Ay190 – Worksheet 7

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Date: January 30, 2014

Measuring π with an MC Experiment

The MC estimate of π with increasing number of points, N, is tabulated in Table . The absolute error versus $\sqrt{N}$ is plotted in Figure 1. The error basically follows $\sim 1/\sqrt{N}$ for not too large N. There is a threshold above which the error cannot be reduced by increasing N.

N	Estimated π	error
1000	3.152	0.0104073464102
2000	3.276	0.13440734641
4000	3.153	0.0114073464102
8000	3.129	-0.0125926535898
16000	3.13025	-0.0113426535898
32000	3.152875	0.0112823464102
64000	3.136375	-0.00521765358979
128000	3.14140625	-0.000186403589793
256000	3.142421875	0.000829221410207
512000	3.1400703125	-0.00152234108979
1024000	3.142890625	0.00129797141021
2048000	3.14418945313	0.00259679953521
4096000	3.14124609375	-0.000346559839793
8192000	3.14158642578	-6.22780854309e-06
16384000	3.14232861328	0.000735959691457
32768000	3.14145458984	-0.000138063746043

Table 1: MC Measurement of π

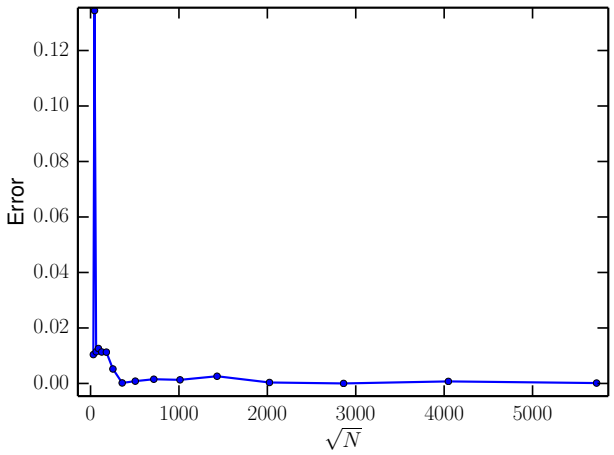

Figure 1: MC Estimation of π

The Birthday Paradox

For number of people N, recall that there are 365 days in a year, the probability that at least two of them have same birthday is

$$P = 1 - \frac{365!/(365-N)!}{365^N}, \tag{1}$$

where the bulk after the minus sign is just the probability that all of them have different birthdays.

WIth MC method, for each N, I do 20000 experiments and get an estimated probability as tabulated below.

N	Probability
21	0.4473
22	0.47475
23	0.50775
24	0.54095
25	0.5763

Table 2: Birthday Paradox

MC Integration

In this section, the intergration

$$I = \int_2^3 f(x)\,\mathrm{d}x = \int_2^3 x^2 + 1\,\mathrm{d}x = \frac{22}{3} \tag{2}$$

will be calculated using MC method.

The results are tabulated in the following table and the error as a function of $\sqrt{N}$ is plotted in Figure 2. As in question 1, the error basically follows $\sim 1/\sqrt{N}$ for not too large N. There is a threshold above which the error cannot be reduced by increasing N.

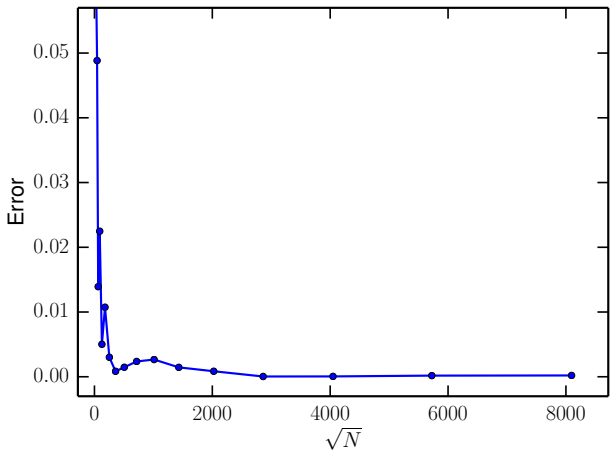

Figure 2: MC Estimation of π