Behavioral Cloning

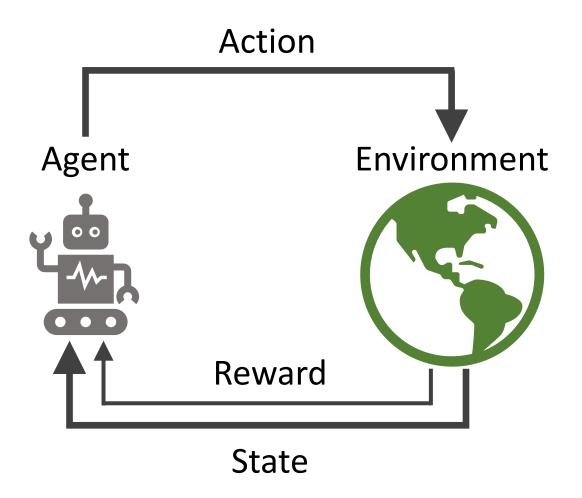
CMPT 729 G100

Jason Peng

Overview

- Behavioral Cloning
- Drift
- Theoretical Analysis
- DAgger
- Applications

Agent-Environment Interface



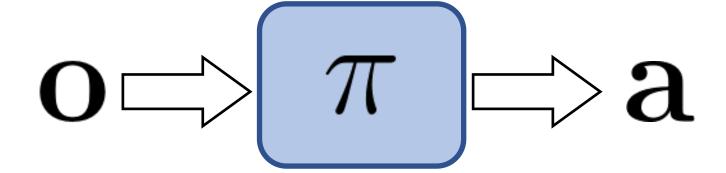
Policy

$$\pi(\mathbf{a}|\mathbf{s})$$

$$\mathbf{s} \Rightarrow \boxed{\pi}$$

Policy

$$\pi(\mathbf{a}|\mathbf{o})$$

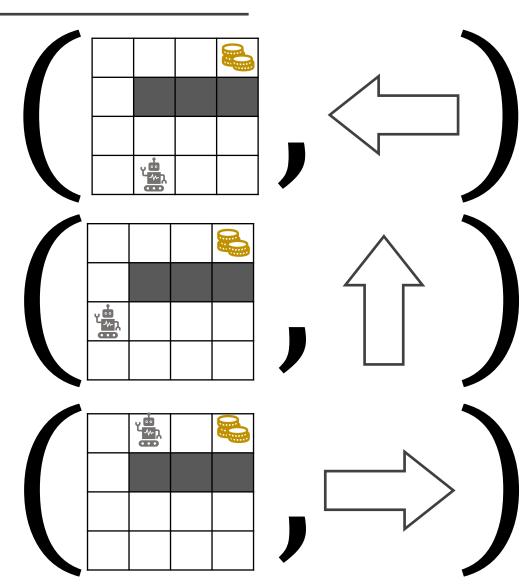


Supervised Learning

$$\{(\mathbf{o}_0, \mathbf{a}_0), (\mathbf{o}_1, \mathbf{a}_1), ...\}$$



Dataset



Supervised Learning

$$\{(\mathbf{o}_0, \mathbf{a}_0), (\mathbf{o}_1, \mathbf{a}_1), ...\}$$



Dataset

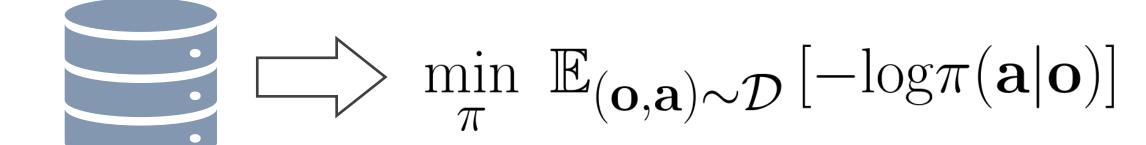


Nvidia Automotive Simulation [NVIDIA]



Supervised Learning

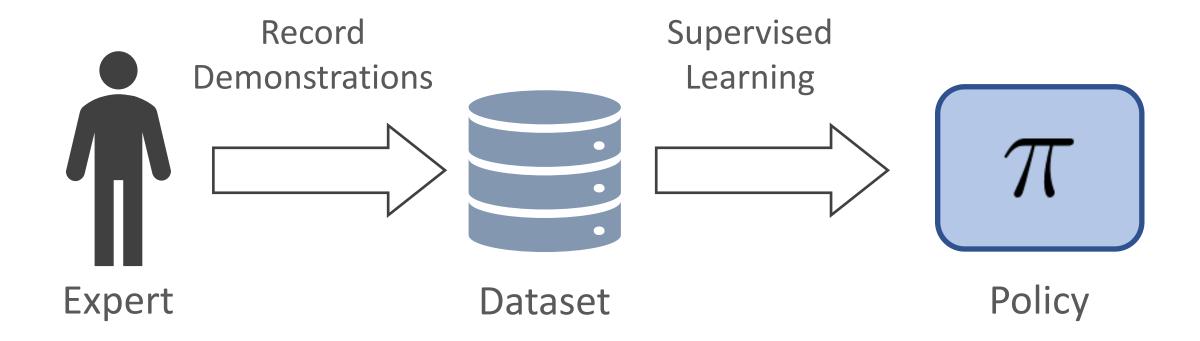
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Dataset

Behavioral Cloning

Behavioral Cloning



Behavioral Cloning



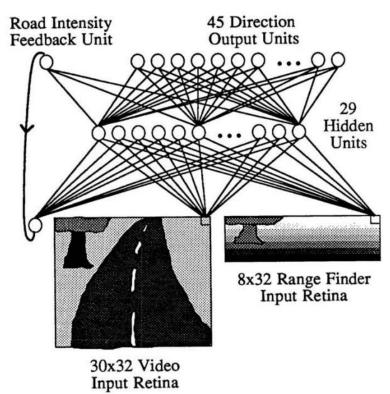
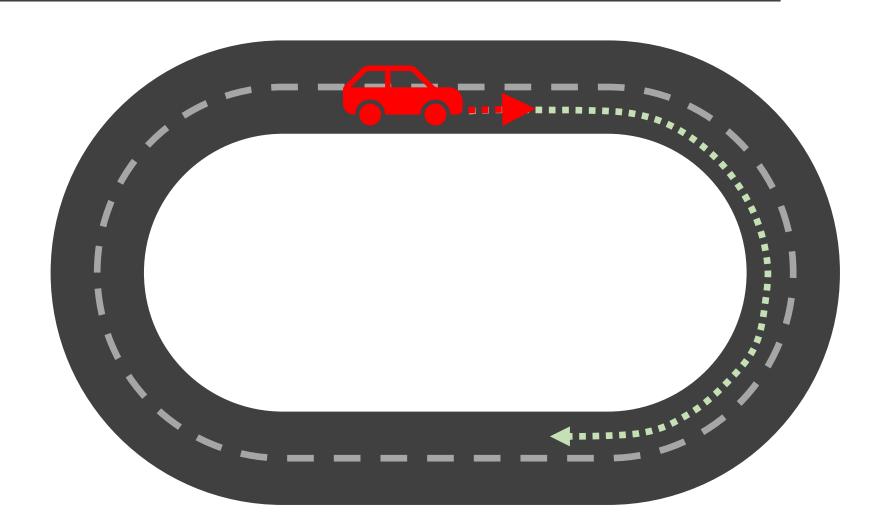
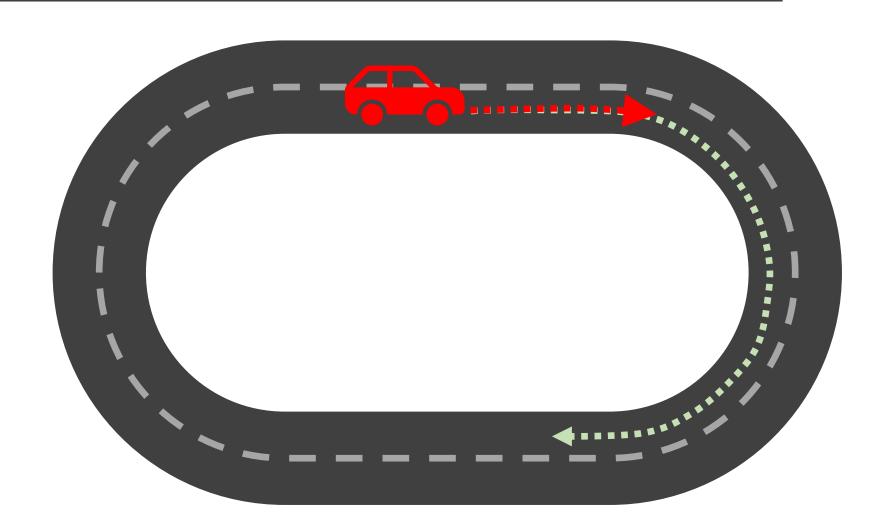
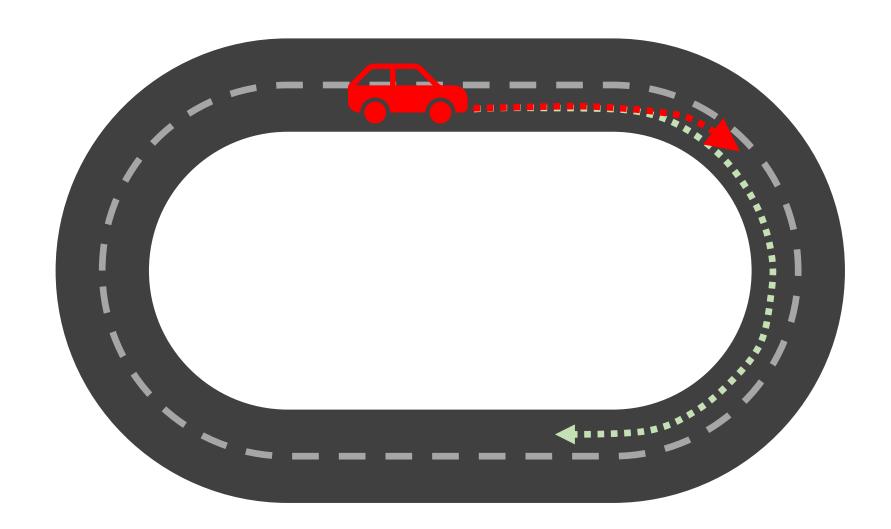


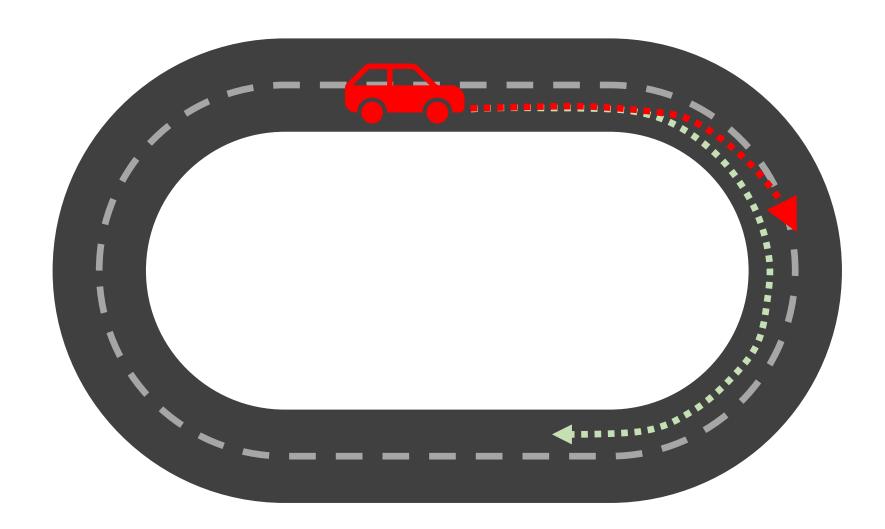
Figure 1: ALVINN Architecture

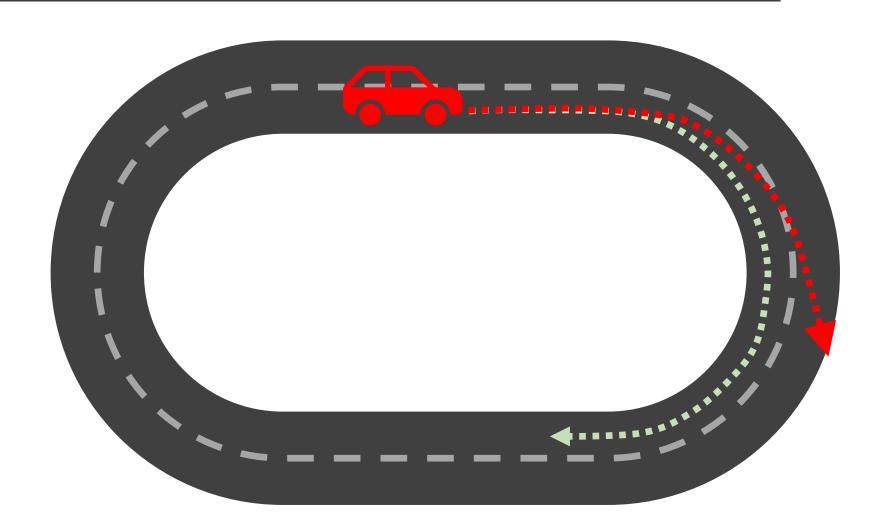
ALVINN: An Autonomous Land Vehicle in a Neural Network [Pomerleau 1989]

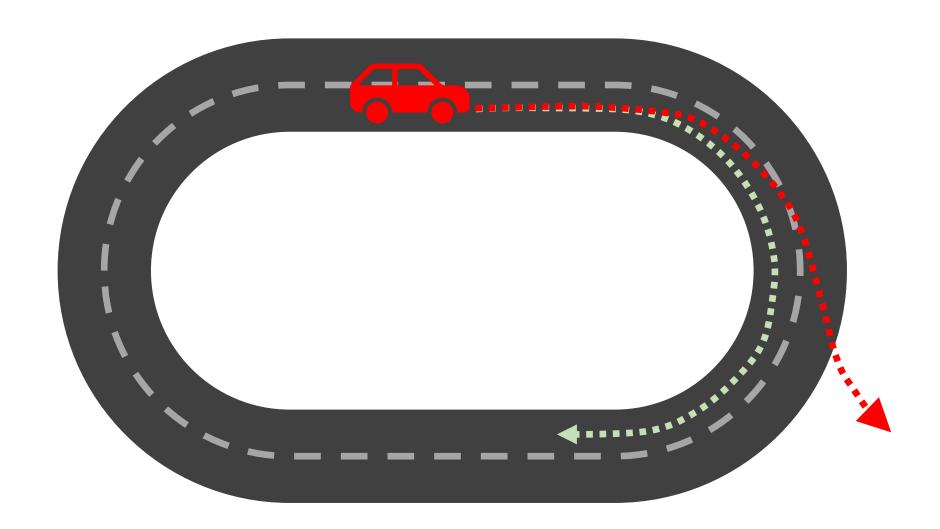


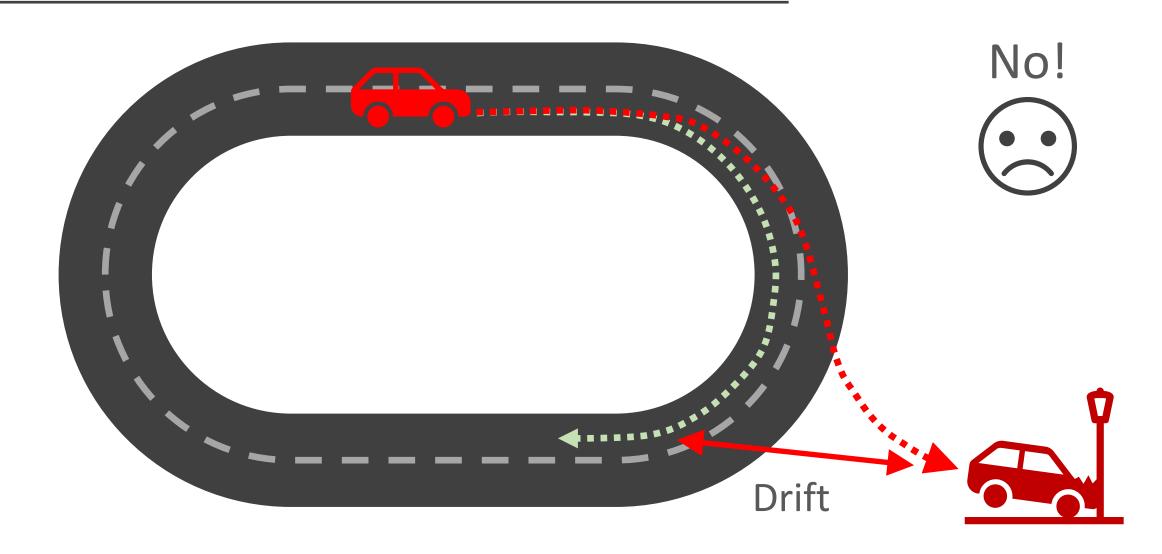












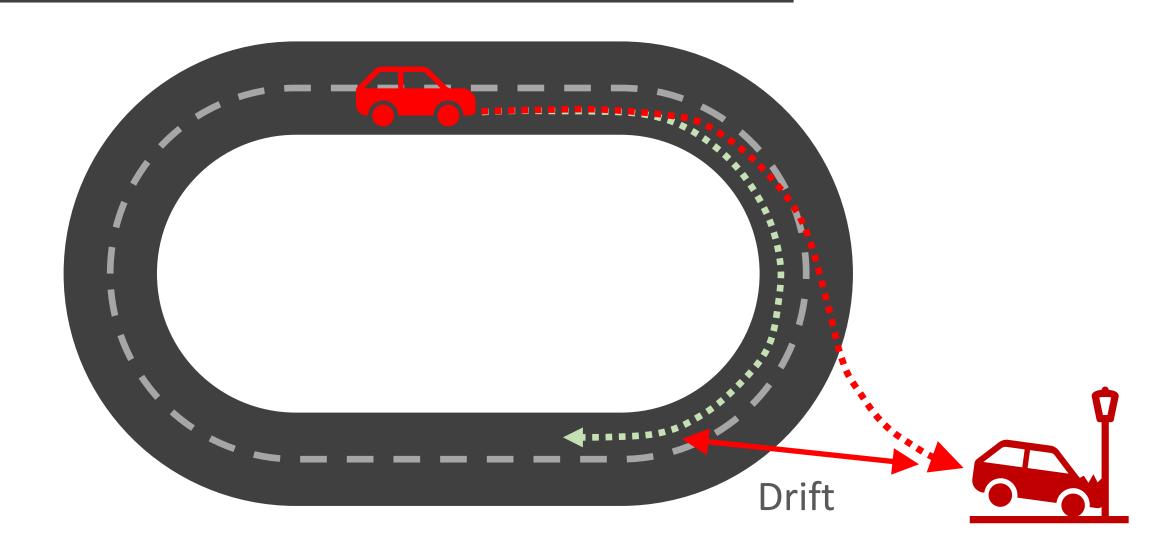
Drift

- Expert is too good
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- Policy inaccuracies
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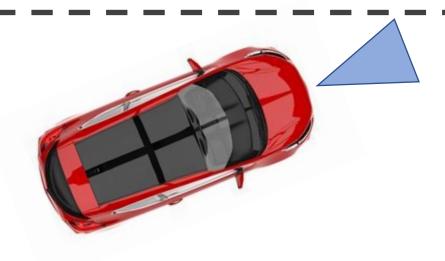
Feedback

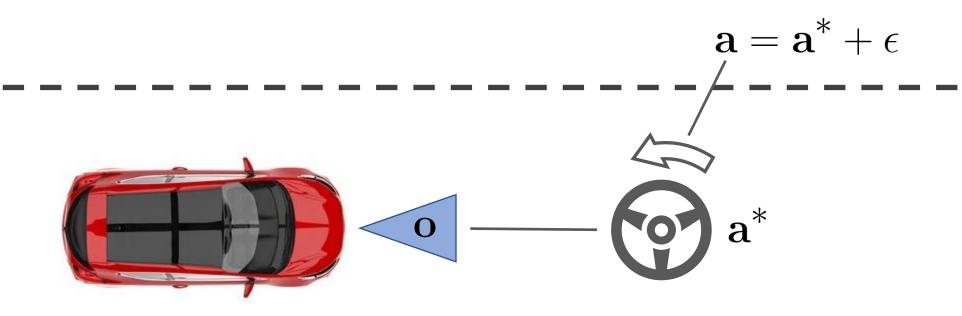


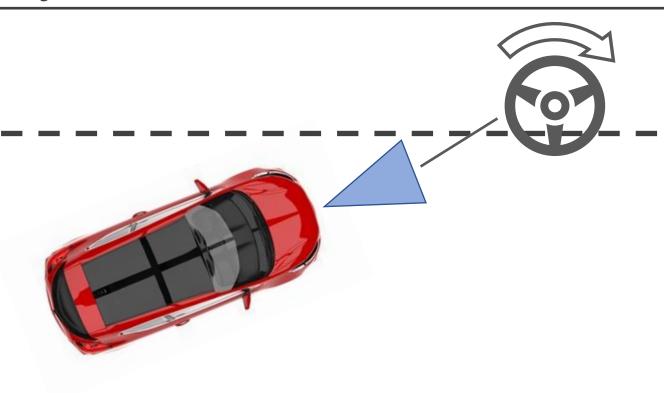
Feedback



Feedback







ALGORITHM 2: BC with Noise Injection

- 1: $\mathcal{D} \leftarrow \emptyset$ initialize dataset
- 2: for timestep t do
- 3: $\mathbf{o_t} \leftarrow \text{record observation}$
- 4: $\mathbf{a}_{\mathbf{t}}^* \leftarrow \text{query expert for an action}$
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- 10: $\pi^{\text{BC}} = \arg\min_{\pi} \mathbb{E}_{(\mathbf{o}_i, \mathbf{a}_i) \sim \mathcal{D}} \left[-\log \pi(\mathbf{a}_i | \mathbf{o}_i) \right]$
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Simple method to get corrective feedback

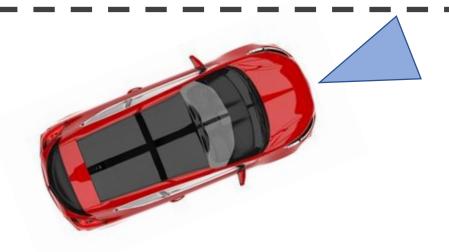


Can work well in practice



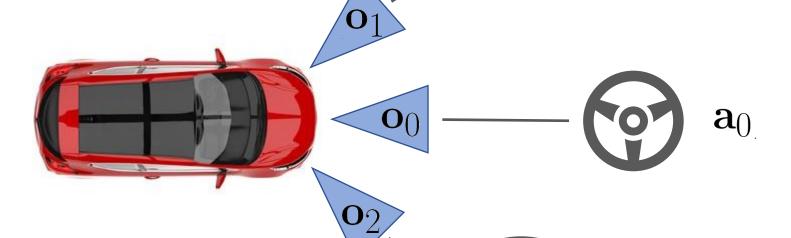
Mark Difficult to pick effective perturbations





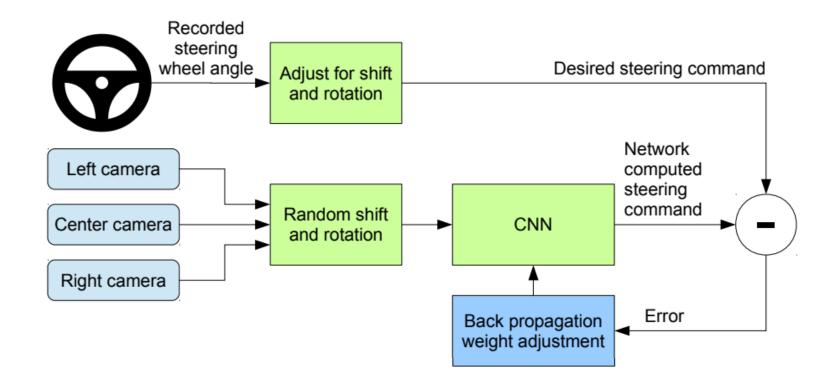


$$\mathbf{a}_1 = \mathbf{a}_0 + \triangle \mathbf{a}_1$$





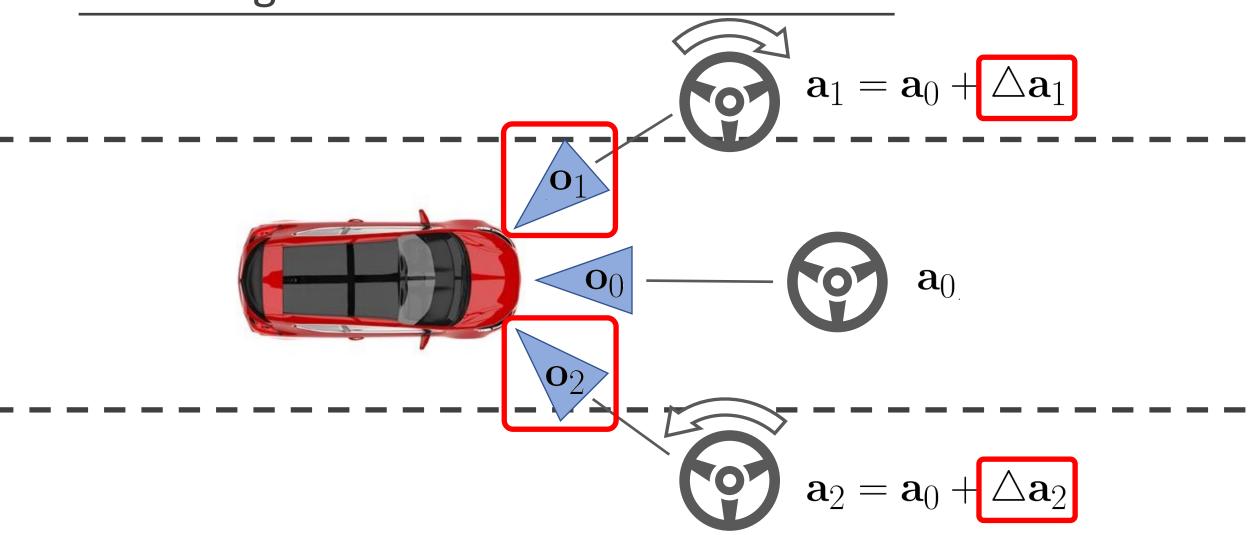
$$\mathbf{a}_2 = \mathbf{a}_0 + \triangle \mathbf{a}_2$$



End to End Learning for Self-Driving Cars [Bojarski et al. 2016]



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Drift

- Expert is too good
- Lack of corrective feedback
- Policy inaccuracies
- Errors compound over time

Analyze the number of mistakes π makes over time

Theorem 1. The number of mistakes grow $O(\epsilon T^2)$

Given dataset sampled from $p_{\mathrm{data}}(\mathbf{s}, \mathbf{a})$

$$\min_{\pi} \mathbb{E}_{(\mathbf{s}, \mathbf{a}) \sim p_{\text{data}}(\mathbf{s}, \mathbf{a})} \left[-\log \pi(\mathbf{a} | \mathbf{s}) \right]$$

Such that

$$\pi\left(\mathbf{a} \neq \pi^*(\mathbf{s})|\mathbf{s}\right) \leq \epsilon \text{ for all } \mathbf{s} \sim p_{\mathrm{data}}(\mathbf{s})$$

i.e. the probability of π making a mistake is bounded.

Cost:
$$c(\mathbf{s}, \mathbf{a}) = \begin{cases} 0 & \text{if } \mathbf{a} = \pi^*(\mathbf{s}) \\ 1 & \text{otherwise} \end{cases}$$

Assume: $\pi (\mathbf{a} \neq \pi^*(\mathbf{s})|\mathbf{s}) \leq \epsilon$ for all $\mathbf{s} \sim p_{\text{data}}(\mathbf{s})$

$$\underline{p_{\pi}^{t}(\mathbf{s})} = (1 - \epsilon)^{t} p_{\text{data}}^{t}(\mathbf{s}) + (1 - (1 - \epsilon)^{t}) p_{\text{mistake}}^{t}(\mathbf{s})$$

probability of being in ${f S}$ after following π for t timesteps

Assume: $\pi (\mathbf{a} \neq \pi^*(\mathbf{s})|\mathbf{s}) \leq \epsilon$ for all $\mathbf{s} \sim p_{\text{data}}(\mathbf{s})$

$$p_{\pi}^{t}(\mathbf{s}) = (1 - \epsilon)^{t} p_{\text{data}}^{t}(\mathbf{s}) + (1 - (1 - \epsilon)^{t}) p_{\text{mistake}}^{t}(\mathbf{s})$$

no mistakes in t timsteps

Assume: $\pi\left(\mathbf{a} \neq \pi^*(\mathbf{s})|\mathbf{s}\right) \leq \epsilon$ for all $\mathbf{s} \sim p_{\text{data}}(\mathbf{s})$

$$p_{\pi}^{t}(\mathbf{s}) = (1 - \epsilon)^{t} p_{\text{data}}^{t}(\mathbf{s}) + (1 - (1 - \epsilon)^{t}) p_{\text{mistake}}^{t}(\mathbf{s})$$

no mistakes in t timsteps

at least 1 mistakes in t timsteps

Assume:
$$\pi\left(\mathbf{a} \neq \pi^*(\mathbf{s})|\mathbf{s}\right) \leq \epsilon$$
 for all $\mathbf{s} \sim p_{\mathrm{data}}(\mathbf{s})$
$$p_{\pi}^t(\mathbf{s}) = (1 - \epsilon)^t p_{\mathrm{data}}^t(\mathbf{s}) + (1 - (1 - \epsilon)^t) p_{\mathrm{mistake}}^t(\mathbf{s})$$

$$\sum_{t} \mathbb{E}_{p_{\pi}^{t}(\mathbf{s})} \mathbb{E}_{\pi(\mathbf{a}|\mathbf{s})} \left[c(\mathbf{s}, \mathbf{a}) \right]$$

expected cost

Assume: $\pi\left(\mathbf{a} \neq \pi^*(\mathbf{s})|\mathbf{s}\right) \leq \epsilon$ for all $\mathbf{s} \sim p_{\mathrm{data}}(\mathbf{s})$ $p_{\pi}^t(\mathbf{s}) = (1 - \epsilon)^t p_{\mathrm{data}}^t(\mathbf{s}) + (1 - (1 - \epsilon)^t) p_{\mathrm{mistake}}^t(\mathbf{s})$ $\sum_t \mathbb{E}_{p_{\pi}^t(\mathbf{s})} \mathbb{E}_{\pi(\mathbf{a}|\mathbf{s})} \left[c(\mathbf{s}, \mathbf{a}) \right]$

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$$= 0$$

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$$= \sum_t \sum_{\mathbf{s}} p_{\mathrm{data}}^t(\mathbf{s}) \mathbb{E}_{\pi(\mathbf{a}|\mathbf{s})} \left[c(\mathbf{s}, \mathbf{a}) \right] + \sum_t \sum_{\mathbf{s}} \left(p_{\pi}^t(\mathbf{s}) - p_{\mathrm{data}}^t(\mathbf{s}) \right) \mathbb{E}_{\pi(\mathbf{a}|\mathbf{s})} \left[c(\mathbf{s}, \mathbf{a}) \right]$$

$$\leq \epsilon T$$

Assume: $\pi\left(\mathbf{a} \neq \pi^*(\mathbf{s})|\mathbf{s}\right) \leq \epsilon$ for all $\mathbf{s} \sim p_{\mathrm{data}}(\mathbf{s})$

$$p_{\pi}^{t}(\mathbf{s}) = (1 - \epsilon)^{t} p_{\text{data}}^{t}(\mathbf{s}) + (1 - (1 - \epsilon)^{t}) p_{\text{mistake}}^{t}(\mathbf{s})$$

$$\sum_{t} \mathbb{E}_{p_{\pi}^{t}(\mathbf{s})} \mathbb{E}_{\pi(\mathbf{a}|\mathbf{s})} [c(\mathbf{s}, \mathbf{a})] = \sum_{t} \sum_{\mathbf{s}} p_{\pi}^{t}(\mathbf{s}) \mathbb{E}_{\pi(\mathbf{a}|\mathbf{s})} [c(\mathbf{s}, \mathbf{a})]$$

$$= \sum_{t} \sum_{\mathbf{s}} \left(p_{\pi}^{t}(\mathbf{s}) - p_{\text{data}}^{t}(\mathbf{s}) + p_{\text{data}}^{t}(\mathbf{s}) \right) \mathbb{E}_{\pi(\mathbf{a}|\mathbf{s})} [c(\mathbf{s}, \mathbf{a})]$$

$$= \sum_{t} \sum_{\mathbf{s}} p_{\text{data}}^{t}(\mathbf{s}) \mathbb{E}_{\pi(\mathbf{a}|\mathbf{s})} [c(\mathbf{s}, \mathbf{a})] + \sum_{t} \sum_{\mathbf{s}} \left(p_{\pi}^{t}(\mathbf{s}) - p_{\text{data}}^{t}(\mathbf{s}) \right) \mathbb{E}_{\pi(\mathbf{a}|\mathbf{s})} [c(\mathbf{s}, \mathbf{a})]$$

$$\leq \epsilon T + \sum_{t} \sum_{\mathbf{s}} \left(p_{\pi}^{t}(\mathbf{s}) - p_{\text{data}}^{t}(\mathbf{s}) \right) \mathbb{E}_{\pi(\mathbf{a}|\mathbf{s})} [c(\mathbf{s}, \mathbf{a})]$$

$$\sum_{\mathbf{s}} p_{\pi}^{t}(\mathbf{s}) = \sum_{\mathbf{s}} (1 - \epsilon)^{t} p_{\text{data}}^{t}(\mathbf{s}) + \left(1 - (1 - \epsilon)^{t}\right) p_{\text{mistake}}^{t}(\mathbf{s})$$
$$\sum_{\mathbf{s}} p_{\pi}^{t}(\mathbf{s}) - (1 - \epsilon)^{t} p_{\text{data}}^{t}(\mathbf{s}) = \sum_{\mathbf{s}} \left(1 - (1 - \epsilon)^{t}\right) p_{\text{mistake}}^{t}(\mathbf{s})$$

$$\begin{split} &\sum_{\mathbf{s}} p_{\pi}^{t}(\mathbf{s}) = \sum_{\mathbf{s}} (1-\epsilon)^{t} p_{\text{data}}^{t}(\mathbf{s}) + \left(1 - (1-\epsilon)^{t}\right) p_{\text{mistake}}^{t}(\mathbf{s}) \\ &\sum_{\mathbf{s}} p_{\pi}^{t}(\mathbf{s}) - (1-\epsilon)^{t} p_{\text{data}}^{t}(\mathbf{s}) = \sum_{\mathbf{s}} \left(1 - (1-\epsilon)^{t}\right) p_{\text{mistake}}^{t}(\mathbf{s}) \\ &\sum_{\mathbf{s}} p_{\pi}^{t}(\mathbf{s}) - (1-\epsilon)^{t} p_{\text{data}}^{t}(\mathbf{s}) - \left(1 - (1-\epsilon)^{t}\right) p_{\text{data}}^{t}(\mathbf{s}) = \sum_{\mathbf{s}} \left(1 - (1-\epsilon)^{t}\right) p_{\text{mistake}}^{t}(\mathbf{s}) - \left(1 - (1-\epsilon)^{t}\right) p_{\text{data}}^{t}(\mathbf{s}) \end{split}$$

$$\sum_{\mathbf{s}} p_{\pi}^{t}(\mathbf{s}) = \sum_{\mathbf{s}} (1 - \epsilon)^{t} p_{\text{data}}^{t}(\mathbf{s}) + \left(1 - (1 - \epsilon)^{t}\right) p_{\text{mistake}}^{t}(\mathbf{s})$$

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$$\sum_{\mathbf{s}} p_{\pi}^{t}(\mathbf{s}) - (1 - \epsilon)^{t} p_{\text{data}}^{t}(\mathbf{s}) - \left(1 - (1 - \epsilon)^{t}\right) p_{\text{data}}^{t}(\mathbf{s}) = \sum_{\mathbf{s}} \left(1 - (1 - \epsilon)^{t}\right) p_{\text{mistake}}^{t}(\mathbf{s}) - \left(1 - (1 - \epsilon)^{t}\right) p_{\text{data}}^{t}(\mathbf{s})$$

$$\sum_{\mathbf{s}} p_{\pi}^{t}(\mathbf{s}) = \sum_{\mathbf{s}} (1 - \epsilon)^{t} p_{\text{data}}^{t}(\mathbf{s}) + \left(1 - (1 - \epsilon)^{t}\right) p_{\text{mistake}}^{t}(\mathbf{s})$$

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$$\sum_{\mathbf{s}} p_{\pi}^{t}(\mathbf{s}) - (1 - \epsilon)^{t} p_{\text{data}}^{t}(\mathbf{s}) - \left(1 - (1 - \epsilon)^{t}\right) p_{\text{data}}^{t}(\mathbf{s}) = \sum_{\mathbf{s}} \left(1 - (1 - \epsilon)^{t}\right) p_{\text{mistake}}^{t}(\mathbf{s}) - \left(1 - (1 - \epsilon)^{t}\right) p_{\text{data}}^{t}(\mathbf{s})$$

$$\sum_{\mathbf{s}} p_{\pi}^{t}(\mathbf{s}) - p_{\text{data}}^{t}(\mathbf{s})$$

$$\begin{split} \sum_{\mathbf{s}} p_{\pi}^{t}(\mathbf{s}) &= \sum_{\mathbf{s}} (1 - \epsilon)^{t} p_{\text{data}}^{t}(\mathbf{s}) + \left(1 - (1 - \epsilon)^{t}\right) p_{\text{mistake}}^{t}(\mathbf{s}) \\ &\sum_{\mathbf{s}} p_{\pi}^{t}(\mathbf{s}) - (1 - \epsilon)^{t} p_{\text{data}}^{t}(\mathbf{s}) = \sum_{\mathbf{s}} \left(1 - (1 - \epsilon)^{t}\right) p_{\text{mistake}}^{t}(\mathbf{s}) \\ &\sum_{\mathbf{s}} p_{\pi}^{t}(\mathbf{s}) - (1 - \epsilon)^{t} p_{\text{data}}^{t}(\mathbf{s}) - \left(1 - (1 - \epsilon)^{t}\right) p_{\text{data}}^{t}(\mathbf{s}) = \sum_{\mathbf{s}} \left(1 - (1 - \epsilon)^{t}\right) p_{\text{mistake}}^{t}(\mathbf{s}) - \left(1 - (1 - \epsilon)^{t}\right) p_{\text{data}}^{t}(\mathbf{s}) \\ &\sum_{\mathbf{s}} p_{\pi}^{t}(\mathbf{s}) - p_{\text{data}}^{t}(\mathbf{s}) = \left(1 - (1 - \epsilon)^{t}\right) \sum_{\mathbf{s}} p_{\text{mistake}}^{t}(\mathbf{s}) - p_{\text{data}}^{t}(\mathbf{s}) \\ &\leq \left(1 - (1 - \epsilon)^{t}\right) \sum_{\mathbf{s}} \left| p_{\text{mistake}}^{t}(\mathbf{s}) - p_{\text{data}}^{t}(\mathbf{s}) \right| \end{split}$$

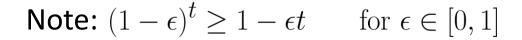
total variation distance ≤ 2

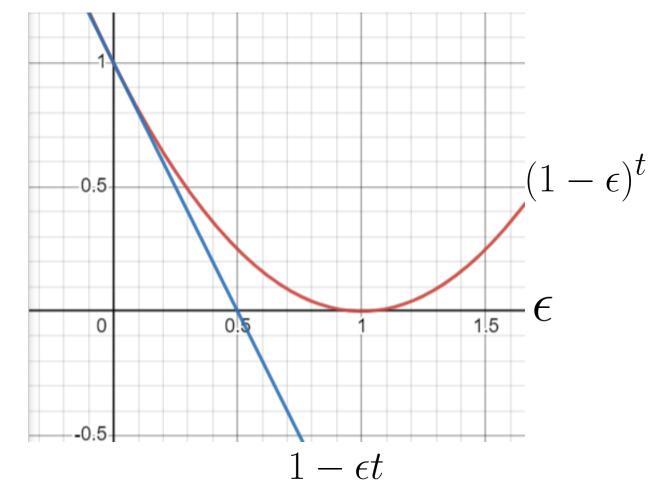
$$\begin{split} \sum_{\mathbf{s}} p_{\pi}^{t}(\mathbf{s}) &= \sum_{\mathbf{s}} (1-\epsilon)^{t} p_{\mathrm{data}}^{t}(\mathbf{s}) + \left(1-(1-\epsilon)^{t}\right) p_{\mathrm{mistake}}^{t}(\mathbf{s}) \\ &\sum_{\mathbf{s}} p_{\pi}^{t}(\mathbf{s}) - (1-\epsilon)^{t} p_{\mathrm{data}}^{t}(\mathbf{s}) = \sum_{\mathbf{s}} \left(1-(1-\epsilon)^{t}\right) p_{\mathrm{mistake}}^{t}(\mathbf{s}) \\ &\sum_{\mathbf{s}} p_{\pi}^{t}(\mathbf{s}) - (1-\epsilon)^{t} p_{\mathrm{data}}^{t}(\mathbf{s}) - \left(1-(1-\epsilon)^{t}\right) p_{\mathrm{data}}^{t}(\mathbf{s}) = \sum_{\mathbf{s}} \left(1-(1-\epsilon)^{t}\right) p_{\mathrm{mistake}}^{t}(\mathbf{s}) - \left(1-(1-\epsilon)^{t}\right) p_{\mathrm{data}}^{t}(\mathbf{s}) \\ &\sum_{\mathbf{s}} p_{\pi}^{t}(\mathbf{s}) - p_{\mathrm{data}}^{t}(\mathbf{s}) = \left(1-(1-\epsilon)^{t}\right) \sum_{\mathbf{s}} p_{\mathrm{mistake}}^{t}(\mathbf{s}) - p_{\mathrm{data}}^{t}(\mathbf{s}) \\ &\leq \left(1-(1-\epsilon)^{t}\right) \sum_{\mathbf{s}} \left| p_{\mathrm{mistake}}^{t}(\mathbf{s}) - p_{\mathrm{data}}^{t}(\mathbf{s}) \right| \\ &\leq 2 \left(1-(1-\epsilon)^{t}\right) \end{split}$$

$$\mathsf{Note:} \ (1-\epsilon)^{t} \geq 1-\epsilon t \qquad \text{for } \epsilon \in [0,1]$$

$$\sum_{\mathbf{s}} p_{\pi}^{t}(\mathbf{s}) - p_{\text{data}}^{t}(\mathbf{s}) \le 2 \left(1 - (1 - \epsilon)^{t} \right)$$
$$\le 2 \left(1 - (1 - \epsilon t) \right)$$
$$\le 2\epsilon t$$

$$\sum_{\mathbf{s}} p_{\pi}^{t}(\mathbf{s}) - p_{\text{data}}^{t}(\mathbf{s}) \le 2\epsilon t$$





$$\sum_{t} \mathbb{E}_{p_{\pi}^{t}(\mathbf{s})} \mathbb{E}_{\pi(\mathbf{a}|\mathbf{s})} \left[c(\mathbf{s}, \mathbf{a}) \right] = \sum_{t} \sum_{\mathbf{s}} p_{\pi}^{t}(\mathbf{s}) \mathbb{E}_{\pi(\mathbf{a}|\mathbf{s})} \left[c(\mathbf{s}, \mathbf{a}) \right]$$

$$\leq \epsilon T + \sum_{t} \sum_{\mathbf{s}} \left(p_{\pi}^{t}(\mathbf{s}) - p_{\text{data}}^{t}(\mathbf{s}) \right) \mathbb{E}_{\pi(\mathbf{a}|\mathbf{s})} \left[c(\mathbf{s}, \mathbf{a}) \right]$$

$$\leq 2\epsilon t \qquad \leq 1$$

$$\sum_{t} \mathbb{E}_{p_{\pi}^{t}(\mathbf{s})} \mathbb{E}_{\pi(\mathbf{a}|\mathbf{s})} \left[c(\mathbf{s}, \mathbf{a}) \right] = \sum_{t} \sum_{\mathbf{s}} p_{\pi}^{t}(\mathbf{s}) \mathbb{E}_{\pi(\mathbf{a}|\mathbf{s})} \left[c(\mathbf{s}, \mathbf{a}) \right]$$

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$$\leq \epsilon T + \sum_{t} 2\epsilon t$$

$$\sum_{t} \mathbb{E}_{p_{\pi}^{t}(\mathbf{s})} \mathbb{E}_{\pi(\mathbf{a}|\mathbf{s})} [c(\mathbf{s}, \mathbf{a})] = \sum_{t} \sum_{\mathbf{s}} p_{\pi}^{t}(\mathbf{s}) \mathbb{E}_{\pi(\mathbf{a}|\mathbf{s})} [c(\mathbf{s}, \mathbf{a})]$$

$$\leq \epsilon T + \sum_{t} \sum_{\mathbf{s}} \left(p_{\pi}^{t}(\mathbf{s}) - p_{\text{data}}^{t}(\mathbf{s}) \right) \mathbb{E}_{\pi(\mathbf{a}|\mathbf{s})} [c(\mathbf{s}, \mathbf{a})]$$

$$\leq \epsilon T + \sum_{t} 2\epsilon t$$

$$\leq \epsilon T + 2\epsilon T^{2} \in O(\epsilon T^{2})$$

Worst Case

$$\sum_{t} \mathbb{E}_{p_{\pi}^{t}(\mathbf{s})} \mathbb{E}_{\pi(\mathbf{a}|\mathbf{s})} \left[c(\mathbf{s}, \mathbf{a}) \right] \le \epsilon T + 2\epsilon T^{2}$$



Worst Case

$$\sum_{t} \mathbb{E}_{p_{\pi}^{t}(\mathbf{s})} \mathbb{E}_{\pi(\mathbf{a}|\mathbf{s})} \left[c(\mathbf{s}, \mathbf{a}) \right] \le \epsilon T + 2\epsilon T^{2}$$



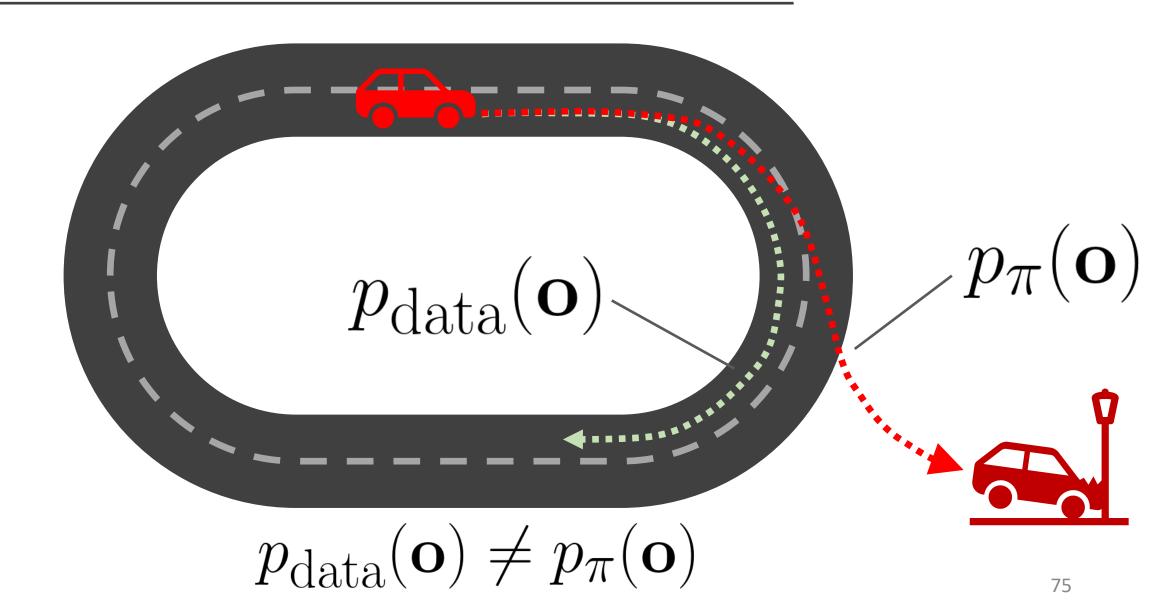
Distribution Shift

$$\sum_{t} \mathbb{E}_{p_{\pi}^{t}(\mathbf{s})} \mathbb{E}_{\pi(\mathbf{a}|\mathbf{s})} \left[c(\mathbf{s}, \mathbf{a}) \right] \leq \epsilon T + 2\epsilon T^{2}$$

$$\sum_{t} \mathbb{E}_{p_{\pi}^{t}(\mathbf{s})} \mathbb{E}_{\pi(\mathbf{a}|\mathbf{s})} \left[c(\mathbf{s}, \mathbf{a}) \right] \leq \epsilon T + \sum_{t} \sum_{\mathbf{s}} \left(p_{\pi}^{t}(\mathbf{s}) - p_{\text{data}}^{t}(\mathbf{s}) \right) \mathbb{E}_{\pi(\mathbf{a}|\mathbf{s})} \left[c(\mathbf{s}, \mathbf{a}) \right]$$

$$p_{\pi}^{t}(\mathbf{s}) \neq p_{\text{data}}^{t}(\mathbf{s})$$

Distribution Shift

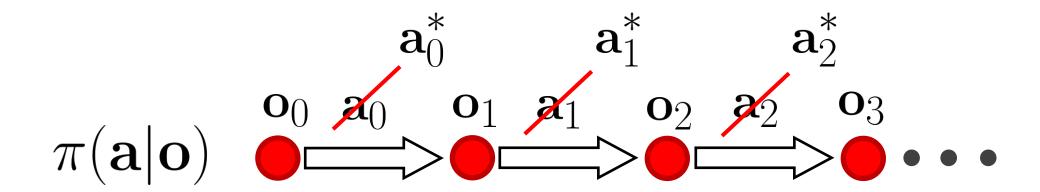


Dataset Aggregation

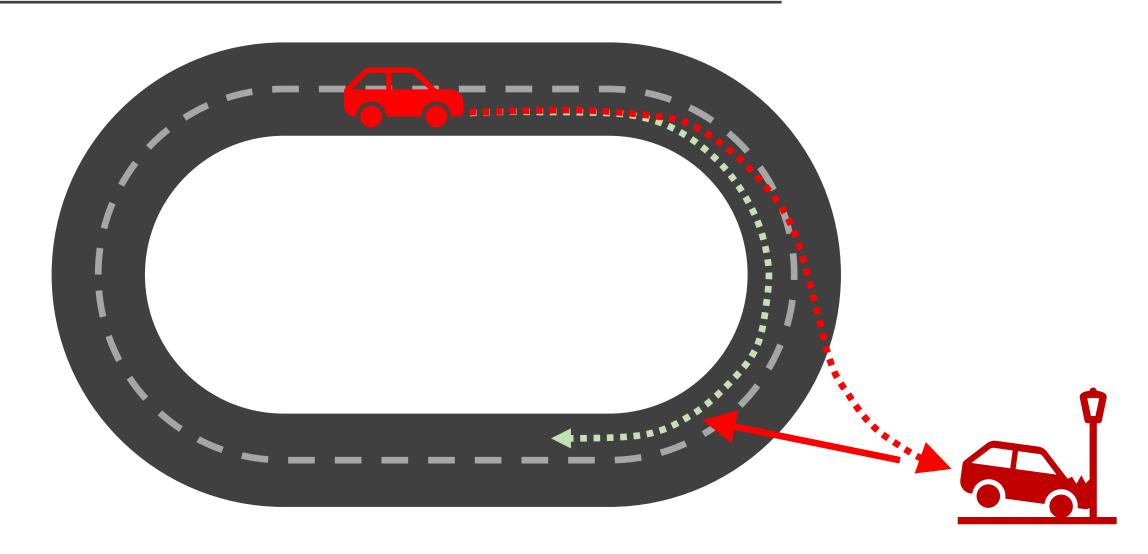
Can we make $p_{\mathrm{data}}(\mathbf{o}) = p_{\pi}(\mathbf{o})$?

Key idea:

- Collect observations from $p_{\pi}(\mathbf{o})$ instead of $p_{\mathrm{data}}(\mathbf{o})$
- Label actions with expert
- DAgger: Dataset Aggregation [Ross et al. 2011]



Train with $(\mathbf{o}_i, \mathbf{a}_i^*)$



ALGORITHM: DAgger

- 1: **for** iteration i = 0, ..., k 1 **do**
- 2: train $\pi(\mathbf{a}|\mathbf{o})$ from dataset $\mathcal{D} = \{\mathbf{o}_0, \mathbf{a}_0, \mathbf{o}_1, \mathbf{a}_0, ...\}$
- 3: run $\pi(\mathbf{a}|\mathbf{o})$ to collect dataset $\mathcal{D}_{\pi} = \{\mathbf{o}_0, \mathbf{o}_1, ...\}$
- 4: Label \mathcal{D}_{π} with actions \mathbf{a}_i from expert
- 5: Aggregate datasets: $\mathcal{D} \leftarrow \mathcal{D} \cup \mathcal{D}_{\pi}$
- 6: end for

ALGORITHM: DAgger

- 1: **for** iteration i = 0, ..., k 1 **do**
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ALGORITHM: DAgger

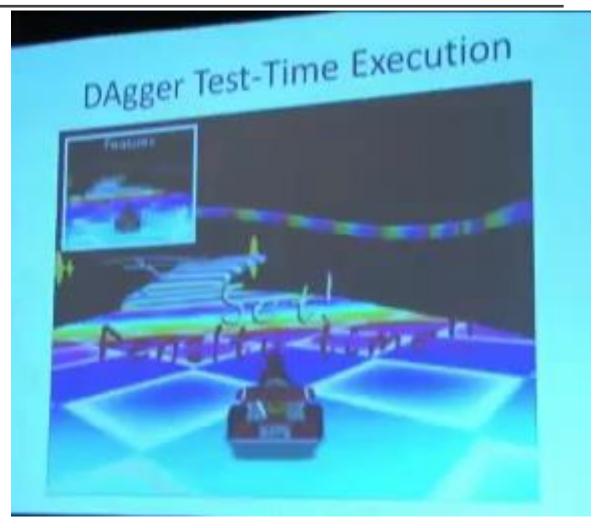
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- 6: end for

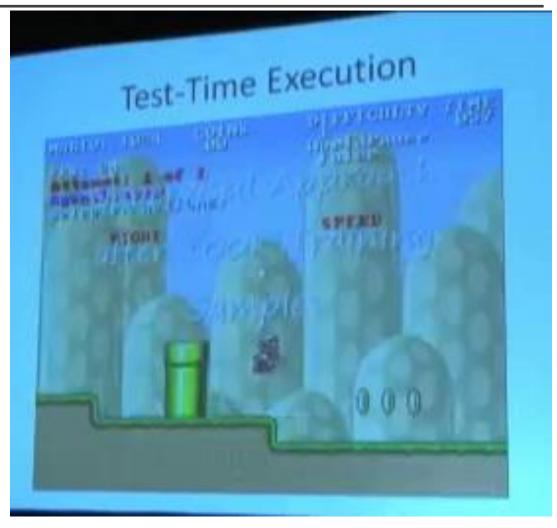
ALGORITHM: DAgger

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- 4: Label \mathcal{D}_{π} with actions \mathbf{a}_i from expert
- 5: Aggregate datasets: $\mathcal{D} \leftarrow \mathcal{D} \cup \mathcal{D}_{\pi}$
- 6: end for





Assume:
$$\pi\left(\mathbf{a} \neq \pi^*(\mathbf{s})|\mathbf{s}\right) \leq \epsilon$$
 for all $\mathbf{s} \sim p_{\mathrm{data}}(\mathbf{s})$
$$p_{\pi}^t(\mathbf{s}) = (1 - \epsilon)^t p_{\mathrm{data}}^t(\mathbf{s}) + (1 - (1 - \epsilon)^t) p_{\mathrm{mistake}}^t(\mathbf{s}) = p_{\mathrm{data}}^t(\mathbf{s})$$

$$p_{\text{data}}(\mathbf{s}) = p_{\pi}(\mathbf{s})!$$

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$$\pi\left(\mathbf{a} \neq \pi^*(\mathbf{s})|\mathbf{s}\right) \leq \epsilon$$
 for all $\mathbf{s} \sim p_{\mathrm{data}}(\mathbf{s})$
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$$p_{\text{data}}(\mathbf{s}) = p_{\pi}(\mathbf{s})!$$

Assume: $\pi\left(\mathbf{a} \neq \pi^*(\mathbf{s})|\mathbf{s}\right) \leq \epsilon$ for all $\mathbf{s} \sim p_{\mathrm{data}}(\mathbf{s})$

$$p_{\text{data}}(\mathbf{s}) = p_{\pi}(\mathbf{s})!$$

$$p_{\pi}^{t}(\mathbf{s}) = p_{\text{data}}^{t}(\mathbf{s})$$

$$\sum_{t} \mathbb{E}_{p_{\pi}^{t}(\mathbf{s})} \mathbb{E}_{\pi(\mathbf{a}|\mathbf{s})} \left[c(\mathbf{s}, \mathbf{a}) \right] = \sum_{t} \mathbb{E}_{p_{\text{data}}^{t}(\mathbf{s})} \mathbb{E}_{\pi(\mathbf{a}|\mathbf{s})} \left[c(\mathbf{s}, \mathbf{a}) \right]$$

Assume: $\pi\left(\mathbf{a} \neq \pi^*(\mathbf{s})|\mathbf{s}\right) \leq \epsilon$ for all $\mathbf{s} \sim p_{\mathrm{data}}(\mathbf{s})$

$$p_{\text{data}}(\mathbf{s}) = p_{\pi}(\mathbf{s})!$$

$$p_{\pi}^{t}(\mathbf{s}) = p_{\text{data}}^{t}(\mathbf{s})$$

$$\sum_{t} \mathbb{E}_{p_{\pi}^{t}(\mathbf{s})} \mathbb{E}_{\pi(\mathbf{a}|\mathbf{s})} \left[c(\mathbf{s}, \mathbf{a}) \right] = \sum_{t} \mathbb{E}_{p_{\text{data}}^{t}(\mathbf{s})} \mathbb{E}_{\pi(\mathbf{a}|\mathbf{s})} \left[c(\mathbf{s}, \mathbf{a}) \right]$$

$$\leq \sum_{t} \epsilon$$

$$\leq \epsilon T \in O(\epsilon T)$$

ALGORITHM: DAgger

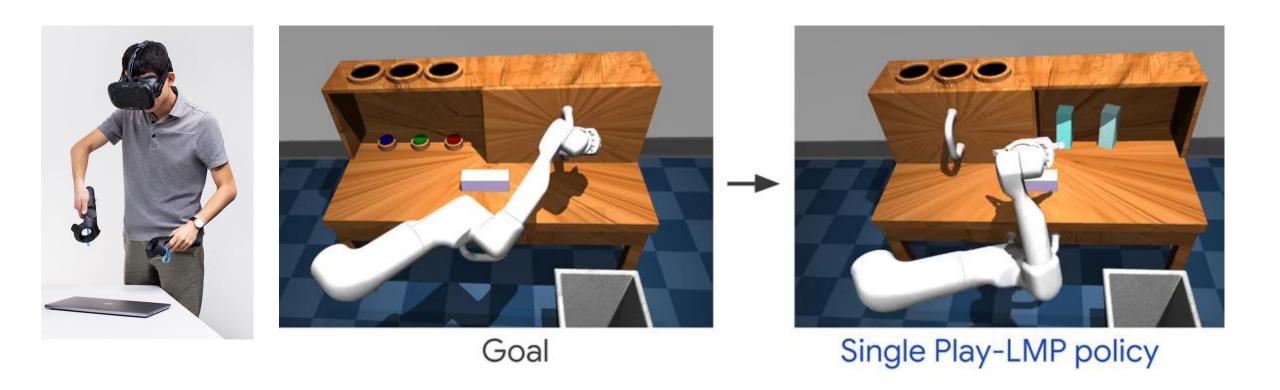
- 1: **for** iteration i = 0, ..., k 1 **do**
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- 6: end for

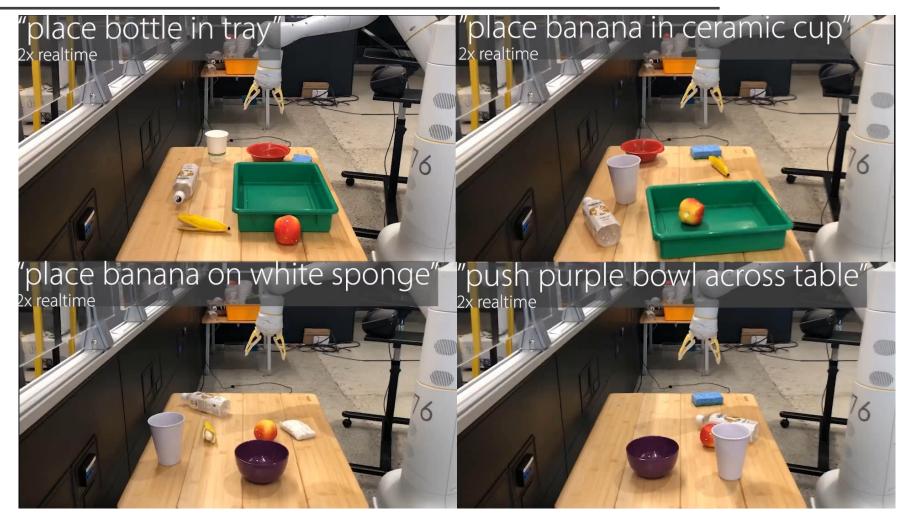
Applications

Applications



Learning Latent Plans from Play [Lynch et al. 2019]

Applications



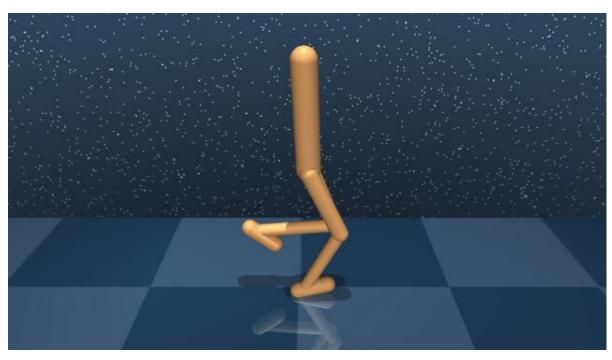
BC-Z: Zero-Shot Task Generalization with Robotic Imitation Learning [Jang et al. 2021]

Summary

- Behavioral Cloning
- Drift
- Theoretical Analysis
- DAgger
- Applications

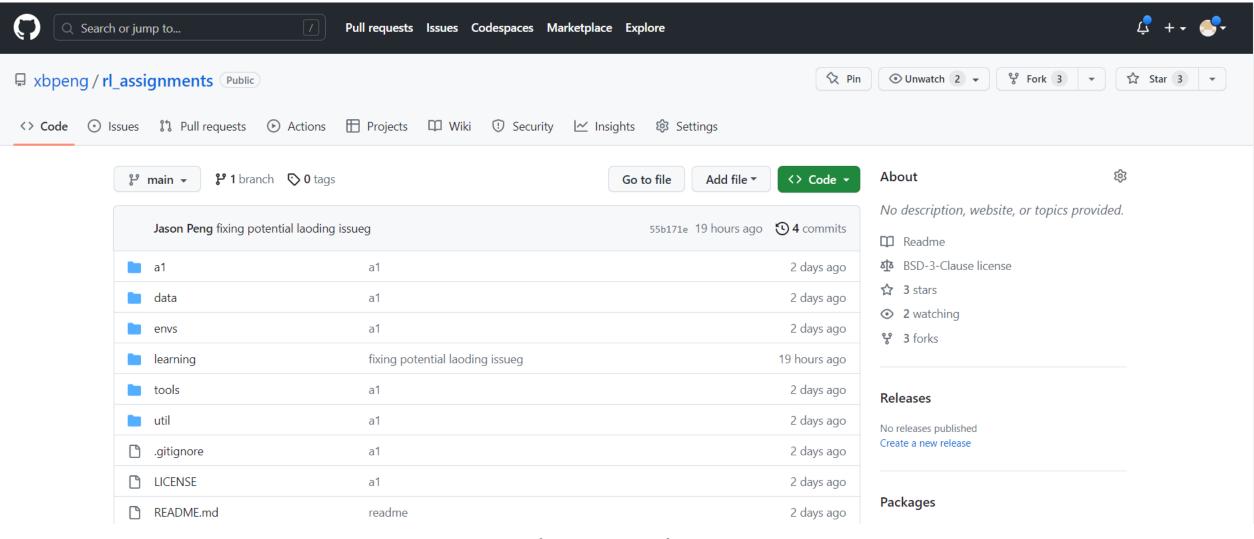
Assignment 1: Behavioral Cloning





Cheetah Walker

Assignment 1: Behavioral Cloning



github.com/xbpeng/rl_assignments