Hashtables

CS 5002: Discrete Math and Data Structures

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Outline

1. Why hashtables?
2. Hash functions
3. Uses for Hash tables
4. Collisions
5. Performance
6. Load Factor
7. Implementation

How much is this onion?

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Comparing Runtimes

Num Items	Simple Search $O(n)$[1]	Binary Search $O(\log n)$	Buddy $O(1)$

[1] assume 1/10 second per comparison

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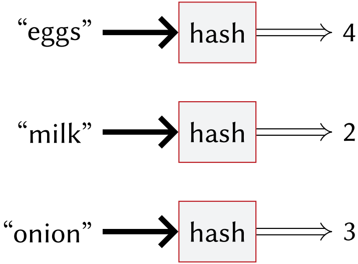
How can we replicate Buddy with a data structure?

(eggs, \$2.49)	(milk, \$4.29)	(onion, \$0.29)

- Each item is essentially 2 items: a product name, and a price
- If we sort by name, we can find a product in $O(\log n)$ time
- How can we get down to $O(1)$?

Hash Function



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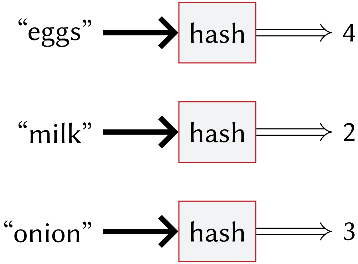

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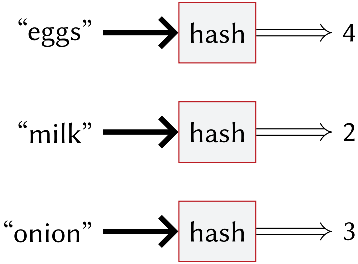

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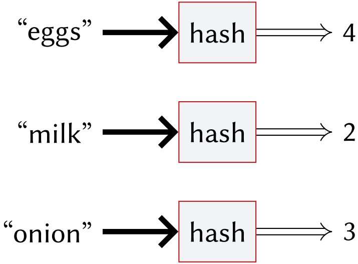

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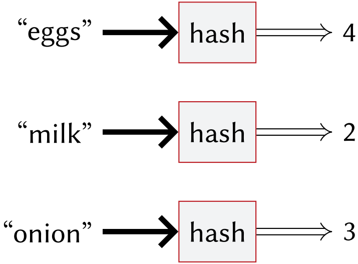

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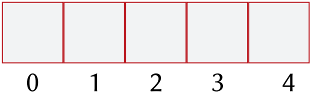
  - Every time you give it “eggs”, it returns ‘4’.
- Must be distinguishing. In the best case, it returns a different value for every distinct input given to it.
  - A function that always returns 1 is not helpful.

How can we use the hash function to make our “Buddy”?

If we have an array:



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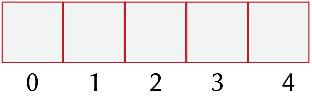

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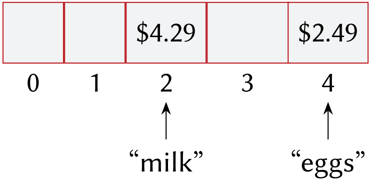
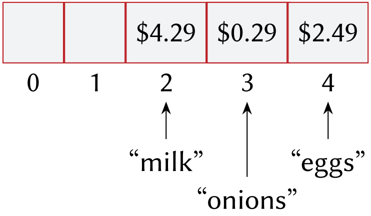
Add milk to the array, using the hash function...



How can we use the hash function to make our “Buddy”?

Add milk to the array, using the hash function…

…and then add onions:



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- The hash function returns a different value for different inputs
- The hash function knows how big the array is, so always returns something within those bounds.

New Data Structure: Hash Table

A ***hash table*** is a combination of a hash function and array.

Other names:

- hash map
- map
- dictionary
- associative array

Which of these hash functions are consistent?

The hash function must be consistent about returning the same output for the same input.

If it doesn't, you won't be able to retrieve the data you store in the hash table.

Which of these is consistent?

1. $f(x) = 1$
2. $f(x) =$rand()
3. $f(x) =$next_empty_slot()
4. $f(x) =$len(x)

Use Cases

- Look up tables
  - Phone book. key = name, value = phone number
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- Prevent duplicate entries
  - Voting roster: ensure the same person doesn't vote twice
- As a cache
  - Store a copy of a web page locally rather than requesting the entire thing over the network
  - key = URL, value = webpage

Hash table operations and vocab

A few hash table-specific terms:

- ***key***
- ***value***
- ***put***: `put(hashtable, key, value)`
- ***get***: `get(hashtable, key)` (returns the value)

Hash Table: `get()` and `put()`

HASH-INSERT(T, key, val)

T[h(key)] = val

HASH-SEARCH(T, key)

return T[h(key)]

Simple Hash Table implementation

```c
#include<stdio.h>
#include<strings.h>

int table[20] = {0};

int hash(char *key){
  return strlen(key);
}

void put(char *key, int value){
  table[hash(key)] = value;
}

int get(char *key){
  return table[hash(key)];
}

int main(){
  put("onion", 3);
  put("tomato", 4);
  put("red pepper", 15);

  printf("value for onion: %d\n", get("onion"));
  printf("value for onion: %d\n", get("tomato"));
  printf("value for onion: %d\n", get("red pepper"));
}

```

Listing 1: hash table implementation

Demo...

Collisions

So far, our understanding of hash tables is predicated on the ***hash function***.

Specifically, that the has function generates a **unique** value for every input.

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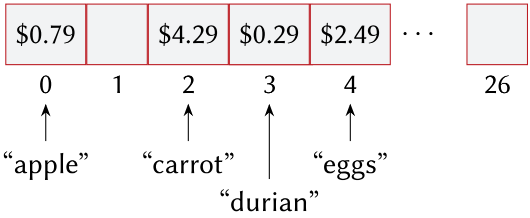

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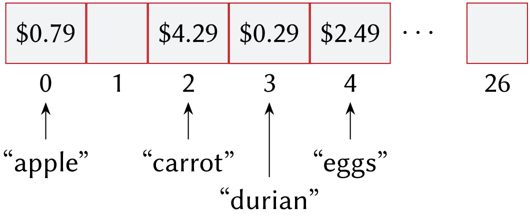
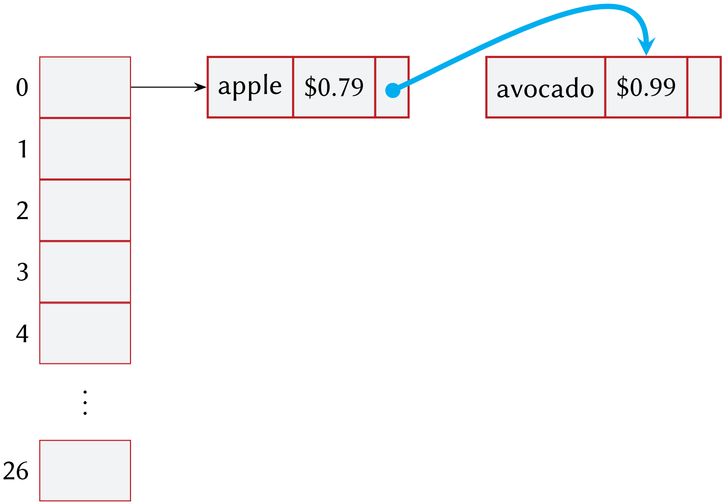


What happens when I go to add "avocado"?

Collisions: One Solution is Chaining



Next time: add slide on probability of collisions. Birthday paradox, Pigeonhole principle

Hash Table: `get()` and `put()` with chaining

HASH-INSERT(T, key, val)

1 insert val at head of list T[h(key)]

HASH-SEARCH(T, key)

1 search for element with key in list T[h(key)]

Hashtable Performance

	Average Case	Worst case
Search (get)	O(1)	
Insert (put)	O(1)	
Delete	O(1)	

In the Average Case, we get a hit right away— don't have to search through the list.

This is equivalent to our Buddy saying "Apples are $0.29".

Hashtable Performance

	Average Case	Worst case
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Delete	O(1)	O(n)

In the Average Case, we get a hit right away— don't have to search through the list.
This is equivalent to our Buddy saying "Apples are $0.29".

In the Worst Case, we need to search through all the items in a chained linked list.

This is when our Buddy says "I don't know– look in the book".

What does our hash table look like...?

Add items: avocado, apple, asparagus, ...

$\Rightarrow$ Your hash function is super important!! (You want one that will map keys evenly)

A good hash function keeps those linked lists small too.

Load factor

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3/5

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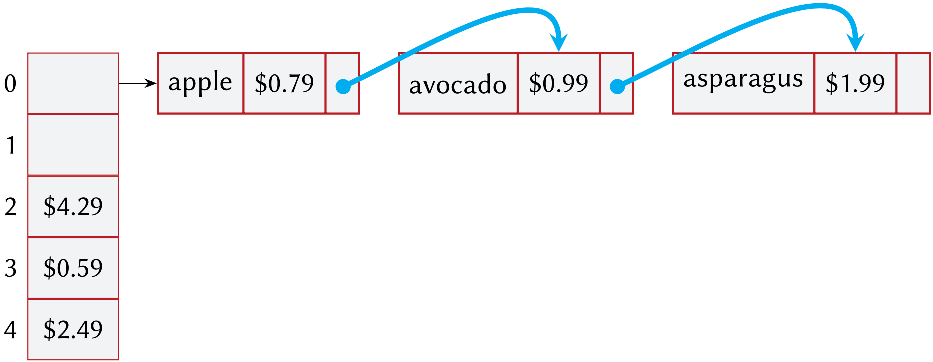

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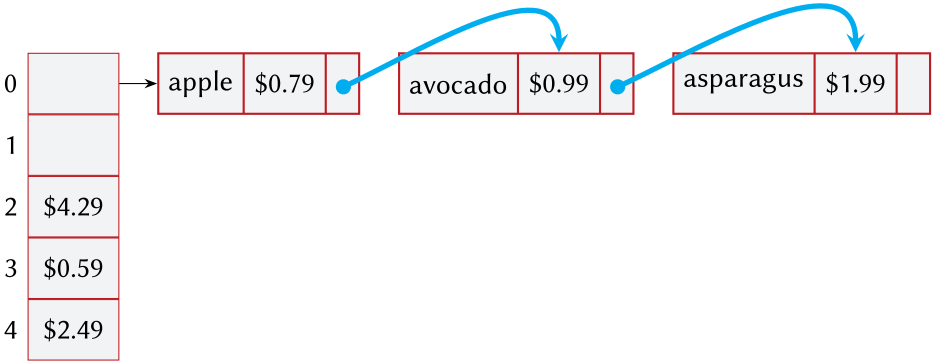

6/5

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- How can we deal with this?

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- Resizing is expensive (in time), but it averages out overall by allowing us an improved access time

Back to the hash function...

A good hash function give us:

		$4.29		$2.49
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(Items distributed, one per bucket)

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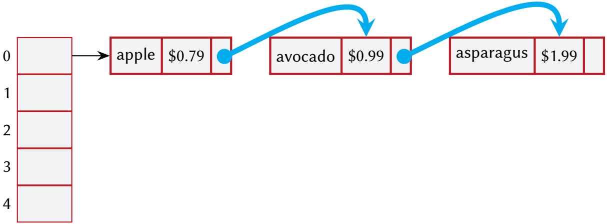

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- Going from a string to a number:
  - `pt` $\Rightarrow$ (112, 116) (ASCII vals) $\Rightarrow (112 \cdot 128) + 116 = 14452$[2]

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Division Method

$$h(k) = k \mod m$$

- m is the size of the table
- k is the key
- Usually pretty fast
- Want to avoid certain values of m:
  - Powers of 2: if $m = 2^p$, $h(k)$ is the p-lowest order bits of k
  - Powers of 10: if keys are decimal numbers
  - $m = 2^p - 1$ when k is a string interpreted in radix-2^p
- General good values of m:
  - Primes not too close to powers of 2

Multiplication Method

$$h(k) = \lfloor m(kA \mod 1) \rfloor$$

- $kA \mod 1$ is the fractional part of kA (or $kA - \lfloor kA \rfloor$)
- Nice becaues the value of m is not critical
- m is typically a power of 2

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Designing a universal class of hash functions

- m: table size, make it prime
- key x is $r + 1$ bytes: $x = \{x_0, x_1, \ldots x_r\}$
- $a = \{a_0, a_1, \ldots a_r\}$ is a sequence elements chosen randomly from the set of indices in m
- a

$$h_a(x) = \sum_{i=0}^{r} a_i x_i \mod m$$

Hash Table: Storage

- If the universe $|U|$ is large, ...
- When the set of K keys stored is much smaller than the universe U of possible keys, storage can be reduced to $\Theta(|K|)$
-

Direct Addressing vs Hash Table

- Direct Address: an element with key k is stored in slot k.
    -
- Hash Table: an element with key k is stored in slot $h(k)$
- h maps the universe U of keys into the slots of a hash table $T[0..m-1]$

Summary

Helpful Notes: `http://www.cs.yale.edu/homes/aspnes/pinewiki/C(2f)HashTables.html?highlight=%28CategoryAlgorithmNotes%29`