Ay190 – Worksheet 3

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1 Integration via Newton-Cotes Formulae

(a) The accurate result is

$$\int_0^\pi \sin x \, \mathrm{d}x = 2. \tag{1}$$

I tabulate the numerical results of mid-point, trapezoidal and Simpson's formulae in Table 1.

(b) The accurate result is

$$\int_0^\pi x \sin x \, \mathrm{d}x = \pi. \tag{2}$$

I tabulate the numerical results of mid-point, trapezoidal and Simpson's formulae in Table 1.

From Table 1, we can see the mid-point and trapezoidal formulae have error of order $O(h^2)$, while the Simpson's rule has an error of order $O(h^4)$. As we expected, when h drops by a factor of 2, the error of mid-point and trapezoidal formulae decrease by a factor of 4, but the error of Simpson's rule reduced by a factor of 16. Note that it is probably not what appears exactly on the lecture notes. The reason is that the error on the lecture note is the error of integration over a specific interval, while here the error is amplified by a factor of $\sim 1/h$ because we divide the interval into $\sim 1/h$ smaller intervals.

	$h = \pi/100$	$h = \pi/200$	$h = \pi/500$	$h = \pi/1000$
		$f(x) = \sin(x)$		
Mid-point	2.00008391911	2.000020769	2.00000330307	2.00000082411
Error	8.39191093354e-05	2.07689960372e-05	3.30307100915e-06	8.24114675613e-07
Trapezoidal	1.99983216389	1.99995846214	1.99999339386	1.99999835177
Error	-0.000167836106008	-4.15378626695e-05	-6.6061387447e-06	-1.64822914717e-06
Simpson	2.0000000007	2.00000000004	2.0	2.0
Error	7.04222014036e-10	4.31357172204e-11	1.09023901018e-12	6.66133814775e-14
		$f(x) = x\sin(x)$		
Mid-point	3.14172447342	3.14162527745	3.14159784204	3.14159394811
Error	0.000131819828693	3.26238626873e-05	5.1884518113e-06	1.29451631237e-06
Trapezoidal	3.14132901725	3.14152740607	3.14158227669	3.14159006456
Error	-0.00026363633882	-6.52475221039e-05	-1.03768984743e-05	-2.58903229344e-06
Simpson	3.1415926547	3.14159265366	3.14159265359	3.14159265359
Error	1.10618758598e-09	6.77577993713e-11	1.71462843923e-12	1.05249142734e-13

Table 1: Intergation via Newton-Cotes Formulae

2 Gaussian Quadrature

(a) I will focus on the integral

$$I = \int_0^\infty \frac{x^2 \, \mathrm{d}x}{e^x + 1}. \tag{3}$$

I tabulate n and results of the intergral in Table 2. It is convergent with increasing n.

n	I
6	1.80269370975
7	1.80305547176
8	1.80312884947
9	1.80311453153
10	1.80309513198
11	1.80308593362
12	1.80308364319
13	1.80308394561
14	1.80308467175
15	1.80308516046
16	1.80308537432
17	1.80308542649
18	1.80308541379
19	1.80308538784

Table 2: Gauss-Laguerre Quadrature

(b) This question depends on the temperature. Here I take $k_B T = 20$ MeV. The result is listed in Table 2. It is easy to check that

$$\sum_{i=0}^{\infty} \left[\frac{\mathrm{d}n_e}{\mathrm{d}E}\right]_i \times \Delta E = n_e \tag{4}$$

is valid.

i	E_i	n_i	$n_i/\Delta E$
1	0	18.8869583882	3.77739167764
2	5	116.955759331	23.3911518662
3	10	273.198503663	54.6397007325
4	15	450.142105167	90.0284210334
5	20	619.214587182	123.842917436
6	25	761.543791108	152.308758222
7	30	867.183979764	173.436795953
8	35	933.337618905	186.667523781
9	40	962.205306939	192.441061388
10	45	958.955799737	191.791159947
11	50	930.091739218	186.018347844
12	55	882.295575664	176.459115133
13	60	821.717024628	164.343404926
14	65	753.607006725	150.721401345
15	70	682.192871201	136.43857424
16	75	610.703792631	122.140758526
17	80	541.477677812	108.295535562
18	85	476.102866841	95.2205733682
19	90	415.565659199	83.1131318397
20	95	360.3875129	72.0775025799
21	100	310.744276751	62.1488553501
22	105	266.565057663	53.3130115325
23	110	227.611293054	45.5222586107
24	115	193.538088272	38.7076176544
25	120	163.940464581	32.7880929163
26	125	138.387231414	27.6774462828
27	130	116.444994465	23.2889988929
28	135	97.6944892251	19.538897845
29	140	81.7410745231	16.3482149046
30	145	68.2208792326	13.6441758465

Table 3: Gauss-Legendre Quadrature