Course notes for EE394V Restructured Electricity Markets: Locational Marginal Pricing

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Unit commitment

(i) Temporal issues,
(ii) Formulation,
(iii) Lagrangian relaxation,
(iv) Duality gaps,
(v) Mixed-integer programming,
(vi) Make-whole payments,
(vii) Anonymity of prices,

(viii) Implications for investment decisions,
(ix) Transmission constraints,
(x) Day-ahead and real-time markets,
(xi) Homework exercises.

10.1 Temporal issues

- So far we have considered particular dispatch intervals.
- Demand has been represented by its assumed known average value over the dispatch interval, ignoring whether this demand was occurring:
  - "now" (that is, in the next few minutes or next dispatch interval), or
  - in the future (such as during an hour of tomorrow).
- Supply has been represented by assuming that unit commitment decisions had already been taken:
  - each generator's commitment status is fixed.
- In this chapter we will generalize this in several ways, by considering:
  - (i) variation of demand over time,
  - (ii) ramp rates,
  - (iii) unit commitment, and
  - (iv) day-ahead and real-time markets.

10.1.1 Variation of demand over time

- Suppose that we are considering the average demand in each dispatch interval, say each hour, tomorrow.
- That is, we are planning **day-ahead**.
- For now, we will continue to ignore unit commitment decisions.
- For each hour $t = 1, \ldots, n_T$, we have a specification or a forecast of the average power demand, $\overline{D}_t$ in dispatch interval t.
- We need to satisfy power balance each hour (and, of course, continuously, but this will be achieved by the **real-time** market).

10.1.2 Ramp-constrained economic dispatch

10.1.2.1 Decision variables

- We generalize our previous formulation so that P_{kt} represents the average power generated by generator $k = 1, \ldots, n_P$ in hour $t = 1, \ldots, n_T$.
- We collect the entries P_{kt} together into a vector $P_k \in \mathbb{R}^{n_T}$.
- As previously, we can also consider the spinning reserve and let S_{kt} be the amount of spinning reserve provided by generator k in hour t.
- We collect the entries S_{kt} together into a vector $S_k \in \mathbb{R}^{n_T}$.
- We collect P_k and S_k together into a vector $x_k \in \mathbb{R}^{2n_T}$.
- We collect the vectors x_k together into a vector $x \in \mathbb{R}^{2n_P \times n_T}$.
- In some examples, we will only consider energy and not reserve, in which case, we re-define $x = P \in \mathbb{R}^{n_P \times n_T}$ and re-define any associated functions, matrices, and vectors appropriately.

10.1.2.2 System constraints

- Typical equality constraints would include average power balance in each hour of tomorrow, which we will represent in the form $Ax = b$.
- For example:
  - for simplicity, if we ignore reserve, then $x = P = \begin{bmatrix} P_1 \\ \vdots \\ P_{n_P} \end{bmatrix} \in \mathbb{R}^{n_P \times n_T}$, with

    $$P_k = \begin{bmatrix} P_{k1} \\ \vdots \\ P_{kn_T} \end{bmatrix} \in \mathbb{R}^{n_T},$$
  - let $\overline{D} \in \mathbb{R}^{n_T}$ be a vector of forecasts of average demand in each hour,
  - let $A = \begin{bmatrix} -\mathbf{I} & \cdots & -\mathbf{I} \end{bmatrix}$ and $b = -\overline{D}$,
  - then $Ax = b$ represents average power balance in each hour.
- Typical inequality constraints would include reserve requirements and transmission constraints in each hour, which we will represent in the form $Cx \leq d$.

10.1.2.3 Generator constraints including ramp-rate constraints

- Each generator k has a feasible operating set $\mathbb{S}_k$.
- In addition to minimum and maximum generation and spinning reserve constraints, there can be **inter-temporal constraints** in the specification of $\mathbb{S}_k$ that limit the change in average production from hour to hour.
- For example, if the ramp-rate limit is 1 MW per minute then the generator constraints for generator k could be:

$$
\begin{array}{ll}
\forall t = 1,\ldots,n_T, & \underline{P}_k \leq P_{kt} \leq \overline{P}_k, \\
\forall t = 1,\ldots,n_T, & 0 \leq S_{kt} \leq 10, \\
\forall t = 1,\ldots,n_T, & \underline{P}_k \leq P_{kt} + S_{kt} \leq \overline{P}_k, \\
\forall t = 1,\ldots,n_T, & P_{k,t-1} - 60 \leq P_{kt} \leq P_{k,t-1} + 60 - S_{k,t},
\end{array}
$$

- where P_{k0} and S_{k0} are the power and reserve for the last hour of today, and
- where we have required that procured spinning reserve be available for deployment within any one 10 minute period throughout the hour.

Generator constraints including ramp-rate constraints, continued

- As previously, we can specify the feasible operating set for generator k in the form:

$$\mathbb{S}_k = \{x_k \in \mathbb{R}^{2n_T} | \underline{\delta}_k \leq \Gamma_k x_k \leq \overline{\delta}_k\},$$

- where $\Gamma_k \in \mathbb{R}^{r_k \times 2n_T}$, $\underline{\delta}_k \in \mathbb{R}^{r_k}$, and $\overline{\delta}_k \in \mathbb{R}^{r_k}$.
- Other formulations of generator constraints besides our example also fit into this form.

10.1.2.4 Generator costs

- Generator k has a cost function f_k for its generation over the hours $t = 1, \ldots, n_T$.
- Typically, if a unit is committed then the production in one hour does not (directly) affect the costs in another hour so that the costs are additively separable across time:

$$\forall x_k, f_k(x_k) = \sum_{t=1}^{n_T} f_{kt}(x_{kt}),$$

- where $x_{kt} = \begin{bmatrix} P_{kt} \\ S_{kt} \end{bmatrix}$.
- Typically, we would expect that f_{kt} does not vary significantly from hour to hour, except for:
  - temperature and pressure related changes, and
  - significant change in fuel availability or cost.
- We will treat start-up costs when we explicitly consider unit commitment.

10.1.2.5 Problem formulation

- The resulting ramp-constrained economic dispatch problem is in the form of our generalized economic dispatch problem:

$$\begin{aligned}&\min_{\forall k, x_k \in \mathbb{S}_k} \{f(x) | Ax = b, Cx \leq d\} \\ &= \min_{x \in \mathbb{R}^{2n_P \times n_T}} \{f(x) | Ax = b, Cx \leq d, \forall k, \underline{\delta}_k \leq \Gamma_k x_k \leq \overline{\delta}_k\}.\end{aligned}$$

- This problem is convex and can be solved with standard algorithms for minimizing convex problems.

10.1.2.6 Example

- Suppose that we have two generators, $n_P = 2$, with costs:

$$\begin{aligned} \forall t, f_{1t}(P_{1t}) &= 2P_{1t}, 100 \leq P_{1t} \leq 300, \\ \forall t, f_{2t}(P_{2t}) &= 5P_{2t}, 100 \leq P_{2t} \leq 300. \end{aligned}$$

- The generators have ramp-rate limits of $\Delta_1 = 200$ MW/h and $\Delta_2 = 100$ MW/h, respectively.
- We consider day-ahead dispatch across two hours, $n_T = 2$, with demands:

t	0	1	2
$\overline{D}_t$	200	400	600

- The $t = 0$ entry in the table is the demand for the last hour of today.
- The $t = 1, 2$ entries are the demands for the first two hours of tomorrow.
- Also, $P_{10} = 100$ MW and $P_{20} = 100$ MW are the generations in the last hour of today.
- We ignore reserve requirements so that the only system constraint is supply-demand balance for power.
- We solve the ramp-constrained economic dispatch problem.

Example, continued

- The generator constraints for generator $k = 1,2$ are:

$$\begin{aligned} \forall t = 1,2, &\quad \underline{P}_k \leq P_{kt} \leq \overline{P}_k, \\ \forall t = 1,2, &\quad P_{k,t-1} - \Delta_k \leq P_{kt} \leq P_{k,t-1} + \Delta_k, \end{aligned}$$

- which we can represent in the form $\mathbb{S}_k = \{x_k \in \mathbb{R}^2 | \underline{\delta}_k \leq \Gamma_k x_k \leq \overline{\delta}_k\}$,
- by defining $\underline{\delta}_k \in \mathbb{R}^4$, $\Gamma_k \in \mathbb{R}^{4\times 2}$, and $\overline{\delta}_k \in \mathbb{R}^4$ as:

$$\underline{\delta}_k = \begin{bmatrix} \underline{P}_k \\ P_{k,0} - \Delta_k \\ \underline{P}_k \\ -\Delta_k \end{bmatrix}, \Gamma_k = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}, \overline{\delta}_k = \begin{bmatrix} \overline{P}_k \\ P_{k,0} + \Delta_k \\ \overline{P}_k \\ \Delta_k \end{bmatrix}.$$

- We label the Lagrange multipliers on these generator inequality constraints as, respectively, $\underline{\mu}_k = \begin{bmatrix} \underline{\mu}^\star_{k1\text{capacity}} \\ \underline{\mu}^\star_{k1\text{ramp}} \\ \underline{\mu}^\star_{k2\text{capacity}} \\ \underline{\mu}^\star_{k2\text{ramp}} \end{bmatrix}, \overline{\mu}_k = \begin{bmatrix} \overline{\mu}^\star_{k1\text{capacity}} \\ \overline{\mu}^\star_{k1\text{ramp}} \\ \overline{\mu}^\star_{k2\text{capacity}} \\ \overline{\mu}^\star_{k2\text{ramp}} \end{bmatrix}.$

Example, continued

- Since generator 1 is the cheaper, so we would prefer to use it instead of generator 2.
- Since the ramp-rate limit for generator 1 is $\Delta_1 = 200$, for hour $t = 1$, we consider setting:

$$\begin{aligned} P_{11} &= P_{10} + \Delta_1, \\ &= 100 + 200, \\ &= 300. \end{aligned}$$

- With $P_{11} = 300$, to meet demand we would have:

$$\begin{aligned} P_{21} &= \overline{D}_1 - P_{11}, \\ &= 400 - 300, \\ &= 100. \end{aligned}$$

Example, continued

- However, we now have a problem in hour $t = 2$, since:
  - generator 1 would be at its maximum,
  - generator 2 can only increase by $\Delta_2 = 100$ from hour 1 to hour 2, so that $P_{22} \leq P_{21} + \Delta_1 = 100 + 100 = 200$ MW, and
  - supply would then be 100 MW less than demand in hour 2.
- Setting $P_{11} = 200$ does not work!

Example, continued

- Instead, we need both generators each producing at their capacity of 300 MW in hour 2 to meet the demand, so that $P_{12} = P_{22} = 300$ MW.
  - Working backwards in time, generator 2 must be producing at least 200 MW in period 1 because of its ramp rate constraint, so $P_{21} \geq 200$ MW.
  - Since generator 2 is the more expensive, we do not want it to produce more than necessary, and so we will try to see if we can set $P_{21} = 200$ MW.
  - In this case, generator 1 must produce $P_{11} = 200$ MW in period 1 to meet demand of $\overline{D}_1 = 400$.
  - This solution satisfies the ramp-rate constraints and is optimal.
- The ramp-constrained economic dispatch solution is:

t	0	1	2
$\overline{D}_t$	200	400	600
$P^{\star}_{1t}$	100	200	300
$P^{\star}_{2t}$	100	200	300

Example, continued

- What are the values of the Lagrange multipliers?
- To answer this question, we will consider several of the first-order necessary conditions.
- Note that the ramp constraints are binding for generator $k = 2$ across two successive pairs of dispatch intervals, from $t = 0$ to $t = 1$ and from $t = 1$ to $t = 2$.
- Also note that generator $k = 1$ is neither at its maximum nor minimum in hour 1, nor are its ramp constraints binding across any periods.
- Therefore, by complementary slackness, all Lagrange multipliers on generator $k = 1$ constraints associated with P_{11} are zero.

Example, continued

- By the first-order necessary conditions for generator 1 associated with P_{11}:

$$\begin{aligned} 0 &= \nabla f_{11}(P_{11}^\star) - \lambda_1^\star - [\Gamma_{11}]^\dagger \underline{\mu}_1^\star + [\Gamma_{11}]^\dagger \overline{\mu}_1^\star, \\ &= \nabla f_{11}(P_{11}^\star) - \lambda_1^\star, \\ &= 2 - \lambda_1^\star, \end{aligned}$$

- where:

  Γ_1 is the generator constraint matrix for generator 1,

  $\Gamma_{11} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}$ is the column of Γ_1 associated with P_{11}, and

  $\underline{\mu}_1^\star = \mathbf{0}$ and $\overline{\mu}_1^\star = \mathbf{0}$ are the Lagrange multipliers on the generator constraints for generator 1.

- That is, $\lambda_1^\star = 2$.

Example, continued

- Generator 2:

  is at its maximum in hour 2,
  has its ramp rate constraint binding from hour 0 to hour 1, and
  has its ramp rate constraint binding from hour 1 to hour 2.

- That is the binding generator constraints for generator 2 are:

$$\begin{aligned}
P_{22} &\leq 300, \text{ (Lagrange multiplier } \overline{\mu}^{\star}_{22\text{capacity}}), \\
P_{21} &\leq P_{20} + \Delta_2, \text{ (Lagrange multiplier } \overline{\mu}^{\star}_{21\text{ramp}}), \\
P_{22} &\leq P_{21} + \Delta_2, \text{ (Lagrange multiplier } \overline{\mu}^{\star}_{22\text{ramp}}).
\end{aligned}$$

- By complementary slackness, all Lagrange multipliers on generator constraints for generator 2 are zero, except for the Lagrange multipliers on these three binding constraints.

Example, continued

- By the first-order necessary conditions for generator 2 associated with P_{21}:

$$
\begin{aligned}
0 &= \nabla f_{21}(P_{21}^{\star}) - \lambda_1^{\star} - [\Gamma_{21}]^{\dagger}\underline{\mu}_2^{\star} + [\Gamma_{21}]^{\dagger}\overline{\mu}_2^{\star},\\
&= \nabla f_{21}(P_{21}^{\star}) - \lambda_1^{\star} - \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}^{\dagger} \begin{bmatrix} \underline{\mu}_{21\text{capacity}}^{\star} \\ \underline{\mu}_{21\text{ramp}}^{\star} \\ \underline{\mu}_{22\text{capacity}}^{\star} \\ \underline{\mu}_{22\text{ramp}}^{\star} \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}^{\dagger} \begin{bmatrix} \overline{\mu}_{21\text{capacity}}^{\star} \\ \overline{\mu}_{21\text{ramp}}^{\star} \\ \overline{\mu}_{22\text{capacity}}^{\star} \\ \overline{\mu}_{22\text{ramp}}^{\star} \end{bmatrix},\\
&= \nabla f_{21}(P_{21}^{\star}) - \lambda_1^{\star} - \underline{\mu}_{21\text{capacity}}^{\star} - \underline{\mu}_{21\text{ramp}}^{\star} + \underline{\mu}_{22\text{ramp}}^{\star}\\
&\qquad + \overline{\mu}_{21\text{capacity}}^{\star} + \overline{\mu}_{21\text{ramp}}^{\star} - \overline{\mu}_{22\text{ramp}}^{\star},\\
&= \nabla f_{21}(P_{21}^{\star}) - \lambda_1^{\star} + \overline{\mu}_{21\text{ramp}}^{\star} - \overline{\mu}_{22\text{ramp}}^{\star},\\
&\quad \text{by complementary slackness},\\
&= 5 - 2 + \overline{\mu}_{21\text{ramp}}^{\star} - \overline{\mu}_{22\text{ramp}}^{\star},
\end{aligned}
$$

Example, continued

- where:

  Γ_2 is the generator constraint matrix for generator 2,

  $\Gamma_{21} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}$ is the column of Γ_2 associated with P_{21}, and

  $\underline{\mu}_2^\star = \begin{bmatrix} \underline{\mu}_{21\text{capacity}}^\star \\ \underline{\mu}_{21\text{ramp}}^\star \\ \underline{\mu}_{22\text{capacity}}^\star \\ \underline{\mu}_{22\text{ramp}}^\star \end{bmatrix}$ and $\overline{\mu}_2^\star = \begin{bmatrix} \overline{\mu}_{21\text{capacity}}^\star \\ \overline{\mu}_{21\text{ramp}}^\star \\ \overline{\mu}_{22\text{capacity}}^\star \\ \overline{\mu}_{22\text{ramp}}^\star \end{bmatrix}$ are the Lagrange multipliers on the generator constraints for generator 2.

- Therefore, $\overline{\mu}_{22\text{ramp}}^\star = \overline{\mu}_{21\text{ramp}}^\star + 3$.

Example, continued

- By the first-order necessary conditions for generator 2 associated with P_{22}:

$$\begin{aligned} 0 &= \nabla f_{22}(P_{22}^{\star}) - \lambda_2^{\star} - [\Gamma_{22}]^{\dagger}\underline{\mu}_2^{\star} + [\Gamma_{22}]^{\dagger}\overline{\mu}_2^{\star}, \\ &= \nabla f_{22}(P_{22}^{\star}) - \lambda_2^{\star} + \overline{\mu}_{22\text{capacity}}^{\star} + \overline{\mu}_{22\text{ramp}}^{\star}, \\ &\quad \text{by complementary slackness,} \\ &= 5 - \lambda_2^{\star} + \overline{\mu}_{22\text{capacity}}^{\star} + \overline{\mu}_{22\text{ramp}}^{\star}, \end{aligned}$$

- where:

  Γ_2 is the generator constraint matrix for generator 2,

  $\Gamma_{22} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$ is the column of Γ_2 associated with P_{22}, and

  $\underline{\mu}_2^{\star}$ and $\overline{\mu}_2^{\star}$ are the Lagrange multipliers on the generator constraints for generator 2.

- Therefore, $\lambda_2^{\star} = 5 + \overline{\mu}_{22\text{capacity}}^{\star} + \overline{\mu}_{22\text{ramp}}^{\star}$.

Example, continued

- Summarizing:

$$\begin{aligned}\overline{\mu}^{\star}_{22\text{ramp}} &= \overline{\mu}^{\star}_{21\text{ramp}} + 3,\\ \lambda^{\star}_{2} &= 5 + \overline{\mu}^{\star}_{22\text{capacity}} + \overline{\mu}^{\star}_{22\text{ramp}}.\end{aligned}$$

- Let's try to find a non-negative solution for these two equations in the four variables $\overline{\mu}^{\star}_{22\text{ramp}}, \overline{\mu}^{\star}_{21\text{ramp}}, \overline{\mu}^{\star}_{22\text{capacity}}$, and $\lambda^{\star}$:

$$\begin{aligned}\text{We set } \overline{\mu}^{\star}_{21\text{ramp}} &= 0, \text{ hypothesizing that constraint is ``just'' binding},\\ \text{Therefore: } \overline{\mu}^{\star}_{22\text{ramp}} &= \overline{\mu}^{\star}_{21\text{ramp}} + 3,\\ &= 3,\\ \text{We set } \overline{\mu}^{\star}_{22\text{capacity}} &= 0, \text{ hypothesizing that constraint is ``just'' binding},\\ \text{Therefore: } \lambda^{\star}_{2} &= 5 + \overline{\mu}^{\star}_{22\text{capacity}} + \overline{\mu}^{\star}_{22\text{ramp}},\\ &= 5 + 0 + 3,\\ &= 8.\end{aligned}$$

Example, continued

- The solution is:

$$\begin{aligned}
\overline{\mu}^{\star}_{21\text{ramp}} &= 0, \\
\overline{\mu}^{\star}_{22\text{ramp}} &= 3, \\
\overline{\mu}^{\star}_{22\text{capacity}} &= 0, \\
\lambda^{\star}_{2} &= 8.
\end{aligned}$$

- These particular values constitute one of multiple solutions for the Lagrange multipliers.
- Any other solution of the two equations having non-negative values for the Lagrange multipliers on the inequality constraints also provides Lagrange multipliers for this problem.

10.1.3 Ramp-constrained offer-based economic dispatch

10.1.3.1 Generator offers

- Generator k makes an offer for its generation.
- The offer is usually required to be separable across hours.
- Sometimes market rules require the offer for each hour t to be fixed independent of t (as in PJM) and sometimes the offer can vary from hour to hour (as in ISO-NE, NYISO, and ERCOT):
  - market rules on fixed versus varying offers can affect the exercise of market power,
  - discussed in market power course, http://users.ece.utexas.edu/~baldick/classes/394V_market_power/EE394V_market_power.html.
- Assuming that offers reflect marginal costs, the offer for generator k is:

$$\nabla f_{kt}, t = 1, \ldots, n_T,$$

- where $x_{kt} = [P_{kt}]$ for simplicity, ignoring reserve and where we will typically assume that the marginal costs do not vary with time, even though the notation allows for such variation.

10.1.3.2 *Offer-based economic dispatch and prices*

- Using the offers, we can solve the first-order necessary and sufficient conditions for offer-based ramp-constrained economic dispatch:

$$\min_{x \in \mathbb{R}^{2n_P \times n_T}} \{ f(x) | Ax = b, Cx \leq d, \forall k, \underline{\delta}_k \leq \Gamma_k x_k \leq \overline{\delta}_k \}.$$

- The solution involves dispatch $x_k^\star$ for each generator k and Lagrange multipliers:

  $\lambda^\star$ and $\mu^\star$ on system constraints, and
  $\underline{\mu}_k^\star$ and $\overline{\mu}_k^\star$ on generator constraints for each generator k.
- By Theorem 8.1, dispatch-supporting prices can be constructed as previously:

$$\pi_{x_k} = -[A_k]^\dagger \lambda^\star - [C_k]^\dagger \mu^\star.$$

- To summarize: the generalization of the problem to include more complicated generator constraints and more complicated system constraints does not fundamentally complicate the pricing rule, so long as the generalized economic dispatch problem is convex:
  - we will qualify this statement in the context of *anticipating* prices.

10.1.3.3 Example

- Continuing with the previous example, assume that the generators offer at their marginal costs in each period:

$$\begin{aligned}\nabla f_{1t}(P_{1t}) &= 2, 100 \leq P_{1t} \leq 300, t = 1,2,\\ \nabla f_{2t}(P_{2t}) &= 5, 100 \leq P_{2t} \leq 300, t = 1,2.\end{aligned}$$

- From the previous analysis, we have that:

t	1	2
$\overline{D}_t$	400	600
$P^{\star}_{1t}$	200	300
$P^{\star}_{2t}$	200	300
$\lambda^{\star}_t$	2	8

- The price for energy in hour $t = 1$ is $\lambda^{\star}_1 = \$2$/MWh:
  - generator 1 with offer price $\nabla f_{11}(P^{\star}_{11}) = \$2$/MWh is marginal, but
  - the price is *lower* than the offer price of $\nabla f_{21}(P^{\star}_{21}) = \$5$/MWh for generator 2.

Example, continued

- Generator 2 is operating *above* its minimum in hour $t = 1$, so it is operating at a loss in hour $t = 1$ and could reduce its operating losses if it operated at its minimum in hour $t = 1$.
- Why would generator operate above its minimum in hour $t = 1$ when the price is only \$2/MWh?
- The price for energy in hour $t = 2$ is $\lambda_2^\star =$ \$8/MWh, which is higher than the higher offer price of both generators!
- The price in hour $t = 2$ is necessary to induce generator 2 to produce at a loss in hour $t = 1$:
  - The infra-marginal rent in hour $t = 2$ equals the loss in hour $t = 1$ for generator 2.
  - Generator 2 is indifferent to any levels of production that involve $P_{22} - P_{21} = \Delta_1$.
  - The prices support the dispatch but do not strictly support dispatch.

Example, continued

- Generator 2 is "marginal" in hour $t = 2$ in that changes to its offer price $\nabla f_{22}(P_{22}^{\star})$ would affect the price $\lambda_2^{\star}$ in hour $t = 2$:
  - the price in hour $t = 2$ is $\lambda_2^{\star} = \nabla f_{22}(P_{22}^{\star}) + (\nabla f_{21}(P_{21}^{\star}) - \nabla f_{11}(P_{11}^{\star}))$,
  - somewhat different to earlier use of "marginal" since offer prices $\nabla f_{22}(P_{22}^{\star})$ and $\nabla f_{21}(P_{21}^{\star})$ of generator 1 are both involved in setting the price for hour $t = 2$.
- Prices are above the highest marginal cost because there are binding ramp-rate constraints.
  - We also saw in Homework Exercise 9.2 that prices can rise above the highest offer price in the presence of binding transmission constraints.

10.1.3.4 Discussion

- This example is somewhat unrealistic for several reasons:
  - Ramp-rate constraints are typically not binding across multiple hours in ERCOT (but increased wind generation may change this in the morning ramp-up of demand and the evening ramp-down of demand).
  - The more expensive generator has the tighter ramp-rate constraint.
  - The ERCOT day-ahead market does not represent ramp-rate constraints.
- This particular example requires *anticipation* across multiple periods to solve:
  - Anticipation across multiple periods is not always necessary for finding the ramp-constrained optimum. (See homework exercise 10.2.)
- As will be discussed in Section 10.2, **day-ahead** markets provide all prices to market participants for a full day at once and can therefore support anticipation:
  - but, as mentioned, the ERCOT day-ahead market does not (currently) represent ramp-rate constraints.

Discussion, continued

- Some **real-time** markets do represent ramp-rate constraints across several (five minute) dispatch intervals:
  - California market,
  - PJM and MISO are implementing.
- The typical arrangement is to solve multi-interval dispatch (and in some cases unit commitment) for several intervals but to only commit to prices and dispatch for the next interval.
- If market participants do not anticipate prices in subsequent intervals (or if these prices are not implemented) then the market cannot incentivize sequences of dispatch through time that involve anticipation:
  - Real-time markets can represent ramp-rate constraints on change in generation between most recent interval and the next interval (see Homework Exercise 10.1), but
  - anticipation is required to incentivize actions when there are binding ramp rate constraints between two successive pairs of dispatch intervals (as was necessary in the ramp-constrained example in Section 10.1.3.3).

Discussion, continued

- Despite the implications of anticipation, the example illustrates that inter-temporal constraints do not *per se* present fundamental difficulties for pricing so long as future prices are anticipated.
- In the next section, we will see that non-convexities introduced by unit commitment decisions do pose difficulties for pricing.

10.2 Formulation of unit commitment

- Now we consider the commitment of generators.
- In a typical **day-ahead** unit commitment problem, the ISO makes decisions today about commitment, dispatch, and prices for tomorrow:
  - a commitment decision for each hour of tomorrow,
  - an energy dispatch decision and ancillary services decisions for each hour of tomorrow, and
  - prices for energy and ancillary services for each hour of tomorrow.
- In contrast to the economic dispatch problems and the generalizations we have considered so far, unit commitment requires **integer** variables to represent the decisions.
- The integer variables present difficulties in two related ways:

  (i) solving the problem, and
  (ii) finding dispatch- (and commitment-) supporting prices.

Formulation of unit commitment, continued

- We will first explicitly formulate the unit commitment problem.
- We will then analyze one particular type of algorithm, **Lagrangian relaxation**, that approximately solves such problems.
- Lagrangian relaxation will help us to understand:
  - the difficulty in solving such problems, and
  - why the previous approach to finding dispatch-supporting prices for convex problems does not (quite) work in the context of unit commitment.
- We will then consider how to provide incentives to generators to commit and dispatch consistently with the commitment and dispatch desired by the ISO.

10.2.1 Decision variables

- For simplicity, assume that generators can provide energy and one type of reserve, so the continuous decision variables for generator k in hour $t = 1, \ldots, n_T$, are $x_{kt} = \begin{bmatrix} P_{kt} \\ S_{kt} \end{bmatrix}$, typically with $n_T = 24$.
- We collect the entries x_{kt} together into a vector $x_k \in \mathbb{Z}^{2n_T}$ and collect the vectors x_k together into a vector $x \in \mathbb{Z}^{2n_P \times n_T}$.
- In addition to these continuous decision variables, we must consider representation of the decision of a generator to be on or off.
- We will represent this with **binary** variables:

$$z_{kt} = \begin{cases} 0, & \text{if generator } k \text{ is off in hour } t, \\ 1, & \text{if generator } k \text{ is on in hour } t. \end{cases}$$

- We collect the entries z_{kt} together into a vector $z_k \in \mathbb{Z}^{n_T}$ and collect the vectors z_k together into a vector $z \in \mathbb{Z}^{n_P \times n_T}$.

Decision variables, continued

- Other more general representations may be necessary in some cases:
  - **combined-cycle** generators typically have multiple operating modes, requiring **integer** variables to represent.
- Various “tricks” are typically used in the specification of problems with integer and binary variables in order to help with solution:
  - some of these tricks are proprietary or not widely known, and
  - we will simply consider a straightforward formulation.

Decision variables, continued

- We can consider the requirement for z_{kt} to be binary as consisting of two requirements:

$$\begin{aligned} z_{kt} &\in \{z_{kt} \in \mathbb{R} | 0 \leq z_{kt} \leq 1\}, \\ z_{kt} &\in \mathbb{Z}. \end{aligned}$$

- The first requirement that z_{kt} be between 0 and 1 is an example of a generator constraint that can be represented with linear inequalities.
  - This fits our previous formulation for economic dispatch.
  - As previously, suitable $\underline{\delta}_k, \overline{\delta}_k$, and Γ_k can be found to express such generator constraints in the form:

$$\underline{\delta}_k \leq \Gamma_k \begin{bmatrix} z_k \\ x_k \end{bmatrix} \leq \overline{\delta}_k.$$

  - For example, the constraint $0 \leq z_{kt} \leq 1$ could be expressed as:

$$[0] \leq [1 \quad \mathbf{0}] \begin{bmatrix} z_k \\ x_k \end{bmatrix} \leq [1].$$

Decision variables, continued

- The requirement that z_{kt} be integer-valued yields a non-convex feasible operating set for each generator:

$$\mathbb{S}_k = \left\{ \begin{bmatrix} z_k \\ x_k \end{bmatrix} \in \mathbb{Z}^{n_T} \times \mathbb{R}^{2n_T} \middle| \underline{\delta}_k \leq \Gamma_k \begin{bmatrix} z_k \\ x_k \end{bmatrix} \leq \overline{\delta}_k \right\}.$$

- This means that the unit commitment problem is a non-convex problem.
- The non-convexity makes solution difficult and complicates the pricing rule as discussed in Section 4.8.

10.2.2 Generator costs

- We now assume that the cost function for generator k depends on both z_k and x_k, so that $f_k : \mathbb{Z}^{n_T} \times \mathbb{R}^{2n_T} \to \mathbb{R}$.
- For convenience, we will sometimes assume that f_k has been extrapolated to a function $f_k : \mathbb{R}^{n_T} \times \mathbb{R}^{2n_T} \to \mathbb{R}$.
- The cost function for generator k represents:

  the cost of producing energy and of providing reserves (already considered in the dispatch problem),

  start-up costs, and

  no-load or **min-load costs** (typically associated with auxiliary costs as illustrated in Figure 5.2).
- Because start-up costs can depend on *changes* in commitment status, the cost function is no longer (completely) additively separable.

Generator costs, continued

- However, costs can usually be considered to be the sum of costs associated with:

  start-up costs, depending only on the integer variables z_k, but not additively separable,

  no-load or min-load costs, additively separable, and depending only on the integer variables $z_{kt}, t = 1, \ldots, n_T$, and

  incremental energy and reserves costs, additively separable, and depending only on the continuous x_{kt} in each period $t = 1, \ldots, n_T$ for which the unit is running.

10.2.2.1 Start-up costs

- Start-up costs depend on z_k and are not additively separable across time:

$$\sum_{t=1}^{n_T} z_{kt}(1 - z_{k,t-1})s_{kt},$$

where:

s_{kt} are the start-up costs for starting up in hour t, ignoring variation of start-up cost with time since last shutdown, and

z_{k0} is the commitment status at the end of today.

- That is, start-up costs are only incurred when a generator was off in hour $t - 1$ and on in hour t.

10.2.2.2 Min-load costs

- Min-load costs are the costs to operate at the minimum capacity, $P_k = \underline{P}_k, S_k = 0$ during an interval when the unit is committed.
- Min-load costs depend on z_k and are additively separable across time:

$$f_k\left(\begin{bmatrix} z_k \\ \mathbf{1}\underline{P}_k \\ \mathbf{0} \end{bmatrix}\right) = \sum_{t=1}^{n_T} z_{kt} \times \underline{f}_{kt} \times \underline{P}_k,$$

where:

$\underline{P}_k$ is the min load, and

$\underline{f}_{kt}$ is the min-load average energy costs per unit energy associated with operating at $\underline{P}_k$.

10.2.2.3 Incremental energy and reserves costs

- Incremental energy and reserves costs above min-load costs depend on x_{kt} in each period for which the unit is running and are additively separable across time:

$$f_k\left(\begin{bmatrix} \mathbf{1} \\ x_k \end{bmatrix}\right) = \sum_{t=1}^{n_T} f_{kt}\left(\begin{bmatrix} z_{kt} \\ x_{kt} \end{bmatrix}\right).$$

10.2.3 Objective

- As previously, we define the objective of the unit commitment problem to be the sum of the cost functions of the generators:

$$\forall z \in \mathbb{Z}^{n_P \times n_T}, x \in \mathbb{R}^{2n_P \times n_T}, f\left(\begin{bmatrix} z \\ x \end{bmatrix}\right) = \sum_{k=1}^{n_P} f_k\left(\begin{bmatrix} z_k \\ x_k \end{bmatrix}\right).$$

10.2.4 System constraints

- Typical equality constraints would include average power balance in each hour of tomorrow, which we will represent in the form $Ax = b$.
- For example, as in Section 10.1.2.2:
  - if we ignore reserve, then $x = P = \begin{bmatrix} P_1 \\ \vdots \\ P_{n_P} \end{bmatrix} \in \mathbb{R}^{n_P \times n_T}$, with
    $$P_k = \begin{bmatrix} P_{k1} \\ \vdots \\ P_{kn_T} \end{bmatrix} \in \mathbb{R}^{n_T},$$
  - let $\overline{D} \in \mathbb{R}^{n_T}$ be a vector of forecasts of average demand in each hour,
  - let $A = \begin{bmatrix} -\mathbf{I} & \cdots & -\mathbf{I} \end{bmatrix}$ and $b = -\overline{D}$,
  - then $Ax = b$ represents average power balance in each hour.
- Typical inequality constraints would include reserve requirements and transmission constraints in each hour, which we will represent in the form $Cx \leq d$.

10.2.5 Problem

- The unit commitment problem is:

$$\min_{\forall k, \left[\begin{smallmatrix} z_k \\ x_k \end{smallmatrix}\right] \in \mathbb{S}_k} \left\{ f\left(\begin{bmatrix} z \\ x \end{bmatrix} \right) \middle| Ax = b, Cx \leq d \right\}$$
$$= \min_{z \in \mathbb{Z}^{n_P \times n_T}, x \in \mathbb{R}^{2n_P \times n_T}} \left\{ f\left(\begin{bmatrix} z \\ x \end{bmatrix} \right) \middle| Ax = b, Cx \leq d, \forall k, \underline{\delta}_k \leq \Gamma_k \begin{bmatrix} z_k \\ x_k \end{bmatrix} \leq \overline{\delta}_k \right\}. \tag{10.1}$$

- In principle, the ISO solves Problem (10.1) for **optimal commitment and dispatch**, which we will denote by $z^\star$ and $x^\star$, respectively.
- In some examples and some of the development, we will only consider energy and not reserve, in which case, $x = P \in \mathbb{R}^{n_P \times n_T}$, as in the example in Section 10.2.4.

10.2.6 Generator offers

- How to specify the offers from generators to the ISO?
- Building on offer-based economic dispatch, we will still assume that the dependence of offers on power and reserves are specified as the gradient of costs with respect to power and reserves.
- We will assume that the dependence of offers on power and reserves are required to be separable across time, so that the offers are:

$$\frac{\partial f_{kt}}{\partial x_{kt}}\left(\begin{bmatrix} 1 \\ \bullet \end{bmatrix}\right)^{\dagger}, t = 1, \ldots, n_T.$$

- Abusing notation somewhat, we will write $\nabla f_{kt}(x_{kt})$ for $\frac{\partial f_{kt}}{\partial x_{kt}}\left(\begin{bmatrix} 1 \\ x_{kt} \end{bmatrix}\right)^{\dagger}$, with the understanding that the offer function dependence on power and reserves is only meaningful for $z_{kt} = 1$.
- That is, the offers are specified by the functions $\nabla f_{kt}, t = 1, \ldots, n_T$.
- We will call this collection of functions the incremental energy and reserve offers.

Generator offers, continued

- To specify the start-up costs, the values of $s_{kt}, t = 1, \ldots, n_T$ are required.
- To specify the min-load costs, the values of the min-load average energy costs $\underline{f}_{kt}, t = 1, \ldots, n_T$ and the value of the min-load $\underline{P}_k$ are required.
- We will assume that the generator specifies a **start-up offer** equal to its start-up costs, and specifies a **min-load offer** equal to its min-load average energy cost.
- The offer cost function can then be reconstructed from the start-up offer, the min-load energy offer, the min-load, and the incremental energy and reserve offers:

$$f_k\left(\begin{bmatrix} z_k \\ x_k \end{bmatrix}\right) = \sum_{t=1}^{n_T} z_{kt}(1 - z_{k,t-1})s_{kt} + \sum_{t=1}^{n_T} z_{kt}\underline{f}_{kt}\underline{P}_k + \sum_{t=1}^{n_T} \int_{y_{kt}=\begin{bmatrix} \underline{P}_k \\ 0 \end{bmatrix}}^{y_{kt}=x_{kt}} z_{kt}[\nabla f_{kt}(y_{kt})]^\dagger \, dy_{kt}.$$

- In contrast to the economic dispatch problem, it is necessary to explicitly represent the cost function (and not just its derivative) in order to:
  - compare alternative costs of committing and dispatching different combinations of generators in Problem (10.1), and
  - (as we will see in Section 10.4.3) to calculate **make-whole** costs.

10.3 Lagrangian relaxation

10.3.1 Description

- Lagrangian relaxation is a computational technique for approximately solving problems such as the unit commitment problem.
- The system constraints are dualized and we maximize the dual:

$$\max_{\lambda,\mu\geq\mathbf{0}}\left\{\min_{\forall k,\left[\begin{smallmatrix}z_k\\x_k\end{smallmatrix}\right]\in\mathbb{S}_k}\left\{f\left(\begin{bmatrix}z\\x\end{bmatrix}\right)+\lambda^\dagger(Ax-b)+\mu^\dagger(Cx-d)\right\}\right\}. \qquad (10.2)$$

- This is analogous to solving the economic dispatch problem by dualizing the system constraints.
- We will consider the use of a similar pricing rule to that used in offer-based economic dispatch, but based on either the current value of the dual variables at a particular iteration or on the maximizer of the dual.
- This separates the unit commitment problem into:
  - a sub-problem for each generator involving profit maximization for the generator given the value of dual variables, and
  - the problem of finding the values of the dual variables that maximize the dual.

Description, continued

- Each generator maximizes its operating profit for the given vector of prices, as specified by the current values of the dual variables.
- The dual variables are updated until a maximum of the dual function is obtained.
- There may be a duality gap.
- If the duality gap is non-zero then an *ad hoc* post-processing step is required to produce a solution that satisfies the system constraints:
  - such *ad hoc* steps are problematic because they present opportunities for market participants to influence outcomes through changes to offers that do not represent economic fundamentals,
  - in practice, all North American ISOs now use mixed-integer programming software to solve unit commitment problems.
- We will see that the maximizer of the dual can nevertheless provide important insights even if it does not yield the optimal unit commitment.

10.3.2 Example

- Recall the previous example in Section 4.8.3 in the context of duality gaps where a single generator was available to meet a demand of $\overline{D} = 3$ MW in the single period $n_T = 1$.
- The generator had two variables associated with its operation:
  - the "unit commitment" variable $z \in \mathbb{Z}$, and
  - the "production" variable $x = P \in \mathbb{R}$.
- The cost function $f : \mathbb{Z} \times \mathbb{R} \to \mathbb{R}$ for the generator is:

$$f\left(\begin{bmatrix} z \\ x \end{bmatrix}\right) = 4z + x, z \in \{0, 1\}, 2z \leq x \leq 4z.$$

- Recall that if the generator were paid λ for its production then its profit maximizing behavior would be:

$$x = \begin{cases} 0, & \text{if } \lambda < 2, \\ 0 \text{ or } 4, & \text{if } \lambda = 2, \\ 4, & \text{if } \lambda > 2. \end{cases}$$

- This meant that no price would equate supply to demand of 3 MW.

Example, continued

- Now consider the case of a generator with cost function:

$$f\left(\begin{bmatrix} z \\ x \end{bmatrix}\right) = 4z + \beta x, z \in \{0, 1\}, 2z \leq x \leq 4z,$$

- where $\beta \geq 0$.
- Suppose that the generator is paid λ for its power production x and that it finds the value of production that maximizes profit specified by:

$$\lambda x - f\left(\begin{bmatrix} z \\ x \end{bmatrix}\right).$$

- We perform similar analysis to previously to find the profit maximizing x (and z).

Example, continued

- To maximize profit $\lambda x - f\left(\begin{bmatrix} z \\ x \end{bmatrix}\right) = (\lambda - \beta)x - 4z$, we must compare:

  the profit for $z = 0$ and $x = 0$, (namely, a profit of 0), to
  the maximum profit over $2 \leq x \leq 4$ for $z = 1$.
- We consider various cases for λ.

$\lambda \leq \beta$

$$\begin{aligned} 0 &> -4, \\ &\geq (\lambda - \beta)x - 4, \text{ for } 2 \leq x \leq 4. \end{aligned}$$

  - So, the profit is maximized for $z = 0, x = 0$.

$\beta < \lambda < 1 + \beta$

  - Then $(\lambda - \beta)x < 4$ for $2 \leq x \leq 4$.

$$0 > (\lambda - \beta)x - 4, \text{ for } 2 \leq x \leq 4.$$

  - So, the profit is again maximized for $z = 0, x = 0$.

Example, continued

$\lambda = 1 + \beta$

- Then $0 > (\lambda - \beta)x - 4$ for $2 \leq x < 4$.
- Also, $0 = (\lambda - \beta)x - 4$ for $x = 4$.
- So, the profit has two maximizers:

  $z = 0, x = 0$, and
  $z = 1, x = 4$.

$\lambda > 1 + \beta$

$$0 < (\lambda - \beta)x - 4, \text{ for } x = 4.$$

- Moreover, the right-hand side increases with increasing x, so it is maximized over $2 \leq x \leq 4$ by $x = 4$.
- So, the profit is maximized for $z = 1, x = 4$.

Example, continued

- Therefore, if the generator were paid λ for its production then its profit maximizing behavior would be:

$$x = \begin{cases} 0, & \text{if } \lambda < 1+\beta, \\ 0 \text{ or } 4, & \text{if } \lambda = 1+\beta, \\ 4, & \text{if } \lambda > 1+\beta. \end{cases}$$

- If we have just one generator having marginal cost β then there will still typically be no price that equates supply to demand, unless demand were changed to $\overline{D} = 0$ or 4.
- The price, $\lambda = 1+\beta$, at which the generator starts to produce depends on β.
- We still typically have a duality gap since the minimum of Problem (10.1) is strictly greater than the maximum of Problem (10.2).

10.3.3 Larger example

- Suppose that we generalize the example problem from Sections 10.3.2 and 4.8.3 to the case where there are multiple generators with different cost characteristics and a larger demand.
- Suppose that demand was $\overline{D} = 303$ MW.
- Assume that there are no reserve requirements, so $x_k = P_k$ for generator k.
- Suppose that there are 100 generators, with generator $k = 1, \ldots, 100$ having cost function:

$$f_k\left(\begin{bmatrix} z_k \\ x_k \end{bmatrix}\right) = 4z_k + \beta_k x_k, z_k \in \{0,1\}, 2z_k \leq x_k \leq 4z_k,$$

- where:

$$\forall k = 1, \ldots, 100, \beta_k = 1 + k/100.$$

- The feasible operating set for each generator k is:

$$\mathbb{S}_k = \left\{ \begin{bmatrix} z_k \\ x_k \end{bmatrix} \middle| z_k \in \{0,1\}, 2z_k \leq x_k \leq 4z_k \right\}.$$

10.3.3.1 Solution

- Each generator has a slightly different operating cost function, with higher values of k associated with more expensive generators.
- The optimal commitment is for:
  - generators $1,\ldots,75$ to be committed and producing at full capacity of 4,
  - generator 76 to be committed and producing 3, and
  - generators $77,\ldots,100$ to be off.
- Minimum cost is therefore:

$$\sum_{k=1}^{75}[4\times 1+(1+k/100)\times 4]+[4\times 1+(1+76/100)\times 3]=723.28.$$

- This is the minimum of Problem (10.1), which we could find in this case because of the simple structure of the problem.
- We will investigate the maximizer of the dual problem, Problem (10.2), and see the insights it provides into the minimum and minimizer of Problem (10.1):
  - these insights from the dual problem are useful even if we cannot find the minimum and minimizer of Problem (10.1)

10.3.3.2 Maximizer of dual

- The dual problem, Problem (10.2), in this case is:

$$\max_{\lambda \in \mathbb{R}} \left\{ \min_{\forall k=1,\ldots,100, \left[\begin{smallmatrix} z_k \\ x_k \end{smallmatrix}\right] \in \mathbb{S}_k} \left\{ f\left(\begin{bmatrix} z \\ x \end{bmatrix}\right) + \lambda \left(\overline{D} - \sum_{k=1}^{100} x_k \right) \right\} \right\}.$$

- Suppose we set λ so that $2 + 75/100 < \lambda < 2 + 76/100$.
  - For example, suppose that we set the price to be $\lambda = 2.755$.
  - Generators $k = 1, \ldots, 75$ will produce 4 MW.
  - Generators $k = 76, \ldots, 100$ will produce nothing.
  - Total production will be 300 MW.
  - The dual function is:

$$\begin{aligned} \mathcal{D}(2.755) &= \sum_{k=1}^{75} [4 \times 1 + (1 + k/100) \times 4] + 2.755 \times \left(\overline{D} - \sum_{k=1}^{75} 4 \right), \\ &= 722.265. \end{aligned}$$

- For values of $\lambda \leq 2 + 75/100$, the value of the dual will be less than or equal to 722.265.

Maximizer of dual, continued

- Now suppose that we set λ so that $2+76/100 < \lambda < 2+77/100$.
  - For example, suppose that we set the price to be $\lambda = 2.765$.
  - Generators $k = 1,\ldots,76$ will produce 4 MW.
  - Generators $k = 77,\ldots,100$ will produce nothing.
  - Total production will be 304 MW.
  - The dual function is:

$$\begin{aligned}\mathcal{D}(2.765) &= \sum_{k=1}^{76}[4\times 1+(1+k/100)\times 4]+2.765\times\left(\overline{D}-\sum_{k=1}^{76}4\right),\\ &= 722.275.\end{aligned}$$

- For values of $\lambda \geq 2+77/100$, the value of the dual will be less than or equal to 722.275.

Maximizer of dual, continued

- The maximizer of the dual, Problem (10.2), occurs for $\lambda^\star = 2.76$:
  - Generators $k = 1, \ldots, 75$ will produce 4 MW.
  - Generator $k = 76$ is indifferent to either not producing or producing 4 MW.
  - Generators $k = 77, \ldots, 100$ will produce nothing.
  - Total production is either 300 or 304 MW.
  - The dual function is:

$$\begin{aligned}
\mathcal{D}(2.76) &= \sum_{k=1}^{75} [4 \times 1 + (1 + k/100) \times 4] + 2.76 \times \left(\overline{D} - \sum_{k=1}^{75} 4\right), \\
&= \sum_{k=1}^{76} [4 \times 1 + (1 + k/100) \times 4] + 2.76 \times \left(\overline{D} - \sum_{k=1}^{76} 4\right), \\
&= 722.28.
\end{aligned}$$

Maximizer of dual, continued

- There is no price were supply equals demand of 303 MW.
- However, the supply-demand constraint is violated by a *relatively* smaller amount than in the smaller example in Sections 10.3.2 and 4.8.3.
- Moreover, the commitment and dispatch decisions for generators $k = 1, \ldots, 75$ and $77, \ldots, 100$ in the generator profit maximization problems are correct given that the price is $\lambda^\star = 2.76$.
- The duality gap is $723.28 - 722.28 = 1$.
- The duality gap is relatively smaller as a fraction of the minimum of the unit commitment problem.

10.4 Duality gaps

10.4.1 Discussion

- In both the first example and the larger example, there is a duality gap.

  The maximum of the dual obtained by dualizing the system constraints is strictly less than the minimum of the primal problem.

  The commitment variables z and the dispatch variables x resulting from the generator profit maximization sub-problems do not satisfy the system constraints.

- However, the duality gap is relatively smaller in the larger example than in the first example and the system constraints are violated by a relatively smaller amount, so the commitment and dispatch values corresponding to the dual maximizer can provide a useful approximate guide to the optimum of the unit commitment Problem (10.1).

Discussion, continued

- If the generator cost characteristics are heterogeneous then the duality gap (and the violation of the system constraints) becomes relatively smaller as the number of generators grows large.
- This is the key to application of Lagrangian relaxation to large-scale systems since the post-processing step to create a feasible solution involves a smaller adjustment for larger systems.
- What are reasons for heterogeneity and homogeneity in the cost functions of generators?

10.4.2 Non-existence of dispatch-supporting prices

- Unfortunately, the non-zero duality gap means that prices on the system constraints alone cannot encourage profit-maximizing generators to commit and dispatch in a way that is (exactly) consistent with optimal commitment and dispatch.
- For each value of the price vector, some system constraint will fail to be satisfied by the resulting profit-maximizing decisions of the generators.

Non-existence of dispatch-supporting prices, continued

- As Stoft argues, by modifying demand slightly we can obtain dispatch supporting prices:
  - if the generation stock is heterogeneous then the modification will be small,
  - in the larger example, the modification would be at most 2 MW,
  - since there are other uncertainties and errors in dispatch, it may be reasonable to ignore the duality gap in this case.
- This is the basis of a principled argument against centralized unit commitment.
- However we will continue to assume that the ISO performs centralized unit commitment:
  - ERCOT and other ISOs optimize the commitment and dispatch in the day-ahead market, reflecting the complexity of the various constraints, particularly transmission constraints.
- In the next section, we will develop an approach to inducing the generators to produce consistent with optimal commitment and dispatch that involves an additional **make-whole** payment to some generators.

10.4.3 Make-whole payment

- As discussed above, the non-zero duality gap means that prices on the system constraints *alone* cannot encourage profit-maximizing generators to *all* commit and dispatch in a way that is consistent with optimal unit commitment.
- However, in the example with $\overline{D} = 303$, all but one of the generators would be committed and dispatched correctly if the price were set equal to the maximizer of the dual $\lambda^\star = 2.76$:
  - generators $k = 1, \ldots, 75$ would produce 4 MW, while
  - generators $k = 77, \ldots, 100$ will produce nothing.
- Generators $k = 1, \ldots, 75$ and $77, \ldots, 100$ would collectively produce a total of 300 MW.

Make-whole payment, continued

- To meet the total demand of $\overline{D} = 303$ MW, generator $k = 76$ should produce 3 MW:
  - the cost for generator $k = 76$ to produce 3 MW is:

$$
\begin{aligned}
f_{76}\left(\begin{bmatrix} z_{76}^{\star} \\ x_{76}^{\star} \end{bmatrix}\right) &= 4z_{76}^{\star} + \beta_{76} x_{76}^{\star}, \\
&= 4 \times 1 + (1 + 76/100) \times 3, \\
&= 9.28.
\end{aligned}
$$

  - with an energy price of $\lambda^{\star} = \$2.76$/MWh, generator $k = 76$ would receive revenues of $\lambda^{\star} \times x_{76}^{\star} = 2.76 \times 3 = 8.28$ if it produced $x_{76}^{\star} = 3$.
  - generator $k = 76$ would need an additional payment of $9.28 - 8.28 = \$1$/h in order to have non-negative profit, based on an energy price of $\lambda^{\star} = \$2.76$/MWh,
  - this difference is equal to the duality gap.

Make-whole payment, continued

- Suppose the ISO determines prices $\lambda^{\star}$ for energy and $\mu^{\star}$ for reserve using some algorithm:
  - for example, the maximizer of the dual from Lagrangian relaxation, which leads to so-called **convex hull pricing**,
  - however, most ISOs currently use a different approach for energy and reserve prices, which we will also describe.
- We consider the profit maximizing response to these prices.
- For some generators, their profit maximizing generation based on these energy and reserves prices will be consistent with optimal commitment and dispatch:
  - these generators are paid based on these energy and reserves prices.
- For the rest of the generators, pay based on:
  - the energy and reserves prices, plus
  - an additional **make-whole** payment that induces them to generate consistent with optimal commitment and dispatch.
- What would the make-whole payment be for generator $k = 76$ to induce it to produce 3 MW, given an energy price of \$2.76/MWh?

Make-whole payment, continued

- We seek a general expression for the make-whole payment that would induce behavior consistent with optimal commitment and dispatch.
- For simplicity, we consider energy prices and ignore reserve prices.
- Suppose the ISO specifies a vector of energy prices $\lambda^\star \in \mathbb{R}^{n_T}$.
- We consider two cases:
  - (i) generator k can choose its commitment $z_k^{\star\star}$ and dispatch $x_k^{\star\star}$ to maximize its operating profit given $\lambda^\star$, and
  - (ii) generator k commits and dispatches consistent with the solution of the ISO optimal commitment $z_k^\star$ and dispatch $P_k^\star$.

10.4.3.1 Generator profit maximization

- Generator k operating profit maximum, given prices $\lambda^\star$, is:

$$\text{Profit}_k^{\star\star}(\lambda^\star) = \max_{\left[\begin{smallmatrix} z_k \\ x_k \end{smallmatrix}\right] \in \mathbb{S}_k} \left\{ [\lambda^\star]^\dagger x_k - f_k\left(\begin{bmatrix} z_k \\ x_k \end{bmatrix} \right) \right\},$$

- where, as previously, the double star refers to generator operating profit maximization.

10.4.3.2 Profit under optimal commitment and dispatch from ISO problem

- Given an energy price $\lambda^\star$ and given that generator k operated according to the optimal commitment $z_k^\star$ and dispatch $x_k^\star$ determined by the ISO, the profit for generator k would be:

$$[\lambda^\star]^\dagger x_k^\star - f_k\left(\begin{bmatrix} z_k^\star \\ x_k^\star \end{bmatrix}\right).$$

- Note that, by definition:

$$\text{Profit}_k^{\star\star}(\lambda^\star) \geq [\lambda^\star]^\dagger x_k^\star - f_k\left(\begin{bmatrix} z_k^\star \\ x_k^\star \end{bmatrix}\right).$$

- Moreover, if:

$$\text{Profit}_k^{\star\star}(\lambda^\star) = [\lambda^\star]^\dagger x_k^\star - f_k\left(\begin{bmatrix} z_k^\star \\ x_k^\star \end{bmatrix}\right).$$

- then the profit maximizing decision of generator k is consistent with the ISO optimal commitment and dispatch:
  - the vector of prices $\lambda^\star$ **supports** the ISO optimal commitment and dispatch.

Make-whole payment, continued

- If $\text{Profit}_k^{\star\star}(\lambda^\star) = [\lambda^\star]^\dagger x_k^\star - f_k\left(\begin{bmatrix} z_k^\star \\ x_k^\star \end{bmatrix}\right)$:
  - then the profit maximizing behavior of generator k in response to $\lambda^\star$ alone is consistent with optimal commitment and dispatch,
  - no make-whole payment is needed.
- If $\text{Profit}_k^{\star\star}(\lambda^\star) > [\lambda^\star]^\dagger x_k^\star - f_k\left(\begin{bmatrix} z_k^\star \\ x_k^\star \end{bmatrix}\right)$:
  - then the profit maximizing behavior of generator k in response to $\lambda^\star$ alone is inconsistent with optimal commitment and dispatch,
  - an additional make-whole payment of $\text{Profit}_k^{\star\star}(\lambda^\star) - \left([\lambda^\star]^\dagger x_k^\star - f_k\left(\begin{bmatrix} z_k^\star \\ x_k^\star \end{bmatrix}\right)\right)$ is necessary to induce behavior that is consistent with optimal commitment and dispatch.
- We can combine both cases by observing that the payment is equal to $\text{Profit}_k^{\star\star}(\lambda^\star) - \left([\lambda^\star]^\dagger x_k^\star - f_k\left(\begin{bmatrix} z_k^\star \\ x_k^\star \end{bmatrix}\right)\right)$ in both cases.

Make-whole payment, continued

- Note that make-whole payment is only made to generator k if generator k commits and dispatches according to $\begin{bmatrix} z_k^\star \\ x_k^\star \end{bmatrix}$.
- By design, the make-whole payment adjusts the profit for generator k so that $\begin{bmatrix} z_k^\star \\ x_k^\star \end{bmatrix}$ is generator k's profit maximizing commitment and dispatch.
- In principle, no additional inducement is necessary for generator k to behave consistently with centralized optimal unit commitment and dispatch:
  - need to set tolerance on commitment and dispatch being "close enough" to $\begin{bmatrix} z_k^\star \\ x_k^\star \end{bmatrix}$ to qualify to receive make-whole payment.

10.4.4 Simplified make-whole payment

- To develop a simplified make-whole payment, observe that there are three possibilities for profit-maximizing behavior $z_k^{\star\star}$ and $P_k^{\star\star}$ by generator k in response to the price $\lambda^\star$:

  (i) the generator does not commit, so that $z_k^{\star\star} = \mathbf{0}$ and $P_k^{\star\star} = \mathbf{0}$ and $0 = \text{Profit}_k^{\star\star}(\lambda^\star) \geq [\lambda^\star]^\dagger x_k^\star - f_k\left(\begin{bmatrix} z_k^\star \\ x_k^\star \end{bmatrix}\right)$,

  (ii) the generator commits and dispatches consistently with ISO optimal commitment and dispatch, so that $z_k^{\star\star} = z_k^\star$ and $P_k^{\star\star} = P_k^\star$, and $\text{Profit}_k^{\star\star}(\lambda^\star) = [\lambda^\star]^\dagger x_k^\star - f_k\left(\begin{bmatrix} z_k^\star \\ x_k^\star \end{bmatrix}\right) > 0$, or

  (iii) the generator commits and dispatches, but inconsistently with ISO optimal commitment and dispatch, so that $z_k^{\star\star} \neq z_k^\star$ and/or $P_k^{\star\star} \neq P_k^\star$, and $\text{Profit}_k^{\star\star}(\lambda^\star) > [\lambda^\star]^\dagger x_k^\star - f_k\left(\begin{bmatrix} z_k^\star \\ x_k^\star \end{bmatrix}\right)$

Simplified make-whole payment, continued

- Note that for the first alternative, a make-whole payment of $f_k\left(\begin{bmatrix} z_k^\star \\ x_k^\star \end{bmatrix}\right) - [\lambda^\star]^\dagger x_k^\star$ would be required to make generator k indifferent between:
  - not committing, and
  - commitment and dispatching consistently with ISO optimal commitment and dispatch.
- In the second alternative, no make-whole payment is required since profit-maximization is consistent with ISO optimal commitment and dispatch.
- So, if we ignore the third alternative, then the make-whole payment can be simplified to:

$$\max\left\{0, f_k\left(\begin{bmatrix} z_k^\star \\ x_k^\star \end{bmatrix}\right) - [\lambda^\star]^\dagger x_k^\star\right\}.$$

Simplified make-whole payment, continued

- The simplified make-whole payment of $\max\left\{0, f_k\left(\begin{bmatrix} z_k^\star \\ x_k^\star \end{bmatrix}\right) - [\lambda^\star]^\dagger x_k^\star\right\}$ is used in ERCOT and other markets, even though it does not have the correct incentives in the case that both:
  $\text{Profit}_k^{\star\star}(\lambda^\star) \neq 0$, and $\text{Profit}_k^{\star\star}(\lambda^\star) > f_k\left(\begin{bmatrix} z_k^\star \\ x_k^\star \end{bmatrix}\right) - [\lambda^\star]^\dagger x_k^\star$.
- To ensure that behavior is actually consistent with optimal commitment and dispatch:
  - make-whole is only paid when generator k commits and dispatches "close enough" to $\begin{bmatrix} z_k^\star \\ x_k^\star \end{bmatrix}$, and
  - generator k is penalized if it deviates too far from $\begin{bmatrix} z_k^\star \\ x_k^\star \end{bmatrix}$.

10.4.5 Make-whole payment in example

- To achieve optimal unit commitment in the example:
  - price energy based on the dual maximizer, $\lambda^\star = \$2.76/\text{MWh}$,
  - profit-maximizing behavior of generators $1,\ldots,75$, and $77,\ldots,100$ in response to this price is to behave consistently with centralized optimal unit commitment and dispatch, but
  - an additional **make-whole payment** is paid to generator 76 of:

$$\begin{aligned}
\text{Profit}^{\star\star}_{76}(\lambda^\star) - \left(\lambda^\star x^\star_{76} - f_{76}\left(\begin{bmatrix} z^\star_{76} \\ x^\star_{76} \end{bmatrix}\right)\right) &= 0 - \left(\lambda^\star x^\star_{76} - f_{76}\left(\begin{bmatrix} z^\star_{76} \\ x^\star_{76} \end{bmatrix}\right)\right), \\
&= \max\left\{0, f_{76}\left(\begin{bmatrix} z^\star_{76} \\ x^\star_{76} \end{bmatrix}\right) - [\lambda^\star]^\dagger x^\star_{76}\right\}, \\
&= 4z^\star_{76} + \beta_{76} x^\star_{76} - \lambda^\star x^\star_{76}, \\
&= 9.28 - 8.28 = 1,
\end{aligned}$$

  - generator 76 requires an additional \$1/h to induce it to generate consistent with optimal commitment and dispatch,
  - the make-whole payment would only be paid if generator 76 commits and dispatches consistently with the optimal commitment and dispatch.

Make-whole payment in example, continued

- Demand pays for:
  - energy based on $\lambda^\star \times \overline{D} = 2.76 \times 303 = \$836.28/\text{h}$, plus
  - the make-whole payment to generator 76 of \$1/h.
- The make-whole payment is charged as an uplift to demand.
- Note that the payments are no longer anonymous since the payment to generator 76 is qualitatively different to the payment to other generators since it involves a make-whole payment.

10.4.6 Demand response

- Demand response can reduce the duality gap (and therefore reduce the make-whole payment required to achieve optimality).
- Suppose that instead of fixed demand of 303 MW, the demand was the sum of:

  a fixed demand of $\overline{D} = 290$ MW, plus
  price-responsive demand ΔD with willingness-to-pay of

$$(2.755 + 10) \text{ \$/MWh} - 1 \text{ \$/h} \times \Delta D, \; 0 \leq \Delta D \leq 20 \text{ MW}.$$

- At a price of $\lambda^{\star} = \$2.755$/MWh, the price-responsive demand would be $\Delta D = 10$ MW, so that total demand would be $290 + 10 = 300$ MW.
- At a price of $\lambda^{\star} = \$2.755$/MWh, the supply equals 300 MW.
- So, supply equals demand and there is no duality gap and no need for a make-whole payment.
- In general, price-responsive demand can reduce the duality gap and reduce the make-whole payments.

Demand response, continued

- This demand response example is somewhat unrealistic in that demand is generally not willing to voluntarily curtail at prices that are close to typical generation marginal costs:
  - we will assume fixed demand in subsequent examples.
- Such price responsiveness does, however, have an important effect in the presence of scarcity and/or market power where offer prices might otherwise rise to far above generation marginal costs.
- See market power course, http://users.ece.utexas.edu/~baldick/classes/394V_market_power/EE394V_market_power.html.
- Moreover, as in Section 8.12.7.7, there may be representation of responsive demand for adequacy reserve.

10.5 Mixed-integer programming

- Commercial software for solving mixed-integer programming problems has become much more capable in the last decade.
- In principle, such algorithms can exactly solve Problem (10.1):

$$\min_{\forall k, \left[\begin{smallmatrix} z_k \\ x_k \end{smallmatrix}\right] \in \mathbb{S}_k} \left\{ f\left(\begin{bmatrix} z \\ x \end{bmatrix} \right) \middle| Ax = b, Cx \leq d \right\}$$
$$= \min_{z \in \mathbb{Z}^{n_P \times n_T}, x \in \mathbb{R}^{2n_P \times n_T}} \left\{ f\left(\begin{bmatrix} z \\ x \end{bmatrix} \right) \middle| Ax = b, Cx \leq d, \forall k, \underline{\delta}_k \leq \Gamma_k \begin{bmatrix} z_k \\ x_k \end{bmatrix} \leq \overline{\delta}_k \right\}.$$

- All US ISOs now use mixed-integer programming algorithms for solving unit commitment.
- However, these algorithms may be terminated before completion to optimality because of excessive computational effort.
- We will nevertheless suppose that the ISO can solve Problem (10.1) and that the minimizer is $z^\star$ and $x^\star$:
  - we already made this assumption implicitly in the discussion of Lagrangian relaxation since we found the optimal commitment and dispatch by inspection.

10.6 Another approach to make-whole payments

10.6.1 Analysis

- Recall that in offer-based economic dispatch we required that the cost function was convex.
- Similarly, the incremental energy and reserves costs in the unit commitment problem are required to be convex.
- Therefore, for fixed z, the cost function $f\left(\begin{bmatrix} z \\ \bullet \end{bmatrix}\right)$ is convex.
- In particular, $f\left(\begin{bmatrix} z^\star \\ \bullet \end{bmatrix}\right)$ is convex.
- We suppose that the cost function f can be *extrapolated* to a function that is convex on $\mathbb{R}^{n_P \times n_T} \times \mathbb{R}^{2n_P \times n_T}$.
  - For example, each f_k is convex if can be expressed as a linear function of $\begin{bmatrix} z_k \\ x_k \end{bmatrix}$,
  - If each f_k is convex then f is convex.

- Consider the following problem:

$$\min_{z\in\mathbb{R}^{n_P\times n_T},x\in\mathbb{R}^{2n_P\times n_T}}\left\{ f\left(\begin{bmatrix} z \\ x \end{bmatrix}\right) \,\middle|\, \begin{array}{c} Ax=b, Cx\leq d, \forall k, \underline{\delta}_k \leq \Gamma_k \begin{bmatrix} z_k \\ x_k \end{bmatrix} \leq \overline{\delta}_k, \\ z_{kt}\leq 0, \forall k,t \text{ such that } z^\star_{kt}=0, \\ z_{kt}\geq 1, \forall k,t \text{ such that } z^\star_{kt}=1 \end{array} \right\}. \tag{10.3}$$

- The feasible set of Problem (10.3) is a subset of the feasible set of Problem (10.1).
- Problem (10.3) is *convex* since:

  we allow $z\in\mathbb{R}^{n_P\times n_T}$ to be chosen from a convex set,
  each entry of z is constrained by the inequalities to be one particular
  fixed value, either 0 or 1, and
  $f\left(\begin{bmatrix} z \\ \bullet \end{bmatrix}\right)$ is convex for fixed z.
- Therefore, by Theorem 4.12, the first-order necessary conditions are satisfied by a minimizer of this problem.

Analysis, continued

- The constraints on z in Problem (10.3) require that $z = z^\star$:
  - we have required that the discrete variables in Problem (10.3) are equal their optimal values from Problem (10.1).
- Moreover, $\begin{bmatrix} z^\star \\ x^\star \end{bmatrix}$ is a minimizer of Problem (10.3), since $\begin{bmatrix} z^\star \\ x^\star \end{bmatrix}$:
  - is feasible for Problem (10.3), and
  - is a minimizer of Problem (10.1), which has a feasible set that contains the feasible set for Problem (10.3).
- Finally, $x^\star$ is also a minimizer of:

$$\min_{x \in \mathbb{R}^{2n_P \times n_T}} \left\{ f\left(\begin{bmatrix} z^\star \\ x \end{bmatrix}\right) \middle| Ax = b, Cx \leq d, \forall k, \underline{\delta}_k \leq \Gamma_k \begin{bmatrix} z_k^\star \\ x_k \end{bmatrix} \leq \overline{\delta}_k \right\}, \quad (10.4)$$

- where the discrete variables are explicitly fixed at their optimal values.

Analysis, continued

- As previously, we define $\lambda^\star$, $\mu^\star$, $\underline{\mu}^\star$, and $\overline{\mu}^\star$ to be the Lagrange multipliers on the system equality and inequality, and the generator lower and upper bound constraints in Problem (10.3):
  - note that $\lambda^\star$ and $\mu^\star$ will typically be different to the values of the dual maximizer calculated in the Lagrangian relaxation approach,
  - however, solving Problem (10.4), where the discrete variables are explicitly fixed at their optimal values, would result in the same values of $\lambda^\star$, $\mu^\star$, $\underline{\mu}^\star$, and $\overline{\mu}^\star$ as in Problem (10.3).
- Additionally, define $\underline{\rho}^\star$ and $\overline{\rho}^\star$ to be the vectors of Lagrange multipliers on the additional constraints in Problem (10.3):

$$\begin{aligned}
\underline{\rho}^\star_{kt} \text{ is the Lagrange multiplier on } z_{kt} \leq 0, \forall k,t \text{ such that } z^\star_{kt} &= 0, \\
\overline{\rho}^\star_{kt} \text{ is the Lagrange multiplier on } z_{kt} \geq 1, \forall k,t \text{ such that } z^\star_{kt} &= 1.
\end{aligned}$$

- We now interpret these constraints to be additional system constraints on additional **commitment commodities** and pay generators based on these constraints and on the associated Lagrange multipliers.
- This will form the basis of an another make-whole payment rule.

Analysis, continued

- That is, we define a pricing rule for both the integer and the continuous decision variables according to:

$$\begin{aligned} \pi_{x_k} &= -[A_k]^\dagger \lambda^\star - [C_k]^\dagger \mu^\star, \\ \pi_{z_k} &= \overline{\rho}_k^\star - \underline{\rho}_k^\star, \end{aligned}$$

- where $\underline{\rho}_k^\star$ and $\overline{\rho}_k^\star$ are the vectors of Lagrange multipliers associated with generator k.
- The payment to a generator again consists of:
  - a payment for energy (and reserves) based on $\lambda^\star$ and $\mu^\star$, and
  - a make-whole payment (or charge) based on $\overline{\rho}_k^\star$ and $\underline{\rho}_k^\star$.
- The make-whole payment is again charged as an uplift to demand.
- As with the previous make-whole payment based on the maximizer of the dual, these prices are not anonymous since the payment to generator k depends on a Lagrange multiplier explicitly associated with a generator constraint for generator k.

Analysis, continued

- Consider the Lagrange multiplier $\overline{\rho}^{\star}_{kt}$ corresponding to a particular constraint $z_{kt} \geq 1$.
- If $\overline{\rho}^{\star}_{kt} \neq 0$ and $z_{kt} = 1$ then generator k receives a contribution to payment of $\overline{\rho}^{\star}_{kt}$.
- Similarly, if $\underline{\rho}^{\star}_{kt} \neq 0$ and $z_{kt} = 1$ then generator k would be charged a penalty.
- Over a day, the total payment is:

$$[\pi_{z_k}]^{\dagger} z_k = \sum_{t=1}^{n_T} (\overline{\rho}^{\star}_{kt} - \underline{\rho}^{\star}_{kt}) z_{kt},$$

- where we define $\overline{\rho}^{\star}_{kt}$ to be zero if $z^{\star}_{kt} = 0$ and define $\underline{\rho}^{\star}_{kt}$ to be zero if $z^{\star}_{kt} = 1$.

10.6.2 Commitment-supporting prices

- We show that the pricing rule results in $z_k^\star$ and $x_k^\star$ being the profit-maximizing commitment and dispatch decisions for generator k.
- Consider the *relaxation* of generator k's profit maximization problem obtained by allowing it to choose continuous values of z_{kt}.
- That is, we relax the feasible operating set to be:

$$\overline{\mathbb{S}}_k = \left\{ \begin{bmatrix} z_k \\ x_k \end{bmatrix} \in \mathbb{R}^{n_T} \times \mathbb{R}^{2n_T} \middle| \underline{\delta}_k \leq \Gamma_k \begin{bmatrix} z_k \\ x_k \end{bmatrix} \leq \overline{\delta}_k \right\}.$$

- We call this the **relaxed generator feasible operating set**.

Commitment-supporting prices, continued

- We consider the solution of the relaxed profit maximization problem for generator k:

$$\max_{\left[\begin{smallmatrix} z_k \\ x_k \end{smallmatrix}\right] \in \overline{\mathbb{S}}_k} \left\{ [\pi_{x_k}]^\dagger x_k + \sum_{t=1}^{n_T} (\overline{\rho}^\star_{kt} - \underline{\rho}^\star_{kt}) z_{kt} - f_k\left(\begin{bmatrix} z_k \\ x_k \end{bmatrix}\right) \right\}, \tag{10.5}$$

- where $\begin{bmatrix} z_k \\ x_k \end{bmatrix}$ can be chosen from the relaxed generator feasible operating set $\overline{\mathbb{S}}_k$, and
- where the price vectors for, respectively, energy and reserves (and other ancillary services) (π_{x_k}), and for commitment variables (π_{z_k}), are:

$$\begin{aligned} \pi_{x_k} &= -[A_k]^\dagger \lambda^\star - [C_k]^\dagger \mu^\star, \\ \pi_{z_k} &= \overline{\rho}^\star_k - \underline{\rho}^\star_k. \end{aligned}$$

- This relaxation of the profit maximization problem is a convex problem since the relaxed generator feasible operating set is convex and the objective is assumed to be convex on this feasible set.

Commitment-supporting prices, continued

- We apply pricing Theorem 8.3 to Problem (10.3).
- Denote the solution of the relaxed generator profit maximization problem, Problem (10.5), by $\begin{bmatrix} z_k^{\star\star} \\ x_k^{\star\star} \end{bmatrix}$.
- We interpret $Ax = b$, $Cx \leq d$, $z_{kt} \leq 0, \forall t$ such that $z_{kt}^{\star} = 0$, and $z_{kt} \geq 1, \forall t$ such that $z_{kt}^{\star} = 1$ as "system constraints" in Problem (10.3).
- Then, by Theorem 8.3, "dispatching" at $\begin{bmatrix} z_k^{\star\star} \\ x_k^{\star\star} \end{bmatrix}$ (assuming it were possible) would yield profits for generator k that are no higher than the profits under the corresponding entries of the solution of the convex Problem (10.3), namely $\begin{bmatrix} z_k^{\star} \\ x_k^{\star} \end{bmatrix}$.
- Therefore, $\begin{bmatrix} z_k^{\star} \\ x_k^{\star} \end{bmatrix}$ is optimal for the relaxed generator profit maximization problem, Problem (10.5).

- But we already know that $z_k^\star, x_k^\star$ is feasible for the un-relaxed profit maximization problem for generator k that must respect the integrality of z_k:

$$\max_{\begin{bmatrix} z_k \\ x_k \end{bmatrix} \in \mathbb{S}_k} \left\{ [\pi_{x_k}]^\dagger x_k + \sum_{t=1}^{n_T} (\overline{\rho}_{kt}^\star - \underline{\rho}_{kt}^\star) z_{kt} - f_k\left(\begin{bmatrix} z_k \\ x_k \end{bmatrix}\right) \right\}. \quad (10.6)$$

- Therefore, $z_k^\star, x_k^\star$ is also optimal for the un-relaxed profit maximization problem for generator k, Problem (10.6), since $\begin{bmatrix} z_k^\star \\ x_k^\star \end{bmatrix} \in \mathbb{S}_k \subset \overline{\mathbb{S}}_k$:

  $\begin{bmatrix} z_k^\star \\ x_k^\star \end{bmatrix}$ maximizes profits for generator k over $\overline{\mathbb{S}}_k$, since we have proved that it is a mazimizer of the relaxed generator profit maximization problem, Problem (10.5),

  also $\begin{bmatrix} z_k^\star \\ x_k^\star \end{bmatrix} \in \mathbb{S}_k \subset \overline{\mathbb{S}}_k$,

  so $\begin{bmatrix} z_k^\star \\ x_k^\star \end{bmatrix}$ also maximizes profit over $\mathbb{S}_k$.

Commitment-supporting prices, continued

- That is, prices based on $\lambda^\star$, $\mu^\star$, $\underline{\rho}^\star$, and $\overline{\rho}^\star$ are dispatch- (and commitment-) supporting prices:
  - There are no choices of $\begin{bmatrix} z_k \\ x_k \end{bmatrix}$ that yield a higher profit to generator k than do the choices $\begin{bmatrix} z_k^\star \\ x_k^\star \end{bmatrix}$.
  - Generator k is motivated by the prices based on $\lambda^\star$, $\mu^\star$, $\underline{\rho}^\star$, and $\overline{\rho}^\star$ to commit and dispatch according to $\begin{bmatrix} z_k^\star \\ x_k^\star \end{bmatrix}$.
- We will refer to prices and make-whole payments based on $\lambda^\star$, $\mu^\star$, $\underline{\rho}^\star$ and $\overline{\rho}^\star$ as **commitment-supporting prices**.
- We will contrast these prices with those obtained from the maximum of the dual from the Lagrangian relaxation formulation.

10.6.3 Example

- Consider the example with 100 heterogeneous generators, marginal costs of the form $\beta_k = 1 + k/100$, and demand $\overline{D} = 303$.
- Consider the first-order necessary conditions in Problem (10.3), where we constrain the commitment decisions with inequalities, for generator $k = 76$:

$$\begin{aligned}
0 &= \nabla f_{76}(x_{76}^{\star}) - \lambda^{\star}, \\
&= \beta_{76} - \lambda^{\star}, \\
&= (1 + 76/100) - \lambda^{\star}, \\
0 &= \frac{\partial f_{76}}{\partial z_{76}}(z_{76}^{\star}) - \overline{\rho}_{76}^{\star}, \\
&= 4 - \overline{\rho}_{76}^{\star},
\end{aligned}$$

- where we have omitted the subscript for time since there is only one period.
- In this case, the commitment-supporting prices for generator $k = 76$ are:
  an energy price of $\lambda^{\star} = \$1.76$/MWh (so generator 76 is marginal), and
  a make-whole payment to generator 76 of $\overline{\rho}_k^{\star} = \$4$/h.

Example, continued

- Now consider the first-order necessary conditions in Problem (10.3) for generators $k = 1, \ldots, 75$:

$$\begin{aligned} 0 &= \nabla f_k(x_k^\star) - \lambda^\star + \overline{\mu}_k^\star, \\ &= \beta_k - \lambda^\star + \overline{\mu}_k^\star, \\ &= (1 + k/100) - \lambda^\star + \overline{\mu}_k^\star, \\ 0 &= \frac{\partial f_k}{\partial z_k}(z_k^\star) - 4\overline{\mu}_k^\star - \overline{\rho}_k^\star, \\ &= 4 - 4\overline{\mu}_k^\star - \overline{\rho}_k^\star, \end{aligned}$$

- where we have again omitted the subscript for time since there is only one period, and
- $\overline{\mu}_k^\star$ is the Lagrange multiplier on the maximum capacity constraint $x_k \leq 4z_k$ for generator k.
- The commitment-supporting prices for generator $k = 1, \ldots, 75$ are:

  an energy price of $\lambda^\star = \$1.76$/MWh, and
  a make-whole payment based on $\overline{\rho}_k^\star = 4 - 4\overline{\mu}_k^\star =$
  $4 - 4(\lambda^\star - (1 + k/100)) = 4 - (76 - k)/25 = 1 + (k-1)/25.$

Example, continued

- Given an energy price of $\lambda^\star = \$1.76/\text{MWh}$, generators $77, \ldots, 100$ will remain off and there is no additional make-whole payment for these generators.
- Demand pays for:
  - energy based on $\lambda^\star \times \overline{D} = 1.76 \times 303 = \$533.28/\text{h}$, plus
  - the make-whole payments to generators $k = 1, \ldots, 76$, of $\sum_{k=1}^{75}(1+(k-1)/25)+4 = 76.48+4 = \$80.48/\text{h}$.
- Compared to the case where the dual maximizer was used to price energy and the make-whole payment was only made to generator 76, the commitment-supporting prices result in:
  - a larger make-whole payment (that are paid to a larger number of market participants), but
  - a smaller total payment by demand.

10.6.4 Discussion

- In this example the prices from Problem (10.3) are very different to the energy prices obtained from the dual maximizer:
  - The maximizer of the dual yielded:
    an energy price of $\lambda^\star = \$2.76$/MWh, plus
    a make-whole payment of \$1/h to generator 76 alone.
  - The commitment-supporting prices yielded:
    an energy price of $\lambda^\star = \$1.76$/MWh, plus
    a make-whole payment of $\overline{\rho}_k^\star = 1 + (k-1)/25$ for *each* committed generator.
- The reason for the commitment payment to *each* generator is that:
  - the marginal costs of all the generators are clustered together, and
  - the average costs of all the generators are clustered together, but
  - for each generator, the marginal cost differs significantly from the average cost due to the large min-load cost.
- If the start-up and min-load costs were relatively smaller and there is a wider dispersion of marginal costs then the difference in energy prices between the two approaches will be smaller.

10.6.5 Example with wider dispersion of marginal costs

- Suppose that demand is again 303 MW.
- Suppose that we again have 100 generators, with generator k having cost function:

$$f_k\left(\begin{bmatrix} z_k \\ x_k \end{bmatrix}\right) = 4z_k + \beta_k x_k, z_k \in \{0,1\}, 2z_k \leq x_k \leq 4z_k,$$

- where:

$$\forall k = 1,\ldots,100, \beta_k = 1 + k.$$

- There is now a wider dispersion of marginal costs and, for each generator, the marginal cost and average cost are not extremely different.

10.6.5.1 Solution

- The optimal commitment is for:
  - generators $1,\ldots,75$ to be committed and producing at full capacity of 4,
  - generator 76 to be committed and producing 3, and
  - generators $77,\ldots,100$ to be off.
- Minimum cost is therefore:

$$\sum_{k=1}^{75}[4\times 1+(1+k)\times 4]+[4\times 1+(1+76)\times 3]=13235.$$

- This is the minimum of Problem (10.1).

10.6.5.2 Maximizer of dual

- We first find the value of $\lambda^\star$ that maximizes the dual.
- The dual problem is:

$$\max_{\lambda\in\mathbb{R}}\left\{\min_{\begin{bmatrix}z_k\\x_k\end{bmatrix}\in\mathbb{S}_k}\left\{f\left(\begin{bmatrix}z\\x\end{bmatrix}\right)+\lambda\left(\overline{D}-\sum_{k=1}^{100}x_k\right)\right\}\right\}.$$

- Suppose we set λ so that $2+75<\lambda<2+76$.
  - For example, suppose that we set the price to be $\lambda=77.5$.
  - Generators $k=1,\ldots,75$ will produce 4 MW.
  - Generators $k=76,\ldots,100$ will produce nothing.
  - Total production will be 300 MW.
- Now suppose that we set λ so that $2+76<\lambda<2+77$.
  - Generators $k=1,\ldots,76$ will produce 4 MW.
  - Generators $k=77,\ldots,100$ will produce nothing.
  - Total production will be 304 MW.

Maximizer of dual, continued

- Again, there is still no price were supply equals demand of 303 MW.
- The maximizer of the dual, Problem (10.2), occurs for $\lambda^\star = 78$ and has value of 13234.
  - Total production is either 300 or 304 MW.
  - There is still no price were supply equals demand of 303 MW.
- The duality gap is $13235 - 13234 = 1$.
- The duality gap is relatively small as a fraction of the minimum of the primal unit commitment problem.
- In this case, an energy price of $\lambda^\star = \$78$/MWh together with a make-whole payment of \$1/h for generator $k = 76$ would provide for optimal unit commitment.
- Demand pays for:
  - energy based on $\lambda^\star \times \overline{D} = 78 \times 303 = \$23,634$/h, plus
  - the make-whole payment of \$1/h.

10.6.5.3 Commitment-supporting prices

- Consider the first-order necessary conditions in Problem (10.3) for generator $k = 76$:

$$
\begin{aligned}
0 &= \nabla f_{76}(x_{76}^{\star}) - \lambda^{\star}, \\
&= \beta_{76} - \lambda^{\star}, \\
&= (1 + 76) - \lambda^{\star}, \\
0 &= \frac{\partial f_{76}}{\partial z_{76}}(z_{76}^{\star}) - \overline{\rho}_{76}^{\star}, \\
&= 4 - \overline{\rho}_{76}^{\star},
\end{aligned}
$$

- where we have again omitted the subscript for time since there is only one period.
- In this case, the commitment-supporting prices for generator $k = 76$ are:

  an energy price of $\lambda^{\star} = \$77/\text{MWh}$, and
  commitment payment to generator 76 of $\overline{\rho}_{k}^{\star} = \$4/\text{h}$, if it commits and generates.

Commitment-supporting prices, continued

- For generators $k = 1, \ldots, 75$, the energy price of $\lambda^\star = 77$ induces each of the generators to be committed, without any additional commitment payment, since the revenue for producing $x_k = 4$ is $77 \times 4 = 308$.
- The production cost is:

$$\begin{aligned} 4z_k + (1+k)x_k &= 8 + 4k, \\ &\leq 308, \end{aligned}$$

- so $\overline{\rho}_k^\star = 0, k = 1, \ldots, 75$.
- In this case, the commitment-supporting prices are:

  an energy price of $\lambda^\star = \$77/\text{MWh}$, and
  commitment payment to generator 76 alone of $\overline{\rho}_{76}^\star = \$4/\text{h}$.
- Demand pays for:
  - energy based on $\lambda^\star \times \overline{D} = 77 \times 303 = \$23,331/\text{h}$, plus
  - the make-whole payment of $4/h.
- The total payment by demand is again less than with the dual maximizer and the make-whole payment is only slightly more than with the dual maximizer.

10.6.5.4 Summary

- With a wider dispersion of marginal costs, the difference between the energy prices in the two formulations is now relatively much smaller.
- However, energy prices tend to vary more with the commitment-supporting prices than with the dual maximizer.
- Moreover, the make-whole payment is made only to one generator in both formulations for the example with the wider dispersion of marginal costs.
- This situation is likely to be typical when marginal costs have wide dispersion and start-up and no-load costs are relatively small.

Summary, continued

- If the small relative difference between the two formulations is typical, then why bother with the complexity of the mixed-integer formulation, or even with centralized unit commitment?
- Why bother with centralized unit commitment?
  - The cost of incorrect decentralized commitment decisions might be large, particularly when transmission constraints are binding.
  - The cost of incorrect decentralized commitment decisions is an empirical question that has not been studied in a systematic way, except for particular case studies such as in the ERCOT “backcast” study.
- Why bother with the mixed-integer formulation?
  - Necessary heuristics in Lagrangian relaxation are very detailed and “brittle,” particularly with transmission constraints.
  - The heuristics are problematic in a market setting, where a particular heuristic may have significant implications for profitability or be vulnerable to “strategic” offers.

10.6.6 Make-whole payments in practice

- The goal of make-whole payments is to ensure that each generator is paid enough to cover its offer costs and so that it commits and dispatches consistently with the optimal commitment and dispatch as determined by the ISO:
  - all centralized unit commitment formulations require an uplift from demand.
- The simplified make-whole payment $\max\left\{0, f_k\left(\begin{bmatrix} z_k^\star \\ x_k^\star \end{bmatrix}\right) - [\lambda^\star]^\dagger x_k^\star\right\}$ can be applied to any pricing rule on energy and reserves in order to induce a particular behavior:
  - make-whole payments are paid to a generator that commits and dispatches according to (or close enough to) $z_k = z_k^\star$ and $x_k = x_k^\star$, but
  - the make-whole payment is withheld and (possibly) a penalty is charged for any generator that deviates significantly from the desired commitment and dispatch,
  - as previously mentioned, the simplified payment is used in practice even though it does not provide the exactly correct incentives.

Make-whole payments, continued

- In ERCOT and other ISOs:
  - commitment $z^\star$ and dispatch $x^\star$ from solution of Problem (10.1),
  - energy and reserves prices based on Lagrange multipliers $\lambda^\star$ and $\mu^\star$ obtained from the solution of the convex problem, Problem (10.4), obtained by fixing the integer variables at their optimal values $z^\star$,
  - make-whole payment (for generators that follow commitment and dispatch from ERCOT) based on a daily calculation of:

$$\max\left\{0, f_k\left(\begin{bmatrix} z_k^\star \\ x_k^\star \end{bmatrix}\right) - [\pi_{x_k}]^\dagger x_k^\star\right\}.$$

- The Midwest ISO has proposed to use prices that are analogous to the prices found in Lagrangian relaxation:
  - commitment $z^\star$ and dispatch $x^\star$ from solution of Problem (10.1),
  - energy and reserves payments based on the the maximizer of the dual problem, Problem (10.2), or an approximation to this problem, and
  - make-whole payment.

10.7 Anonymity of prices

- An important role for prices is to inform potential entrants to the market about whether new entry would be profitable.
- Prices π_{x_k} on system equality and inequality constraints are paid for energy and reserves produced by everyone and anyone and are said to be **anonymous**.
- However, make-whole payments, including payments based on commitment-supporting prices for commitment decision variables, are not *anonymous*.
- Non-anonymity makes it harder for a potential entrant to determine if new entry would be profitable, particularly if the make-whole payments are not disclosed publicly.
  - It is difficult for a new entrant to understand if it would be profitable to enter at the current prices.

10.8 Implications for investment decisions

- Even if the make-whole payments are disclosed, make-whole payments can distort investment decisions.
- Make-whole payments contribute to the infra-marginal rents of some generators.
- These rents are not also available to everyone else.
- The incentives for building new capacity may be depressed.

10.9 Transmission constraints

- In the discussion so far we have not considered transmission constraints.
- However, transmission constraints can limit the dispatch decisions.
- We can expand the formulation to include the transmission constraints in the system constraints.
- We could consider prices based on either:
  - the dual maximizer (as proposed for the Midwest ISO), or
  - energy and reserves prices obtained from the solution of the convex problem, Problem (10.4), having the integer variables fixed at their optimal values (as used in ERCOT and other ISOs).
- In both cases, a make-whole payment may be necessary.
- In practice, transmission-constrained unit commitment can be an extremely difficult problem to solve.

10.9.1 Example

- We consider day-ahead dispatch across two hours, $n_T = 2$, with demands:

t	0	1	2
$\overline{D}_t$	90	110	125

- The $t = 0$ entry in the table is the demand for the last hour of today.
- The $t = 1,2$ entries are the demands for the first two hours of tomorrow.
- Also, $P_{10} = 90$ MW and $P_{20} = 0$ MW are the generations in the last hour of today, with generator 2 out-of-service at the end of today.
- We ignore reserves, min-load costs, and ramp-rate constraints.
- The offers are specified by:

$$\begin{aligned}
\forall t = 1,2, s_{1t} = 1000, \forall P_{1t} \in [0,200], \nabla f_{1t}(P_{1t}) &= \$25/\text{MWh}, \\
\forall t = 1,2, s_{2t} = 1000, \forall P_{2t} \in [0,50], \nabla f_{2t}(P_{2t}) &= \$35/\text{MWh}.
\end{aligned}$$

Example, continued

- The generators are located in the following one-line two-bus system.
- We use the DC power flow approximation and the transmission line has transmission capacity of 100 MW.
- We solve the transmission constrained, offer-based unit commitment for this system.
- We will consider energy prices based on Problem (10.4).
- Make-whole payments will be based on $\max\left\{0, f_k\left(\begin{bmatrix} z_k^\star \\ x_k^\star \end{bmatrix}\right) - [\pi_{x_k}]^\dagger x_k^\star\right\}$.

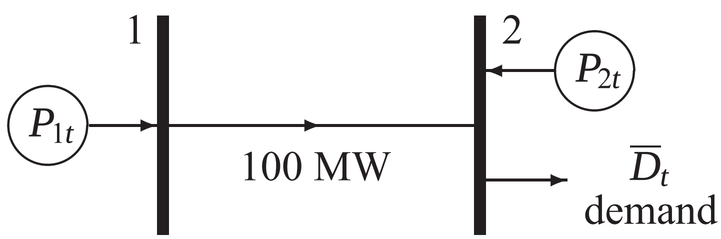

Fig. 10.1. One-line two-bus network.

Example, continued

- Because of the transmission constraint, it will be necessary to commit generator 2 and run it during periods 1 and 2.
- The optimal offer-based commitment and dispatch is:

t	0	1	2
$\overline{D}_t$	90	110	125
$z_{1t}^{\star}$	1	1	1
$P_{1t}^{\star}$	90	100	100
$z_{2t}^{\star}$	0	1	1
$P_{2t}^{\star}$	0	10	25

- We calculate the locational marginal prices using commitment variables fixed at their optimal values:

t	0	1	2
$\overline{D}_t$	90	110	125
$\lambda_{1t}^{\star}$	25	25	25
$\lambda_{2t}^{\star}$	25	35	35

Example, continued

- Since generator 1 is already committed at the start of the day, and since the revenue (just) covers its incremental energy costs, there is no make-whole payment for generator 1.
- Generator 2 must be started, but the revenue only just covers its incremental energy costs.
- Therefore, the make-whole payment to generator 2 is equal to its start-up cost of $s_{21} = \$1000$.

10.10 Day-ahead and real-time markets

- As discussed in Section 8.12.7.1, randomness and uncertainty is pervasive in supply and demand of electricity.
- A sequence of markets can be used to help cope with randomness.
- Successive markets can compensate for and correct the forecast errors and other errors from previous markets.
- For example:
  - the day-ahead commitment and dispatch market sets up (financial) agreements a day in advance to generate based on forecasts or specifications of demand a day in advance.
  - the real-time market deals with the deviations of actual supply and demand from day-ahead specifications.

Day-ahead and real-time markets, continued

- For each commodity (besides the commitment commodities) there is a day-ahead price and a real-time price.
- The day-ahead price applies to commodities bought and sold in the day-ahead market.
- The day-ahead market is technically a **short-term forward market**.
- Participation in the day-ahead market entails a **forward financial commitment** to produce or consume in real-time:
  - a financial commitment means that either the action is carried out physically, or the actual deviation from the financial commitment is purchased from the real-time market.
- That is, payment in the real-time market applies to the **deviations** from day-ahead positions.

10.10.1 Example

- Suppose that the ISO day-ahead market yields a day-ahead price of \$50/MWh in a particular hour and generation of 40 MW by a particular generator.
- In the real-time market, if the real-time price is \$60/MWh and the generator actually produces 45 MW, then its payment is:

$$\begin{aligned}
\text{(DA quantity)(DA price)} + \text{(RT quantity - DA quantity)(RT price)}, & \\
&= 40 \times 50 + (45 - 40) \times 60, \\
&= \$2300/\text{h}.
\end{aligned}$$

10.10.2 Consistency between day-ahead and real-time markets

- It is desirable that each market represents similar system constraints:
  - Otherwise there would be price differences between the two markets even in the absence of randomness.
- Conceptual difficulties:
  - Ancillary services, such as reserves, acquired in the day-ahead market are commitments to be available in real-time.
  - What happens when reserves are actually deployed?
  - What happens if a generator is paid to commit in day-ahead but does not actually commit?

10.11 Summary

- In this chapter we have considered temporal issues.
- We formulated the unit commitment problem.
- We investigated the duality gap in the problem and the implications for commitment-supporting prices.
- We briefly considered day-ahead and real-time markets.
- We considered reserves.

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Homework exercises

10.1 Suppose that we have two generators, $n_P = 2$, with offers:

$$\begin{aligned} \forall t, \nabla f_{1t}(P_{1t}) &= 2, 100 \leq P_{1t} \leq 400, \\ \forall t, \nabla f_{2t}(P_{2t}) &= 5, 100 \leq P_{2t} \leq 300. \end{aligned}$$

The generators have ramp-rate limits of $\Delta_1 = 50$ MW/h and $\Delta_2 = 100$ MW/h, respectively. We consider day-ahead dispatch across five hours, $n_T = 5$, with demands:

t	0	1	2	3	4	5
$\overline{D}_t$	250	350	400	425	450	475

The $t = 0$ entry in the table is the demand for the last hour of today. Also, $P_{10} = 150$ MW and $P_{20} = 100$ MW. We ignore reserves.

(i) Solve the ramp-constrained economic dispatch problem.
(ii) What price is paid for energy in each hour?
(iii) What do you notice about the relationship between demand and prices?

10.2 Suppose that we have two generators, $n_P = 2$, with offers:

$$\begin{aligned} \forall t, \nabla f_{1t}(P_{1t}) &= 2, 200 \le P_{1t} \le 400, \\ \forall t, \nabla f_{2t}(P_{2t}) &= 3, 50 \le P_{2t} \le 150. \end{aligned}$$

There are no ramp-rate limits nor min-load costs, but the start-up costs are:

$$\begin{aligned} s_{1t} &= 1000, t = 1, \ldots, n_T, \\ s_{2t} &= 200, t = 1, \ldots, n_T. \end{aligned}$$

We consider day-ahead dispatch across eight hours, $n_T = 8$, with demands:

t	0	1	2	3	4	5	6	7	8
$\overline{D}_t$	200	350	500	350	200	350	500	350	200

The $t = 0$ entry in the table is the demand for the last hour of today. Also, $P_{10} = 200$ MW, and $P_{20} = 0$ MW with generator 2 out-of-service at the end of today. We ignore both ramp-rates and reserves.

(i) Find the maximizer of the dual obtained by dualizing the demand constraint in each hour. (Hint: What price will induce generator 2 to be indifferent between being off and being on at full capacity in periods 2 and 6. What is the price in the other periods?)
(ii) Solve the unit commitment problem.
(iii) What are the energy prices obtained from the solution of the convex problem obtained by fixing the integer variables at their optimal values and optimizing P_{1t} and P_{2t}?
(iv) What is the make-whole payment for each generator based on prices from Part (iii)?