Ay190 – Worksheet 11

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Advection Equation

(1) See the implemented code.

(2) In all the sections, I adopt $\Delta t = 0.1$ and $\Delta x = 0.1$. First, I take $v = 0.9$ and thus $\alpha = 0.9 < 1$. One snapshot at $t = 540$ is showed at upper-left panel in Figure 1. In all the plots, numerical solutions are plotted in solid blue lines, analytical solutions are plotted in solid red lines and errors are plotted in solid green lines. For $\alpha = 0.9$, the solution is stable.

Then switch σ to a value five times smaller than the original one. A snapshot at $t = 270$ is plotted in the upper-right panel in Figure 1. The error is highly amplified around the peak.

Finally, change v to 1.5 and thus $\alpha = 1.5 > 1$ and get back to the original σ value. A snapshot at $t = 78$ is plotted in lower panel in Figure 1. The solution starts to become unstable. The unstability just above $x = 0$ may originate from the boundary condition we adopt.

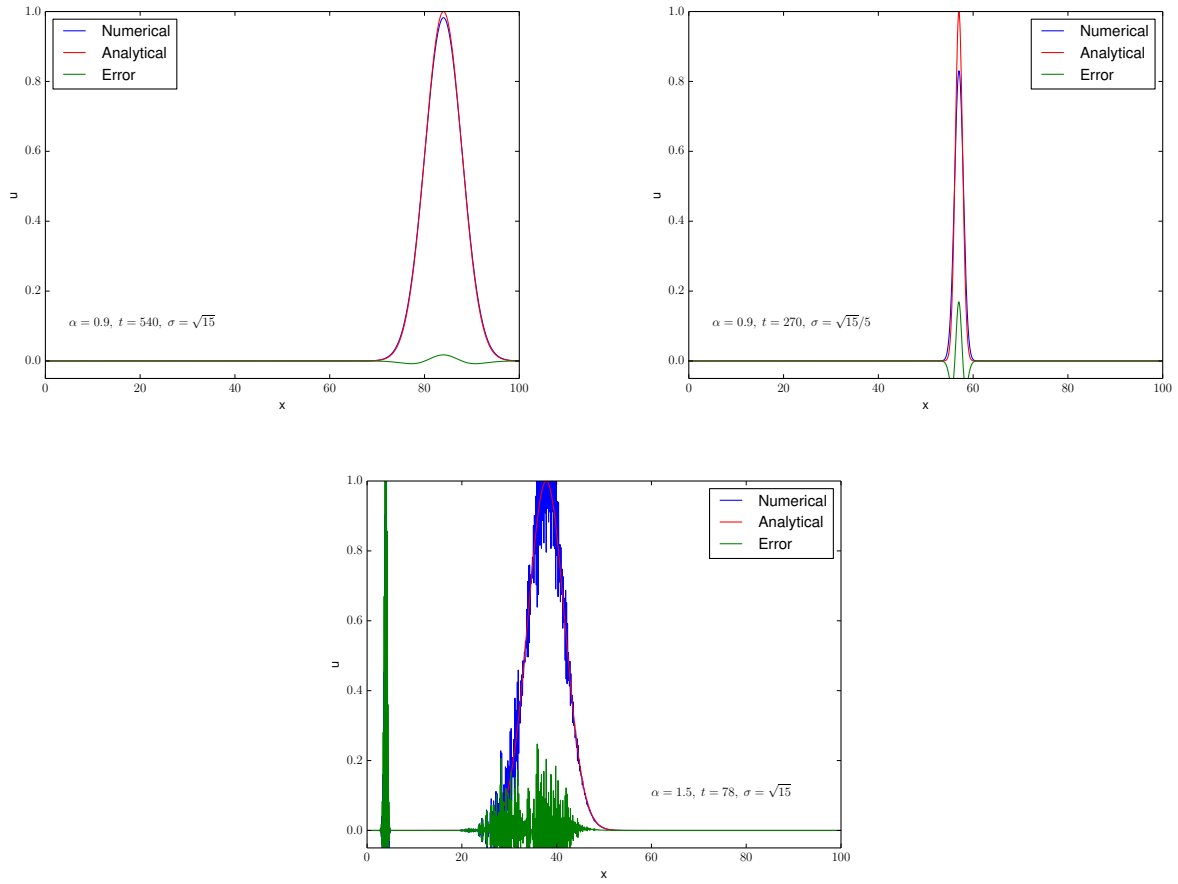


Figure 1: Upwind method

(3) In this section, I take $v = 0.9$ ($\alpha = 0.9$). Two snapshots at $t = 90$ and $t = 108$ are plotted in Figure 2. We

can see that the solution becomes stable at some time between those despite $\alpha < 1$.

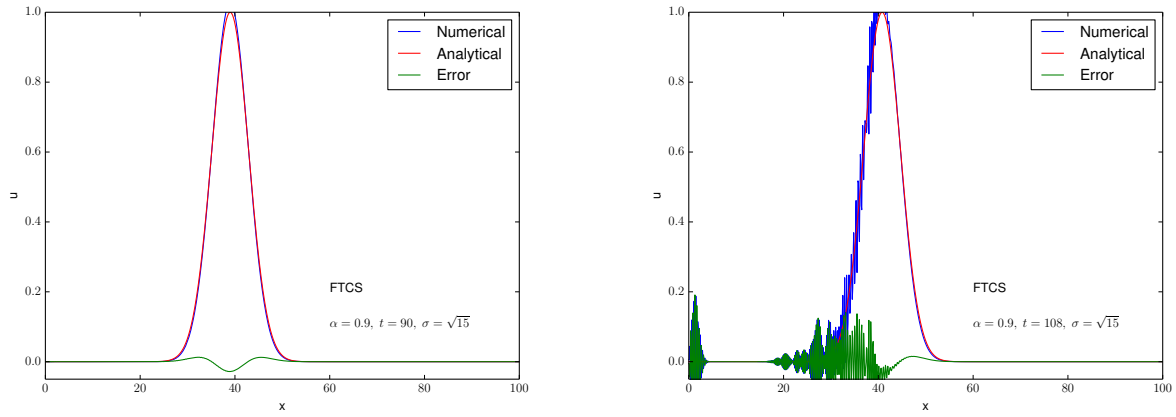


Figure 2: FTCS method

(4) I take $v = 0.9$ and $v = 1.5$, respectively as what I have done in (2). For $\alpha = 0.9$, a snapshot at $t = 540$ is plotted in Figure 3. For $\alpha = 1.5$, two snapshot at $t = 78$ and $t = 135$ are plotted. The solution is unstable for $\alpha > 1$ similar to the upwind case.

(5) Three snapshots for Leapfrog method are plotted in Figure 4. This method is unstable for both $\alpha = 0.9$ and $\alpha = 1.5$.

Two snapshots for $\alpha = 0.9$ and $\alpha = 1.5$ are plotted in Figure 5. In the former case, the error is much smaller than any other cases. And it seems that the solution becomes unstable much faster than other cases for $\alpha > 1$.

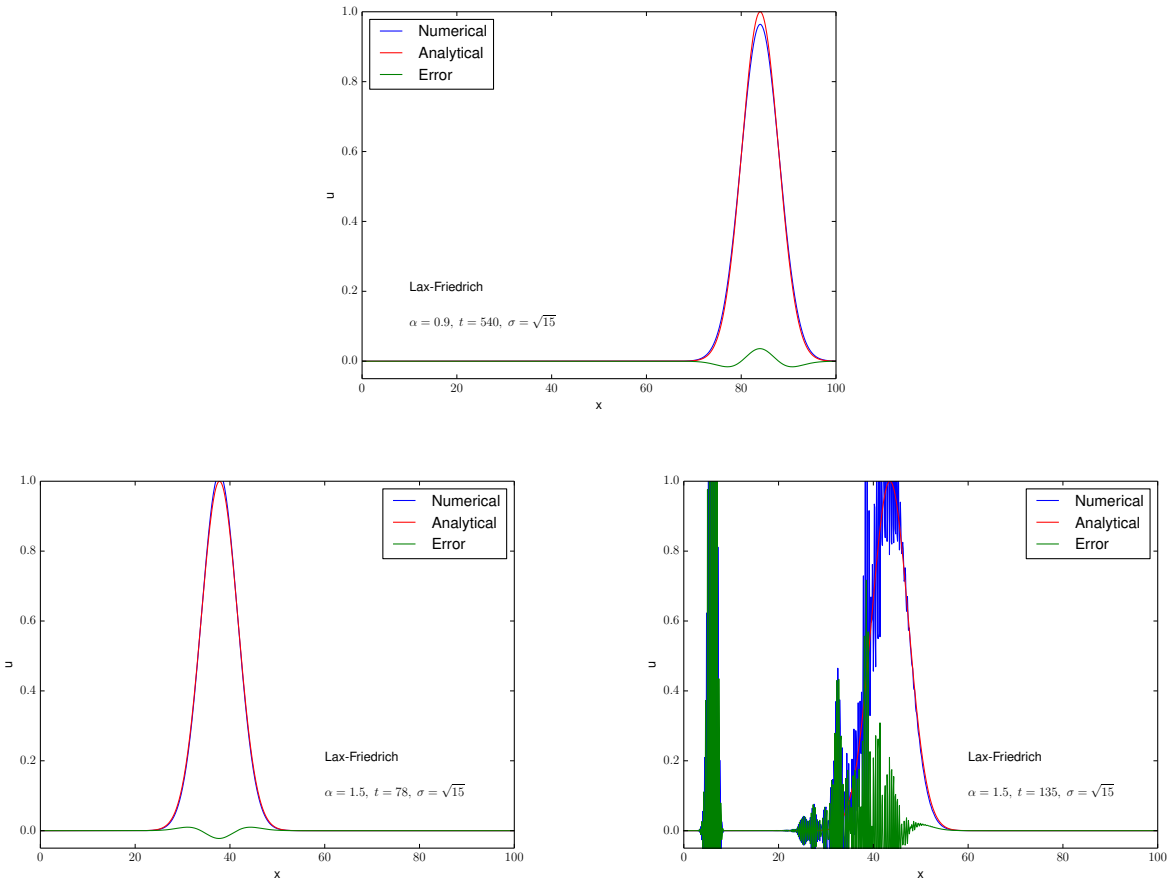
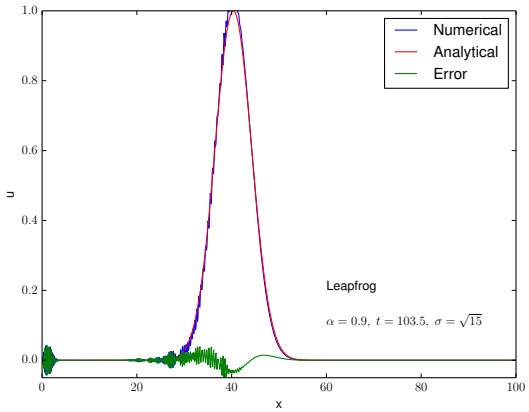
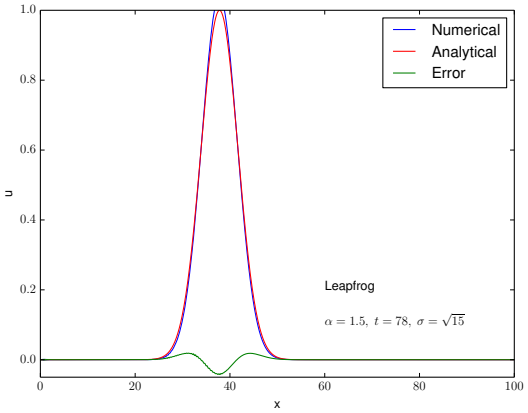


Figure 3: Lax-Friedrich method





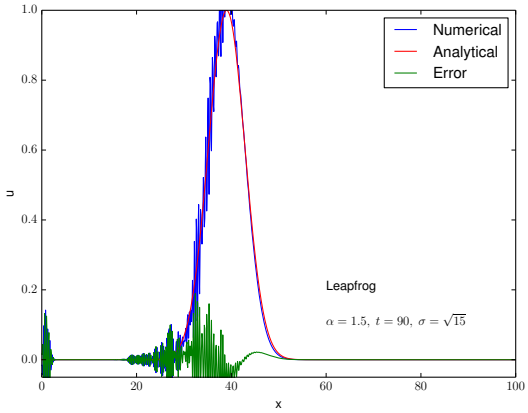
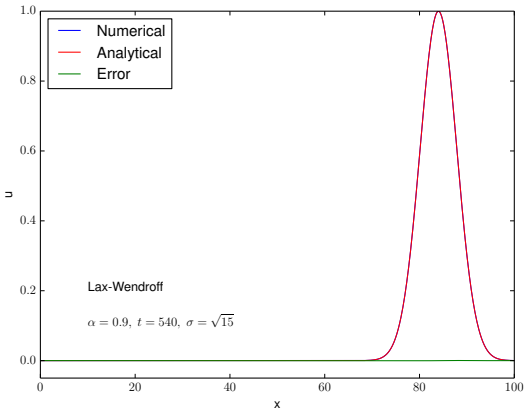


Figure 4: Leapfrog method



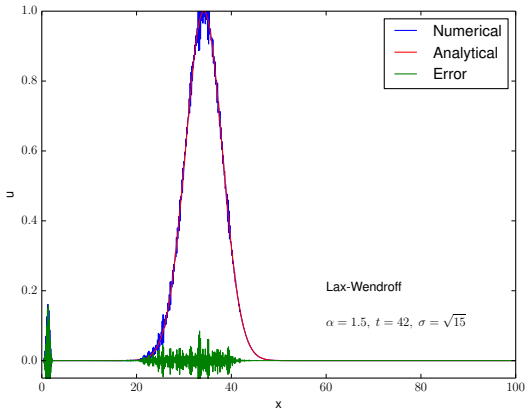


Figure 5: Lax-Wendroff method