$$

Critical Idea

- Choose x, B so that we create the same payoff structure as the option, i.e.,

$$\begin{aligned} x\cdot u \;+\; B\cdot r &= C_u \\ x\cdot d \;+\; B\cdot r &= C_d \end{aligned}$$

- Solving the Linear System

$$x^* = \frac{C_u - C_d}{u-d}, \qquad B^* = \frac{u\cdot C_d - d\cdot C_u}{(u-d)r}$$

Critical Idea, continued

- What should the price of the option be?
- By No-Arbitrage

$$\underbrace{C}_{\substack{\text{Cost of}\\ \text{Option}}} = \underbrace{x^* + B^*}_{\text{Cost of Portfolio}}$$

- Since they have identical payoffs, they should have the same cost; otherwise there exists an arbitrage opportunity.

The Price

- By no arbitrage:

$$
\begin{aligned}
C &= \frac{C_u - C_d}{u-d} + \frac{u \cdot C_d - d \cdot C_u}{(u-d)r} \\
&= \frac{1}{r}\left[p \cdot C_u + (1-p) C_d\right] \quad , \quad p = \frac{r-d}{u-d}
\end{aligned}
$$

- Price of option is the present value of the expected payoff of the option, but not using the original probability q, but probability p, the risk neutral measure.

Multiple periods

- Consider now T periods
- Payoff of European call option

$$f(S_T) \;=\; (S_T - K)^+ \;=\; \max(S_T - K, 0)$$

- Price

$$C = \frac{1}{r^T} \sum_{n=0}^{T} \binom{T}{n} p^n \, (1-p)^{T-n} \, (u^n \, d^{T-n} \, S - K)^+$$

- Independent of probability q

Limitations

- Options depending on many securities problematic computationally.
- Modeling transaction costs, liquidity issues and market restrictions problematic conceptually and computationally.

The Model for stock returns

- Consider discrete time $\{0, 1, 2, \ldots, T\}$.
- Let $\widetilde{r}_t^S$ be the return at t; i.e., the return from period $[t, t+1)$.
- Assuming $\{\widetilde{r}_1^S, \widetilde{r}_2^S, \ldots, \widetilde{r}_\tau^S\}$ are independent, we have from the central limit theorem,

$$\frac{\sum_{i=1}^{\tau} \log\left(1 + \widetilde{r}_i^S\right) - \tau \cdot \mu_{\log}}{\sigma_{\log} \cdot \sqrt{\tau}} \sim N(0, 1),$$

where $\mu_{\log}$, $\sigma_{\log}$ are mean and standard deviation of $\log\left(1 + \widetilde{r}_i^S\right)$, respectively.

The Model for stock returns, continued

- The CLT motivates us to consider constraints of the form

$$\left|\frac{\log \widetilde{R}^S_\tau - \tau \cdot \mu_{\log}}{\sigma_{\log} \cdot \sqrt{\tau}}\right| \leq \Gamma_\tau \qquad \forall \tau,$$

- $\widetilde{R}^S_\tau = \prod_{i=1}^{\tau}(1 + \widetilde{r}^S_i)$, is the cumulative return up to time τ and Γ_τ is some parameter.
- Other constraints can be based on the the assumption of a bounded support

$$\mu_r - \Gamma_\tau \sigma_r \leq \frac{\widetilde{R}^S_\tau}{\widetilde{R}^S_{\tau-1}} \leq \mu_r + \Gamma_\tau \sigma_r \qquad \forall \tau.$$

The Model for stock returns, continued

- Uncertainty set for stock returns:

$$\mathbb{U}^1 = \left\{ \widetilde{R}_t^S \;\middle|\; \begin{array}{ll} \underline{R_t^S} \le \widetilde{R}_t^S \le \overline{R_t^S}, & \forall t = 1 \dots T \\ \underline{r_t^S} \cdot \widetilde{R} \le \widetilde{R}_t^S \le \overline{r_t^S} \cdot \widetilde{R}_{t-1}^S, & \forall t = 1 \dots T \\ \underline{R_{t,\tau}^S} \cdot \widetilde{R}_\tau \le \widetilde{R}_t \le \overline{R_{t,\tau}^S} \cdot \widetilde{R}_\tau, & \forall \{(t,\tau) | \tau < t,\ t = 1 \dots T\} \end{array} \right\}$$

where $\underline{R_t^S} = e^{t \cdot \mu_{\log} - \Gamma \cdot \sqrt{t} \cdot \sigma_{\log}}$, $\overline{R_t^S} = e^{t \cdot \mu_{\log} + \Gamma \cdot \sqrt{t} \cdot \sigma_{\log}}$, $\underline{r_t^S} = \mu_r - \Gamma_t \cdot \sigma_r$,
$\overline{r_t^S} = \mu_r + \Gamma_t \cdot \sigma_r$,
$\underline{R_{t,\tau}^S} = (t - \tau) \cdot \mu_r - \Gamma_t \cdot \sigma_r \cdot \sqrt{t - \tau}$ and
$\overline{\overline{R_{t,\tau}^S}} = (t - \tau) \cdot \mu_r + \Gamma_t \cdot \sigma_r \cdot \sqrt{t - \tau}$.

The idea of $\epsilon-$arbitrage

- Replicating portfolios and incomplete markets
    - Exact replication may not be possible.
- The idea of $\epsilon-$arbitrage.
    - Compute the best possible replicating portfolio, when the stock returns lie in an uncertainty set.
    - The resulting replication error stands for the $\epsilon-$arbitrage.
- ϵ can be seen as a measure of incompleteness of the market.

The Problem of Optimal Replication

- Given $P\left(S_T, K\right)$ is the payoff of the option.
- Define x_t^S and x_t^B are the amounts invested in the stock and the bond during the period $[t, t+1)$.
- W_T is the value of the replicating portfolio.

$$\begin{aligned}
\min_{\left\{x_t^S, x_t^B, y_t\right\}} \max_{\left\{\widetilde{R}_t^S \in \mathbb{U}^1\right\}} \quad & \left|P\left(S_T, K\right) - W_T\right| \\
\text{s.t.} \quad & W_T = x_T^S + x_T^B \\
& x_t^S = \left(1 + \widetilde{r}_{t-1}^S\right)\left(x_{t-1}^S + y_{t-1}\right), \forall t = 1, \ldots, T, \\
& x_t^B = \left(1 + r_{t-1}^B\right)\left(x_{t-1}^B - y_{t-1}\right), \forall t = 1, \ldots, T,
\end{aligned}$$

Robust Options Pricing

- European Call option: $P\left(\widetilde{S}, K\right) = \left(\widetilde{S}_T - K\right)^+$.
- The optimization problem

$$
\begin{aligned}
\min_{X} \max_{U^1} \quad & \left| \left(\widetilde{S}_T - K\right)^+ - W_T \right| \\
\text{s.t.} \quad & W_T = x_T^S + x_T^B, \\
& x_t^S = \left(1 + \widetilde{r}_{t-1}^S\right)\left(x_{t-1}^S + y_{t-1}\right), \forall t = 1 \dots T, \\
& x_t^B = \left(1 + r_{t-1}^B\right)\left(x_{t-1}^B - y_{t-1}\right), \forall t = 1 \dots T.
\end{aligned}
$$

Robust Options Pricing, continued

- Variable transformations :

$$\alpha_t^S = \frac{x_t^S}{R_t^S},\ \alpha_t^B = \frac{x_t^B}{R_t^B},\ \beta_t = \frac{y_t}{R_t^S},\ \text{where } \widetilde{R}_t^S = \prod_{i=0}^{t-1}\left(1+\widetilde{r}_i^S\right),\ \text{and } R_t^B = \prod_{i=0}^{t-1}\left(1+r_i^B\right).$$

- After substitution:

$$\begin{array}{cl} \min\limits_{\{\alpha_t^S,\alpha_t^B,\beta_t\}} \max\limits_{\{\tilde{R}_t^S\in\mathbb{U}^1\}} & \left|\left(S_0\widetilde{R}_T^S - K\right)^+ - \left(\widetilde{R}_T^S\alpha_T^S + R_T^B\alpha_T^B\right)\right| \\ \text{s.t.} & \alpha_t^S = \alpha_{t-1}^S + \beta_{t-1},\quad \forall t = 1,\dots,T, \\ & \alpha_t^B = \alpha_{t-1}^B - \beta_{t-1}\dfrac{\widetilde{R}_{t-1}^S}{R_{t-1}^B},\ \forall t = 1,\dots,T. \end{array}$$

Robust Options Pricing, continued

- Substituting all intermediate α_t^B, α_t^S:

$$\min_{\{\alpha_t^S,\alpha_t^B,\beta_t\}} \max_{\{\tilde{R}_t^S \in \mathbb{U}^1\}} \left| \left(S_0 \widetilde{R}_T^S - K\right)^+ - \left(\alpha_0^S + \sum_{t=1}^{T} \beta_{t-1}\right) \widetilde{R}_T^S - \alpha_0^B R_T^B + \sum_{t=0}^{T-1} \beta_t \frac{\mathrm{R}_\mathrm{T}^\mathrm{B}}{\mathrm{R}_\mathrm{t}^\mathrm{B}} \widetilde{R}_t^S \right|.$$

- Inner problem:

$$\begin{aligned}
\min \quad & \kappa \\
\text{s.t.} \quad & \kappa \geq \left(S_0 \widetilde{R}_T^S - K\right)^+ - \left(\alpha_0^S + \sum_{t=1}^{T} \beta_{t-1}\right) \widetilde{R}_T^S - \alpha_0^B R_T^B + \sum_{t=0}^{T-1} \beta_t \frac{R_T^B}{R_t^B} \widetilde{R}_t^S, \quad \forall \tilde{R}_t^S \in \mathbb{U}^1, \\
& \kappa \geq -\left(\left(S_0 \widetilde{R}_T^S - K\right)^+ - \left(\alpha_0^S + \sum_{t=1}^{T} \beta_{t-1}\right) \widetilde{R}_T^S - \alpha_0^B R_T^B + \sum_{t=0}^{T-1} \beta_t \frac{R_T^B}{R_t^B} \widetilde{R}_t^S \right), \ \forall \tilde{R}_t^S \in \mathbb{U}^1.
\end{aligned}$$

Robust Options Pricing, continued

- Model the piecewise-linear function $\left(S_0 \widetilde{R}_T^S - K\right)^+$,
- We partition the uncertainty set $\mathbb{U}^1$

$$\mathbb{U}_a^1 = \mathbb{U}^1 \cap \left\{ \tilde{R}_T^S \geq \frac{K}{S_0} \right\}, \; \mathbb{U}_b^1 = \mathbb{U}^1 \cap \left\{ \tilde{R}_T^S \leq \frac{K}{S_0} \right\}.$$

Robust Options Pricing, continued

- Using this partition, we obtain the following equivalent formulation

$$\min_{\{\alpha_0^S, \alpha_0^B, \beta_t\}} \epsilon$$

s.t.

$$\epsilon \geq \left(S_0 \widetilde{R}_T^S - K\right) - \left(\alpha_0^S + \sum_{t=0}^{T-1} \beta_t\right) \widetilde{R}_T^S - \alpha_0^B R_T^B + \sum_{t=0}^{T-1} \beta_t \frac{R_T^{\mathrm{B}}}{R_t^B} \widetilde{R}_t^S, \qquad \forall \tilde{R}_t^S \in \mathbb{U}_a^1,$$

$$\epsilon \geq -\left(\left(S_0 \widetilde{R}_T^S - K\right) - \left(\alpha_0^S + \sum_{t=0}^{T-1} \beta_t\right) \widetilde{R}_T^S - \alpha_0^B R_T^B + \sum_{t=0}^{T-1} \beta_t \frac{R_T^{\mathrm{B}}}{R_t^B} \widetilde{R}_t^S\right), \qquad \forall \tilde{R}_t^S \in \mathbb{U}_a^1,$$

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$$\epsilon \geq -\left(-\left(\alpha_0^S + \sum_{t=0}^{T-1} \beta_t\right) \widetilde{R}_T^S - \alpha_0^B R_T^B + \sum_{t=0}^{T-1} \beta_t \frac{R_T^{\mathrm{B}}}{R_t^B} \widetilde{R}_t^S\right), \qquad \forall \tilde{R}_t^S \in \mathbb{U}_b^1.$$

Robust Options Pricing, continued

- Can be converted to an equivalent linear optimization problem.
- Resulting size: $16T + 4$ decision variables and $4T + 4$ constraints.

Pricing Asian options

- Asian Call option: $P\left(\widetilde{S}, K\right) = \left(S_0 \widetilde{R}^S_{\text{ave}} - K\right)^+$, where $\widetilde{R}^S_{\text{ave}} = \sum_{t=1}^{T} \frac{\widetilde{R}^S_t}{T}$.
- Again leads to a linear formulation with size that scales linearly in T.

Pricing Lookback options

- $P\left(\widetilde{S},K\right)=\left(S_0\widetilde{R}^S_{\max}-K\right)^+,\ \widetilde{R}^S_{\max}=\max_{t=1\dots T}\left\{\widetilde{R}^S_t\right\}.$
- We obtain the following formulation

$$\min_{\{\alpha_0^S,\alpha_0^B,\beta_t\}} \quad \epsilon$$

s.t.

$$\forall k=1\dots T$$

$$\epsilon \geq \left(S_0\tilde{R}^S_k-K\right)-\alpha_0^S\tilde{R}^S_T-\alpha_0^B R^B_T+\sum_{t=1}^{T}\beta_{t-1}\left(\frac{R^B_T}{R^B_t}\tilde{R}^S_t-\tilde{R}^S_T\right) \quad \forall \mathbb{U}^1_k \cap \left\{\tilde{R}^S_k \geq \frac{K}{S_0}\right\}$$

$$\epsilon \geq -\left(S_0\tilde{R}^S_k-K\right)+\alpha_0^S\tilde{R}^S_T+\alpha_0^B R^B_T-\textstyle\sum_{t=1}^{T}\beta_{t-1}\left(\frac{R^B_T}{R^B_t}\tilde{R}^S_t-\tilde{R}^S_T\right) \quad \forall \mathbb{U}^1_k \cap \left\{\tilde{R}^S_k \geq \frac{K}{S_0}\right\}$$

$$\epsilon \geq -\left(\alpha_0^S+\sum_{t=1}^{T}\beta_{t-1}\right)\tilde{R}^S_T-\alpha_0^B R^B_T+\sum_{t=1}^{T}\beta_{t-1}\frac{R^B_T}{R^B_t}\tilde{R}^S_t \quad \forall \mathbb{U}^1_k \cap \left\{\tilde{R}^S_k \leq \frac{K}{S_0}\right\}$$

$$\epsilon \geq -\left(-\left(\alpha_0^S+\sum_{t=1}^{T}\beta_{t-1}\right)\tilde{R}^S_T-\alpha_0^B R^B_T+\sum_{t=1}^{T}\beta_{t-1}\frac{R^B_T}{R^B_t}\tilde{R}^S_t\right) \quad \forall \mathbb{U}^1_k \cap \left\{\tilde{R}^S_k \leq \frac{K}{S_0}\right\}$$

Pricing American Put options

- Can exercise the option at any time up to the time of exercise T.
- The payoff is then given by $P\left(\widetilde{S}, K\right) = \left(K - \widetilde{S}_\tau\right)$, where τ is the time of exercise.
  - τ depends on the utility of the option holder.
- In the absence of any information about the utility function of the option holder, we seek to find a replicating portfolio for all possible exercise policies.

Pricing American Put options

- Proceeding as before, we obtain the following formulation

$$\begin{aligned} \min_{\{\alpha_t^S, \alpha_t^B, \beta_t\}} \max_{\{\tilde{R}_t^S\} \in \mathbb{U}^1} \max_{\tau = 1, \ldots, T} \quad & \left| \left(K - S_0 \widetilde{R}_\tau \right)^+ - \left(\widetilde{R}_\tau^S \alpha_\tau^S + R_\tau^B \alpha_\tau^B \right) \right| \\ \text{s.t.} \quad & \alpha_t^S = \alpha_{t-1}^S + \beta_{t-1}, \ \ \forall t = 1, \ldots, T, \\ & \alpha_t^B = \alpha_{t-1}^B - \beta_{t-1} \frac{\widetilde{R}_t^S}{R_t^B}, \ \forall t = 1, \ldots, T, \end{aligned}$$

  where τ is the time of exercise.
- The size of the resulting formulation will be quadratic in T.

Pricing Barrier options

- These options become inactive as soon as a condition($\mathbb{C}$) of the form $S_t \leq a$ or $S_t \geq b$ is reached.
- The problem of optimal replication then reduces to

$$\min_{\{x_t^S, x_t^B, y_t\}} \max_{\{\widetilde{R}_t^S \in \mathbb{U}^1 \cap \mathbb{C}\}} |P(S_T, K) - W_T|$$

  because if $\mathbb{C}$ is not satisfied the option ceases to exist and one need not worry about replicating its payoff.
- We obtain a linear formulation that scales linearly in T.

Pricing Multidimensional options

- Pricing options that depend on M underlying assets is difficult to price, for large M, using current methods because of
  - unavailability of an analytic solution and
  - the curse-of-dimensionality which prevents one from using dynamic programming.
- Proceeding as before, we seek to obtain the optimal solution of the following optimization problem

$$
\begin{aligned}
\min_{\{x_t^m, y_t^m\}} \quad & \max_{\{\tilde{r}_t^m\} \in \mathbb{U}^M} \left| P_f\left(\{S^i\}_{i=1,\dots,M}, K\right) - W_T \right| \\
\text{s.t.} \quad & W_T = \sum_{m=0}^{M} x_T^m, \\
& x_t^m = \left(1 + \tilde{r}_{t-1}^m\right) \cdot \left(x_{t-1}^m + y_{t-1}^m\right) \quad \forall t = 1, \dots, T,\ \forall m = 1, \dots, M, \\
& x_t^0 = \left(1 + \tilde{r}_{t-1}^m\right) \cdot \left(x_{t-1}^0 - \sum_{m=1}^{M} y_{t-1}^m\right) \quad \forall t = 1, \dots, T.
\end{aligned}
$$

Multidimensional options

- $\mathbf{C}$: the covariance matrix of the single period returns.
- Uncertainty set $\mathbb{U}^M$

$$\mathbb{U}^M = \left\{ \widetilde{\mathbf{R}}_t \;\middle|\; \begin{array}{cc} \|\mathbf{C}(\widetilde{\mathbf{R}}_1 - \overline{\mathbf{R}}_1)\| \leq \Gamma, & \\ \underline{r_t^m}\widetilde{R}_{t-1}^m \leq \widetilde{R}_t^m \leq \overline{r_t^m}\widetilde{R}_{t-1}^m & \forall t = 2, \ldots, T, \forall m = 1, \ldots, M, \\ \underline{R_t^m} \leq \widetilde{R}_t^m \leq \overline{R_t^m}, & \forall t = 1, \ldots, T, \forall m = 1, \ldots, M. \end{array} \right\}.$$

- By choosing the ℓ_1 or ℓ_∞ we obtain LO formulations.

Computational Complexity

- Methodology scales polynomially with the dimension of the option and the discretization, unlike DP.

Option Type	European	Asian	Lookback	American	Index	American Index
Size	$O(T)$	$O(T)$	$O(T^2)$	$O(T^2)$	$O(M \cdot T)$	$O(M \cdot T^2)$

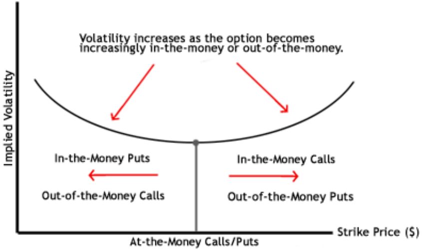
T : the number of time periods the option is written for.
M: the number of different assets required to define the option.
Size : Number of variables and constraints.

Modeling Flexibility

- Modeling many types of options - Barrier, Lookback etc.
- Modeling implied volatility smile
- Modeling transaction costs and other market restrictions, still obtaining LO formulations.

Modeling Implied Volatility smile

- *Implied volatility* of an option is simply that volatility that makes the Black-Scholes (BS) model price exactly equal to the observed market price.
  - Expect flat lines, if market participants use BS Model.
- Plotted across strike prices, they exhibit *smiles* or *smirks*.



Modeling the implied volatility smile

- Risk Aversion as an explanation for the Implied Volatility Smile.
- Lower value for Γ implies that the user is willing to take higher risk by ignoring the variability of stock prices.
- Higher value of Γ indicates that the user seeks a price that will allow him to replicate the payoff of the option for a larger range of stock prices.
- Historical evidence.
  - No smile was observed before the crash of 1987.
  - After the crash, the smile started appearing.

Modeling the implied volatility smile, continued

Risk Aversion as an explanation for the Implied Volatility Smile

- Empirical results indicate that a quadratic dependence of $\Gamma_{implied}$ with $\frac{K}{S_0}$ would be adequate to characterize the risk aversion of investor towards different strike prices.
- The following function is used to describe the relationship:

$$\Gamma(K) = \theta_0 + \theta_1 \frac{K - S_0}{S_0} + \theta_2 \left(\frac{K - S_0}{S_0} \right)^2, \ \theta_2 \geq 0.$$

- The quantity $\frac{K - S_0}{S_0}$ captures the distance between the strike and the spot price and is also called as *moneyness* in the literature.

Experiments setup

- We perform the following experiments:
  - Experiment 1 : Compare with actual market prices for European call options.
  - Experiment 2 : Compare with actual market prices for American put options.

Experiments setup

- All the experiments have a training stage and a testing stage.
  - In the training stage, we choose a random set of strike prices and calibrate (compute $\theta_0, \theta_1, \theta_2$) our model to it.
  - In the testing stage, we use our model to price the options for the remaining strikes.

Experiment 1

- The underlying security is Microsoft stock.
- The number of periods T = 18 weeks.
- The initial price of underlying security S_0= 21.4.
- Strike price of options K: ranges from 2.5 to 30.

Experiment 1

Out of Sample

No.	T	K/S	$\Gamma_{implied}$	Mkt Price	Model Price	Error
1	18	0.654	2.45	7.475	7.48	0.005
2	18	0.794	2.056	4.8	4.797	-0.003
3	18	0.888	1.75	3.25	3.232	-0.018
4	18	0.981	1.66	1.97	1.968	-0.002
5	18	1.028	1.65	1.47	1.462	-0.008
6	18	1.121	1.73	0.735	0.749	0.014

In Sample

No.	T	K/S	$\Gamma_{implied}$	Mkt Price	Model Price	Error
1	18	0.607	2.77	8.425	8.42	-0.005
2	18	0.701	2.25	6.55	6.561	0.011
3	18	0.841	1.921	4	3.984	-0.016
4	18	0.935	1.69	2.56	2.556	-0.004
5	18	1.285	2.19	0.155	0.152	-0.003

Experiment 2

- The underlying security is MSFT stock.
- The number of periods T = 25 weeks.
- The initial price of underlying security $S_0 = 24.8$.
- Strike price of options K: ranges from 7.5 to 50.

Experiment 2

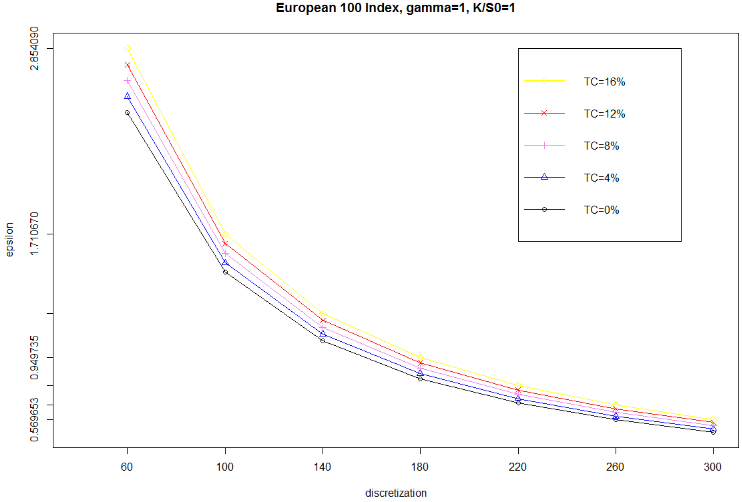
Out of Sample

No.	T	K/S	$\Gamma_{implied}$	Mkt Price	Model Price	Error
1	25	0.605	2.62	0.17	0.201	0.031
2	25	0.806	1.83	0.695	0.589	-0.106
3	25	0.968	1.6	1.895	1.764	-0.132
4	25	1.008	1.59	2.365	2.266	-0.099
5	25	1.21	1.9	5.85	5.939	0.089
6	25	1.411	2.87	10.5	10.703	0.203
7	25	1.815	7.7	20.45	20.303	-0.147

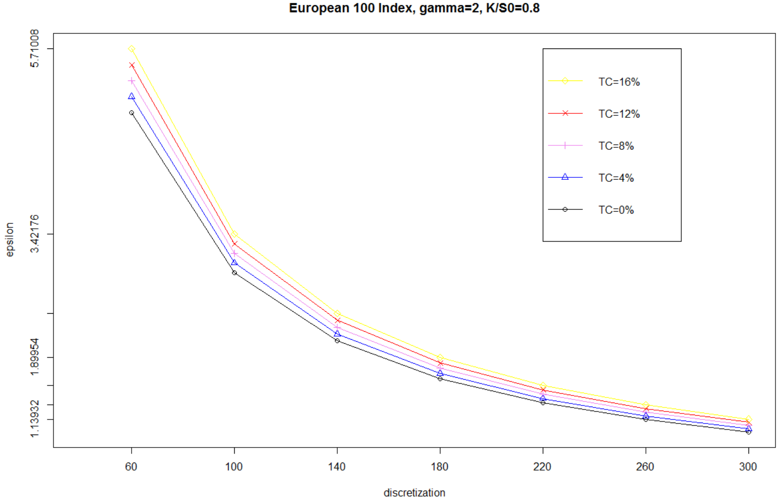
In Sample

No.	T	K/S	$\Gamma_{implied}$	Mkt Price	Model Price	Error
1	25	0.504	3.24	0.065	0.17	0.105
2	25	0.706	2.15	0.34	0.305	-0.035
3	25	0.766	1.94	0.525	0.442	-0.083
4	25	0.847	1.74	0.905	0.778	-0.127

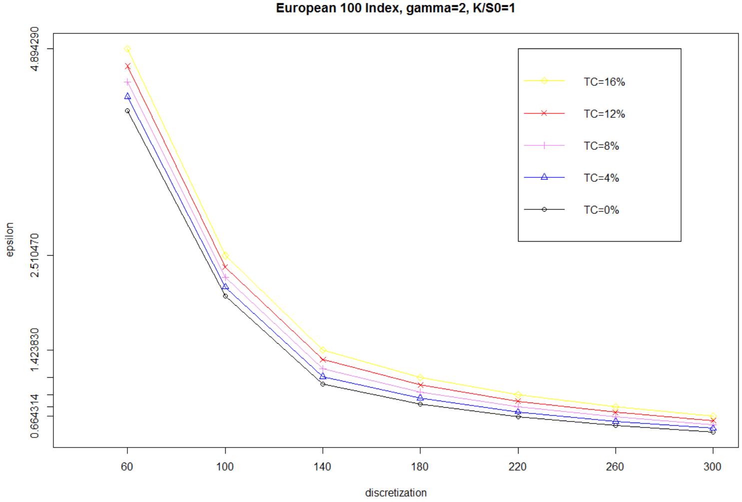
Dependence on Discretization and Transaction Costs



Dependence on Discretization and Transaction Costs



Dependence on Discretization and Transaction Costs



Conclusions

- Tractable approach to price options while accounting for the risk attitudes of the option writer.
- Approach scales polynomially (as opposed to exponentially) with the dimension of the original pricing problem.
- We provide a potential explanation for the phenomenon of the implied volatility smile, and support experimental evidence for the same.